## Particle spectrograph

## Wave operator and propagator

|                               | $\sigma_{1}^{\#1}{}_{\alpha\beta}$                               | $\sigma_{1}^{\#2}{}_{\alpha\beta}$                     | $\tau_{1}^{\#1}{}_{\alpha\beta}$                | $\sigma_{1^{-}\alpha}^{\#1}$       | $\sigma_{1}^{\#2}{}_{\alpha}$                                | $\tau_{1^{-}\alpha}^{\#1}$ | $\tau_{1}^{\#2}{}_{\alpha}$                      |
|-------------------------------|--|--|---|------------------------------------|--|----------------------------|--|
| $r_1^{#1} + \alpha \beta$     | 0  |  | $-\frac{i\sqrt{2}k}{t_1+k^2t_1}$                | 0                                  | 0  | 0                          | 0  |
| $r_1^{#2} + \alpha \beta$     | $-\frac{\sqrt{2}}{t_1+k^2t_1}$                                   | $\frac{-2 k^2 (2 r_1 + r_5) + t_1}{(1 + k^2)^2 t_1^2}$ | $\frac{-2ik^3(2r_1+r_5)+ikt_1}{(1+k^2)^2t_1^2}$ | 0                                  | 0  | 0                          | 0  |
| $\frac{1}{1+} + \alpha \beta$ | $\tau_{1}^{\#1} + \alpha \beta \frac{i\sqrt{2}k}{t_1 + k^2 t_1}$ | $\frac{i(2k^3(2r_1+r_5)-kt_1)}{(1+k^2)^2t_1^2}$        | $\frac{-2k^4(2r_1+r_5)+k^2t_1}{(1+k^2)^2t_1^2}$ | 0                                  | 0  | 0                          | 0  |
| $\sigma_{1}^{\#1} +^{lpha}$   | 0  | 0  | 0   | 0                                  | $\frac{\sqrt{2}}{t_1 + 2k^2t_1}$                             | 0                          | $\frac{2ik}{t_1 + 2k^2t_1}$                      |
| $\sigma_1^{\#2} +^{\alpha}$   | 0  | 0  | 0   | $\frac{\sqrt{2}}{t_1 + 2 k^2 t_1}$ | $\frac{-2 k^2 (r_1 + r_5) + t_1}{(t_1 + 2 k^2 t_1)^2}$       | 0                          | $-\frac{i\sqrt{2}}{(t_1+2k^2t_1)^2}$             |
| $\tau_1^{\#_1} + ^\alpha$     | 0  | 0  | 0   | 0                                  | 0  | 0                          | 0  |
| $\tau_{1}^{#2} + \alpha$      | 0  | 0  | 0   | $-\frac{2ik}{t_1+2k^2t_1}$         | $\frac{i\sqrt{2}k(2k^2(r_1+r_5)\cdot t_1)}{(t_1+2k^2t_1)^2}$ | 0                          | $\frac{-4k^4(r_1+r_5)+2k^2t_1}{(t_1+2k^2t_1)^2}$ |

|  | $\sigma_{2}^{\#1}{}_{lphaeta}$      | $	au_{2}^{\#1}{}_{lphaeta}$          | $\sigma_{2-\alpha\beta\chi}^{\#1}$ |
|--|-------------------------------------|--------------------------------------|------------------------------------|
| $\sigma_{2}^{\#1}\dagger^{lphaeta}$        | $\frac{2}{(1+2k^2)^2t_1}$           | $-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$ | 0                                  |
| $	au_2^{\#1} \dagger^{lphaeta}$            | $\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$ | $\frac{4k^2}{(1+2k^2)^2t_1}$         | 0                                  |
| $\sigma_2^{\#1} \dagger^{\alpha\beta\chi}$ | 0                                   | 0                                    | $\frac{2}{2 k^2 r_1 + t_1}$        |

|                           | $\omega_0^{\sharp 1}$ | $f_{0}^{\#1}$     | $f_{0}^{#2}$ | $\omega_0^{\#1}$ |
|---------------------------|-----------------------|-------------------|--------------|------------------|
| $\omega_{0^{+}}^{\#1}$ †  | -t <sub>1</sub>       | $i \sqrt{2} kt_1$ | 0            | 0                |
| $f_{0^{+}}^{#1}\dagger$   | $-i \sqrt{2} kt_1$    | $-2 k^2 t_1$      | 0            | 0                |
| $f_{0}^{#2} \dagger$      | 0                     | 0                 | 0            | 0                |
| $\omega_{0}^{\sharp 1}$ † | 0                     | 0                 | 0            | $-t_1$           |

| Quadratic (free) action   |
|---|
| Sp. ==  |
| $\iiint_{6}^{1} (-6t_{1} \omega_{\alpha}^{\alpha \prime} \omega_{\kappa\alpha}^{\kappa} - 6t_{1} \omega_{\kappa\lambda}^{\kappa\lambda} \omega_{\kappa\lambda}^{\prime} + 6f^{\alpha\beta} \tau_{\alpha\beta} + 6\omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - 6r_{5}\partial_{\iota}\omega^{\kappa\lambda}_{\kappa}$   |
| $\partial' \omega_{\lambda}^{\ \alpha} - 4  r_1  \partial^{\beta} \omega^{\theta \alpha}_{\ \kappa}  \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - 4  r_1  \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa}  \partial_{\kappa} \omega^{\alpha\beta\theta} + 4  r_1  \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa}  \partial_{\kappa} \omega^{\theta\alpha\beta} -$ |
| $6 r_5 \partial_\alpha \omega_\lambda^{\ \alpha}_{\ \ \theta} \partial_\kappa \omega^{\theta \kappa \lambda} + 6 r_5 \partial_\theta \omega_\lambda^{\ \alpha}_{\ \ \alpha} \partial_\kappa \omega^{\theta \kappa \lambda}_{\ \ \alpha} - 6 r_5 \partial_\alpha \omega_\lambda^{\ \alpha}_{\ \ \theta} \partial_\kappa \omega^{\kappa \lambda \theta}_{\ \ \lambda} +$        |
| $12r_5\partial_\theta\omega_^\alpha_{\alpha}\partial_\kappa\omega^{\kappa\lambda\theta} - 3t_1\partial^\alpha f_{\beta}\partial^\kappa f_{\alpha}^{\theta} - 3t_1\partial^\alpha f_{\beta}\partial^\kappa f_{\theta}^{\theta} - 3t_1\partial^\alpha f^\lambda_{\lambda}\partial^\kappa f_{\lambda} +$   |
| $6t_1\omega_{\kappa\alpha}^{}\partial^\kappa f^{\prime}{}_{} + 6t_1\omega_{\kappa\lambda}^{}\partial^\kappa f^{\prime}{}_{} + 12t_1\partial^\alpha f_{}\partial^\kappa f^{\prime}{}_{} - 6t_1\partial_\kappa f^{\lambda}{}_{}\partial^\kappa f^{\prime}{}_{} +$   |
| $12t_1\ \omega_{_{/K}\theta}\ \partial^{K}f^{'\theta}\text{-}6t_1\ \omega_{_{'\alpha}}^{\ \alpha}\ \partial^{K}f^{'}_{\ \ _{K}}\text{-}6t_1\ \omega_{_{/\lambda}}^{\ \ \lambda}\ \partial^{K}f^{'}_{\ \ _{K}}+3t_1\partial^{\alpha}f^{\lambda}_{\ \ _{K}}\partial^{K}f_{\lambda\alpha}+$  |
| $3t_1\partial_\kappa f_{\theta}^{\lambda}\partial^\kappa f_{\lambda}^{\theta} + 3t_1\partial_\kappa f^{\lambda}_{\theta}\partial^\kappa f_{\lambda}^{\theta} - 6t_1\partial^\alpha f^{\lambda}_{\alpha}\partial^\kappa f_{\lambda\kappa} + 4r_1\partial_\kappa \omega^{\alpha\beta\theta}\partial^\kappa \omega_{\alpha\beta\theta} -$  |
| $4 r_1 \partial_{\kappa} \omega^{\theta \alpha \beta} \partial^{\kappa} \omega_{\alpha \beta \theta} + 4 r_1 \partial^{\beta} \omega_{\alpha}^{\ \alpha \lambda} \partial_{\lambda} \omega_{\alpha \beta}^{\ \prime} - 16 r_1 \partial^{\beta} \omega_{\alpha}^{\ \lambda \alpha} \partial_{\lambda} \omega_{\alpha \beta}^{\ \prime} +$                                      |
| $6  r_5  \partial_\alpha \omega_\lambda^{\ \alpha}_{\ \ \theta}  \partial^\lambda \omega^{\theta \kappa}_{\ \kappa} - 6  r_5  \partial_\theta \omega_\lambda^{\ \alpha}_{\ \alpha}  \partial^\lambda \omega^{\theta \kappa}_{\ \kappa}) [t,  \varkappa,  y,  z]  dz  dy  dx  dt$  |

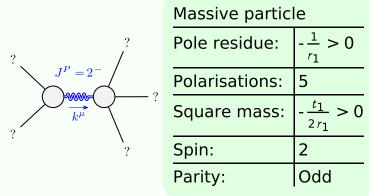
| Source constraints/gauge generators                                 |                |  |  |  |  |
|---|----------------|--|--|--|--|
| SO(3) irreps  | Multiplicities |  |  |  |  |
| $\tau_{0+}^{\#2} == 0$  | 1              |  |  |  |  |
| $\tau_{0+}^{\#1} - 2  i  k  \sigma_{0+}^{\#1} == 0$                 | 1              |  |  |  |  |
| $\tau_1^{\#2\alpha} + 2ik \sigma_1^{\#2\alpha} == 0$                | 3              |  |  |  |  |
| $\tau_1^{\#1\alpha} == 0$   | 3              |  |  |  |  |
| $\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$  | 3              |  |  |  |  |
| $\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$ | 5              |  |  |  |  |
| Total constraints:  | 16             |  |  |  |  |

| _                    | $\sigma_{0}^{\sharp 1}$                | $	au_{0}^{\#1}$                    | $	au_{0}^{\#2}$ | $\sigma_0^{\#1}$ |
|----------------------|--|------------------------------------|-----------------|------------------|
| $\sigma_{0}^{\#1}$ † | $-\frac{1}{(1+2k^2)^2t_1}$             | $\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$ | 0               | 0                |
| $\tau_{0}^{\#1}$ †   | $-\frac{i \sqrt{2} k}{(1+2k^2)^2 t_1}$ | $-\frac{2k^2}{(1+2k^2)^2t_1}$      | 0               | 0                |
| $	au_{0}^{\#2}$ †    | 0                                      | 0                                  | 0               | 0                |
| $\sigma_0^{\#1}$ †   | 0                                      | 0                                  | 0               | $-\frac{1}{t_1}$ |

| $\omega_{2^{-}}^{\#1}\alpha\beta\chi$               | 0                              | 0                            | $k^2 r_1 + \frac{t_1}{2}$              |
|---|--------------------------------|------------------------------|--|
| $\omega_{2}^{\#1}_{+lphaeta}f_{2}^{\#1}_{+lphaeta}$ | $-\frac{ikt_1}{\sqrt{2}}$      | $k^2 t_1$                    | 0                                      |
| $\omega_{2}^{\#1}{}_{\alpha\beta}$                  | $\frac{t_1}{2}$                | $\frac{i  k  t_1}{\sqrt{2}}$ | 0                                      |
| ·   | $\omega_2^{#1} + \alpha \beta$ | $f_2^{\#1} + ^{\alpha\beta}$ | $\omega_{2^{-}}^{\#1} +^{lphaeta\chi}$ |

|   | $\omega_{1}^{\sharp 1}{}_{lphaeta}$ | $\omega_{1}^{\#2}{}_{\alpha\beta}$ | $f_{1^{+}\alpha\beta}^{\#1}$ | $\omega_{1^{-}\ lpha}^{\#1}$      | $\omega_{1-\alpha}^{\#2}$ | $f_{1-\alpha}^{\#1}$ | $f_{1-\alpha}^{\#2}$ |
|---|-------------------------------------|------------------------------------|------------------------------|-----------------------------------|---------------------------|----------------------|----------------------|
| $\omega_1^{\sharp 1} \dagger^{lpha eta}$                                  | $k^2 (2r_1 + r_5) - \frac{t_1}{2}$  | $-\frac{t_1}{\sqrt{2}}$            | $-\frac{ikt_1}{\sqrt{2}}$    | 0                                 | 0                         | 0                    | 0                    |
| $\omega_{\scriptscriptstyle 1}^{\scriptscriptstyle \#2}\dagger^{lphaeta}$ | $-\frac{t_1}{\sqrt{2}}$             | 0                                  | 0                            | 0                                 | 0                         | 0                    | 0                    |
| $f_{1}^{\#1}\dagger^{\alpha\beta}$  | $\frac{ikt_1}{\sqrt{2}}$            | 0                                  | 0                            | 0                                 | 0                         | 0                    | 0                    |
| $\omega_1^{\#1}$ † $^{lpha}$  | 0                                   | 0                                  | 0                            | $k^2 (r_1 + r_5) - \frac{t_1}{2}$ | $\frac{t_1}{\sqrt{2}}$    | 0                    | īkt <sub>1</sub>     |
| $\omega_1^{\#2}\dagger^{\alpha}$  | 0                                   | 0                                  | 0                            | $\frac{t_1}{\sqrt{2}}$            | 0                         | 0                    | 0                    |
| $f_{1}^{#1} \dagger^{\alpha}$   | 0                                   | 0                                  | 0                            | 0                                 | 0                         | 0                    | 0                    |
| $f_{1}^{#2} \dagger^{\alpha}$   | 0                                   | 0                                  | 0                            | - ī k t 1                         | 0                         | 0                    | 0                    |

## Massive and massless spectra



(No massless particles)

## Unitarity conditions

 $r_1 < 0 \&\& t_1 > 0$