

# Particle spectrograph

## Wave operator and propagator

Quadratic (free) action

$$S_F = \int \int \int (\frac{1}{6}$$

$$(-6t_1\omega_{\kappa\alpha}^{\prime}\omega_{\kappa\alpha}^{\kappa}-2(t_1-2t_2)\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\kappa\lambda}^{\prime}+2t_1\omega_{\kappa\lambda}^{\prime}\omega_{\kappa\lambda}^{\prime}+2t_2\omega_{\kappa\lambda}^{\prime}\omega_{\kappa\lambda}^{\kappa\lambda}+6f^{\alpha\beta}\tau_{\alpha\beta}+6\omega_{\alpha\beta\chi}^{\alpha}\sigma_{\alpha\beta\chi}+12r_1\partial_l\omega_{\lambda}^{\kappa\lambda}\partial_l\omega_{\lambda}^{\alpha}-4r_1\partial_l\omega_{\lambda}^{\beta\theta}\omega_{\kappa}^{\theta\alpha}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}+4r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}-4r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\kappa}\omega_{\alpha\beta}^{\theta\theta}-2r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\kappa}\omega_{\alpha\beta}^{\theta\theta}+4r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\lambda}^{\alpha}\omega_{\alpha}^{\theta\kappa\lambda}-12r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\alpha}\omega_{\alpha}^{\theta\kappa\lambda}+12r_1\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\alpha}^{\theta\kappa\lambda}-24r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\kappa\lambda\theta}-2t_1\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}+t_2\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}^{\theta}-4t_1\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}^{\theta}-t_2\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}^{\theta}-2t_1\partial^{\alpha}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\alpha}^{\lambda}+t_2\partial^{\alpha}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\alpha}^{\lambda}+6t_1\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\prime}^{\prime}+6t_1\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\prime}^{\prime}+12t_1\partial^{\alpha}f_{\kappa\alpha}\partial^{\kappa}f_{\prime}^{\prime}-6t_1\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\prime}^{\prime}+2t_1\omega_{l\theta\kappa}\partial^{\kappa}f^{\prime\theta}+2t_2\omega_{l\theta\kappa}\partial^{\kappa}f^{\prime\theta}+8t_1\omega_{l\kappa\theta}\partial^{\kappa}f^{\prime\theta}-4t_2\omega_{l\kappa\theta}\partial^{\kappa}f^{\prime\theta}-2t_1\omega_{\theta l\kappa}\partial^{\kappa}f^{\prime\theta}-2t_2\omega_{\theta l\kappa}\partial^{\kappa}f^{\prime\theta}+4t_1\omega_{\theta\kappa l}\partial^{\kappa}f^{\prime\theta}+4t_2\omega_{\theta\kappa l}\partial^{\kappa}f^{\prime\theta}-6t_1\omega_{\prime\alpha}^{\alpha}\partial^{\kappa}f_{\kappa}^{\prime}-6t_1\omega_{\prime\lambda}^{\lambda}\partial^{\kappa}f_{\kappa}^{\prime}+2t_1\partial^{\alpha}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\kappa}^{\prime}-t_2\partial^{\alpha}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\theta}^{\theta}+4t_1\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+t_2\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}-6t_1\partial^{\alpha}f_{\lambda}^{\theta}\partial^{\kappa}f_{\alpha}^{\lambda}+4r_1\partial_{\kappa}\omega_{\lambda}^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+2r_2\partial_{\kappa}\omega_{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}-4r_1\partial_{\kappa}\omega_{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+4r_2\partial_{\kappa}\omega_{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+4r_1\partial^{\beta}\omega_{\prime}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\prime}-4r_2\partial^{\beta}\omega_{\prime}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\prime}-16r_1\partial_l\omega_{\alpha\beta}^{\prime\prime}\partial^{\beta}\omega_{\prime}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\prime}+4r_2\partial_l\omega_{\alpha\beta}^{\prime\prime}\partial^{\beta}\omega_{\prime}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\prime}-12r_1\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\lambda}^{\theta\kappa}+12r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\alpha}^{\theta\kappa}))[t,x,y,z]dzdxdydt$$

$\sigma_{1+}^{\#1} \dagger \alpha\beta$	$\sigma_{1+}^{\#2} \alpha\beta$	$\tau_{1+}^{\#1} \alpha\beta$	$\sigma_{1-}^{\#1} \alpha$	$\sigma_{1-}^{\#2} \alpha$	$\tau_{1-}^{\#1} \alpha$	$\tau_{1-}^{\#2} \alpha$
$\sigma_{1+}^{\#1} \dagger \alpha\beta$	$\frac{2(t_1+t_2)}{3t_1t_2}$	$\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	0	0	0
$\sigma_{1+}^{\#2} \dagger \alpha\beta$	$\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{t_1+4t_2}{3(1+k^2)^2t_1t_2}$	$\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0
$\tau_{1+}^{\#1} \dagger \alpha\beta$	$-\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$-\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	$\frac{k^2(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0
$\sigma_{1-}^{\#1} \dagger \alpha$	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	0	$\frac{2ik}{t_1+2k^2t_1}$
$\sigma_{1-}^{\#2} \dagger \alpha$	0	0	0	$-\frac{\sqrt{2}}{t_1+2k^2t_1}$	0	$\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
$\tau_{1-}^{\#1} \dagger \alpha$	0	0	0	0	0	0
$\tau_{1-}^{\#2} \dagger \alpha$	0	0	0	$-\frac{2ik}{t_1+2k^2t_1}$	0	$\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$

$\omega_{1+}^{\#1} \alpha\beta$	$\omega_{1+}^{\#2} \alpha\beta$	$f_{1+}^{\#1} \alpha\beta$	$\omega_{1-}^{\#1} \alpha$	$\omega_{1-}^{\#2} \alpha$	$f_{1-}^{\#1} \alpha$	$f_{1-}^{\#2} \alpha$
$\omega_{1+}^{\#1} \dagger \alpha\beta$	$\frac{1}{6}(t_1+4t_2)$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	0	0	0	0
$\omega_{1+}^{\#2} \dagger \alpha\beta$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$\frac{t_1+t_2}{3}$	0	0	0	0
$f_{1+}^{\#1} \dagger \alpha\beta$	$\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	$-\frac{1}{3}ik(t_1+t_2)$	0	0	0	0
$\omega_{1-}^{\#1} \dagger \alpha$	0	0	$-k^2r_1-\frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$ik t_1$
$\omega_{1-}^{\#2} \dagger \alpha$	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
$f_{1-}^{\#1} \dagger \alpha$	0	0	0	0	0	0
$f_{1-}^{\#2} \dagger \alpha$	0	0	$-ik t_1$	0	0	0

Source constraints/gauge generators

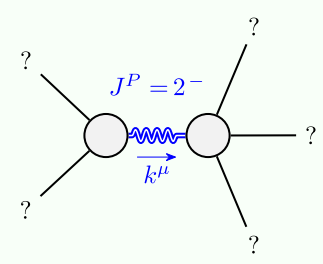
SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	16

$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	$-\frac{1}{(1+2k^2)^2t_1}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
$\tau_{0+}^{\#1} \dagger$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0
$\tau_{0+}^{\#2} \dagger$	0	0	0
$\sigma_{0-}^{\#1} \dagger$	0	0	$\frac{1}{k^2r_2+t_2}$

$\omega_{2+}^{\#1} \alpha\beta$	$f_{2+}^{\#1} \alpha\beta$	$\omega_{2-}^{\#1} \alpha\beta\chi$
$\omega_{2+}^{\#1} \dagger \alpha\beta$	$\frac{t_1}{2}$	$-\frac{ik t_1}{\sqrt{2}}$
$f_{2+}^{\#1} \dagger \alpha\beta$	$\frac{ik t_1}{\sqrt{2}}$	$k^2 t_1$
$\omega_{2-}^{\#1} \dagger \alpha\beta\chi$	0	$k^2 r_1 + \frac{t_1}{2}$

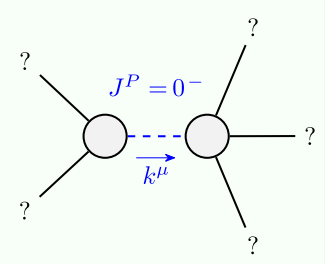
$\omega_{0+}^{\#1}$	$f_{0+}^{\#1}$	$f_{0+}^{\#2}$	$\omega_{0-}^{\#1}$
$\omega_{0+}^{\#1} \dagger$	$-t_1$	$i\sqrt{2}kt_1$	0
$f_{0+}^{\#1} \dagger$	$-i\sqrt{2}kt_1$	$-2k^2t_1$	0
$f_{0+}^{\#2} \dagger$	0	0	0
$\omega_{0-}^{\#1} \dagger$	0	0	$k^2r_2+t_2$

## Massive and massless spectra



Massive particle

Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd



Massive particle

Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

## Unitarity conditions

$$r_1 < 0 \&\& r_2 < 0 \&\& t_1 > 0 \&\& t_2 > 0$$