

Particle spectrograph

Wave operator and propagator

Source constraints			Fundamental fields	Multiplicities
SO(3) irreps				
$\sigma_0^{\#1} == 0$			$\epsilon \Pi_{a\beta\gamma\delta} \partial^{\delta} \sigma^{a\beta\chi} == 0$	1
$\tau_0^{\#2} == 0$			$\partial_{\beta} \partial_a \tau^{a\beta} == 0$	1
$\tau_{0+}^{\#1} - 2 \, i \, k \, \sigma_{0+}^{\#1} == 0$			$\partial_{\beta} \partial_a \tau^{a\beta} == \partial_{\beta} \partial^{\beta} \tau^a{}_a + 2 \, \partial_{\chi} \partial^{\chi} \partial_{\sigma} \sigma^a{}_{\alpha}{}^{\beta}$	1
$\tau_1^{\#2\alpha} + 2 \, i \, k \, \sigma_1^{\#2\alpha} == 0$			$\partial_{\chi} \partial_{\beta} \partial^a \tau^{\beta\chi} == \partial_{\chi} \partial^{\chi} \partial_{\beta} \tau^{a\beta} + 2 \, \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial_{\beta} \sigma^{a\beta\chi}$	3
$\tau_1^{\#1\alpha} == 0$			$\partial_{\chi} \partial_{\beta} \partial^a \tau^{\beta\chi} == \partial_{\chi} \partial^{\chi} \partial_{\beta} \tau^{\beta\alpha}$	3
$\tau_1^{\#1\alpha\beta} - 2 \, i \, k \, \sigma_1^{\#1\alpha\beta} == 0$			$\partial_{\chi} \partial^a \tau^{\beta\chi} + \partial_{\chi} \partial^{\beta} \tau^{\chi\alpha} + \partial_{\chi} \partial^{\chi} \tau^{a\beta} +$ $2 \, \partial_{\delta} \partial_{\chi} \partial^a \sigma^{\beta\chi\delta} + 2 \, \partial_{\delta} \partial^{\delta} \partial_{\chi} \sigma^{a\chi\beta} ==$ $\partial_{\chi} \partial^a \tau^{\chi\beta} + \partial_{\chi} \partial^{\beta} \tau^{\alpha\chi} + \partial_{\chi} \partial^{\chi} \tau^{\beta\alpha} +$ $2 \, \partial_{\delta} \partial_{\chi} \partial_{\chi} \partial^{\beta} \sigma^{a\chi\delta} + 2 \, \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\beta\chi\alpha}$	3
$\tau_2^{\#1\alpha\beta} - 2 \, i \, k \, \sigma_2^{\#1\alpha\beta} == 0$			$\partial_{\chi} \sigma^{a\beta\chi} + \partial_{\chi} \sigma^{\beta\chi\alpha} == \partial_{\chi} \sigma^{a\chi\beta}$	3
$\tau_2^{\#1\alpha\beta} - 2 \, i \, k \, \sigma_2^{\#1\alpha\beta} == 0$			$-i \, (4 \, \partial_{\delta} \partial_{\chi} \partial^{\beta} \partial^a \tau^{\chi\delta} + 2 \, \partial_{\delta} \partial^{\delta} \partial_{\beta} \partial^a \tau^{\chi}{}_{{\chi}} -$ $3 \, \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^a \tau^{\beta\chi} - 3 \, \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^a \tau^{\chi\beta} -$ $3 \, \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\beta} \tau^{\alpha\chi} - 3 \, \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\beta} \tau^{\chi\alpha} +$ $3 \, \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\chi} \tau^{a\beta} + 3 \, \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\chi} \tau^{\beta\alpha} +$ $4 \, i \, k^{\chi} \, \partial_{\varepsilon} \partial_{\chi} \partial^{\beta} \partial^a \sigma^{\delta\varepsilon}{}_{{\delta}} -$ $6 \, i \, k^{\chi} \, \partial_{\varepsilon} \partial_{\delta} \partial_{\chi} \partial^a \sigma^{\beta\delta\varepsilon}{}_{{\varepsilon}} -$ $6 \, i \, k^{\chi} \, \partial_{\varepsilon} \partial_{\delta} \partial_{\chi} \partial^{\beta} \sigma^{\alpha\delta\varepsilon}{}_{{\varepsilon}} +$ $2 \, \eta^{\alpha\beta} \, \partial_{\varepsilon} \partial^{\varepsilon} \partial_{\delta} \partial_{\chi} \tau^{\chi\delta} +$ $6 \, i \, k^{\chi} \, \partial_{\varepsilon} \partial^{\varepsilon} \partial_{\delta} \partial_{\chi} \sigma^{a\delta\beta}{}_{{\beta}} +$ $6 \, i \, k^{\chi} \, \partial_{\varepsilon} \partial^{\varepsilon} \partial_{\delta} \partial_{\chi} \sigma^{\beta\delta\alpha}{}_{{\alpha}} -$ $2 \, \eta^{\alpha\beta} \, \partial_{\varepsilon} \partial^{\varepsilon} \partial_{\delta} \partial^{\chi}{}_{{\chi}} \tau^{\chi}{}_{{\chi}} -$ $4 \, i \, \eta^{\alpha\beta} \, k^{\chi} \, \partial_{\phi} \partial^{\phi} \partial_{\varepsilon} \partial_{\chi} \sigma^{\delta\varepsilon}{}_{{\delta}}) == 0$	5
Total constraints/gauge generators:				20

Quadratic (free) action

$$S = \iiint \Big(\frac{1}{3} (3 t_1 \mathcal{A}^{\alpha}{}_a \mathcal{A}^{\theta}{}_{,\theta} + 3 f^{a\beta}{}_{\tau} \tau_{a\beta} + 3 \mathcal{A}^{a\beta\chi} \sigma_{a\beta\chi} -$$

$$6 t_1 \mathcal{A}^{\theta}{}_{\alpha} \partial_{,\theta} f^{\alpha}{}_{,\theta} + 6 t_1 \mathcal{A}^{\theta}{}_{,\theta} \partial_{,\theta} f^{\alpha}{}_{{\alpha}} - 3 t_1 \partial_{,\theta} f^{\theta}{}_{\theta} \partial_{,\theta} f^{\alpha}{}_{{\alpha}} -$$

$$6 r_1 \partial_{\beta} \mathcal{A}^{\theta}{}_{,\theta} \partial' \mathcal{A}^{\alpha\beta}{}_{{\alpha}} + 6 r_1 \partial_{,\theta} \mathcal{A}^{\theta}{}_{\beta} \partial' \mathcal{A}^{\alpha\beta}{}_{{\alpha}} -$$

$$3 t_1 \partial_{,\theta} f^{\alpha}{}_{,\theta} \partial_{\theta f}{}^{\theta}{}_{{\alpha}} + 6 t_1 \partial_{,\theta} f^{\alpha}{}_{{\alpha}} \partial_{\theta f}{}^{\theta}{}_{{\theta}} + 6 r_1 \partial_{\alpha} \mathcal{A}^{\alpha\beta}{}_{,\beta} \partial_{\theta} \mathcal{A}^{\theta}{}_{,\beta} -$$

$$12 r_1 \partial' \mathcal{A}^{\alpha\beta}{}_{{\alpha}} \partial_{\theta} \mathcal{A}^{\theta}{}_{,\beta} - 6 r_1 \partial_{\alpha} \mathcal{A}^{\alpha\beta}{}_{,\beta} \partial_{\theta} \mathcal{A}^{\theta}{}_{,\beta} +$$

$$12 r_1 \partial' \mathcal{A}^{\alpha\beta}{}_{{\alpha}} \partial_{\theta} \mathcal{A}^{\theta}{}_{,\beta} + 2 t_1 \mathcal{A}^{\theta}{}_{,\theta} \partial^{\theta} f^{\alpha}{}_{,\theta} - 2 t_1 \partial_{\alpha f}{}^{\theta}{}_{,\theta} \partial^{\theta} f^{\alpha}{}_{,\theta} -$$

$$2 t_1 \partial_{\alpha f}{}^{\theta}{}_{,\theta} \partial^{\theta} f^{\alpha}{}_{,\theta} + t_1 \partial_{,\theta} f^{\alpha}{}_{\alpha\theta} \partial^{\theta} f^{\alpha}{}_{,\theta} + 2 t_1 \partial_{\theta f}{}^{\alpha}{}_{,\theta} \partial^{\theta} f^{\alpha}{}_{,\theta} +$$

$$t_1 \partial_{\theta f}{}^{\alpha}{}_{,\theta} \partial^{\theta} f^{\alpha}{}_{,\theta} + t_1 \mathcal{A}^{\alpha\theta}{}_{,\theta} (\mathcal{A}^{\alpha\theta}{}_{,\theta} + 2 \partial^{\theta} f^{\alpha}{}_{,\theta}) - 4 r_1 \partial_{\beta} \mathcal{A}^{\alpha\theta}{}_{,\theta} \partial^{\theta} \mathcal{A}^{\alpha\beta}{}_{,\beta} +$$

$$t_1 \mathcal{A}^{\alpha\theta}{}_{,\theta} (\mathcal{A}^{\alpha\theta}{}_{,\theta} + 4 \partial^{\theta} f^{\alpha}{}_{,\theta}) - 8 r_1 \partial_{\beta} \mathcal{A}^{\alpha\theta}{}_{,\theta} \partial^{\theta} \mathcal{A}^{\alpha\beta}{}_{,\beta} -$$

$$2 r_1 \partial_{\beta} \mathcal{A}^{\alpha\theta}{}_{,\theta} \partial^{\theta} \mathcal{A}^{\alpha\beta}{}_{,\beta} - 8 r_1 \partial_{\beta} \mathcal{A}^{\alpha\theta}{}_{,\theta} \partial^{\theta} \mathcal{A}^{\alpha\beta}{}_{,\beta} -$$

$$2 r_1 \partial_{,\theta} \mathcal{A}^{\alpha\beta}{}_{\alpha\beta\theta} \partial^{\theta} \mathcal{A}^{\alpha\beta}{}_{,\theta} + 2 r_1 \partial_{\theta} \mathcal{A}^{\alpha\beta}{}_{,\theta} \partial^{\theta} \mathcal{A}^{\alpha\beta}{}_{,\theta} +$$

$$2 r_1 \partial_{\theta} \mathcal{A}^{\alpha\beta}{}_{\alpha\beta\theta} \partial^{\theta} \mathcal{A}^{\alpha\beta}{}_{,\theta}) [t, x, y, z] d t d y d x d t$$

$\sigma_1^{\#1} \dagger^{\alpha\beta}$	$\sigma_1^{\#2} \dagger^{\alpha\beta}$	$\tau_1^{\#1} \dagger^{\alpha\beta}$	$\sigma_1^{\#1} \dagger^{\alpha}$	$\sigma_1^{\#2} \dagger^{\alpha}$	$\tau_1^{\#1} \dagger^{\alpha}$	$\tau_1^{\#2} \dagger^{\alpha}$
$\frac{6}{(3+2k^2)^2} t_1$	$-\frac{6\sqrt{2}}{(3+2k^2)^2} t_1$	$-\frac{6i\sqrt{2}k}{(3+2k^2)^2} t_1$	0	0	0	0
$-\frac{6\sqrt{2}}{(3+2k^2)^2} t_1$	$\frac{12}{(3+2k^2)^2} t_1$	$\frac{12ik}{(3+2k^2)^2} t_1$	0	0	0	0
$\frac{6i\sqrt{2}k}{(3+2k^2)^2} t_1$	$-\frac{12ik}{(3+2k^2)^2} t_1$	$\frac{12k^2}{(3+2k^2)^2} t_1$	0	0	0	0
0	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2} t_1$	0	$\frac{2ik}{t_1+2k^2} t_1$
0	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2} t_1$	0	$\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
0	0	0	0	0	0	0
0	0	0	$-\frac{2ik}{t_1+2k^2} t_1$	$-\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$	0	$\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$

$\mathcal{A}_1^{\#1} \dagger^{\alpha\beta}$	$\mathcal{A}_1^{\#2} \dagger^{\alpha\beta}$	$f_1^{\#1} \dagger^{\alpha\beta}$	$\mathcal{A}_1^{\#1} \dagger^{\alpha}$	$\mathcal{A}_1^{\#2} \dagger^{\alpha}$	$f_1^{\#1} \dagger^{\alpha}$	$f_1^{\#2} \dagger^{\alpha}$
$\frac{t_1}{6}$	$-\frac{t_1}{3\sqrt{2}}$	$-\frac{ikt_1}{3\sqrt{2}}$	0	0	0	0
$-\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	$\frac{ikt_1}{3}$	0	0	0	0
$\frac{ikt_1}{3\sqrt{2}}$	$-\frac{1}{3} \frac{ikt_1}{\sqrt{2}}$	$\frac{k^2t_1}{3}$	0	0	0	0
0	0	0	$-k^2r_1 - \frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$i k t_1$
0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
0	0	0	0	0	0	0
0	0	0	$-i k t_1$	0	0	0

$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$\frac{2}{(1+2k^2)^2} t_1$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2} t_1$	0
$\frac{2i\sqrt{2}k}{(1+2k^2)^2} t_1$	$\frac{4k^2}{(1+2k^2)^2} t_1$	0
0	0	$\frac{2}{2k^2r_1+t_1}$

$\mathcal{A}_{2+}^{\#1} \dagger^{\alpha\beta}$	$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\mathcal{A}_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$\frac{ikt_1}{\sqrt{2}}$	k^2t_1	0
0	0	$k^2r_1 + \frac{t_1}{2}$

$\mathcal{A}_0^{\#1} \dagger$	$f_0^{\#1} \dagger$	$f_0^{\#2} \dagger$	$\mathcal{A}_0^{\#1} \dagger$
$-t_1$	$i\sqrt{2}kt_1$	0	0
$-i\sqrt{2}kt_1$	$-2k^2t_1$	0	0
0	0	0	0
0	0	0	0

Massive and massless spectra

Diagram illustrating a massive particle (wavy line) interacting with four massless particles (straight lines). The diagram is labeled with $J^P = 2^-$ and k^μ .

Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0 \ \&\& \ t_1 > 0$