Cellular Automata

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Outline

The Basics

- Spatial Structure

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- Local Interactions

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- Local Interactions
- Cell State

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- Cell State
- Cell Transitions

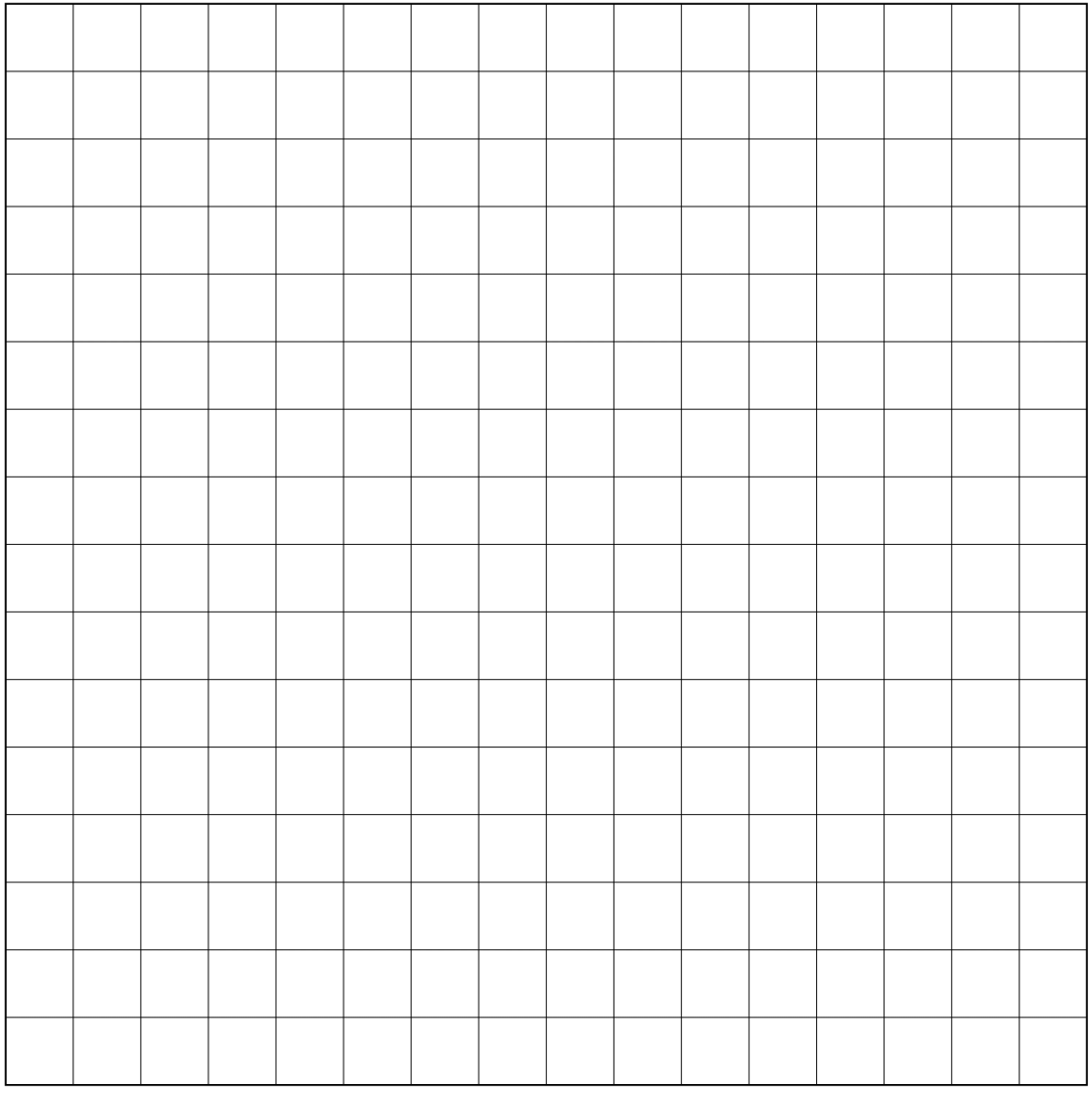
Spatial Structure

- Grid - Like a city

Spatial Structure

- Grid - Like a city
- Agents - Like a home in the city

Spatial Structure



Local Interactions

Agents may only interact with a set number of agents around them. This region is called a neighborhood.

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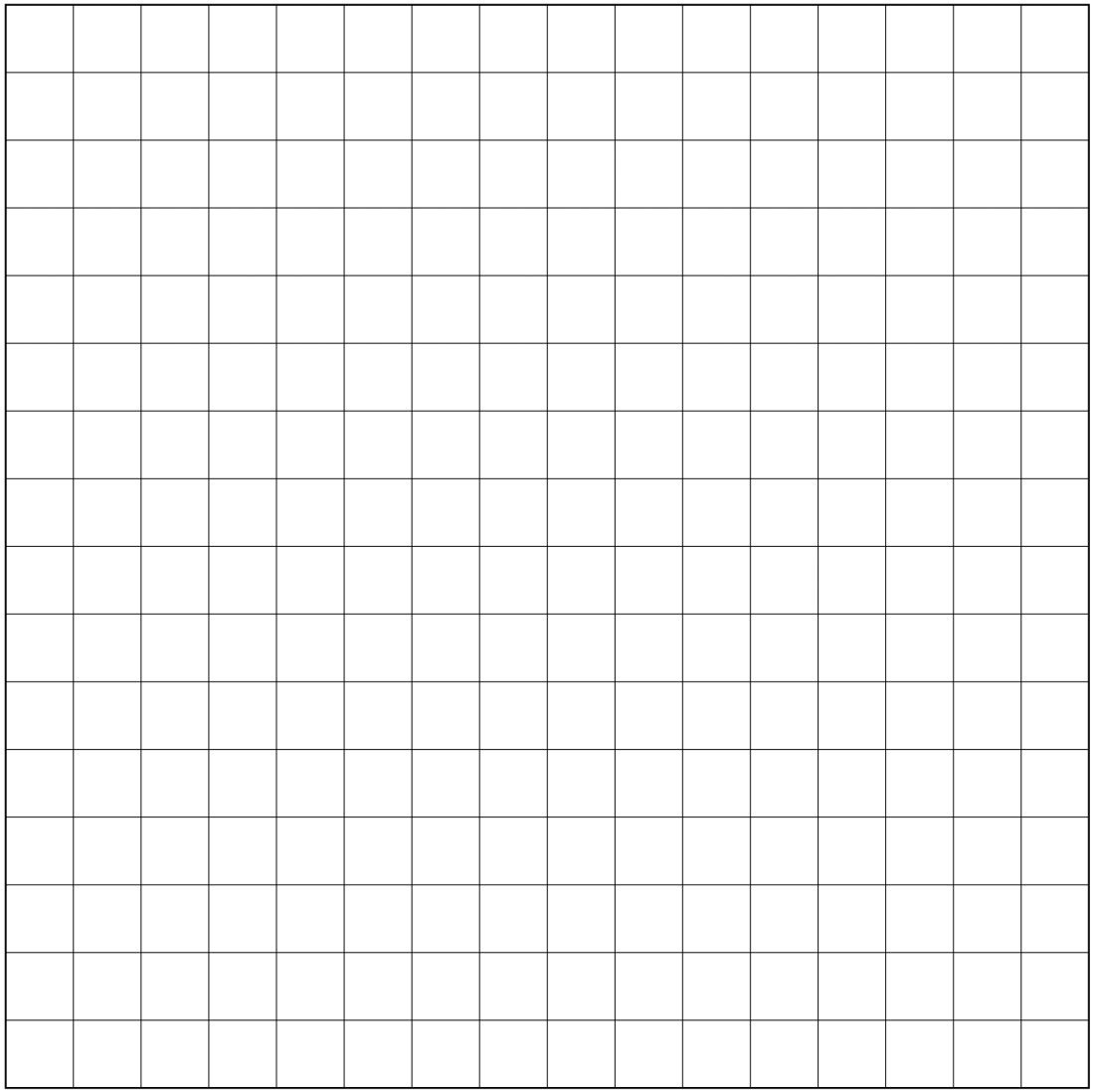
Locality indicates how many agents a single agent interacts with. High locality indicates more interactions (Think of how a rumor spreads).

Local Interactions



Cell State

Cells have a state

- Binary

Cell State

Cells have a state

- Binary
- Any number of states

Cell Transitions

CA have discrete chunks of time called rounds. Each round produces the next "generation" of cells.

Cell Transitions

The next state of a cell is a function of:

- It's current state

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Think of your opinion on a topic changing by being surrounded by people with a different view.

Elementary Cellular Automaton

What is the simplest...

- Grid of cells?

Elementary Cellular Automaton

What is the simplest...

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- Useful number of states?

Elementary Cellular Automaton

What is the simplest...

- Grid of cells?
- Useful number of states?
- Neighborhood?

Elementary Cellular Automaton

What is the simplest...

- Grid of cells - an array
- Useful number of states - binary
- Neighborhood - 2 adjacent cells of the agent

Elementary Cellular Automaton (Exercise)

How do we describe the state of cell c_t as a function of the neighbors of c_t at time $t-1$?

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- There are $2^3 = 8$ neighborhood combinations for any given neighborhood.
- Simply create an assignment for each combination!
- This assignment is called a rule set (how many possible rule sets?)

Example Program

- http://mathworld.wolfram.com/ElementaryCellularAutomaton.html
- https://github.com/westonkd/Completeness/tree/CA

Interesting Tidbits

- Rule 30 - Random number generator for Wolfram language

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- Rule 22 - Sierpinski triangle

2D CA Example - Conway's Game of Life

Conway's Game of Life

1. Rules

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2. Entities

Conway's Game of Life

1. Rules
2. Entities
3. Results

Conway's Game of Life

1. Rules
2. Entities
3. Results
4. Computational tidbits

Conway's Game of Life Rules

- 2 states - dead or alive

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- 2 states - dead or alive
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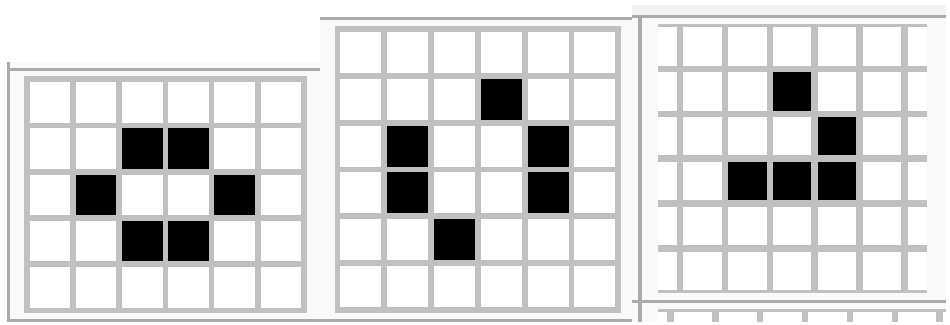
2. Any cell with 2 or 3 live neighbors lives on to the next generation.
3. A cell with more than 3 live neighbors dies (over crowding).
4. A dead cell with exactly 3 live neighbors becomes alive (reproduction).

Practice



Conway's Game of Life Entities

1. Still lives - entity stays the same through generations

`https://en.wikipedia.org/wiki/Conway%27s_Game_of_Life`

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Conway's Game of Life Entities

1. Still lives - entity stays the same through generations
2. Oscillators - entity changes shape and returns to original position (periods)
3. Spaceships - Moving oscillators

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Conway's Game of Life Results

1. Fade away completely

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Conway's Game of Life Results

1. Fade away completely
2. Stable configuration
3. Oscillating phase
4. ...Gosper glider gun

Demo

- http://www.bitstorm.org/gameoflife/

Conway's Game of Life Turing Completeness

- Conway's Game of Life is Turing Complete
- `http://rendell-attic.org/gol/tm.htm`
- `https://www.youtube.com/watch?v=My8AsV7bA94`

Conway's Game of Life Decidability

The question - "Given a starting pattern and an ending pattern, will the starting pattern ever reach the ending pattern?"

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- Undecidable - halting problem

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- Undecidable - halting problem
- "Indeed, since the game of life includes a pattern that is equivalent to a UTM (universal Turing machine), this "deciding" algorithm, if existed, could have been used to solve the halting problem, by taking the initial pattern as the one corresponding to a UTM+input and the later pattern as the one corresponding to a halting state of the machine with an empty tape (as one can modify the Turing machine to always erase the tape before halting). However the halting problem is provably undecidable and so such an algorithm does not exist" (Wikipedia).

CAs in the Wild

- Image processing (pixels)

CAs in the Wild

- Image processing (pixels)
- Nature

CAs in the Wild

- Image processing (pixels)
- Nature
- Migration patterns

CAs in the Wild

Extra: 3D CAs

http://cubes.io/

Going Further

- Crazy grids

Going Further

- Crazy grids
- Probability

Going Further

- Crazy grids
- Probability
- History

Sources

- https://en.wikipedia.org/wiki/Conway%27s_Game_of_Life
- https://www.youtube.com/watch?v=W1zKu3fDQR8
- https://www.youtube.com/watch?v=EyrwOf239M4
- https://www3.nd.edu/~mtns/papers/17761_4.pdf
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