EXAM Three - **Name** Solutions

Physics 151 – Dr. Kim

November 18th, 2016 – 1:00 pm

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Signature ____________________

100 points total

Linear momentum

$$m\vec{\mathbf{v}} = \vec{\mathbf{p}}$$

Impulse

$$\vec{\mathbf{I}} = \Delta\vec{\mathbf{p}}$$

Center of mass

$$\vec{\mathbf{r}}_{com} = \frac{\sum m_i \vec{\mathbf{r}}_i}{M}$$

Torque

$$\tau = Fd = Fr\sin\phi$$

$$\sum \tau = I\alpha$$

Moment of Inertia

$$I = \sum m_i r_i^2$$

Parallel-Axis Theorem

$$I_{new} = I_{CM} + MD^2$$

Rotational kinetic energy

$$K = \tfrac{1}{2}I\omega^2$$

1) A 30.0-kg girl is standing on a 90-kg plank. Both are originally at rest on a frozen lake that constitutes a frictionless, flat surface. The girl begins to walk along the plank at a constant velocity of 1.50 $\hat{\imath}$ m/s relative to the plank.

(5 pts) What is the velocity of the plank relative to the ice surface?

a) $\frac{3}{4}\hat{\imath}$ m/s

b) $-\frac{1}{2}\hat{\imath}$ m/s

c) $-\frac{3}{2}\hat{\imath}$ m/s

d) $\frac{3}{8}\hat{\imath}$ m/s

e) $-\frac{3}{8}\hat{\imath}$ m/s

Horizontal Momentum Conserved

$P_{gi} + P_{pi} = P_{gf} + P_{pf}$

$v_{gi} = v_{gp} + v_{pi}$

$0 = m_g v_{gi} + m_p v_{pi}$

$= m_g(v_{pi} + v_{gp}) + m_p v_{pi}$

$v_{pi} = -\left(\frac{m_g}{m_g + m_p}\right) v_{gp}$

$\frac{-30}{120} v_{gp} = \frac{-1}{4}\,\frac{3}{2} = \frac{-3}{8}$

(5 pts) What is the girl's velocity relative to the ice surface?

and

$v_{gi} = v_{gp} + v_{pi}$

$\left(\frac{3}{2}\right) + \left(-\frac{3}{8}\right) = \frac{9}{8}$

a) $\frac{9}{8}\hat{\imath}$ m/s

b) $-\frac{9}{8}\hat{\imath}$ m/s

c) $\frac{3}{2}\hat{\imath}$ m/s

d) $\frac{13}{8}\hat{\imath}$ m/s

e) $-\frac{3}{8}\hat{\imath}$ m/s

2) (5 pts) Suppose a rod is **nonuniform** such that its mass per unit length varies linearly with x according to the expression $\lambda = \alpha x$ where α is a constant. Find the x coordinate of the center of mass as a fraction of L.

$\lambda = \frac{M}{L}$ $dx\,\lambda = dm$

We know $X_{cm} = \frac{1}{M}\int_0^L x\,dm = \frac{1}{M}\int_0^L x\lambda\,dx = \frac{1}{M}\int_0^L \alpha x^2 dx = \frac{\alpha}{3M} L^3$

a) $\frac{L}{4}$

b) $\frac{L}{3}$

c) $\frac{4L}{3}$

Total Mass $M = \int_0^L dm = \int_0^L \lambda\,dx = \int_0^L \alpha x\,dx = \frac{\alpha L^2}{2}$

d) $\frac{3L}{4}$

e) $\frac{2L}{3}$

$X_{cm} = \frac{\alpha L^3}{3} \frac{2}{\alpha L^2} = \frac{2L}{3}$

3) A car of mass m_1 traveling initially with a speed of v_1 in an easterly direction crashes into the back of a truck of mass m_2 moving in the same direction at speed v_2. The velocity of the car right after the collision is v_3 to the east.

(5 pts) What is the speed v_4 of the truck right after the collision?

$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4$

$v_4 = \frac{m_1(v_1 - v_3) + m_2 v_2}{m_2}$

a) $\frac{m_1(v_1+v_2) + \cancel{m_2 v_3}}{m_2}$

b) $\frac{m_1(v_3-v_1) + \cancel{m_2 v_3}}{m_2}$

c) $\frac{m_1(v_1-v_3) + m_2 v_2}{m_2}$

d) $\frac{m_1(v_1+v_2) + \cancel{m_2 v_3}}{m_1}$

e) $\frac{m_2(v_1-v_3) + \cancel{m_1 v_3}}{m_1}$

(5 pts) What is the change in mechanical energy of the car–truck system in the collision?
Use: m_1= 4 kg, m_2= 2 kg, v_1= 1 m/s, v_2= 3 m/s, v_3= 2 m/s, v_4= 1 m/s

$\Delta KE = K_f - K_i$

$\Delta KE = \frac{m_2}{2}(v_4^2 - v_2^2) + \frac{m_1}{2}(v_3^2 - v_1^2)$

$\frac{2}{2}(1^2 - 3^2) + \frac{4}{2}(4-1)$

$-8J + 6J$

$\boxed{KE\Delta = -2J}$

a) 1 J

b) $\frac{3}{2}$ J

c) −1 J

d) 2 J

e) −2 J

Find the velocity of their center of mass before and after the collision using the values from the previous problem.

(5 pts) Before

$(m_1+m_2)v_{cm} = m_1v_1 + m_2v_2$

$v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1+m_2}$

$= \frac{(4)(1)+(2)(3)}{4+2}$

$= \frac{10}{6}$

$\frac{5}{3}$

a) $\frac{10}{3}$ m/s

b) $\frac{5}{3}$ m/s

c) $\frac{4}{3}$ m/s

d) 3 m/s

e) $\frac{5}{2}$ m/s

(5 pts) After

$V_{cm} = \frac{m_1v_3 + m_2v_4}{m_1+m_2}$

$v_{cm} = \frac{(4)(2)+(2)(1)}{4+2}$

$= \frac{10}{6}$

$v_{cm} = \frac{5}{3}$

a) $\frac{10}{3}$ m/s

b) $\frac{5}{3}$ m/s

c) $\frac{4}{3}$ m/s

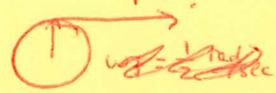
d) 3 m/s

e) $\frac{5}{2}$ m/s

V_{cm} is the Same Before | After Collision

4) (10 pts) A 100-kg merry-go-round in the shape of a uniform, solid, horizontal disk of radius 4.0 m is set in motion by wrapping a rope about the rim of the disk and pulling on the rope. What constant force would have to be exerted on the rope to bring the merry-go-round from rest to an angular speed of 0.50 rad/s in 2.00 s?

$F = ?$



$I = \int_0^R r^2\,dm$ $\quad M = \rho dV$ $\quad dm = \rho dV$ $\quad V = \pi r^2 d$ $\quad dV = 2\pi r d\,dr$ $\longrightarrow I = \frac{1}{2}MR^2$

$I = 2\pi\rho d \int_0^R r^3\,dr$

$= 2\pi\rho d \frac{R^4}{4} = \frac{2\pi d M}{\pi R^2 d}\frac{R^4}{4} = \frac{1}{2}MR^2$

$M = (\pi R^2 d)\rho$

$\tau = I\alpha$

$\tau = \vec{r}\times\vec{F} = R\cdot F$

$F = \frac{I\alpha}{R}$

$= \frac{(\frac{1}{2}MR^2)}{R}\cdot\frac{1}{4}$

$F = \frac{MR}{8} = \frac{(100)(4)}{8} = 50\ N$

$\omega_f(2) = \frac{1}{2}$ rad/sec

$\omega_f = \omega_i + \alpha t$

$\alpha = \frac{\omega_f}{t} = \frac{\frac{1}{2}}{2}$

$= \frac{1}{4}$

a) 100 N

b) 50 N

c) 200 N

d) 100π N

e) 50π N

5) (10 pts) A horizontal 50- kg merry-go-round is a solid disk of radius 2.00 m and is started from rest by a constant horizontal force of 10.0 N applied tangentially to the edge of the disk. Find the kinetic energy of the disk after 5.0 s.

$F = 10 \quad R = 2 \quad M = 50$

$KE = \frac{1}{2}I\omega^2$

$KE = \frac{1}{2}I[\omega(5)]^2$

$= \frac{1}{2}\left(\frac{1}{2}(50)(2)^2\right)(1)^2$

$KE = \frac{1}{4}(50)4 = 50\ J$

$I = \frac{1}{2}MR^2$

$\tau = I\alpha \quad \alpha = \frac{RF}{I}$

$\tau = r\times F = R\cdot F$

$\omega_f = \omega_i + \alpha t$

$\omega(5) = \frac{RF}{I}(5) = \frac{5RF}{\frac{1}{2}MR^2}$

$= \frac{10F}{MR} = \frac{100}{(50)(2)}$

$\omega(5) = 1$ rad/sec

a) 50 J

b) 25 J

c) 200 J

d) 100 J

e) 150 J

6) A car accelerates uniformly from rest and reaches a speed of 10.0 m/s in 8.0 s. Assume the diameter of a tire is 40 cm.

(5 pts) Find the number of revolutions the tire makes during this motion, assuming that no slipping occurs.

Uniform accel means

$a = \frac{v_f}{t} = \frac{10}{8} = \frac{5}{4}$ $\alpha = \frac{a}{r} = \frac{\frac{5}{4}}{\frac{1}{5}} = \frac{25}{4}$

$\theta = \omega_i t + \alpha \frac{t^2}{2} = \frac{25}{4}\frac{(8)^2}{2} = \frac{(25)(2)(8)}{2} = \cancel{400}\,200 \text{ rad} = \frac{\cancel{400}\,200 \text{ rad}}{} \frac{1 \text{ rev}}{2\pi \text{ rad}}$

$1 \text{ rev} = 2\pi \text{ rad}$

$= \cancel{200}\cdot\frac{100}{\pi} \text{ rev}$

a) $\frac{50}{\pi}$ rev

b) $\frac{100}{\pi}$ rev

c) $\frac{200}{\pi}$ rev

d) $\frac{25}{\pi}$ rev

e) 400π rev

(5 pts) What is the final angular speed of the tire in revolutions per second?

$\omega_f = \frac{v}{r} = \frac{10 \text{ m/s}}{\frac{1}{5} \text{ m}} = 50 \frac{\text{rad}}{\text{sec}} = \frac{50}{2\pi} \frac{\text{rev}}{\text{s}} = \frac{25}{\pi} \frac{\text{rev}}{\text{s}}$

a) $\frac{50}{\pi}$ rev/s

b) $\frac{100}{\pi}$ rev/s

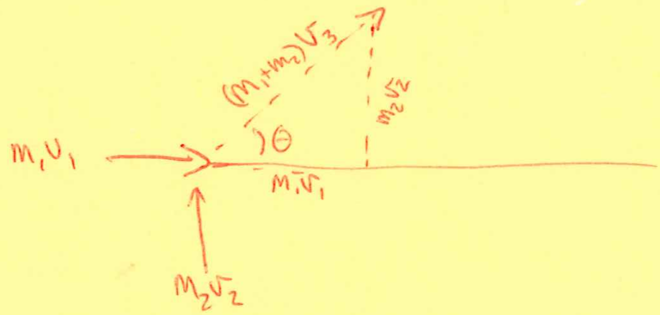
c) $\frac{200}{\pi}$ rev/s

d) $\frac{25}{\pi}$ rev/s

e) 400π rev/s

7) A car of mass m_1 traveling east with a speed of v_1 collides at an intersection with a truck of mass m_2 traveling north at a speed of v_2. Assume the vehicles stick together after the collision.

(5 pts) Find the angle θ that the wreckage travels after the collision, where east is taken as zero.



a) $Tan^{-1}\left[\cancel{\frac{m_1v_1}{m_2v_2}}\right]$

b) $\cancel{Csc}^{-1}\left[\frac{m_2v_2}{m_1v_1}\right]$

c) $Tan^{-1}\left[\frac{m_2v_2}{m_1v_1}\right]$

d) $\cancel{Csc^{-1}}\left[\frac{m_2v_2}{2m_1v_1}\right]$

e) $\cancel{Tan^{-1}}\left[\frac{m_2v_2}{2m_1v_1}\right]$

$\tan\theta = \frac{m_2v_2}{m_1v_1}$

$\sqrt{v_1^2+v_2^2}$

$\theta = \tan^{-1}\left(\frac{m_2v_2}{m_1v_1}\right)$

$\cot\theta = \frac{m_1v_1}{V_2v_2}$

$\csc^{-1}\left(\frac{(m_1+m_2)v_3}{m_2v_2}\right)$

(5 pts) Find the magnitude of the velocity of the wreckage after the collision

$V_3 = ?$ $\quad m_1v_1 + m_2v_2 = (m_1+m_2)v_3$

a) $\frac{m_1v_1}{[m_1+m_2]Sin\theta}$

b) $\frac{m_2v_2}{[m_1+m_2]Sin\theta}$

c) $\frac{\cancel{m_1}v_2}{[m_1+m_2]Sin\theta}$

d) $\frac{m_2v_2}{[m_1+m_2]Cos\theta}$

e) $\frac{2\cancel{m_2}v_2}{[m_1+m_2]Sin\theta}$

$(m_1+m_2)v_3\sin\theta = m_2v_2 \quad v_3 = \frac{m_2v_2}{(m_1+m_2)\sin\theta}$

$m_1+m_2v_3\cos\theta = m_1v_1$

8) Consider two objects with $m_1 > m_2$ connected by a light string that passes over a pulley having a moment of inertia of I about its axis of rotation as in the figure below. The string does not slip on the pulley or stretch. The pulley turns without friction. The two objects are released from rest separated by a vertical distance $2h$. (Use any variable or symbol stated above along with the following as necessary: g and R.)

(5 pts) Find the translational speeds of the masses as they pass each other.

a. $\sqrt{gh\,\frac{m_1-m_2}{m_1+m_2+\frac{I}{R^2}}}$

b. $\sqrt{2gh\,\frac{m_2\cancel{+}m_1}{m_1+m_2+\frac{I}{R^2}}}$

c. $\sqrt{4gh\,\frac{m_1-m_2}{m_1+m_2+\frac{I}{R^2}}}$

d. $\sqrt{2gh\,\frac{m_1-m_2}{m_1+m_2+\frac{I}{R^2}}}$

e. $\sqrt{gh\,\frac{m_2\cancel{-}m_1}{m_1+m_2+\frac{I}{R^2}}}$

$E_i = E_f$

$PE_i + KE_i = PE_f + KE_f$

$(m_1gh - m_2gh) = m_1g(h_{pass} - m_2gh_{pass}) + KE_1 + KE_2 + KE_{rot}$

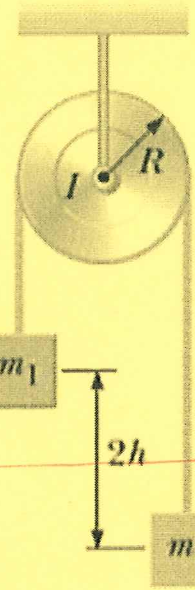
No friction means

$\Delta KE + \Delta PE = 0$

$K_f - K_i = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$

$\frac{1}{2}\left(m_1+m_2+\frac{I}{R^2}\right)v^2 + m_2gh - m_1gh = 0$

$V = \sqrt{\frac{2(m_1-m_2)gh}{m_1+m_2+I/R^2}}$?



when do they pass? must be at h

(5 pts) Find the angular speed of the pulley at this time.

a) $\sqrt{\cancel{R^2}gh\,\frac{m_1-m_2}{m_1+m_2+\frac{I}{R^2}}}$

b) $\sqrt{2\cancel{R^2}gh\,\frac{m_2-m_1}{m_1+m_2+I}}$

c) $\sqrt{4gh\,\frac{m_1-m_2}{R^2m_1+R^2m_2+I}}$

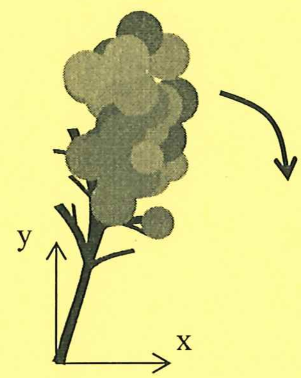
d) $\sqrt{2gh\,\frac{m_1-m_2}{R^2m_1+R^2m_2+I}}$

e) $\sqrt{gh\,\frac{(m_2-m_1)}{\cancel{R^2}(m_1+m_2+I)}}$

$\omega = \frac{V}{r} = \frac{1}{R}\sqrt{\frac{2(m_1-m_2)gh}{(m_1+m_2+I/R^2)}}$

9) An old tree of mass m and height h falls over with its base as a pivot point. The tree was initially at rest and oriented vertically (parallel to the y-direction). The tree can be considered a thin rod which pivots about one end, so that its moment of inertia is $I = mh^2/3$. When the tree is horizontal (i.e. hits the ground), find:

(5 pts) Its angular speed



a) $\sqrt{\frac{6g}{h}}$

b) $\sqrt{\frac{2g}{h}}$

c) $\sqrt{\frac{3g}{h}}$

d) $\sqrt{\frac{2h}{g}}$

e) $\sqrt{\frac{g}{h}}$

$PE_i + KE_i = PE_f + KE_f$

$mg\left(\frac{h}{2}\right) = 0 + \frac{1}{2}I\omega^2$

$\omega = \sqrt{\frac{mgh}{I}} = \sqrt{\frac{mgh}{\frac{1}{3}mh^2}} = \sqrt{\frac{3g}{h}}$

-(5 pts) The magnitude of its angular acceleration

a) $\frac{6g}{h}$

b) $\frac{2g}{h}$

c) $\frac{3g}{2h}$

d) $\frac{g}{h}$

e) $\frac{2g}{3h}$

$\tau = I\alpha$

$\tau = r \times F = \left(\frac{h}{2}\right) mg = I\alpha$

$\alpha = \frac{\frac{h}{2} mg}{I} = \frac{hmg}{2\left[\frac{1}{3}mh^2\right]} = \frac{3g}{2h}$

(5 pts) The magnitude of the translational acceleration of the center of mass

a) g

b) $\frac{3g}{4}$

c) $\frac{3g}{2h}$

d) $\frac{g}{2}$

e) $\frac{g}{3}$

$a = \alpha \cdot (r) = \alpha \left(\frac{h}{2}\right) = \frac{3g}{2h} \cdot \frac{h}{2} = \frac{3g}{4}$