Gaussian Process Convolution Model

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28 January 2018

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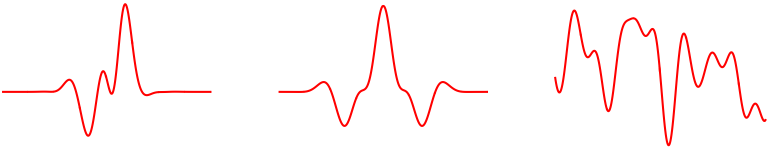

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$$x \sim \mathcal{GP}(0, \delta) \implies \text{“}hx\text{”} \sim \mathcal{GP}(0, \text{“}hh^\mathsf{T}\text{”})$$

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- Joint distribution:

$$p(f, h, \underset{\uparrow}{u}, x, \underset{\uparrow}{z}) = p(f \mid h, x)p(h \mid u)p(u)p(x \mid z)p(z).$$

inducing points for h and x resp.

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- Approximate posterior:

$$q(f, h, u, x, z) = p(f \mid h, x) p(h \mid u) q(u) p(x \mid z) q(z).$$

Extension: Improved Inference

- Mean-field approximate posterior:

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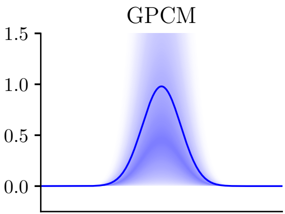
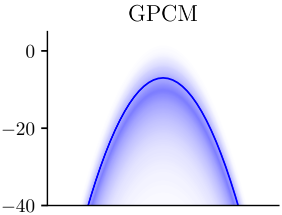
- MCMC to sample from q^*.

Extension: Causality

Prior over kernel:



Prior over PSD:



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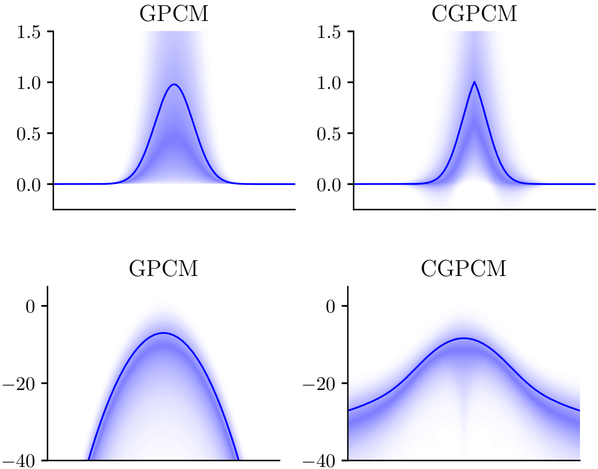

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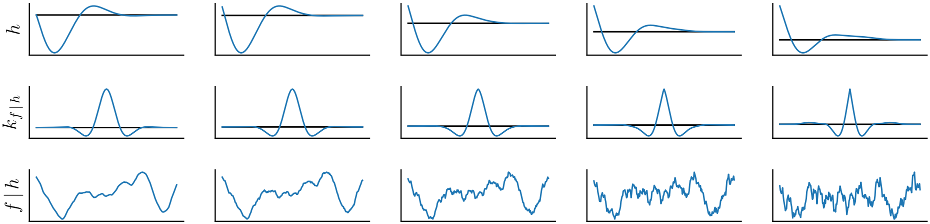
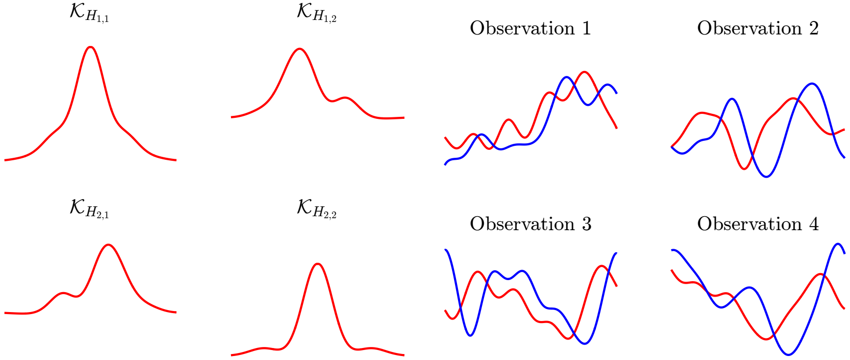


Gaussian Process Convolution Model

Extension: Multiple Outputs



But what about the *kernel of the kernel*?

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And the *kernel of the kernel of the kernel*?

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$$\vdots$$

And the *kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel of the kernel*?

Extension: Deep Kernel Model

Model (N-Deep Kernel Model)

$$
\begin{aligned}
h_0 &\sim \mathcal{GP}(0, k_h), \\
h_1 \,|\, h_0 &\sim \mathcal{GP}(0, h_0 * R h_0), \\
&\vdots \\
h_N \,|\, h_{N-1} &\sim \mathcal{GP}(0, h_{N-1} * R h_{N-1}), \\
f \,|\, h_N &= h_N.
\end{aligned}
$$

Extension: Deep Kernel Model