# Particle spectrograph

## Wave operator and propagator

| | $\sigma_{1^+}^{\#1}{}^{\alpha\beta}$ | $\sigma_{1^+}^{\#2}{}^{\alpha\beta}$ | $\tau_{1^+}^{\#1}{}^{\alpha\beta}$ | $\sigma_{1^-}^{\#1}{}_{\alpha}$ | $\sigma_{1^-}^{\#2}{}_{\alpha}$ | $\tau_{1^-}^{\#1}{}_{\alpha}$ | $\tau_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1}{}^{\dagger\alpha\beta}$ | 0 | $-\frac{\sqrt{2}}{t_1+k^2 t_1}$ | $-\frac{i\sqrt{2}\,k}{t_1+k^2 t_1}$ | 0 | $\frac{\sqrt{2}}{t_1+2k^2 t_1}$ | 0 | $\frac{2ik}{t_1+2k^2 t_1}$ |
| $\sigma_{1^+}^{\#2}{}^{\dagger\alpha\beta}$ | $-\frac{\sqrt{2}}{t_1+k^2 t_1}$ | $\frac{1}{(1+k^2)^2 t_1}$ | $\frac{ik}{(1+k^2)^2 t_1}$ | 0 | $\frac{1}{(1+2k^2)^2 t_1}$ | 0 | $\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ |
| $\tau_{1^+}^{\#1}{}^{\dagger\alpha\beta}$ | $-\frac{i\sqrt{2}\,k}{t_1+k^2 t_1}$ | $-\frac{ik}{(1+k^2)^2 t_1}$ | $\frac{k^2}{(1+k^2)^2 t_1}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1}{}^{\dagger\alpha}$ | 0 | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2 t_1}$ | 0 | $\frac{2ik}{t_1+2k^2 t_1}$ |
| $\sigma_{1^-}^{\#2}{}^{\dagger\alpha}$ | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2 t_1}$ | $\frac{1}{(1+2k^2)^2 t_1}$ | 0 | $\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ |
| $\tau_{1^-}^{\#1}{}^{\dagger\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2}{}^{\dagger\alpha}$ | 0 | 0 | 0 | $-\frac{2ik}{t_1+2k^2 t_1}$ | $-\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | 0 | $\frac{2k^2}{(1+2k^2)^2 t_1}$ |

Quadratic (free) action

$$S_F = \iiint\!\!\iint \left(\tfrac{1}{6}(-6t_1\,\omega_l^{\alpha\prime}\,\omega_{\kappa\alpha}{}^{\kappa} - 6t_1\,\omega_l^{\kappa\lambda}\,\omega_{\kappa\lambda}{}' + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + 4r_2\,\partial^\beta\omega^{\theta\alpha}{}_{\kappa}\,\partial_\theta\omega_{\alpha\beta}{}^{\kappa} - 2r_2\,\partial_\theta\omega_{\alpha\beta}{}^{\kappa}\,\partial_\kappa\omega^{\alpha\beta\theta} - 4r_2\,\partial_\theta\omega_{\alpha\beta}{}^{\kappa}\,\partial_\kappa\omega^{\theta\alpha\beta} - 3t_1\,\partial^\alpha f_{\theta\kappa}\,\partial^\kappa f_\alpha{}^\theta - 3t_1\,\partial^\alpha f^\lambda{}_\kappa\,\partial^\kappa f_{\alpha\lambda} + 6t_1\,\omega_{\kappa\alpha}{}^{\alpha}\,\partial^\kappa f'_l + 12t_1\,\partial^\alpha f_{\kappa\alpha}\,\partial^\kappa f'_l - 6t_1\,\partial_\kappa f^\lambda{}_l\,\partial^\kappa f'_\lambda + 12t_1\,\omega_{l\kappa\theta}\,\partial^\kappa f'^\theta - 6t_1\,\omega_{l\lambda}{}^{\lambda}\,\partial^\kappa f'_\kappa + 3t_1\,\partial^\alpha f^\lambda{}_\kappa\,\partial^\kappa f_{\lambda\alpha} + 3t_1\,\partial_\kappa f^\lambda{}_\theta\,\partial^\kappa f_\lambda{}^\theta + 6t_1\,\omega_{l\alpha}{}^{\alpha}\,\partial^\kappa f'_\kappa - 6t_1\,\omega_{\kappa\lambda}{}^{\lambda}\,\partial^\kappa f'_l + 4r_2\,\partial^\beta\omega_l^{\alpha\lambda}\,\partial_\lambda\omega_{\alpha\beta}{}' + 2r_2\,\partial_\kappa\omega^{\alpha\beta\theta}\,\partial^\kappa\omega_{\alpha\beta\theta} + 4r_2\,\partial_\kappa\omega^{\theta\alpha\beta}\,\partial^\kappa\omega_{\alpha\beta\theta}{}')\right)[t,x,y,z]\,dt\,dx\,dy\,dz$$

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}{}^{\dagger}$ | $-\frac{1}{(1+2k^2)^2 t_1}$ | $\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | 0 | 0 |
| $\tau_{0^+}^{\#1}{}^{\dagger}$ | $-\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | $-\frac{2k^2}{(1+2k^2)^2 t_1}$ | 0 | 0 |
| $\tau_{0^+}^{\#2}{}^{\dagger}$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1}{}^{\dagger}$ | 0 | 0 | 0 | $\frac{1}{k^2 r_2 - t_1}$ |

| | $\sigma_{2^+}^{\#1}{}_{\alpha\beta}$ | $\tau_{2^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1}{}^{\dagger\alpha\beta}$ | $\frac{2}{(1+2k^2)^2 t_1}$ | $-\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | 0 |
| $\tau_{2^+}^{\#1}{}^{\dagger\alpha\beta}$ | $\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | $\frac{4k^2}{(1+2k^2)^2 t_1}$ | 0 |
| $\sigma_{2^-}^{\#1}{}^{\dagger\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{t_1}$ |

| | $\omega_{2^+}^{\#1}{}_{\alpha\beta}$ | $f_{2^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1}{}^{\dagger\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{ikt_1}{\sqrt{2}}$ | 0 |
| $f_{2^+}^{\#1}{}^{\dagger\alpha\beta}$ | $\frac{ikt_1}{\sqrt{2}}$ | $k^2 t_1$ | 0 |
| $\omega_{2^-}^{\#1}{}^{\dagger\alpha\beta\chi}$ | 0 | 0 | $\frac{t_1}{2}$ |

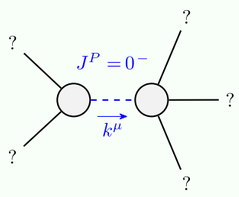
| | $\omega_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^+}^{\#2}{}_{\alpha\beta}$ | $f_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^-}^{\#1}{}_{\alpha}$ | $\omega_{1^-}^{\#2}{}_{\alpha}$ | $f_{1^-}^{\#1}{}_{\alpha}$ | $f_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1}{}^{\dagger\alpha\beta}$ | $-\frac{t_1}{2}$ | $-\frac{t_1}{\sqrt{2}}$ | 0 | $-\frac{t_1}{2}$ | $\frac{t_1}{\sqrt{2}}$ | 0 | $i\,k\,t_1$ |
| $\omega_{1^+}^{\#2}{}^{\dagger\alpha\beta}$ | $-\frac{t_1}{\sqrt{2}}$ | 0 | 0 | $\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 |
| $f_{1^+}^{\#1}{}^{\dagger\alpha\beta}$ | 0 | 0 | 0 | 0 | 0 | 0 | $-i\,k\,t_1$ |
| $\omega_{1^-}^{\#1}{}^{\dagger\alpha}$ | $-\frac{t_1}{2}$ | $\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#2}{}^{\dagger\alpha}$ | $\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#1}{}^{\dagger\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2}{}^{\dagger\alpha}$ | $i\,k\,t_1$ | 0 | 0 | 0 | 0 | 0 | 0 |

| | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|
| $\omega_{0^+}^{\#1}{}^{\dagger}$ | 0 | 0 | 0 |
| $f_{0^+}^{\#1}{}^{\dagger}$ | $i\sqrt{2}\,k\,t_1$ | $-2k^2 t_1$ | 0 |
| $f_{0^+}^{\#2}{}^{\dagger}$ | $-t_1$ | $-i\sqrt{2}\,k\,t_1$ | 0 |
| $\omega_{0^+}^{\#1}{}^{\dagger}$ | 0 | 0 | $k^2 r_2 - t_1$ |

Source constraints/gauge generators

| SO(3) irreps | Multiplicities |
|---|---|
| $\tau_{0^+}^{\#2} == 0$ | 1 |
| $\tau_{0^+}^{\#1} - 2\,i\,i\,k\,\sigma_{0^+}^{\#1} == 0$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} + 2\,i\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + i\,i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$ | 3 |
| $\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$ | 5 |
| Total constraints: | 16 |

## Massive and massless spectra




Massive particle

| | |
|---|---|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $\frac{t_1}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

(No massless particles)

## Unitarity conditions

$r_2 < 0 \;\&\&\; t_1 < 0$