MATH 1051H - S61 - Lecture 09

Decision Errors

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- In the court system innocent people are sometimes wrongly convicted and the guilty sometimes walk free.
- Similarly, we can make a wrong decision in statistical hypothesis tests as well.
- The difference is that we have the tools necessary to quantify how often we make errors in statistics.

Decision errors (cont.)

There are two competing hypotheses: the null and the alternative. In a hypothesis test, we make a decision about which might be true, but our choice might be incorrect.

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- A **Type 1 Error** is rejecting the null hypothesis when H_0 is true.
- A **Type 2 Error** is failing to reject the null hypothesis when H_A is true.
- We (almost) never know if H_0 or H_A is true, but we need to consider all possibilities.

Hypothesis Test as a trial

If we again think of a hypothesis test as a criminal trial then it makes sense to frame the verdict in terms of the null and alternative hypotheses:

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\begin{aligned}
H_0 &: \text{ Defendant is innocent} \\
H_A &: \text{ Defendant is guilty}
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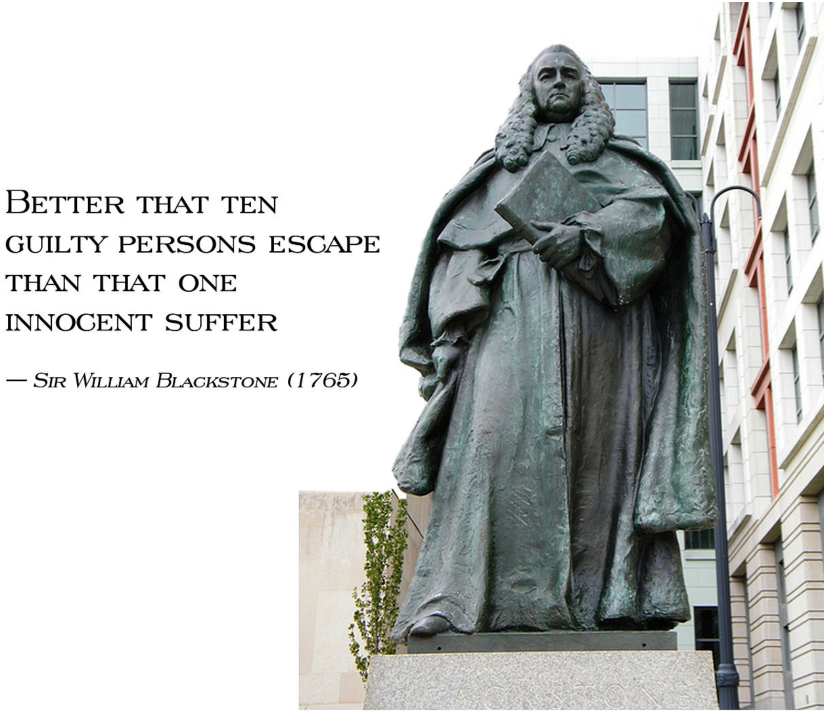
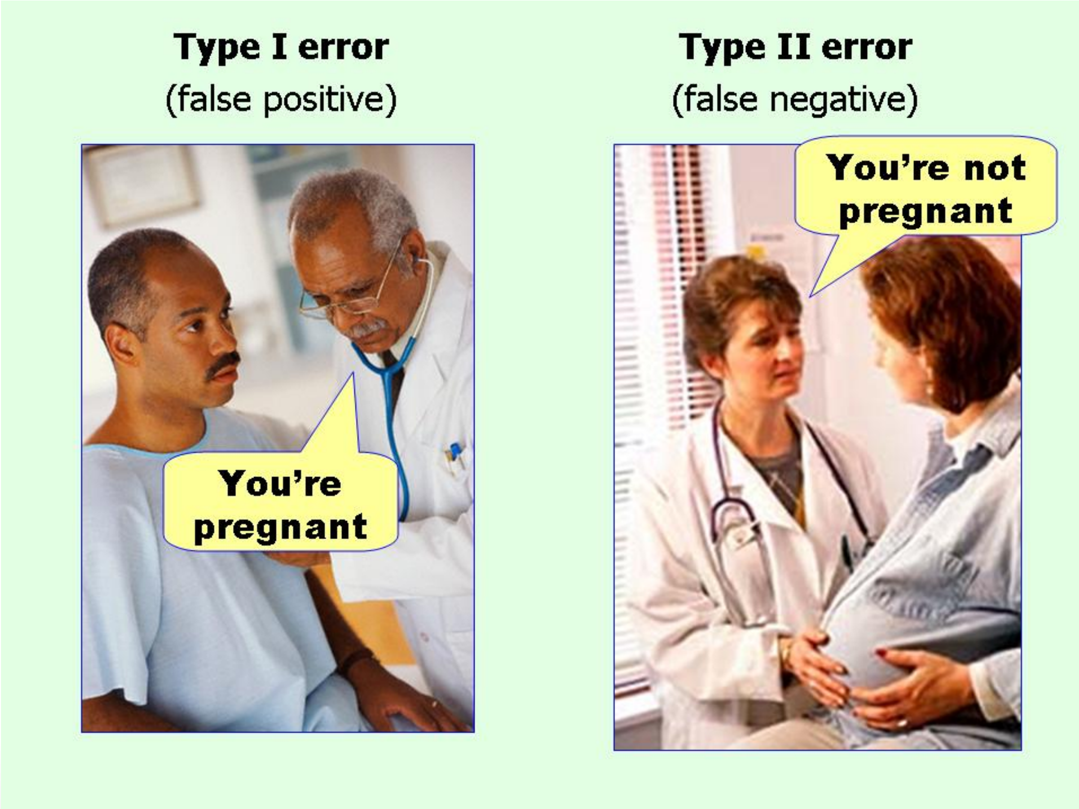
Which error do you think is the worse error to make?

Hypothesis Test as a trial



https://en.wikipedia.org/wiki/Blackstone%27s_ratio

Another way to remember



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  - but the diagnosis was “positive” (the alternative)
  - this is equivalent to **declaring the defendent guilty, when they are actually innocent**

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- So for the woman in the right panel, being told “you are not pregnant” means **fail to reject the null** - there is no evidence against the null state
    - this is obviously incorrect (poor woman!)
    - therefore it is **false**
    - the diagnosis was “negative” (against the alternative)
    - this is equivalent to **declaring the defendent innocent, when they are actually guilty**

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$$P\left(\text{Type 1 error } \middle| H_0 \text{ true}\right) = \alpha$$

- This is why we prefer small values of α - **increasing α increases the Type 1 error rate**.

Choosing a significance level

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- If making a Type 1 Error is dangerous or especially costly, we should choose a small significance level (e.g. 0.01). Under this scenario we want to be very cautious about rejecting the null hypothesis, so we demand very strong evidence favoring H_A before we would reject H_0.
- If a Type 2 Error is relatively more dangerous or much more costly than a Type 1 Error, then we should choose a higher significance level (e.g. 0.10). Here we want to be cautious about failing to reject H_0 when the null is actually false.

Recap: Hypothesis testing framework

- Set the hypotheses.
- Check assumptions and conditions.
- Calculate a **test statistic** and a p-value.
- Make a decision, and interpret it in context of the research question.

Recap: Hypothesis testing for a population mean

- Set the hypotheses
  - $H_0 : \mu = \text{null value}$
  - $H_A : \mu <$ or $>$ or $\neq$ null value
- Calculate the point estimate
- Check assumptions and conditions
  - Independence: random sample/assignment, 10% condition when sampling without replacement
  - Normality: nearly normal population or $n \geq 30$, no extreme skew – **or use the t distribution** (next chapter)

Recap: Hypothesis testing for a population mean

- Calculate a **test statistic** and a p-value (draw a picture!)

$$Z = \frac{\bar{x} - \mu}{SE}, \text{ where } SE = \frac{s}{\sqrt{n}}$$

- Make a decision, and interpret it in context
  - If p-value $< \alpha$, reject H_0, data provide evidence for H_A
  - If p-value $> \alpha$, do not reject H_0, data do not provide evidence for H_A

Next Lecture

In the next lecture, we will tie hypothesis testing back to linear regression, and show you how they are connected … and set ourselves up for the final week, where we'll do some more variations!