In[∘]:= **Get@FileNameJoin@{NotebookDirectory[], "Calibration.m"};**

```
First we import some formatting...

...okay, that's better, from now on any commentary written inside this
  Calibration.m wrapper will present as blue text (i.e. this text is not
  part of PSALTer, it is just a use-case). Next we load the PSALTer package:

------------------------------------------------------------

Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}

CopyRight (C) 2003-2020, Jose M. Martin-Garcia, under the General Public License.

Connecting to external linux executable...

Connection established.

------------------------------------------------------------

Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}

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------------------------------------------------------------

Package xAct`xPert`  version 1.0.6, {2018, 2, 28}

CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia
  and Guillermo A. Mena Marugan, under the General Public License.

** Variable $PrePrint assigned value ScreenDollarIndices

** Variable $CovDFormat changed from Prefix to Postfix

** Option AllowUpperDerivatives of ContractMetric changed from False to True

** Option MetricOn of MakeRule changed from None to All

** Option ContractMetrics of MakeRule changed from False to True

------------------------------------------------------------

Package xAct`Invar`  version 2.0.5, {2013, 7, 1}

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  D. Yllanes and R. Portugal, under the General Public License.

** DefConstantSymbol: Defining constant symbol sigma.

** DefConstantSymbol: Defining constant symbol dim.

** Option CurvatureRelations of DefCovD changed from True to False

** Variable $CommuteCovDsOnScalars changed from True to False

------------------------------------------------------------

Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}

CopyRight (C) 2005-2021, David Yllanes and
  Jose M. Martin-Garcia, under the General Public License.
```

Package xAct`SymManipulator` version 0.9.5, {2021, 9, 14}

CopyRight (C) 2011-2021, Thomas Bäckdahl, under the General Public License.

Package xAct`xTras` version 1.4.2, {2014, 10, 30}

CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.

** Variable $CovDFormat changed from Postfix to Prefix

** Option CurvatureRelations of DefCovD changed from False to True

----

Package xAct`HiGGS` version 2.0.0-developer, {2023, 2, 22}

CopyRight © 2022, Will E. V. Barker and Manuel Hohmann, under the General Public License.

HiGGS incorporates code by Cyril Pitrou.

Package xAct`PSALTer` version 1.0.0-developer, {2023, 2, 22}

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These packages come with ABSOLUTELY NO WARRANTY; for details type Disclaimer[]. This is free software, and you are welcome to redistribute it under certain conditions. See the General Public License for details.

Now we set up the general Lagrangian:

$$-\lambda_{\cdot}\,\mathcal{R}^{\mathfrak{i}\mathfrak{j}}{}_{\mathfrak{i}\mathfrak{j}}+\left(\frac{r_{1}}{3}+\frac{r_{2}}{6}\right)\mathcal{R}_{\mathfrak{i}\mathfrak{j}\mathfrak{k}\mathfrak{l}}\,\mathcal{R}^{\mathfrak{i}\mathfrak{j}\mathfrak{k}\mathfrak{l}}+\left(\frac{2r_{1}}{3}-\frac{2r_{2}}{3}\right)\mathcal{R}_{\mathfrak{i}\mathfrak{k}\mathfrak{j}\mathfrak{l}}\,\mathcal{R}^{\mathfrak{i}\mathfrak{j}\mathfrak{k}\mathfrak{l}}+$$

$$\left(r_{4}+r_{5}\right)\mathcal{R}_{\mathfrak{i}}{}^{\mathfrak{l}}{}_{\mathfrak{j}\mathfrak{l}}\,\mathcal{R}^{\mathfrak{i}\mathfrak{k}\mathfrak{j}}{}_{\mathfrak{k}}+\left(r_{4}-r_{5}\right)\mathcal{R}^{\mathfrak{i}\mathfrak{k}\mathfrak{j}}{}_{\mathfrak{k}}\,\mathcal{R}_{\mathfrak{j}}{}^{\mathfrak{l}}{}_{\mathfrak{i}\mathfrak{l}}+\left(\frac{r_{1}}{3}+\frac{r_{2}}{6}-r_{3}\right)\mathcal{R}^{\mathfrak{i}\mathfrak{j}\mathfrak{k}\mathfrak{l}}\,\mathcal{R}_{\mathfrak{k}\mathfrak{l}\mathfrak{i}\mathfrak{j}}+$$

$$\left(\frac{\lambda_{\cdot}}{4}+\frac{t_{1}}{3}+\frac{t_{2}}{12}\right)\mathcal{T}_{\mathfrak{i}\mathfrak{j}\mathfrak{k}}\,\mathcal{T}^{\mathfrak{i}\mathfrak{j}\mathfrak{k}}+\left(-\frac{\lambda_{\cdot}}{2}-\frac{t_{1}}{3}+\frac{t_{2}}{6}\right)\mathcal{T}^{\mathfrak{i}\mathfrak{j}\mathfrak{k}}\,\mathcal{T}_{\mathfrak{j}\mathfrak{k}\mathfrak{i}}+\left(-\lambda_{\cdot}-\frac{t_{1}}{3}+\frac{2t_{3}}{3}\right)\mathcal{T}_{\mathfrak{i}}{}^{\mathfrak{j}\mathfrak{i}}\,\mathcal{T}_{\mathfrak{k}\mathfrak{j}}{}^{\mathfrak{k}}$$

We also knock up some simple tools to linearise the Lagrangian:

** DefConstantSymbol: Defining constant symbol PerturbativeParameter.

Now we would like to check the basic Einstein-Cartan theory. Here is the full nonlinear Lagrangian:

$$t_{1}\,\mathcal{R}^{\mathfrak{i}\mathfrak{j}}{}_{\mathfrak{i}\mathfrak{j}}$$

To use PSALTer, you have to first linearise
this Lagrangian to second order around the desired vacuum:

$$\dot{t}_1\, \mathcal{A}_{aji}\, \mathcal{A}^{aij} + \dot{t}_1\, \mathcal{A}^{ai}{}_{a}\, \mathcal{A}_{i}{}^{j}{}_{j} + 2\dot{t}_1\, f^{ai}\, \partial_i \mathcal{A}_{a}{}^{j}{}_{j} - 2\dot{t}_1\, \partial_i \mathcal{A}^{ai}{}_{a} - 2\dot{t}_1\, f^{ai}\, \partial_j \mathcal{A}_{a}{}^{j}{}_{i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & \frac{i k \dot{t}_1}{\sqrt{2}} & 0 \\ -\frac{i k \dot{t}_1}{\sqrt{2}} & -\dot{t}_1 & -i\sqrt{\frac{3}{2}}\, k \dot{t}_1 \\ 0 & i\sqrt{\frac{3}{2}}\, k \dot{t}_1 & 0 \end{pmatrix}, \begin{pmatrix} -\dot{t}_1 \end{pmatrix}, \begin{pmatrix} 0 & \frac{i k \dot{t}_1}{\sqrt{2}} & 0 \\ -\frac{i k \dot{t}_1}{\sqrt{2}} & -\frac{\dot{t}_1}{2} & -\frac{\dot{t}_1}{\sqrt{2}} \\ 0 & -\frac{\dot{t}_1}{\sqrt{2}} & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{\dot{t}_1}{2} & i k \dot{t}_1 & \frac{\dot{t}_1}{\sqrt{2}} \\ 0 & -i k \dot{t}_1 & 0 & 0 \\ 0 & \frac{\dot{t}_1}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \frac{i k \dot{t}_1}{\sqrt{2}} \\ -\frac{i k \dot{t}_1}{\sqrt{2}} & \frac{\dot{t}_1}{2} \end{pmatrix}, \begin{pmatrix} \frac{\dot{t}_1}{2} \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{ \overset{0^+}{\cdot}\tau^{\flat\parallel} == \overset{0^+}{\cdot}\tau^{\flat\perp},\ i\ \overset{1^+}{\cdot}\tau^{\flat\parallel}{}^{ab} == k\ \overset{1^+}{\cdot}\sigma^{\flat\perp}{}^{ab},\ i\ \overset{1^-}{\cdot}\tau^{\flat\perp}{}^{a} == 2k\ \overset{1^-}{\cdot}\sigma^{\flat\perp}{}^{a},\ \overset{1^-}{\cdot}\tau^{\flat\parallel}{}^{a} == 0 \right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{1}{8k^2 \dot{t}_1} & \frac{i}{2\sqrt{2}\, k \dot{t}_1} & \frac{\sqrt{3}}{8k^2 \dot{t}_1} \\ -\frac{i}{2\sqrt{2}\, k \dot{t}_1} & 0 & -\frac{i\sqrt{\frac{3}{2}}}{2k \dot{t}_1} \\ \frac{\sqrt{3}}{8k^2 \dot{t}_1} & \frac{i\sqrt{\frac{3}{2}}}{2k \dot{t}_1} & \frac{3}{8k^2 \dot{t}_1} \end{pmatrix}, \begin{pmatrix} -\frac{1}{\dot{t}_1} \end{pmatrix}, \begin{pmatrix} \frac{k^2}{(1+k^2)^2 \dot{t}_1} & \frac{i\sqrt{2}\, k}{\dot{t}_1 + k^2 \dot{t}_1} & -\frac{i k}{(1+k^2)^2 \dot{t}_1} \\ -\frac{i\sqrt{2}\, k}{\dot{t}_1 + k^2 \dot{t}_1} & 0 & -\frac{\sqrt{2}}{\dot{t}_1 + k^2 \dot{t}_1} \\ \frac{i k}{(1+k^2)^2 \dot{t}_1} & -\frac{\sqrt{2}}{\dot{t}_1 + k^2 \dot{t}_1} & \frac{1}{(1+k^2)^2 \dot{t}_1} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{2ik}{\dot{t}_1 + 2k^2 \dot{t}_1} & \frac{\sqrt{2}}{\dot{t}_1 + 2k^2 \dot{t}_1} \\ 0 & -\frac{2ik}{\dot{t}_1 + 2k^2 \dot{t}_1} & \frac{2k^2}{(1+2k^2)^2 \dot{t}_1} & -\frac{i\sqrt{2}\, k}{(1+2k^2)^2 \dot{t}_1} \\ 0 & \frac{\sqrt{2}}{\dot{t}_1 + 2k^2 \dot{t}_1} & \frac{i\sqrt{2}\, k}{(1+2k^2)^2 \dot{t}_1} & \frac{1}{(1+2k^2)^2 \dot{t}_1} \end{pmatrix}, \begin{pmatrix} -\frac{1}{k^2 \dot{t}_1} & \frac{i\sqrt{2}}{k \dot{t}_1} \\ -\frac{i\sqrt{2}}{k \dot{t}_1} & 0 \end{pmatrix}, \begin{pmatrix} \frac{2}{\dot{t}_1} \end{pmatrix}\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{9p^2}{\dot{t}_1}, -\frac{9p^2}{\dot{t}_1}\right\}$$

Overall unitarity conditions:

$$\left(p < 0 \,\&\&\, t_{\underset{1}{\cdot}} < 0\right) \,||\, \left(p > 0 \,\&\&\, t_{\underset{1}{\cdot}} < 0\right)$$

Okay, so that is the end of the PSALTer output for Einstein-Cartan gravity. What we find are no propagating massive modes, but instead two degrees of freedom in the massive sector. The no-ghost conditions on these massless d.o.f restrict the sign in front of the Einstein-Hilbert term to be negative (which is what we expect for our conventions).

Using Karananas' coefficients, it is particularly easy to also look at GR, instead of Einstein-Cartan theory. The difference here is that the quadratic torsion coefficients are manually removed. Here is the nonlinear Lagrangian:

$$-\lambda_{\cdot}\, \mathcal{R}^{ij}{}_{ij} + \frac{1}{4}\lambda_{\cdot}\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} + \frac{1}{2}\lambda_{\cdot}\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik} + \lambda_{\cdot}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{k}{}_{jk}$$

Here is the linearised theory:

$$-2\lambda_{\cdot}\, \mathcal{A}_{\alpha'}{}^{i}{}_{i}\, \partial_\alpha f^{\alpha\alpha'} - 2\lambda_{\cdot}\, f^{\alpha\alpha'}\, \partial_{\alpha'}\mathcal{A}_{\alpha}{}^{i}{}_{i} + 2\lambda_{\cdot}\, \partial_{\alpha'}\mathcal{A}^{\alpha\alpha'}{}_{\alpha} + 2\lambda_{\cdot}\, \mathcal{A}_{\alpha'}{}^{i}{}_{i}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha} - \lambda_{\cdot}\, \partial_{\alpha'} f^{i}{}_{i}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha} + 2\lambda_{\cdot}\, f^{\alpha\alpha'}\, \partial_i \mathcal{A}_{\alpha}{}^{i}{}_{\alpha'} - \lambda_{\cdot}\, \partial_\alpha f^{\alpha\alpha'}\, \partial_i f^{i}{}_{\alpha'} + 2\lambda_{\cdot}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha}\, \partial_i f^{i}{}_{\alpha'} + 2\lambda_{\cdot}\, \mathcal{A}_{\alpha' i\alpha}\, \partial^i f^{\alpha\alpha'} - \lambda_{\cdot}\, \partial_\alpha f_{\alpha' i}\, \partial^i f^{\alpha\alpha'} + \frac{1}{2}\lambda_{\cdot}\, \partial_\alpha f_{i\alpha'}\, \partial^i f^{\alpha\alpha'} - \frac{1}{2}\lambda_{\cdot}\, \partial_{\alpha'} f_{\alpha i}\, \partial^i f^{\alpha\alpha'} + \frac{1}{2}\lambda_{\cdot}\, \partial_i f_{\alpha\alpha'}\, \partial^i f^{\alpha\alpha'} + \frac{1}{2}\lambda_{\cdot}\, \partial_i f_{\alpha'\alpha}\, \partial^i f^{\alpha\alpha'}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{ \begin{pmatrix} -2k^2\lambda_{\cdot} & -\frac{3 i k \lambda_{\cdot}}{\sqrt{2}} & 0 \\ \frac{3 i k \lambda_{\cdot}}{\sqrt{2}} & 0 & i\sqrt{\frac{3}{2}}\, k\lambda_{\cdot} \\ 0 & -i\sqrt{\frac{3}{2}}\, k\lambda_{\cdot} & 0 \end{pmatrix}, (0), \begin{pmatrix} 0 & -i\sqrt{2}\, k\lambda_{\cdot} & 0 \\ i\sqrt{2}\, k\lambda_{\cdot} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i k\lambda_{\cdot} & 0 & 0 \\ i k\lambda_{\cdot} & 0 & -i k\lambda_{\cdot} & 0 \\ 0 & i k\lambda_{\cdot} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2\lambda_{\cdot} & 0 \\ 0 & 0 \end{pmatrix}, (0) \right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^-}_{\cdot}\sigma^{\flat\parallel} == 0,\ {}^{1^+}_{\cdot}\sigma^{\flat\perp}{}^{ab} == 0,\ {}^{1^-}_{\cdot}\sigma^{\flat\perp}{}^{a} == 0,\ {}^{1^-}_{\cdot}\tau^{\flat\parallel}{}^{a} + {}^{1^-}_{\cdot}\tau^{\flat\perp}{}^{a} == 0,\ {}^{2^+}_{\cdot}\sigma^{\flat\parallel}{}^{ab} == 0,\ {}^{2^-}_{\cdot}\sigma^{\flat\parallel}{}^{abc} == 0 \right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{1}{2k^2\lambda} & 0 & \frac{\sqrt{3}}{2k^2\lambda} \\ 0 & 0 & \frac{i\sqrt{\frac{2}{3}}}{k\lambda} \\ \frac{\sqrt{3}}{2k^2\lambda} & -\frac{i\sqrt{\frac{2}{3}}}{k\lambda} & -\frac{3}{2k^2\lambda} \end{pmatrix}, (0), \begin{pmatrix} 0 & -\frac{i}{\sqrt{2}k\lambda} & 0 \\ \frac{i}{\sqrt{2}k\lambda} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i}{2k\lambda} & 0 & 0 \\ \frac{i}{2k\lambda} & 0 & -\frac{i}{2k\lambda} & 0 \\ 0 & \frac{i}{2k\lambda} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2\lambda} & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{p^2}{\lambda}, \frac{p^2}{\lambda}\right\}$$

Overall unitarity conditions:

$$(p < 0 \,\&\&\, \lambda > 0) \,||\, (p > 0 \,\&\&\, \lambda > 0)$$

Thus, the conclusions are the same, as expected.

We are now ready to check that PSALTer is getting the physics right by running it on the 58 theories in arXiv:1910.14197.

Performing the survey

Case 1

Now for a new theory. Here is the full nonlinear Lagrangian for Case 1 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{2} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \left(\frac{r_{3}}{2} + r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} +$$
$$\frac{1}{6}\left(r_{2} - 6 r_{3}\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij} + \frac{1}{2}\left(r_{3} - 2 r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12} t_{2} \mathcal{T}_{ijk} \mathcal{T}^{ijk} - \frac{1}{6} t_{2} \mathcal{T}^{ijk} \mathcal{T}_{jik}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$
\begin{aligned}
&\frac{1}{3} t_{\dot{2}}\, \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i}\, \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} t_{\dot{2}}\, \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'}\, \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + \left(-\frac{r_{\dot{3}}}{2} + r_{\dot{5}}\right) \partial_{\mathfrak{a}'}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} + \\
&\left(-\frac{r_{\dot{3}}}{2} - r_{\dot{5}}\right) \partial_{i}\mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} - \frac{2}{3} t_{\dot{2}}\, \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'} + \frac{2}{3} t_{\dot{2}}\, \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'} - \frac{2}{3} t_{\dot{2}}\, \mathcal{A}_{\mathfrak{a}'i\mathfrak{a}}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'} + \\
&\frac{1}{3} t_{\dot{2}}\, \partial_{\mathfrak{a}}f_{\mathfrak{a}'i}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'} - \frac{1}{6} t_{\dot{2}}\, \partial_{\mathfrak{a}}f_{i\mathfrak{a}'}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'} - \frac{1}{6} t_{\dot{2}}\, \partial_{\mathfrak{a}'}f_{\mathfrak{a}i}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'} + \frac{1}{6} t_{\dot{2}}\, \partial_{i}f_{\mathfrak{a}\mathfrak{a}'}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'} - \frac{1}{6} t_{\dot{2}}\, \partial_{i}f_{\mathfrak{a}'\mathfrak{a}}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'} + \\
&\left(-\frac{r_{\dot{3}}}{2} - r_{\dot{5}}\right) \partial_{\mathfrak{a}}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\, \partial_{j}\mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{i} + \left(r_{\dot{3}} + 2 r_{\dot{5}}\right) \partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}\, \partial_{j}\mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{i} + \left(-\frac{r_{\dot{3}}}{2} + r_{\dot{5}}\right) \partial_{\mathfrak{a}}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{\mathfrak{a}'} + \\
&\left(r_{\dot{3}} - 2 r_{\dot{5}}\right) \partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{\mathfrak{a}'} + \frac{4}{3} r_{\dot{2}}\, \partial_{\mathfrak{a}'}\mathcal{A}_{\mathfrak{a}ij}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} r_{\dot{2}}\, \partial_{\mathfrak{a}'}\mathcal{A}_{\mathfrak{a}ji}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + \\
&\frac{2}{3}\left(r_{\dot{2}} - 6 r_{\dot{3}}\right) \partial_{\mathfrak{a}'}\mathcal{A}_{ij\mathfrak{a}}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{1}{3} r_{\dot{2}}\, \partial_{i}\mathcal{A}_{\mathfrak{a}\mathfrak{a}'j}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + \frac{1}{3} r_{\dot{2}}\, \partial_{j}\mathcal{A}_{\mathfrak{a}\mathfrak{a}'i}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} r_{\dot{2}}\, \partial_{j}\mathcal{A}_{\mathfrak{a}i\mathfrak{a}'}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}
\end{aligned}
$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$
\left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^{2} r_{\dot{2}} + t_{\dot{2}}\right), \begin{pmatrix} \frac{k^{2} t_{\dot{2}}}{3} & \frac{1}{3} i \sqrt{2}\, k\, t_{\dot{2}} & \frac{i k t_{\dot{2}}}{3} \\ -\frac{1}{3} i \sqrt{2}\, k\, t_{\dot{2}} & \frac{1}{2}\left(2 k^{2}\left(2 r_{\dot{3}} + r_{\dot{5}}\right) + \frac{4 t_{\dot{2}}}{3}\right) & \frac{\sqrt{2}\, t_{\dot{2}}}{3} \\ -\frac{1}{3} i k\, t_{\dot{2}} & \frac{\sqrt{2}\, t_{\dot{2}}}{3} & \frac{t_{\dot{2}}}{3} \end{pmatrix}, \right.
$$

$$
\left. \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} k^{2}\left(r_{\dot{3}} + 2 r_{\dot{5}}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{3 k^{2} r_{\dot{3}}}{2} \end{pmatrix}, (0) \right\}
$$

Gauge constraints on source currents:

$$
\left\{ {}^{0^{+}}_{\cdot}\tau^{\flat\perp} == 0,\ {}^{0^{+}}_{\cdot}\sigma^{\flat\parallel} == 0,\ {}^{0^{+}}_{\cdot}\tau^{\flat\parallel} == 0,\ -i\ {}^{1^{+}}_{\cdot}\tau^{\flat\parallel}{}^{\mathfrak{a}\mathfrak{b}} == k\ {}^{1^{+}}_{\cdot}\sigma^{\flat\perp}{}^{\mathfrak{a}\mathfrak{b}}, \right.
$$

$$
\left. {}^{1^{-}}_{\cdot}\sigma^{\flat\perp}{}^{\mathfrak{a}} == 0,\ {}^{1^{-}}_{\cdot}\tau^{\flat\perp}{}^{\mathfrak{a}} == 0,\ {}^{1^{-}}_{\cdot}\tau^{\flat\parallel}{}^{\mathfrak{a}} == 0,\ {}^{2^{+}}_{\cdot}\tau^{\flat\parallel}{}^{\mathfrak{a}\mathfrak{b}} == 0,\ {}^{2^{-}}_{\cdot}\sigma^{\flat\parallel}{}^{\mathfrak{a}\mathfrak{b}\mathfrak{c}} == 0 \right\}
$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_{2} + t_{2}} \end{pmatrix}, \begin{pmatrix} \frac{3 k^2 (2 r_{3} + r_{5}) + 2 t_{2}}{(1+k^2)^2 (2 r_{3} + r_{5}) t_{2}} & -\frac{i \sqrt{2}}{k (1+k^2)(2 r_{3} + r_{5})} & \frac{i (3 k^2 (2 r_{3} + r_{5}) + 2 t_{2})}{k (1+k^2)^2 (2 r_{3} + r_{5}) t_{2}} \\ \frac{i \sqrt{2}}{k (1+k^2)(2 r_{3} + r_{5})} & \frac{1}{k^2 (2 r_{3} + r_{5})} & -\frac{\sqrt{2}}{k^2 (1+k^2)(2 r_{3} + r_{5})} \\ -\frac{i (3 k^2 (2 r_{3} + r_{5}) + 2 t_{2})}{k (1+k^2)^2 (2 r_{3} + r_{5}) t_{2}} & -\frac{\sqrt{2}}{k^2 (1+k^2)(2 r_{3} + r_{5})} & \frac{3 k^2 (2 r_{3} + r_{5}) + 2 t_{2}}{(k+k^3)^2 (2 r_{3} + r_{5}) t_{2}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{2}{k^2 (r_{3} + 2 r_{5})} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3 k^2 r_{3}} \end{pmatrix}, (0)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\left\{-\frac{45 r_{3}^2 + 20 r_{3} r_{5} + 4 r_{5}^2}{r_{3} (2 r_{3} + r_{5})(r_{3} + 2 r_{5})}, -\frac{45 r_{3}^2 + 20 r_{3} r_{5} + 4 r_{5}^2}{r_{3} (2 r_{3} + r_{5})(r_{3} + 2 r_{5})}\right\}$$

Overall unitarity conditions:

$$\left(r_{2} < 0 \,\&\&\, r_{3} < 0 \,\&\&\, r_{5} < -\frac{r_{3}}{2} \,\&\&\, t_{2} > 0\right) \,||$$

$$\left(r_{2} < 0 \,\&\&\, r_{3} < 0 \,\&\&\, r_{5} > -2 r_{3} \,\&\&\, t_{2} > 0\right) \,||\, \left(r_{2} < 0 \,\&\&\, r_{3} > 0 \,\&\&\, -2 r_{3} < r_{5} < -\frac{r_{3}}{2} \,\&\&\, t_{2} > 0\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{2} < 0 \,\&\&\, r_{3} < 0 \,\&\&\, r_{5} < -\frac{r_{3}}{2} \,\&\&\, t_{2} > 0\right) \,||$$

$$\left(r_{2} < 0 \,\&\&\, r_{3} < 0 \,\&\&\, r_{5} > -2 r_{3} \,\&\&\, t_{2} > 0\right) \,||\, \left(r_{2} < 0 \,\&\&\, r_{3} > 0 \,\&\&\, -2 r_{3} < r_{5} < -\frac{r_{3}}{2} \,\&\&\, t_{2} > 0\right)$$

Okay, that concludes the analysis of this theory.

Case 2

Now for a new theory. Here is the full nonlinear Lagrangian for Case 2 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{2} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \left(\frac{r_{3}}{2} + r_{5}\right) \mathcal{R}^{ijk}_{i} \mathcal{R}_{jkl}^{l} + \frac{1}{6}\left(r_{2} - 6 r_{3}\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij} + \frac{1}{2}\left(r_{3} - 2 r_{5}\right) \mathcal{R}^{ijk}_{i} \mathcal{R}_{kjl}^{l} + \frac{1}{12} t_{2} \mathcal{T}_{ijk} \mathcal{T}^{ijk} - \frac{1}{6} t_{2} \mathcal{T}^{ijk} \mathcal{T}_{jik} - \frac{2}{3} t_{3} \mathcal{T}^{ij}_{i} \mathcal{T}^{k}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{2} \mathcal{A}_{\alpha\alpha'i} \mathcal{A}^{\alpha\alpha'i} - \frac{2}{3} t_{2} \mathcal{A}_{\alpha i\alpha'} \mathcal{A}^{\alpha\alpha'i} - \frac{2}{3} t_{3} \mathcal{A}^{\alpha\alpha'}_{\alpha} \mathcal{A}_{\alpha'i}^{i} + \frac{4}{3} t_{3} \mathcal{A}_{\alpha'i}^{i} \partial_{\alpha} f^{\alpha\alpha'} - \frac{4}{3} t_{3} \mathcal{A}_{\alpha'i}^{i} \partial^{\alpha'} f^{\alpha}_{\alpha} + \frac{2}{3} t_{3} \partial_{\alpha'} f^{i}_{i} \partial^{\alpha'} f^{\alpha}_{\alpha} + \frac{2}{3} t_{3} \partial_{\alpha} f^{\alpha\alpha'} \partial_{i} f^{i}_{\alpha'} - \frac{4}{3} t_{3} \partial^{\alpha'} f^{\alpha}_{\alpha} \partial_{i} f^{i}_{\alpha'} + \left(-\frac{r_{3}}{2} + r_{5}\right) \partial_{\alpha'} \mathcal{A}_{ij}^{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}_{\alpha} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_{i} \mathcal{A}_{\alpha'j}^{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}_{\alpha} - \frac{2}{3} t_{2} \mathcal{A}_{\alpha\alpha'i} \partial^{i} f^{\alpha\alpha'} + \frac{2}{3} t_{2} \mathcal{A}_{\alpha i\alpha'} \partial^{i} f^{\alpha\alpha'} - \frac{2}{3} t_{2} \mathcal{A}_{\alpha'i\alpha} \partial^{i} f^{\alpha\alpha'} + \frac{1}{3} t_{2} \partial_{\alpha} f_{\alpha'i} \partial^{i} f^{\alpha\alpha'} - \frac{1}{6} t_{2} \partial_{\alpha} f_{i\alpha'} \partial^{i} f^{\alpha\alpha'} - \frac{1}{6} t_{2} \partial_{\alpha'} f_{\alpha i} \partial^{i} f^{\alpha\alpha'} + \frac{1}{6} t_{2} \partial_{i} f_{\alpha\alpha'} \partial^{i} f^{\alpha\alpha'} - \frac{1}{6} t_{2} \partial_{i} f_{\alpha'\alpha} \partial^{i} f^{\alpha\alpha'} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_{\alpha} \mathcal{A}^{\alpha\alpha'i} \partial_{j} \mathcal{A}_{\alpha'i}^{j} + \left(r_{3} + 2 r_{5}\right) \partial^{i} \mathcal{A}^{\alpha\alpha'}_{\alpha} \partial_{j} \mathcal{A}_{\alpha'i}^{j} + \left(-\frac{r_{3}}{2} + r_{5}\right) \partial_{\alpha} \mathcal{A}^{\alpha\alpha'i} \partial_{j} \mathcal{A}_{i\alpha'}^{j} + \left(r_{3} - 2 r_{5}\right) \partial^{i} \mathcal{A}^{\alpha\alpha'}_{\alpha} \partial_{j} \mathcal{A}_{i\alpha'}^{j} + \frac{4}{3} r_{2} \partial_{\alpha'} \mathcal{A}_{\alpha ij} \partial^{j} \mathcal{A}^{\alpha\alpha'i} - \frac{2}{3} r_{2} \partial_{\alpha'} \mathcal{A}_{\alpha ji} \partial^{j} \mathcal{A}^{\alpha\alpha'i} + \frac{2}{3}\left(r_{2} - 6 r_{3}\right) \partial_{\alpha'} \mathcal{A}_{ij\alpha} \partial^{j} \mathcal{A}^{\alpha\alpha'i} - \frac{1}{3} r_{2} \partial_{i} \mathcal{A}_{\alpha\alpha'j} \partial^{j} \mathcal{A}^{\alpha\alpha'i} + \frac{1}{3} r_{2} \partial_{j} \mathcal{A}_{\alpha\alpha'i} \partial^{j} \mathcal{A}^{\alpha\alpha'i} - \frac{2}{3} r_{2} \partial_{j} \mathcal{A}_{\alpha i\alpha'} \partial^{j} \mathcal{A}^{\alpha\alpha'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2 k^{2} t_{3} & i \sqrt{2}\, k t_{3} & 0 \\ -i \sqrt{2}\, k t_{3} & t_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^{2} r_{2} + t_{2}\right), \begin{pmatrix} \frac{k^{2} t_{2}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{2} & \frac{i k t_{2}}{3} \\ -\frac{1}{3} i \sqrt{2}\, k t_{2} & \frac{1}{2}\left(2 k^{2}\left(2 r_{3} + r_{5}\right) + \frac{4 t_{2}}{3}\right) & \frac{\sqrt{2}\, t_{2}}{3} \\ -\frac{1}{3} i k t_{2} & \frac{\sqrt{2}\, t_{2}}{3} & \frac{t_{2}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2 k^{2} t_{3}}{3} & \frac{2 i k t_{3}}{3} & 0 & -\frac{1}{3} i \sqrt{2}\, k t_{3} \\ -\frac{2}{3} i k t_{3} & k^{2}\left(\frac{r_{3}}{2} + r_{5}\right) + \frac{2 t_{3}}{3} & 0 & -\frac{\sqrt{2}\, t_{3}}{3} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{3} & -\frac{\sqrt{2}\, t_{3}}{3} & 0 & \frac{t_{3}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{3 k^{2} r_{3}}{2} \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}_{\bullet}\tau^{\flat\parallel} == 2\,k\,{}^{0^+}_{\bullet}\sigma^{\flat\parallel},\ -i\,{}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab},\right.$$
$$\left. i\,{}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a} == 2\,k\,{}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ {}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 0,\ {}^{2^-}_{\bullet}\sigma^{\flat\parallel}{}^{abc} == 0\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_{3}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & \frac{1}{(1+2k^2)^2 t_{3}} & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \begin{pmatrix} \frac{1}{k^2 r_{2}+t_{2}} \end{pmatrix},\ \begin{pmatrix} \frac{3k^2(2r_{3}+r_{5})+2t_{2}}{(1+k^2)^2(2r_{3}+r_{5})t_{2}} & -\frac{i\sqrt{2}}{k(1+k^2)(2r_{3}+r_{5})} & \frac{i(3k^2(2r_{3}+r_{5})+2t_{2})}{k(1+k^2)^2(2r_{3}+r_{5})t_{2}} \\ \frac{i\sqrt{2}}{k(1+k^2)(2r_{3}+r_{5})} & \frac{1}{k^2(2r_{3}+r_{5})} & -\frac{\sqrt{2}}{k^2(1+k^2)(2r_{3}+r_{5})} \\ -\frac{i(3k^2(2r_{3}+r_{5})+2t_{2})}{k(1+k^2)^2(2r_{3}+r_{5})t_{2}} & -\frac{\sqrt{2}}{k^2(1+k^2)(2r_{3}+r_{5})} & \frac{3k^2(2r_{3}+r_{5})+2t_{2}}{(k+k^3)^2(2r_{3}+r_{5})t_{2}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6k^2(r_{3}+2r_{5})+8t_{3}}{(1+2k^2)^2(r_{3}+2r_{5})t_{3}} & -\frac{4i}{k(1+2k^2)(r_{3}+2r_{5})} & 0 & -\frac{i\sqrt{2}(3k^2(r_{3}+2r_{5})+4t_{3})}{k(1+2k^2)^2(r_{3}+2r_{5})t_{3}} \\ \frac{4i}{k(1+2k^2)(r_{3}+2r_{5})} & \frac{2}{k^2(r_{3}+2r_{5})} & 0 & \frac{2\sqrt{2}}{k^2(1+2k^2)(r_{3}+2r_{5})} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}(3k^2(r_{3}+2r_{5})+4t_{3})}{k(1+2k^2)^2(r_{3}+2r_{5})t_{3}} & \frac{2\sqrt{2}}{k^2(1+2k^2)(r_{3}+2r_{5})} & 0 & \frac{3k^2(r_{3}+2r_{5})+4t_{3}}{(k+2k^3)^2(r_{3}+2r_{5})t_{3}} \end{pmatrix},\ \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3k^2 r_{3}} \end{pmatrix},\ \begin{pmatrix} 0 \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\},\ \left\{-\frac{t_{2}}{r_{2}}\right\},\ \{\},\ \{\},\ \{\},\ \{\}\right\}$$

Massive pole residues:

$$\left\{\{\},\ \left\{-\frac{1}{r_{2}}\right\},\ \{\},\ \{\},\ \{\},\ \{\}\right\}$$

Massless eigenvalues:

$$\left\{-\frac{403\,r_{3}^2+172\,r_{3}\,r_{5}+28\,r_{5}^2}{6\,r_{3}\,(2\,r_{3}+r_{5})(r_{3}+2\,r_{5})},\ -\frac{403\,r_{3}^2+172\,r_{3}\,r_{5}+28\,r_{5}^2}{6\,r_{3}\,(2\,r_{3}+r_{5})(r_{3}+2\,r_{5})}\right\}$$

Overall unitarity conditions:

$$\left(r_{2}<0\ \&\&\ r_{3}<0\ \&\&\ r_{5}<-\frac{r_{3}}{2}\ \&\&\ t_{2}>0\right)\ ||$$
$$\left(r_{2}<0\ \&\&\ r_{3}<0\ \&\&\ r_{5}>-2\,r_{3}\ \&\&\ t_{2}>0\right)\ ||\ \left(r_{2}<0\ \&\&\ r_{3}>0\ \&\&\ -2\,r_{3}<r_{5}<-\frac{r_{3}}{2}\ \&\&\ t_{2}>0\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(\dot{r}_2 < 0 \,\&\&\, \dot{r}_3 < 0 \,\&\&\, \dot{r}_5 < -\frac{\dot{r}_3}{2} \,\&\&\, \dot{t}_2 > 0\right) \,||\, \left(\dot{r}_2 < 0 \,\&\&\, \dot{r}_3 < 0 \,\&\&\, \dot{r}_5 > -2\,\dot{r}_3 \,\&\&\, \dot{t}_2 > 0\right) \,||\, \left(\dot{r}_2 < 0 \,\&\&\, \dot{r}_3 > 0 \,\&\&\, -2\,\dot{r}_3 < \dot{r}_5 < -\frac{\dot{r}_3}{2} \,\&\&\, \dot{t}_2 > 0\right)$$

Okay, that concludes the analysis of this theory.

Case 3

Now for a new theory. Here is the full nonlinear Lagrangian for Case 3 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\dot{r}_2\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} - \frac{2}{3}\dot{r}_2\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} + \dot{r}_5\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6}\dot{r}_2\, \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} - \dot{r}_5\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4}\dot{t}_1\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} + \frac{1}{2}\dot{t}_1\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik} + \frac{1}{3}\dot{t}_1\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\dot{t}_1\, \mathcal{A}_{aia'}\, \mathcal{A}^{aa'i} + \frac{1}{3}\dot{t}_1\, \mathcal{A}^{aa'}{}_{a}\, \mathcal{A}_{a'}{}^{i}{}_{i} - \frac{2}{3}\dot{t}_1\, \mathcal{A}_{a'}{}^{i}{}_{i}\, \partial_a f^{aa'} + \frac{2}{3}\dot{t}_1\, \mathcal{A}_{a'}{}^{i}{}_{i}\, \partial^{a'} f^{a}{}_{a} - \frac{1}{3}\dot{t}_1\, \partial_{a'} f^{i}{}_{i}\, \partial^{a'} f^{a}{}_{a} - \frac{1}{3}\dot{t}_1\, \partial_a f^{aa'}\, \partial_i f^{i}{}_{a'} + \frac{2}{3}\dot{t}_1\, \partial^{a'} f^{a}{}_{a}\, \partial_i f^{i}{}_{a'} + \dot{r}_5\, \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^i \mathcal{A}^{aa'}{}_{a} - \dot{r}_5\, \partial_i \mathcal{A}_{a'}{}^{j}{}_{j}\, \partial^i \mathcal{A}^{aa'}{}_{a} + 2\dot{t}_1\, \mathcal{A}_{a'ia}\, \partial^i f^{aa'} - \dot{t}_1\, \partial_a f_{a'i}\, \partial^i f^{aa'} + \frac{1}{2}\dot{t}_1\, \partial_a f_{ia'}\, \partial^i f^{aa'} - \frac{1}{2}\dot{t}_1\, \partial_{a'} f_{ai}\, \partial^i f^{aa'} + \frac{1}{2}\dot{t}_1\, \partial_i f_{aa'}\, \partial^i f^{aa'} + \frac{1}{2}\dot{t}_1\, \partial_i f_{a'a}\, \partial^i f^{aa'} - \dot{r}_5\, \partial_a \mathcal{A}^{aa'i}\, \partial_j \mathcal{A}_{a'}{}^{j}{}_{i} + 2\dot{r}_5\, \partial^i \mathcal{A}^{aa'}{}_{a}\, \partial_j \mathcal{A}_{a'}{}^{j}{}_{i} + \dot{r}_5\, \partial_a \mathcal{A}^{aa'i}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{a'} - 2\dot{r}_5\, \partial^i \mathcal{A}^{aa'}{}_{a}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{a'} + \frac{4}{3}\dot{r}_2\, \partial_{a'} \mathcal{A}_{aij}\, \partial^j \mathcal{A}^{aa'i} - \frac{2}{3}\dot{r}_2\, \partial_{a'} \mathcal{A}_{aji}\, \partial^j \mathcal{A}^{aa'i} + \frac{2}{3}\dot{r}_2\, \partial_{a'} \mathcal{A}_{ija}\, \partial^j \mathcal{A}^{aa'i} - \frac{1}{3}\dot{r}_2\, \partial_i \mathcal{A}_{aa'j}\, \partial^j \mathcal{A}^{aa'i} + \frac{1}{3}\dot{r}_2\, \partial_j \mathcal{A}_{aa'i}\, \partial^j \mathcal{A}^{aa'i} - \frac{2}{3}\dot{r}_2\, \partial_j \mathcal{A}_{aia'}\, \partial^j \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_2 - t_1\right), \begin{pmatrix} 0 & -\frac{i k t_1}{\sqrt{2}} & 0 \\ \frac{i k t_1}{\sqrt{2}} & \frac{1}{2}\left(2 k^2 r_5 - t_1\right) & -\frac{t_1}{\sqrt{2}} \\ 0 & -\frac{t_1}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2 k^2 t_1}{3} & -\frac{1}{3} i k t_1 & 0 & -\frac{1}{3} i \sqrt{2}\, k t_1 \\ \frac{i k t_1}{3} & k^2 r_5 + \frac{t_1}{6} & 0 & \frac{t_1}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_1 & \frac{t_1}{3\sqrt{2}} & 0 & \frac{t_1}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_1 & \frac{i k t_1}{\sqrt{2}} \\ -\frac{i k t_1}{\sqrt{2}} & \frac{t_1}{2} \end{pmatrix}, \left(\frac{t_1}{2}\right)\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\bullet}\sigma^{\flat\parallel} == 0,\ {}^{0^+}_{\bullet}\tau^{\flat\parallel} == 0,\ -i\, {}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\, {}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab},\ i\, {}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a} == 2 k\, {}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ -i\, {}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 2 k\, {}^{2^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab} \right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_2 - t_1}\right), \begin{pmatrix} \frac{-2 k^4 r_5 + k^2 t_1}{(1+k^2)^2 t_1^2} & -\frac{i \sqrt{2}\, k}{t_1 + k^2 t_1} & -\frac{i\left(2 k^3 r_5 - k t_1\right)}{(1+k^2)^2 t_1^2} \\ \frac{i \sqrt{2}\, k}{t_1 + k^2 t_1} & 0 & -\frac{\sqrt{2}}{t_1 + k^2 t_1} \\ \frac{i\left(2 k^3 r_5 - k t_1\right)}{(1+k^2)^2 t_1^2} & -\frac{\sqrt{2}}{t_1 + k^2 t_1} & \frac{-2 k^2 r_5 + t_1}{(1+k^2)^2 t_1^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6 k^2 r_5 + t_1}{\left(1+2 k^2\right)^2 r_5 t_1} & \frac{i}{k r_5 + 2 k^3 r_5} & 0 & -\frac{i\left(6 k^2 r_5 + t_1\right)}{\sqrt{2}\, k \left(1+2 k^2\right)^2 r_5 t_1} \\ -\frac{i}{k r_5 + 2 k^3 r_5} & \frac{1}{k^2 r_5} & 0 & -\frac{1}{\sqrt{2}\left(k^2 r_5 + 2 k^4 r_5\right)} \\ 0 & 0 & 0 & 0 \\ \frac{i\left(6 k^2 r_5 + t_1\right)}{\sqrt{2}\, k \left(1+2 k^2\right)^2 r_5 t_1} & -\frac{1}{\sqrt{2}\left(k^2 r_5 + 2 k^4 r_5\right)} & 0 & \frac{6 k^2 r_5 + t_1}{2\left(k+2 k^3\right)^2 r_5 t_1} \end{pmatrix}, \begin{pmatrix} \frac{4 k^2}{\left(1+2 k^2\right)^2 t_1} & \frac{2 i \sqrt{2}\, k}{\left(1+2 k^2\right)^2 t_1} \\ -\frac{2 i \sqrt{2}\, k}{\left(1+2 k^2\right)^2 t_1} & \frac{2}{\left(1+2 k^2\right)^2 t_1} \end{pmatrix}, \left(\frac{2}{t_1}\right)\right\}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\left\{-\frac{7\,t_{1}^{2}+2\,r_{5}\,t_{1}\,p^{2}+4\,r_{5}^{2}\,p^{4}}{2\,r_{5}\,t_{1}^{2}},\,-\frac{7\,t_{1}^{2}+2\,r_{5}\,t_{1}\,p^{2}+4\,r_{5}^{2}\,p^{4}}{2\,r_{5}\,t_{1}^{2}}\right\}$$

Overall unitarity conditions:

$p\in\mathbb{R}\ \&\&\ r_{2}<0\ \&\&\ r_{5}<0\ \&\&\ t_{1}<0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{2}<0\ \&\&\ r_{5}<0\ \&\&\ t_{1}<0$

Okay, that concludes the analysis of this theory.

Case 4

Now for a new theory. Here is the full nonlinear Lagrangian for Case 4 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}r_{1}\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl}+\frac{2}{3}r_{1}\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl}+r_{5}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl}-\frac{2}{3}r_{1}\,\mathcal{R}^{ijkl}\,\mathcal{R}_{klij}-r_{5}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl}+\frac{1}{4}t_{1}\,\mathcal{T}_{ijk}\,\mathcal{T}^{ijk}+\frac{1}{2}t_{1}\,\mathcal{T}^{ijk}\,\mathcal{T}_{jik}+\frac{1}{3}t_{1}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&t_{1}\,\mathcal{A}_{\alpha i\alpha'}\,\mathcal{A}^{\alpha\alpha' i}+\frac{1}{3}t_{1}\,\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\,\mathcal{A}_{\alpha'}{}^{i}{}_{i}-\frac{2}{3}t_{1}\,\mathcal{A}_{\alpha'}{}^{i}{}_{i}\,\partial_{\alpha}f^{\alpha\alpha'}+\frac{2}{3}t_{1}\,\mathcal{A}_{\alpha'}{}^{i}{}_{i}\,\partial^{\alpha'}f^{\alpha}{}_{\alpha}-\\
&\frac{1}{3}t_{1}\,\partial_{\alpha'}f^{i}{}_{i}\,\partial^{\alpha'}f^{\alpha}{}_{\alpha}-\frac{1}{3}t_{1}\,\partial_{\alpha}f^{\alpha\alpha'}\,\partial_{i}f^{i}{}_{\alpha'}+\frac{2}{3}t_{1}\,\partial^{\alpha'}f^{\alpha}{}_{\alpha}\,\partial_{i}f^{i}{}_{\alpha'}+r_{5}\,\partial_{\alpha'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}-\\
&r_{5}\,\partial_{i}\mathcal{A}_{\alpha'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}+2\,t_{1}\,\mathcal{A}_{\alpha' i\alpha}\,\partial^{i}f^{\alpha\alpha'}-t_{1}\,\partial_{\alpha}f_{\alpha' i}\,\partial^{i}f^{\alpha\alpha'}+\frac{1}{2}t_{1}\,\partial_{\alpha}f_{i\alpha'}\,\partial^{i}f^{\alpha\alpha'}-\frac{1}{2}t_{1}\,\partial_{\alpha'}f_{\alpha i}\,\partial^{i}f^{\alpha\alpha'}+\\
&\frac{1}{2}t_{1}\,\partial_{i}f_{\alpha\alpha'}\,\partial^{i}f^{\alpha\alpha'}+\frac{1}{2}t_{1}\,\partial_{i}f_{\alpha'\alpha}\,\partial^{i}f^{\alpha\alpha'}-r_{5}\,\partial_{\alpha}\mathcal{A}^{\alpha\alpha' i}\,\partial_{j}\mathcal{A}_{\alpha'}{}^{j}{}_{i}+2\,r_{5}\,\partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\,\partial_{j}\mathcal{A}_{\alpha'}{}^{j}{}_{i}+\\
&r_{5}\,\partial_{\alpha}\mathcal{A}^{\alpha\alpha' i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{\alpha'}-2\,r_{5}\,\partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{\alpha'}-\frac{4}{3}r_{1}\,\partial_{\alpha'}\mathcal{A}_{\alpha ij}\,\partial^{j}\mathcal{A}^{\alpha\alpha' i}+\frac{2}{3}r_{1}\,\partial_{\alpha'}\mathcal{A}_{\alpha ji}\,\partial^{j}\mathcal{A}^{\alpha\alpha' i}-\\
&\frac{8}{3}r_{1}\,\partial_{\alpha'}\mathcal{A}_{ij\alpha}\,\partial^{j}\mathcal{A}^{\alpha\alpha' i}-\frac{2}{3}r_{1}\,\partial_{i}\mathcal{A}_{\alpha\alpha' j}\,\partial^{j}\mathcal{A}^{\alpha\alpha' i}+\frac{2}{3}r_{1}\,\partial_{j}\mathcal{A}_{\alpha\alpha' i}\,\partial^{j}\mathcal{A}^{\alpha\alpha' i}+\frac{2}{3}r_{1}\,\partial_{j}\mathcal{A}_{\alpha i\alpha'}\,\partial^{j}\mathcal{A}^{\alpha\alpha' i}
\end{aligned}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_{1} \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & \frac{1}{2}\left(2 k^2 \left(2 r_{1} + r_{5}\right) - t_{1}\right) & -\frac{t_{1}}{\sqrt{2}} \\ 0 & -\frac{t_{1}}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2 k^2 t_{1}}{3} & -\frac{1}{3} i k t_{1} & 0 & -\frac{1}{3} i \sqrt{2}\, k t_{1} \\ \frac{i k t_{1}}{3} & k^2 \left(r_{1} + r_{5}\right) + \frac{t_{1}}{6} & 0 & \frac{t_{1}}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{1} & \frac{t_{1}}{3\sqrt{2}} & 0 & \frac{t_{1}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2 k^2 r_{1} + t_{1}\right) \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\bullet}\sigma^{\flat\parallel} == 0,\ {}^{0^+}_{\bullet}\tau^{\flat\parallel} == 0,\ -i\ {}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\ {}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab},\ i\ {}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a} == 2 k\ {}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ -i\ {}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 2 k\ {}^{2^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab} \right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{-2 k^4 \left(2 r_{1} + r_{5}\right) + k^2 t_{1}}{\left(1+k^2\right)^2 t_{1}^2} & -\frac{i \sqrt{2}\, k}{t_{1} + k^2 t_{1}} & \frac{-2 i k^3 \left(2 r_{1} + r_{5}\right) + i k t_{1}}{\left(1+k^2\right)^2 t_{1}^2} \\ \frac{i \sqrt{2}\, k}{t_{1} + k^2 t_{1}} & 0 & -\frac{\sqrt{2}}{t_{1} + k^2 t_{1}} \\ \frac{i \left(2 k^3 \left(2 r_{1} + r_{5}\right) - k t_{1}\right)}{\left(1+k^2\right)^2 t_{1}^2} & -\frac{\sqrt{2}}{t_{1} + k^2 t_{1}} & \frac{-2 k^2 \left(2 r_{1} + r_{5}\right) + t_{1}}{\left(1+k^2\right)^2 t_{1}^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6 k^2 \left(r_{1} + r_{5}\right) + t_{1}}{\left(1+2 k^2\right)^2 \left(r_{1} + r_{5}\right) t_{1}} & \frac{i}{k \left(1+2 k^2\right)\left(r_{1} + r_{5}\right)} & 0 & -\frac{i \left(6 k^2 \left(r_{1} + r_{5}\right) + t_{1}\right)}{\sqrt{2}\, k \left(1+2 k^2\right)^2 \left(r_{1} + r_{5}\right) t_{1}} \\ -\frac{i}{k \left(1+2 k^2\right)\left(r_{1} + r_{5}\right)} & \frac{1}{k^2 \left(r_{1} + r_{5}\right)} & 0 & -\frac{1}{\sqrt{2} \left(k^2 + 2 k^4\right)\left(r_{1} + r_{5}\right)} \\ 0 & 0 & 0 & 0 \\ \frac{i \left(6 k^2 \left(r_{1} + r_{5}\right) + t_{1}\right)}{\sqrt{2}\, k \left(1+2 k^2\right)^2 \left(r_{1} + r_{5}\right) t_{1}} & -\frac{1}{\sqrt{2} \left(k^2 + 2 k^4\right)\left(r_{1} + r_{5}\right)} & 0 & \frac{6 k^2 \left(r_{1} + r_{5}\right) + t_{1}}{2 \left(k + 2 k^3\right)^2 \left(r_{1} + r_{5}\right) t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{4 k^2}{\left(1+2 k^2\right)^2 t_{1}} & \frac{2 i \sqrt{2}\, k}{\left(1+2 k^2\right)^2 t_{1}} \\ -\frac{2 i \sqrt{2}\, k}{\left(1+2 k^2\right)^2 t_{1}} & \frac{2}{\left(1+2 k^2\right)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{2 k^2 r_{1} + t_{1}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{1}}{2 r_{1}}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{1}}\right\}\right\}$$

Massless eigenvalues:

$$\left\{-\frac{7 t_1^2+2 r_1 t_1 p^2+2 r_5 t_1 p^2+4 r_1^2 p^4+8 r_1 r_5 p^4+4 r_5^2 p^4}{2\left(r_1+r_5\right) t_1^2},\; -\frac{7 t_1^2+2 r_1 t_1 p^2+2 r_5 t_1 p^2+4 r_1^2 p^4+8 r_1 r_5 p^4+4 r_5^2 p^4}{2\left(r_1+r_5\right) t_1^2}\right\}$$

Overall unitarity conditions:

$$p \in \mathbb{R} \;\&\&\; r_1 < 0 \;\&\&\; r_5 < -r_1 \;\&\&\; t_1 > 0$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_1 < 0 \;\&\&\; r_5 < -r_1 \;\&\&\; t_1 > 0$$

Okay, that concludes the analysis of this theory.

Case 5

Now for a new theory. Here is the full nonlinear Lagrangian for Case 5 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_1 \mathcal{R}_{ijkl} \mathcal{R}^{ijkl}+\frac{2}{3} r_1 \mathcal{R}_{ikjl} \mathcal{R}^{ijkl}+r_5 \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl}-\frac{2}{3} r_1 \mathcal{R}^{ijkl} \mathcal{R}_{klij}- r_5 \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl}+\frac{1}{3} t_1 \mathcal{T}_{ijk} \mathcal{T}^{ijk}+\frac{1}{3} t_1 \mathcal{T}^{ijk} \mathcal{T}_{jik}+t_1 \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3} t_1 \mathcal{A}_{aa'i} \mathcal{A}^{aa'i}+\frac{1}{3} t_1 \mathcal{A}_{aia'} \mathcal{A}^{aa'i}+t_1 \mathcal{A}^{aa'}{}_{a} \mathcal{A}_{a'}{}^{i}{}_{i}-2 t_1 \mathcal{A}_{a'}{}^{i}{}_{i} \partial_a f^{aa'}+2 t_1 \mathcal{A}_{a'}{}^{i}{}_{i} \partial^{a'} f^{a}{}_{a}- \\
&t_1 \partial_{a'} f^{i}{}_{i} \partial^{a'} f^{a}{}_{a}-t_1 \partial_a f^{aa'} \partial_i f^{i}{}_{a'}+2 t_1 \partial^{a'} f^{a}{}_{a} \partial_i f^{i}{}_{a'}+r_5 \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^i \mathcal{A}^{aa'}{}_{a}-r_5 \partial_i \mathcal{A}_{a'}{}^{j}{}_{j} \partial^i \mathcal{A}^{aa'}{}_{a}- \\
&\frac{2}{3} t_1 \mathcal{A}_{aa'i} \partial^i f^{aa'}+\frac{2}{3} t_1 \mathcal{A}_{aia'} \partial^i f^{aa'}+\frac{4}{3} t_1 \mathcal{A}_{a'ia} \partial^i f^{aa'}-\frac{2}{3} t_1 \partial_a f_{a'i} \partial^i f^{aa'}+\frac{1}{3} t_1 \partial_a f_{ia'} \partial^i f^{aa'}- \\
&\frac{2}{3} t_1 \partial_{a'} f_{ai} \partial^i f^{aa'}+\frac{2}{3} t_1 \partial_i f_{aa'} \partial^i f^{aa'}+\frac{1}{3} t_1 \partial_i f_{a'a} \partial^i f^{aa'}-r_5 \partial_a \mathcal{A}^{aa'i} \partial_j \mathcal{A}_{a'}{}^{j}{}_{i}+2 r_5 \partial^i \mathcal{A}^{aa'}{}_{a} \partial_j \mathcal{A}_{a'}{}^{j}{}_{i}+ \\
&r_5 \partial_a \mathcal{A}^{aa'i} \partial_j \mathcal{A}_{i}{}^{j}{}_{a'}-2 r_5 \partial^i \mathcal{A}^{aa'}{}_{a} \partial_j \mathcal{A}_{i}{}^{j}{}_{a'}-\frac{4}{3} r_1 \partial_{a'} \mathcal{A}_{aij} \partial^j \mathcal{A}^{aa'i}+\frac{2}{3} r_1 \partial_{a'} \mathcal{A}_{aji} \partial^j \mathcal{A}^{aa'i}- \\
&\frac{8}{3} r_1 \partial_{a'} \mathcal{A}_{ija} \partial^j \mathcal{A}^{aa'i}-\frac{2}{3} r_1 \partial_i \mathcal{A}_{aa'j} \partial^j \mathcal{A}^{aa'i}+\frac{2}{3} r_1 \partial_j \mathcal{A}_{aa'i} \partial^j \mathcal{A}^{aa'i}+\frac{2}{3} r_1 \partial_j \mathcal{A}_{aia'} \partial^j \mathcal{A}^{aa'i}
\end{aligned}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_{\dot 1} & -i\sqrt{2}\,k\,t_{\dot 1} & 0 \\ i\sqrt{2}\,k\,t_{\dot 1} & -t_{\dot 1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} \frac{k^2 t_{\dot 1}}{3} & -\frac{i\,k\,t_{\dot 1}}{3\sqrt{2}} & \frac{i\,k\,t_{\dot 1}}{3} \\ \frac{i\,k\,t_{\dot 1}}{3\sqrt{2}} & \frac{1}{2}\left(2k^2\left(2r_{\dot 1}+r_{\dot 5}\right)+\frac{t_{\dot 1}}{3}\right) & -\frac{t_{\dot 1}}{3\sqrt{2}} \\ -\frac{1}{3}\,i\,k\,t_{\dot 1} & -\frac{t_{\dot 1}}{3\sqrt{2}} & \frac{t_{\dot 1}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & -i\,k\,t_{\dot 1} & 0 & 0 \\ i\,k\,t_{\dot 1} & k^2\left(r_{\dot 1}+r_{\dot 5}\right)-\frac{t_{\dot 1}}{2} & 0 & \frac{t_{\dot 1}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{\dot 1}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_{\dot 1} & \frac{i\,k\,t_{\dot 1}}{\sqrt{2}} \\ -\frac{i\,k\,t_{\dot 1}}{\sqrt{2}} & \frac{t_{\dot 1}}{2} \end{pmatrix}, \left(\frac{1}{2}\left(2k^2 r_{\dot 1}+t_{\dot 1}\right)\right)\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}_{\bullet}\tau^{\flat\parallel} == 2k\,{}^{0^+}_{\bullet}\sigma^{\flat\parallel},\ {}^{0^-}_{\bullet}\sigma^{\flat\parallel} == 0,\ -i\,{}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab},\right.$$

$$\left. i\,{}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a} == 2k\,{}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ -i\,{}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 2k\,{}^{2^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these a–matrices, which are functionally analogous to the inverse b–matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{(1+2k^2)^2 t_{\dot 1}} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot 1}} & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot 1}} & -\frac{1}{(1+2k^2)^2 t_{\dot 1}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} \frac{6k^2\left(2r_{\dot 1}+r_{\dot 5}\right)+t_{\dot 1}}{2(1+k^2)^2\left(2r_{\dot 1}+r_{\dot 5}\right)t_{\dot 1}} & \frac{i}{\sqrt{2}\,(k+k^3)\left(2r_{\dot 1}+r_{\dot 5}\right)} & \frac{i\left(6k^2\left(2r_{\dot 1}+r_{\dot 5}\right)+t_{\dot 1}\right)}{2k(1+k^2)^2\left(2r_{\dot 1}+r_{\dot 5}\right)t_{\dot 1}} \\ -\frac{i}{\sqrt{2}\,(k+k^3)\left(2r_{\dot 1}+r_{\dot 5}\right)} & \frac{1}{k^2\left(2r_{\dot 1}+r_{\dot 5}\right)} & \frac{1}{\sqrt{2}\,(k^2+k^4)\left(2r_{\dot 1}+r_{\dot 5}\right)} \\ -\frac{i\left(6k^2\left(2r_{\dot 1}+r_{\dot 5}\right)+t_{\dot 1}\right)}{2k(1+k^2)^2\left(2r_{\dot 1}+r_{\dot 5}\right)t_{\dot 1}} & \frac{1}{\sqrt{2}\,(k^2+k^4)\left(2r_{\dot 1}+r_{\dot 5}\right)} & \frac{6k^2\left(2r_{\dot 1}+r_{\dot 5}\right)+t_{\dot 1}}{2(k+k^3)^2\left(2r_{\dot 1}+r_{\dot 5}\right)t_{\dot 1}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{-4k^4\left(r_{\dot 1}+r_{\dot 5}\right)+2k^2 t_{\dot 1}}{\left(t_{\dot 1}+2k^2 t_{\dot 1}\right)^2} & -\frac{2\,i\,k}{t_{\dot 1}+2k^2 t_{\dot 1}} & 0 & \frac{i\sqrt{2}\,k\left(2k^2\left(r_{\dot 1}+r_{\dot 5}\right)-t_{\dot 1}\right)}{\left(t_{\dot 1}+2k^2 t_{\dot 1}\right)^2} \\ \frac{2\,i\,k}{t_{\dot 1}+2k^2 t_{\dot 1}} & 0 & 0 & \frac{\sqrt{2}}{t_{\dot 1}+2k^2 t_{\dot 1}} \\ 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k\left(2k^2\left(r_{\dot 1}+r_{\dot 5}\right)-t_{\dot 1}\right)}{\left(t_{\dot 1}+2k^2 t_{\dot 1}\right)^2} & \frac{\sqrt{2}}{t_{\dot 1}+2k^2 t_{\dot 1}} & 0 & \frac{-2k^2\left(r_{\dot 1}+r_{\dot 5}\right)+t_{\dot 1}}{\left(t_{\dot 1}+2k^2 t_{\dot 1}\right)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_{\dot 1}} & \frac{2\,i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot 1}} \\ -\frac{2\,i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot 1}} & \frac{2}{(1+2k^2)^2 t_{\dot 1}} \end{pmatrix}, \left(\frac{2}{2k^2 r_{\dot 1}+t_{\dot 1}}\right)\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{\dot 1}}{2r_{\dot 1}}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{\dot 1}}\right\}\right\}$$

Massless eigenvalues:

$$\left\{\frac{9 t_1^2 + 4 r_1 t_1 p^2 + 2 r_5 t_1 p^2 + 8 r_1^2 p^4 + 8 r_1 r_5 p^4 + 2 r_5^2 p^4}{\left(2 r_1 + r_5\right) t_1^2},\right.$$
$$\left.\frac{9 t_1^2 + 4 r_1 t_1 p^2 + 2 r_5 t_1 p^2 + 8 r_1^2 p^4 + 8 r_1 r_5 p^4 + 2 r_5^2 p^4}{\left(2 r_1 + r_5\right) t_1^2}\right\}$$

Overall unitarity conditions:

$$p \in \mathbb{R} \,\&\&\, r_1 < 0 \,\&\&\, r_5 > -2 r_1 \,\&\&\, t_1 > 0$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_1 < 0 \,\&\&\, r_5 > -2 r_1 \,\&\&\, t_1 > 0$$

Okay, that concludes the analysis of this theory.

Case 6

Now for a new theory. Here is the full nonlinear Lagrangian for Case 6 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_2 \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_2 \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \left(2 r_3 + r_5\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6}\left(r_2 - 6 r_3\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij} +$$
$$\left(2 r_3 - r_5\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4} t_1 \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{2} t_1 \mathcal{T}^{ijk} \mathcal{T}_{jik} + \frac{1}{3} t_1 \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_1 \mathcal{A}_{\alpha i \alpha'} \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3} t_1 \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \mathcal{A}_{\alpha'}{}^{i}{}_{i} - \frac{2}{3} t_1 \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial_{\alpha} f^{\alpha\alpha'} + \frac{2}{3} t_1 \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha} -$$
$$\frac{1}{3} t_1 \partial_{\alpha'} f^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha} - \frac{1}{3} t_1 \partial_{\alpha} f^{\alpha\alpha'} \partial_{i} f^{i}{}_{\alpha'} + \frac{2}{3} t_1 \partial^{\alpha'} f^{\alpha}{}_{\alpha} \partial_{i} f^{i}{}_{\alpha'} + \left(-2 r_3 + r_5\right) \partial_{\alpha'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} +$$
$$\left(-2 r_3 - r_5\right) \partial_{i} \mathcal{A}_{\alpha'}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} + 2 t_1 \mathcal{A}_{\alpha' i \alpha} \partial^{i} f^{\alpha\alpha'} - t_1 \partial_{\alpha} f_{\alpha' i} \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_1 \partial_{\alpha} f_{i\alpha'} \partial^{i} f^{\alpha\alpha'} - \frac{1}{2} t_1 \partial_{\alpha'} f_{\alpha i} \partial^{i} f^{\alpha\alpha'} +$$
$$\frac{1}{2} t_1 \partial_{i} f_{\alpha\alpha'} \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_1 \partial_{i} f_{\alpha'\alpha} \partial^{i} f^{\alpha\alpha'} + \left(-2 r_3 - r_5\right) \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i} \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i} + 2\left(2 r_3 + r_5\right) \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i} +$$
$$\left(-2 r_3 + r_5\right) \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'} + \left(4 r_3 - 2 r_5\right) \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'} + \frac{4}{3} r_2 \partial_{\alpha'} \mathcal{A}_{\alpha ij} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_2 \partial_{\alpha'} \mathcal{A}_{\alpha ji} \partial^{j} \mathcal{A}^{\alpha\alpha' i} +$$
$$\frac{2}{3}\left(r_2 - 6 r_3\right) \partial_{\alpha'} \mathcal{A}_{ij\alpha} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{1}{3} r_2 \partial_{i} \mathcal{A}_{\alpha\alpha' j} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3} r_2 \partial_{j} \mathcal{A}_{\alpha\alpha' i} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_2 \partial_{j} \mathcal{A}_{\alpha i\alpha'} \partial^{j} \mathcal{A}^{\alpha\alpha' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 6 k^2 r_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 r_{2} - t_{1} \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & \frac{1}{2}\left(2 k^2 \left(2 r_{3} + r_{5}\right) - t_{1}\right) & -\frac{t_{1}}{\sqrt{2}} \\ 0 & -\frac{t_{1}}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2 k^2 t_{1}}{3} & -\frac{1}{3} i k t_{1} & 0 & -\frac{1}{3} i \sqrt{2} k t_{1} \\ \frac{i k t_{1}}{3} & k^2 \left(2 r_{3} + r_{5}\right) + \frac{t_{1}}{6} & 0 & \frac{t_{1}}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2} k t_{1} & \frac{t_{1}}{3\sqrt{2}} & 0 & \frac{t_{1}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \begin{pmatrix} \frac{t_{1}}{2} \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}\tau^{\flat\perp} == 0,\ {}^{0^+}\tau^{\flat\parallel} == 0,\ -i\ {}^{1^+}\tau^{\flat\parallel\,ab} == k\ {}^{1^+}\sigma^{\flat\perp\,ab},\ i\ {}^{1^-}\tau^{\flat\parallel\,a} == 2 k\ {}^{1^-}\sigma^{\flat\perp\,a},\ {}^{1^-}\tau^{\flat\perp\,a} == 0,\ -i\ {}^{2^+}\tau^{\flat\parallel\,ab} == 2 k\ {}^{2^+}\sigma^{\flat\parallel\,ab} \right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{6 k^2 r_{3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_{2} - t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{-2 k^4 \left(2 r_{3}+r_{5}\right)+k^2 t_{1}}{(1+k^2)^2 t_{1}^2} & -\frac{i \sqrt{2} k}{t_{1}+k^2 t_{1}} & \frac{-2 i k^3 \left(2 r_{3}+r_{5}\right)+i k t_{1}}{(1+k^2)^2 t_{1}^2} \\ \frac{i \sqrt{2} k}{t_{1}+k^2 t_{1}} & 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} \\ \frac{i \left(2 k^3 \left(2 r_{3}+r_{5}\right)-k t_{1}\right)}{(1+k^2)^2 t_{1}^2} & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2 k^2 \left(2 r_{3}+r_{5}\right)+t_{1}}{(1+k^2)^2 t_{1}^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6 k^2 \left(2 r_{3}+r_{5}\right)+t_{1}}{\left(1+2 k^2\right)^2 \left(2 r_{3}+r_{5}\right) t_{1}} & \frac{i}{k \left(1+2 k^2\right)\left(2 r_{3}+r_{5}\right)} & 0 & -\frac{i \left(6 k^2 \left(2 r_{3}+r_{5}\right)+t_{1}\right)}{\sqrt{2}\, k \left(1+2 k^2\right)^2 \left(2 r_{3}+r_{5}\right) t_{1}} \\ -\frac{i}{k \left(1+2 k^2\right)\left(2 r_{3}+r_{5}\right)} & \frac{1}{k^2 \left(2 r_{3}+r_{5}\right)} & 0 & -\frac{1}{\sqrt{2} \left(k^2+2 k^4\right)\left(2 r_{3}+r_{5}\right)} \\ 0 & 0 & 0 & 0 \\ \frac{i \left(6 k^2 \left(2 r_{3}+r_{5}\right)+t_{1}\right)}{\sqrt{2}\, k \left(1+2 k^2\right)^2 \left(2 r_{3}+r_{5}\right) t_{1}} & -\frac{1}{\sqrt{2} \left(k^2+2 k^4\right)\left(2 r_{3}+r_{5}\right)} & 0 & \frac{6 k^2 \left(2 r_{3}+r_{5}\right)+t_{1}}{2 \left(k+2 k^3\right)^2 \left(2 r_{3}+r_{5}\right) t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{4 k^2}{\left(1+2 k^2\right)^2 t_{1}} & \frac{2 i \sqrt{2} k}{\left(1+2 k^2\right)^2 t_{1}} \\ -\frac{2 i \sqrt{2} k}{\left(1+2 k^2\right)^2 t_{1}} & \frac{2}{\left(1+2 k^2\right)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{t_{1}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_{1}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\left\{-\frac{7 t_{\cdot 1}^{2}+4 r_{\cdot 3} t_{\cdot 1} p^{2}+2 r_{\cdot 5} t_{\cdot 1} p^{2}+16 r_{\cdot 3}^{2} p^{4}+16 r_{\cdot 3} r_{\cdot 5} p^{4}+4 r_{\cdot 5}^{2} p^{4}}{2\left(2 r_{\cdot 3}+r_{\cdot 5}\right) t_{\cdot 1}^{2}},\right.$$
$$\left.-\frac{7 t_{\cdot 1}^{2}+4 r_{\cdot 3} t_{\cdot 1} p^{2}+2 r_{\cdot 5} t_{\cdot 1} p^{2}+16 r_{\cdot 3}^{2} p^{4}+16 r_{\cdot 3} r_{\cdot 5} p^{4}+4 r_{\cdot 5}^{2} p^{4}}{2\left(2 r_{\cdot 3}+r_{\cdot 5}\right) t_{\cdot 1}^{2}}\right\}$$

Overall unitarity conditions:

$$\left(p \mid r_{\cdot 3}\right) \in \mathbb{R} \,\&\&\, r_{\cdot 2}<0 \,\&\&\, r_{\cdot 5}<-2 r_{\cdot 3} \,\&\&\, t_{\cdot 1}<0$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\cdot 3} \in \mathbb{R} \,\&\&\, r_{\cdot 2}<0 \,\&\&\, r_{\cdot 5}<-2 r_{\cdot 3} \,\&\&\, t_{\cdot 1}<0$$

Okay, that concludes the analysis of this theory.

Case 7

Now for a new theory. Here is the full nonlinear Lagrangian for Case 7 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{\cdot 1} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl}+\frac{2}{3} r_{\cdot 1} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl}+\left(-2 r_{\cdot 1}+2 r_{\cdot 3}+r_{\cdot 5}\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl}+\frac{1}{3}\left(r_{\cdot 1}-3 r_{\cdot 3}\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij}+$$
$$\left(-2 r_{\cdot 1}+2 r_{\cdot 3}-r_{\cdot 5}\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl}+\frac{1}{4} t_{\cdot 1} \mathcal{T}_{ijk} \mathcal{T}^{ijk}+\frac{1}{2} t_{\cdot 1} \mathcal{T}^{ijk} \mathcal{T}_{jik}+\frac{1}{3} t_{\cdot 1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{\cdot 1} \mathcal{A}_{\alpha i \alpha'} \mathcal{A}^{\alpha\alpha' i}+\frac{1}{3} t_{\cdot 1} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \mathcal{A}_{\alpha'}{}^{i}{}_{i}-\frac{2}{3} t_{\cdot 1} \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial_{\alpha} f^{\alpha\alpha'}+\frac{2}{3} t_{\cdot 1} \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha}-$$
$$\frac{1}{3} t_{\cdot 1} \partial_{\alpha'} f^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha}-\frac{1}{3} t_{\cdot 1} \partial_{\alpha} f^{\alpha\alpha'} \partial_{i} f^{i}{}_{\alpha'}+\frac{2}{3} t_{\cdot 1} \partial^{\alpha'} f^{\alpha}{}_{\alpha} \partial_{i} f^{i}{}_{\alpha'}+\left(2 r_{\cdot 1}-2 r_{\cdot 3}+r_{\cdot 5}\right) \partial_{\alpha'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha}+$$
$$\left(2 r_{\cdot 1}-2 r_{\cdot 3}-r_{\cdot 5}\right) \partial_{i} \mathcal{A}_{\alpha'}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha}+2 t_{\cdot 1} \mathcal{A}_{\alpha' i \alpha} \partial^{i} f^{\alpha\alpha'}-t_{\cdot 1} \partial_{\alpha} f_{\alpha' i} \partial^{i} f^{\alpha\alpha'}+\frac{1}{2} t_{\cdot 1} \partial_{\alpha} f_{i\alpha'} \partial^{i} f^{\alpha\alpha'}-$$
$$\frac{1}{2} t_{\cdot 1} \partial_{\alpha'} f_{\alpha i} \partial^{i} f^{\alpha\alpha'}+\frac{1}{2} t_{\cdot 1} \partial_{i} f_{\alpha\alpha'} \partial^{i} f^{\alpha\alpha'}+\frac{1}{2} t_{\cdot 1} \partial_{i} f_{\alpha'\alpha} \partial^{i} f^{\alpha\alpha'}+\left(2 r_{\cdot 1}-2 r_{\cdot 3}-r_{\cdot 5}\right) \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i} \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i}+$$
$$\left(-4 r_{\cdot 1}+4 r_{\cdot 3}+2 r_{\cdot 5}\right) \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i}+\left(2 r_{\cdot 1}-2 r_{\cdot 3}+r_{\cdot 5}\right) \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'}-$$
$$2\left(2 r_{\cdot 1}-2 r_{\cdot 3}+r_{\cdot 5}\right) \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'}-\frac{4}{3} r_{\cdot 1} \partial_{\alpha'} \mathcal{A}_{\alpha ij} \partial^{j} \mathcal{A}^{\alpha\alpha' i}+\frac{2}{3} r_{\cdot 1} \partial_{\alpha'} \mathcal{A}_{\alpha ji} \partial^{j} \mathcal{A}^{\alpha\alpha' i}+$$
$$\frac{4}{3}\left(r_{\cdot 1}-3 r_{\cdot 3}\right) \partial_{\alpha'} \mathcal{A}_{ij\alpha} \partial^{j} \mathcal{A}^{\alpha\alpha' i}-\frac{2}{3} r_{\cdot 1} \partial_{i} \mathcal{A}_{\alpha\alpha' j} \partial^{j} \mathcal{A}^{\alpha\alpha' i}+\frac{2}{3} r_{\cdot 1} \partial_{j} \mathcal{A}_{\alpha\alpha' i} \partial^{j} \mathcal{A}^{\alpha\alpha' i}+\frac{2}{3} r_{\cdot 1} \partial_{j} \mathcal{A}_{\alpha i\alpha'} \partial^{j} \mathcal{A}^{\alpha\alpha' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 6k^2\left(-r_{1}+r_{3}\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_{1} \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & \frac{1}{2}\left(2k^2\left(2r_{3}+r_{5}\right)-t_{1}\right) & -\frac{t_{1}}{\sqrt{2}} \\ 0 & -\frac{t_{1}}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2 t_{1}}{3} & -\frac{1}{3} i k t_{1} & 0 & -\frac{1}{3} i \sqrt{2}\, k t_{1} \\ \frac{i k t_{1}}{3} & \frac{1}{6}\left(-6k^2\left(r_{1}-2r_{3}-r_{5}\right)+t_{1}\right) & 0 & \frac{t_{1}}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{1} & \frac{t_{1}}{3\sqrt{2}} & 0 & \frac{t_{1}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2k^2 r_{1}+t_{1}\right) \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}\tau^{\flat\perp} == 0,\ {}^{0^+}\tau^{\flat\parallel} == 0,\ -i\ {}^{1^+}\tau^{\flat\parallel}{}^{ab} == k\ {}^{1^+}\sigma^{\flat\perp}{}^{ab},\ i\ {}^{1^-}\tau^{\flat\parallel}{}^{a} == 2k\ {}^{1^-}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}\tau^{\flat\perp}{}^{a} == 0,\ -i\ {}^{2^+}\tau^{\flat\parallel}{}^{ab} == 2k\ {}^{2^+}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{6k^2\left(-r_{1}+r_{3}\right)} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{-2k^4\left(2r_{3}+r_{5}\right)+k^2 t_{1}}{(1+k^2)^2 t_{1}^2} & -\frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & \frac{-2ik^3\left(2r_{3}+r_{5}\right)+ikt_{1}}{(1+k^2)^2 t_{1}^2} \\ \frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} \\ \frac{i\left(2k^3\left(2r_{3}+r_{5}\right)-kt_{1}\right)}{(1+k^2)^2 t_{1}^2} & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2k^2\left(2r_{3}+r_{5}\right)+t_{1}}{(1+k^2)^2 t_{1}^2} \end{pmatrix},\right.$$

$$\begin{pmatrix} \frac{\frac{1}{-r_{1}+2r_{3}+r_{5}}+\frac{6k^2}{t_{1}}}{\left(1+2k^2\right)^2} & \frac{i}{k\left(1+2k^2\right)\left(-r_{1}+2r_{3}+r_{5}\right)} & 0 & -\frac{i\left(6k^2\left(r_{1}-2r_{3}-r_{5}\right)-t_{1}\right)}{\sqrt{2}\,k\left(1+2k^2\right)^2\left(r_{1}-2r_{3}-r_{5}\right)t_{1}} \\ \frac{i}{k\left(1+2k^2\right)\left(r_{1}-2r_{3}-r_{5}\right)} & \frac{1}{k^2\left(-r_{1}+2r_{3}+r_{5}\right)} & 0 & \frac{1}{\sqrt{2}\left(k^2+2k^4\right)\left(r_{1}-2r_{3}-r_{5}\right)} \\ 0 & 0 & 0 & 0 \\ \frac{i\left(6k^2\left(r_{1}-2r_{3}-r_{5}\right)-t_{1}\right)}{\sqrt{2}\,k\left(1+2k^2\right)^2\left(r_{1}-2r_{3}-r_{5}\right)t_{1}} & \frac{1}{\sqrt{2}\left(k^2+2k^4\right)\left(r_{1}-2r_{3}-r_{5}\right)} & 0 & \frac{\frac{1}{-r_{1}+2r_{3}+r_{5}}+\frac{6k^2}{t_{1}}}{2\left(k+2k^3\right)^2} \end{pmatrix},$$

$$\left.\begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_{1}} & \frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} \\ -\frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} & \frac{2}{\left(1+2k^2\right)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{2k^2 r_{1}+t_{1}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\},\ \{\},\ \{\},\ \{\},\ \{\},\ \left\{-\frac{t_{1}}{2r_{1}}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\},\ \{\},\ \{\},\ \{\},\ \{\},\ \left\{-\frac{1}{r_{1}}\right\}\right\}$$

Massless eigenvalues:

$$\left\{\frac{1}{2\left(\dot{r}_1-2\dot{r}_3-\dot{r}_5\right)\dot{t}_1^{2}}\left(7\dot{t}_1^{2}-2\dot{r}_1\dot{t}_1 p^2+4\dot{r}_3\dot{t}_1 p^2+2\dot{r}_5\dot{t}_1 p^2+4\dot{r}_1^{2}p^4-16\dot{r}_1\dot{r}_3 p^4+16\dot{r}_3^{2}p^4-8\dot{r}_1\dot{r}_5 p^4+16\dot{r}_3\dot{r}_5 p^4+4\dot{r}_5^{2}p^4\right),\right.$$
$$\left.\frac{1}{2\left(\dot{r}_1-2\dot{r}_3-\dot{r}_5\right)\dot{t}_1^{2}}\left(7\dot{t}_1^{2}-2\dot{r}_1\dot{t}_1 p^2+4\dot{r}_3\dot{t}_1 p^2+2\dot{r}_5\dot{t}_1 p^2+4\dot{r}_1^{2}p^4-16\dot{r}_1\dot{r}_3 p^4+16\dot{r}_3^{2}p^4-8\dot{r}_1\dot{r}_5 p^4+16\dot{r}_3\dot{r}_5 p^4+4\dot{r}_5^{2}p^4\right)\right\}$$

Overall unitarity conditions:

$$\left(p\,\middle|\,\dot{r}_3\right)\in\mathbb{R}\ \&\&\ \dot{r}_1<0\ \&\&\ \dot{r}_5<\dot{r}_1-2\dot{r}_3\ \&\&\ \dot{t}_1>0$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\dot{r}_3\in\mathbb{R}\ \&\&\ \dot{r}_1<0\ \&\&\ \dot{r}_5<\dot{r}_1-2\dot{r}_3\ \&\&\ \dot{t}_1>0$$

Okay, that concludes the analysis of this theory.

Case 8

Now for a new theory. Here is the full nonlinear Lagrangian for Case 8 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}\dot{r}_1\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl}+\frac{2}{3}\dot{r}_1\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl}+\dot{r}_5\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl}-\frac{2}{3}\dot{r}_1\,\mathcal{R}^{ijkl}\,\mathcal{R}_{klij}-\dot{r}_5\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl}-\frac{2}{3}\dot{t}_3\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$-\frac{2}{3}\dot{t}_3\,\mathcal{A}^{aa'}{}_{a}\,\mathcal{A}_{a'}{}^{i}{}_{i}+\frac{4}{3}\dot{t}_3\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial_a f^{aa'}-\frac{4}{3}\dot{t}_3\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}+\frac{2}{3}\dot{t}_3\,\partial_{a'}f^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}+\frac{2}{3}\dot{t}_3\,\partial_a f^{aa'}\,\partial_i f^{i}{}_{a'}-$$
$$\frac{4}{3}\dot{t}_3\,\partial^{a'}f^{a}{}_{a}\,\partial_i f^{i}{}_{a'}+\dot{r}_5\,\partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}-\dot{r}_5\,\partial_i\mathcal{A}_{a'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}-\dot{r}_5\,\partial_a\mathcal{A}^{aa'i}\,\partial_j\mathcal{A}_{a'}{}^{j}{}_{i}+2\dot{r}_5\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_j\mathcal{A}_{a'}{}^{j}{}_{i}+$$
$$\dot{r}_5\,\partial_a\mathcal{A}^{aa'i}\,\partial_j\mathcal{A}_{i}{}^{j}{}_{a'}-2\dot{r}_5\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_j\mathcal{A}_{i}{}^{j}{}_{a'}-\frac{4}{3}\dot{r}_1\,\partial_{a'}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\dot{r}_1\,\partial_{a'}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{aa'i}-$$
$$\frac{8}{3}\dot{r}_1\,\partial_{a'}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{aa'i}-\frac{2}{3}\dot{r}_1\,\partial_i\mathcal{A}_{aa'j}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\dot{r}_1\,\partial_j\mathcal{A}_{aa'i}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\dot{r}_1\,\partial_j\mathcal{A}_{aia'}\,\partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2k^2 t_3 & i\sqrt{2}\,k t_3 & 0 \\ -i\sqrt{2}\,k t_3 & t_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} 0 & 0 & 0 \\ 0 & k^2\left(2r_1+r_5\right) & 0 \\ 0 & 0 & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2 t_3}{3} & \frac{2ikt_3}{3} & 0 & -\frac{1}{3}i\sqrt{2}\,kt_3 \\ -\frac{2}{3}ikt_3 & k^2\left(r_1+r_5\right)+\frac{2t_3}{3} & 0 & -\frac{\sqrt{2}\,t_3}{3} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3}i\sqrt{2}\,kt_3 & -\frac{\sqrt{2}\,t_3}{3} & 0 & \frac{t_3}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \left(k^2 r_1\right)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}_{\bullet}\tau^{\flat\parallel} == 2k\,{}^{0^+}_{\bullet}\sigma^{\flat\parallel},\ {}^{0^-}_{\bullet}\sigma^{\flat\parallel} == 0,\ {}^{1^+}_{\bullet}\sigma^{\flat\perp\,\mathfrak{a}\mathfrak{b}} == 0,\right.$$

$$\left.{}^{1^+}_{\bullet}\tau^{\flat\parallel\,\mathfrak{a}\mathfrak{b}} == 0,\ i\,{}^{1^-}_{\bullet}\tau^{\flat\parallel\,\mathfrak{a}} == 2k\,{}^{1^-}_{\bullet}\sigma^{\flat\perp\,\mathfrak{a}},\ {}^{1^-}_{\bullet}\tau^{\flat\perp\,\mathfrak{a}} == 0,\ {}^{2^+}_{\bullet}\sigma^{\flat\parallel\,\mathfrak{a}\mathfrak{b}} == 0,\ {}^{2^+}_{\bullet}\tau^{\flat\parallel\,\mathfrak{a}\mathfrak{b}} == 0\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{\left(1+2k^2\right)^2 t_3} & \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_3} & 0 \\ -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_3} & \frac{1}{\left(1+2k^2\right)^2 t_3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{k^2\left(2r_1+r_5\right)} & 0 \\ 0 & 0 & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6k^2\left(r_1+r_5\right)+4t_3}{\left(1+2k^2\right)^2\left(r_1+r_5\right)t_3} & -\frac{2i}{k\left(1+2k^2\right)\left(r_1+r_5\right)} & 0 & -\frac{i\sqrt{2}\left(3k^2\left(r_1+r_5\right)+2t_3\right)}{k\left(1+2k^2\right)^2\left(r_1+r_5\right)t_3} \\ \frac{2i}{k\left(1+2k^2\right)\left(r_1+r_5\right)} & \frac{1}{k^2\left(r_1+r_5\right)} & 0 & \frac{\sqrt{2}}{k^2\left(1+2k^2\right)\left(r_1+r_5\right)} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\left(3k^2\left(r_1+r_5\right)+2t_3\right)}{k\left(1+2k^2\right)^2\left(r_1+r_5\right)t_3} & \frac{\sqrt{2}}{k^2\left(1+2k^2\right)\left(r_1+r_5\right)} & 0 & \frac{3k^2\left(r_1+r_5\right)+2t_3}{\left(k+2k^3\right)^2\left(r_1+r_5\right)t_3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_1}\right)\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{-5r_1^2-4r_1r_5-3r_5^2}{r_1\left(r_1+r_5\right)\left(2r_1+r_5\right)}, \frac{-5r_1^2-4r_1r_5-3r_5^2}{r_1\left(r_1+r_5\right)\left(2r_1+r_5\right)}\right\}$$

Overall unitarity conditions:

$$\left(r_1 < 0\ \&\&\ \left(r_5 < -r_1 \,\|\, r_5 > -2r_1\right)\right) \,\|\, \left(r_1 > 0\ \&\&\ -2r_1 < r_5 < -r_1\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{1} < 0 \,\&\&\, \left(r_{5} < -r_{1} \,\|\, r_{5} > -2\, r_{1}\right)\right) \| \left(r_{1} > 0 \,\&\&\, -2\, r_{1} < r_{5} < -r_{1}\right)$$

Okay, that concludes the analysis of this theory.

Case 9

Now for a new theory. Here is the full nonlinear Lagrangian for Case 9 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{1} \mathcal{R}_{ij\hbar l} \mathcal{R}^{ij\hbar l} + \frac{2}{3} r_{1} \mathcal{R}_{i\hbar jl} \mathcal{R}^{ij\hbar l} + r_{5} \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{j}{}^{l}{}_{\hbar l} - \frac{2}{3} r_{1} \mathcal{R}^{ij\hbar l} \mathcal{R}_{\hbar lij} - r_{5} \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{\hbar}{}^{l}{}_{jl}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$r_{5} \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{aa'}{}_{a} - r_{5} \partial_{i} \mathcal{A}_{a'}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{aa'}{}_{a} - r_{5} \partial_{a} \mathcal{A}^{aa'i} \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} + 2\, r_{5} \partial^{i} \mathcal{A}^{aa'}{}_{a} \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} + r_{5} \partial_{a} \mathcal{A}^{aa'i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} - 2\, r_{5} \partial^{i} \mathcal{A}^{aa'}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} - \frac{4}{3} r_{1} \partial_{a'} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{1} \partial_{a'} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{aa'i} - \frac{8}{3} r_{1} \partial_{a'} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{1} \partial_{i} \mathcal{A}_{aa'j} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{aa'i} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{aia'} \partial^{j} \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} 0 & 0 & 0 \\ 0 & k^2 \left(2\, r_{1} + r_{5}\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & k^2 \left(r_{1} + r_{5}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \left(k^2\, r_{1}\right) \right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\cdot}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\cdot}\sigma^{\flat\parallel} == 0,\ {}^{0^+}_{\cdot}\tau^{\flat\parallel} == 0,\ {}^{0^-}_{\cdot}\sigma^{\flat\parallel} == 0,\ {}^{1^+}_{\cdot}\sigma^{\flat\perp}{}^{ab} == 0,\ {}^{1^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == 0,\ {}^{1^-}_{\cdot}\sigma^{\flat\perp}{}^{a} == 0,\ {}^{1^-}_{\cdot}\tau^{\flat\perp}{}^{a} == 0,\ {}^{1^-}_{\cdot}\tau^{\flat\parallel}{}^{a} == 0,\ {}^{2^+}_{\cdot}\sigma^{\flat\parallel}{}^{ab} == 0,\ {}^{2^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == 0 \right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{k^2 \left(2\, r_{1} + r_{5}\right)} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{k^2 \left(r_{1} + r_{5}\right)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2\, r_{1}}\right) \right\}$$

Square masses:

$\{\{\}, \{\}, \{\}, \{\}, \{\}, \{\}\}$

Massive pole residues:

$\{\{\}, \{\}, \{\}, \{\}, \{\}, \{\}\}$

Massless eigenvalues:

$$\left\{\frac{-4\dot{r}_1^2-4\dot{r}_1\dot{r}_5-3\dot{r}_5^2}{\dot{r}_1\left(\dot{r}_1+\dot{r}_5\right)\left(2\dot{r}_1+\dot{r}_5\right)}, \frac{-4\dot{r}_1^2-4\dot{r}_1\dot{r}_5-3\dot{r}_5^2}{\dot{r}_1\left(\dot{r}_1+\dot{r}_5\right)\left(2\dot{r}_1+\dot{r}_5\right)}\right\}$$

Overall unitarity conditions:

$$\left(\dot{r}_1<0\ \&\&\left(\dot{r}_5<-\dot{r}_1\ \|\ \dot{r}_5>-2\dot{r}_1\right)\right)\ \|\ \left(\dot{r}_1>0\ \&\&-2\dot{r}_1<\dot{r}_5<-\dot{r}_1\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(\dot{r}_1<0\ \&\&\left(\dot{r}_5<-\dot{r}_1\ \|\ \dot{r}_5>-2\dot{r}_1\right)\right)\ \|\ \left(\dot{r}_1>0\ \&\&-2\dot{r}_1<\dot{r}_5<-\dot{r}_1\right)$$

Okay, that concludes the analysis of this theory.

Case 10

Now for a new theory. Here is the full nonlinear Lagrangian for Case 10 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\left(\frac{\dot{r}_3}{2}+\dot{r}_5\right)\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl}-\dot{r}_3\,\mathcal{R}^{ijkl}\,\mathcal{R}_{klij}+\frac{1}{2}\left(\dot{r}_3-2\dot{r}_5\right)\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\left(-\frac{\dot{r}_3}{2}+\dot{r}_5\right)\partial_{\alpha'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}+\left(-\frac{\dot{r}_3}{2}-\dot{r}_5\right)\partial_{i}\mathcal{A}_{\alpha'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}+\left(-\frac{\dot{r}_3}{2}-\dot{r}_5\right)\partial_{\alpha}\mathcal{A}^{\alpha\alpha'i}\,\partial_{j}\mathcal{A}_{\alpha'}{}^{j}{}_{i}+$$
$$\left(\dot{r}_3+2\dot{r}_5\right)\partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\,\partial_{j}\mathcal{A}_{\alpha'}{}^{j}{}_{i}+\left(-\frac{\dot{r}_3}{2}+\dot{r}_5\right)\partial_{\alpha}\mathcal{A}^{\alpha\alpha'i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{\alpha'}+\left(\dot{r}_3-2\dot{r}_5\right)\partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{\alpha'}-4\dot{r}_3\,\partial_{\alpha'}\mathcal{A}_{ij\alpha}\,\partial^{j}\mathcal{A}^{\alpha\alpha'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}, (0), \begin{pmatrix}0&0&0\\0&k^2\left(2\dot{r}_3+\dot{r}_5\right)&0\\0&0&0\end{pmatrix}, \begin{pmatrix}0&0&0&0\\0&\frac{1}{2}k^2\left(\dot{r}_3+2\dot{r}_5\right)&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}, \begin{pmatrix}0&0\\0&-\frac{3k^2\dot{r}_3}{2}\end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\bullet}\sigma^{\flat\parallel} == 0,\ {}^{0^+}_{\bullet}\tau^{\flat\parallel} == 0,\ {}^{0^-}_{\bullet}\sigma^{\flat\parallel} == 0,\ {}^{1^+}_{\bullet}\sigma^{\flat\perp\, ab} == 0,\right.$$
$$\left.{}^{1^+}_{\bullet}\tau^{\flat\parallel\, ab} == 0,\ {}^{1^-}_{\bullet}\sigma^{\flat\perp\, a} == 0,\ {}^{1^-}_{\bullet}\tau^{\flat\perp\, a} == 0,\ {}^{1^-}_{\bullet}\tau^{\flat\parallel\, a} == 0,\ {}^{2^+}_{\bullet}\tau^{\flat\parallel\, ab} == 0,\ {}^{2^-}_{\bullet}\sigma^{\flat\parallel\, abc} == 0\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{k^2\left(2\dot{r}_3+\dot{r}_5\right)} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{2}{k^2\left(\dot{r}_3+2\dot{r}_5\right)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3k^2\dot{r}_3} \end{pmatrix}, (0)\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{33\dot{r}_3^2+20\dot{r}_3\dot{r}_5+4\dot{r}_5^2}{\dot{r}_3\left(2\dot{r}_3+\dot{r}_5\right)\left(\dot{r}_3+2\dot{r}_5\right)}, -\frac{33\dot{r}_3^2+20\dot{r}_3\dot{r}_5+4\dot{r}_5^2}{\dot{r}_3\left(2\dot{r}_3+\dot{r}_5\right)\left(\dot{r}_3+2\dot{r}_5\right)}\right\}$$

Overall unitarity conditions:

$$\left(\dot{r}_3 < 0 \,\&\&\, \left(\dot{r}_5 < -\frac{\dot{r}_3}{2} \,\|\, \dot{r}_5 > -2\dot{r}_3\right)\right) \,\|\, \left(\dot{r}_3 > 0 \,\&\&\, -2\dot{r}_3 < \dot{r}_5 < -\frac{\dot{r}_3}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(\dot{r}_3 < 0 \,\&\&\, \left(\dot{r}_5 < -\frac{\dot{r}_3}{2} \,\|\, \dot{r}_5 > -2\dot{r}_3\right)\right) \,\|\, \left(\dot{r}_3 > 0 \,\&\&\, -2\dot{r}_3 < \dot{r}_5 < -\frac{\dot{r}_3}{2}\right)$$

Okay, that concludes the analysis of this theory.

Case 11

Now for a new theory. Here is the full nonlinear Lagrangian for Case 11 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\dot{r}_2\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl} - \frac{2}{3}\dot{r}_2\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl} +$$
$$\left(\frac{\dot{r}_3}{2}+\dot{r}_5\right)\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6}\left(\dot{r}_2-6\dot{r}_3\right)\mathcal{R}^{ijkl}\,\mathcal{R}_{klij} + \frac{1}{2}\left(\dot{r}_3-2\dot{r}_5\right)\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\left(-\frac{r_{\cdot 3}}{2}+r_{\cdot 5}\right)\partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}+\left(-\frac{r_{\cdot 3}}{2}-r_{\cdot 5}\right)\partial_{i}\mathcal{A}_{a'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}+\left(-\frac{r_{\cdot 3}}{2}-r_{\cdot 5}\right)\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}+$$
$$\left(r_{\cdot 3}+2\,r_{\cdot 5}\right)\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}+\left(-\frac{r_{\cdot 3}}{2}+r_{\cdot 5}\right)\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}+\left(r_{\cdot 3}-2\,r_{\cdot 5}\right)\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}+$$
$$\frac{4}{3}\,r_{\cdot 2}\,\partial_{a'}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{aa'i}-\frac{2}{3}\,r_{\cdot 2}\,\partial_{a'}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\left(r_{\cdot 2}-6\,r_{\cdot 3}\right)\partial_{a'}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{aa'i}-$$
$$\frac{1}{3}\,r_{\cdot 2}\,\partial_{i}\mathcal{A}_{aa'j}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{1}{3}\,r_{\cdot 2}\,\partial_{j}\mathcal{A}_{aa'i}\,\partial^{j}\mathcal{A}^{aa'i}-\frac{2}{3}\,r_{\cdot 2}\,\partial_{j}\mathcal{A}_{aia'}\,\partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix},\ \begin{pmatrix}k^2\,r_{\cdot 2}\end{pmatrix},\ \begin{pmatrix}0&0&0\\0&k^2\left(2\,r_{\cdot 3}+r_{\cdot 5}\right)&0\\0&0&0\end{pmatrix},\ \begin{pmatrix}0&0&0&0\\0&\frac{1}{2}\,k^2\left(r_{\cdot 3}+2\,r_{\cdot 5}\right)&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\ \begin{pmatrix}0&0\\0&-\frac{3\,k^2\,r_{\cdot 3}}{2}\end{pmatrix},\ \begin{pmatrix}0\end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\bullet}\tau^{\flat\perp}==0,\ {}^{0^+}_{\bullet}\sigma^{\flat\parallel}==0,\ {}^{0^+}_{\bullet}\tau^{\flat\parallel}==0,\ {}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab}==0,\ {}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab}==0,\right.$$
$$\left.{}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a}==0,\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a}==0,\ {}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a}==0,\ {}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab}==0,\ {}^{2^-}_{\bullet}\sigma^{\flat\parallel}{}^{abc}==0\right\}$$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally
analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix},\ \begin{pmatrix}\frac{1}{k^2\,r_{\cdot 2}}\end{pmatrix},\ \begin{pmatrix}0&0&0\\0&\frac{1}{k^2\left(2\,r_{\cdot 3}+r_{\cdot 5}\right)}&0\\0&0&0\end{pmatrix},\ \begin{pmatrix}0&0&0&0\\0&\frac{2}{k^2\left(r_{\cdot 3}+2\,r_{\cdot 5}\right)}&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\ \begin{pmatrix}0&0\\0&-\frac{2}{3\,k^2\,r_{\cdot 3}}\end{pmatrix},\ \begin{pmatrix}0\end{pmatrix}\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{33\,r_{\cdot 3}^2+20\,r_{\cdot 3}\,r_{\cdot 5}+4\,r_{\cdot 5}^2}{r_{\cdot 3}\left(2\,r_{\cdot 3}+r_{\cdot 5}\right)\left(r_{\cdot 3}+2\,r_{\cdot 5}\right)},\ -\frac{33\,r_{\cdot 3}^2+20\,r_{\cdot 3}\,r_{\cdot 5}+4\,r_{\cdot 5}^2}{r_{\cdot 3}\left(2\,r_{\cdot 3}+r_{\cdot 5}\right)\left(r_{\cdot 3}+2\,r_{\cdot 5}\right)}\right\}$$

Overall unitarity conditions:

$$\left(r_{\cdot 3}<0\ \&\&\left(r_{\cdot 5}<-\frac{r_{\cdot 3}}{2}\ \|\ r_{\cdot 5}>-2\,r_{\cdot 3}\right)\right)\|\left(r_{\cdot 3}>0\ \&\&-2\,r_{\cdot 3}<r_{\cdot 5}<-\frac{r_{\cdot 3}}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{\dot 3} < 0 \,\&\&\, \left(r_{\dot 5} < -\frac{r_{\dot 3}}{2} \,\|\, r_{\dot 5} > -2\,r_{\dot 3}\right)\right) \,\|\, \left(r_{\dot 3} > 0 \,\&\&\, -2\,r_{\dot 3} < r_{\dot 5} < -\frac{r_{\dot 3}}{2}\right)$$

Okay, that concludes the analysis of this theory.

Case 12

Now for a new theory. Here is the full nonlinear Lagrangian for Case 12 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\left(\frac{r_{\dot 3}}{2} + r_{\dot 5}\right) \mathcal{R}^{ij}{}_{i}{}^{\hbar}\,\mathcal{R}_{j}{}^{l}{}_{\hbar l} - r_{\dot 3}\,\mathcal{R}^{ij\hbar l}\,\mathcal{R}_{\hbar l ij} + \frac{1}{2}\left(r_{\dot 3} - 2\,r_{\dot 5}\right) \mathcal{R}^{ij}{}_{i}{}^{\hbar}\,\mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{12}\,t_{\dot 2}\,\mathcal{T}_{ij\hbar}\,\mathcal{T}^{ij\hbar} - \frac{1}{6}\,t_{\dot 2}\,\mathcal{T}^{ij\hbar}\,\mathcal{T}_{ji\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\,t_{\dot 2}\,\mathcal{A}_{\mathfrak{a}\mathfrak{a}'i}\,\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3}\,t_{\dot 2}\,\mathcal{A}_{\mathfrak{a}i\mathfrak{a}'}\,\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + \left(-\frac{r_{\dot 3}}{2} + r_{\dot 5}\right) \partial_{\mathfrak{a}'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} + \left(-\frac{r_{\dot 3}}{2} - r_{\dot 5}\right) \partial_{i}\mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} -$$
$$\frac{2}{3}\,t_{\dot 2}\,\mathcal{A}_{\mathfrak{a}\mathfrak{a}'i}\,\partial^{i}f^{\mathfrak{a}\mathfrak{a}'} + \frac{2}{3}\,t_{\dot 2}\,\mathcal{A}_{\mathfrak{a}i\mathfrak{a}'}\,\partial^{i}f^{\mathfrak{a}\mathfrak{a}'} - \frac{2}{3}\,t_{\dot 2}\,\mathcal{A}_{\mathfrak{a}'i\mathfrak{a}}\,\partial^{i}f^{\mathfrak{a}\mathfrak{a}'} + \frac{1}{3}\,t_{\dot 2}\,\partial_{\mathfrak{a}}f_{\mathfrak{a}'i}\,\partial^{i}f^{\mathfrak{a}\mathfrak{a}'} - \frac{1}{6}\,t_{\dot 2}\,\partial_{\mathfrak{a}}f_{i\mathfrak{a}'}\,\partial^{i}f^{\mathfrak{a}\mathfrak{a}'} -$$
$$\frac{1}{6}\,t_{\dot 2}\,\partial_{\mathfrak{a}'}f_{\mathfrak{a}i}\,\partial^{i}f^{\mathfrak{a}\mathfrak{a}'} + \frac{1}{6}\,t_{\dot 2}\,\partial_{i}f_{\mathfrak{a}\mathfrak{a}'}\,\partial^{i}f^{\mathfrak{a}\mathfrak{a}'} - \frac{1}{6}\,t_{\dot 2}\,\partial_{i}f_{\mathfrak{a}'\mathfrak{a}}\,\partial^{i}f^{\mathfrak{a}\mathfrak{a}'} + \left(-\frac{r_{\dot 3}}{2} - r_{\dot 5}\right) \partial_{\mathfrak{a}}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\,\partial_{j}\mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{i} +$$
$$\left(r_{\dot 3} + 2\,r_{\dot 5}\right) \partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}\,\partial_{j}\mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{i} + \left(-\frac{r_{\dot 3}}{2} + r_{\dot 5}\right) \partial_{\mathfrak{a}}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{\mathfrak{a}'} + \left(r_{\dot 3} - 2\,r_{\dot 5}\right) \partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{\mathfrak{a}'} - 4\,r_{\dot 3}\,\partial_{\mathfrak{a}'}\mathcal{A}_{ij\mathfrak{a}}\,\partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} t_{\dot 2} \end{pmatrix}, \begin{pmatrix} \frac{k^2 t_{\dot 2}}{3} & \frac{1}{3}\, i \sqrt{2}\, k\, t_{\dot 2} & \frac{i k t_{\dot 2}}{3} \\ -\frac{1}{3}\, i \sqrt{2}\, k\, t_{\dot 2} & \frac{1}{2}\left(2 k^2 \left(2 r_{\dot 3} + r_{\dot 5}\right) + \frac{4 t_{\dot 2}}{3}\right) & \frac{\sqrt{2}\, t_{\dot 2}}{3} \\ -\frac{1}{3}\, i k\, t_{\dot 2} & \frac{\sqrt{2}\, t_{\dot 2}}{3} & \frac{t_{\dot 2}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} k^2 \left(r_{\dot 3} + 2 r_{\dot 5}\right) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{3 k^2 r_{\dot 3}}{2} \end{pmatrix}, \begin{pmatrix} 0 \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\cdot}\tau^{\flat\perp} == 0,\ \overset{0^+}{\cdot}\sigma^{\flat\parallel} == 0,\ \overset{0^+}{\cdot}\tau^{\flat\parallel} == 0,\ -i\,\overset{1^+}{\cdot}\tau^{\flat\parallel}{}^{\mathfrak{a}\mathfrak{b}} == k\,\overset{1^+}{\cdot}\sigma^{\flat\perp}{}^{\mathfrak{a}\mathfrak{b}},\right.$$
$$\left.\overset{1^-}{\cdot}\sigma^{\flat\perp}{}^{\mathfrak{a}} == 0,\ \overset{1^-}{\cdot}\tau^{\flat\perp}{}^{\mathfrak{a}} == 0,\ \overset{1^-}{\cdot}\tau^{\flat\parallel}{}^{\mathfrak{a}} == 0,\ \overset{2^+}{\cdot}\tau^{\flat\parallel}{}^{\mathfrak{a}\mathfrak{b}} == 0,\ \overset{2^-}{\cdot}\sigma^{\flat\parallel}{}^{\mathfrak{a}\mathfrak{b}\mathfrak{c}} == 0\right\}$$

The Drazin (Moore–Penrose) inverses of these a–matrices, which are functionally analogous to the inverse b–matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{t_{2}} \end{pmatrix}, \begin{pmatrix} \frac{3k^2(2r_{3}+r_{5})+2t_{2}}{(1+k^2)^2(2r_{3}+r_{5})t_{2}} & -\frac{i\sqrt{2}}{k(1+k^2)(2r_{3}+r_{5})} & \frac{i(3k^2(2r_{3}+r_{5})+2t_{2})}{k(1+k^2)^2(2r_{3}+r_{5})t_{2}} \\ \frac{i\sqrt{2}}{k(1+k^2)(2r_{3}+r_{5})} & \frac{1}{k^2(2r_{3}+r_{5})} & -\frac{\sqrt{2}}{k^2(1+k^2)(2r_{3}+r_{5})} \\ -\frac{i(3k^2(2r_{3}+r_{5})+2t_{2})}{k(1+k^2)^2(2r_{3}+r_{5})t_{2}} & -\frac{\sqrt{2}}{k^2(1+k^2)(2r_{3}+r_{5})} & \frac{3k^2(2r_{3}+r_{5})+2t_{2}}{(k+k^3)^2(2r_{3}+r_{5})t_{2}} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{2}{k^2(r_{3}+2r_{5})} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3k^2 r_{3}} \end{pmatrix}, \begin{pmatrix} 0 \end{pmatrix}\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{45r_{3}^2+20r_{3}r_{5}+4r_{5}^2}{r_{3}(2r_{3}+r_{5})(r_{3}+2r_{5})}, -\frac{45r_{3}^2+20r_{3}r_{5}+4r_{5}^2}{r_{3}(2r_{3}+r_{5})(r_{3}+2r_{5})}\right\}$$

Overall unitarity conditions:

$$\left(r_{3}<0\,\&\&\left(r_{5}<-\frac{r_{3}}{2}\,\|\,r_{5}>-2r_{3}\right)\right)\|\left(r_{3}>0\,\&\&-2r_{3}<r_{5}<-\frac{r_{3}}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{3}<0\,\&\&\left(r_{5}<-\frac{r_{3}}{2}\,\|\,r_{5}>-2r_{3}\right)\right)\|\left(r_{3}>0\,\&\&-2r_{3}<r_{5}<-\frac{r_{3}}{2}\right)$$

Okay, that concludes the analysis of this theory.

Case 13

Now for a new theory. Here is the full nonlinear Lagrangian for Case 13 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}r_{1}\mathcal{R}_{ijkl}\mathcal{R}^{ijkl}+\frac{2}{3}r_{1}\mathcal{R}_{ikjl}\mathcal{R}^{ijkl}+(-2r_{1}+2r_{3}+r_{5})\mathcal{R}^{ij}{}_{i}{}^{k}\mathcal{R}_{j}{}^{l}{}_{kl}+$$
$$\frac{1}{3}(r_{1}-3r_{3})\mathcal{R}^{ijkl}\mathcal{R}_{klij}+(-2r_{1}+2r_{3}-r_{5})\mathcal{R}^{ij}{}_{i}{}^{k}\mathcal{R}_{k}{}^{l}{}_{jl}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\left(2 r_{\dot 1}-2 r_{\dot 3}+r_{\dot 5}\right)\partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}+\left(2 r_{\dot 1}-2 r_{\dot 3}-r_{\dot 5}\right)\partial_{i}\mathcal{A}_{a'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}+$$
$$\left(2 r_{\dot 1}-2 r_{\dot 3}-r_{\dot 5}\right)\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}+\left(-4 r_{\dot 1}+4 r_{\dot 3}+2 r_{\dot 5}\right)\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}+\left(2 r_{\dot 1}-2 r_{\dot 3}+r_{\dot 5}\right)\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}-$$
$$2\left(2 r_{\dot 1}-2 r_{\dot 3}+r_{\dot 5}\right)\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}-\frac{4}{3} r_{\dot 1}\,\partial_{a'}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3} r_{\dot 1}\,\partial_{a'}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{aa'i}+$$
$$\frac{4}{3}\left(r_{\dot 1}-3 r_{\dot 3}\right)\partial_{a'}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{aa'i}-\frac{2}{3} r_{\dot 1}\,\partial_{i}\mathcal{A}_{aa'j}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3} r_{\dot 1}\,\partial_{j}\mathcal{A}_{aa'i}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3} r_{\dot 1}\,\partial_{j}\mathcal{A}_{aia'}\,\partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}0 & 0 & 0\\ 0 & 6k^{2}\left(-r_{\dot 1}+r_{\dot 3}\right) & 0\\ 0 & 0 & 0\end{pmatrix},\ (0),\ \begin{pmatrix}0 & 0 & 0\\ 0 & k^{2}\left(2 r_{\dot 3}+r_{\dot 5}\right) & 0\\ 0 & 0 & 0\end{pmatrix},\ \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & k^{2}\left(-r_{\dot 1}+2 r_{\dot 3}+r_{\dot 5}\right) & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix},\ \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix},\ \left(k^{2} r_{\dot 1}\right)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\cdot}\tau^{\flat\perp}==0,\ {}^{0^+}_{\cdot}\tau^{\flat\parallel}==0,\ {}^{0^-}_{\cdot}\sigma^{\flat\parallel}==0,\ {}^{1^+}_{\cdot}\sigma^{\flat\perp}{}^{ab}==0,\ {}^{1^+}_{\cdot}\tau^{\flat\parallel}{}^{ab}==0,\right.$$
$$\left.{}^{1^-}_{\cdot}\sigma^{\flat\perp}{}^{a}==0,\ {}^{1^-}_{\cdot}\tau^{\flat\perp}{}^{a}==0,\ {}^{1^-}_{\cdot}\tau^{\flat\parallel}{}^{a}==0,\ {}^{2^+}_{\cdot}\sigma^{\flat\parallel}{}^{ab}==0,\ {}^{2^+}_{\cdot}\tau^{\flat\parallel}{}^{ab}==0\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}0 & 0 & 0\\ 0 & \frac{1}{6k^{2}\left(-r_{\dot 1}+r_{\dot 3}\right)} & 0\\ 0 & 0 & 0\end{pmatrix},\ (0),\ \begin{pmatrix}0 & 0 & 0\\ 0 & \frac{1}{k^{2}\left(2 r_{\dot 3}+r_{\dot 5}\right)} & 0\\ 0 & 0 & 0\end{pmatrix},\ \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & \frac{1}{k^{2}\left(-r_{\dot 1}+2 r_{\dot 3}+r_{\dot 5}\right)} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix},\ \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix},\ \left(\frac{1}{k^{2} r_{\dot 1}}\right)\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{8 r_{\dot 1}^{2}-16 r_{\dot 1} r_{\dot 3}+12 r_{\dot 3}^{2}-8 r_{\dot 1} r_{\dot 5}+12 r_{\dot 3} r_{\dot 5}+3 r_{\dot 5}^{2}}{r_{\dot 1}\left(r_{\dot 1}-2 r_{\dot 3}-r_{\dot 5}\right)\left(2 r_{\dot 3}+r_{\dot 5}\right)},\ \frac{8 r_{\dot 1}^{2}-16 r_{\dot 1} r_{\dot 3}+12 r_{\dot 3}^{2}-8 r_{\dot 1} r_{\dot 5}+12 r_{\dot 3} r_{\dot 5}+3 r_{\dot 5}^{2}}{r_{\dot 1}\left(r_{\dot 1}-2 r_{\dot 3}-r_{\dot 5}\right)\left(2 r_{\dot 3}+r_{\dot 5}\right)}\right\}$$

Overall unitarity conditions:

$$r_{\dot 3}\in\mathbb{R}\ \&\&\left(\left(r_{\dot 1}<0\ \&\&\left(r_{\dot 5}<r_{\dot 1}-2 r_{\dot 3}\ \|\ r_{\dot 5}>-2 r_{\dot 3}\right)\right)\|\left(r_{\dot 1}>0\ \&\&-2 r_{\dot 3}<r_{\dot 5}<r_{\dot 1}-2 r_{\dot 3}\right)\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\dot 3} \in \mathbb{R} \,\&\&\, \left(\left(r_{\dot 1} < 0 \,\&\&\, \left(r_{\dot 5} < r_{\dot 1} - 2 r_{\dot 3} \,\|\, r_{\dot 5} > -2 r_{\dot 3}\right)\right) \,\|\, \left(r_{\dot 1} > 0 \,\&\&\, -2 r_{\dot 3} < r_{\dot 5} < r_{\dot 1} - 2 r_{\dot 3}\right)\right)$

Okay, that concludes the analysis of this theory.

Case 14

Now for a new theory. Here is the full nonlinear Lagrangian for
Case 14 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\left(\frac{r_{\dot 3}}{2} + r_{\dot 5}\right) \mathcal{R}^{ijk}{}_{i} \, \mathcal{R}_{j}{}^{l}{}_{kl} - r_{\dot 3} \, \mathcal{R}^{ijkl} \, \mathcal{R}_{klij} + \frac{1}{2}\left(r_{\dot 3} - 2 r_{\dot 5}\right) \mathcal{R}^{ijk}{}_{i} \, \mathcal{R}_{k}{}^{l}{}_{jl} - \frac{2}{3} t_{\dot 3} \, \mathcal{T}^{i}{}_{i}{}^{j} \, \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise
this Lagrangian to second order around the desired vacuum:

$$-\frac{2}{3} t_{\dot 3} \, \mathcal{A}^{aa'}{}_{a} \, \mathcal{A}_{a'}{}^{i}{}_{i} + \frac{4}{3} t_{\dot 3} \, \mathcal{A}_{a'}{}^{i}{}_{i} \, \partial_{a} f^{aa'} - \frac{4}{3} t_{\dot 3} \, \mathcal{A}_{a'}{}^{i}{}_{i} \, \partial^{a'} f^{a}{}_{a} + \frac{2}{3} t_{\dot 3} \, \partial_{a'} f^{i}{}_{i} \, \partial^{a'} f^{a}{}_{a} + \frac{2}{3} t_{\dot 3} \, \partial_{a} f^{aa'} \, \partial_{i} f^{i}{}_{a'} - \frac{4}{3} t_{\dot 3} \, \partial^{a'} f^{a}{}_{a} \, \partial_{i} f^{i}{}_{a'} + \left(-\frac{r_{\dot 3}}{2} + r_{\dot 5}\right) \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j} \, \partial^{i} \mathcal{A}^{aa'}{}_{a} + \left(-\frac{r_{\dot 3}}{2} - r_{\dot 5}\right) \partial_{i} \mathcal{A}_{a'}{}^{j}{}_{j} \, \partial^{i} \mathcal{A}^{aa'}{}_{a} + \left(-\frac{r_{\dot 3}}{2} - r_{\dot 5}\right) \partial_{a} \mathcal{A}^{aa'i} \, \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} + \left(r_{\dot 3} + 2 r_{\dot 5}\right) \partial^{i} \mathcal{A}^{aa'}{}_{a} \, \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} + \left(-\frac{r_{\dot 3}}{2} + r_{\dot 5}\right) \partial_{a} \mathcal{A}^{aa'i} \, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} + \left(r_{\dot 3} - 2 r_{\dot 5}\right) \partial^{i} \mathcal{A}^{aa'}{}_{a} \, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} - 4 r_{\dot 3} \, \partial_{a'} \mathcal{A}_{ija} \, \partial^{j} \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2 k^2 t_{\dot 3} & i \sqrt{2} \, k \, t_{\dot 3} & 0 \\ -i \sqrt{2} \, k \, t_{\dot 3} & t_{\dot 3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} 0 & 0 & 0 \\ 0 & k^2 \left(2 r_{\dot 3} + r_{\dot 5}\right) & 0 \\ 0 & 0 & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2 k^2 t_{\dot 3}}{3} & \frac{2 i k t_{\dot 3}}{3} & 0 & -\frac{1}{3} i \sqrt{2} \, k \, t_{\dot 3} \\ -\frac{2}{3} i k t_{\dot 3} & k^2 \left(\frac{r_{\dot 3}}{2} + r_{\dot 5}\right) + \frac{2 t_{\dot 3}}{3} & 0 & -\frac{\sqrt{2} \, t_{\dot 3}}{3} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2} \, k \, t_{\dot 3} & -\frac{\sqrt{2} \, t_{\dot 3}}{3} & 0 & \frac{t_{\dot 3}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{3 k^2 r_{\dot 3}}{2} \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{ \overset{0^+}{\cdot}\tau^{\flat\perp} == 0, \; -i \, \overset{0^+}{\cdot}\tau^{\flat\parallel} == 2 k \, \overset{0^+}{\cdot}\sigma^{\flat\parallel}, \; \overset{0^-}{\cdot}\sigma^{\flat\parallel} == 0, \; \overset{1^+}{\cdot}\sigma^{\flat\perp}{}^{ab} == 0, \right.$$
$$\left. \overset{1^+}{\cdot}\tau^{\flat\parallel}{}^{ab} == 0, \; i \, \overset{1^-}{\cdot}\tau^{\flat\parallel}{}^{a} == 2 k \, \overset{1^-}{\cdot}\sigma^{\flat\perp}{}^{a}, \; \overset{1^-}{\cdot}\tau^{\flat\perp}{}^{a} == 0, \; \overset{2^+}{\cdot}\tau^{\flat\parallel}{}^{ab} == 0, \; \overset{2^-}{\cdot}\sigma^{\flat\parallel}{}^{abc} == 0 \right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2 k^2}{(1+2 k^2)^2 \dot{t}_3} & \frac{i \sqrt{2}\, k}{(1+2 k^2)^2 \dot{t}_3} & 0 \\ -\frac{i \sqrt{2}\, k}{(1+2 k^2)^2 \dot{t}_3} & \frac{1}{(1+2 k^2)^2 \dot{t}_3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, (\,0\,), \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{k^2 (2 \dot{r}_3+\dot{r}_5)} & 0 \\ 0 & 0 & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6 k^2 (\dot{r}_3+2 \dot{r}_5)+8 \dot{t}_3}{(1+2 k^2)^2 (\dot{r}_3+2 \dot{r}_5) \dot{t}_3} & -\frac{4 i}{k (1+2 k^2) (\dot{r}_3+2 \dot{r}_5)} & 0 & -\frac{i \sqrt{2} (3 k^2 (\dot{r}_3+2 \dot{r}_5)+4 \dot{t}_3)}{k (1+2 k^2)^2 (\dot{r}_3+2 \dot{r}_5) \dot{t}_3} \\ \frac{4 i}{k (1+2 k^2) (\dot{r}_3+2 \dot{r}_5)} & \frac{2}{k^2 (\dot{r}_3+2 \dot{r}_5)} & 0 & \frac{2 \sqrt{2}}{k^2 (1+2 k^2) (\dot{r}_3+2 \dot{r}_5)} \\ 0 & 0 & 0 & 0 \\ \frac{i \sqrt{2} (3 k^2 (\dot{r}_3+2 \dot{r}_5)+4 \dot{t}_3)}{k (1+2 k^2)^2 (\dot{r}_3+2 \dot{r}_5) \dot{t}_3} & \frac{2 \sqrt{2}}{k^2 (1+2 k^2) (\dot{r}_3+2 \dot{r}_5)} & 0 & \frac{3 k^2 (\dot{r}_3+2 \dot{r}_5)+4 \dot{t}_3}{(k+2 k^3)^2 (\dot{r}_3+2 \dot{r}_5) \dot{t}_3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3 k^2 \dot{r}_3} \end{pmatrix}, (\,0\,)\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{-445 \dot{r}_3^2 - 268 \dot{r}_3 \dot{r}_5 - 52 \dot{r}_5^2}{12 \dot{r}_3 (2 \dot{r}_3 + \dot{r}_5)(\dot{r}_3 + 2 \dot{r}_5)}, \frac{-445 \dot{r}_3^2 - 268 \dot{r}_3 \dot{r}_5 - 52 \dot{r}_5^2}{12 \dot{r}_3 (2 \dot{r}_3 + \dot{r}_5)(\dot{r}_3 + 2 \dot{r}_5)}\right\}$$

Overall unitarity conditions:

$$\left(\dot{r}_3 < 0 \,\&\&\, \left(\dot{r}_5 < -\frac{\dot{r}_3}{2} \,||\, \dot{r}_5 > -2 \dot{r}_3\right)\right) || \left(\dot{r}_3 > 0 \,\&\&\, -2 \dot{r}_3 < \dot{r}_5 < -\frac{\dot{r}_3}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(\dot{r}_3 < 0 \,\&\&\, \left(\dot{r}_5 < -\frac{\dot{r}_3}{2} \,||\, \dot{r}_5 > -2 \dot{r}_3\right)\right) || \left(\dot{r}_3 > 0 \,\&\&\, -2 \dot{r}_3 < \dot{r}_5 < -\frac{\dot{r}_3}{2}\right)$$

Okay, that concludes the analysis of this theory.

Case 15

Now for a new theory. Here is the full nonlinear Lagrangian for Case 15 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\left(\frac{\underset{\cdot}{r}_{3}}{2}+\underset{\cdot}{r}_{5}\right)\mathcal{R}^{ij\ \ \hbar}_{\ \ i}\ \mathcal{R}_{j\ \ \hbar l}^{\ \ l}-\underset{\cdot}{r}_{3}\,\mathcal{R}^{ij\hbar l}\ \mathcal{R}_{\hbar l ij}+\frac{1}{2}\left(\underset{\cdot}{r}_{3}-2\underset{\cdot}{r}_{5}\right)\mathcal{R}^{ij\ \ \hbar}_{\ \ i}\ \mathcal{R}_{\hbar\ \ jl}^{\ \ l}+$$
$$\frac{1}{12}\underset{\cdot}{t}_{2}\,\mathcal{T}_{ij\hbar}\ \mathcal{T}^{ij\hbar}-\frac{1}{6}\underset{\cdot}{t}_{2}\,\mathcal{T}^{ij\hbar}\ \mathcal{T}_{ji\hbar}-\frac{2}{3}\underset{\cdot}{t}_{3}\,\mathcal{T}^{i\ \ j}_{\ \ i}\ \mathcal{T}^{\hbar}_{\ \ j\hbar}$$

To use PSALTer, you have to first linearise
 this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\underset{\cdot}{t}_{2}\,\mathcal{A}_{aa'i}\ \mathcal{A}^{aa'i}-\frac{2}{3}\underset{\cdot}{t}_{2}\,\mathcal{A}_{aia'}\ \mathcal{A}^{aa'i}-\frac{2}{3}\underset{\cdot}{t}_{3}\,\mathcal{A}^{aa'}_{\ \ \ a}\ \mathcal{A}_{a'\ \ i}^{\ \ i}+\frac{4}{3}\underset{\cdot}{t}_{3}\,\mathcal{A}_{a'\ \ i}^{\ \ i}\ \partial_{a}f^{aa'}-\frac{4}{3}\underset{\cdot}{t}_{3}\,\mathcal{A}_{a'\ \ i}^{\ \ i}\ \partial^{a'}f^{a}_{\ \ a}+$$
$$\frac{2}{3}\underset{\cdot}{t}_{3}\,\partial_{a'}f^{i}_{\ \ i}\ \partial^{a'}f^{a}_{\ \ a}+\frac{2}{3}\underset{\cdot}{t}_{3}\,\partial_{a}f^{aa'}\ \partial_{i}f^{i}_{\ \ a'}-\frac{4}{3}\underset{\cdot}{t}_{3}\,\partial^{a'}f^{a}_{\ \ a}\ \partial_{i}f^{i}_{\ \ a'}+\left(-\frac{\underset{\cdot}{r}_{3}}{2}+\underset{\cdot}{r}_{5}\right)\partial_{a'}\mathcal{A}_{i\ \ j}^{\ \ j}\ \partial^{i}\mathcal{A}^{aa'}_{\ \ \ a}+$$
$$\left(-\frac{\underset{\cdot}{r}_{3}}{2}-\underset{\cdot}{r}_{5}\right)\partial_{i}\mathcal{A}_{a'\ \ j}^{\ \ j}\ \partial^{i}\mathcal{A}^{aa'}_{\ \ \ a}-\frac{2}{3}\underset{\cdot}{t}_{2}\,\mathcal{A}_{aa'i}\ \partial^{i}f^{aa'}+\frac{2}{3}\underset{\cdot}{t}_{2}\,\mathcal{A}_{aia'}\ \partial^{i}f^{aa'}-\frac{2}{3}\underset{\cdot}{t}_{2}\,\mathcal{A}_{a'ia}\ \partial^{i}f^{aa'}+\frac{1}{3}\underset{\cdot}{t}_{2}\,\partial_{a}f_{a'i}\ \partial^{i}f^{aa'}-$$
$$\frac{1}{6}\underset{\cdot}{t}_{2}\,\partial_{a}f_{ia'}\ \partial^{i}f^{aa'}-\frac{1}{6}\underset{\cdot}{t}_{2}\,\partial_{a'}f_{ai}\ \partial^{i}f^{aa'}+\frac{1}{6}\underset{\cdot}{t}_{2}\,\partial_{i}f_{aa'}\ \partial^{i}f^{aa'}-\frac{1}{6}\underset{\cdot}{t}_{2}\,\partial_{i}f_{a'a}\ \partial^{i}f^{aa'}+\left(-\frac{\underset{\cdot}{r}_{3}}{2}-\underset{\cdot}{r}_{5}\right)\partial_{a}\mathcal{A}^{aa'i}\ \partial_{j}\mathcal{A}_{a'\ \ i}^{\ \ j}+$$
$$\left(\underset{\cdot}{r}_{3}+2\underset{\cdot}{r}_{5}\right)\partial^{i}\mathcal{A}^{aa'}_{\ \ \ a}\ \partial_{j}\mathcal{A}_{a'\ \ i}^{\ \ j}+\left(-\frac{\underset{\cdot}{r}_{3}}{2}+\underset{\cdot}{r}_{5}\right)\partial_{a}\mathcal{A}^{aa'i}\ \partial_{j}\mathcal{A}_{i\ \ a'}^{\ \ j}+\left(\underset{\cdot}{r}_{3}-2\underset{\cdot}{r}_{5}\right)\partial^{i}\mathcal{A}^{aa'}_{\ \ \ a}\ \partial_{j}\mathcal{A}_{i\ \ a'}^{\ \ j}-4\underset{\cdot}{r}_{3}\,\partial_{a'}\mathcal{A}_{ija}\ \partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
 the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2k^{2}\underset{\cdot}{t}_{3} & i\sqrt{2}\,k\underset{\cdot}{t}_{3} & 0 \\ -i\sqrt{2}\,k\underset{\cdot}{t}_{3} & \underset{\cdot}{t}_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \begin{pmatrix}\underset{\cdot}{t}_{2}\end{pmatrix},\ \begin{pmatrix} \frac{k^{2}\underset{\cdot}{t}_{2}}{3} & \frac{1}{3}i\sqrt{2}\,k\underset{\cdot}{t}_{2} & \frac{ik\underset{\cdot}{t}_{2}}{3} \\ -\frac{1}{3}i\sqrt{2}\,k\underset{\cdot}{t}_{2} & \frac{1}{2}\left(2k^{2}\left(2\underset{\cdot}{r}_{3}+\underset{\cdot}{r}_{5}\right)+\frac{4\underset{\cdot}{t}_{2}}{3}\right) & \frac{\sqrt{2}\,\underset{\cdot}{t}_{2}}{3} \\ -\frac{1}{3}ik\underset{\cdot}{t}_{2} & \frac{\sqrt{2}\,\underset{\cdot}{t}_{2}}{3} & \frac{\underset{\cdot}{t}_{2}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^{2}\underset{\cdot}{t}_{3}}{3} & \frac{2ik\underset{\cdot}{t}_{3}}{3} & 0 & -\frac{1}{3}i\sqrt{2}\,k\underset{\cdot}{t}_{3} \\ -\frac{2}{3}ik\underset{\cdot}{t}_{3} & k^{2}\left(\frac{\underset{\cdot}{r}_{3}}{2}+\underset{\cdot}{r}_{5}\right)+\frac{2\underset{\cdot}{t}_{3}}{3} & 0 & -\frac{\sqrt{2}\,\underset{\cdot}{t}_{3}}{3} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3}i\sqrt{2}\,k\underset{\cdot}{t}_{3} & -\frac{\sqrt{2}\,\underset{\cdot}{t}_{3}}{3} & 0 & \frac{\underset{\cdot}{t}_{3}}{3} \end{pmatrix},\ \begin{pmatrix} 0 & 0 \\ 0 & -\frac{3k^{2}\underset{\cdot}{r}_{3}}{2} \end{pmatrix},\ \begin{pmatrix}0\end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^{+}}{\underset{\cdot}{}}\tau^{\flat\perp}==0,\ -i\,\overset{0^{+}}{\underset{\cdot}{}}\tau^{\flat\parallel}==2k\,\overset{0^{+}}{\underset{\cdot}{}}\sigma^{\flat\parallel},\ -i\,\overset{1^{+}}{\underset{\cdot}{}}\tau^{\flat\parallel\,ab}==k\,\overset{1^{+}}{\underset{\cdot}{}}\sigma^{\flat\perp\,ab},\right.$$
$$\left.i\,\overset{1^{-}}{\underset{\cdot}{}}\tau^{\flat\parallel\,a}==2k\,\overset{1^{-}}{\underset{\cdot}{}}\sigma^{\flat\perp\,a},\ \overset{1^{-}}{\underset{\cdot}{}}\tau^{\flat\perp\,a}==0,\ \overset{2^{+}}{\underset{\cdot}{}}\tau^{\flat\parallel\,ab}==0,\ \overset{2^{-}}{\underset{\cdot}{}}\sigma^{\flat\parallel\,abc}==0\right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
 analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2 k^2}{\left(1+2 k^2\right)^2 t_{\cdot 3}} & \frac{i \sqrt{2} k}{\left(1+2 k^2\right)^2 t_{\cdot 3}} & 0 \\ -\frac{i \sqrt{2} k}{\left(1+2 k^2\right)^2 t_{\cdot 3}} & \frac{1}{\left(1+2 k^2\right)^2 t_{\cdot 3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{t_{\cdot 2}} \end{pmatrix}, \begin{pmatrix} \frac{3 k^2 \left(2 r_{\cdot 3}+r_{\cdot 5}\right)+2 t_{\cdot 2}}{(1+k^2)^2 \left(2 r_{\cdot 3}+r_{\cdot 5}\right) t_{\cdot 2}} & -\frac{i \sqrt{2}}{k (1+k^2) \left(2 r_{\cdot 3}+r_{\cdot 5}\right)} & \frac{i \left(3 k^2 \left(2 r_{\cdot 3}+r_{\cdot 5}\right)+2 t_{\cdot 2}\right)}{k (1+k^2)^2 \left(2 r_{\cdot 3}+r_{\cdot 5}\right) t_{\cdot 2}} \\ \frac{i \sqrt{2}}{k (1+k^2) \left(2 r_{\cdot 3}+r_{\cdot 5}\right)} & \frac{1}{k^2 \left(2 r_{\cdot 3}+r_{\cdot 5}\right)} & -\frac{\sqrt{2}}{k^2 (1+k^2) \left(2 r_{\cdot 3}+r_{\cdot 5}\right)} \\ -\frac{i \left(3 k^2 \left(2 r_{\cdot 3}+r_{\cdot 5}\right)+2 t_{\cdot 2}\right)}{k (1+k^2)^2 \left(2 r_{\cdot 3}+r_{\cdot 5}\right) t_{\cdot 2}} & -\frac{\sqrt{2}}{k^2 (1+k^2) \left(2 r_{\cdot 3}+r_{\cdot 5}\right)} & \frac{3 k^2 \left(2 r_{\cdot 3}+r_{\cdot 5}\right)+2 t_{\cdot 2}}{(k+k^3)^2 \left(2 r_{\cdot 3}+r_{\cdot 5}\right) t_{\cdot 2}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6 k^2 \left(r_{\cdot 3}+2 r_{\cdot 5}\right)+8 t_{\cdot 3}}{\left(1+2 k^2\right)^2 \left(r_{\cdot 3}+2 r_{\cdot 5}\right) t_{\cdot 3}} & -\frac{4 i}{k \left(1+2 k^2\right) \left(r_{\cdot 3}+2 r_{\cdot 5}\right)} & 0 & -\frac{i \sqrt{2} \left(3 k^2 \left(r_{\cdot 3}+2 r_{\cdot 5}\right)+4 t_{\cdot 3}\right)}{k \left(1+2 k^2\right)^2 \left(r_{\cdot 3}+2 r_{\cdot 5}\right) t_{\cdot 3}} \\ \frac{4 i}{k \left(1+2 k^2\right) \left(r_{\cdot 3}+2 r_{\cdot 5}\right)} & \frac{2}{k^2 \left(r_{\cdot 3}+2 r_{\cdot 5}\right)} & 0 & \frac{2 \sqrt{2}}{k^2 \left(1+2 k^2\right) \left(r_{\cdot 3}+2 r_{\cdot 5}\right)} \\ 0 & 0 & 0 & 0 \\ \frac{i \sqrt{2} \left(3 k^2 \left(r_{\cdot 3}+2 r_{\cdot 5}\right)+4 t_{\cdot 3}\right)}{k \left(1+2 k^2\right)^2 \left(r_{\cdot 3}+2 r_{\cdot 5}\right) t_{\cdot 3}} & \frac{2 \sqrt{2}}{k^2 \left(1+2 k^2\right) \left(r_{\cdot 3}+2 r_{\cdot 5}\right)} & 0 & \frac{3 k^2 \left(r_{\cdot 3}+2 r_{\cdot 5}\right)+4 t_{\cdot 3}}{\left(k+2 k^3\right)^2 \left(r_{\cdot 3}+2 r_{\cdot 5}\right) t_{\cdot 3}} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3 k^2 r_{\cdot 3}} \end{pmatrix}, \begin{pmatrix} 0 \end{pmatrix}\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{403 r_{\cdot 3}^2+172 r_{\cdot 3} r_{\cdot 5}+28 r_{\cdot 5}^2}{6 r_{\cdot 3} \left(2 r_{\cdot 3}+r_{\cdot 5}\right) \left(r_{\cdot 3}+2 r_{\cdot 5}\right)}, -\frac{403 r_{\cdot 3}^2+172 r_{\cdot 3} r_{\cdot 5}+28 r_{\cdot 5}^2}{6 r_{\cdot 3} \left(2 r_{\cdot 3}+r_{\cdot 5}\right) \left(r_{\cdot 3}+2 r_{\cdot 5}\right)}\right\}$$

Overall unitarity conditions:

$$\left(r_{\cdot 3} < 0 \,\&\&\, \left(r_{\cdot 5} < -\frac{r_{\cdot 3}}{2} \,\|\, r_{\cdot 5} > -2 r_{\cdot 3}\right)\right) \| \left(r_{\cdot 3} > 0 \,\&\&\, -2 r_{\cdot 3} < r_{\cdot 5} < -\frac{r_{\cdot 3}}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{\cdot 3} < 0 \,\&\&\, \left(r_{\cdot 5} < -\frac{r_{\cdot 3}}{2} \,\|\, r_{\cdot 5} > -2 r_{\cdot 3}\right)\right) \| \left(r_{\cdot 3} > 0 \,\&\&\, -2 r_{\cdot 3} < r_{\cdot 5} < -\frac{r_{\cdot 3}}{2}\right)$$

Okay, that concludes the analysis of this theory.

Case 16

Now for a new theory. Here is the full nonlinear Lagrangian for Case 16 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\cdot 2}\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} - \frac{2}{3} r_{\cdot 2}\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} + \left(\frac{r_{\cdot 3}}{2} + r_{\cdot 5}\right) \mathcal{R}^{ij\ \ k}_{\ \ i}\, \mathcal{R}_{j\ \ kl}^{\ \ l} +$$
$$\frac{1}{6}\left(r_{\cdot 2} - 6\, r_{\cdot 3}\right) \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + \frac{1}{2}\left(r_{\cdot 3} - 2\, r_{\cdot 5}\right) \mathcal{R}^{ij\ \ k}_{\ \ i}\, \mathcal{R}_{k\ \ jl}^{\ \ l} - \frac{2}{3} t_{\cdot 3}\, \mathcal{T}^{i\ \ j}_{\ \ i}\, \mathcal{T}^{k}_{\ \ jk}$$

To use PSALTer, you have to first linearise
 this Lagrangian to second order around the desired vacuum:

$$-\frac{2}{3} t_{\cdot 3}\, \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}_{\ \ \ \ \mathfrak{a}}\, \mathcal{A}_{\mathfrak{a}'\ \ i}^{\ \ \ i} + \frac{4}{3} t_{\cdot 3}\, \mathcal{A}_{\mathfrak{a}'\ \ i}^{\ \ \ i}\, \partial_{\mathfrak{a}} f^{\mathfrak{a}\mathfrak{a}'} - \frac{4}{3} t_{\cdot 3}\, \mathcal{A}_{\mathfrak{a}'\ \ i}^{\ \ \ i}\, \partial^{\mathfrak{a}'} f^{\mathfrak{a}}_{\ \ \mathfrak{a}} + \frac{2}{3} t_{\cdot 3}\, \partial_{\mathfrak{a}'} f^{i}_{\ \ i}\, \partial^{\mathfrak{a}'} f^{\mathfrak{a}}_{\ \ \mathfrak{a}} +$$
$$\frac{2}{3} t_{\cdot 3}\, \partial_{\mathfrak{a}} f^{\mathfrak{a}\mathfrak{a}'}\, \partial_{i} f^{i}_{\ \ \mathfrak{a}'} - \frac{4}{3} t_{\cdot 3}\, \partial^{\mathfrak{a}'} f^{\mathfrak{a}}_{\ \ \mathfrak{a}}\, \partial_{i} f^{i}_{\ \ \mathfrak{a}'} + \left(-\frac{r_{\cdot 3}}{2} + r_{\cdot 5}\right) \partial_{\mathfrak{a}'} \mathcal{A}_{i\ \ j}^{\ \ j}\, \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}_{\ \ \ \ \mathfrak{a}} + \left(-\frac{r_{\cdot 3}}{2} - r_{\cdot 5}\right) \partial_{i} \mathcal{A}_{\mathfrak{a}'\ \ j}^{\ \ \ j}\, \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}_{\ \ \ \ \mathfrak{a}} +$$
$$\left(-\frac{r_{\cdot 3}}{2} - r_{\cdot 5}\right) \partial_{\mathfrak{a}} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\, \partial_{j} \mathcal{A}_{\mathfrak{a}'\ \ i}^{\ \ \ j} + \left(r_{\cdot 3} + 2\, r_{\cdot 5}\right) \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}_{\ \ \ \ \mathfrak{a}}\, \partial_{j} \mathcal{A}_{\mathfrak{a}'\ \ i}^{\ \ \ j} + \left(-\frac{r_{\cdot 3}}{2} + r_{\cdot 5}\right) \partial_{\mathfrak{a}} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\, \partial_{j} \mathcal{A}_{i\ \ \mathfrak{a}'}^{\ \ j} +$$
$$\left(r_{\cdot 3} - 2\, r_{\cdot 5}\right) \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}_{\ \ \ \ \mathfrak{a}}\, \partial_{j} \mathcal{A}_{i\ \ \mathfrak{a}'}^{\ \ j} + \frac{4}{3} r_{\cdot 2}\, \partial_{\mathfrak{a}'} \mathcal{A}_{\mathfrak{a}ij}\, \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} r_{\cdot 2}\, \partial_{\mathfrak{a}'} \mathcal{A}_{\mathfrak{a}ji}\, \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} +$$
$$\frac{2}{3}\left(r_{\cdot 2} - 6\, r_{\cdot 3}\right) \partial_{\mathfrak{a}'} \mathcal{A}_{ij\mathfrak{a}}\, \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{1}{3} r_{\cdot 2}\, \partial_{i} \mathcal{A}_{\mathfrak{a}\mathfrak{a}'j}\, \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + \frac{1}{3} r_{\cdot 2}\, \partial_{j} \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i}\, \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} r_{\cdot 2}\, \partial_{j} \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'}\, \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated with
 the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2k^2 t_{\cdot 3} & i\sqrt{2}\, k\, t_{\cdot 3} & 0 \\ -i\sqrt{2}\, k\, t_{\cdot 3} & t_{\cdot 3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{\cdot 2}\right), \begin{pmatrix} 0 & 0 & 0 \\ 0 & k^2\left(2\, r_{\cdot 3} + r_{\cdot 5}\right) & 0 \\ 0 & 0 & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2 t_{\cdot 3}}{3} & \frac{2ik\, t_{\cdot 3}}{3} & 0 & -\frac{1}{3} i\sqrt{2}\, k\, t_{\cdot 3} \\ -\frac{2}{3} ik\, t_{\cdot 3} & k^2\left(\frac{r_{\cdot 3}}{2} + r_{\cdot 5}\right) + \frac{2 t_{\cdot 3}}{3} & 0 & -\frac{\sqrt{2}\, t_{\cdot 3}}{3} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i\sqrt{2}\, k\, t_{\cdot 3} & -\frac{\sqrt{2}\, t_{\cdot 3}}{3} & 0 & \frac{t_{\cdot 3}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{3k^2 r_{\cdot 3}}{2} \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ -i\, {}^{0^+}_{\bullet}\tau^{\flat\parallel} == 2k\, {}^{0^+}_{\bullet}\sigma^{\flat\parallel},\ {}^{1^+}_{\bullet}\sigma^{\flat\perp\, \mathfrak{a}\mathfrak{b}} == 0,\ {}^{1^+}_{\bullet}\tau^{\flat\parallel\, \mathfrak{a}\mathfrak{b}} == 0,\right.$$
$$\left. i\, {}^{1^-}_{\bullet}\tau^{\flat\parallel\, \mathfrak{a}} == 2k\, {}^{1^-}_{\bullet}\sigma^{\flat\perp\, \mathfrak{a}},\ {}^{1^-}_{\bullet}\tau^{\flat\perp\, \mathfrak{a}} == 0,\ {}^{2^+}_{\bullet}\tau^{\flat\parallel\, \mathfrak{a}\mathfrak{b}} == 0,\ {}^{2^-}_{\bullet}\sigma^{\flat\parallel\, \mathfrak{a}\mathfrak{b}\mathfrak{c}} == 0\right\}$$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally
 analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_{\cdot 3}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\cdot 3}} & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\cdot 3}} & \frac{1}{(1+2k^2)^2 t_{\cdot 3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{\cdot 2}}\right), \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{k^2(2r_{\cdot 3}+r_{\cdot 5})} & 0 \\ 0 & 0 & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6k^2(r_{\cdot 3}+2r_{\cdot 5})+8t_{\cdot 3}}{(1+2k^2)^2(r_{\cdot 3}+2r_{\cdot 5})t_{\cdot 3}} & -\frac{4i}{k(1+2k^2)(r_{\cdot 3}+2r_{\cdot 5})} & 0 & -\frac{i\sqrt{2}\,(3k^2(r_{\cdot 3}+2r_{\cdot 5})+4t_{\cdot 3})}{k(1+2k^2)^2(r_{\cdot 3}+2r_{\cdot 5})t_{\cdot 3}} \\ \frac{4i}{k(1+2k^2)(r_{\cdot 3}+2r_{\cdot 5})} & \frac{2}{k^2(r_{\cdot 3}+2r_{\cdot 5})} & 0 & \frac{2\sqrt{2}}{k^2(1+2k^2)(r_{\cdot 3}+2r_{\cdot 5})} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,(3k^2(r_{\cdot 3}+2r_{\cdot 5})+4t_{\cdot 3})}{k(1+2k^2)^2(r_{\cdot 3}+2r_{\cdot 5})t_{\cdot 3}} & \frac{2\sqrt{2}}{k^2(1+2k^2)(r_{\cdot 3}+2r_{\cdot 5})} & 0 & \frac{3k^2(r_{\cdot 3}+2r_{\cdot 5})+4t_{\cdot 3}}{(k+2k^3)^2(r_{\cdot 3}+2r_{\cdot 5})t_{\cdot 3}} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3k^2 r_{\cdot 3}} \end{pmatrix}, (0)\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{-445 r_{\cdot 3}^2 - 268 r_{\cdot 3} r_{\cdot 5} - 52 r_{\cdot 5}^2}{12 r_{\cdot 3}(2r_{\cdot 3}+r_{\cdot 5})(r_{\cdot 3}+2r_{\cdot 5})}, \frac{-445 r_{\cdot 3}^2 - 268 r_{\cdot 3} r_{\cdot 5} - 52 r_{\cdot 5}^2}{12 r_{\cdot 3}(2r_{\cdot 3}+r_{\cdot 5})(r_{\cdot 3}+2r_{\cdot 5})}\right\}$$

Overall unitarity conditions:

$$\left(r_{\cdot 3} < 0 \;\&\&\; \left(r_{\cdot 5} < -\frac{r_{\cdot 3}}{2} \;\|\; r_{\cdot 5} > -2r_{\cdot 3}\right)\right) \;\|\; \left(r_{\cdot 3} > 0 \;\&\&\; -2r_{\cdot 3} < r_{\cdot 5} < -\frac{r_{\cdot 3}}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{\cdot 3} < 0 \;\&\&\; \left(r_{\cdot 5} < -\frac{r_{\cdot 3}}{2} \;\|\; r_{\cdot 5} > -2r_{\cdot 3}\right)\right) \;\|\; \left(r_{\cdot 3} > 0 \;\&\&\; -2r_{\cdot 3} < r_{\cdot 5} < -\frac{r_{\cdot 3}}{2}\right)$$

Okay, that concludes the analysis of this theory.

Case 17

Now for a new theory. Here is the full nonlinear Lagrangian for Case 17 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$r_{\cdot 5}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl} - r_{\cdot 5}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4}t_{\cdot 1}\,\mathcal{T}_{ijk}\,\mathcal{T}^{ijk} + \frac{1}{2}t_{\cdot 1}\,\mathcal{T}^{ijk}\,\mathcal{T}_{jik} + \frac{1}{3}t_{\cdot 1}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise
this Lagrangian to second order around the desired vacuum:

$$\underset{1}{t}\,\mathcal{A}_{a i a'}\,\mathcal{A}^{a a' i}+\frac{1}{3}\underset{1}{t}\,\mathcal{A}^{a a'}{}_{a}\,\mathcal{A}_{a'}{}^{i}{}_{i}-\frac{2}{3}\underset{1}{t}\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial_{a}f^{a a'}+\frac{2}{3}\underset{1}{t}\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}-\frac{1}{3}\underset{1}{t}\,\partial_{a'}f^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}-\frac{1}{3}\underset{1}{t}\,\partial_{a}f^{a a'}\,\partial_{i}f^{i}{}_{a'}+\frac{2}{3}\underset{1}{t}\,\partial^{a'}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{a'}+\underset{5}{r}\,\partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{a a'}{}_{a}-\underset{5}{r}\,\partial_{i}\mathcal{A}_{a'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{a a'}{}_{a}+2\underset{1}{t}\,\mathcal{A}_{a' i a}\,\partial^{i}f^{a a'}-\underset{1}{t}\,\partial_{a}f_{a' i}\,\partial^{i}f^{a a'}+\frac{1}{2}\underset{1}{t}\,\partial_{a}f_{i a'}\,\partial^{i}f^{a a'}-\frac{1}{2}\underset{1}{t}\,\partial_{a'}f_{a i}\,\partial^{i}f^{a a'}+\frac{1}{2}\underset{1}{t}\,\partial_{i}f_{a a'}\,\partial^{i}f^{a a'}+\frac{1}{2}\underset{1}{t}\,\partial_{i}f_{a' a}\,\partial^{i}f^{a a'}-\underset{5}{r}\,\partial_{a}\mathcal{A}^{a a' i}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}+2\underset{5}{r}\,\partial^{i}\mathcal{A}^{a a'}{}_{a}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}+\underset{5}{r}\,\partial_{a}\mathcal{A}^{a a' i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}-2\underset{5}{r}\,\partial^{i}\mathcal{A}^{a a'}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix},\ \left(-\underset{1}{t}\right),\ \begin{pmatrix}0&-\frac{i k \underset{1}{t}}{\sqrt{2}}&0\\ \frac{i k \underset{1}{t}}{\sqrt{2}}&\frac{1}{2}\left(2k^{2}\underset{5}{r}-\underset{1}{t}\right)&-\frac{\underset{1}{t}}{\sqrt{2}}\\ 0&-\frac{\underset{1}{t}}{\sqrt{2}}&0\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}\frac{2k^{2}\underset{1}{t}}{3}&-\frac{1}{3}i k\underset{1}{t}&0&-\frac{1}{3}i\sqrt{2}\,k\underset{1}{t}\\ \frac{i k\underset{1}{t}}{3}&k^{2}\underset{5}{r}+\frac{\underset{1}{t}}{6}&0&\frac{\underset{1}{t}}{3\sqrt{2}}\\ 0&0&0&0\\ \frac{1}{3}i\sqrt{2}\,k\underset{1}{t}&\frac{\underset{1}{t}}{3\sqrt{2}}&0&\frac{\underset{1}{t}}{3}\end{pmatrix},\ \begin{pmatrix}k^{2}\underset{1}{t}&\frac{i k\underset{1}{t}}{\sqrt{2}}\\ -\frac{i k\underset{1}{t}}{\sqrt{2}}&\frac{\underset{1}{t}}{2}\end{pmatrix},\ \left(\frac{\underset{1}{t}}{2}\right)\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^{+}}{\cdot}\tau^{\flat\perp}==0,\ \overset{0^{+}}{\cdot}\sigma^{\flat\parallel}==0,\ \overset{0^{+}}{\cdot}\tau^{\flat\parallel}==0,\ -i\,\overset{1^{+}}{\cdot}\tau^{\flat\parallel}{}^{ab}==k\,\overset{1^{+}}{\cdot}\sigma^{\flat\perp}{}^{ab},\ i\,\overset{1^{-}}{\cdot}\tau^{\flat\parallel}{}^{a}==2k\,\overset{1^{-}}{\cdot}\sigma^{\flat\perp}{}^{a},\ \overset{1^{-}}{\cdot}\tau^{\flat\perp}{}^{a}==0,\ -i\,\overset{2^{+}}{\cdot}\tau^{\flat\parallel}{}^{ab}==2k\,\overset{2^{+}}{\cdot}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(-\frac{1}{t_1}\right), \begin{pmatrix} \frac{-2k^4 r_5 + k^2 t_1}{(1+k^2)^2 t_1^2} & -\frac{i\sqrt{2}\,k}{t_1 + k^2 t_1} & -\frac{i\left(2k^3 r_5 - k t_1\right)}{(1+k^2)^2 t_1^2} \\ \frac{i\sqrt{2}\,k}{t_1 + k^2 t_1} & 0 & -\frac{\sqrt{2}}{t_1 + k^2 t_1} \\ \frac{i\left(2k^3 r_5 - k t_1\right)}{(1+k^2)^2 t_1^2} & -\frac{\sqrt{2}}{t_1 + k^2 t_1} & \frac{-2k^2 r_5 + t_1}{(1+k^2)^2 t_1^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6k^2 r_5 + t_1}{\left(1+2k^2\right)^2 r_5 t_1} & \frac{i}{k r_5 + 2k^3 r_5} & 0 & -\frac{i\left(6k^2 r_5 + t_1\right)}{\sqrt{2}\,k\left(1+2k^2\right)^2 r_5 t_1} \\ -\frac{i}{k r_5 + 2k^3 r_5} & \frac{1}{k^2 r_5} & 0 & -\frac{1}{\sqrt{2}\left(k^2 r_5 + 2k^4 r_5\right)} \\ 0 & 0 & 0 & 0 \\ \frac{i\left(6k^2 r_5 + t_1\right)}{\sqrt{2}\,k\left(1+2k^2\right)^2 r_5 t_1} & -\frac{1}{\sqrt{2}\left(k^2 r_5 + 2k^4 r_5\right)} & 0 & \frac{6k^2 r_5 + t_1}{2\left(k+2k^3\right)^2 r_5 t_1} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_1} & \frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} \\ -\frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} & \frac{2}{\left(1+2k^2\right)^2 t_1} \end{pmatrix}, \left(\frac{2}{t_1}\right)\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{7 t_1^2 + 2 r_5 t_1 p^2 + 4 r_5^2 p^4}{2 r_5 t_1^2}, -\frac{7 t_1^2 + 2 r_5 t_1 p^2 + 4 r_5^2 p^4}{2 r_5 t_1^2}\right\}$$

Overall unitarity conditions:

$$p \in \mathbb{R} \,\&\&\, r_5 < 0 \,\&\&\, \left(t_1 < 0 \,||\, t_1 > 0\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$t_1 \neq 0 \,\&\&\, r_5 < 0$$

Okay, that concludes the analysis of this theory.

Case 18

Now for a new theory. Here is the full nonlinear Lagrangian for Case 18 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$r_5 \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} - r_5 \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{3} t_1 \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{3} t_1 \mathcal{T}^{ijk} \mathcal{T}_{jik} + t_1 \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{1} \mathcal{A}_{a a' i} \mathcal{A}^{a a' i} + \frac{1}{3} t_{1} \mathcal{A}_{a i a'} \mathcal{A}^{a a' i} + t_{1} \mathcal{A}^{a a'}{}_{a} \mathcal{A}_{a'}{}^{i}{}_{i} - 2 t_{1} \mathcal{A}_{a'}{}^{i}{}_{i} \partial_{a} f^{a a'} +$$
$$2 t_{1} \mathcal{A}_{a'}{}^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - t_{1} \partial_{a'} f^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - t_{1} \partial_{a} f^{a a'} \partial_{i} f^{i}{}_{a'} + 2 t_{1} \partial^{a'} f^{a}{}_{a} \partial_{i} f^{i}{}_{a'} + r_{5} \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{a a'}{}_{a} -$$
$$r_{5} \partial_{i} \mathcal{A}_{a'}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{a a'}{}_{a} - \frac{2}{3} t_{1} \mathcal{A}_{a a' i} \partial^{i} f^{a a'} + \frac{2}{3} t_{1} \mathcal{A}_{a i a'} \partial^{i} f^{a a'} + \frac{4}{3} t_{1} \mathcal{A}_{a' i a} \partial^{i} f^{a a'} -$$
$$\frac{2}{3} t_{1} \partial_{a} f_{a' i} \partial^{i} f^{a a'} + \frac{1}{3} t_{1} \partial_{a} f_{i a'} \partial^{i} f^{a a'} - \frac{2}{3} t_{1} \partial_{a'} f_{a i} \partial^{i} f^{a a'} + \frac{2}{3} t_{1} \partial_{i} f_{a a'} \partial^{i} f^{a a'} + \frac{1}{3} t_{1} \partial_{i} f_{a' a} \partial^{i} f^{a a'} -$$
$$r_{5} \partial_{a} \mathcal{A}^{a a' i} \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} + 2 r_{5} \partial^{i} \mathcal{A}^{a a'}{}_{a} \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} + r_{5} \partial_{a} \mathcal{A}^{a a' i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} - 2 r_{5} \partial^{i} \mathcal{A}^{a a'}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2 k^2 t_{1} & -i \sqrt{2}\, k t_{1} & 0 \\ i \sqrt{2}\, k t_{1} & -t_{1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0),\right.$$

$$\begin{pmatrix} \frac{k^2 t_{1}}{3} & -\frac{i k t_{1}}{3 \sqrt{2}} & \frac{i k t_{1}}{3} \\ \frac{i k t_{1}}{3\sqrt{2}} & \frac{1}{6}\left(6 k^2 r_{5} + t_{1}\right) & -\frac{t_{1}}{3\sqrt{2}} \\ -\frac{1}{3} i k t_{1} & -\frac{t_{1}}{3\sqrt{2}} & \frac{t_{1}}{3} \end{pmatrix}, \begin{pmatrix} 0 & -i k t_{1} & 0 & 0 \\ i k t_{1} & k^2 r_{5} - \frac{t_{1}}{2} & 0 & \frac{t_{1}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{1}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \left(\frac{t_{1}}{2}\right)\Bigg\}$$

Gauge constraints on source currents:

$$\left\{ \overset{0^+}{\tau}{}^{\flat\perp} == 0,\ -i\, \overset{0^+}{\tau}{}^{\flat\parallel} == 2 k\, \overset{0^+}{\sigma}{}^{\flat\parallel},\ \overset{0^-}{\sigma}{}^{\flat\parallel} == 0,\ -i\, \overset{1^+}{\tau}{}^{\flat\parallel\, a b} == k\, \overset{1^+}{\sigma}{}^{\flat\perp\, a b},\right.$$
$$\left. i\, \overset{1^-}{\tau}{}^{\flat\parallel\, a} == 2 k\, \overset{1^-}{\sigma}{}^{\flat\perp\, a},\ \overset{1^-}{\tau}{}^{\flat\perp\, a} == 0,\ -i\, \overset{2^+}{\tau}{}^{\flat\parallel\, a b} == 2 k\, \overset{2^+}{\sigma}{}^{\flat\parallel\, a b} \right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2 k^2}{(1+2k^2)^2 t_{1}} & -\frac{i \sqrt{2}\, k}{(1+2k^2)^2 t_{1}} & 0 \\ \frac{i \sqrt{2}\, k}{(1+2k^2)^2 t_{1}} & -\frac{1}{(1+2k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} \frac{6 k^2 r_{5} + t_{1}}{2 (1+k^2)^2 r_{5} t_{1}} & \frac{i}{\sqrt{2}\left(k r_{5} + k^3 r_{5}\right)} & \frac{i\left(6 k^2 r_{5} + t_{1}\right)}{2 k (1+k^2)^2 r_{5} t_{1}} \\ -\frac{i}{\sqrt{2}\left(k r_{5} + k^3 r_{5}\right)} & \frac{1}{k^2 r_{5}} & \frac{1}{\sqrt{2}\left(k^2 r_{5} + k^4 r_{5}\right)} \\ -\frac{i\left(6 k^2 r_{5} + t_{1}\right)}{2 k (1+k^2)^2 r_{5} t_{1}} & \frac{1}{\sqrt{2}\left(k^2 r_{5} + k^4 r_{5}\right)} & \frac{6 k^2 r_{5} + t_{1}}{2 (k+k^3)^2 r_{5} t_{1}} \end{pmatrix},\right.$$

$$\begin{pmatrix} \frac{-4 k^4 r_{5} + 2 k^2 t_{1}}{\left(t_{1} + 2 k^2 t_{1}\right)^2} & -\frac{2 i k}{t_{1} + 2 k^2 t_{1}} & 0 & \frac{i \sqrt{2}\, k \left(2 k^2 r_{5} - t_{1}\right)}{\left(t_{1} + 2 k^2 t_{1}\right)^2} \\ \frac{2 i k}{t_{1} + 2 k^2 t_{1}} & 0 & 0 & \frac{\sqrt{2}}{t_{1} + 2 k^2 t_{1}} \\ 0 & 0 & 0 & 0 \\ -\frac{i \sqrt{2}\, k \left(2 k^2 r_{5} - t_{1}\right)}{\left(t_{1} + 2 k^2 t_{1}\right)^2} & \frac{\sqrt{2}}{t_{1} + 2 k^2 t_{1}} & 0 & \frac{-2 k^2 r_{5} + t_{1}}{\left(t_{1} + 2 k^2 t_{1}\right)^2} \end{pmatrix}, \begin{pmatrix} \frac{4 k^2}{(1+2k^2)^2 t_{1}} & \frac{2 i \sqrt{2}\, k}{(1+2k^2)^2 t_{1}} \\ -\frac{2 i \sqrt{2}\, k}{(1+2k^2)^2 t_{1}} & \frac{2}{(1+2k^2)^2 t_{1}} \end{pmatrix}, \left(\frac{2}{t_{1}}\right)\Bigg\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{9 t_{1}^{2}+2 r_{5} t_{1} p^{2}+2 r_{5}^{2} p^{4}}{r_{5} t_{1}^{2}}, \frac{9 t_{1}^{2}+2 r_{5} t_{1} p^{2}+2 r_{5}^{2} p^{4}}{r_{5} t_{1}^{2}}\right\}$$

Overall unitarity conditions:

$$p \in \mathbb{R} \;\&\&\; r_{5} > 0 \;\&\&\; \left(t_{1} < 0 \;\|\; t_{1} > 0\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$t_{1} \neq 0 \;\&\&\; r_{5} > 0$$

Okay, that concludes the analysis of this theory.

Case 19

Now for a new theory. Here is the full nonlinear Lagrangian for Case 19 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\left(2 r_{3}+r_{5}\right) \mathcal{R}^{ij\hbar}{}_{i}\, \mathcal{R}_{j}{}^{l}{}_{\hbar l}-r_{3}\, \mathcal{R}^{ij\hbar l}\, \mathcal{R}_{\hbar l i j}+$$
$$\left(2 r_{3}-r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{\hbar}\, \mathcal{R}_{\hbar}{}^{l}{}_{jl}+\frac{1}{4} t_{1}\, \mathcal{T}_{ij\hbar}\, \mathcal{T}^{ij\hbar}+\frac{1}{2} t_{1}\, \mathcal{T}^{ij\hbar}\, \mathcal{T}_{ji\hbar}+\frac{1}{3} t_{1}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{\hbar}{}_{j\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{1}\, \mathcal{A}_{\alpha i \alpha'}\, \mathcal{A}^{\alpha\alpha' i}+\frac{1}{3} t_{1}\, \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \mathcal{A}_{\alpha'}{}^{i}{}_{i}-\frac{2}{3} t_{1}\, \mathcal{A}_{\alpha'}{}^{i}{}_{i}\, \partial_{\alpha} f^{\alpha\alpha'}+\frac{2}{3} t_{1}\, \mathcal{A}_{\alpha'}{}^{i}{}_{i}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha}-$$
$$\frac{1}{3} t_{1}\, \partial_{\alpha'} f^{i}{}_{i}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha}-\frac{1}{3} t_{1}\, \partial_{\alpha} f^{\alpha\alpha'}\, \partial_{i} f^{i}{}_{\alpha'}+\frac{2}{3} t_{1}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha}\, \partial_{i} f^{i}{}_{\alpha'}+\left(-2 r_{3}+r_{5}\right) \partial_{\alpha'} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha}+$$
$$\left(-2 r_{3}-r_{5}\right) \partial_{i} \mathcal{A}_{\alpha'}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha}+2 t_{1}\, \mathcal{A}_{\alpha' i \alpha}\, \partial^{i} f^{\alpha\alpha'}-t_{1}\, \partial_{\alpha} f_{\alpha' i}\, \partial^{i} f^{\alpha\alpha'}+\frac{1}{2} t_{1}\, \partial_{\alpha} f_{i\alpha'}\, \partial^{i} f^{\alpha\alpha'}-\frac{1}{2} t_{1}\, \partial_{\alpha'} f_{\alpha i}\, \partial^{i} f^{\alpha\alpha'}+$$
$$\frac{1}{2} t_{1}\, \partial_{i} f_{\alpha\alpha'}\, \partial^{i} f^{\alpha\alpha'}+\frac{1}{2} t_{1}\, \partial_{i} f_{\alpha'\alpha}\, \partial^{i} f^{\alpha\alpha'}+\left(-2 r_{3}-r_{5}\right) \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i}\, \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i}+2\left(2 r_{3}+r_{5}\right) \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i}+$$
$$\left(-2 r_{3}+r_{5}\right) \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'}+\left(4 r_{3}-2 r_{5}\right) \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'}-4 r_{3}\, \partial_{\alpha'} \mathcal{A}_{ij\alpha}\, \partial^{j} \mathcal{A}^{\alpha\alpha' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a–matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 6k^2 r_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_{1} \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & \frac{1}{2}\left(2k^2\left(2 r_{3} + r_{5}\right) - t_{1}\right) & -\frac{t_{1}}{\sqrt{2}} \\ 0 & -\frac{t_{1}}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2 t_{1}}{3} & -\frac{1}{3} i k t_{1} & 0 & -\frac{1}{3} i \sqrt{2}\, k t_{1} \\ \frac{i k t_{1}}{3} & k^2\left(2 r_{3} + r_{5}\right) + \frac{t_{1}}{6} & 0 & \frac{t_{1}}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{1} & \frac{t_{1}}{3\sqrt{2}} & 0 & \frac{t_{1}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \begin{pmatrix} \frac{t_{1}}{2} \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\cdot}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\cdot}\tau^{\flat\parallel} == 0,\ -i\, {}^{1^+}_{\cdot}\tau^{\flat\parallel\,ab} == k\, {}^{1^+}_{\cdot}\sigma^{\flat\perp\,ab},\ i\, {}^{1^-}_{\cdot}\tau^{\flat\parallel\,a} == 2k\, {}^{1^-}_{\cdot}\sigma^{\flat\perp\,a},\ {}^{1^-}_{\cdot}\tau^{\flat\perp\,a} == 0,\ -i\, {}^{2^+}_{\cdot}\tau^{\flat\parallel\,ab} == 2k\, {}^{2^+}_{\cdot}\sigma^{\flat\parallel\,ab} \right\}$$

The Drazin (Moore–Penrose) inverses of these a–matrices, which are functionally
analogous to the inverse b–matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{6k^2 r_{3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{-2k^4\left(2 r_{3}+r_{5}\right)+k^2 t_{1}}{\left(1+k^2\right)^2 t_{1}^2} & -\frac{i\sqrt{2}\, k}{t_{1}+k^2 t_{1}} & \frac{-2 i k^3\left(2 r_{3}+r_{5}\right)+i k t_{1}}{\left(1+k^2\right)^2 t_{1}^2} \\ \frac{i\sqrt{2}\, k}{t_{1}+k^2 t_{1}} & 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} \\ \frac{i\left(2k^3\left(2 r_{3}+r_{5}\right)-k t_{1}\right)}{\left(1+k^2\right)^2 t_{1}^2} & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2k^2\left(2 r_{3}+r_{5}\right)+t_{1}}{\left(1+k^2\right)^2 t_{1}^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6k^2\left(2 r_{3}+r_{5}\right)+t_{1}}{\left(1+2k^2\right)^2\left(2 r_{3}+r_{5}\right) t_{1}} & \frac{i}{k\left(1+2k^2\right)\left(2 r_{3}+r_{5}\right)} & 0 & -\frac{i\left(6k^2\left(2 r_{3}+r_{5}\right)+t_{1}\right)}{\sqrt{2}\, k\left(1+2k^2\right)^2\left(2 r_{3}+r_{5}\right) t_{1}} \\ -\frac{i}{k\left(1+2k^2\right)\left(2 r_{3}+r_{5}\right)} & \frac{1}{k^2\left(2 r_{3}+r_{5}\right)} & 0 & -\frac{1}{\sqrt{2}\left(k^2+2k^4\right)\left(2 r_{3}+r_{5}\right)} \\ 0 & 0 & 0 & 0 \\ \frac{i\left(6k^2\left(2 r_{3}+r_{5}\right)+t_{1}\right)}{\sqrt{2}\, k\left(1+2k^2\right)^2\left(2 r_{3}+r_{5}\right) t_{1}} & -\frac{1}{\sqrt{2}\left(k^2+2k^4\right)\left(2 r_{3}+r_{5}\right)} & 0 & \frac{6k^2\left(2 r_{3}+r_{5}\right)+t_{1}}{2\left(k+2k^3\right)^2\left(2 r_{3}+r_{5}\right) t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_{1}} & \frac{2 i\sqrt{2}\, k}{\left(1+2k^2\right)^2 t_{1}} \\ -\frac{2 i\sqrt{2}\, k}{\left(1+2k^2\right)^2 t_{1}} & \frac{2}{\left(1+2k^2\right)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{t_{1}} \end{pmatrix}\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{7 t_{1}^{2}+4 r_{3} t_{1} p^{2}+2 r_{5} t_{1} p^{2}+16 r_{3}^{2} p^{4}+16 r_{3} r_{5} p^{4}+4 r_{5}^{2} p^{4}}{2\left(2 r_{3}+r_{5}\right) t_{1}^{2}},\right.$$

$$\left.-\frac{7 t_{1}^{2}+4 r_{3} t_{1} p^{2}+2 r_{5} t_{1} p^{2}+16 r_{3}^{2} p^{4}+16 r_{3} r_{5} p^{4}+4 r_{5}^{2} p^{4}}{2\left(2 r_{3}+r_{5}\right) t_{1}^{2}}\right\}$$

Overall unitarity conditions:

$$\left(p \mid r_{3}\right) \in \mathbb{R} \;\&\&\; r_{5}<-2 r_{3} \;\&\&\; \left(t_{1}<0 \,\|\, t_{1}>0\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{3} \in \mathbb{R} \;\&\&\; t_{1} \neq 0 \;\&\&\; r_{5}<-2 r_{3}$$

Okay, that concludes the analysis of this theory.

Case 20

Now for a new theory. Here is the full nonlinear Lagrangian for Case 20 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl}-\frac{2}{3} r_{2} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl}+\frac{1}{6} r_{2} \mathcal{R}^{ijkl} \mathcal{R}_{klij}+$$
$$\frac{1}{12}\left(4 t_{1}+t_{2}\right) \mathcal{T}_{ijk} \mathcal{T}^{ijk}+\frac{1}{6}\left(2 t_{1}-t_{2}\right) \mathcal{T}^{ijk} \mathcal{T}_{jik}+\frac{1}{3}\left(t_{1}-2 t_{3}\right) \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\left(t_{1}+t_{2}\right) \mathcal{A}_{\alpha\alpha' i} \mathcal{A}^{\alpha\alpha' i}+\frac{1}{3}\left(t_{1}-2 t_{2}\right) \mathcal{A}_{\alpha i\alpha'} \mathcal{A}^{\alpha\alpha' i}+\frac{1}{3}\left(t_{1}-2 t_{3}\right) \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \mathcal{A}_{\alpha'}{}^{i}{}_{i}-\frac{2}{3}\left(t_{1}-2 t_{3}\right) \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial_{\alpha} f^{\alpha\alpha'}+$$
$$\frac{2}{3}\left(t_{1}-2 t_{3}\right) \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha}+\frac{1}{3}\left(-t_{1}+2 t_{3}\right) \partial_{\alpha'} f^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha}+\frac{1}{3}\left(-t_{1}+2 t_{3}\right) \partial_{\alpha} f^{\alpha\alpha'} \partial_{i} f^{i}{}_{\alpha'}+$$
$$\frac{2}{3}\left(t_{1}-2 t_{3}\right) \partial^{\alpha'} f^{\alpha}{}_{\alpha} \partial_{i} f^{i}{}_{\alpha'}-\frac{2}{3}\left(t_{1}+t_{2}\right) \mathcal{A}_{\alpha\alpha' i} \partial^{i} f^{\alpha\alpha'}+\frac{2}{3}\left(t_{1}+t_{2}\right) \mathcal{A}_{\alpha i\alpha'} \partial^{i} f^{\alpha\alpha'}+\frac{2}{3}\left(2 t_{1}-t_{2}\right) \mathcal{A}_{\alpha' i\alpha} \partial^{i} f^{\alpha\alpha'}+$$
$$\frac{1}{3}\left(-2 t_{1}+t_{2}\right) \partial_{\alpha} f_{\alpha' i} \partial^{i} f^{\alpha\alpha'}+\frac{1}{6}\left(2 t_{1}-t_{2}\right) \partial_{\alpha} f_{i\alpha'} \partial^{i} f^{\alpha\alpha'}+\frac{1}{6}\left(-4 t_{1}-t_{2}\right) \partial_{\alpha'} f_{\alpha i} \partial^{i} f^{\alpha\alpha'}+$$
$$\frac{1}{6}\left(4 t_{1}+t_{2}\right) \partial_{i} f_{\alpha\alpha'} \partial^{i} f^{\alpha\alpha'}+\frac{1}{6}\left(2 t_{1}-t_{2}\right) \partial_{i} f_{\alpha'\alpha} \partial^{i} f^{\alpha\alpha'}+\frac{4}{3} r_{2} \partial_{\alpha'} \mathcal{A}_{\alpha ij} \partial^{j} \mathcal{A}^{\alpha\alpha' i}-\frac{2}{3} r_{2} \partial_{\alpha'} \mathcal{A}_{\alpha ji} \partial^{j} \mathcal{A}^{\alpha\alpha' i}+$$
$$\frac{2}{3} r_{2} \partial_{\alpha'} \mathcal{A}_{ij\alpha} \partial^{j} \mathcal{A}^{\alpha\alpha' i}-\frac{1}{3} r_{2} \partial_{i} \mathcal{A}_{\alpha\alpha' j} \partial^{j} \mathcal{A}^{\alpha\alpha' i}+\frac{1}{3} r_{2} \partial_{j} \mathcal{A}_{\alpha\alpha' i} \partial^{j} \mathcal{A}^{\alpha\alpha' i}-\frac{2}{3} r_{2} \partial_{j} \mathcal{A}_{\alpha i\alpha'} \partial^{j} \mathcal{A}^{\alpha\alpha' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2k^2 t_{3} & i\sqrt{2}\,k\,t_{3} & 0 \\ -i\sqrt{2}\,k\,t_{3} & t_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{2} + t_{2}\right), \begin{pmatrix} \frac{1}{3}k^2\left(t_{1}+t_{2}\right) & -\frac{i k\left(t_{1}-2t_{2}\right)}{3\sqrt{2}} & \frac{1}{3}i k\left(t_{1}+t_{2}\right) \\ \frac{i k\left(t_{1}-2t_{2}\right)}{3\sqrt{2}} & \frac{1}{6}\left(t_{1}+4t_{2}\right) & \frac{-t_{1}+2t_{2}}{3\sqrt{2}} \\ -\frac{1}{3}i k\left(t_{1}+t_{2}\right) & \frac{-t_{1}+2t_{2}}{3\sqrt{2}} & \frac{t_{1}+t_{2}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2}{3}k^2\left(t_{1}+t_{3}\right) & -\frac{1}{3}i k\left(t_{1}-2t_{3}\right) & 0 & -\frac{1}{3}i\sqrt{2}\,k\left(t_{1}+t_{3}\right) \\ \frac{1}{3}i k\left(t_{1}-2t_{3}\right) & \frac{1}{6}\left(t_{1}+4t_{3}\right) & 0 & \frac{t_{1}-2t_{3}}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3}i\sqrt{2}\,k\left(t_{1}+t_{3}\right) & \frac{t_{1}-2t_{3}}{3\sqrt{2}} & 0 & \frac{t_{1}+t_{3}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \left(\frac{t_{1}}{2}\right)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}\tau^{\flat\parallel} == 2k\,{}^{0^+}\sigma^{\flat\parallel},\ -i\,{}^{1^+}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}\sigma^{\flat\perp}{}^{ab},\ i\,{}^{1^-}\tau^{\flat\parallel}{}^{a} == 2k\,{}^{1^-}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}\tau^{\flat\perp}{}^{a} == 0,\ -i\,{}^{2^+}\tau^{\flat\parallel}{}^{ab} == 2k\,{}^{2^+}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{\left(1+2k^2\right)^2 t_{3}} & \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{3}} & 0 \\ -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{3}} & \frac{1}{\left(1+2k^2\right)^2 t_{3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{2}+t_{2}}\right), \begin{pmatrix} \frac{k^2\left(t_{1}+4t_{2}\right)}{3\left(1+k^2\right)^2 t_{1} t_{2}} & \frac{i\sqrt{2}\,k\left(t_{1}-2t_{2}\right)}{3\left(1+k^2\right) t_{1} t_{2}} & \frac{i k\left(t_{1}+4t_{2}\right)}{3\left(1+k^2\right)^2 t_{1} t_{2}} \\ -\frac{i\sqrt{2}\,k\left(t_{1}-2t_{2}\right)}{3\left(1+k^2\right) t_{1} t_{2}} & \frac{2\left(t_{1}+t_{2}\right)}{3 t_{1} t_{2}} & \frac{\sqrt{2}\left(t_{1}-2t_{2}\right)}{3\left(1+k^2\right) t_{1} t_{2}} \\ -\frac{i k\left(t_{1}+4t_{2}\right)}{3\left(1+k^2\right)^2 t_{1} t_{2}} & \frac{\sqrt{2}\left(t_{1}-2t_{2}\right)}{3\left(1+k^2\right) t_{1} t_{2}} & \frac{t_{1}+4t_{2}}{3\left(1+k^2\right)^2 t_{1} t_{2}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2\left(t_{1}+4t_{3}\right)}{3\left(1+2k^2\right)^2 t_{1} t_{3}} & \frac{2 i k t_{1}-4 i k t_{3}}{3 t_{1} t_{3}+6k^2 t_{1} t_{3}} & 0 & -\frac{i\sqrt{2}\,k\left(t_{1}+4t_{3}\right)}{3\left(1+2k^2\right)^2 t_{1} t_{3}} \\ -\frac{2 i k t_{1}-4 i k t_{3}}{3 t_{1} t_{3}+6k^2 t_{1} t_{3}} & \frac{2\left(t_{1}+t_{3}\right)}{3 t_{1} t_{3}} & 0 & -\frac{\sqrt{2}\left(t_{1}-2t_{3}\right)}{3\left(1+2k^2\right) t_{1} t_{3}} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k\left(t_{1}+4t_{3}\right)}{3\left(1+2k^2\right)^2 t_{1} t_{3}} & -\frac{\sqrt{2}\left(t_{1}-2t_{3}\right)}{3\left(1+2k^2\right) t_{1} t_{3}} & 0 & \frac{t_{1}+4t_{3}}{3\left(1+2k^2\right)^2 t_{1} t_{3}} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_{1}} & \frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} \\ -\frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} & \frac{2}{\left(1+2k^2\right)^2 t_{1}} \end{pmatrix}, \left(\frac{2}{t_{1}}\right)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall unitarity conditions:

$\underset{2}{\underset{\cdot}{r}} < 0 \,\&\&\, \underset{2}{\underset{\cdot}{t}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$\underset{2}{\underset{\cdot}{r}} < 0 \,\&\&\, \underset{2}{\underset{\cdot}{t}} > 0$

Okay, that concludes the analysis of this theory.

Case 21

Now for a new theory. Here is the full nonlinear Lagrangian for Case 21 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\underset{2}{\underset{\cdot}{r}}\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl} - \frac{2}{3}\underset{2}{\underset{\cdot}{r}}\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl} + \frac{1}{6}\underset{2}{\underset{\cdot}{r}}\,\mathcal{R}^{ijkl}\,\mathcal{R}_{klij} + \frac{1}{4}\underset{1}{\underset{\cdot}{t}}\,\mathcal{T}_{ijk}\,\mathcal{T}^{ijk} + \frac{1}{2}\underset{1}{\underset{\cdot}{t}}\,\mathcal{T}^{ijk}\,\mathcal{T}_{jik} + \frac{1}{3}\left(\underset{1}{\underset{\cdot}{t}} - 2\underset{3}{\underset{\cdot}{t}}\right)\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\underset{1}{\underset{\cdot}{t}}\,\mathcal{A}_{a i a'}\,\mathcal{A}^{a a' i} + \frac{1}{3}\left(\underset{1}{\underset{\cdot}{t}} - 2\underset{3}{\underset{\cdot}{t}}\right)\mathcal{A}^{a a'}{}_{a}\,\mathcal{A}_{a'}{}^{i}{}_{i} - \frac{2}{3}\left(\underset{1}{\underset{\cdot}{t}} - 2\underset{3}{\underset{\cdot}{t}}\right)\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial_{a}f^{a a'} +$$
$$\frac{2}{3}\left(\underset{1}{\underset{\cdot}{t}} - 2\underset{3}{\underset{\cdot}{t}}\right)\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a} + \frac{1}{3}\left(-\underset{1}{\underset{\cdot}{t}} + 2\underset{3}{\underset{\cdot}{t}}\right)\partial_{a'}f^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a} + \frac{1}{3}\left(-\underset{1}{\underset{\cdot}{t}} + 2\underset{3}{\underset{\cdot}{t}}\right)\partial_{a}f^{a a'}\,\partial_{i}f^{i}{}_{a'} +$$
$$\frac{2}{3}\left(\underset{1}{\underset{\cdot}{t}} - 2\underset{3}{\underset{\cdot}{t}}\right)\partial^{a'}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{a'} + 2\underset{1}{\underset{\cdot}{t}}\,\mathcal{A}_{a' i a}\,\partial^{i}f^{a a'} - \underset{1}{\underset{\cdot}{t}}\,\partial_{a}f_{a' i}\,\partial^{i}f^{a a'} + \frac{1}{2}\underset{1}{\underset{\cdot}{t}}\,\partial_{a}f_{i a'}\,\partial^{i}f^{a a'} - \frac{1}{2}\underset{1}{\underset{\cdot}{t}}\,\partial_{a'}f_{a i}\,\partial^{i}f^{a a'} +$$
$$\frac{1}{2}\underset{1}{\underset{\cdot}{t}}\,\partial_{i}f_{a a'}\,\partial^{i}f^{a a'} + \frac{1}{2}\underset{1}{\underset{\cdot}{t}}\,\partial_{i}f_{a' a}\,\partial^{i}f^{a a'} + \frac{4}{3}\underset{2}{\underset{\cdot}{r}}\,\partial_{a'}\mathcal{A}_{a i j}\,\partial^{j}\mathcal{A}^{a a' i} - \frac{2}{3}\underset{2}{\underset{\cdot}{r}}\,\partial_{a'}\mathcal{A}_{a j i}\,\partial^{j}\mathcal{A}^{a a' i} +$$
$$\frac{2}{3}\underset{2}{\underset{\cdot}{r}}\,\partial_{a'}\mathcal{A}_{i j a}\,\partial^{j}\mathcal{A}^{a a' i} - \frac{1}{3}\underset{2}{\underset{\cdot}{r}}\,\partial_{i}\mathcal{A}_{a a' j}\,\partial^{j}\mathcal{A}^{a a' i} + \frac{1}{3}\underset{2}{\underset{\cdot}{r}}\,\partial_{j}\mathcal{A}_{a a' i}\,\partial^{j}\mathcal{A}^{a a' i} - \frac{2}{3}\underset{2}{\underset{\cdot}{r}}\,\partial_{j}\mathcal{A}_{a i a'}\,\partial^{j}\mathcal{A}^{a a' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2k^2 t_3 & i\sqrt{2}\,k\,t_3 & 0 \\ -i\sqrt{2}\,k\,t_3 & t_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_2 - t_1\right), \begin{pmatrix} 0 & -\frac{i\,k\,t_1}{\sqrt{2}} & 0 \\ \frac{i\,k\,t_1}{\sqrt{2}} & -\frac{t_1}{2} & -\frac{t_1}{\sqrt{2}} \\ 0 & -\frac{t_1}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2}{3}k^2\left(t_1+t_3\right) & -\frac{1}{3}\,i\,k\left(t_1-2t_3\right) & 0 & -\frac{1}{3}\,i\sqrt{2}\,k\left(t_1+t_3\right) \\ \frac{1}{3}\,i\,k\left(t_1-2t_3\right) & \frac{1}{6}\left(t_1+4t_3\right) & 0 & \frac{t_1-2t_3}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3}\,i\sqrt{2}\,k\left(t_1+t_3\right) & \frac{t_1-2t_3}{3\sqrt{2}} & 0 & \frac{t_1+t_3}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_1 & \frac{i\,k\,t_1}{\sqrt{2}} \\ -\frac{i\,k\,t_1}{\sqrt{2}} & \frac{t_1}{2} \end{pmatrix}, \left(\frac{t_1}{2}\right)\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\cdot}\tau^{\flat\perp} == 0,\ -i\,\overset{0^+}{\cdot}\tau^{\flat\|} == 2k\,\overset{0^+}{\cdot}\sigma^{\flat\|},\ -i\,\overset{1^+}{\cdot}\tau^{\flat\|ab} == k\,\overset{1^+}{\cdot}\sigma^{\flat\perp ab},\ i\,\overset{1^-}{\cdot}\tau^{\flat\|a} == 2k\,\overset{1^-}{\cdot}\sigma^{\flat\perp a},\ \overset{1^-}{\cdot}\tau^{\flat\perp a} == 0,\ -i\,\overset{2^+}{\cdot}\tau^{\flat\|ab} == 2k\,\overset{2^+}{\cdot}\sigma^{\flat\|ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{\left(1+2k^2\right)^2 t_3} & \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_3} & 0 \\ -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_3} & \frac{1}{\left(1+2k^2\right)^2 t_3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_2 - t_1}\right), \begin{pmatrix} \frac{k^2}{\left(1+k^2\right)^2 t_1} & -\frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & \frac{i\,k}{\left(1+k^2\right)^2 t_1} \\ \frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & 0 & -\frac{\sqrt{2}}{t_1+k^2 t_1} \\ -\frac{i\,k}{\left(1+k^2\right)^2 t_1} & -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{1}{\left(1+k^2\right)^2 t_1} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2\left(t_1+4t_3\right)}{3\left(1+2k^2\right)^2 t_1 t_3} & \frac{2ikt_1-4ikt_3}{3t_1t_3+6k^2t_1t_3} & 0 & -\frac{i\sqrt{2}\,k\left(t_1+4t_3\right)}{3\left(1+2k^2\right)^2 t_1 t_3} \\ -\frac{2ikt_1-4ikt_3}{3t_1t_3+6k^2t_1t_3} & \frac{2\left(t_1+t_3\right)}{3t_1t_3} & 0 & -\frac{\sqrt{2}\left(t_1-2t_3\right)}{3\left(1+2k^2\right)t_1t_3} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k\left(t_1+4t_3\right)}{3\left(1+2k^2\right)^2 t_1 t_3} & -\frac{\sqrt{2}\left(t_1-2t_3\right)}{3\left(1+2k^2\right)t_1t_3} & 0 & \frac{t_1+4t_3}{3\left(1+2k^2\right)^2 t_1 t_3} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_1} & \frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} \\ -\frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} & \frac{2}{\left(1+2k^2\right)^2 t_1} \end{pmatrix}, \left(\frac{2}{t_1}\right)\right\}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall unitarity conditions:

$r_{\underset{2}{\cdot}} < 0 \,\&\&\, t_{\underset{1}{\cdot}} < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\underset{2}{\cdot}} < 0 \,\&\&\, t_{\underset{1}{\cdot}} < 0$

Okay, that concludes the analysis of this theory.

Case 22

Now for a new theory. Here is the full nonlinear Lagrangian for Case 22 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\underset{2}{\cdot}} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{\underset{2}{\cdot}} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \frac{1}{6} r_{\underset{2}{\cdot}} \mathcal{R}^{ijkl} \mathcal{R}_{klij} + \frac{1}{12}\left(4 t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{6}\left(2 t_{\underset{1}{\cdot}} - t_{\underset{2}{\cdot}}\right) \mathcal{T}^{ijk} \mathcal{T}_{jik} + t_{\underset{1}{\cdot}} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\left(t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \mathcal{A}_{aa'i} \mathcal{A}^{aa'i} + \frac{1}{3}\left(t_{\underset{1}{\cdot}} - 2 t_{\underset{2}{\cdot}}\right) \mathcal{A}_{aia'} \mathcal{A}^{aa'i} + t_{\underset{1}{\cdot}} \mathcal{A}^{aa'}{}_{a} \mathcal{A}_{a'}{}^{i}{}_{i} - 2 t_{\underset{1}{\cdot}} \mathcal{A}_{a'}{}^{i}{}_{i} \partial_{a} f^{aa'} + 2 t_{\underset{1}{\cdot}} \mathcal{A}_{a'}{}^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - t_{\underset{1}{\cdot}} \partial_{a'} f^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - t_{\underset{1}{\cdot}} \partial_{a} f^{aa'} \partial_{i} f^{i}{}_{a'} + 2 t_{\underset{1}{\cdot}} \partial^{a'} f^{a}{}_{a} \partial_{i} f^{i}{}_{a'} - \frac{2}{3}\left(t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \mathcal{A}_{aa'i} \partial^{i} f^{aa'} + \frac{2}{3}\left(t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \mathcal{A}_{aia'} \partial^{i} f^{aa'} + \frac{2}{3}\left(2 t_{\underset{1}{\cdot}} - t_{\underset{2}{\cdot}}\right) \mathcal{A}_{a'ia} \partial^{i} f^{aa'} + \frac{1}{3}\left(-2 t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \partial_{a} f_{a'i} \partial^{i} f^{aa'} + \frac{1}{6}\left(2 t_{\underset{1}{\cdot}} - t_{\underset{2}{\cdot}}\right) \partial_{a} f_{ia'} \partial^{i} f^{aa'} + \frac{1}{6}\left(-4 t_{\underset{1}{\cdot}} - t_{\underset{2}{\cdot}}\right) \partial_{a'} f_{ai} \partial^{i} f^{aa'} + \frac{1}{6}\left(4 t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \partial_{i} f_{aa'} \partial^{i} f^{aa'} + \frac{1}{6}\left(2 t_{\underset{1}{\cdot}} - t_{\underset{2}{\cdot}}\right) \partial_{i} f_{a'a} \partial^{i} f^{aa'} + \frac{4}{3} r_{\underset{2}{\cdot}} \partial_{a'} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{\underset{2}{\cdot}} \partial_{a'} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{\underset{2}{\cdot}} \partial_{a'} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{aa'i} - \frac{1}{3} r_{\underset{2}{\cdot}} \partial_{i} \mathcal{A}_{aa'j} \partial^{j} \mathcal{A}^{aa'i} + \frac{1}{3} r_{\underset{2}{\cdot}} \partial_{j} \mathcal{A}_{aa'i} \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{\underset{2}{\cdot}} \partial_{j} \mathcal{A}_{aia'} \partial^{j} \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_{\dot 1} & -i\sqrt{2}\,k\,t_{\dot 1} & 0 \\ i\sqrt{2}\,k\,t_{\dot 1} & -t_{\dot 1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{\dot 2} + t_{\dot 2}\right),\right.$$

$$\begin{pmatrix} \frac{1}{3}k^2\left(t_{\dot 1}+t_{\dot 2}\right) & -\frac{i k\left(t_{\dot 1}-2t_{\dot 2}\right)}{3\sqrt{2}} & \frac{1}{3}i k\left(t_{\dot 1}+t_{\dot 2}\right) \\ \frac{i k\left(t_{\dot 1}-2t_{\dot 2}\right)}{3\sqrt{2}} & \frac{1}{6}\left(t_{\dot 1}+4t_{\dot 2}\right) & \frac{-t_{\dot 1}+2t_{\dot 2}}{3\sqrt{2}} \\ -\frac{1}{3}i k\left(t_{\dot 1}+t_{\dot 2}\right) & \frac{-t_{\dot 1}+2t_{\dot 2}}{3\sqrt{2}} & \frac{t_{\dot 1}+t_{\dot 2}}{3} \end{pmatrix}, \begin{pmatrix} 0 & -i k t_{\dot 1} & 0 & 0 \\ i k t_{\dot 1} & -\frac{t_{\dot 1}}{2} & 0 & \frac{t_{\dot 1}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{\dot 1}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_{\dot 1} & \frac{i k t_{\dot 1}}{\sqrt{2}} \\ -\frac{i k t_{\dot 1}}{\sqrt{2}} & \frac{t_{\dot 1}}{2} \end{pmatrix}, \left(\frac{t_{\dot 1}}{2}\right)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\cdot}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}_{\cdot}\tau^{\flat\parallel} == 2k\,{}^{0^+}_{\cdot}\sigma^{\flat\parallel},\ -i\,{}^{1^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}_{\cdot}\sigma^{\flat\perp}{}^{ab},\ i\,{}^{1^-}_{\cdot}\tau^{\flat\parallel}{}^{a} == 2k\,{}^{1^-}_{\cdot}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\cdot}\tau^{\flat\perp}{}^{a} == 0,\ -i\,{}^{2^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == 2k\,{}^{2^+}_{\cdot}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{\left(1+2k^2\right)^2 t_{\dot 1}} & -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{\dot 1}} & 0 \\ \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{\dot 1}} & -\frac{1}{\left(1+2k^2\right)^2 t_{\dot 1}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{\dot 2}+t_{\dot 2}}\right), \begin{pmatrix} \frac{k^2\left(t_{\dot 1}+4t_{\dot 2}\right)}{3\left(1+k^2\right)^2 t_{\dot 1}t_{\dot 2}} & \frac{i\sqrt{2}\,k\left(t_{\dot 1}-2t_{\dot 2}\right)}{3\left(1+k^2\right)t_{\dot 1}t_{\dot 2}} & \frac{i k\left(t_{\dot 1}+4t_{\dot 2}\right)}{3\left(1+k^2\right)^2 t_{\dot 1}t_{\dot 2}} \\ -\frac{i\sqrt{2}\,k\left(t_{\dot 1}-2t_{\dot 2}\right)}{3\left(1+k^2\right)t_{\dot 1}t_{\dot 2}} & \frac{2\left(t_{\dot 1}+t_{\dot 2}\right)}{3t_{\dot 1}t_{\dot 2}} & \frac{\sqrt{2}\left(t_{\dot 1}-2t_{\dot 2}\right)}{3\left(1+k^2\right)t_{\dot 1}t_{\dot 2}} \\ -\frac{i k\left(t_{\dot 1}+4t_{\dot 2}\right)}{3\left(1+k^2\right)^2 t_{\dot 1}t_{\dot 2}} & \frac{\sqrt{2}\left(t_{\dot 1}-2t_{\dot 2}\right)}{3\left(1+k^2\right)t_{\dot 1}t_{\dot 2}} & \frac{t_{\dot 1}+4t_{\dot 2}}{3\left(1+k^2\right)^2 t_{\dot 1}t_{\dot 2}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2}{\left(1+2k^2\right)^2 t_{\dot 1}} & -\frac{2ik}{t_{\dot 1}+2k^2 t_{\dot 1}} & 0 & -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{\dot 1}} \\ \frac{2ik}{t_{\dot 1}+2k^2 t_{\dot 1}} & 0 & 0 & \frac{\sqrt{2}}{t_{\dot 1}+2k^2 t_{\dot 1}} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{\dot 1}} & \frac{\sqrt{2}}{t_{\dot 1}+2k^2 t_{\dot 1}} & 0 & \frac{1}{\left(1+2k^2\right)^2 t_{\dot 1}} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_{\dot 1}} & \frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{\dot 1}} \\ -\frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{\dot 1}} & \frac{2}{\left(1+2k^2\right)^2 t_{\dot 1}} \end{pmatrix}, \left(\frac{2}{t_{\dot 1}}\right)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\dot 2}}{r_{\dot 2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\dot 2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{\dot 2} < 0\ \&\&\ t_{\dot 2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\dot 2} < 0 \,\&\&\, t_{\dot 2} > 0$$

Okay, that concludes the analysis of this theory.

Case 23

Now for a new theory. Here is the full nonlinear Lagrangian for Case 23 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\dot 2} \mathcal{R}_{ijhl} \mathcal{R}^{ijhl} - \frac{2}{3} r_{\dot 2} \mathcal{R}_{ihjl} \mathcal{R}^{ijhl} + \frac{1}{6} r_{\dot 2} \mathcal{R}^{ijhl} \mathcal{R}_{hlij} + \frac{1}{4} t_{\dot 1} \mathcal{T}_{ijh} \mathcal{T}^{ijh} + \frac{1}{2} t_{\dot 1} \mathcal{T}^{ijh} \mathcal{T}_{jih} + t_{\dot 1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{h}{}_{jh}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{\dot 1} \mathcal{A}_{aia'} \mathcal{A}^{aa'i} + t_{\dot 1} \mathcal{A}^{aa'}{}_{a} \mathcal{A}_{a'}{}^{i}{}_{i} - 2 t_{\dot 1} \mathcal{A}_{a'}{}^{i}{}_{i} \partial_a f^{aa'} + 2 t_{\dot 1} \mathcal{A}_{a'}{}^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - t_{\dot 1} \partial_{a'} f^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - t_{\dot 1} \partial_a f^{aa'} \partial_i f^{i}{}_{a'} + 2 t_{\dot 1} \partial^{a'} f^{a}{}_{a} \partial_i f^{i}{}_{a'} + 2 t_{\dot 1} \mathcal{A}_{a'ia} \partial^i f^{aa'} - t_{\dot 1} \partial_a f_{a'i} \partial^i f^{aa'} + \frac{1}{2} t_{\dot 1} \partial_a f_{ia'} \partial^i f^{aa'} - \frac{1}{2} t_{\dot 1} \partial_{a'} f_{ai} \partial^i f^{aa'} + \frac{1}{2} t_{\dot 1} \partial_i f_{aa'} \partial^i f^{aa'} + \frac{1}{2} t_{\dot 1} \partial_i f_{a'a} \partial^i f^{aa'} + \frac{4}{3} r_{\dot 2} \partial_{a'} \mathcal{A}_{aij} \partial^j \mathcal{A}^{aa'i} - \frac{2}{3} r_{\dot 2} \partial_{a'} \mathcal{A}_{aji} \partial^j \mathcal{A}^{aa'i} + \frac{2}{3} r_{\dot 2} \partial_{a'} \mathcal{A}_{ija} \partial^j \mathcal{A}^{aa'i} - \frac{1}{3} r_{\dot 2} \partial_i \mathcal{A}_{aa'j} \partial^j \mathcal{A}^{aa'i} + \frac{1}{3} r_{\dot 2} \partial_j \mathcal{A}_{aa'i} \partial^j \mathcal{A}^{aa'i} - \frac{2}{3} r_{\dot 2} \partial_j \mathcal{A}_{aia'} \partial^j \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{ \begin{pmatrix} -2k^2 t_{\dot 1} & -i\sqrt{2}\, k\, t_{\dot 1} & 0 \\ i\sqrt{2}\, k\, t_{\dot 1} & -t_{\dot 1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{\dot 2} - t_{\dot 1} \right), \begin{pmatrix} 0 & -\frac{i k t_{\dot 1}}{\sqrt{2}} & 0 \\ \frac{i k t_{\dot 1}}{\sqrt{2}} & -\frac{t_{\dot 1}}{2} & -\frac{t_{\dot 1}}{\sqrt{2}} \\ 0 & -\frac{t_{\dot 1}}{\sqrt{2}} & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i k t_{\dot 1} & 0 & 0 \\ i k t_{\dot 1} & -\frac{t_{\dot 1}}{2} & 0 & \frac{t_{\dot 1}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{\dot 1}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_{\dot 1} & \frac{i k t_{\dot 1}}{\sqrt{2}} \\ -\frac{i k t_{\dot 1}}{\sqrt{2}} & \frac{t_{\dot 1}}{2} \end{pmatrix}, \left(\frac{t_{\dot 1}}{2} \right) \right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ -i\, {}^{0^+}_{\bullet}\tau^{\flat\parallel} == 2k\, {}^{0^+}_{\bullet}\sigma^{\flat\parallel},\ -i\, {}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\, {}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab},\ i\, {}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a} == 2k\, {}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ -i\, {}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 2k\, {}^{2^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab} \right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{(1+2k^2)^2 t_{1}} & -\frac{i\sqrt{2}k}{(1+2k^2)^2 t_{1}} & 0 \\ \frac{i\sqrt{2}k}{(1+2k^2)^2 t_{1}} & -\frac{1}{(1+2k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_{2} - t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{k^2}{(1+k^2)^2 t_{1}} & -\frac{i\sqrt{2}k}{t_{1}+k^2 t_{1}} & \frac{ik}{(1+k^2)^2 t_{1}} \\ \frac{i\sqrt{2}k}{t_{1}+k^2 t_{1}} & 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} \\ -\frac{ik}{(1+k^2)^2 t_{1}} & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{1}{(1+k^2)^2 t_{1}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_{1}} & -\frac{2ik}{t_{1}+2k^2 t_{1}} & 0 & -\frac{i\sqrt{2}k}{(1+2k^2)^2 t_{1}} \\ \frac{2ik}{t_{1}+2k^2 t_{1}} & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}k}{(1+2k^2)^2 t_{1}} & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & 0 & \frac{1}{(1+2k^2)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_{1}} & \frac{2i\sqrt{2}k}{(1+2k^2)^2 t_{1}} \\ -\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_{1}} & \frac{2}{(1+2k^2)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{t_{1}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_{1}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{2} < 0 \,\&\&\, t_{1} < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{2} < 0 \,\&\&\, t_{1} < 0$

Okay, that concludes the analysis of this theory.

Case 24

Now for a new theory. Here is the full nonlinear Lagrangian for Case 24 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ijhl} \mathcal{R}^{ijhl} - \frac{2}{3} r_{2} \mathcal{R}_{ihjl} \mathcal{R}^{ijhl} + r_{5} \mathcal{R}^{ij}{}_{i}{}^{h} \mathcal{R}_{j}{}^{l}{}_{hl} + \frac{1}{6} r_{2} \mathcal{R}^{ijhl} \mathcal{R}_{hlij} - r_{5} \mathcal{R}^{ij}{}_{i}{}^{h} \mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{12} t_{2} \mathcal{T}_{ijh} \mathcal{T}^{ijh} - \frac{1}{6} t_{2} \mathcal{T}^{ijh} \mathcal{T}_{jih} - \frac{2}{3} t_{3} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{h}{}_{jh}$$

To use PSALTer, you have to first linearise
this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{2}\, \mathcal{A}_{aa'i}\, \mathcal{A}^{aa'i} - \frac{2}{3} t_{2}\, \mathcal{A}_{aia'}\, \mathcal{A}^{aa'i} - \frac{2}{3} t_{3}\, \mathcal{A}^{aa'}{}_{a}\, \mathcal{A}_{a'}{}^{i}{}_{i} + \frac{4}{3} t_{3}\, \mathcal{A}_{a'}{}^{i}{}_{i}\, \partial_{a} f^{aa'} - \frac{4}{3} t_{3}\, \mathcal{A}_{a'}{}^{i}{}_{i}\, \partial^{a'} f^{a}{}_{a} +$$
$$\frac{2}{3} t_{3}\, \partial_{a'} f^{i}{}_{i}\, \partial^{a'} f^{a}{}_{a} + \frac{2}{3} t_{3}\, \partial_{a} f^{aa'}\, \partial_{i} f^{i}{}_{a'} - \frac{4}{3} t_{3}\, \partial^{a'} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{a'} + r_{5}\, \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{aa'}{}_{a} - r_{5}\, \partial_{i} \mathcal{A}_{a'}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{aa'}{}_{a} -$$
$$\frac{2}{3} t_{2}\, \mathcal{A}_{aa'i}\, \partial^{i} f^{aa'} + \frac{2}{3} t_{2}\, \mathcal{A}_{aia'}\, \partial^{i} f^{aa'} - \frac{2}{3} t_{2}\, \mathcal{A}_{a'ia}\, \partial^{i} f^{aa'} + \frac{1}{3} t_{2}\, \partial_{a} f_{a'i}\, \partial^{i} f^{aa'} - \frac{1}{6} t_{2}\, \partial_{a} f_{ia'}\, \partial^{i} f^{aa'} -$$
$$\frac{1}{6} t_{2}\, \partial_{a'} f_{ai}\, \partial^{i} f^{aa'} + \frac{1}{6} t_{2}\, \partial_{i} f_{aa'}\, \partial^{i} f^{aa'} - \frac{1}{6} t_{2}\, \partial_{i} f_{a'a}\, \partial^{i} f^{aa'} - r_{5}\, \partial_{a} \mathcal{A}^{aa'i}\, \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} + 2 r_{5}\, \partial^{i} \mathcal{A}^{aa'}{}_{a}\, \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} +$$
$$r_{5}\, \partial_{a} \mathcal{A}^{aa'i}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} - 2 r_{5}\, \partial^{i} \mathcal{A}^{aa'}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} + \frac{4}{3} r_{2}\, \partial_{a'} \mathcal{A}_{aij}\, \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{2}\, \partial_{a'} \mathcal{A}_{aji}\, \partial^{j} \mathcal{A}^{aa'i} +$$
$$\frac{2}{3} r_{2}\, \partial_{a'} \mathcal{A}_{ija}\, \partial^{j} \mathcal{A}^{aa'i} - \frac{1}{3} r_{2}\, \partial_{i} \mathcal{A}_{aa'j}\, \partial^{j} \mathcal{A}^{aa'i} + \frac{1}{3} r_{2}\, \partial_{j} \mathcal{A}_{aa'i}\, \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{2}\, \partial_{j} \mathcal{A}_{aia'}\, \partial^{j} \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2 k^2 t_{3} & i \sqrt{2}\, k t_{3} & 0 \\ -i \sqrt{2}\, k t_{3} & t_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{2} + t_{2}\right), \begin{pmatrix} \frac{k^2 t_{2}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{2} & \frac{i k t_{2}}{3} \\ -\frac{1}{3} i \sqrt{2}\, k t_{2} & \frac{1}{2}\left(2 k^2 r_{5} + \frac{4 t_{2}}{3}\right) & \frac{\sqrt{2}\, t_{2}}{3} \\ -\frac{1}{3} i k t_{2} & \frac{\sqrt{2}\, t_{2}}{3} & \frac{t_{2}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2 k^2 t_{3}}{3} & \frac{2 i k t_{3}}{3} & 0 & -\frac{1}{3} i \sqrt{2}\, k t_{3} \\ -\frac{2}{3} i k t_{3} & k^2 r_{5} + \frac{2 t_{3}}{3} & 0 & -\frac{\sqrt{2}\, t_{3}}{3} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{3} & -\frac{\sqrt{2}\, t_{3}}{3} & 0 & \frac{t_{3}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}\tau^{\flat\perp} == 0,\ -i\, {}^{0^+}\tau^{\flat\parallel} == 2 k\, {}^{0^+}\sigma^{\flat\parallel},\ -i\, {}^{1^+}\tau^{\flat\parallel\, ab} == k\, {}^{1^+}\sigma^{\flat\perp\, ab},\right.$$
$$\left. i\, {}^{1^-}\tau^{\flat\parallel\, a} == 2 k\, {}^{1^-}\sigma^{\flat\perp\, a},\ {}^{1^-}\tau^{\flat\perp\, a} == 0,\ {}^{2^+}\sigma^{\flat\parallel\, ab} == 0,\ {}^{2^+}\tau^{\flat\parallel\, ab} == 0,\ {}^{2^-}\sigma^{\flat\parallel\, abc} == 0 \right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_3} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3} & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3} & \frac{1}{(1+2k^2)^2 t_3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_2 + t_2} \end{pmatrix}, \begin{pmatrix} \frac{3k^2 r_5 + 2t_2}{(1+k^2)^2 r_5 t_2} & -\frac{i\sqrt{2}}{k r_5 + k^3 r_5} & \frac{i(3k^2 r_5 + 2t_2)}{k(1+k^2)^2 r_5 t_2} \\ \frac{i\sqrt{2}}{k r_5 + k^3 r_5} & \frac{1}{k^2 r_5} & -\frac{\sqrt{2}}{k^2 r_5 + k^4 r_5} \\ -\frac{i(3k^2 r_5 + 2t_2)}{k(1+k^2)^2 r_5 t_2} & -\frac{\sqrt{2}}{k^2 r_5 + k^4 r_5} & \frac{3k^2 r_5 + 2t_2}{(k+k^3)^2 r_5 t_2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6k^2 r_5 + 4t_3}{(1+2k^2)^2 r_5 t_3} & -\frac{2i}{k r_5 + 2k^3 r_5} & 0 & -\frac{i\sqrt{2}(3k^2 r_5 + 2t_3)}{k(1+2k^2)^2 r_5 t_3} \\ \frac{2i}{k r_5 + 2k^3 r_5} & \frac{1}{k^2 r_5} & 0 & \frac{\sqrt{2}}{k^2 r_5 + 2k^4 r_5} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}(3k^2 r_5 + 2t_3)}{k(1+2k^2)^2 r_5 t_3} & \frac{\sqrt{2}}{k^2 r_5 + 2k^4 r_5} & 0 & \frac{3k^2 r_5 + 2t_3}{(k+2k^3)^2 r_5 t_3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_2}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$r_2 < 0 \,\&\&\, t_2 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_2 < 0 \,\&\&\, t_2 > 0$

Okay, that concludes the analysis of this theory.

Case 25

Now for a new theory. Here is the full nonlinear Lagrangian for Case 25 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\underset{2}{\bullet}} \mathcal{R}_{ij\hbar l} \mathcal{R}^{ij\hbar l} - \frac{2}{3} r_{\underset{2}{\bullet}} \mathcal{R}_{i\hbar jl} \mathcal{R}^{ij\hbar l} + \frac{1}{6} r_{\underset{2}{\bullet}} \mathcal{R}^{ij\hbar l} \mathcal{R}_{\hbar lij} +$$
$$\frac{1}{12} t_{\underset{2}{\bullet}} \mathcal{T}_{ij\hbar} \mathcal{T}^{ij\hbar} - \frac{1}{6} t_{\underset{2}{\bullet}} \mathcal{T}^{ij\hbar} \mathcal{T}_{ji\hbar} - \frac{2}{3} t_{\underset{3}{\bullet}} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{\hbar}{}_{j\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{\underset{2}{\bullet}} \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} t_{\underset{2}{\bullet}} \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} t_{\underset{3}{\bullet}} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} \mathcal{A}_{\mathfrak{a}'}{}^{i}{}_{i} + \frac{4}{3} t_{\underset{3}{\bullet}} \mathcal{A}_{\mathfrak{a}'}{}^{i}{}_{i} \partial_{\mathfrak{a}} f^{\mathfrak{a}\mathfrak{a}'} -$$
$$\frac{4}{3} t_{\underset{3}{\bullet}} \mathcal{A}_{\mathfrak{a}'}{}^{i}{}_{i} \partial^{\mathfrak{a}'} f^{\mathfrak{a}}{}_{\mathfrak{a}} + \frac{2}{3} t_{\underset{3}{\bullet}} \partial_{\mathfrak{a}'} f^{i}{}_{i} \partial^{\mathfrak{a}'} f^{\mathfrak{a}}{}_{\mathfrak{a}} + \frac{2}{3} t_{\underset{3}{\bullet}} \partial_{\mathfrak{a}} f^{\mathfrak{a}\mathfrak{a}'} \partial_{i} f^{i}{}_{\mathfrak{a}'} - \frac{4}{3} t_{\underset{3}{\bullet}} \partial^{\mathfrak{a}'} f^{\mathfrak{a}}{}_{\mathfrak{a}} \partial_{i} f^{i}{}_{\mathfrak{a}'} - \frac{2}{3} t_{\underset{2}{\bullet}} \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} +$$
$$\frac{2}{3} t_{\underset{2}{\bullet}} \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} - \frac{2}{3} t_{\underset{2}{\bullet}} \mathcal{A}_{\mathfrak{a}'i\mathfrak{a}} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \frac{1}{3} t_{\underset{2}{\bullet}} \partial_{\mathfrak{a}} f_{\mathfrak{a}'i} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} - \frac{1}{6} t_{\underset{2}{\bullet}} \partial_{\mathfrak{a}} f_{i\mathfrak{a}'} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} - \frac{1}{6} t_{\underset{2}{\bullet}} \partial_{\mathfrak{a}'} f_{\mathfrak{a}i} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} +$$
$$\frac{1}{6} t_{\underset{2}{\bullet}} \partial_{i} f_{\mathfrak{a}\mathfrak{a}'} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} - \frac{1}{6} t_{\underset{2}{\bullet}} \partial_{i} f_{\mathfrak{a}'\mathfrak{a}} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \frac{4}{3} r_{\underset{2}{\bullet}} \partial_{\mathfrak{a}'} \mathcal{A}_{\mathfrak{a}ij} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} r_{\underset{2}{\bullet}} \partial_{\mathfrak{a}'} \mathcal{A}_{\mathfrak{a}ji} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} +$$
$$\frac{2}{3} r_{\underset{2}{\bullet}} \partial_{\mathfrak{a}'} \mathcal{A}_{ij\mathfrak{a}} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{1}{3} r_{\underset{2}{\bullet}} \partial_{i} \mathcal{A}_{\mathfrak{a}\mathfrak{a}'j} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + \frac{1}{3} r_{\underset{2}{\bullet}} \partial_{j} \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} r_{\underset{2}{\bullet}} \partial_{j} \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{ \begin{pmatrix} 2k^2 t_{\underset{3}{\bullet}} & i\sqrt{2}\,k t_{\underset{3}{\bullet}} & 0 \\ -i\sqrt{2}\,k t_{\underset{3}{\bullet}} & t_{\underset{3}{\bullet}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{\underset{2}{\bullet}} + t_{\underset{2}{\bullet}} \right), \right.$$

$$\left. \begin{pmatrix} \frac{k^2 t_{\underset{2}{\bullet}}}{3} & \frac{1}{3} i\sqrt{2}\,k t_{\underset{2}{\bullet}} & \frac{i k t_{\underset{2}{\bullet}}}{3} \\ -\frac{1}{3} i\sqrt{2}\,k t_{\underset{2}{\bullet}} & \frac{2 t_{\underset{2}{\bullet}}}{3} & \frac{\sqrt{2}\, t_{\underset{2}{\bullet}}}{3} \\ -\frac{1}{3} i k t_{\underset{2}{\bullet}} & \frac{\sqrt{2}\, t_{\underset{2}{\bullet}}}{3} & \frac{t_{\underset{2}{\bullet}}}{3} \end{pmatrix}, \begin{pmatrix} \frac{2k^2 t_{\underset{3}{\bullet}}}{3} & \frac{2 i k t_{\underset{3}{\bullet}}}{3} & 0 & -\frac{1}{3} i\sqrt{2}\,k t_{\underset{3}{\bullet}} \\ -\frac{2}{3} i k t_{\underset{3}{\bullet}} & \frac{2 t_{\underset{3}{\bullet}}}{3} & 0 & -\frac{\sqrt{2}\, t_{\underset{3}{\bullet}}}{3} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i\sqrt{2}\,k t_{\underset{3}{\bullet}} & -\frac{\sqrt{2}\, t_{\underset{3}{\bullet}}}{3} & 0 & \frac{t_{\underset{3}{\bullet}}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, (0) \right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ -i\, {}^{0^+}_{\bullet}\tau^{\flat\parallel} == 2k\, {}^{0^+}_{\bullet}\sigma^{\flat\parallel},\ -i\, {}^{1^+}_{\bullet}\tau^{\flat\parallel\,ab} == k\, {}^{1^+}_{\bullet}\sigma^{\flat\perp\,ab},\ -i\, {}^{1^+}_{\bullet}\tau^{\flat\parallel\,ab} == k\, {}^{1^+}_{\bullet}\sigma^{\flat\parallel\,ab}, \right.$$
$$\left. i\, {}^{1^-}_{\bullet}\tau^{\flat\parallel\,a} == 2k\, {}^{1^-}_{\bullet}\sigma^{\flat\perp\,a},\ {}^{1^-}_{\bullet}\tau^{\flat\perp\,a} == 0,\ -i\, {}^{1^-}_{\bullet}\tau^{\flat\parallel\,a} == k\, {}^{1^-}_{\bullet}\sigma^{\flat\parallel\,a},\ {}^{2^+}_{\bullet}\sigma^{\flat\parallel\,ab} == 0,\ {}^{2^+}_{\bullet}\tau^{\flat\parallel\,ab} == 0,\ {}^{2^-}_{\bullet}\sigma^{\flat\parallel\,abc} == 0 \right\}$$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_{\dot{3}}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot{3}}} & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot{3}}} & \frac{1}{(1+2k^2)^2 t_{\dot{3}}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{\dot{2}} + t_{\dot{2}}}\right), \begin{pmatrix} \frac{3k^2}{(3+k^2)^2 t_{\dot{2}}} & \frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{\dot{2}}} & \frac{3ik}{(3+k^2)^2 t_{\dot{2}}} \\ -\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{\dot{2}}} & \frac{6}{(3+k^2)^2 t_{\dot{2}}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{\dot{2}}} \\ -\frac{3ik}{(3+k^2)^2 t_{\dot{2}}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{\dot{2}}} & \frac{3}{(3+k^2)^2 t_{\dot{2}}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6k^2}{(3+2k^2)^2 t_{\dot{3}}} & \frac{6ik}{(3+2k^2)^2 t_{\dot{3}}} & 0 & -\frac{3i\sqrt{2}\,k}{(3+2k^2)^2 t_{\dot{3}}} \\ -\frac{6ik}{(3+2k^2)^2 t_{\dot{3}}} & \frac{6}{(3+2k^2)^2 t_{\dot{3}}} & 0 & -\frac{3\sqrt{2}}{(3+2k^2)^2 t_{\dot{3}}} \\ 0 & 0 & 0 & 0 \\ \frac{3i\sqrt{2}\,k}{(3+2k^2)^2 t_{\dot{3}}} & -\frac{3\sqrt{2}}{(3+2k^2)^2 t_{\dot{3}}} & 0 & \frac{3}{(3+2k^2)^2 t_{\dot{3}}} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\dot{2}}}{r_{\dot{2}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\dot{2}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$r_{\dot{2}} < 0 \,\&\&\, t_{\dot{2}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\dot{2}} < 0 \,\&\&\, t_{\dot{2}} > 0$

Okay, that concludes the analysis of this theory.

Case 26

Now for a new theory. Here is the full nonlinear Lagrangian for Case 26 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\dot{2}}\, \mathcal{R}_{ij\hbar l}\, \mathcal{R}^{ij\hbar l} - \frac{2}{3} r_{\dot{2}}\, \mathcal{R}_{i\hbar jl}\, \mathcal{R}^{ij\hbar l} + \frac{1}{6} r_{\dot{2}}\, \mathcal{R}^{ij\hbar l}\, \mathcal{R}_{\hbar lij} + \frac{1}{12} t_{\dot{2}}\, \mathcal{T}_{ij\hbar}\, \mathcal{T}^{ij\hbar} - \frac{1}{6} t_{\dot{2}}\, \mathcal{T}^{ij\hbar}\, \mathcal{T}_{ji\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{\dot 2}\, \mathcal{A}_{a a' i}\, \mathcal{A}^{a a' i} - \frac{2}{3} t_{\dot 2}\, \mathcal{A}_{a i a'}\, \mathcal{A}^{a a' i} - \frac{2}{3} t_{\dot 2}\, \mathcal{A}_{a a' i}\, \partial^{i} f^{a a'} + \frac{2}{3} t_{\dot 2}\, \mathcal{A}_{a i a'}\, \partial^{i} f^{a a'} - \frac{2}{3} t_{\dot 2}\, \mathcal{A}_{a' i a}\, \partial^{i} f^{a a'} + \frac{1}{3} t_{\dot 2}\, \partial_{a} f_{a' i}\, \partial^{i} f^{a a'} - \frac{1}{6} t_{\dot 2}\, \partial_{a} f_{i a'}\, \partial^{i} f^{a a'} - \frac{1}{6} t_{\dot 2}\, \partial_{a'} f_{a i}\, \partial^{i} f^{a a'} + \frac{1}{6} t_{\dot 2}\, \partial_{i} f_{a a'}\, \partial^{i} f^{a a'} - \frac{1}{6} t_{\dot 2}\, \partial_{i} f_{a' a}\, \partial^{i} f^{a a'} + \frac{4}{3} r_{\dot 2}\, \partial_{a'} \mathcal{A}_{a i j}\, \partial^{j} \mathcal{A}^{a a' i} - \frac{2}{3} r_{\dot 2}\, \partial_{a'} \mathcal{A}_{a j i}\, \partial^{j} \mathcal{A}^{a a' i} + \frac{2}{3} r_{\dot 2}\, \partial_{a'} \mathcal{A}_{i j a}\, \partial^{j} \mathcal{A}^{a a' i} - \frac{1}{3} r_{\dot 2}\, \partial_{i} \mathcal{A}_{a a' j}\, \partial^{j} \mathcal{A}^{a a' i} + \frac{1}{3} r_{\dot 2}\, \partial_{j} \mathcal{A}_{a a' i}\, \partial^{j} \mathcal{A}^{a a' i} - \frac{2}{3} r_{\dot 2}\, \partial_{j} \mathcal{A}_{a i a'}\, \partial^{j} \mathcal{A}^{a a' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{\dot 2} + t_{\dot 2}\right), \begin{pmatrix} \frac{k^2 t_{\dot 2}}{3} & \frac{1}{3} i \sqrt{2}\, k\, t_{\dot 2} & \frac{i k t_{\dot 2}}{3} \\ -\frac{1}{3} i \sqrt{2}\, k\, t_{\dot 2} & \frac{2 t_{\dot 2}}{3} & \frac{\sqrt{2}\, t_{\dot 2}}{3} \\ -\frac{1}{3} i k\, t_{\dot 2} & \frac{\sqrt{2}\, t_{\dot 2}}{3} & \frac{t_{\dot 2}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\bullet}\tau^{\flat\perp} == 0,\ \overset{0^+}{\bullet}\sigma^{\flat\parallel} == 0,\ \overset{0^+}{\bullet}\tau^{\flat\parallel} == 0,\ -i\ \overset{1^+}{\bullet}\tau^{\flat\parallel\,ab} == k\ \overset{1^+}{\bullet}\sigma^{\flat\perp\,ab},\ -i\ \overset{1^+}{\bullet}\tau^{\flat\parallel\,ab} == k\ \overset{1^+}{\bullet}\sigma^{\flat\parallel\,ab},\ \overset{1^-}{\bullet}\sigma^{\flat\perp\,a} == 0,\ \overset{1^-}{\bullet}\tau^{\flat\perp\,a} == 0,\ \overset{1^-}{\bullet}\sigma^{\flat\parallel\,a} == 0,\ \overset{1^-}{\bullet}\tau^{\flat\parallel\,a} == 0,\ \overset{2^+}{\bullet}\sigma^{\flat\parallel\,ab} == 0,\ \overset{2^+}{\bullet}\tau^{\flat\parallel\,ab} == 0,\ \overset{2^-}{\bullet}\sigma^{\flat\parallel\,abc} == 0\right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{\dot 2} + t_{\dot 2}}\right), \begin{pmatrix} \frac{3 k^2}{(3+k^2)^2 t_{\dot 2}} & \frac{3 i \sqrt{2}\, k}{(3+k^2)^2 t_{\dot 2}} & \frac{3 i k}{(3+k^2)^2 t_{\dot 2}} \\ -\frac{3 i \sqrt{2}\, k}{(3+k^2)^2 t_{\dot 2}} & \frac{6}{(3+k^2)^2 t_{\dot 2}} & \frac{3 \sqrt{2}}{(3+k^2)^2 t_{\dot 2}} \\ -\frac{3 i k}{(3+k^2)^2 t_{\dot 2}} & \frac{3 \sqrt{2}}{(3+k^2)^2 t_{\dot 2}} & \frac{3}{(3+k^2)^2 t_{\dot 2}} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\dot 2}}{r_{\dot 2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\dot 2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{\dot 2} < 0\ \&\&\ t_{\dot 2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\dot{2}} < 0 \,\&\&\, t_{\dot{2}} > 0$

Okay, that concludes the analysis of this theory.

Case 27

Now for a new theory. Here is the full nonlinear Lagrangian for Case 27 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\dot{2}} \mathcal{R}_{ij\hbar l} \mathcal{R}^{ij\hbar l} - \frac{2}{3} r_{\dot{2}} \mathcal{R}_{i\hbar jl} \mathcal{R}^{ij\hbar l} + \frac{1}{6}\left(r_{\dot{2}} - 6 r_{\dot{3}}\right) \mathcal{R}^{ij\hbar l} \mathcal{R}_{\hbar lij} + r_{\dot{3}} \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{12} t_{\dot{2}} \mathcal{T}_{ij\hbar} \mathcal{T}^{ij\hbar} - \frac{1}{6} t_{\dot{2}} \mathcal{T}^{ij\hbar} \mathcal{T}_{ji\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{\dot{2}} \mathcal{A}_{aa'i} \mathcal{A}^{aa'i} - \frac{2}{3} t_{\dot{2}} \mathcal{A}_{aia'} \mathcal{A}^{aa'i} - r_{\dot{3}} \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{aa'}{}_{a} - \frac{2}{3} t_{\dot{2}} \mathcal{A}_{aa'i} \partial^{i} f^{aa'} + \frac{2}{3} t_{\dot{2}} \mathcal{A}_{aia'} \partial^{i} f^{aa'} - \frac{2}{3} t_{\dot{2}} \mathcal{A}_{a'ia} \partial^{i} f^{aa'} + \frac{1}{3} t_{\dot{2}} \partial_{a} f_{a'i} \partial^{i} f^{aa'} - \frac{1}{6} t_{\dot{2}} \partial_{a} f_{ia'} \partial^{i} f^{aa'} - \frac{1}{6} t_{\dot{2}} \partial_{a'} f_{ai} \partial^{i} f^{aa'} + \frac{1}{6} t_{\dot{2}} \partial_{i} f_{aa'} \partial^{i} f^{aa'} - \frac{1}{6} t_{\dot{2}} \partial_{i} f_{a'a} \partial^{i} f^{aa'} - r_{\dot{3}} \partial_{a} \mathcal{A}^{aa'i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} + 2 r_{\dot{3}} \partial^{i} \mathcal{A}^{aa'}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} + \frac{4}{3} r_{\dot{2}} \partial_{a'} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{\dot{2}} \partial_{a'} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3}\left(r_{\dot{2}} - 6 r_{\dot{3}}\right) \partial_{a'} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{aa'i} - \frac{1}{3} r_{\dot{2}} \partial_{i} \mathcal{A}_{aa'j} \partial^{j} \mathcal{A}^{aa'i} + \frac{1}{3} r_{\dot{2}} \partial_{j} \mathcal{A}_{aa'i} \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{\dot{2}} \partial_{j} \mathcal{A}_{aia'} \partial^{j} \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{\dot{2}} + t_{\dot{2}}\right), \begin{pmatrix} \frac{k^2 t_{\dot{2}}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{\dot{2}} & \frac{i k t_{\dot{2}}}{3} \\ -\frac{1}{3} i \sqrt{2}\, k t_{\dot{2}} & \frac{1}{2}\left(3 k^2 r_{\dot{3}} + \frac{4 t_{\dot{2}}}{3}\right) & \frac{\sqrt{2}\, t_{\dot{2}}}{3} \\ -\frac{1}{3} i k t_{\dot{2}} & \frac{\sqrt{2}\, t_{\dot{2}}}{3} & \frac{t_{\dot{2}}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{3 k^2 r_{\dot{3}}}{2} \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\cdot}\tau^{\flat\perp} == 0,\ \overset{0^+}{\cdot}\sigma^{\flat\parallel} == 0,\ \overset{0^+}{\cdot}\tau^{\flat\parallel} == 0,\ -i\, \overset{1^+}{\cdot}\tau^{\flat\parallel ab} == k\, \overset{1^+}{\cdot}\sigma^{\flat\perp ab},\ \overset{1^-}{\cdot}\sigma^{\flat\perp a} == 0,\ \overset{1^-}{\cdot}\tau^{\flat\perp a} == 0,\ \overset{1^-}{\cdot}\sigma^{\flat\parallel a} == 0,\ \overset{1^-}{\cdot}\tau^{\flat\parallel a} == 0,\ \overset{2^+}{\cdot}\tau^{\flat\parallel ab} == 0,\ \overset{2^-}{\cdot}\sigma^{\flat\parallel abc} == 0\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*–matrices, which are functionally analogous to the inverse *b*–matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{2} + t_{2}}\right), \begin{pmatrix} \frac{9 k^2 r_{3} + 4 t_{2}}{3 (1+k^2)^2 r_{3} t_{2}} & -\frac{2 i \sqrt{2}}{3 k r_{3} + 3 k^3 r_{3}} & \frac{i (9 k^2 r_{3} + 4 t_{2})}{3 k (1+k^2)^2 r_{3} t_{2}} \\ \frac{2 i \sqrt{2}}{3 k r_{3} + 3 k^3 r_{3}} & \frac{2}{3 k^2 r_{3}} & -\frac{2 \sqrt{2}}{3 k^2 r_{3} + 3 k^4 r_{3}} \\ -\frac{i (9 k^2 r_{3} + 4 t_{2})}{3 k (1+k^2)^2 r_{3} t_{2}} & -\frac{2 \sqrt{2}}{3 k^2 r_{3} + 3 k^4 r_{3}} & \frac{9 k^2 r_{3} + 4 t_{2}}{3 (k+k^3)^2 r_{3} t_{2}} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3 k^2 r_{3}} \end{pmatrix}, (0)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{2} < 0 \,\&\&\, t_{2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{2} < 0 \,\&\&\, t_{2} > 0$

Okay, that concludes the analysis of this theory.

Case 28

Now for a new theory. Here is the full nonlinear Lagrangian for Case 28 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ij\hbar l} \mathcal{R}^{ij\hbar l} - \frac{2}{3} r_{2} \mathcal{R}_{i\hbar jl} \mathcal{R}^{ij\hbar l} + r_{5} \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{j}{}^{l}{}_{\hbar l} + \frac{1}{6} r_{2} \mathcal{R}^{ij\hbar l} \mathcal{R}_{\hbar lij} - r_{5} \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{12} t_{2} \mathcal{T}_{ij\hbar} \mathcal{T}^{ij\hbar} - \frac{1}{6} t_{2} \mathcal{T}^{ij\hbar} \mathcal{T}_{ji\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{2}\, \mathcal{A}_{\alpha\alpha' i}\, \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} t_{2}\, \mathcal{A}_{\alpha i \alpha'}\, \mathcal{A}^{\alpha\alpha' i} + r_{5}\, \partial_{\alpha'}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha} - r_{5}\, \partial_{i}\mathcal{A}_{\alpha'}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha} -$$
$$\frac{2}{3} t_{2}\, \mathcal{A}_{\alpha\alpha' i}\, \partial^{i} f^{\alpha\alpha'} + \frac{2}{3} t_{2}\, \mathcal{A}_{\alpha i \alpha'}\, \partial^{i} f^{\alpha\alpha'} - \frac{2}{3} t_{2}\, \mathcal{A}_{\alpha' i \alpha}\, \partial^{i} f^{\alpha\alpha'} + \frac{1}{3} t_{2}\, \partial_{\alpha} f_{\alpha' i}\, \partial^{i} f^{\alpha\alpha'} - \frac{1}{6} t_{2}\, \partial_{\alpha} f_{i\alpha'}\, \partial^{i} f^{\alpha\alpha'} -$$
$$\frac{1}{6} t_{2}\, \partial_{\alpha'} f_{\alpha i}\, \partial^{i} f^{\alpha\alpha'} + \frac{1}{6} t_{2}\, \partial_{i} f_{\alpha\alpha'}\, \partial^{i} f^{\alpha\alpha'} - \frac{1}{6} t_{2}\, \partial_{i} f_{\alpha'\alpha}\, \partial^{i} f^{\alpha\alpha'} - r_{5}\, \partial_{\alpha}\mathcal{A}^{\alpha\alpha' i}\, \partial_{j}\mathcal{A}_{\alpha'}{}^{j}{}_{i} + 2 r_{5}\, \partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \partial_{j}\mathcal{A}_{\alpha'}{}^{j}{}_{i} +$$
$$r_{5}\, \partial_{\alpha}\mathcal{A}^{\alpha\alpha' i}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{\alpha'} - 2 r_{5}\, \partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{\alpha'} + \frac{4}{3} r_{2}\, \partial_{\alpha'}\mathcal{A}_{\alpha i j}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_{2}\, \partial_{\alpha'}\mathcal{A}_{\alpha j i}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i} +$$
$$\frac{2}{3} r_{2}\, \partial_{\alpha'}\mathcal{A}_{i j \alpha}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i} - \frac{1}{3} r_{2}\, \partial_{i}\mathcal{A}_{\alpha\alpha' j}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i} + \frac{1}{3} r_{2}\, \partial_{j}\mathcal{A}_{\alpha\alpha' i}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_{2}\, \partial_{j}\mathcal{A}_{\alpha i \alpha'}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{2} + t_{2}\right), \begin{pmatrix} \frac{k^2 t_{2}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{2} & \frac{i k t_{2}}{3} \\ -\frac{1}{3} i \sqrt{2}\, k t_{2} & \frac{1}{2}\left(2 k^2 r_{5} + \frac{4 t_{2}}{3}\right) & \frac{\sqrt{2}\, t_{2}}{3} \\ -\frac{1}{3} i k t_{2} & \frac{\sqrt{2}\, t_{2}}{3} & \frac{t_{2}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & k^2 r_{5} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}\tau^{\flat\perp} == 0,\ {}^{0^+}\sigma^{\flat\parallel} == 0,\ {}^{0^+}\tau^{\flat\parallel} == 0,\ -i\ {}^{1^+}\tau^{\flat\parallel\, ab} == k\ {}^{1^+}\sigma^{\flat\perp\, ab},\right.$$
$$\left. {}^{1^-}\sigma^{\flat\perp\, a} == 0,\ {}^{1^-}\tau^{\flat\perp\, a} == 0,\ {}^{1^-}\tau^{\flat\parallel\, a} == 0,\ {}^{2^+}\sigma^{\flat\parallel\, ab} == 0,\ {}^{2^+}\tau^{\flat\parallel\, ab} == 0,\ {}^{2^-}\sigma^{\flat\parallel\, abc} == 0 \right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{2} + t_{2}}\right), \begin{pmatrix} \frac{3 k^2 r_{5} + 2 t_{2}}{(1+k^2)^2 r_{5} t_{2}} & -\frac{i \sqrt{2}}{k r_{5} + k^3 r_{5}} & \frac{i\left(3 k^2 r_{5} + 2 t_{2}\right)}{k (1+k^2)^2 r_{5} t_{2}} \\ \frac{i \sqrt{2}}{k r_{5} + k^3 r_{5}} & \frac{1}{k^2 r_{5}} & -\frac{\sqrt{2}}{k^2 r_{5} + k^4 r_{5}} \\ -\frac{i\left(3 k^2 r_{5} + 2 t_{2}\right)}{k (1+k^2)^2 r_{5} t_{2}} & -\frac{\sqrt{2}}{k^2 r_{5} + k^4 r_{5}} & \frac{3 k^2 r_{5} + 2 t_{2}}{(k+k^3)^2 r_{5} t_{2}} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{k^2 r_{5}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$r_{\cdot 2} < 0 \&\& t_{\cdot 2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\cdot 2} < 0 \&\& t_{\cdot 2} > 0$

Okay, that concludes the analysis of this theory.

Case 29

Now for a new theory. Here is the full nonlinear Lagrangian for Case 29 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\left(2 r_{\cdot 1}+r_{\cdot 2}\right) \mathcal{R}_{ijkl}\ \mathcal{R}^{ijkl}+\frac{2}{3}\left(r_{\cdot 1}-r_{\cdot 2}\right) \mathcal{R}_{ikjl}\ \mathcal{R}^{ijkl}-2 r_{\cdot 1}\ \mathcal{R}^{ij\ \ k}_{\ \ i}\ \mathcal{R}_{j\ kl}^{\ l}+\frac{1}{6}\left(-4 r_{\cdot 1}+r_{\cdot 2}\right) \mathcal{R}^{ijkl}\ \mathcal{R}_{klij}+ 2 r_{\cdot 1}\ \mathcal{R}^{ij\ \ k}_{\ \ i}\ \mathcal{R}_{k\ jl}^{\ l}+\frac{1}{12} t_{\cdot 2}\ \mathcal{T}_{ijk}\ \mathcal{T}^{ijk}-\frac{1}{6} t_{\cdot 2}\ \mathcal{T}^{ijk}\ \mathcal{T}_{jik}-\frac{2}{3} t_{\cdot 3}\ \mathcal{T}^{i\ \ j}_{\ i}\ \mathcal{T}^{k}_{\ jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{\cdot 2}\ \mathcal{A}_{aa'i}\ \mathcal{A}^{aa'i}-\frac{2}{3} t_{\cdot 2}\ \mathcal{A}_{aia'}\ \mathcal{A}^{aa'i}-\frac{2}{3} t_{\cdot 3}\ \mathcal{A}^{aa'}_{\ \ \ a}\ \mathcal{A}_{a'\ i}^{\ \ i}+\frac{4}{3} t_{\cdot 3}\ \mathcal{A}_{a'\ i}^{\ \ i}\ \partial_{a} f^{aa'}- \frac{4}{3} t_{\cdot 3}\ \mathcal{A}_{a'\ i}^{\ \ i}\ \partial^{a'} f^{a}_{\ a}+\frac{2}{3} t_{\cdot 3}\ \partial_{a'} f^{i}_{\ i}\ \partial^{a'} f^{a}_{\ a}+\frac{2}{3} t_{\cdot 3}\ \partial_{a} f^{aa'}\ \partial_{i} f^{i}_{\ a'}-\frac{4}{3} t_{\cdot 3}\ \partial^{a'} f^{a}_{\ a}\ \partial_{i} f^{i}_{\ a'}-2 r_{\cdot 1}\ \partial_{a'} \mathcal{A}_{i\ j}^{\ j}\ \partial^{i} \mathcal{A}^{aa'}_{\ \ \ a}+ 2 r_{\cdot 1}\ \partial_{i} \mathcal{A}_{a'\ j}^{\ \ j}\ \partial^{i} \mathcal{A}^{aa'}_{\ \ \ a}-\frac{2}{3} t_{\cdot 2}\ \mathcal{A}_{aa'i}\ \partial^{i} f^{aa'}+\frac{2}{3} t_{\cdot 2}\ \mathcal{A}_{aia'}\ \partial^{i} f^{aa'}-\frac{2}{3} t_{\cdot 2}\ \mathcal{A}_{a'ia}\ \partial^{i} f^{aa'}+ \frac{1}{3} t_{\cdot 2}\ \partial_{a} f_{a'i}\ \partial^{i} f^{aa'}-\frac{1}{6} t_{\cdot 2}\ \partial_{a} f_{ia'}\ \partial^{i} f^{aa'}-\frac{1}{6} t_{\cdot 2}\ \partial_{a'} f_{ai}\ \partial^{i} f^{aa'}+\frac{1}{6} t_{\cdot 2}\ \partial_{i} f_{aa'}\ \partial^{i} f^{aa'}-\frac{1}{6} t_{\cdot 2}\ \partial_{i} f_{a'a}\ \partial^{i} f^{aa'}+ 2 r_{\cdot 1}\ \partial_{a} \mathcal{A}^{aa'i}\ \partial_{j} \mathcal{A}_{a'\ i}^{\ \ j}-4 r_{\cdot 1}\ \partial^{i} \mathcal{A}^{aa'}_{\ \ \ a}\ \partial_{j} \mathcal{A}_{a'\ i}^{\ \ j}-2 r_{\cdot 1}\ \partial_{a} \mathcal{A}^{aa'i}\ \partial_{j} \mathcal{A}_{i\ a'}^{\ j}+4 r_{\cdot 1}\ \partial^{i} \mathcal{A}^{aa'}_{\ \ \ a}\ \partial_{j} \mathcal{A}_{i\ a'}^{\ j}- \frac{4}{3}\left(r_{\cdot 1}-r_{\cdot 2}\right) \partial_{a'} \mathcal{A}_{aij}\ \partial^{j} \mathcal{A}^{aa'i}+\frac{2}{3}\left(r_{\cdot 1}-r_{\cdot 2}\right) \partial_{a'} \mathcal{A}_{aji}\ \partial^{j} \mathcal{A}^{aa'i}+\frac{2}{3}\left(-4 r_{\cdot 1}+r_{\cdot 2}\right) \partial_{a'} \mathcal{A}_{ija}\ \partial^{j} \mathcal{A}^{aa'i}+ \frac{1}{3}\left(-2 r_{\cdot 1}-r_{\cdot 2}\right) \partial_{i} \mathcal{A}_{aa'j}\ \partial^{j} \mathcal{A}^{aa'i}+\frac{1}{3}\left(2 r_{\cdot 1}+r_{\cdot 2}\right) \partial_{j} \mathcal{A}_{aa'i}\ \partial^{j} \mathcal{A}^{aa'i}+\frac{2}{3}\left(r_{\cdot 1}-r_{\cdot 2}\right) \partial_{j} \mathcal{A}_{aia'}\ \partial^{j} \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2k^2 t_{3} & i\sqrt{2}\,k\,t_{3} & 0 \\ -i\sqrt{2}\,k\,t_{3} & t_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{2} + t_{2}\right), \begin{pmatrix} \frac{k^2 t_{2}}{3} & \frac{1}{3} i\sqrt{2}\,k\,t_{2} & \frac{i k t_{2}}{3} \\ -\frac{1}{3} i\sqrt{2}\,k\,t_{2} & \frac{2 t_{2}}{3} & \frac{\sqrt{2}\,t_{2}}{3} \\ -\frac{1}{3} i k\,t_{2} & \frac{\sqrt{2}\,t_{2}}{3} & \frac{t_{2}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2 t_{3}}{3} & \frac{2 i k t_{3}}{3} & 0 & -\frac{1}{3} i\sqrt{2}\,k\,t_{3} \\ -\frac{2}{3} i k\,t_{3} & -k^2 r_{1} + \frac{2 t_{3}}{3} & 0 & -\frac{\sqrt{2}\,t_{3}}{3} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i\sqrt{2}\,k\,t_{3} & -\frac{\sqrt{2}\,t_{3}}{3} & 0 & \frac{t_{3}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \left(k^2 r_{1}\right)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}_{\bullet}\tau^{\flat\parallel} == 2k\,{}^{0^+}_{\bullet}\sigma^{\flat\parallel},\ -i\,{}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab},\right.$$
$$\left.-i\,{}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab},\ i\,{}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a} == 2k\,{}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ {}^{2^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab} == 0,\ {}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 0\right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_{3}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & \frac{1}{(1+2k^2)^2 t_{3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{2} + t_{2}}\right), \begin{pmatrix} \frac{3k^2}{(3+k^2)^2 t_{2}} & \frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{2}} & \frac{3ik}{(3+k^2)^2 t_{2}} \\ -\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{2}} & \frac{6}{(3+k^2)^2 t_{2}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{2}} \\ -\frac{3ik}{(3+k^2)^2 t_{2}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{2}} & \frac{3}{(3+k^2)^2 t_{2}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6k^2 r_{1} - 4 t_{3}}{(1+2k^2)^2 r_{1} t_{3}} & \frac{2i}{k r_{1} + 2k^3 r_{1}} & 0 & -\frac{i\sqrt{2}\left(3k^2 r_{1} - 2 t_{3}\right)}{k(1+2k^2)^2 r_{1} t_{3}} \\ -\frac{2i}{k r_{1} + 2k^3 r_{1}} & -\frac{1}{k^2 r_{1}} & 0 & -\frac{\sqrt{2}}{k^2 r_{1} + 2k^4 r_{1}} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\left(3k^2 r_{1} - 2 t_{3}\right)}{k(1+2k^2)^2 r_{1} t_{3}} & -\frac{\sqrt{2}}{k^2 r_{1} + 2k^4 r_{1}} & 0 & \frac{3k^2 r_{1} - 2 t_{3}}{(k+2k^3)^2 r_{1} t_{3}} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{1}}\right)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall unitarity conditions:

$r_{\dot{2}} < 0 \;\&\&\; t_{\dot{2}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\dot{2}} < 0 \;\&\&\; t_{\dot{2}} > 0$

Okay, that concludes the analysis of this theory.

Case 30

Now for a new theory. Here is the full nonlinear Lagrangian for Case 30 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\left(2 r_{\dot{1}}+r_{\dot{2}}\right) \mathcal{R}_{ij\mathfrak{k}l}\, \mathcal{R}^{ij\mathfrak{k}l}+\frac{2}{3}\left(r_{\dot{1}}-r_{\dot{2}}\right) \mathcal{R}_{i\mathfrak{k}jl}\, \mathcal{R}^{ij\mathfrak{k}l}-2 r_{\dot{1}}\, \mathcal{R}^{ij}{}_{i}{}^{\mathfrak{k}}\, \mathcal{R}_{j}{}^{l}{}_{\mathfrak{k}l}+ \frac{1}{6}\left(-4 r_{\dot{1}}+r_{\dot{2}}\right) \mathcal{R}^{ij\mathfrak{k}l}\, \mathcal{R}_{\mathfrak{k}lij}+2 r_{\dot{1}}\, \mathcal{R}^{ij}{}_{i}{}^{\mathfrak{k}}\, \mathcal{R}_{\mathfrak{k}}{}^{l}{}_{jl}+\frac{1}{12} t_{\dot{2}}\, \mathcal{T}_{ij\mathfrak{k}}\, \mathcal{T}^{ij\mathfrak{k}}-\frac{1}{6} t_{\dot{2}}\, \mathcal{T}^{ij\mathfrak{k}}\, \mathcal{T}_{ji\mathfrak{k}}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\frac{1}{3} t_{\dot{2}}\, \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i}\, \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}-\frac{2}{3} t_{\dot{2}}\, \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'}\, \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}-2 r_{\dot{1}}\, \partial_{\mathfrak{a}'}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}+2 r_{\dot{1}}\, \partial_{i}\mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}- \\ &\frac{2}{3} t_{\dot{2}}\, \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'}+\frac{2}{3} t_{\dot{2}}\, \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'}-\frac{2}{3} t_{\dot{2}}\, \mathcal{A}_{\mathfrak{a}'i\mathfrak{a}}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'}+\frac{1}{3} t_{\dot{2}}\, \partial_{\mathfrak{a}}f_{\mathfrak{a}'i}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'}- \\ &\frac{1}{6} t_{\dot{2}}\, \partial_{\mathfrak{a}}f_{i\mathfrak{a}'}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'}-\frac{1}{6} t_{\dot{2}}\, \partial_{\mathfrak{a}'}f_{\mathfrak{a}i}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'}+\frac{1}{6} t_{\dot{2}}\, \partial_{i}f_{\mathfrak{a}\mathfrak{a}'}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'}-\frac{1}{6} t_{\dot{2}}\, \partial_{i}f_{\mathfrak{a}'\mathfrak{a}}\, \partial^{i}f^{\mathfrak{a}\mathfrak{a}'}+ \\ &2 r_{\dot{1}}\, \partial_{\mathfrak{a}}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\, \partial_{j}\mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{i}-4 r_{\dot{1}}\, \partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}\, \partial_{j}\mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{i}-2 r_{\dot{1}}\, \partial_{\mathfrak{a}}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{\mathfrak{a}'}+4 r_{\dot{1}}\, \partial^{i}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{\mathfrak{a}'}- \\ &\frac{4}{3}\left(r_{\dot{1}}-r_{\dot{2}}\right) \partial_{\mathfrak{a}'}\mathcal{A}_{\mathfrak{a}ij}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}+\frac{2}{3}\left(r_{\dot{1}}-r_{\dot{2}}\right) \partial_{\mathfrak{a}'}\mathcal{A}_{\mathfrak{a}ji}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}+\frac{2}{3}\left(-4 r_{\dot{1}}+r_{\dot{2}}\right) \partial_{\mathfrak{a}'}\mathcal{A}_{ij\mathfrak{a}}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}+ \\ &\frac{1}{3}\left(-2 r_{\dot{1}}-r_{\dot{2}}\right) \partial_{i}\mathcal{A}_{\mathfrak{a}\mathfrak{a}'j}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}+\frac{1}{3}\left(2 r_{\dot{1}}+r_{\dot{2}}\right) \partial_{j}\mathcal{A}_{\mathfrak{a}\mathfrak{a}'i}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}+\frac{2}{3}\left(r_{\dot{1}}-r_{\dot{2}}\right) \partial_{j}\mathcal{A}_{\mathfrak{a}i\mathfrak{a}'}\, \partial^{j}\mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\end{aligned}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}, \left(k^{2} r_{\dot{2}}+t_{\dot{2}}\right), \begin{pmatrix}\frac{k^{2} t_{\dot{2}}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{\dot{2}} & \frac{i k t_{\dot{2}}}{3}\\ -\frac{1}{3} i \sqrt{2}\, k t_{\dot{2}} & \frac{2 t_{\dot{2}}}{3} & \frac{\sqrt{2}\, t_{\dot{2}}}{3}\\ -\frac{1}{3} i k t_{\dot{2}} & \frac{\sqrt{2}\, t_{\dot{2}}}{3} & \frac{t_{\dot{2}}}{3}\end{pmatrix}, \begin{pmatrix}0&0&0&0\\0&-k^{2} r_{\dot{1}}&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}, \begin{pmatrix}0&0\\0&0\end{pmatrix}, \left(k^{2} r_{\dot{1}}\right)\right\}$$

Gauge constraints on source currents:

$$\Big\{ {}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\bullet}\sigma^{\flat\parallel} == 0,\ {}^{0^+}_{\bullet}\tau^{\flat\parallel} == 0,\ -i\ {}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\ {}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab},$$
$$-i\ {}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\ {}^{1^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab},\ {}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a} == 0,\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ {}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a} == 0,\ {}^{2^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab} == 0,\ {}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 0\Big\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_2 + t_2} \end{pmatrix}, \begin{pmatrix} \frac{3k^2}{(3+k^2)^2 t_2} & \frac{3i\sqrt{2}k}{(3+k^2)^2 t_2} & \frac{3ik}{(3+k^2)^2 t_2} \\ -\frac{3i\sqrt{2}k}{(3+k^2)^2 t_2} & \frac{6}{(3+k^2)^2 t_2} & \frac{3\sqrt{2}}{(3+k^2)^2 t_2} \\ -\frac{3ik}{(3+k^2)^2 t_2} & \frac{3\sqrt{2}}{(3+k^2)^2 t_2} & \frac{3}{(3+k^2)^2 t_2} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{k^2 r_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_1} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_2}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_2 < 0 \,\&\&\, t_2 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_2 < 0 \,\&\&\, t_2 > 0$

Okay, that concludes the analysis of this theory.

Case 31

Now for a new theory. Here is the full nonlinear Lagrangian for Case 31 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_2\, \mathcal{R}_{ijhl}\, \mathcal{R}^{ijhl} - \frac{2}{3} r_2\, \mathcal{R}_{ihjl}\, \mathcal{R}^{ijhl} + \frac{1}{6}\left(r_2 - 6 r_3\right) \mathcal{R}^{ijhl}\, \mathcal{R}_{hlij} +$$
$$4 r_3\, \mathcal{R}^{ij}{}_{i}{}^{h}\, \mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{12} t_2\, \mathcal{T}_{ijh}\, \mathcal{T}^{ijh} - \frac{1}{6} t_2\, \mathcal{T}^{ijh}\, \mathcal{T}_{jih}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{\cdot 2}\, \mathcal{A}_{a a' i}\, \mathcal{A}^{a a' i} - \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{a i a'}\, \mathcal{A}^{a a' i} - 4 r_{\cdot 3}\, \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{a a'}{}_{a} -$$
$$\frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{a a' i}\, \partial^{i} f^{a a'} + \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{a i a'}\, \partial^{i} f^{a a'} - \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{a' i a}\, \partial^{i} f^{a a'} + \frac{1}{3} t_{\cdot 2}\, \partial_{a} f_{a' i}\, \partial^{i} f^{a a'} -$$
$$\frac{1}{6} t_{\cdot 2}\, \partial_{a} f_{i a'}\, \partial^{i} f^{a a'} - \frac{1}{6} t_{\cdot 2}\, \partial_{a'} f_{a i}\, \partial^{i} f^{a a'} + \frac{1}{6} t_{\cdot 2}\, \partial_{i} f_{a a'}\, \partial^{i} f^{a a'} - \frac{1}{6} t_{\cdot 2}\, \partial_{i} f_{a' a}\, \partial^{i} f^{a a'} -$$
$$4 r_{\cdot 3}\, \partial_{a} \mathcal{A}^{a a' i}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} + 8 r_{\cdot 3}\, \partial^{i} \mathcal{A}^{a a'}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} + \frac{4}{3} r_{\cdot 2}\, \partial_{a'} \mathcal{A}_{a i j}\, \partial^{j} \mathcal{A}^{a a' i} - \frac{2}{3} r_{\cdot 2}\, \partial_{a'} \mathcal{A}_{a j i}\, \partial^{j} \mathcal{A}^{a a' i} +$$
$$\frac{2}{3} \left(r_{\cdot 2} - 6 r_{\cdot 3}\right) \partial_{a'} \mathcal{A}_{i j a}\, \partial^{j} \mathcal{A}^{a a' i} - \frac{1}{3} r_{\cdot 2}\, \partial_{i} \mathcal{A}_{a a' j}\, \partial^{j} \mathcal{A}^{a a' i} + \frac{1}{3} r_{\cdot 2}\, \partial_{j} \mathcal{A}_{a a' i}\, \partial^{j} \mathcal{A}^{a a' i} - \frac{2}{3} r_{\cdot 2}\, \partial_{j} \mathcal{A}_{a i a'}\, \partial^{j} \mathcal{A}^{a a' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 6 k^2 r_{\cdot 3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{\cdot 2} + t_{\cdot 2}\right), \begin{pmatrix} \frac{k^2 t_{\cdot 2}}{3} & \frac{1}{3} i \sqrt{2}\, k\, t_{\cdot 2} & \frac{i k t_{\cdot 2}}{3} \\ -\frac{1}{3} i \sqrt{2}\, k\, t_{\cdot 2} & \frac{2 t_{\cdot 2}}{3} & \frac{\sqrt{2}\, t_{\cdot 2}}{3} \\ -\frac{1}{3} i k\, t_{\cdot 2} & \frac{\sqrt{2}\, t_{\cdot 2}}{3} & \frac{t_{\cdot 2}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\cdot}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\cdot}\tau^{\flat\parallel} == 0,\ -i\, {}^{1^+}_{\cdot}\tau^{\flat\parallel\, ab} == k\, {}^{1^+}_{\cdot}\sigma^{\flat\perp\, ab},\ -i\, {}^{1^+}_{\cdot}\tau^{\flat\parallel\, ab} == k\, {}^{1^+}_{\cdot}\sigma^{\flat\parallel\, ab},\ {}^{1^-}_{\cdot}\sigma^{\flat\perp\, a} == 0,\right.$$
$$\left. {}^{1^-}_{\cdot}\tau^{\flat\perp\, a} == 0,\ {}^{1^-}_{\cdot}\sigma^{\flat\parallel\, a} == 0,\ {}^{1^-}_{\cdot}\tau^{\flat\parallel\, a} == 0,\ {}^{2^+}_{\cdot}\sigma^{\flat\parallel\, ab} == 0,\ {}^{2^+}_{\cdot}\tau^{\flat\parallel\, ab} == 0,\ {}^{2^-}_{\cdot}\sigma^{\flat\parallel\, abc} == 0 \right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{6 k^2 r_{\cdot 3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{\cdot 2} + t_{\cdot 2}}\right), \begin{pmatrix} \frac{3 k^2}{(3+k^2)^2 t_{\cdot 2}} & \frac{3 i \sqrt{2}\, k}{(3+k^2)^2 t_{\cdot 2}} & \frac{3 i k}{(3+k^2)^2 t_{\cdot 2}} \\ -\frac{3 i \sqrt{2}\, k}{(3+k^2)^2 t_{\cdot 2}} & \frac{6}{(3+k^2)^2 t_{\cdot 2}} & \frac{3 \sqrt{2}}{(3+k^2)^2 t_{\cdot 2}} \\ -\frac{3 i k}{(3+k^2)^2 t_{\cdot 2}} & \frac{3 \sqrt{2}}{(3+k^2)^2 t_{\cdot 2}} & \frac{3}{(3+k^2)^2 t_{\cdot 2}} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\cdot 2}}{r_{\cdot 2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\cdot 2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{\underset{2}{\cdot}} < 0 \;\&\&\; t_{\underset{2}{\cdot}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\underset{2}{\cdot}} < 0 \;\&\&\; t_{\underset{2}{\cdot}} > 0$

Okay, that concludes the analysis of this theory.

Case 32

Now for a new theory. Here is the full nonlinear Lagrangian for Case 32 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\underset{2}{\cdot}} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{\underset{2}{\cdot}} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \frac{1}{6} r_{\underset{2}{\cdot}} \mathcal{R}^{ijkl} \mathcal{R}_{klij} + \frac{1}{12}\left(4 t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{6}\left(2 t_{\underset{1}{\cdot}} - t_{\underset{2}{\cdot}}\right) \mathcal{T}^{ijk} \mathcal{T}_{jik} + \frac{1}{3} t_{\underset{1}{\cdot}} \mathcal{T}^{i\;\;j}_{\;\;i} \mathcal{T}^{k}_{\;\;jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\left(t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \mathcal{A}_{\alpha\alpha' i} \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3}\left(t_{\underset{1}{\cdot}} - 2 t_{\underset{2}{\cdot}}\right) \mathcal{A}_{\alpha i \alpha'} \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3} t_{\underset{1}{\cdot}} \mathcal{A}^{\alpha\alpha'}_{\;\;\;\;\alpha} \mathcal{A}_{\alpha'\;\;i}^{\;\;i} - \frac{2}{3} t_{\underset{1}{\cdot}} \mathcal{A}_{\alpha'\;\;i}^{\;\;i} \partial_{\alpha} f^{\alpha\alpha'} + \frac{2}{3} t_{\underset{1}{\cdot}} \mathcal{A}_{\alpha'\;\;i}^{\;\;i} \partial^{\alpha'} f^{\alpha}_{\;\;\alpha} - \frac{1}{3} t_{\underset{1}{\cdot}} \partial_{\alpha'} f^{i}_{\;\;i} \partial^{\alpha'} f^{\alpha}_{\;\;\alpha} - \frac{1}{3} t_{\underset{1}{\cdot}} \partial_{\alpha} f^{\alpha\alpha'} \partial_{i} f^{i}_{\;\;\alpha'} + \frac{2}{3} t_{\underset{1}{\cdot}} \partial^{\alpha'} f^{\alpha}_{\;\;\alpha} \partial_{i} f^{i}_{\;\;\alpha'} - \frac{2}{3}\left(t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \mathcal{A}_{\alpha\alpha' i} \partial^{i} f^{\alpha\alpha'} + \frac{2}{3}\left(t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \mathcal{A}_{\alpha i \alpha'} \partial^{i} f^{\alpha\alpha'} + \frac{2}{3}\left(2 t_{\underset{1}{\cdot}} - t_{\underset{2}{\cdot}}\right) \mathcal{A}_{\alpha' i \alpha} \partial^{i} f^{\alpha\alpha'} + \frac{1}{3}\left(-2 t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \partial_{\alpha} f_{\alpha' i} \partial^{i} f^{\alpha\alpha'} + \frac{1}{6}\left(2 t_{\underset{1}{\cdot}} - t_{\underset{2}{\cdot}}\right) \partial_{\alpha} f_{i\alpha'} \partial^{i} f^{\alpha\alpha'} + \frac{1}{6}\left(-4 t_{\underset{1}{\cdot}} - t_{\underset{2}{\cdot}}\right) \partial_{\alpha'} f_{\alpha i} \partial^{i} f^{\alpha\alpha'} + \frac{1}{6}\left(4 t_{\underset{1}{\cdot}} + t_{\underset{2}{\cdot}}\right) \partial_{i} f_{\alpha\alpha'} \partial^{i} f^{\alpha\alpha'} + \frac{1}{6}\left(2 t_{\underset{1}{\cdot}} - t_{\underset{2}{\cdot}}\right) \partial_{i} f_{\alpha'\alpha} \partial^{i} f^{\alpha\alpha'} + \frac{4}{3} r_{\underset{2}{\cdot}} \partial_{\alpha'} \mathcal{A}_{\alpha ij} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_{\underset{2}{\cdot}} \partial_{\alpha'} \mathcal{A}_{\alpha ji} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{2}{3} r_{\underset{2}{\cdot}} \partial_{\alpha'} \mathcal{A}_{ij\alpha} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{1}{3} r_{\underset{2}{\cdot}} \partial_{i} \mathcal{A}_{\alpha\alpha' j} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3} r_{\underset{2}{\cdot}} \partial_{j} \mathcal{A}_{\alpha\alpha' i} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_{\underset{2}{\cdot}} \partial_{j} \mathcal{A}_{\alpha i \alpha'} \partial^{j} \mathcal{A}^{\alpha\alpha' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 r_{2} + t_{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{3} k^2 \left(t_{1} + t_{2}\right) & -\frac{i k \left(t_{1} - 2 t_{2}\right)}{3 \sqrt{2}} & \frac{1}{3} i k \left(t_{1} + t_{2}\right) \\ \frac{i k \left(t_{1} - 2 t_{2}\right)}{3 \sqrt{2}} & \frac{1}{6} \left(t_{1} + 4 t_{2}\right) & \frac{-t_{1} + 2 t_{2}}{3 \sqrt{2}} \\ -\frac{1}{3} i k \left(t_{1} + t_{2}\right) & \frac{-t_{1} + 2 t_{2}}{3 \sqrt{2}} & \frac{t_{1} + t_{2}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2 k^2 t_{1}}{3} & -\frac{1}{3} i k t_{1} & 0 & -\frac{1}{3} i \sqrt{2} k t_{1} \\ \frac{i k t_{1}}{3} & \frac{t_{1}}{6} & 0 & \frac{t_{1}}{3 \sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2} k t_{1} & \frac{t_{1}}{3 \sqrt{2}} & 0 & \frac{t_{1}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \begin{pmatrix} \frac{t_{1}}{2} \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}\tau^{\flat\perp} == 0,\ {}^{0^+}\sigma^{\flat\parallel} == 0,\ {}^{0^+}\tau^{\flat\parallel} == 0,\ -i\ {}^{1^+}\tau^{\flat\parallel\, ab} == k\ {}^{1^+}\sigma^{\flat\perp\, ab},\right.$$

$$\left. i\ {}^{1^-}\tau^{\flat\parallel\, a} == 2 k\ {}^{1^-}\sigma^{\flat\perp\, a},\ {}^{1^-}\tau^{\flat\perp\, a} == 0,\ i\ {}^{1^-}\tau^{\flat\parallel\, a} == 2 k\ {}^{1^-}\sigma^{\flat\parallel\, a},\ -i\ {}^{2^+}\tau^{\flat\parallel\, ab} == 2 k\ {}^{2^+}\sigma^{\flat\parallel\, ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*–matrices, which are functionally analogous to the inverse *b*–matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_{2} + t_{2}} \end{pmatrix}, \begin{pmatrix} \frac{k^2 \left(t_{1} + 4 t_{2}\right)}{3 \left(1 + k^2\right)^2 t_{1} t_{2}} & \frac{i \sqrt{2} k \left(t_{1} - 2 t_{2}\right)}{3 \left(1 + k^2\right) t_{1} t_{2}} & \frac{i k \left(t_{1} + 4 t_{2}\right)}{3 \left(1 + k^2\right)^2 t_{1} t_{2}} \\ -\frac{i \sqrt{2} k \left(t_{1} - 2 t_{2}\right)}{3 \left(1 + k^2\right) t_{1} t_{2}} & \frac{2 \left(t_{1} + t_{2}\right)}{3 t_{1} t_{2}} & \frac{\sqrt{2} \left(t_{1} - 2 t_{2}\right)}{3 \left(1 + k^2\right) t_{1} t_{2}} \\ -\frac{i k \left(t_{1} + 4 t_{2}\right)}{3 \left(1 + k^2\right)^2 t_{1} t_{2}} & \frac{\sqrt{2} \left(t_{1} - 2 t_{2}\right)}{3 \left(1 + k^2\right) t_{1} t_{2}} & \frac{t_{1} + 4 t_{2}}{3 \left(1 + k^2\right)^2 t_{1} t_{2}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{24 k^2}{\left(3 + 4 k^2\right)^2 t_{1}} & -\frac{12 i k}{\left(3 + 4 k^2\right)^2 t_{1}} & 0 & -\frac{12 i \sqrt{2} k}{\left(3 + 4 k^2\right)^2 t_{1}} \\ \frac{12 i k}{\left(3 + 4 k^2\right)^2 t_{1}} & \frac{6}{\left(3 + 4 k^2\right)^2 t_{1}} & 0 & \frac{6 \sqrt{2}}{\left(3 + 4 k^2\right)^2 t_{1}} \\ 0 & 0 & 0 & 0 \\ \frac{12 i \sqrt{2} k}{\left(3 + 4 k^2\right)^2 t_{1}} & \frac{6 \sqrt{2}}{\left(3 + 4 k^2\right)^2 t_{1}} & 0 & \frac{12}{\left(3 + 4 k^2\right)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{4 k^2}{\left(1 + 2 k^2\right)^2 t_{1}} & \frac{2 i \sqrt{2} k}{\left(1 + 2 k^2\right)^2 t_{1}} \\ -\frac{2 i \sqrt{2} k}{\left(1 + 2 k^2\right)^2 t_{1}} & \frac{2}{\left(1 + 2 k^2\right)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{t_{1}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$r_{2} < 0 \,\&\&\, t_{2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{2} < 0 \,\&\&\, t_{2} > 0$

Okay, that concludes the analysis of this theory.

Case 33

Now for a new theory. Here is the full nonlinear Lagrangian for Case 33 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ij\mathfrak{k}l} \mathcal{R}^{ij\mathfrak{k}l} - \frac{2}{3} r_{2} \mathcal{R}_{i\mathfrak{k}jl} \mathcal{R}^{ij\mathfrak{k}l} + \frac{1}{6} r_{2} \mathcal{R}^{ij\mathfrak{k}l} \mathcal{R}_{\mathfrak{k}lij} + \frac{1}{4} t_{1} \mathcal{T}_{ij\mathfrak{k}} \mathcal{T}^{ij\mathfrak{k}} + \frac{1}{2} t_{1} \mathcal{T}^{ij\mathfrak{k}} \mathcal{T}_{ji\mathfrak{k}} + \frac{1}{3} t_{1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{\mathfrak{k}}{}_{j\mathfrak{k}}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{1} \mathcal{A}_{aia'} \mathcal{A}^{aa'i} + \frac{1}{3} t_{1} \mathcal{A}^{aa'}{}_{a} \mathcal{A}_{a'}{}^{i}{}_{i} - \frac{2}{3} t_{1} \mathcal{A}_{a'}{}^{i}{}_{i} \partial_{a} f^{aa'} + \frac{2}{3} t_{1} \mathcal{A}_{a'}{}^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - \frac{1}{3} t_{1} \partial_{a'} f^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - \frac{1}{3} t_{1} \partial_{a} f^{aa'} \partial_{i} f^{i}{}_{a'} + \frac{2}{3} t_{1} \partial^{a'} f^{a}{}_{a} \partial_{i} f^{i}{}_{a'} + 2 t_{1} \mathcal{A}_{a'ia} \partial^{i} f^{aa'} - t_{1} \partial_{a} f_{a'i} \partial^{i} f^{aa'} + \frac{1}{2} t_{1} \partial_{a} f_{ia'} \partial^{i} f^{aa'} - \frac{1}{2} t_{1} \partial_{a'} f_{ai} \partial^{i} f^{aa'} + \frac{1}{2} t_{1} \partial_{i} f_{aa'} \partial^{i} f^{aa'} + \frac{1}{2} t_{1} \partial_{i} f_{a'a} \partial^{i} f^{aa'} + \frac{4}{3} r_{2} \partial_{a'} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{2} \partial_{a'} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{2} \partial_{a'} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{aa'i} - \frac{1}{3} r_{2} \partial_{i} \mathcal{A}_{aa'j} \partial^{j} \mathcal{A}^{aa'i} + \frac{1}{3} r_{2} \partial_{j} \mathcal{A}_{aa'i} \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{2} \partial_{j} \mathcal{A}_{aia'} \partial^{j} \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{2} - t_{1}\right), \begin{pmatrix} 0 & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & -\frac{t_{1}}{2} & -\frac{t_{1}}{\sqrt{2}} \\ 0 & -\frac{t_{1}}{\sqrt{2}} & 0 \end{pmatrix}, \begin{pmatrix} \frac{2 k^2 t_{1}}{3} & -\frac{1}{3} i k t_{1} & 0 & -\frac{1}{3} i \sqrt{2}\, k t_{1} \\ \frac{i k t_{1}}{3} & \frac{t_{1}}{6} & 0 & \frac{t_{1}}{3 \sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{1} & \frac{t_{1}}{3 \sqrt{2}} & 0 & \frac{t_{1}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \left(\frac{t_{1}}{2}\right)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}\tau^{\flat\perp} == 0,\ {}^{0^+}\sigma^{\flat\parallel} == 0,\ {}^{0^+}\tau^{\flat\parallel} == 0,\ -i\,{}^{1^+}\tau^{\flat\parallel\,ab} == k\,{}^{1^+}\sigma^{\flat\perp\,ab},\right.$$
$$\left. i\,{}^{1^-}\tau^{\flat\parallel\,a} == 2\,k\,{}^{1^-}\sigma^{\flat\perp\,a},\ {}^{1^-}\tau^{\flat\perp\,a} == 0,\ i\,{}^{1^-}\tau^{\flat\parallel\,a} == 2\,k\,{}^{1^-}\sigma^{\flat\parallel\,a},\ -i\,{}^{2^+}\tau^{\flat\parallel\,ab} == 2\,k\,{}^{2^+}\sigma^{\flat\parallel\,ab}\right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_{2} - t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{k^2}{(1+k^2)^2 t_{1}} & -\frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & \frac{i\,k}{(1+k^2)^2 t_{1}} \\ \frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} \\ -\frac{i\,k}{(1+k^2)^2 t_{1}} & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{1}{(1+k^2)^2 t_{1}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{24\,k^2}{(3+4\,k^2)^2 t_{1}} & -\frac{12\,i\,k}{(3+4\,k^2)^2 t_{1}} & 0 & -\frac{12\,i\sqrt{2}\,k}{(3+4\,k^2)^2 t_{1}} \\ \frac{12\,i\,k}{(3+4\,k^2)^2 t_{1}} & \frac{6}{(3+4\,k^2)^2 t_{1}} & 0 & \frac{6\sqrt{2}}{(3+4\,k^2)^2 t_{1}} \\ 0 & 0 & 0 & 0 \\ \frac{12\,i\sqrt{2}\,k}{(3+4\,k^2)^2 t_{1}} & \frac{6\sqrt{2}}{(3+4\,k^2)^2 t_{1}} & 0 & \frac{12}{(3+4\,k^2)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{4\,k^2}{(1+2\,k^2)^2 t_{1}} & \frac{2\,i\sqrt{2}\,k}{(1+2\,k^2)^2 t_{1}} \\ -\frac{2\,i\sqrt{2}\,k}{(1+2\,k^2)^2 t_{1}} & \frac{2}{(1+2\,k^2)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{t_{1}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_{1}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{2} < 0\ \&\&\ t_{1} < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{2} < 0\ \&\&\ t_{1} < 0$

Okay, that concludes the analysis of this theory.

Case 34

Now for a new theory. Here is the full nonlinear Lagrangian for
Case 34 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\cdot 2}\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} - \frac{2}{3} r_{\cdot 2}\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} + r_{\cdot 3}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6}\left(r_{\cdot 2} - 6 r_{\cdot 3}\right) \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + 3 r_{\cdot 3}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12} t_{\cdot 2}\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} - \frac{1}{6} t_{\cdot 2}\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik}$$

To use PSALTer, you have to first linearise
this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{\cdot 2}\, \mathcal{A}_{aa'i}\, \mathcal{A}^{aa'i} - \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{aia'}\, \mathcal{A}^{aa'i} - 3 r_{\cdot 3}\, \partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{aa'}{}_{a} - r_{\cdot 3}\, \partial_{i}\mathcal{A}_{a'}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{aa'}{}_{a} - \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{aa'i}\, \partial^{i} f^{aa'} + \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{aia'}\, \partial^{i} f^{aa'} - \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{a'ia}\, \partial^{i} f^{aa'} + \frac{1}{3} t_{\cdot 2}\, \partial_{a} f_{a'i}\, \partial^{i} f^{aa'} - \frac{1}{6} t_{\cdot 2}\, \partial_{a} f_{ia'}\, \partial^{i} f^{aa'} - \frac{1}{6} t_{\cdot 2}\, \partial_{a'} f_{ai}\, \partial^{i} f^{aa'} + \frac{1}{6} t_{\cdot 2}\, \partial_{i} f_{aa'}\, \partial^{i} f^{aa'} - \frac{1}{6} t_{\cdot 2}\, \partial_{i} f_{a'a}\, \partial^{i} f^{aa'} - r_{\cdot 3}\, \partial_{a}\mathcal{A}^{aa'i}\, \partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i} + 2 r_{\cdot 3}\, \partial^{i}\mathcal{A}^{aa'}{}_{a}\, \partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i} - 3 r_{\cdot 3}\, \partial_{a}\mathcal{A}^{aa'i}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'} + 6 r_{\cdot 3}\, \partial^{i}\mathcal{A}^{aa'}{}_{a}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'} + \frac{4}{3} r_{\cdot 2}\, \partial_{a'}\mathcal{A}_{aij}\, \partial^{j}\mathcal{A}^{aa'i} - \frac{2}{3} r_{\cdot 2}\, \partial_{a'}\mathcal{A}_{aji}\, \partial^{j}\mathcal{A}^{aa'i} + \frac{2}{3}\left(r_{\cdot 2} - 6 r_{\cdot 3}\right) \partial_{a'}\mathcal{A}_{ija}\, \partial^{j}\mathcal{A}^{aa'i} - \frac{1}{3} r_{\cdot 2}\, \partial_{i}\mathcal{A}_{aa'j}\, \partial^{j}\mathcal{A}^{aa'i} + \frac{1}{3} r_{\cdot 2}\, \partial_{j}\mathcal{A}_{aa'i}\, \partial^{j}\mathcal{A}^{aa'i} - \frac{2}{3} r_{\cdot 2}\, \partial_{j}\mathcal{A}_{aia'}\, \partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 6 k^2 r_{\cdot 3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 r_{\cdot 2} + t_{\cdot 2} \end{pmatrix}, \begin{pmatrix} \frac{k^2 t_{\cdot 2}}{3} & \frac{1}{3} i \sqrt{2}\, k\, t_{\cdot 2} & \frac{i k t_{\cdot 2}}{3} \\ -\frac{1}{3} i \sqrt{2}\, k\, t_{\cdot 2} & \frac{1}{2}\left(2 k^2 r_{\cdot 3} + \frac{4 t_{\cdot 2}}{3}\right) & \frac{\sqrt{2}\, t_{\cdot 2}}{3} \\ -\frac{1}{3} i k\, t_{\cdot 2} & \frac{\sqrt{2}\, t_{\cdot 2}}{3} & \frac{t_{\cdot 2}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & k^2 r_{\cdot 3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\cdot}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\cdot}\tau^{\flat\parallel} == 0,\ -i\, {}^{1^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == k\, {}^{1^+}_{\cdot}\sigma^{\flat\perp}{}^{ab},\ {}^{1^-}_{\cdot}\sigma^{\flat\perp}{}^{a} == 0,\ {}^{1^-}_{\cdot}\tau^{\flat\perp}{}^{a} == 0,\ {}^{1^-}_{\cdot}\tau^{\flat\parallel}{}^{a} == 0,\ {}^{2^+}_{\cdot}\sigma^{\flat\parallel}{}^{ab} == 0,\ {}^{2^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == 0,\ {}^{2^-}_{\cdot}\sigma^{\flat\parallel}{}^{abc} == 0\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{6 k^2 r_{\cdot 3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_{\cdot 2} + t_{\cdot 2}} \end{pmatrix}, \begin{pmatrix} \frac{3 k^2 r_{\cdot 3} + 2 t_{\cdot 2}}{(1+k^2)^2 r_{\cdot 3} t_{\cdot 2}} & -\frac{i \sqrt{2}}{k r_{\cdot 3} + k^3 r_{\cdot 3}} & \frac{i\left(3 k^2 r_{\cdot 3} + 2 t_{\cdot 2}\right)}{k (1+k^2)^2 r_{\cdot 3} t_{\cdot 2}} \\ \frac{i \sqrt{2}}{k r_{\cdot 3} + k^3 r_{\cdot 3}} & \frac{1}{k^2 r_{\cdot 3}} & -\frac{\sqrt{2}}{k^2 r_{\cdot 3} + k^4 r_{\cdot 3}} \\ -\frac{i\left(3 k^2 r_{\cdot 3} + 2 t_{\cdot 2}\right)}{k (1+k^2)^2 r_{\cdot 3} t_{\cdot 2}} & -\frac{\sqrt{2}}{k^2 r_{\cdot 3} + k^4 r_{\cdot 3}} & \frac{3 k^2 r_{\cdot 3} + 2 t_{\cdot 2}}{(k+k^3)^2 r_{\cdot 3} t_{\cdot 2}} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{k^2 r_{\cdot 3}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{\dot{t}_2}{\dot{r}_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{\dot{r}_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$\dot{r}_2 < 0 \,\&\&\, \dot{t}_2 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$\dot{r}_2 < 0 \,\&\&\, \dot{t}_2 > 0$

Okay, that concludes the analysis of this theory.

Case 35

Now for a new theory. Here is the full nonlinear Lagrangian for Case 35 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\dot{r}_2\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} - \frac{2}{3}\dot{r}_2\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} - \frac{3}{2}\dot{r}_3\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6}\left(\dot{r}_2 - 6\dot{r}_3\right)\mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + \frac{5}{2}\dot{r}_3\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12}\dot{t}_2\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} - \frac{1}{6}\dot{t}_2\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3}\dot{t}_2\, \mathcal{A}_{\alpha\alpha' i}\, \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3}\dot{t}_2\, \mathcal{A}_{\alpha i\alpha'}\, \mathcal{A}^{\alpha\alpha' i} - \frac{5}{2}\dot{r}_3\, \partial_{\alpha'}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha} + \frac{3}{2}\dot{r}_3\, \partial_{i}\mathcal{A}_{\alpha'}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha} - \\
&\frac{2}{3}\dot{t}_2\, \mathcal{A}_{\alpha\alpha' i}\, \partial^{i}f^{\alpha\alpha'} + \frac{2}{3}\dot{t}_2\, \mathcal{A}_{\alpha i\alpha'}\, \partial^{i}f^{\alpha\alpha'} - \frac{2}{3}\dot{t}_2\, \mathcal{A}_{\alpha' i\alpha}\, \partial^{i}f^{\alpha\alpha'} + \frac{1}{3}\dot{t}_2\, \partial_{\alpha}f_{\alpha' i}\, \partial^{i}f^{\alpha\alpha'} - \frac{1}{6}\dot{t}_2\, \partial_{\alpha}f_{i\alpha'}\, \partial^{i}f^{\alpha\alpha'} - \\
&\frac{1}{6}\dot{t}_2\, \partial_{\alpha'}f_{\alpha i}\, \partial^{i}f^{\alpha\alpha'} + \frac{1}{6}\dot{t}_2\, \partial_{i}f_{\alpha\alpha'}\, \partial^{i}f^{\alpha\alpha'} - \frac{1}{6}\dot{t}_2\, \partial_{i}f_{\alpha'\alpha}\, \partial^{i}f^{\alpha\alpha'} + \frac{3}{2}\dot{r}_3\, \partial_{\alpha}\mathcal{A}^{\alpha\alpha' i}\, \partial_{j}\mathcal{A}_{\alpha'}{}^{j}{}_{i} - 3\dot{r}_3\, \partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \partial_{j}\mathcal{A}_{\alpha'}{}^{j}{}_{i} - \\
&\frac{5}{2}\dot{r}_3\, \partial_{\alpha}\mathcal{A}^{\alpha\alpha' i}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{\alpha'} + 5\dot{r}_3\, \partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{\alpha'} + \frac{4}{3}\dot{r}_2\, \partial_{\alpha'}\mathcal{A}_{\alpha ij}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i} - \frac{2}{3}\dot{r}_2\, \partial_{\alpha'}\mathcal{A}_{\alpha ji}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i} + \\
&\frac{2}{3}\left(\dot{r}_2 - 6\dot{r}_3\right)\partial_{\alpha'}\mathcal{A}_{ij\alpha}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i} - \frac{1}{3}\dot{r}_2\, \partial_{i}\mathcal{A}_{\alpha\alpha' j}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i} + \frac{1}{3}\dot{r}_2\, \partial_{j}\mathcal{A}_{\alpha\alpha' i}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i} - \frac{2}{3}\dot{r}_2\, \partial_{j}\mathcal{A}_{\alpha i\alpha'}\, \partial^{j}\mathcal{A}^{\alpha\alpha' i}
\end{aligned}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{\dot{2}} + t_{\dot{2}}\right), \begin{pmatrix} \frac{k^2 t_{\dot{2}}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{\dot{2}} & \frac{i k t_{\dot{2}}}{3} \\ -\frac{1}{3} i \sqrt{2}\, k t_{\dot{2}} & \frac{2 t_{\dot{2}}}{3} & \frac{\sqrt{2}\, t_{\dot{2}}}{3} \\ -\frac{1}{3} i k t_{\dot{2}} & \frac{\sqrt{2}\, t_{\dot{2}}}{3} & \frac{t_{\dot{2}}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{3 k^2 r_{\dot{3}}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{3 k^2 r_{\dot{3}}}{2} \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\bullet}\sigma^{\flat\parallel} == 0,\ {}^{0^+}_{\bullet}\tau^{\flat\parallel} == 0,\ -i\ {}^{1^+}_{\bullet}\tau^{\flat\parallel\,ab} == k\ {}^{1^+}_{\bullet}\sigma^{\flat\perp\,ab},\right.$$
$$\left. -i\ {}^{1^+}_{\bullet}\tau^{\flat\parallel\,ab} == k\ {}^{1^+}_{\bullet}\sigma^{\flat\parallel\,ab},\ {}^{1^-}_{\bullet}\sigma^{\flat\perp\,a} == 0,\ {}^{1^-}_{\bullet}\tau^{\flat\perp\,a} == 0,\ {}^{1^-}_{\bullet}\tau^{\flat\parallel\,a} == 0,\ {}^{2^+}_{\bullet}\tau^{\flat\parallel\,ab} == 0,\ {}^{2^-}_{\bullet}\sigma^{\flat\parallel\,abc} == 0 \right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{\dot{2}} + t_{\dot{2}}}\right), \begin{pmatrix} \frac{3 k^2}{(3+k^2)^2 t_{\dot{2}}} & \frac{3 i \sqrt{2}\, k}{(3+k^2)^2 t_{\dot{2}}} & \frac{3 i k}{(3+k^2)^2 t_{\dot{2}}} \\ -\frac{3 i \sqrt{2}\, k}{(3+k^2)^2 t_{\dot{2}}} & \frac{6}{(3+k^2)^2 t_{\dot{2}}} & \frac{3 \sqrt{2}}{(3+k^2)^2 t_{\dot{2}}} \\ -\frac{3 i k}{(3+k^2)^2 t_{\dot{2}}} & \frac{3 \sqrt{2}}{(3+k^2)^2 t_{\dot{2}}} & \frac{3}{(3+k^2)^2 t_{\dot{2}}} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{2}{3 k^2 r_{\dot{3}}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3 k^2 r_{\dot{3}}} \end{pmatrix}, (0)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\dot{2}}}{r_{\dot{2}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\dot{2}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{\dot{2}} < 0 \,\&\&\, t_{\dot{2}} > 0$

So, that's the end of the PSALTer output for this theory. You
can check the particle content against TABLE IV. in arXiv:1910.14197.
If you take the overall unitarity conditions from the final column in
TABLE V., and decompose them using Mathematica's Reduce function, you
get the following (to be compared with the PSALTer conditions above):

$r_{\dot{2}} < 0 \,\&\&\, t_{\dot{2}} > 0$

Okay, that concludes the analysis of this theory.

Case 36

Now for a new theory. Here is the full nonlinear Lagrangian for
Case 36 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\left(2\dot{r}_1+\dot{r}_2\right)\mathcal{R}_{ij\hbar l}\ \mathcal{R}^{ij\hbar l}+\frac{2}{3}\left(\dot{r}_1-\dot{r}_2\right)\mathcal{R}_{i\hbar jl}\ \mathcal{R}^{ij\hbar l}-2\dot{r}_1\,\mathcal{R}^{ij\ \hbar}_{\ \ i}\ \mathcal{R}_{j\ \hbar l}^{\ l}+\frac{1}{6}\left(2\dot{r}_1+\dot{r}_2-6\dot{r}_3\right)\mathcal{R}^{ij\hbar l}\ \mathcal{R}_{\hbar lij}+\left(-2\dot{r}_1+4\dot{r}_3\right)\mathcal{R}^{ij\ \hbar}_{\ \ i}\ \mathcal{R}_{\hbar\ jl}^{\ l}+\frac{1}{12}\dot{t}_2\,\mathcal{T}_{ij\hbar}\ \mathcal{T}^{ij\hbar}-\frac{1}{6}\dot{t}_2\,\mathcal{T}^{ij\hbar}\ \mathcal{T}_{ji\hbar}$$

To use PSALTer, you have to first linearise
this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\dot{t}_2\,\mathcal{A}_{aa'i}\ \mathcal{A}^{aa'i}-\frac{2}{3}\dot{t}_2\,\mathcal{A}_{aia'}\ \mathcal{A}^{aa'i}+2\left(\dot{r}_1-2\dot{r}_3\right)\partial_{a'}\mathcal{A}_{i\ \,j}^{\ j}\ \partial^{i}\mathcal{A}^{aa'}{}_{a}+2\dot{r}_1\,\partial_{i}\mathcal{A}_{a'\ \,j}^{\ j}\ \partial^{i}\mathcal{A}^{aa'}{}_{a}-\frac{2}{3}\dot{t}_2\,\mathcal{A}_{aa'i}\ \partial^{i}f^{aa'}+\frac{2}{3}\dot{t}_2\,\mathcal{A}_{aia'}\ \partial^{i}f^{aa'}-\frac{2}{3}\dot{t}_2\,\mathcal{A}_{a'ia}\ \partial^{i}f^{aa'}+\frac{1}{3}\dot{t}_2\,\partial_{a}f_{a'i}\ \partial^{i}f^{aa'}-\frac{1}{6}\dot{t}_2\,\partial_{a}f_{ia'}\ \partial^{i}f^{aa'}-\frac{1}{6}\dot{t}_2\,\partial_{a'}f_{ai}\ \partial^{i}f^{aa'}+\frac{1}{6}\dot{t}_2\,\partial_{i}f_{aa'}\ \partial^{i}f^{aa'}-\frac{1}{6}\dot{t}_2\,\partial_{i}f_{a'a}\ \partial^{i}f^{aa'}+2\dot{r}_1\,\partial_{a}\mathcal{A}^{aa'i}\ \partial_{j}\mathcal{A}_{a'\ \,i}^{\ j}-4\dot{r}_1\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\ \partial_{j}\mathcal{A}_{a'\ \,i}^{\ j}+2\left(\dot{r}_1-2\dot{r}_3\right)\partial_{a}\mathcal{A}^{aa'i}\ \partial_{j}\mathcal{A}_{i\ \,a'}^{\ j}+\left(-4\dot{r}_1+8\dot{r}_3\right)\partial^{i}\mathcal{A}^{aa'}{}_{a}\ \partial_{j}\mathcal{A}_{i\ \,a'}^{\ j}-\frac{4}{3}\left(\dot{r}_1-\dot{r}_2\right)\partial_{a'}\mathcal{A}_{aij}\ \partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\left(\dot{r}_1-\dot{r}_2\right)\partial_{a'}\mathcal{A}_{aji}\ \partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\left(2\dot{r}_1+\dot{r}_2-6\dot{r}_3\right)\partial_{a'}\mathcal{A}_{ija}\ \partial^{j}\mathcal{A}^{aa'i}+\frac{1}{3}\left(-2\dot{r}_1-\dot{r}_2\right)\partial_{i}\mathcal{A}_{aa'j}\ \partial^{j}\mathcal{A}^{aa'i}+\frac{1}{3}\left(2\dot{r}_1+\dot{r}_2\right)\partial_{j}\mathcal{A}_{aa'i}\ \partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\left(\dot{r}_1-\dot{r}_2\right)\partial_{j}\mathcal{A}_{aia'}\ \partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}0&0&0\\0&6k^2\left(-\dot{r}_1+\dot{r}_3\right)&0\\0&0&0\end{pmatrix},\ \left(k^2\dot{r}_2+\dot{t}_2\right),\ \begin{pmatrix}\frac{k^2\dot{t}_2}{3}&\frac{1}{3}i\sqrt{2}\,k\dot{t}_2&\frac{ik\dot{t}_2}{3}\\-\frac{1}{3}i\sqrt{2}\,k\dot{t}_2&\frac{2\dot{t}_2}{3}&\frac{\sqrt{2}\,\dot{t}_2}{3}\\-\frac{1}{3}ik\dot{t}_2&\frac{\sqrt{2}\,\dot{t}_2}{3}&\frac{\dot{t}_2}{3}\end{pmatrix},\ \begin{pmatrix}0&0&0&0\\0&-k^2\dot{r}_1&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\ \begin{pmatrix}0&0\\0&0\end{pmatrix},\ \left(k^2\dot{r}_1\right)\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\cdot}\tau^{\flat\perp}==0,\ \overset{0^+}{\cdot}\tau^{\flat\parallel}==0,\ -i\ \overset{1^+}{\cdot}\tau^{\flat\parallel}{}^{ab}==k\ \overset{1^+}{\cdot}\sigma^{\flat\perp}{}^{ab},\ -i\ \overset{1^+}{\cdot}\tau^{\flat\parallel}{}^{ab}==k\ \overset{1^+}{\cdot}\sigma^{\flat\parallel}{}^{ab},\ \overset{1^-}{\cdot}\sigma^{\flat\perp}{}^{a}==0,\ \overset{1^-}{\cdot}\tau^{\flat\perp}{}^{a}==0,\ \overset{1^-}{\cdot}\tau^{\flat\parallel}{}^{a}==0,\ \overset{2^+}{\cdot}\sigma^{\flat\parallel}{}^{ab}==0,\ \overset{2^+}{\cdot}\tau^{\flat\parallel}{}^{ab}==0\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}0&0&0\\0&\frac{1}{6k^2\left(-\dot{r}_1+\dot{r}_3\right)}&0\\0&0&0\end{pmatrix},\ \left(\frac{1}{k^2\dot{r}_2+\dot{t}_2}\right),\ \begin{pmatrix}\frac{3k^2}{(3+k^2)^2\dot{t}_2}&\frac{3i\sqrt{2}\,k}{(3+k^2)^2\dot{t}_2}&\frac{3ik}{(3+k^2)^2\dot{t}_2}\\-\frac{3i\sqrt{2}\,k}{(3+k^2)^2\dot{t}_2}&\frac{6}{(3+k^2)^2\dot{t}_2}&\frac{3\sqrt{2}}{(3+k^2)^2\dot{t}_2}\\-\frac{3ik}{(3+k^2)^2\dot{t}_2}&\frac{3\sqrt{2}}{(3+k^2)^2\dot{t}_2}&\frac{3}{(3+k^2)^2\dot{t}_2}\end{pmatrix},\ \begin{pmatrix}0&0&0&0\\0&-\frac{1}{k^2\dot{r}_1}&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix},\ \begin{pmatrix}0&0\\0&0\end{pmatrix},\ \left(\frac{1}{k^2\dot{r}_1}\right)\right\}$$

Square masses:

$$\left\{\{\},\left\{-\frac{t_{\cdot 2}}{r_{\cdot 2}}\right\},\{\},\{\},\{\},\{\}\right\}$$

Massive pole residues:

$$\left\{\{\},\left\{-\frac{1}{r_{\cdot 2}}\right\},\{\},\{\},\{\},\{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{\cdot 2} < 0 \,\&\&\, t_{\cdot 2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\cdot 2} < 0 \,\&\&\, t_{\cdot 2} > 0$

Okay, that concludes the analysis of this theory.

Case 37

Now for a new theory. Here is the full nonlinear Lagrangian for Case 37 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\cdot 2}\, \mathcal{R}_{ij\hbar l}\, \mathcal{R}^{ij\hbar l} - \frac{2}{3} r_{\cdot 2}\, \mathcal{R}_{i\hbar jl}\, \mathcal{R}^{ij\hbar l} - \frac{3}{2} r_{\cdot 3}\, \mathcal{R}^{ij}{}_{i}{}^{\hbar}\, \mathcal{R}_{j}{}^{l}{}_{\hbar l} + \frac{1}{6}\left(r_{\cdot 2} - 6 r_{\cdot 3}\right) \mathcal{R}^{ij\hbar l}\, \mathcal{R}_{\hbar l ij} + \frac{5}{2} r_{\cdot 3}\, \mathcal{R}^{ij}{}_{i}{}^{\hbar}\, \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{12} t_{\cdot 2}\, \mathcal{T}_{ij\hbar}\, \mathcal{T}^{ij\hbar} - \frac{1}{6} t_{\cdot 2}\, \mathcal{T}^{ij\hbar}\, \mathcal{T}_{ji\hbar} - \frac{2}{3} t_{\cdot 3}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{\hbar}{}_{j\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{\cdot 2}\, \mathcal{A}_{aa'i}\, \mathcal{A}^{aa'i} - \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{aia'}\, \mathcal{A}^{aa'i} - \frac{2}{3} t_{\cdot 3}\, \mathcal{A}^{aa'}{}_{a}\, \mathcal{A}_{a'}{}^{i}{}_{i} + \frac{4}{3} t_{\cdot 3}\, \mathcal{A}_{a'}{}^{i}{}_{i}\, \partial_{a} f^{aa'} - \frac{4}{3} t_{\cdot 3}\, \mathcal{A}_{a'}{}^{i}{}_{i}\, \partial^{a'} f^{a}{}_{a} + \frac{2}{3} t_{\cdot 3}\, \partial_{a'} f^{i}{}_{i}\, \partial^{a'} f^{a}{}_{a} + \frac{2}{3} t_{\cdot 3}\, \partial_{a} f^{aa'}\, \partial_{i} f^{i}{}_{a'} - \frac{4}{3} t_{\cdot 3}\, \partial^{a'} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{a'} - \frac{5}{2} r_{\cdot 3}\, \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{aa'}{}_{a} + \frac{3}{2} r_{\cdot 3}\, \partial_{i} \mathcal{A}_{a'}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{aa'}{}_{a} - \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{aa'i}\, \partial^{i} f^{aa'} + \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{aia'}\, \partial^{i} f^{aa'} - \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{a'ia}\, \partial^{i} f^{aa'} + \frac{1}{3} t_{\cdot 2}\, \partial_{a} f_{a'i}\, \partial^{i} f^{aa'} - \frac{1}{6} t_{\cdot 2}\, \partial_{a} f_{ia'}\, \partial^{i} f^{aa'} - \frac{1}{6} t_{\cdot 2}\, \partial_{a'} f_{ai}\, \partial^{i} f^{aa'} + \frac{1}{6} t_{\cdot 2}\, \partial_{i} f_{aa'}\, \partial^{i} f^{aa'} - \frac{1}{6} t_{\cdot 2}\, \partial_{i} f_{a'a}\, \partial^{i} f^{aa'} + \frac{3}{2} r_{\cdot 3}\, \partial_{a} \mathcal{A}^{aa'i}\, \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} - 3 r_{\cdot 3}\, \partial^{i} \mathcal{A}^{aa'}{}_{a}\, \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} - \frac{5}{2} r_{\cdot 3}\, \partial_{a} \mathcal{A}^{aa'i}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} + 5 r_{\cdot 3}\, \partial^{i} \mathcal{A}^{aa'}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} + \frac{4}{3} r_{\cdot 2}\, \partial_{a'} \mathcal{A}_{aij}\, \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{\cdot 2}\, \partial_{a'} \mathcal{A}_{aji}\, \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3}\left(r_{\cdot 2} - 6 r_{\cdot 3}\right) \partial_{a'} \mathcal{A}_{ija}\, \partial^{j} \mathcal{A}^{aa'i} - \frac{1}{3} r_{\cdot 2}\, \partial_{i} \mathcal{A}_{aa'j}\, \partial^{j} \mathcal{A}^{aa'i} + \frac{1}{3} r_{\cdot 2}\, \partial_{j} \mathcal{A}_{aa'i}\, \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{\cdot 2}\, \partial_{j} \mathcal{A}_{aia'}\, \partial^{j} \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2k^2 t_{\underset{3}{\cdot}} & i\sqrt{2}\,k\,t_{\underset{3}{\cdot}} & 0 \\ -i\sqrt{2}\,k\,t_{\underset{3}{\cdot}} & t_{\underset{3}{\cdot}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{\underset{2}{\cdot}} + t_{\underset{2}{\cdot}}\right), \begin{pmatrix} \frac{k^2 t_{\underset{2}{\cdot}}}{3} & \frac{1}{3} i\sqrt{2}\,k\,t_{\underset{2}{\cdot}} & \frac{i k t_{\underset{2}{\cdot}}}{3} \\ -\frac{1}{3} i\sqrt{2}\,k\,t_{\underset{2}{\cdot}} & \frac{2t_{\underset{2}{\cdot}}}{3} & \frac{\sqrt{2}\,t_{\underset{2}{\cdot}}}{3} \\ -\frac{1}{3} i k\,t_{\underset{2}{\cdot}} & \frac{\sqrt{2}\,t_{\underset{2}{\cdot}}}{3} & \frac{t_{\underset{2}{\cdot}}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2 t_{\underset{3}{\cdot}}}{3} & \frac{2 i k t_{\underset{3}{\cdot}}}{3} & 0 & -\frac{1}{3} i\sqrt{2}\,k\,t_{\underset{3}{\cdot}} \\ -\frac{2}{3} i k\,t_{\underset{3}{\cdot}} & \frac{1}{6}\left(-9k^2 r_{\underset{3}{\cdot}} + 4t_{\underset{3}{\cdot}}\right) & 0 & -\frac{\sqrt{2}\,t_{\underset{3}{\cdot}}}{3} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i\sqrt{2}\,k\,t_{\underset{3}{\cdot}} & -\frac{\sqrt{2}\,t_{\underset{3}{\cdot}}}{3} & 0 & \frac{t_{\underset{3}{\cdot}}}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{3k^2 r_{\underset{3}{\cdot}}}{2} \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\cdot}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}_{\cdot}\tau^{\flat\parallel} == 2k\,{}^{0^+}_{\cdot}\sigma^{\flat\parallel},\ -i\,{}^{1^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}_{\cdot}\sigma^{\flat\perp}{}^{ab},\right.$$
$$\left.-i\,{}^{1^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}_{\cdot}\sigma^{\flat\parallel}{}^{ab},\ i\,{}^{1^-}_{\cdot}\tau^{\flat\parallel}{}^{a} == 2k\,{}^{1^-}_{\cdot}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\cdot}\tau^{\flat\perp}{}^{a} == 0,\ {}^{2^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == 0,\ {}^{2^-}_{\cdot}\sigma^{\flat\parallel}{}^{abc} == 0\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_{\underset{3}{\cdot}}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\underset{3}{\cdot}}} & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\underset{3}{\cdot}}} & \frac{1}{(1+2k^2)^2 t_{\underset{3}{\cdot}}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{\underset{2}{\cdot}} + t_{\underset{2}{\cdot}}}\right), \begin{pmatrix} \frac{3k^2}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & \frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & \frac{3ik}{(3+k^2)^2 t_{\underset{2}{\cdot}}} \\ -\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & \frac{6}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{\underset{2}{\cdot}}} \\ -\frac{3ik}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & \frac{3}{(3+k^2)^2 t_{\underset{2}{\cdot}}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2\left(9k^2 r_{\underset{3}{\cdot}} - 4t_{\underset{3}{\cdot}}\right)}{3(1+2k^2)^2 r_{\underset{3}{\cdot}} t_{\underset{3}{\cdot}}} & \frac{4i}{3k r_{\underset{3}{\cdot}} + 6k^3 r_{\underset{3}{\cdot}}} & 0 & -\frac{i\sqrt{2}\left(9k^2 r_{\underset{3}{\cdot}} - 4t_{\underset{3}{\cdot}}\right)}{3k(1+2k^2)^2 r_{\underset{3}{\cdot}} t_{\underset{3}{\cdot}}} \\ -\frac{4i}{3k r_{\underset{3}{\cdot}} + 6k^3 r_{\underset{3}{\cdot}}} & -\frac{2}{3k^2 r_{\underset{3}{\cdot}}} & 0 & -\frac{2\sqrt{2}}{3k^2 r_{\underset{3}{\cdot}} + 6k^4 r_{\underset{3}{\cdot}}} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\left(9k^2 r_{\underset{3}{\cdot}} - 4t_{\underset{3}{\cdot}}\right)}{3k(1+2k^2)^2 r_{\underset{3}{\cdot}} t_{\underset{3}{\cdot}}} & -\frac{2\sqrt{2}}{3k^2 r_{\underset{3}{\cdot}} + 6k^4 r_{\underset{3}{\cdot}}} & 0 & \frac{9k^2 r_{\underset{3}{\cdot}} - 4t_{\underset{3}{\cdot}}}{3(k+2k^3)^2 r_{\underset{3}{\cdot}} t_{\underset{3}{\cdot}}} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3k^2 r_{\underset{3}{\cdot}}} \end{pmatrix}, (0)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\underset{2}{\cdot}}}{r_{\underset{2}{\cdot}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\underset{2}{\cdot}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$\dot{r}_2 < 0 \,\&\&\, \dot{t}_2 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$\dot{r}_2 < 0 \,\&\&\, \dot{t}_2 > 0$

Okay, that concludes the analysis of this theory.

Case 38

Now for a new theory. Here is the full nonlinear Lagrangian for Case 38 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\dot{r}_2 \mathcal{R}_{ijkl}\mathcal{R}^{ijkl} - \frac{2}{3}\dot{r}_2 \mathcal{R}_{ikjl}\mathcal{R}^{ijkl} + \frac{1}{6}\left(\dot{r}_2 - 6\dot{r}_3\right)\mathcal{R}^{ijkl}\mathcal{R}_{klij} + 4\dot{r}_3 \mathcal{R}^{ij\;\;k}_{\;\;\;i}\mathcal{R}_{k\;\;jl}^{\;\;l} + \frac{1}{12}\left(4\dot{t}_1 + \dot{t}_2\right)\mathcal{T}_{ijk}\mathcal{T}^{ijk} + \frac{1}{6}\left(2\dot{t}_1 - \dot{t}_2\right)\mathcal{T}^{ijk}\mathcal{T}_{jik} + \frac{1}{3}\dot{t}_1 \mathcal{T}^{i\;\;j}_{\;\;i}\mathcal{T}^{k}_{\;\;jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3}\left(\dot{t}_1 + \dot{t}_2\right)\mathcal{A}_{\alpha\alpha' i}\mathcal{A}^{\alpha\alpha' i} + \frac{1}{3}\left(\dot{t}_1 - 2\dot{t}_2\right)\mathcal{A}_{\alpha i\alpha'}\mathcal{A}^{\alpha\alpha' i} + \frac{1}{3}\dot{t}_1 \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\mathcal{A}_{\alpha'}{}^{i}{}_{i} - \frac{2}{3}\dot{t}_1 \mathcal{A}_{\alpha'}{}^{i}{}_{i}\partial_{\alpha}f^{\alpha\alpha'} + \\
&\frac{2}{3}\dot{t}_1 \mathcal{A}_{\alpha'}{}^{i}{}_{i}\partial^{\alpha'}f^{\alpha}{}_{\alpha} - \frac{1}{3}\dot{t}_1 \partial_{\alpha'}f^{i}{}_{i}\partial^{\alpha'}f^{\alpha}{}_{\alpha} - \frac{1}{3}\dot{t}_1 \partial_{\alpha}f^{\alpha\alpha'}\partial_{i}f^{i}{}_{\alpha'} + \frac{2}{3}\dot{t}_1 \partial^{\alpha'}f^{\alpha}{}_{\alpha}\partial_{i}f^{i}{}_{\alpha'} - 4\dot{r}_3 \partial_{\alpha'}\mathcal{A}_{i}{}^{j}{}_{j}\partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha} - \\
&\frac{2}{3}\left(\dot{t}_1 + \dot{t}_2\right)\mathcal{A}_{\alpha\alpha' i}\partial^{i}f^{\alpha\alpha'} + \frac{2}{3}\left(\dot{t}_1 + \dot{t}_2\right)\mathcal{A}_{\alpha i\alpha'}\partial^{i}f^{\alpha\alpha'} + \frac{2}{3}\left(2\dot{t}_1 - \dot{t}_2\right)\mathcal{A}_{\alpha' i\alpha}\partial^{i}f^{\alpha\alpha'} + \frac{1}{3}\left(-2\dot{t}_1 + \dot{t}_2\right)\partial_{\alpha}f_{\alpha' i}\partial^{i}f^{\alpha\alpha'} + \\
&\frac{1}{6}\left(2\dot{t}_1 - \dot{t}_2\right)\partial_{\alpha}f_{i\alpha'}\partial^{i}f^{\alpha\alpha'} + \frac{1}{6}\left(-4\dot{t}_1 - \dot{t}_2\right)\partial_{\alpha'}f_{\alpha i}\partial^{i}f^{\alpha\alpha'} + \frac{1}{6}\left(4\dot{t}_1 + \dot{t}_2\right)\partial_{i}f_{\alpha\alpha'}\partial^{i}f^{\alpha\alpha'} + \frac{1}{6}\left(2\dot{t}_1 - \dot{t}_2\right)\partial_{i}f_{\alpha'\alpha}\partial^{i}f^{\alpha\alpha'} - \\
&4\dot{r}_3 \partial_{\alpha}\mathcal{A}^{\alpha\alpha' i}\partial_{j}\mathcal{A}_{i}{}^{j}{}_{\alpha'} + 8\dot{r}_3 \partial^{i}\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\partial_{j}\mathcal{A}_{i}{}^{j}{}_{\alpha'} + \frac{4}{3}\dot{r}_2 \partial_{\alpha'}\mathcal{A}_{\alpha ij}\partial^{j}\mathcal{A}^{\alpha\alpha' i} - \frac{2}{3}\dot{r}_2 \partial_{\alpha'}\mathcal{A}_{\alpha ji}\partial^{j}\mathcal{A}^{\alpha\alpha' i} + \\
&\frac{2}{3}\left(\dot{r}_2 - 6\dot{r}_3\right)\partial_{\alpha'}\mathcal{A}_{ij\alpha}\partial^{j}\mathcal{A}^{\alpha\alpha' i} - \frac{1}{3}\dot{r}_2 \partial_{i}\mathcal{A}_{\alpha\alpha' j}\partial^{j}\mathcal{A}^{\alpha\alpha' i} + \frac{1}{3}\dot{r}_2 \partial_{j}\mathcal{A}_{\alpha\alpha' i}\partial^{j}\mathcal{A}^{\alpha\alpha' i} - \frac{2}{3}\dot{r}_2 \partial_{j}\mathcal{A}_{\alpha i\alpha'}\partial^{j}\mathcal{A}^{\alpha\alpha' i}
\end{aligned}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 6k^2 r_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 r_{2} + t_{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{3} k^2 \left(t_{1} + t_{2}\right) & -\frac{i k \left(t_{1} - 2 t_{2}\right)}{3\sqrt{2}} & \frac{1}{3} i k \left(t_{1} + t_{2}\right) \\ \frac{i k \left(t_{1} - 2 t_{2}\right)}{3\sqrt{2}} & \frac{1}{6}\left(t_{1} + 4 t_{2}\right) & \frac{-t_{1} + 2 t_{2}}{3\sqrt{2}} \\ -\frac{1}{3} i k \left(t_{1} + t_{2}\right) & \frac{-t_{1} + 2 t_{2}}{3\sqrt{2}} & \frac{t_{1} + t_{2}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2 k^2 t_{1}}{3} & -\frac{1}{3} i k t_{1} & 0 & -\frac{1}{3} i \sqrt{2}\, k t_{1} \\ \frac{i k t_{1}}{3} & \frac{t_{1}}{6} & 0 & \frac{t_{1}}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{1} & \frac{t_{1}}{3\sqrt{2}} & 0 & \frac{t_{1}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \begin{pmatrix} \frac{t_{1}}{2} \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\cdot}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\cdot}\tau^{\flat\parallel} == 0,\ -i\ {}^{1^+}_{\cdot}\tau^{\flat\parallel\, ab} == k\ {}^{1^+}_{\cdot}\sigma^{\flat\perp\, ab},\right.$$

$$\left. i\ {}^{1^-}_{\cdot}\tau^{\flat\parallel\, a} == 2k\ {}^{1^-}_{\cdot}\sigma^{\flat\perp\, a},\ {}^{1^-}_{\cdot}\tau^{\flat\perp\, a} == 0,\ i\ {}^{1^-}_{\cdot}\tau^{\flat\parallel\, a} == 2k\ {}^{1^-}_{\cdot}\sigma^{\flat\parallel\, a},\ -i\ {}^{2^+}_{\cdot}\tau^{\flat\parallel\, ab} == 2k\ {}^{2^+}_{\cdot}\sigma^{\flat\parallel\, ab} \right\}$$

The Drazin (Moore–Penrose) inverses of these *a*–matrices, which are functionally analogous to the inverse *b*–matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{6k^2 r_{3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_{2} + t_{2}} \end{pmatrix}, \begin{pmatrix} \frac{k^2\left(t_{1} + 4 t_{2}\right)}{3\left(1+k^2\right)^2 t_{1} t_{2}} & \frac{i\sqrt{2}\, k\left(t_{1} - 2 t_{2}\right)}{3\left(1+k^2\right) t_{1} t_{2}} & \frac{i k\left(t_{1} + 4 t_{2}\right)}{3\left(1+k^2\right)^2 t_{1} t_{2}} \\ -\frac{i\sqrt{2}\, k\left(t_{1} - 2 t_{2}\right)}{3\left(1+k^2\right) t_{1} t_{2}} & \frac{2\left(t_{1} + t_{2}\right)}{3 t_{1} t_{2}} & \frac{\sqrt{2}\left(t_{1} - 2 t_{2}\right)}{3\left(1+k^2\right) t_{1} t_{2}} \\ -\frac{i k\left(t_{1} + 4 t_{2}\right)}{3\left(1+k^2\right)^2 t_{1} t_{2}} & \frac{\sqrt{2}\left(t_{1} - 2 t_{2}\right)}{3\left(1+k^2\right) t_{1} t_{2}} & \frac{t_{1} + 4 t_{2}}{3\left(1+k^2\right)^2 t_{1} t_{2}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{24 k^2}{\left(3+4k^2\right)^2 t_{1}} & -\frac{12 i k}{\left(3+4k^2\right)^2 t_{1}} & 0 & -\frac{12 i\sqrt{2}\, k}{\left(3+4k^2\right)^2 t_{1}} \\ \frac{12 i k}{\left(3+4k^2\right)^2 t_{1}} & \frac{6}{\left(3+4k^2\right)^2 t_{1}} & 0 & \frac{6\sqrt{2}}{\left(3+4k^2\right)^2 t_{1}} \\ 0 & 0 & 0 & 0 \\ \frac{12 i\sqrt{2}\, k}{\left(3+4k^2\right)^2 t_{1}} & \frac{6\sqrt{2}}{\left(3+4k^2\right)^2 t_{1}} & 0 & \frac{12}{\left(3+4k^2\right)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_{1}} & \frac{2 i\sqrt{2}\, k}{\left(1+2k^2\right)^2 t_{1}} \\ -\frac{2 i\sqrt{2}\, k}{\left(1+2k^2\right)^2 t_{1}} & \frac{2}{\left(1+2k^2\right)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{t_{1}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$r_{\underset{2}{\cdot}} < 0 \,\&\&\, t_{\underset{2}{\cdot}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\underset{2}{\cdot}} < 0 \,\&\&\, t_{\underset{2}{\cdot}} > 0$

Okay, that concludes the analysis of this theory.

Case 39

Now for a new theory. Here is the full nonlinear Lagrangian for Case 39 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\underset{2}{\cdot}} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{\underset{2}{\cdot}} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \frac{1}{6} \left(r_{\underset{2}{\cdot}} - 6 r_{\underset{3}{\cdot}}\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij} + 4 r_{\underset{3}{\cdot}} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4} t_{\underset{1}{\cdot}} \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{2} t_{\underset{1}{\cdot}} \mathcal{T}^{ijk} \mathcal{T}_{jik} + \frac{1}{3} t_{\underset{1}{\cdot}} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{\underset{1}{\cdot}} \mathcal{A}_{\alpha i \alpha'} \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3} t_{\underset{1}{\cdot}} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \mathcal{A}_{\alpha'}{}^{i}{}_{i} - \frac{2}{3} t_{\underset{1}{\cdot}} \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial_{\alpha} f^{\alpha\alpha'} + \frac{2}{3} t_{\underset{1}{\cdot}} \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha} - \frac{1}{3} t_{\underset{1}{\cdot}} \partial_{\alpha'} f^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha} - \frac{1}{3} t_{\underset{1}{\cdot}} \partial_{\alpha} f^{\alpha\alpha'} \partial_{i} f^{i}{}_{\alpha'} + \frac{2}{3} t_{\underset{1}{\cdot}} \partial^{\alpha'} f^{\alpha}{}_{\alpha} \partial_{i} f^{i}{}_{\alpha'} - 4 r_{\underset{3}{\cdot}} \partial_{\alpha'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} + 2 t_{\underset{1}{\cdot}} \mathcal{A}_{\alpha' i \alpha} \partial^{i} f^{\alpha\alpha'} - t_{\underset{1}{\cdot}} \partial_{\alpha} f_{\alpha' i} \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{\underset{1}{\cdot}} \partial_{\alpha} f_{i\alpha'} \partial^{i} f^{\alpha\alpha'} - \frac{1}{2} t_{\underset{1}{\cdot}} \partial_{\alpha'} f_{\alpha i} \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{\underset{1}{\cdot}} \partial_{i} f_{\alpha\alpha'} \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{\underset{1}{\cdot}} \partial_{i} f_{\alpha'\alpha} \partial^{i} f^{\alpha\alpha'} - 4 r_{\underset{3}{\cdot}} \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'} + 8 r_{\underset{3}{\cdot}} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'} + \frac{4}{3} r_{\underset{2}{\cdot}} \partial_{\alpha'} \mathcal{A}_{\alpha ij} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_{\underset{2}{\cdot}} \partial_{\alpha'} \mathcal{A}_{\alpha ji} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{2}{3} \left(r_{\underset{2}{\cdot}} - 6 r_{\underset{3}{\cdot}}\right) \partial_{\alpha'} \mathcal{A}_{ij\alpha} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{1}{3} r_{\underset{2}{\cdot}} \partial_{i} \mathcal{A}_{\alpha\alpha' j} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3} r_{\underset{2}{\cdot}} \partial_{j} \mathcal{A}_{\alpha\alpha' i} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_{\underset{2}{\cdot}} \partial_{j} \mathcal{A}_{\alpha i \alpha'} \partial^{j} \mathcal{A}^{\alpha\alpha' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 6k^2 r_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 r_2 - t_1 \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_1}{\sqrt{2}} & 0 \\ \frac{i k t_1}{\sqrt{2}} & -\frac{t_1}{2} & -\frac{t_1}{\sqrt{2}} \\ 0 & -\frac{t_1}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2 t_1}{3} & -\frac{1}{3} i k t_1 & 0 & -\frac{1}{3} i \sqrt{2} k t_1 \\ \frac{i k t_1}{3} & \frac{t_1}{6} & 0 & \frac{t_1}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2} k t_1 & \frac{t_1}{3\sqrt{2}} & 0 & \frac{t_1}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_1 & \frac{i k t_1}{\sqrt{2}} \\ -\frac{i k t_1}{\sqrt{2}} & \frac{t_1}{2} \end{pmatrix}, \begin{pmatrix} \frac{t_1}{2} \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}\tau^{\flat\perp} == 0,\ {}^{0^+}\tau^{\flat\parallel} == 0,\ -i\, {}^{1^+}\tau^{\flat\parallel\, ab} == k\, {}^{1^+}\sigma^{\flat\perp\, ab},\right.$$

$$\left. i\, {}^{1^-}\tau^{\flat\parallel\, a} == 2k\, {}^{1^-}\sigma^{\flat\perp\, a},\ {}^{1^-}\tau^{\flat\perp\, a} == 0,\ i\, {}^{1^-}\tau^{\flat\parallel\, a} == 2k\, {}^{1^-}\sigma^{\flat\parallel\, a},\ -i\, {}^{2^+}\tau^{\flat\parallel\, ab} == 2k\, {}^{2^+}\sigma^{\flat\parallel\, ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{6k^2 r_3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_2 - t_1} \end{pmatrix}, \begin{pmatrix} \frac{k^2}{(1+k^2)^2 t_1} & -\frac{i\sqrt{2}k}{t_1 + k^2 t_1} & \frac{i k}{(1+k^2)^2 t_1} \\ \frac{i\sqrt{2}k}{t_1+k^2 t_1} & 0 & -\frac{\sqrt{2}}{t_1+k^2 t_1} \\ -\frac{i k}{(1+k^2)^2 t_1} & -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{1}{(1+k^2)^2 t_1} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{24k^2}{(3+4k^2)^2 t_1} & -\frac{12 i k}{(3+4k^2)^2 t_1} & 0 & -\frac{12 i\sqrt{2}k}{(3+4k^2)^2 t_1} \\ \frac{12 i k}{(3+4k^2)^2 t_1} & \frac{6}{(3+4k^2)^2 t_1} & 0 & \frac{6\sqrt{2}}{(3+4k^2)^2 t_1} \\ 0 & 0 & 0 & 0 \\ \frac{12 i\sqrt{2}k}{(3+4k^2)^2 t_1} & \frac{6\sqrt{2}}{(3+4k^2)^2 t_1} & 0 & \frac{12}{(3+4k^2)^2 t_1} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_1} & \frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1} \\ -\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1} & \frac{2}{(1+2k^2)^2 t_1} \end{pmatrix}, \begin{pmatrix} \frac{2}{t_1} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$\dot{r}_2 < 0 \,\&\&\, \dot{t}_1 < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$\dot{r}_2 < 0 \,\&\&\, \dot{t}_1 < 0$

Okay, that concludes the analysis of this theory.

Case 40

Now for a new theory. Here is the full nonlinear Lagrangian for Case 40 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\dot{r}_2\, \mathcal{R}_{ij\hbar l}\, \mathcal{R}^{ij\hbar l} - \frac{2}{3}\dot{r}_2\, \mathcal{R}_{i\hbar jl}\, \mathcal{R}^{ij\hbar l} + \frac{1}{6}\left(\dot{r}_2 - 6\dot{r}_3\right)\mathcal{R}^{ij\hbar l}\, \mathcal{R}_{\hbar lij} + 2\dot{r}_4\, \mathcal{R}^{ij\hbar}{}_{i}\, \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{12}\dot{t}_2\, \mathcal{T}_{ij\hbar}\, \mathcal{T}^{ij\hbar} - \frac{1}{6}\dot{t}_2\, \mathcal{T}^{ij\hbar}\, \mathcal{T}_{ji\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\dot{t}_2\, \mathcal{A}_{aa'i}\, \mathcal{A}^{aa'i} - \frac{2}{3}\dot{t}_2\, \mathcal{A}_{aia'}\, \mathcal{A}^{aa'i} - 2\dot{r}_4\, \partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{aa'}{}_{a} - \frac{2}{3}\dot{t}_2\, \mathcal{A}_{aa'i}\, \partial^{i}f^{aa'} + \frac{2}{3}\dot{t}_2\, \mathcal{A}_{aia'}\, \partial^{i}f^{aa'} - \frac{2}{3}\dot{t}_2\, \mathcal{A}_{a'ia}\, \partial^{i}f^{aa'} + \frac{1}{3}\dot{t}_2\, \partial_{a}f_{a'i}\, \partial^{i}f^{aa'} - \frac{1}{6}\dot{t}_2\, \partial_{a}f_{ia'}\, \partial^{i}f^{aa'} - \frac{1}{6}\dot{t}_2\, \partial_{a'}f_{ai}\, \partial^{i}f^{aa'} + \frac{1}{6}\dot{t}_2\, \partial_{i}f_{aa'}\, \partial^{i}f^{aa'} - \frac{1}{6}\dot{t}_2\, \partial_{i}f_{a'a}\, \partial^{i}f^{aa'} - 2\dot{r}_4\, \partial_{a}\mathcal{A}^{aa'i}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'} + 4\dot{r}_4\, \partial^{i}\mathcal{A}^{aa'}{}_{a}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'} + \frac{4}{3}\dot{r}_2\, \partial_{a'}\mathcal{A}_{aij}\, \partial^{j}\mathcal{A}^{aa'i} - \frac{2}{3}\dot{r}_2\, \partial_{a'}\mathcal{A}_{aji}\, \partial^{j}\mathcal{A}^{aa'i} + \frac{2}{3}\left(\dot{r}_2 - 6\dot{r}_3\right)\partial_{a'}\mathcal{A}_{ija}\, \partial^{j}\mathcal{A}^{aa'i} - \frac{1}{3}\dot{r}_2\, \partial_{i}\mathcal{A}_{aa'j}\, \partial^{j}\mathcal{A}^{aa'i} + \frac{1}{3}\dot{r}_2\, \partial_{j}\mathcal{A}_{aa'i}\, \partial^{j}\mathcal{A}^{aa'i} - \frac{2}{3}\dot{r}_2\, \partial_{j}\mathcal{A}_{aia'}\, \partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & -2k^2\left(\dot{r}_3 - 2\dot{r}_4\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2\dot{r}_2 + \dot{t}_2\right), \begin{pmatrix} \frac{k^2\dot{t}_2}{3} & \frac{1}{3}i\sqrt{2}\,k\,\dot{t}_2 & \frac{i k\,\dot{t}_2}{3} \\ -\frac{1}{3}i\sqrt{2}\,k\,\dot{t}_2 & \frac{1}{2}\left(k^2\left(4\dot{r}_3 - 2\dot{r}_4\right) + \frac{4\dot{t}_2}{3}\right) & \frac{\sqrt{2}\,\dot{t}_2}{3} \\ -\frac{1}{3}i\,k\,\dot{t}_2 & \frac{\sqrt{2}\,\dot{t}_2}{3} & \frac{\dot{t}_2}{3} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & k^2\left(-2\dot{r}_3 + \dot{r}_4\right) \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\bullet}\tau^{\flat\parallel} == 0,\ -i\ {}^{1^+}_{\bullet}\tau^{\flat\parallel\,ab} == k\ {}^{1^+}_{\bullet}\sigma^{\flat\perp\,ab},\ {}^{1^-}_{\bullet}\sigma^{\flat\perp\,a} == 0,\ {}^{1^-}_{\bullet}\tau^{\flat\perp\,a} == 0,\ {}^{1^-}_{\bullet}\sigma^{\flat\parallel\,a} == 0,\ {}^{1^-}_{\bullet}\tau^{\flat\parallel\,a} == 0,\ {}^{2^+}_{\bullet}\tau^{\flat\parallel\,ab} == 0,\ {}^{2^-}_{\bullet}\sigma^{\flat\parallel\,abc} == 0\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{-2k^2 r_{3}+4k^2 r_{4}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_{2}+t_{2}} \end{pmatrix},\right.$$

$$\begin{pmatrix} \frac{\frac{1}{r_{3}-\frac{r_{4}}{2}}+\frac{3k^2}{t_{2}}}{(1+k^2)^2} & -\frac{i\sqrt{2}}{k(1+k^2)\left(2r_{3}-r_{4}\right)} & \frac{i\left(k^2\left(6r_{3}-3r_{4}\right)+2t_{2}\right)}{k(1+k^2)^2\left(2r_{3}-r_{4}\right)t_{2}} \\ \frac{i\sqrt{2}}{k(1+k^2)\left(2r_{3}-r_{4}\right)} & \frac{1}{k^2\left(2r_{3}-r_{4}\right)} & -\frac{\sqrt{2}}{k^2(1+k^2)\left(2r_{3}-r_{4}\right)} \\ -\frac{i\left(k^2\left(6r_{3}-3r_{4}\right)+2t_{2}\right)}{k(1+k^2)^2\left(2r_{3}-r_{4}\right)t_{2}} & -\frac{\sqrt{2}}{k^2(1+k^2)\left(2r_{3}-r_{4}\right)} & \frac{k^2\left(6r_{3}-3r_{4}\right)+2t_{2}}{(k+k^3)^2\left(2r_{3}-r_{4}\right)t_{2}} \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \left.\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{k^2\left(-2r_{3}+r_{4}\right)} \end{pmatrix}, \begin{pmatrix} 0 \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{2} < 0\ \&\&\ t_{2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{2} < 0\ \&\&\ t_{2} > 0$

Okay, that concludes the analysis of this theory.

Case 41

Now for a new theory. Here is the full nonlinear Lagrangian for
Case 41 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\dot{r}_{2}\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl}-\frac{2}{3}\dot{r}_{2}\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl}+\frac{1}{6}\left(\dot{r}_{2}-6\dot{r}_{3}\right)\mathcal{R}^{ijkl}\,\mathcal{R}_{klij}+\dot{r}_{3}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl}+\frac{1}{12}\dot{t}_{2}\,\mathcal{T}_{ijk}\,\mathcal{T}^{ijk}-\frac{1}{6}\dot{t}_{2}\,\mathcal{T}^{ijk}\,\mathcal{T}_{jik}-\frac{2}{3}\dot{t}_{3}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise
this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\dot{t}_{2}\,\mathcal{A}_{aa'i}\,\mathcal{A}^{aa'i}-\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{aia'}\,\mathcal{A}^{aa'i}-\frac{2}{3}\dot{t}_{3}\,\mathcal{A}^{aa'}{}_{a}\,\mathcal{A}_{a'}{}^{i}{}_{i}+\frac{4}{3}\dot{t}_{3}\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial_{a}f^{aa'}-\frac{4}{3}\dot{t}_{3}\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}+\frac{2}{3}\dot{t}_{3}\,\partial_{a'}f^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}+\frac{2}{3}\dot{t}_{3}\,\partial_{a}f^{aa'}\,\partial_{i}f^{i}{}_{a'}-\frac{4}{3}\dot{t}_{3}\,\partial^{a'}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{a'}-\dot{r}_{3}\,\partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}-\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{aa'i}\,\partial^{i}f^{aa'}+\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{aia'}\,\partial^{i}f^{aa'}-\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{a'ia}\,\partial^{i}f^{aa'}+\frac{1}{3}\dot{t}_{2}\,\partial_{a}f_{a'i}\,\partial^{i}f^{aa'}-\frac{1}{6}\dot{t}_{2}\,\partial_{a}f_{ia'}\,\partial^{i}f^{aa'}-\frac{1}{6}\dot{t}_{2}\,\partial_{a'}f_{ai}\,\partial^{i}f^{aa'}+\frac{1}{6}\dot{t}_{2}\,\partial_{i}f_{aa'}\,\partial^{i}f^{aa'}-\frac{1}{6}\dot{t}_{2}\,\partial_{i}f_{a'a}\,\partial^{i}f^{aa'}-\dot{r}_{3}\,\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}+2\dot{r}_{3}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}+\frac{4}{3}\dot{r}_{2}\,\partial_{a'}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{aa'i}-\frac{2}{3}\dot{r}_{2}\,\partial_{a'}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\left(\dot{r}_{2}-6\dot{r}_{3}\right)\partial_{a'}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{aa'i}-\frac{1}{3}\dot{r}_{2}\,\partial_{i}\mathcal{A}_{aa'j}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{1}{3}\dot{r}_{2}\,\partial_{j}\mathcal{A}_{aa'i}\,\partial^{j}\mathcal{A}^{aa'i}-\frac{2}{3}\dot{r}_{2}\,\partial_{j}\mathcal{A}_{aia'}\,\partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2k^{2}\dot{t}_{3} & i\sqrt{2}\,k\,\dot{t}_{3} & 0 \\ -i\sqrt{2}\,k\,\dot{t}_{3} & \dot{t}_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix},\ \left(k^{2}\dot{r}_{2}+\dot{t}_{2}\right),\ \begin{pmatrix} \frac{k^{2}\dot{t}_{2}}{3} & \frac{1}{3}i\sqrt{2}\,k\,\dot{t}_{2} & \frac{i\,k\,\dot{t}_{2}}{3} \\ -\frac{1}{3}i\sqrt{2}\,k\,\dot{t}_{2} & \frac{1}{2}\left(3k^{2}\dot{r}_{3}+\frac{4\dot{t}_{2}}{3}\right) & \frac{\sqrt{2}\,\dot{t}_{2}}{3} \\ -\frac{1}{3}i\,k\,\dot{t}_{2} & \frac{\sqrt{2}\,\dot{t}_{2}}{3} & \frac{\dot{t}_{2}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^{2}\dot{t}_{3}}{3} & \frac{2i\,k\,\dot{t}_{3}}{3} & 0 & -\frac{1}{3}i\sqrt{2}\,k\,\dot{t}_{3} \\ -\frac{2}{3}i\,k\,\dot{t}_{3} & \frac{2\dot{t}_{3}}{3} & 0 & -\frac{\sqrt{2}\,\dot{t}_{3}}{3} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3}i\sqrt{2}\,k\,\dot{t}_{3} & -\frac{\sqrt{2}\,\dot{t}_{3}}{3} & 0 & \frac{\dot{t}_{3}}{3} \end{pmatrix},\ \begin{pmatrix} 0 & 0 \\ 0 & -\frac{3k^{2}\dot{r}_{3}}{2} \end{pmatrix},\ (0)\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^{+}}{\cdot}\tau^{\flat\perp}==0,\ -i\,\overset{0^{+}}{\cdot}\tau^{\flat\parallel}==2k\,\overset{0^{+}}{\cdot}\sigma^{\flat\parallel},\ -i\,\overset{1^{+}}{\cdot}\tau^{\flat\parallel}{}^{ab}==k\,\overset{1^{+}}{\cdot}\sigma^{\flat\perp}{}^{ab},\ i\,\overset{1^{-}}{\cdot}\tau^{\flat\parallel}{}^{a}==2k\,\overset{1^{-}}{\cdot}\sigma^{\flat\perp}{}^{a},\ \overset{1^{-}}{\cdot}\tau^{\flat\perp}{}^{a}==0,\ -i\,\overset{1^{-}}{\cdot}\tau^{\flat\parallel}{}^{a}==k\,\overset{1^{-}}{\cdot}\sigma^{\flat\parallel}{}^{a},\ \overset{2^{+}}{\cdot}\tau^{\flat\parallel}{}^{ab}==0,\ \overset{2^{-}}{\cdot}\sigma^{\flat\parallel}{}^{abc}==0\right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_{3}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & \frac{1}{(1+2k^2)^2 t_{3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_{2}+t_{2}} \end{pmatrix}, \begin{pmatrix} \frac{9k^2 r_{3}+4t_{2}}{3(1+k^2)^2 r_{3} t_{2}} & -\frac{2i\sqrt{2}}{3k r_{3}+3k^3 r_{3}} & \frac{i(9k^2 r_{3}+4t_{2})}{3k(1+k^2)^2 r_{3} t_{2}} \\ \frac{2i\sqrt{2}}{3k r_{3}+3k^3 r_{3}} & \frac{2}{3k^2 r_{3}} & -\frac{2\sqrt{2}}{3k^2 r_{3}+3k^4 r_{3}} \\ -\frac{i(9k^2 r_{3}+4t_{2})}{3k(1+k^2)^2 r_{3} t_{2}} & -\frac{2\sqrt{2}}{3k^2 r_{3}+3k^4 r_{3}} & \frac{9k^2 r_{3}+4t_{2}}{3(k+k^3)^2 r_{3} t_{2}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{6k^2}{(3+2k^2)^2 t_{3}} & \frac{6ik}{(3+2k^2)^2 t_{3}} & 0 & -\frac{3i\sqrt{2}\,k}{(3+2k^2)^2 t_{3}} \\ -\frac{6ik}{(3+2k^2)^2 t_{3}} & \frac{6}{(3+2k^2)^2 t_{3}} & 0 & -\frac{3\sqrt{2}}{(3+2k^2)^2 t_{3}} \\ 0 & 0 & 0 & 0 \\ \frac{3i\sqrt{2}\,k}{(3+2k^2)^2 t_{3}} & -\frac{3\sqrt{2}}{(3+2k^2)^2 t_{3}} & 0 & \frac{3}{(3+2k^2)^2 t_{3}} \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -\frac{2}{3k^2 r_{3}} \end{pmatrix}, \begin{pmatrix} 0 \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{2} < 0 \;\&\&\; t_{2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{2} < 0 \;\&\&\; t_{2} > 0$

Okay, that concludes the analysis of this theory.

Case 42

Now for a new theory. Here is the full nonlinear Lagrangian for Case 42 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ijhl} \mathcal{R}^{ijhl} - \frac{2}{3} r_{2} \mathcal{R}_{ihjl} \mathcal{R}^{ijhl} + r_{5} \mathcal{R}^{ij}{}_{i}{}^{h} \mathcal{R}_{j}{}^{l}{}_{hl} + \frac{1}{6} r_{2} \mathcal{R}^{ijhl} \mathcal{R}_{hlij} - r_{5} \mathcal{R}^{ij}{}_{i}{}^{h} \mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{4} t_{1} \mathcal{T}_{ijh} \mathcal{T}^{ijh} + \frac{1}{2} t_{1} \mathcal{T}^{ijh} \mathcal{T}_{jih} + \frac{1}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{h}{}_{jh}$$

To use PSALTer, you have to first linearise
this Lagrangian to second order around the desired vacuum:

$$
\begin{aligned}
&t_{1}\,\mathcal{A}_{a i a'}\,\mathcal{A}^{a a' i}+\frac{1}{3}\left(t_{1}-2t_{3}\right)\mathcal{A}^{a a'}{}_{a}\,\mathcal{A}_{a'}{}^{i}{}_{i}-\frac{2}{3}\left(t_{1}-2t_{3}\right)\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial_{a}f^{a a'}+\frac{2}{3}\left(t_{1}-2t_{3}\right)\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}+\\
&\frac{1}{3}\left(-t_{1}+2t_{3}\right)\partial_{a'}f^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}+\frac{1}{3}\left(-t_{1}+2t_{3}\right)\partial_{a}f^{a a'}\,\partial_{i}f^{i}{}_{a'}+\frac{2}{3}\left(t_{1}-2t_{3}\right)\partial^{a'}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{a'}+\\
&r_{5}\,\partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{a a'}{}_{a}-r_{5}\,\partial_{i}\mathcal{A}_{a'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{a a'}{}_{a}+2t_{1}\,\mathcal{A}_{a' i a}\,\partial^{i}f^{a a'}-t_{1}\,\partial_{a}f_{a' i}\,\partial^{i}f^{a a'}+\frac{1}{2}t_{1}\,\partial_{a}f_{i a'}\,\partial^{i}f^{a a'}-\\
&\frac{1}{2}t_{1}\,\partial_{a'}f_{a i}\,\partial^{i}f^{a a'}+\frac{1}{2}t_{1}\,\partial_{i}f_{a a'}\,\partial^{i}f^{a a'}+\frac{1}{2}t_{1}\,\partial_{i}f_{a' a}\,\partial^{i}f^{a a'}-r_{5}\,\partial_{a}\mathcal{A}^{a a' i}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}+2r_{5}\,\partial^{i}\mathcal{A}^{a a'}{}_{a}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}+\\
&r_{5}\,\partial_{a}\mathcal{A}^{a a' i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}-2r_{5}\,\partial^{i}\mathcal{A}^{a a'}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}+\frac{4}{3}r_{2}\,\partial_{a'}\mathcal{A}_{a i j}\,\partial^{j}\mathcal{A}^{a a' i}-\frac{2}{3}r_{2}\,\partial_{a'}\mathcal{A}_{a j i}\,\partial^{j}\mathcal{A}^{a a' i}+\\
&\frac{2}{3}r_{2}\,\partial_{a'}\mathcal{A}_{i j a}\,\partial^{j}\mathcal{A}^{a a' i}-\frac{1}{3}r_{2}\,\partial_{i}\mathcal{A}_{a a' j}\,\partial^{j}\mathcal{A}^{a a' i}+\frac{1}{3}r_{2}\,\partial_{j}\mathcal{A}_{a a' i}\,\partial^{j}\mathcal{A}^{a a' i}-\frac{2}{3}r_{2}\,\partial_{j}\mathcal{A}_{a i a'}\,\partial^{j}\mathcal{A}^{a a' i}
\end{aligned}
$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$
\left\{\begin{pmatrix} 2k^2 t_{3} & i\sqrt{2}\,k\,t_{3} & 0\\ -i\sqrt{2}\,k\,t_{3} & t_{3} & 0\\ 0 & 0 & 0\end{pmatrix},\ \begin{pmatrix} k^2 r_{2}-t_{1}\end{pmatrix},\ \begin{pmatrix} 0 & -\frac{i k t_{1}}{\sqrt{2}} & 0\\ \frac{i k t_{1}}{\sqrt{2}} & \frac{1}{2}\left(2k^2 r_{5}-t_{1}\right) & -\frac{t_{1}}{\sqrt{2}}\\ 0 & -\frac{t_{1}}{\sqrt{2}} & 0\end{pmatrix},\right.
$$

$$
\left.\begin{pmatrix} \frac{2}{3}k^2\left(t_{1}+t_{3}\right) & -\frac{1}{3}i k\left(t_{1}-2t_{3}\right) & 0 & -\frac{1}{3}i\sqrt{2}\,k\left(t_{1}+t_{3}\right)\\ \frac{1}{3}i k\left(t_{1}-2t_{3}\right) & \frac{1}{6}\left(6k^2 r_{5}+t_{1}+4t_{3}\right) & 0 & \frac{t_{1}-2t_{3}}{3\sqrt{2}}\\ 0 & 0 & 0 & 0\\ \frac{1}{3}i\sqrt{2}\,k\left(t_{1}+t_{3}\right) & \frac{t_{1}-2t_{3}}{3\sqrt{2}} & 0 & \frac{t_{1}+t_{3}}{3}\end{pmatrix},\ \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}}\\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2}\end{pmatrix},\ \begin{pmatrix}\frac{t_{1}}{2}\end{pmatrix}\right\}
$$

Gauge constraints on source currents:

$$
\left\{{}^{0^+}_{\cdot}\tau^{\flat\perp}==0,\ -i\,{}^{0^+}_{\cdot}\tau^{\flat\parallel}==2k\,{}^{0^+}_{\cdot}\sigma^{\flat\parallel},\ -i\,{}^{1^+}_{\cdot}\tau^{\flat\parallel}{}^{ab}==k\,{}^{1^+}_{\cdot}\sigma^{\flat\perp}{}^{ab},\ i\,{}^{1^-}_{\cdot}\tau^{\flat\parallel}{}^{a}==2k\,{}^{1^-}_{\cdot}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\cdot}\tau^{\flat\perp}{}^{a}==0,\ -i\,{}^{2^+}_{\cdot}\tau^{\flat\parallel}{}^{ab}==2k\,{}^{2^+}_{\cdot}\sigma^{\flat\parallel}{}^{ab}\right\}
$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_3} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3} & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3} & \frac{1}{(1+2k^2)^2 t_3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_2 - t_1} \end{pmatrix}, \begin{pmatrix} \frac{-2k^4 r_5 + k^2 t_1}{(1+k^2)^2 t_1^2} & -\frac{i\sqrt{2}\,k}{t_1 + k^2 t_1} & -\frac{i\left(2k^3 r_5 - k t_1\right)}{(1+k^2)^2 t_1^2} \\ \frac{i\sqrt{2}\,k}{t_1 + k^2 t_1} & 0 & -\frac{\sqrt{2}}{t_1 + k^2 t_1} \\ \frac{i\left(2k^3 r_5 - k t_1\right)}{(1+k^2)^2 t_1^2} & -\frac{\sqrt{2}}{t_1 + k^2 t_1} & \frac{-2k^2 r_5 + t_1}{(1+k^2)^2 t_1^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2\left(6k^2 r_5 + t_1 + 4t_3\right)}{(1+2k^2)^2\left(3t_1 t_3 + 2k^2 r_5 (t_1+t_3)\right)} & \frac{2ik\left(t_1 - 2t_3\right)}{(1+2k^2)\left(3t_1 t_3 + 2k^2 r_5 (t_1+t_3)\right)} & 0 & -\frac{i\sqrt{2}\,k\left(6k^2 r_5 + t_1 + 4t_3\right)}{(1+2k^2)^2\left(3t_1 t_3 + 2k^2 r_5 (t_1+t_3)\right)} \\ -\frac{2ik\left(t_1 - 2t_3\right)}{(1+2k^2)\left(3t_1 t_3 + 2k^2 r_5 (t_1+t_3)\right)} & \frac{2\left(t_1 + t_3\right)}{3t_1 t_3 + 2k^2 r_5 (t_1+t_3)} & 0 & -\frac{\sqrt{2}\left(t_1 - 2t_3\right)}{(1+2k^2)\left(3t_1 t_3 + 2k^2 r_5 (t_1+t_3)\right)} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k\left(6k^2 r_5 + t_1 + 4t_3\right)}{(1+2k^2)^2\left(3t_1 t_3 + 2k^2 r_5 (t_1+t_3)\right)} & -\frac{\sqrt{2}\left(t_1 - 2t_3\right)}{(1+2k^2)\left(3t_1 t_3 + 2k^2 r_5 (t_1+t_3)\right)} & 0 & \frac{6k^2 r_5 + t_1 + 4t_3}{(1+2k^2)^2\left(3t_1 t_3 + 2k^2 r_5 (t_1+t_3)\right)} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_1} & \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} \\ -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & \frac{2}{(1+2k^2)^2 t_1} \end{pmatrix}, \begin{pmatrix} \frac{2}{t_1} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_1}{r_2}\right\}, \{\}, \left\{-\frac{3t_1 t_3}{2r_5 t_1 + 2r_5 t_3}\right\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \left\{\frac{6t_1 t_3\left(t_1 + t_3\right) - 3r_5\left(t_1^2 + 2t_3^2\right)}{2r_5\left(t_1 + t_3\right)\left(-3t_1 t_3 + r_5\left(t_1 + t_3\right)\right)}\right\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$r_2 < 0 \,\&\&\, r_5 < 0 \,\&\&\, t_1 < 0 \,\&\&\, 0 < t_3 < -t_1$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_2 < 0 \,\&\&\, r_5 < 0 \,\&\&\, t_1 < 0 \,\&\&\, 0 < t_3 < -t_1$

Okay, that concludes the analysis of this theory.

Case 43

Now for a new theory. Here is the full nonlinear Lagrangian for Case 43 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{2} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + r_{5} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6} r_{2} \mathcal{R}^{ijkl} \mathcal{R}_{klij} - r_{5} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12}\left(4 t_{1} + t_{2}\right) \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{6}\left(2 t_{1} - t_{2}\right) \mathcal{T}^{ijk} \mathcal{T}_{jik} + t_{1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise
this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3}\left(t_{1}+t_{2}\right) \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + \frac{1}{3}\left(t_{1}-2 t_{2}\right) \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + t_{1} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} \mathcal{A}_{\mathfrak{a}'}{}^{i}{}_{i} - 2 t_{1} \mathcal{A}_{\mathfrak{a}'}{}^{i}{}_{i} \partial_{\mathfrak{a}} f^{\mathfrak{a}\mathfrak{a}'} + \\
&2 t_{1} \mathcal{A}_{\mathfrak{a}'}{}^{i}{}_{i} \partial^{\mathfrak{a}'} f^{\mathfrak{a}}{}_{\mathfrak{a}} - t_{1} \partial_{\mathfrak{a}'} f^{i}{}_{i} \partial^{\mathfrak{a}'} f^{\mathfrak{a}}{}_{\mathfrak{a}} - t_{1} \partial_{\mathfrak{a}} f^{\mathfrak{a}\mathfrak{a}'} \partial_{i} f^{i}{}_{\mathfrak{a}'} + 2 t_{1} \partial^{\mathfrak{a}'} f^{\mathfrak{a}}{}_{\mathfrak{a}} \partial_{i} f^{i}{}_{\mathfrak{a}'} + r_{5} \partial_{\mathfrak{a}'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} - \\
&r_{5} \partial_{i} \mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} - \frac{2}{3}\left(t_{1}+t_{2}\right) \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \frac{2}{3}\left(t_{1}+t_{2}\right) \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \frac{2}{3}\left(2 t_{1}-t_{2}\right) \mathcal{A}_{\mathfrak{a}'i\mathfrak{a}} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \\
&\frac{1}{3}\left(-2 t_{1}+t_{2}\right) \partial_{\mathfrak{a}} f_{\mathfrak{a}'i} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \frac{1}{6}\left(2 t_{1}-t_{2}\right) \partial_{\mathfrak{a}} f_{i\mathfrak{a}'} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \frac{1}{6}\left(-4 t_{1}-t_{2}\right) \partial_{\mathfrak{a}'} f_{\mathfrak{a}i} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \\
&\frac{1}{6}\left(4 t_{1}+t_{2}\right) \partial_{i} f_{\mathfrak{a}\mathfrak{a}'} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \frac{1}{6}\left(2 t_{1}-t_{2}\right) \partial_{i} f_{\mathfrak{a}'\mathfrak{a}} \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} - r_{5} \partial_{\mathfrak{a}} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} \partial_{j} \mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{i} + 2 r_{5} \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} \partial_{j} \mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{i} + \\
&r_{5} \partial_{\mathfrak{a}} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\mathfrak{a}'} - 2 r_{5} \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\mathfrak{a}'} + \frac{4}{3} r_{2} \partial_{\mathfrak{a}'} \mathcal{A}_{\mathfrak{a}ij} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} r_{2} \partial_{\mathfrak{a}'} \mathcal{A}_{\mathfrak{a}ji} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + \\
&\frac{2}{3} r_{2} \partial_{\mathfrak{a}'} \mathcal{A}_{ij\mathfrak{a}} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{1}{3} r_{2} \partial_{i} \mathcal{A}_{\mathfrak{a}\mathfrak{a}'j} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + \frac{1}{3} r_{2} \partial_{j} \mathcal{A}_{\mathfrak{a}\mathfrak{a}'i} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} - \frac{2}{3} r_{2} \partial_{j} \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'} \partial^{j} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}
\end{aligned}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with
the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2 k^{2} t_{1} & -i \sqrt{2}\, k t_{1} & 0 \\ i \sqrt{2}\, k t_{1} & -t_{1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^{2} r_{2} + t_{2}\right),\right.$$

$$\begin{pmatrix} \frac{1}{3} k^{2}\left(t_{1}+t_{2}\right) & -\frac{i k\left(t_{1}-2 t_{2}\right)}{3 \sqrt{2}} & \frac{1}{3} i k\left(t_{1}+t_{2}\right) \\ \frac{i k\left(t_{1}-2 t_{2}\right)}{3 \sqrt{2}} & \frac{1}{6}\left(6 k^{2} r_{5}+t_{1}+4 t_{2}\right) & \frac{-t_{1}+2 t_{2}}{3 \sqrt{2}} \\ -\frac{1}{3} i k\left(t_{1}+t_{2}\right) & \frac{-t_{1}+2 t_{2}}{3 \sqrt{2}} & \frac{t_{1}+t_{2}}{3} \end{pmatrix}, \begin{pmatrix} 0 & -i k t_{1} & 0 & 0 \\ i k t_{1} & k^{2} r_{5}-\frac{t_{1}}{2} & 0 & \frac{t_{1}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{1}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^{2} t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \left(\frac{t_{1}}{2}\right)\Bigg\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^{+}}\tau^{\flat\perp} == 0,\ -i\,{}^{0^{+}}\tau^{\flat\parallel} == 2 k\,{}^{0^{+}}\sigma^{\flat\parallel},\ -i\,{}^{1^{+}}\tau^{\flat\parallel\mathfrak{a}\mathfrak{b}} == k\,{}^{1^{+}}\sigma^{\flat\perp\mathfrak{a}\mathfrak{b}},\ i\,{}^{1^{-}}\tau^{\flat\parallel\mathfrak{a}} == 2 k\,{}^{1^{-}}\sigma^{\flat\perp\mathfrak{a}},\ {}^{1^{-}}\tau^{\flat\perp\mathfrak{a}} == 0,\ -i\,{}^{2^{+}}\tau^{\flat\parallel\mathfrak{a}\mathfrak{b}} == 2 k\,{}^{2^{+}}\sigma^{\flat\parallel\mathfrak{a}\mathfrak{b}}\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{(1+2k^2)^2 t_1} & -\frac{i\sqrt{2}k}{(1+2k^2)^2 t_1} & 0 \\ \frac{i\sqrt{2}k}{(1+2k^2)^2 t_1} & -\frac{1}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_2+t_2}\right), \begin{pmatrix} \frac{k^2(6k^2 r_5+t_1+4t_2)}{(1+k^2)^2(3t_1 t_2+2k^2 r_5(t_1+t_2))} & \frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1 t_2+2k^2 r_5(t_1+t_2))} & \frac{ik(6k^2 r_5+t_1+4t_2)}{(1+k^2)^2(3t_1 t_2+2k^2 r_5(t_1+t_2))} \\ -\frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1 t_2+2k^2 r_5(t_1+t_2))} & \frac{2(t_1+t_2)}{3t_1 t_2+2k^2 r_5(t_1+t_2)} & \frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1 t_2+2k^2 r_5(t_1+t_2))} \\ -\frac{ik(6k^2 r_5+t_1+4t_2)}{(1+k^2)^2(3t_1 t_2+2k^2 r_5(t_1+t_2))} & \frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1 t_2+2k^2 r_5(t_1+t_2))} & \frac{6k^2 r_5+t_1+4t_2}{(1+k^2)^2(3t_1 t_2+2k^2 r_5(t_1+t_2))} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{-4k^4 r_5+2k^2 t_1}{(t_1+2k^2 t_1)^2} & -\frac{2ik}{t_1+2k^2 t_1} & 0 & \frac{i\sqrt{2}k(2k^2 r_5-t_1)}{(t_1+2k^2 t_1)^2} \\ \frac{2ik}{t_1+2k^2 t_1} & 0 & 0 & \frac{\sqrt{2}}{t_1+2k^2 t_1} \\ 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}k(2k^2 r_5-t_1)}{(t_1+2k^2 t_1)^2} & \frac{\sqrt{2}}{t_1+2k^2 t_1} & 0 & \frac{-2k^2 r_5+t_1}{(t_1+2k^2 t_1)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_1} & \frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1} \\ -\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1} & \frac{2}{(1+2k^2)^2 t_1} \end{pmatrix}, \left(\frac{2}{t_1}\right)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_2}{r_2}\right\}, \left\{-\frac{3 t_1 t_2}{2 r_5 t_1 + 2 r_5 t_2}\right\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \left\{\frac{-3 t_1 t_2 (t_1 + t_2) + 3 r_5 (t_1^2 + 2 t_2^2)}{r_5 (t_1 + t_2)(-3 t_1 t_2 + 2 r_5 (t_1 + t_2))}\right\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$r_2 < 0 \,\&\&\, r_5 > 0 \,\&\&\, t_1 < 0 \,\&\&\, t_2 > -t_1$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_2 < 0 \,\&\&\, r_5 > 0 \,\&\&\, t_1 < 0 \,\&\&\, t_2 > -t_1$

Okay, that concludes the analysis of this theory.

Case 44

Now for a new theory. Here is the full nonlinear Lagrangian for Case 44 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{1} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} + \frac{2}{3} r_{1} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + r_{5} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} - \frac{2}{3} r_{1} \mathcal{R}^{ijkl} \mathcal{R}_{klij} - r_{5} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}^{l}{}_{kjl} + \frac{1}{4} t_{1} \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{2} t_{1} \mathcal{T}^{ijk} \mathcal{T}_{jik} + \frac{1}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{1} \mathcal{A}_{a i a'} \mathcal{A}^{a a' i} + \frac{1}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}^{a a'}{}_{a} \mathcal{A}_{a'}{}^{i}{}_{i} - \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}_{a'}{}^{i}{}_{i} \partial_{a} f^{a a'} + \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}_{a'}{}^{i}{}_{i} \partial^{a'} f^{a}{}_{a} +$$
$$\frac{1}{3}\left(-t_{1} + 2 t_{3}\right) \partial_{a'} f^{i}{}_{i} \partial^{a'} f^{a}{}_{a} + \frac{1}{3}\left(-t_{1} + 2 t_{3}\right) \partial_{a} f^{a a'} \partial_{i} f^{i}{}_{a'} + \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \partial^{a'} f^{a}{}_{a} \partial_{i} f^{i}{}_{a'} +$$
$$r_{5} \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{a a'}{}_{a} - r_{5} \partial_{i} \mathcal{A}_{a'}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{a a'}{}_{a} + 2 t_{1} \mathcal{A}_{a' i a} \partial^{i} f^{a a'} - t_{1} \partial_{a} f_{a' i} \partial^{i} f^{a a'} + \frac{1}{2} t_{1} \partial_{a} f_{i a'} \partial^{i} f^{a a'} -$$
$$\frac{1}{2} t_{1} \partial_{a'} f_{a i} \partial^{i} f^{a a'} + \frac{1}{2} t_{1} \partial_{i} f_{a a'} \partial^{i} f^{a a'} + \frac{1}{2} t_{1} \partial_{i} f_{a' a} \partial^{i} f^{a a'} - r_{5} \partial_{a} \mathcal{A}^{a a' i} \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} + 2 r_{5} \partial^{i} \mathcal{A}^{a a'}{}_{a} \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} +$$
$$r_{5} \partial_{a} \mathcal{A}^{a a' i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} - 2 r_{5} \partial^{i} \mathcal{A}^{a a'}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} - \frac{4}{3} r_{1} \partial_{a'} \mathcal{A}_{a i j} \partial^{j} \mathcal{A}^{a a' i} + \frac{2}{3} r_{1} \partial_{a'} \mathcal{A}_{a j i} \partial^{j} \mathcal{A}^{a a' i} -$$
$$\frac{8}{3} r_{1} \partial_{a'} \mathcal{A}_{i j a} \partial^{j} \mathcal{A}^{a a' i} - \frac{2}{3} r_{1} \partial_{i} \mathcal{A}_{a a' j} \partial^{j} \mathcal{A}^{a a' i} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{a a' i} \partial^{j} \mathcal{A}^{a a' i} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{a i a'} \partial^{j} \mathcal{A}^{a a' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2 k^2 t_{3} & i \sqrt{2}\, k t_{3} & 0 \\ -i \sqrt{2}\, k t_{3} & t_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_{1} \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & \frac{1}{2}\left(2 k^2 \left(2 r_{1} + r_{5}\right) - t_{1}\right) & -\frac{t_{1}}{\sqrt{2}} \\ 0 & -\frac{t_{1}}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2}{3} k^2 \left(t_{1} + t_{3}\right) & -\frac{1}{3} i k \left(t_{1} - 2 t_{3}\right) & 0 & -\frac{1}{3} i \sqrt{2}\, k \left(t_{1} + t_{3}\right) \\ \frac{1}{3} i k \left(t_{1} - 2 t_{3}\right) & \frac{1}{6}\left(6 k^2 \left(r_{1} + r_{5}\right) + t_{1} + 4 t_{3}\right) & 0 & \frac{t_{1} - 2 t_{3}}{3 \sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k \left(t_{1} + t_{3}\right) & \frac{t_{1} - 2 t_{3}}{3 \sqrt{2}} & 0 & \frac{t_{1} + t_{3}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2 k^2 r_{1} + t_{1}\right) \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}\tau^{\flat\perp} == 0,\ -i\, {}^{0^+}\tau^{\flat\parallel} == 2 k\, {}^{0^+}\sigma^{\flat\parallel},\ -i\, {}^{1^+}\tau^{\flat\parallel}{}^{ab} == k\, {}^{1^+}\sigma^{\flat\perp}{}^{ab},\ i\, {}^{1^-}\tau^{\flat\parallel}{}^{a} == 2 k\, {}^{1^-}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}\tau^{\flat\perp}{}^{a} == 0,\ -i\, {}^{2^+}\tau^{\flat\parallel}{}^{ab} == 2 k\, {}^{2^+}\sigma^{\flat\parallel}{}^{ab} \right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_{3}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & \frac{1}{(1+2k^2)^2 t_{3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(-\frac{1}{t_{1}}\right), \begin{pmatrix} \frac{-2k^4(2r_{1}+r_{5})+k^2 t_{1}}{(1+k^2)^2 t_{1}^2} & -\frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & \frac{-2ik^3(2r_{1}+r_{5})+ik\,t_{1}}{(1+k^2)^2 t_{1}^2} \\ \frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} \\ \frac{i(2k^3(2r_{1}+r_{5})-k\,t_{1})}{(1+k^2)^2 t_{1}^2} & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2k^2(2r_{1}+r_{5})+t_{1}}{(1+k^2)^2 t_{1}^2} \end{pmatrix},\right.$$

$$\begin{pmatrix} \frac{2k^2(6k^2(r_{1}+r_{5})+t_{1}+4t_{3})}{(1+2k^2)^2(3t_{1}t_{3}+2k^2(r_{1}+r_{5})(t_{1}+t_{3}))} & \frac{2ik(t_{1}-2t_{3})}{(1+2k^2)(3t_{1}t_{3}+2k^2(r_{1}+r_{5})(t_{1}+t_{3}))} & 0 & -\frac{i\sqrt{2}\,k(6k^2(r_{1}+r_{5})+t_{1}+4t_{3})}{(1+2k^2)^2(3t_{1}t_{3}+2k^2(r_{1}+r_{5})(t_{1}+t_{3}))} \\ -\frac{2ik(t_{1}-2t_{3})}{(1+2k^2)(3t_{1}t_{3}+2k^2(r_{1}+r_{5})(t_{1}+t_{3}))} & \frac{2(t_{1}+t_{3})}{3t_{1}t_{3}+2k^2(r_{1}+r_{5})(t_{1}+t_{3})} & 0 & -\frac{\sqrt{2}(t_{1}-2t_{3})}{(1+2k^2)(3t_{1}t_{3}+2k^2(r_{1}+r_{5})(t_{1}+t_{3}))} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k(6k^2(r_{1}+r_{5})+t_{1}+4t_{3})}{(1+2k^2)^2(3t_{1}t_{3}+2k^2(r_{1}+r_{5})(t_{1}+t_{3}))} & -\frac{\sqrt{2}(t_{1}-2t_{3})}{(1+2k^2)(3t_{1}t_{3}+2k^2(r_{1}+r_{5})(t_{1}+t_{3}))} & 0 & \frac{6k^2(r_{1}+r_{5})+t_{1}+4t_{3}}{(1+2k^2)^2(3t_{1}t_{3}+2k^2(r_{1}+r_{5})(t_{1}+t_{3}))} \end{pmatrix},$$

$$\left.\begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_{1}} & \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} \\ -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & \frac{2}{(1+2k^2)^2 t_{1}} \end{pmatrix}, \left(\frac{2}{2k^2 r_{1}+t_{1}}\right)\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \left\{-\frac{3t_{1}t_{3}}{2(r_{1}+r_{5})(t_{1}+t_{3})}\right\}, \{\}, \left\{-\frac{t_{1}}{2r_{1}}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \left\{-\frac{3(-2t_{1}t_{3}(t_{1}+t_{3})+r_{1}(t_{1}^2+2t_{3}^2)+r_{5}(t_{1}^2+2t_{3}^2))}{2(r_{1}+r_{5})(t_{1}+t_{3})(-3t_{1}t_{3}+r_{1}(t_{1}+t_{3})+r_{5}(t_{1}+t_{3}))}\right\}, \{\}, \left\{-\frac{1}{r_{1}}\right\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$$r_{1} < 0 \,\&\&\, r_{5} < -r_{1} \,\&\&\, t_{1} > 0 \,\&\&\, (t_{3} < -t_{1} \,\|\, t_{3} > 0)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$(r_{1} < 0 \,\&\&\, r_{5} < -r_{1} \,\&\&\, t_{1} > 0 \,\&\&\, t_{3} < -t_{1}) \,\|\, (r_{1} < 0 \,\&\&\, r_{5} < -r_{1} \,\&\&\, t_{1} > 0 \,\&\&\, t_{3} > 0)$$

Okay, that concludes the analysis of this theory.

Case 45

Now for a new theory. Here is the full nonlinear Lagrangian for Case 45 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}\underset{1}{r}\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl}+\frac{2}{3}\underset{1}{r}\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl}+\underset{5}{r}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl}-\frac{2}{3}\underset{1}{r}\,\mathcal{R}^{ijkl}\,\mathcal{R}_{klij}-\underset{5}{r}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl}+\frac{1}{12}\left(4\underset{1}{t}+\underset{2}{t}\right)\mathcal{T}_{ijk}\,\mathcal{T}^{ijk}+\frac{1}{6}\left(2\underset{1}{t}-\underset{2}{t}\right)\mathcal{T}^{ijk}\,\mathcal{T}_{jik}+\underset{1}{t}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\left(\underset{1}{t}+\underset{2}{t}\right)\mathcal{A}_{aa'i}\,\mathcal{A}^{aa'i}+\frac{1}{3}\left(\underset{1}{t}-2\underset{2}{t}\right)\mathcal{A}_{aia'}\,\mathcal{A}^{aa'i}+\underset{1}{t}\,\mathcal{A}^{aa'}{}_{a}\,\mathcal{A}_{a'}{}^{i}{}_{i}-2\underset{1}{t}\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial_{a}f^{aa'}+2\underset{1}{t}\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}-\underset{1}{t}\,\partial_{a'}f^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}-\underset{1}{t}\,\partial_{a}f^{aa'}\,\partial_{i}f^{i}{}_{a'}+2\underset{1}{t}\,\partial^{a'}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{a'}+\underset{5}{r}\,\partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}-\underset{5}{r}\,\partial_{i}\mathcal{A}_{a'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}-\frac{2}{3}\left(\underset{1}{t}+\underset{2}{t}\right)\mathcal{A}_{aa'i}\,\partial^{i}f^{aa'}+\frac{2}{3}\left(\underset{1}{t}+\underset{2}{t}\right)\mathcal{A}_{aia'}\,\partial^{i}f^{aa'}+\frac{2}{3}\left(2\underset{1}{t}-\underset{2}{t}\right)\mathcal{A}_{a'ia}\,\partial^{i}f^{aa'}+\frac{1}{3}\left(-2\underset{1}{t}+\underset{2}{t}\right)\partial_{a}f_{a'i}\,\partial^{i}f^{aa'}+\frac{1}{6}\left(2\underset{1}{t}-\underset{2}{t}\right)\partial_{a}f_{ia'}\,\partial^{i}f^{aa'}+\frac{1}{6}\left(-4\underset{1}{t}-\underset{2}{t}\right)\partial_{a'}f_{ai}\,\partial^{i}f^{aa'}+\frac{1}{6}\left(4\underset{1}{t}+\underset{2}{t}\right)\partial_{i}f_{aa'}\,\partial^{i}f^{aa'}+\frac{1}{6}\left(2\underset{1}{t}-\underset{2}{t}\right)\partial_{i}f_{a'a}\,\partial^{i}f^{aa'}-\underset{5}{r}\,\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}+2\underset{5}{r}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}+\underset{5}{r}\,\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}-2\underset{5}{r}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}-\frac{4}{3}\underset{1}{r}\,\partial_{a'}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\underset{1}{r}\,\partial_{a'}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{aa'i}-\frac{8}{3}\underset{1}{r}\,\partial_{a'}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{aa'i}-\frac{2}{3}\underset{1}{r}\,\partial_{i}\mathcal{A}_{aa'j}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{aa'i}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{aia'}\,\partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}-2k^{2}\underset{1}{t} & -i\sqrt{2}\,k\underset{1}{t} & 0\\ i\sqrt{2}\,k\underset{1}{t} & -\underset{1}{t} & 0\\ 0 & 0 & 0\end{pmatrix},\left(\underset{2}{t}\right),\begin{pmatrix}\frac{1}{3}k^{2}\left(\underset{1}{t}+\underset{2}{t}\right) & -\frac{ik\left(\underset{1}{t}-2\underset{2}{t}\right)}{3\sqrt{2}} & \frac{1}{3}ik\left(\underset{1}{t}+\underset{2}{t}\right)\\ \frac{ik\left(\underset{1}{t}-2\underset{2}{t}\right)}{3\sqrt{2}} & \frac{1}{6}\left(6k^{2}\left(2\underset{1}{r}+\underset{5}{r}\right)+\underset{1}{t}+4\underset{2}{t}\right) & \frac{-\underset{1}{t}+2\underset{2}{t}}{3\sqrt{2}}\\ -\frac{1}{3}ik\left(\underset{1}{t}+\underset{2}{t}\right) & \frac{-\underset{1}{t}+2\underset{2}{t}}{3\sqrt{2}} & \frac{\underset{1}{t}+\underset{2}{t}}{3}\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}0 & -ik\underset{1}{t} & 0 & 0\\ ik\underset{1}{t} & k^{2}\left(\underset{1}{r}+\underset{5}{r}\right)-\frac{\underset{1}{t}}{2} & 0 & \frac{\underset{1}{t}}{\sqrt{2}}\\ 0 & 0 & 0 & 0\\ 0 & \frac{\underset{1}{t}}{\sqrt{2}} & 0 & 0\end{pmatrix},\begin{pmatrix}k^{2}\underset{1}{t} & \frac{ik\underset{1}{t}}{\sqrt{2}}\\ -\frac{ik\underset{1}{t}}{\sqrt{2}} & \frac{\underset{1}{t}}{2}\end{pmatrix},\left(\frac{1}{2}\left(2k^{2}\underset{1}{r}+\underset{1}{t}\right)\right)\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\cdot}\tau^{\flat\perp}==0,\ -i\,\overset{0^+}{\cdot}\tau^{\flat\parallel}==2k\,\overset{0^+}{\cdot}\sigma^{\flat\parallel},\ -i\,\overset{1^+}{\cdot}\tau^{\flat\parallel}{}^{ab}==k\,\overset{1^+}{\cdot}\sigma^{\flat\perp}{}^{ab},\ i\,\overset{1^-}{\cdot}\tau^{\flat\parallel}{}^{a}==2k\,\overset{1^-}{\cdot}\sigma^{\flat\perp}{}^{a},\ \overset{1^-}{\cdot}\tau^{\flat\perp}{}^{a}==0,\ -i\,\overset{2^+}{\cdot}\tau^{\flat\parallel}{}^{ab}==2k\,\overset{2^+}{\cdot}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{(1+2k^2)^2 t_1} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & -\frac{1}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{t_2} \end{pmatrix},\right.$$

$$\begin{pmatrix} \frac{k^2\left(6k^2(2r_1+r_5)+t_1+4t_2\right)}{(1+k^2)^2\left(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)\right)} & \frac{i\sqrt{2}\,k(t_1-2t_2)}{(1+k^2)\left(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)\right)} & \frac{i\,k\left(6k^2(2r_1+r_5)+t_1+4t_2\right)}{(1+k^2)^2\left(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)\right)} \\ -\frac{i\sqrt{2}\,k(t_1-2t_2)}{(1+k^2)\left(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)\right)} & \frac{2(t_1+t_2)}{3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)} & \frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)\left(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)\right)} \\ -\frac{i\,k\left(6k^2(2r_1+r_5)+t_1+4t_2\right)}{(1+k^2)^2\left(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)\right)} & \frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)\left(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)\right)} & \frac{6k^2(2r_1+r_5)+t_1+4t_2}{(1+k^2)^2\left(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)\right)} \end{pmatrix},$$

$$\left.\begin{pmatrix} \frac{-4k^4(r_1+r_5)+2k^2t_1}{(t_1+2k^2t_1)^2} & -\frac{2ik}{t_1+2k^2t_1} & 0 & \frac{i\sqrt{2}\,k\left(2k^2(r_1+r_5)-t_1\right)}{(t_1+2k^2t_1)^2} \\ \frac{2ik}{t_1+2k^2t_1} & 0 & 0 & \frac{\sqrt{2}}{t_1+2k^2t_1} \\ 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k\left(2k^2(r_1+r_5)-t_1\right)}{(t_1+2k^2t_1)^2} & \frac{\sqrt{2}}{t_1+2k^2t_1} & 0 & \frac{-2k^2(r_1+r_5)+t_1}{(t_1+2k^2t_1)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_1} & \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} \\ -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & \frac{2}{(1+2k^2)^2 t_1} \end{pmatrix}, \begin{pmatrix} \frac{2}{2k^2r_1+t_1} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \left\{-\frac{3t_1t_2}{2(2r_1+r_5)(t_1+t_2)}\right\}, \{\}, \{\}, \left\{-\frac{t_1}{2r_1}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \left\{\frac{-3t_1t_2(t_1+t_2)+6r_1(t_1^2+2t_2^2)+3r_5(t_1^2+2t_2^2)}{(2r_1+r_5)(t_1+t_2)\left(-3t_1t_2+4r_1(t_1+t_2)+2r_5(t_1+t_2)\right)}\right\}, \{\}, \{\}, \left\{-\frac{1}{r_1}\right\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$$r_1 < 0 \;\&\&\; r_5 > -2r_1 \;\&\&\; t_1 > 0 \;\&\&\; -t_1 < t_2 < 0$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$\dot{r}_1 < 0 \,\&\&\, \dot{r}_5 > -2\dot{r}_1 \,\&\&\, \dot{t}_1 > 0 \,\&\&\, -\dot{t}_1 < \dot{t}_2 < 0$

Okay, that concludes the analysis of this theory.

Case 46

Now for a new theory. Here is the full nonlinear Lagrangian for Case 46 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\left(2\dot{r}_1+\dot{r}_2\right)\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl}+\frac{2}{3}\left(\dot{r}_1-\dot{r}_2\right)\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl}-2\dot{r}_1\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl}+\frac{1}{6}\left(-4\dot{r}_1+\dot{r}_2\right)\mathcal{R}^{ijkl}\,\mathcal{R}_{klij}+2\dot{r}_1\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl}+\frac{1}{12}\left(4\dot{t}_1+\dot{t}_2\right)\mathcal{T}_{ijk}\,\mathcal{T}^{ijk}+\frac{1}{6}\left(2\dot{t}_1-\dot{t}_2\right)\mathcal{T}^{ijk}\,\mathcal{T}_{jik}+\dot{t}_1\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\left(\dot{t}_1+\dot{t}_2\right)\mathcal{A}_{aa'i}\,\mathcal{A}^{aa'i}+\frac{1}{3}\left(\dot{t}_1-2\dot{t}_2\right)\mathcal{A}_{aia'}\,\mathcal{A}^{aa'i}+\dot{t}_1\,\mathcal{A}^{aa'}{}_{a}\,\mathcal{A}_{a'}{}^{i}{}_{i}-2\dot{t}_1\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial_{a}f^{aa'}+2\dot{t}_1\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}-\dot{t}_1\,\partial_{a'}f^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}-\dot{t}_1\,\partial_{a}f^{aa'}\,\partial_{i}f^{i}{}_{a'}+2\dot{t}_1\,\partial^{a'}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{a'}-2\dot{r}_1\,\partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}+2\dot{r}_1\,\partial_{i}\mathcal{A}_{a'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}-\frac{2}{3}\left(\dot{t}_1+\dot{t}_2\right)\mathcal{A}_{aa'i}\,\partial^{i}f^{aa'}+\frac{2}{3}\left(\dot{t}_1+\dot{t}_2\right)\mathcal{A}_{aia'}\,\partial^{i}f^{aa'}+\frac{2}{3}\left(2\dot{t}_1-\dot{t}_2\right)\mathcal{A}_{a'ia}\,\partial^{i}f^{aa'}+\frac{1}{3}\left(-2\dot{t}_1+\dot{t}_2\right)\partial_{a}f_{a'i}\,\partial^{i}f^{aa'}+\frac{1}{6}\left(2\dot{t}_1-\dot{t}_2\right)\partial_{a}f_{ia'}\,\partial^{i}f^{aa'}+\frac{1}{6}\left(-4\dot{t}_1-\dot{t}_2\right)\partial_{a'}f_{ai}\,\partial^{i}f^{aa'}+\frac{1}{6}\left(4\dot{t}_1+\dot{t}_2\right)\partial_{i}f_{aa'}\,\partial^{i}f^{aa'}+\frac{1}{6}\left(2\dot{t}_1-\dot{t}_2\right)\partial_{i}f_{a'a}\,\partial^{i}f^{aa'}+2\dot{r}_1\,\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}-4\dot{r}_1\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}-2\dot{r}_1\,\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}+4\dot{r}_1\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}-\frac{4}{3}\left(\dot{r}_1-\dot{r}_2\right)\partial_{a'}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\left(\dot{r}_1-\dot{r}_2\right)\partial_{a'}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\left(-4\dot{r}_1+\dot{r}_2\right)\partial_{a'}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{1}{3}\left(-2\dot{r}_1-\dot{r}_2\right)\partial_{i}\mathcal{A}_{aa'j}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{1}{3}\left(2\dot{r}_1+\dot{r}_2\right)\partial_{j}\mathcal{A}_{aa'i}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\left(\dot{r}_1-\dot{r}_2\right)\partial_{j}\mathcal{A}_{aia'}\,\partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_{\dot 1} & -i\sqrt{2}\,k\,t_{\dot 1} & 0 \\ i\sqrt{2}\,k\,t_{\dot 1} & -t_{\dot 1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_{\dot 2} + t_{\dot 2}\right), \begin{pmatrix} \frac{1}{3}k^2\left(t_{\dot 1}+t_{\dot 2}\right) & -\frac{i k\left(t_{\dot 1}-2t_{\dot 2}\right)}{3\sqrt{2}} & \frac{1}{3} i k\left(t_{\dot 1}+t_{\dot 2}\right) \\ \frac{i k\left(t_{\dot 1}-2t_{\dot 2}\right)}{3\sqrt{2}} & \frac{1}{6}\left(t_{\dot 1}+4t_{\dot 2}\right) & \frac{-t_{\dot 1}+2t_{\dot 2}}{3\sqrt{2}} \\ -\frac{1}{3} i k\left(t_{\dot 1}+t_{\dot 2}\right) & \frac{-t_{\dot 1}+2t_{\dot 2}}{3\sqrt{2}} & \frac{t_{\dot 1}+t_{\dot 2}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & -i k t_{\dot 1} & 0 & 0 \\ i k t_{\dot 1} & -k^2 r_{\dot 1} - \frac{t_{\dot 1}}{2} & 0 & \frac{t_{\dot 1}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{\dot 1}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_{\dot 1} & \frac{i k t_{\dot 1}}{\sqrt{2}} \\ -\frac{i k t_{\dot 1}}{\sqrt{2}} & \frac{t_{\dot 1}}{2} \end{pmatrix}, \left(\frac{1}{2}\left(2k^2 r_{\dot 1} + t_{\dot 1}\right)\right)\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\cdot}\tau^{\flat\perp} == 0,\ -i\,\overset{0^+}{\cdot}\tau^{\flat\parallel} == 2k\,\overset{0^+}{\cdot}\sigma^{\flat\parallel},\ -i\,\overset{1^+}{\cdot}\tau^{\flat\parallel}{}^{ab} == k\,\overset{1^+}{\cdot}\sigma^{\flat\perp}{}^{ab},\ i\,\overset{1^-}{\cdot}\tau^{\flat\parallel}{}^{a} == 2k\,\overset{1^-}{\cdot}\sigma^{\flat\perp}{}^{a},\ \overset{1^-}{\cdot}\tau^{\flat\perp}{}^{a} == 0,\ -i\,\overset{2^+}{\cdot}\tau^{\flat\parallel}{}^{ab} == 2k\,\overset{2^+}{\cdot}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{(1+2k^2)^2 t_{\dot 1}} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot 1}} & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot 1}} & -\frac{1}{(1+2k^2)^2 t_{\dot 1}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_{\dot 2}+t_{\dot 2}}\right), \begin{pmatrix} \frac{k^2\left(t_{\dot 1}+4t_{\dot 2}\right)}{3(1+k^2)^2 t_{\dot 1} t_{\dot 2}} & \frac{i\sqrt{2}\,k\left(t_{\dot 1}-2t_{\dot 2}\right)}{3(1+k^2) t_{\dot 1} t_{\dot 2}} & \frac{i k\left(t_{\dot 1}+4t_{\dot 2}\right)}{3(1+k^2)^2 t_{\dot 1} t_{\dot 2}} \\ -\frac{i\sqrt{2}\,k\left(t_{\dot 1}-2t_{\dot 2}\right)}{3(1+k^2) t_{\dot 1} t_{\dot 2}} & \frac{2\left(t_{\dot 1}+t_{\dot 2}\right)}{3 t_{\dot 1} t_{\dot 2}} & \frac{\sqrt{2}\left(t_{\dot 1}-2t_{\dot 2}\right)}{3(1+k^2) t_{\dot 1} t_{\dot 2}} \\ -\frac{i k\left(t_{\dot 1}+4t_{\dot 2}\right)}{3(1+k^2)^2 t_{\dot 1} t_{\dot 2}} & \frac{\sqrt{2}\left(t_{\dot 1}-2t_{\dot 2}\right)}{3(1+k^2) t_{\dot 1} t_{\dot 2}} & \frac{t_{\dot 1}+4t_{\dot 2}}{3(1+k^2)^2 t_{\dot 1} t_{\dot 2}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2\left(2k^2 r_{\dot 1}+t_{\dot 1}\right)}{\left(t_{\dot 1}+2k^2 t_{\dot 1}\right)^2} & -\frac{2ik}{t_{\dot 1}+2k^2 t_{\dot 1}} & 0 & -\frac{i\sqrt{2}\,k\left(2k^2 r_{\dot 1}+t_{\dot 1}\right)}{\left(t_{\dot 1}+2k^2 t_{\dot 1}\right)^2} \\ \frac{2ik}{t_{\dot 1}+2k^2 t_{\dot 1}} & 0 & 0 & \frac{\sqrt{2}}{t_{\dot 1}+2k^2 t_{\dot 1}} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k\left(2k^2 r_{\dot 1}+t_{\dot 1}\right)}{\left(t_{\dot 1}+2k^2 t_{\dot 1}\right)^2} & \frac{\sqrt{2}}{t_{\dot 1}+2k^2 t_{\dot 1}} & 0 & \frac{2k^2 r_{\dot 1}+t_{\dot 1}}{\left(t_{\dot 1}+2k^2 t_{\dot 1}\right)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_{\dot 1}} & \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot 1}} \\ -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot 1}} & \frac{2}{(1+2k^2)^2 t_{\dot 1}} \end{pmatrix}, \left(\frac{2}{2k^2 r_{\dot 1}+t_{\dot 1}}\right)\right\}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\dot 2}}{r_{\dot 2}}\right\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{\dot 1}}{2r_{\dot 1}}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\dot 2}}\right\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{\dot 1}}\right\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall unitarity conditions:

$r_{1} < 0 \,\&\&\, r_{2} < 0 \,\&\&\, t_{1} > 0 \,\&\&\, t_{2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{1} < 0 \,\&\&\, r_{2} < 0 \,\&\&\, t_{1} > 0 \,\&\&\, t_{2} > 0$

Okay, that concludes the analysis of this theory.

Case 47

Now for a new theory. Here is the full nonlinear Lagrangian for Case 47 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$r_{5}\, \mathcal{R}^{ij}{}_{i}{}^{\mathfrak{k}}\, \mathcal{R}_{j}{}^{l}{}_{\mathfrak{k}l} - r_{5}\, \mathcal{R}^{ij}{}_{i}{}^{\mathfrak{k}}\, \mathcal{R}_{\mathfrak{k}}{}^{l}{}_{jl} + \frac{1}{4} t_{1}\, \mathcal{T}_{ij\mathfrak{k}}\, \mathcal{T}^{ij\mathfrak{k}} + \frac{1}{2} t_{1}\, \mathcal{T}^{ij\mathfrak{k}}\, \mathcal{T}_{ji\mathfrak{k}} + \frac{1}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{\mathfrak{k}}{}_{j\mathfrak{k}}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
& t_{1}\, \mathcal{A}_{\mathfrak{a}i\mathfrak{a}'}\, \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i} + \frac{1}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}\, \mathcal{A}_{\mathfrak{a}'}{}^{i}{}_{i} - \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}_{\mathfrak{a}'}{}^{i}{}_{i}\, \partial_{\mathfrak{a}} f^{\mathfrak{a}\mathfrak{a}'} + \\
& \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}_{\mathfrak{a}'}{}^{i}{}_{i}\, \partial^{\mathfrak{a}'} f^{\mathfrak{a}}{}_{\mathfrak{a}} + \frac{1}{3}\left(-t_{1} + 2 t_{3}\right) \partial_{\mathfrak{a}'} f^{i}{}_{i}\, \partial^{\mathfrak{a}'} f^{\mathfrak{a}}{}_{\mathfrak{a}} + \frac{1}{3}\left(-t_{1} + 2 t_{3}\right) \partial_{\mathfrak{a}} f^{\mathfrak{a}\mathfrak{a}'}\, \partial_{i} f^{i}{}_{\mathfrak{a}'} + \\
& \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \partial^{\mathfrak{a}'} f^{\mathfrak{a}}{}_{\mathfrak{a}}\, \partial_{i} f^{i}{}_{\mathfrak{a}'} + r_{5}\, \partial_{\mathfrak{a}'} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} - r_{5}\, \partial_{i} \mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}} + 2 t_{1}\, \mathcal{A}_{\mathfrak{a}'i\mathfrak{a}}\, \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} - \\
& t_{1}\, \partial_{\mathfrak{a}} f_{\mathfrak{a}'i}\, \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \frac{1}{2} t_{1}\, \partial_{\mathfrak{a}} f_{i\mathfrak{a}'}\, \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} - \frac{1}{2} t_{1}\, \partial_{\mathfrak{a}'} f_{\mathfrak{a}i}\, \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \frac{1}{2} t_{1}\, \partial_{i} f_{\mathfrak{a}\mathfrak{a}'}\, \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} + \frac{1}{2} t_{1}\, \partial_{i} f_{\mathfrak{a}'\mathfrak{a}}\, \partial^{i} f^{\mathfrak{a}\mathfrak{a}'} - \\
& r_{5}\, \partial_{\mathfrak{a}} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\, \partial_{j} \mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{i} + 2 r_{5}\, \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}\, \partial_{j} \mathcal{A}_{\mathfrak{a}'}{}^{j}{}_{i} + r_{5}\, \partial_{\mathfrak{a}} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'i}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\mathfrak{a}'} - 2 r_{5}\, \partial^{i} \mathcal{A}^{\mathfrak{a}\mathfrak{a}'}{}_{\mathfrak{a}}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\mathfrak{a}'}
\end{aligned}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2k^2 t_{3} & i\sqrt{2}\,k\,t_{3} & 0 \\ -i\sqrt{2}\,k\,t_{3} & t_{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_{1} \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & \frac{1}{2}\left(2k^2 r_{5} - t_{1}\right) & -\frac{t_{1}}{\sqrt{2}} \\ 0 & -\frac{t_{1}}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2}{3}k^2\left(t_{1}+t_{3}\right) & -\frac{1}{3}\,i\,k\left(t_{1}-2t_{3}\right) & 0 & -\frac{1}{3}\,i\sqrt{2}\,k\left(t_{1}+t_{3}\right) \\ \frac{1}{3}\,i\,k\left(t_{1}-2t_{3}\right) & \frac{1}{6}\left(6k^2 r_{5}+t_{1}+4t_{3}\right) & 0 & \frac{t_{1}-2t_{3}}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3}\,i\sqrt{2}\,k\left(t_{1}+t_{3}\right) & \frac{t_{1}-2t_{3}}{3\sqrt{2}} & 0 & \frac{t_{1}+t_{3}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \begin{pmatrix} \frac{t_{1}}{2} \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}\tau^{\flat\parallel} == 2k\,{}^{0^+}\sigma^{\flat\parallel},\ -i\,{}^{1^+}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}\sigma^{\flat\perp}{}^{ab},\ i\,{}^{1^-}\tau^{\flat\parallel}{}^{a} == 2k\,{}^{1^-}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}\tau^{\flat\perp}{}^{a} == 0,\ -i\,{}^{2^+}\tau^{\flat\parallel}{}^{ab} == 2k\,{}^{2^+}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{\left(1+2k^2\right)^2 t_{3}} & \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{3}} & 0 \\ -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{3}} & \frac{1}{\left(1+2k^2\right)^2 t_{3}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{-2k^4 r_{5}+k^2 t_{1}}{\left(1+k^2\right)^2 t_{1}^2} & -\frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & -\frac{i\left(2k^3 r_{5}-k t_{1}\right)}{\left(1+k^2\right)^2 t_{1}^2} \\ \frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} \\ \frac{i\left(2k^3 r_{5}-k t_{1}\right)}{\left(1+k^2\right)^2 t_{1}^2} & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2k^2 r_{5}+t_{1}}{\left(1+k^2\right)^2 t_{1}^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2\left(6k^2 r_{5}+t_{1}+4t_{3}\right)}{\left(1+2k^2\right)^2\left(3t_{1}t_{3}+2k^2 r_{5}\left(t_{1}+t_{3}\right)\right)} & \frac{2ik\left(t_{1}-2t_{3}\right)}{\left(1+2k^2\right)\left(3t_{1}t_{3}+2k^2 r_{5}\left(t_{1}+t_{3}\right)\right)} & 0 & -\frac{i\sqrt{2}\,k\left(6k^2 r_{5}+t_{1}+4t_{3}\right)}{\left(1+2k^2\right)^2\left(3t_{1}t_{3}+2k^2 r_{5}\left(t_{1}+t_{3}\right)\right)} \\ -\frac{2ik\left(t_{1}-2t_{3}\right)}{\left(1+2k^2\right)\left(3t_{1}t_{3}+2k^2 r_{5}\left(t_{1}+t_{3}\right)\right)} & \frac{2\left(t_{1}+t_{3}\right)}{3t_{1}t_{3}+2k^2 r_{5}\left(t_{1}+t_{3}\right)} & 0 & -\frac{\sqrt{2}\left(t_{1}-2t_{3}\right)}{\left(1+2k^2\right)\left(3t_{1}t_{3}+2k^2 r_{5}\left(t_{1}+t_{3}\right)\right)} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k\left(6k^2 r_{5}+t_{1}+4t_{3}\right)}{\left(1+2k^2\right)^2\left(3t_{1}t_{3}+2k^2 r_{5}\left(t_{1}+t_{3}\right)\right)} & -\frac{\sqrt{2}\left(t_{1}-2t_{3}\right)}{\left(1+2k^2\right)\left(3t_{1}t_{3}+2k^2 r_{5}\left(t_{1}+t_{3}\right)\right)} & 0 & \frac{6k^2 r_{5}+t_{1}+4t_{3}}{\left(1+2k^2\right)^2\left(3t_{1}t_{3}+2k^2 r_{5}\left(t_{1}+t_{3}\right)\right)} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_{1}} & \frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} \\ -\frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} & \frac{2}{\left(1+2k^2\right)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{t_{1}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \left\{-\frac{3t_{1}t_{3}}{2r_{5}t_{1}+2r_{5}t_{3}}\right\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \left\{\frac{6t_{1}t_{3}\left(t_{1}+t_{3}\right)-3r_{5}\left(t_{1}^2+2t_{3}^2\right)}{2r_{5}\left(t_{1}+t_{3}\right)\left(-3t_{1}t_{3}+r_{5}\left(t_{1}+t_{3}\right)\right)}\right\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$$r_5 < 0 \,\&\&\, \left(\left(t_1 < 0 \,\&\&\, 0 < t_3 < -t_1\right) \,\|\, \left(t_1 > 0 \,\&\&\, \left(t_3 < -t_1 \,\|\, t_3 > 0\right)\right)\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_5 < 0 \,\&\&\, t_1 < 0 \,\&\&\, 0 < t_3 < -t_1\right) \,\|\, \left(r_5 < 0 \,\&\&\, t_1 > 0 \,\&\&\, t_3 < -t_1\right) \,\|\, \left(r_5 < 0 \,\&\&\, t_1 > 0 \,\&\&\, t_3 > 0\right)$$

Okay, that concludes the analysis of this theory.

Case 48

Now for a new theory. Here is the full nonlinear Lagrangian for Case 48 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$r_5\, \mathcal{R}^{ij\hbar}{}_{i}\, \mathcal{R}_{j}{}^{l}{}_{\hbar l} - r_5\, \mathcal{R}^{ij\hbar}{}_{i}\, \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{12}\left(4 t_1 + t_2\right) \mathcal{T}_{ij\hbar}\, \mathcal{T}^{ij\hbar} + \frac{1}{6}\left(2 t_1 - t_2\right) \mathcal{T}^{ij\hbar}\, \mathcal{T}_{ji\hbar} + t_1\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{\hbar}{}_{j\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3}\left(t_1 + t_2\right) \mathcal{A}_{\alpha\alpha' i}\, \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3}\left(t_1 - 2 t_2\right) \mathcal{A}_{\alpha i\alpha'}\, \mathcal{A}^{\alpha\alpha' i} + t_1\, \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \mathcal{A}_{\alpha'}{}^{i}{}_{i} - \\
&2 t_1\, \mathcal{A}_{\alpha'}{}^{i}{}_{i}\, \partial_{\alpha} f^{\alpha\alpha'} + 2 t_1\, \mathcal{A}_{\alpha'}{}^{i}{}_{i}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha} - t_1\, \partial_{\alpha'} f^{i}{}_{i}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha} - t_1\, \partial_{\alpha} f^{\alpha\alpha'}\, \partial_i f^{i}{}_{\alpha'} + 2 t_1\, \partial^{\alpha'} f^{\alpha}{}_{\alpha}\, \partial_i f^{i}{}_{\alpha'} + \\
&r_5\, \partial_{\alpha'} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} - r_5\, \partial_i \mathcal{A}_{\alpha'}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} - \frac{2}{3}\left(t_1 + t_2\right) \mathcal{A}_{\alpha\alpha' i}\, \partial^{i} f^{\alpha\alpha'} + \frac{2}{3}\left(t_1 + t_2\right) \mathcal{A}_{\alpha i\alpha'}\, \partial^{i} f^{\alpha\alpha'} + \\
&\frac{2}{3}\left(2 t_1 - t_2\right) \mathcal{A}_{\alpha' i\alpha}\, \partial^{i} f^{\alpha\alpha'} + \frac{1}{3}\left(-2 t_1 + t_2\right) \partial_{\alpha} f_{\alpha' i}\, \partial^{i} f^{\alpha\alpha'} + \frac{1}{6}\left(2 t_1 - t_2\right) \partial_{\alpha} f_{i\alpha'}\, \partial^{i} f^{\alpha\alpha'} + \\
&\frac{1}{6}\left(-4 t_1 - t_2\right) \partial_{\alpha'} f_{\alpha i}\, \partial^{i} f^{\alpha\alpha'} + \frac{1}{6}\left(4 t_1 + t_2\right) \partial_i f_{\alpha\alpha'}\, \partial^{i} f^{\alpha\alpha'} + \frac{1}{6}\left(2 t_1 - t_2\right) \partial_i f_{\alpha'\alpha}\, \partial^{i} f^{\alpha\alpha'} - \\
&r_5\, \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i}\, \partial_j \mathcal{A}_{\alpha'}{}^{j}{}_{i} + 2 r_5\, \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \partial_j \mathcal{A}_{\alpha'}{}^{j}{}_{i} + r_5\, \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{\alpha'} - 2 r_5\, \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{\alpha'}
\end{aligned}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_{1} & -i\sqrt{2}\,k\,t_{1} & 0 \\ i\sqrt{2}\,k\,t_{1} & -t_{1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} t_{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{3}k^2\left(t_{1}+t_{2}\right) & -\frac{i\,k\left(t_{1}-2t_{2}\right)}{3\sqrt{2}} & \frac{1}{3}i\,k\left(t_{1}+t_{2}\right) \\ \frac{i\,k\left(t_{1}-2t_{2}\right)}{3\sqrt{2}} & \frac{1}{6}\left(6k^2 r_{5}+t_{1}+4t_{2}\right) & \frac{-t_{1}+2t_{2}}{3\sqrt{2}} \\ -\frac{1}{3}i\,k\left(t_{1}+t_{2}\right) & \frac{-t_{1}+2t_{2}}{3\sqrt{2}} & \frac{t_{1}+t_{2}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & -i\,k\,t_{1} & 0 & 0 \\ i\,k\,t_{1} & k^2 r_{5}-\frac{t_{1}}{2} & 0 & \frac{t_{1}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{1}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i\,k\,t_{1}}{\sqrt{2}} \\ -\frac{i\,k\,t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \begin{pmatrix} \frac{t_{1}}{2} \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\cdot}\tau^{\flat\perp} == 0,\ -i\,\overset{0^+}{\cdot}\tau^{\flat\parallel} == 2k\,\overset{0^+}{\cdot}\sigma^{\flat\parallel},\ -i\,\overset{1^+}{\cdot}\tau^{\flat\parallel}{}^{ab} == k\,\overset{1^+}{\cdot}\sigma^{\flat\perp}{}^{ab},\ i\,\overset{1^-}{\cdot}\tau^{\flat\parallel}{}^{a} == 2k\,\overset{1^-}{\cdot}\sigma^{\flat\perp}{}^{a},\ \overset{1^-}{\cdot}\tau^{\flat\perp}{}^{a} == 0,\ -i\,\overset{2^+}{\cdot}\tau^{\flat\parallel}{}^{ab} == 2k\,\overset{2^+}{\cdot}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{\left(1+2k^2\right)^2 t_{1}} & -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} & 0 \\ \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} & -\frac{1}{\left(1+2k^2\right)^2 t_{1}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{t_{2}} \end{pmatrix}, \begin{pmatrix} \frac{k^2\left(6k^2 r_{5}+t_{1}+4t_{2}\right)}{\left(1+k^2\right)^2\left(3t_{1}t_{2}+2k^2 r_{5}\left(t_{1}+t_{2}\right)\right)} & \frac{i\sqrt{2}\,k\left(t_{1}-2t_{2}\right)}{\left(1+k^2\right)\left(3t_{1}t_{2}+2k^2 r_{5}\left(t_{1}+t_{2}\right)\right)} & \frac{i\,k\left(6k^2 r_{5}+t_{1}+4t_{2}\right)}{\left(1+k^2\right)^2\left(3t_{1}t_{2}+2k^2 r_{5}\left(t_{1}+t_{2}\right)\right)} \\ -\frac{i\sqrt{2}\,k\left(t_{1}-2t_{2}\right)}{\left(1+k^2\right)\left(3t_{1}t_{2}+2k^2 r_{5}\left(t_{1}+t_{2}\right)\right)} & \frac{2\left(t_{1}+t_{2}\right)}{3t_{1}t_{2}+2k^2 r_{5}\left(t_{1}+t_{2}\right)} & \frac{\sqrt{2}\left(t_{1}-2t_{2}\right)}{\left(1+k^2\right)\left(3t_{1}t_{2}+2k^2 r_{5}\left(t_{1}+t_{2}\right)\right)} \\ -\frac{i\,k\left(6k^2 r_{5}+t_{1}+4t_{2}\right)}{\left(1+k^2\right)^2\left(3t_{1}t_{2}+2k^2 r_{5}\left(t_{1}+t_{2}\right)\right)} & \frac{\sqrt{2}\left(t_{1}-2t_{2}\right)}{\left(1+k^2\right)\left(3t_{1}t_{2}+2k^2 r_{5}\left(t_{1}+t_{2}\right)\right)} & \frac{6k^2 r_{5}+t_{1}+4t_{2}}{\left(1+k^2\right)^2\left(3t_{1}t_{2}+2k^2 r_{5}\left(t_{1}+t_{2}\right)\right)} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{-4k^4 r_{5}+2k^2 t_{1}}{\left(t_{1}+2k^2 t_{1}\right)^2} & -\frac{2\,i\,k}{t_{1}+2k^2 t_{1}} & 0 & \frac{i\sqrt{2}\,k\left(2k^2 r_{5}-t_{1}\right)}{\left(t_{1}+2k^2 t_{1}\right)^2} \\ \frac{2\,i\,k}{t_{1}+2k^2 t_{1}} & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} \\ 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k\left(2k^2 r_{5}-t_{1}\right)}{\left(t_{1}+2k^2 t_{1}\right)^2} & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & 0 & \frac{-2k^2 r_{5}+t_{1}}{\left(t_{1}+2k^2 t_{1}\right)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_{1}} & \frac{2\,i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} \\ -\frac{2\,i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} & \frac{2}{\left(1+2k^2\right)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{t_{1}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \left\{-\frac{3t_{1}t_{2}}{2r_{5}t_{1}+2r_{5}t_{2}}\right\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \left\{\frac{-3t_{1}t_{2}\left(t_{1}+t_{2}\right)+3r_{5}\left(t_{1}^2+2t_{2}^2\right)}{r_{5}\left(t_{1}+t_{2}\right)\left(-3t_{1}t_{2}+2r_{5}\left(t_{1}+t_{2}\right)\right)}\right\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall unitarity conditions:

$$r_{5} > 0 \,\&\&\, \left(\left(t_{1} < 0 \,\&\&\, \left(t_{2} < 0 \,\|\, t_{2} > -t_{1}\right)\right) \,\|\, \left(t_{1} > 0 \,\&\&\, -t_{1} < t_{2} < 0\right)\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{5} > 0 \,\&\&\, \left(\left(t_{1} < 0 \,\&\&\, \left(t_{2} < 0 \,\|\, t_{2} > -t_{1}\right)\right) \,\|\, \left(t_{1} > 0 \,\&\&\, -t_{1} < t_{2} < 0\right)\right)$$

Okay, that concludes the analysis of this theory.

Case 49

Now for a new theory. Here is the full nonlinear Lagrangian for Case 49 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2}\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} - \frac{2}{3} r_{2}\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} + r_{5}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6} r_{2}\, \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} - r_{5}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4} t_{1}\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} + \frac{1}{2} t_{1}\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik} + t_{1}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{1}\, \mathcal{A}_{\alpha i \alpha'}\, \mathcal{A}^{\alpha\alpha' i} + t_{1}\, \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \mathcal{A}_{\alpha'}{}^{i}{}_{i} - 2 t_{1}\, \mathcal{A}_{\alpha'}{}^{i}{}_{i}\, \partial_{\alpha} f^{\alpha\alpha'} + 2 t_{1}\, \mathcal{A}_{\alpha'}{}^{i}{}_{i}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha} - t_{1}\, \partial_{\alpha'} f^{i}{}_{i}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha} - t_{1}\, \partial_{\alpha} f^{\alpha\alpha'}\, \partial_{i} f^{i}{}_{\alpha'} + 2 t_{1}\, \partial^{\alpha'} f^{\alpha}{}_{\alpha}\, \partial_{i} f^{i}{}_{\alpha'} + r_{5}\, \partial_{\alpha'} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} - r_{5}\, \partial_{i} \mathcal{A}_{\alpha'}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} + 2 t_{1}\, \mathcal{A}_{\alpha' i \alpha}\, \partial^{i} f^{\alpha\alpha'} - t_{1}\, \partial_{\alpha} f_{\alpha' i}\, \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{1}\, \partial_{\alpha} f_{i\alpha'}\, \partial^{i} f^{\alpha\alpha'} - \frac{1}{2} t_{1}\, \partial_{\alpha'} f_{\alpha i}\, \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{1}\, \partial_{i} f_{\alpha\alpha'}\, \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{1}\, \partial_{i} f_{\alpha'\alpha}\, \partial^{i} f^{\alpha\alpha'} - r_{5}\, \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i}\, \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i} + 2 r_{5}\, \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i} + r_{5}\, \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'} - 2 r_{5}\, \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'} + \frac{4}{3} r_{2}\, \partial_{\alpha'} \mathcal{A}_{\alpha ij}\, \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_{2}\, \partial_{\alpha'} \mathcal{A}_{\alpha ji}\, \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{2}{3} r_{2}\, \partial_{\alpha'} \mathcal{A}_{ij\alpha}\, \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{1}{3} r_{2}\, \partial_{i} \mathcal{A}_{\alpha\alpha' j}\, \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3} r_{2}\, \partial_{j} \mathcal{A}_{\alpha\alpha' i}\, \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_{2}\, \partial_{j} \mathcal{A}_{\alpha i\alpha'}\, \partial^{j} \mathcal{A}^{\alpha\alpha' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_1 & -i\sqrt{2}\,k\,t_1 & 0 \\ i\sqrt{2}\,k\,t_1 & -t_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(k^2 r_2 - t_1\right),\right.$$

$$\begin{pmatrix} 0 & -\frac{i k t_1}{\sqrt{2}} & 0 \\ \frac{i k t_1}{\sqrt{2}} & \frac{1}{2}\left(2k^2 r_5 - t_1\right) & -\frac{t_1}{\sqrt{2}} \\ 0 & -\frac{t_1}{\sqrt{2}} & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i k t_1 & 0 & 0 \\ i k t_1 & k^2 r_5 - \frac{t_1}{2} & 0 & \frac{t_1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_1}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_1 & \frac{i k t_1}{\sqrt{2}} \\ -\frac{i k t_1}{\sqrt{2}} & \frac{t_1}{2} \end{pmatrix}, \left.\left(\frac{t_1}{2}\right)\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}_{\bullet}\tau^{\flat\parallel} == 2k\,{}^{0^+}_{\bullet}\sigma^{\flat\parallel},\ -i\,{}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab},\ i\,{}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a} == 2k\,{}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ -i\,{}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 2k\,{}^{2^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab} \right\}$$

The Drazin (Moore–Penrose) inverses of these a–matrices, which are functionally analogous to the inverse b–matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{(1+2k^2)^2 t_1} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & -\frac{1}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \left(\frac{1}{k^2 r_2 - t_1}\right), \begin{pmatrix} \frac{-2k^4 r_5 + k^2 t_1}{(1+k^2)^2 t_1^2} & -\frac{i\sqrt{2}\,k}{t_1 + k^2 t_1} & -\frac{i\left(2k^3 r_5 - k t_1\right)}{(1+k^2)^2 t_1^2} \\ \frac{i\sqrt{2}\,k}{t_1 + k^2 t_1} & 0 & -\frac{\sqrt{2}}{t_1 + k^2 t_1} \\ \frac{i\left(2k^3 r_5 - k t_1\right)}{(1+k^2)^2 t_1^2} & -\frac{\sqrt{2}}{t_1 + k^2 t_1} & \frac{-2k^2 r_5 + t_1}{(1+k^2)^2 t_1^2} \end{pmatrix},\right.$$

$$\begin{pmatrix} \frac{-4k^4 r_5 + 2k^2 t_1}{\left(t_1 + 2k^2 t_1\right)^2} & -\frac{2 i k}{t_1 + 2k^2 t_1} & 0 & \frac{i\sqrt{2}\,k\left(2k^2 r_5 - t_1\right)}{\left(t_1 + 2k^2 t_1\right)^2} \\ \frac{2 i k}{t_1 + 2k^2 t_1} & 0 & 0 & \frac{\sqrt{2}}{t_1 + 2k^2 t_1} \\ 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k\left(2k^2 r_5 - t_1\right)}{\left(t_1 + 2k^2 t_1\right)^2} & \frac{\sqrt{2}}{t_1 + 2k^2 t_1} & 0 & \frac{-2k^2 r_5 + t_1}{\left(t_1 + 2k^2 t_1\right)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_1} & \frac{2 i\sqrt{2}\,k}{(1+2k^2)^2 t_1} \\ -\frac{2 i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & \frac{2}{(1+2k^2)^2 t_1} \end{pmatrix}, \left.\left(\frac{2}{t_1}\right)\right\}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_2 < 0\ \&\&\ t_1 < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\dot{2}} < 0 \,\&\&\, t_{\dot{1}} < 0$

Okay, that concludes the analysis of this theory.

Case 50

Now for a new theory. Here is the full nonlinear Lagrangian for Case 50 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{\dot{1}} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} + \frac{2}{3} r_{\dot{1}} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} - r_{\dot{1}} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} - \frac{2}{3} r_{\dot{1}} \mathcal{R}^{ijkl} \mathcal{R}_{klij} + r_{\dot{1}} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4} t_{\dot{1}} \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{2} t_{\dot{1}} \mathcal{T}^{ijk} \mathcal{T}_{jik} + \frac{1}{3}\left(t_{\dot{1}} - 2 t_{\dot{3}}\right) \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{\dot{1}} \mathcal{A}_{\alpha i \alpha'} \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3}\left(t_{\dot{1}} - 2 t_{\dot{3}}\right) \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \mathcal{A}_{\alpha'}{}^{i}{}_{i} - \frac{2}{3}\left(t_{\dot{1}} - 2 t_{\dot{3}}\right) \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial_{\alpha} f^{\alpha\alpha'} + \frac{2}{3}\left(t_{\dot{1}} - 2 t_{\dot{3}}\right) \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha} +$$
$$\frac{1}{3}\left(-t_{\dot{1}} + 2 t_{\dot{3}}\right) \partial_{\alpha'} f^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha} + \frac{1}{3}\left(-t_{\dot{1}} + 2 t_{\dot{3}}\right) \partial_{\alpha} f^{\alpha\alpha'} \partial_{i} f^{i}{}_{\alpha'} + \frac{2}{3}\left(t_{\dot{1}} - 2 t_{\dot{3}}\right) \partial^{\alpha'} f^{\alpha}{}_{\alpha} \partial_{i} f^{i}{}_{\alpha'} -$$
$$r_{\dot{1}} \partial_{\alpha'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} + r_{\dot{1}} \partial_{i} \mathcal{A}_{\alpha'}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} + 2 t_{\dot{1}} \mathcal{A}_{\alpha' i \alpha} \partial^{i} f^{\alpha\alpha'} - t_{\dot{1}} \partial_{\alpha} f_{\alpha' i} \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{\dot{1}} \partial_{\alpha} f_{i\alpha'} \partial^{i} f^{\alpha\alpha'} -$$
$$\frac{1}{2} t_{\dot{1}} \partial_{\alpha'} f_{\alpha i} \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{\dot{1}} \partial_{i} f_{\alpha\alpha'} \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{\dot{1}} \partial_{i} f_{\alpha'\alpha} \partial^{i} f^{\alpha\alpha'} + r_{\dot{1}} \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i} \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i} - 2 r_{\dot{1}} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i} -$$
$$r_{\dot{1}} \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'} + 2 r_{\dot{1}} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'} - \frac{4}{3} r_{\dot{1}} \partial_{\alpha'} \mathcal{A}_{\alpha ij} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{2}{3} r_{\dot{1}} \partial_{\alpha'} \mathcal{A}_{\alpha ji} \partial^{j} \mathcal{A}^{\alpha\alpha' i} -$$
$$\frac{8}{3} r_{\dot{1}} \partial_{\alpha'} \mathcal{A}_{ij\alpha} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_{\dot{1}} \partial_{i} \mathcal{A}_{\alpha\alpha' j} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{2}{3} r_{\dot{1}} \partial_{j} \mathcal{A}_{\alpha\alpha' i} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{2}{3} r_{\dot{1}} \partial_{j} \mathcal{A}_{\alpha i \alpha'} \partial^{j} \mathcal{A}^{\alpha\alpha' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 2k^2 t_3 & i\sqrt{2}\,k\,t_3 & 0 \\ -i\sqrt{2}\,k\,t_3 & t_3 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_1 \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_1}{\sqrt{2}} & 0 \\ \frac{i k t_1}{\sqrt{2}} & \frac{1}{2}\left(2k^2 r_1 - t_1\right) & -\frac{t_1}{\sqrt{2}} \\ 0 & -\frac{t_1}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2}{3}k^2\left(t_1+t_3\right) & -\frac{1}{3}i k\left(t_1-2t_3\right) & 0 & -\frac{1}{3}i\sqrt{2}\,k\left(t_1+t_3\right) \\ \frac{1}{3}i k\left(t_1-2t_3\right) & \frac{1}{6}\left(t_1+4t_3\right) & 0 & \frac{t_1-2t_3}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3}i\sqrt{2}\,k\left(t_1+t_3\right) & \frac{t_1-2t_3}{3\sqrt{2}} & 0 & \frac{t_1+t_3}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_1 & \frac{i k t_1}{\sqrt{2}} \\ -\frac{i k t_1}{\sqrt{2}} & \frac{t_1}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2k^2 r_1 + t_1\right) \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\bullet}\tau^{\flat\perp} == 0,\ -i\,\overset{0^+}{\bullet}\tau^{\flat\parallel} == 2k\,\overset{0^+}{\bullet}\sigma^{\flat\parallel},\ -i\,\overset{1^+}{\bullet}\tau^{\flat\parallel\,ab} == k\,\overset{1^+}{\bullet}\sigma^{\flat\perp\,ab},\ i\,\overset{1^-}{\bullet}\tau^{\flat\parallel\,a} == 2k\,\overset{1^-}{\bullet}\sigma^{\flat\perp\,a},\ \overset{1^-}{\bullet}\tau^{\flat\perp\,a} == 0,\ -i\,\overset{2^+}{\bullet}\tau^{\flat\parallel\,ab} == 2k\,\overset{2^+}{\bullet}\sigma^{\flat\parallel\,ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these a–matrices, which are functionally analogous to the inverse b–matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} \frac{2k^2}{\left(1+2k^2\right)^2 t_3} & \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_3} & 0 \\ -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_3} & \frac{1}{\left(1+2k^2\right)^2 t_3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{t_1} \end{pmatrix}, \begin{pmatrix} \frac{-2k^4 r_1 + k^2 t_1}{\left(1+k^2\right)^2 t_1^2} & -\frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & -\frac{i\left(2k^3 r_1 - k t_1\right)}{\left(1+k^2\right)^2 t_1^2} \\ \frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & 0 & -\frac{\sqrt{2}}{t_1+k^2 t_1} \\ \frac{i\left(2k^3 r_1 - k t_1\right)}{\left(1+k^2\right)^2 t_1^2} & -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{-2k^2 r_1 + t_1}{\left(1+k^2\right)^2 t_1^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2\left(t_1+4t_3\right)}{3\left(1+2k^2\right)^2 t_1 t_3} & \frac{2ikt_1-4ikt_3}{3t_1t_3+6k^2t_1t_3} & 0 & -\frac{i\sqrt{2}\,k\left(t_1+4t_3\right)}{3\left(1+2k^2\right)^2 t_1 t_3} \\ -\frac{2ikt_1-4ikt_3}{3t_1t_3+6k^2t_1t_3} & \frac{2\left(t_1+t_3\right)}{3t_1t_3} & 0 & -\frac{\sqrt{2}\left(t_1-2t_3\right)}{3\left(1+2k^2\right)t_1t_3} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k\left(t_1+4t_3\right)}{3\left(1+2k^2\right)^2 t_1 t_3} & -\frac{\sqrt{2}\left(t_1-2t_3\right)}{3\left(1+2k^2\right)t_1t_3} & 0 & \frac{t_1+4t_3}{3\left(1+2k^2\right)^2 t_1 t_3} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_1} & \frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} \\ -\frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} & \frac{2}{\left(1+2k^2\right)^2 t_1} \end{pmatrix}, \begin{pmatrix} \frac{2}{2k^2 r_1 + t_1} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_1}{2r_1}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_1}\right\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall unitarity conditions:

$r_{1} < 0 \&\& t_{1} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{1} < 0 \&\& t_{1} > 0$

Okay, that concludes the analysis of this theory.

Case 51

Now for a new theory. Here is the full nonlinear Lagrangian for Case 51 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{1} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} + \frac{2}{3} r_{1} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} - 2 r_{1} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} - \frac{2}{3} r_{1} \mathcal{R}^{ijkl} \mathcal{R}_{klij} + 2 r_{1} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12} \left(4 t_{1} + t_{2}\right) \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{6} \left(2 t_{1} - t_{2}\right) \mathcal{T}^{ijk} \mathcal{T}_{jik} + t_{1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} \left(t_{1} + t_{2}\right) \mathcal{A}_{aa'i} \mathcal{A}^{aa'i} + \frac{1}{3} \left(t_{1} - 2 t_{2}\right) \mathcal{A}_{aia'} \mathcal{A}^{aa'i} + t_{1} \mathcal{A}^{aa'}{}_{a} \mathcal{A}_{a'}{}^{i}{}_{i} - 2 t_{1} \mathcal{A}_{a'}{}^{i}{}_{i} \partial_{a} f^{aa'} + 2 t_{1} \mathcal{A}_{a'}{}^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - t_{1} \partial_{a'} f^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - t_{1} \partial_{a} f^{aa'} \partial_{i} f^{i}{}_{a'} + 2 t_{1} \partial^{a'} f^{a}{}_{a} \partial_{i} f^{i}{}_{a'} - 2 r_{1} \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{aa'}{}_{a} + 2 r_{1} \partial_{i} \mathcal{A}_{a'}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{aa'}{}_{a} - \frac{2}{3} \left(t_{1} + t_{2}\right) \mathcal{A}_{aa'i} \partial^{i} f^{aa'} + \frac{2}{3} \left(t_{1} + t_{2}\right) \mathcal{A}_{aia'} \partial^{i} f^{aa'} + \frac{2}{3} \left(2 t_{1} - t_{2}\right) \mathcal{A}_{a'ia} \partial^{i} f^{aa'} + \frac{1}{3} \left(-2 t_{1} + t_{2}\right) \partial_{a} f_{a'i} \partial^{i} f^{aa'} + \frac{1}{6} \left(2 t_{1} - t_{2}\right) \partial_{a} f_{ia'} \partial^{i} f^{aa'} + \frac{1}{6} \left(-4 t_{1} - t_{2}\right) \partial_{a'} f_{ai} \partial^{i} f^{aa'} + \frac{1}{6} \left(4 t_{1} + t_{2}\right) \partial_{i} f_{aa'} \partial^{i} f^{aa'} + \frac{1}{6} \left(2 t_{1} - t_{2}\right) \partial_{i} f_{a'a} \partial^{i} f^{aa'} + 2 r_{1} \partial_{a} \mathcal{A}^{aa'i} \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} - 4 r_{1} \partial^{i} \mathcal{A}^{aa'}{}_{a} \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} - 2 r_{1} \partial_{a} \mathcal{A}^{aa'i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} + 4 r_{1} \partial^{i} \mathcal{A}^{aa'}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} - \frac{4}{3} r_{1} \partial_{a'} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{1} \partial_{a'} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{aa'i} - \frac{8}{3} r_{1} \partial_{a'} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{1} \partial_{i} \mathcal{A}_{aa'j} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{aa'i} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{aia'} \partial^{j} \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_1 & -i\sqrt{2}\,k\,t_1 & 0 \\ i\sqrt{2}\,k\,t_1 & -t_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} t_2 \end{pmatrix}, \begin{pmatrix} \frac{1}{3}k^2\left(t_1+t_2\right) & -\frac{i k\left(t_1-2t_2\right)}{3\sqrt{2}} & \frac{1}{3}i k\left(t_1+t_2\right) \\ \frac{i k\left(t_1-2t_2\right)}{3\sqrt{2}} & \frac{1}{6}\left(t_1+4t_2\right) & \frac{-t_1+2t_2}{3\sqrt{2}} \\ -\frac{1}{3}i k\left(t_1+t_2\right) & \frac{-t_1+2t_2}{3\sqrt{2}} & \frac{t_1+t_2}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & -i k t_1 & 0 & 0 \\ i k t_1 & -k^2 r_1-\frac{t_1}{2} & 0 & \frac{t_1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_1}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_1 & \frac{i k t_1}{\sqrt{2}} \\ -\frac{i k t_1}{\sqrt{2}} & \frac{t_1}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2k^2 r_1+t_1\right) \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\cdot}\tau^{\flat\perp} == 0,\ -i\,\overset{0^+}{\cdot}\tau^{\flat\parallel} == 2k\,\overset{0^+}{\cdot}\sigma^{\flat\parallel},\ -i\,\overset{1^+}{\cdot}\tau^{\flat\parallel}{}^{ab} == k\,\overset{1^+}{\cdot}\sigma^{\flat\perp}{}^{ab},\ i\,\overset{1^-}{\cdot}\tau^{\flat\parallel}{}^{a} == 2k\,\overset{1^-}{\cdot}\sigma^{\flat\perp}{}^{a},\ \overset{1^-}{\cdot}\tau^{\flat\perp}{}^{a} == 0,\ -i\,\overset{2^+}{\cdot}\tau^{\flat\parallel}{}^{ab} == 2k\,\overset{2^+}{\cdot}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{\left(1+2k^2\right)^2 t_1} & -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} & 0 \\ \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} & -\frac{1}{\left(1+2k^2\right)^2 t_1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{t_2} \end{pmatrix}, \begin{pmatrix} \frac{k^2\left(t_1+4t_2\right)}{3\left(1+k^2\right)^2 t_1 t_2} & \frac{i\sqrt{2}\,k\left(t_1-2t_2\right)}{3\left(1+k^2\right) t_1 t_2} & \frac{i k\left(t_1+4t_2\right)}{3\left(1+k^2\right)^2 t_1 t_2} \\ -\frac{i\sqrt{2}\,k\left(t_1-2t_2\right)}{3\left(1+k^2\right) t_1 t_2} & \frac{2\left(t_1+t_2\right)}{3 t_1 t_2} & \frac{\sqrt{2}\left(t_1-2t_2\right)}{3\left(1+k^2\right) t_1 t_2} \\ -\frac{i k\left(t_1+4t_2\right)}{3\left(1+k^2\right)^2 t_1 t_2} & \frac{\sqrt{2}\left(t_1-2t_2\right)}{3\left(1+k^2\right) t_1 t_2} & \frac{t_1+4t_2}{3\left(1+k^2\right)^2 t_1 t_2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2\left(2k^2 r_1+t_1\right)}{\left(t_1+2k^2 t_1\right)^2} & -\frac{2 i k}{t_1+2k^2 t_1} & 0 & -\frac{i\sqrt{2}\,k\left(2k^2 r_1+t_1\right)}{\left(t_1+2k^2 t_1\right)^2} \\ \frac{2 i k}{t_1+2k^2 t_1} & 0 & 0 & \frac{\sqrt{2}}{t_1+2k^2 t_1} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k\left(2k^2 r_1+t_1\right)}{\left(t_1+2k^2 t_1\right)^2} & \frac{\sqrt{2}}{t_1+2k^2 t_1} & 0 & \frac{2k^2 r_1+t_1}{\left(t_1+2k^2 t_1\right)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_1} & \frac{2 i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} \\ -\frac{2 i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} & \frac{2}{\left(1+2k^2\right)^2 t_1} \end{pmatrix}, \begin{pmatrix} \frac{2}{2k^2 r_1+t_1} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_1}{2 r_1}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_1}\right\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$\dot{r}_1 < 0 \,\&\&\, \dot{t}_1 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$\dot{r}_1 < 0 \,\&\&\, \dot{t}_1 > 0$

Okay, that concludes the analysis of this theory.

Case 52

Now for a new theory. Here is the full nonlinear Lagrangian for Case 52 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}\dot{r}_1 \mathcal{R}_{ij\hbar l}\mathcal{R}^{ij\hbar l} + \frac{2}{3}\dot{r}_1 \mathcal{R}_{i\hbar jl}\mathcal{R}^{ij\hbar l} + \dot{r}_5 \mathcal{R}^{ij}{}_{i}{}^{\hbar}\mathcal{R}_{j}{}^{l}{}_{\hbar l} - \frac{2}{3}\dot{r}_1 \mathcal{R}^{ij\hbar l}\mathcal{R}_{\hbar lij} - \dot{r}_5 \mathcal{R}^{ij}{}_{i}{}^{\hbar}\mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{4}\dot{t}_1 \mathcal{T}_{ij\hbar}\mathcal{T}^{ij\hbar} + \frac{1}{2}\dot{t}_1 \mathcal{T}^{ij\hbar}\mathcal{T}_{ji\hbar} + \dot{t}_1 \mathcal{T}^{i}{}_{i}{}^{j}\mathcal{T}^{\hbar}{}_{j\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\dot{t}_1 \mathcal{A}_{\alpha i\alpha'}\mathcal{A}^{\alpha\alpha' i} + \dot{t}_1 \mathcal{A}^{\alpha\alpha'}{}_{\alpha}\mathcal{A}_{\alpha'}{}^{i}{}_{i} - 2\dot{t}_1 \mathcal{A}_{\alpha'}{}^{i}{}_{i}\partial_{\alpha} f^{\alpha\alpha'} + 2\dot{t}_1 \mathcal{A}_{\alpha'}{}^{i}{}_{i}\partial^{\alpha'} f^{\alpha}{}_{\alpha} - \dot{t}_1 \partial_{\alpha'} f^{i}{}_{i}\partial^{\alpha'} f^{\alpha}{}_{\alpha} - \dot{t}_1 \partial_{\alpha} f^{\alpha\alpha'}\partial_i f^{i}{}_{\alpha'} + 2\dot{t}_1 \partial^{\alpha'} f^{\alpha}{}_{\alpha}\partial_i f^{i}{}_{\alpha'} + \dot{r}_5 \partial_{\alpha'}\mathcal{A}_{i}{}^{j}{}_{j}\partial^i\mathcal{A}^{\alpha\alpha'}{}_{\alpha} - \dot{r}_5 \partial_i\mathcal{A}_{\alpha'}{}^{j}{}_{j}\partial^i\mathcal{A}^{\alpha\alpha'}{}_{\alpha} + 2\dot{t}_1 \mathcal{A}_{\alpha' i\alpha}\partial^i f^{\alpha\alpha'} - \dot{t}_1 \partial_{\alpha} f_{\alpha' i}\partial^i f^{\alpha\alpha'} + \frac{1}{2}\dot{t}_1 \partial_{\alpha} f_{i\alpha'}\partial^i f^{\alpha\alpha'} - \frac{1}{2}\dot{t}_1 \partial_{\alpha'} f_{\alpha i}\partial^i f^{\alpha\alpha'} + \frac{1}{2}\dot{t}_1 \partial_i f_{\alpha\alpha'}\partial^i f^{\alpha\alpha'} + \frac{1}{2}\dot{t}_1 \partial_i f_{\alpha'\alpha}\partial^i f^{\alpha\alpha'} - \dot{r}_5 \partial_{\alpha}\mathcal{A}^{\alpha\alpha' i}\partial_j\mathcal{A}_{\alpha'}{}^{j}{}_{i} + 2\dot{r}_5 \partial^i\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\partial_j\mathcal{A}_{\alpha'}{}^{j}{}_{i} + \dot{r}_5 \partial_{\alpha}\mathcal{A}^{\alpha\alpha' i}\partial_j\mathcal{A}_{i}{}^{j}{}_{\alpha'} - 2\dot{r}_5 \partial^i\mathcal{A}^{\alpha\alpha'}{}_{\alpha}\partial_j\mathcal{A}_{i}{}^{j}{}_{\alpha'} - \frac{4}{3}\dot{r}_1 \partial_{\alpha'}\mathcal{A}_{\alpha ij}\partial^j\mathcal{A}^{\alpha\alpha' i} + \frac{2}{3}\dot{r}_1 \partial_{\alpha'}\mathcal{A}_{\alpha ji}\partial^j\mathcal{A}^{\alpha\alpha' i} - \frac{8}{3}\dot{r}_1 \partial_{\alpha'}\mathcal{A}_{ij\alpha}\partial^j\mathcal{A}^{\alpha\alpha' i} - \frac{2}{3}\dot{r}_1 \partial_i\mathcal{A}_{\alpha\alpha' j}\partial^j\mathcal{A}^{\alpha\alpha' i} + \frac{2}{3}\dot{r}_1 \partial_j\mathcal{A}_{\alpha\alpha' i}\partial^j\mathcal{A}^{\alpha\alpha' i} + \frac{2}{3}\dot{r}_1 \partial_j\mathcal{A}_{\alpha i\alpha'}\partial^j\mathcal{A}^{\alpha\alpha' i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_1 & -i\sqrt{2}\,k\,t_1 & 0 \\ i\sqrt{2}\,k\,t_1 & -t_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_1 \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i\,k\,t_1}{\sqrt{2}} & 0 \\ \frac{i\,k\,t_1}{\sqrt{2}} & \frac{1}{2}\left(2k^2\left(2r_1+r_5\right)-t_1\right) & -\frac{t_1}{\sqrt{2}} \\ 0 & -\frac{t_1}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & -i\,k\,t_1 & 0 & 0 \\ i\,k\,t_1 & k^2\left(r_1+r_5\right)-\frac{t_1}{2} & 0 & \frac{t_1}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_1}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_1 & \frac{i\,k\,t_1}{\sqrt{2}} \\ -\frac{i\,k\,t_1}{\sqrt{2}} & \frac{t_1}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2k^2 r_1+t_1\right) \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}\tau^{\flat\parallel} == 2k\,{}^{0^+}\sigma^{\flat\parallel},\ -i\,{}^{1^+}\tau^{\flat\parallel\,ab} == k\,{}^{1^+}\sigma^{\flat\perp\,ab},\ i\,{}^{1^-}\tau^{\flat\parallel\,a} == 2k\,{}^{1^-}\sigma^{\flat\perp\,a},\ {}^{1^-}\tau^{\flat\perp\,a} == 0,\ -i\,{}^{2^+}\tau^{\flat\parallel\,ab} == 2k\,{}^{2^+}\sigma^{\flat\parallel\,ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{\left(1+2k^2\right)^2 t_1} & -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} & 0 \\ \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} & -\frac{1}{\left(1+2k^2\right)^2 t_1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{t_1} \end{pmatrix}, \begin{pmatrix} \frac{-2k^4\left(2r_1+r_5\right)+k^2 t_1}{\left(1+k^2\right)^2 t_1^2} & -\frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & \frac{-2i\,k^3\left(2r_1+r_5\right)+i\,k\,t_1}{\left(1+k^2\right)^2 t_1^2} \\ \frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & 0 & -\frac{\sqrt{2}}{t_1+k^2 t_1} \\ \frac{i\left(2k^3\left(2r_1+r_5\right)-k\,t_1\right)}{\left(1+k^2\right)^2 t_1^2} & -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{-2k^2\left(2r_1+r_5\right)+t_1}{\left(1+k^2\right)^2 t_1^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{-4k^4\left(r_1+r_5\right)+2k^2 t_1}{\left(t_1+2k^2 t_1\right)^2} & -\frac{2i\,k}{t_1+2k^2 t_1} & 0 & \frac{i\sqrt{2}\,k\left(2k^2\left(r_1+r_5\right)-t_1\right)}{\left(t_1+2k^2 t_1\right)^2} \\ \frac{2i\,k}{t_1+2k^2 t_1} & 0 & 0 & \frac{\sqrt{2}}{t_1+2k^2 t_1} \\ 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k\left(2k^2\left(r_1+r_5\right)-t_1\right)}{\left(t_1+2k^2 t_1\right)^2} & \frac{\sqrt{2}}{t_1+2k^2 t_1} & 0 & \frac{-2k^2\left(r_1+r_5\right)+t_1}{\left(t_1+2k^2 t_1\right)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{\left(1+2k^2\right)^2 t_1} & \frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} \\ -\frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_1} & \frac{2}{\left(1+2k^2\right)^2 t_1} \end{pmatrix}, \begin{pmatrix} \frac{2}{2k^2 r_1+t_1} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\},\{\},\{\},\{\},\{\},\left\{-\frac{t_1}{2r_1}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\},\{\},\{\},\{\},\{\},\left\{-\frac{1}{r_1}\right\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$\underset{1}{r} < 0 \,\&\&\, \underset{1}{t} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$\underset{1}{r} < 0 \,\&\&\, \underset{1}{t} > 0$

Okay, that concludes the analysis of this theory.

Case 53

Now for a new theory. Here is the full nonlinear Lagrangian for Case 53 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}\underset{1}{r}\,\mathcal{R}_{ij\mathfrak{k}l}\,\mathcal{R}^{ij\mathfrak{k}l}+\frac{2}{3}\underset{1}{r}\,\mathcal{R}_{i\mathfrak{k}jl}\,\mathcal{R}^{ij\mathfrak{k}l}-\underset{1}{r}\,\mathcal{R}^{ij}{}_{i}{}^{\mathfrak{k}}\,\mathcal{R}_{j}{}^{l}{}_{\mathfrak{k}l}-\frac{2}{3}\underset{1}{r}\,\mathcal{R}^{ij\mathfrak{k}l}\,\mathcal{R}_{\mathfrak{k}lij}+ \underset{1}{r}\,\mathcal{R}^{ij}{}_{i}{}^{\mathfrak{k}}\,\mathcal{R}_{\mathfrak{k}}{}^{l}{}_{jl}+\frac{1}{4}\underset{1}{t}\,\mathcal{T}_{ij\mathfrak{k}}\,\mathcal{T}^{ij\mathfrak{k}}+\frac{1}{2}\underset{1}{t}\,\mathcal{T}^{ij\mathfrak{k}}\,\mathcal{T}_{ji\mathfrak{k}}+\underset{1}{t}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{\mathfrak{k}}{}_{j\mathfrak{k}}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\underset{1}{t}\,\mathcal{A}_{aia'}\,\mathcal{A}^{aa'i}+\underset{1}{t}\,\mathcal{A}^{aa'}{}_{a}\,\mathcal{A}_{a'}{}^{i}{}_{i}-2\underset{1}{t}\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial_{a}f^{aa'}+2\underset{1}{t}\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}- \underset{1}{t}\,\partial_{a'}f^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a}-\underset{1}{t}\,\partial_{a}f^{aa'}\,\partial_{i}f^{i}{}_{a'}+2\underset{1}{t}\,\partial^{a'}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{a'}-\underset{1}{r}\,\partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}+\underset{1}{r}\,\partial_{i}\mathcal{A}_{a'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}+ 2\underset{1}{t}\,\mathcal{A}_{a'ia}\,\partial^{i}f^{aa'}-\underset{1}{t}\,\partial_{a}f_{a'i}\,\partial^{i}f^{aa'}+\frac{1}{2}\underset{1}{t}\,\partial_{a}f_{ia'}\,\partial^{i}f^{aa'}-\frac{1}{2}\underset{1}{t}\,\partial_{a'}f_{ai}\,\partial^{i}f^{aa'}+ \frac{1}{2}\underset{1}{t}\,\partial_{i}f_{aa'}\,\partial^{i}f^{aa'}+\frac{1}{2}\underset{1}{t}\,\partial_{i}f_{a'a}\,\partial^{i}f^{aa'}+\underset{1}{r}\,\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}-2\underset{1}{r}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i}- \underset{1}{r}\,\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}+2\underset{1}{r}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'}-\frac{4}{3}\underset{1}{r}\,\partial_{a'}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\underset{1}{r}\,\partial_{a'}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{aa'i}- \frac{8}{3}\underset{1}{r}\,\partial_{a'}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{aa'i}-\frac{2}{3}\underset{1}{r}\,\partial_{i}\mathcal{A}_{aa'j}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{aa'i}\,\partial^{j}\mathcal{A}^{aa'i}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{aia'}\,\partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_{\dot{1}} & -i\sqrt{2}\,k\,t_{\dot{1}} & 0 \\ i\sqrt{2}\,k\,t_{\dot{1}} & -t_{\dot{1}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_{\dot{1}} \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 \\ \frac{i k t_{\dot{1}}}{\sqrt{2}} & \frac{1}{2}\left(2k^2 r_{\dot{1}} - t_{\dot{1}}\right) & -\frac{t_{\dot{1}}}{\sqrt{2}} \\ 0 & -\frac{t_{\dot{1}}}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & -i k t_{\dot{1}} & 0 & 0 \\ i k t_{\dot{1}} & -\frac{t_{\dot{1}}}{2} & 0 & \frac{t_{\dot{1}}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{\dot{1}}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_{\dot{1}} & \frac{i k t_{\dot{1}}}{\sqrt{2}} \\ -\frac{i k t_{\dot{1}}}{\sqrt{2}} & \frac{t_{\dot{1}}}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2k^2 r_{\dot{1}} + t_{\dot{1}}\right) \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\bullet}\tau^{\flat\perp} == 0,\ -i\,\overset{0^+}{\bullet}\tau^{\flat\parallel} == 2k\,\overset{0^+}{\bullet}\sigma^{\flat\parallel},\ -i\,\overset{1^+}{\bullet}\tau^{\flat\parallel}{}^{ab} == k\,\overset{1^+}{\bullet}\sigma^{\flat\perp}{}^{ab},\ i\,\overset{1^-}{\bullet}\tau^{\flat\parallel}{}^{a} == 2k\,\overset{1^-}{\bullet}\sigma^{\flat\perp}{}^{a},\ \overset{1^-}{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ -i\,\overset{2^+}{\bullet}\tau^{\flat\parallel}{}^{ab} == 2k\,\overset{2^+}{\bullet}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{(1+2k^2)^2 t_{\dot{1}}} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot{1}}} & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot{1}}} & -\frac{1}{(1+2k^2)^2 t_{\dot{1}}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{t_{\dot{1}}} \end{pmatrix}, \begin{pmatrix} \frac{-2k^4 r_{\dot{1}} + k^2 t_{\dot{1}}}{(1+k^2)^2 t_{\dot{1}}^2} & -\frac{i\sqrt{2}\,k}{t_{\dot{1}} + k^2 t_{\dot{1}}} & -\frac{i\left(2k^3 r_{\dot{1}} - k t_{\dot{1}}\right)}{(1+k^2)^2 t_{\dot{1}}^2} \\ \frac{i\sqrt{2}\,k}{t_{\dot{1}} + k^2 t_{\dot{1}}} & 0 & -\frac{\sqrt{2}}{t_{\dot{1}} + k^2 t_{\dot{1}}} \\ \frac{i\left(2k^3 r_{\dot{1}} - k t_{\dot{1}}\right)}{(1+k^2)^2 t_{\dot{1}}^2} & -\frac{\sqrt{2}}{t_{\dot{1}} + k^2 t_{\dot{1}}} & \frac{-2k^2 r_{\dot{1}} + t_{\dot{1}}}{(1+k^2)^2 t_{\dot{1}}^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2}{(1+2k^2)^2 t_{\dot{1}}} & -\frac{2ik}{t_{\dot{1}} + 2k^2 t_{\dot{1}}} & 0 & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot{1}}} \\ \frac{2ik}{t_{\dot{1}} + 2k^2 t_{\dot{1}}} & 0 & 0 & \frac{\sqrt{2}}{t_{\dot{1}} + 2k^2 t_{\dot{1}}} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot{1}}} & \frac{\sqrt{2}}{t_{\dot{1}} + 2k^2 t_{\dot{1}}} & 0 & \frac{1}{(1+2k^2)^2 t_{\dot{1}}} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_{\dot{1}}} & \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot{1}}} \\ -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot{1}}} & \frac{2}{(1+2k^2)^2 t_{\dot{1}}} \end{pmatrix}, \begin{pmatrix} \frac{2}{2k^2 r_{\dot{1}} + t_{\dot{1}}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\},\ \{\},\ \{\},\ \{\},\ \{\},\ \left\{-\frac{t_{\dot{1}}}{2 r_{\dot{1}}}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\},\ \{\},\ \{\},\ \{\},\ \{\},\ \left\{-\frac{1}{r_{\dot{1}}}\right\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall unitarity conditions:

$\underset{1}{r} < 0 \&\& \underset{1}{t} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$\underset{1}{r} < 0 \&\& \underset{1}{t} > 0$

Okay, that concludes the analysis of this theory.

Case 54

Now for a new theory. Here is the full nonlinear Lagrangian for Case 54 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}\underset{1}{r}\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl} + \frac{2}{3}\underset{1}{r}\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl} - 2\underset{1}{r}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl} - \frac{2}{3}\underset{1}{r}\,\mathcal{R}^{ijkl}\,\mathcal{R}_{klij} + 2\underset{1}{r}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4}\underset{1}{t}\,\mathcal{T}_{ijk}\,\mathcal{T}^{ijk} + \frac{1}{2}\underset{1}{t}\,\mathcal{T}^{ijk}\,\mathcal{T}_{jik} + \underset{1}{t}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\underset{1}{t}\,\mathcal{A}_{aia'}\,\mathcal{A}^{aa'i} + \underset{1}{t}\,\mathcal{A}^{aa'}{}_{a}\,\mathcal{A}_{a'}{}^{i}{}_{i} - 2\underset{1}{t}\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial_{a}f^{aa'} + 2\underset{1}{t}\,\mathcal{A}_{a'}{}^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a} - \underset{1}{t}\,\partial_{a'}f^{i}{}_{i}\,\partial^{a'}f^{a}{}_{a} - \underset{1}{t}\,\partial_{a}f^{aa'}\,\partial_{i}f^{i}{}_{a'} + 2\underset{1}{t}\,\partial^{a'}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{a'} - 2\underset{1}{r}\,\partial_{a'}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a} + 2\underset{1}{r}\,\partial_{i}\mathcal{A}_{a'}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{aa'}{}_{a} + 2\underset{1}{t}\,\mathcal{A}_{a'ia}\,\partial^{i}f^{aa'} - \underset{1}{t}\,\partial_{a}f_{a'i}\,\partial^{i}f^{aa'} + \frac{1}{2}\underset{1}{t}\,\partial_{a}f_{ia'}\,\partial^{i}f^{aa'} - \frac{1}{2}\underset{1}{t}\,\partial_{a'}f_{ai}\,\partial^{i}f^{aa'} + \frac{1}{2}\underset{1}{t}\,\partial_{i}f_{aa'}\,\partial^{i}f^{aa'} + \frac{1}{2}\underset{1}{t}\,\partial_{i}f_{a'a}\,\partial^{i}f^{aa'} + 2\underset{1}{r}\,\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i} - 4\underset{1}{r}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{a'}{}^{j}{}_{i} - 2\underset{1}{r}\,\partial_{a}\mathcal{A}^{aa'i}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'} + 4\underset{1}{r}\,\partial^{i}\mathcal{A}^{aa'}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{a'} - \frac{4}{3}\underset{1}{r}\,\partial_{a'}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{aa'i} + \frac{2}{3}\underset{1}{r}\,\partial_{a'}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{aa'i} - \frac{8}{3}\underset{1}{r}\,\partial_{a'}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{aa'i} - \frac{2}{3}\underset{1}{r}\,\partial_{i}\mathcal{A}_{aa'j}\,\partial^{j}\mathcal{A}^{aa'i} + \frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{aa'i}\,\partial^{j}\mathcal{A}^{aa'i} + \frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{aia'}\,\partial^{j}\mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_{\underset{1}{\bullet}} & -i\sqrt{2}\,k\,t_{\underset{1}{\bullet}} & 0 \\ i\sqrt{2}\,k\,t_{\underset{1}{\bullet}} & -t_{\underset{1}{\bullet}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_{\underset{1}{\bullet}} \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i\,k\,t_{\underset{1}{\bullet}}}{\sqrt{2}} & 0 \\ \frac{i\,k\,t_{\underset{1}{\bullet}}}{\sqrt{2}} & -\frac{t_{\underset{1}{\bullet}}}{2} & -\frac{t_{\underset{1}{\bullet}}}{\sqrt{2}} \\ 0 & -\frac{t_{\underset{1}{\bullet}}}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & -i\,k\,t_{\underset{1}{\bullet}} & 0 & 0 \\ i\,k\,t_{\underset{1}{\bullet}} & -k^2 r_{\underset{1}{\bullet}} - \frac{t_{\underset{1}{\bullet}}}{2} & 0 & \frac{t_{\underset{1}{\bullet}}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{\underset{1}{\bullet}}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_{\underset{1}{\bullet}} & \frac{i\,k\,t_{\underset{1}{\bullet}}}{\sqrt{2}} \\ -\frac{i\,k\,t_{\underset{1}{\bullet}}}{\sqrt{2}} & \frac{t_{\underset{1}{\bullet}}}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2k^2 r_{\underset{1}{\bullet}} + t_{\underset{1}{\bullet}}\right) \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{\overset{0^+}{\bullet}\tau^{\flat\perp} == 0,\ -i\,\overset{0^+}{\bullet}\tau^{\flat\parallel} == 2k\,\overset{0^+}{\bullet}\sigma^{\flat\parallel},\ -i\,\overset{1^+}{\bullet}\tau^{\flat\parallel\,ab} == k\,\overset{1^+}{\bullet}\sigma^{\flat\perp\,ab},\ i\,\overset{1^-}{\bullet}\tau^{\flat\parallel\,a} == 2k\,\overset{1^-}{\bullet}\sigma^{\flat\perp\,a},\ \overset{1^-}{\bullet}\tau^{\flat\perp\,a} == 0,\ -i\,\overset{2^+}{\bullet}\tau^{\flat\parallel\,ab} == 2k\,\overset{2^+}{\bullet}\sigma^{\flat\parallel\,ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{(1+2k^2)^2 t_{\underset{1}{\bullet}}} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\underset{1}{\bullet}}} & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\underset{1}{\bullet}}} & -\frac{1}{(1+2k^2)^2 t_{\underset{1}{\bullet}}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{t_{\underset{1}{\bullet}}} \end{pmatrix}, \begin{pmatrix} \frac{k^2}{(1+k^2)^2 t_{\underset{1}{\bullet}}} & -\frac{i\sqrt{2}\,k}{t_{\underset{1}{\bullet}}+k^2 t_{\underset{1}{\bullet}}} & \frac{i\,k}{(1+k^2)^2 t_{\underset{1}{\bullet}}} \\ \frac{i\sqrt{2}\,k}{t_{\underset{1}{\bullet}}+k^2 t_{\underset{1}{\bullet}}} & 0 & -\frac{\sqrt{2}}{t_{\underset{1}{\bullet}}+k^2 t_{\underset{1}{\bullet}}} \\ -\frac{i\,k}{(1+k^2)^2 t_{\underset{1}{\bullet}}} & -\frac{\sqrt{2}}{t_{\underset{1}{\bullet}}+k^2 t_{\underset{1}{\bullet}}} & \frac{1}{(1+k^2)^2 t_{\underset{1}{\bullet}}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2\left(2k^2 r_{\underset{1}{\bullet}}+t_{\underset{1}{\bullet}}\right)}{\left(t_{\underset{1}{\bullet}}+2k^2 t_{\underset{1}{\bullet}}\right)^2} & -\frac{2\,i\,k}{t_{\underset{1}{\bullet}}+2k^2 t_{\underset{1}{\bullet}}} & 0 & -\frac{i\sqrt{2}\,k\left(2k^2 r_{\underset{1}{\bullet}}+t_{\underset{1}{\bullet}}\right)}{\left(t_{\underset{1}{\bullet}}+2k^2 t_{\underset{1}{\bullet}}\right)^2} \\ \frac{2\,i\,k}{t_{\underset{1}{\bullet}}+2k^2 t_{\underset{1}{\bullet}}} & 0 & 0 & \frac{\sqrt{2}}{t_{\underset{1}{\bullet}}+2k^2 t_{\underset{1}{\bullet}}} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k\left(2k^2 r_{\underset{1}{\bullet}}+t_{\underset{1}{\bullet}}\right)}{\left(t_{\underset{1}{\bullet}}+2k^2 t_{\underset{1}{\bullet}}\right)^2} & \frac{\sqrt{2}}{t_{\underset{1}{\bullet}}+2k^2 t_{\underset{1}{\bullet}}} & 0 & \frac{2k^2 r_{\underset{1}{\bullet}}+t_{\underset{1}{\bullet}}}{\left(t_{\underset{1}{\bullet}}+2k^2 t_{\underset{1}{\bullet}}\right)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_{\underset{1}{\bullet}}} & \frac{2\,i\sqrt{2}\,k}{(1+2k^2)^2 t_{\underset{1}{\bullet}}} \\ -\frac{2\,i\sqrt{2}\,k}{(1+2k^2)^2 t_{\underset{1}{\bullet}}} & \frac{2}{(1+2k^2)^2 t_{\underset{1}{\bullet}}} \end{pmatrix}, \begin{pmatrix} \frac{2}{2k^2 r_{\underset{1}{\bullet}}+t_{\underset{1}{\bullet}}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{\underset{1}{\bullet}}}{2\,r_{\underset{1}{\bullet}}}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{\underset{1}{\bullet}}}\right\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall unitarity conditions:

$r_{\dot{1}} < 0 \,\&\&\, t_{\dot{1}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\dot{1}} < 0 \,\&\&\, t_{\dot{1}} > 0$

Okay, that concludes the analysis of this theory.

Case 55

Now for a new theory. Here is the full nonlinear Lagrangian for Case 55 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{\dot{1}} \mathcal{R}_{ij\hbar l} \mathcal{R}^{ij\hbar l} + \frac{2}{3} r_{\dot{1}} \mathcal{R}_{i\hbar jl} \mathcal{R}^{ij\hbar l} - r_{\dot{1}} \mathcal{R}^{ij\ \ \hbar}_{\ \ i} \mathcal{R}_{j\ \ \hbar l}^{\ \ l} - \frac{2}{3} r_{\dot{1}} \mathcal{R}^{ij\hbar l} \mathcal{R}_{\hbar lij} + r_{\dot{1}} \mathcal{R}^{ij\ \ \hbar}_{\ \ i} \mathcal{R}_{\hbar\ \ jl}^{\ \ l} + \frac{1}{4} t_{\dot{1}} \mathcal{T}_{ij\hbar} \mathcal{T}^{ij\hbar} + \frac{1}{2} t_{\dot{1}} \mathcal{T}^{ij\hbar} \mathcal{T}_{ji\hbar} + \frac{1}{3} t_{\dot{1}} \mathcal{T}^{i\ \ j}_{\ \ i} \mathcal{T}^{\hbar}_{\ \ j\hbar}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{\dot{1}} \mathcal{A}_{aia'} \mathcal{A}^{aa'i} + \frac{1}{3} t_{\dot{1}} \mathcal{A}^{aa'}_{\ \ \ a} \mathcal{A}_{a'\ \ i}^{\ \ i} - \frac{2}{3} t_{\dot{1}} \mathcal{A}_{a'\ \ i}^{\ \ i} \partial_a f^{aa'} + \frac{2}{3} t_{\dot{1}} \mathcal{A}_{a'\ \ i}^{\ \ i} \partial^{a'} f^{a}_{\ \ a} - \frac{1}{3} t_{\dot{1}} \partial_{a'} f^{i}_{\ \ i} \partial^{a'} f^{a}_{\ \ a} - \frac{1}{3} t_{\dot{1}} \partial_a f^{aa'} \partial_i f^{i}_{\ \ a'} + \frac{2}{3} t_{\dot{1}} \partial^{a'} f^{a}_{\ \ a} \partial_i f^{i}_{\ \ a'} - r_{\dot{1}} \partial_{a'} \mathcal{A}_{i\ \ j}^{\ \ j} \partial^i \mathcal{A}^{aa'}_{\ \ \ a} + r_{\dot{1}} \partial_i \mathcal{A}_{a'\ \ j}^{\ \ j} \partial^i \mathcal{A}^{aa'}_{\ \ \ a} + 2 t_{\dot{1}} \mathcal{A}_{a'ia} \partial^i f^{aa'} - t_{\dot{1}} \partial_a f_{a'i} \partial^i f^{aa'} + \frac{1}{2} t_{\dot{1}} \partial_a f_{ia'} \partial^i f^{aa'} - \frac{1}{2} t_{\dot{1}} \partial_{a'} f_{ai} \partial^i f^{aa'} + \frac{1}{2} t_{\dot{1}} \partial_i f_{aa'} \partial^i f^{aa'} + \frac{1}{2} t_{\dot{1}} \partial_i f_{a'a} \partial^i f^{aa'} + r_{\dot{1}} \partial_a \mathcal{A}^{aa'i} \partial_j \mathcal{A}_{a'\ \ i}^{\ \ j} - 2 r_{\dot{1}} \partial^i \mathcal{A}^{aa'}_{\ \ \ a} \partial_j \mathcal{A}_{a'\ \ i}^{\ \ j} - r_{\dot{1}} \partial_a \mathcal{A}^{aa'i} \partial_j \mathcal{A}_{i\ \ a'}^{\ \ j} + 2 r_{\dot{1}} \partial^i \mathcal{A}^{aa'}_{\ \ \ a} \partial_j \mathcal{A}_{i\ \ a'}^{\ \ j} - \frac{4}{3} r_{\dot{1}} \partial_{a'} \mathcal{A}_{aij} \partial^j \mathcal{A}^{aa'i} + \frac{2}{3} r_{\dot{1}} \partial_{a'} \mathcal{A}_{aji} \partial^j \mathcal{A}^{aa'i} - \frac{8}{3} r_{\dot{1}} \partial_{a'} \mathcal{A}_{ija} \partial^j \mathcal{A}^{aa'i} - \frac{2}{3} r_{\dot{1}} \partial_i \mathcal{A}_{aa'j} \partial^j \mathcal{A}^{aa'i} + \frac{2}{3} r_{\dot{1}} \partial_j \mathcal{A}_{aa'i} \partial^j \mathcal{A}^{aa'i} + \frac{2}{3} r_{\dot{1}} \partial_j \mathcal{A}_{aia'} \partial^j \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_{\dot{1}} \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 \\ \frac{i k t_{\dot{1}}}{\sqrt{2}} & \frac{1}{2}\left(2 k^2 r_{\dot{1}} - t_{\dot{1}}\right) & -\frac{t_{\dot{1}}}{\sqrt{2}} \\ 0 & -\frac{t_{\dot{1}}}{\sqrt{2}} & 0 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} \frac{2 k^2 t_{\dot{1}}}{3} & -\frac{1}{3} i k t_{\dot{1}} & 0 & -\frac{1}{3} i \sqrt{2}\, k t_{\dot{1}} \\ \frac{i k t_{\dot{1}}}{3} & \frac{t_{\dot{1}}}{6} & 0 & \frac{t_{\dot{1}}}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{\dot{1}} & \frac{t_{\dot{1}}}{3\sqrt{2}} & 0 & \frac{t_{\dot{1}}}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_{\dot{1}} & \frac{i k t_{\dot{1}}}{\sqrt{2}} \\ -\frac{i k t_{\dot{1}}}{\sqrt{2}} & \frac{t_{\dot{1}}}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2 k^2 r_{\dot{1}} + t_{\dot{1}}\right) \end{pmatrix} \right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\cdot}\tau^{\flat\perp} == 0,\ {}^{0^+}_{\cdot}\sigma^{\flat\parallel} == 0,\ {}^{0^+}_{\cdot}\tau^{\flat\parallel} == 0,\ -i\, {}^{1^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == k\, {}^{1^+}_{\cdot}\sigma^{\flat\perp}{}^{ab}, \right.$$
$$\left. i\, {}^{1^-}_{\cdot}\tau^{\flat\parallel}{}^{a} == 2 k\, {}^{1^-}_{\cdot}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\cdot}\tau^{\flat\perp}{}^{a} == 0,\ i\, {}^{1^-}_{\cdot}\tau^{\flat\parallel}{}^{a} == 2 k\, {}^{1^-}_{\cdot}\sigma^{\flat\parallel}{}^{a},\ -i\, {}^{2^+}_{\cdot}\tau^{\flat\parallel}{}^{ab} == 2 k\, {}^{2^+}_{\cdot}\sigma^{\flat\parallel}{}^{ab} \right\}$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{t_{\dot{1}}} \end{pmatrix}, \begin{pmatrix} \frac{-2 k^4 r_{\dot{1}} + k^2 t_{\dot{1}}}{(1+k^2)^2 t_{\dot{1}}^2} & -\frac{i \sqrt{2}\, k}{t_{\dot{1}} + k^2 t_{\dot{1}}} & -\frac{i\left(2 k^3 r_{\dot{1}} - k t_{\dot{1}}\right)}{(1+k^2)^2 t_{\dot{1}}^2} \\ \frac{i \sqrt{2}\, k}{t_{\dot{1}} + k^2 t_{\dot{1}}} & 0 & -\frac{\sqrt{2}}{t_{\dot{1}} + k^2 t_{\dot{1}}} \\ \frac{i\left(2 k^3 r_{\dot{1}} - k t_{\dot{1}}\right)}{(1+k^2)^2 t_{\dot{1}}^2} & -\frac{\sqrt{2}}{t_{\dot{1}} + k^2 t_{\dot{1}}} & \frac{-2 k^2 r_{\dot{1}} + t_{\dot{1}}}{(1+k^2)^2 t_{\dot{1}}^2} \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} \frac{24 k^2}{(3+4k^2)^2 t_{\dot{1}}} & -\frac{12 i k}{(3+4k^2)^2 t_{\dot{1}}} & 0 & -\frac{12 i \sqrt{2}\, k}{(3+4k^2)^2 t_{\dot{1}}} \\ \frac{12 i k}{(3+4k^2)^2 t_{\dot{1}}} & \frac{6}{(3+4k^2)^2 t_{\dot{1}}} & 0 & \frac{6\sqrt{2}}{(3+4k^2)^2 t_{\dot{1}}} \\ 0 & 0 & 0 & 0 \\ \frac{12 i \sqrt{2}\, k}{(3+4k^2)^2 t_{\dot{1}}} & \frac{6\sqrt{2}}{(3+4k^2)^2 t_{\dot{1}}} & 0 & \frac{12}{(3+4k^2)^2 t_{\dot{1}}} \end{pmatrix}, \begin{pmatrix} \frac{4 k^2}{(1+2k^2)^2 t_{\dot{1}}} & \frac{2 i \sqrt{2}\, k}{(1+2k^2)^2 t_{\dot{1}}} \\ -\frac{2 i \sqrt{2}\, k}{(1+2k^2)^2 t_{\dot{1}}} & \frac{2}{(1+2k^2)^2 t_{\dot{1}}} \end{pmatrix}, \begin{pmatrix} \frac{2}{2 k^2 r_{\dot{1}} + t_{\dot{1}}} \end{pmatrix} \right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{\dot{1}}}{2 r_{\dot{1}}}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{\dot{1}}}\right\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{\dot{1}} < 0 \&\& t_{\dot{1}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\dot{1}} < 0 \&\& t_{\dot{1}} > 0$

Okay, that concludes the analysis of this theory.

Case 56

Now for a new theory. Here is the full nonlinear Lagrangian for Case 56 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{\dot{1}} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} + \frac{2}{3} r_{\dot{1}} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} - 2 r_{\dot{1}} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} - \frac{2}{3} r_{\dot{1}} \mathcal{R}^{ijkl} \mathcal{R}_{klij} + 2 r_{\dot{1}} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{3} t_{\dot{1}} \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{3} t_{\dot{1}} \mathcal{T}^{ijk} \mathcal{T}_{jik} + t_{\dot{1}} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{\dot{1}} \mathcal{A}_{aa'i} \mathcal{A}^{aa'i} + \frac{1}{3} t_{\dot{1}} \mathcal{A}_{aia'} \mathcal{A}^{aa'i} + t_{\dot{1}} \mathcal{A}^{aa'}{}_{a} \mathcal{A}_{a'}{}^{i}{}_{i} - 2 t_{\dot{1}} \mathcal{A}_{a'}{}^{i}{}_{i} \partial_{a} f^{aa'} + 2 t_{\dot{1}} \mathcal{A}_{a'}{}^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - t_{\dot{1}} \partial_{a'} f^{i}{}_{i} \partial^{a'} f^{a}{}_{a} - t_{\dot{1}} \partial_{a} f^{aa'} \partial_{i} f^{i}{}_{a'} + 2 t_{\dot{1}} \partial^{a'} f^{a}{}_{a} \partial_{i} f^{i}{}_{a'} - 2 r_{\dot{1}} \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{aa'}{}_{a} + 2 r_{\dot{1}} \partial_{i} \mathcal{A}_{a'}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{aa'}{}_{a} - \frac{2}{3} t_{\dot{1}} \mathcal{A}_{aa'i} \partial^{i} f^{aa'} + \frac{2}{3} t_{\dot{1}} \mathcal{A}_{aia'} \partial^{i} f^{aa'} + \frac{4}{3} t_{\dot{1}} \mathcal{A}_{a'ia} \partial^{i} f^{aa'} - \frac{2}{3} t_{\dot{1}} \partial_{a} f_{a'i} \partial^{i} f^{aa'} + \frac{1}{3} t_{\dot{1}} \partial_{a} f_{ia'} \partial^{i} f^{aa'} - \frac{2}{3} t_{\dot{1}} \partial_{a'} f_{ai} \partial^{i} f^{aa'} + \frac{2}{3} t_{\dot{1}} \partial_{i} f_{aa'} \partial^{i} f^{aa'} + \frac{1}{3} t_{\dot{1}} \partial_{i} f_{a'a} \partial^{i} f^{aa'} + 2 r_{\dot{1}} \partial_{a} \mathcal{A}^{aa'i} \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} - 4 r_{\dot{1}} \partial^{i} \mathcal{A}^{aa'}{}_{a} \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i} - 2 r_{\dot{1}} \partial_{a} \mathcal{A}^{aa'i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} + 4 r_{\dot{1}} \partial^{i} \mathcal{A}^{aa'}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'} - \frac{4}{3} r_{\dot{1}} \partial_{a'} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{\dot{1}} \partial_{a'} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{aa'i} - \frac{8}{3} r_{\dot{1}} \partial_{a'} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{aa'i} - \frac{2}{3} r_{\dot{1}} \partial_{i} \mathcal{A}_{aa'j} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{\dot{1}} \partial_{j} \mathcal{A}_{aa'i} \partial^{j} \mathcal{A}^{aa'i} + \frac{2}{3} r_{\dot{1}} \partial_{j} \mathcal{A}_{aia'} \partial^{j} \mathcal{A}^{aa'i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_{\bullet 1} & -i\sqrt{2}\,k\,t_{\bullet 1} & 0 \\ i\sqrt{2}\,k\,t_{\bullet 1} & -t_{\bullet 1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} \frac{k^2 t_{\bullet 1}}{3} & -\frac{i k t_{\bullet 1}}{3\sqrt{2}} & \frac{i k t_{\bullet 1}}{3} \\ \frac{i k t_{\bullet 1}}{3\sqrt{2}} & \frac{t_{\bullet 1}}{6} & -\frac{t_{\bullet 1}}{3\sqrt{2}} \\ -\frac{1}{3} i k t_{\bullet 1} & -\frac{t_{\bullet 1}}{3\sqrt{2}} & \frac{t_{\bullet 1}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & -i k t_{\bullet 1} & 0 & 0 \\ i k t_{\bullet 1} & -k^2 r_{\bullet 1} - \frac{t_{\bullet 1}}{2} & 0 & \frac{t_{\bullet 1}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{\bullet 1}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_{\bullet 1} & \frac{i k t_{\bullet 1}}{\sqrt{2}} \\ -\frac{i k t_{\bullet 1}}{\sqrt{2}} & \frac{t_{\bullet 1}}{2} \end{pmatrix}, \left(\frac{1}{2}\left(2k^2 r_{\bullet 1} + t_{\bullet 1}\right)\right)\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ -i\ {}^{0^+}_{\bullet}\tau^{\flat\parallel} == 2k\ {}^{0^+}_{\bullet}\sigma^{\flat\parallel},\ {}^{0^-}_{\bullet}\sigma^{\flat\parallel} == 0,\ -i\ {}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\ {}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab},\right.$$

$$\left. i\ {}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 2k\ {}^{1^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab},\ i\ {}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a} == 2k\ {}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ -i\ {}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 2k\ {}^{2^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{(1+2k^2)^2 t_{\bullet 1}} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\bullet 1}} & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\bullet 1}} & -\frac{1}{(1+2k^2)^2 t_{\bullet 1}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, (0), \begin{pmatrix} \frac{12k^2}{(3+2k^2)^2 t_{\bullet 1}} & -\frac{6i\sqrt{2}\,k}{(3+2k^2)^2 t_{\bullet 1}} & \frac{12 i k}{(3+2k^2)^2 t_{\bullet 1}} \\ \frac{6i\sqrt{2}\,k}{(3+2k^2)^2 t_{\bullet 1}} & \frac{6}{(3+2k^2)^2 t_{\bullet 1}} & -\frac{6\sqrt{2}}{(3+2k^2)^2 t_{\bullet 1}} \\ -\frac{12 i k}{(3+2k^2)^2 t_{\bullet 1}} & -\frac{6\sqrt{2}}{(3+2k^2)^2 t_{\bullet 1}} & \frac{12}{(3+2k^2)^2 t_{\bullet 1}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2\left(2k^2 r_{\bullet 1}+t_{\bullet 1}\right)}{\left(t_{\bullet 1}+2k^2 t_{\bullet 1}\right)^2} & -\frac{2 i k}{t_{\bullet 1}+2k^2 t_{\bullet 1}} & 0 & -\frac{i\sqrt{2}\,k\left(2k^2 r_{\bullet 1}+t_{\bullet 1}\right)}{\left(t_{\bullet 1}+2k^2 t_{\bullet 1}\right)^2} \\ \frac{2 i k}{t_{\bullet 1}+2k^2 t_{\bullet 1}} & 0 & 0 & \frac{\sqrt{2}}{t_{\bullet 1}+2k^2 t_{\bullet 1}} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k\left(2k^2 r_{\bullet 1}+t_{\bullet 1}\right)}{\left(t_{\bullet 1}+2k^2 t_{\bullet 1}\right)^2} & \frac{\sqrt{2}}{t_{\bullet 1}+2k^2 t_{\bullet 1}} & 0 & \frac{2k^2 r_{\bullet 1}+t_{\bullet 1}}{\left(t_{\bullet 1}+2k^2 t_{\bullet 1}\right)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_{\bullet 1}} & \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{\bullet 1}} \\ -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{\bullet 1}} & \frac{2}{(1+2k^2)^2 t_{\bullet 1}} \end{pmatrix}, \left(\frac{2}{2k^2 r_{\bullet 1}+t_{\bullet 1}}\right)\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{\bullet 1}}{2 r_{\bullet 1}}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{\bullet 1}}\right\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall unitarity conditions:

$r_{1} < 0 \,\&\&\, t_{1} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{1} < 0 \,\&\&\, t_{1} > 0$

Okay, that concludes the analysis of this theory.

Case 57

Now for a new theory. Here is the full nonlinear Lagrangian for Case 57 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$
\begin{aligned}
&\frac{1}{6}\left(2 r_{1}+r_{2}\right) \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl}+\frac{2}{3}\left(r_{1}-r_{2}\right) \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl}-2 r_{1}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{j}{}^{l}{}_{kl}+\\
&\frac{1}{6}\left(-4 r_{1}+r_{2}\right) \mathcal{R}^{ijkl}\, \mathcal{R}_{klij}+2 r_{1}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl}+\frac{1}{3} t_{1}\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk}+\frac{1}{3} t_{1}\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik}+t_{1}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{k}{}_{jk}
\end{aligned}
$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$
\begin{aligned}
&\frac{1}{3} t_{1}\, \mathcal{A}_{aa'i}\, \mathcal{A}^{aa'i}+\frac{1}{3} t_{1}\, \mathcal{A}_{aia'}\, \mathcal{A}^{aa'i}+t_{1}\, \mathcal{A}^{aa'}{}_{a}\, \mathcal{A}_{a'}{}^{i}{}_{i}-2 t_{1}\, \mathcal{A}_{a'}{}^{i}{}_{i}\, \partial_{a} f^{aa'}+\\
&2 t_{1}\, \mathcal{A}_{a'}{}^{i}{}_{i}\, \partial^{a'} f^{a}{}_{a}-t_{1}\, \partial_{a'} f^{i}{}_{i}\, \partial^{a'} f^{a}{}_{a}-t_{1}\, \partial_{a} f^{aa'}\, \partial_{i} f^{i}{}_{a'}+2 t_{1}\, \partial^{a'} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{a'}-2 r_{1}\, \partial_{a'} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{aa'}{}_{a}+\\
&2 r_{1}\, \partial_{i} \mathcal{A}_{a'}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{aa'}{}_{a}-\frac{2}{3} t_{1}\, \mathcal{A}_{aa'i}\, \partial^{i} f^{aa'}+\frac{2}{3} t_{1}\, \mathcal{A}_{aia'}\, \partial^{i} f^{aa'}+\frac{4}{3} t_{1}\, \mathcal{A}_{a'ia}\, \partial^{i} f^{aa'}-\\
&\frac{2}{3} t_{1}\, \partial_{a} f_{a'i}\, \partial^{i} f^{aa'}+\frac{1}{3} t_{1}\, \partial_{a} f_{ia'}\, \partial^{i} f^{aa'}-\frac{2}{3} t_{1}\, \partial_{a'} f_{ai}\, \partial^{i} f^{aa'}+\frac{2}{3} t_{1}\, \partial_{i} f_{aa'}\, \partial^{i} f^{aa'}+\frac{1}{3} t_{1}\, \partial_{i} f_{a'a}\, \partial^{i} f^{aa'}+\\
&2 r_{1}\, \partial_{a} \mathcal{A}^{aa'i}\, \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i}-4 r_{1}\, \partial^{i} \mathcal{A}^{aa'}{}_{a}\, \partial_{j} \mathcal{A}_{a'}{}^{j}{}_{i}-2 r_{1}\, \partial_{a} \mathcal{A}^{aa'i}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'}+4 r_{1}\, \partial^{i} \mathcal{A}^{aa'}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{a'}-\\
&\frac{4}{3}\left(r_{1}-r_{2}\right) \partial_{a'} \mathcal{A}_{aij}\, \partial^{j} \mathcal{A}^{aa'i}+\frac{2}{3}\left(r_{1}-r_{2}\right) \partial_{a'} \mathcal{A}_{aji}\, \partial^{j} \mathcal{A}^{aa'i}+\frac{2}{3}\left(-4 r_{1}+r_{2}\right) \partial_{a'} \mathcal{A}_{ija}\, \partial^{j} \mathcal{A}^{aa'i}+\\
&\frac{1}{3}\left(-2 r_{1}-r_{2}\right) \partial_{i} \mathcal{A}_{aa'j}\, \partial^{j} \mathcal{A}^{aa'i}+\frac{1}{3}\left(2 r_{1}+r_{2}\right) \partial_{j} \mathcal{A}_{aa'i}\, \partial^{j} \mathcal{A}^{aa'i}+\frac{2}{3}\left(r_{1}-r_{2}\right) \partial_{j} \mathcal{A}_{aia'}\, \partial^{j} \mathcal{A}^{aa'i}
\end{aligned}
$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2 t_{1} & -i\sqrt{2}\,k\,t_{1} & 0 \\ i\sqrt{2}\,k\,t_{1} & -t_{1} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 r_{2} \end{pmatrix}, \begin{pmatrix} \frac{k^2 t_{1}}{3} & -\frac{i k t_{1}}{3\sqrt{2}} & \frac{i k t_{1}}{3} \\ \frac{i k t_{1}}{3\sqrt{2}} & \frac{t_{1}}{6} & -\frac{t_{1}}{3\sqrt{2}} \\ -\frac{1}{3} i k t_{1} & -\frac{t_{1}}{3\sqrt{2}} & \frac{t_{1}}{3} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} 0 & -i k t_{1} & 0 & 0 \\ i k t_{1} & -k^2 r_{1} - \frac{t_{1}}{2} & 0 & \frac{t_{1}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ 0 & \frac{t_{1}}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2 t_{1} & \frac{i k t_{1}}{\sqrt{2}} \\ -\frac{i k t_{1}}{\sqrt{2}} & \frac{t_{1}}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2k^2 r_{1} + t_{1}\right) \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\left\{{}^{0^+}_{\bullet}\tau^{\flat\perp} == 0,\ -i\,{}^{0^+}_{\bullet}\tau^{\flat\parallel} == 2k\,{}^{0^+}_{\bullet}\sigma^{\flat\parallel},\ -i\,{}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == k\,{}^{1^+}_{\bullet}\sigma^{\flat\perp}{}^{ab},\right.$$
$$\left. i\,{}^{1^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 2k\,{}^{1^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab},\ i\,{}^{1^-}_{\bullet}\tau^{\flat\parallel}{}^{a} == 2k\,{}^{1^-}_{\bullet}\sigma^{\flat\perp}{}^{a},\ {}^{1^-}_{\bullet}\tau^{\flat\perp}{}^{a} == 0,\ -i\,{}^{2^+}_{\bullet}\tau^{\flat\parallel}{}^{ab} == 2k\,{}^{2^+}_{\bullet}\sigma^{\flat\parallel}{}^{ab}\right\}$$

The Drazin (Moore–Penrose) inverses of these *a*–matrices, which are functionally analogous to the inverse *b*–matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{2k^2}{(1+2k^2)^2 t_{1}} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & -\frac{1}{(1+2k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2 r_{2}} \end{pmatrix}, \begin{pmatrix} \frac{12k^2}{(3+2k^2)^2 t_{1}} & -\frac{6i\sqrt{2}\,k}{(3+2k^2)^2 t_{1}} & \frac{12 i k}{(3+2k^2)^2 t_{1}} \\ \frac{6i\sqrt{2}\,k}{(3+2k^2)^2 t_{1}} & \frac{6}{(3+2k^2)^2 t_{1}} & -\frac{6\sqrt{2}}{(3+2k^2)^2 t_{1}} \\ -\frac{12 i k}{(3+2k^2)^2 t_{1}} & -\frac{6\sqrt{2}}{(3+2k^2)^2 t_{1}} & \frac{12}{(3+2k^2)^2 t_{1}} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2\left(2k^2 r_{1}+t_{1}\right)}{\left(t_{1}+2k^2 t_{1}\right)^2} & -\frac{2 i k}{t_{1}+2k^2 t_{1}} & 0 & -\frac{i\sqrt{2}\,k\left(2k^2 r_{1}+t_{1}\right)}{\left(t_{1}+2k^2 t_{1}\right)^2} \\ \frac{2 i k}{t_{1}+2k^2 t_{1}} & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} \\ 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k\left(2k^2 r_{1}+t_{1}\right)}{\left(t_{1}+2k^2 t_{1}\right)^2} & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & 0 & \frac{2k^2 r_{1}+t_{1}}{\left(t_{1}+2k^2 t_{1}\right)^2} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_{1}} & \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} \\ -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & \frac{2}{(1+2k^2)^2 t_{1}} \end{pmatrix}, \begin{pmatrix} \frac{2}{2k^2 r_{1}+t_{1}} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{1}}{2 r_{1}}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{1}}\right\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall unitarity conditions:

$r_{\dot{1}} < 0 \,\&\&\, t_{\dot{1}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\dot{1}} < 0 \,\&\&\, t_{\dot{1}} > 0$

Okay, that concludes the analysis of this theory.

Case 58

Now for a new theory. Here is the full nonlinear Lagrangian for Case 58 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{\dot{1}} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} + \frac{2}{3} r_{\dot{1}} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} - r_{\dot{1}} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{3}\left(r_{\dot{1}} - 3 r_{\dot{3}}\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij} + \left(-3 r_{\dot{1}} + 4 r_{\dot{3}}\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4} t_{\dot{1}} \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{2} t_{\dot{1}} \mathcal{T}^{ijk} \mathcal{T}_{jik} + \frac{1}{3} t_{\dot{1}} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
& t_{\dot{1}} \mathcal{A}_{\alpha i \alpha'} \mathcal{A}^{\alpha\alpha' i} + \frac{1}{3} t_{\dot{1}} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \mathcal{A}_{\alpha'}{}^{i}{}_{i} - \frac{2}{3} t_{\dot{1}} \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial_{\alpha} f^{\alpha\alpha'} + \frac{2}{3} t_{\dot{1}} \mathcal{A}_{\alpha'}{}^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha} - \frac{1}{3} t_{\dot{1}} \partial_{\alpha'} f^{i}{}_{i} \partial^{\alpha'} f^{\alpha}{}_{\alpha} - \\
& \frac{1}{3} t_{\dot{1}} \partial_{\alpha} f^{\alpha\alpha'} \partial_{i} f^{i}{}_{\alpha'} + \frac{2}{3} t_{\dot{1}} \partial^{\alpha'} f^{\alpha}{}_{\alpha} \partial_{i} f^{i}{}_{\alpha'} + \left(3 r_{\dot{1}} - 4 r_{\dot{3}}\right) \partial_{\alpha'} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} + r_{\dot{1}} \partial_{i} \mathcal{A}_{\alpha'}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} + \\
& 2 t_{\dot{1}} \mathcal{A}_{\alpha' i \alpha} \partial^{i} f^{\alpha\alpha'} - t_{\dot{1}} \partial_{\alpha} f_{\alpha' i} \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{\dot{1}} \partial_{\alpha} f_{i\alpha'} \partial^{i} f^{\alpha\alpha'} - \frac{1}{2} t_{\dot{1}} \partial_{\alpha'} f_{\alpha i} \partial^{i} f^{\alpha\alpha'} + \frac{1}{2} t_{\dot{1}} \partial_{i} f_{\alpha\alpha'} \partial^{i} f^{\alpha\alpha'} + \\
& \frac{1}{2} t_{\dot{1}} \partial_{i} f_{\alpha'\alpha} \partial^{i} f^{\alpha\alpha'} + r_{\dot{1}} \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i} \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i} - 2 r_{\dot{1}} \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \partial_{j} \mathcal{A}_{\alpha'}{}^{j}{}_{i} + \left(3 r_{\dot{1}} - 4 r_{\dot{3}}\right) \partial_{\alpha} \mathcal{A}^{\alpha\alpha' i} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'} + \\
& \left(-6 r_{\dot{1}} + 8 r_{\dot{3}}\right) \partial^{i} \mathcal{A}^{\alpha\alpha'}{}_{\alpha} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{\alpha'} - \frac{4}{3} r_{\dot{1}} \partial_{\alpha'} \mathcal{A}_{\alpha ij} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{2}{3} r_{\dot{1}} \partial_{\alpha'} \mathcal{A}_{\alpha ji} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \\
& \frac{4}{3}\left(r_{\dot{1}} - 3 r_{\dot{3}}\right) \partial_{\alpha'} \mathcal{A}_{ij\alpha} \partial^{j} \mathcal{A}^{\alpha\alpha' i} - \frac{2}{3} r_{\dot{1}} \partial_{i} \mathcal{A}_{\alpha\alpha' j} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{2}{3} r_{\dot{1}} \partial_{j} \mathcal{A}_{\alpha\alpha' i} \partial^{j} \mathcal{A}^{\alpha\alpha' i} + \frac{2}{3} r_{\dot{1}} \partial_{j} \mathcal{A}_{\alpha i \alpha'} \partial^{j} \mathcal{A}^{\alpha\alpha' i}
\end{aligned}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 6k^2\left(-r_1+r_3\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -t_1 \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i k t_1}{\sqrt{2}} & 0 \\ \frac{i k t_1}{\sqrt{2}} & \frac{1}{2}\left(2k^2 r_1 - t_1\right) & -\frac{t_1}{\sqrt{2}} \\ 0 & -\frac{t_1}{\sqrt{2}} & 0 \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{2k^2 t_1}{3} & -\frac{1}{3} i k t_1 & 0 & -\frac{1}{3} i \sqrt{2}\, k t_1 \\ \frac{i k t_1}{3} & \frac{t_1}{6} & 0 & \frac{t_1}{3\sqrt{2}} \\ 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_1 & \frac{t_1}{3\sqrt{2}} & 0 & \frac{t_1}{3} \end{pmatrix}, \begin{pmatrix} k^2 t_1 & \frac{i k t_1}{\sqrt{2}} \\ -\frac{i k t_1}{\sqrt{2}} & \frac{t_1}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2}\left(2k^2 r_1 + t_1\right) \end{pmatrix}\right\}$$

Gauge constraints on source currents:

$$\Big\{ {}^{0^+}\tau^{\flat\perp} == 0,\ {}^{0^+}\tau^{\flat\parallel} == 0,\ -i\, {}^{1^+}\tau^{\flat\parallel\, ab} == k\, {}^{1^+}\sigma^{\flat\perp\, ab},$$
$$i\, {}^{1^-}\tau^{\flat\parallel\, a} == 2k\, {}^{1^-}\sigma^{\flat\perp\, a},\ {}^{1^-}\tau^{\flat\perp\, a} == 0,\ i\, {}^{1^-}\tau^{\flat\parallel\, a} == 2k\, {}^{1^-}\sigma^{\flat\parallel\, a},\ -i\, {}^{2^+}\tau^{\flat\parallel\, ab} == 2k\, {}^{2^+}\sigma^{\flat\parallel\, ab}\Big\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{6k^2\left(-r_1+r_3\right)} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -\frac{1}{t_1} \end{pmatrix}, \begin{pmatrix} \frac{-2k^4 r_1 + k^2 t_1}{(1+k^2)^2 t_1^2} & -\frac{i\sqrt{2}\, k}{t_1 + k^2 t_1} & -\frac{i\left(2k^3 r_1 - k t_1\right)}{(1+k^2)^2 t_1^2} \\ \frac{i\sqrt{2}\, k}{t_1 + k^2 t_1} & 0 & -\frac{\sqrt{2}}{t_1 + k^2 t_1} \\ \frac{i\left(2k^3 r_1 - k t_1\right)}{(1+k^2)^2 t_1^2} & -\frac{\sqrt{2}}{t_1 + k^2 t_1} & \frac{-2k^2 r_1 + t_1}{(1+k^2)^2 t_1^2} \end{pmatrix},\right.$$

$$\left.\begin{pmatrix} \frac{24k^2}{(3+4k^2)^2 t_1} & -\frac{12 i k}{(3+4k^2)^2 t_1} & 0 & -\frac{12 i \sqrt{2}\, k}{(3+4k^2)^2 t_1} \\ \frac{12 i k}{(3+4k^2)^2 t_1} & \frac{6}{(3+4k^2)^2 t_1} & 0 & \frac{6\sqrt{2}}{(3+4k^2)^2 t_1} \\ 0 & 0 & 0 & 0 \\ \frac{12 i \sqrt{2}\, k}{(3+4k^2)^2 t_1} & \frac{6\sqrt{2}}{(3+4k^2)^2 t_1} & 0 & \frac{12}{(3+4k^2)^2 t_1} \end{pmatrix}, \begin{pmatrix} \frac{4k^2}{(1+2k^2)^2 t_1} & \frac{2 i \sqrt{2}\, k}{(1+2k^2)^2 t_1} \\ -\frac{2 i \sqrt{2}\, k}{(1+2k^2)^2 t_1} & \frac{2}{(1+2k^2)^2 t_1} \end{pmatrix}, \begin{pmatrix} \frac{2}{2k^2 r_1 + t_1} \end{pmatrix}\right\}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_1}{2 r_1}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_1}\right\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall unitarity conditions:

$r_{\dot{1}} < 0 \,\&\&\, t_{\dot{1}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$r_{\dot{1}} < 0 \,\&\&\, t_{\dot{1}} > 0$

Okay, that concludes the analysis of this theory.

How long did this take?

Okay, that's all the cases. You can see from the timing below (in seconds) that each theory takes about a minute to process:

```
{{61.6409, Null}, {67.7372, Null}, {61.1294, Null}, {64.431, Null},
 {68.0213, Null}, {61.6632, Null}, {65.4262, Null}, {58.411, Null}, {50.2841, Null},
 {49.3045, Null}, {50.2744, Null}, {60.7454, Null}, {51.4626, Null},
 {59.8304, Null}, {69.1, Null}, {61.262, Null}, {69.252, Null}, {63.4502, Null},
 {66.8088, Null}, {73.8237, Null}, {68.7843, Null}, {68.9477, Null}, {68.7547, Null},
 {63.8486, Null}, {64.0068, Null}, {65.01, Null}, {57.6974, Null}, {63.3222, Null},
 {66.2249, Null}, {60.4557, Null}, {66.8323, Null}, {74.4611, Null}, {60.608, Null},
 {62.4298, Null}, {67.0808, Null}, {62.4554, Null}, {63.8561, Null}, {76.0623, Null},
 {71.4196, Null}, {61.4373, Null}, {64.1609, Null}, {73.5379, Null}, {76.3775, Null},
 {80.9002, Null}, {81.5834, Null}, {72.724, Null}, {74.3604, Null}, {76.6509, Null},
 {64.5455, Null}, {77.4446, Null}, {74.709, Null}, {73.2531, Null}, {75.4617, Null},
 {73.4976, Null}, {76.4416, Null}, {79.2649, Null}, {79.6963, Null}, {79.8686, Null}}
```