Lesson 2: A Few More Basics

Basic Error Propagation

So far we have looked at getting the uncertainties and errors in individual quantities. Of course many times we have to combine separate results together in order to get our final result. There are special rules for combining the errors of different quantities and this process is known as *error propagation*. We will spend an entire lesson on this topic soon, but for now let's focus on the most basic rule. Let's say I have a very basic function where I am only multiplying a physical parameter by a constant,

$$A = cB$$

If we know the error in B, then the we simply multiply the error by c to get the error in A.

$$\sigma_A = c \times \sigma_B$$

For instance, imagine I took the data from lab M1 and plotted the ball's y-position vs. time-squared. Given the formula $y = \frac{1}{2}gt^2$ the slope of such a plot should equal $\frac{1}{2}g$. Let's say I get a value for the slope of 4.9 ± 0.0463 (ignoring formatting rules for the moment). You can get an estimation of g if you multiply the slope by 2. This means that the plot gives an error value for g of about 0.0926. Formally I could say that the graph gives me the result of g = 9.8 ± 0.2 m/s^2 with a 95% confidence interval.

Many times equations are plotted in ways that may at first seem unusual. By plotting variables so that we have a linear relationship, and that the slope differs from our desired result only by a constant factor, we can easily obtain results with uncertainties. Let's now discuss how we can find the error in the slope.

Quick Error Calculation for Linear Plot

In the coming weeks we will look at advanced Excel techniques that will allow you to get equations for a wide variety of functions as well as the errors that go along with the parameters of those equations. However, many times we simply need the slope value and it can be handy to know how to calculate the uncertainty of the slope straight from the graph. This can be achieved with the following formula.

$$\sigma_{slope} = slope \times \sqrt{\frac{{}^{1}\!/_{R^2} - 1}{N-2}}$$

So this formula can give the standard error of the slope once you have the slope itself, the R^2 value, and the number of data points in the set. Most of the time in this class it will be fine to just use the rounded values of the slope and R^2 that are displayed on the graph. You can get more exact values by using the following two formulas. These can also be handy if you just want the information without actually plotting a graph.

=slope(known y's, known x's)

=rsq(known y's, known x's)

Degrees of Freedom

As we saw at the end of the first lesson, there is an increase in our error if we are only measuring a *sample* of the full population of data (remember this is always the case in Physics). This effect was made manifest by changing the denominator of the standard deviation function to *N-1* from *N*. In general we say that the denominator of both equations is the number of *degrees of freedom* in the calculation. If you measure the full population, then the degrees of freedom equals the number of data points. If you take a sample, then the degrees of freedom is reduced by one for calculating the standard deviation. In general the degrees of freedom is calculated as the number of data points minus the number of parameters that must be estimated in doing the calculation. As an example, when we calculate the standard deviation, we must first estimate the true value, which we do so by calculating the average of our data set. This is the only estimation in the calculation; the only other numbers in the calculation are the data points we have measured. So one estimation reduces the degrees of freedom by 1.

Now look at the equation for the error in the slope. Notice that in order to calculate this error, we first have to calculate the slope itself and the R^2 value. So in this calculation there are two estimates that are made and so the degrees of freedom is $N-2$. The number of degrees of freedom not only effects the equations of error themselves, but it also effects how we adjust for different confidence intervals.

Small Number Statistics

So far we have been using the values in Table 1 of the first lesson to convert our errors into uncertainties at specific confidence intervals. This assumes that our data will fall under the normal distribution. Unfortunately, this assumption is not a very good one when we are only taking small amounts of data. With the experiments that you perform in lab, you end up taking a fairly small number of data points simply because we only have three hours in which to conduct the experiment. Luckily this issue is easily solved by having a different table of multipliers.

Table two (on the next page) lists the numbers needed to convert errors into uncertainties for a variety of confidence intervals. Note that it is dependent on the number of degrees of freedom in the estimation, not the number of data points. The numbers in this table are calculated from the Student's T-Distribution. As the number of *df* increases, the Student's Distribution will approach the Normal Distribution, and at $N = \infty$ the two are equivalent. In practice, it is common to use the normal distribution once the *df* reaches 100. You will notice that the multipliers on the last row of Table 2 match the numbers on Table 1.

Table 2

df	68.27%	90%	95%	99%
3	1.20	2.35	3.18	5.84
4	1.14	2.13	2.78	4.60
5	1.11	2.02	2.57	4.03
6	1.09	1.94	2.45	3.71
7	1.08	1.89	2.36	3.50
8	1.07	1.86	2.31	3.36
9	1.06	1.83	2.26	3.25
10	1.05	1.81	2.23	3.17
11	1.05	1.80	2.20	3.11
12	1.04	1.78	2.18	3.05
13	1.04	1.77	2.16	3.01
14	1.04	1.76	2.14	2.98
15	1.03	1.75	2.13	2.95
16	1.03	1.75	2.12	2.92
17	1.03	1.74	2.11	2.90
18	1.03	1.73	2.10	2.88
19	1.03	1.73	2.09	2.86
20	1.03	1.72	2.09	2.85
21	1.02	1.72	2.08	2.83
22	1.02	1.72	2.07	2.82
23	1.02	1.71	2.07	2.81
24	1.02	1.71	2.06	2.80
25	1.02	1.71	2.06	2.79
26	1.02	1.71	2.06	2.78
27	1.02	1.70	2.05	2.77
28	1.02	1.70	2.05	2.76
29	1.02	1.70	2.05	2.76
30	1.02	1.70	2.04	2.75
40	1.01	1.68	2.02	2.70
50	1.01	1.68	2.01	2.68
60	1.01	1.67	2.00	2.66
70	1.01	1.67	1.99	2.65
80	1.01	1.66	1.99	2.64
90	1.01	1.66	1.99	2.63
>=100	1.00	1.64	1.96	2.58