Completeness: 3Color

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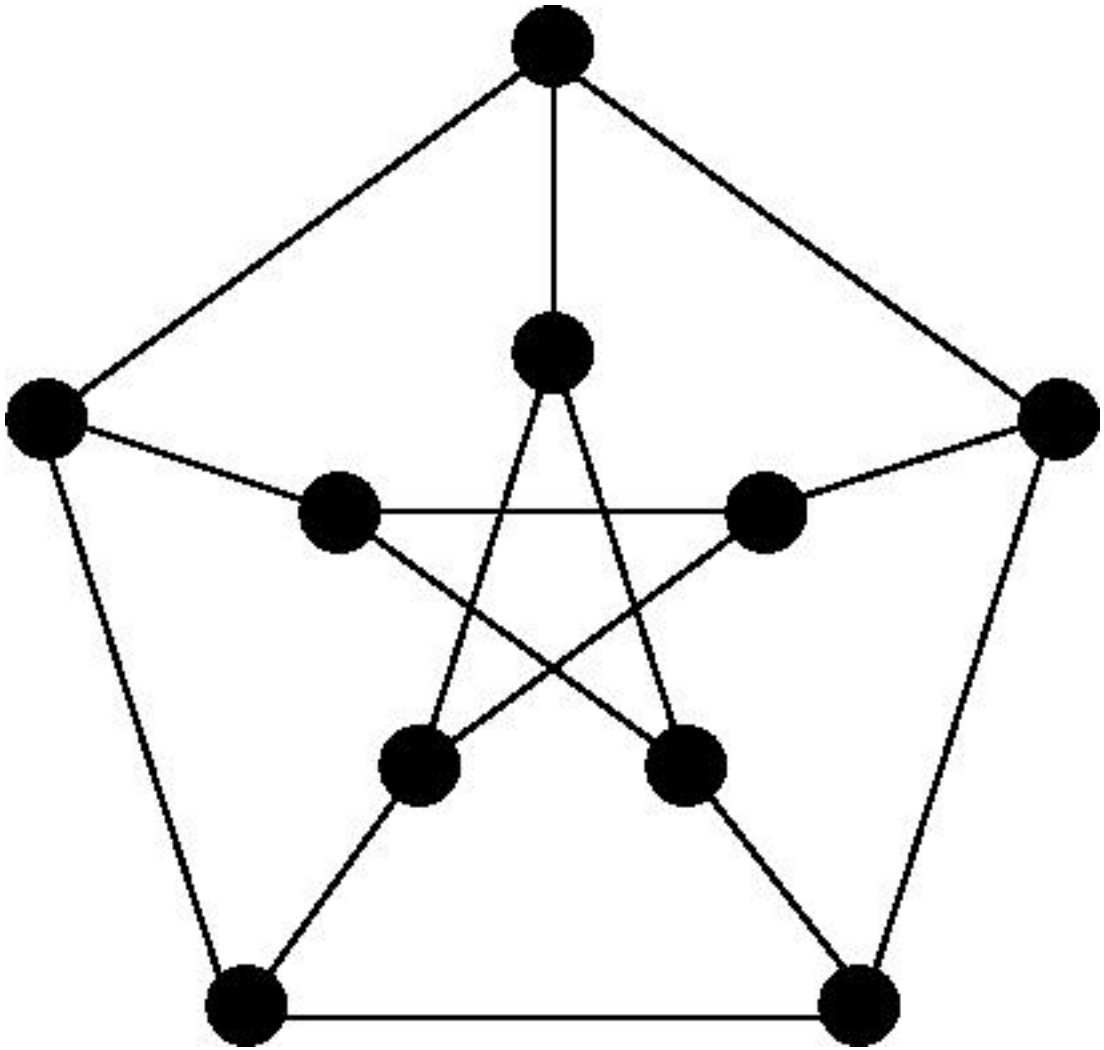
Outline

Description

$\textit{3COLOR} = \{\langle G \rangle \mid$ the nodes of G can be colored with three colors such that no two adjacent nodes are the same color $\}$

Example



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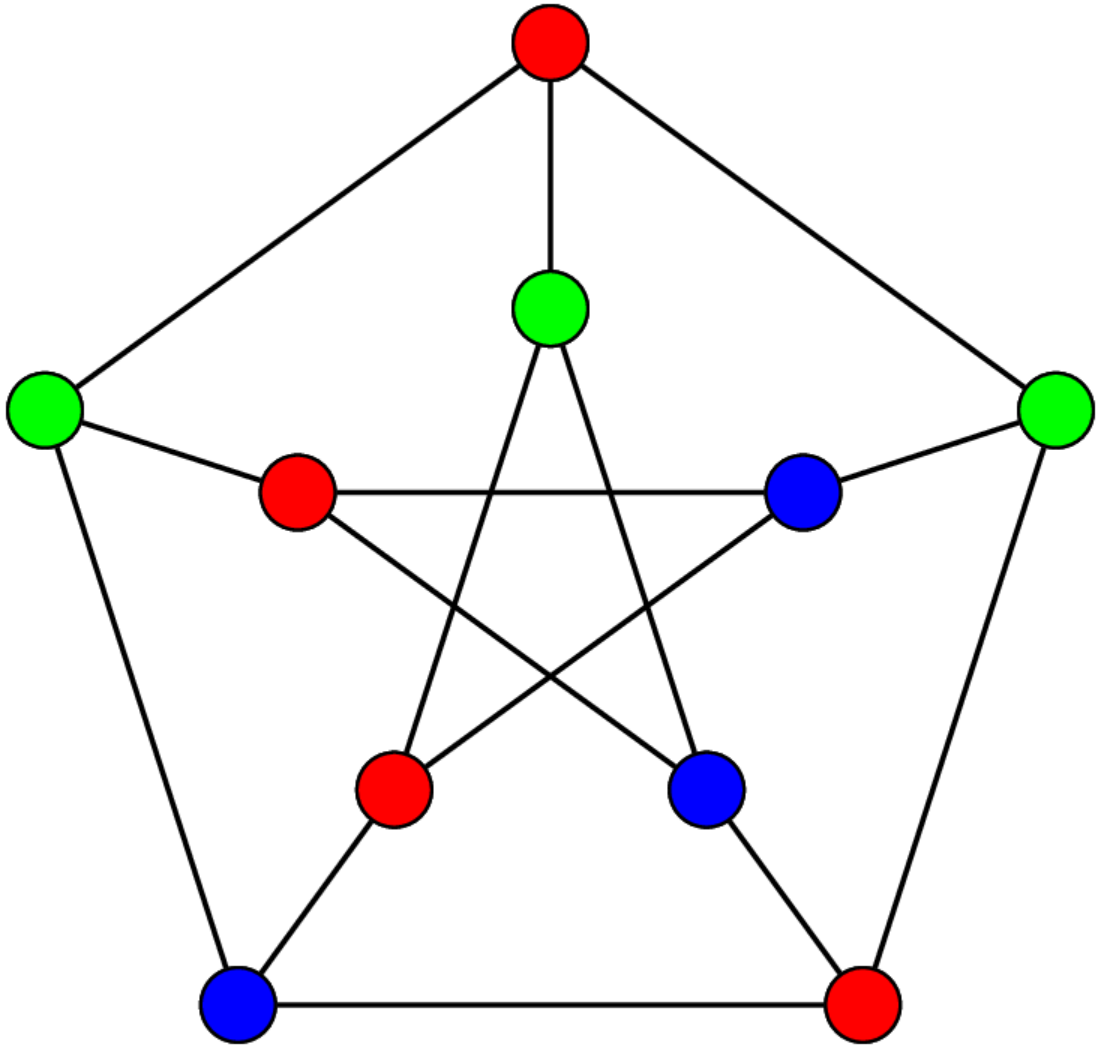

The Problem

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- This is tough to decide, but easy to verify!

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- Step 3 has largest time complexity of $O(n^2)$. 3COLOR is in NP because it can be verified in polynomial time.

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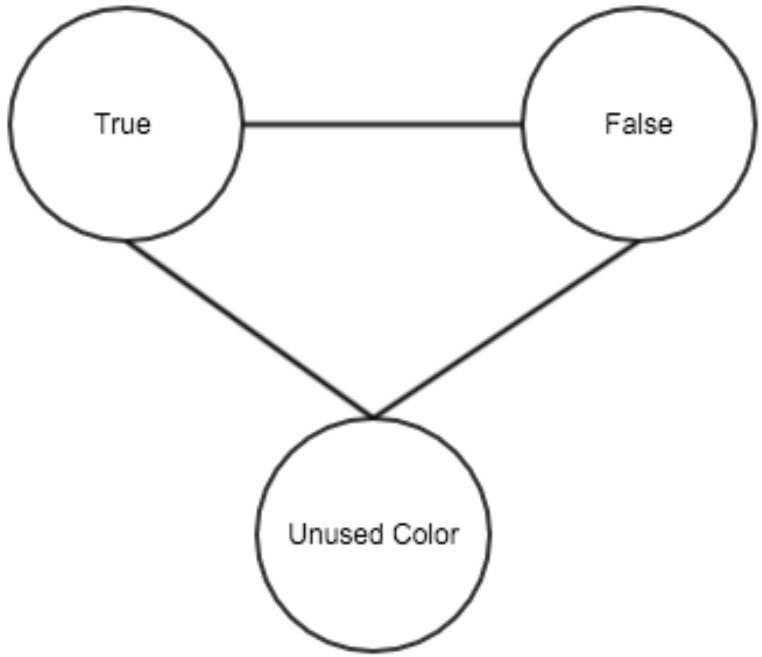
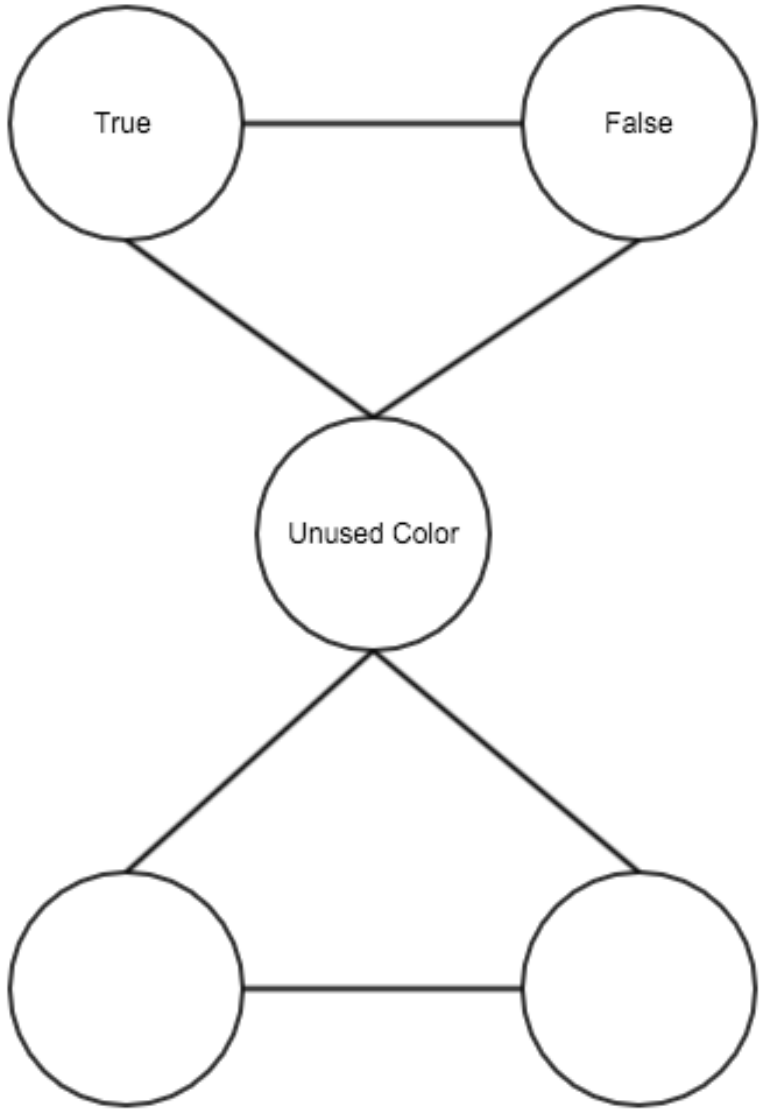
2. Force variables to be true or false
3. Use these subgraphs to create a graph that is 3 colorable iff variables are satisfiable

Constructing the Reduction - Truthiness



Constructing the Reduction - Variables



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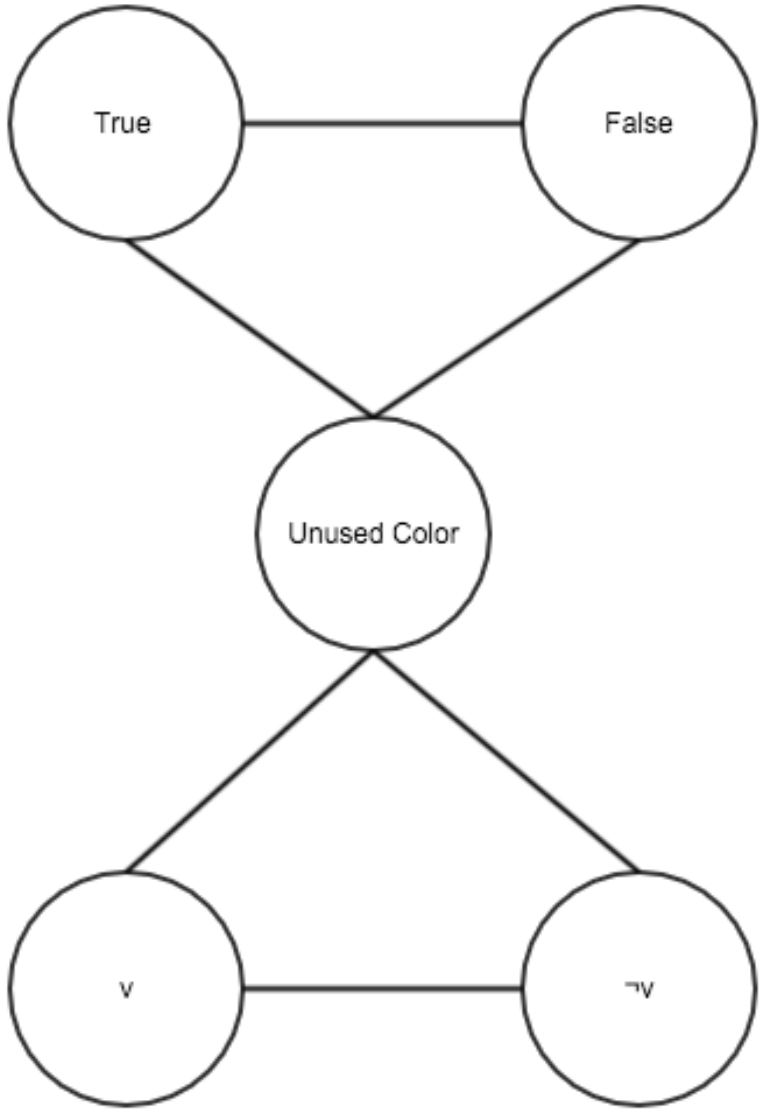
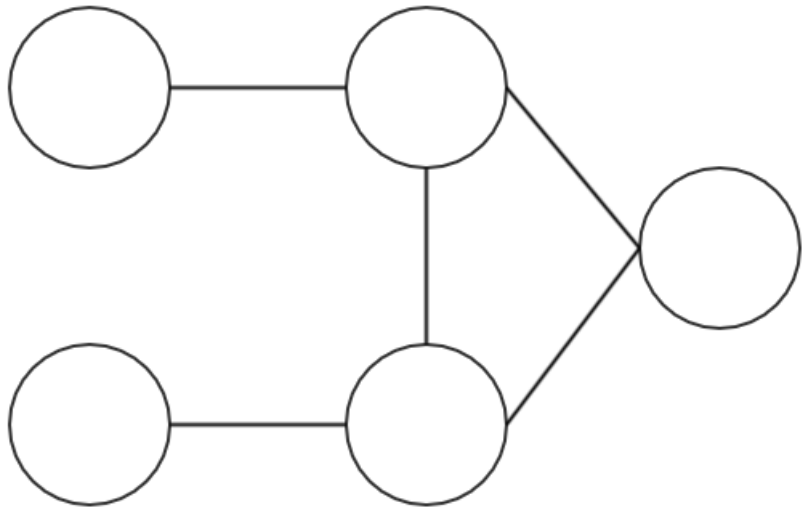
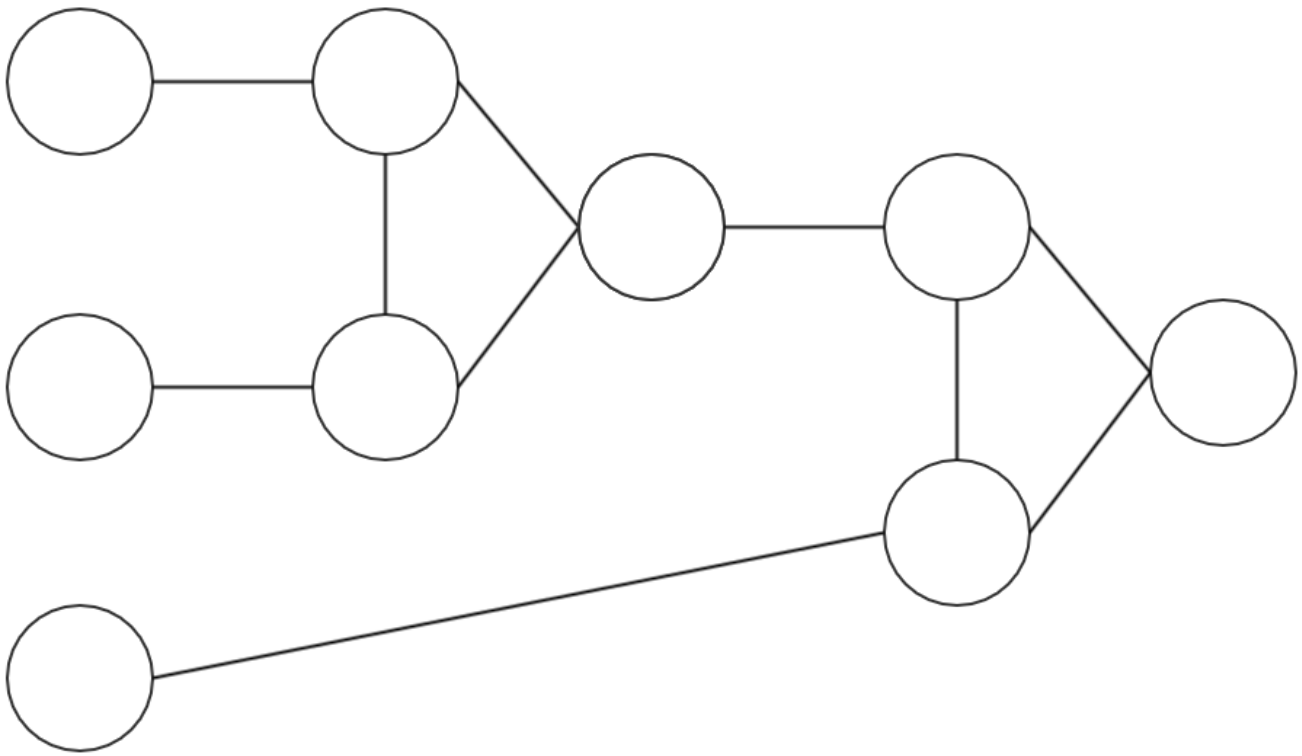


Constructing the Reduction - OR

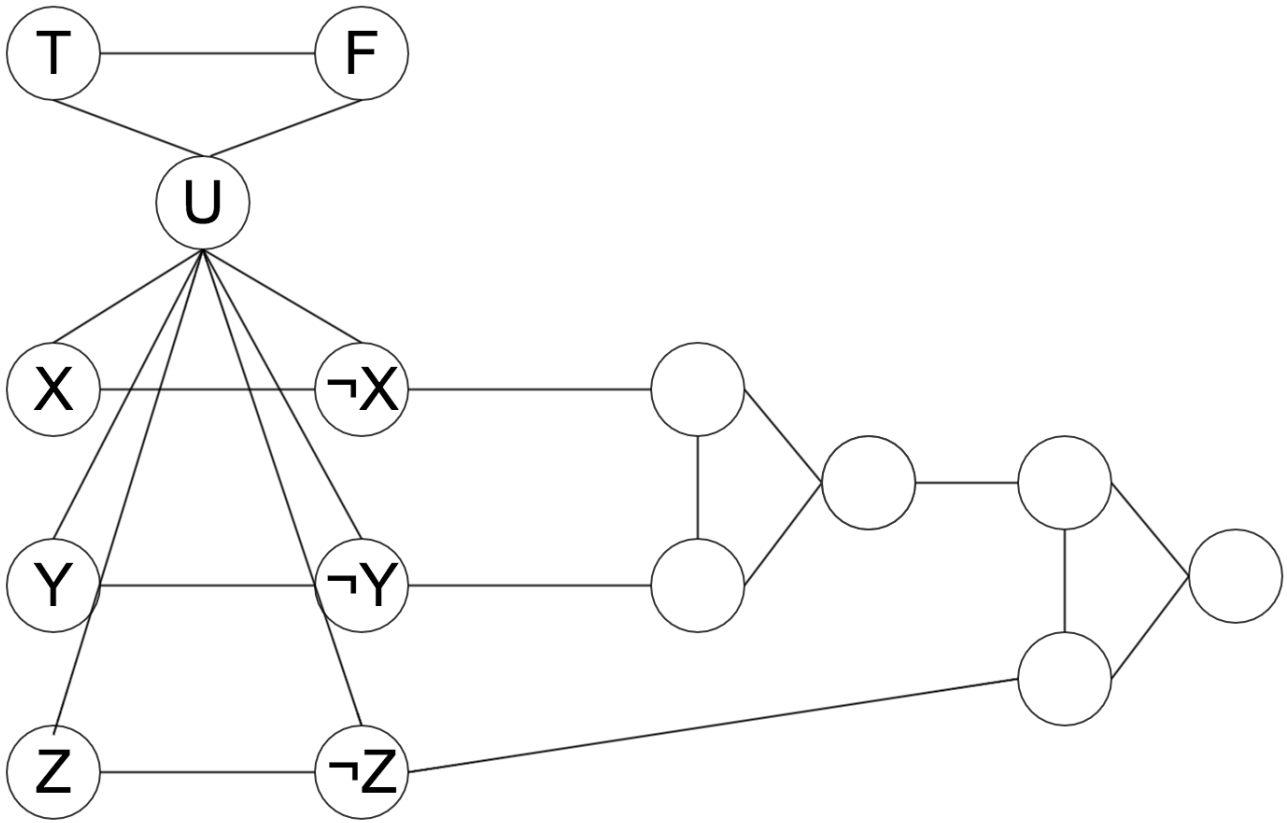


$x \vee y$

Constructing the Reduction - OR



Constructing the Reduction - Clause



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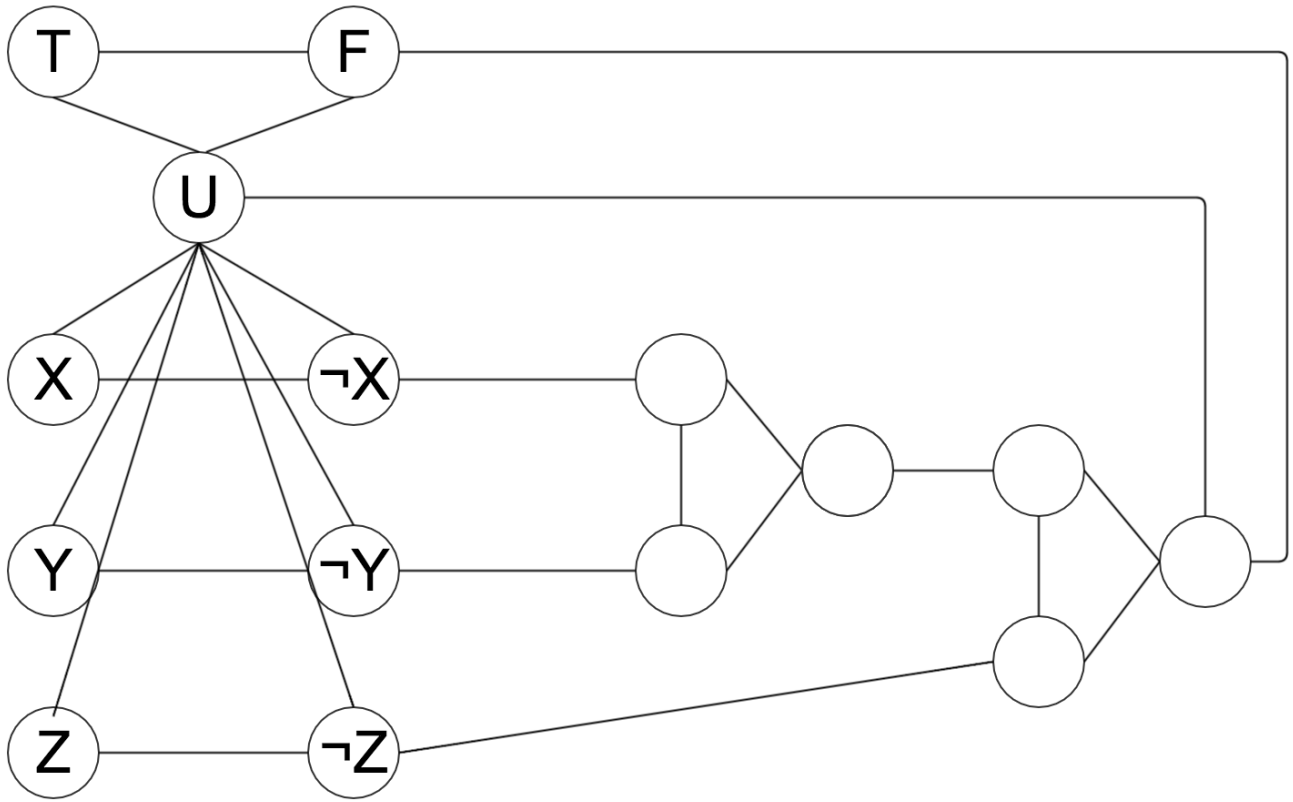

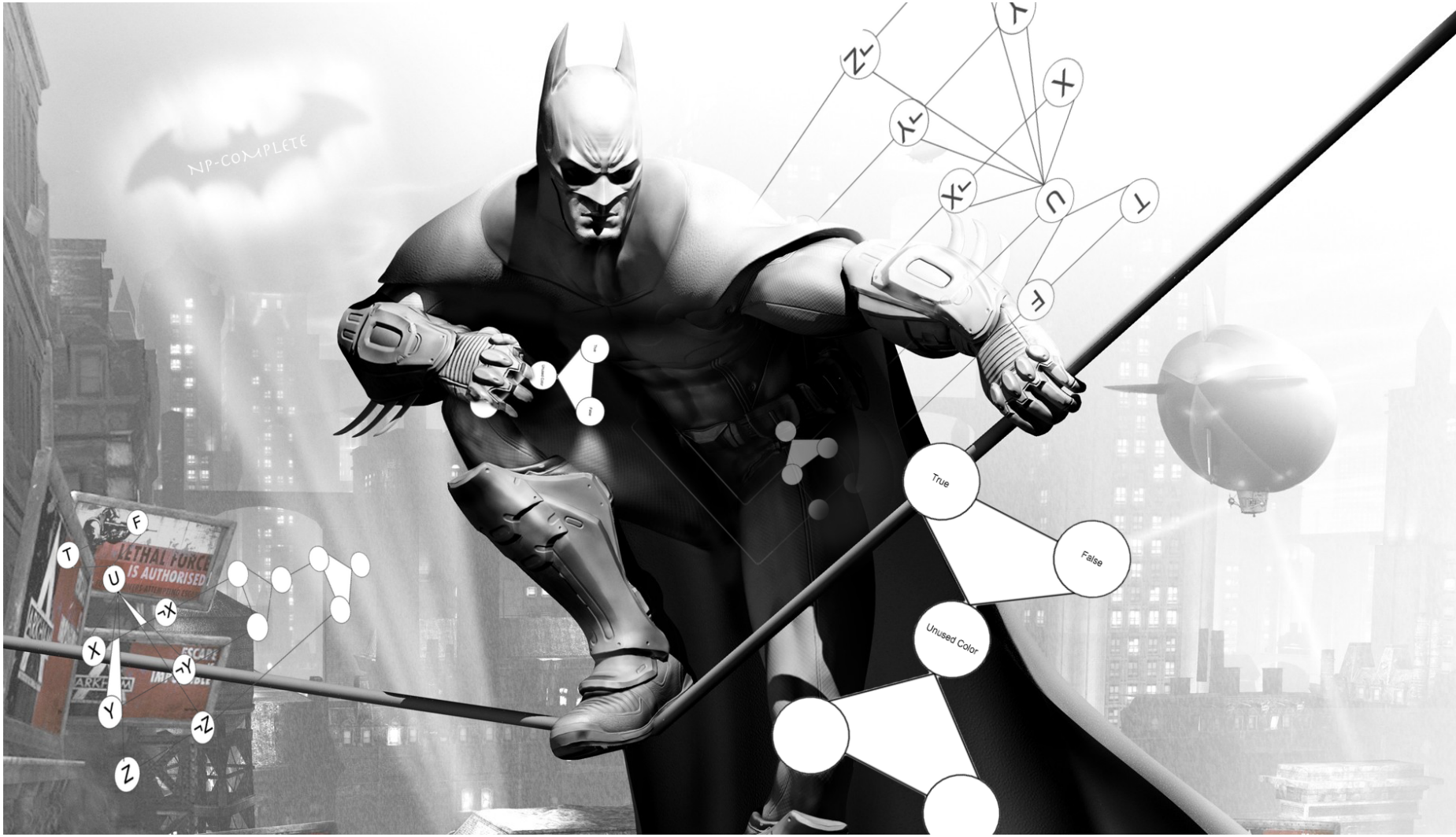
NP-COMPLETE
LETHAL FORCE
IS AUTHORISED
ESCAPE
IMPOSSIBLE
MARKET
F
T
U
¬X
X
¬Y
Y
¬Z
Z
¬Z
¬Y
X
¬X
U
T
F
True
False
Unused Color

Transformation

How to construct the transformation

Transformation - Forward

Forward proof

Transformation - Backward

Backwards proof

Transformation - P

poly time? See small.ppt

Sources