Lecture 15

Inverse Normal Problems

The Inverse Problem

As we've seen in the previous, we often need to take Z-scores and find probabilities from them. Sometimes this is in a normal problem, sometimes an approximation, and so on.

What if you **wanted to go backward**? What would this look like?

Inverse Problem Statement

What if I told you "the probability of this event happening is 0.5". What would the Z-score of such a setup be?

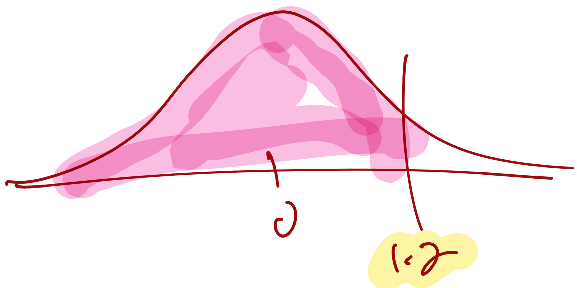
var

$$P(Z \leq \boxed{z_0}) = 0.5$$

What's the unknown here? $\boxed{z_0}$!

unknown

Forward: $P[Z \leq 1.2] = ??$



How do we solve for z_0?

- trial and error?
- use tables?
- **use R!**

Using R to Find z_0

Every family:
p??
q??

```r
qnorm(p = 0.5, mean = 0, sd = 1, lower.tail = TRUE)
```

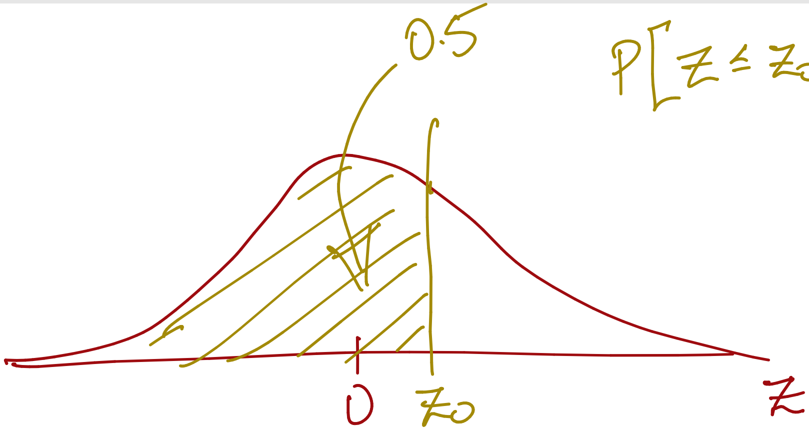
```
## [1] 0
```

Does this make sense? This is saying that $z_0 = 0$ has probability to its left of 0.5.

```r
pnorm(q = 0, mean = 0, sd = 1, lower.tail = TRUE)
```

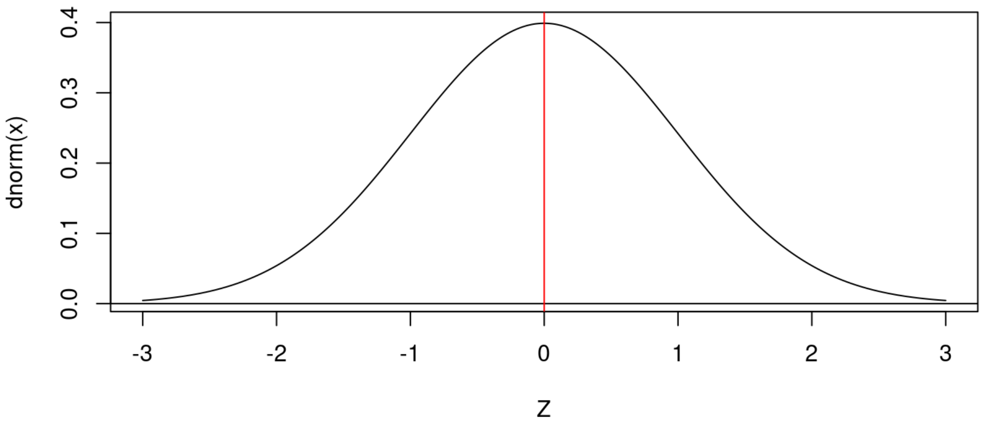
```
## [1] 0.5
```

$P[Z \leq z_0]$



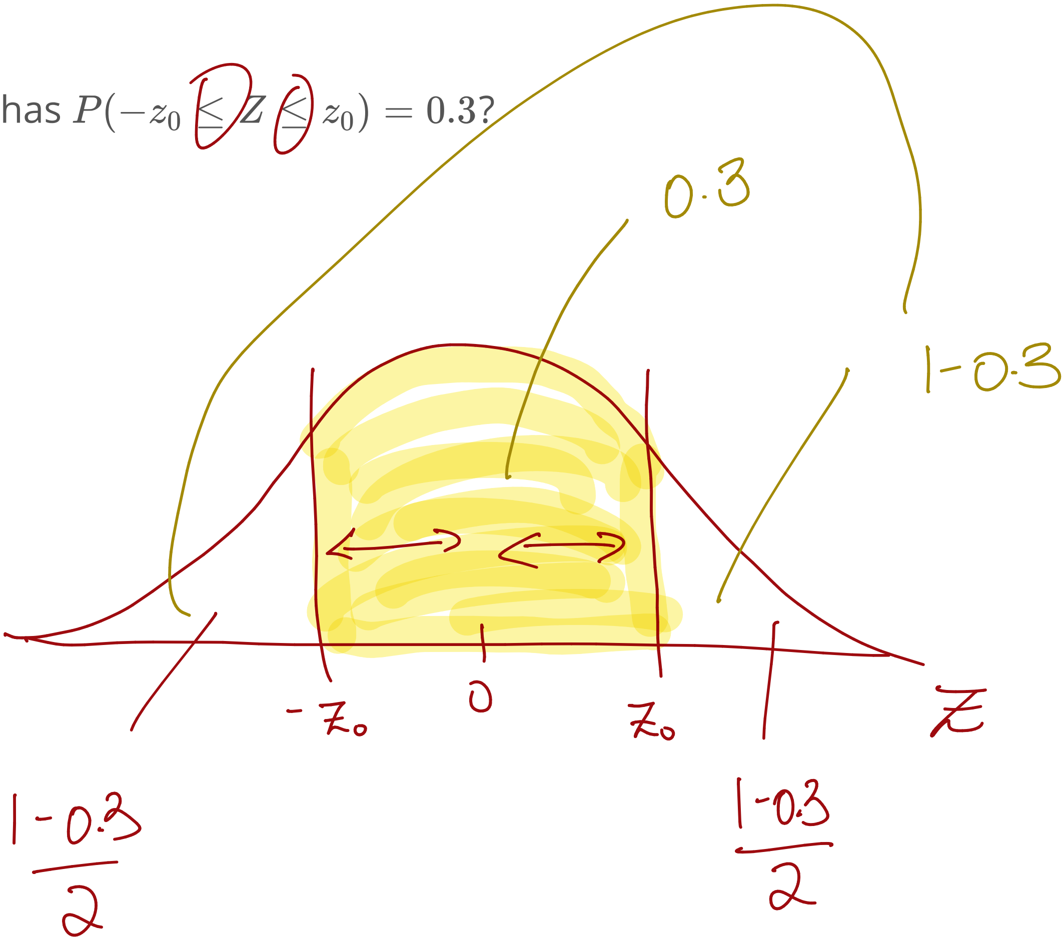
Checking Again

```
x <- seq(from = -3, to = 3, by = 0.01)
plot(x, dnorm(x), type = "l", xlab = "Z")
abline(h = 0)
abline(v = 0, col = "red")
```



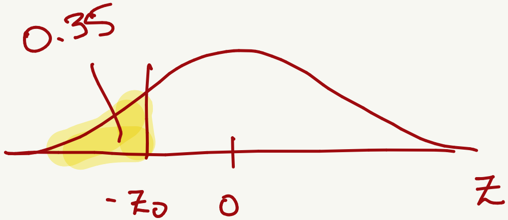
Practice

What value of z_0 has $P(-z_0 \leq Z \leq z_0) = 0.3$?

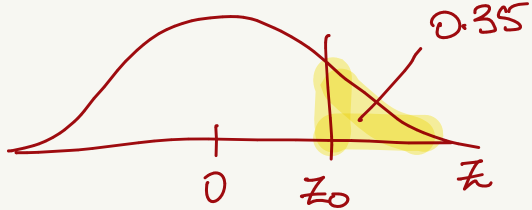


lower.tail = TRUE

lower.tail = FALSE



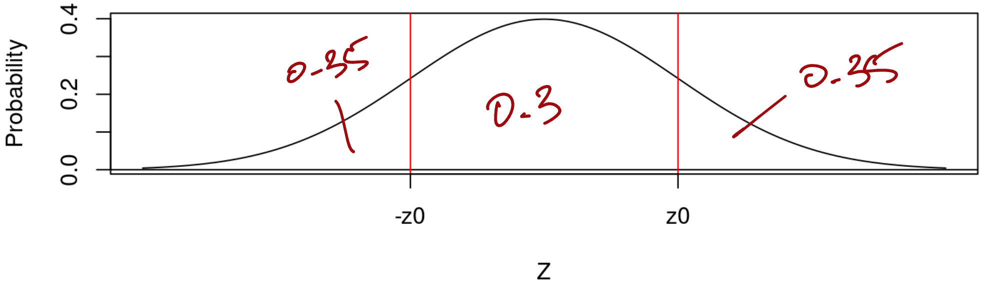
1



2

Practice

What value of z_0 has $P(-z_0 \leq Z \leq z_0) = 0.3$?



So what do we actually need to run?

Practice

```
qnorm(p = 0.15, mean = 0, sd = 1, lower.tail = TRUE)
```

```
## [1] -1.036433
```

−0.3853

This is **-z0**, because we used 0.15 area to the left, and **lower.tail = TRUE**.

```
qnorm(p = 0.15, mean = 0, sd = 1, lower.tail = FALSE)
```

```
## [1] 1.036433
```

0.3853

And this is z_0, because **lower.tail = FALSE**.

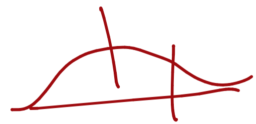
More Practice

We’re now going to solve a series of problems in all the variations of this that we can do.

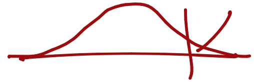
Problem Type 1: Simple Left

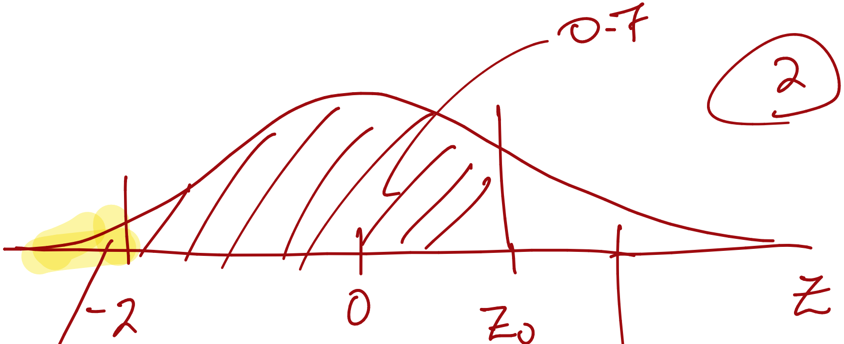
What value of z_0 has $P(-2 \leq Z \leq z_0) = 0.7$?

1



2





pnorm(-2)

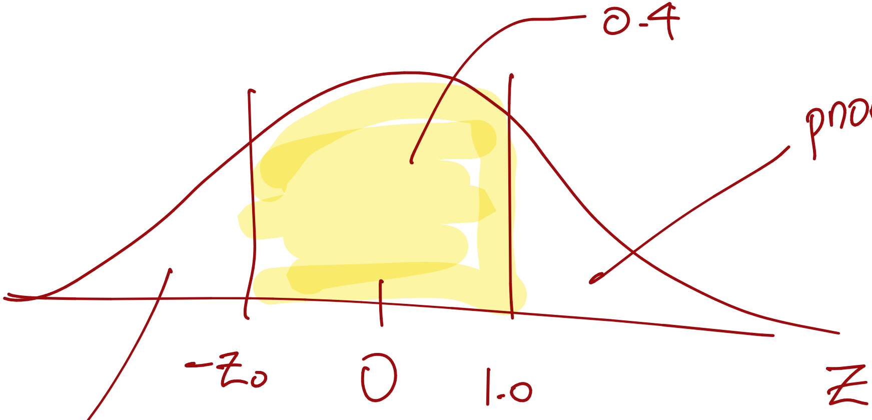
0.02275

0.27725

$z_0 = 0.591$

Problem Type 2: Simple Right

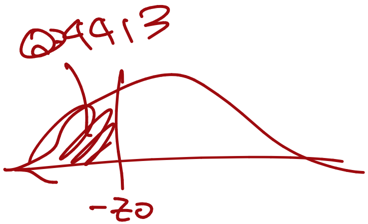
What value of z_0 has $P(-z_0 \leq Z \leq 1.0) = 0.4$?

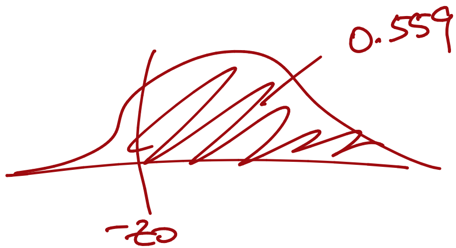


pnorm(1.0, lower.tail = FALSE)

$\boxed{0.159}$

0.4413





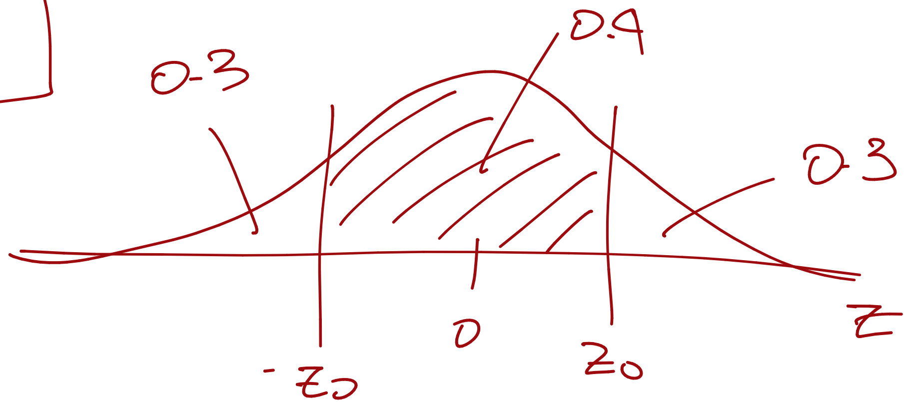
$\therefore -z_0 = -0.1476$

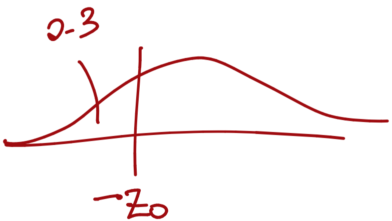
$\boxed{z_0 = 0.1476}$

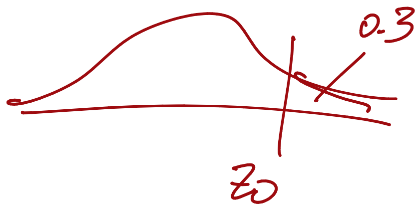
Problem Type 3: Symmetric, known

What value of z_0 has $P(-z_0 \leq Z \leq z_0) = 0.4$?

$\boxed{z_0 = 0.5244}$

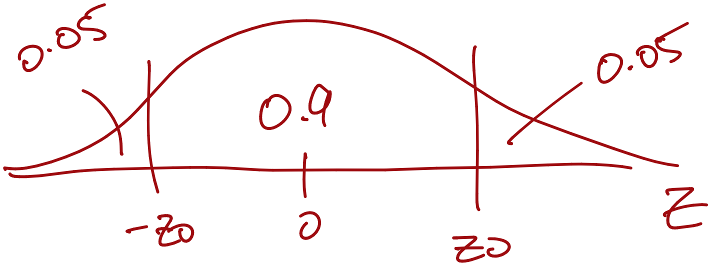






Problem Type 4: Symmetric, Indirect information

If the region from $-z_0$ to z_0 is 90% of the area under a standard normal curve, find z_0.

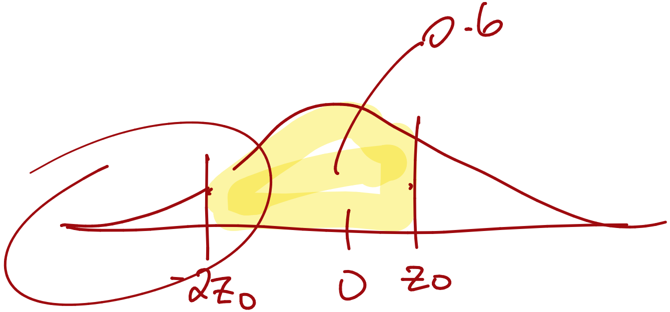


qnorm(0.95)

$\boxed{z_0 = 1.645}$

Problem Type 5: Not Symmetric, known

What value of z_0 has $P(-2 \cdot z_0 \leq Z \leq z_0) = 0.6$?

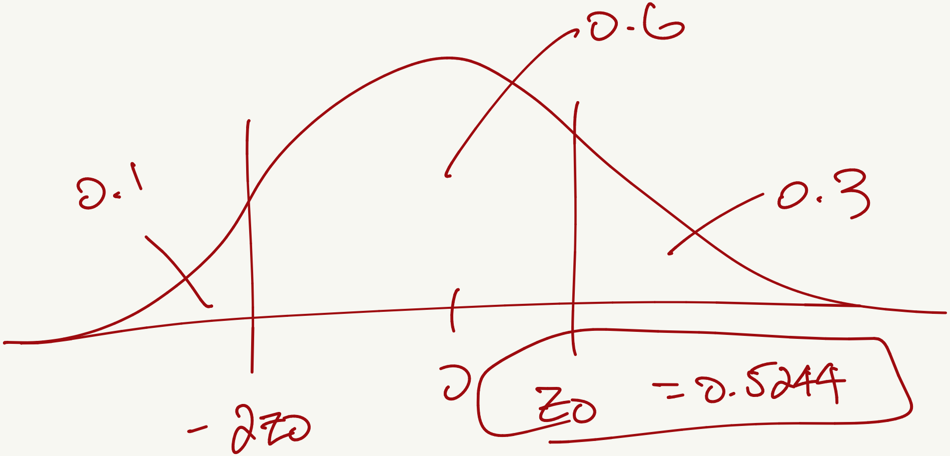


Can't solve as is!

$\boxed{\text{Q5b}}$

One more piece :

$$P[Z \le -2z_0] = 0.1$$

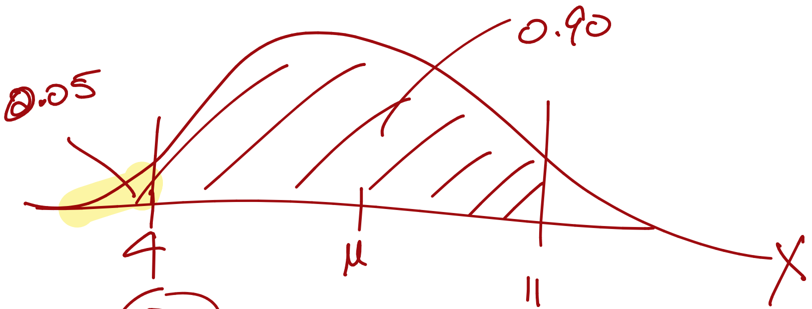


$$\boxed{z_0 = 0.5244}$$

qnorm(0.3, lower.tail = FALSE)

Problem Type 6: Not Symmetric, Indirect information

If the 90th percentile of your distribution is equal to 11, and the 5th percentile is equal to 4, find μ and σ.



$$Z = \frac{X-\mu}{\sigma}$$

? 0 ? Z

-1.645 1.282

qnorm(0.05)

qnorm(0.90)

$$-1.645 = \frac{4-\mu}{\sigma}$$

$$1.282 = \frac{11-\mu}{\sigma}$$

$\therefore$ (1) $-1.645\sigma = 4-\mu$

(2) $1.282\sigma = 11-\mu$