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Determining Planck's Constant with LEDs

Kevin Sohn (260782138), Lambert Francis (260861226)

McGill University Department of Physics

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Abstract

' In this experiment, we determined Planck's constant using LEDs of different colour and the conservation of energy. By plotting turn-on voltage vs. light frequency, we extracted Planck's constant from the slope of the equation $V = \frac{h}{e}f$, where h is Planck's constant, e is the charge of an electron, f is the frequency of emitted light, and V is the turn-on voltage of the LED. Two fits were performed: a fixed y-intercept linear fit of the form, $V = \frac{h}{e}f$, and a free y-intercept linear fit of the form, $V = \frac{h}{e}f + b$, where b is a constant offset. h was determined to be $(5.13 \pm 0.03) \times 10^{-34}$ Js in the fixed intercept fit and $(7.1 \pm 0.2) \times 10^{-34}$ Js in the free intercept fit. Both h's were more than 3σ from the accepted value. We believe this discrepancy is due to systematic errors present in the experimental setup and underestimation of our uncertainty.

1 Introduction

LEDs are ubiquitous in modern times. They are widespread due to their low energy consumption and high efficiency, unlike its older counterpart, the light bulb [1]. LEDs consist of two different semiconductors: n-type (negative-type) and p-type (positive-type) [2]. The n-type possess excess electrons (net negative charge) and the p-type possess electron holes (net positive charge). The combination of these two semiconductors in junction create a potential difference, causing excess electrons to flow from the n-type to the p-type. Eventually, the electron holes near the junction will disappear, creating a potential barrier that prevents more electron flow [3].

Unless an external voltage is applied to the LED, no current will be generated. However, the voltage must be supplied in the correct orientation; the negative terminal joined to the n-type and the positive terminal joined to the p-type. Otherwise, the potential barrier will increase rather than decrease. This direction dependence classifies the LED as a diode: a circuit element that admits current in only one direction [3].

If the supplied voltage is large enough to overcome the barrier, the electrons will flow once more from the n-type to the p-type. Crossing the barrier causes electrons to drop in energy level, conserving energy in the form of emitted photons. We call the critical voltage at which the LED lights up the turn-on voltage. The energy loss of the electron at the turn-on voltage must equal the energy of the photon emitted, giving the equation

$$E = hf = eV, \tag{1}$$

where h is Planck's constant, f is the photon's frequency, e is the charge of the electron, and V is the turn-on voltage.

The goal of this experiment was to determine Planck's constant by measuring the turn-on voltages of LEDs of different wavelengths. From the wavelengths, we calculate frequency using the equation

$$f = \frac{c}{\lambda}, \tag{2}$$

where c is the speed of light and λ is emitted wavelength of light. By plotting voltage vs.

frequency, we extract Planck's constant from the slope.

2 Materials and Methods

The experimental setup consisted of a DC variable power source, LEDs tuned to different wavelengths of light, a breadboard, a voltmeter, and a spectrometer. A diagram of the experimental setup can be found in Figure 1.

Firstly, the power source was linked to the breadboard to power the LEDs. Then, a voltmeter was linked to the circuit to measure turn-on voltage accurately. To find the turn-on voltage, the voltage level was raised until the LED lit up; Then, we took turns dialing down the voltage level and deciding when the LED was off by eye. The difference between our turn-on voltages was taken as the uncertainty in the wavelength measurement. The spectrometer was not used to measure the turn-on voltage because it was too insensitive to the LED light compared to our eyes. However, it was used to determine the wavelength of the LED light. A fiber optic cable was used to funnel light into the spectrometer.

Planck's constant was determined from the slope of the turn-on voltage vs. LED light frequency graph.

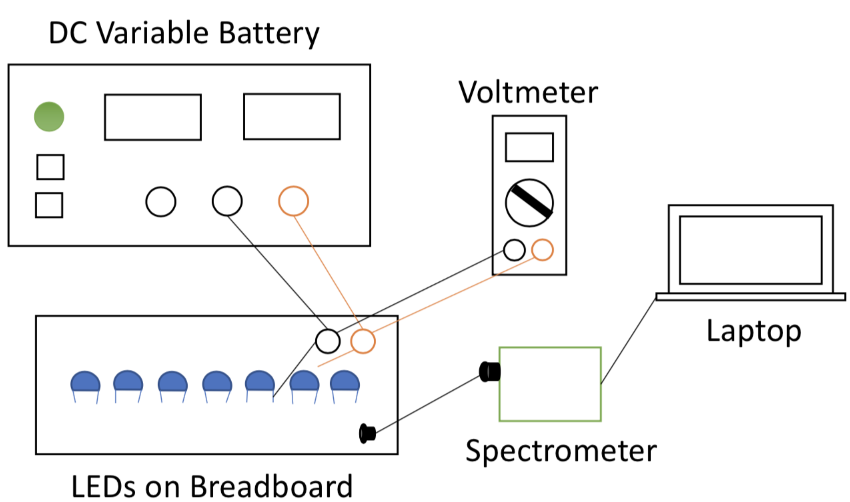


Figure 1: Schematic of the experimental setup.

3 Results

The last digit displayed by the voltmeter was $\pm$ 0.001 V, however, our individual determinations of turn-on voltage typically differed by 0.03 V, so we took that as our voltage uncertainty. The wavelength uncertainty was determined to be $\sigma_\lambda = 2\ nm$ by taking repeated spectrometer readings and taking the maximum range of the peak values. The uncertainty in frequency was determined by propagating the uncertainty in wavelength with the equation, $\sigma_f = \sqrt{(\frac{c}{\lambda^2})^2 \cdot \sigma_\lambda^2}$, determined by employing the differential uncertainty propagation approach on Eq. (2).

The uncertainty in the slope ($\sigma_{\frac{h}{e}}$) was determined from the covariance matrix returned by scipy.optimize; σ_h was then calculated by multiplying this value by the electric charge, e. All calculations were done with Python in Jupyter Notebook. The code can be found in Appendix B.

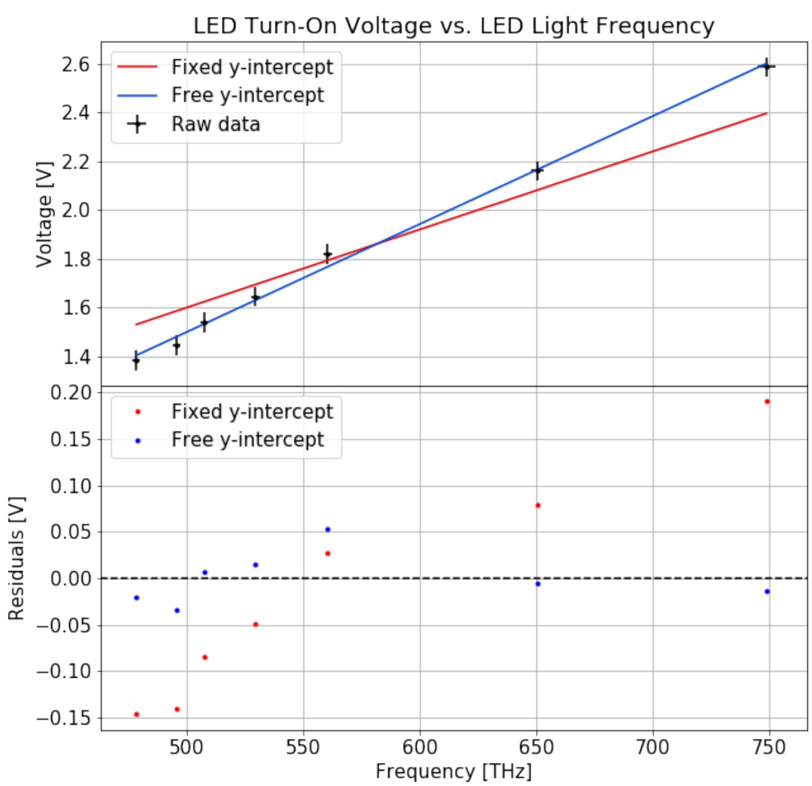


Figure 2: The top plot shows turn-on voltage vs. LED light frequency. The free and fixed y-intercept best fit lines are shown for comparison. The bottom plot shows the residuals of both fits. Linear fits were performed with scipy.optimize.

Fit Type	Planck's Constant (Js)
Fixed y-intercept	$(5.13 \pm 0.03) \times 10^{-34}$
Free y-intercept	$(7.1 \pm 0.2) \times 10^{-34}$

Table 1: Extracted Planck's constant values from Fig. 2

Planck's constant was extracted by rearranging Eq. (1) into $V = \frac{h}{e} f$ and plotting the turn-on voltage against frequency. Then, the slope ($\frac{h}{e}$) was extracted from the fixed y-intercept best fit line. By multiplying the slope by the electric charge, $e = 1.602 \times 10^{-19}\ C$, we obtained h. An additional linear fit was performed with a free y-intercept parameter to compare the resulting value of h.

4 Discussion

Our experimentally determined Planck's constant, $(5.13 \pm 0.03) \times 10^{-34}\ Js$, was more than 5σ below the accepted value of $6.626 \times 10^{-34} Js$ [4]. However, our free y-parameter fit yielded $(7.1 \pm 0.2) \times 10^{-34}\ Js$, a result within 2σ of the accepted value.

We suspect the difference in the two fits is due to internal resistance of the LEDs. Internal resistance would cause over-estimation of the turn-on voltage because a higher voltage must be applied to overcome the inherent energy loss; This would lead us to read a higher turn-on voltage, resulting in a higher estimate for h. These ideas are supported by the fact that allowing a free y-intercept parameter (to account for the voltage offset) resulted in a much better fit to the data as seen in Fig. 2.

Underestimation of the uncertainty may also be a factor in the inconsistency. Turn-on voltage was determined by eye and is susceptible to human observers imagining that the LED is faintly on. The eye is a relatively imprecise detector and it is likely that we chose a low uncertainty based on our method of measurement.

5 Conclusions

In this experiment, we found that it was possible make a reasonable measurement of Planck's constant using only LEDs, a variable power supply, and a voltmeter. While our value ($h = (5.13 \pm 0.03) \times 10^{-34}\ Js$) was inconsistent with the accepted value [4], the value that accounted for the voltage offset ($h = (7.1 \pm 0.2) \times 10^{-34}$) was consistent. It only differed from the accepted value by about 13% in magnitude, demonstrating that LEDs can indeed be used to make a reasonable estimate for h.

While our initial determination for Planck's constant was statistically inconsistent with the accepted value, our adjusted value was reasonable; We believe this discrepancy is due to a combination of uncertainty underestimation and internal resistance present in the LEDs.

This experiment could be improved significantly by reducing the internal resistance of the LEDs, using more precise equipment, and estimating the uncertainties more appropriately. For example, by taking repeated measurements of the voltage and the wavelength, the mean and standard error could have been calculated to provide a more rigorous value and uncertainty of each measured quantity.

Author Contribution Statement: K.S and L.F contributed equally to the experiment and the report.

References

[1] "LED Lights - How it Works - History." [Online]. Available: https://edisontechcenter.org/LED.html 1

[2] "How do diodes and light-emitting diodes (LEDs) work?" [Online]. Available: https://www.explainthatstuff.com/diodes.html 1

[3] "Introduction to Diodes And Rectifiers — Diodes and Rectifiers — Electronics Textbook." [Online]. Available: https://www.allaboutcircuits.com/textbook/semiconductors/chpt-3/introduction-to-diodes-and-rectifiers/ 1

[4] "88.10 – Planck's constant demonstration." [Online]. Available: http://web.physics.ucsb.edu/{~}lecturedemonstrations/Composer/Pages/88.10.html 4, 5

A Lab Notebook

Lab 3 – LED

☆ Measure wide enough frequency to get a nice spectrum of the EM field.

$$E = hf = eV \Rightarrow h = \frac{eV}{f} ; \quad V = \text{turn-on}$$

- Using a voltmeter to measure voltage across the circuit ⇒ more accurate than the display on the power source
  ↳ error = ± 0.001 V → digital

- Using a spectrometer to measure peak wavelength accurately.
  ↳ figure out freq. with $f = \frac{c}{\lambda}$
  ↳ $\lambda_{error} = \pm\ 0.01$ nm → digital

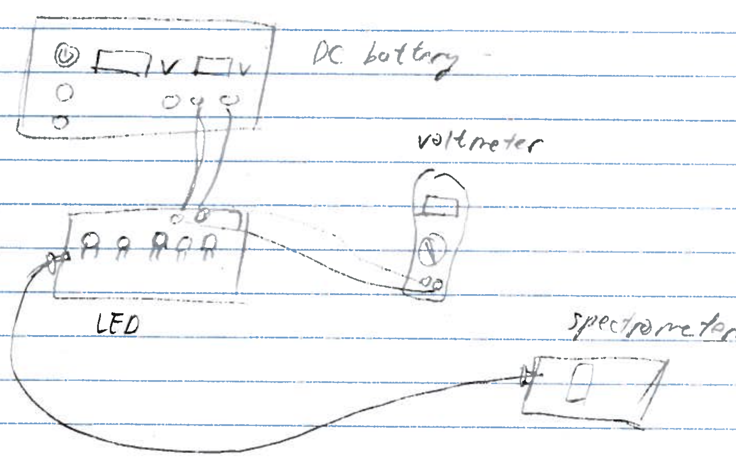
☆ Maybe 10 measurements for each LED to bring down the uncertainty by one magnitude.

- Measuring turn-on voltage by eye because fiber optic is not sensitive enough.
  ↳ V_{error} depends on our agreement.

Sources of error:
- human eye measurement
  ↳ eyes not the same sensitivit
  ↳ always over the true value
- Internal resistance of LED making turn-on voltage higher
- Measuring λ at diff. intensit

☆ Ignore double-peak LED for convience

colour	Wavelength (nm)	Turn-on Voltage (v)
red	627.03 / 624.4e ± 2	− 1.384 ± 0.02
~~purple~~		~~− 2.283 ± 0.01~~ → Double peak
orange	605.41 / 603.04 ± 2	− 1.445 ± 0.01
yellow	590.97 / 588.70 ± 2	− 1.540 ± 0.01
yel-green	566.58 / 574.78 ± 2	− 1.645 ± 0.01
green	535.21 / 541.47 ± 2	− 1.820 ± 0.01
blue	461.19 / 466.02 ± 2	− 2.160 ± 0.01
UV	400.69 / 401.80 ± 2	− 2.586 ± 0.02
red	628.61 / 623.49	−1.401 ± 0.02
orange	606.19 / 603.72	−1.449 ± 0.01
yellow	591.36 / 589.25	−1.548 ± 0.01
yel-green	566.19 / 570.25	−1.649 ± 0.01
green	534.07 / 540.34	−1.819 ± 0.01
blue	460.83 / 465.85	−2.135 ± 0.01
UV	404.98 / 402.30	−2.600 ± 0.02



B Python Code

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
import scipy.optimize as opt
```

In [3]:

```
wavelen = np.array([627.03, 605.41, 590.97, 566.58, 535.21, 461.
19, 400.69]) # nm
volt = np.array([1.384, 1.445, 1.540, 1.645, 1.820, 2.160, 2.586
]) # V

wavelen = wavelen * 10**(-9) # m
c = 3e8 # m/s
freq = (c/wavelen)*10**(-12) # THz
print(freq)
e = 1.602e-19 # C

wavelen_err = 2e-9 # m
freq_err = np.sqrt((c/wavelen**2)**2 * wavelen_err**2)*10**(-12)
# THz
volt_err = np.ones(len(volt))*0.03 # V
```

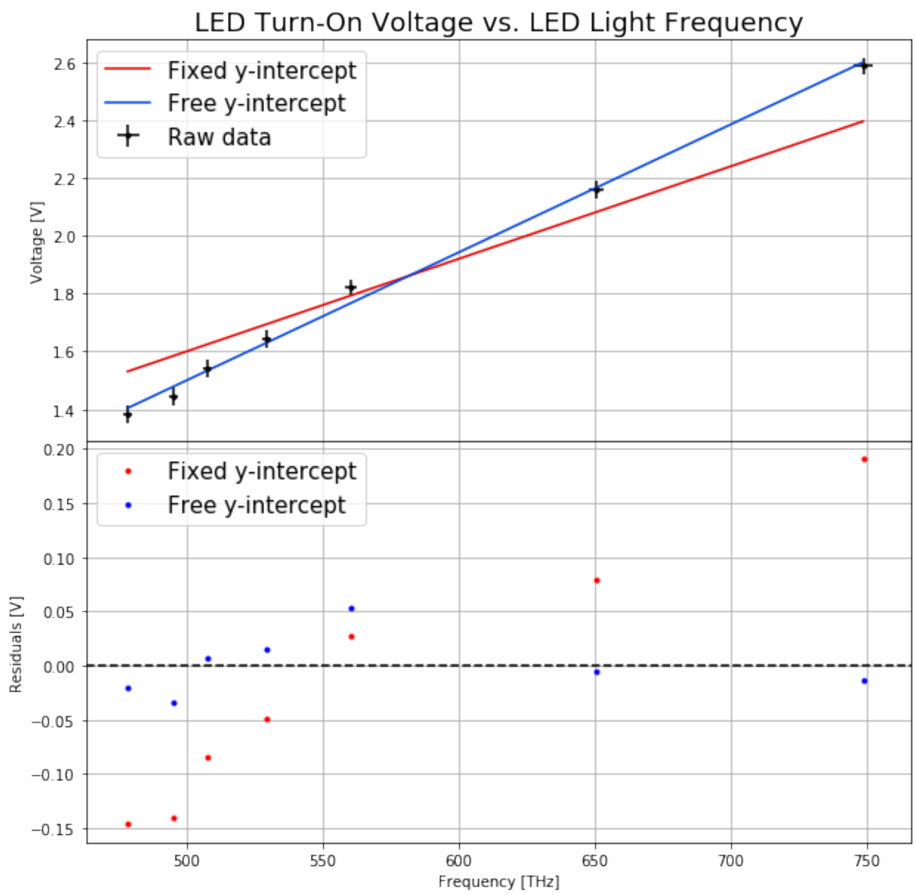
```
[478.44600737 495.53195355 507.63998172 529.49274595
560.52764335
 650.4911208  748.70847788]
```

In [4]:

```
def f1(x, m):
    return x*m

def f2(x, m, b):
    return x*m + b

params1, cov1 = opt.curve_fit(f1, freq, volt, absolute_sigma=Tru
e, sigma=volt_err)
params2, cov2 = opt.curve_fit(f2, freq, volt, absolute_sigma=Tru
e, sigma=volt_err)

# plotting
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(10, 10), sharex=Tr
```

```
ue)

plt.rc('font', size = 15)
plt.rc('xtick', labelsize = 15)
plt.rc('ytick', labelsize = 15)
plt.subplots_adjust(hspace=0)

# raw data
ax1.errorbar(freq, volt, xerr = freq_err, yerr = volt_err, fmt =
'.', label = "Raw data", color='xkcd:black')
ax1.set_title('LED Turn-On Voltage vs. LED Light Frequency')
ax2.set_xlabel('Frequency [THz]')
ax1.set_ylabel('Voltage [V]')

# fits
ax1.plot(freq, f1(freq, *params1), label='Fixed y-intercept', co
lor='xkcd:red')
ax1.plot(freq, f2(freq, *params2), label='Free y-intercept', col
or='xkcd:blue')
ax1.legend()
ax1.grid()

# residuals
res1 = volt - f1(freq, *params1)
res2 = volt - f2(freq, *params2)
ax2.plot(freq, res1, '.', label='Fixed y-intercept', color='red'
)
ax2.plot(freq, res2, '.', label='Free y-intercept', color='blue'
)
ax2.axhline(y=0, linestyle='--', color='k')
ax2.set_ylabel(r'Residuals [V]')
ax2.grid()
ax2.legend()

print(params1[0]*e*10**(-12))
print(params2[0]*e*10**(-12))
print(np.sqrt(cov1[0][0])*e*10**(-12))
print(np.sqrt(cov2[0][0])*e*10**(-12))
```

```
5.125733365117461e-34
7.087753760562653e-34
3.161829958621039e-36
1.9971322757652748e-35
```



$h_{free} = (7.1 \pm 0.2) \times 10^{-34}\ Js$ $h_{free} = (7.1 \pm 0.2) \times 10^{-34}\ Js$

$h_{fixed} = (5.13 \pm 0.03) \times 10^{-34}\ Js$ $h_{fixed} = (5.13 \pm 0.03) \times 10^{-34}\ Js$