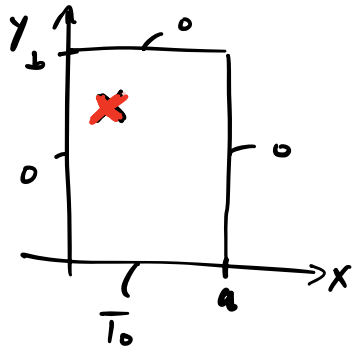
EXAMPLE 2

BC's: $T(0,y) = 0$

$T(x,b) = 0$

$T(a,y) = 0$

$T(x,0) = T_0$



① SCHEMATIC/ BCs

T/T_0 at $x/a = 1/4$, $y/b = 0.75$

$b/a = 2$

② SoV. $X(x) = C_1 \sin(\lambda x) + C_2 \cos(\lambda x)$

$Y(y) = C_3 \cdot e^{\lambda y} + C_4 e^{-\lambda y}$

③ APPLY BCs

a) $T(0,y) = 0$

$C_1 \cdot 0 + C_2 \cos(0) = 0$ ($\cos(0) = 1$)

$C_2 = 0$

b) $T(x,b) = 0$

$X(x) = C_1 \sin \lambda x \qquad Y(y) = C_3 e^{\lambda b} + C_4 e^{-\lambda b}$

$(C_1 \sin \lambda x)(C_3 e^{\lambda b} + C_4 e^{-\lambda b}) = 0$

$C_4 = -C_3 \dfrac{\sinh(\lambda b)}{\cosh(\lambda b)}$

$T(x,y) = C_1 \sin \lambda x \cdot C_3 \left[\sinh(\lambda y) - \dfrac{\sinh(\lambda b)}{\cosh(\lambda b)} \cdot \cosh(\lambda y) \right]$

$= C_5 \cdot \sin \lambda x \cdot \underbrace{\left[\cosh(\lambda b) \cdot \sinh(\lambda y) - \sinh(\lambda b) \cdot \cosh(\lambda y) \right]}_{= \sinh(\lambda(b-y))}$

c) $T(a,y) = 0$

$$0 = C_5 \cdot \sin(\lambda a) \cdot \sinh(\lambda(b-y))$$

$$\sin(\lambda a) = 0$$

$$\lambda_n = \frac{n\pi}{a}, \quad n = 1, 2, 3 \ldots$$

d) $T(x, 0) = T_0$

④ Solving:

$$T(x,y) = \frac{2T_0}{\pi} \sum_{n=1}^{\infty} C_5 \sin(\lambda_n x) \sinh(\lambda_n (b-y))$$

$$\frac{T(x,y)}{T_0} = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n} \cdot \frac{\sin(n\pi x/a) \sinh[\lambda_n (b-y)]}{\sinh(n\pi b/a)}$$

$$b - y = 2a - 0.75b$$
$$= 2a \cdot 0.75(2a)$$
$$= 2a(0.25)$$
$$= a/2$$

$$* \quad \sinh[\lambda_n (b-y)]$$
$$= \sinh\left[\frac{n\pi}{a}(b-y)\right]$$
$$= \sinh[n\pi/2]$$