Completeness: 3Color

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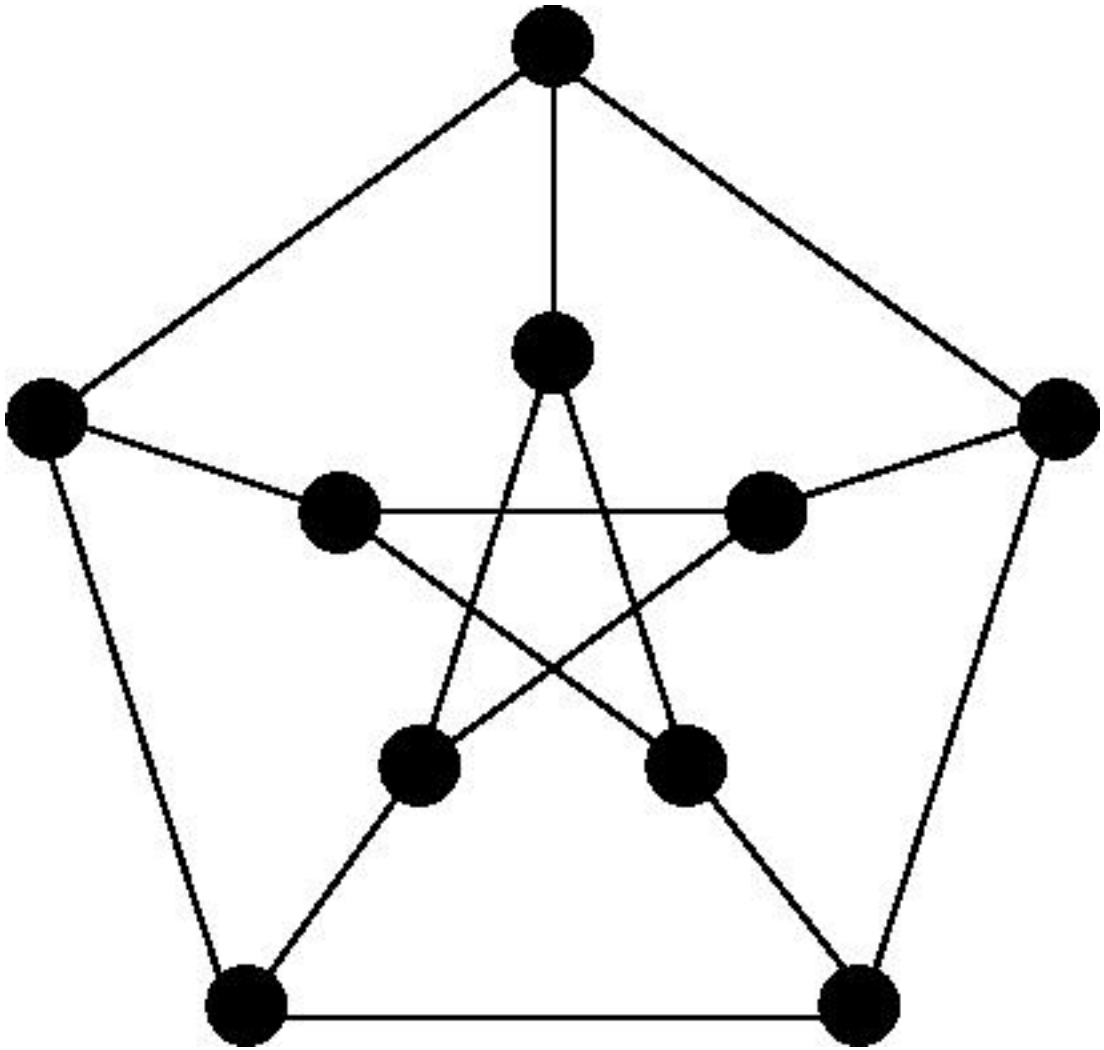
Outline

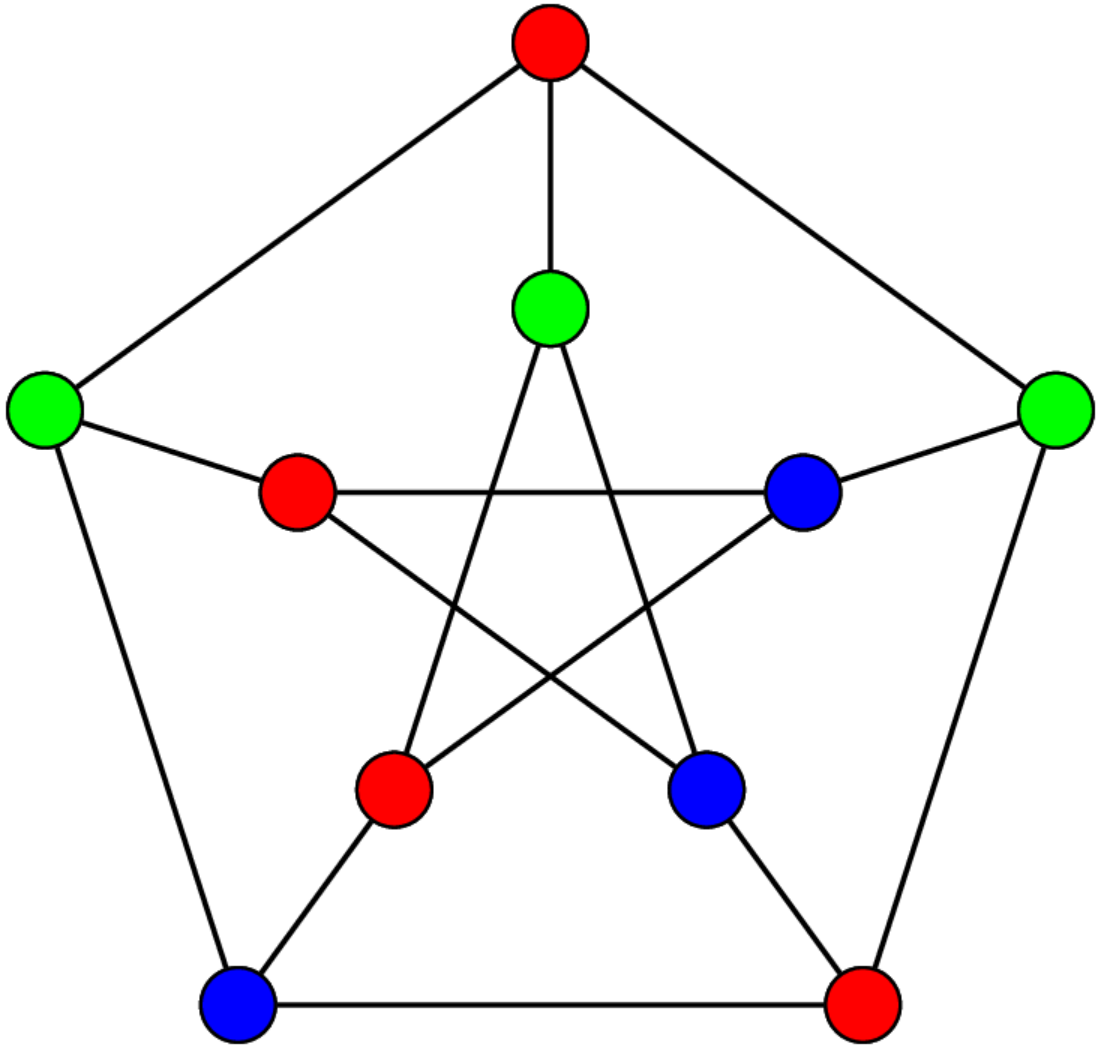
Description

$\textit{3COLOR} = \{\langle G \rangle \mid$ the nodes of G can be colored with three colors such that no two adjacent nodes are the same color $\}$

Example



Example



The Problem

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- This is tough to decide, but easy to verify!

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- Step 3 has largest time complexity of $O(n^2)$. 3COLOR is in NP because it can be verified in polynomial time.

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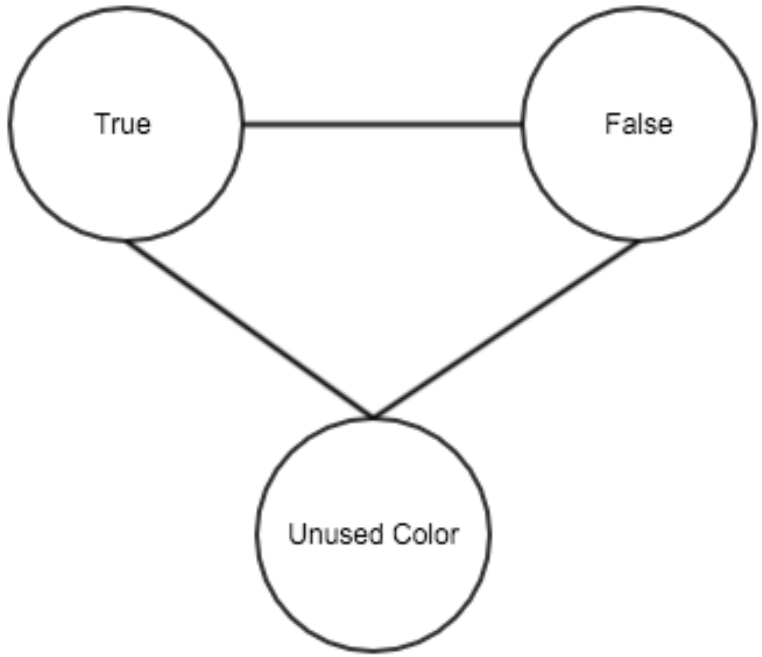
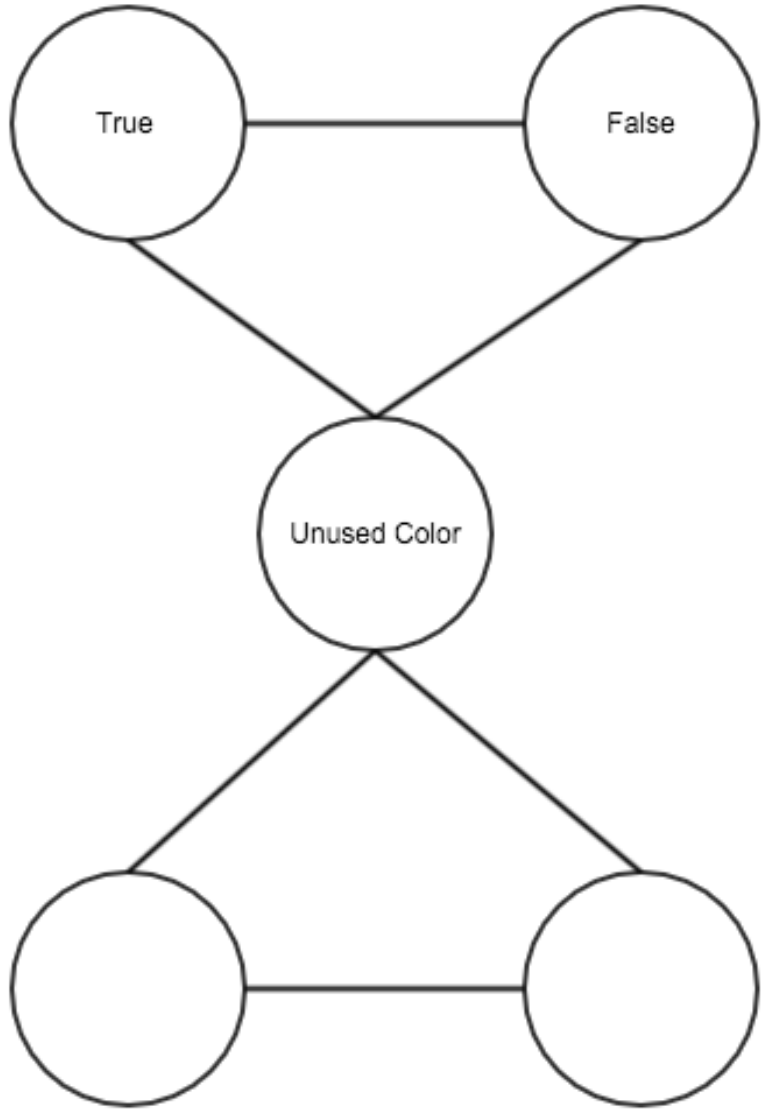
Construct a transformation f from $3SAT$ to $3COLOR$.

1. Establish Truthiness
2. Force variables to be true or false
3. Use these subgraphs to create a graph that is 3 colorable iff variables are satisfiable

Constructing the Reduction - Truthiness



Constructing the Reduction - Variables



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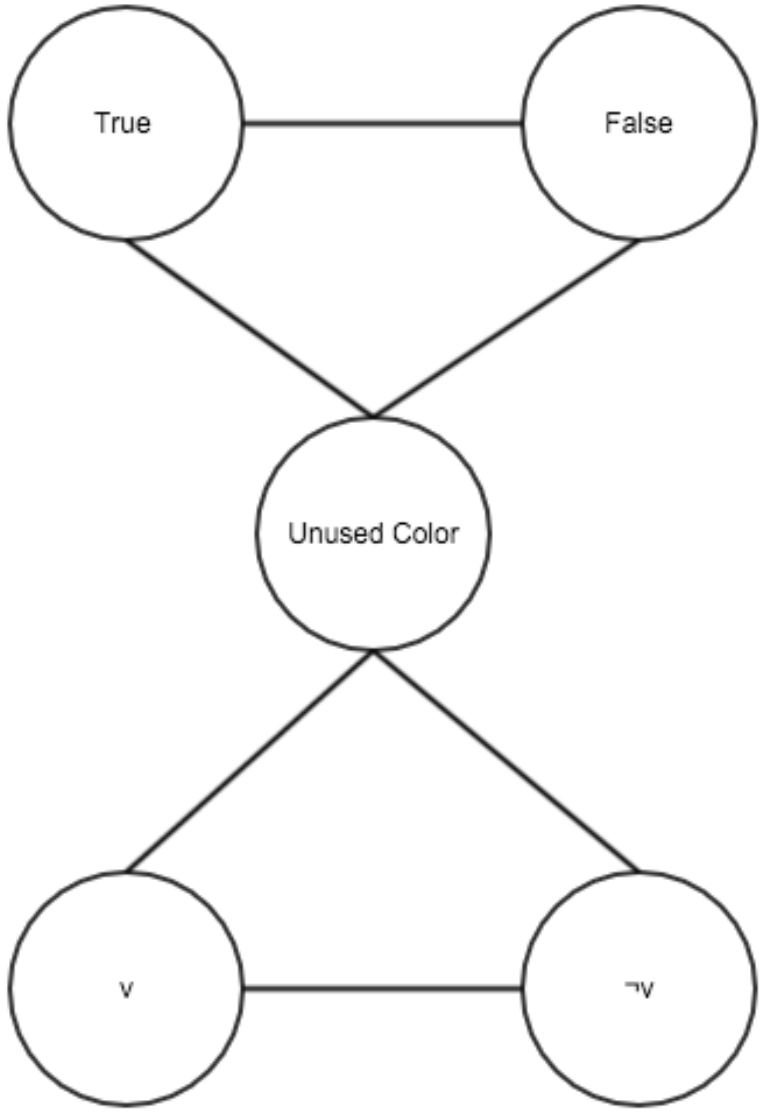
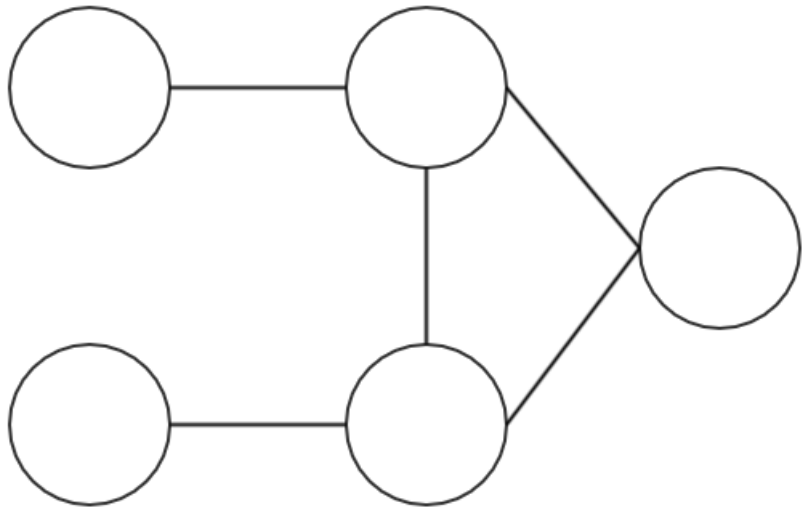
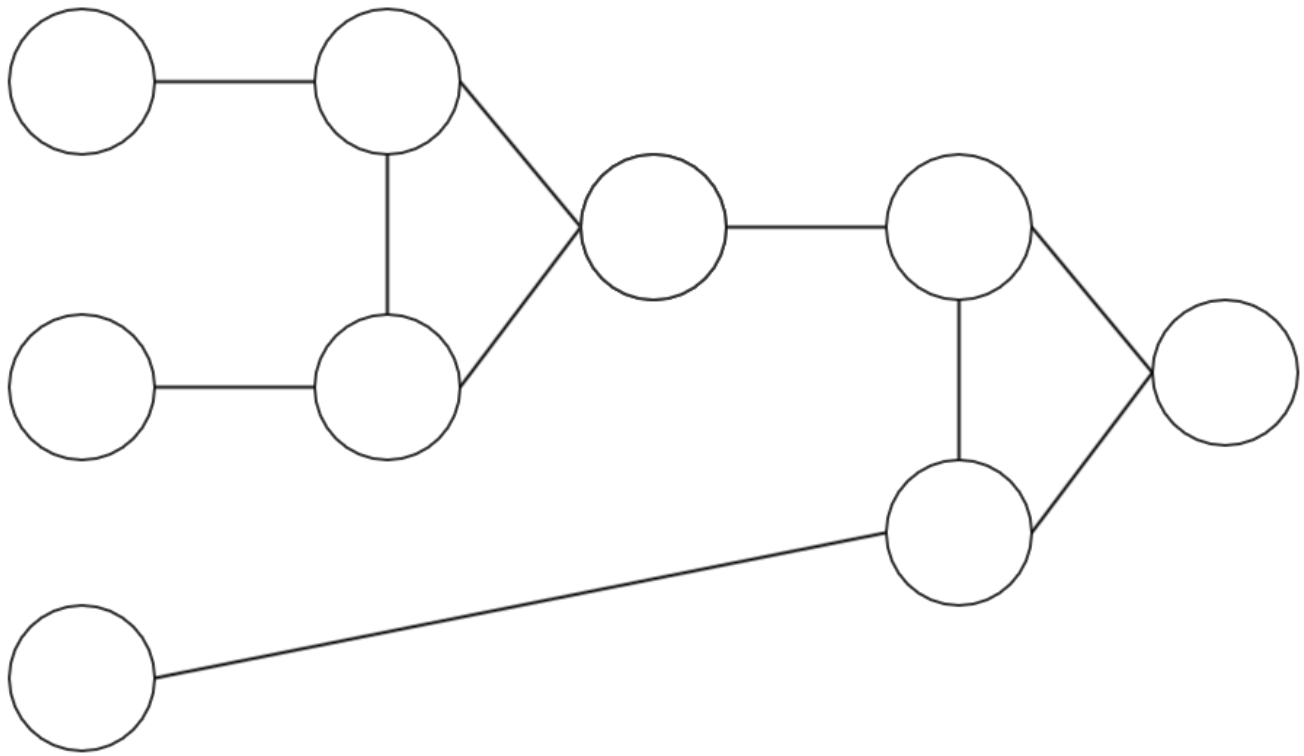


Constructing the Reduction - OR

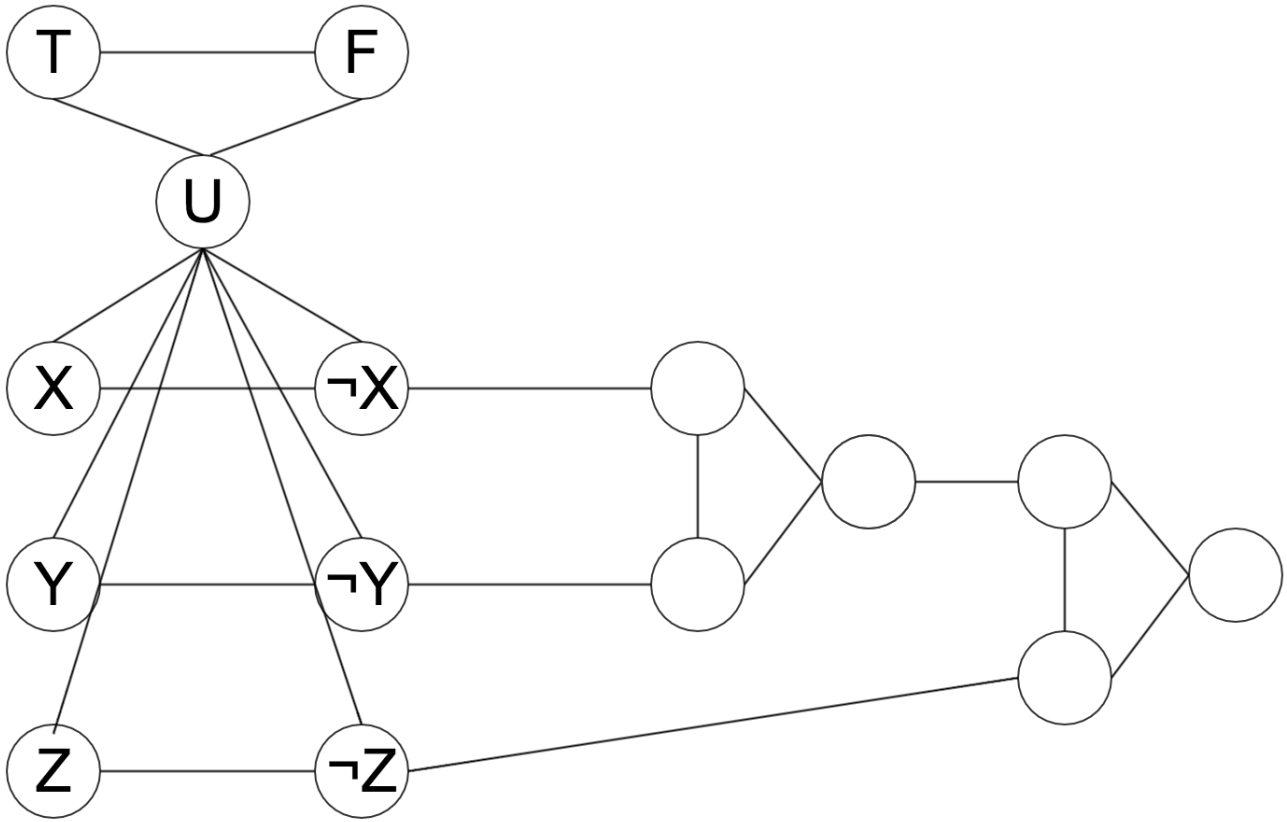


$x \vee y$

Constructing the Reduction - OR



Constructing the Reduction - Clause



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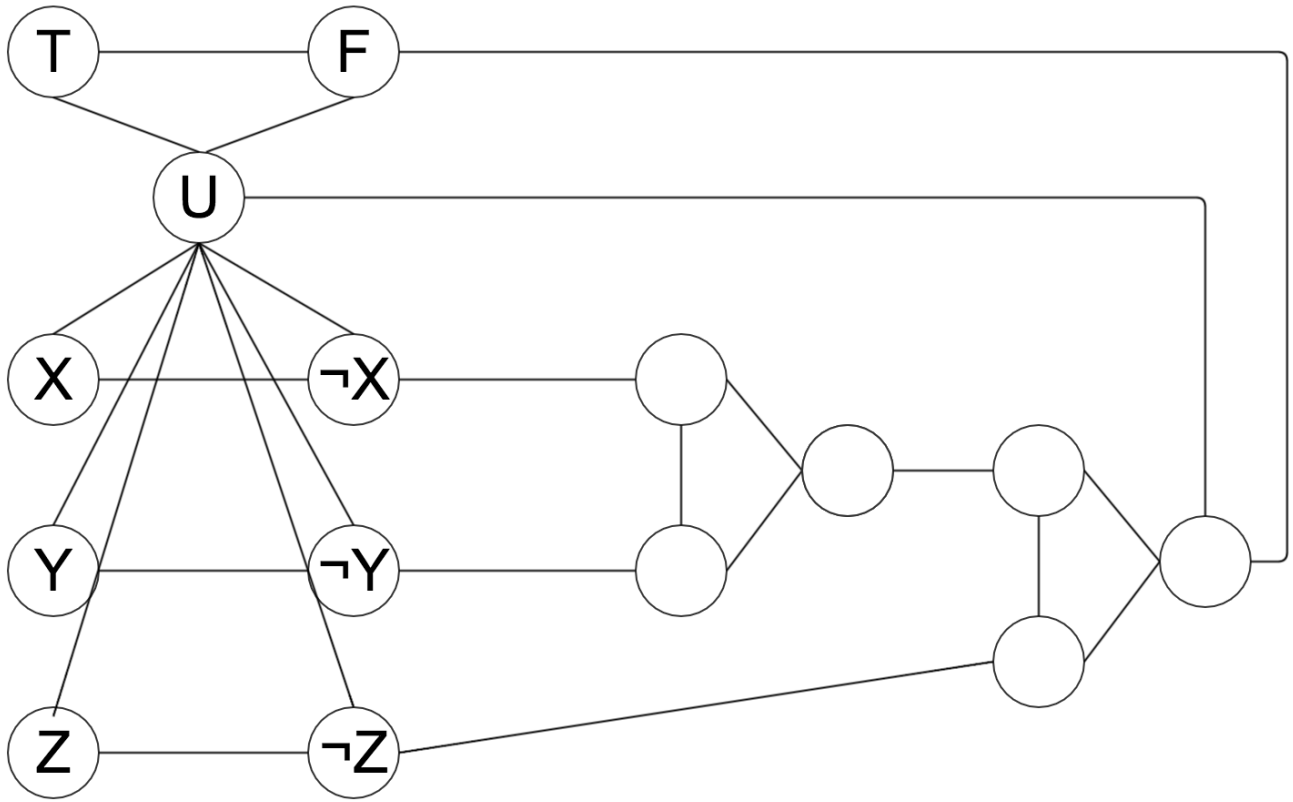

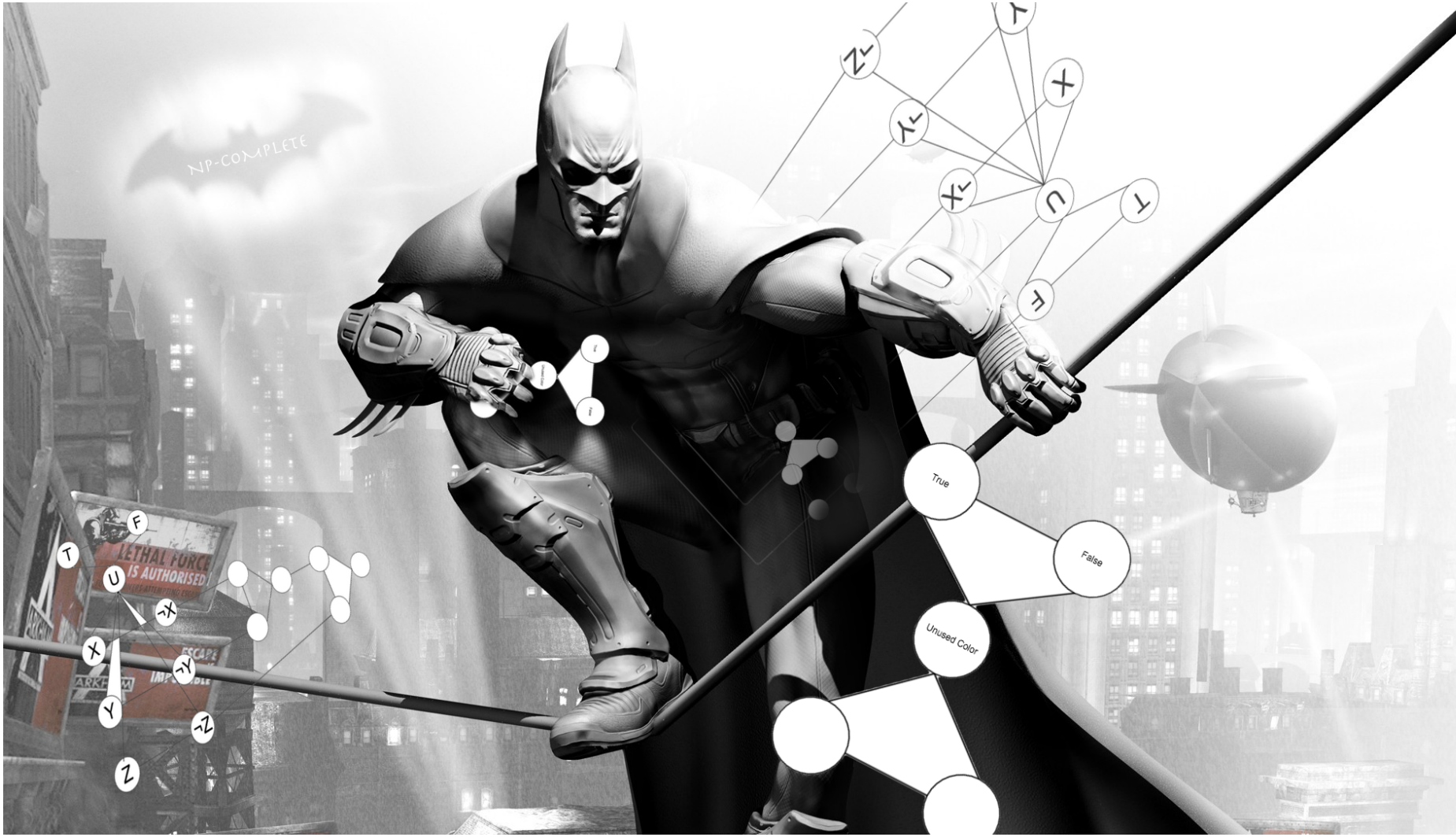
NP-COMPLETE
LETHAL FORCE
IS AUTHORISED
ESCAPE
IMPOSSIBLE
F
T
U
¬X
X
¬Y
Y
¬Z
Z
¬Z
Y
X
¬Y
¬X
U
T
F
True
False
Unused Color

Transformation

TODO: Formal write it

1. Construct the truthiness subgraph T

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4. For each variable in the Boolean statement:

- Add nodes v and v_0 connected by an edge
- Connect nodes v and v_0 to the "unused" end of t
- Connect node v_0 to one input of the clause's 3 way OR gate O_i

Transformation - Forward

Forward proof

Transformation - Backward

Backwards proof

Transformation - P

poly time? See small.ppt

Sources