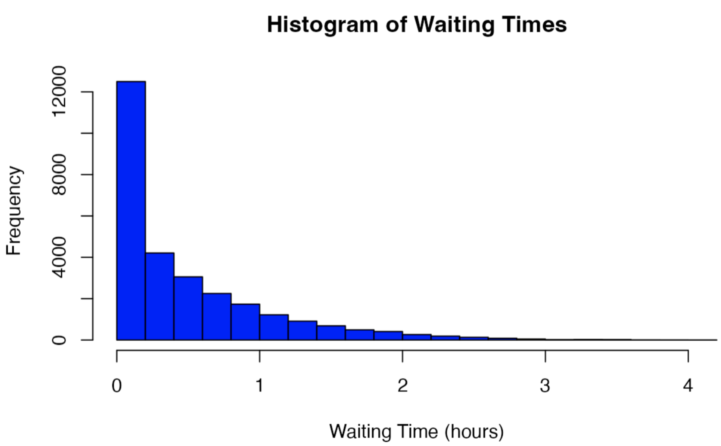
Weston Slayton and Egan Barzilay

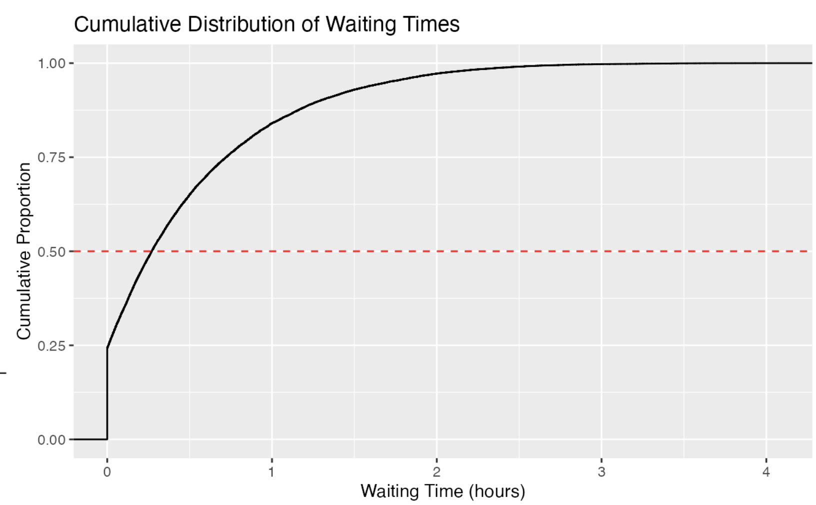
3 May, 2024

Final Project Report

In scenario 1, we have a one table restaurant open from 10am-10pm with one chef. We model the arrival of customers as a poisson process with an arrival rate of $\lambda_A = 5$ customers per hour. We also represent the service time with an exponential distribution with a rate $\lambda_S = 6$, meaning we expect it to take 10 minutes to serve a customer. Afterwards, we produce wait times by checking if the previous customer is still being served or is still waiting in line. Additionally, we don't allow people to be sat past 10pm.

In the conditions of scenario 1 over 500 days, there's a mean wait time of 29 minutes with variance of 20 minutes. Below shows an approximate wait time pdf (in histogram format, not normalizing for AUC = 1), as well as a CDF including CDF results for plugged in times from 0 to 60 minutes, in percentage format:

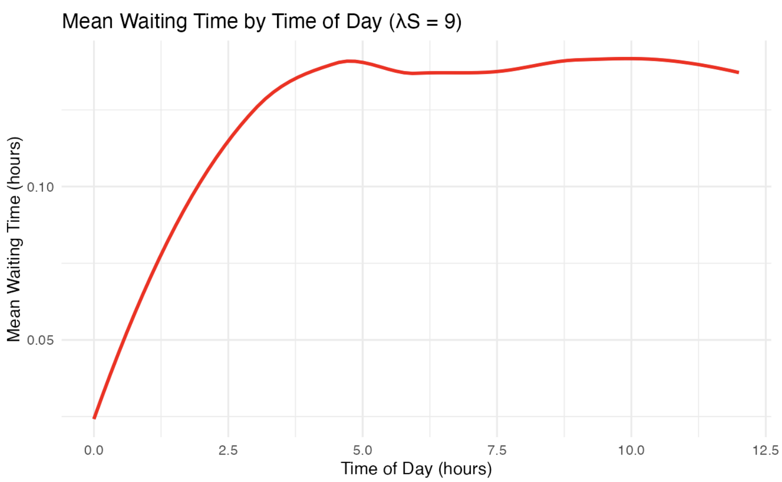


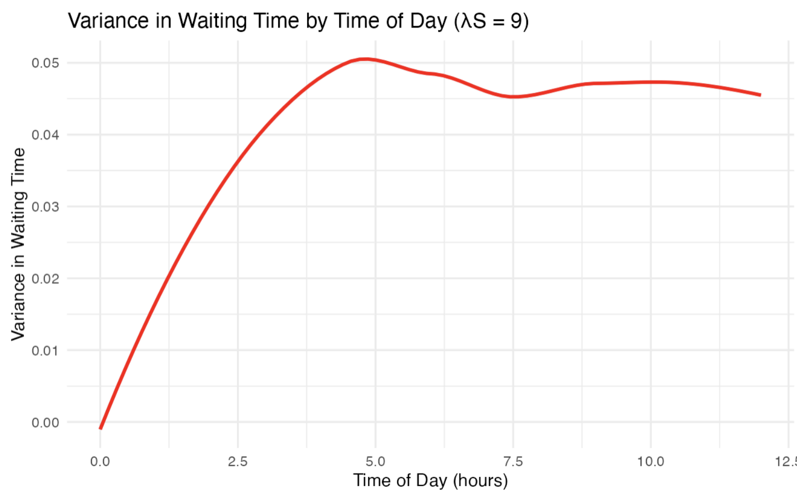


0 minutes	10 minutes	20 minutes	30 minutes	40 minutes	50 minutes	60 minutes
24.21615	41.25557	54.59339	65.03999	72.84663	78.93694	84.03638

These values are for wait times of <= 0 minutes, <= 10 minutes, etc… as percentages of people

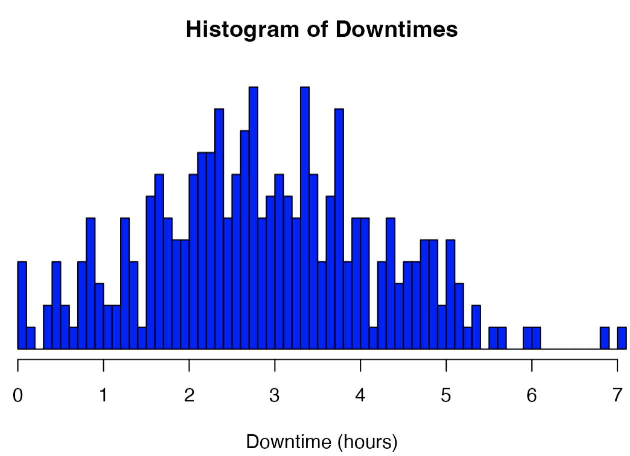
What we can see in the graph above is that about 25% of customers do not have to wait at all in this scenario. Among people that do wait, the mean wait time is 38 minutes, with a variance of 21 minutes. We can visualize mean wait time over the 12 hour operating time, starting from arrival times at hour 0 (10 am) to hour 12 (10 pm). Increasing the service speed allows us to see the leveling out of wait times past opening, as we can keep up with demand:





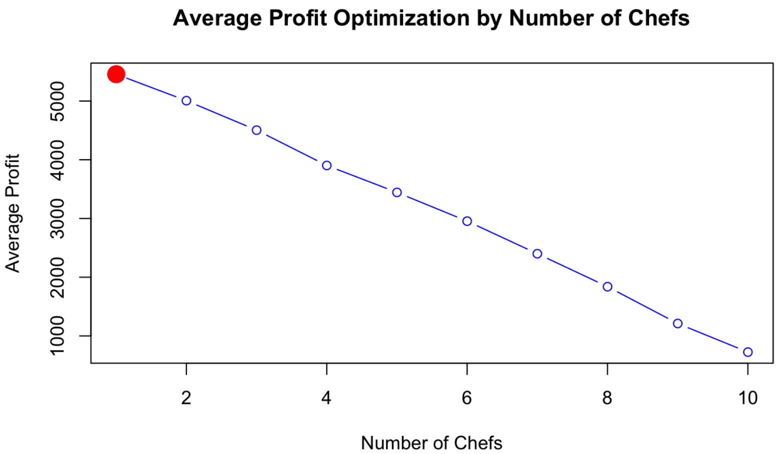
When the restaurant first opens, the table is open, so it makes sense that there is a low average wait time. As the day progresses the average wait time increases as more people show up, eventually reaching a steady state. Additionally, the variance in wait times is lower in the morning, as it's more probable that people don't have to wait for a table. It's useful to note that we can't keep up with demand (and reach a leveling out) without increasing service speed. This shows the importance of service speed in decreasing mean wait times, which we'll revisit later.

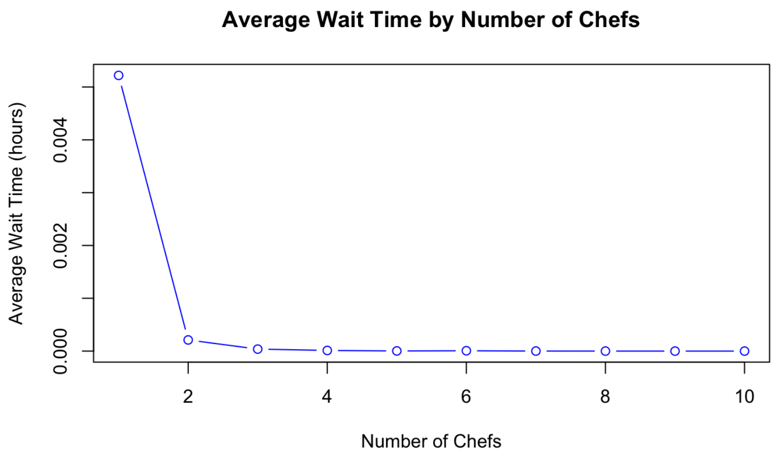
Returning back to $\lambda s=6$, we have a mean downtime throughout the day (no customers being served) of 2 hours and 53 minutes, with a variance of 1 hour 46 minutes. This is a relatively well-sized amount of downtime for 12 hours. We can visualize a histogram of downtimes over 1,000 days (Right).



Scenario 2:

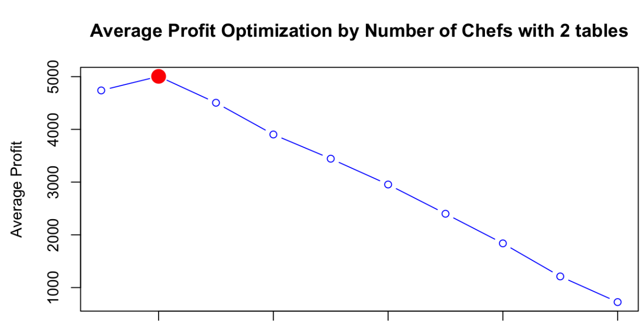
In this scenario, we now have 5 tables and we model customer arrivals with a poisson process with rate $\lambda_A = 10$ customers / hour. We model their service time as an exponential distribution with a rate $\lambda_S = 3L$ which depends on the number of chefs, L, that are working. We pay chefs \$40 an hour and charge each customer \$50 per meal. Under these conditions, we've plotted average profit and wait time as we vary the number of chefs (L):

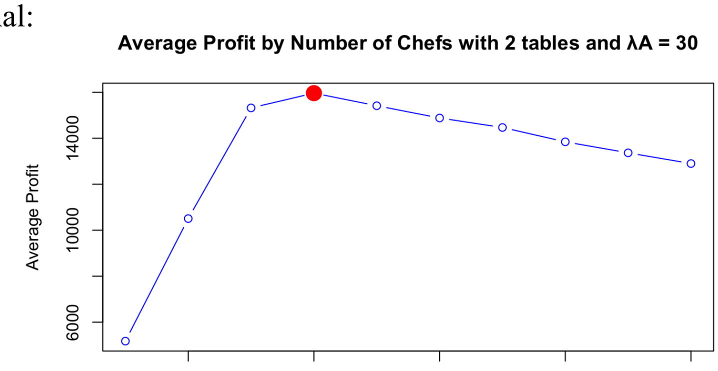




On the right, we see that with 1 chef, mean wait times are close to 0 (14 seconds with variance 2 seconds) and approximately 0 for two or more chefs. This is the reason that profits go down as chefs increase; increasing chefs doesn't allow us to meet more demand, as there aren't people waiting to begin with. Adding chefs would only increase costs, which is seen in the steady decrease in profits as the number of chefs go up. With 1 chef: mean daily profit is \$5,446.90 with standard deviation of \$518.96.

1 chef being optimal is not always the case. If we reduce the number of tables to 2 and hold all the other parameters the same, we see that 2 chefs is optimal. If we have a really busy day and $\lambda_A = 30$, we see that 4 chefs is optimal:

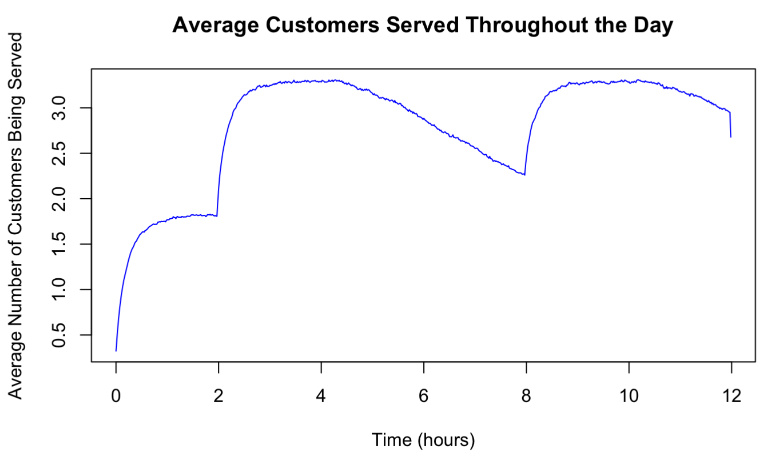


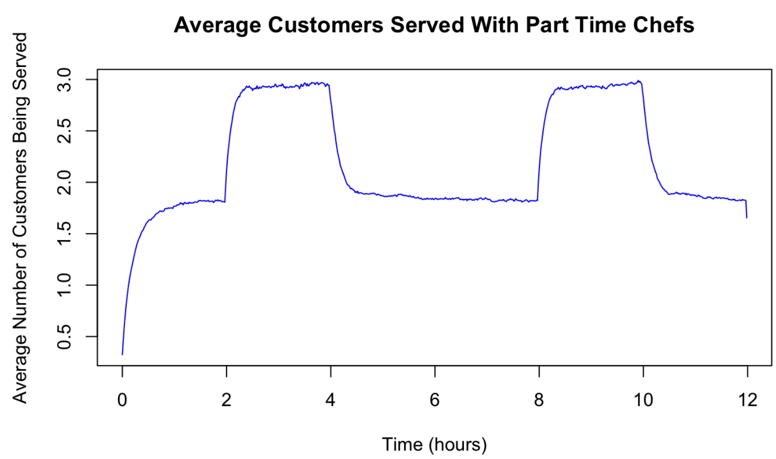


As our capacity to serve customers decreases (less tables) or demand increases (more arrivals), increasing the number of chefs allows us to serve more customers that would've otherwise waited in line. Under conditions such that higher revenues outweigh the costs of hiring more chefs, having more than 1 chef is optimal.

Now, adding more nuance to scenario 2, suppose we adjust our arrival distribution to have more customers / hour in peak hours: $\lambda_A = 30$ from 12-2pm and 6-8pm, representing lunch and dinner. With 1 chef and 2 tables, we can't deal with the demand at peak hours, and wait times pile up to a mean of 2 hours 47 minutes.

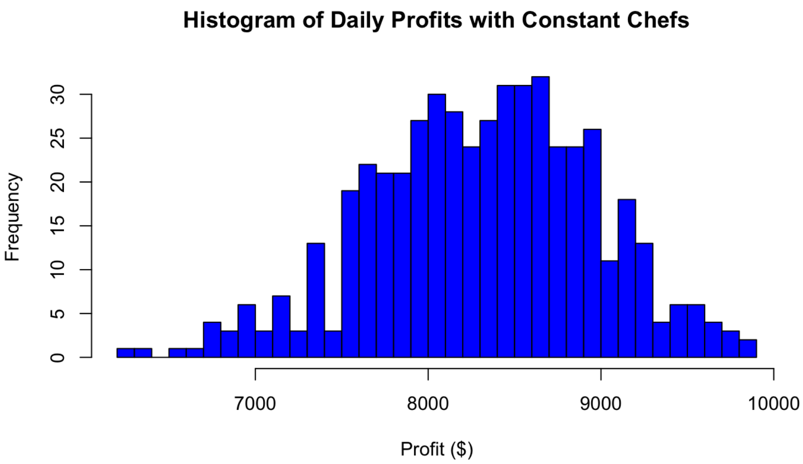
To deal with increased demand from peak hours, it may make sense to hire more chefs. We can try two approaches:

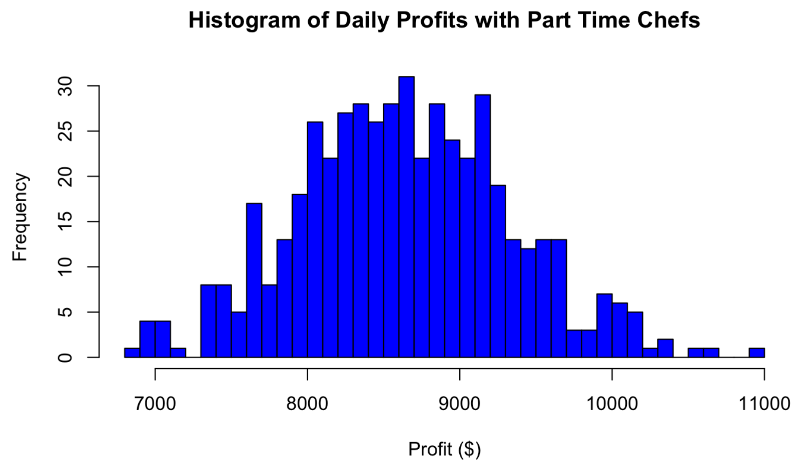




On the left, we hire 2 chefs for the entire day, the optimal number of chefs for profit when we have a λ_A of 10 and 2 tables. This reduces mean wait time to 43 minutes, though there isn't much downtime in off-peak hours (mean customers being served stays high from 12pm on). On the right, we hire an additional 2 chefs during peak hours. This allows us to fully meet the demand of lunch and dinner and steadily return to downtimes (less customer served) during off-peak hours. Mean wait times fall all the way down to 6 minutes, with a standard deviation of 9 minutes.

To analyze the profitability of these two approaches for dealing with peak hours, we can plot a histogram of daily profits over 500 days:





When we keep the chefs constant at 2, we observe a daily mean profit of $8,323, with a standard deviation of $647. Our minimum profit is $6,290 and our maximum profit is $9,890. When we hire an additional 2 chefs for lunch and dinner, we observe a daily mean profit of $8,630, with a standard deviation of $699. Our minimum profit is $6,820 and our maximum profit is $10,920. These results show a $307 dollar increase in mean profits with only a $52 increase in standard deviation. Therefore, with part-time chefs we are also able to meet total customer demand, increasing our profits with low variance, despite higher wages.

In a more realistic scenario 2 where we have varying rates of customers depending on the time of day, hiring part-time chefs during these hours is optimal for restaurant downtime, customer satisfaction (lower waiting times), and profits.