Name: Key

Midterm Exam 1, February 2, 2017

Physics 152-000

THE HONOR CODE IS IN EFFECT FOR THIS EXAM – IT IS YOUR RESPONSIBILITY TO MAINTAIN HONESTY AND FAIRNESS.

Instructions:

1. When told to begin, please write your name on the top of every page.
2. **Write neatly and show your solution methods clearly**.
3. You will be graded on how you got your answers. Little or no credit will be given for answers that do not show how you got them.
4. Partial credit will be given if you have minor errors, but not for answers that incorrectly solve the problem.
5. Do your work for each problem on the page for that problem.
6. Point totals are noted by each question.
7. This exam is closed book and closed notes. You have up to 70 minutes to complete this exam. You must stop and turn in your exam when I announce the exam is over.

Good Luck!

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Emory Honor Pledge:

I, ______________________________ **(print name)**, by signing this examination, acknowledge that I have abided by the provisions and spirit of the **EMORY COLLEGE HONOR CODE** in my completion of this examination. This exam represents my own work, and I have neither given or received aid in completing this exam.

Signature: ______________________________

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1.	**2.**	**3.**	**4.**	**Total:**
out of 25	**out of 25**	**out of 30**	**out of 20**	**Out of 100**

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Permittivity constant	$\varepsilon_0 = 8.854 \times 10^{-12}\ C^2 / N\ m^2$ $k = 8.99 \times 10^9\ N\ m^2 / C^2$	
Elementary charge	$e = 1.602 \times 10^{-19}\ C$	
Electron mass	$m_e = 9.109 \times 10^{-31}\ kg$	
Proton mass	$m_p = 1.673 \times 10^{-27}\ kg$	
Neutron mass	$m_n = 1.675 \times 10^{-27}\ kg$	
circumference of a circle	$2 \pi r$	$1\ km = 10^3\ m$
area of a circle	πr^2	$1\ mm = 10^{-3}\ m$
surface area of sphere	$4 \pi r^2$	$1\ \mu m = 10^{-6}\ m$
volume of a sphere	$4/3\ \pi r^3$	$1\ nm = 10^{-9}\ m$
volume of a cylinder	$\pi r^2 L$	
area of a triangle	½ base x height	

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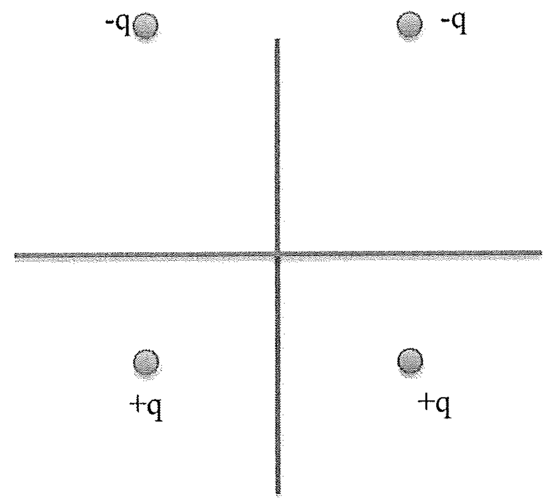
1) Four point charges have Cartesian coordinates of (±a, ±a); in other words they form the corners of a square of side length 2a, centered on the origin, as shown in the figure. Note the sign of each charge in the figure. Each charge has been numbered to help with your notation. In deriving your answers, please use $\cos(45)=\sin(45)=\frac{\sqrt{2}}{2}$ (i.e. do NOT use a decimal).



a) Find $\vec{E}$ at the origin (magnitude and direction) (10 points).

E_x will cancel, so we need only find E_y for each & add. They all have the same

$$E_y = \frac{kq}{2a^2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}\frac{kq}{a^2}$$

$$\Rightarrow E_t = \frac{\sqrt{2}\,kq}{a^2}$$

b) Find V at the origin. (8 points)

$$V = \sum \frac{kq}{r} = \frac{k}{\sqrt{2a^2}} \sum q = 0$$

c) Write and expression for V at point P whose coordinates are (0, y). (7 points)

$$V_1 = V_2 = \frac{kq}{\sqrt{a^2+(y+a)^2}} \qquad V_3 = V_4 = -\frac{kq}{\sqrt{a^2+(y-a)^2}}$$

$$V = 2kq\left(\frac{1}{[a^2+(y+a)^2]^{1/2}} - \frac{1}{[a^2+(y-a)^2]^{1/2}}\right.$$

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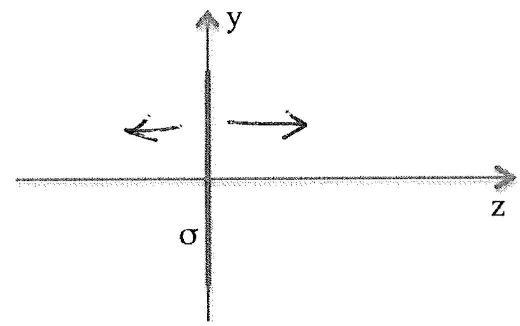
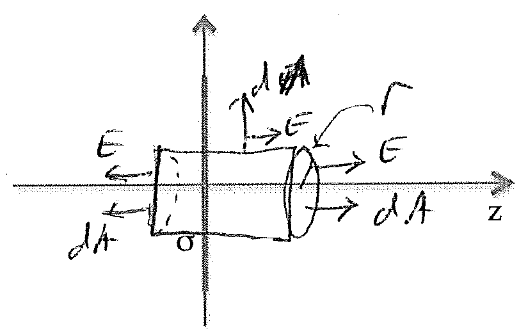
2) An infinite conducting plane in the x-y plane has uniform charge density σ. (Note: $\sigma>0$).

a. Find the direction of $\vec{E}$ everywhere by symmetry arguments and use arrows to draw the direction of $\vec{E}$ on the diagram below, which shows the plane looking along the x-axis. *Make sure to consider each side of the plane*, i.e. draw arrows for both z>0 and z<0. (4 points)



b. Using Gauss's Law:

i. Draw an appropriate Gaussian surface to find the magnitude of E. Draw and label arrows noting the directions of E and dA on each part of the surface in your diagram. (6 points)



ii. Use Gauss's Law to solve for the magnitude of E. (show all of your work) (5 points)

$$\oint \vec{E}\cdot d\vec{A} = \int_{\substack{\text{top}\\ \text{bottom}}} \vec{E}\cdot d\vec{A} + \int_{\substack{\text{curved}\\ \text{parts}}} \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$= 2\int E\,dA + 0 \qquad = \frac{\pi r^2 \sigma}{\epsilon_0}$$

$$= 2E\int dA$$

$$= 2\pi r^2 E$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

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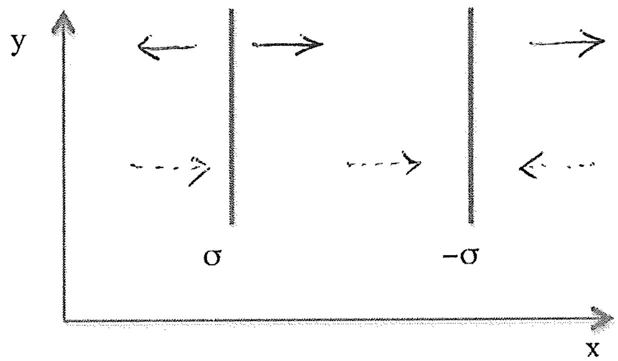
(Problem 2 continued)

c. Consider two parallel infinite planes (perpendicular to the page). One has charge density σ and the other has charge density - σ. (Hint: The two planes are independent of each other, so you may use superposition)



i. Find the magnitude and direction of of E between the two planes. (5 points) *To justify your answer you may either provide a one-sentence explanation supporting your answer, or you can show how you used Gauss's Law to derive your answer.*

$$\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

can just add fields from each plane by superposition.

ii. Find the magnitude of E to the right of the two plates. (5 points) *To justify your answer you may either provide a one-sentence explanation supporting your answer, or you can show how you used Gauss's Law to derive your answer.*

$$\frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

as above.

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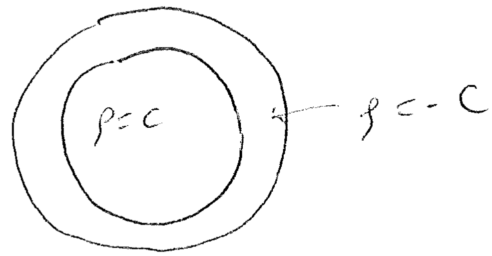
3) A solid non-conducting sphere of radius R has a charge density ρ which varies with the distance r from its center as follows, where c is a constant with appropriate dimensions:

$\rho = +c$ for $0 \le r \le \frac{a}{\sqrt[3]{2}}$

$\rho = -c$ for $\frac{a}{\sqrt[3]{2}} \le r \le a$

$\rho = 0$ for $a < r$



a) Find the total charge in the inner part of the sphere (where $r \le \frac{a}{\sqrt[3]{2}}$). (5 points)

$$\frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi \frac{a^3}{2} c = 2\pi c \frac{a^3}{3}$$

b) Find the total charge in the outer part of the sphere (where $\frac{a}{\sqrt[3]{2}} \le r \le a$). (5 points)

$$\frac{4}{3}\pi \left(a^3 - \frac{a^3}{2}\right)(-c) = -2\pi \frac{c a^3}{3}$$

(Same magnitude as (a) but opposite sign.

c) Find E for r >a. (5 points)

$$\int E \cdot da = \frac{q_{in}}{\epsilon_0} = 0 \quad \Rightarrow E = 0$$

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Problem 3 (continued)

d) Find E for $r \le \frac{a}{\sqrt[3]{2}}$ (5 points)

$$\oint E \cdot dA = E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} \frac{4}{3}\pi r^3 c$$

$$\boxed{E = \frac{c}{3\epsilon_0} r}$$

e) Find E for $\frac{a}{\sqrt[3]{2}} \le r \le a$ (5 points)

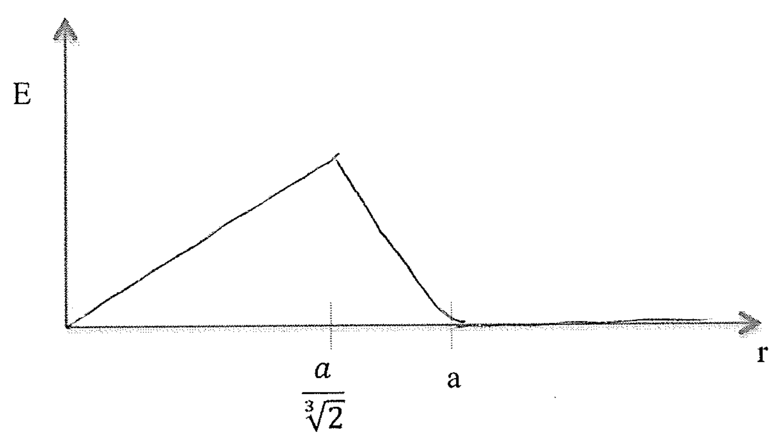
$$E \cdot 4\pi r^2 = \frac{q_{in}}{\epsilon_0}$$

$$q_{in} = \frac{4}{3}\pi \left[\frac{c a^3}{2} - c\left(r^3 - \frac{a^3}{2}\right) \right]$$

$$= \frac{4}{3}\pi c \left(a^3 - r^3 \right)$$

$$E = \frac{c}{3\epsilon_0} \frac{(a^3 - r^3)}{r^2}$$

f) Plot E vs r. (5 points)



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4) Two points in space each contain point charges. Point A has coordinates (2m, 5m) and charge +2q. Point B has coordinates (6m, 8m) and charge -2.5q.

a. Calculate the unit vector that is directed from point A **to** point B. Your answer should use vector notation ($\hat{\imath}$ and $\hat{\jmath}$). (5 points)

$$\vec{r} = 4\hat{i} + 3\hat{j} \qquad |r| = \sqrt{16+9} = 5$$

$$\hat{r} = \frac{1}{5}(4\hat{i} + 3\hat{j})$$

b. Write an expression for the force on the charge at point B due to the charge at point A using appropriate symbols and your vector from part (a). (5 points)

$$F = \frac{k q_1 q_2}{r^2}\hat{r} = -\frac{1}{4\pi\epsilon_0}\frac{5q^2}{25m^2}\frac{1}{5}(4\hat{i} + 3\hat{j})$$

$$= \frac{-q^2}{4\pi\epsilon_0\, 25m^2}(4\hat{i} + 3\hat{j})$$

c. If you move the two charges twice as far apart, will the potential energy of the charge configuration increase or decrease? (5 points)

Increase

d. What is the magnitude of the change in potential energy of this charge configuration when you move the two charges twice as far apart? (5 points) *Note: Derive an appropriate expression. You do not need a numerical answer.*

$$\Delta PE = q\Delta V = q_A q_B K_e\left(\frac{1}{5m} - \frac{1}{10m}\right)$$

$$= 5q^2 K_e\left(\frac{1}{10m}\right)$$

$$= \frac{K_e q^2}{2m}$$