Particle spectrograph

Wave operator and propagator

Quadratic (free) action

$$\mathcal{S} == \iiiint \left(\tfrac{1}{6}\left(6 t_1 \, \omega^{\alpha l}{}_{,\theta} \, \omega_{\alpha l}{}^{,\theta} + 6 f^{\alpha\beta}{}_{,\theta}\, \tau_{\alpha\beta} + 6\, \omega^{\alpha\beta\chi}\, \sigma_{\alpha\beta\chi} - 12 t_1 \, \omega_{\alpha}{}^{\theta}{}_{,\theta}\, \partial_l f^{\alpha l} + 12 t_1 \right.\right.$$
$$\omega_{l,\theta}{}^{\theta}\, \partial_l f^{\alpha}{}_{\alpha} - 6 t_1 \,\partial_l f^{\theta}{}_{\alpha}\, \partial^l f^{\alpha}{}_{\theta} - 12 r_1\, \partial_l \omega_{l,\theta}{}^{\theta}\, \partial^l \omega^{\alpha\beta}{}_{\alpha} +12 r_1 \,\partial_l \omega_{\beta,\theta}{}^{\theta}\, \partial^l \omega^{\alpha\beta}{}_{\alpha} -$$
$$6 t_1\, \partial_l f^{\alpha l}\, \partial_\theta f^{\theta}{}_{\alpha} + 12 t_1\, \partial_l f^{\alpha}{}_{\alpha}\, \partial_\theta f^{\theta}{}_{l} + 12 r_1\, \partial_\alpha \omega^{\alpha\beta l}\, \partial_\theta \omega_{l,\beta}{}^{\theta} - 24 r_1\, \partial^l \omega^{\alpha\beta}{}_{\alpha}\, \partial_\theta \omega_{\beta,l}{}^{\theta} -$$
$$12 r_1 \,\partial_\alpha \omega^{\alpha\beta l}{}_{,\theta}\, \partial_\theta \omega_{l,\beta}{}^{\theta} + 24 r_1\, \partial_\theta \omega^{\alpha\beta}{}_{\beta} + 4 t_1 \,\omega_{l\theta\alpha}\, \partial^l f^{\alpha l} + 4 t_1 \,\omega_{l\theta\alpha}\, \partial^\theta f^{\alpha l} +$$
$$4 t_1 \,\partial_\alpha f_{l\theta}\, \partial^\theta f^{\alpha l} + 2 t_2 \,\partial_\alpha f_{l\theta}\, \partial^\theta f^{\alpha l} - 4 t_1\, \partial_\alpha f_{\theta l}\, \partial^\theta f^{\alpha l} + 4 t_1 \,\partial_\theta f_{\alpha l}\, \partial^\theta f^{\alpha l} - t_2\, \partial_\alpha f_{\theta l}\, \partial^\theta f^{\alpha l} +$$
$$2 t_1 \,\partial_l f_{\alpha\theta}\, \partial^\theta f^{\alpha l} - t_2\, \partial_\theta f_{l\alpha}\, \partial^\theta f^{\alpha l} + t_2\, \partial_\theta f_{\alpha l}\, \partial^\theta f^{\alpha l} +$$
$$2 t_1\, \partial_l f_{l\alpha}\, \partial^\theta f^{\alpha l} + 2\,(t_1 + t_2)\, \omega_{\alpha l\theta}\,(\omega^{\alpha l\theta} + 2\,\partial^\theta f^{\alpha l}) +$$
$$2\,\omega_{\alpha\theta l}\,((t_1 - 2 t_2)\,\omega^{\alpha l\theta} + 2\,(2 t_1 - t_2)\, \partial^\theta f^{\alpha l}) - 8 r_1 \,\partial_\beta \omega_{\alpha l\theta}\, \partial^\theta \omega^{\alpha\beta l} +$$
$$8 r_2\, \partial_\beta \omega_{\alpha l\theta}\, \partial^\theta \omega^{\alpha\beta l} + 4 r_1\, \partial_\beta \omega_{l\theta\alpha}\, \partial^\theta \omega^{\alpha\beta l} - 4 r_2\, \partial_\beta \omega_{l\theta\alpha}\, \partial^\theta \omega^{\alpha\beta l} -$$
$$16 r_1\, \partial_\beta \omega_{l\theta\alpha}\, \partial^\theta \omega^{\alpha\beta l} - 4 r_1\, \partial_l \omega_{\alpha\beta\theta}\, \partial^\theta \omega^{\alpha\beta l} +$$
$$2 r_2\, \partial_l \omega_{\alpha\beta\theta}\, \partial^\theta \omega^{\alpha\beta l} + 4 r_1\, \partial_\theta \omega_{\alpha\beta l}\, \partial^\theta \omega^{\alpha\beta l} + 2 r_2\, \partial_\theta \omega_{\alpha\beta l}\, \partial^\theta \omega^{\alpha\beta l} +$$
$$4 r_1\, \partial_\theta \omega_{\alpha l\beta}\, \partial^\theta \omega^{\alpha\beta l} - 4 r_2\, \partial_\theta \omega_{\alpha l\beta}\, \partial^\theta \omega^{\alpha\beta l}\right)\right)[t, x, y, z]\, dz\, dy\, dx\, dt$$

Matrix 1 (blue/pink):

	$\sigma_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\sigma_{1^+}^{\#2}\dagger^{\alpha\beta}$	$\tau_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\sigma_{1^-}^{\#1}\dagger^{\alpha}$	$\sigma_{1^-}^{\#2}\dagger^{\alpha}$	$\tau_{1^-}^{\#1}\dagger^{\alpha}$	$\tau_{1^-}^{\#2}\dagger^{\alpha}$
$\tau_{1^-}^{\#2}{}_{\alpha}$	0	0	0	$\frac{2 i k}{t_1 + 2 k^2 t_1}$	$\frac{i\sqrt{2}\, k\,(2 k^2 r_1 + t_1)}{(t_1 + 2 k^2 t_1)^2}$	0	$\frac{2 k^2\,(2 k^2 r_1 + t_1)}{(t_1 + 2 k^2 t_1)^2}$
$\tau_{1^-}^{\#1}{}_{\alpha}$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#2}{}_{\alpha}$	0	0	0	$\frac{\sqrt{2}}{t_1 + 2 k^2 t_1}$	$\frac{2 k^2 r_1 + t_1}{(t_1 + 2 k^2 t_1)^2}$	0	$-\frac{i\sqrt{2}\, k\,(2 k^2 r_1 + t_1)}{(t_1 + 2 k^2 t_1)^2}$
$\sigma_{1^-}^{\#1}{}_{\alpha}$	0	0	0	0	$\frac{\sqrt{2}}{t_1 + 2 k^2 t_1}$	0	$-\frac{2 i k}{t_1 + 2 k^2 t_1}$
$\tau_{1^+}^{\#1}{}_{\alpha\beta}$	$\frac{i\sqrt{2}\, k\,(t_1 - 2 t_2)}{t_1 t_2}$	$\frac{i k\,(t_1 + 4 t_2)}{3\,(1 + k^2)\, t_1 t_2}$	$\frac{k^2\,(t_1 + 4 t_2)}{3\,(1 + k^2)\, t_1 t_2}$	0	0	0	0
$\sigma_{1^+}^{\#2}{}_{\alpha\beta}$	$\frac{\sqrt{2}\,(t_1 - 2 t_2)}{3\,(1 + k^2)\, t_1 t_2}$	$\frac{t_1 + 4 t_2}{3\,(1 + k^2)^2\, t_1 t_2}$	$\frac{i k\,(t_1 + 4 t_2)}{3\,(1 + k^2)^2\, t_1 t_2}$	0	0	0	0
$\sigma_{1^+}^{\#1}{}_{\alpha\beta}$	$\frac{2\,(t_1 + t_2)}{3 t_1 t_2}$	$\frac{\sqrt{2}\,(t_1 - 2 t_2)}{3\,(1 + k^2)\, t_1 t_2}$	$-\frac{i\sqrt{2}\, k\,(t_1 - 2 t_2)}{3\,(1 + k^2)\, t_1 t_2}$	0	0	0	0

Matrix 2:

	$\omega_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\omega_{1^+}^{\#2}\dagger^{\alpha\beta}$	$f_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\omega_{1^-}^{\#1}\dagger^{\alpha}$	$\omega_{1^-}^{\#2}\dagger^{\alpha}$	$f_{1^-}^{\#1}\dagger^{\alpha}$	$f_{1^-}^{\#2}\dagger^{\alpha}$
$\tau_{1^-}^{\#2}{}_{\alpha}$	0	0	0	$i k t_1$	0	0	0
$\omega_{1^-}^{\#2}{}_{\alpha}$	0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
$\omega_{1^-}^{\#1}{}_{\alpha}$	0	0	0	$-k^2 r_1 - \frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$-i k t_1$
$f_{1^+}^{\#1}{}_{\alpha\beta}$	$-\frac{i k\,(t_1 - 2 t_2)}{3\sqrt{2}}$	$\frac{1}{3} i k\,(t_1 + t_2)$	$\frac{1}{3} k^2\,(t_1 + t_2)$	0	0	0	0
$\omega_{1^+}^{\#2}{}_{\alpha\beta}$	$-\frac{t_1 - 2 t_2}{3\sqrt{2}}$	$\frac{t_1 + t_2}{3}$	$-\frac{1}{3} i k\,(t_1 + t_2)$	0	0	0	0
$\omega_{1^+}^{\#1}{}_{\alpha\beta}$	$\frac{1}{6}\,(t_1 + 4 t_2)$	$-\frac{t_1 - 2 t_2}{3\sqrt{2}}$	$\frac{i k\,(t_1 - 2 t_2)}{3\sqrt{2}}$	0	0	0	0

Matrix 3:

	$\omega_{2^+}^{\#1}{}_{\alpha\beta}$	$f_{2^+}^{\#1}{}_{\alpha\beta}$	$\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\omega_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{i k t_1}{\sqrt{2}}$	0
$f_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{i k t_1}{\sqrt{2}}$	$k^2 t_1$	0
$\omega_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$	0	0	$k^2 r_1 + \frac{t_1}{2}$

Matrix 4:

	$\sigma_{2^+}^{\#1}{}_{\alpha\beta}$	$\tau_{2^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\sigma_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{2}{(1 + 2 k^2)^2\, t_1}$	$-\frac{2 i\sqrt{2}\, k}{(1 + 2 k^2)^2\, t_1}$	0
$\tau_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{2 i\sqrt{2}\, k}{(1 + 2 k^2)^2\, t_1}$	$\frac{4 k^2}{(1 + 2 k^2)^2\, t_1}$	0
$\sigma_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{2 k^2 r_1 + t_1}$

Matrix 5 (0 sector):

	$\omega_{0^+}^{\#1}\dagger$	$f_{0^+}^{\#1}\dagger$	$f_{0^+}^{\#2}\dagger$	$\omega_{0^-}^{\#1}\dagger$
$\omega_{0^-}^{\#1}$	0	0	0	$k^2 r_2 + t_2$
$f_{0^+}^{\#2}$	0	0	0	0
$f_{0^+}^{\#1}$	$i k\sqrt{2}\, t_1$	$-2 k^2 t_1$	0	0
$\omega_{0^+}^{\#1}$	$-t_1$	$-i\sqrt{2}\, k t_1$	0	0

Matrix 6 (0 sector, pink):

	$\sigma_{0^+}^{\#1}\dagger$	$\tau_{0^+}^{\#1}\dagger$	$\tau_{0^+}^{\#2}\dagger$	$\sigma_{0^-}^{\#1}\dagger$
$\sigma_{0^-}^{\#1}$	0	0	0	$\frac{1}{k^2 r_2 + t_2}$
$\tau_{0^+}^{\#2}$	0	0	0	0
$\tau_{0^+}^{\#1}$	$\frac{i\sqrt{2}\, k}{(1 + 2 k^2)^2\, t_1}$	$\frac{2 k^2}{(1 + 2 k^2)^2\, t_1}$	0	0
$\sigma_{0^+}^{\#1}$	$\frac{1}{(1 + 2 k^2)^2\, t_1}$	$-\frac{i\sqrt{2}\, k}{(1 + 2 k^2)^2\, t_1}$	0	0

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau_{0^+}^{\#2} == 0$	1
$\tau_{0^+}^{\#1} - 2 i k\, \sigma_{0^+}^{\#1} == 0$	1
$\tau_{1^-}^{\#2\,\alpha} + 2 i k\, \sigma_{1^-}^{\#2\,\alpha} == 0$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	3
$\tau_{1^+}^{\#1\,\alpha\beta} + i k\, \sigma_{1^+}^{\#2\,\alpha\beta} == 0$	3
$\tau_{2^+}^{\#1\,\alpha\beta} - 2 i k\, \sigma_{2^+}^{\#1\,\alpha\beta} == 0$	5
Total constraints:	16

Massive and massless spectra

Massive particle ($J^P = 2^-$)

Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2 r_1} > 0$
Spin:	2
Parity:	Odd

Massive particle ($J^P = 0^-$)

Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)
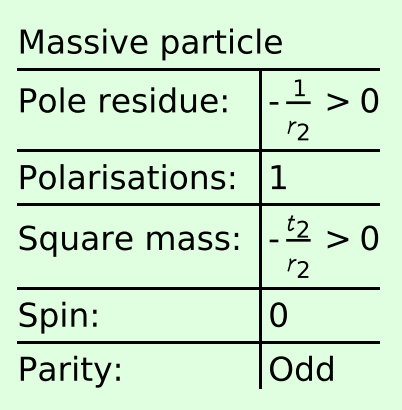
Unitarity conditions

$r_1 < 0\ \&\&\ r_2 < 0\ \&\&\ t_1 > 0\ \&\&\ t_2 > 0$