ProMCDA: A Python package for Probabilistic Multi-Criteria Decision Analysis

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Summary

Multi-Criteria Decision Analysis (MCDA) is a formal process used to assist decision-makers in structuring complex decision problems and providing recommendations based on a comprehensive evaluation of alternatives. This evaluation is conducted by selecting relevant criteria and subcriteria, which are then aggregated according to the preferences of the decision-makers to produce a ranking or classification of the alternatives (Roy (1996); Bouyssou et al. (2006)). A wide range of MCDA methods are available in the literature for integrating information to classify alternatives into preference classes or rank them from best to worst (Cinelli et al. (2022)). Among these, composite indicators (CIs) are commonly used synthetic measures for ranking and benchmarking alternatives across complex concepts (Greco et al. (2019)). Examples of CI applications include environmental quality assessment (Oţoiu and Grădinaru (2018)), resilience of energy supply (Gasser et al. (2020)), sustainability (Volkart et al. (2016)), and global competitiveness (Klaus Schwab (2018)).

However, the final ranking of alternatives in MCDA can be influenced by various factors such as uncertainty in the criteria, the choice of weights assigned to them, and the selection of methods for normalization and aggregation to construct CIs (Cinelli et al. (2020)). To address these challenges, the `ProMCDA` Python module has been developed to allow decision-makers to explore the sensitivity and robustness of CI results in a user-friendly manner. This tool facilitates sensitivity analysis related to the choice of normalization and aggregation methods and accounts for uncertainty in criteria and weights, providing a systematic approach to understanding the impact of these factors on decision outcomes.

Statement of need

Several MCDA tools are available in the literature. For example, the Python library `pymcdm` (Kizielewicz, Shekhovtsov, and Sałabun (2023)) provides a broad collection of different MCDA methods, including those commonly used to construct CIs. The `pyDecision` library (Pereira, Basilio, and Santos (2024)) offers a large collection of MCDA methods and allows users to compare outcomes of different methods interactively, thanks to integration with ChatGPT. In R, the package `COINr` enables users to develop CIs with all standard operations, including criteria selection, data treatment, normalization, aggregation, and sensitivity analysis (Becker et al. (2022)). Other packages, such as `compind`, focus specifically on weighting and aggregation (Fusco, Vidoli, and Sahoo (2018)), while MATLAB tools like CIAO (Lindén et al. (2021)) offer specialized capabilities for parts of CI development.

The Python module `Decisi-o-Rama` (Chacon-Hurtado and Scholten (2021)) focuses on implementing Multi-Attribute Utility Theory (MAUT) to normalize criteria, considering a hierarchical criteria structure and uncertain criteria, and aggregate the results using different aggregation methods. Additionally, the web-based MCDA Index Tool supports sensitivity analysis based on various combinations of normalization functions and aggregation methods.

While these tools provide valuable functionalities, `ProMCDA` differentiates itself by adopting a fully probabilistic approach to perform MCDA for CIs, providing sensitivity and robustness analysis of the ranking results. The sensitivity of the MCDA scores arises from the use of various combinations of normalization/aggregation functions (Cinelli et al. (2020)) that can be used in the evaluation process. Meanwhile, uncertainty stems from the variability associated with the criteria values (Stewart and Durbach (2016)) or the randomness that may be associated with their weights (Lahdelma, Hokkanen, and Salminen (1998)). `ProMCDA` is unique in combining all these different sources of variability and providing a systematic analysis.

The tool is designed for use by both researchers and practitioners in operations research. Its approach offers a broad range of potential applications, including sustainability, healthcare, and risk assessment, among others. `ProMCDA` has been developed as a core methodology for the development of a decision support system for forest management (FutureForest). However, the tool is versatile and can be used in any other domain involving multi-criteria decision-making.

Overview

`ProMCDA` is a Python module that allows users to construct CIs while considering uncertainties associated with criteria, weights, and the choice of normalization and aggregation methods. The module's evaluation process is divided into two main steps: - **Data Normalization:** Ensuring all data values are on the same scale. - **Data Aggregation:** Estimating a single composite indicator from all criteria.

`ProMCDA` receives all necessary input information via a configuration file in JSON format (for more details, see the README). The alternatives are represented as rows in an input matrix (CSV file format), with criteria values in columns. The tool offers the flexibility to conduct sensitivity analysis by comparing the different scores associated with alternatives using various combinations of normalization and aggregation functions. `ProMCDA` currently implements four normalization and four aggregation functions, as described in Table 1 and Table 2, respectively. However, the user can run `ProMCDA` with a specific pair of normalization and aggregation functions, thus switching off the sensitivity analysis.

Normalization methods		Formula	Description	Comments
Linear scale	**Min-max**	$N_{ia} = \frac{x_{ia} - \min(x_i)}{\max(x_i) - \min(x_i)}$	It applies a linear transformation to rescale the data in a specified range (typically 0-1).	Most common normalization method used. The order and proximity of the data points are maintained. Outliers can have a significant impact on the transformation. Loss of information: it compresses the range of the original data.
	Standardization (z-score)	$N_{ia} = \frac{x_{ia} - x_{ia=\bar{a}}}{\sigma_{ia=\bar{a}}}$	It applies a linear transformation with mean of 0 and standard deviation of 1.	The order and proximity of the data points are maintained. The standardized data is not bounded. High values have a great impact on the result, which is desirable if the wish is to reward exceptional behaviour. It preserves the shape and distribution of the original data. Loss of information: none.
Ratio scale	**Target**	$N_{ia} = \frac{x_{ia}}{\max\,(x_i)}$	It normalizes the upper limit to 1.	The order and proximity of the data points are maintained. No fixed range. Sensitive to outliers. It can be useful when the maximum value is of particular interest or importance in the analysis. Loss of information: it can reduce the relative differences between values, potentially compressing the data.
Ordinal	**Rank**	$N_{ia} = rank(x_{ia})$	The data points are ranked based on their relative values.	The order and proximity of the data points are maintained. It does not impose a fixed range on the transformed data. It eliminates magnitude differences. It can be useful when the exact values are not important, but rather the relative positions or comparisons between values. Robust to outliers.

Legend

N_{ia}: the normalized value of indicator *i* for alternative *a*.
x_{ia}: the value of indicator i for alternative *a*.
$x_{ia=\bar{a}}$the average value of indicator *i* across all alternatives.
$\sigma_{ia=\bar{a}}$: the standard deviation of indicator *i* across all alternatives.
$min(x_i)$: the minimum value of indicator *i* across all alternatives.
$max(x_i)$: the maximum value of indicator *i* across all alternatives.

Figure 1: Normalization functions used in `ProMCDA`.

The user can bypass both the sensitivity and robustness analysis when running `ProMCDA`.

Sensitivity Analysis: `ProMCDA` provides a default sensitivity analysis based on the predefined normalization and aggregation pairs. However, users can specify the pair of functions they want to use and switch this analysis off.

Robustness Analysis: `ProMCDA` also allows for robustness analysis by introducing randomness to either the weights or the criteria in order to make the results as transparent as possible and avoid a lack of distinction between the effect of one or the other. Randomly sampling the weights or the criteria values is done using a Monte Carlo method.

The randomness in the weights can be applied to one weight at a time or to all weights simultaneously. In both cases, by default, the weights are sampled from a uniform distribution [0-1]. If the user decides to analyse the robustness of the criteria, they have to provide the parameters defining the marginal distribution (i.e., a probability density function, pdf) that best describes the criteria rather

Aggregation methods	Formula	Level of compensation	Comments
Additive (weighted arithmetic mean)	$score_a = \sum_{i=1}^{n} N_{ia}\, w_i$	Full	Most common aggregation method used. It is a linear combination. It amplifies the effect of the higher values. Commonly used in situations where variables are considered equally important.
Geometric (weighted geometric mean)	$score_a = \prod_{i=1}^{n} {N_{ia}}_i^{w_i}$	Partial	The indicators values should be larger than 0. It is a non-linear combination. The impact of each variable's value is not proportional to its magnitude, and the relative contribution of each variable depends on the other variables involved. It amplifies the impact of variables with small values. The method is commonly used in situations where the interaction or joint effect of variables is of interest.
Harmonic	$score_a = \frac{\sum_{i=1}^{n} w_i}{\sum_{i=1}^{n} \frac{w_i}{N_{ia}}}$	Partial (less than Geometric)	The indicators values should strictly be larger than 0. It is a non-linear combination. The impact of each value is not proportional to its magnitude, and the relative contribution of each variable depends on the other variables involved. Insensitivity to extreme values. It is primarily used in situations where smaller values are considered more important or when dealing with ratios or rates.
Minimum	$N_{ia} = min(N_{1a}, N_{2a}, \ldots, N_{na})$	None	The worst performing indicator equals the final score. Suitable if the DM is interested in an assessment driven by the worst performing indicator.
Legend $score_a$: the composite score for alternative *a*. n: the number of indicators. w_i: the weight of indicator *i*. N_{ia}: the normalized value of indicator *i* for alternative *a*.			

Figure 2: Aggregation functions used in `ProMCDA`. The sum of the weights is normalized to 1.

than the criteria values. This means that if a pdf described by 2 parameters characterizes a criterion, two columns should be allocated in the input CSV file for it. In `ProMCDA` 4 different pdfs describing the criteria uncertainty are considered:

- *uniform*, which is described by 2 parameters, i.e., minimum and maximum
- *normal*, which is described by 2 parameters, i.e., mean and standard deviation
- *lognormal*, which is described by 2 parameters, i.e., log(mean) and log(standard deviation)
- *Poisson*, which is described by 1 parameter, i.e., the rate.

Once the pdf for each criterion is selected and the input parameters are in place in the input CSV file, `ProMCDA` randomly samples n-values of each criterion per alternative from the given pdf and assesses the score and ranking of the alternatives by considering robustness at the criteria level. The number of samples is defined in the configuration file by the user.

Once the pdfs for each criterion are selected and the input parameters are in the input CSV file, `ProMCDA` randomly samples n-values of each criterion per alternative from the given pdf to evaluate the alternatives' scores and rankings, taking into account robustness at the criteria level.

Finally, in all possible cases (i.e., a simple MCDA, MCDA with sensitivity analysis for the different normalization/aggregation functions used, MCDA with robustness investigation related either to randomness on the weights or on the indicators), `ProMCDA` will output a CSV file with the scores/average scores and their plots. For a quick overview of the functionality of `ProMCDA`, refer to Table 3. For more details, refer to the README.

Possible usages of ProMCDA	Specifications	Notes
Simple MCDA No sensitivity nor robustness analysis is performed.	The specific pair normalization /aggregation to be used for the evaluation of the alternatives.	For a fully controlled MCDA.
Sensitivity analysis Focus is on the role of the normalization and aggregation functions.	All normalization and aggregation pairs are used for the evaluation of the alternatives.	Each pair normalization/aggregation will produce different scores for every alternative. **The sensitivity analysis can be associated with the robustness analysis due to the weights or the indicators.**
Robustness analysis of one weight at time Focus is on the role of one indicator and its relative weight at time.	One single weight at time is sampled from the uniform distribution [0,1].	This run can help investigate the importance of each indicator for the final scores. Average results are reported a number-of-indicator times. ***This robustness analysis cannot be used together with the robustness analysis associated with the indicators.***
Robustness analysis of all weights Focus is on the role of the weights.	All weights are sampled from the uniform distribution [0,1].	This run can help understanding the overall impact of the uncertainty due to the weights. ***This robustness analysis cannot be used together with the robustness analysis associated with the indicators.***
Robustness analysis of the indicators Focus is on the role of the uncertainty of the indicators.	All indicators, whose values are distributed as a non-exact pdf, are randomly sampled. *ProMCDA* needs N-values for each indicator per alternative to build N random input-matrices.	This run let the user analyse the impact of the uncertainty on the indicators for the final scores. ***This robustness analysis cannot be used together with the robustness analysis associated to the weights.***

Figure 3: Overview on the functionalities of `ProMCDA`.

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