

B505 Applied Algorithms
HW 2

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1.

2.

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given A[1...n] containing all integers from 0 to n except one number:
  increase size of A by 1
  A[n+1] ← -1
  m ← n + 1
  for i ← 0 to n:
    while (A[i + 1] != i) and (A[i + 1] != -1):
      swap A[A[i+1] + 1], A[i+1]
    if A[i + 1] == -1:
      m ← i

  return m
```

3.

a)

(1, 5), (2, 5), (3, 4), (3, 5), (4, 5)

b)

The array with the elements in reverse order: {n, ..., 2, 1}.

There are $(n * (n - 1)) / 2$ inversions.

c)

Every inversion will require a swap and comparison operation.

The running time will be directly proportional to the number of inversions.

4.

given sorted lists A[1...n] and B[1...m]:

i ← min(n, k)

j ← min(m, k)

while i + j > k:

if A[i] < B[j]:

j ← j - 1

else if B[j] < A[i]:

i ← i - 1

if A[i] >= B[j]:

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return A[i]
else:
return B[j]

```

5.

(a) $T(n) = T(n/2) + n$

$a = 1, n = n, b = 2, d = 1$

$d \stackrel{?}{=} \log_b(a)$

$1 \stackrel{?}{=} \log_2(1)$

$1 > 0$

$T(n) = O(n)$

(b) $T(n) = T(n/5) + n^2$

$a = 1, n = n, b = 5, d = 2$

$d \stackrel{?}{=} \log_b(a)$

$2 \stackrel{?}{=} \log_5(1)$

$2 > 0$

$T(n) = O(n^2)$

(c) $T(n) = T(n/3) + \text{constant}$

$a = 1, n = n, b = 3, d = 0$

$d \stackrel{?}{=} \log_b(a)$

$0 \stackrel{?}{=} \log_3(1)$

$0 = 0$

$T(n) = O(\log(n))$

6.

In case a. an insertion sort will likely be better because it will require only moving the few inverted datapoints.

Using a merge sort in case a. will require still merging the entire array even though it is mostly already sorted.

In case b. a merge sort will be better because there will likely be many inversions.