

Lecture 15

Inverse Normal Problems

The Inverse Problem

As we've seen in the previous, we often need to take Z -scores and find probabilities from them. Sometimes this is in a normal problem, sometimes an approximation, and so on.

What if you **wanted to go backward**? What would this look like?

Inverse Problem Statement

What if I told you “the probability of this event happening is 0.5”. What would the Z -score of such a setup be?

$$P(Z \leq z_0) = 0.5$$

What's the unknown here? z_0 !

How do we solve for z_0 ?

- trial and error?
- use tables?
- **use R!**

Using R to Find z_0

```
qnorm(p = 0.5, mean = 0, sd = 1, lower.tail = TRUE)
```

```
## [1] 0
```

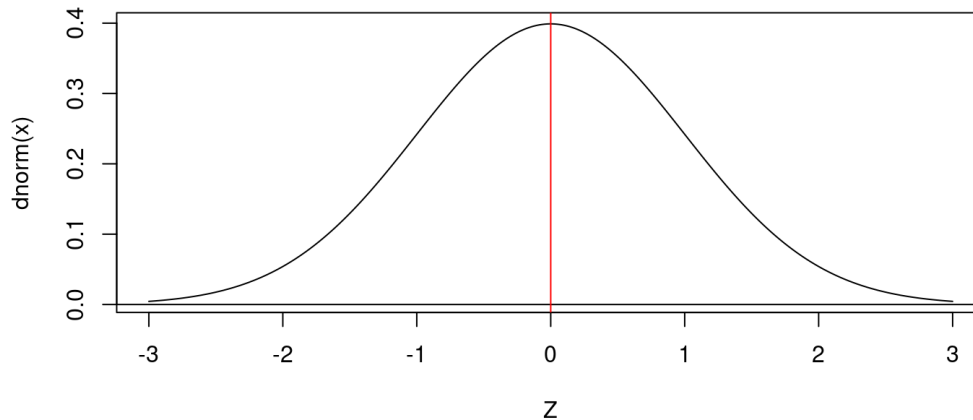
Does this make sense? This is saying that $z_0 = 0$ has probability to its left of 0.5.

```
pnorm(q = 0, mean = 0, sd = 1, lower.tail = TRUE)
```

```
## [1] 0.5
```

Checking Again

```
x <- seq(from = -3, to = 3, by = 0.01)
plot(x, dnorm(x), type = "l", xlab = "Z")
abline(h = 0)
abline(v = 0, col = "red")
```

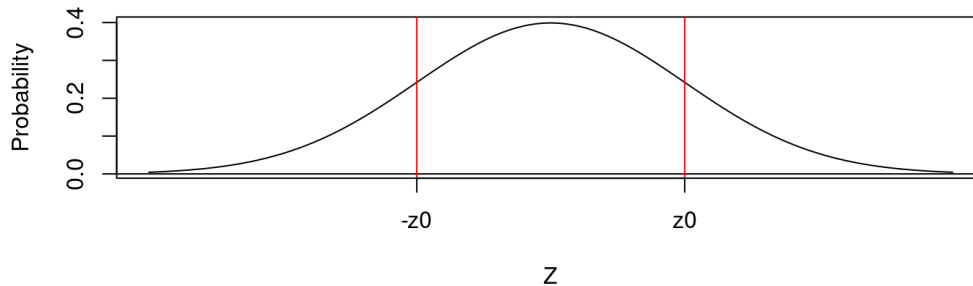


Practice

What value of z_0 has $P(-z_0 \leq Z \leq z_0) = 0.3$?

Practice

What value of z_0 has $P(-z_0 \leq Z \leq z_0) = 0.3$?



So what do we actually need to run?

Practice

```
qnorm(p = 0.15, mean = 0, sd = 1, lower.tail = TRUE)
```

```
## [1] -1.036433
```

This is $-z_0$, because we used 0.15 area to the left, and `lower.tail = TRUE`.

```
qnorm(p = 0.15, mean = 0, sd = 1, lower.tail = FALSE)
```

```
## [1] 1.036433
```

And this is z_0 , because `lower.tail = FALSE`.

More Practice

We're now going to solve a series of problems in all the variations of this that we can do.

Problem Type 1: Simple Left

What value of z_0 has $P(-2 \leq Z \leq z_0) = 0.7$?

Problem Type 2: Simple Right

What value of z_0 has $P(-z_0 \leq Z \leq 1.0) = 0.4$?

Problem Type 3: Symmetric, known

What value of z_0 has $P(-z_0 \leq Z \leq z_0) = 0.4$?

Problem Type 4: Symmetric, Indirect information

If the region from $-z_0$ to z_0 is 90% of the area under a standard normal curve, find z_0 .

Problem Type 5: Not Symmetric, known

What value of z_0 has $P(-2 \cdot z_0 \leq Z \leq z_0) = 0.6$?

Problem Type 6: Not Symmetric, Indirect information

If the 90th percentile of your distribution is equal to 11, and the 5th percentile is equal to 4, find μ and σ .

