

Let G be a directed graph with a set of vertices V and a set of edges E such that $G = (V, E)$.

Let s be the source vertex and t be the target vertex of the flow in G .

Consider the max-flow f computed for G using the F-F Algorithm:

For the solution graph G_{sol} ,

Let A be the subset of V such that all vertices in A are reachable from s , the source.

Let A^* be the subset of V such $V - A = A^*$.

Suppose $f(x) =$ the capacity of a cut from s to t such that $f(x) = c(A, A^*)$ where

$c(S, T) =$ the sum of capacities for all vertex-pairs (u, v) where u is an element of S , and v is an element of T , and S is the set of all the vertices reachable by s and T is all the vertices reachable by t .

Then, $f(x) = (\text{the flow out from } A) - (\text{the flow in from } A^*)$ for a given subset of vertices, A .

Thus, for $f(x) = c(A, A^*)$ the following needs to hold.:

1. All outgoing edges must be at a full capacity
2. All incoming edges must have 0 flow

Consider two cases:

1. In G , there is an outgoing edge (d, e) where d is an element of A , and e is an element of A^* such that $f((d, e)) < c(d, e)$. Thus, there exists a forward edge from d to e in G_{sol} . Therefore there exists a path from s to e , which is a contradiction. Hence, any outgoing edge must be at full capacity.

2. In G , there exists an incoming edge (d, e) , such that d is an element of A and e is an element of A^* , where (d, e) has a non-zero flow. This implies that there exists an edge from d to e in G_{sol} . Thus, there exists a path from s to e , which is a contradiction. Therefore, any incoming edge must have zero flow.

These two cases show that the capacity of cut described above is the same as the flow of G .