Introductory Probability

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Probability

- Statements involving probability are not hard to find. A weather forecaster may give a 60% chance of rain tomorrow, or a doctor may report that 95% of people taking a particular medicine are relieved of their symptoms within three days.
- Probability is a numerical measure between 0 and 1 which measures how likely an event is to occur.
- Consider the rain example above. According to the weather forecaster, the probability that it rains tomorrow is 0.6.

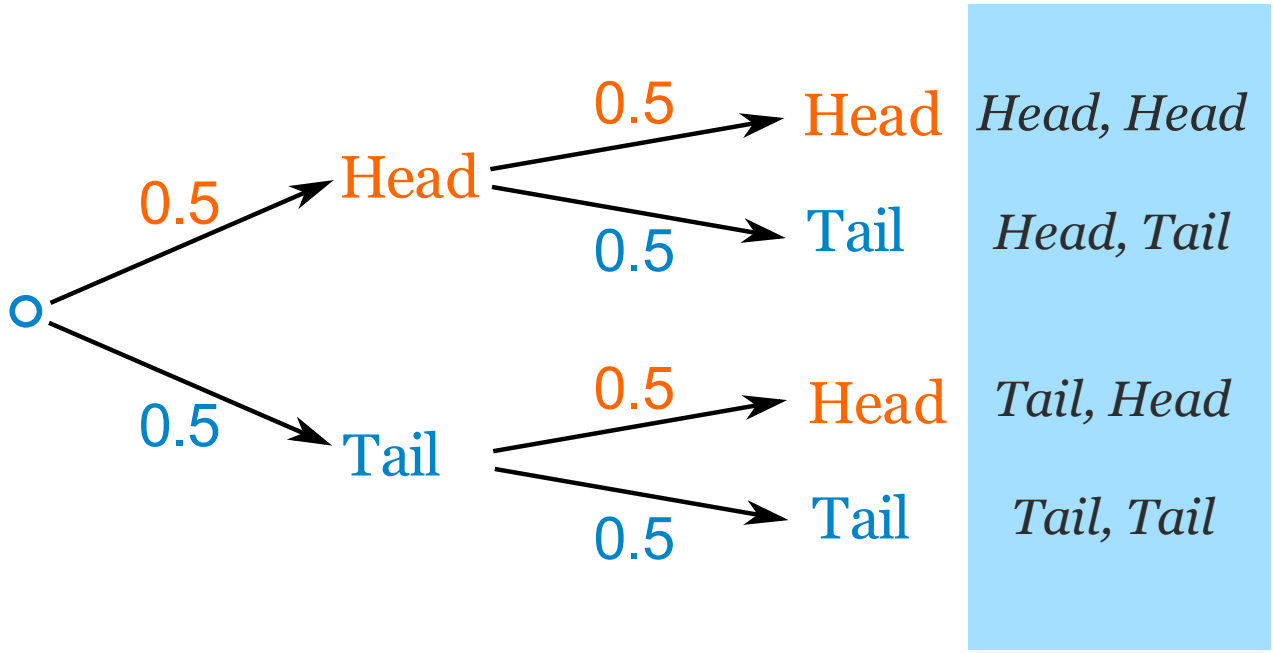
Examples of Experiments

Experiment	Possible Outcomes	Sample Space
Roll a die and record the number	1,2,3,4,5,6	S = {1,2,3,4,5,6}
Pick a card and record its suit	Heart, diamond, spade and club	S = {heart, diamond, spade, club}
Toss a coin twice and record what appears	HH, HT, TH, TT	S = {HH, HT, TH, TT}
Count the number of die rolls preceding a 6	0,1,2,3,4,5,6,7,8,9,10,11...	$S = \mathbb{N}$ (all positive integers)

For experiment 1 in the table, we could define an event A to occur when an even number appears on the die, i.e. A = {2,4,6}. Other events can similarly be defined. For example, B = {2,3,4} or C = {1,2,6}.

Tree Diagrams

We can use *tree diagrams* to represent the outcomes of an experiment and help us to work out the sample space.

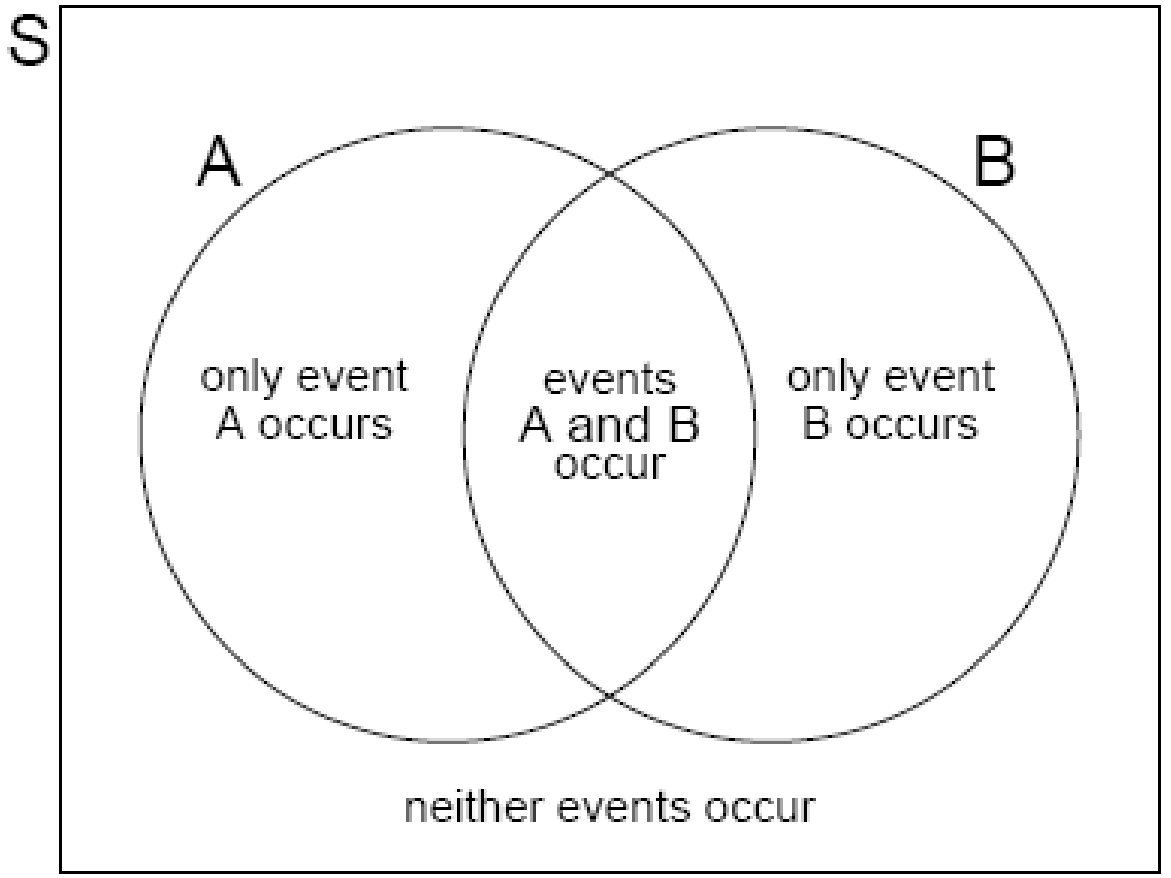


Set Theory and Venn Diagrams

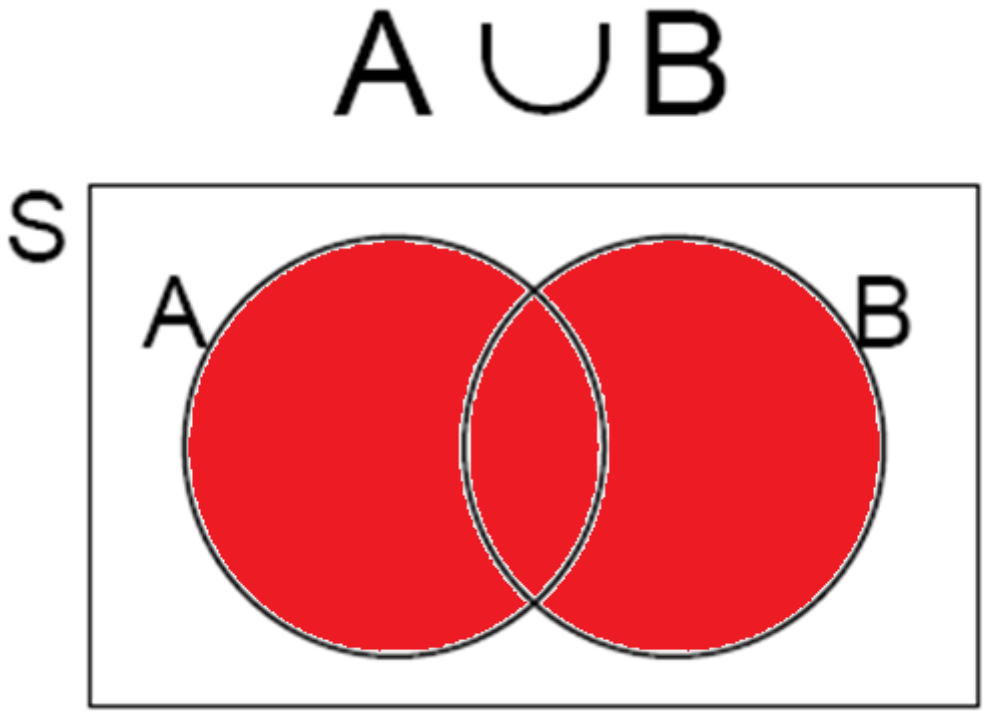
Set theory is used to relate events to each other and Venn diagrams are used to illustrate these relationships.

Consider two events A and B that may occur from an experiment.

The Venn diagram shows all possible outcomes of the experiment (i.e. the sample space).



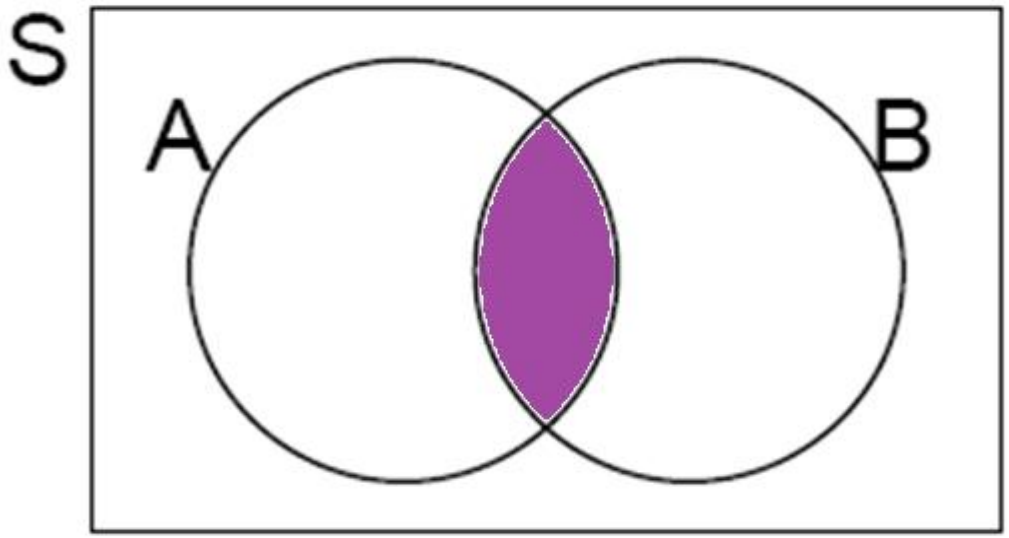
A union B



A or B or both occur

A intersection B

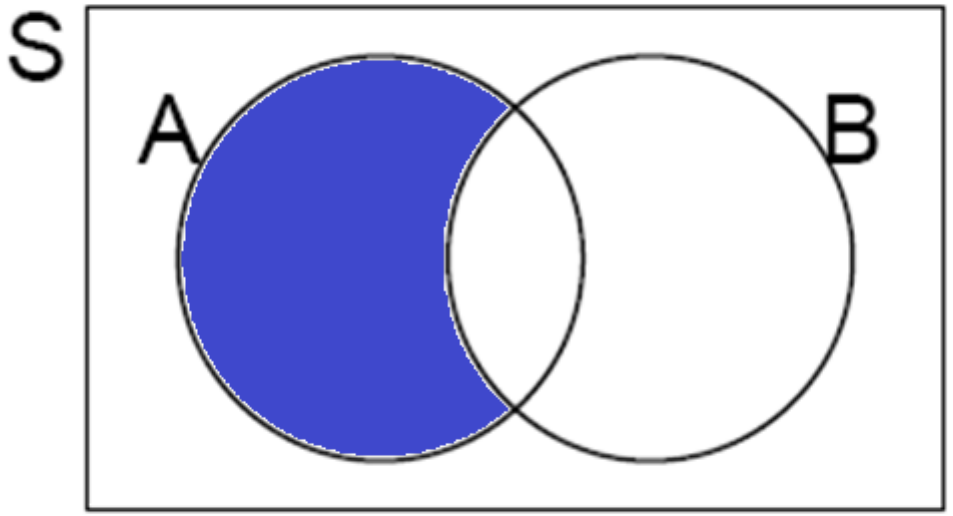
$A \cap B$



A and B both occur

Only A

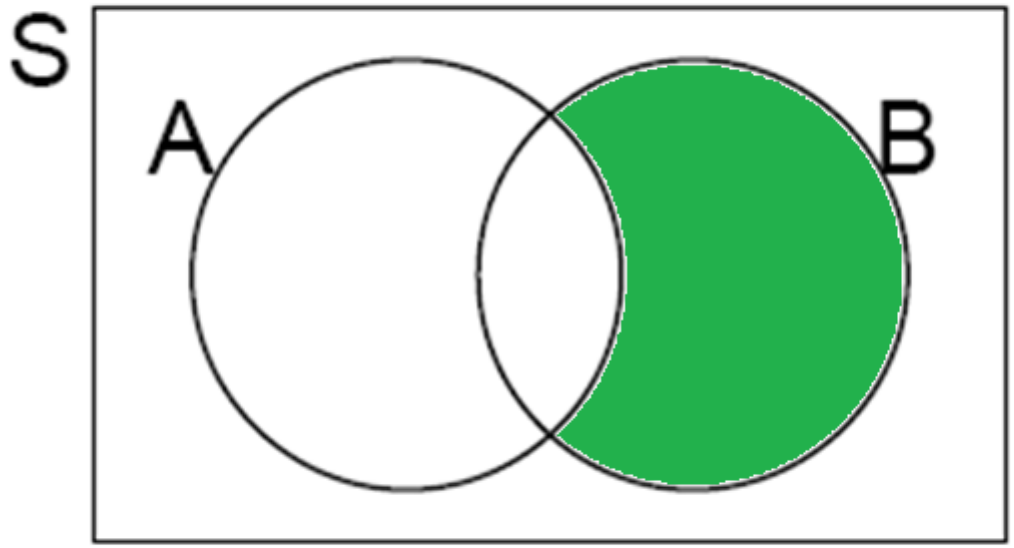
$A \cap \overline{B}$



A and not B
A, but not B

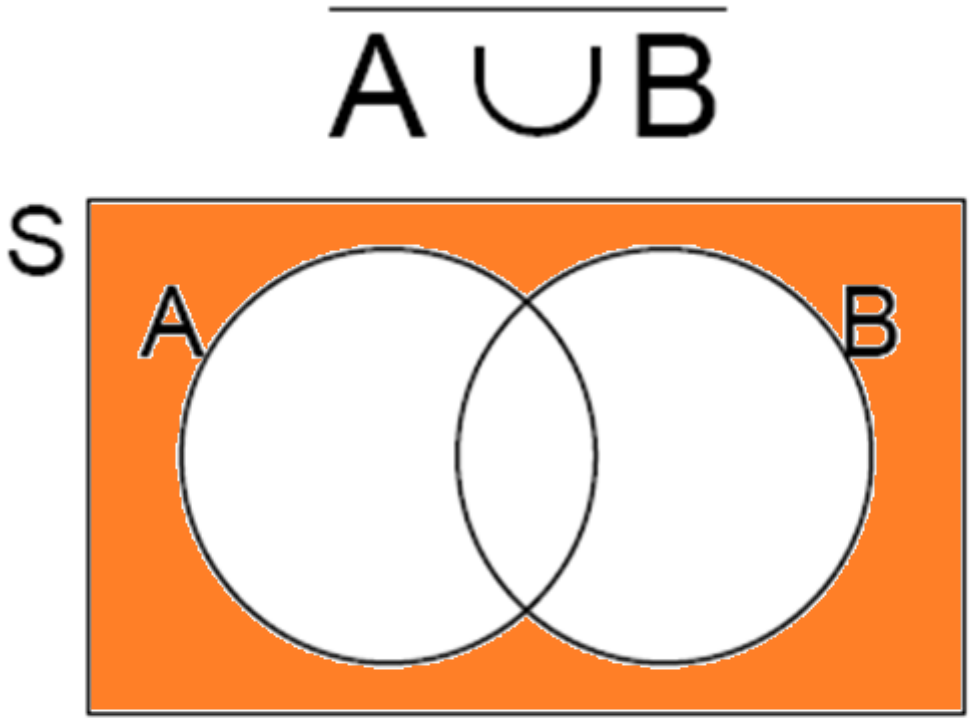
Only B

$\bar{A} \cap B$



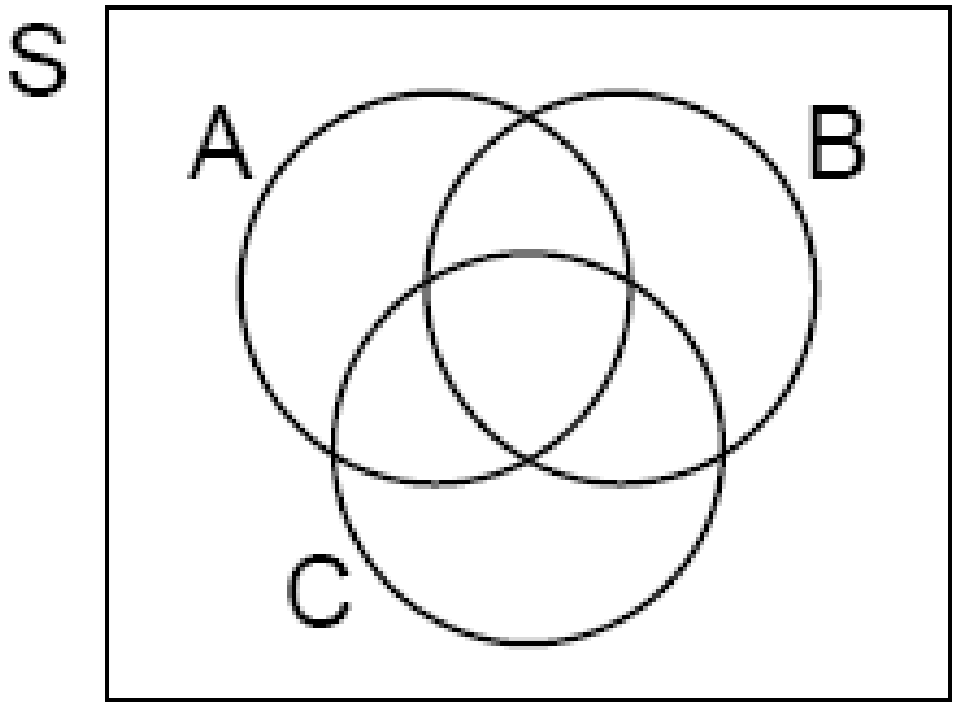
Not A and B
Not A, but B

Neither



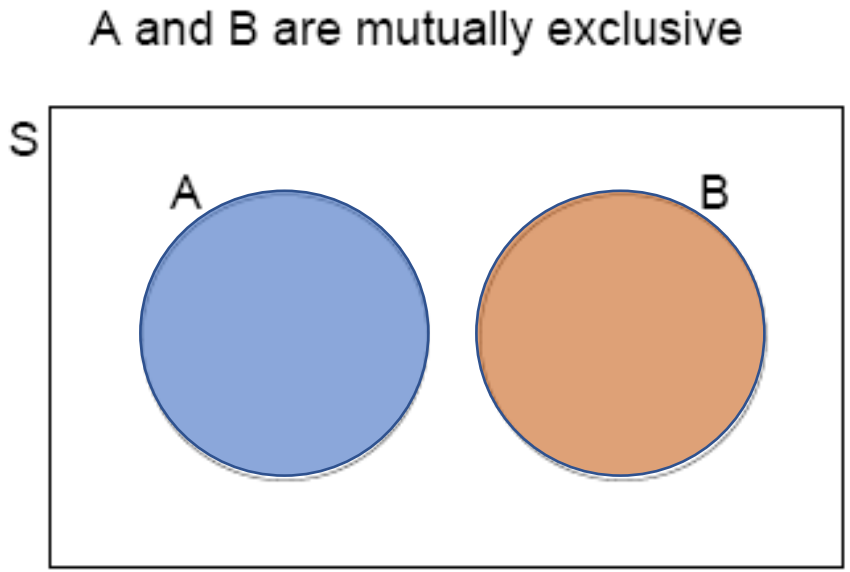
Not A or B

Extension to more states!



Mutually exclusive events

Events A and B are mutually exclusive if they cannot occur at the same time.



Mutually exhaustive events

Events A and B are mutually exhaustive if at least one is certain to occur.