MATH 1051H S61 - Lecture 13

Confidence Intervals for t Cases

Context

Confidence interval, a general formula

$$\text{point estimate} \pm z^\star \cdot SE$$

Conditions when the point estimate = $\bar{x}$:

- **Independence:** Observations in the sample must be independent
  - random sample/assignment
  - if sampling without replacement, $n <$ 10% of population
- **Sample size / skew:** $n \geq 30$ and population distribution should not be extremely skewed

Note: We will discuss working with samples where $n < 30$ **now**.

Confidence Intervals, $n < 30$

When we were doing hypothesis tests, and had small samples, do you remember what we did? We said that the issue was the SE … and then we used a t!

That's all we do here. If $n < 30$ and you don't know σ, you use a t distribution for your confidence interval.

But basically nothing changes … instead of using $z^\star$, you use $t^\star$ instead. That's it.

Let's just solve a couple of problems, and see how it works.

Problem 1: Highway Speeds

Southbound traffic on the I-280 highway near Cupertino, California had its speed monitored at 3:30pm on a Wednesday. The sample of 12 cars had mean 97.6 km/hr with standard deviation 6.56 km/hr. Test the highway patrol's claim that the average speed on this highway at this time of day is lower than the speed limit of 105 km/hr.

Also compute a 99% confidence interval for the mean speed.

Problem 1: Information

What do we have from this problem?

- $n = 12$
- $\bar{x} = 97.6$
- $s = 6.56$
- $H_A : \mu < 105$
- $\alpha = 0.01$ (confidence: 99%)

Problem 1: Statistic

We're doing a hypothesis test:

$$H_0 : \mu = 105 \qquad \text{versus} \qquad H_A : \mu < 105.$$

Compute the test statistic:

$$t_{\text{test}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} = \frac{97.6 - 105}{\frac{6.56}{\sqrt{12}}} = -3.907676$$

Problem 1: ME

When computing a confidence interval, we use the formula

$$\text{point estimate} \pm t^\star \cdot \text{SE}$$

where we use $t^\star$ instead of $z^\star$ because $n < 30$. Thus,

$$\text{ME} = t^\star \cdot \text{SE} = t^\star \cdot 1.893709$$

```r
qt(p = 0.99 + 0.01/2, df = 12 - 1, lower.tail = TRUE)
```

```
## [1] 3.105807
```

$$\text{ME} = 3.1058 \cdot 1.8937 = 5.8815.$$

Problem 1: Conclusion (Test)

We found a test statistic of -3.9077, so the p-value is (since the alternative is $\mu < 105$):

```
pt(-3.9077, df = 12 - 1, lower.tail = TRUE)
```

```
## [1] 0.001221992
```

and we reject the null, and conclude that the average speed **is** less than 105 km/hr.

Problem 1: Conclusion (CI)

Similarly, we can form the 99% confidence interval, which is

$$97.6 \pm \mathrm{ME} = 97.6 \pm 5.8815$$

which is

```
97.6 + c(-1, 1) * 5.8815
```

```
## [1]  91.7185 103.4815
```

and thus the 99% confidence interval for the true mean speed is (91.7, 103.5) km/hr.

Problem 2: Pennies

Before 1983, US pennies were made with 97% copper and 3% zinc. After 1983, they were converted to 3% copper and 97% zinc to make them cheaper to manufacture. A simple random sample of 25 post-1983 pennies had an average weight of 2.49910g, with standard deviation 0.01648g. The US Mint specifies that post-1983 pennies should be manufactured with mean weight 2.500g. At a 95% level, do you believe that pennies are actually being manufactured with mean weight of 2.500g?

Compute a 93% confidence interval for the mean weight of post-1983 pennies.

Problem 2: Information

What do we have from this problem?

- $n = 25$
- $\bar{x} = 2.49910$
- $s = 0.01648$
- $H_A : \mu \neq 2.500$
- $\alpha = 0.07$ (confidence: 93%)

Problem 2: Statistic

We're doing a hypothesis test:

$$H_0 : \mu = 2.500 \qquad \text{versus} \qquad H_A : \mu \neq 2.500.$$

Compute the test statistic:

$$t_{\text{test}} = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} = \frac{2.49910 - 2.500}{\frac{0.01648}{\sqrt{25}}} = -0.2730583$$

Problem 2: ME

When computing a confidence interval, we use the formula

$$\text{point estimate} \pm t^\star \cdot \text{SE}$$

where we use $t^\star$ instead of $z^\star$ because $n < 30$. Thus,

$$\text{ME} = t^\star \cdot \text{SE} = t^\star \cdot 0.003296.$$

```
qt(p = 0.93 + 0.07/2, df = 25 - 1, lower.tail = TRUE)
```

```
## [1] 1.896457
```

$$\text{ME} = 1.896457 \cdot 0.003296 = 0.00625.$$

Problem 2: Conclusion (Test)

We found a test statistic of -0.27306, so the p-value is (since the alternative is $\mu \neq 2.500$):

```
pt(-0.27306, df = 25 - 1, lower.tail = TRUE) * 2
```

```
## [1] 0.787143
```

and thus we fail to reject the null, and conclude that we do not have evidence to conclude that the average weight of pennies is anything but 2.5000 g.

Problem 2: Conclusion (CI)

Similarly, we can form the 93% confidence interval, which is

$$2.49910 \pm \mathrm{ME} = 2.49910 \pm 0.00625$$

which is

```
2.49910 + c(-1, 1) * 0.00625
```

```
## [1] 2.49285 2.50535
```

and thus the 93% confidence interval for the true mean penny weight is (2.493, 2.505) g.