

Evaluation report of polar codes

In this report the performance of polar codes will be evaluated. A specific decoder implementation with the use of likelihood caches will be used for the evaluation. According to ¹ a complexity of $O(N \log N)$ should be achieved.

For each evaluation random messages will be generated and compared after the decoding. The following parameters are relevant for those evaluations:

Blocklength:

Blocklength (Length of code word) is defined by the amount of bits which will be transferred through the binary erasure channel.

Information bitrate:

The relation between information bits and parity bits. (Information bitrate of 0.5 and blocklength of 8 would mean 4 bits of parity and 4 bits of information. Information bitrate of 0.25 and a blocklength of 8 would mean 2 information bits and 6 parity bits)

Epsilon:

Epsilon defines the erasure rate of the binary erasure channel. (An Epsilon of 0.25 would mean that 25% of the code word will be erased)

Error rate:

The rate of unsuccessful decoded messages (code words which could not be decoded). An error rate of 0.5 would mean, that 50% of all code words could not be decoded.

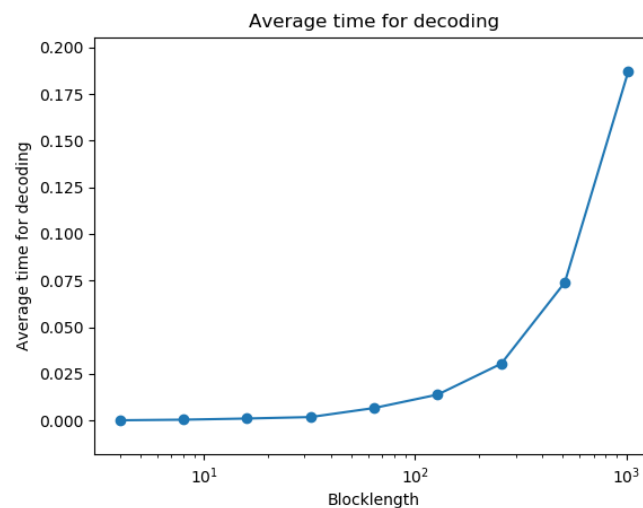
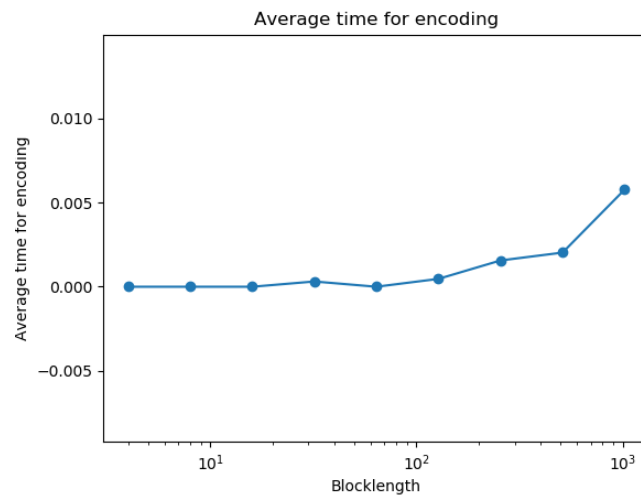
Time:

Time will be measured in seconds.

¹ <http://ipgdemos.epfl.ch/polarcodetutorial/>

Time and space complexity:

In the first evaluation the complexity of the decoder ($O(N \log N)$) will be evaluated. Every blocklength between 4 and 1024 will be tested with 100 iterations per blocklength. Information bitrate will be 0.5 and an epsilon of 0.25. The X-Axis (Blocklength) is scaled logarithmic:



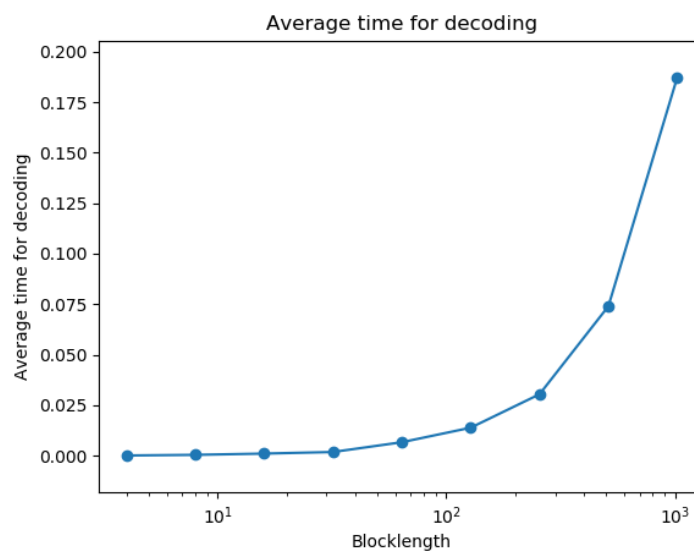
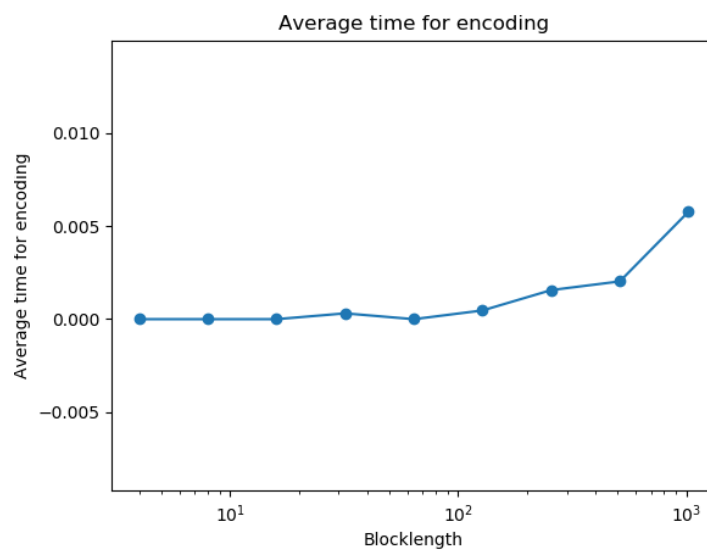
The results show that the complexity of encoding is almost linear ($O(N \log N)$). (Note that the complexity appears as polynomial because of the logarithmic scale)

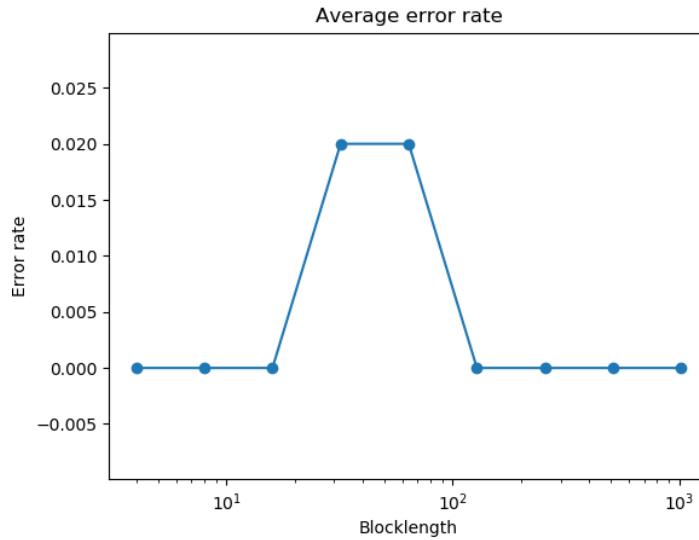
Blocklengths:

In this test the influence of the blocklength on the error rate will be evaluated.

Run1:

| | |
|----------------------|------|
| Information Bitrate: | 0.25 |
| Epsilon: | 0.7 |

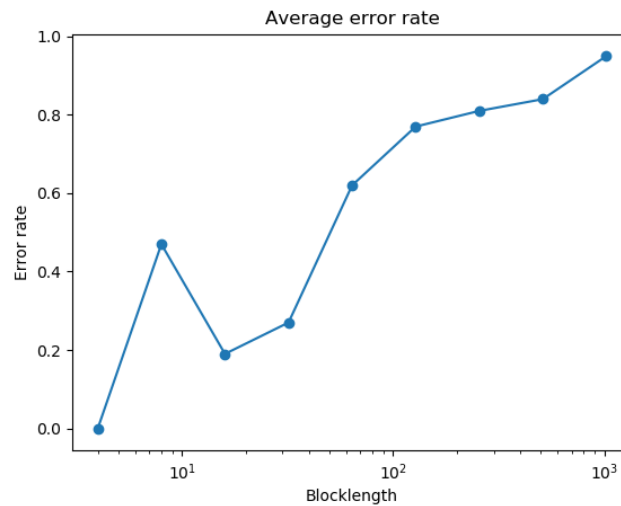




The time for encoding and decoding is similar to the previous test. Therefore the complexity of $O(N \log N)$ could be proven. There is a small rise of errors of blocklengths 32 and 64. After the error rate drops again to 0. In further tests with this configuration, a fluctuation of error rates before the blocklength of 64 could have always been observed and then the error rate always drops to 0.

Run2:

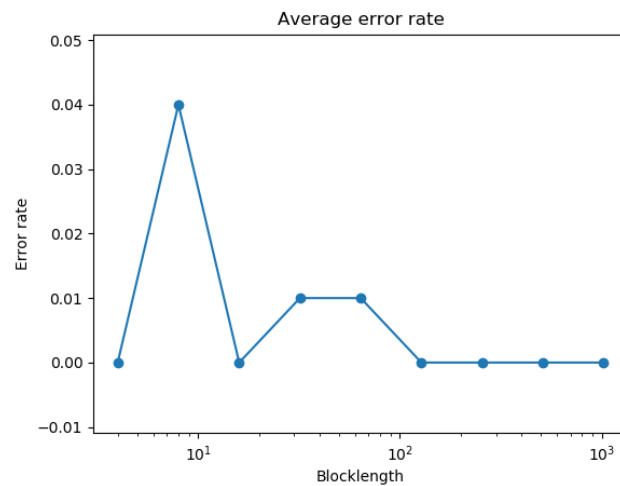
| | |
|----------------------|------|
| Information Bitrate: | 0.75 |
| Epsilon: | 0.2 |



In this scenario the error rate fluctuates at the beginning and after a blocklength of 32 it rises constantly. This shows that having a lower information bitrate (More parity checks per code word) has a larger impact of transferring successful message over the BEC than the erasure rate itself.

Run3:

| | |
|----------------------|-----|
| Information Bitrate: | 0.2 |
| Epsilon: | 0.5 |

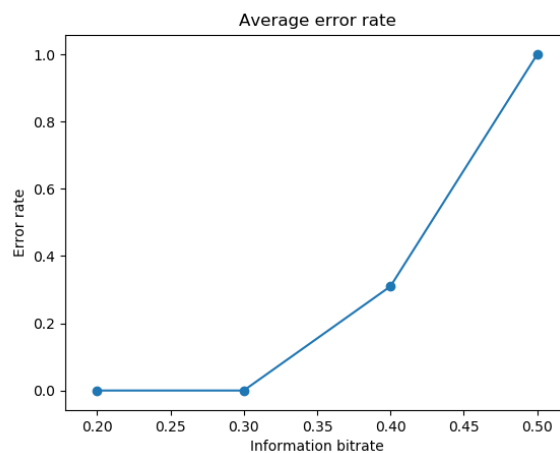


This configuration proves that in order to have an efficient transmission a low information bitrate is necessary. It also proves, that the channel capacity improves with the blocklength (The longer the length, the more mistakes can be corrected).

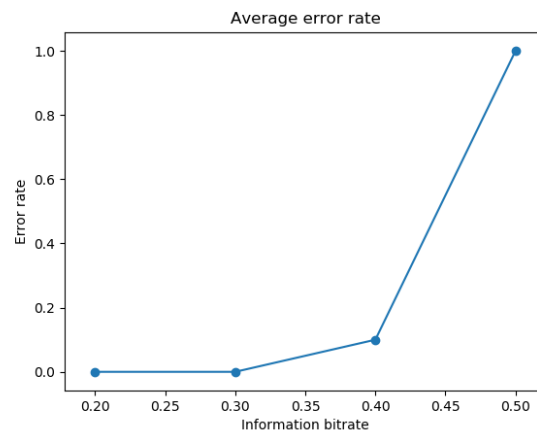
Finding best information bitrate:

In this evaluation a various blocklengths had been chosen. The Epsilon is fixed on 0.5. The information bit rate rises from 0.2 to 0.5 in order to find the best performance.

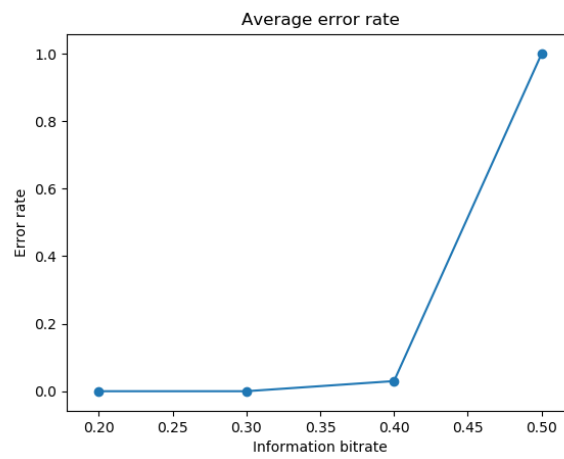
Blocklength: 2048 (2^{11})



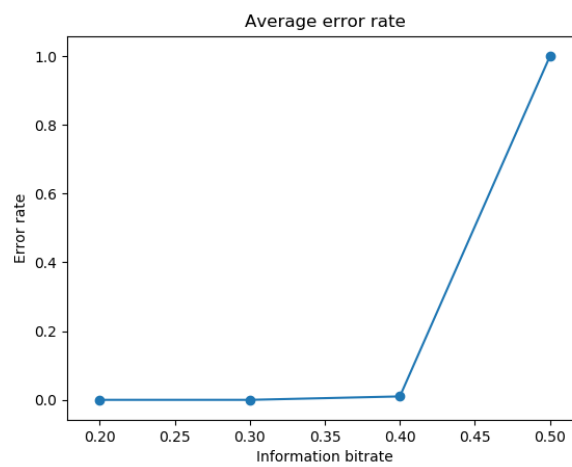
Blocklength: 4096 (2^{12})

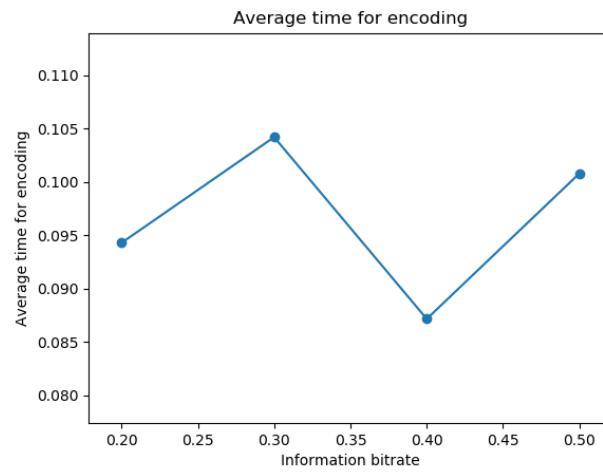
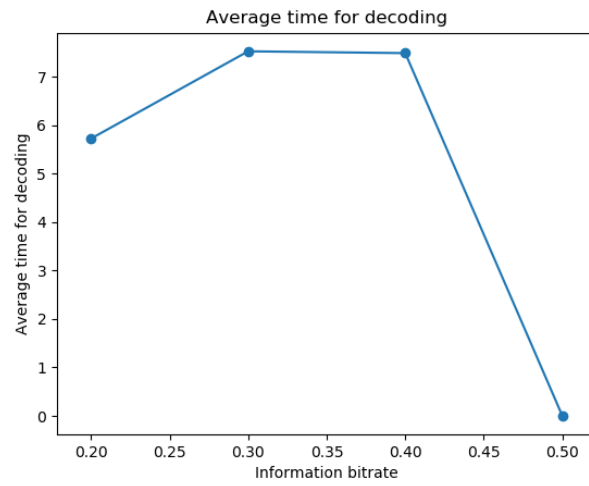


Blocklength: 8192 (2^{13})



Blocklength 8192 (2^{14}):

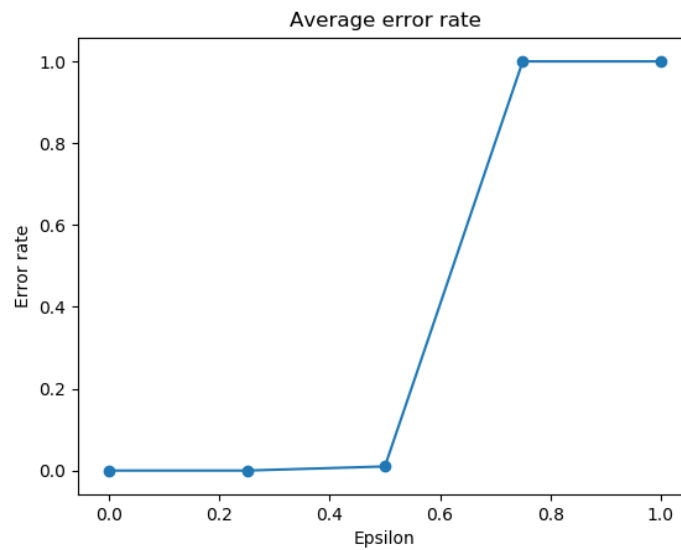




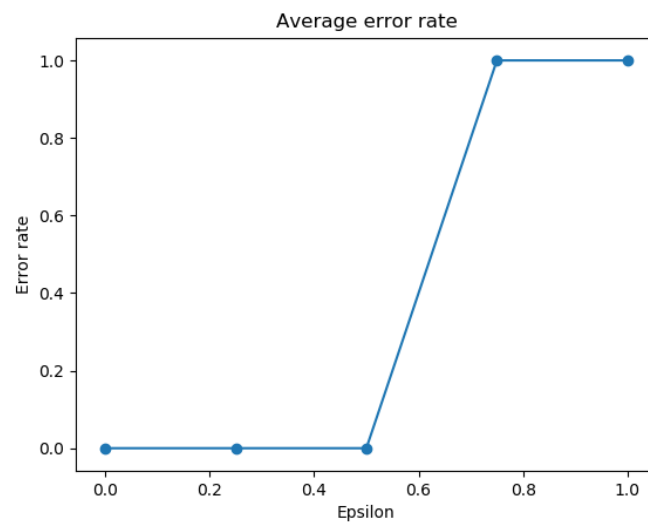
Varying epsilon rate:

In this evaluation the bitrate will be fixed on 0.3 and the epsilon erasure rate will slowly rise from 0 to 1. The test will be performed for different blocklengths:

Blocklength 128 (2^7)



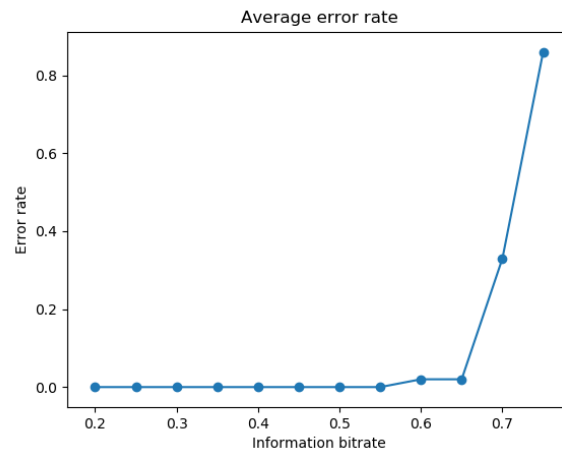
Blocklength: 8192 (2^{13})



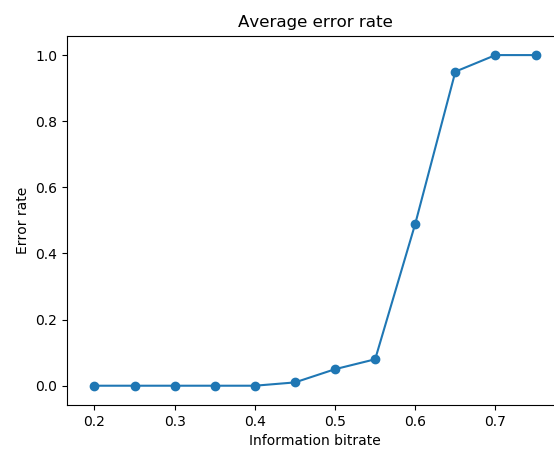
Ideal parameters:

The goal of this test is to find ideal parameters for the most efficient use of polar codes for a blocklength of 256. Different erasure rates will be tested to find the best bitrate configuration.

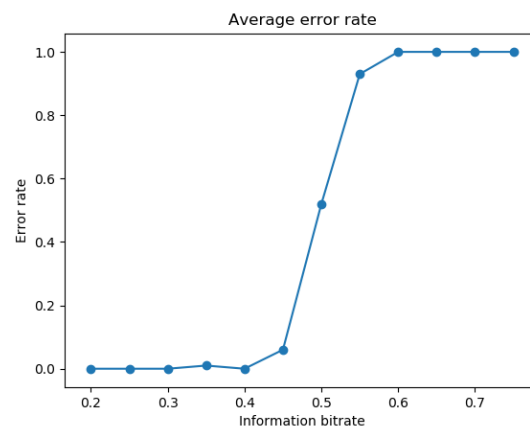
Epsilon: 0.2



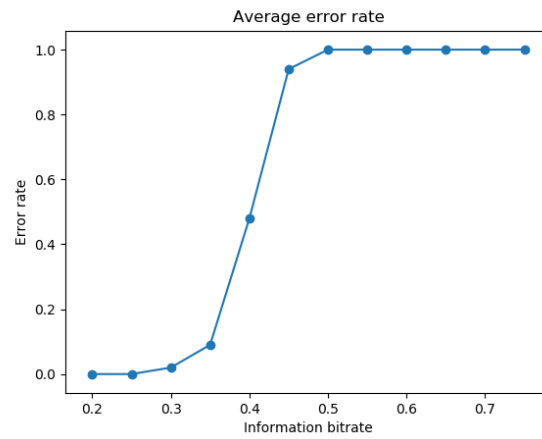
Epsilon: 0.3



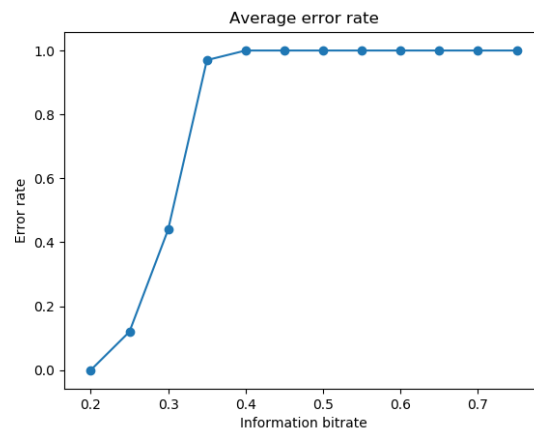
Epsilon: 0.4



Epsilon: 0.5



Epsilon: 0.6



The following table visualizes the ideal parameters for blocklengths of 256:

| Epsilon: | Ideal information bitrate: | Acceptable information bitrate: |
|----------|----------------------------|---------------------------------|
| 0.2 | 0.55 | 0.65 |
| 0.3 | 0.4 | 0.55 |
| 0.4 | 0.3 | 0.45 |
| 0.5 | 0.25 | 0.3 |
| 0.6 | 0.2 | 0.25 |