# Particle spectrograph

## Wave operator and propagator

### Source constraints

| SO(3) irreps | Fundamental fields | Multiplicities |
|---|---|---|
| $\tau_{0^+}^{\#2} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$ | 1 |
| $\tau_{0^+}^{\#1} - 2\,i\,k\,\sigma_{0^+}^{\#1} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha + 2\,\partial_\chi\partial^\chi\partial_\beta\sigma^{\alpha\beta}{}_\alpha$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#1\,\alpha\beta} == 0$ | $\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} + \partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} + \partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta}$ | 3 |
| $\sigma_{1^+}^{\#1\,\alpha\beta} == \sigma_{1^+}^{\#2\,\alpha\beta}$ | $3\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta} == 3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha}$ | 3 |
| $\tau_{2^+}^{\#1\,\alpha\beta} == 0$ | $4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} == 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi$ | 5 |
| $\sigma_{2^+}^{\#1\,\alpha\beta} == 0$ | $3\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\sigma^{\chi\delta}{}_\chi == 2\,\partial_\delta\partial^\beta\partial^\alpha\sigma^{\chi\delta}{}_\chi + 3\,(\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha})$ | 5 |
| Total constraints/gauge generators: | | 24 |

### Quadratic (free) action

$$\begin{aligned}\mathcal{S} == \int\int\int\int (&\tfrac{1}{6}\,(-4\,t_3\,\omega^{\alpha\prime}{}_\alpha\,\omega_\prime{}^\kappa{}_\kappa + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + 8\,t_3\,\omega_\alpha{}^\kappa{}_\kappa\,\partial_\prime f^{\alpha\prime} - \\ &8\,t_3\,\omega_\prime{}^\kappa{}_\kappa\,\partial^\prime f^\alpha{}_\alpha + 4\,t_3\,\partial_\prime f^\kappa{}_\kappa\,\partial^\prime f^\alpha{}_\alpha - 12\,r_1\,\partial_\beta\omega_\prime{}^\theta{}_\theta\,\partial^\prime\omega^{\alpha\beta}{}_\alpha + \\ &12\,r_1\,\partial_\prime\omega_\beta{}^\theta{}_\theta\,\partial^\prime\omega^{\alpha\beta}{}_\alpha + 12\,r_1\,\partial_\alpha\omega^{\alpha\beta\prime}\,\partial_\theta\omega_\beta{}^\theta{}_\prime - \\ &24\,r_1\,\partial^\prime\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_\beta{}^\theta{}_\prime - 12\,r_1\,\partial_\alpha\omega^{\alpha\beta\prime}\,\partial_\theta\omega_\prime{}^\theta{}_\beta + \\ &24\,r_1\,\partial^\prime\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_\prime{}^\theta{}_\beta + 4\,t_2\,\omega_{\prime\theta\alpha}\,\partial^\theta f^{\alpha\prime} + 2\,t_2\,\partial_\alpha f_{\prime\theta}\,\partial^\theta f^{\alpha\prime} - \\ &t_2\,\partial_\alpha f_{\theta\prime}\,\partial^\theta f^{\alpha\prime} - t_2\,\partial_\prime f_{\alpha\theta}\,\partial^\theta f^{\alpha\prime} + t_2\,\partial_\theta f_{\alpha\prime}\,\partial^\theta f^{\alpha\prime} - \\ &t_2\,\partial_\theta f_{\prime\alpha}\,\partial^\theta f^{\alpha\prime} - 4\,t_2\,\omega_{\alpha\theta\prime}\,(\omega^{\alpha\prime\theta} + \partial^\theta f^{\alpha\prime}) + \\ &2\,t_2\,\omega_{\alpha\prime\theta}\,(\omega^{\alpha\prime\theta} + 2\,\partial^\theta f^{\alpha\prime}) - 8\,r_1\,\partial_\beta\omega_{\alpha\prime\theta}\,\partial^\theta\omega^{\alpha\beta\prime} + \\ &8\,r_2\,\partial_\beta\omega_{\alpha\prime\theta}\,\partial^\theta\omega^{\alpha\beta\prime} + 4\,r_1\,\partial_\beta\omega_{\alpha\theta\prime}\,\partial^\theta\omega^{\alpha\beta\prime} - \\ &4\,r_2\,\partial_\beta\omega_{\alpha\theta\prime}\,\partial^\theta\omega^{\alpha\beta\prime} - 16\,r_1\,\partial_\beta\omega_{\prime\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\prime} + \\ &4\,r_2\,\partial_\beta\omega_{\prime\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\prime} - 4\,r_1\,\partial_\prime\omega_{\alpha\beta\theta}\,\partial^\theta\omega^{\alpha\beta\prime} - \\ &2\,r_2\,\partial_\prime\omega_{\alpha\beta\theta}\,\partial^\theta\omega^{\alpha\beta\prime} + 4\,r_1\,\partial_\theta\omega_{\alpha\beta\prime}\,\partial^\theta\omega^{\alpha\beta\prime} + \\ &2\,r_2\,\partial_\theta\omega_{\alpha\beta\prime}\,\partial^\theta\omega^{\alpha\beta\prime} + 4\,r_1\,\partial_\theta\omega_{\alpha\prime\beta}\,\partial^\theta\omega^{\alpha\beta\prime} - \\ &4\,r_2\,\partial_\theta\omega_{\alpha\prime\beta}\,\partial^\theta\omega^{\alpha\beta\prime} + 4\,t_3\,\partial_\prime f^{\alpha\prime}\,\partial_\kappa f_\alpha{}^\kappa - \\ &8\,t_3\,\partial^\prime f^\alpha{}_\alpha\,\partial_\kappa f_\prime{}^\kappa))[t, x, y, z]\,dz\,dy\,dx\,dt\end{aligned}$$

| | $\sigma_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^+}^{\#2\,\alpha\beta}$ | $\tau_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^-}^{\#1\,\alpha}$ | $\sigma_{1^-}^{\#2\,\alpha}$ | $\tau_{1^-}^{\#1\,\alpha}$ | $\tau_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1\,\alpha\beta}\dagger$ | $\frac{6}{(3+k^2)^2 t_2}$ | $\frac{3\sqrt{2}}{(3+k^2)^2 t_2}$ | $\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2\,\alpha\beta}\dagger$ | $\frac{3\sqrt{2}}{(3+k^2)^2 t_2}$ | $\frac{3}{(3+k^2)^2 t_2}$ | $\frac{3ik}{(3+k^2)^2 t_2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1\,\alpha\beta}\dagger$ | $-\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_2}$ | $-\frac{3ik}{(3+k^2)^2 t_2}$ | $\frac{3k^2}{(3+k^2)^2 t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1\,\alpha}\dagger$ | 0 | 0 | 0 | $-\frac{1}{k^2 r_1}$ | $-\frac{\sqrt{2}}{k^2 r_1 + 2k^4 r_1}$ | 0 | $-\frac{2i}{k r_1 + 2k^3 r_1}$ |
| $\sigma_{1^-}^{\#2\,\alpha}\dagger$ | 0 | 0 | 0 | $-\frac{\sqrt{2}}{k^2 r_1 + 2k^4 r_1}$ | $\frac{3k^2 r_1 - 2t_3}{(k+2k^3)^2 r_1 t_3}$ | 0 | $\frac{i\sqrt{2}\,(3k^2 r_1 - 2t_3)}{k(1+2k^2)^2 r_1 t_3}$ |
| $\tau_{1^-}^{\#1\,\alpha}\dagger$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2\,\alpha}\dagger$ | 0 | 0 | 0 | $\frac{2i}{k r_1 + 2k^3 r_1}$ | $-\frac{i\sqrt{2}\,(3k^2 r_1 - 2t_3)}{k(1+2k^2)^2 r_1 t_3}$ | 0 | $\frac{6k^2 r_1 - 4t_3}{(1+2k^2)^2 r_1 t_3}$ |

| | $\omega_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^+}^{\#2\,\alpha\beta}$ | $f_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^-}^{\#1\,\alpha}$ | $\omega_{1^-}^{\#2\,\alpha}$ | $f_{1^-}^{\#1\,\alpha}$ | $f_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1\,\alpha\beta}\dagger$ | $\frac{2t_2}{3}$ | $\frac{\sqrt{2}\,t_2}{3}$ | $\frac{1}{3}i\sqrt{2}\,k t_2$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2\,\alpha\beta}\dagger$ | $\frac{\sqrt{2}\,t_2}{3}$ | $\frac{t_2}{3}$ | $\frac{ik t_2}{3}$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1\,\alpha\beta}\dagger$ | $-\frac{1}{3}i\sqrt{2}\,k t_2$ | $-\frac{1}{3}ik t_2$ | $\frac{k^2 t_2}{3}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1\,\alpha}\dagger$ | 0 | 0 | 0 | $-k^2 r_1 + \frac{2t_3}{3}$ | $-\frac{\sqrt{2}\,t_3}{3}$ | 0 | $-\frac{2}{3}ik t_3$ |
| $\omega_{1^-}^{\#2\,\alpha}\dagger$ | 0 | 0 | 0 | $-\frac{\sqrt{2}\,t_3}{3}$ | $\frac{t_3}{3}$ | 0 | $\frac{1}{3}i\sqrt{2}\,k t_3$ |
| $f_{1^-}^{\#1\,\alpha}\dagger$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2\,\alpha}\dagger$ | 0 | 0 | 0 | $\frac{2ikt_3}{3}$ | $-\frac{1}{3}i\sqrt{2}\,k t_3$ | 0 | $\frac{2k^2 t_3}{3}$ |

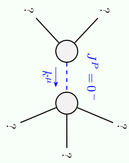
| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^+}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}\dagger$ | $\frac{1}{(1+2k^2)^2 t_3}$ | $\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3}$ | 0 | 0 |
| $\tau_{0^+}^{\#1}\dagger$ | $-\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3}$ | $\frac{2k^2}{(1+2k^2)^2 t_3}$ | 0 | 0 |
| $\tau_{0^+}^{\#2}\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | $\frac{1}{k^2 r_1 + t_2}$ |

| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^+}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1}\dagger$ | $t_3$ | $-i\sqrt{2}\,k t_3$ | 0 | 0 |
| $f_{0^+}^{\#1}\dagger$ | $i\sqrt{2}\,k t_3$ | $2k^2 t_3$ | 0 | 0 |
| $f_{0^+}^{\#2}\dagger$ | 0 | 0 | 0 | 0 |
| $\omega_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | $k^2 r_2 + t_2$ |

| | $\omega_{2^+}^{\#1\,\alpha\beta}$ | $f_{2^+}^{\#1\,\alpha\beta}$ | $\omega_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1\,\alpha\beta}\dagger$ | 0 | 0 | 0 |
| $f_{2^+}^{\#1\,\alpha\beta}\dagger$ | 0 | 0 | 0 |
| $\omega_{2^-}^{\#1\,\alpha\beta\chi}\dagger$ | 0 | 0 | $k^2 r_1$ |

| | $\sigma_{2^+}^{\#1\,\alpha\beta}$ | $\tau_{2^+}^{\#1\,\alpha\beta}$ | $\sigma_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1\,\alpha\beta}\dagger$ | 0 | 0 | 0 |
| $\tau_{2^+}^{\#1\,\alpha\beta}\dagger$ | 0 | 0 | 0 |
| $\sigma_{2^-}^{\#1\,\alpha\beta\chi}\dagger$ | 0 | 0 | $\frac{1}{k^2 r_1}$ |

# Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $-\frac{t_2}{2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

(No massless particles)

# Unitarity conditions

$r_2 < 0\ \&\&\ t_2 > 0$