Gaussian Process Convolution Model

Wessel Bruinsma

1 November 2018

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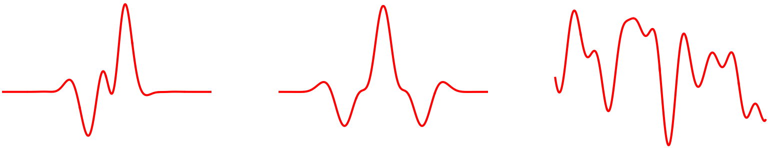
$$h \sim \mathcal{GP}(0, k_h), \qquad k \,|\, h = h * Rh, \qquad f \,|\, k \sim \mathcal{GP}(0, k):$$



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$$x \sim \mathcal{GP}(0, \delta) \implies \text{“}hx\text{”} \sim \mathcal{GP}(0, \text{“}hh^{\mathsf{T}}\text{”})$$

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Gaussian Process Convolution Model

Inference

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- Joint distribution:

$$p(f, h, \underset{\uparrow}{u}, x, \underset{\uparrow}{z}) = p(f \mid h, x)p(h \mid u)p(u)p(x \mid z)p(z).$$

inducing points for h and x resp.

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- Approximate posterior:

$$q(f, h, u, x, z) = p(f \mid h, x) p(h \mid u) q(u) p(x \mid z) q(z).$$

- Mean-field approximate posterior:

$$q(f, h, u, x, z) = p(f \mid h, x)p(h \mid u)q(u)p(x \mid z)q(z).$$

Extension: Improved Inference

- Mean-field approximate posterior:

$$q(f, h, u, x, z) = p(f \mid h, x)p(h \mid u)q(u)p(x \mid z)q(z).$$

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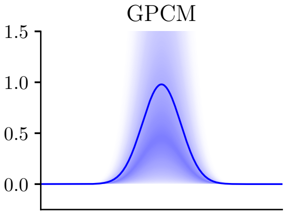
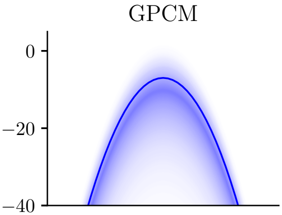
- MCMC to sample from q^*.

Extension: Causality

Prior over kernel:



Prior over PSD:



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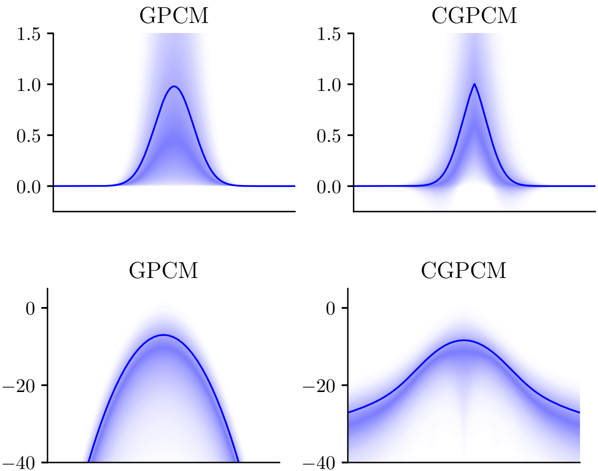
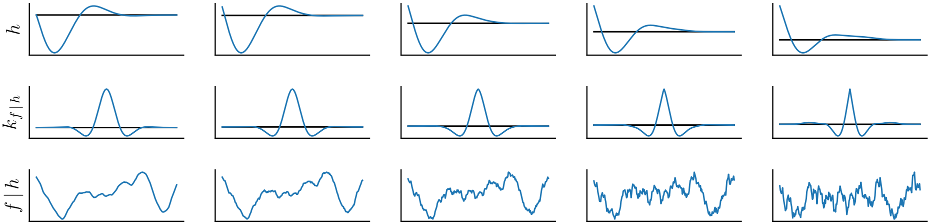
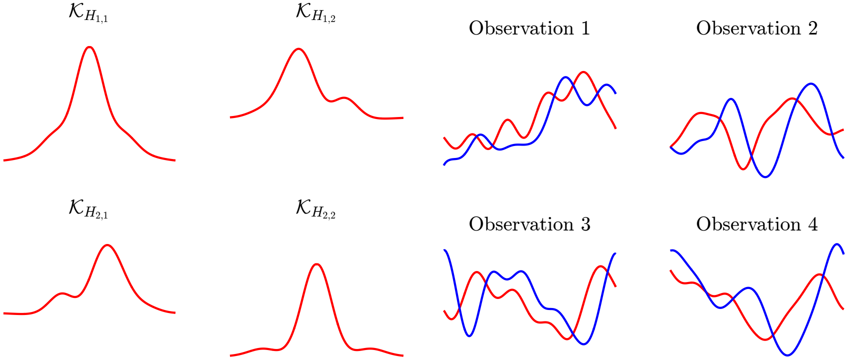


Extension: Causality



Extension: Multiple Outputs



But what about the *kernel of the kernel*?

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Extension: Deep Kernel Model

Model (N-Deep Kernel Model)

$$h_0 \sim \mathcal{GP}(0, k_h),$$

$$h_1 \,|\, h_0 \sim \mathcal{GP}(0, h_0 * Rh_0),$$

$$\vdots$$

$$h_N \,|\, h_{N-1} \sim \mathcal{GP}(0, h_{N-1} * Rh_{N-1}),$$

$$f \,|\, h_N = h_N.$$

Gaussian Process Convolution Model

Extension: Deep Kernel Model

Observation ($N = 1$)	Kernel	PSD	Observation ($N = 20$)	Kernel	PSD