Vector Exercises (Pre-Reading for Data Science MSc)

1. Define a vector in $\mathbb{R}^n$ and give an example in $\mathbb{R}^3$.

$$\mathbb{R}^n \rightarrow \mathbf{v} = (v_1, v_2, v_3 \ldots, v_n) \quad v_i \in \mathbb{R}$$

$$\mathbb{R}^3 \rightarrow \mathbf{v} = (2, -1, 3)$$

2. Compute

$$\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 1 \end{bmatrix}$$

3. Compute

$$3 \times \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 12 \end{bmatrix}$$

4. Form a linear combination:

$$2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 3 \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

5. For $v = (3, 4)$, compute its magnitude $||v||$.

$$||v|| = 5$$

6. Compute the unit vector of

$$\mathbf{v} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}.$$

$$\|v\| = \sqrt{6^2 + 8^2} = 10$$

$$\frac{1}{10}\begin{bmatrix} 6 \\ 8 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$$

7. Compute the dot product of

$$\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ -5 \\ 6 \end{bmatrix}.$$

$$ab = 1(4) + 2(-5) + 3(6)$$
$$= 12$$

8. For $\mathbf{a} = (1, 0, 1)$ and $\mathbf{b} = (0, 1, 1)$, compute the angle between them.

$$\cos\theta = \frac{ab}{\|a\|\,\|b\|}$$

$$ab = 1(0) + 0(1) + 1(1) = 1$$

$$\|a\| = \sqrt{2}$$
$$\|b\| = \sqrt{2}$$

$$\cos\theta = \frac{1}{2}$$
$$\theta = 60°$$

9. Compute the cosine similarity between

$$\mathbf{a} = (2, 0, 1), \quad \mathbf{b} = (1, 1, 0).$$

$$\cos\theta = \frac{ab}{\|a\|\|b\|}$$

$ab = 2$

$\|a\| = \sqrt{5}$

$\|b\| = \sqrt{2}$

$\therefore \cos\theta = \frac{2}{\sqrt{10}} = 0.63$

10. Suppose a user's genre vector is $u = (3, 2, 0)$ and a film's genre vector is $f = (1, 1, 1)$. Compute their cosine similarity.

$$\cos\theta = \frac{uf}{\|u\|\|f\|}$$

$uf = 5$

$\|u\| = \sqrt{13}$

$\|f\| = \sqrt{3}$

$\cos\theta = \frac{5}{\sqrt{39}} = 0.801$