AEM 566 Project 3

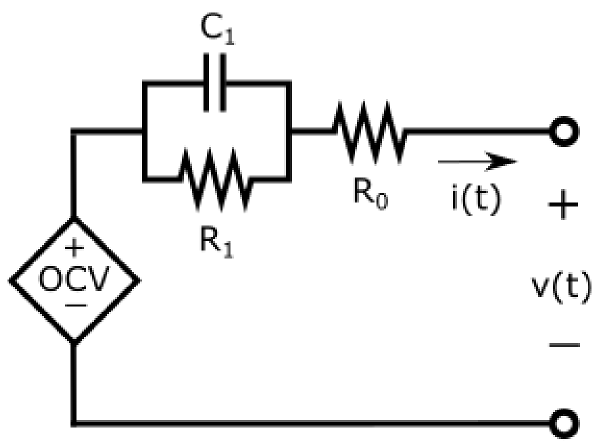
Battery Equivalent Circuit Model Design

Learning Objective

This project is intended to introduce the setup, use, and analysis of the nonlinear least-squares optimal parameter estimation for designing a battery equivalent circuit model.

Dynamical System

To that end, consider the following equivalent circuit diagram of a battery.



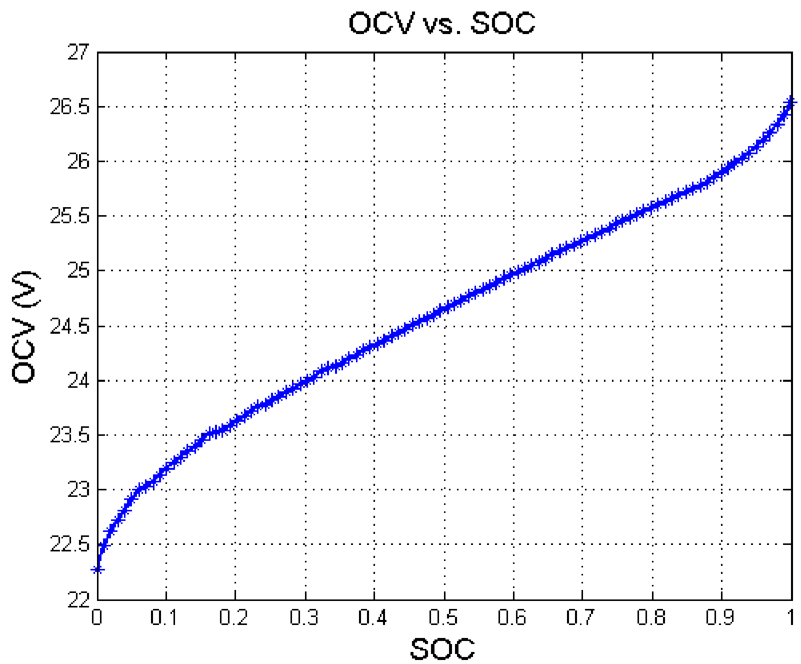
where R_0 is the internal resistance, $\tau_1 = R_1 C_1$ is one dynamic element of the battery (there may be multiple of these), $v(t)$ is the voltage observation, $i(t)$ is the current observation, and OCV is the **open circuit voltage** (OCV) of the battery. In addition, consider the **State-Of-Charge (SOC)** of the battery, SOC, i.e. the percentage of current remaining in the battery relative to the total capacity, Q, which can be written mathematically as

$$SOC(t) = SOC(0) - \frac{\int_0^t i(t)dt}{Q} \tag{1}$$

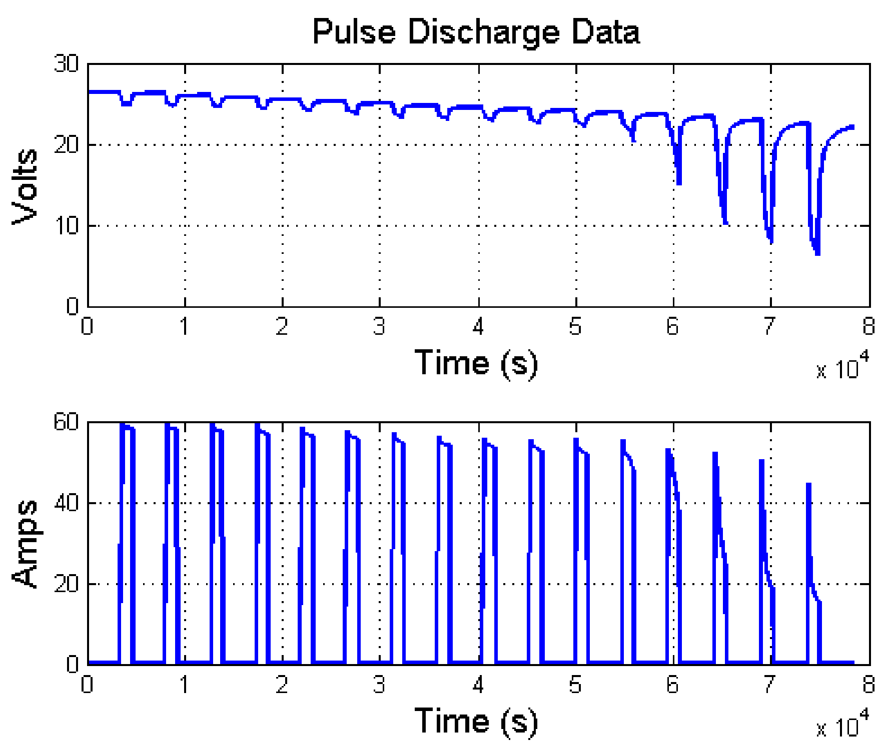
where one can use numerical integration, e.g. a trapezoidal method, to approximate this integral. Note that $i(t)$ should be positive for a discharging battery. Note that the total capacity can be estimated by extracting all of the current from the battery, i.e.

$$Q = \int_0^\infty i(t)dt \tag{2}$$

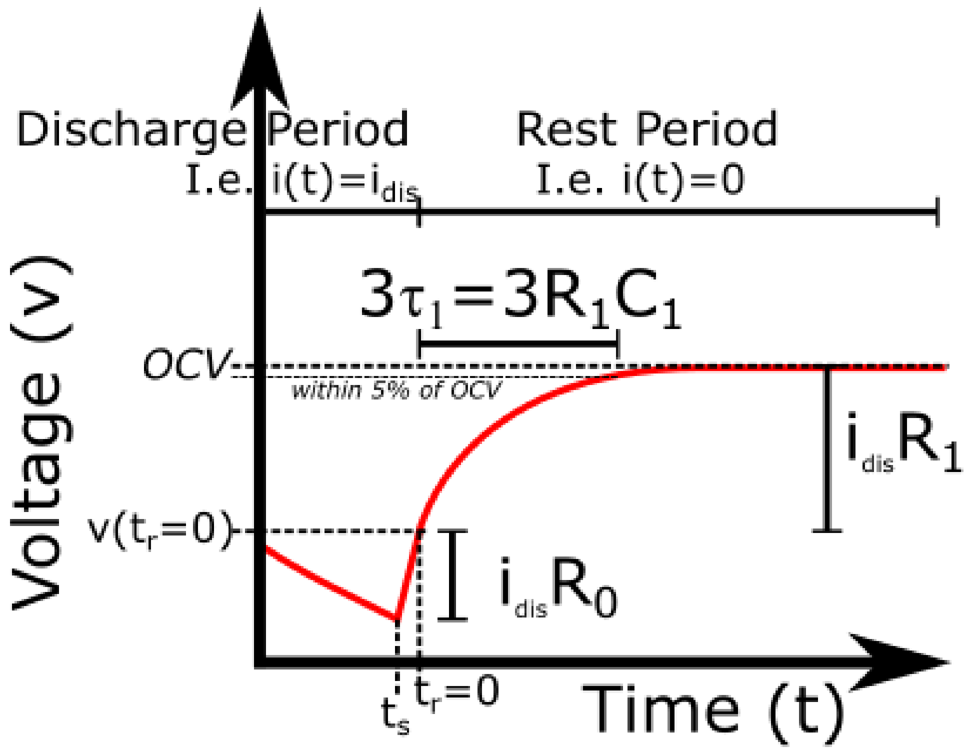
Empirically, it is known that the OCV is a nonlinear function of the SOC as shown in the following figure.



which remains relatively constant over the lifetime of the battery. This OCV-SOC curve can be obtained using a **pulse discharge test** where one can measure the OCV after a long rest period and smooth the samples over multiple tests.



The purpose of this project is to estimate the circuit parameters, including the OCV-SOC curve, from the pulse discharge tests where each rest period allows one to model the parameters shown above nominally by a exponential decay



where a rest period corresponds to no load being applied to the battery, i.e. the current is approximately 0. Here the first initial increase in voltage from a load to a rest is approximately $i_{dis}R_0$, the time to reach 95% of the final value is $3\times\tau_1$, and the final voltage is approximately the OCV.

By inspection, one can see that an estimate of the internal resistance can computed approximately for small time intervals $t_r - t_s$ from the pulse discharge data as

$$v(t_r = 0) - v(t_s) = i_{dis}R_0 \tag{3}$$

and the dynamic resistance as

$$0.95(OCV - v(t_r = 0)) = i_{dis}R_1 \tag{4}$$

and the time constant as

$$3\tau_1 = t \text{ where first } v(t_r) \geq 0.95OCV \tag{5}$$

However, the number of RC elements is **unknown** for batteries in general thereby the total increase in voltage may occur due to multiple exponential terms, i.e. $i_{dis}R_1$, $i_{dis}R_2$, ... Thus, one could attempt to fit the time series data of one rest period to an exponential decaying function for $t_r \geq 0$

with any number of terms, i.e.

$$v(t_r) = OCV - i(t_s)R_1 \exp\left(\frac{-t_r}{\tau_1}\right) - i(t_s)R_2 \exp\left(\frac{-t_r}{\tau_2}\right) - \cdots \tag{6}$$

where $\tau_i = R_i C_i$ is the time constant of the i^{th} exponential, t_r is the time across one rest period starting at 0 for the initial rest, t_s is the single time step immediately before the rest period. Note that the OCV is the value of $v(t_r)$ as $t \to \infty$ when no current is being discharged.

Project Assignment and Deliverables

Do: the following tasks in MATLAB or Python.

1. load the battery data for the pulse discharge tests in the accompanying CSV file;
2. numerically integrate the current to obtain the SOC for the entire data set
   - the SOC at each rest period will be used to construct the $OCV - SOC$ curve;
3. partition the data into each rest period using the current measurement as the trigger,
   - where one can see where the current is set back close to zero instead of the discharge amperage;
4. for each rest period, in each pulse discharge data set,
   - compute three sets of optimal battery parameters, i.e. OCV, R_0, R_1, τ_1, ... for the exponential decaying function using nonlinear least-squares with 1, 2, and 3 exponential terms, e.g., MATLAB's `lsqnonlin` or SciPy's `optimize.least_squares`,
   - note that initial guesses for the parameters can be obtained using reasonable values from the graphical model shown,
   - compare the models through a comparative analysis of the observed residuals between the models and across the entire pulse discharge test;
5. plot the estimated parameters as a function of SOC over each pulse discharge test and comment on their variety with SOC;
   - comment on the improvement from using additional RC elements which make the model more complicated, both in terms of residuals and
6. estimate the OCV-SOC curve for each battery and compare for both batteries,
   - use the OCV from each of the exponential function fits.

Deliver: in the Blackboard assignment, all files to run your MATLAB or Python script(s). There is no need to zip your files.