1. What did you use for data sources? Do you have any concerns about these sources?

The analysis relies on four sources of data, three of which are from the 2020 Census and accompanying 2021 Gazetteer. The last data source was a file of latitude/longitude coordinates per FIPS code, made available as part of the Plotly user guide.

There are two concerns to track with this data:

1. Long-term stability of FIPS code assignments. The FIPS code was used as the primary key across all datasets, and while pulling this analysis together it seems that FIPS code structures evolve over time. This is of especial concern with the Plotly-sourced file, since it lacked documentation
2. Accuracy of 2020 census data. While the data is the best available, any subsequent challenges to its validity would impact the downstream analytics.

2. How did you define your objective function? To what extent were you able to accommodate both racial balance and population objectives?

My ingoing approach was to define the objective function in two iterations. The first iteration would ignore racial balance and only optimize by minimizing population dispersion while forcing districts made of contiguous counties. This first iteration would define a floor for optimal population packing, which would be passed to the second iteration. The second iteration would optimize on racial balance, while adding a constraint that forced population dispersion to be within an acceptable tolerance of the first iteration's optimal solution.

In practice, optimizing on racial balance proved too computationally intensive, given Ohio's 16 target districts. Racial balance was alternatively tested as a series of constraints, while maintaining geographic proximity as the optimization function and recognizing this threatened district contiguity. This approach was marginally successful, but even weak constraints (e.g., setting a cap that no district can be >90% white) resulted in some districts being spread into multiple disconnected islands.

First iteration objective functions to make compact districts (note: key features shown. Other features necessary for successful execution excluded from this summary):

Successful (Hess Moment of Inertia)	**Unsuccessful (Minimize "cut lines")**
$\min \sum_{i,j} w_{ij} d_{ij}$ $d_{ij} = \begin{cases} 1 & \text{if vertex i is assigned to district j} \\ 0 & \text{otherwise} \end{cases}$ $w_{ij} = pop_i * dis_{ij}^2$ $TPop_{LB} \leq TPop_k \leq TPop_{UB}$ Source: (Validi et al., 2020)	$cl_{ij} = \begin{cases} 1 & \text{if counties i and j are neighbors i} \\ 0 & \text{otherwise} \end{cases}$ $\min \sum_{i,j} cl_{ij}$ $\text{for all districts k} \begin{cases} cl_{ij} \geq d_{ik} - d_{jk} \\ cl_{ij} \geq d_{jk} - d_{ik} \end{cases}$ $TPop_{LB} \leq TPop_k \leq TPop_{UB}$ Source: (Becker & Solomon, 2020)

Second iteration objective functions to enforce racial balance

Unsuccessful objective function	**(Marginally Successful) Add constraints to force racial balance**
Minimize absolute sum of errors $\min \sum_k \Delta^* WPop_k$ $\Delta^* WPop_k \geq WPop_k - WPop_{OH}$ $\Delta^* WPop_k \geq WPop_{OH} - WPop_k$	*Add ceiling to allowable district white %:* $WPop_k \leq TgtWhitePct * TPop_k$
	Add allowable white population range $WPop_{LB} \leq WPop_k \leq WPop_{UB}$

3. How did you formulate the set covering and adjacency constraints?

The two-stage optimization approach was selected because of the high likelihood to produce contiguous districts in the first iteration and then loosen the requirement in exchange for improved racial balance. In practice, the first iteration achieved this objective.

The key concept of the Hess Moment of Inertia approach is that each district has a centroid county *j*, and all of the counties *i* assigned to that district are represented as $d_{ij} = 1$ .This allows for assignment of constraints to enforce every county to be assigned to a district (e.g., $\sum_j d_{ij} = 1$). This approach breaks down, however, when there are counties with populations larger than the acceptable upper range. The d_{ij} decision is kept binary, while an additional variable a_{ij} representing allocation of county i to district (centered on) j is introduced.

Larger counties are allowed to allocate a fraction of their population to multiple districts.

IF $TPop_i \leq TPop_{UB}$	**IF** $TPop_i \geq TPop_{UB}$
$\sum_j d_{ij} = 1$ $a_{ij} = d_{ij}$	$\sum_j d_{ij} \leq MaxDistricts$ $a_{ij} \geq d_{ij} * Allocation_{LB}$ $d_{ij} \geq a_{ij}$ $\sum_j a_{ij} = 1$
$TPop_{LB} \leq \sum_j a_{ij} * TPop_i \leq TPop_{UB}$	

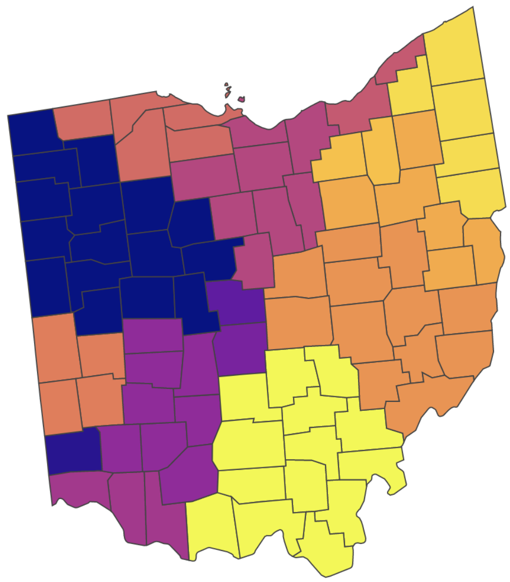
4. What is the optimal redistricting solution for your state?

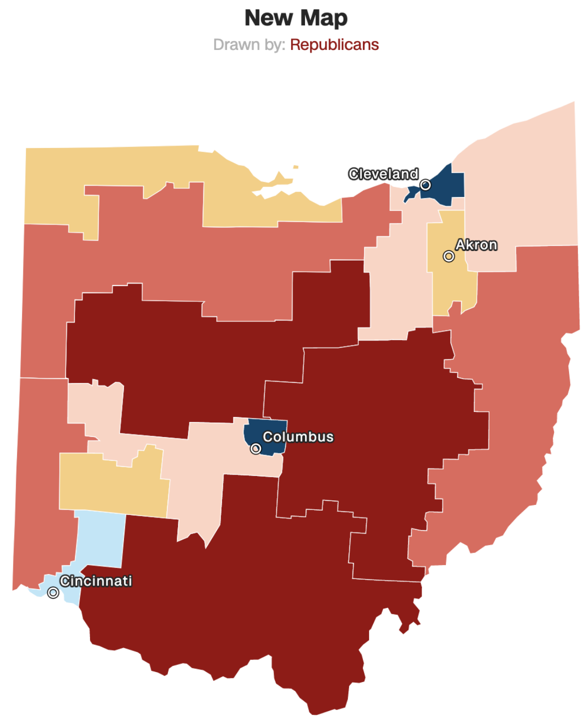
The analysis performed here is limited at the county level. At that level of resolution, there is very strong tension between ensuring districts "look' cohesive and ensuring districts represent state-level levels of racial balance. Of the results produced via linear programming, the best approach is the one that aims purely to ensure districts are generally similar in size. The fact that Ohio has three pockets of racial diversity in a sea of white people means that attempts to racially balance districts at the county level will look highly gerrymandered.

5. Which map is better, the algorithmic one or the one being proposed by lawmakers?

The proposed Ohio map still fails the "sniff test" of whether the districts look appropriately apportioned, and the adjustments from the old map eliminate the only majority-black district in the state. The algorithmically-generated map looks more balanced, although it also would leave Ohio with only majority-white districts.

Algorithmically-Drawn Map	**Proposed Map**





Source: (Boschma et al., 2022)

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