

Lecture 16

Case Study: Gender Discrimination

Gender Discrimination

- In 1972, as a part of a study on gender discrimination, 48 male bank supervisors were each given the same personnel file and asked to judge whether the person should be promoted to a branch manager job that was described as “routine”.
- The files were identical except that half of the supervisors had files showing the person was male while the other half had files showing the person was female.
- It was randomly determined which supervisors got “male” applications and which got “female” applications.
- Of the 48 files reviewed, 35 were promoted.
- The study is testing whether females are unfairly discriminated against.
- Is this an observational study or an experiment?

B.Rosen and T. Jerdee (1974), “Influence of sex role stereotypes on personnel decisions”, J.Applied Psychology, 59:9-14.

Data

At a first glance, does there appear to be a relationship between promotion and gender?

```
## Registered S3 method overwritten by 'rvest':  
##   method      from  
##   read_xml.response xml2
```

	<u>Promoted</u>		Total
	Yes	No	
<u>Gender</u>			
Male	21	3	24
Female	14	10	24
Total	35	13	48

% of males promoted: $21 / 24 = 0.875$

% of females promoted: $14 / 24 = 0.583$

Practice

We saw a difference of almost 30% (29.2% to be exact) between the proportion of male and female files that are promoted. Based on this information, which of the below is true?

1. If we were to repeat the experiment we will definitely see that more female files get promoted. This was a fluke.
2. Promotion is dependent on gender, males are more likely to be promoted, and hence there is gender discrimination against women in promotion decisions.
3. The difference in the proportions of promoted male and female files is due to chance, this is not evidence of gender discrimination against women in promotion decisions.
4. Women are less qualified than men, and this is why fewer females get promoted.

Practice

We saw a difference of almost 30% (29.2% to be exact) between the proportion of male and female files that are promoted. Based on this information, which of the below is true?

1. If we were to repeat the experiment we will definitely see that more female files get promoted. This was a fluke.
2. *Promotion is dependent on gender, males are more likely to be promoted, and hence there is gender discrimination against women in promotion decisions.* Maybe!
3. *The difference in the proportions of promoted male and female files is due to chance, this is not evidence of gender discrimination against women in promotion decisions.* Maybe!
4. Women are less qualified than men, and this is why fewer females get promoted.

Two Competing Claims

"There is nothing going on." (**Null Hypothesis**)

Promotion and gender are **independent**.

No gender discrimination.

Observed difference in proportions is simply due to chance.

versus

"There is something going on." (**Alternative Hypothesis**)

Promotion and gender are **dependent**.

There is gender discrimination.

Observed difference in proportions is not due to chance.

We Will Return!

We will continue with the concept of hypothesis testing later in this lecture, and over the following weeks. First, let's develop some other ideas.

Variability in estimates

Young, Underemployed and Optimistic

Coming of Age, Slowly, in a Tough Economy

Young adults hit hard by the recession. A plurality of the public (41%) believes young adults, rather than middle-aged or older adults, are having the toughest time in today's economy. An analysis of government economic data suggests that this perception is correct. The recent indicators on the nation's labor market show a decline in the

Tough economic times altering young adults' daily lives, long-term plans. While negative trends in the labor market have been felt most acutely by the youngest workers, many adults in their late 20s and early 30s have also felt the impact of the weak economy. Among all 18- to 34-year-olds, fully half (49%) say they have taken a job they didn't want just to pay the bills, with 24% saying they have taken an unpaid job to gain work experience. And more than one-third (35%) say that, as a result of the poor economy, they have gone back to school. Their personal lives have also been affected: 31% have postponed either getting married or having a baby (22% say they have postponed having a baby and 20% have put off getting married). One-in-four (24%) say they have moved back in with their parents after living on their own.

<http://pewresearch.org/pubs/2191/young-adults-workers-labor-market-pay-careers-advancement-recession>

Margin of error

The general public survey is based on telephone interviews conducted Dec. 6-19, 2011, with a nationally representative sample of 2,048 adults ages 18 and older living in the continental United States, including an oversample of 346 adults ages 18 to 34. A total of 769 interviews were completed with respondents contacted by landline telephone and 1,279 with those contacted on their cellular phone. Data are weighted to produce a final sample that is representative of the general population of adults in the continental United States. Survey interviews were conducted under the direction of Princeton Survey Research Associates International, in English and Spanish. Margin of sampling error is plus or minus 2.9 percentage points for results based on the total sample and 4.4 percentage points for adults ages 18-34 at the 95% confidence level.

- 41% \pm 2.9%: We are 95% confident that 38.1% to 43.9% of the public believe young adults, rather than middle-aged or older adults, are having the toughest time in today's economy.
- 49% \pm 4.4%: We are 95% confident that 44.6% to 53.4% of 18-34 years olds have taken a job they didn't want just to pay the bills.

Parameter estimation

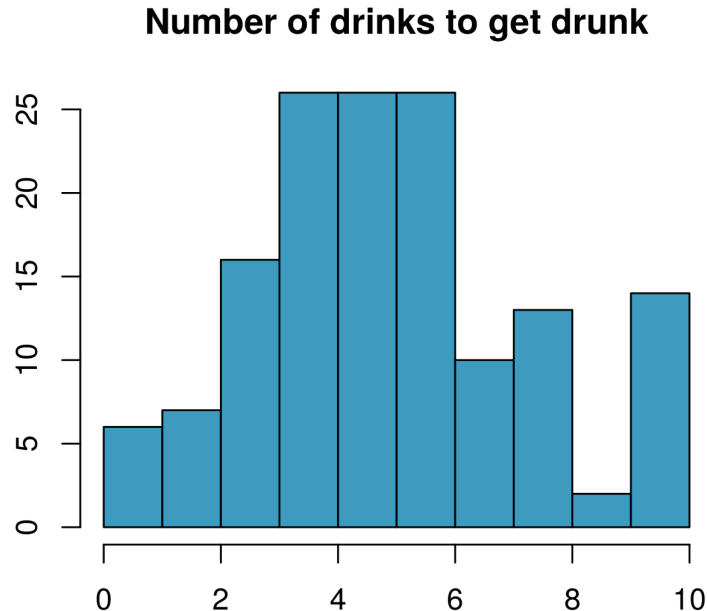
- We are often interested in **population parameters**.
- Since complete populations are difficult (or impossible) to collect data on, we use **sample statistics** as **point estimates** for the unknown population parameters of interest.
- Sample statistics vary from sample to sample.
- Quantifying how sample statistics vary provides a way to estimate the **margin of error** associated with our point estimate.
- But before we get to quantifying the variability among samples, let's try to understand how and why point estimates vary from sample to sample.

Parameter estimation

Suppose we randomly sample 1,000 adults from each state in the US. Would you expect the sample means of their heights to be the same, somewhat different, or very different?

Not the same, but only somewhat different.

The following histogram shows the distribution of number of drinks it takes a group of college students to get drunk. We will assume that this is our population of interest. If we randomly select observations from this data set, which values are most likely to be selected (which are least likely)?



Suppose that you don't have access to the population data. In order to estimate the average number of drinks it takes these college students to get drunk, you might sample from the population and use your sample mean as the best guess for the unknown population mean.

- Sample, with replacement, ten students from the population, and record the number of drinks it takes them to get drunk.
- Find the sample mean.
- Plot the distribution of the sample averages obtained by members of the class.

1	7	16	3	31	5	46	4	61	10	76	6	91	4	106	6	121	6	136	6
2	5	17	10	32	9	47	3	62	7	77	6	92	0.5	107	2	122	5	137	7
3	4	18	8	33	7	48	3	63	4	78	5	93	3	108	5	123	3	138	3
4	4	19	5	34	5	49	6	64	5	79	4	94	3	109	1	124	2	139	10
5	6	20	10	35	5	50	8	65	6	80	5	95	5	110	5	125	2	140	4
6	2	21	6	36	7	51	8	66	6	81	6	96	6	111	5	126	5	141	4
7	3	22	2	37	4	52	8	67	6	82	5	97	4	112	4	127	10	142	6
8	5	23	6	38	0	53	2	68	7	83	6	98	4	113	4	128	4	143	6
9	5	24	7	39	4	54	4	69	7	84	8	99	2	114	9	129	1	144	4
10	6	25	3	40	3	55	8	70	5	85	4	100	5	115	4	130	4	145	5
11	1	26	6	41	6	56	3	71	10	86	10	101	4	116	3	131	10	146	5
12	10	27	5	42	10	57	5	72	3	87	5	102	7	117	3	132	8		
13	4	28	8	43	3	58	5	73	5.5	88	10	103	6	118	4	133	10		
14	4	29	0	44	6	59	8	74	7	89	8	104	8	119	4	134	6		
15	6	30	8	45	10	60	4	75	10	90	5	105	3	120	8	135	6		

Example: List of random numbers: 59, 121, 88, 46, 58, 72, 82, 81, 5, 10

1	7	16	3	31	5	46	4	61	10	76	6	91	4	106	6	121	6	136	6
2	5	17	10	32	9	47	3	62	7	77	6	92	0.5	107	2	122	5	137	7
3	4	18	8	33	7	48	3	63	4	78	5	93	3	108	5	123	3	138	3
4	4	19	5	34	5	49	6	64	5	79	4	94	3	109	1	124	2	139	10
5	6	20	10	35	5	50	8	65	6	80	5	95	5	110	5	125	2	140	4
6	2	21	6	36	7	51	8	66	6	81	6	96	6	111	5	126	5	141	4
7	3	22	2	37	4	52	8	67	6	82	5	97	4	112	4	127	10	142	6
8	5	23	6	38	0	53	2	68	7	83	6	98	4	113	4	128	4	143	6
9	5	24	7	39	4	54	4	69	7	84	8	99	2	114	9	129	1	144	4
10	6	25	3	40	3	55	8	70	5	85	4	100	5	115	4	130	4	145	5
11	1	26	6	41	6	56	3	71	10	86	10	101	4	116	3	131	10	146	5
12	10	27	5	42	10	57	5	72	3	87	5	102	7	117	3	132	8		
13	4	28	8	43	3	58	5	73	5.5	88	10	103	6	118	4	133	10		
14	4	29	0	44	6	59	8	74	7	89	8	104	8	119	4	134	6		
15	6	30	8	45	10	60	4	75	10	90	5	105	3	120	8	135	6		

Sample mean: $\frac{8+6+10+4+5+3+5+6+6+6}{10} = 5.9$

Sampling distribution

What you just constructed is called a *sampling distribution*.

What is the shape and center of this distribution? Based on this distribution, what do you think is the true population average?

Sampling distribution

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Approximately 5.39, the true population mean.

Sampling distributions - via CLT

Central limit theorem

Central limit theorem The distribution of the sample mean is well approximated by a normal model:

$$\bar{x} \sim \mathcal{N} \left(\text{mean} = \mu, \text{SE} = \frac{\sigma}{\sqrt{n}} \right),$$

where SE represents **standard error**, which is defined as the standard deviation of the sampling distribution. If σ is unknown, use s (recall: standard deviation of sample).

Central limit theorem

- It wasn't a coincidence that the sampling distribution we saw earlier was symmetric, and centered at the true population mean.
- We won't go through a detailed proof of why $SE = \frac{\sigma}{\sqrt{n}}$, but note that as n increases SE decreases.
 - As the sample size increases we would expect samples to yield more consistent sample means, hence the variability among the sample means would be lower.

CLT - conditions

Certain conditions must be met for the CLT to apply:

- **Independence:** Sampled observations must be independent. This is difficult to verify, but is more likely if
 - random sampling/assignment is used, and
 - if sampling without replacement, $n < 10\%$ of the population.

CLT - conditions

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- **Independence:** Sampled observations must be independent. This is difficult to verify, but is more likely if
 - random sampling/assignment is used, and
 - if sampling without replacement, $n < 10\%$ of the population.
- **Sample size/skew:** Either the population distribution is normal, or if the population distribution is skewed, the sample size is large. This is also difficult to verify for the population, but we can check it using the sample data, and assume that the sample mirrors the population.
 - the more skewed the population distribution, the larger sample size we need for the CLT to apply
 - for moderately skewed distributions $n > 30$ is a widely used rule of thumb

Confidence intervals

Why do we report confidence intervals?

- A plausible range of values for the population parameter is called a **confidence interval**.
- Using only a sample statistic to estimate a parameter is like **fishing with a spear** in a murky lake, and using a confidence interval is like **fishing with a net**.
- We can throw a spear where we saw a fish but we will probably miss. If we toss a net in that area, we have a good chance of catching the fish.

So the analogy: if we report a point estimate, we probably won't hit the exact population parameter. If we report a range of plausible values we have a good shot at capturing the parameter.

Average number of exclusive relationships

A random sample of 50 college students were asked how many exclusive relationships they have been in so far. This sample yielded a mean of 3.2 and a standard deviation of 1.74. Estimate the true average number of exclusive relationships using this sample.

$$\bar{x} = 3.2 \quad s = 1.74$$

The approximate 95% confidence interval is defined as

$$\text{point estimate} \pm 2 \times \text{SE}$$

$$SE = \frac{s}{\sqrt{n}} = \frac{1.74}{\sqrt{50}} \approx 0.25$$

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$$\begin{aligned}\bar{x} \pm 2 \times SE &= 3.2 \pm 2 \times 0.25 \\ &= (3.2 - 0.5, 3.2 + 0.5) \\ &= (2.7, 3.7)\end{aligned}$$

Practice

Which of the following is the correct interpretation of this confidence interval?

We are 95% confident that

- the average number of exclusive relationships college students in this sample have been in is between 2.7 and 3.7.
- college students on average have been in between 2.7 and 3.7 exclusive relationships.
- a randomly chosen college student has been in 2.7 to 3.7 exclusive relationships.
- 95% of college students have been in 2.7 to 3.7 exclusive relationships.

Practice

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- *college students on average have been in between 2.7 and 3.7 exclusive relationships.*
- a randomly chosen college student has been in 2.7 to 3.7 exclusive relationships.
- 95% of college students have been in 2.7 to 3.7 exclusive relationships.

A more accurate interval

Confidence interval, a general formula

$$\text{point estimate} \pm z^* \cdot SE$$

Conditions when the point estimate = \bar{x} :

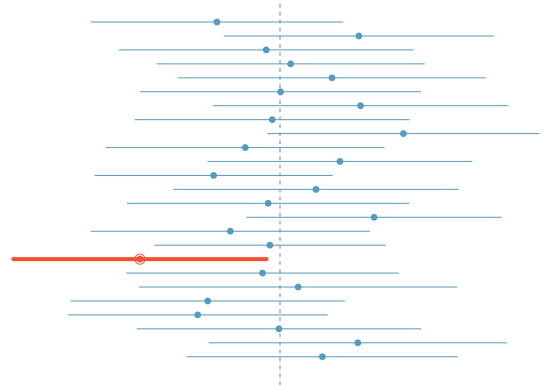
- **Independence:** Observations in the sample must be independent
 - random sample/assignment
 - if sampling without replacement, $n < 10\%$ of population
- **Sample size / skew:** $n \geq 30$ and population distribution should not be extremely skewed

Note: We will discuss working with samples where $n < 30$ later.

Capturing the population parameter

What does 95% confident mean?

- Suppose we took many samples and built a confidence interval from each sample using the equation $\text{point estimate} \pm 2 \cdot SE$.
- Then about 95% of those intervals would contain the true population μ .
- The figure to the right shows this process with 25 samples, where 24 of the resulting confidence intervals contain the true average number of exclusive relationships, and one does not.



Width of an interval

If we want to be more certain that we capture the population parameter, *i.e.*, increase our confidence level, should we use a wider interval or a smaller interval?

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A wider interval.

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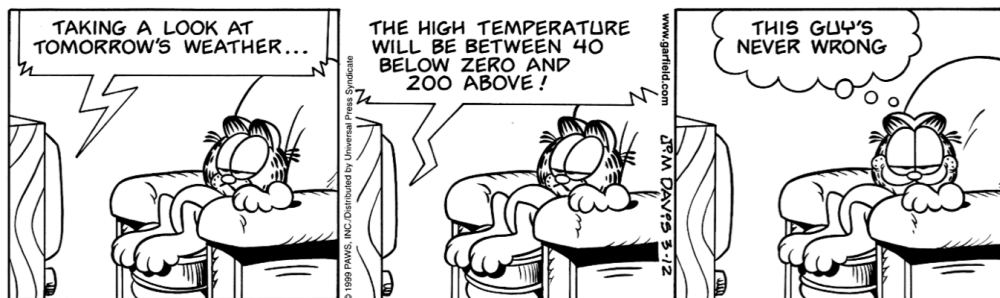
Can you see any drawbacks to using a wider interval?

Width of an interval

If we want to be more certain that we capture the population parameter, i.e. increase our confidence level, should we use a wider interval or a smaller interval?

A wider interval.

Can you see any drawbacks to using a wider interval?



If the interval is too wide it may not be very informative.

http://web.as.uky.edu/statistics/users/earo227/misc/garfield_weather.gif

Changing the confidence level

$$\text{point estimate} \pm z^* \cdot SE$$

- In a confidence interval, $z^* \cdot SE$ is called the **margin of error** (ME), and for a given sample, the margin of error changes as the confidence level changes.
- In order to change the confidence level we need to adjust z^* in the above formula.
- Commonly used confidence levels in practice are 90%, 95%, 98%, and 99%.
- For a 95% confidence interval, $z^* = 1.96$.
- However, using the standard normal (z) distribution, it is possible to find the appropriate z^* for any confidence level.

Practice

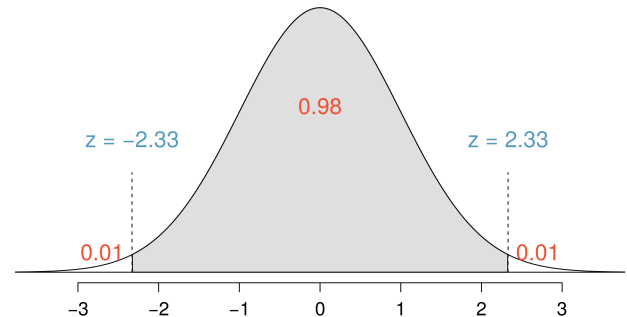
Which of the below Z scores is the appropriate z^* when calculating a 98% confidence interval?

- $Z = 2.05$
- $Z = 1.96$
- $Z = 2.33$
- $Z = -2.33$
- $Z = -1.65$

Practice

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Hypothesis Testing

Hypothesis Tests as a Trial

Hypothesis testing is very much like a court trial.

- H_0 : defendant is innocent (English common law; Justinian Codes, UN Declaration of Human Rights), *versus* H_A : defendant is guilty
- We then present the evidence – collect data
- Then we judge the evidence: “Could these data plausibly have happened by chance if the null hypothesis were true?”
 - If they were very unlikely to have occurred, then the evidence raises more than *a reasonable doubt* in our minds about the null hypothesis
- Ultimately, we must make a decision: how unlikely is **unlikely**?



Image from http://www.nwherald.com/_internal/cimg!0/oo1il4sf8zzaqbboq25oenvbg99wpot

A Hypothesis Test as a Trial (continued)

- If the evidence is not strong enough to reject the assumption of innocence, the jury returns with a verdict of “not guilty”.
 - The jury does not say that the defendant is innocent, just that there is not enough evidence to convict.
 - The defendant may, in fact, be innocent, but the jury has no way of being sure.
- Said statistically, we **fail to reject the null hypothesis**.
 - We never declare the null hypothesis to be true, because we simply do not know whether it's true or not.
 - Therefore we never “accept the null hypothesis”.

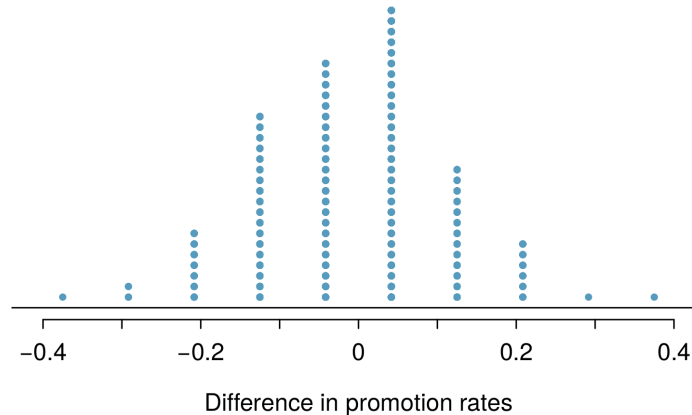
A Hypothesis Test as a Trial (continued)

- In a trial, the burden of proof is on the prosecution.
- In a hypothesis test, the burden of proof is on the unusual claim.
- The null hypothesis is the ordinary state of affairs (the status quo), so it's the alternative hypothesis that we consider unusual and for which we must gather evidence.

Recap: Concept of Hypothesis Testing

- We start with a **null hypothesis** (H_0) that represents the status quo.
- We also have an **alternative hypothesis** (H_A) that represents our research question, i.e. what we're testing for.
- We conduct a hypothesis test under the assumption that the null hypothesis is true, either via simulation (today) or theoretical methods (later in the course).
- If the test results suggest that the data do not provide convincing evidence for the alternative hypothesis, we stick with the null hypothesis. If they do, then we reject the null hypothesis in favor of the alternative.

Back to the example of gender discrimination ...



Since it was quite unlikely to obtain results like the actual data or something more extreme in the simulations (male promotions being 30% or more higher than female promotions), we would decide to reject the null hypothesis in favor of the alternative.

Testing Hypotheses: CIs

Testing hypotheses using confidence intervals

Earlier we calculated a 95% confidence interval for the average number of exclusive relationships college students have been in to be (2.7, 3.7). Based on this confidence interval, do these data support the hypothesis that college students on average have been in more than 3 exclusive relationships?

- The associated hypotheses are:
 - $H_0: \mu = 3$: College students have been in 3 exclusive relationships, on average
 - $H_A: \mu > 3$: College students have been in more than 3 exclusive relationships, on average

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- Since the null value is included in the interval, we do not reject the null hypothesis in favor of the alternative.
- This is a quick-and-dirty approach for hypothesis testing. However it doesn't tell us the likelihood of certain outcomes under the null hypothesis, i.e., the p -value, based on which we can make a decision on the hypotheses.

Hypothesis Testing

Concept of Hypothesis Testing

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Recall

Confidence intervals for the population mean μ from large samples have the form

$$\bar{x} \pm \text{ME} = \bar{x} \pm z^* \cdot \text{SE}$$

and explicitly, the Standard Error, SE, is

$$\text{SE} = \frac{\sigma}{\sqrt{n}}.$$

Examples

Example 1

Maple Leaf receives a large shipment of turkeys carcasses for packaging and sale, and the manager wants to determine if the true mean weight of the turkeys meets their requirement of 3.7 kg per turkey, on average. A random sample of 36 turkeys yields a sample mean weight of 3.6 kg., with a sample standard deviation of 0.61 kg. Does the shipment satisfy Maple Leaf's requirement? (Note: it would be very costly to reject a shipment incorrectly.) Complete a test of hypothesis using a confidence interval.

Example 1

Key information:

- $\mu = 3.7$
- $\bar{x} = 3.6$
- $s = 0.61$
- $n = 36$

Example 1: the CI

Compute the confidence interval:

$$\bar{x} \pm z^* \cdot \frac{s}{\sqrt{n}} = 3.6 \pm z^* \cdot \frac{0.61}{\sqrt{36}}$$

What is z^* ? The question says that a “mistake will be very costly” - so we really don’t want to make a mistake! This corresponds to a high degree of confidence, so let’s say 99%, which is $\alpha/2 = 0.005$. Then:

```
qnorm(1 - 0.01/2)
```

```
## [1] 2.575829
```

Example 1: finishing

$$\bar{x} \pm z^* \cdot \frac{s}{\sqrt{n}} = 3.6 \pm 2.576 \cdot \frac{0.61}{\sqrt{36}} = (3.3381, 3.8619)$$

Since this confidence interval **includes** the null hypothesis of 3.7, we **fail to reject the null hypothesis**, and this shipment probably meets the company's requirement.

Number of university applications

A survey asked how many universities students applied to, and 206 students responded to this question. This sample yielded an average of 9.7 university applications with a standard deviation of 7. A government website states that counselors recommend students apply to roughly 8 universities. Do these data provide convincing evidence that the average number of universities all Trent students apply to is *higher* than recommended?

Setting the hypotheses

- The **parameter of interest** is the average number of schools applied to by **all** Trent students.
- There may be two explanations why our sample mean is higher than the recommended 8 schools.
 - The true population mean is different.
 - The true population mean is 8, and the difference between the true population mean and the sample mean is simply due to natural sampling variability.

Setting the hypotheses (ctd.)

- We start with the assumption the average number of schools Trent students apply to is 8 (as recommended)

$$H_0 : \mu = 8$$

- We test the claim that the average number of schools Trent students apply to is greater than 8

$$H_A : \mu > 8$$

Number of university applications - conditions

Which of the following is *not* a condition that needs to be met to proceed with this hypothesis test?

- Students in the sample should be independent of each other with respect to how many schools they applied to.
- Sampling should have been done randomly.
- The sample size should be less than 10% of the population of all Trent students.
- There should be at least 10 successes and 10 failures in the sample.
- The distribution of the number of schools students apply to should not be extremely skewed.

Using a CI

What's the key information again?

- $n = 206$
- $\bar{x} = 9.7$
- $s = 7$
- $\mu = 8$

$$\bar{x} \pm z^* \cdot \frac{s}{\sqrt{n}} = 9.7 \pm 1.96 \cdot \frac{7}{\sqrt{206}} = (8.744, 10.656)$$

So, since 8 is **not** included in this interval, we **do have evidence to reject the null**, and we can conclude that it appears that Trent students are applying to too many universities, based on the standard recommendation.

Next Idea

There is another way we can do these hypothesis tests, which we'll talk about next class.