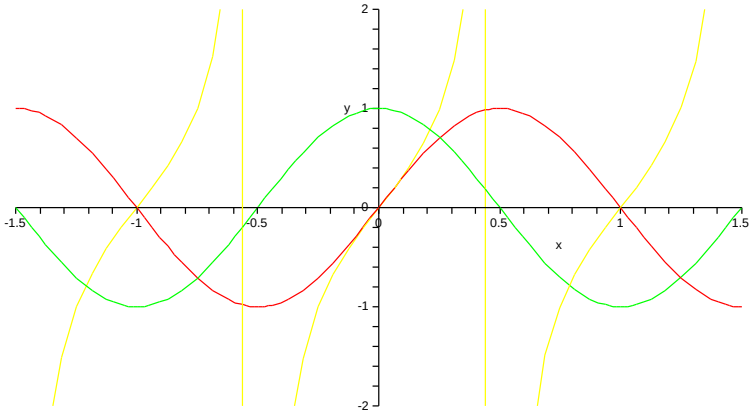
Trigonometrische und Hyperbolische Funktionen

Beziehung zur Exponentialfunktion		
$\cos x = \dfrac{e^{ix}+e^{-ix}}{2}$	$\cosh x = \dfrac{e^{x}+e^{-x}}{2}$	$\operatorname{arcosh} y = \log\bigl(y+\sqrt{y^2-1}\,\bigr)$
$\sin x = \dfrac{e^{ix}-e^{-ix}}{2i}$	$\sinh x = \dfrac{e^{x}-e^{-x}}{2}$	$\operatorname{arsinh} y = \log\bigl(y+\sqrt{y^2+1}\,\bigr)$
$\tan x = \dfrac{e^{ix}-e^{-ix}}{e^{ix}+e^{-ix}}\cdot\dfrac{1}{i}$	$\tanh x = \dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$	$\operatorname{artanh} y = \dfrac{1}{2}\log\dfrac{1+y}{1-y}$

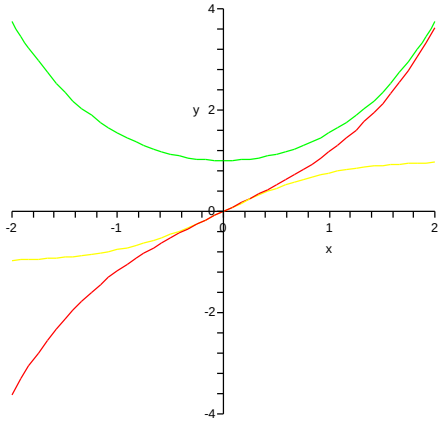
Reihenentwicklungen
$e^x = 1+x+\dfrac{x^2}{2}+\dfrac{x^3}{6}+\dfrac{x^4}{24}+\ldots = \displaystyle\sum_{k=0}^{\infty}\frac{x^k}{k!}$
$\cos x = 1-\dfrac{x^2}{2}+\dfrac{x^4}{24}-\ldots = \displaystyle\sum_{k=0}^{\infty}(-1)^k\cdot\frac{x^{2k}}{(2k)!}$
$\sin x = x-\dfrac{x^3}{6}+\dfrac{x^5}{120}-\ldots = \displaystyle\sum_{k=0}^{\infty}(-1)^k\cdot\frac{x^{2k+1}}{(2k+1)!}$
$\cosh x = 1+\dfrac{x^2}{2}+\dfrac{x^4}{24}+\ldots = \displaystyle\sum_{k=0}^{\infty}\frac{x^{2k}}{(2k)!}$
$\sinh x = x+\dfrac{x^3}{6}+\dfrac{x^5}{120}+\ldots = \displaystyle\sum_{k=0}^{\infty}\frac{x^{2k+1}}{(2k+1)!}$
$\log(1+x) = x-\dfrac{x}{2}+\dfrac{x^2}{3}-\ldots = \displaystyle\sum_{k=1}^{\infty}(-1)^{k-1}\cdot\frac{x^k}{k}$
$\arctan x = x-\dfrac{x^3}{3}+\dfrac{x^5}{5}-\ldots = \displaystyle\sum_{k=0}^{\infty}(-1)^k\cdot\frac{x^{2k+1}}{2k+1}$
$\operatorname{artanh} x = x+\dfrac{x^3}{3}+\dfrac{x^5}{5}+\ldots = \displaystyle\sum_{k=0}^{\infty}\frac{x^{2k+1}}{2k+1}$

Additionstheoreme, Pythagoras	
$e^{x+y} = e^x \cdot e^y$	$\log xy = \log x + \log y$
$\sin(x+y) = \sin x \cos y + \cos x \sin y$	$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$
$\cos(x+y) = \cos x \cos y - \sin x \sin y$	$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$
$\sin(2x) = 2 \sin x \cos x$	$\sinh(2x) = 2 \sinh x \cosh x$
$\cos(2x) = \cos^2 x - \sin^2 x$	$\cosh(2x) = \cosh^2 x + \sinh^2 x$
$\cos^2 x + \sin^2 x = 1$	$\cosh^2 x - \sinh^2 x = 1$

Definitions- und Zielbereiche	**Bezeichnung**
$\log y : \quad]0, \infty[\longrightarrow]-\infty, \infty[$	Natürlicher Logarithmus
$\arcsin y : \quad [-1, 1] \longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$	Arcus Sinus
$\arccos y : \quad [-1, 1] \longrightarrow [0, \pi]$	Arcus Cosinus
$\arctan y : \quad]-\infty, \infty[\longrightarrow]-\frac{\pi}{2}, \frac{\pi}{2}[$	Arcus Tangens
$\operatorname{arsinh} y : \quad]-\infty, \infty[\longrightarrow]-\infty, \infty[$	Area Sinus hyperbolicus
$\operatorname{arcosh} y : \quad [1, \infty[\longrightarrow [0, \infty[$	Area Cosinus hyperbolicus
$\operatorname{artanh} y : \quad]-1, 1[\longrightarrow]-\infty, \infty[$	Area Tangens hyperbolicus



sin, cos, tan



sinh, cosh, tanh