

Particle spectrograph

Wave operator and propagator

	$\sigma_{1^+ \alpha \beta}^{\#1}$	$\sigma_{1^+ \alpha \beta}^{\#2}$	$\tau_{1^+ \alpha \beta}^{\#1}$	$\sigma_{1^- \alpha}^{\#1}$	$\sigma_{1^- \alpha}^{\#2}$	$\tau_{1^- \alpha}^{\#1}$	$\tau_{1^- \alpha}^{\#2}$
$\sigma_{1^+ \dagger \alpha \beta}^{\#1}$	0	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$-\frac{i \sqrt{2} k}{t_1+k^2 t_1}$	0	0	0	0
$\sigma_{1^+ \dagger \alpha \beta}^{\#2}$	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$\frac{-2 k^2 (2 r_1+r_5)+t_1}{(1+k^2)^2 t_1^2}$	$\frac{-2 i k^3 (2 r_1+r_5)+i k t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\tau_{1^+ \dagger \alpha \beta}^{\#1}$	$\frac{i \sqrt{2} k}{t_1+k^2 t_1}$	$\frac{i (2 k^3 (2 r_1+r_5)-k t_1)}{(1+k^2)^2 t_1^2}$	$\frac{-2 k^4 (2 r_1+r_5)+k^2 t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\sigma_{1^+ \dagger \alpha}^{\#1}$	0	0	0	$\frac{2 (t_1+t_3)}{3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3)}$	$-\frac{\sqrt{2} (t_1-2 t_3)}{(1+2 k^2) (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$	0	$-\frac{2 i k (t_1-2 t_3)}{(1+2 k^2) (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$
$\sigma_{1^+ \dagger \alpha}^{\#2}$	0	0	0	$-\frac{\sqrt{2} (t_1-2 t_3)}{(1+2 k^2) (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$	$\frac{6 k^2 (r_1+r_5)+t_1+4 t_3}{(1+2 k^2)^2 (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$	0	$\frac{i \sqrt{2} k (6 k^2 (r_1+r_5)+t_1+4 t_3)}{(1+2 k^2)^2 (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$
$\tau_{1^+ \dagger \alpha}^{\#1}$	0	0	0	0	0	0	0
$\tau_{1^+ \dagger \alpha}^{\#2}$	0	0	0	$\frac{2 i k (t_1-2 t_3)}{(1+2 k^2) (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$	$-\frac{i \sqrt{2} k (6 k^2 (r_1+r_5)+t_1+4 t_3)}{(1+2 k^2)^2 (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$	0	$\frac{2 k^2 (6 k^2 (r_1+r_5)+t_1+4 t_3)}{(1+2 k^2)^2 (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$

Source constraints		
SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0^+}^{\#2} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha \beta} == 0$	1
$\tau_{0^+}^{\#1} - 2 \, i \, k \, \sigma_{0^+}^{\#1} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha \beta} == \partial_\beta \partial^\beta \tau^\alpha_\alpha + 2 \, \partial_\chi \partial^X \partial_\beta \sigma^{\alpha \beta}_\alpha$	1
$\tau_1^{\#2 \alpha} + 2 \, i \, k \, \sigma_1^{\#2 \alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta X} == \partial_\chi \partial^X \partial_\beta \tau^{\alpha \beta} + 2 \, \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha \beta X}$	3
$\tau_1^{\#1 \alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta X} == \partial_\chi \partial^X \partial_\beta \tau^{\beta \alpha}$	3
$\tau_{1^+}^{\#1 \alpha \beta} + i \, k \, \sigma_{1^+}^{\#2 \alpha \beta} == 0$	$\partial_\chi \partial^\alpha \tau^{\beta X} + \partial_\chi \partial^\beta \tau^{X \alpha} + \partial_\chi \partial^X \tau^{\alpha \beta} +$ $2 \, \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta X \delta} + 2 \, \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha \beta X} ==$ $\partial_\chi \partial^\alpha \tau^{X \beta} + \partial_\chi \partial^\beta \tau^{\alpha X} +$ $\partial_\chi \partial^X \tau^{\beta \alpha} + 2 \, \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha X \delta}$	3
$\tau_{2^+}^{\#1 \alpha \beta} - 2 \, i \, k \, \sigma_{2^+}^{\#1 \alpha \beta} == 0$	$-i \, (4 \, \partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{X \delta} + 2 \, \partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^X_\chi -$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta X} - 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{X \beta} -$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha X} - 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{X \alpha} +$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^X \tau^{\alpha \beta} + 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^X \tau^{\beta \alpha} +$ $4 \, i \, k^X \, \partial_\epsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta \epsilon}_\delta -$ $6 \, i \, k^X \, \partial_\epsilon \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta \delta \epsilon} -$ $6 \, i \, k^X \, \partial_\epsilon \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha \delta \epsilon} +$ $2 \, \eta^{\alpha \beta} \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{X \delta} +$ $6 \, i \, k^X \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\alpha \delta \beta} +$ $6 \, i \, k^X \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\beta \delta \alpha} -$ $2 \, \eta^{\alpha \beta} \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \tau^X_\chi -$ $4 \, i \, \eta^{\alpha \beta} \, k^X \, \partial_\phi \partial^\phi \partial_\epsilon \partial_\chi \sigma^{\delta \epsilon}_\delta) == 0$	5
Total constraints/gauge generators:		16

	$\mathcal{A}_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\mathcal{A}_{0^+}^{\#1}$	$\mathcal{A}_{2^+ \alpha \beta}^{\#1}$	$f_{2^+ \alpha \beta}^{\#1}$	$\mathcal{A}_{2^- \alpha \beta X}^{\#1}$
$\mathcal{A}_{0^+ \dagger}^{\#1}$	t_3	$-i \sqrt{2} k t_3$	0	0	$\frac{t_1}{2}$	$-\frac{i k t_1}{\sqrt{2}}$	0
$f_{0^+ \dagger}^{\#1}$	$i \sqrt{2} k t_3$	$2 k^2 t_3$	0	0	$\frac{i k t_1}{\sqrt{2}}$	$k^2 t_1$	0
$f_{0^+ \dagger}^{\#2}$	0	0	0	0	0	0	0
$\mathcal{A}_{0^+}^{\#1 \dagger}$	0	0	0	$-t_1$	0	0	$k^2 r_1 + \frac{t_1}{2}$
	$\mathcal{A}_{1^+ \alpha \beta}^{\#1}$	$\mathcal{A}_{1^+ \alpha \beta}^{\#2}$	$f_{1^+ \alpha \beta}^{\#1}$	$\mathcal{A}_{1^- \alpha}^{\#1}$	$\mathcal{A}_{1^- \alpha}^{\#2}$	$f_{1^- \alpha}^{\#1}$	$f_{1^- \alpha}^{\#2}$
$\mathcal{A}_{1^+ \dagger}^{\#1 \alpha \beta}$	$k^2 (2 r_1 + r_5) - \frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{i k t_1}{\sqrt{2}}$	0	0	0	0
$\mathcal{A}_{1^+ \dagger}^{\#2 \alpha \beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1^+ \dagger}^{\#1 \alpha \beta}$	$\frac{i k t_1}{\sqrt{2}}$	0	0	0	0	0	0
$\mathcal{A}_{1^+}^{\#1 \dagger \alpha}$	0	0	0	$\frac{1}{6} (6 k^2 (r_1 + r_5) + t_1 + 4 t_3)$	$\frac{t_1-2 t_3}{3 \sqrt{2}}$	0	$\frac{1}{3} i k (t_1 - 2 t_3)$
$\mathcal{A}_{1^+}^{\#2 \dagger \alpha}$	0	0	0	$\frac{t_1-2 t_3}{3 \sqrt{2}}$	$\frac{t_1+t_3}{3}$	0	$\frac{1}{3} i \sqrt{2} k (t_1 + t_3)$
$f_{1^+}^{\#1 \dagger \alpha}$	0	0	0	0	0	0	0
$f_{1^+}^{\#2 \dagger \alpha}$	0	0	0	$-\frac{1}{3} i k (t_1 - 2 t_3)$	$-\frac{1}{3} i \sqrt{2} k (t_1 + t_3)$	0	$\frac{2}{3} k^2 (t_1 + t_3)$

Quadratic (free) action
$S ==$ $\iiint (\frac{1}{6} (2 (t_1 - 2 t_3) \, \mathcal{A}^{\alpha \iota}_\alpha \, \mathcal{A}_{\iota \, \theta}^\theta + 6 f^{\alpha \beta} \, \tau_{\alpha \beta} + 6 \, \mathcal{A}^{\alpha \beta X} \, \sigma_{\alpha \beta X} - 4 t_1 \, \mathcal{A}_\alpha^\theta \, \partial_{\iota} f^{\alpha \iota} +$ $8 t_3 \, \mathcal{A}_\alpha^\theta \, \partial_{\iota} f^{\alpha \iota} + 4 t_1 \, \mathcal{A}_{\iota \, \theta}^\theta \, \partial' f^\alpha_\alpha - 8 t_3 \, \mathcal{A}_{\iota \, \theta}^\theta \, \partial' f^\alpha_\alpha -$ $2 t_1 \, \partial_{\iota} f^\theta_\theta \, \partial' f^\alpha_\alpha + 4 t_3 \, \partial_{\iota} f^\theta_\theta \, \partial' f^\alpha_\alpha - 2 t_1 \, \partial_{\iota} f^{\alpha \iota} \, \partial_\theta f^\theta_\alpha +$ $4 t_3 \, \partial_{\iota} f^{\alpha \iota} \, \partial_\theta f^\theta_\alpha + 4 t_1 \, \partial' f^\alpha_\alpha \, \partial_\theta f_{\iota \, \theta}^\theta - 8 t_3 \, \partial' f^\alpha_\alpha \, \partial_\theta f_{\iota \, \theta}^\theta -$ $6 t_1 \, \partial_\alpha f_{\iota \, \theta} \, \partial^\theta f^{\alpha \iota} - 3 t_1 \, \partial_\alpha f_{\theta \iota} \, \partial^\theta f^{\alpha \iota} + 3 t_1 \, \partial_{\iota} f_{\alpha \theta} \, \partial^\theta f^{\alpha \iota} +$ $3 t_1 \, \partial_\theta f_{\alpha \iota} \, \partial^\theta f^{\alpha \iota} + 3 t_1 \, \partial_\theta f_{\iota \alpha} \, \partial^\theta f^{\alpha \iota} +$ $6 t_1 \, \mathcal{A}_{\alpha \theta \iota} (\mathcal{A}^{\alpha \iota \theta} + 2 \, \partial^\theta f^{\alpha \iota}) - 8 r_1 \, \partial_\beta \mathcal{A}_{\alpha \iota \theta} \, \partial^\theta \mathcal{A}^{\alpha \beta \iota} +$ $4 r_1 \, \partial_\beta \mathcal{A}_{\alpha \theta \iota} \, \partial^\theta \mathcal{A}^{\alpha \beta \iota} - 16 r_1 \, \partial_\beta \mathcal{A}_{\iota \theta \alpha} \, \partial^\theta \mathcal{A}^{\alpha \beta \iota} -$ $4 r_1 \, \partial_{\iota} \mathcal{A}_{\alpha \beta \theta} \, \partial^\theta \mathcal{A}^{\alpha \beta \iota} + 4 r_1 \, \partial_\theta \mathcal{A}_{\alpha \beta \iota} \, \partial^\theta \mathcal{A}^{\alpha \beta \iota} +$ $4 r_1 \, \partial_\theta \mathcal{A}_{\alpha \beta} \, \partial^\theta \mathcal{A}^{\alpha \beta \iota} + 6 r_5 \, \partial_{\iota} \mathcal{A}_{\theta \, \kappa}^\kappa \, \partial^\theta \mathcal{A}^{\alpha \iota}_\alpha -$ $6 r_5 \, \partial_\theta \mathcal{A}_{\iota \, \kappa}^\kappa \, \partial^\theta \mathcal{A}^{\alpha \iota}_\alpha - 6 r_5 \, \partial_\alpha \mathcal{A}^{\alpha \iota \theta} \, \partial_\kappa \mathcal{A}_{\iota \, \theta}^\kappa +$ $12 r_5 \, \partial^\theta \mathcal{A}^{\alpha \iota}_\alpha \, \partial_\kappa \mathcal{A}_{\iota \, \theta}^\kappa + 6 r_5 \, \partial_\alpha \mathcal{A}^{\alpha \iota \theta} \, \partial_\kappa \mathcal{A}_{\theta \, \iota}^\kappa -$ $12 r_5 \, \partial^\theta \mathcal{A}^{\alpha \iota}_\alpha \, \partial_\kappa \mathcal{A}_{\theta \, \iota}^\kappa)) [t, \, x, \, y, \, z] \, d z \, d y \, d x \, d t$

$\sigma_{2^+ \alpha \beta}^{\#1 \dagger \alpha \beta X}$	$\tau_{2^+ \alpha \beta}^{\#1}$	$\sigma_{2^+ \alpha \beta}^{\#2}$
0	$-\frac{2 i \sqrt{2} k}{(1+2 k^2)^2 t_1}$	$\frac{2}{(1+2 k^2)^2 t_1}$
0	$\frac{4 k^2}{(1+2 k^2)^2 t_1}$	0
$\frac{2}{2 k^2 r_1+t_1}$	0	0

$\sigma_{0^+}^{\#1 \dagger}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^+}^{\#1}$
0	0	0
0	0	0
$-\frac{1}{t_1}$	0	0
0	0	0
0	0	0
0	0	0

Massive and massless spectra

Massive particle	
Pole residue:	$-\frac{3 (-2 t_1 t_3 (t_1+t_3)+r_1 (t_1^2+2 t_3^2)+r_5 (t_1^2+2 t_3^2))}{2 (r_1+r_5) (t_1+t_3) (-3 t_1 t_3+r_1 (t_1+t_3)+r_5 (t_1+t_3))} > 0$
Polarisations:	3
Square mass:	$-\frac{3 t_1 t_3}{2 (r_1+r_5) (t_1+t_3)} > 0$
Spin:	1
Parity:	Odd

Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2 r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0 \ \&\& \ r_5 < -r_1 \ \&\& \ t_1 > 0 \ \&\& \ t_3 < -t_1 || t_3 > 0$