**Tables**

Table 1: Normalization functions.

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| **Normalization methods** | | **Formula** | **Description** | **Comments** |
| *Linear scale* | **Min-max** |  | It applies a linear transformation to rescale the data in a specified range (typically 0-1). | Most common normalization method used. The order and proximity of the data points are maintained. Outliers can have a significant impact on the transformation. Loss of information: it compresses the range of the original data. |
| **Standardization**  **(z-score)** |  | It applies a linear transformation with mean of 0 and standard deviation of 1. | The order and proximity of the data points are maintained. The standardized data is not bounded. High values have a great impact on the result, which is desirable if the wish is to reward exceptional behaviour. It preserves the shape and distribution of the original data. Loss of information: none. |
| *Ratio scale* | **Target** |  | It normalizes the upper limit to 1. | The order and proximity of the data points are maintained. No fixed range. Sensitive to outliers. It can be useful when the maximum value is of particular interest or importance in the analysis. Loss of information: it can reduce the relative differences between values, potentially compressing the data. |
| *Ordinal* | **Rank** |  | The data points are ranked based on their relative values. | The order and proximity of the data points are maintained. It does not impose a fixed range on the transformed data. It eliminates magnitude differences. It can be useful when the exact values are not important, but rather the relative positions or comparisons between values. Robust to outliers. |
| **Legend**  : the normalized value of indicator *i* for alternative *a.*  : the value of indicator i for alternative *a.*  the average value of indicator *i* across all alternatives.  : the standard deviation of indicator *i* across all alternatives.  : the minimum value of indicator *i* across all alternatives.  : the maximum value of indicator *i* across all alternatives. | | | | |

Table 2: Aggregation functions. As reference see Langhans et al. [2014]. The sum of the weights is considered normalized to 1.

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| **Aggregation methods** | **Formula** | | **Level of compensation** | **Comments** |
| **Additive**  **(weighted arithmetic mean)** |  | Full | | Most common aggregation method used. It is a linear combination. It amplifies the effect of the higher values. Commonly used in situations where variables are considered equally important. |
| **Geometric**  **(weighted geometric mean)** |  | Partial | | The indicators values should be larger than 0. It is a non-linear combination. The impact of each variable's value is not proportional to its magnitude, and the relative contribution of each variable depends on the other variables involved. It amplifies the impact of variables with small values. The method is commonly used in situations where the interaction or joint effect of variables is of interest. |
| **Harmonic** |  | Partial  (less than Geometric) | | The indicators values should strictly be larger than 0. It is a non-linear combination. The impact of each value is not proportional to its magnitude, and the relative contribution of each variable depends on the other variables involved. Insensitivity to extreme values. It is primarily used in situations where smaller values are considered more important or when dealing with ratios or rates. |
| **Minimum** |  | None | | The worst performing indicator equals the final score. Suitable if the DM is interested in an assessment driven by the worst performing indicator. |
| **Legend**  : the composite score for alternative *a*.  : the number of indicators*.*  : the weight of indicator *i*.  : the normalized value of indicator *i* for alternative *a.* | | | | |

Table 3: ProMCDA usages.

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| **Possible usages of ProMCDA** | **Specifications** | **Notes** |
| **Simple MCDA**  No sensitivity nor robustness analysis is performed. | The specific pair normalization / aggregation to be used for the evaluation of the alternatives. | For a fully controlled MCDA. |
| **Sensitivity analysis**  Focus is on the role of the normalization and aggregation functions. | All normalization and aggregation pairs are used for the evaluation of the alternatives. | Each pair normalization/aggregation will produce different scores for every alternative.  **The sensitivity analysis can be associated with the robustness analysis due to the weights or the indicators.** |
| **Robustness analysis of one weight at time**  Focus is on the role of one indicator and its relative weight at time. | One single weight at time is sampled from the uniform distribution [0,1]. | This run can help investigate the importance of each indicator for the final scores. Average results are reported a number-of-indicator times.  ***This robustness analysis cannot be used together with the robustness analysis associated with the indicators.*** |
| **Robustness analysis of all weights**  Focus is on the role of the weights. | All weights are sampled from the uniform distribution [0,1]. | This run can help understanding the overall impact of the uncertainty due to the weights.  ***This robustness analysis cannot be used together with the robustness analysis associated with the indicators.*** |
| **Robustness analysis of the indicators**  Focus is on the role of the uncertainty of the indicators. | All indicators, whose values are distributed as a non-exact pdf, are randomly sampled. *ProMCDA* needs N-values for each indicator per alternative to build N random input-matrices. | This run let the user analyse the impact of the uncertainty on the indicators for the final scores.  ***This robustness analysis cannot be used together with the robustness analysis associated to the weights.*** |