

# Particle spectrograph

## Wave operator and propagator

Quadratic (free) action

$$S_F ==$$
$$\int \int \int (\frac{1}{6} (4 t_3 \omega_{\kappa \alpha}^{\alpha \iota} \omega_{\kappa \alpha}^{\kappa} + 4 t_2 \omega_{\kappa \lambda}^{\kappa \lambda} \omega_{\kappa \lambda}^{\iota} + 2 t_2 \omega_{\kappa \lambda}^{\kappa \lambda} \omega_{\kappa \lambda}^{\iota} + 6 f^{\alpha \beta} \tau_{\alpha \beta} + 6 \omega^{\alpha \beta \chi} \sigma_{\alpha \beta \chi} + 9 r_3 \partial_{\iota} \omega_{\kappa}^{\kappa \lambda} \partial' \omega_{\lambda}^{\alpha} + 4 r_2 \partial^{\beta} \omega_{\kappa}^{\theta \alpha} \partial_{\theta} \omega_{\alpha \beta}^{\kappa} - 2 r_2 \partial_{\theta} \omega_{\alpha \beta}^{\kappa} \partial_{\kappa} \omega^{\alpha \beta \theta} - 4 r_2 \partial_{\theta} \omega_{\alpha \beta}^{\kappa} \partial_{\kappa} \omega^{\theta \alpha \beta} + 15 r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\theta \kappa \lambda} - 15 r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\theta \kappa \lambda} - 18 r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\kappa \lambda \theta} + t_2 \partial^{\alpha} f_{\theta \kappa} \partial^{\kappa} f_{\alpha}^{\theta} - t_2 \partial^{\alpha} f_{\kappa \theta} \partial^{\kappa} f_{\alpha}^{\theta} + t_2 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\alpha \lambda} - 4 t_3 \omega_{\kappa \alpha}^{\alpha} \partial^{\kappa} f_{\iota}^{\iota} - 4 t_3 \omega_{\kappa \lambda}^{\lambda} \partial^{\kappa} f_{\iota}^{\iota} - 8 t_3 \partial^{\alpha} f_{\kappa \alpha} \partial^{\kappa} f_{\iota}^{\iota} + 4 t_3 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\iota}^{\iota} + 2 t_2 \omega_{\iota \theta \kappa} \partial^{\kappa} f^{\iota \theta} - 4 t_2 \omega_{\iota \kappa \theta} \partial^{\kappa} f^{\iota \theta} - 2 t_2 \omega_{\theta \iota \kappa} \partial^{\kappa} f^{\iota \theta} + 4 t_2 \omega_{\theta \kappa \iota} \partial^{\kappa} f^{\iota \theta} + 4 t_3 \omega_{\iota \alpha}^{\alpha} \partial^{\kappa} f_{\kappa}^{\iota} + 4 t_3 \omega_{\iota \lambda}^{\lambda} \partial^{\kappa} f_{\kappa}^{\iota} - t_2 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\lambda \alpha} - t_2 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} + t_2 \partial_{\kappa} f_{\lambda}^{\theta} \partial^{\kappa} f_{\theta}^{\lambda} + 4 t_3 \partial^{\alpha} f_{\lambda}^{\alpha} \partial^{\kappa} f_{\alpha}^{\theta} + 2 r_2 \partial_{\kappa} \omega^{\alpha \beta \theta} \partial^{\kappa} \omega_{\alpha \beta \theta} + 4 r_2 \partial_{\kappa} \omega^{\theta \alpha \beta} \partial^{\kappa} \omega_{\alpha \beta \theta} - 4 r_2 \partial^{\beta} \omega_{\lambda}^{\alpha \lambda} \partial_{\lambda} \omega_{\alpha \beta}^{\iota} + 4 r_2 \partial^{\beta} \omega_{\lambda}^{\lambda \alpha} \partial_{\lambda} \omega_{\alpha \beta}^{\iota} - 24 r_3 \partial^{\beta} \omega_{\lambda}^{\lambda \alpha} \partial_{\lambda} \omega_{\alpha \beta}^{\iota} - 15 r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial^{\iota} \omega_{\theta}^{\theta \kappa} + 15 r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\iota} \omega_{\alpha}^{\theta \kappa} )) [t, x, y, z] d z d y d x d t$$

	$\sigma_{1^{+}\alpha\beta}^{\#1}$	$\sigma_{1^{+}\alpha\beta}^{\#2}$	$\tau_{1^{+}\alpha\beta}^{\#1}$	$\sigma_{1^{-}\alpha}^{\#1}$	$\sigma_{1^{-}\alpha}^{\#2}$	$\tau_{1^{-}\alpha}^{\#1}$	$\tau_{1^{-}\alpha}^{\#2}$
$\sigma_{1^{+}}^{\#1} \dagger^{\alpha\beta}$	$\frac{6}{(3+k^2)^2} t_2$	$\frac{3\sqrt{2}}{(3+k^2)^2} t_2$	$\frac{3i\sqrt{2}k}{(3+k^2)^2} t_2$	0	0	0	0
$\sigma_{1^{+}}^{\#2} \dagger^{\alpha\beta}$	$\frac{3\sqrt{2}}{(3+k^2)^2} t_2$	$\frac{3}{(3+k^2)^2} t_2$	$\frac{3ik}{(3+k^2)^2} t_2$	0	0	0	0
$\tau_{1^{+}}^{\#1} \dagger^{\alpha\beta}$	$-\frac{3i\sqrt{2}k}{(3+k^2)^2} t_2$	$-\frac{3ik}{(3+k^2)^2} t_2$	$\frac{3k^2}{(3+k^2)^2} t_2$	0	0	0	0
$\sigma_{1^{-}}^{\#1} \dagger^{\alpha}$	0	0	0	$-\frac{2}{3k^2} r_3$	$-\frac{2\sqrt{2}}{3k^2 r_3 + 6k^4} r_3$	0	$-\frac{4i}{3kr_3 + 6k^3} r_3$
$\sigma_{1^{-}}^{\#2} \dagger^{\alpha}$	0	0	0	$-\frac{2\sqrt{2}}{3k^2 r_3 + 6k^4} r_3$	$\frac{9k^2 r_3 - 4t_3}{3(k+2k^3)^2 r_3 t_3}$	0	$\frac{i\sqrt{2}(9k^2 r_3 - 4t_3)}{3k(1+2k^2)^2 r_3 t_3}$
$\tau_{1^{-}}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$\tau_{1^{-}}^{\#2} \dagger^{\alpha}$	0	0	0	$\frac{4i}{3kr_3 + 6k^3} r_3$	$-\frac{i\sqrt{2}(9k^2 r_3 - 4t_3)}{3k(1+2k^2)^2 r_3 t_3}$	0	$\frac{2(9k^2 r_3 - 4t_3)}{3(1+2k^2)^2 r_3 t_3}$

	$\omega_{1^{+}\alpha\beta}^{\#1}$	$\omega_{1^{+}\alpha\beta}^{\#2}$	$f_{1^{+}\alpha\beta}^{\#1}$	$\omega_{1^{-}\alpha}^{\#1}$	$\omega_{1^{-}\alpha}^{\#2}$	$f_{1^{-}\alpha}^{\#1}$	$f_{1^{-}\alpha}^{\#2}$
$\omega_{1^{+}}^{\#1} \dagger^{\alpha\beta}$	$\frac{2t_2}{3}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{1}{3}i\sqrt{2}kt_2$	0	0	0	0
$\omega_{1^{+}}^{\#2} \dagger^{\alpha\beta}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{t_2}{3}$	$\frac{ikt_2}{3}$	0	0	0	0
$f_{1^{+}}^{\#1} \dagger^{\alpha\beta}$	$-\frac{1}{3}i\sqrt{2}kt_2$	$-\frac{1}{3}ikt_2$	$\frac{k^2t_2}{3}$	0	0	0	0
$\omega_{1^{-}}^{\#1} \dagger^{\alpha}$	0	0	0	$\frac{1}{6}(-9k^2r_3 + 4t_3)$	$-\frac{\sqrt{2}t_3}{3}$	0	$-\frac{2}{3}ikt_3$
$\omega_{1^{-}}^{\#2} \dagger^{\alpha}$	0	0	0	$-\frac{\sqrt{2}t_3}{3}$	$\frac{t_3}{3}$	0	$\frac{1}{3}i\sqrt{2}kt_3$
$f_{1^{-}}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$f_{1^{-}}^{\#2} \dagger^{\alpha}$	0	0	0	$\frac{2ikt_3}{3}$	$-\frac{1}{3}i\sqrt{2}kt_3$	0	$\frac{2k^2t_3}{3}$

$$\omega_{2^{+}\alpha\beta}^{\#1} \quad f_{2^{+}\alpha\beta}^{\#1} \quad \omega_{2^{-}\alpha\beta\chi}^{\#1}$$

$\omega_{2^{+}}^{\#1} \dagger^{\alpha\beta}$	$-\frac{3k^2r_3}{2}$	0	0
$f_{2^{+}}^{\#1} \dagger^{\alpha\beta}$	0	0	0
$\omega_{2^{-}}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	0

$$\sigma_{2^{+}\alpha\beta}^{\#1} \quad \tau_{2^{+}\alpha\beta}^{\#1} \quad \sigma_{2^{-}\alpha\beta\chi}^{\#1}$$

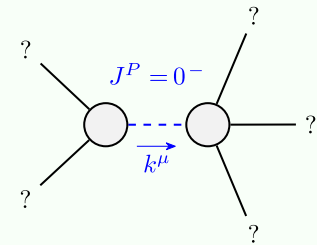
$\sigma_{2^{+}}^{\#1} \dagger^{\alpha\beta}$	$-\frac{2}{3k^2} r_3$	0	0
$\tau_{2^{+}}^{\#1} \dagger^{\alpha\beta}$	0	0	0
$\sigma_{2^{-}}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	0

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau_{0^{+}}^{\#2} == 0$	1
$\tau_{0^{+}}^{\#1} - 2i k \sigma_{0^{+}}^{\#1} == 0$	1
$\tau_{1^{-}}^{\#2\alpha} + 2i k \sigma_{1^{-}}^{\#2\alpha} == 0$	3
$\tau_{1^{-}}^{\#1\alpha} == 0$	3
$\tau_{1^{+}}^{\#1\alpha\beta} + i k \sigma_{1^{+}}^{\#1\alpha\beta} == 0$	3
$\sigma_{1^{+}}^{\#1\alpha\beta} == \sigma_{1^{+}}^{\#2\alpha\beta}$	3
$\sigma_{2^{-}}^{\#1\alpha\beta\chi} == 0$	5
$\tau_{2^{+}}^{\#1\alpha\beta} == 0$	5
Total constraints:	24

$\sigma_{0^{+}}^{\#1} \dagger$	$\frac{1}{(1+2k^2)^2} t_3$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2} t_3$	0	0
$\tau_{0^{+}}^{\#1} \dagger$	$\frac{i\sqrt{2}k}{(1+2k^2)^2} t_3$	$\frac{2k^2}{(1+2k^2)^2} t_3$	0	0
$\tau_{0^{+}}^{\#2} \dagger$	0	0	0	0
$\sigma_{0^{-}}^{\#1} \dagger$	0	0	0	$\frac{1}{k^2 r_2 + t_2}$
$\omega_{0^{+}}^{\#1} \dagger$	$t_3$	$-i\sqrt{2}kt_3$	0	0
$f_{0^{+}}^{\#1} \dagger$	$i\sqrt{2}kt_3$	$2k^2t_3$	0	0
$f_{0^{+}}^{\#2} \dagger$	0	0	0	0
$\omega_{0^{-}}^{\#1} \dagger$	0	0	0	$k^2r_2 + t_2$

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(seipited sseasew on)

## Unitarity conditions

$$r_2 < 0 \&\& t_2 > 0$$