Particle spectrograph

Wave operator and propagator

Source constraints		
SO(3) irreps	Fundamental fields	Multiplicities
$\tau^{\#2}_{0^+} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$	1
$\tau^{\#1}_{0^+} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha$	1
$\sigma^{\#1}_{0^+} == 0$	$\partial_\beta\sigma^{\alpha\beta}{}_\alpha == 0$	1
$\tau^{\#2}_{1^-}{}^\alpha == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta}$	3
$\tau^{\#1}_{1^-}{}^\alpha == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$	3
$\sigma^{\#2}_{1^-}{}^\alpha == 0$	$\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi} == 0$	3
$\tau^{\#1}_{1^+}{}^{\alpha\beta} + i\,k\,\sigma^{\#2}_{1^+}{}^{\alpha\beta} == 0$	$\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} + 2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$	3
$\sigma^{\#1}_{2^-}{}^{\alpha\beta\chi} == 0$	$3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\alpha\sigma^{\beta\delta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\alpha\sigma^{\beta\delta}{}_\delta + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\delta\chi} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\chi\delta\alpha} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\chi\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\beta\delta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\delta\beta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\beta\chi\alpha} + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\alpha\sigma^{\delta\epsilon}{}_\delta + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\beta\delta\epsilon} + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\alpha\delta}{}_\delta == 3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\beta\sigma^{\alpha\delta}{}_\delta + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\delta\chi} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\chi\delta\beta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\beta\delta\alpha} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\beta\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\chi\beta} + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\beta\sigma^{\delta\epsilon}{}_\delta + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\alpha\delta\epsilon} + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\beta\delta}{}_\delta$	5
$\tau^{\#1}_{2^+}{}^{\alpha\beta} == 0$	$4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} == 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi$	5
Total constraints/gauge generators:		25

Quadratic (free) action

$$S == \iiiint\Big(\tfrac{1}{6}\big(6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 3\,r_3\,\partial_\beta\omega_\iota{}^\theta{}_\theta\,\partial^\iota\omega^{\alpha\beta}{}_\alpha - 3\,r_3\,\partial_\iota\omega_\beta{}^\theta{}_\theta\,\partial^\iota\omega^{\alpha\beta}{}_\alpha - 3\,r_3\,\partial_\alpha\omega^{\alpha\beta\iota}\,\partial_\theta\omega_\beta{}^\theta{}_\iota + 6\,r_3\,\partial^\iota\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_\beta{}^\theta{}_\iota - 3\,r_3\,\partial_\alpha\omega^{\alpha\beta\iota}\,\partial_\theta\omega_\iota{}^\theta{}_\beta + 6\,r_3\,\partial^\iota\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_\iota{}^\theta{}_\beta + 4\,t_2\,\omega_{\iota\theta\alpha}\,\partial^\theta f^{\alpha\iota} + 2\,t_2\,\partial_\alpha f_{\iota\theta}\,\partial^\theta f^{\alpha\iota} - t_2\,\partial_\alpha f_{\theta\iota}\,\partial^\theta f^{\alpha\iota} - t_2\,\partial_\iota f_{\alpha\theta}\,\partial^\theta f^{\alpha\iota} + t_2\,\partial_\theta f_{\alpha\iota}\,\partial^\theta f^{\alpha\iota} - t_2\,\partial_\theta f_{\iota\alpha}\,\partial^\theta f^{\alpha\iota} - 4\,t_2\,\omega_{\alpha\theta\iota}\,(\omega^{\alpha\iota\theta} + \partial^\theta f^{\alpha\iota}) + 2\,t_2\,\omega_{\alpha\iota\theta}\,(\omega^{\alpha\iota\theta} + 2\,\partial^\theta f^{\alpha\iota}) + 8\,r_2\,\partial_\beta\omega_{\alpha\iota\theta}\,\partial^\theta\omega^{\alpha\beta\iota} - 4\,r_2\,\partial_\beta\omega_{\alpha\theta\iota}\,\partial^\theta\omega^{\alpha\beta\iota} - 4\,r_2\,\partial_\beta\omega_{\iota\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\iota} - 24\,r_3\,\partial_\beta\omega_{\iota\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\iota} - 2\,r_2\,\partial_\iota\omega_{\alpha\beta\theta}\,\partial^\theta\omega^{\alpha\beta\iota} + 2\,r_2\,\partial_\theta\omega_{\alpha\beta\iota}\,\partial^\theta\omega^{\alpha\beta\iota} + 4\,r_2\,\partial_\theta\omega_{\alpha\iota\beta}\,\partial^\theta\omega^{\alpha\beta\iota} + 6\,r_5\,\partial_\iota\omega_\theta{}^\kappa{}_\kappa\,\partial^\theta\omega^{\alpha\iota}{}_\alpha - 6\,r_5\,\partial_\theta\omega_\iota{}^\kappa{}_\kappa\,\partial^\theta\omega^{\alpha\iota}{}_\alpha - 6\,r_5\,\partial_\alpha\omega^{\alpha\iota\theta}\,\partial_\kappa\omega_\iota{}^\kappa{}_\theta + 12\,r_5\,\partial^\theta\omega^{\alpha\iota}{}_\alpha\,\partial_\kappa\omega_\iota{}^\kappa{}_\theta + 6\,r_5\,\partial_\alpha\omega^{\alpha\iota\theta}\,\partial_\kappa\omega_\theta{}^\kappa{}_\iota - 12\,r_5\,\partial^\theta\omega^{\alpha\iota}{}_\alpha\,\partial_\kappa\omega_\theta{}^\kappa{}_\iota\big)\Big)[t, x, y, z]\,dz\,dy\,dx\,dt$$

	$\sigma^{\#1}_{1^+}{}_{\alpha\beta}$	$\sigma^{\#2}_{1^+}{}_{\alpha\beta}$	$\tau^{\#1}_{1^+}{}_{\alpha\beta}$	$\sigma^{\#1}_{1^-}{}_\alpha$	$\sigma^{\#2}_{1^-}{}_\alpha$	$\tau^{\#1}_{1^-}{}_\alpha$	$\tau^{\#2}_{1^-}{}_\alpha$
$\sigma^{\#1}_{1^+}\dagger^{\alpha\beta}$	$\frac{1}{k^2(2r_3+r_5)}$	$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$	$-\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$	0	0	0	0
$\sigma^{\#2}_{1^+}\dagger^{\alpha\beta}$	$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$	$\frac{3k^2(2r_3+r_5)+2t_2}{(k+k^3)^2(2r_3+r_5)t_2}$	$\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$	0	0	0	0
$\tau^{\#1}_{1^+}\dagger^{\alpha\beta}$	$\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$	$-\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$	$\frac{3k^2(2r_3+r_5)+2t_2}{(1+k^2)^2(2r_3+r_5)t_2}$	0	0	0	0
$\sigma^{\#1}_{1^-}\dagger^\alpha$	0	0	0	$\frac{2}{k^2(r_3+2r_5)}$	0	0	0
$\sigma^{\#2}_{1^-}\dagger^\alpha$	0	0	0	0	0	0	0
$\tau^{\#1}_{1^-}\dagger^\alpha$	0	0	0	0	0	0	0
$\tau^{\#2}_{1^-}\dagger^\alpha$	0	0	0	0	0	0	0

	$\omega^{\#1}_{1^+}{}_{\alpha\beta}$	$\omega^{\#2}_{1^+}{}_{\alpha\beta}$	$f^{\#1}_{1^+}{}_{\alpha\beta}$	$\omega^{\#1}_{1^-}{}_\alpha$	$\omega^{\#2}_{1^-}{}_\alpha$	$f^{\#1}_{1^-}{}_\alpha$	$f^{\#2}_{1^-}{}_\alpha$
$\omega^{\#1}_{1^+}\dagger^{\alpha\beta}$	$k^2(2r_3+r_5)+\frac{2t_2}{3}$	$\frac{\sqrt{2}\,t_2}{3}$	$\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2$	0	0	0	0
$\omega^{\#2}_{1^+}\dagger^{\alpha\beta}$	$\frac{\sqrt{2}\,t_2}{3}$	$\frac{t_2}{3}$	$\frac{i\,k\,t_2}{3}$	0	0	0	0
$f^{\#1}_{1^+}\dagger^{\alpha\beta}$	$-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2$	$-\frac{1}{3}\,i\,k\,t_2$	$\frac{k^2\,t_2}{3}$	0	0	0	0
$\omega^{\#1}_{1^-}\dagger^\alpha$	0	0	0	$\frac{1}{2}k^2(r_3+2r_5)$	0	0	0
$\omega^{\#2}_{1^-}\dagger^\alpha$	0	0	0	0	0	0	0
$f^{\#1}_{1^-}\dagger^\alpha$	0	0	0	0	0	0	0
$f^{\#2}_{1^-}\dagger^\alpha$	0	0	0	0	0	0	0

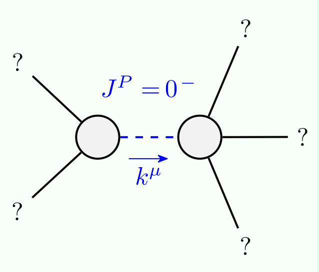
	$\sigma^{\#1}_{0^+}$	$\tau^{\#1}_{0^+}$	$\tau^{\#2}_{0^+}$	$\sigma^{\#1}_{0^-}$
$\sigma^{\#1}_{0^+}\dagger$	0	0	0	0
$\tau^{\#1}_{0^+}\dagger$	0	0	0	0
$\tau^{\#2}_{0^+}\dagger$	0	0	0	0
$\sigma^{\#1}_{0^-}\dagger$	0	0	0	$\frac{1}{k^2 r_2+t_2}$

	$\omega^{\#1}_{2^+}{}_{\alpha\beta}$	$f^{\#1}_{2^+}{}_{\alpha\beta}$	$\omega^{\#1}_{2^-}{}_{\alpha\beta\chi}$
$\omega^{\#1}_{2^+}\dagger^{\alpha\beta}$	$-\frac{3k^2 r_3}{2}$	0	0
$f^{\#1}_{2^+}\dagger^{\alpha\beta}$	0	0	0
$\omega^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$	0	0	0

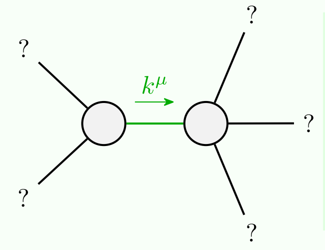
	$\sigma^{\#1}_{2^+}{}_{\alpha\beta}$	$\tau^{\#1}_{2^+}{}_{\alpha\beta}$	$\sigma^{\#1}_{2^-}{}_{\alpha\beta\chi}$
$\sigma^{\#1}_{2^+}\dagger^{\alpha\beta}$	$-\frac{2}{3k^2 r_3}$	0	0
$\tau^{\#1}_{2^+}\dagger^{\alpha\beta}$	0	0	0
$\sigma^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$	0	0	0

	$\omega^{\#1}_{0^+}$	$f^{\#1}_{0^+}$	$f^{\#2}_{0^+}$	$\omega^{\#1}_{0^-}$
$\omega^{\#1}_{0^+}\dagger$	0	0	0	0
$f^{\#1}_{0^+}\dagger$	0	0	0	0
$f^{\#2}_{0^+}\dagger$	0	0	0	0
$\omega^{\#1}_{0^-}\dagger$	0	0	0	$k^2 r_2+t_2$

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd



Quadratic pole	
Pole residue:	$-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2} > 0$
Polarisations:	2

Unitarity conditions

$r_2 < 0 \,\&\&\, r_3 < 0 \,\&\&\, r_5 < -\frac{r_3}{2} \,\&\&\, t_2 > 0 \,\|\, r_2 < 0 \,\&\&\, r_3 < 0 \,\&\&\, r_5 > -2r_3 \,\&\&\, t_2 > 0 \,\|\, r_2 < 0 \,\&\&\, r_3 > 0 \,\&\&\, -2r_3 < r_5 < -\frac{r_3}{2} \,\&\&\, t_2 > 0$