Gaussian Process Convolution Model

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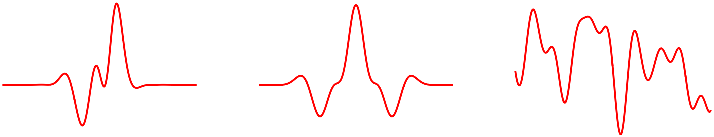
$$h \sim \mathcal{GP}(0, k_h), \qquad k \mid h = h * Rh, \qquad f \mid k \sim \mathcal{GP}(0, k):$$



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$$x \sim \mathcal{GP}(0, \delta) \implies \text{“}hx\text{”} \sim \mathcal{GP}(0, \text{“}hh^{\mathsf{T}}\text{”})$$

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Model (GPCM (Tobar et al., 2015), Equivalent Formulation)

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- Joint distribution:

$$p(f, h, \underset{\uparrow}{u}, x, \underset{\uparrow}{z}) = p(f \mid h, x)p(h \mid u)p(u)p(x \mid z)p(z).$$

inducing points for h and x resp.

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- Approximate posterior:

$$q(f, h, u, x, z) = p(f \mid h, x)p(h \mid u)q(u)p(x \mid z)q(z).$$

Extension: Improved Inference

- Mean-field approximate posterior:

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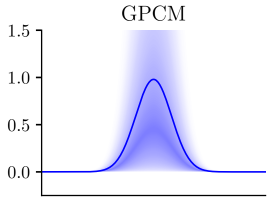
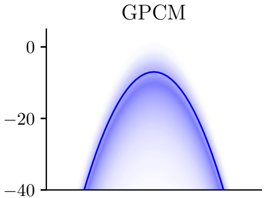
- MCMC to sample from q^*.

Extension: Causality

Prior over kernel:



Prior over PSD:



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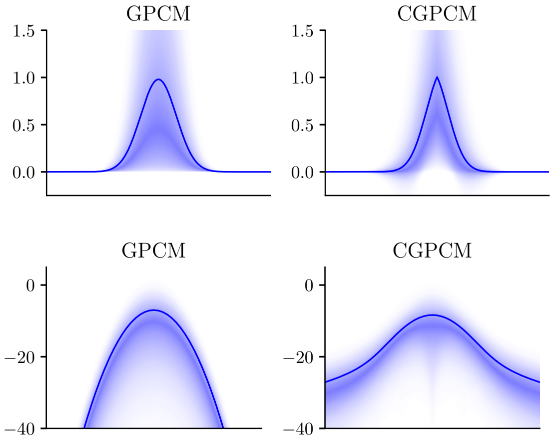
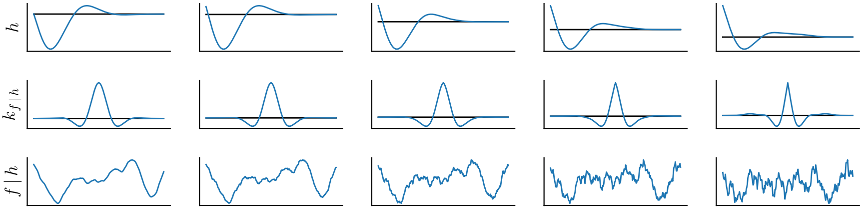
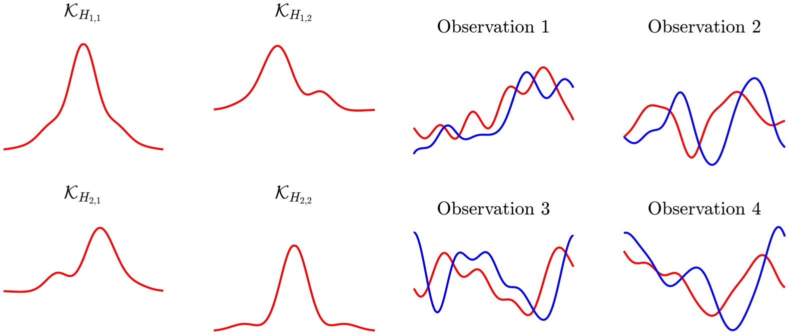


Extension: Causality



Extension: Multiple Outputs



But what about the *kernel of the kernel*?

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And the *kernel of the kernel of the kernel*?

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$$\vdots$$

And the *kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel*?

Extension: Deep Kernel Model

Model (N-Deep Kernel Model)

$$\begin{aligned} h_0 &\sim \mathcal{GP}(0, k_h), \\ h_1 \,|\, h_0 &\sim \mathcal{GP}(0, h_0 * R h_0), \\ &\vdots \\ h_N \,|\, h_{N-1} &\sim \mathcal{GP}(0, h_{N-1} * R h_{N-1}), \\ f \,|\, h_N &= h_N. \end{aligned}$$

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