CAP5415-Computer Vision
Lecture 2-Filtering

Ulas Bagci

bagci@ucf.edu

Outline

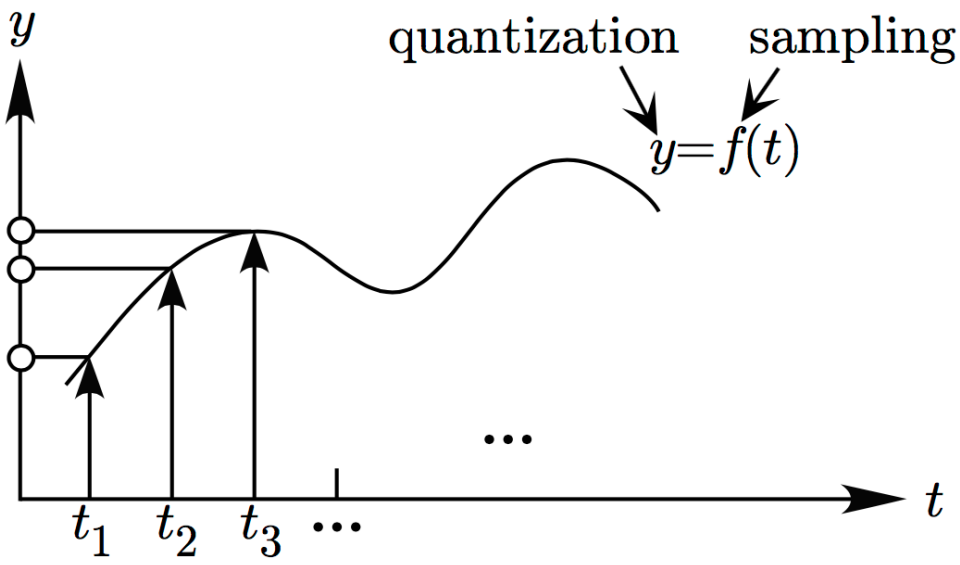
- Image as a function
- Extracting useful information from Images
  - Edges
  - Smoothing/Removing noise
  - Convolution/Correlation
  - Image Derivatives/Gradient
  - Histogram
- Some coding examples

- *Read Szeliski, Chapter 3.*
- *Read Shah, Chapter 2.*
- *Read/Program CV with Python, Chapters 1 and 2.*

PICTURES

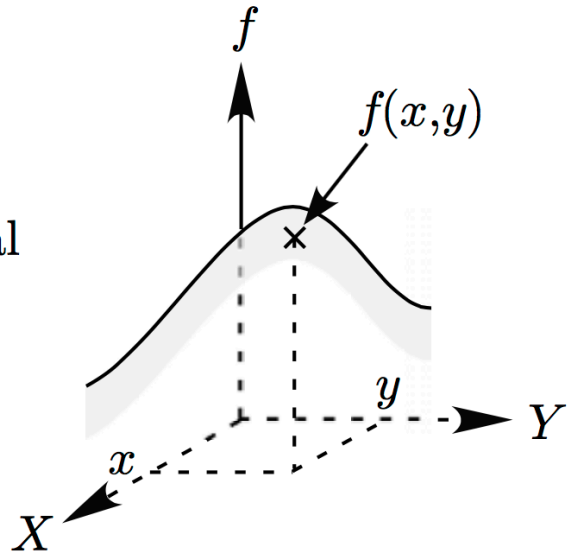
- Computers use discrete form of the pictures
- The process transforming *continuous space* into *discrete space* is called digitization

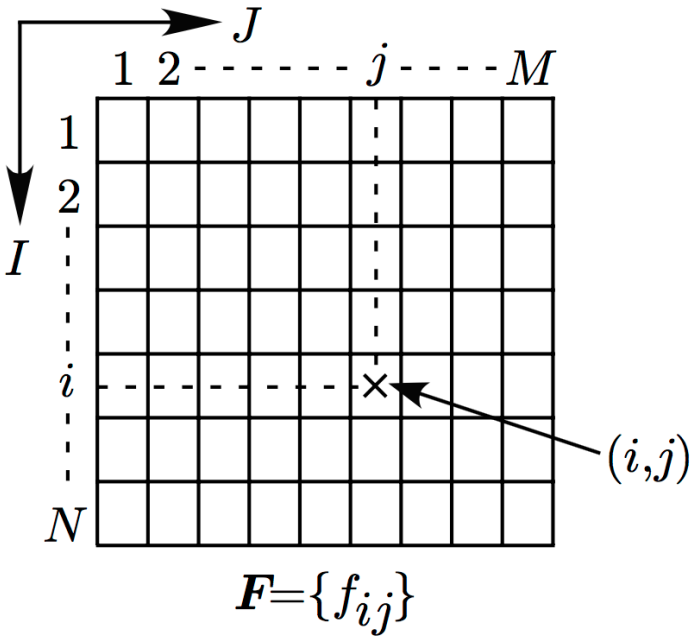
one-dimensional



continuous signal → digitized signal

two-dimensional

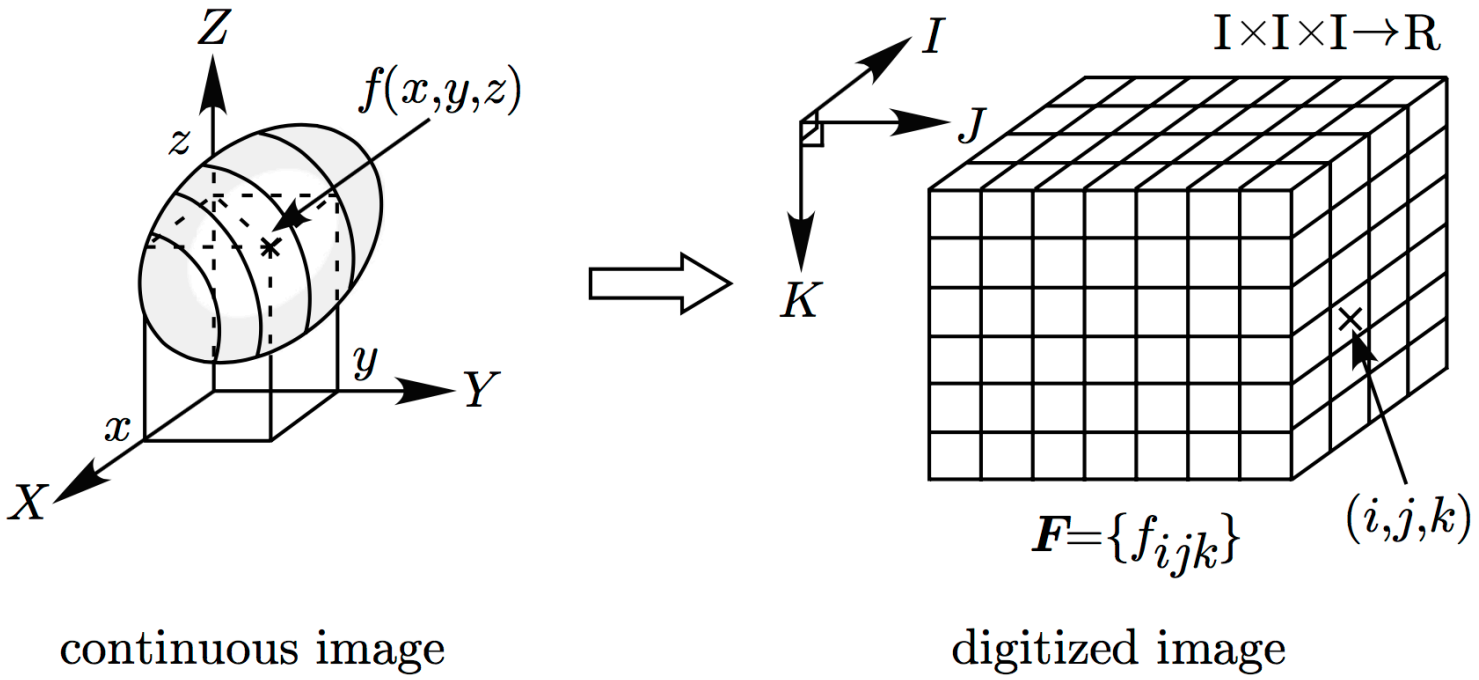




$\boldsymbol{F}=\{f_{ij}\}$

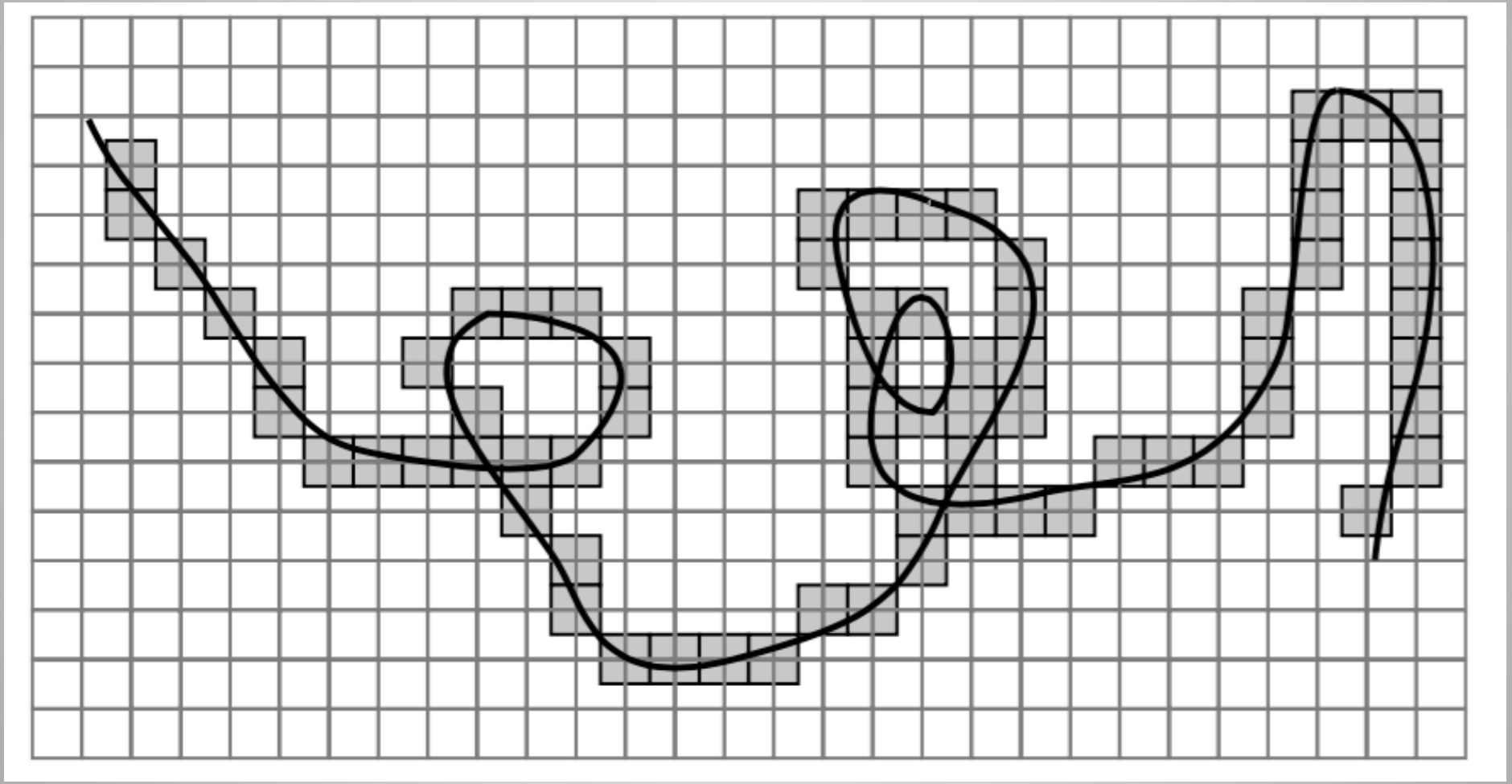
Digitization/Sampling of 3D Image

three-dimensional



continuous image digitized image

Digitization of an arc



Definition

- A (2D) picture *P* is a function defined on a (finite) rectangular subset *G* of a regular planar orthogonal array. G is called (2D) grid, and an element of G is called pixel. P assigns a value of *P(p)* to each $p \in G$

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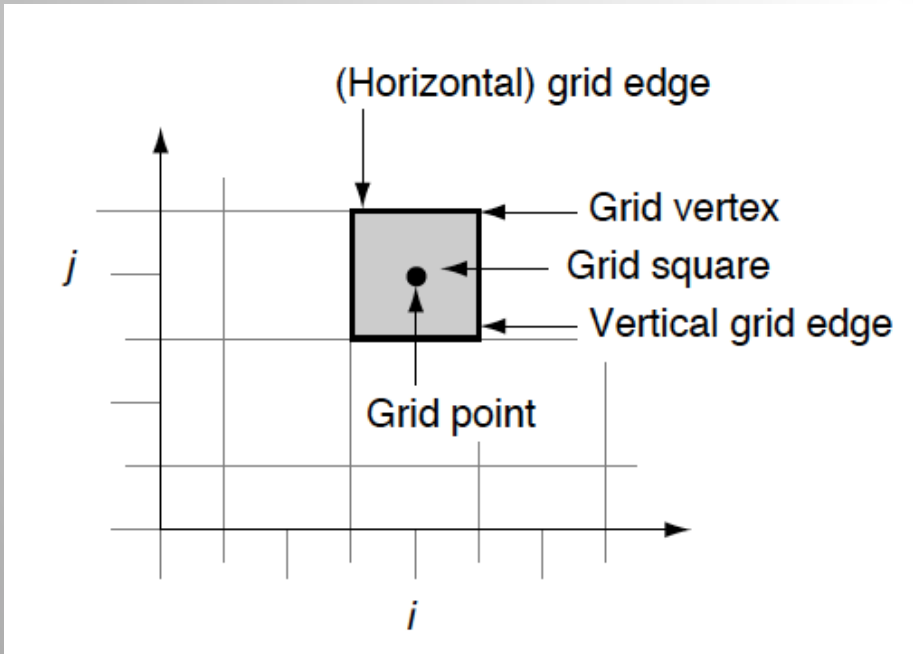
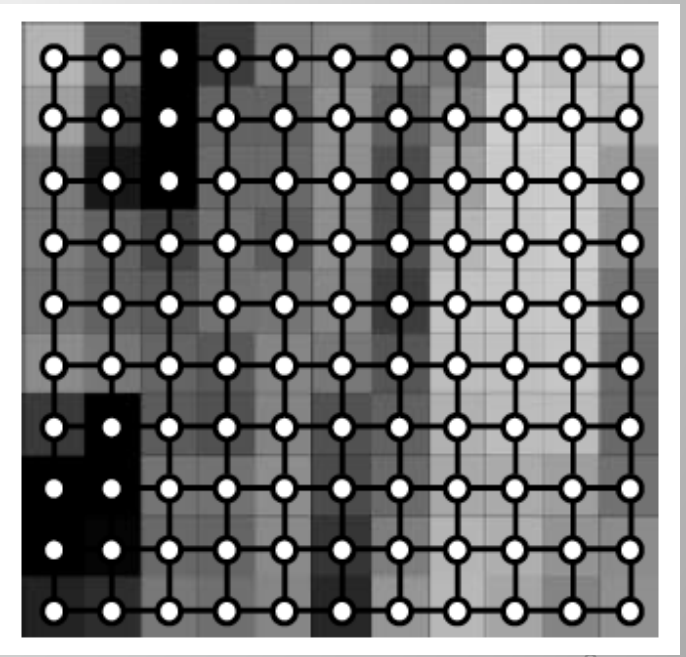
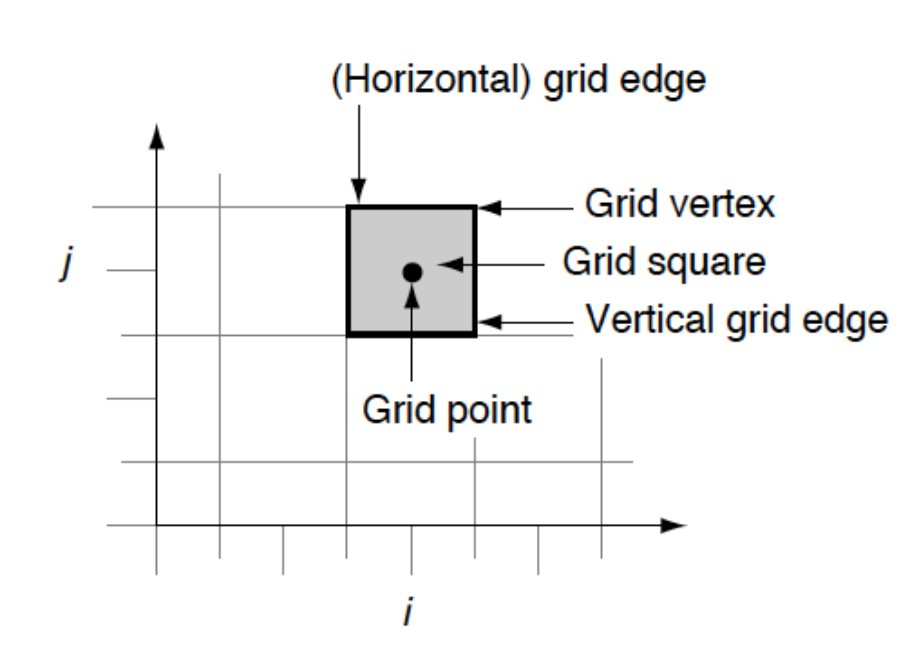
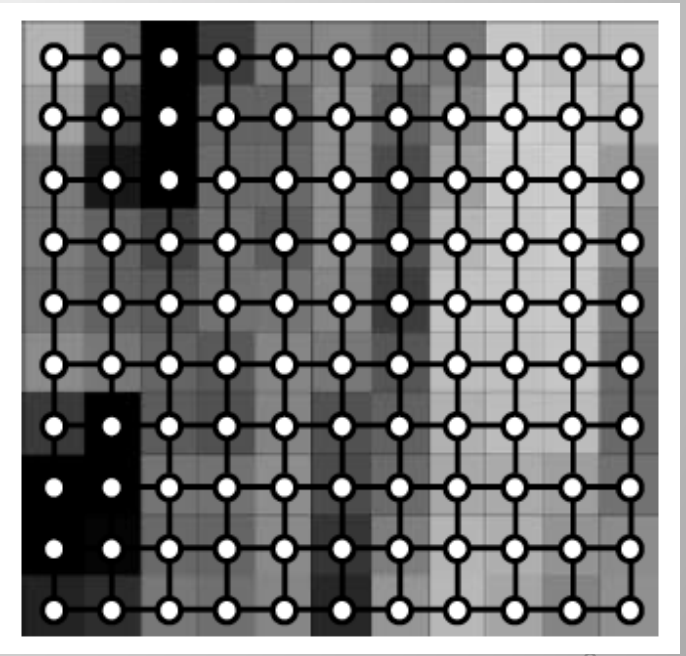




Definition

- Pictures are not only sampled, they are also quantized: they may have only a finite number of possible values (i.e., 0 to 255, 0-1, …)



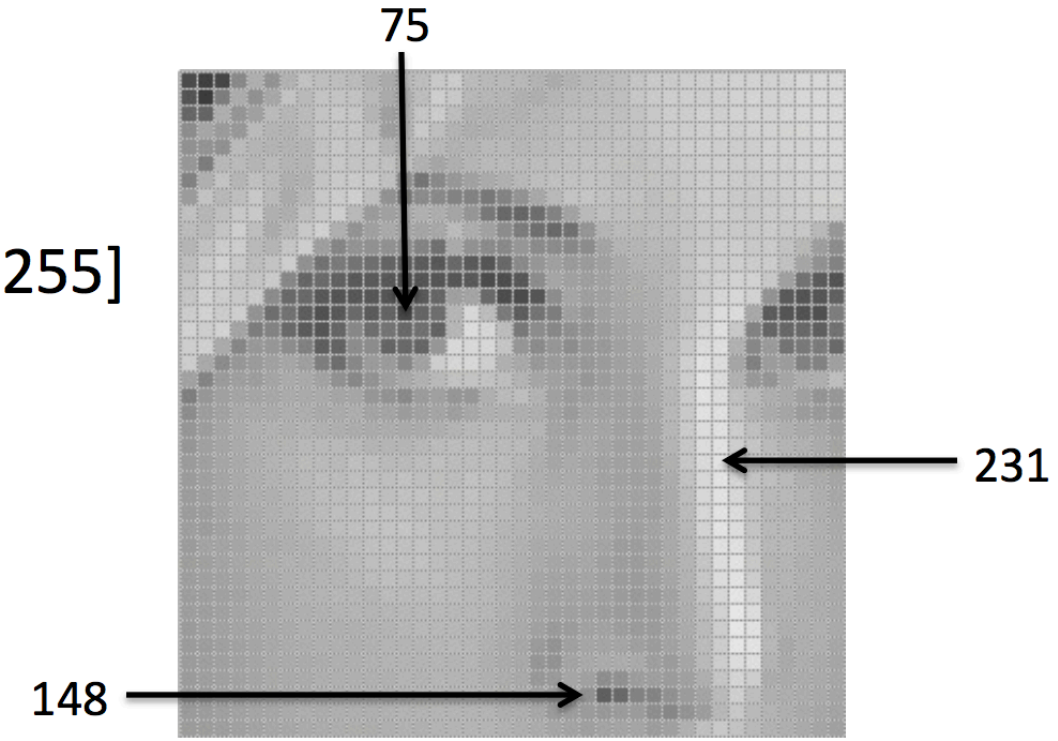


Resolution

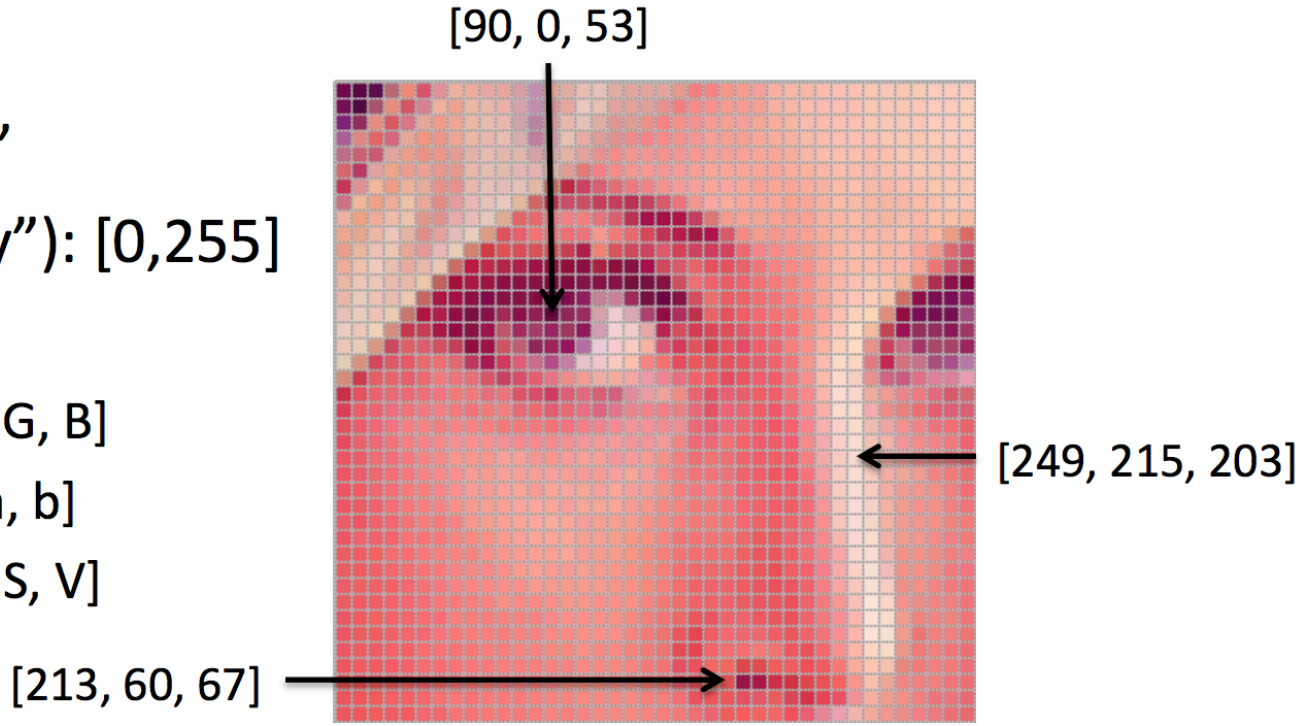
- is a display parameter, defined in dots per inch (DPI) or equivalent measures of spatial pixel density, and its standard value for recent screen technologies is 72 dpi. Recent printer resolutions are in 300 dpi and/or 600 dpi.

- An image contains discrete number of pixels
  - A simple example
  - Pixel value:
    - "grayscale"

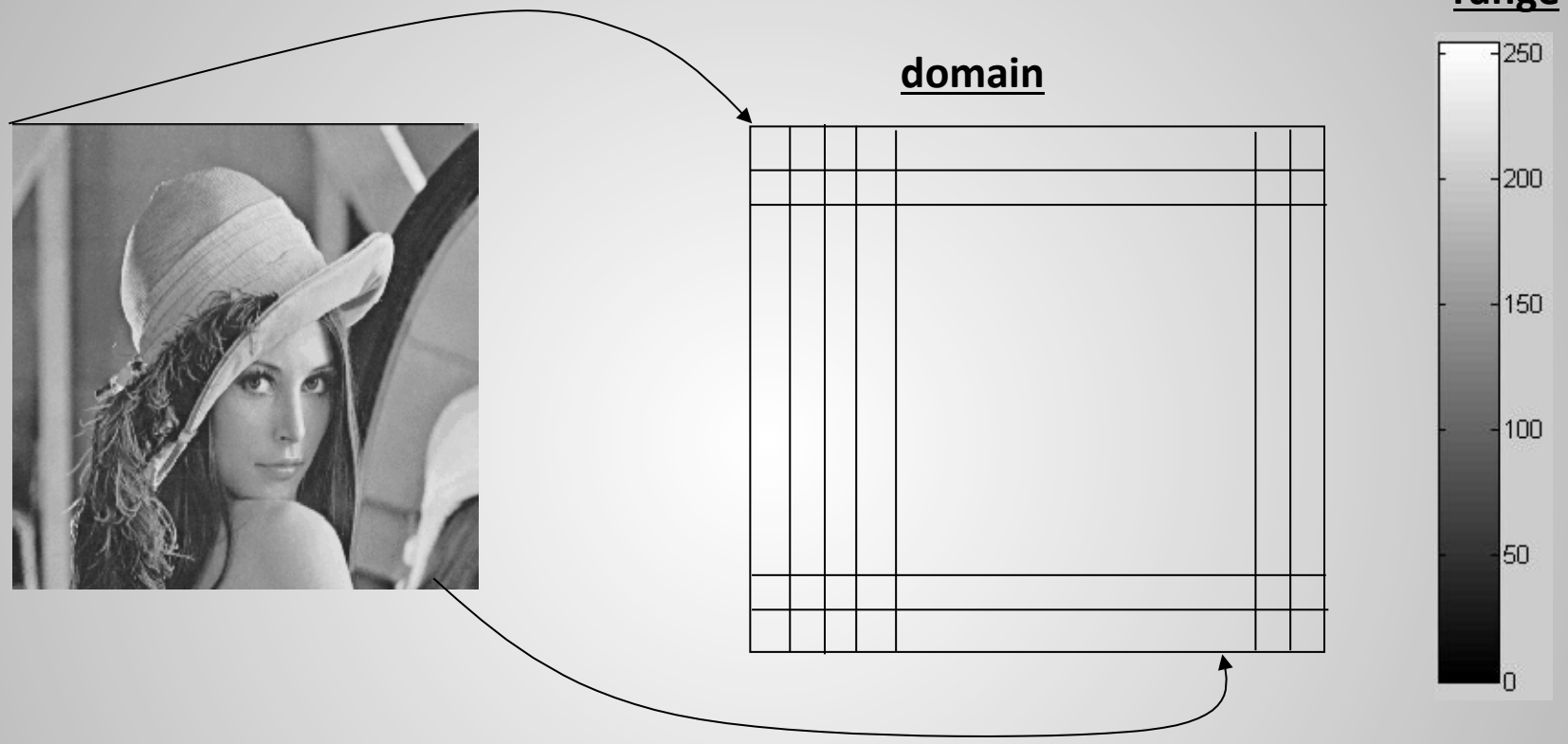
    (or "intensity"): [0,255]



- An image contains discrete number of pixels
  - A simple example
  - Pixel value:
    - "grayscale" (or "intensity"): [0,255]
    - "color"
      - RGB: [R, G, B]
      - Lab: [L, a, b]
      - HSV: [H, S, V]



Source: F.F. Li


range
domain
250
200
150
100
50
0

RGB Channels

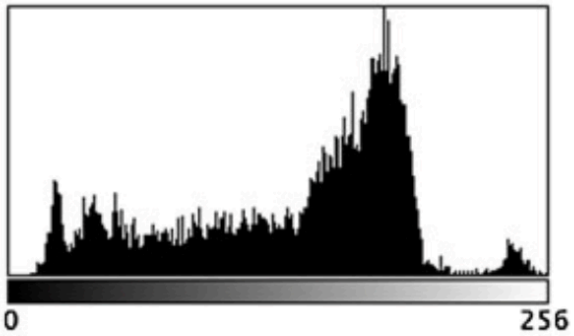
Filters

- Forming a new image whose pixel values are transformed from original pixel values
- Goal is to extract useful information from images, or transform images into another domain where we can modify/enhance image properties
  - Edges, corners, blobs, regions,….
  - Super-resolution, in-painting, denoising,..
  - Texture
  - Patterns
  - Photo editing, tone mapping, abstraction,…
- Linear filtering: The value of output → is calculated by a linear combination of density values on an input image in the neighborhood

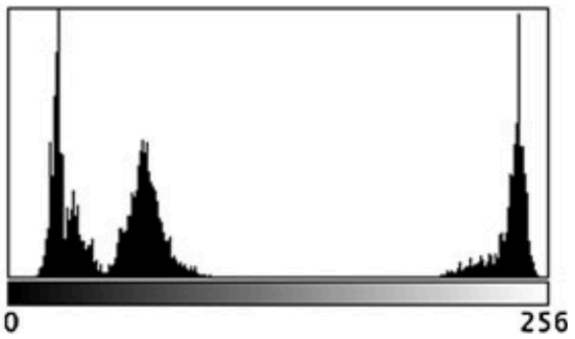
Histogram

- Histogram of an image provides the frequency of the brightness (intensity) value in the image.
  - Pseudo-Code
  1. Assign zero values to all elements of the array *h*
  2. For all pixels *(x,y)* of the image *A*, increment *h(A(x,y))* by 1

Histogram Example

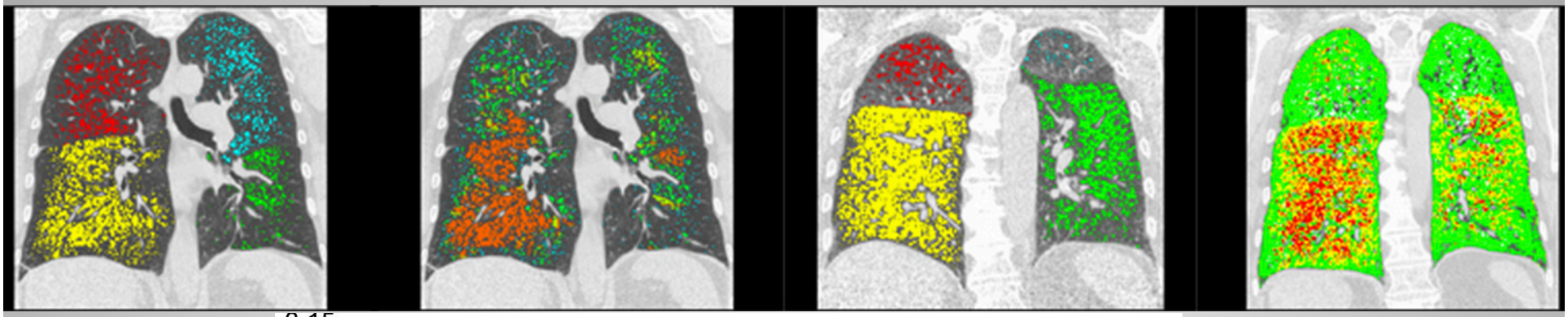


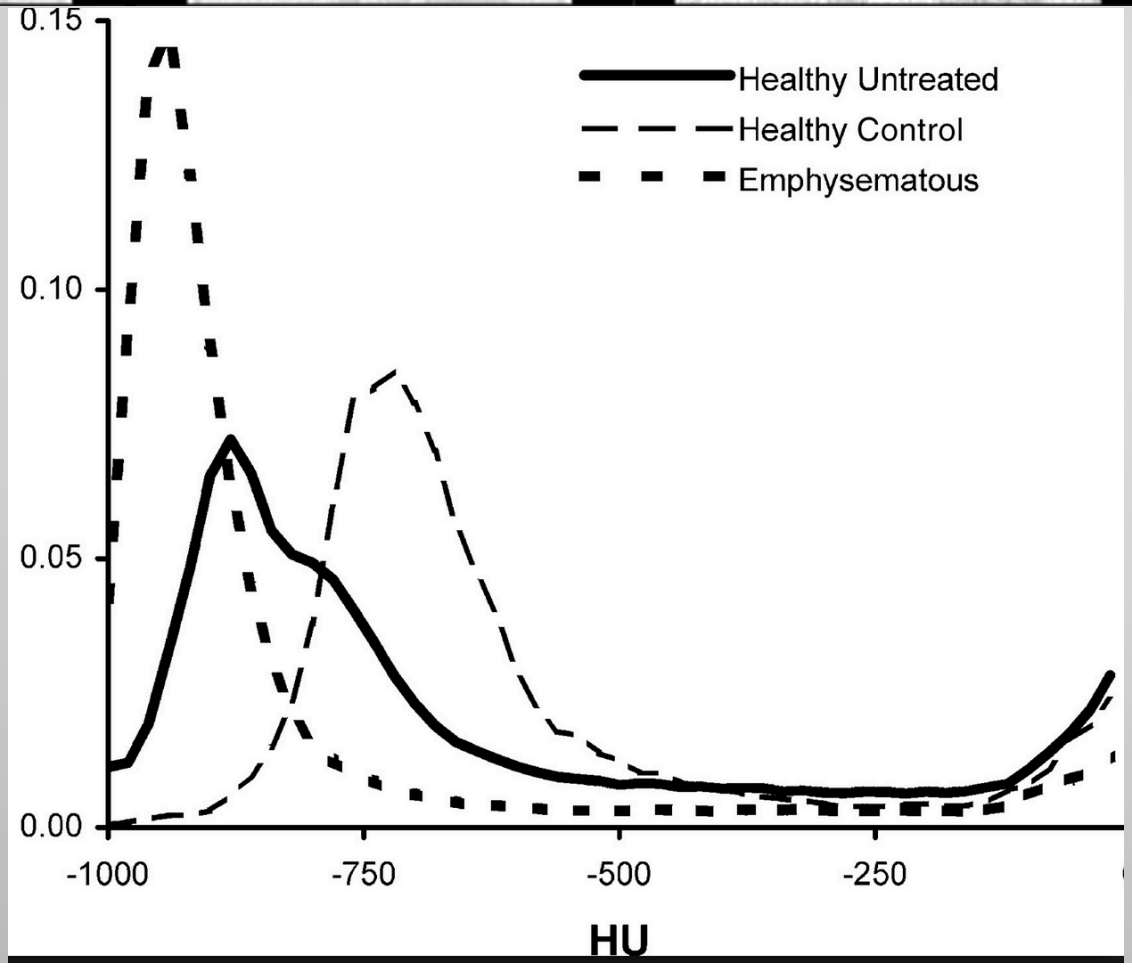
Count: 10192 Min: 9
Mean: 133.711 Max: 255
StdDev: 55.391 Mode: 178 (180)



Count: 10192 Min: 11
Mean: 104.637 Max: 254
StdDev: 89.862 Mode: 23 (440)

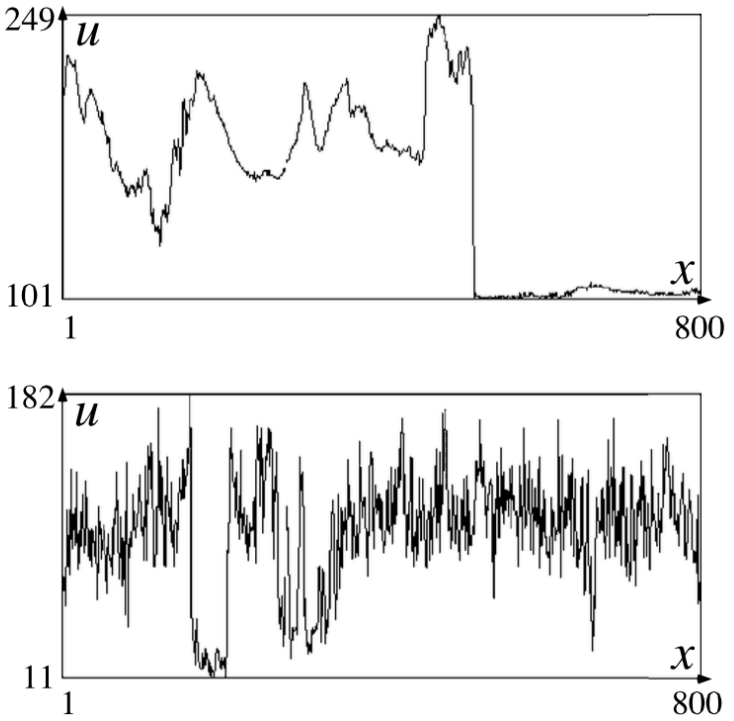
Use ImageJ and/or FIJI
Credit: Klette 2012.





Nat Medicine
IMBIO

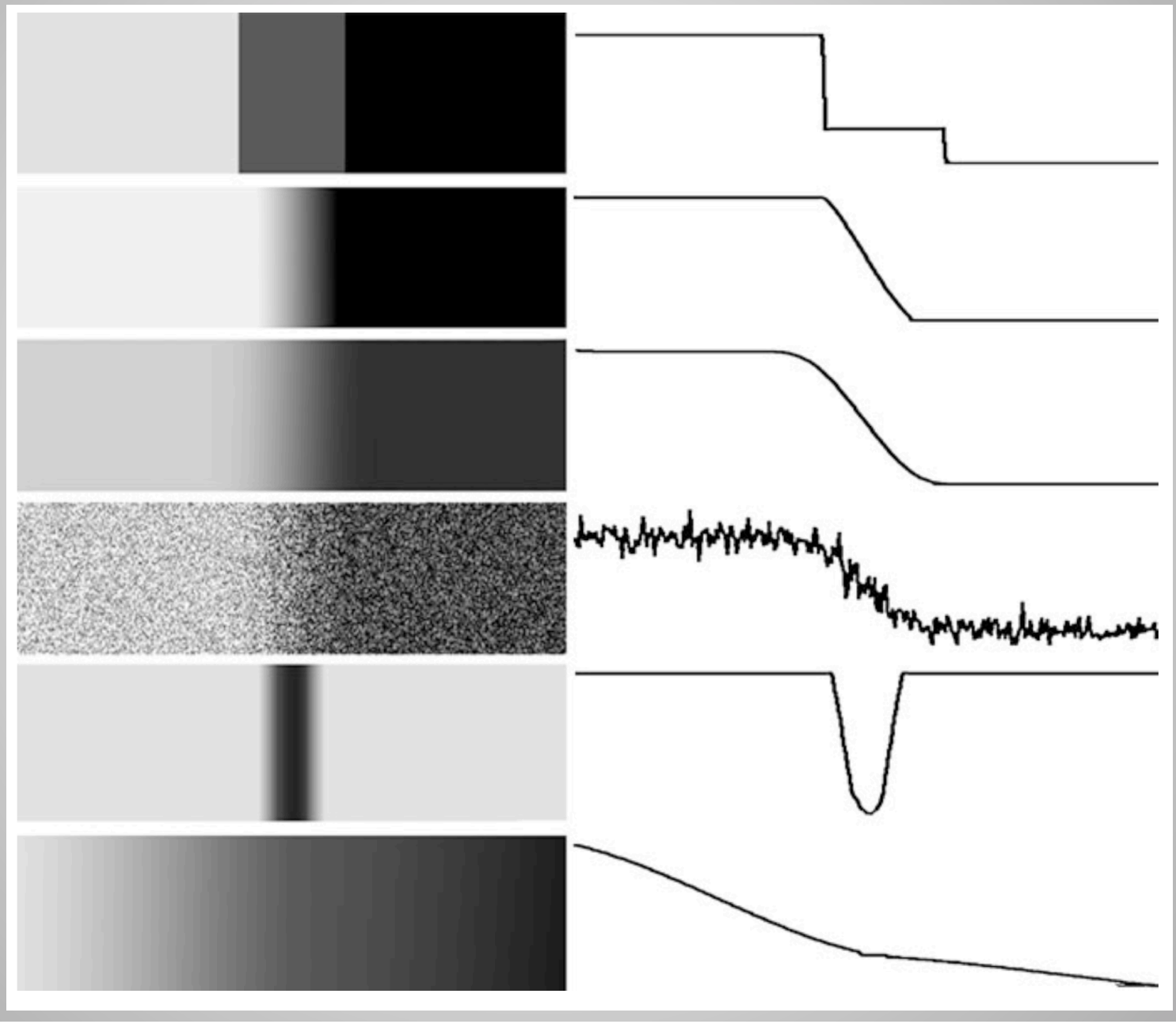
Intensity profiles for selected (two) rows



EDGES

- Discontinuities in images are features that are often useful for initializing an image analysis procedure.
- Edges are important information for understanding an image; by moving "non-edge" data we also simplify the data.

Edge Models



Derivatives and Average

- Derivative: rate of change
  - Speed is a rate of change of a distance, X=V.t
  - Acceleration is a rate of change of speed, V=a.t
- Average: mean
  - Dividing the sum of N values by N

Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

$$y = \sin x + e^{-x}$$

$$\frac{dy}{dx} = \cos x + (-1)e^{-x}$$

Discrete Derivative

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x)$$

$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$

Discrete Derivative / Finite Difference

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$ Backward difference

$$\frac{df}{dx} = f(x) - f(x+1) = f'(x)$$ Forward difference

$$\frac{df}{dx} = f(x+1) - f(x-1) = f'(x)$$ Central difference

Example: Finite Difference

$$f(x) = 10 \quad 15 \quad 10 \quad 10 \quad 25 \quad 20 \quad 20 \quad 20$$
$$f'(x) = \quad 0 \quad 5 \quad -5 \quad 0 \quad 15 \quad -5 \quad 0 \quad 0$$
$$f''(x) = \quad 0 \quad 5 \quad 10 \quad 5 \quad 15 \quad -20 \quad 5 \quad 0$$

Derivative Masks

Backward difference	[-1 1]
Forward difference	[1 -1]
Central difference	[-1 0 1]

Derivative in 2-D

Given function $f(x, y)$

Gradient vector $$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f(x, y)}{\partial x} \\ \frac{\partial f(x, y)}{\partial y} \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

Gradient magnitude $$\left|\nabla f(x, y)\right| = \sqrt{f_x^2 + f_y^2}$$

Gradient direction $$\theta = \tan^{-1} \frac{f_x}{f_y}$$

Derivative of Images

Derivative masks $f_x \Rightarrow \frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$ $f_y \Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

$$I = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \qquad I_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Correlation (linear relationship)

$$f \otimes h = \sum_k \sum_l f(k,l)h(k,l)$$

f = Image

h = Kernel

f

f_1	f_2	f_3
f_4	f_5	f_6
f_7	f_8	f_9

$\otimes$

h

h_1	h_2	h_3
h_4	h_5	h_6
h_7	h_8	h_9

$$f \otimes h = f_1h_1 + f_2h_2 + f_3h_3 + f_4h_4 + f_5h_5 + f_6h_6 + f_7h_7 + f_8h_8 + f_9h_9$$

Convolution

$$f * h = \sum_k \sum_l f(k,l)h(-k,-l)$$

f = Image

h = Kernel

h

h_1	h_2	h_3
h_4	h_5	h_6
h_7	h_8	h_9

$X - flip$

h_7	h_8	h_9
h_4	h_5	h_6
h_1	h_2	h_3

$Y - flip$

f

f_1	f_2	f_3
f_4	f_5	f_6
f_7	f_8	f_9

$*$

h_9	h_8	h_7
h_6	h_5	h_4
h_3	h_2	h_1

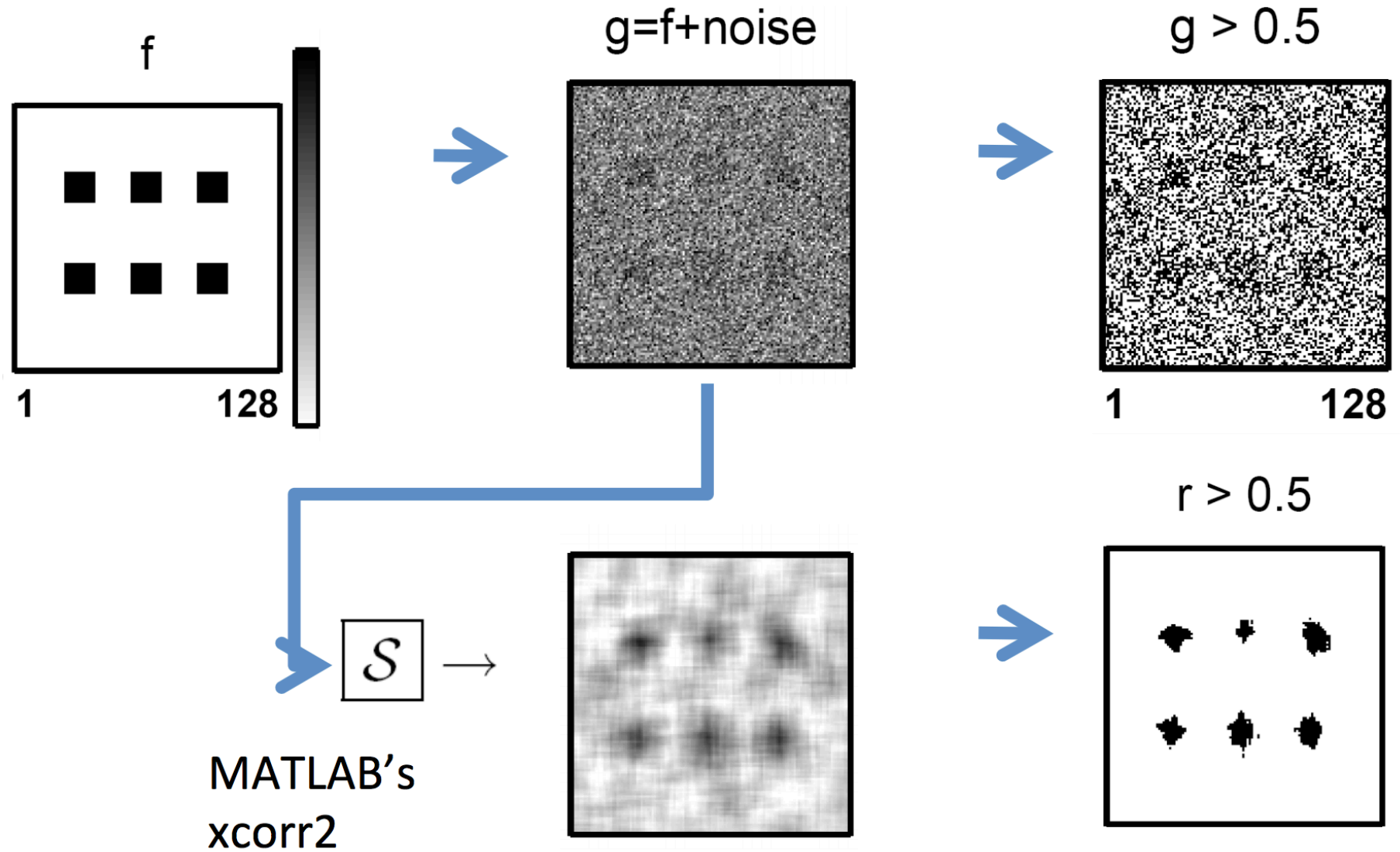
$$f * h = f_1h_9 + f_2h_8 + f_3h_7 + f_4h_6 + f_5h_5 + f_6h_4 + f_7h_3 + f_8h_2 + f_9h_1$$

Correlation and Convolution

- Convolution is associative

$$F * (G * I) = (F * G) * I$$

(Cross) correlation – example



Courtesy of J. Fessler

Correlation and Convolution

- Convolution is a filtering operation, expresses the amount of overlap of one function as it is shifted over another function

- Correlation compares the similarity of two sets of data. (relatedness of the signals!)

Averages

- Mean

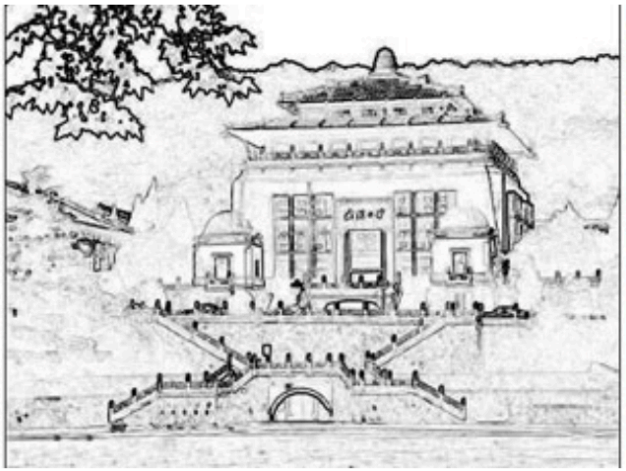
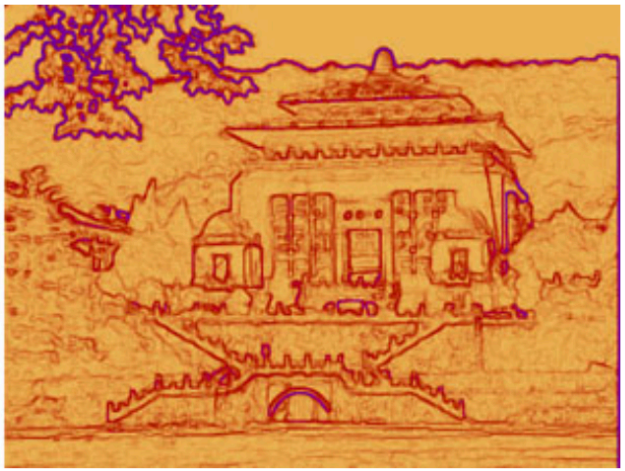
$$I = \frac{I_1 + I_2 + \ldots I_n}{n} = \frac{\sum_{i=1}^{n} I_i}{n}$$

- Weighted mean

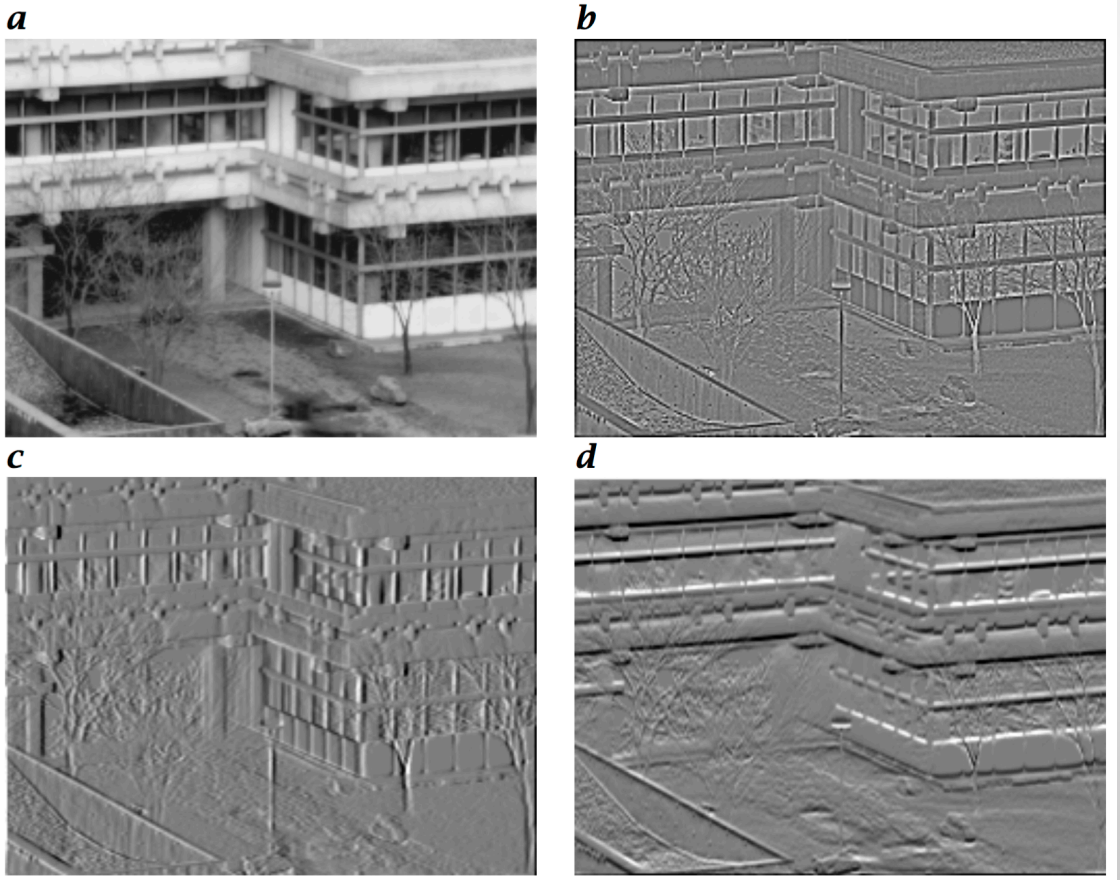
$$I = \frac{w_1 I_1 + w_2 I_2 + \ldots + w_n I_n}{n} = \frac{\sum_{i=1}^{n} w_i I_i}{n}$$

Laplacian-Second Order Derivative

- Will be doing in your PA#1

Wuhan University.



a. Original image
b. Laplacian operator
c. Horizontal derivative
d. Vertical derivative

Noise and Denoising

- Low-pass filtering is normally thought of as the elimination of signal component with high spatial frequencies.

➔

Salt-pepper

Noise

- $I_{observed} = I_{original} + n$ additive
- $I_{observed} = I_{original} * n$ multiplicative

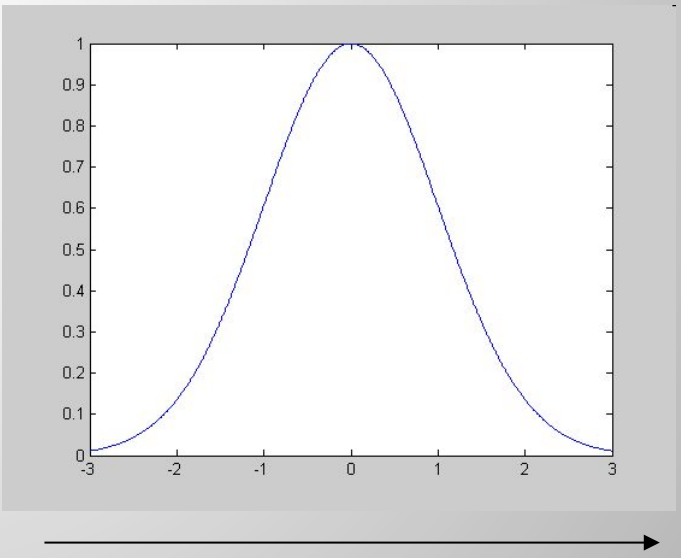
- Light/brightness variations, camera hardware/ lenses, surface reflectance

- Random in nature, occurring with some probability
- It has a distribution (Gaussian (white), Poisson,..)

Gaussian Noise

$$n(x,y) \approx g(n) = e^{\frac{-n^2}{2\sigma^2}}$$

$g(n)$

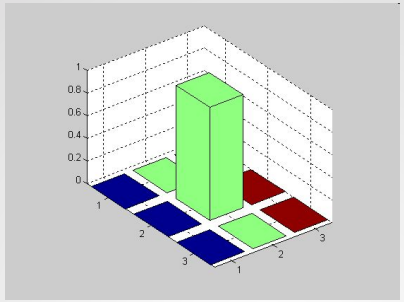


Probability Distribution
n is a random variable

Why Gaussian Assumption?

- Most common natural model
- Smooth function, it has infinite number of derivatives
- It is Symmetric
- Fourier Transform of Gaussian is Gaussian.
- Convolution of a Gaussian with itself is a Gaussian.
- Gaussian is separable; 2D convolution can be performed by two 1-D convolutions
- There are cells in eye that perform Gaussian filtering.

Filtering Examples - 1



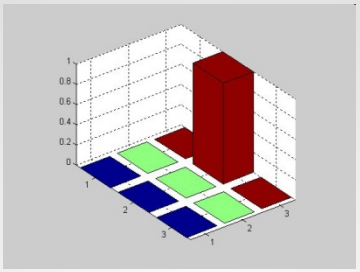
*

0	0	0
0	1	0
0	0	0

=

Filtering Examples - 2

*



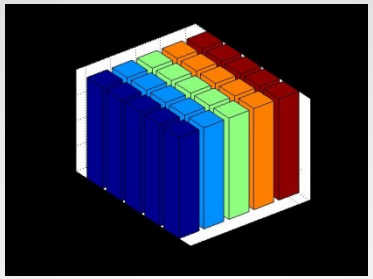
0	0	0
0	0	1
0	0	0

=

Filtering Examples - 3

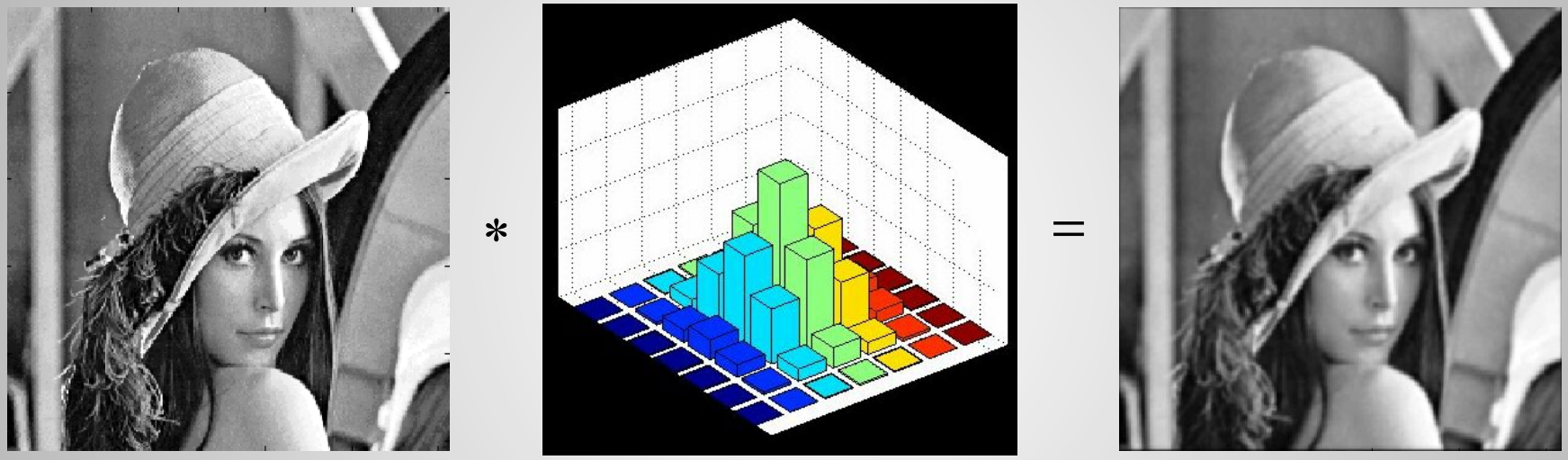
$$* \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

Filtering Examples - 4



$$* \frac{1}{25} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} =$$

Filtering Examples - 5



Gaussian Smoothing

Filtering Examples - 6

Gaussian Smoothing

Smoothing by Averaging

Filtering Examples - 7

After additive Gaussian Noise

After Averaging

After Gaussian Smoothing

Example: box filter

What does it do?

- Replaces each pixel with an average of its neighborhood
- Achieve smoothing effect (remove sharp features)

$$g[\cdot,\cdot]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Slide credit: David Lowe (UBC)

Image filtering

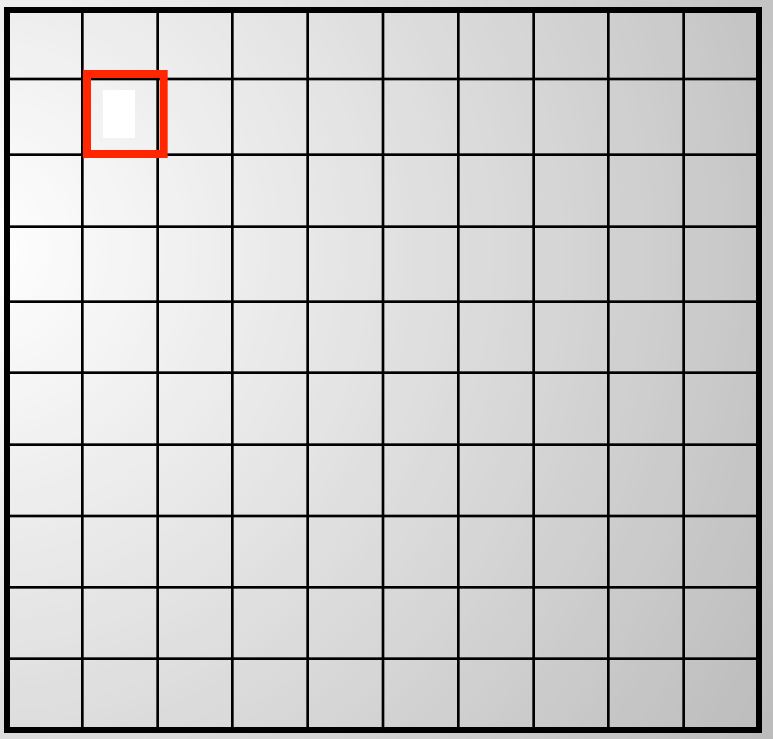
$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$



$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10							

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20						

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30					

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
				?					

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
						?			
				50					

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k,n+l]$$

Credit: S. Seitz

Image filtering

$g[\cdot,\cdot]$ $\frac{1}{9}$

1	1	1
1	1	1
1	1	1

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

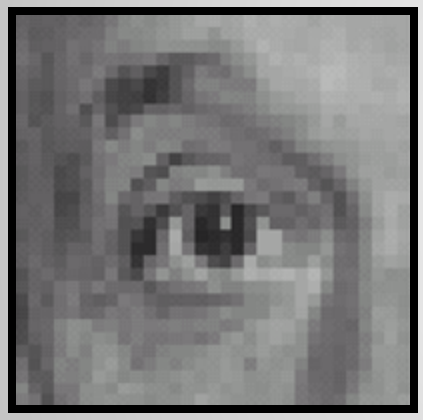
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m,n] = \sum_{k,l} g[k,l]\, f[m+k, n+l]$$

Credit: S. Seitz

Smoothing with Box Filter

Practice with kernels

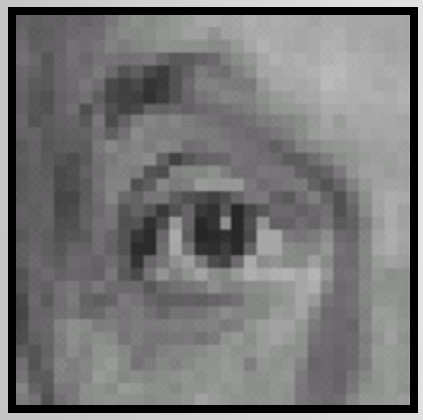


Original

0	0	0
0	1	0
0	0	0

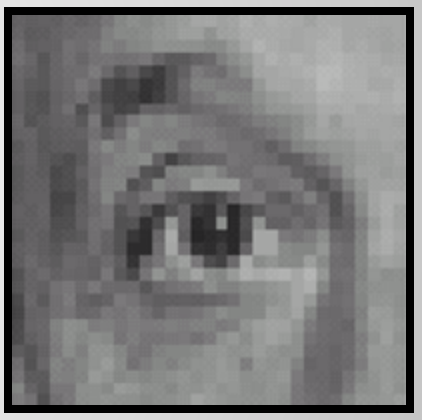
?

Practice with kernels



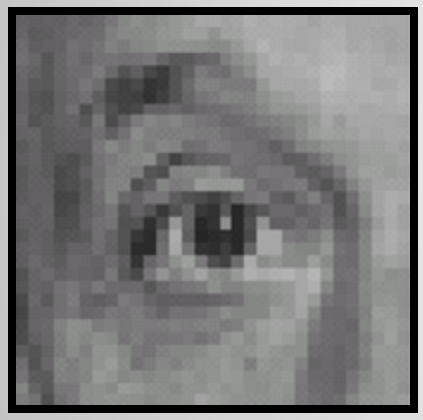
Original

0	0	0
0	1	0
0	0	0



Filtered
(no change)

Practice with kernels

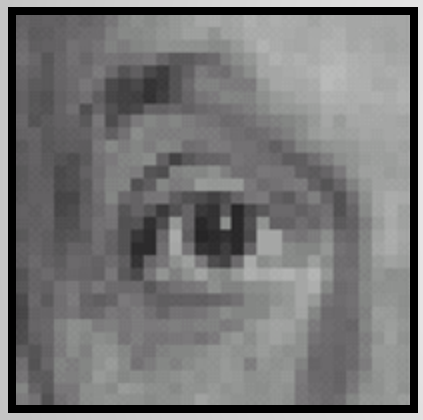


Original

0	0	0
0	0	1
0	0	0

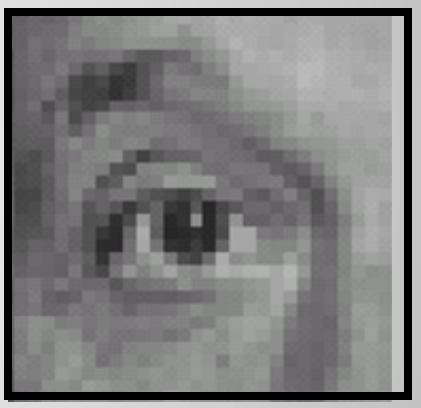
?

Practice with kernels



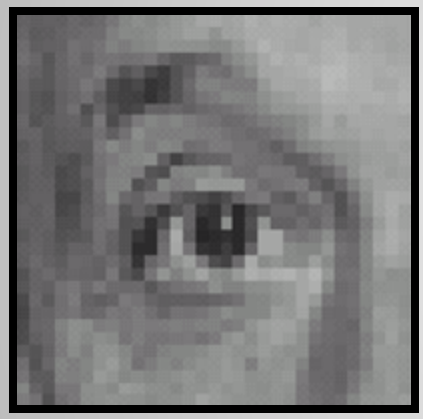
Original

0	0	0
0	0	1
0	0	0



Shifted left
By 1 pixel

Practice with kernels

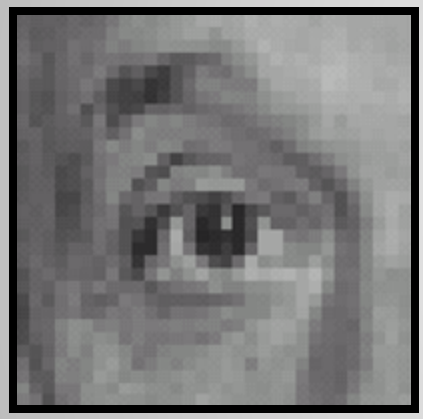


Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad ?$$

(Note that filter sums to 1)

Practice with kernels



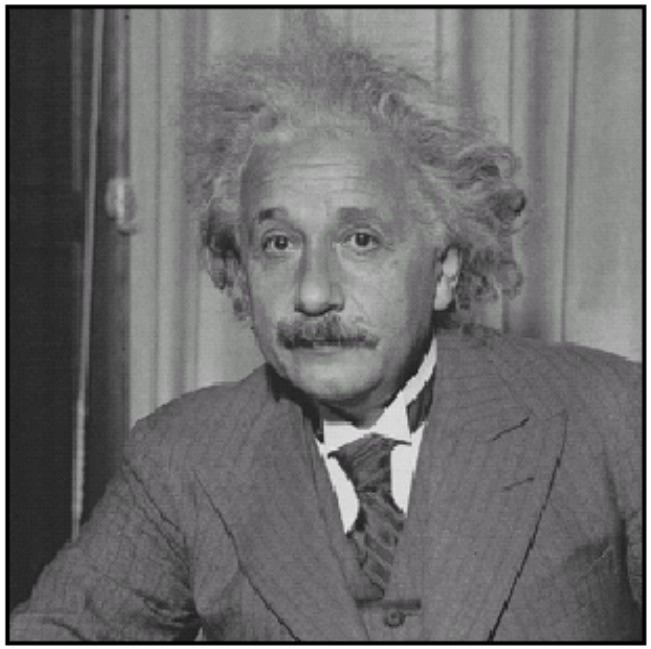
Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

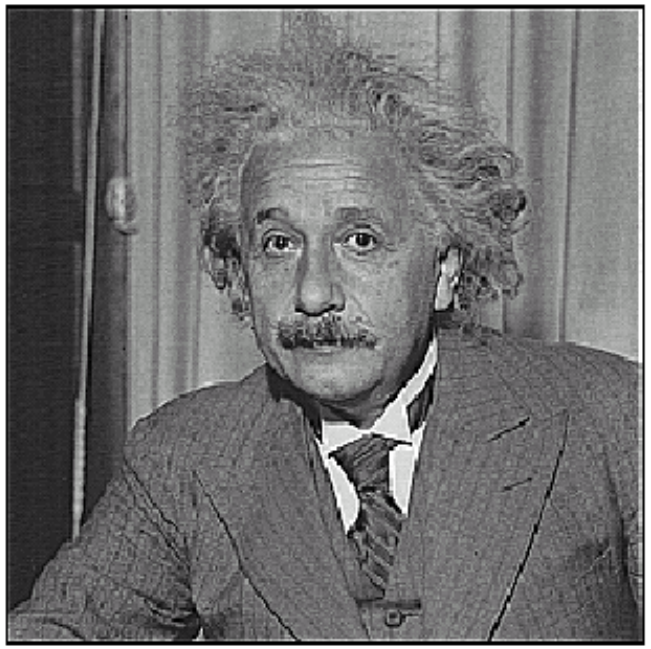
Sharpening filter

- Accentuates differences with local average

Sharpening Filter

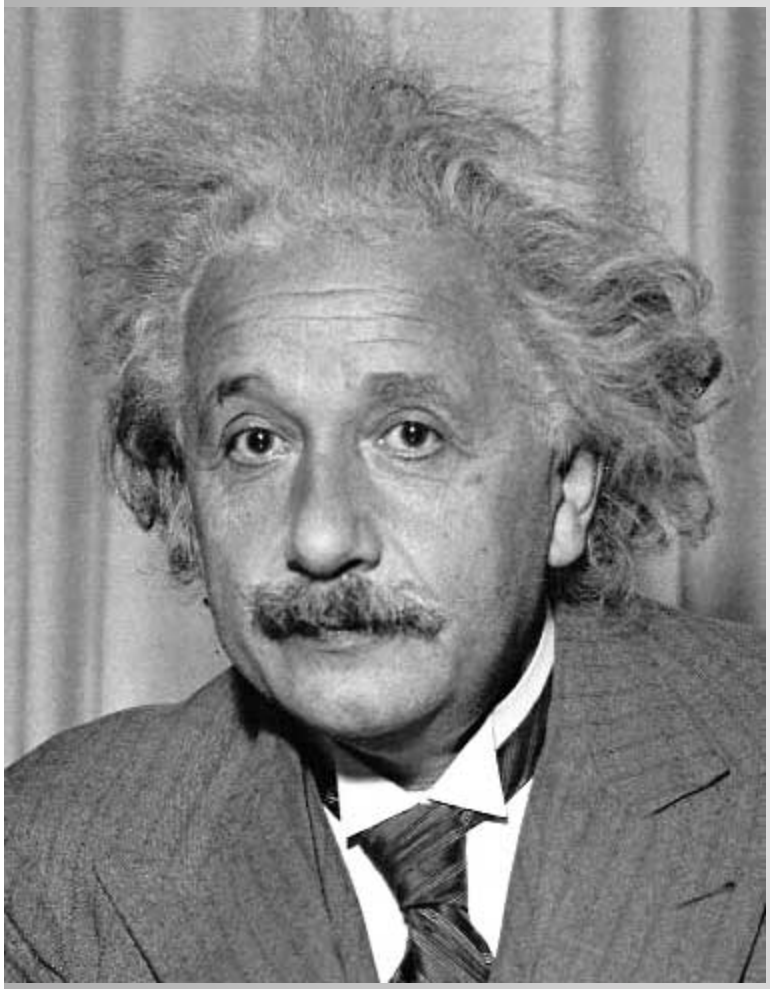


before



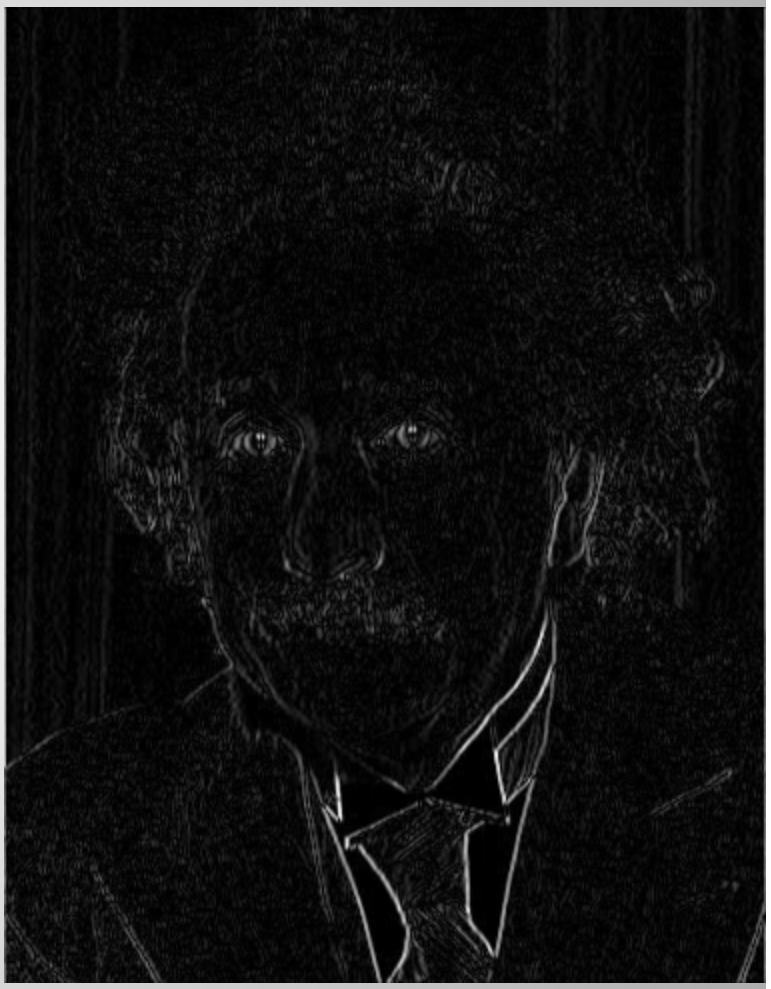
after

Sobel Filtering



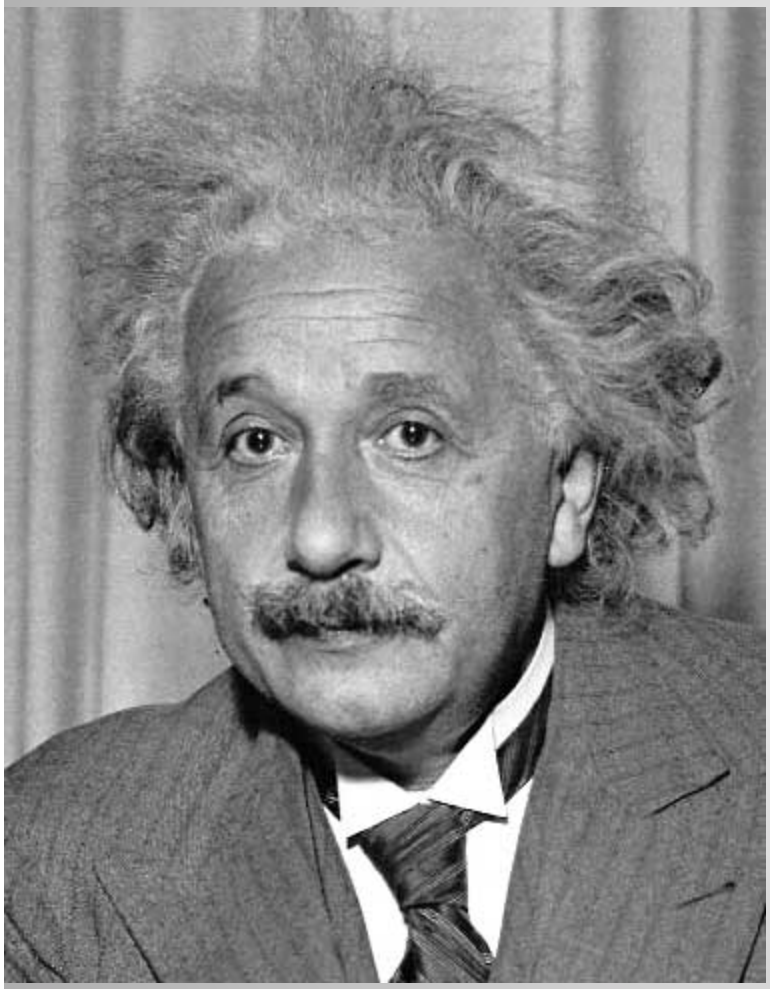
1	0	-1
2	0	-2
1	0	-1

Sobel



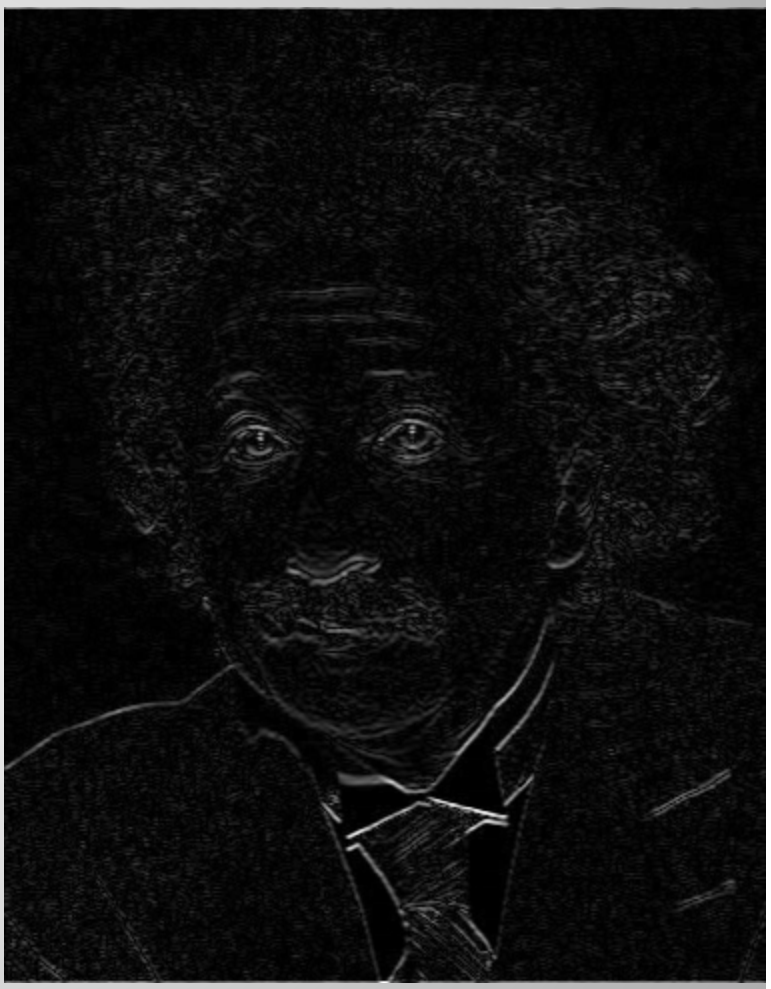
Vertical Edge
(absolute value)

Sobel Filtering



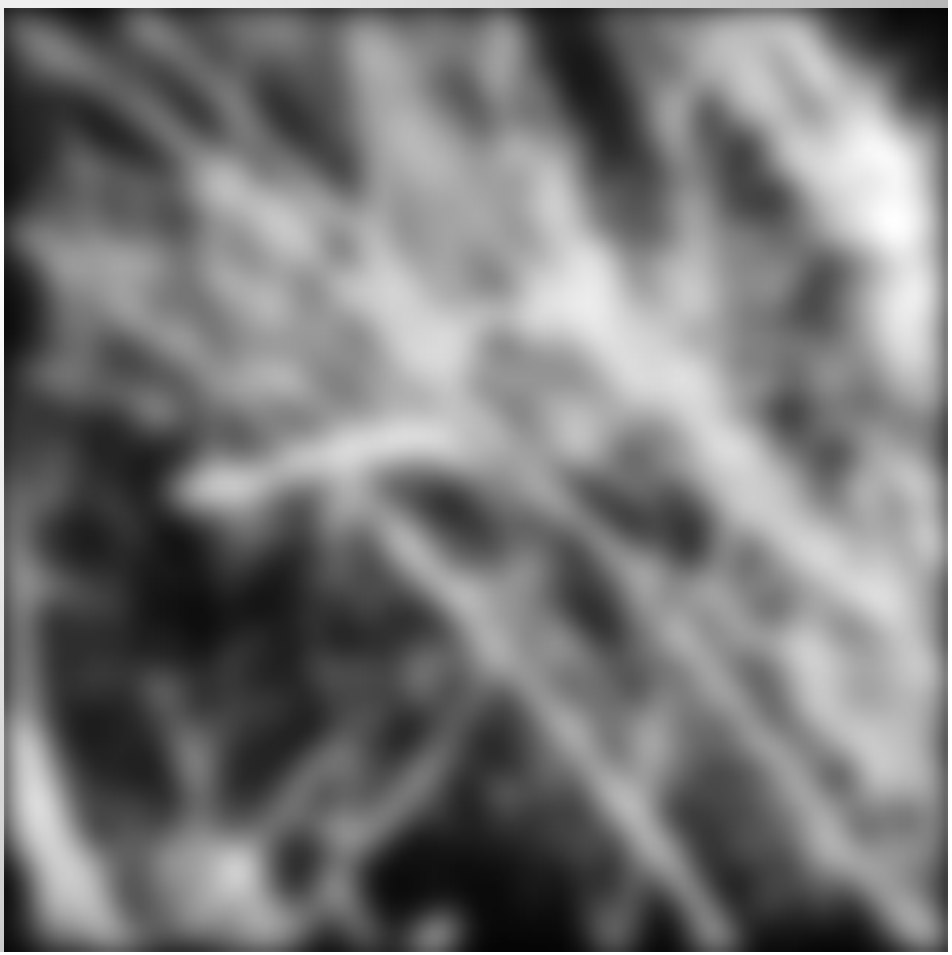
1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

Smoothing with Gaussian Kernel



Smoothing with Box Kernel

Key Properties of Linear Filters

Linearity:

$\texttt{filter}(f_1 + f_2) = \texttt{filter}(f_1) + \texttt{filter}(f_2)$

Shift invariance: same behavior regardless of pixel location

$\texttt{filter}(\texttt{shift}(f)) = \texttt{shift}(\texttt{filter}(f))$

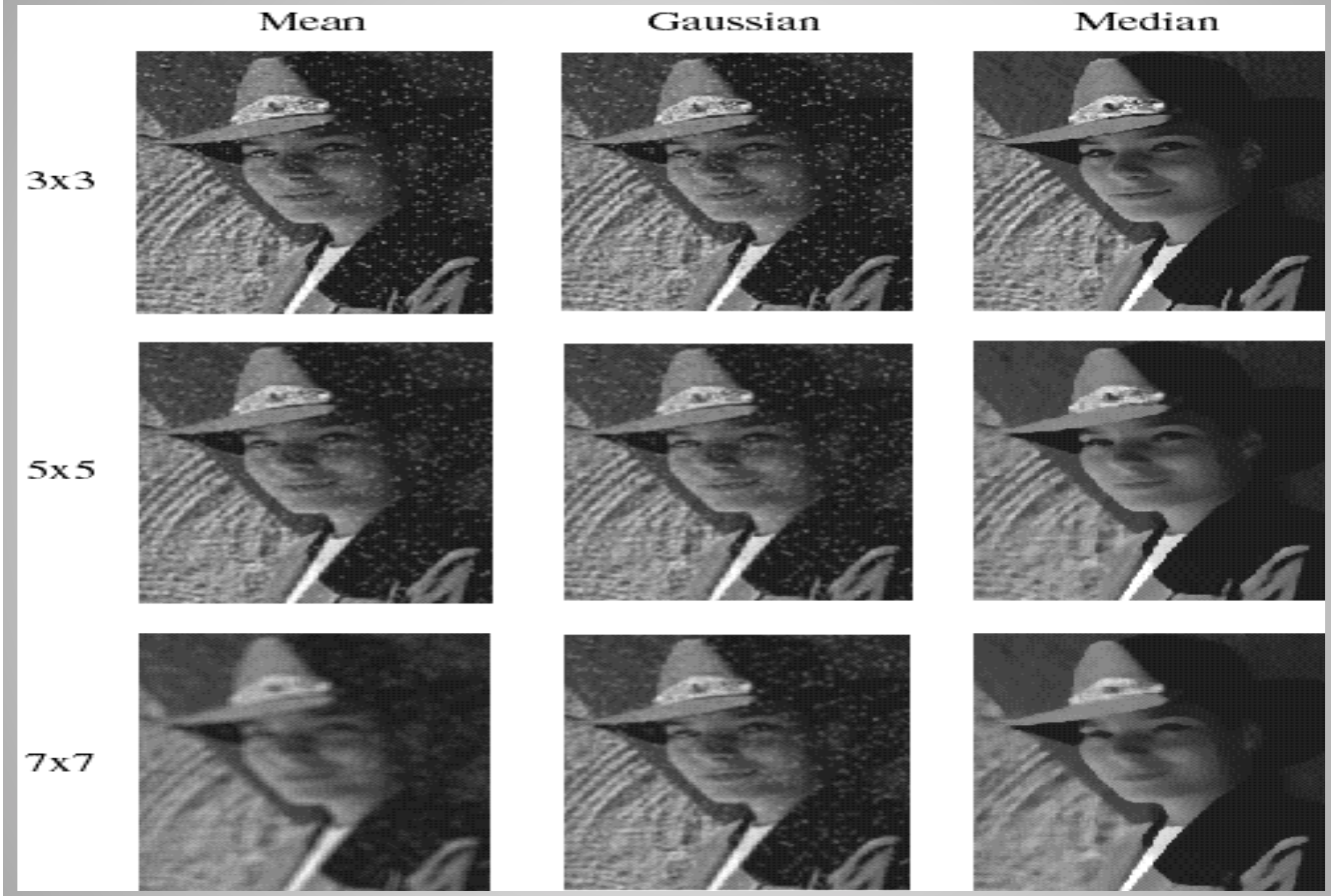
Any linear, shift-invariant operator can be represented as a convolution

More properties

- Commutative: $a * b = b * a$
  - Conceptually no difference between filter and signal
  - But particular filtering implementations might break this equality
- Associative: $a * (b * c) = (a * b) * c$
  - Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - This is equivalent to applying one filter: a * $(b_1 * b_2 * b_3)$
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
- Scalars factor out: $ka * b = a * kb = k\,(a * b)$
- Identity: unit impulse $e = [0, 0, 1, 0, 0]$,
  $a * e = a$

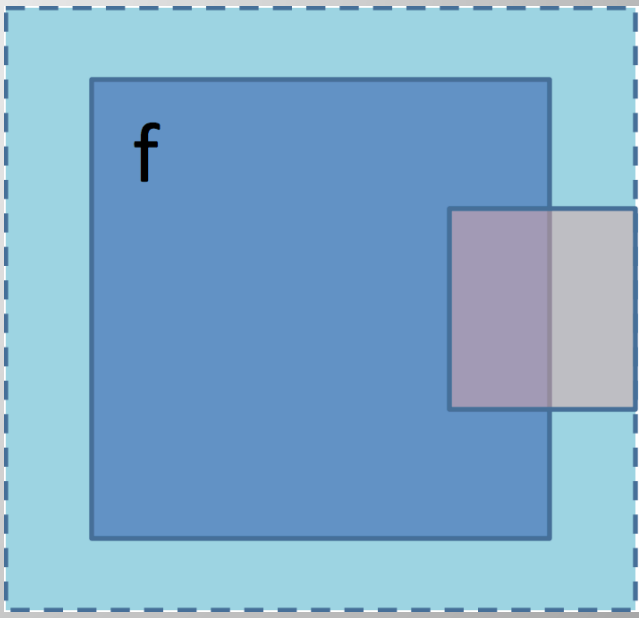
Median Filters

- A **Median Filter** operates over a window by selecting the median intensity in the window.
- What advantage does a median filter have over a mean filter?
- Is a median filter a kind of convolution?


Mean
Gaussian
Median
3x3
5x5
7x7

Practical issues

- How big should the filter be?
  - Values at edges should be near zero
  - Rule of thumb for Gaussian: set filter half-width to about 3*sigma
- Zero Padding and Edge Effects
  - Kernel should include edges
  - Mirrorring is another way



Programming Assignment #1

- Image display and filtering
- Deadline: September 10, 2015.

- REMINDERS
  - Deadline for first week's activity quiz is tomorrow.
  - Deadline for PA0 (Bonus) is 3rd September, 2015.
  - Submit online.