# Particle spectrograph

## Wave operator and propagator

Source constraints

| SO(3) irreps | Fundamental fields | Multiplicities |
|---|---|---|
| $\tau^{\#2}_{0^+} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$ | 1 |
| $\tau^{\#1}_{0^+} - 2\,i\,k\,\sigma^{\#1}_{0^+} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha + 2\,\partial_\chi\partial^\chi\partial_\beta\sigma^{\alpha\beta}{}_\alpha$ | 1 |
| $\tau^{\#2\,\alpha}_{1^-} + 2\,i\,k\,\sigma^{\#2\,\alpha}_{1^-} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$ | 3 |
| $\tau^{\#1\,\alpha}_{1^-} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$ | 3 |
| $\tau^{\#1\,\alpha\beta}_{1^+} + i\,k\,\sigma^{\#2\,\alpha\beta}_{1^+} == 0$ | $\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} + 2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$ | 3 |
| $\tau^{\#1\,\alpha\beta}_{2^+} - 2\,i\,k\,\sigma^{\#1\,\alpha\beta}_{2^+} == 0$ | $-i\,(4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} + 4\,i\,k^\chi\,\partial_\epsilon\partial_\chi\partial^\beta\partial^\alpha\sigma^{\delta\epsilon}{}_\delta - 6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\delta\epsilon} - 6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} + 6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\alpha\delta\beta} + 6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\beta\delta\alpha} - 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi - 4\,i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\chi\sigma^{\delta\epsilon}{}_\delta) == 0$ | 5 |
| Total constraints/gauge generators: | | 16 |

| | $\sigma^{\#1}_{1^+\,\alpha\beta}$ | $\sigma^{\#2}_{1^+\,\alpha\beta}$ | $\tau^{\#1}_{1^+\,\alpha\beta}$ | $\sigma^{\#1}_{1^-\,\alpha}$ | $\sigma^{\#2}_{1^-\,\alpha}$ | $\tau^{\#1}_{1^-\,\alpha}$ | $\tau^{\#2}_{1^-\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma^{\#1}_{1^+}\dagger^{\alpha\beta}$ | 0 | $-\frac{\sqrt{2}}{t_1+k^2 t_1}$ | $-\frac{i\sqrt{2}\,k}{t_1+k^2 t_1}$ | 0 | 0 | 0 | 0 |
| $\sigma^{\#2}_{1^+}\dagger^{\alpha\beta}$ | $-\frac{\sqrt{2}}{t_1+k^2 t_1}$ | $\frac{-2k^2(2r_1+r_5)+t_1}{(1+k^2)^2 t_1{}^2}$ | $\frac{-2ik^3(2r_1+r_5)+ikt_1}{(1+k^2)^2 t_1{}^2}$ | 0 | 0 | 0 | 0 |
| $\tau^{\#1}_{1^+}\dagger^{\alpha\beta}$ | $\frac{i\sqrt{2}\,k}{t_1+k^2 t_1}$ | $\frac{i(2k^3(2r_1+r_5)-kt_1)}{(1+k^2)^2 t_1{}^2}$ | $\frac{-2k^4(2r_1+r_5)+k^2 t_1}{(1+k^2)^2 t_1{}^2}$ | 0 | 0 | 0 | 0 |
| $\sigma^{\#1}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2 t_1}$ | 0 | $\frac{2ik}{t_1+2k^2 t_1}$ |
| $\sigma^{\#2}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2 t_1}$ | $\frac{-2k^2(r_1+r_5)+t_1}{(t_1+2k^2 t_1)^2}$ | 0 | $-\frac{i\sqrt{2}\,k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2 t_1)^2}$ |
| $\tau^{\#1}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau^{\#2}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | $-\frac{2ik}{t_1+2k^2 t_1}$ | $\frac{i\sqrt{2}\,k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2 t_1)^2}$ | 0 | $\frac{-4k^4(r_1+r_5)+2k^2 t_1}{(t_1+2k^2 t_1)^2}$ |

| | $\sigma^{\#1}_{2^+\,\alpha\beta}$ | $\tau^{\#1}_{2^+\,\alpha\beta}$ | $\sigma^{\#1}_{2^-\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\frac{2}{(1+2k^2)^2 t_1}$ | $-\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | 0 |
| $\tau^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | $\frac{4k^2}{(1+2k^2)^2 t_1}$ | 0 |
| $\sigma^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{2k^2 r_1+t_1}$ |

| | $\omega^{\#1}_{0^+}$ | $f^{\#1}_{0^+}$ | $f^{\#2}_{0^+}$ | $\omega^{\#1}_{0^-}$ |
|---|---|---|---|---|
| $\omega^{\#1}_{0^+}\dagger$ | $-t_1$ | $i\sqrt{2}\,k t_1$ | 0 | 0 |
| $f^{\#1}_{0^+}\dagger$ | $-i\sqrt{2}\,k t_1$ | $-2k^2 t_1$ | 0 | 0 |
| $f^{\#2}_{0^+}\dagger$ | 0 | 0 | 0 | 0 |
| $\omega^{\#1}_{0^-}\dagger$ | 0 | 0 | 0 | $-t_1$ |

| | $\sigma^{\#1}_{0^+}$ | $\tau^{\#1}_{0^+}$ | $\tau^{\#2}_{0^+}$ | $\sigma^{\#1}_{0^-}$ |
|---|---|---|---|---|
| $\sigma^{\#1}_{0^+}\dagger$ | $-\frac{1}{(1+2k^2)^2 t_1}$ | $\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | 0 | 0 |
| $\tau^{\#1}_{0^+}\dagger$ | $-\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | $-\frac{2k^2}{(1+2k^2)^2 t_1}$ | 0 | 0 |
| $\tau^{\#2}_{0^+}\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma^{\#1}_{0^-}\dagger$ | 0 | 0 | 0 | $-\frac{1}{t_1}$ |

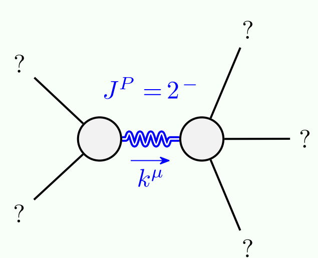
Quadratic (free) action

$$
\begin{aligned}
S == \iiiint\Big(& f^{\alpha\beta}\ \tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\ \sigma_{\alpha\beta\chi} + \\
& \tfrac{1}{2} t_1 \big(2\ \omega^{\alpha\iota}{}_\alpha\ \omega_\iota{}^\theta{}_\theta - 4\ \omega_\alpha{}^\theta{}_\theta\,\partial_\iota f^{\alpha\iota} + 4\ \omega_\iota{}^\theta{}_\theta\,\partial^\iota f^\alpha{}_\alpha - \\
& 2\,\partial_\iota f^\theta{}_\theta\partial^\iota f^\alpha{}_\alpha - 2\,\partial_\iota f^{\alpha\iota}\partial_\theta f_\alpha{}^\theta + 4\,\partial^\iota f^\alpha{}_\alpha\partial_\theta f_\iota{}^\theta - 2\,\partial_\alpha f_{\iota\theta}\\
& \partial^\theta f^{\alpha\iota} - \partial_\alpha f_{\theta\iota}\partial^\theta f^{\alpha\iota} + \partial_\iota f_{\alpha\theta}\partial^\theta f^{\alpha\iota} + \partial_\theta f_{\alpha\iota}\partial^\theta f^{\alpha\iota} + \\
& \partial_\theta f_{\iota\alpha}\partial^\theta f^{\alpha\iota} + 2\ \omega_{\alpha\theta\iota}\ (\omega^{\alpha\iota\theta} + 2\,\partial^\theta f^{\alpha\iota})\big) - \\
& \tfrac{2}{3} r_1 \big(2\,\partial_\beta\omega_{\alpha\iota\theta} - \partial_\beta\omega_{\alpha\theta\iota} + 4\,\partial_\beta\omega_{\iota\theta\alpha} + \partial_\iota\omega_{\alpha\beta\theta} - \\
& \partial_\theta\omega_{\alpha\beta\iota} - \partial_\theta\omega_{\alpha\iota\beta}\big)\,\partial^\theta\omega^{\alpha\beta\iota} + \\
& r_5\big(\partial_\iota\omega_\theta{}^\kappa{}_\kappa\partial^\theta\omega^{\alpha\iota}{}_\alpha - \partial_\theta\omega_\iota{}^\kappa{}_\kappa\partial^\theta\omega^{\alpha\iota}{}_\alpha - (\partial_\alpha\omega^{\alpha\iota\theta} - 2\,\partial^\theta\omega^{\alpha\iota}{}_\alpha)\\
& (\partial_\kappa\omega_\iota{}^\kappa{}_\theta - \partial_\kappa\omega_\theta{}^\kappa{}_\iota)\big)\Big)[t, x, y, z]\, dz\, dy\, dx\, dt
\end{aligned}
$$

| | $\omega^{\#1}_{2^+\,\alpha\beta}$ | $f^{\#1}_{2^+\,\alpha\beta}$ | $\omega^{\#1}_{2^-\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{ikt_1}{\sqrt{2}}$ | 0 |
| $f^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\frac{ikt_1}{\sqrt{2}}$ | $k^2 t_1$ | 0 |
| $\omega^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$ | 0 | 0 | $k^2 r_1 + \frac{t_1}{2}$ |

| | $\omega^{\#1}_{1^+\,\alpha\beta}$ | $\omega^{\#2}_{1^+\,\alpha\beta}$ | $f^{\#1}_{1^+\,\alpha\beta}$ | $\omega^{\#1}_{1^-\,\alpha}$ | $\omega^{\#2}_{1^-\,\alpha}$ | $f^{\#1}_{1^-\,\alpha}$ | $f^{\#2}_{1^-\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega^{\#1}_{1^+}\dagger^{\alpha\beta}$ | $k^2(2r_1+r_5) - \frac{t_1}{2}$ | $-\frac{t_1}{\sqrt{2}}$ | $-\frac{ikt_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $\omega^{\#2}_{1^+}\dagger^{\alpha\beta}$ | $-\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $f^{\#1}_{1^+}\dagger^{\alpha\beta}$ | $\frac{ikt_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega^{\#1}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | $k^2(r_1+r_5) - \frac{t_1}{2}$ | $\frac{t_1}{\sqrt{2}}$ | 0 | $i k t_1$ |
| $\omega^{\#2}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 |
| $f^{\#1}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f^{\#2}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | $-i k t_1$ | 0 | 0 | 0 |

## Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_1} > 0$ |
| Polarisations: | 5 |
| Square mass: | $-\frac{t_1}{2 r_1} > 0$ |
| Spin: | 2 |
| Parity: | Odd |

(No massless particles)

## Unitarity conditions

$r_1 < 0$ && $t_1 > 0$