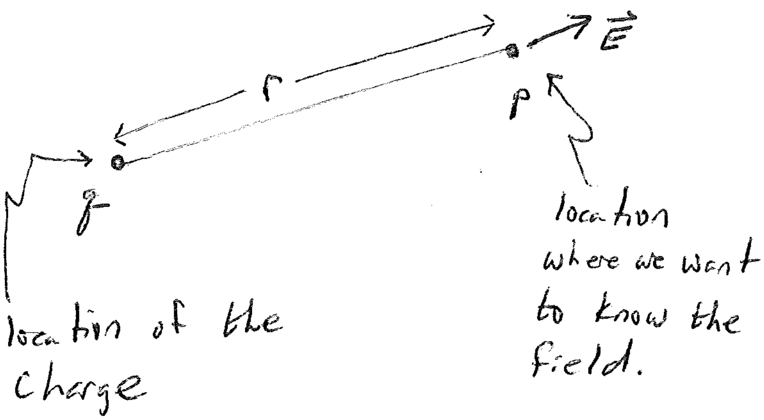
Some notes about using Coulomb's Law to find $\vec{E}$

$$\vec{E} = \frac{k_e q}{r^2}\hat{r}$$

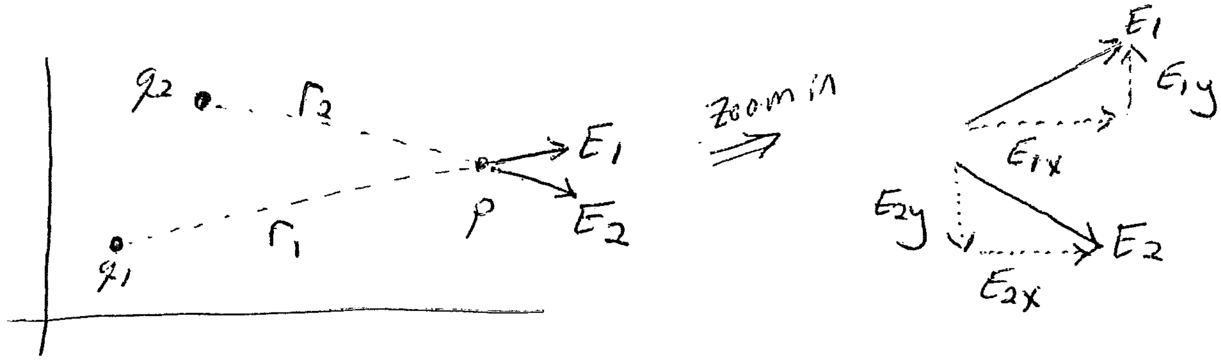
where
- q is the charge that creates the field
- r is distance from the charge to the location we calculate the E-field
- $\hat{r}$ is a unit vector in the direction of $\vec{E}$. For (+) charge it points away from the charge. For (−) charge it points towards the charge.



In the diagram above, I have drawn $\vec{E}$ assuming that q is positive, so $\vec{E}$ (and $\hat{r}$) point away from the charge.

If we have more than one charge, we need to find $\vec{E}$ (magnitude <u>and</u> direction) from <u>each</u> charge, and then use <u>vector addition</u> to find the net $\vec{E}$ at point p.

example: 2 discrete charges, q_1 and q_2



Steps:
1) identify all the charges
2) find r_i for each charge
3) find $|E_i|$ for each charge (at point P)
4) find direction of E, and then decompose into E_x and E_y so you can use vector addition.
5) add components of E.

1) two charges, q_1 + q_2
2) diagram shows r_1 + r_2
3) $|E_1| = \dfrac{k_e q_1}{r_1^2}$ $\qquad |E_2| = \dfrac{k_e q_2}{r_2^2}$
4) Use trig or vector notation to find $E_{1x}, E_{1y}, E_{2x}, E_{2y}$
5) $\vec{E} = (E_{1x} + E_{2x})\hat{i} + (E_{1y} + E_{2y})\hat{j}$

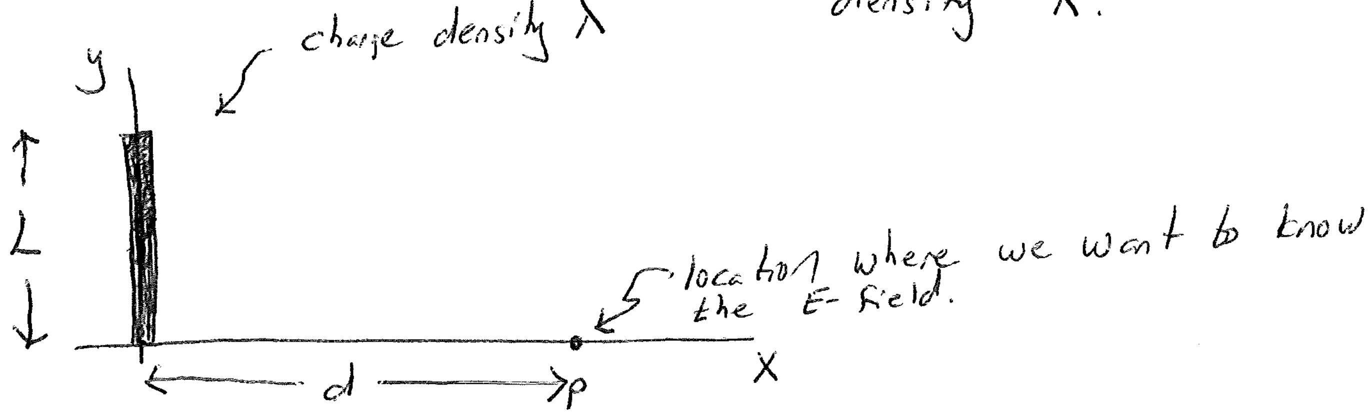
Note: This procedure doesn't change when you have additional charges. Just keep adding....

example: <u>Continuous charge distribution</u>

Notes: 1) when we have continuous charge distributions, the procedure outlined in previous example does not change!!!

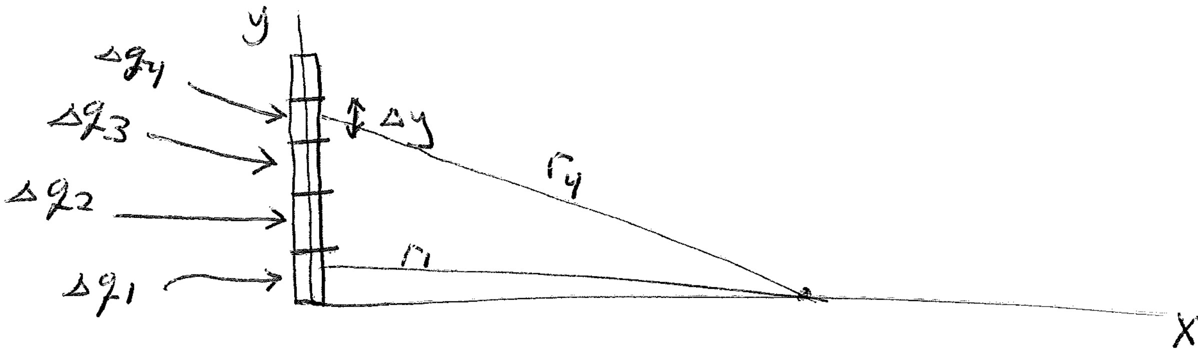
2) Integration is how we <u>add</u>, but integration does <u>not</u> by itself take care of the <u>vector addition</u>. It is our job to make sure the quantities we add are already in proper component form before we "add" - i.e. integrate.

<u>example</u> (as shown in class) rod of length L, charge density λ.



Step 1 identify all the charges: We do this by breaking the continuous object into tiny pieces (conceptually), each of charge dq, and then each dq creates its own field at point P.

in this case



as $\Delta y \to 0$, $\Delta q \to$ "dq", but still think of each as a "point charge". Note $\Delta q = \lambda\, dy$

Step 2
Diagram shows r_1 and r_4. Others left off so it doesn't get too cluttered, but you can see how to identify each r_i

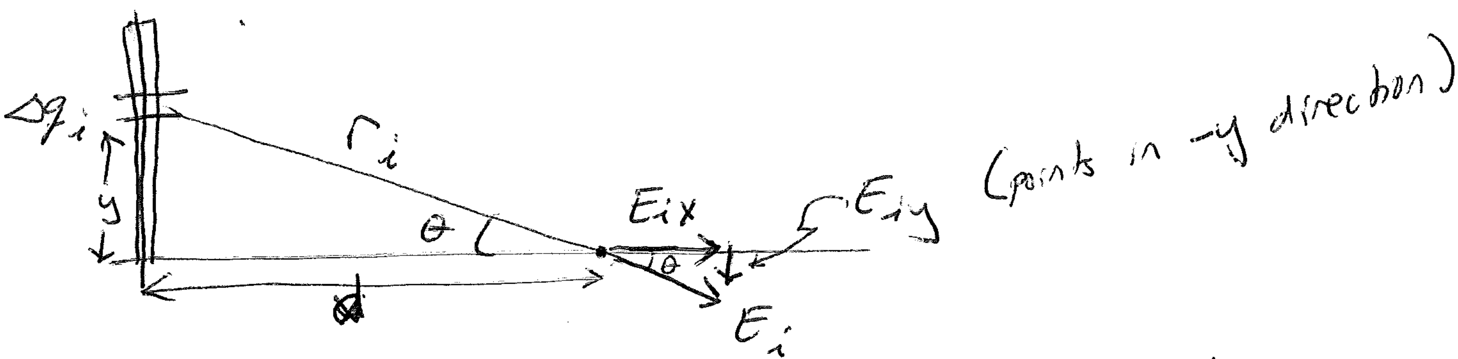
Step 3 $|E_i| = \dfrac{k_e \Delta q_i}{r_i^2}$

Step 4

I can't integrate until I get vector components. I want to add $E_{1x} + E_{2x} + \cdots$

and $E_{1y} + E_{2y} + E_{3y} + \cdots\cdots$

I do not want to add $E_{1x} + E_{1y} + E_{2x} + E_{2y} + \cdots$



$E_{ix} = E_i \cos\theta$ $\qquad$ $\cos\theta = \dfrac{d}{r_i}$

$E_{iy} = E_i \sin\theta$ $\qquad$ $-\sin\theta = \dfrac{-y}{r_i}$

in our example, $x = d$ so $\cos\theta = \dfrac{d}{r_i}$

" " " y varied from 0 to L,

so $\sin\theta = \dfrac{y}{r_i}$

Step 5

$$E_{ix} = \frac{K_e \Delta q_i}{r_i^2} \cos\theta \, \hat{\imath}$$

$$= \frac{K_e \Delta q_i}{r_i^2} \frac{d}{r_i} \, \hat{\imath}$$

recall $\Delta q_i = \lambda \, dy$, $r_i = (y^2 + d^2)^{1/2}$

$$E_{ix} = \frac{K_e \lambda \, dy \, d}{(d^2+y^2)^{3/2}} \, \hat{\imath}$$

add components (i.e. integrate)

$$\Rightarrow \vec{E}_{X\,net} = \int E_{ix} = \hat{\imath} \int_0^L K_e \lambda d \frac{dy}{(d^2+y^2)^{3/2}}$$

$\#$

$$E_{iy} = \frac{K_e \Delta q_i}{r_i^2} (-\sin\theta) \, \hat{\jmath} = -\frac{K_e \lambda \, dy}{r_i^2} \frac{y}{r_i} \, \hat{\jmath}$$

$$\vec{E}_{y\,net} = \int E_{iy} = -K_e \lambda \hat{\jmath} \int_0^L \frac{y \, dy}{(d^2+y^2)^{3/2}}$$

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note: sometimes we write this all at once:

$$\vec{E}_{tot} = \vec{E}_{X\,tot} + \vec{E}_{y\,tot} = \int E_x \hat{\imath} + \int E_y \hat{\jmath}$$

$$= \int (E_x \hat{\imath} + E_y \hat{\jmath})$$