McGill Physical Journal

Demonstrating Quantized Energy Levels for Argon Gas with the Franck-Hertz Experiment

Kevin Sohn (260782138), Lambert Francis (260861226)

McGill University Department of Physics

June 15, 2022

Abstract

In this experiment, we determined the excitation energy of argon in electron volts by recording the accelerating voltage where current spikes occurred, then calculating the mean voltage difference between the peaks and troughs. We also estimated Planck's constant by plugging in the determined excitation energy and the ultraviolet emission line of argon into Planck's equation, $E = h\frac{c}{\lambda}$. The excitation energy and Planck's constant were determined to be $E = 11.5 \pm 0.1\ eV$ and $h = (6.57 \pm 0.08) \times 10^{-34}\ Js$, respectively. Both of these values were within 1σ of their accepted values, showing consistency with the literature. Repetitions of the experiment with different accelerating and filament voltages had no significant effect on our determined values.

1 Introduction

In 1914, James Franck and Gustav Hertz conducted an experiment that demonstrated the quantized nature of energy levels in mercury atoms [1]. They designed a tube filled with mercury vapour that consisted of an anode, a cathode, and a grid. The cathode produced electrons which were accelerated by the potential difference generated from the presence of the cathode and the grid. This caused collisions to occur between mercury atoms and electrons. The anode was reverse biased, meaning it possessed negative charge with respect to the electrons. By noticing periodic drops in current at the anode, Frank and Hertz found that, in the case of mercury, electrons only lost energy at multiples of 4.9 eV [2]. They later showed that this energy value corresponded to an ultraviolet line in the emission spectrum of mercury, adding experimental proof to Bohr's model of the atom [3].

The basis of the experiment boils down to conservation of energy. If the electron lacks sufficient kinetic energy to increase the energy level of a shell electron, they simply bounce off the atom elastically, retaining enough kinetic energy to overcome the reverse bias and register as current. However, if the electron has kinetic energy greater than or equal to the energy threshold (4.9 eV for mercury), they undergo inelastic collision, transferring energy to the shell electron exactly equal to the threshold amount, allowing for a rise in energy level. If the electron loses enough kinetic energy, it is unable to overcome the reverse bias and reach the anode, causing a current drop.

When the shell electron falls back down to the ground energy level, a photon is emitted at a specific wavelength with its energy equal to the difference in energy levels. This relation is described by Planck's equation,

$$E = hf = \frac{hc}{\lambda}, \tag{1}$$

where E is the difference in energy levels, h is Planck's constant, c is the speed of light, and λ is the wavelength of light emitted.

In this experiment, we explore the footsteps of Franck and Hertz using a modern version of their setup to observe the quantized energy levels of argon, instead of mercury.

2 Materials and Methods

The experimental setup consisted of a DC current amplifier, two adjustable DC power sources, an argon Franck-Hertz tube, and a PASCO interface. A diagram of the experimental setup can be found in Figure 1. The experiment was repeated with three different settings to determine if the parameters affected the excitation energy determination. The settings are listed in Table 1.

Settings	V_H (V)	V_{G2A} (V)	V_{G1K} (V)	V_{G2K} (V)
Trial 1	2.4	11	1.5	0-80
Trial 2	2.4	12	1.5	0-80
Trial 3	2.9	11	1.5	0-80

Table 1: Settings used for each trial of the experiment, where V_H is filament voltage, V_{G2A} is the retarding voltage, V_{G1K} is the voltage between the first grid and the cathode, and V_{G2K} is the accelerating voltage.

All of the power supplies and amplifiers were turned off and set to 0 V before connecting them to the argon Frank-Hertz tube as shown in Figure 1. After the wires were connected, the settings for trial 1, listed in Table 1, were applied to the apparatus, and the argon tube was allowed to warm up for about 15 minutes. The warm up is crucial; otherwise, the current will not be stable for the first trial and will cause inaccurate readings.

Once the setup was completed, the experiment consisted of recording the current in the anode at ≈ 0.5 V intervals of the accelerating voltage (V_{G2K}), ranging from $0 - 80$ V. Going any higher in voltage would damage the argon tube. The small voltage increment provides higher resolution and more accurate location of the peaks and troughs than a larger increment. Current vs. accelerating voltage was then plotted in Capstone. The peaks and troughs were found using the Capstone delta tool, which snapped to the local extrema. This procedure was repeated for the second and third trial, changing only the initial settings as indicated by Table 1. Afterwards, the excitation energy (E) was determined by performing a weighted mean on the results of the trials. Using this excitation energy, Planck's constant was calculated with Eq. (1).

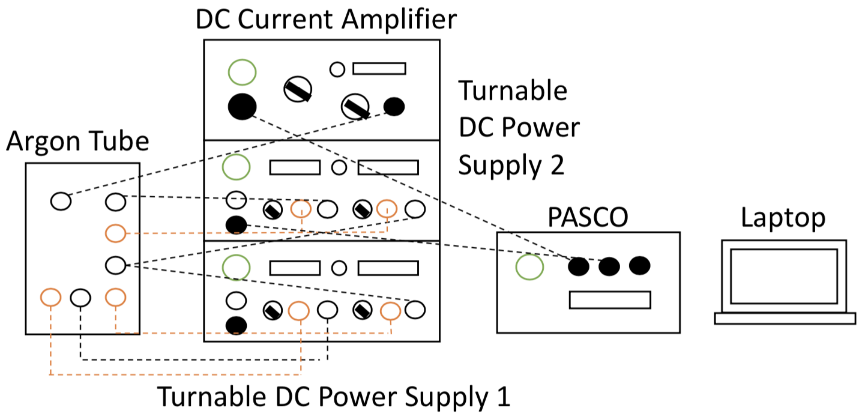


Figure 1: Schematic of the experimental setup.

3 Results

The voltage of the current peaks and troughs found in Figure 2 were taken to have uncertainties of $\pm 0.1\ V$ because it was measured digitally. All of the analysis was performed with Python in Jupyter notebook and the code can be found in Appendix B.

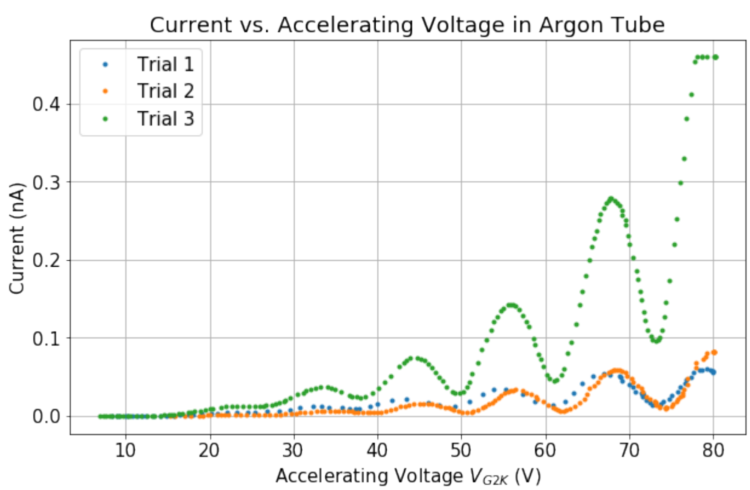


Figure 2: Plot of the raw data obtained from the PASCO interface sampled at an interval of $\approx 0.5\ V$. Each colour represents a trial of the experiment performed with different settings. The settings are found in Table 1. We observe the current peaks and troughs lining up for trials with the same V_{G2A} (Trials 1 and 3), and out of phase by $\approx 1\ V$ for trials with a $1\ V$ offset on V_{G2A} (Trial 2).

The excitation energies for each trial were calculated by computing the mean voltage difference between adjacent current peaks and troughs, respectively. The uncertainty in the mean was computed using the standard error formula, $\sigma_{mean} = \frac{\sigma}{\sqrt{N}}$, where σ is the standard deviation of the sample and N is the number of measurements of the sample.

Settings	Excitation Energy (eV)
Trial 1	11.5 ± 0.3
Trial 2	11.5 ± 0.2
Trial 3	11.6 ± 0.2

Table 2: Excitation energies of argon extracted from Figure 2 for each trial of the experiment.

By taking the weighted mean [4] of the results in Table 2, the overall excitation energy for argon was determined to be 11.5 ± 0.1 eV. The uncertainty for this value was calculated using a combination of the addition and multiplication error propagation formulas, $\sigma_V = \frac{\sqrt{\sigma_1^2+\sigma_2^2+\sigma_3^2}}{3}$ [5]. Using the literature ultraviolet wavelength emitted by argon, $\lambda_{arg} =$ 106.7 nm [6], we calculate Planck's constant using Eq. (1) to be $h = (6.57 \pm 0.08) \times 10^{-34}$ Js. The uncertainty in h was calculated by $\sigma_h = \sqrt{(\frac{\lambda}{c})^2\sigma_E^2}$, obtained from using the differential calculus method on Eq. (1).

4 Discussion

Our experimentally determined excitation energy for argon, 11.5 ± 0.1 eV, was within 1σ of the accepted value, 11.54835433 eV [7]. Our value of Planck's constant, $h = (6.57 \pm 0.08) \times 10^{-34}$ Js, was also within 1σ of the accepted value, $6.62607015 \times 10^{-34}$ Js [8].

There were no significant differences between the excitation energy values obtained from each trial; all three trials had an excitation energy within 1σ of the others. This indicates that changing the settings had no effect on the required energy threshold to boost a shell electron up an energy level. This makes sense intuitively; increasing the filament voltage would simply cause the filament to produce more electrons, leading to higher current because more electrons are accelerated to reach the anode. However, this has no influence on the amount of energy required to raise a shell electron up an energy level. Changing the retarding voltage by 1 V corresponded to an ≈ 1 V shift in the overall curve as shown in Figure 2. The Trial 2 curve being shifted $\approx 1V$ to the right relative to Trial 1 and Trial 3 makes sense because each electron would need to be accelerated by one additional volt to overcome the reverse bias. However, since that is true for every electron, the difference between peaks and troughs would remain the same.

Therefore, we conclude that changing the filament voltage and the retarding voltage has no effect on the distance between adjacent peaks or troughs, as observed.

5 Conclusions

In this paper, we showed that electrons inhabit quantized energy states for argon and used this relationship to estimate Planck's constant. Our data agreed with the accepted values from literature; our determinations of argon's excitation energy, $E = 11.5 \pm 0.1$ eV, and Planck's constant, $h = (6.57 \pm 0.08) \times 10^{-34}$ Js, were both within 1σ of their accepted values.

Our measurements agreed with accepted values to the quoted uncertainties. In future experiments, we hope to measure to greater precision and further confirm these accepted values. To do this, we suggest using a more sensitive and reliable device to measure the voltage because the displays on the power supplies are known to be less accurate than voltmeters. Furthermore, we suggest recording data continuously rather than manually in fixed increments. This would allow us to determine the peaks more reliably if our data points do not fall directly on the peaks; the Capstone delta tool likely introduced systematic error as it snapped to points that did not always lie directly on the extrema. An experiment designed to take these factors into account would be able to determine the excitation energy for argon and the Planck's constant to higher precision.

Author Contribution Statement: K.S and L.F contributed equally to the experiment and the report.

References

[1] “Franck-Hertz Experiment (Theory) : Modern Physics Virtual Lab : Physical Sciences : Amrita Vishwa Vidyapeetham Virtual Lab.” [Online]. Available: https://vlab.amrita.edu/?sub=1{&}brch=195{&}sim=355{&}cnt=1 1

[2] “Franck-Hertz Experiment.” [Online]. Available: http://hyperphysics.phy-astr.gsu.edu/hbase/FrHz.html 1

[3] “The Franck-Hertz experiment supports Bohr’s model.” [Online]. Available: http://spiff.rit.edu/classes/phys314/lectures/fh/fh.html 1

[4] “Weighted Mean.” [Online]. Available: https://www.mathsisfun.com/data/weighted-mean.html 4

[5] “Error Analysis.” [Online]. Available: http://lectureonline.cl.msu.edu/{~}mmp/labs/error/e2.htm 4

[6] “Argon resonance lines emitted by pure Ar plasma. — Download Scientific Diagram.” [Online]. Available: https://www.researchgate.net/figure/Argon-resonance-lines-emitted-by-pure-Ar-plasma{_}fig2{_}257953656 4

[7] A. Kramida, Y. Ralchenko, J. Reader, and The NIST ASD Team (2018), “NIST Atomic Spectra Database (ver. 5.6.1), [Online]. Available: http://physics.nist.gov/asd [11-Aug-2014]. National Institute of Standards and Technology, Gaithersburg, MD,” *NIST Atomic Spectra Database (ver. 5.6.1)*, vol. [Online], p. http://physics.nist.gov/asd, 2019. [Online]. Available: http://physics.nist.gov/asdhttps://doi.org/10.18434/T4W30F 4

[8] “CODATA Value: Planck constant.” [Online]. Available: https://physics.nist.gov/cgi-bin/cuu/Value?h 4

A Lab Notebook

Lab 4 Frank-Hertz

☆ Check for current dips every ~4.9V increase.

↳ should only dip at quantized E-states

$E = \frac{hc}{\lambda} = h\nu$, h = Planck's const.

c = speed of light

λ = photon wavelength

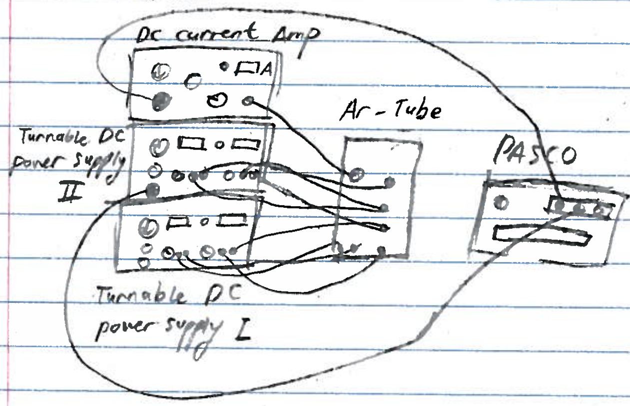
ν = photon frequency

$V_H = 2.4\ V$ → Filament voltage

$V_{G1K} = 1.5V$

$V_{G2K} = 0 \sim 85V$

$V_{G2A} = 11\ V$



Error sources: Miscalibrated 0-level.

Not warmed-up enough

Not precise enough measurement of peaks.

↳ Delta tool snapping off peak

↳ not high enough resolution

→ $V_{G2A} = 11\,V$

peak / trough voltage (V)	Voltage Uncertainty (V)
34.3 / 40.5	± 0.1
44.2 / 50.3	± 0.1
56.2 / 62.2	± 0.1
68.4 / 74.3	± 0.1
79.4 /	± 0.1

34.2
44.3
56.3
68.3
79.3

Trial # 2, checking error

33.4 / 37.4
43.5 / 49.2
55.3 / 60.9
66.9 / 72.7
79.2 /

part 2 : $V_{G2A} = 12\,V$

peak / trough voltage (V)	voltage uncertainty
35.6 / 40.7	± 0.1
45.6 / 52.3	± 0.1
57.3 / 63.0	± 0.1
69.0 / 74.5	± 0.1
79.5 /	± 0.1

34.4 / 39.1
45.0 / 50.6
56.5 / 62.2
68.3 / 74.3
80.1 /

Part 3: $V_H = 2.9V$ / $V_{G2A} = 11\,V$

34.1 / 39.3	± 0.1
45.2 / 51.1	± 0.1
57.7 / 62.3	± 0.1
68.9 / 73.7	± 0.1
79.5 /	± 0.1

33.2 / 38.0
43.9 / 49.8
55.9 / 61.0
67.9 / 73.1
78.7 /

B Python Code

In [2]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

data1 = pd.read_csv('part1repeat.csv')
data2 = pd.read_csv('part2repeat.csv')
data3 = pd.read_csv('part3repeat.csv')
```

In [3]:

```
display(data1)
data1
```

	Time (s)	Current (A)	Voltage (V)	Electron Current (x10^-10 A)	Peak Voltage (V)	Diff between Peaks (V)	Trough Voltage (V)	be Tr
0	11.7	-5.570000e-11	80.0	0.56	33.4	10.1	37.4	
1	11.9	-5.650000e-11	80.0	0.57	43.5	11.8	49.2	
2	12.1	-5.740000e-11	80.0	0.57	55.3	11.6	60.9	
3	12.4	-5.860000e-11	79.7	0.59	66.9	NaN	72.7	
4	12.7	-5.950000e-11	79.2	0.60	NaN	NaN	NaN	
...	...	...	...	...	...	...	...	
81	35.1	1.590000e-13	10.2	0.00	NaN	NaN	NaN	
82	35.3	1.740000e-13	9.9	0.00	NaN	NaN	NaN	
83	35.6	1.590000e-13	9.3	0.00	NaN	NaN	NaN	
84	35.9	1.500000e-13	8.9	0.00	NaN	NaN	NaN	
85	36.2	1.650000e-13	8.4	0.00	NaN	NaN	NaN	

86 rows × 8 columns

Out[3]:

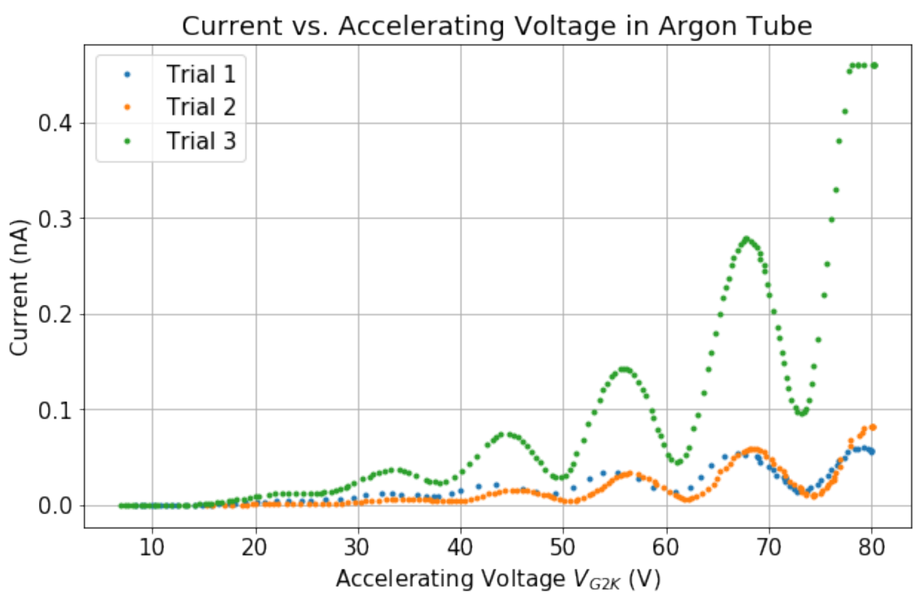
	Time (s)	Current (A)	Voltage (V)	Electron Current (x10^-10 A)	Peak Voltage (V)	Diff between Peaks (V)	Trough Voltage (V)	be Tr
0	11.7	-5.570000e-11	80.0	0.56	33.4	10.1	37.4	
1	11.9	-5.650000e-11	80.0	0.57	43.5	11.8	49.2	
2	12.1	-5.740000e-11	80.0	0.57	55.3	11.6	60.9	
3	12.4	-5.860000e-11	79.7	0.59	66.9	NaN	72.7	
4	12.7	-5.950000e-11	79.2	0.60	NaN	NaN	NaN	
...	...	...	...	...	...	...	...	
81	35.1	1.590000e-13	10.2	0.00	NaN	NaN	NaN	
82	35.3	1.740000e-13	9.9	0.00	NaN	NaN	NaN	
83	35.6	1.590000e-13	9.3	0.00	NaN	NaN	NaN	
84	35.9	1.500000e-13	8.9	0.00	NaN	NaN	NaN	
85	36.2	1.650000e-13	8.4	0.00	NaN	NaN	NaN	

86 rows × 8 columns

In [9]:

```
fig, (ax1) = plt.subplots(figsize=(10, 6))
plt.rc("font", size = 15)
plt.rc("xtick", labelsize = 15)
plt.rc("ytick", labelsize = 15)

ax1.plot(data1['Voltage (V)'], -data1['Current (A)']/10**-9, '.'
, label='Trial 1')
ax1.plot(data2['Voltage (V)'], -data2['Current (A)']/10**-9, '.'
, label='Trial 2')
ax1.plot(data3['Voltage (V)'], -data3['Current (A)']/10**-9, '.'
, label='Trial 3')
ax1.set_title('Current vs. Accelerating Voltage in Argon Tube')
ax1.set_xlabel(r'Accelerating Voltage $V_{G2K}$ (V)')
ax1.set_ylabel('Current (nA)')
ax1.legend()
ax1.grid()
```



In [5]:

```
# Average distance between peaks and troughs for each trial

diff1 = data1.head(3)['Diff between Peaks (V)'].append(data1.hea
d(3)['Diff between Troughs (V)'])
mean1 = diff1.mean()

diff2 = data2.head(3)['Diff between Peaks (V)'].append(data2.hea
d(3)['Diff between Troughs (V)'])
mean2 = diff2.mean()

diff3 = data3.head(3)['Diff between Peaks (V)'].append(data3.hea
d(3)['Diff between Troughs (V)'])
mean3 = diff3.mean()

std1 = np.std(diff1, ddof = 1)
std2 = np.std(diff2, ddof = 1)
std3 = np.std(diff3, ddof = 1)

# calculating standard error
err1 = std1/np.sqrt(len(diff1))
err2 = std2/np.sqrt(len(diff2))
err3 = std3/np.sqrt(len(diff3))

print(std1, std2, std3)
print(mean1, mean2, mean3)
print(err1, err2, err3)
```

```
0.6742897497861488 0.5036533199202272 0.560951572479
0037
11.466666666666667 11.516666666666667 11.63333333333
3333
0.2752776376275012 0.20561560684388186 0.22900752049
756903
```

In [6]:

```
# Calculating weights for weighted mean
w1 = 1/err1**2
w2 = 1/err2**2
w3 = 1/err3**2

# Weighted mean
excite_E = (mean1*w1 + mean2*w2 + mean3*w3) / (w1 + w2 + w3)
# Standard error prop. formula for addition and multiplication w
ith a constant
excite_E_err = np.sqrt(err1**2 + err2**2 + err3**2) / 3

print(excite_E)
print(excite_E_err)
```

```
11.544649963228258
0.1376388188137506
```

In [7]:

```
data5 = data1.head(5)
display(data5)
```

	Time (s)	Current (A)	Voltage (V)	Electron Current (x10^-10 A)	Peak Voltage (V)	Diff between Peaks (V)	Trough Voltage (V)	betv Tro
0	11.7	-5.570000e-11	80.0	0.56	33.4	10.1	37.4	
1	11.9	-5.650000e-11	80.0	0.57	43.5	11.8	49.2	
2	12.1	-5.740000e-11	80.0	0.57	55.3	11.6	60.9	
3	12.4	-5.860000e-11	79.7	0.59	66.9	NaN	72.7	
4	12.7	-5.950000e-11	79.2	0.60	NaN	NaN	NaN	

In [8]:

```
excite_E_J = 1.6e-19*excite_E # multiplying by fundamental charg
e of electron
excite_E_J_err = excite_E_err * 1.6e-19 # J
c=3e8 # speed of light in m/s
literature_wav = 106.7e-9 # m

h = excite_E_J*literature_wav/c
print(h)

error_h = np.sqrt((literature_wav/c)**2 * excite_E_J_err**2)
print(error_h)
```

```
6.56967547240776e-34
7.832566382627832e-36
```