

# Particle spectrograph

## Wave operator and propagator

|   | $\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$ | $\tau_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{1-}^{\#1} \dagger^{\alpha}$ | $\sigma_{1-}^{\#2} \dagger^{\alpha}$ | $\tau_{1-}^{\#1} \dagger^{\alpha}$ | $\tau_{1-}^{\#2} \dagger^{\alpha}$  |
|---|---|---|---|--------------------------------------|--------------------------------------|------------------------------------|-------------------------------------|
| $\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\frac{6}{(3+k^2)^2}t_2$                  | $\frac{3\sqrt{2}}{(3+k^2)^2}t_2$          | $\frac{3i\sqrt{2}k}{(3+k^2)^2}t_2$      | 0                                    | 0                                    | 0                                  | 0                                   |
| $\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$ | $\frac{3\sqrt{2}}{(3+k^2)^2}t_2$          | $\frac{3}{(3+k^2)^2}t_2$                  | $\frac{3ik}{(3+k^2)^2}t_2$              | 0                                    | 0                                    | 0                                  | 0                                   |
| $\tau_{1+}^{\#1} \dagger^{\alpha\beta}$   | $-\frac{3i\sqrt{2}k}{(3+k^2)^2}t_2$       | $-\frac{3ik}{(3+k^2)^2}t_2$               | $\frac{3k^2}{(3+k^2)^2}t_2$             | 0                                    | 0                                    | 0                                  | 0                                   |
| $\sigma_{1-}^{\#1} \dagger^{\alpha}$      | 0   | 0   | 0                                       | $\frac{6}{(3+2k^2)^2}t_3$            | $-\frac{3\sqrt{2}}{(3+2k^2)^2}t_3$   | 0                                  | $-\frac{6ik}{(3+2k^2)^2}t_3$        |
| $\sigma_{1-}^{\#2} \dagger^{\alpha}$      | 0   | 0   | 0                                       | $-\frac{3\sqrt{2}}{(3+2k^2)^2}t_3$   | $\frac{3}{(3+2k^2)^2}t_3$            | 0                                  | $\frac{3i\sqrt{2}k}{(3+2k^2)^2}t_3$ |
| $\tau_{1-}^{\#1} \dagger^{\alpha}$        | 0   | 0   | 0                                       | 0                                    | 0                                    | 0                                  | 0                                   |
| $\tau_{1-}^{\#2} \dagger^{\alpha}$        | 0   | 0   | 0                                       | $\frac{6ik}{(3+2k^2)^2}t_3$          | $-\frac{3i\sqrt{2}k}{(3+2k^2)^2}t_3$ | 0                                  | $\frac{6k^2}{(3+2k^2)^2}t_3$        |

|   | $\omega_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{1+}^{\#2} \dagger^{\alpha\beta}$ | $f_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{1-}^{\#1} \dagger^{\alpha}$ | $\omega_{1-}^{\#2} \dagger^{\alpha}$ | $f_{1-}^{\#1} \dagger^{\alpha}$ | $f_{1-}^{\#2} \dagger^{\alpha}$ |
|---|---|---|--------------------------------------|--------------------------------------|--------------------------------------|---------------------------------|---------------------------------|
| $\omega_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\frac{2t_2}{3}$                          | $\frac{\sqrt{2}t_2}{3}$                   | $\frac{1}{3}i\sqrt{2}kt_2$           | 0                                    | 0                                    | 0                               | 0                               |
| $\omega_{1+}^{\#2} \dagger^{\alpha\beta}$ | $\frac{\sqrt{2}t_2}{3}$                   | $\frac{t_2}{3}$                           | $\frac{ikt_2}{3}$                    | 0                                    | 0                                    | 0                               | 0                               |
| $f_{1+}^{\#1} \dagger^{\alpha\beta}$      | $-\frac{1}{3}i\sqrt{2}kt_2$               | $-\frac{1}{3}ikt_2$                       | $\frac{k^2t_2}{3}$                   | 0                                    | 0                                    | 0                               | 0                               |
| $\omega_{1-}^{\#1} \dagger^{\alpha}$      | 0   | 0   | 0                                    | $\frac{2t_3}{3}$                     | $-\frac{\sqrt{2}t_3}{3}$             | 0                               | $-\frac{2}{3}ikt_3$             |
| $\omega_{1-}^{\#2} \dagger^{\alpha}$      | 0   | 0   | 0                                    | $-\frac{\sqrt{2}t_3}{3}$             | $\frac{t_3}{3}$                      | 0                               | $\frac{1}{3}i\sqrt{2}kt_3$      |
| $f_{1-}^{\#1} \dagger^{\alpha}$           | 0   | 0   | 0                                    | 0                                    | 0                                    | 0                               | 0                               |
| $f_{1-}^{\#2} \dagger^{\alpha}$           | 0   | 0   | 0                                    | $\frac{2ikt_3}{3}$                   | $-\frac{1}{3}i\sqrt{2}kt_3$          | 0                               | $\frac{2k^2t_3}{3}$             |

### Quadratic (free) Lagrangian density

$$\frac{2}{3}t_3\omega_{\kappa}^{\alpha\lambda}\omega_{\kappa\alpha}^{\kappa}+\frac{2}{3}t_2\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\kappa\lambda}^{\kappa\lambda}+\frac{1}{3}t_2\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\kappa\lambda}^{\kappa\lambda}+f_{\alpha\beta}^{\alpha\beta}\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}_{\alpha\beta\chi}\sigma_{\alpha\beta\chi}+\frac{2}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\theta\alpha}+\frac{1}{6}t_2\partial_2^{\alpha}f_{\theta\kappa}^{\alpha}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{6}t_2\partial_2^{\alpha}f_{\kappa\theta}^{\alpha}\partial^{\kappa}f_{\alpha}^{\theta}+\frac{1}{6}t_2\partial_2^{\alpha}f_{\alpha}^{\kappa}\partial^{\kappa}f_{\alpha\lambda}^{\lambda}-\frac{2}{3}t_3\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\kappa}^{\lambda}-\frac{2}{3}t_3\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\kappa}^{\lambda}-\frac{4}{3}t_3\partial_3^{\alpha}f_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\kappa}^{\lambda}+\frac{2}{3}t_3\partial_3^{\alpha}f_{\lambda}^{\alpha}\partial^{\kappa}f_{\lambda}^{\lambda}+\frac{1}{3}t_2\omega_{\theta\kappa}^{\kappa}\partial^{\kappa}f_{\theta}^{\lambda}-\frac{2}{3}t_2\omega_{\kappa\theta}^{\theta}\partial^{\kappa}f_{\theta}^{\lambda}-\frac{1}{3}t_2\omega_{\theta\kappa}^{\kappa}\partial^{\kappa}f_{\theta}^{\lambda}+\frac{2}{3}t_2\omega_{\theta\kappa}^{\kappa}\partial^{\kappa}f_{\theta}^{\lambda}+\frac{2}{3}t_3\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\kappa}^{\lambda}+\frac{2}{3}t_3\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\kappa}^{\lambda}-\frac{1}{6}t_2\partial_2^{\alpha}f_{\kappa}^{\lambda}\partial^{\kappa}f_{\lambda}^{\alpha}-\frac{1}{6}t_2\partial_2^{\alpha}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{1}{6}t_2\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{2}{3}t_3\partial_3^{\alpha}f_{\alpha}^{\lambda}\partial^{\kappa}f_{\lambda}^{\alpha}+\frac{1}{3}r_2\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+\frac{2}{3}r_2\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}-\frac{2}{3}r_2\partial_2^{\beta}\omega_{\alpha\beta}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda\alpha}+\frac{2}{3}r_2\partial_2^{\beta}\omega_{\alpha\beta}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda\alpha}$$

|   |   |                                      |   |
|---|---|--------------------------------------|---|
|   | $\omega_{2+}^{\#1} \dagger^{\alpha\beta}$ | $f_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$ |
| $\omega_{2+}^{\#1} \dagger^{\alpha\beta}$     | 0   | 0                                    | 0   |
| $f_{2+}^{\#1} \dagger^{\alpha\beta}$          | 0   | 0                                    | 0   |
| $\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$ | 0   | 0                                    | 0   |

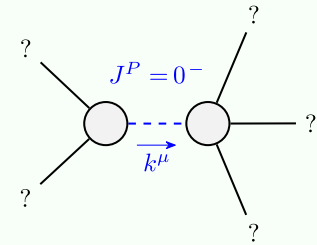
|   |   |   |   |
|---|---|---|---|
|   | $\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\tau_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$ |
| $\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$     | 0   | 0                                       | 0   |
| $\tau_{2+}^{\#1} \dagger^{\alpha\beta}$       | 0   | 0                                       | 0   |
| $\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$ | 0   | 0                                       | 0   |

| Source constraints/gauge generators                                |                |
|--|----------------|
| SO(3) irreps   | Multiplicities |
| $\tau_{0+}^{\#2} == 0$   | 1              |
| $\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$                      | 1              |
| $\tau_{1-}^{\#2\alpha} - ik\sigma_{1-}^{\#1\alpha} == 0$           | 3              |
| $\tau_{1-}^{\#1\alpha} == 0$                                       | 3              |
| $\sigma_{1-}^{\#1\alpha} + 2\sigma_{1-}^{\#2\alpha} == 0$          | 3              |
| $\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#1\alpha\beta} == 0$ | 3              |
| $\sigma_{1+}^{\#1\alpha\beta} == \sigma_{1+}^{\#2\alpha\beta}$     | 3              |
| $\sigma_{2-}^{\#1\alpha\beta\chi} == 0$                            | 5              |
| $\tau_{2+}^{\#1\alpha\beta} == 0$                                  | 5              |
| $\sigma_{2+}^{\#1\alpha\beta} == 0$                                | 5              |
| Total constraints:   | 32             |

|                     |                     |                  |                |                     |
|---------------------|---------------------|------------------|----------------|---------------------|
|                     | $\omega_{0+}^{\#1}$ | $f_{0+}^{\#1}$   | $f_{0+}^{\#2}$ | $\omega_{0-}^{\#1}$ |
| $\omega_{0+}^{\#1}$ | $t_3$               | $-i\sqrt{2}kt_3$ | 0              | 0                   |
| $f_{0+}^{\#1}$      | $i\sqrt{2}kt_3$     | $2k^2t_3$        | 0              | 0                   |
| $f_{0+}^{\#2}$      | 0                   | 0                | 0              | 0                   |
| $\omega_{0-}^{\#1}$ | 0                   | 0                | 0              | $k^2r_2+t_2$        |

|                     |                                    |                                     |                   |                        |
|---------------------|------------------------------------|-------------------------------------|-------------------|------------------------|
|                     | $\sigma_{0+}^{\#1}$                | $\tau_{0+}^{\#1}$                   | $\tau_{0+}^{\#2}$ | $\sigma_{0-}^{\#1}$    |
| $\sigma_{0+}^{\#1}$ | $\frac{1}{(1+2k^2)^2}t_3$          | $-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_3$ | 0                 | 0                      |
| $\tau_{0+}^{\#1}$   | $\frac{i\sqrt{2}k}{(1+2k^2)^2}t_3$ | $\frac{2k^2}{(1+2k^2)^2}t_3$        | 0                 | 0                      |
| $\tau_{0+}^{\#2}$   | 0                                  | 0                                   | 0                 | 0                      |
| $\sigma_{0-}^{\#1}$ | 0                                  | 0                                   | 0                 | $\frac{1}{k^2r_2+t_2}$ |

## Massive and massless spectra



| Massive particle |                        |
|------------------|------------------------|
| Pole residue:    | $-\frac{1}{r_2} > 0$   |
| Polarisations:   | 1                      |
| Square mass:     | $-\frac{t_2}{r_2} > 0$ |
| Spin:            | 0                      |
| Parity:          | Odd                    |

(No massless particles)

## Unitarity conditions

$$r_2 < 0 \&\& t_2 > 0$$