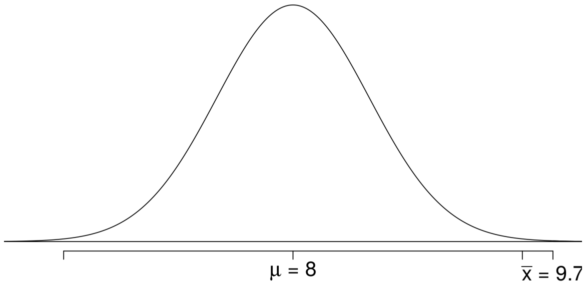
Lecture 17

Formal testing using p-values

Test statistic

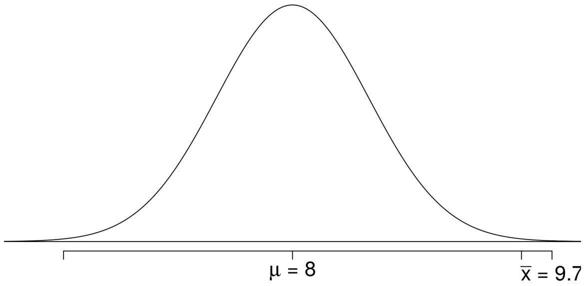


statistically significant

$$\bar{x} \sim N\left(\mu = 8, SE = \frac{7}{\sqrt{206}} \approx 0.5\right)$$

$$Z = \frac{9.7 - 8}{0.5} = 3.4$$

Test statistic



$$\bar{x} \sim N\left(\mu = 8, SE = \frac{7}{\sqrt{206}} \approx 0.5\right)$$

$$Z = \frac{9.7 - 8}{0.5} = 3.4$$

statistically significant

Yes, and we can quantify how unusual it is using a p-value.

p-values

- p-value

p-values

- **p-value**
- **low** α

  reject H_0

p-values

- **p-value**
- **low** α

  reject H_0
- **high** α

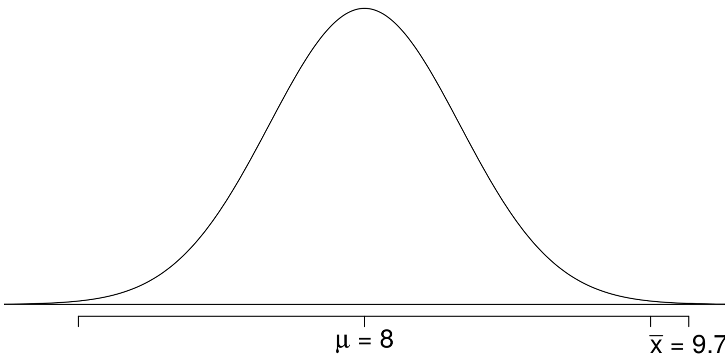
  do not reject H_0

Number of university applications - p-value

p-value:

H_A

H_0



$$P(\bar{x} > 9.7 \mid \mu = 8) = P(Z > 3.4) = 0.0003$$

Number of university applications - Making a decision

-

Number of university applications - Making a decision

Number of university applications - Making a decision

-
    -
    -
- **low** **reject H_0**

Number of university applications - Making a decision

-
  -
  -
- low reject H_0
-

Number of university applications - Making a decision

-
    -
    -
- low reject H_0
-
- not due to chance

less than

- H_0
- H_0
- H_0
- H_0
- H_0

Two-sided hypothesis testing with p-values

-

different

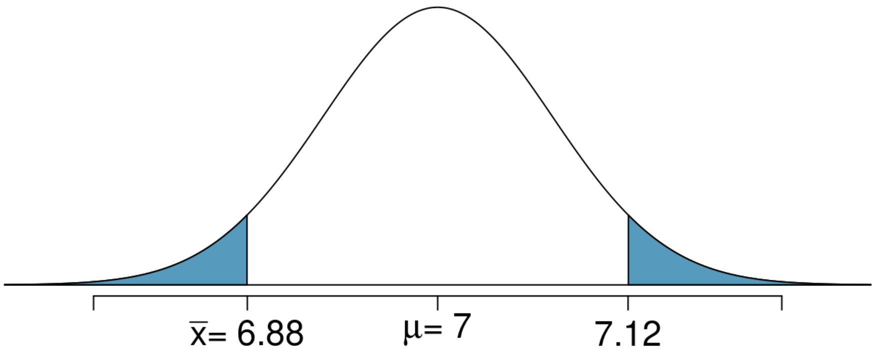
$$H_0 : \mu = 7$$
$$H_A : \mu \neq 7$$

Two-sided hypothesis testing with p-values

- different

- would change as well

$$= 0.0485 \times 2 = 0.097$$

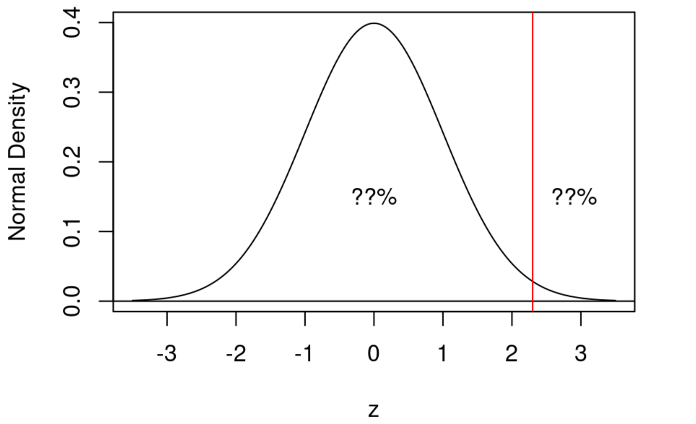


Computing the -value

pnorm()

Example **test statistic**

$$H_0 : \mu = 5 \qquad \text{versus} \qquad H_A : \mu > 5$$



Example, continued

```
pnorm(2.3, mean = 0, sd = 1, lower.tail = FALSE)
```

```
## [1] 0.01072411
```

one-tailed hypothesis test

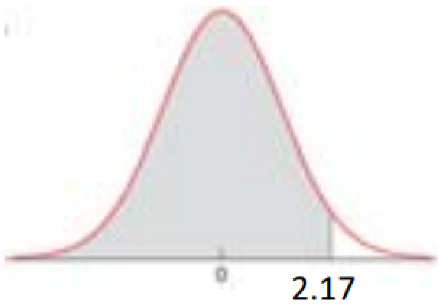
$$0.011 < 0.05$$

$$\mu > 5$$

The Alternative Hypothesis …

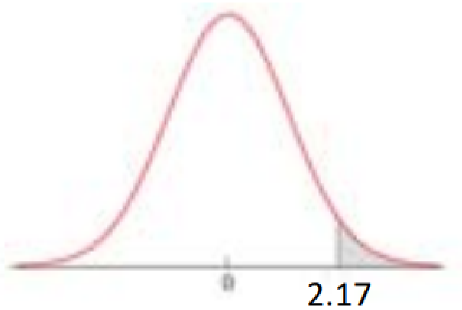
Two-tailed H_A
(≠)

Find the area to the right of z and multiply by 2, (or to the left of z if *z* were negative and multiply by 2).



Left-tailed H_A
(<)

Find the area to the left of z.



Right-tailed H_A
(>)

Find the area to the right of z.

Decision Errors

Decision errors

-

Decision errors

-
-

Decision errors

Decision errors

Decision errors (cont.)

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	
	H_A true		✓

Decision errors (cont.)

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type 1 Error*
	H_A true	*Type 2 Error*	✓

Decision errors (cont.)

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	Type 1 Error
	H_A true	Type 2 Error	✓

- **Type 1 Error** H_0

Decision errors (cont.)

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	*Type 1 Error*
	H_A true	*Type 2 Error*	✓

- **Type 1 Error** H_0
- **Type 2 Error** H_A

Decision errors (cont.)

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	✓	Type 1 Error
	H_A true	Type 2 Error	✓

- **Type 1 Error** H_0
- **Type 2 Error** H_A
- H_0 H_A

Hypothesis Test as a trial

$$
\begin{aligned}
H_0 &: \text{ Defendant is innocent} \\
H_A &: \text{ Defendant is guilty}
\end{aligned}
$$

Hypothesis Test as a trial

$$
\begin{aligned}
H_0 &: \text{ Defendant is innocent} \\
H_A &: \text{ Defendant is guilty}
\end{aligned}
$$

-
-

Hypothesis Test as a trial

$$\begin{aligned} H_0 &: \text{ Defendant is innocent} \\ H_A &: \text{ Defendant is guilty} \end{aligned}$$

-
    - Type 2 error
-
    - Type 1 error

Hypothesis Test as a trial

$$
\begin{aligned}
H_0 &: \text{ Defendant is innocent} \\
H_A &: \text{ Defendant is guilty}
\end{aligned}
$$

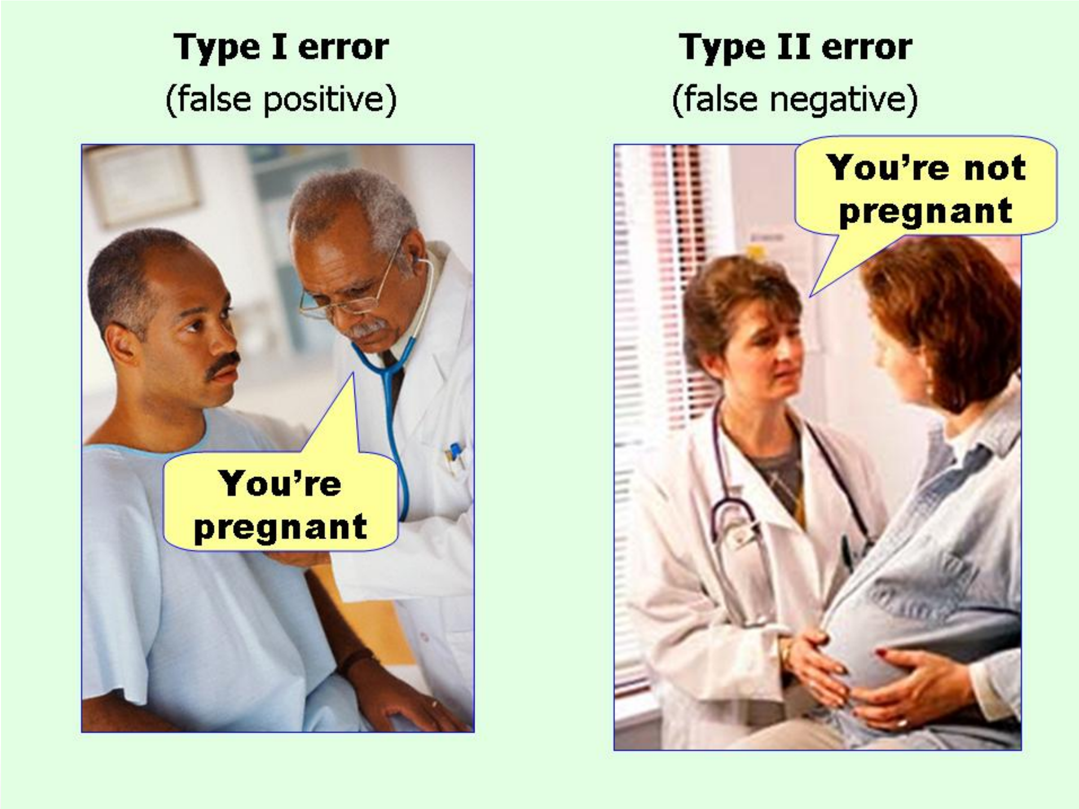
-
    - Type 2 error
-
    - Type 1 error

Hypothesis Test as a trial

Better that ten guilty persons escape than that one innocent suffer

— *Sir William Blackstone (1765)*

Another way to remember



Another way to remember

- **medical test**

Another way to remember

- **medical test**
- **reject the null**

Another way to remember

- **medical test**
- **reject the null**
  -

Another way to remember

- medical test
- reject the null
  -
  - false

Another way to remember

- medical test
- reject the null
    -
    - false
    -
    - declaring the defendent guilty, when they are actually innocent

Another way to remember

- **medical test**
- **fail to reject the null**

Another way to remember

- **medical test**
- **fail to reject the null**
  -

Another way to remember

- medical test
- fail to reject the null
    -
    - false

Another way to remember

- medical test
- fail to reject the null
  -
  - false
  -
  - declaring the defendent innocent, when they are actually guilty

Type 1 error rate

- H_0
  significance level $\alpha = 0.05$

Type 1 error rate

- H_0
  significance level $\alpha = 0.05$
- H_0

Type 1 error rate

- H_0
  significance level $\alpha = 0.05$
- H_0
-

$$P\left(\text{Type 1 error } \middle| H_0 \text{ true}\right) = \alpha$$

Type 1 error rate

- H_0
  significance level $\alpha = 0.05$
- H_0
-

$$P\left(\text{Type 1 error } \middle| H_0 \text{ true}\right) = \alpha$$

- α increasing α increases the Type 1 error rate

Choosing a significance level

-

Choosing a significance level

-
-

Choosing a significance level

-
-
-

H_A H_0

Choosing a significance level

-

-

-

H_A H_0

-

H_0

Recap: Hypothesis testing framework

-
-
- test statistic
-

Recap: Hypothesis testing for a population mean

-
    - $H_0 : \mu = \text{null value}$
    - $H_A : \mu < \quad > \quad \neq$
-
-
    -
    - $n \geq 30$ or use the t distribution

Recap: Hypothesis testing for a population mean

- test statistic

$$Z = \frac{\bar{x} - \mu}{SE}, \text{ where } SE = \frac{s}{\sqrt{n}}$$

-
  - $< \alpha$ H_0 H_A
  - $> \alpha$ H_0 H_A