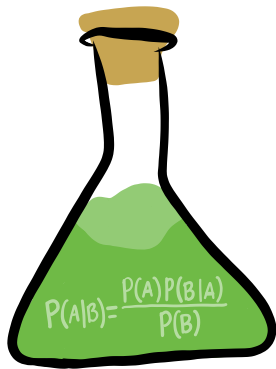


**Wessel Bruinsma**

2 November 2018

# A BAYESIAN TRUTH SERUM



**Wessel Bruinsma**

2 November 2018

Prelec, D. (2004). A Bayesian Truth Serum for Subjective Data.  
*Science*, 306(5695), 462–466.

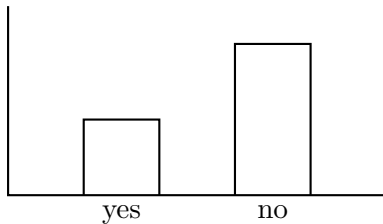
“Do you like this painting?”



“Do you like this painting?”



common prediction

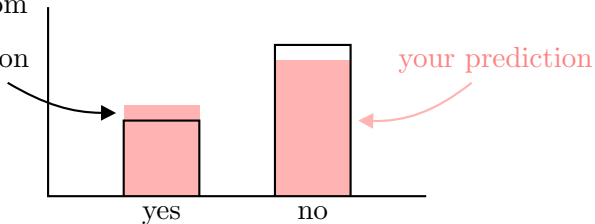


“Do you like this painting?”



common prediction

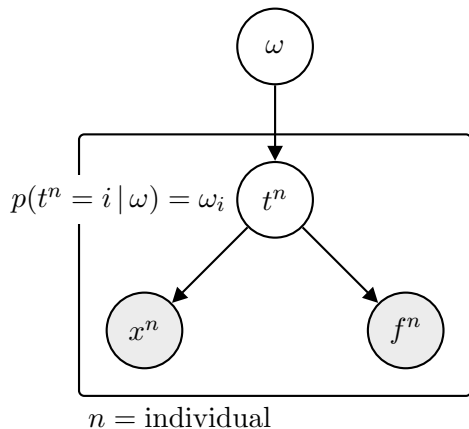
update from  
your opinion



“Do you like this painting?”



Your opinion is the opinion that you believe has the highest probability of being *more common than commonly predicted*.



$\omega$  = distribution of opinions

$t^n$  = opinion

$x^n$  = answer

$f^n$  = prediction of  
frequencies of answers



$$\text{🧪}(x = i, f) = \overbrace{\log \frac{\langle x \rangle_i}{\langle f \rangle_i}}^{\text{information score}} - \overbrace{\sum_j \langle x \rangle_j \log \frac{f_j}{\langle x \rangle_j}}^{\text{prediction penalty}}$$

$\langle x \rangle_i$  = average of  $(\mathbb{1}(x^n = i))$

$\langle f \rangle$  = geometric average of  $(f^n)$



PROPOSITION. Suppose that a respondent holds opinion  $t$ , answers  $x$ , and predicts  $f$ ; and everyone else answers and predicts honestly. Then the respondent does best also by answering and predicting honestly:

$$\max_{(x,f)} \mathbb{E} \left[ \text{🧪}(x, f) \mid t \right] = (t, p(t' \mid t)).$$

Prelec, D. (2004). A Bayesian Truth Serum for Subjective Data. *Science*, 306(5695), 462–466.

- Your opinion is the opinion that you believe has the highest probability of being *more common than commonly predicted*.

$$\text{🧪}(x_i, f) = \overbrace{\log \frac{\langle x \rangle_i}{\langle f \rangle_i}}^{\text{information score}} - \overbrace{\sum_j \langle x \rangle_j \log \frac{f_j}{\langle x \rangle_j}}^{\text{prediction penalty}}$$

- Truth telling is a Bayesian Nash equilibrium.

These slides: [goo.gl/UsG2WG](https://goo.gl/UsG2WG).

Writeup: [goo.gl/ZBJikX](https://goo.gl/ZBJikX).