# Spectral Methods in Gaussian Modelling

# Topic 4: Spectrum Estimation

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21 January 2019

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# Spectrum Estimation



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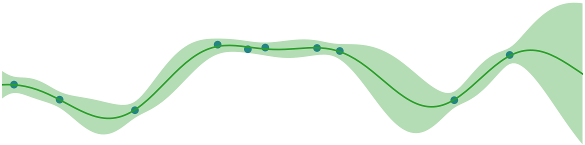
- Estimation difficult: samples limited, noisy, and nonuniform.



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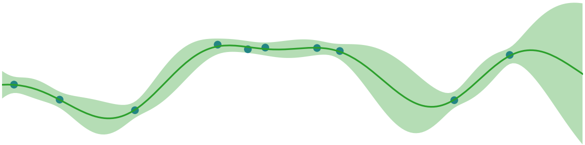

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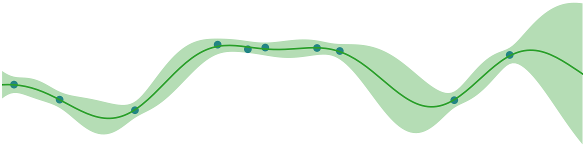

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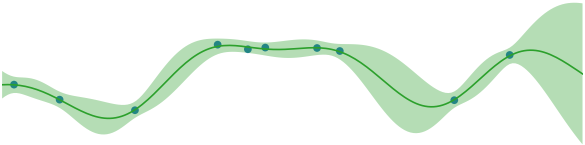

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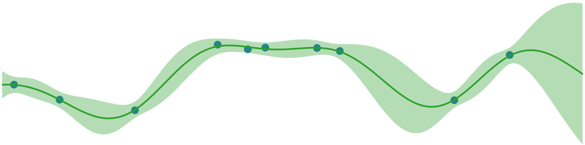

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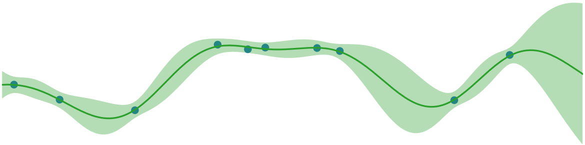

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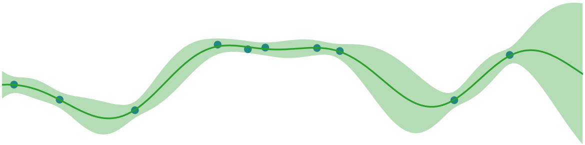

- Estimators:
  - parametric methods (SSA, SMK),
  - nonparametric methods (GPCM).
- Novel model by Tobar (2018).

# Bayesian Nonparametric Spectral Estimation



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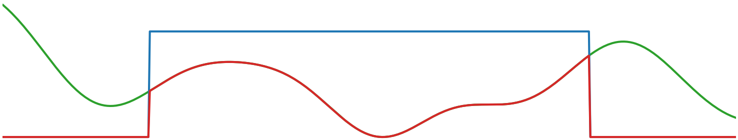

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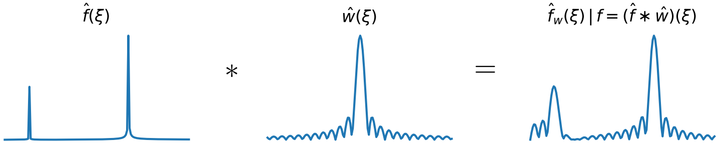

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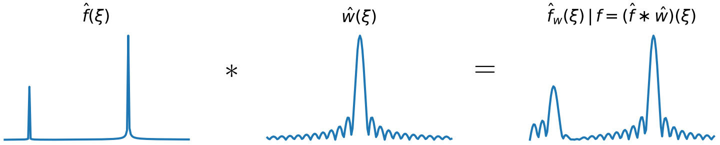


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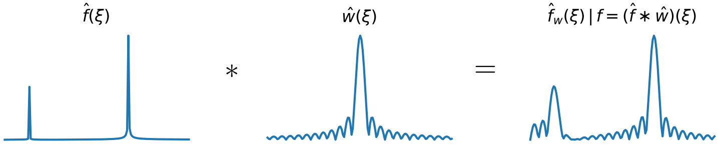


- Window functions:
  - algorithmic purposes (here: ensure integrability),
  - acquisition devices (e.g., sampling: $w = Ш$).

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    - $k =$ SMK or EQ in simple cases.

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$$k_{\mathrm{Re}\ \hat{f}_w}(\xi,\xi')$$

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$$k_{y(\mathrm{Im}\ \hat{f}_w)}(t,\xi)$$

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\begin{aligned}
k_{\mathrm{Re}\ \hat{f}_w}(\xi,\xi') &= \frac{1}{2}(k_{\hat{f}_w}(\xi,\xi') + k_{\hat{f}_w}(\xi,-\xi')),\\
k_{y(\mathrm{Re}\ \hat{f}_w)}(t,\xi) &= \mathrm{Re}\ k_{y\hat{f}_w}(t,\xi),\\
k_{\mathrm{Im}\ \hat{f}_w}(\xi,\xi') &= \frac{1}{2}(k_{\hat{f}_w}(\xi,\xi') - k_{\hat{f}_w}(\xi,-\xi')),\\
k_{y(\mathrm{Im}\ \hat{f}_w)}(t,\xi) &= \mathrm{Im}\ k_{y\hat{f}_w}(t,\xi).
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$k_{\hat{f}_w}(\xi, \xi')$

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k_{\hat{f}_w}(\xi, \xi') &= \mathbb{E}[\hat{f}^*_w(\xi)\hat{f}_w(\xi')] \\
&= \mathbb{E}[\mathcal{F}_{t,t'}\{f(t)f(t')w(t)w(t')\}(-\xi, \xi')] \\
&= (\mathcal{F}_{t,t'}\{k(t-t')\}(u, u') * \mathcal{F}_{t,t'}\{w(t)w(t')\}(u, u'))(-\xi, \xi'),
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$$k_{\hat{f}_w}(\xi, \xi') = \mathcal{N}\big(\xi - \xi'; 0, \alpha\big)\Big(\hat{k}(u) * \mathcal{N}\big(u; 0, \tfrac{1}{4}\alpha\big)\Big)\big(\tfrac{1}{2}(\xi + \xi')\big)$$

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# Interpretation of Posterior Mean



$$\mathbb{E}[\hat{f}_w(\xi) \,|\, e] = \int \hat{k}(u) \left( \sum_{i=1}^{N} e^{-2\pi\iota u t_i} (K_e^{-1} e)_i \right) \mathcal{N}\big(u; \xi, \tfrac{1}{2}\alpha\big) \, \mathrm{d}u.$$

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- Interpretation: **DFT** of **whitened** observations, **weighted** by prior, then **smoothed** due to window.
- If prior uninformative, $K_e \approx I$, then weighted DFT in the limit:

$$\lim_{\alpha \to 0} \mathbb{E}[\hat{f}_w(\xi) \,|\, e] \approx \hat{k}(\xi) \sum_{i=1}^{N} e^{-2\pi\iota\xi t_i} e_i.$$

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Lomb–Scargle:

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- Gaussian prior on $A$ and $B$: LS recovers BNSE in the limit.

# Experiment: Two Heart Rate Signals



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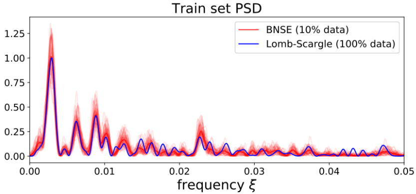


(Figure taken from Tobar (2018).)

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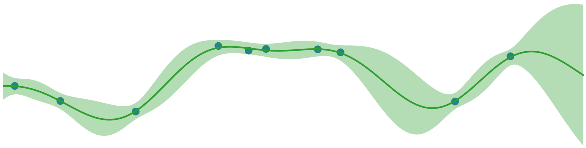

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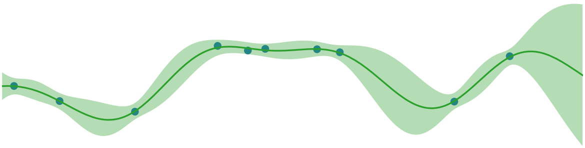

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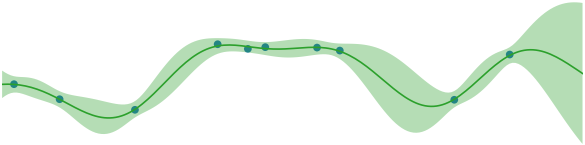

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$\Rightarrow$ Can optimise to find periodicities.