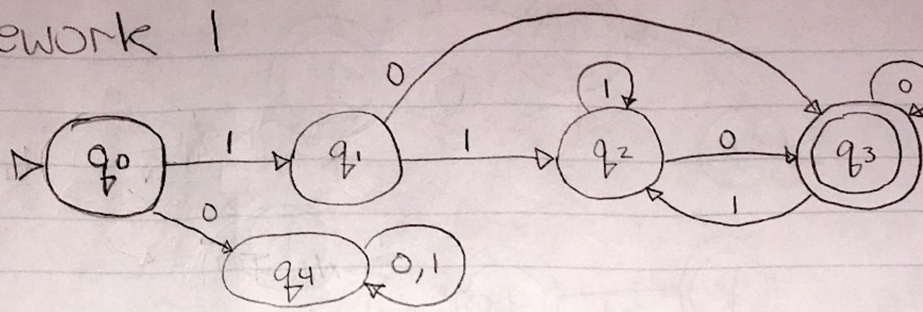
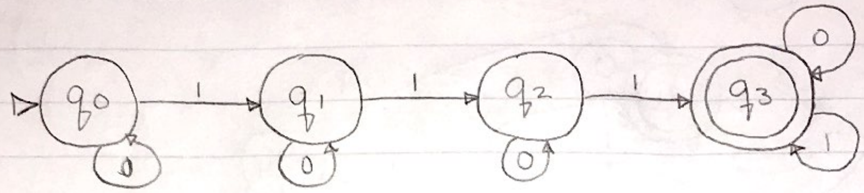
Homework 1

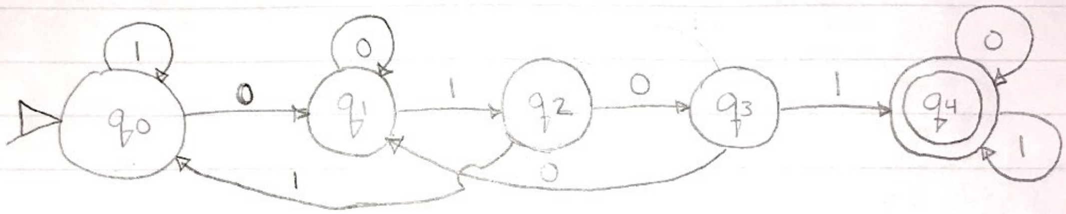
1. a.)



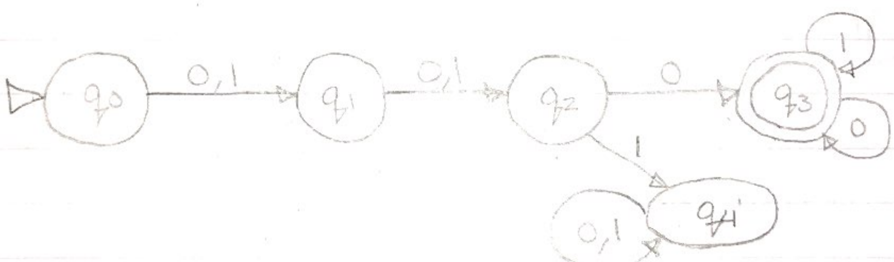
b.)



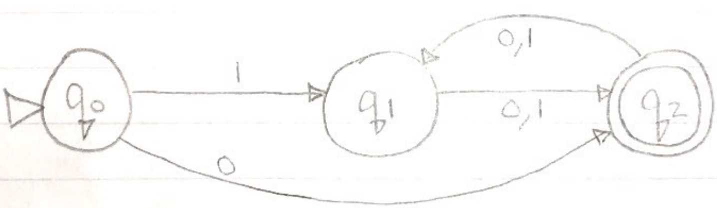
c.)



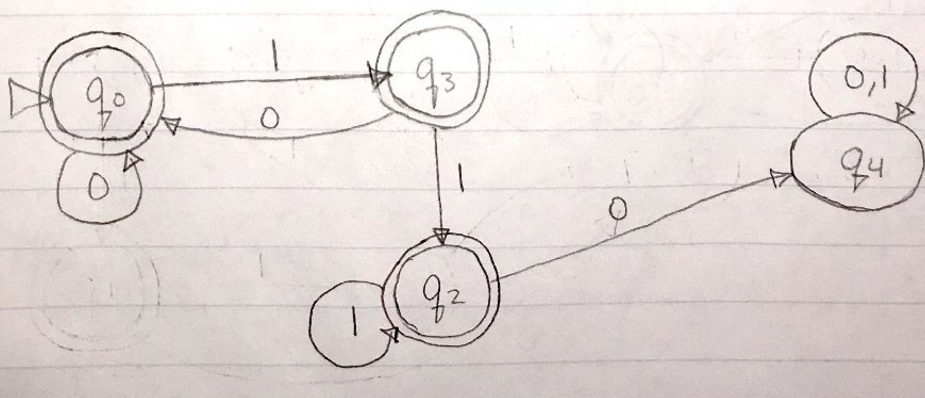
d.)



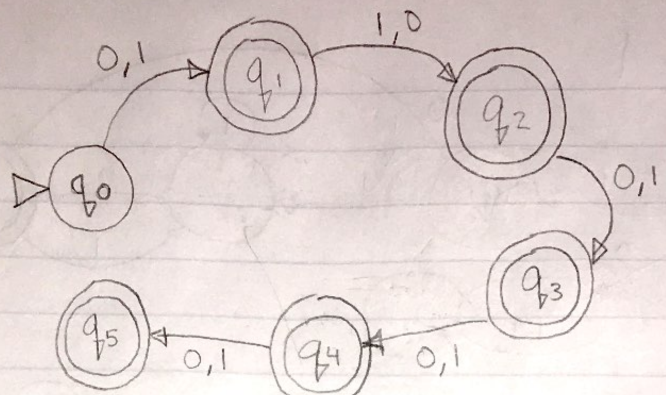
e.)



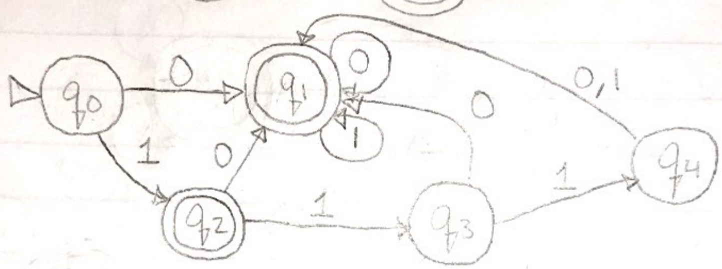
F.)



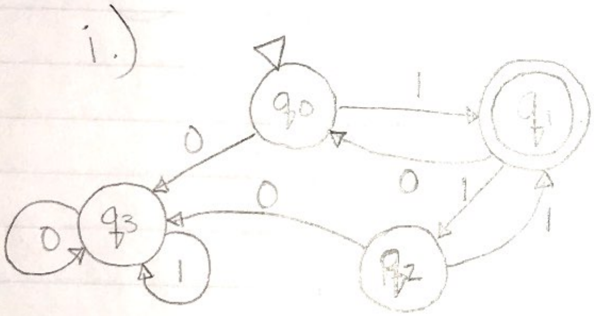
1. g.)


0,1
q1
1,0
q2
q0
0,1
q3
q5
0,1
q4
0,1

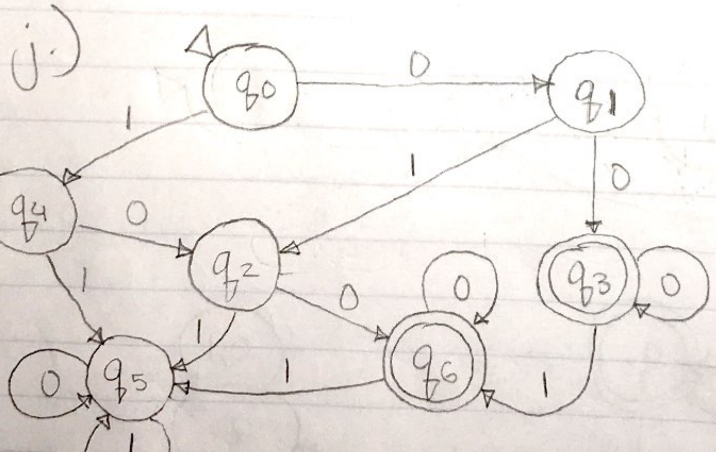
h.)


q0
0
q1
0
1
0,1
0
1
0
q4
1
1
q2
q3

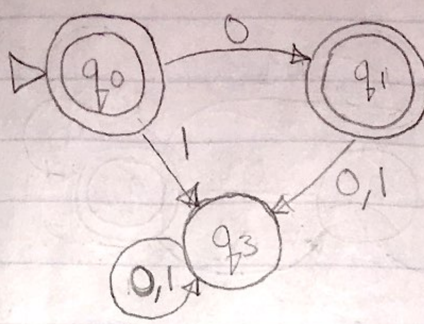
i.)


q0
1
q1
0
0
1
0
q3
0
1
q2
1

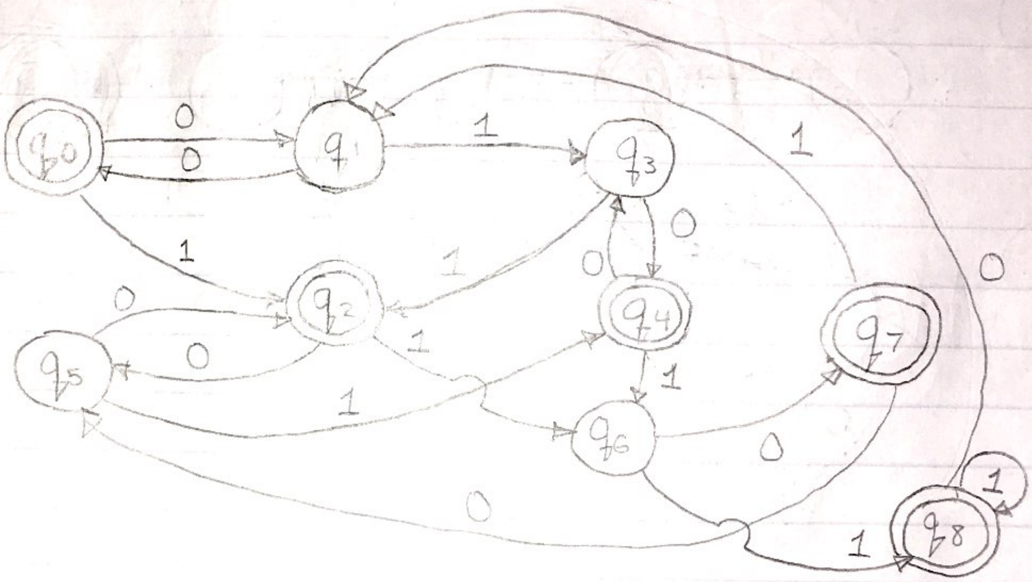
j.)


q0
0
q1
1
1
0
q4
0
q2
1
0
0
q3
0
1
q5
0
1
q6
1
1

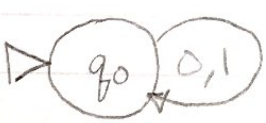
1. k.) $\{\varepsilon, 0\}$



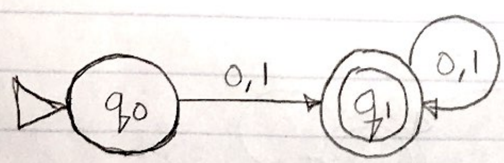
l.)



m.) Empty set

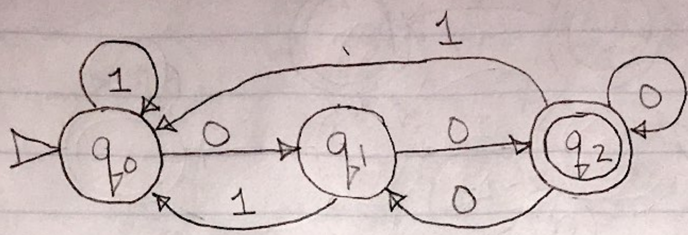


n.)

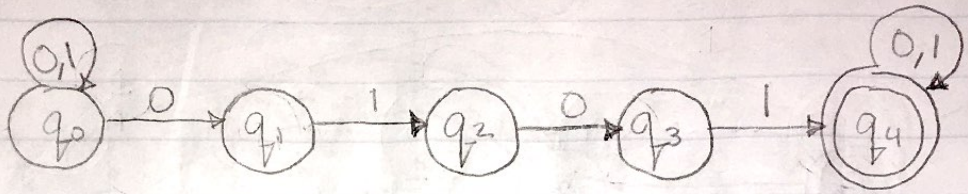


2. [1.7.]

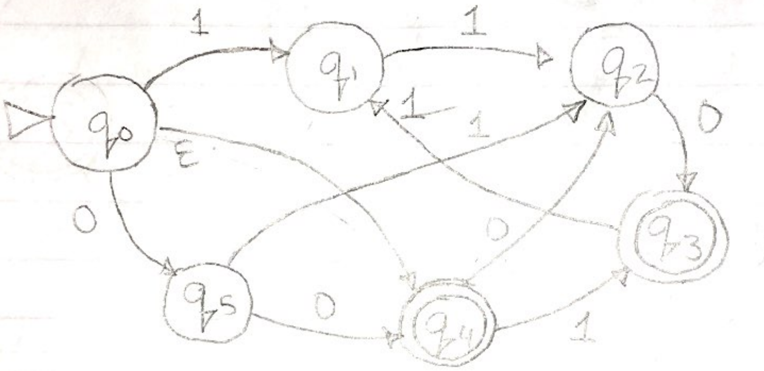
a.)



b.)



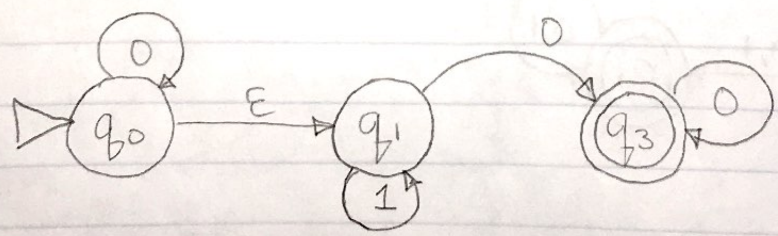
c.)



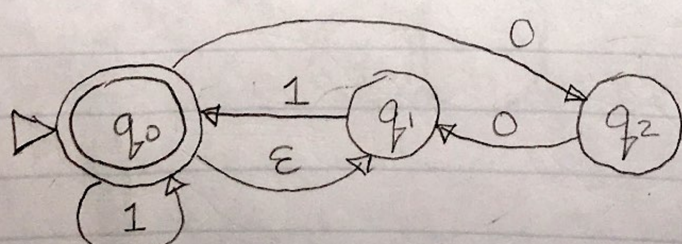
// zero 0s as even?

d.)

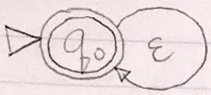
e.)



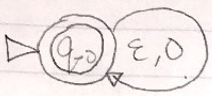
F.)



g.)



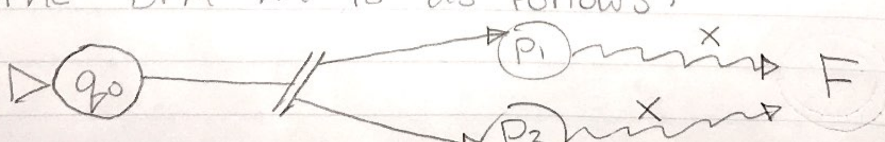
h.)



c.) M is minimal iff $\nexists\, M' \ni |M'| < |M|$. Assume M is minimal! Since no two states are indistinguishable, $\nexists$ two paths to $p \ni \delta^*(q_0, ax_p) \in F, \delta^*(q_0, bx_p) \in F$, where $a \neq b$. If M were not minimal, then there would exist 2 strings $a, b \in \Sigma^* \ni \delta^*(q_0, ax_p) \in F$, & $\delta^*(q_0, bx_p) \in F$, which is a contradiction. Therefore, M must be minimal.

Assume;

5.) If $\delta^*(p_1, x) \in F$, then $\delta^*(p_2, x) \in F$.
If $\delta^*(p_2, x) \in F$, then $\delta^*(p_1, x) \in F$.
The DFA M is as follows;



Since p_1 & p_2 are reachable from q_0, $\exists\, a, b \in \Sigma^* \ni \delta^*(q_0, a) = p_1$ & $\delta^*(q_0, b) = p_2$. Then $\delta^*(q_0, ax) \in F$ & $\delta^*(q_0, bx) \in F$. So, we can make DFA M', $\ni \forall c \in \Sigma^*$ $\delta^*(q_0, cx)$ takes us to F by the implication that paths from q_0 to F by ax & bx are indistinguishable.

We can then remove p_1 & p_2, replaced by p', & $|M'|$ will be $\leq |M| - 1$.