APPM 2460 **Homework** **Week 4**

Submit a published pdf of your script and any other supporting code needed to solve the following problem to Canvas by Monday, February 10 at 11:59 p.m.

The *Binomial Distribution* is a mathematical function that provides the probability of outcomes for experiments satisfying the following conditions:

- It is possible to perform n trials of the experiment.
- In each trial of the experiment there are only two possible outcomes. Call these outcomes S and F.
- The probability of S is constant from one trial to the next and is denoted p. (By the laws of probability, F has a constant probability $1 - p$.)

The mathematical function that governs this probability distribution is

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

1. The *expected value* of a random variable is the average of all possible values that the variable could take. To compute the expected value of a binomial random variable, we must evaluate

$$\mathbb{E}[X] = \sum_{x=0}^{N} x \cdot f(x) = \sum_{x=0}^{N} x \cdot \binom{n}{x} p^x (1-p)^{n-x}$$

   (a) Write a function called `bin_mean` that computes $\mathbb{E}[X]$. This function should take in variables n (the number of trials of the experiment), p (the probability of S), and N (the upper bound of the sum, since we cannot run the sum all the way to infinity).[1]

   (b) Evaluate `bin_mean` if $n = 100$, $p = 0.3$ and $N = 75$.

   (c) If X is a binomial random variable, we should get $\mathbb{E}[X] = n \cdot p$. Does this match the answer you found in (b)?

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2. The *variance* of a random variable measures how different the outcomes of an experiment are.

   (a) One term needed to calculate variance is

$$\mathbb{E}[X^2] = \sum_{x=0}^{N} x^2 \cdot f(x) = \sum_{x=0}^{N} x^2 \cdot \binom{n}{x} p^x (1-p)^{n-x}$$

   Make a copy of your code `bin_mean`, name the copy `bin_xsq`, and adjust this code to compute $\mathbb{E}[X^2]$ instead of $\mathbb{E}[X]$.

   (b) By definition, $Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$. Use `bin_mean` and `bin_xsq` together to compute variance if $n = 100$, $p = 0.3$ and $N = 75$.

   (c) If X is a binomial random variable, we should get $Var(X) = n \cdot p \cdot (1 - p)$. Does this match the answer you found in (b)?

[1] Hint: To compute $\binom{n}{k}$ you *must* use the code you wrote for homework 3. *DO NOT* use a built-in Matlab function.