UNIVERSITY OF WATERLOO

Faculty of Engineering

MTE 219 Bridge Project
Optimum Design of a Truss

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Group 2

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Executive Summary

a. Purpose

The objective of the bridge project is to demonstrate and apply concepts taught in previous applicable course and more specifically Mechanics of Deformable Solids (MTE 219) through the construction of a balsa wood bridge. The bridge is to be designed as an optimal 3D truss to achieve a maximum strength to weight ratio. The challenge is to apply force analysis and knowledge of failure modes. [1]

b. Methodology

The process of developing the bridge involved three major stages: designing, testing and optimization. This falls under the Design Process map, where the process is clearly defined [2]. Through the first stage, various trusses would be conceptualized along with methods of reinforcing and specific features. After picking a design, the testing stage would involve the construction of the truss and loading it to failure. This would outline various difficulties in construction, and refine the design and build. The testing would also reveal the weaknesses of the bridge and its over-designed sections. Through several iterations of this optimization the bridge design would achieve a much higher load to weight ratio.

c. Results / Conclusions

After many changes were made, the final bridge was recreated using the remaining materials and the methods learnt. The glue was given 24 hours to set and the bridge was ready for demo day. The achieved performance value ended up being approximately 603. This fell short of the expected value, and it was later found out that a member was sanded too thin where the wood was unknowingly weak (Figure 2, Appendix IV).

The project offered many challenges in terms of applying the entire design process and seeing a solution come to fruition from concept. It also showed the gap between paper and application. Engineering doesn’t end with a design, it merely starts.

1. Introduction

The group consisted of three members. The planning and organization of tasks being an important part of every project, deadlines and deliverables were set at the first meeting. The team successfully designed individually and collectively before the first building session. Calculations were completed before the first cut and every member had a large role in every stage of the design process.

a. Review Problem

In this Mechanics of Deformable Solids course (MTE 219), the term project is to design and build a bridge that meets specific constraints, including materials given, dimensions and component types (such as two force members and pin joints). The performance is based on the largest load to weight ratio. [1]

b. Design Constraints

According to the Project Outline [1] there are several constraints placed on the design of the Bridge, they are summarized below.

The Bridge must have:

I. a length of 40±1 cm measured from pin to pin
II. a height of 10±0.5 cm measured from the highest to lowest pin
III. a width of 8±0.5 cm measured to the outermost face of members at each end
IV. a pin located in the centre of the Bridge (20cm from each end) from which the bridge will be loaded
V. a Performance Value (PV) greater than 100 as defined by the formula

$$PV = \frac{Max\ Applied\ Load\ (grams)}{Mass\ of\ the\ Bridge\ (grams)}$$

The structure can only be composed of two force members and pin joints (freely rotating). With the limited quantity of material, the mass of the bridge could become a criterion in the design process with objective of minimization. It can also be noted that the problem is not limited by two-dimensional symmetry.

c. Design Criteria

An optimal design would have the greatest Performance Value (PV) as a result of the greatest load supported by the smallest mass paired with a degree of creativity. The marking scheme for the PV is as follows:

Range of PV	Marks out of 7
100 < PV < 150	3
150 < PV < 200	5
200 < PV	7

Table 1 - Grading scheme for bridges based on PV [1]

Achieving the initial PV of 200 was for achieving full marks in the design. But as the designs reached higher PV values their limitations would be revealed and create an opportunity for learning. An intrinsic reason for going beyond the minimum requirements.

Increasing the complexity of the bride can yield a better PV, but the probability of variation and error in fabrication increases. The difficulty of the analysis would also increase. Therefore keeping the bridge and build simple is a favourable criteria in design when it comes to performance. However, simple and performing designs can be lacking in creativity and would not provide as much content for the report.

Due to the limited resources, a bridge design that saved in use of materials would allow several iterations of the bridge. That way it can be loaded and broken to see what its flaws are and where improvements need to be made. This is a necessary process in refining and optimizing the bridge.

d. Material Properties

The materials supplied for this project were a limited number of balsa wood sticks and hardwood dowels. Their given properties are defined in the tables below.

Thickness	3.175 mm (⅛”)
Width	9.525 mm (⅜”)
Length	914.4 mm (36”)
Density [kg/m^3]	179
Elastic Modulus [GPa]	1.5
Normal Strength [MPa]	10
Shear Strength [MPa]	1

Table 2 – The physical properties of the provided balsa wood sticks. [1]

Diameter	3.175 mm (⅛”)
Length	914.4 mm (36”)
Density [kg/m^3]	680
Elastic Modulus [GPa]	16
Normal Strength [MPa]	171
Shear Strength [MPa]	32

Table 3 – The physical properties of the provided hardwood dowels. [1]

Realistically the difference between two sticks can be very large, for example, one of the measured sticks had a mass of 8 grams whereas the next heaviest was only 3.6 grams. This inconsistency requires extra care when choosing which piece of wood to use in each section of the bridge.

2. Preliminary Design

a. Conceptual Designs

The first design, appropriately named the complex truss [Figure 1. Appendix I] was created very early on in the design process and did not match up to the criteria well. This design was created primarily to combat buckling loads and was conceptualized prior to the idea of using I-beams. The design featured many joints with unreinforced members to reduce length of individual members and therefore increase buckling strength. The design although being strong against buckling required the use of more material and would have been difficult to construct as mentioned in the Design Criteria. The assembly of the design would have revealed further flaws if it had been constructed as the assembly order and number of holes would have revealed a weakness to twisting as well as pushing the limits of the eight centimetre width constraint of the bridge.

During the testing and construction of what would become our final design, a very effective and simple design was being made by other groups which only involved six members and was with no creativity called the single triangle truss [Figure 2. Appendix I]. This light weight design used a small number of pins and members but relied primarily on reinforcing the compression members. These spanned the entire forty centimetre constraint of the design, and as such their required reinforcements would use more material over a larger dimension, adding to the bridge’s overall weight. Though the construction was much simpler, the design had a potential for early and unpredicted failure especially in twisting. This was later observed amongst a large portion of the other teams, some of which resolved this by bringing the lower members together at the pin. Although effective, the simplicity of this design can be its downfall. It lacks in creativity and, as an easy win, would reduce the challenge that a more complex bridge can offer.

After more iterations of other designs and brainstorming concepts to improve strength, the Simple Truss [Figure 3. Appendix I] structure was designed. This design found a balance between a light, easy to make bridge and a more complex, interesting design. It also made room for optimization and reinforcement. After running the calculations, the simple truss was found to have a very high theoretical PV of over two thousand. The calculations clearly favoured this design compared to previous models. The smaller number of members and pins would make its construction and assembling simpler, wherein less material would be used and the propagation of error would be reduced. While comparing design concepts a deciding factor was that this bridge would be more creative, challenging and reliable.

b. Concept evaluation and design selection

The development and iterations of designs showed the relevance of the criteria in place for the optimal truss design. Firstly ease of construction is essential as a more complex design that is poorly constructed could fail much earlier than expected and with the unreliable properties of the balsa wood could lead to early failure. The creativity of the design was mentioned in the outline but did not seem very relevant at first. It was later observed that this may be relevant as a majority of designs looked near identical. Although it may have relevant it did not seem appropriate to weigh this criteria high relative to the others. Optimization is crucial in improving a designs performance and can make all the difference during loading. With this in mind bridges with more room for reinforcements and simple joints would score higher in this regard. The most essential part of every design would be the load to weight ratio and as a result this criteria was the primary decision making factor.

Criteria	Complex Truss	Single Triangle Truss	Simple Truss
Ease of Construction [10%]	15	90	70
Creativity [10%]	75	10	60
Optimization Potential and Flexibility [30%]	20	100	90
Load to Weight Ratio (PV) [50%]	55	80	90
Score	42.5	80	85
Decision	N	N	Y

Table 4 – The Decision Matrix used when deciding on which bridge concept to pursue.

3. Design Analysis and Optimization

a. Force Analysis

The bridge was designed in order to optimize the amount of force it could support while adding the least amount of weight to the design. [1]

Preliminary force analysis was completed with Force Effect [3], a free software from Autodesk. This allowed the designers to play with geometry to attempt balancing forces in similar members before completing extensive analysis.

The final bridge was optimized to minimize the tension in all members after member tearing had been identified as the main cause of failure. The geometry was then optimized with simple trigonometry and an Excel worksheet to ensure that all the tension members were under equal force.

Figure 1 shows the distribution of forces throughout the structure and the final lengths of all the members. The alphabetical identification system shown will be used throughout analysis.

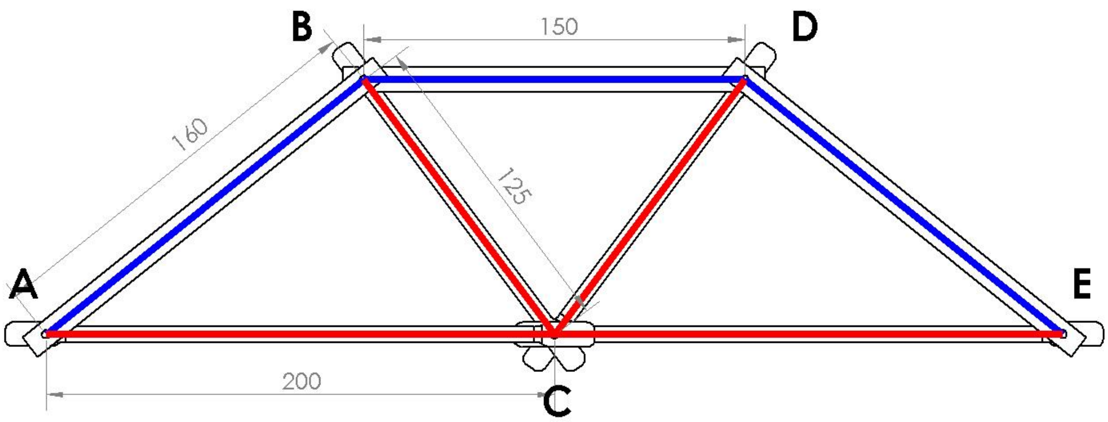

Figure 1 – The final bridge design. Members in blue are under compression; members in red are in tension.

Once the geometry had been confirmed hand calculations were done to confirm the results from Force Effect [3]. The following table shows the forces that were found in each member after this force analysis. Detailed calculations can be found in Appendix II.

For this 2D analysis a force of P, equal to one half of the total applied load (the load is split between the identical halves of the bridge), was applied at C and support reactions were supplied at A and E.

Due to the symmetrical nature of the bridge the support reactions on each end were assumed to be equal. Forces in mirrored members are also assumed equal.

Members	**Length (mm)**	**Force (N)**
AB, DE	160	0.800P [C]
BD	150	1.000P [C]
AC, CE	200	0.625P [T]
BC, CD	125	0.625P [T]

Table 5 – The forces experienced by members with regards to P.

With force analysis complete, each member was then analysed for multiple failure modes to see where the bridge would fail first.

Tension members were analysed for member rupture, member tearing and bearing stresses. The equations for each are given below. As a result of optimization all members in tension have the same forces in them. See Table 6 for the results of the tension member analysis.

$$Member\ Rupture,\ F_{max} = Normal\ Stress \cdot Min.\ Area\ = (\sigma_{max})_{member} \cdot (W_{member} - d_{hole}) \cdot t_{member}$$

$$Member\ Tearing,\ F_{max} = Shear\ Stress \cdot Area\ = (\gamma_{max})_{member} \cdot 2 \cdot b_{hole} \cdot t_{member}$$

$$Bearing\ Stress,\ F_{max} = Min\ Normal\ Stress \cdot Area\ = MIN(\sigma_{max})_{pin}^{member} \cdot d_{hole} \cdot t_{member}$$

Member	**Member Rupture**	**Member Tearing**	**Bearing Stress**	**Max Load**
AC, BC,CD,CE	206.375	203.2	201.6	645.16

Table 6 – Tension member failure analysis. All forces in N.

Compression members were analysed for member buckling and bearing stresses, see Table 7 for results.

$$Member\ Buckling,\ F_{max} = \frac{\pi^2 EI_{min}}{l_{effective}^2}$$

Member	**Buckling Force**	**Bearing Stress**	**Max Load**
AB, DE	1103.01	201.6	504.0
BD	1255.55	201.6	403.2

Table 7 – Compression members' failure analysis. All forces in N.

Pins were analysed for pin shearing. The loaded pin at Point C was also tested for pin bending. See Table 8 for the results of pin analysis.

$$Pin\ Shear,\ F_{max} = Shear\ Stress \cdot Area = (\gamma_{max})_{pin} \cdot \frac{\pi}{4} \cdot d_{pin}^{2}$$

$$Pin\ Bending,\ F_{max} = \frac{(\sigma_{max})_{pin} \cdot \pi \cdot c_{pin}^{3}}{l_{pin}}$$

Pin	Pin Shear	Pin Bending	Max Load
C	253.4	3605.6	506.8

Table 8 – Pin failure analysis. All forces in N.

This analysis shows the bridge should fail in bearing stress at over 400 N. It, however, failed much sooner than this due to material and construction fallacies.

b. Design Optimization

An Excel spreadsheet was created to quickly run the calculations. This allowed different dimensions to be quickly compared to help in the optimization process. The spreadsheet avoided extensive calculations and provided a user-friendly method of stress and failure analysis. This was essential further into the project where small adjustments were made and when problems arose.

Tension members were thickened at the ends by gluing on a measured piece of balsa wood to help prevent member tearing and bearing stresses. These members were also sanded down in the centre to remove excess material and for a smooth finish.

The I-beam compression members were drilled out along the centre to remove excess weight. This middle connecting section of the I-beam does not contribute much to the moment of inertia along either axis. Even when removing the middle part of the I-beam entirely it is still theoretically strong enough. Sections were left to connect the two sides so that the beam still functioned as a fixed support at the pins and thus halved the effective length for buckling and quadrupled the strength of the beam.
In initial testing the loaded pin proved to be the point of failure. To counter this members around it were moved closer together and the pin was thickened with balsa wood. Though this improved the strength of the bridge considerably it was still the point of failure.

For the final design the middle pin was over-designed to ensure it would not be the point of failure. Six additional dowels were glued around the loaded section raising the pin bending failure to a theoretical value over 3600 N.

Extra care was taken during assembly to ease pins into the holes and ensure that no stresses were being artificially induced. The final bridge was only assembled a single time to avoid wear on members. A single pin had to be removed and replaced for transportation of the bridge.

All told these optimizations reduced the weight from 20.6 grams to 19.9 grams. A larger improvement was expected but wood density was not controlled in the initial testing and thus is an unknown in the equation. The original wood may have been lighter or stronger. The main difference in strength from the provided values to the actual wood is most likely included in the differences in densities.

c. Final Design

Three dimensional models of the bridge were created prior to construction and later updated to match the final product as reinforcements and optimizations were made. This allowed construction methods and assembly order to be confirmed before the limited material was modified for a design.

Final dimensioned drawings of the bridge can be found in Appendix IV.

4. Construction

The construction of the bridge is a critical step for the competition as poor construction could lead to premature failure relative to the calculations.

The first main concern for the construction of the bridge was the positioning and centring of the holes. Small errors could cause misalignment with adjacent members. When measuring and cutting the pieces, two millimetres of tolerance were given because of the damage caused while using the box cutter. The members were then sanded down to the perfect length. This perfect length was measured with the pin to pin distance between holes, as it was a critical distance. Initial attempts at marking the exact centre of holes before drilling proved to be ineffective because of the drill type and the softness of the wood. In practice, lining up the drill by eye seemed much more accurate, especially while making corrections during the process. To further guide the holes, a pencil was used to make a small depression where the hole should be, this acted as a pilot hole to help guide the drill-bit.

Another concern when drilling was the splintering of the wood as the drill pushed through the final bit. The splintering was reduced through drilling slowly and through other more proactive means. One method in reducing splintering was the use of masking tape on the surface of the wood, this reduced splintering but was too time consuming for such a minor improvement. This method was shortly discarded.

The assembly order of the bridge's members is important when considering the distribution of load and the twisting experienced. After re-evaluating the design a final and improved assembly order was developed that removed are concerns of failure due to twisting.

The hardwood dowels also presented concerns through various iterations of the design. The dowels were not round as initially assumed and during the first assembly this caused the holes to expand and tear. The issue was resolved by using a larger drill-bit and eventually a handmade drill-bit made out of a recycled pin.

During the testing of the first bridge the dowels were identified as the main source of failure. As the bridge was loaded the pins began to bend and eventually fail. The first fix was increasing the diameter of the pin by gluing rings of balsa wood around it. The bridge withstood till the weight was large enough to crush the balsa wood and return to the original failure mode. The solution to the pin bending was to glue six sections of dowel around the loading point, increasing the diameter and maintaining its strength. Due to limited materials there was no opportunity to empirically test this reinforcement method before the competition. Assuming that they would support the bending load as did the balsa wood and not be crushed was sufficient.

Through calculations, certain failure points were identified to be much more critical than others such as pull-out and rupture. The issue of pull-out along with the related bearing stress was reinforced by increasing the thickness of the members at the joints as well as increasing their length beyond the joint. The thickness was increased by adding additional small segments of balsa wood. This method of failure would exist only on members under tension, therefore the reinforcement of these failure modes was concentrated on tensile members to minimize weight.

To reinforce against buckling, the main factor in calculations was the second moment of area. A few different beam designs were considered but the final decision was the I-beam. This beam not only provided us with large strength against buckling but added strength against twisting. The beam created two contact points that retained stiffness in that section. The forces responsible for twisting would then be distributed into a shearing load along the glue that held the I-beam together. The major drawback of the I-beam is its weight.

The final portion of the construction process was reducing the weight without reducing the maximum load. It was determined that the strength of the tensile members was more than sufficient as rupture would occur much later on. Therefore to reduce weight the tensile members were sanded down along the centre of their lengths. This sanding process contained a major flaw as certain members were unintentionally sanded too close to the ends creating a weak point in the design.

Another method of reducing weight, although marginally, was to sand the corners of the members as they do not contribute to the ultimate strength.

As mentioned earlier the I-beams were over designed and hand a lot of room for weight reduction. After some quick calculations it was determined that the central portion of the beam was not required but the stiffness of the beam was still dependent on it. Considering this, the weight of the I-beam was reduced significantly by drilling large holes along its length.

5. Conclusion

On the day of testing the final bridge failed due to plate rupture in a tension member at 12 kg of load, the collateral damage broke four more of the eight tension members (see Figure 2. Appendix IV). This was significantly lower than expected due to the large uncertainties in manufacturing, material, and assembly. In the end however, it came down to a member that had an unknowable fragility, it ruptured where it should not have if its tensile strength was as calculated. This highlights the risk and uncertainty of using wood as a material, something that is far from homogeneity. Only after breaking the final bridge did the realization of the power of uncertainty settle in. A lot of work was put into being meticulous and factoring in as much as possible. Thanks to Force Effect [3] and the calculation spreadsheet, the numbers were easy to derive. Valuable as these resources were, they had to be tied in with experimental testing and iterations of the design process. Even when these methods were reasonably exhausted, after crafting the bridge with care and letting the glue set, the final bridge was hampered by the available resources.

6. Recommendations

After completing the final iteration of the simple truss and witnessing the variety of other concepts used by other teams, concepts for further improvement were generated. The use of more than one pin at least at the loading point greatly increases the bridges capacity in regards to bending at the loading point. Also the simpler Single Triangle bridge and its great capacity for optimization could prove to be more advantages in regards to conserving material, allowing for more robust reinforcements and more design iterations. It is crucial that the materials are inspected upon distribution to ensure that the strongest material is reserved for the final design. Completing calculations of conceptualized designs beforehand proved very effective as it allowed more accurate predictions of failure modes and rejection of ineffective designs. The pursuit of simpler designs is not only advantageous in reducing material consumption but improving ease of construction.

7. References

[1] *MTE 219 Project. Project Outline* [Online]. Avalable:
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[4] Christopher Kohar, private communication, March, 2014.

[5] M. Vable, "Stress," in *Mechanics of Materials Online Book*, 2nd ed. Houghton, MI: Michigan Technological University, 2009, Ch. 1, pp. 6-7

Appendix I: Concept Rough Sketches

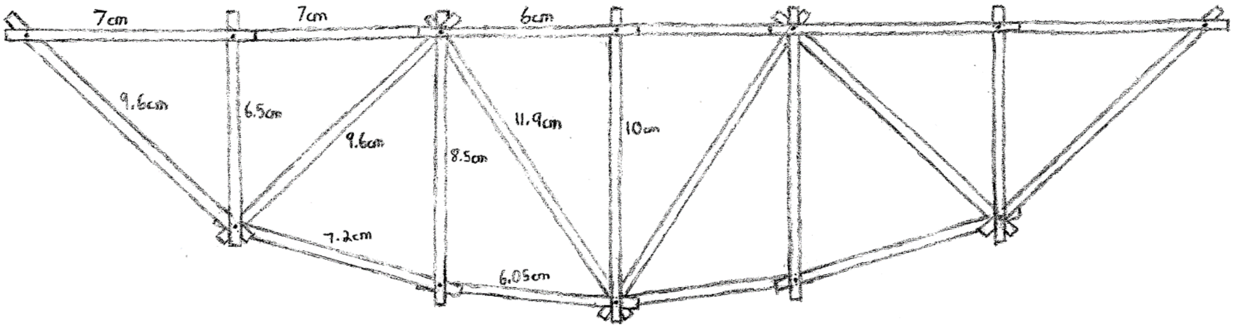

Figure 1 - Complex Truss

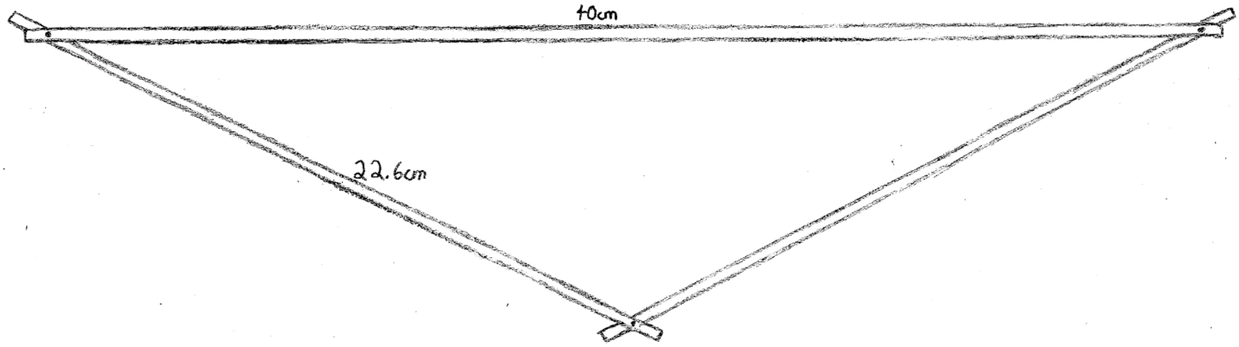

Figure 2 – Single Triangle

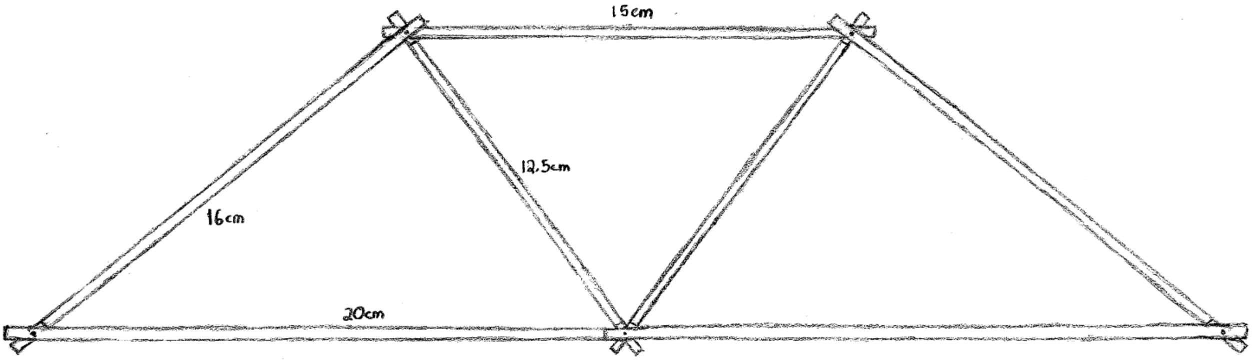

Figure 3 – Simple Truss

Appendix II: Force Calculations

The following encompasses calculations completed to find the forces in each member. These results are summarized in Design Analysis & Optimization, Table 4. All members are characterized as shown in Design Analysis & Optimization, Figure 1.

X is defined as horizontal. Y is defined as vertical. X/Y Components of any force (ZZ) are given as ZZx and ZZy.

Applied Force at C (P)
Reaction Force at A (RA) = Reaction Force at E (RE)

Force in Member AB (AB) = Force in Member DE (DE)
Force in Member AC (AC) = Force in Member CE (CE)
Force in Member BC (BC) = Force in Member CD (CD)
Force in Member BD (BD)

RA + RE = -P, RA = RE
RA = -0.5P
AB = RA/sin($\tan^{-1}$(100mm/(200mm-(150mm/2))))
AB = DE = -0.800P [C]

ABy = RA = -0.5P
AC = ABx = SQRT(AB^2 - ABy^2)
AC = CE = 0.625P [T]

BCy + CDy = P, BC = CD
BCy = 0.5P
BC = BCy/cos($\tan^{-1}$((150mm/2)/100mm))
BC = CD = 0.625P [T]

BCx = SQRT(BC^2 - BCy^2) = 0.375P
$(\sum F_x)_B$ = ABx + BCx + BD = 0
BD = -(ABx + BCx) = -(0.625P + 0.375P)
BD = -1.000P [C]

Due to there being two sides to the truss in the final 3D structure each of these forces are halved in reality meaning that P is actually representative of half of the applied load.

Appendix III: Member Failure Calculations

Tension Members (AC, BC, CD, CE)

- Thickness (t) at the holes is assumed to be 6.35mm even though some locations were thicker to aid in the assembly process and solidify the bridge.
- Three millimetres was removed from the width (W) in the middle of the tension members to bring Member Rupture to the same point as Tearing and Bearing Stresses.

Member Rupture:

Rupture Area $= MIN(W_{Middle} * t_{Middle}, (W_{Hole} - d_{Hole}) * t_{Hole})$
$= MIN(6.5mm * 3.175mm, (9.525mm - 3.175mm) * 6.35mm)$
$= MIN(20.6375mm^2, 40.3225mm^2)$
$= 20.6375mm^2$

$(F_{MAX})_{RUPTURE}$ $= (Normal\ Stress)_{MEMBER} * Rupture\ Area$
$= 10\ MPa * 20.6375mm^2$
$= 206.375\ N$

Member Tearing:

Tearing Area $= 2 * b * t$
$= 2 * 16mm * 3.175mm$
$= 101.6mm^2$

$(F_{MAX})_{TEARING}$ $= (Shear\ Stress)_{MEMBER} * Tearing\ Area$
$= 1\ MPa * 101.6mm^2$
$= 203.2\ N$

Bearing Stress:

Bearing Area $= d_{Hole} * t_{Hole}$
$= 3.175mm * 6.35mm$
$= 20.16125mm^2$

$(F_{MAX})_{BEARING}$ $= MIN(Normal\ Stress) * Bearing\ Area$
$= 10\ MPa * 20.16125mm^2$
$= 201.6\ N$

From calculations in Appendix II it is known that; AC = BC = CD = CE = 0.625P. And P = ½ Total Load.

Therefore the following should be true:

$(F_{MAX})_{AC} = (F_{MAX})_{BC} = (F_{MAX})_{CD} = (F_{MAX})_{CE}$ = MIN(Member Rupture, Member Tearing, Bearing Stress)

$(F_{MAX})_{TENSION\ MEMBERS} = 201.6\ N$

$(Total\ Load)_{TENSION\ MEMBERS}$ $= 2 * ((F_{MAX})_{TENSION\ MEMBERS} / 0.625)$
$= 645.16\ N$

Compression Members (AB, BD, DE)

- The average width of the drilled out middle section was calculated as a better representation of the true moment of inertia [4]

Buckling in AB & DE:

Average Width = (Original Area - Removed Area / Original Area) * Original Width
= ((9.525mm * 130mm) - (9 holes * Pi / 4 * (6.35mm)^2) / A_o * 9.525mm
= 7.33mm

(Moment of Inertia)$_{xx}$ = 2 * (1/12 * b_1 * h_1^3) + (1/12 * b_2 * h_2^3)
= 2 * 1/12 * 3.175mm * (9.525mm)^3 + 1/12 * 7.33mm * (3.175mm)^3
= 457.286mm^4 + 19.550mm^4
= 476.836mm^4

(Moment of Inertia)$_{YY}$ = 2 * ((1/12 * h_1 * b_1^3) + A_1 * d_1^2) + (1/12 * h_2 * b_2^3)
= 2 * (1/12 * 9.175mm * (3.175mm)^3 + 3.175mm * 9.525mm * ((9.525mm+3.175mm)/2)^2) + (1/12 * 3.175mm * (7.33mm)^3
= 2 * (25.40mm^4 + 1219.43mm^4) + 104.20mm^4
= 2593.86mm^4

(Effective Length)$_{FIXED\text{-}FIXED}$ = 0.5 * L_{AB}
= 0.5 * 160mm
= 80mm

$((F_{MAX})_{BUCKLING})_{AB}$ = Pi^2 * E_{MEMBER} * MIN(Moment of Inertia) / (Effective Length)^2
= (3.14159)^2 * 1.5 GPa * 476.836mm^4 / (80mm)^2
= 1103.01 N

Bearing Stress in AB & DE:

Bearing Area = d_{Hole} * t_{Hole}
= 3.175mm * 6.35mm
= 20.16125mm^2

$(F_{MAX})_{BEARING}$ = MIN(Normal Stress) * Bearing Area
= 10 MPa * 20.16125mm^2
= 201.6 N

Buckling in BD:

Average Width = (Original Area - Removed Area / Original Area) * Original Width
= ((9.525mm * 120mm) - (8 holes * Pi / 4 * (6.35mm)^2) / A_o * 9.525mm
= 7.41mm

(Moment of Inertia)$_{xx}$ = 2 * (1/12 * b_1 * h_1^3) + (1/12 * b_2 * h_2^3)
= 2 * 1/12 * 3.175mm * $(9.525mm)^3$ + 1/12 * 7.41mm * $(3.175mm)^3$
= 457.286mm^4 + 19.764mm^4
= 477.05mm^4

(Moment of Inertia)$_{YY}$ = 2 * ((1/12 * h_1 * b_1^3) + A_1 * d_1^2) + (1/12 * h_2 * b_2^3)
= 2 * (1/12 * 9.175mm * $(3.175mm)^3$ + 3.175mm * 9.525mm * $((9.525mm+3.175mm)/2)^2$) + (1/12 * 3.175mm * $(7.41mm)^3$
= 2 * (25.40mm^4 + 1219.43mm^4) + 107.65mm^4
= 2597.31mm^4

(Effective Length)$_{FIXED-FIXED}$ = 0.5 * L_{BD}
= 0.5 * 150mm
= 75mm

((F$_{MAX}$)$_{BUCKLING}$)$_{BD}$ = Pi^2 * E_{MEMBER} * MIN(Moment of Inertia) / (Effective Length)^2
= $(3.14159)^2$ * 1.5 GPa * 477.05mm^4 / $(75mm)^2$
= 1255.55 N

Bearing Stress in BD:

Bearing Area = d_{Hole} * t_{Hole}
= 3.175mm * 6.35mm
= 20.16125mm^2

(F$_{MAX}$)$_{BEARING}$ = MIN(Normal Stress) * Bearing Area
= 10 MPa * 20.16125mm^2
= 201.6 N

From Appendix II it is known that AB = DE = 0.800P and BD = 1.000P. Therefore since both members fail at the same force in Bearing Stress member BD will be focused since it sees more force (1.000P vs 0.800P) then AB or DE.

$(F_{MAX})_{COMPRESSION}$ = $(F_{MAX})_{BEARING}$
= 201.6 N

Again, the Total Load is twice P therefore,

(Total Load)$_{COMPRESSION\ MEMBERS}$ = 2 * ($(F_{MAX})_{COMPRESSION}$ / 1.000)
= 403.2 N

Pins (A, B, C, D, E)

- Assuming worst case scenario of single shear.
- Maximum shear at loaded pin between reinforcements and tension pins.

Pin Shear:

Area = Pi / 4 * d^2
= Pi / 4 * $(3.175\text{mm})^2$
= 7.917mm^2

$(F_{MAX})_{PIN\ SHEAR}$ = $(\text{Shear Stress})_{PINS}$ * Area
= 32 MPa * 7.917mm^2
= 253.4 N

Pin Bending:

Moment of Inertia = 7 * (Pi/64 * d^4) + 4 * (Area * distance^2)
= 34.917mm^4 + 239.434mm^4
= 274.351mm^4

Furthest Point = 3.175mm / 2 + 3.175mm * sin60°
= 4.3371mm

Length = 12mm

$(F_{MAX})_{PIN\ BENDING}$ = $(\text{Normal Stress})_{PIN}$ * 4 * Moment of Inertia / (Length * Furthest Point)
= 171 MPa * 4 *
= 3605.637 N

Each of the compression members is an I-beam assembled with other members between the two sides of the I and thus is in Double Shear. The outer tension members at Pin C and inner tension members at Pin B are under Single Shear. However, all of the shears from the members attached to Pin C are additive and greater than the shear at the individual pins themselves. [5] A shear force equal to P is seen between the reinforcement at the middle of Pin C and the tension member.
Therefore pin failure should occur when,

$(\text{Total Load})_{PIN}$ = 2 * ($(F_{MAX})_{PIN\ SHEAR}$ / 1.000)
= 506.8 N

Appendix IV: Final Bridge

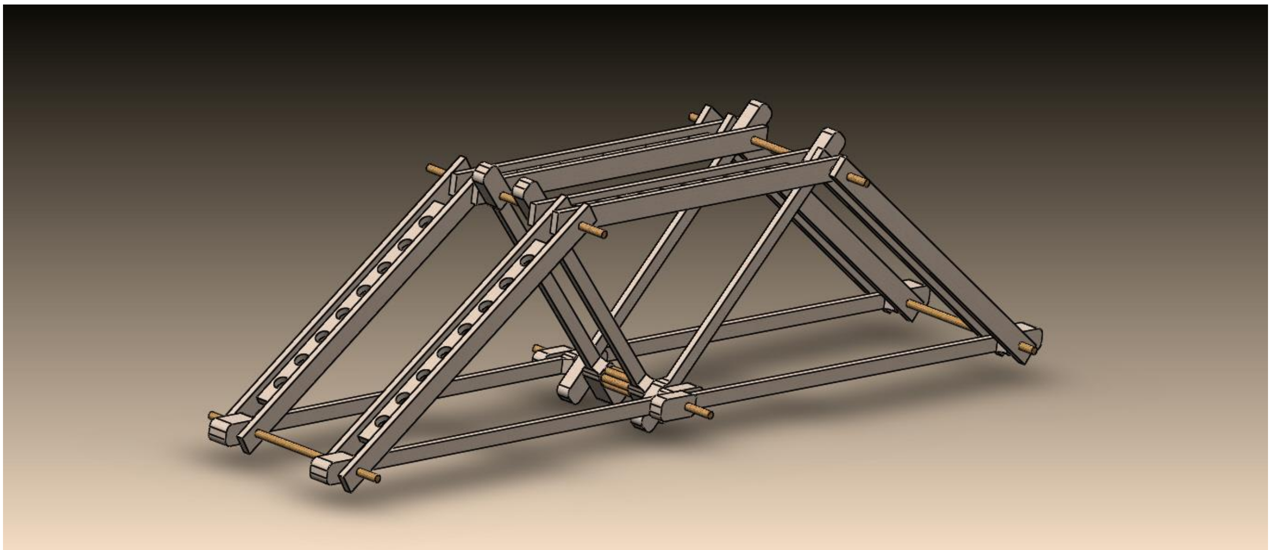

Figure 1 – The completed bridge design.

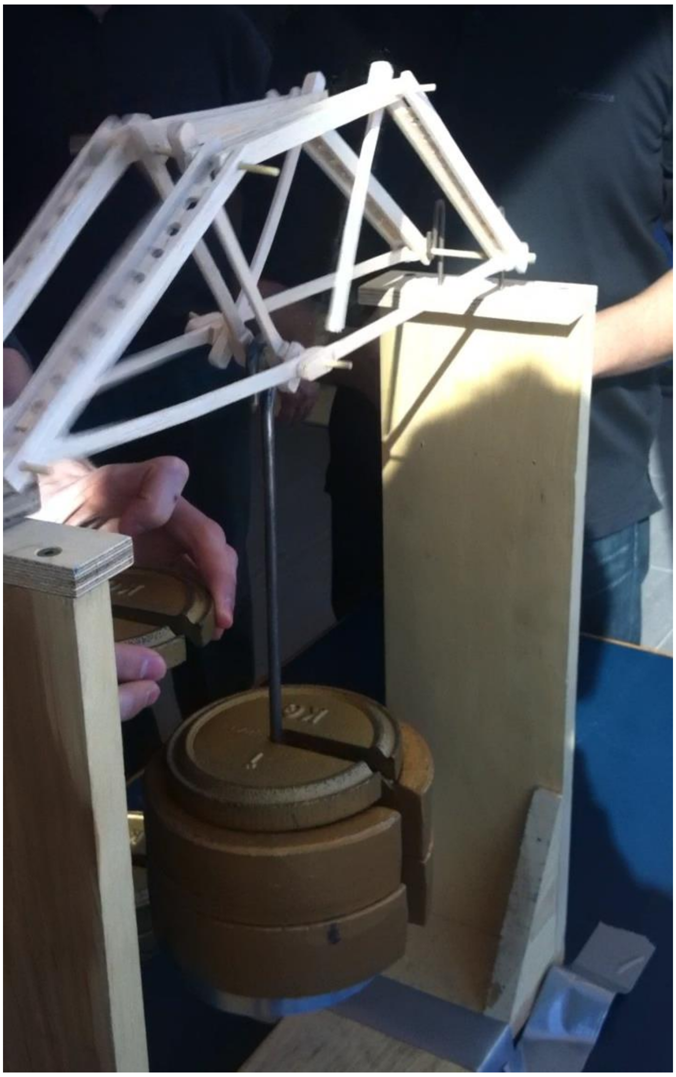

Figure 2 – The bridge at failure. Notice the tension member ruptured at Pin C.