Completeness: 3Color

Weston Dransfield

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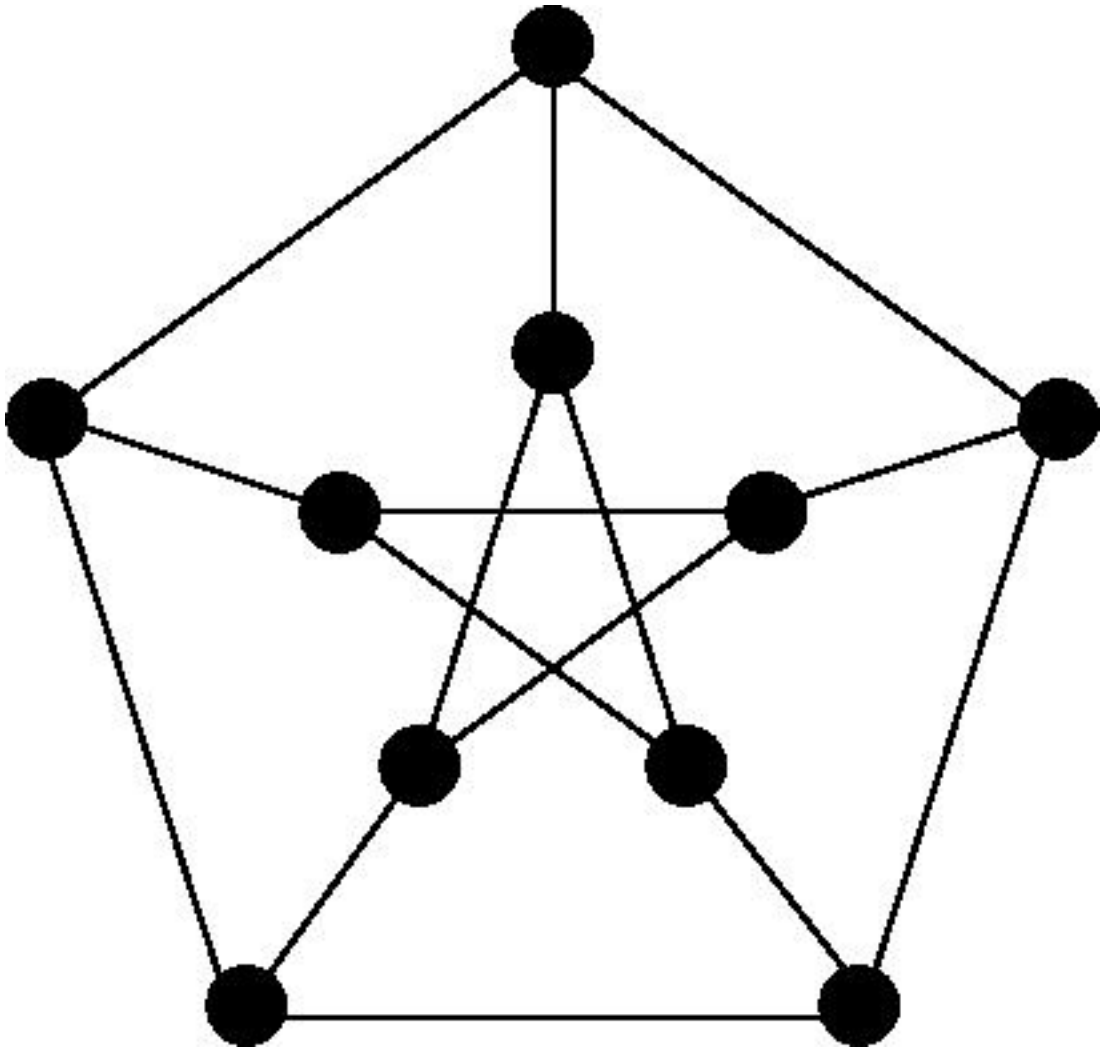
Outline

Description

$\textit{3COLOR} = \{\langle G \rangle \mid$ the nodes of G can be colored with three colors such that no two adjacent nodes are the same color $\}$

Example



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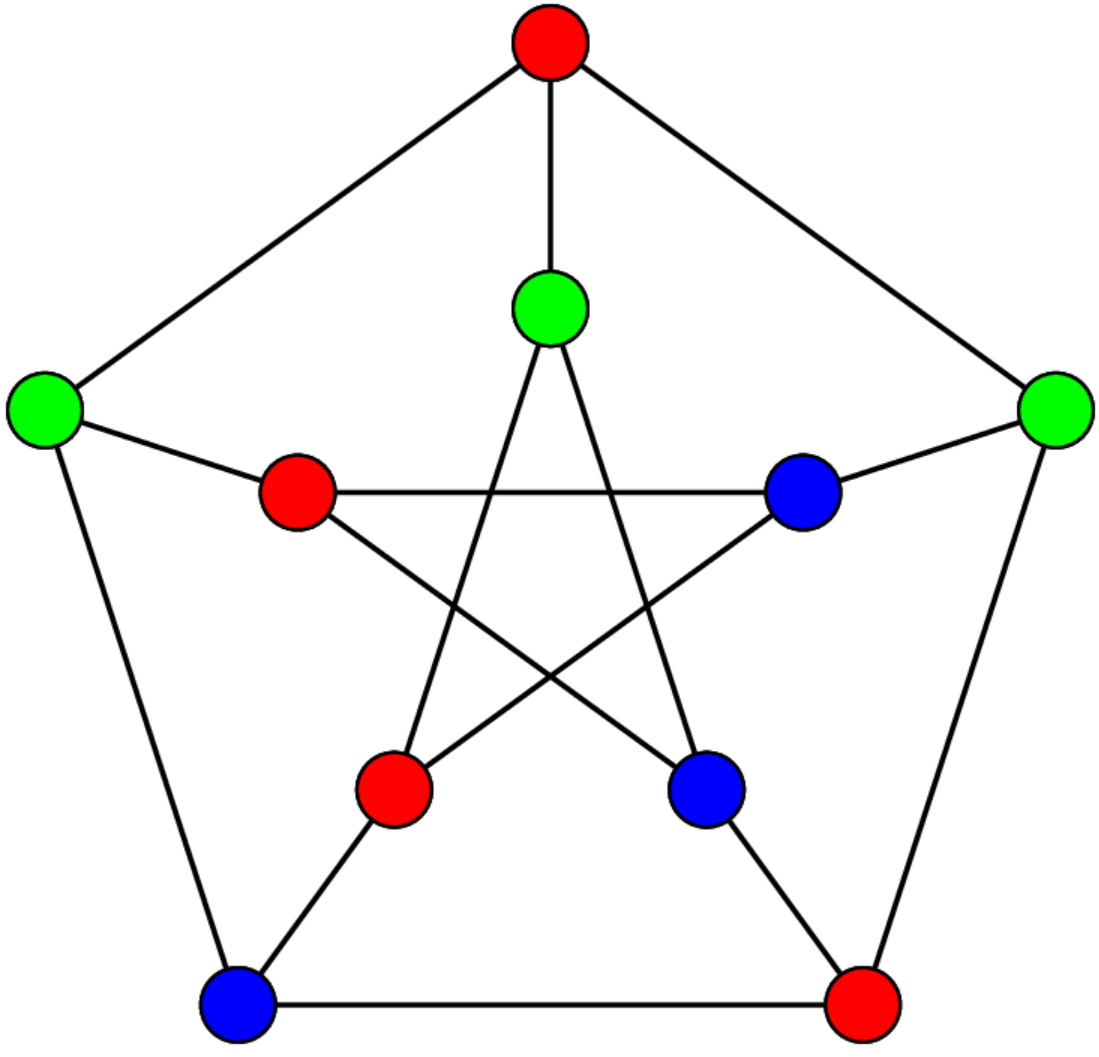

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- This is tough to decide, but easy to verify!

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- Step 3 has largest time complexity of $O(n^2)$. 3COLOR is in NP because it can be verified in polynomial time.