Homework 2

1. Given grammar: $S \to aSA \mid AB \mid aB$

$$A \to abb \mid \varepsilon$$
$$B \to bbA \mid AA$$

(1)
$$S_0 \to S$$
$$S \to aSA \mid AB \mid aB \mid aS \mid B$$
$$A \to abb \mid \boxed{\cancel{\varepsilon}}$$
$$B \to bbA \mid AA \mid A \mid \varepsilon \mid bb$$

$$S_0 \to S$$
$$S \to aSA \mid AB \mid aB \mid aS \mid B \mid A \mid a \mid \varepsilon$$
$$A \to abb$$
$$B \to bbA \mid AA \mid bb \mid A \mid \boxed{\cancel{\varepsilon}}$$

$$S_0 \to S \mid \varepsilon$$
$$S \to aSA \mid AB \mid aB \mid aS \mid B \mid A \mid a \mid \cancel{\varepsilon}$$
$$A \to abb$$
$$B \to bbA \mid AA \mid bb \mid A$$

(2)
$$S_0 \to S \mid \varepsilon$$
$$S \to aSA \mid AB \mid aB \mid aS \mid B \mid A \mid a$$
$$A \to abb$$
$$B \to bbA \mid AA \mid bb \mid A$$

$$S_0 \to aSA \mid AB \mid aB \mid aS \mid bbA \mid AA \mid bb \mid abb \mid a \mid \varepsilon$$
$$S \to aSA \mid AB \mid aB \mid aS \mid bbA \mid AA \mid bb \mid abb \mid a$$
$$A \to abb$$
$$B \to bbA \mid AA \mid bb \mid abb$$

1.

(3) $S_0 \rightarrow aSA \mid AB \mid aB \mid aS \mid bbA \mid AA \mid bb \mid abb \mid a \mid \varepsilon$ // bb...

$S \rightarrow aSA \mid AB \mid aB \mid aS \mid bbA \mid AA \mid bb \mid abb \mid a$

$A \rightarrow abb$

$B \rightarrow bbA \mid AA \mid bb \mid abb$

$C \rightarrow DD$

$D \rightarrow b$

$S_0 \rightarrow aSA \mid AB \mid aB \mid aS \mid CA \mid AA \mid C \mid aC \mid a \mid \varepsilon$ // Clean up...

$S \rightarrow aSA \mid AB \mid aB \mid aS \mid CA \mid AA \mid C \mid aC \mid a \mid \varepsilon$

$A \rightarrow aC$

$B \rightarrow CA \mid AA \mid C \mid aC$

$C \rightarrow DD$

$D \rightarrow b$

$S_0 \rightarrow aSA \mid AB \mid aB \mid aS \mid CA \mid AA \mid DD \mid aC \mid a \mid \varepsilon$ // a...

$S \rightarrow aSA \mid AB \mid aB \mid aS \mid CA \mid AA \mid DD \mid aC \mid a \mid \varepsilon$

$A \rightarrow aC$

$B \rightarrow CA \mid AA \mid DD \mid aC$

$C \rightarrow DD$

$D \rightarrow b$

$E \rightarrow a$

$S_0 \rightarrow ESA \mid AB \mid EB \mid ES \mid CA \mid AA \mid DD \mid EC \mid a \mid \varepsilon$ // SA...

$S \rightarrow ESA \mid AB \mid EB \mid ES \mid CA \mid AA \mid DD \mid EC \mid a \mid \varepsilon$

$A \rightarrow EC$

$B \rightarrow CA \mid AA \mid DD \mid EC$

$C \rightarrow DD$

$D \rightarrow b$

$E \rightarrow a$

$F \rightarrow SA$

1. (3) $S_0 \rightarrow EF \mid AB \mid EB \mid ES \mid CA \mid AA \mid DD \mid EC \mid a \mid \varepsilon$

$S \rightarrow EF \mid AB \mid EB \mid ES \mid CA \mid AA \mid DD \mid EC \mid a \mid \varepsilon$

$A \rightarrow EC$

$B \rightarrow CA \mid AA \mid DD \mid EC$

$C \rightarrow DD$

$D \rightarrow b$

$E \rightarrow a$

$F \rightarrow SA$ // CNF achieved!

2. a.) Given PDA P & input string $x = aabbabaababa$

$\boxed{x = \cancel{a}\cancel{a}\cancel{b}\cancel{b}\cancel{a}\cancel{b}\cancel{a}\cancel{a}\cancel{b}\cancel{a}\cancel{b}a}$

Input Consumed	Resulting state	Resulting Stack
ε	2	\$
a	2	a\$
a	2	aa\$
b	3	a\$
b	2	aa\$
a	2	aaa\$
b	3	aa\$
a	3	a\$
a	3	\$
b	2	a\$
a	2	aa\$
b	3	a\$
a	3	\$
ε	4	—

aabbabaababa

$P = (\{1,2,3,4\}, \{a,b\}, \{a,\$\}, \delta, 1, 4)$

2.b.) PDA P is given, generate (CFG) G:

$A_{14} \rightarrow A_{23}$ // Pushed \$ to 2, popped \$ from 3

↳ (\$) A_{23} (\$)!!!

$A_{22} \rightarrow A_{22}A_{22} \mid \cancel{A_{23}A_{32}} \mid \varepsilon$

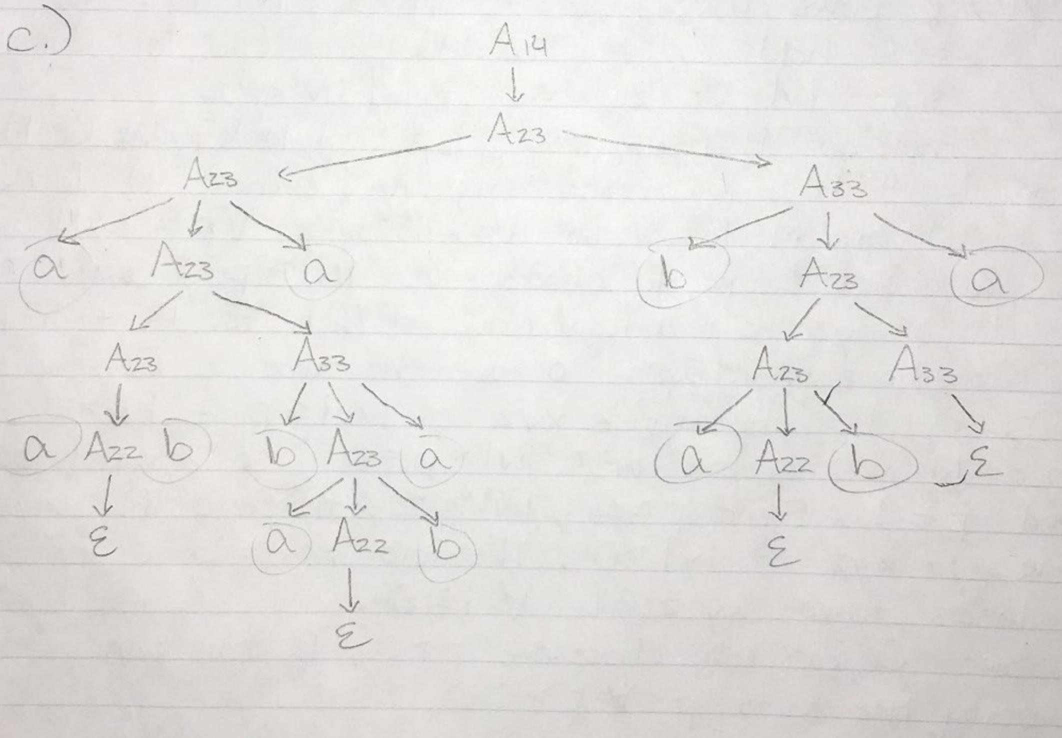
$A_{33} \rightarrow A_{33}A_{33} \mid \cancel{A_{32}A_{23}} \mid \varepsilon \mid bA_{22}b \mid bA_{23}a$

$A_{23} \rightarrow \cancel{A_{22}A_{23}} \mid \cancel{A_{23}A_{33}} \mid aA_{22}b \mid aA_{23}a$

$\cancel{A_{32} \rightarrow A_{33}A_{32} \mid A_{32}A_{22}}$

$$\boxed{\begin{aligned} A_{14} &\rightarrow A_{23} \\ A_{22} &\rightarrow \varepsilon \\ A_{23} &\rightarrow A_{23}A_{33} \mid aA_{22}b \mid aA_{23}a \\ A_{33} &\rightarrow \varepsilon \mid A_{33}A_{33} \mid bA_{22}b \mid bA_{23}a \end{aligned}}$$

c.)



4. Given $L = \{a^n \mid n \geq 0\}$

Show: C has winning strategy $\Rightarrow$ R has winning strategy

Assume C has a winning strategy. Then C must pick $p > 0$ since if $p = 0$, N may pick $w \ni |w| = 0$, in which C has no legal move & loses at (3). So w/ $p > 0$, this implies N picks $w \ni |w| \geq 1$. There are 2 possibilities for C to win:

i.) C picks $u, v, x, y, z \ni |y| = 0, |v| \geq 1$. Then $|vxy| \leq p \geq 1$ & $|vy| > 0$. Since $|y| = 0$, N cannot pick $i \geq 0 \ni uv^ixy^iz$ & therefore N loses.

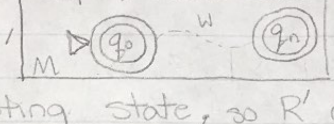
ii.) C picks $u, v, x, y, z \ni |y| \geq 1, |v| = 0$. The same holds for the conditions of (3) for C in i.) Since $|v| = 0$, $\nexists\, i \geq 0 \ni \exists\, uv^ixy^iz$ & therefore N loses, since $\nexists$ a legal last move for N!

For R to have a winning strategy, either N loses at (2) or N loses at (4). Since $\forall p \geq 0$, N will always be able to choose $w \ni |w| \geq p$ & $w \in L$, so N will always have a legal move at (2). For R to win, the conditions for choosing p are the same as C, since if $p = 0$, R may not be able to pick $xyz \ni |xy| \leq p$ & $|y| > 0$ since N may choose $w \ni |w| = p = 0$, so it must be that $p \geq 1$. Since $p \geq 1$, $|w| \geq 1$, therefore R may pick $w = xyz \ni |y| > 0$. Therefore $|xy| \leq p$ & N loses since $xy^iz \in L \;\forall i \geq 0$.

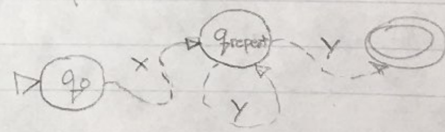
$\therefore$ Since C can win choosing $p > 0$, R can win using the same p $\forall p > 0$.

5.) I claim if L is regular, then R' has a winning strategy.

Assume L is regular, then $\exists$ DFA $M \ni L = L(M)$.
Then $\exists$ 2 possibilities: either $\exists\, w \ni |w| \geq p$ or $\nexists\, w \ni |w| \geq p$. If R' picks $p \ni \nexists\, w$ where $w \in L$, & $|w| \geq p$, then R' wins.
If $\exists\, w \in L \ni |w| \geq p$, then there are 2 possible DFAs: For $p = 0$,



the start state is also an accepting state, so R' can win w/ $|w| = 0$ or $|w| > 0$
For $p > 0$, $x = \varepsilon$, $z = \varepsilon$, $x, z = \varepsilon$, or $x, y \neq \varepsilon$, which satisfies the condition $|xz| \leq p$ (assuming $|y| > 0$) & $w \in L$. Choosing p to be the number of states in DFA M accepting L, we know M accepts w. Let $|w| = n \ni n \geq p$. Thus, the ~~sequence~~ # of states visited by reading w is $n+1$. Then $n + 1 > p \;\forall\, n \geq p$. Thus, we know at least 1 state is revisited. Letting y be the part of w where the repeated state is visited twice, the DFA $M =$



$\therefore\ \forall\, i \geq 0$, xy^iz will always be in L since consuming y will take you back to q_{repeat}.