

# Completeness: 3Color

Weston Dransfield

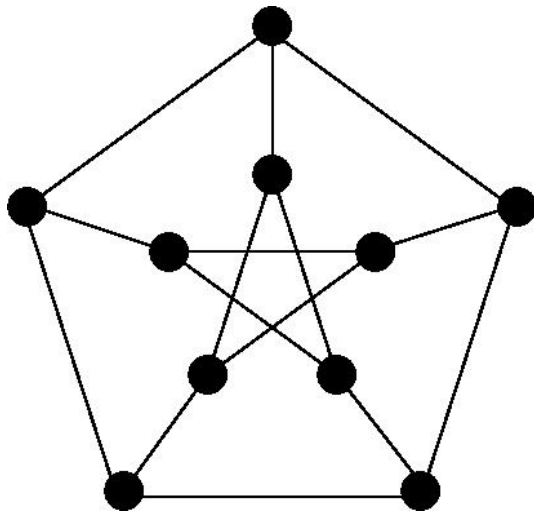
March 12, 2016

# Outline

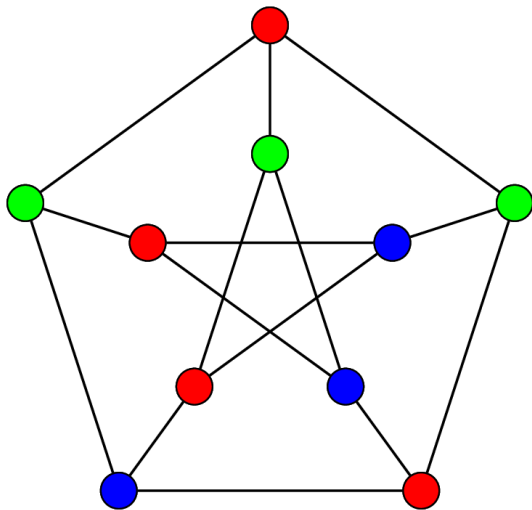
# Description

**3COLOR** =  $\{\langle G \rangle \mid \text{the nodes of } G \text{ can be colored with three colors such that no two adjacent nodes are the same color}\}$

# Example



# Example



# The Problem

Is a given graph  $G$  a member of the *3COLOR*?

# The Problem

Is a given graph  $G$  a member of the *3COLOR*?

- ▶ This is tough to decide, but easy to verify!

# The Verifier

$V =$  "On input  $\langle G, c \rangle$ ,

1. Check that  $c$  includes 3 colors.



# The Verifier

$V =$  "On input  $\langle G, c \rangle$ ,

1. Check that  $c$  includes 3 colors.
2. Color each node of  $G$  as specified by  $c$ .

# The Verifier

$V =$  "On input  $\langle G, c \rangle$ ,

1. Check that  $c$  includes 3 colors.
2. Color each node of  $G$  as specified by  $c$ .
3. For each node, check that each adjacent node is not the same color.

# The Verifier

$V =$  "On input  $\langle G, c \rangle$ ,

1. Check that  $c$  includes 3 colors.
2. Color each node of  $G$  as specified by  $c$ .
3. For each node, check that each adjacent node is not the same color.
4. If all checks pass accept, otherwise reject."

# The Verifier

$V =$  "On input  $\langle G, c \rangle$ ,

1. Check that  $c$  includes 3 colors.
  2. Color each node of  $G$  as specified by  $c$ .
  3. For each node, check that each adjacent node is not the same color.
  4. If all checks pass accept, otherwise reject."
- Step 3 has largest time complexity of  $O(n^2)$ . 3COLOR is in NP because it can be verified in polynomial time.

# Constructing the Reduction

Construct a transformation  $f$  from  $3SAT$  to  $3COLOR$ .

# Constructing the Reduction

Construct a transformation  $f$  from  $3SAT$  to  $3COLOR$ .

1. Establish Truthiness

# Constructing the Reduction

Construct a transformation  $f$  from  $3SAT$  to  $3COLOR$ .

1. Establish Truthiness
2. Force variables to be true or false

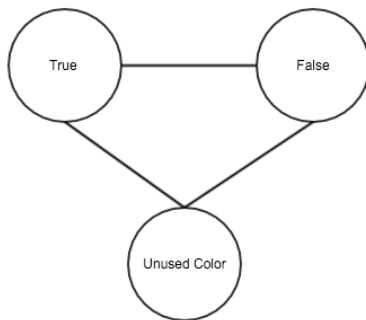
# Constructing the Reduction

Construct a transformation  $f$  from  $3SAT$  to  $3COLOR$ .

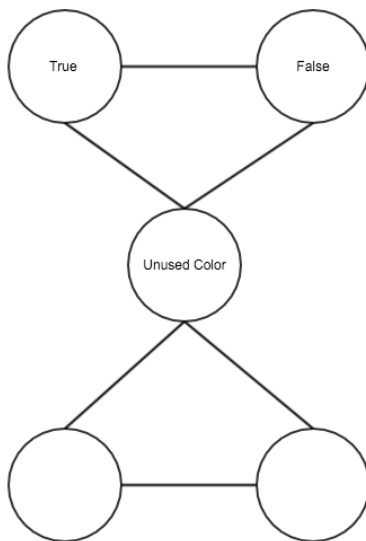
1. Establish Truthiness
2. Force variables to be true or false
3. Use these subgraphs to create a graph that is 3 colorable iff variables are satisfiable



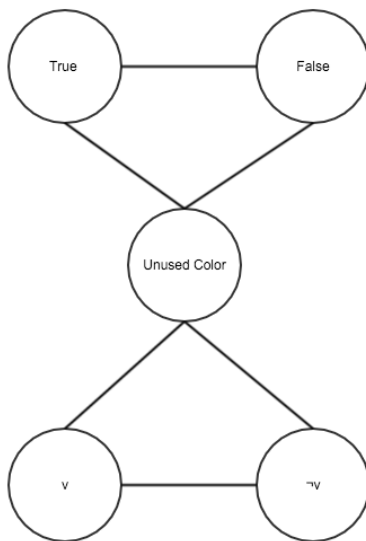
# Constructing the Reduction - Truthiness



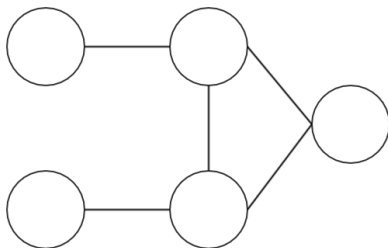
# Constructing the Reduction - Variables



# Constructing the Reduction - Variables

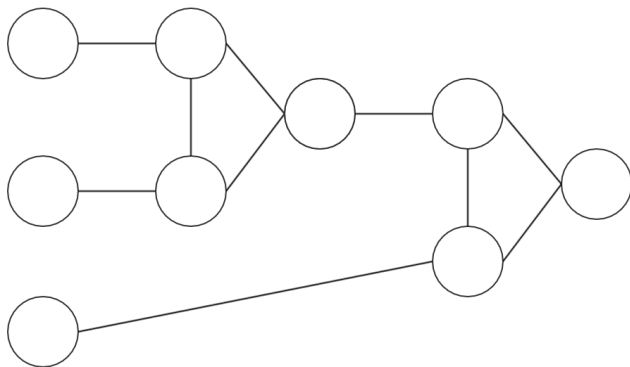


# Constructing the Reduction - OR

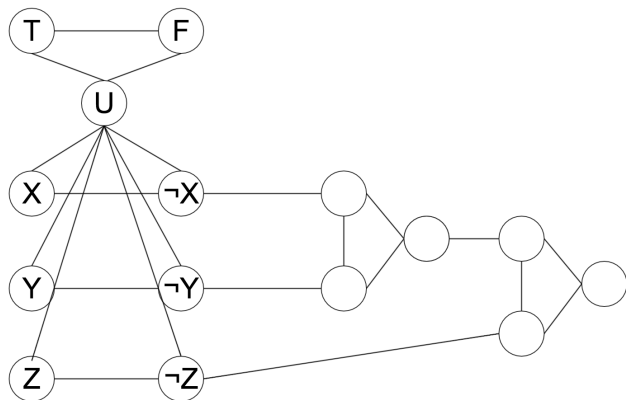


$$x \vee y$$

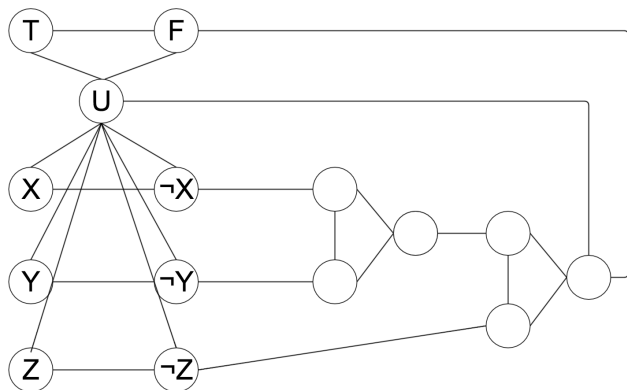
# Constructing the Reduction - OR



# Constructing the Reduction - Clause



# Constructing the Reduction - Clause



# Transformation

How to construct the transformation



# Transformation - Forward

Forward proof

# Transformation - Backward

Backwards proof

# Transformation - P

poly time? See small.ppt

# Sources