

GAUSSIAN PROCESS CONVOLUTION MODEL

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21 October 2018

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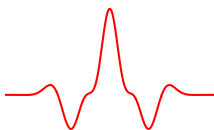
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$$k \mid h = h * Rh,$$

$$f \mid k \sim \mathcal{GP}(0, k):$$



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$$x \sim \mathcal{GP}(0, \delta) \implies "hx" \sim \mathcal{GP}(0, "hh^T")$$

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- Joint distribution:

$$p(f, h, \underset{\uparrow}{u}, x, \underset{\uparrow}{z}) = p(f \mid h, x) p(h \mid \underset{\uparrow}{u}) p(\underset{\uparrow}{u}) p(x \mid \underset{\uparrow}{z}) p(\underset{\uparrow}{z}).$$

inducing points for h and x resp.

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- Approximate posterior:

$$q(f, h, \underset{\uparrow}{u}, x, \underset{\uparrow}{z}) = p(f | h, x) p(h | \underset{\uparrow}{u}) q(\underset{\uparrow}{u}) p(x | \underset{\uparrow}{z}) q(\underset{\uparrow}{z}).$$

Extension: Improved Inference

- Mean-field approximate posterior:

$$q(f, h, \mathbf{u}, x, \mathbf{z}) = p(f \mid h, x)p(h \mid \mathbf{u})q(\mathbf{u})p(x \mid \mathbf{z})q(\mathbf{z}).$$

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$$q(f, h, \mathbf{u}, x, \mathbf{z}) = p(f \mid h, x)p(h \mid \mathbf{u})p(x \mid \mathbf{z})q(\mathbf{u}, \mathbf{z}).$$

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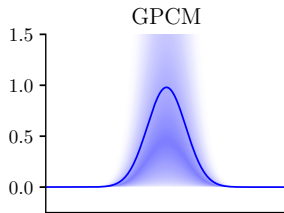
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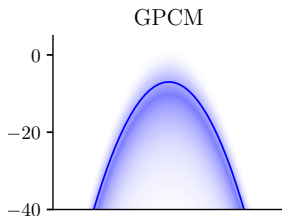
- MCMC to sample from q^* .

Extension: Causality

Prior over kernel:

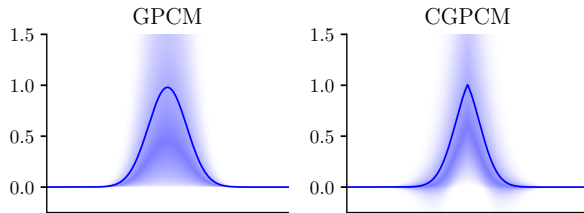


Prior over PSD:

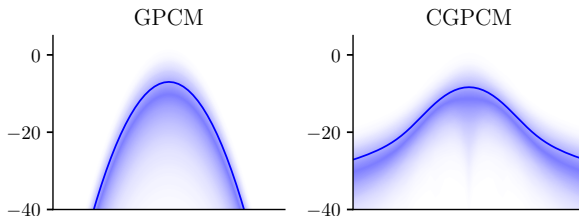


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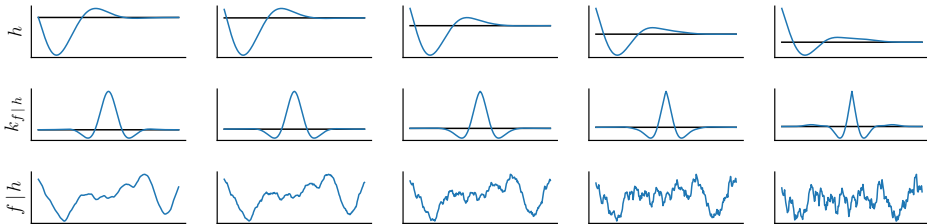
Prior over kernel:



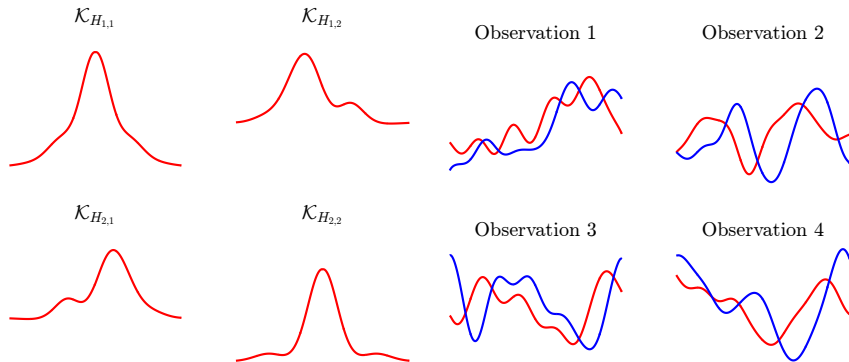
Prior over PSD:



Extension: Causality



Extension: Multiple Outputs



But what about the *kernel of the kernel*?

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And the *kernel of the kernel of the kernel*?

Extension: Deep Kernel Model

Model (N -Deep Kernel Model)

$$h_0 \sim \mathcal{GP}(0, k_h),$$

$$h_1 | h_0 \sim \mathcal{GP}(0, h_0 * Rh_0),$$

$$\vdots$$

$$h_N | h_{N-1} \sim \mathcal{GP}(0, h_{N-1} * Rh_{N-1}),$$

$$f | h_N = h_N.$$

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