# Particle spectrograph

## Wave operator and propagator

Quadratic (free) action

$$S_F == \iiint (\tfrac{1}{6}(4\,t_2\,\omega_{\iota}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}'+2\,t_2\,\omega_{\kappa\lambda}{}'\,\omega^{\kappa\lambda}{}_{\iota}+6\,f^{\alpha\beta}\,\tau_{\alpha\beta}+6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}+9\,r_3\,\partial_{\iota}\omega^{\kappa\lambda}{}_{\kappa}\partial'\omega'_{\lambda}{}^{\alpha}+4\,r_2\,\partial^{\beta}\omega^{\theta\alpha}{}_{\kappa}\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}-6\,r_2\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta}-4\,r_2\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta}+15\,r_3\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\partial_{\kappa}\omega^{\theta\kappa\lambda}-2\,r_2\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\partial_{\kappa}\omega^{\theta\kappa\lambda}+9\,r_3\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\partial_{\kappa}\omega^{\kappa\lambda\theta}-18\,r_3\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}+15\,r_3\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}+t_2\,\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}{}^{\theta}+t_2\,\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}{}^{\theta}-t_2\,\partial^{\alpha}f_{\kappa}{}^{\lambda}\partial^{\kappa}f_{\alpha\lambda}+2\,t_2\,\omega_{\iota\theta\kappa}\partial^{\kappa}f'^{\iota\theta}-2\,t_2\,\omega_{\iota\kappa\theta}\partial^{\kappa}f'^{\iota\theta}+4\,t_2\,\omega_{\theta\kappa\iota}\partial^{\kappa}f'^{\iota\theta}-t_2\,\partial^{\alpha}f^{\lambda}{}_{\kappa}\partial^{\kappa}f_{\lambda\alpha}+t_2\,\partial_{\kappa}f^{\lambda}{}_{\theta}\partial^{\kappa}f_{\lambda}{}^{\theta}+2\,r_2\,\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+4\,r_2\,\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}+4\,r_2\,\partial^{\beta}\omega_{\iota}{}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}{}'+4\,r_2\,\partial^{\beta}\omega_{\iota}{}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}{}'-24\,r_3\,\partial^{\beta}\omega_{\iota}{}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}{}'+15\,r_3\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa})) [t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt$$

| | $\omega_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^+}^{\#2\,\alpha\beta}$ | $f_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^-}^{\#1\,\alpha}$ | $\omega_{1^-}^{\#2\,\alpha}$ | $f_{1^-}^{\#1\,\alpha}$ | $f_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $\frac{2\,t_2}{3}$ | $\frac{\sqrt{2}\,t_2}{3}$ | $-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2\,\dagger\,\alpha\beta}$ | $\frac{\sqrt{2}\,t_2}{3}$ | $\frac{t_2}{3}$ | $\frac{i\,k\,t_2}{3}$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2$ | $\frac{i\,k\,t_2}{3}$ | $\frac{k^2\,t_2}{3}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | $-\frac{3\,k^2\,r_3}{2}$ | 0 | 0 | 0 |
| $\omega_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | $\sigma_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^+}^{\#2\,\alpha\beta}$ | $\tau_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^-}^{\#1\,\alpha}$ | $\sigma_{1^-}^{\#2\,\alpha}$ | $\tau_{1^-}^{\#1\,\alpha}$ | $\tau_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $\frac{6}{(3+k^2)^2\,t_2}$ | $\frac{3\,\sqrt{2}}{(3+k^2)^2\,t_2}$ | $-\frac{3\,i\,\sqrt{2}\,k}{(3+k^2)^2\,t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2\,\dagger\,\alpha\beta}$ | $\frac{3\,\sqrt{2}}{(3+k^2)^2\,t_2}$ | $\frac{3}{(3+k^2)^2\,t_2}$ | $-\frac{3\,i\,k}{(3+k^2)^2\,t_2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $-\frac{3\,i\,\sqrt{2}\,k}{(3+k^2)^2\,t_2}$ | $-\frac{3\,i\,k}{(3+k^2)^2\,t_2}$ | $\frac{3\,k^2}{(3+k^2)^2\,t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | $-\frac{2}{3\,k^2\,r_3}$ | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | $\omega_{2^+}^{\#1\,\alpha\beta}$ | $f_{2^+}^{\#1\,\alpha\beta}$ | $\omega_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | $-\frac{3\,k^2\,r_3}{2}$ | 0 | 0 |
| $f_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $\omega_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$ | 0 | 0 | 0 |

| | $\sigma_{2^+}^{\#1\,\alpha\beta}$ | $\tau_{2^+}^{\#1\,\alpha\beta}$ | $\sigma_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | $-\frac{2}{3\,k^2\,r_3}$ | 0 | 0 |
| $\tau_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $\sigma_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$ | 0 | 0 | 0 |

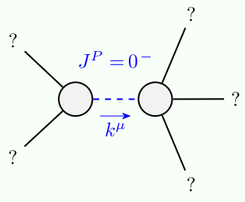
| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1\,\dagger}$ | 0 | 0 | 0 | 0 |
| $f_{0^+}^{\#1\,\dagger}$ | 0 | 0 | 0 | 0 |
| $f_{0^+}^{\#2\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1\,\dagger}$ | 0 | 0 | 0 | $k^2\,r_2+t_2$ |

Source constraints/gauge generators

| SO(3) irreps | Multiplicities |
|---|---|
| $\tau_{0^+}^{\#2} == 0$ | 1 |
| $\tau_{0^+}^{\#1} == 0$ | 1 |
| $\sigma_{0^+}^{\#1} == 0$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} == 0$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | 3 |
| $\sigma_{1^-}^{\#2\,\alpha} == 0$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#1\,\alpha\beta} == 0$ | 3 |
| $\sigma_{1^+}^{\#1\,\alpha\beta} == \sigma_{1^+}^{\#2\,\alpha\beta}$ | 3 |
| $\sigma_{2^-}^{\#1\,\alpha\beta\chi} == 0$ | 5 |
| $\tau_{2^+}^{\#1\,\alpha\beta} == 0$ | 5 |
| Total constraints: | 28 |

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#1\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#2\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1\,\dagger}$ | 0 | 0 | 0 | $\frac{1}{k^2\,r_2+t_2}$ |

## Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $-\frac{t_2}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

(No massless particles)

## Unitarity conditions

$r_2 < 0 \;\&\&\; t_2 > 0$