MATH 1051H-A: Lecture #04

Examining Numerical Data

Scatterplots

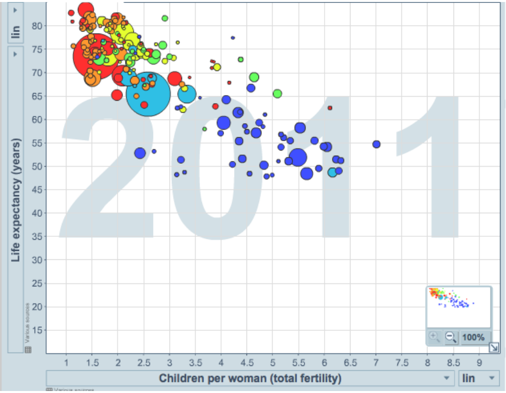
Scatterplots are useful for visualizing the relationship between two numerical variables.

Do life expectancy and total fertility appear to be associated or independent?

They appear to be linearly and negatively associated: as fertility increases, life expectancy decreases.

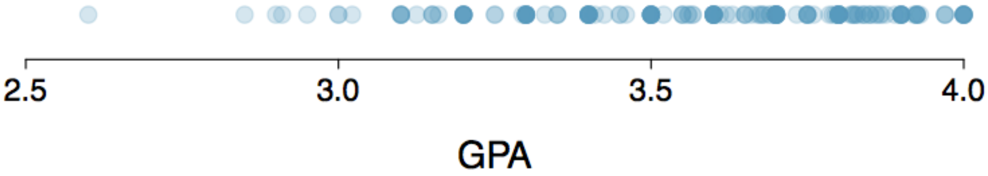
Was the relationship the same throughout the years, or did it change?

The relationship changed over the years.



Dot Plots

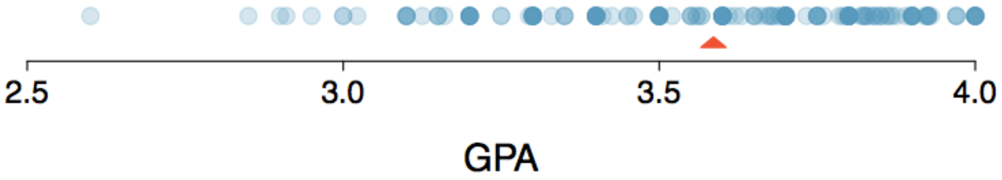
Useful for visualizing one numerical variable. Darker colors represent areas where there are more observations.



How would you describe the distribution of GPAs in this data set?

Make sure to say something about the center, shape, and spread of the distribution.

Dot Plots and Mean



The **mean**, also called the average (marked with a triangle in the plot), is one way to measure the center of a **distribution** of data.

The mean GPA is 3.59.

Mean

The **sample mean**, denoted as $\bar{x}$, can be calculated as

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

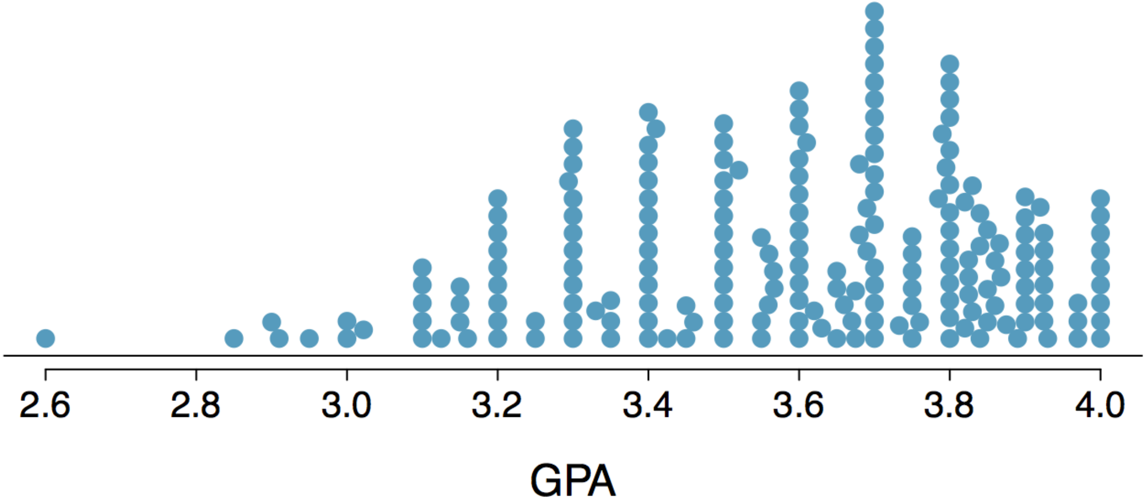
where $x_1, x_2, \cdots, x_n$ represent the n observed values.

The **population mean** is also computed the same way but is denoted as μ. It is often not possible to calculate μ since population data are rarely available.

The sample mean is a **sample statistic**, and serves as a **point estimate** of the population mean. This estimate may not be perfect, but if the sample is good (representative of the population), it is usually a pretty good estimate.

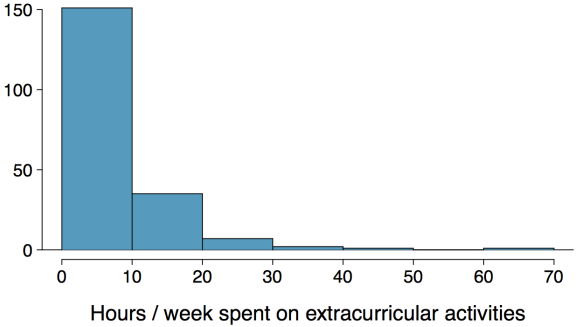
Stacked Dot Plot

Higher bars represent areas where there are more observations, makes it a little easier to judge the center and the shape of the distribution.



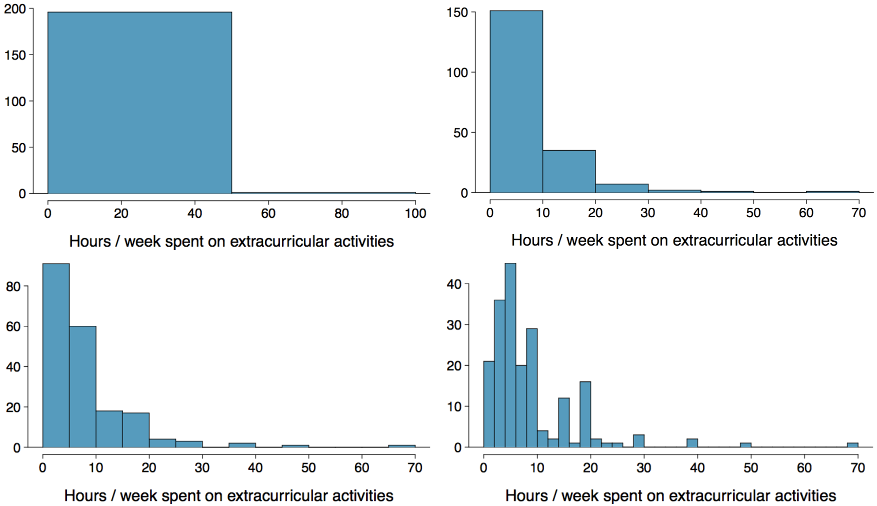
Histograms — Extracurricular Hours

- Histograms provide a view of the **data density**. Higher bars represent where the data are relatively more common.
- Histograms are especially convenient for describing the **shape** of the data distribution.
- The chosen **bin width** can alter the story the histogram is telling.



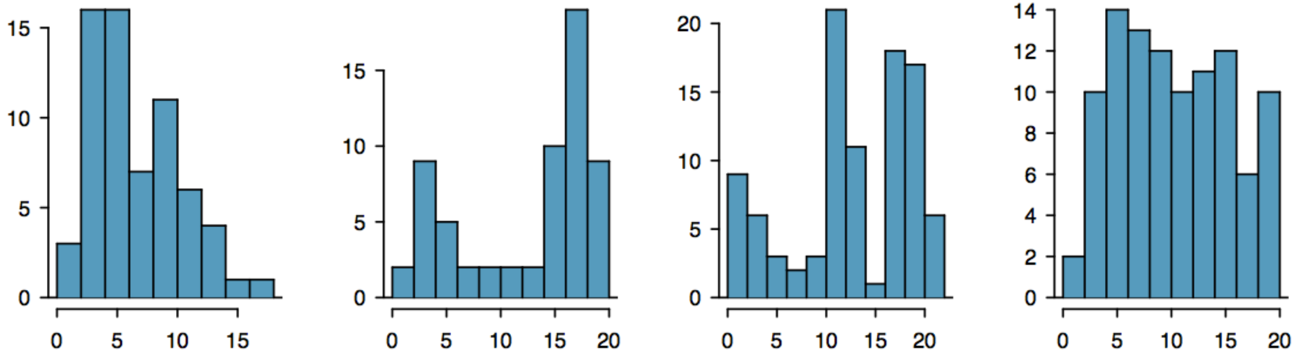
Bin Width

Which one(s) of these histograms are useful? Which reveal too much about the data? Which hide too much?



Shape of a Distribution: Modality

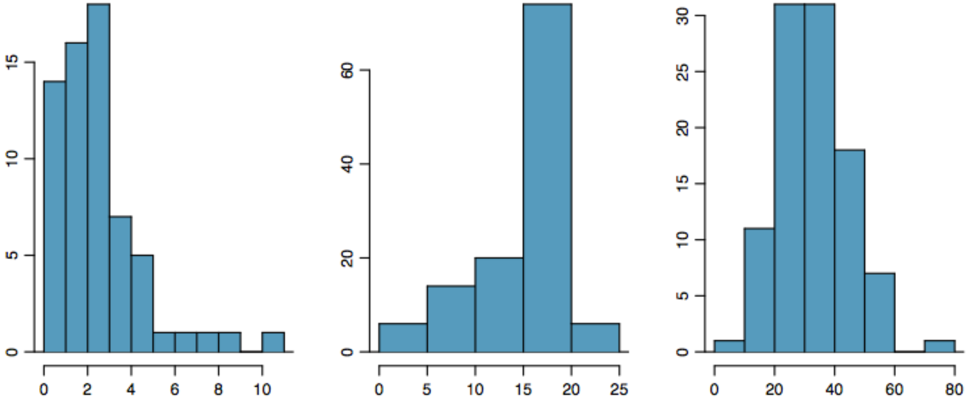
Does the histogram have a single prominent peak (**unimodal**), several prominent peaks (**bimodal/multimodal**), or no apparent peaks (**uniform**)?



Note: In order to determine modality, step back and imagine a smooth curve over the histogram – imagine that the bars are wooden blocks and you drop a limp spaghetti over them, the shape the spaghetti would take could be viewed as a smooth curve.

Shape of a Distribution: Skewness

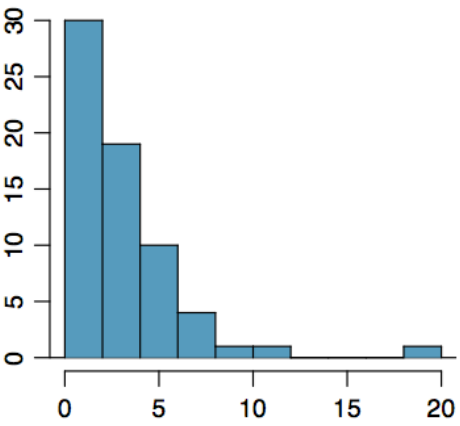
Is the histogram **right skewed**, **left skewed** or **symmetric**?

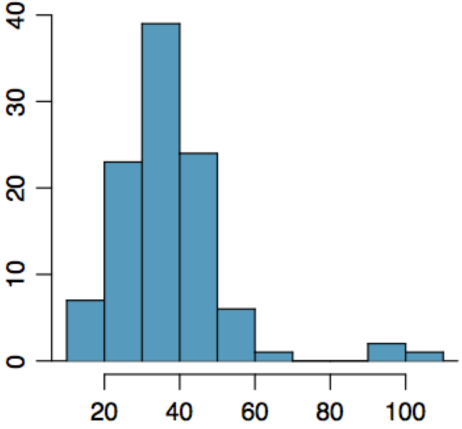


Histograms are said to be skewed to the side of the **long tail**.

Shape of a Distribution: Unusual Observations

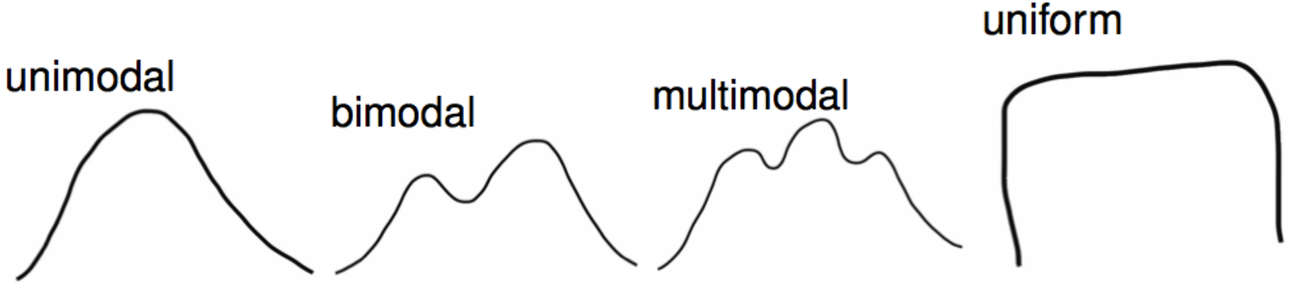
Are there any unusual observations or potential **outliers**?



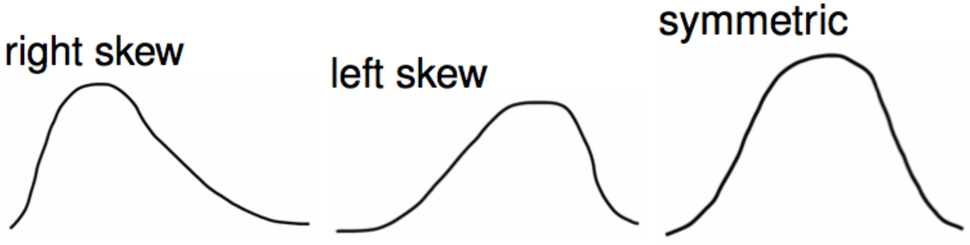


Commonly observed shapes of distributions

Modality



Skewness



Practice

Which of these variables do you expect to be uniformly distributed?

1. weights of adult females
2. salaries of a random sample of people from North Carolina
3. house prices
4. birthdays of classmates (day of the month)

Practice

Which of these variables do you expect to be uniformly distributed?

1. weights of adult females
2. salaries of a random sample of people from North Carolina
3. house prices
4. *birthdays of classmates (day of the month)*

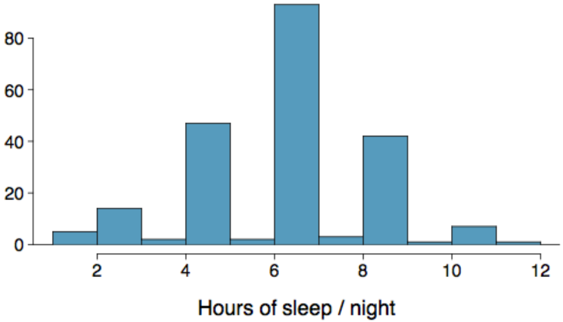
Are you typical?

How useful are centers alone for conveying the true characteristics of a distribution?

Variance

Variance is roughly the average squared deviation from the mean.

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$



- The sample mean is $\bar{x} = 6.71$, and the sample size is $n = 217$.
- The variance of the amount of sleep students get per night can be calculated as:

$$\begin{aligned} s^2 &= \frac{1}{217-1}\left[(5-6.71)^2 + (9-6.71)^2 + \cdots + (7-6.71)^2\right] \\ &= 4.11 \text{ hours}^2 \end{aligned}$$

Variance (continued)

Why do we use the squared deviation in the calculation of variance?

- To get rid of negatives so that observations equally distant from the mean are weighed equally.
- To weigh larger deviations more heavily.

Standard Deviation

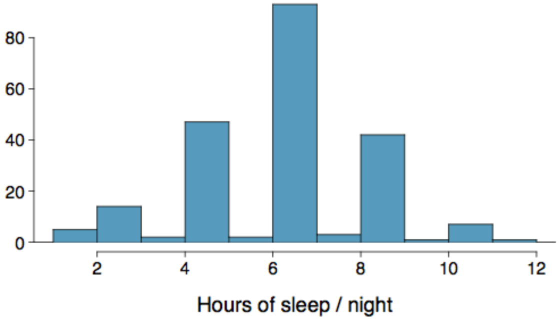
The **standard deviation** is the square root of the variance, and has the same units as the data.

$$s = \sqrt{s^2}$$

The standard deviation of the amount of sleep students get per night can be calculated as:

$$s = \sqrt{4.11} = 2.03 \text{ hours}$$

We can see that all of the data are within 3 standard deviations of the mean of $\bar{x} = 6.17$.



Median

The median is the value that splits the data in half when ordered in ascending order.

$$0, 1, 2, 3, 4$$

If there are an even number of observations, then the median is the average of the two values in the middle.

$$0, 1, \underline{2, 3}, 4, 5 \rightarrow \frac{2+3}{2} = 2.5$$

Since the median is the midpoint of the data, 50% of the values are below it. Hence, it is also the 50th percentile.

Percentile

A **percentile** is the the smallest value from an ordered list of numbers which is greater than or equal to that percentage of list elements.

Example: The 42^{nd} percentile of the numbers $\{1, 2, 3, \cdots, 99, 100\}$ is 42.

It can become quite complicated when there aren't an even multiple of 100 items!

Q1, Q3 and IQR

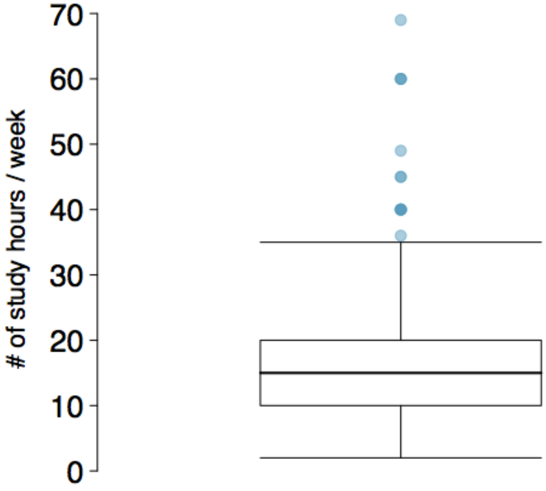
- The 25th percentile is also called the first quartile, **Q1**.
- The 50th percentile is also called the median.
- The 75th percentile is also called the third quartile, **Q3**.

Between Q1 and Q3 is the middle 50% of the data. The range these data span is called the **interquartile range**, or the IQR.

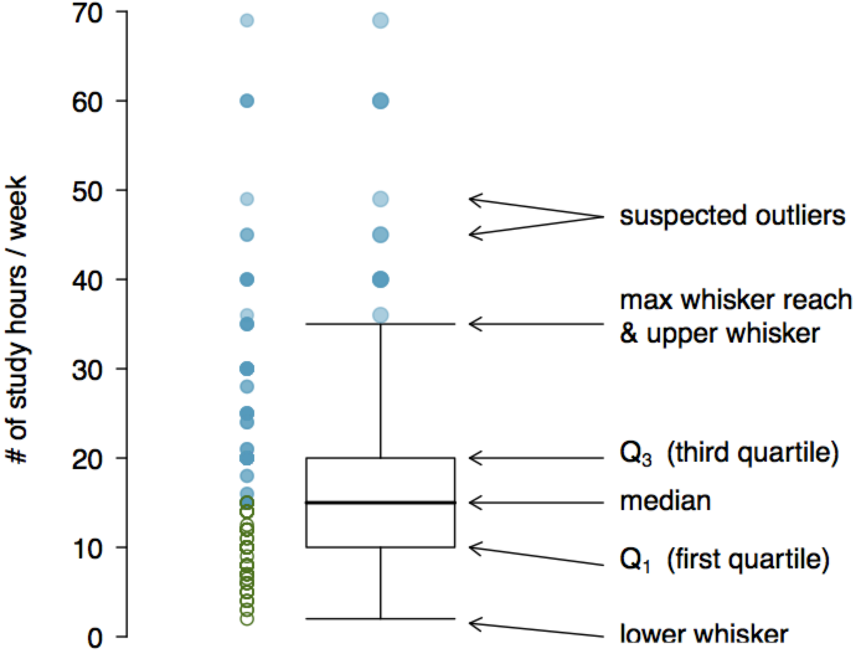
$$\mathrm{IQR} = \mathrm{Q3} - \mathrm{Q1}$$

Box Plot

The box in a **box plot** represents the middle 50% of the data, and the thick line in the box is the median.



Anatomy of a Box Plot



Whiskers and Outliers

The **whiskers** of a box plot can extend up to $1.5 \times \mathrm{IQR}$ away from the quartiles.

- max upper whisker reach = $\mathrm{Q3} + 1.5 \times \mathrm{IQR}$
- max lower whisker reach = $\mathrm{Q1} - 1.5 \times \mathrm{IQR}$

Example: IQR: 20 - 10 = 10

- max upper whisker reach = $20 + 1.5 \times 10 = 35$
- max lower whisker reach = $10 - 1.5 \times 10 = -5$

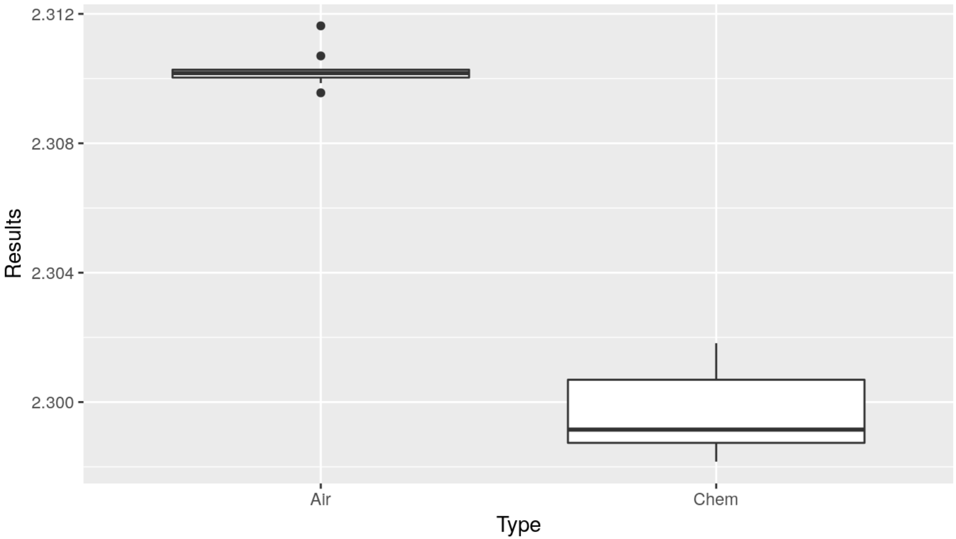
A potential outlier is defined as an observation beyond the maximum reach of the whiskers. It is an observation that appears extreme relative to the rest of the data.

Outliers (continued)

Why is it important to look for outliers?

- Identify extreme skew in the distribution.
- Identify data collection and entry errors.
- Provide insight into interesting features of the data.

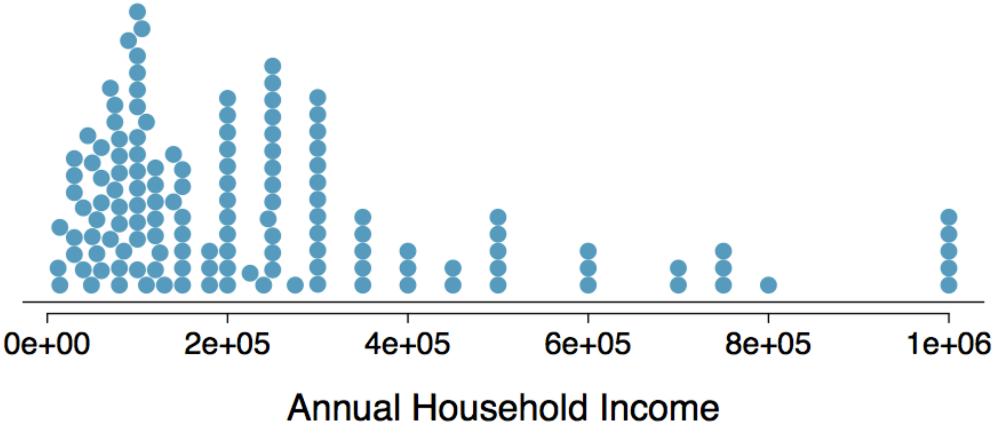
Lord Rayleigh & Nitrogen



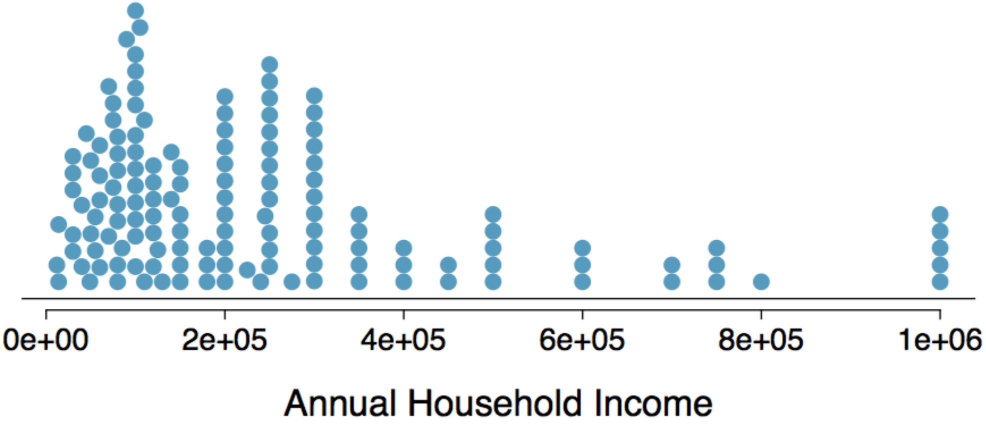
This set of data helped Raleigh conclude that another gas was present in the atmosphere (1894), and led to the discovery of argon (1895). He was awarded the 1904 Nobel Prize in Physics for this discovery.

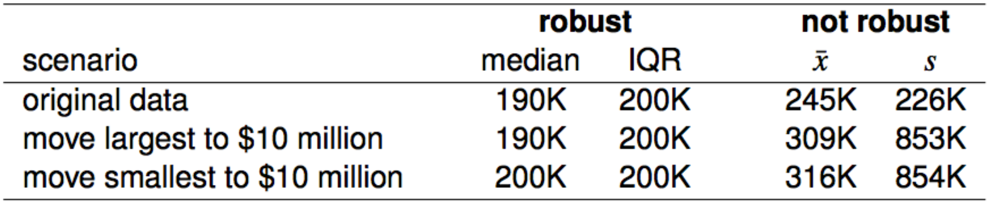
Extreme Observations

How would sample statistics such as mean, median, SD, and IQR of household income be affected if the largest value was replaced with $10 million? What if the smallest value was replaced with $10 million?



Robust Statistics





	robust		not robust	
scenario	median	IQR	$\bar{x}$	s
original data	190K	200K	245K	226K
move largest to $10 million	190K	200K	309K	853K
move smallest to $10 million	200K	200K	316K	854K

Robust Statistics

Median and IQR are more robust to skewness and outliers than mean and SD. Therefore,

- for skewed distributions it is often more helpful to use median and IQR to describe the center and spread
- for symmetric distributions it is often more helpful to use the mean and SD to describe the center and spread

If you would like to estimate the typical household income for a student, would you be more interested in the mean or median income?

Robust Statistics

Median and IQR are more robust to skewness and outliers than mean and SD. Therefore,

- for skewed distributions it is often more helpful to use median and IQR to describe the center and spread
- for symmetric distributions it is often more helpful to use the mean and SD to describe the center and spread

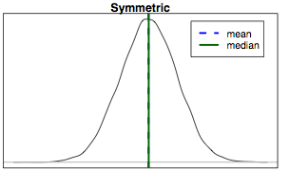
If you would like to estimate the typical household income for a student, would you be more interested in the mean or median income?

Median

Mean versus Median

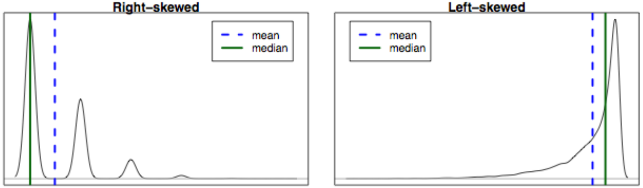
If the distribution is symmetric, center is often defined as the mean:

- mean ≈ median



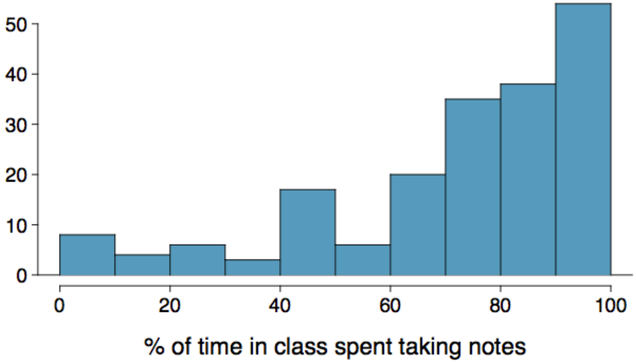
If the distribution is skewed or has extreme outliers, center is often defined as the median

- Right-skewed: mean > median
- Left-skewed: mean < median



Practice

Which is most likely true for the distribution of percentage of time actually spent taking notes in class versus on Facebook, Twitter, etc.?



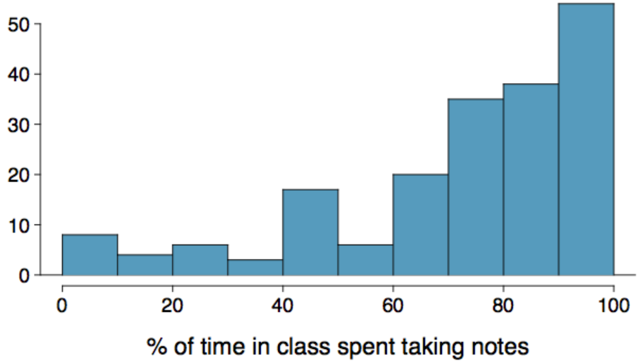
1. mean > median
2. mean < median
3. mean ≈ median
4. impossible to tell

Practice

Which is most likely true for the distribution of percentage of time actually spent taking notes in class versus on Facebook, Twitter, etc.?

If we compute, the mean = 80% and the median = 76%. So …

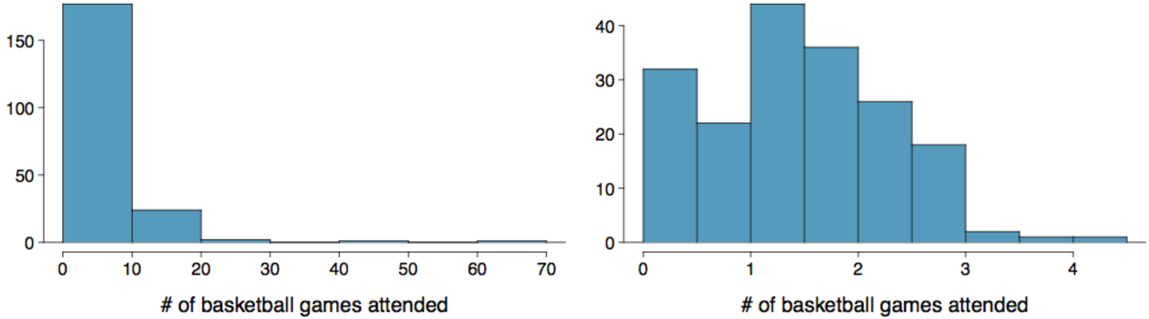
1. mean > median
2. *mean < median*
3. mean ≈ median
4. impossible to tell



Extremely Skewed Data

When data are extremely skewed, transforming them might make modeling easier. A common transformation is the **log transformation**.

The histograms on the left shows the distribution of number of basketball games attended by students. The histogram on the right shows the distribution of log of number of games attended.



Pros and Cons of Transformations

Skewed data are easier to model with when they are transformed because outliers tend to become far less prominent after an appropriate transformation.

# of games	70	50	25	$\cdots$
# of games	4.25	3.91	3.22	$\cdots$

However, results of an analysis might be difficult to interpret because the log of a measured variable is usually meaningless.

What other variables would you expect to be extremely skewed?

Pros and Cons of Transformations

Skewed data are easier to model with when they are transformed because outliers tend to become far less prominent after an appropriate transformation.

# of games	70	50	25	. . .
# of games	4.25	3.91	3.22	. . .

However, results of an analysis might be difficult to interpret because the log of a measured variable is usually meaningless.

What other variables would you expect to be extremely skewed?

Salary, housing prices, ability to throw a football, . . .

Considering Categorical Data

Contingency Tables

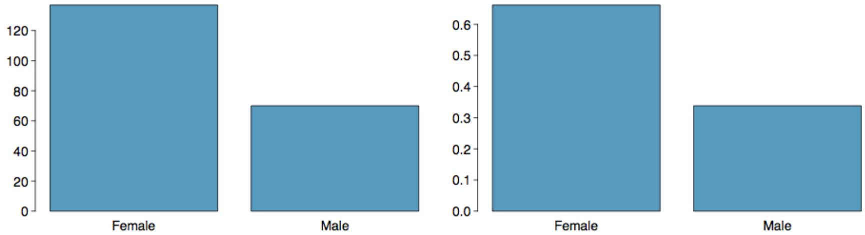
A table that summarizes data for two categorical variables is called a **contingency table**.

The contingency table below shows the distribution of students' genders and whether or not they are looking for a spouse while in college.

	looking for spouse		
	No	Yes	Total
gender			
Female	86	51	137
Male	52	18	70
Total	138	69	207

Bar Plots

A **bar plot** is a common way to display a single categorical variable. A bar plot where proportions instead of frequencies are shown is called a **relative frequency bar plot**.



How are bar plots/charts different than histograms?

- Bar plots are used for displaying distributions of categorical variables, while histograms are used for numerical variables.
- The x-axis in a histogram is a number line, hence the order of the bars cannot be changed, while in a bar plot the categories can be listed in any order (though some orderings make more sense than others, especially for ordinal variables.)

How are bar plots/charts different than histograms

	Histogram	Bar Plots/Charts
Data:	Numerical	Categorical
Bar Width:	Bin Width	Can be any width (all same)
Bars:	Must touch if data exists	Do **not** touch
Horizontal Labels:	Numerical order	No natural order, many choices possible

Choosing the Appropriate Proportion

Does there appear to be a relationship between gender and whether the student is looking for a spouse in college?

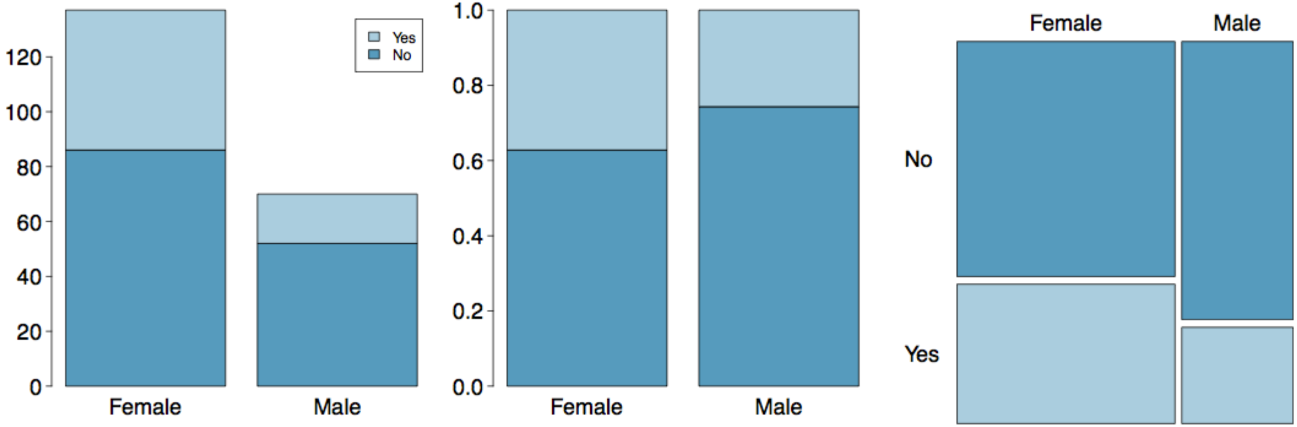
	looking for spouse		
	No	Yes	Total
gender			
Female	86	51	137
Male	52	18	70
Total			
	138	69	207

To answer this question we examine the row proportions:

- % Females looking for a spouse: 51 / 137 ~ 0.37
- % Males looking for a spouse: 18 / 70 ~ 0.26

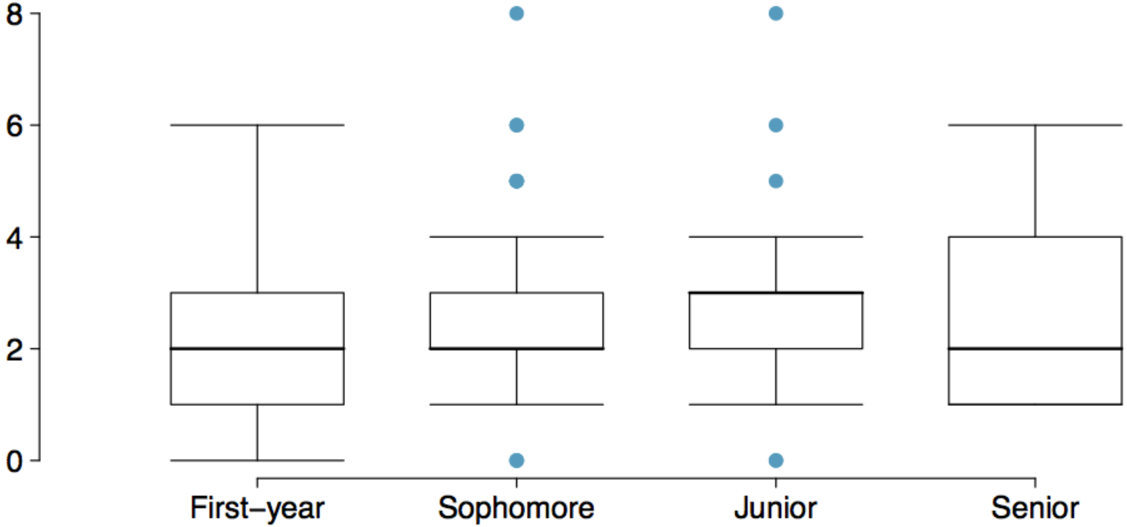
Segmented Bar and Mosaic Plots

What are the differences between the three visualizations shown below?



Comparing Numerical Data Across Groups

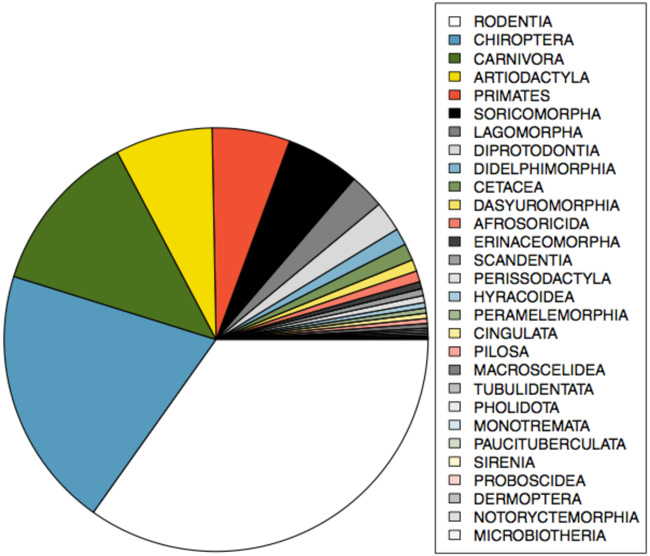
Does there appear to be a relationship between class year and the number of clubs students are in?



Pie Charts

Can you tell which order encompasses the lowest percentage of mammal species?

Source: http://www.bucknell.edu/msw3



Pie Charts are Horrible

If I ever see you using a pie chart, I will come up to you and slam a lemon meringue in your face.

- Humans are **bad, bad, bad** at comparing angles!
- Bar charts are better at doing what pie charts are supposed to do
- The only good thing about pie charts is that they look like pies!

“In a sense, it might be construed as an insult to a man's intelligence to show him a pie chart.” K.G. Karsten, Charts and Graphs (1923)

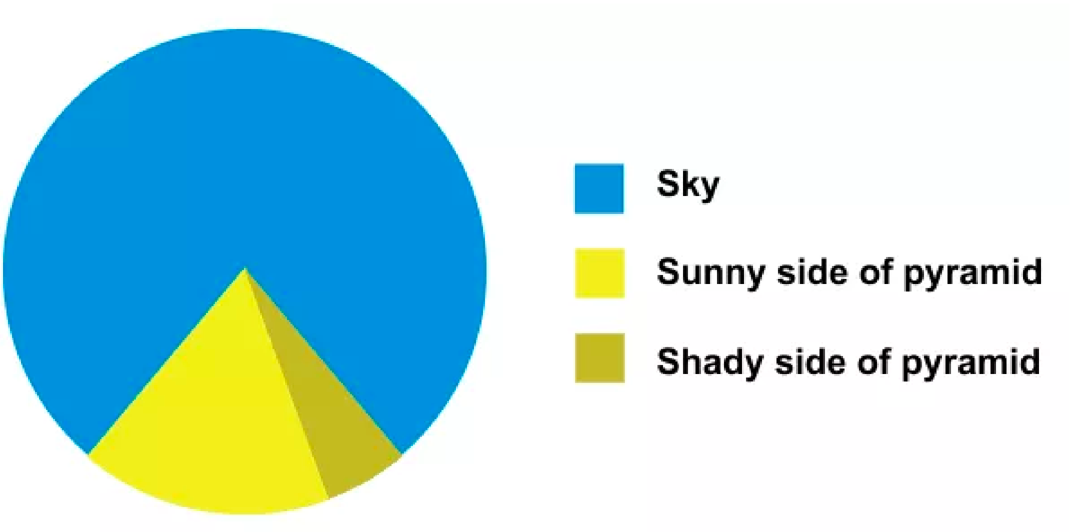


Image from: https://priceonomics.com/should-you-ever-use-a-pie-chart/.

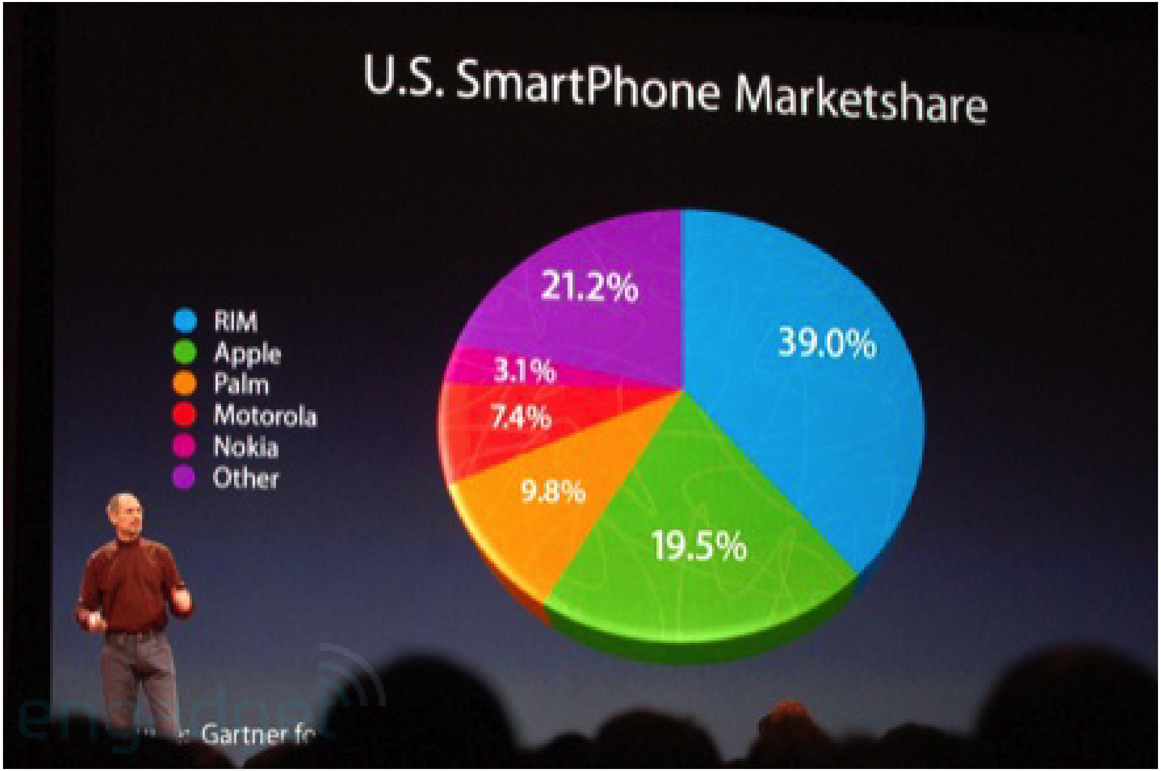
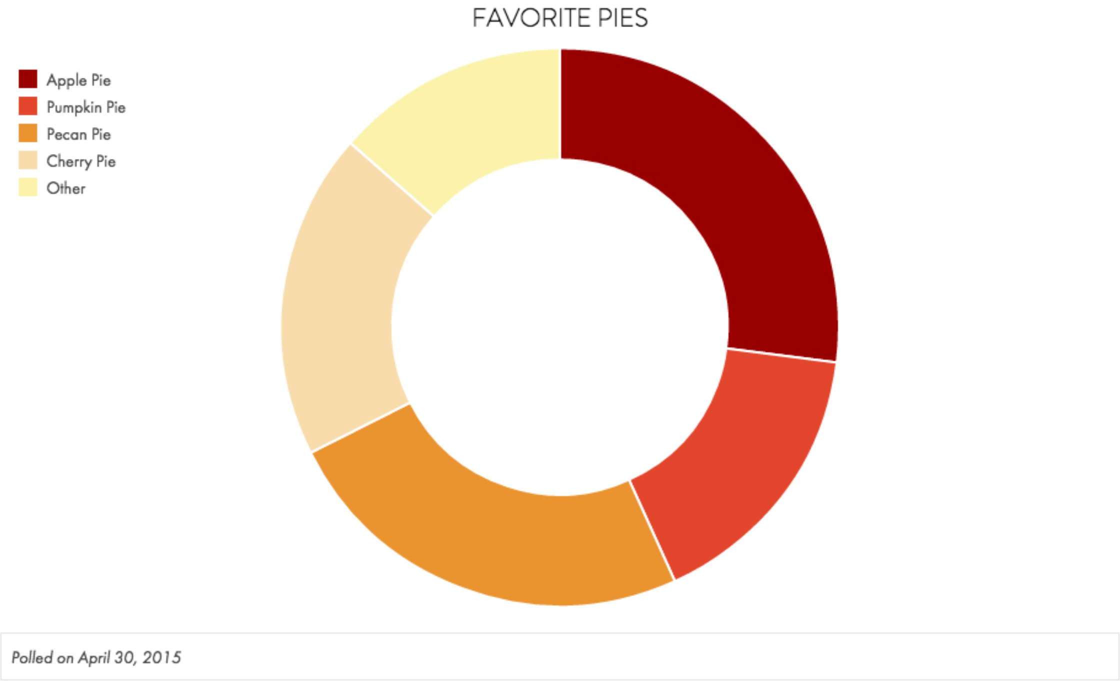


Image from: https://www.edwardtufte.com.



FAVORITE PIES

Polled on April 30, 2015

Image from: Squarespace.com.

Practice 1

In a symmetric distribution,

1. The mean and median are approximately the same.
2. The mean tends to be greater than the median.
3. The mean tends to be less than the median.
4. We do not have enough information to determine the relative sizes of the mean and median.

Practice 1 - Solution

In a symmetric distribution,

1. *The mean and median are approximately the same.*
2. The mean tends to be greater than the median.
3. The mean tends to be less than the median.
4. We do not have enough information to determine the relative sizes of the mean and median.

Practice 2

In a right-skewed distribution,

1. The mean and median are approximately the same.
2. The mean tends to be greater than the median.
3. The mean tends to be less than the median.
4. We do not have enough information to determine the relative sizes of the mean and median.

Practice 2 - Solution

In a right-skewed distribution,

1. The mean and median are approximately the same.
2. *The mean tends to be greater than the median.*
3. The mean tends to be less than the median.
4. We do not have enough information to determine the relative sizes of the mean and median.

Practice 3

Which of the following measures is most commonly used to describe the centre of a symmetric distribution?

1. Mean
2. Median
3. Standard deviation
4. Interquartile range

Practice 3 - Solution

Which of the following measures is most commonly used to describe the centre of a symmetric distribution?

1. *Mean*
2. Median
3. Standard deviation
4. Interquartile range

Practice 4

Which of the following measures is most commonly used to describe the centre of a skewed distribution?

1. Mean
2. Median
3. Standard deviation
4. Interquartile range

Practice 4 - Solution

Which of the following measures is most commonly used to describe the centre of a skewed distribution?

1. Mean
2. *Median*
3. Standard deviation
4. Interquartile range