1. Create a model for the problem and solve it.

This model is intended to identify the currency trades that achieve target levels of EUR and JPY while maintaining minimum levels for all other currencies, through the lowest-cost set of trades. Note that lowest-cost set of trades is equivalent to maximizing the USD-denominated value of final currency holdings.

Define Variables:

$C = \{USD, GBP, EUR, HKD, JPY\}$

$StartCurr_i$ as the starting volume of currency i before any trades	$EndCurr_i$ as the ending volume of currency C after all trades
$Exch_{i,j}$ as the exchange rate to convert from currency i into currency j	$Trade_{i,j}$ as the volume of currency i being converted into currency j. Denominated in currency i

Maximize USD-denominated end currencies:

$$\sum_{i \in C} Exch_{i,USD} * EndCurr_i$$

Where EndCurr is the sum of starting balance, outflows, and exchange-rate-impacted inflows:

$$EndCurr_i = StartCurr_i - \sum_{j \in C} Trade_{i,j} + \sum_{j \in C} Exch_{j,i} * Trade_{j,i}$$

Such that:

$$EndCurr_{EUR} \geq 8,000,000$$
$$EndCurr_{JPY} \geq 54,000,000$$
$$EndCurr_{USD} \geq 250,000$$
$$Exch_{GBP,USD} * EndCurr_{GBP} \geq 250,000$$
$$Exch_{HKD,USD} * EndCurr_{HKD} \geq 250,000$$

2. What is the optimal trading plan?

Trading Plan (denominated in the "From" currency)								
	To ->						**Ending balance**	
From	**USD**	**EUR**	**GBP**	**HKD**	**JPY**		**In USD**	**In native currency**
USD	$0	$2,945,103	$0	$0	$0		$296,273	$296,273
EUR	€0	€0	€0	€0	€0		$7,779,200	€8,000,000
GBP	£0	£0	£0	£0	£839,672		$250,000	£160,328
HKD	$9,689,170	$0	$0	$0	$0		$250,000	$1,951,295
JPY	¥0	¥0	¥0	¥131,314,060	¥0		$455,220	¥54,000,000

3. What is the optimal transaction cost (in equivalent USD)?

The trading plan identified in item #2 results in a USD-denominated value of $9,030,693. This represents a **$27,867** reduction from the original USD-denominated value of $9,058,560.

4. Suppose another executive thinks that holding $250,000 USD in each currency is excessive and wants to lower the amount to $50,000 USD in each currency. Does this help to lower the transaction cost? Why or why not?

Topline, loosening constraints in an optimization problem will axiomatically maintain or improve the optimization outcome. Without even considering the specifics, we can conclude that reducing the minimum holdings will not *increase* the currency exchange expense. Further, the fact that the optimal trading solution in part #2 show multiple currencies sitting at the minimum value, we can expect with high confidence that the cost will decline.

Updating the model's constraints with new minima impacts the trading plan, with higher volumes in GBP->JPY, HKD->USD, and JPY->HKD resulting in higher USD balances and lower GBP and HKD balances.

Trading Plan (denominated in the "From" currency)								
	To ->						**Ending balance**	
From	**USD**	**EUR**	**GBP**	**HKD**	**JPY**		**In USD**	**In native currency**
USD	$0	$2,945,103	$0	$0	$0		$696,280	$696,280
EUR	€0	€0	€0	€0	€0		$7,779,200	€8,000,000
GBP	£0	£0	£0	£0	£967,934		$50,000	£32,065
HKD	$12,811,295	$0	$0	$0	$0		$50,000	$390,258
JPY	¥0	¥0	¥0	¥155,038,810	¥0		$455,220	¥54,000,000

The trading plan results in a USD-denominated value of $9,030,700. This represents a **$27,860** reduction from the original USD-denominated value. Reducing the minima results in a **$7** improvement in exchange expenses.

5. Suppose the exchange rate for converting USD to GBP increased from 0.6409 to 0.6414. What happens to the optimal solution in this case?

This adjustment to the exchange rate creates an unbounded problem: we make infinite money by exchanging our USD to GBP and back to USD, earning a positive 0.014% return on each roundtrip pair of trades.

In transparency, the submitted code file has an iteration of the optimization problem with the updated currency exchange rates, which throws an "Unbounded" error.