Name: Wesley Harmon

MATH 315 FALL 2016 ESTIMATING DERIVATIVES

THE EMORY COLLEGE HONOR CODE IS IN EFFECT. NO BOOKS, NOTES, OR ELECTRONIC DEVICES. SHOW YOUR WORK!. SHOW THE REASONING FOR YOUR SOLUTIONS AND IDENTIFY ALL VARIABLES TO RECEIVE THE FULL CREDIT! PLACE YOUR ANSWERS IN THE SPACE PROVIDED.

Recall that a Taylor Series expansion of $f(z)$ about a point c is defined as:

$$f(z) = f(c) + (z-c)f'(c) + (z-c)^2\frac{f''(c)}{2!} + (z-c)^3\frac{f'''(c)}{3!} + (z-c)^4\frac{f^{(4)}(c)}{4!} + \cdots$$

(1) Use this to write down series formulae for for $f(x+h)$ ①, $f(x-h)$ ②, and $f(x+2h)$ ③, where we assume h is a small (less than 1) positive number.

① $z = x+h$, $c = x$, $z-c = h$

$$F(x+h) = F(x) + hF'(x) + \underbrace{h^2\frac{F''(x)}{2!} + h^3\frac{F'''(x)}{3!} + h^4\frac{F^{(4)}(x)}{4!} \cdots}_{\text{can truncate since } h \text{ is small}} \Rightarrow F(x+h) \approx F(x) + hF'(x)$$

② $z = x-h$, $c = x$, $z-c = -h$

$$F(x-h) = F(x) - hF'(x) + \underbrace{(-h)^2\frac{F''(x)}{2!} - h^3\frac{F'''(x)}{3!} + (-h)^4\frac{F^{(4)}(x)}{4!} \cdots}_{\text{can truncate since } h \text{ is small}} \Rightarrow F(x-h) \approx F(x) - hF'(x)$$

③ $z = x+2h$, $c = x$, $\frac{z-c}{2} = h$, $z-c = 2h$

$$F(x+2h) = F(x) + 2hF'(x) + \underbrace{\frac{4h^2F''(x)}{2!} + \frac{8h^3F'''(x)}{3!} + \frac{16h^4F^{(4)}(x)}{4!} \cdots}_{\text{can truncate since } h \text{ is small}} \Rightarrow F(x+2h) \approx F(x) + 2hF'(x)$$

(2) Use the formulae in part (a) to derive the approximation

$$f'(x) \approx \frac{4f(x+h) - 3f(x) - f(x+2h)}{2h}$$

Try ① + ② + ③ // Introduces unwanted terms & we need the term $4F(x+h)$

~~$F(x+h) + F(x-h) + F(x+2h) \approx 3F(x) + 2hF'(x) + \frac{2h^2F''(x)}{2!} + \frac{4h^2F''(x)}{2!}$~~

~~$F(x+2h) \approx F(x) + 2hF'(x) + 2h^2F''(x) + \frac{4}{3}h^3F'''(x)$~~

Try ~~4·① + ②~~

$4\cdot① - 1\cdot②$, gives us the approximation we seek (While also canceling terms w/ h^2)

$$4F(x+h) - F(x+2h) \approx 4F(x) - F(x) + 4hF'(x) - 2hF'(x) + 0 - \ldots$$

~~4f(x+h)~~

$$4F(x+h) - F(x+2h) \approx 3F(x) + 2hF'(x)$$ // Solve for $F'(x)$

$$\frac{4F(x+h) - F(x+2h) - 3F(x)}{2h} \approx \frac{2hF'(x)}{2h}$$ // Divide by $2h$ & rearrange

$$F'(x) \approx \frac{4F(x+h) - 3F(x) - F(x+2h)}{2h}$$

truncate higher order terms that were not canceled