MAST7866
Axioms of Probability

Jian Zhang

University of Kent

The Axioms of Probability

The axioms of probability, from which the theory of probability is developed:

1. For any event A, $0 \leq \Pr(A) \leq 1$.

2. For the event S, $\Pr(S) = 1$.

3. For any two events A and B that cannot occur at the same time

$$\Pr(A \cup B) = \Pr(A) + \Pr(B).$$

Interpretation of Probabilities

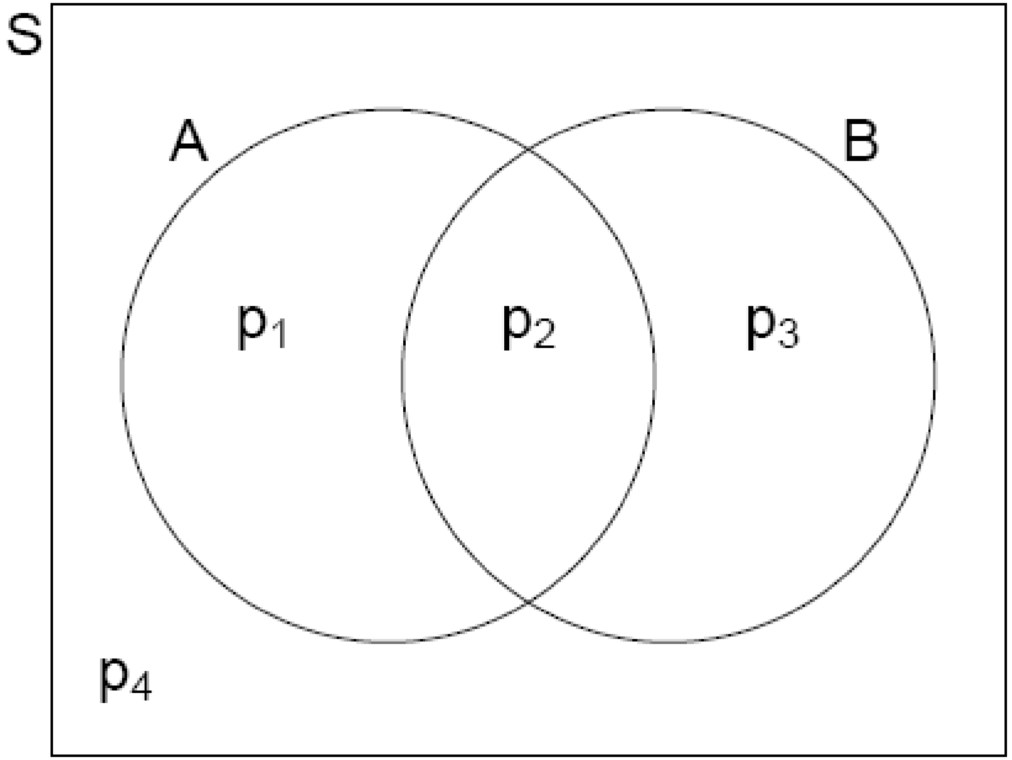
The closer a probability is to 1, the more likely the event is to occur.

For an event A, Pr(A) = 1 means that A is certain to occur.

For an event B, Pr(B) = 0 means that B has no chance of occurring.

Deductions from axioms

There are many results that can be deduced from the axioms.

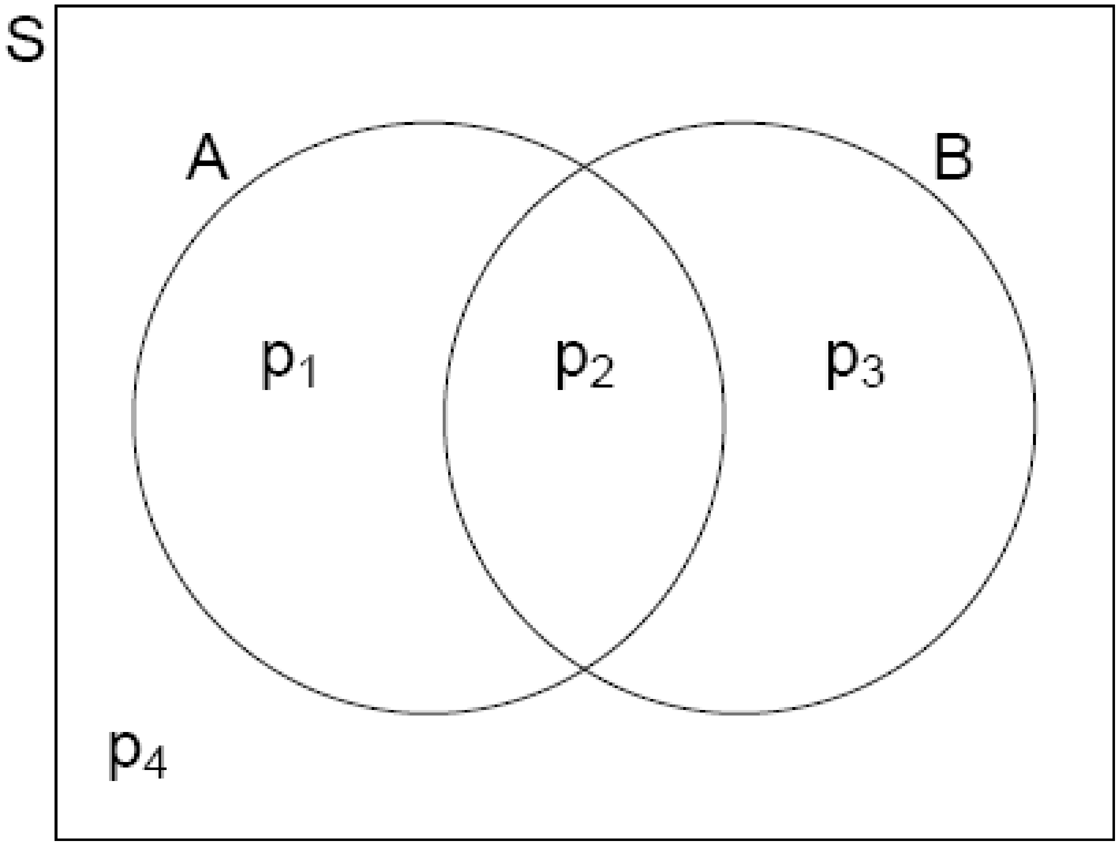


Let
$$p_1 = \Pr(A \cap \bar{B})$$
$$p_2 = \Pr(A \cap B)$$
$$p_3 = \Pr(\bar{A} \cap B)$$
$$p_4 = \Pr(\overline{A \cup B})$$

Note
$$\Pr(S) = 1$$
$$p_1 + p_2 + p_3 + p_4 = 1$$

Complements

- $\Pr(\bar{A}) = 1 - \Pr(A)$

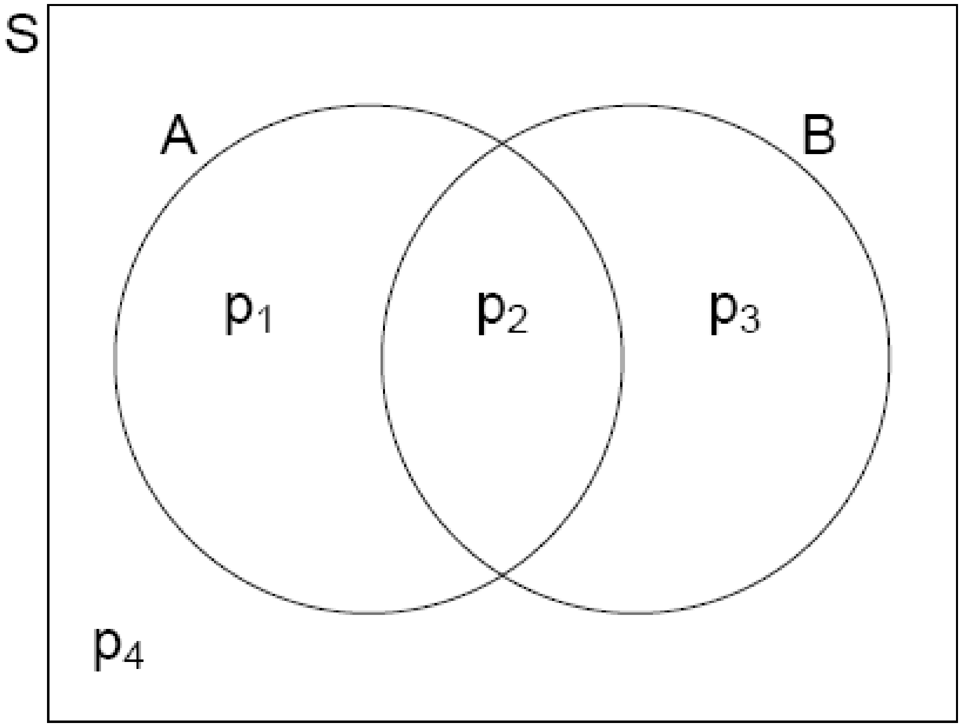


$\Pr(A) = p_1 + p_2$

$\Pr(\bar{A}) = p_3 + p_4$

As $p_1 + p_2 + p_3 + p_4 = 1$ then $p_3 + p_4 = 1 - p_1 - p_2$

$$\begin{aligned}\Pr(\bar{A}) &= p_3 + p_4 = 1 - p_1 - p_2 \\ &= 1 - (p_1 + p_2) \\ &= 1 - \Pr(A)\end{aligned}$$



Probability for Union

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

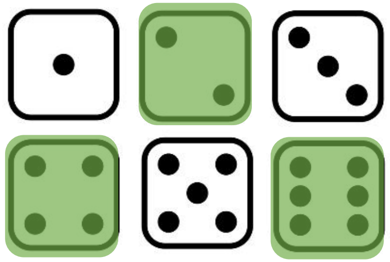
$$\begin{aligned}
\Pr(A) &= p_1 + p_2 \\
\Pr(B) &= p_2 + p_3 \\
\Pr(A \cap B) &= p_2 \\
\Pr(A) + \Pr(B) - \Pr(A \cap B) &= p_1 + p_2 + p_2 + p_3 - p_2 \\
&= p_1 + p_2 + p_3 \\
&= \Pr(A \cup B)
\end{aligned}$$

Calculating Probabilities

- For an experiment with a finite number of equally likely outcomes, the probability that event A occurs is worked out using

$$\Pr(A) = \frac{\text{number of outcomes favourable to A}}{\text{total number of outcomes}}$$

- What is the probability of rolling an **even number** on a standard die?



Conditional Probability

- We use the following notation to denote conditional probability.

- Pr(A|B) denotes the conditional probability that *event A occurs given that event B occurs*.

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Independent Events

- Two events, A and B, are independent if the occurrence of one event does not affect the probability of the other occurring.

- For independent events A and B,

$$\Pr(A \mid B) = \Pr(A)$$

$$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$$