MAST7866
Continuous Random variables and probability distributions

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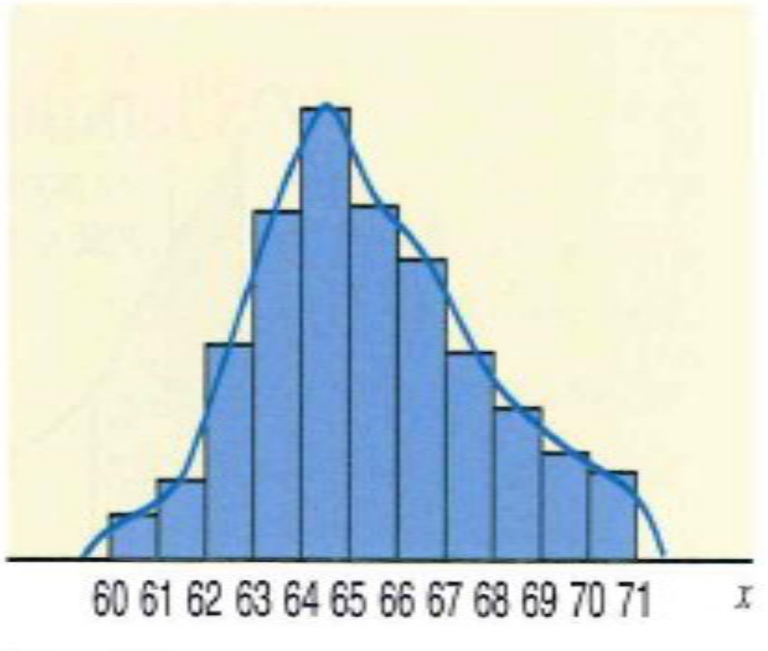
Continuous probability distribution

- A continuous random variable is a random variable whose values are not countable.
- Suppose 5000 female students are enrolled at a university, and x is the continuous random variable that represents the height of a randomly selected female student.

- The table below lists the frequency and relative frequency distributions of x.

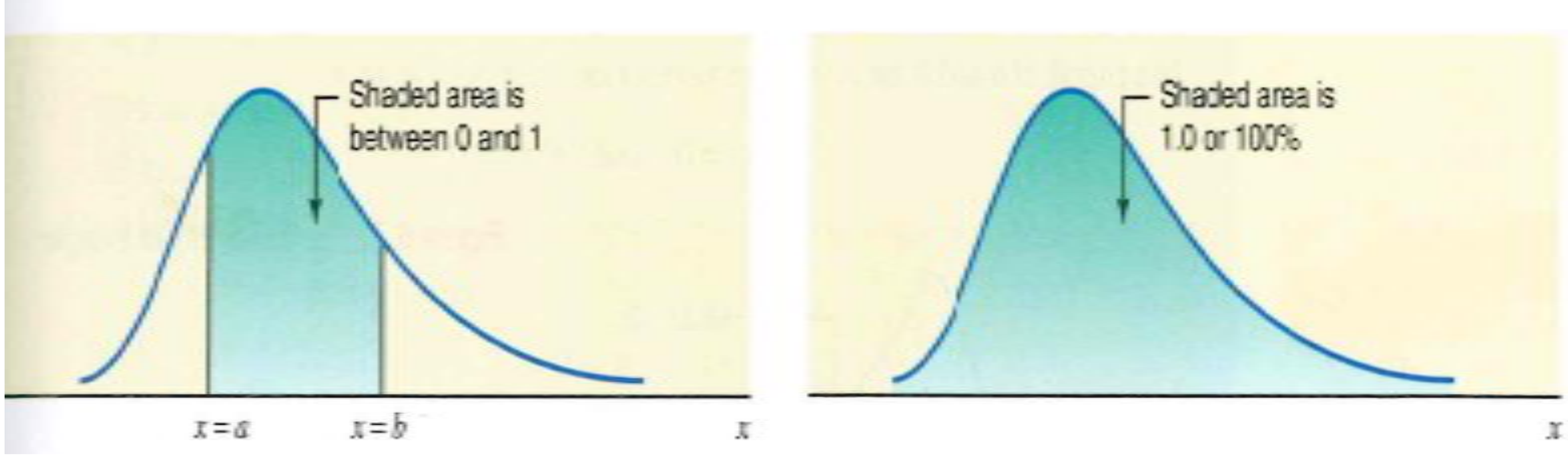
Height of a female student (inches), x	f	Relative frequency
60 to less than 61	90	0.018
61 to less than 62	170	0.034
62 to less than 63	460	0.092
63 to less than 64	750	0.150
64 to less than 65	970	0.194
65 to less than 66	760	0.152
66 to less than 67	640	0.128
67 to less than 68	440	0.088
68 to less than 69	320	0.064
69 to less than 70	220	0.044
70 to less than 71	180	0.036
	N = 5000	Sum = 1.000

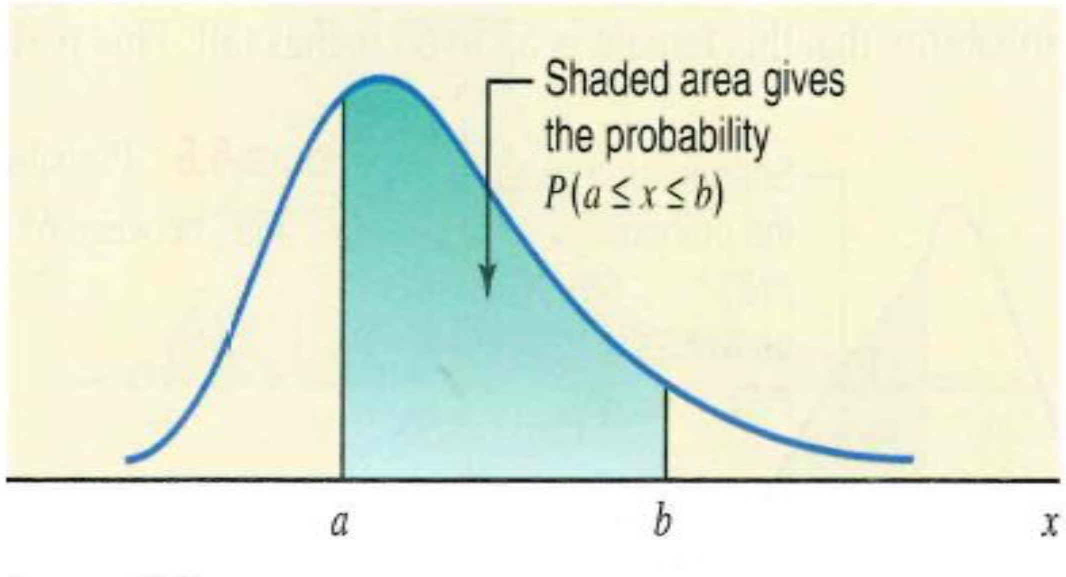
- The plot below show a histogram of the relative frequencies of these data.



- The relative frequencies can be used as the probabilities of the respective classes. The probability distribution curve of a continuous random variable is also called its **probability density function.**

- The probability distribution of a continuous random variable possesses the following two characteristics:
  - The probability that x assumes a value in any interval lies in the range 0 to 1.
  - The total probability of all the (mutually exclusive) intervals within which x can assume a value is 1.





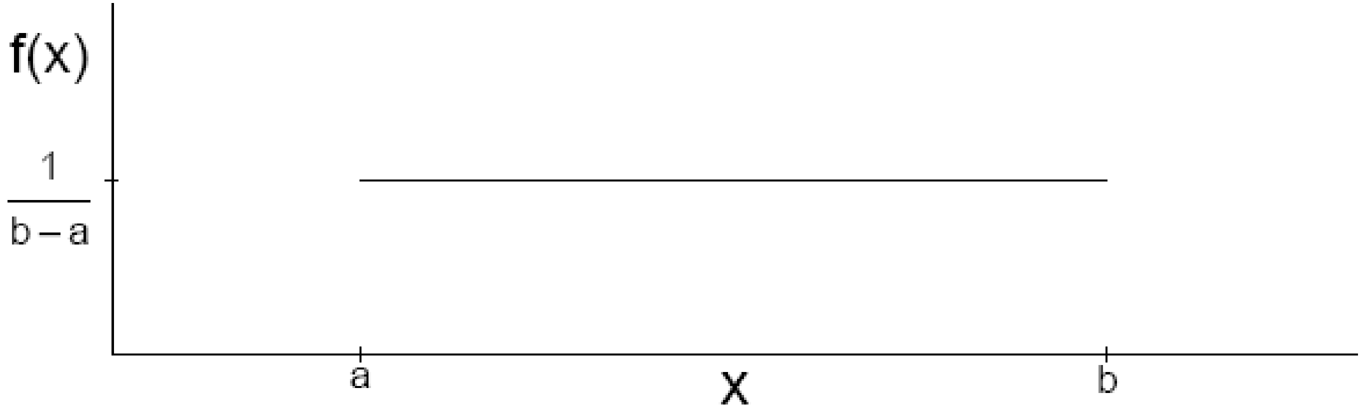
- The probability that a continuous random variable x assumes a value within a certain interval is given by the area under the curve between the two limits of the interval.
- For a continuous probability distribution, the probability is always calculated for an interval.
- The probability that a continuous random variable x assumes a single value is always zero.

The uniform distribution

- The uniform distribution describes a continuous random variable X which is equally likely to take any value in its range.

- If X is uniformly distributed over the range (a,b), then we write $X \sim U(a,b)$.

The pdf of the uniform distribution



$$f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise,} \end{cases}$$

Mean and variance of Uniform

- The mean and variance of a uniformly distributed random variable X~U(a,b) are given by

$$E(X) = \frac{(a+b)}{2}$$

$$Var(X) = \frac{1}{12}(b-a)^2$$

The Normal Distribution

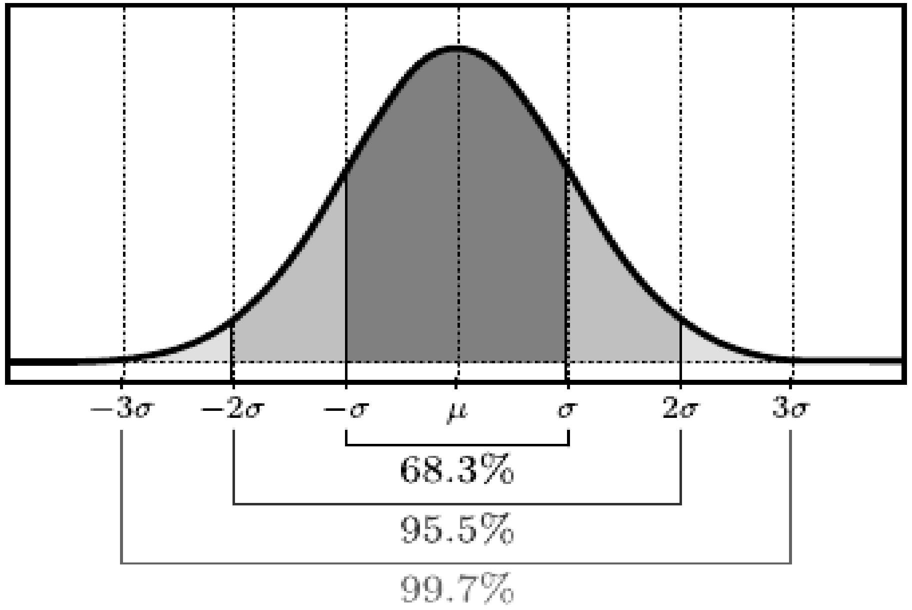
- The normal distribution is one of the many probability distributions that a continuous random variable can possess.
- The normal distribution is the most important and most widely used of all probability distributions as a large number of phenomena in the real world are approximately normally distributed.
- A normal probability distribution is a bell-shaped curve and its mean is denoted by μ and its standard deviation by σ.

The Normal Distribution (continued)

- Note that not all bell-shaped curves represent a normal distribution, only a specific kind of bell-shaped curve represents a normal curve.

- The bell-shaped curve is such that:
  - The total area under the curve is 1.
  - The curve is symmetric about the mean.
  - The two tails of the curve extend indefinitely.

- Although the normal curve never meets the horizontal axis, beyond the points represented by $\mu - 3\sigma$ and $\mu + 3\sigma$ it becomes so close to the axis that the area under the curve beyond those points in both directions is very small. The actual area in each tail is 0.0013.



- The mean, μ and standard deviation, σ are the parameters of the normal distribution. The value of μ determines the centre of the curve and the σ gives normal curves with different height/spread.