In[1]:=
```
Get@FileNameJoin@{NotebookDirectory[], "Calibration.m"};
```

PSALTer Calibration

During the calibration run, we need to write some commentary, which will appear in this green text, or as numbered equations/expressions with a green background. The output of the PSALTer package (specifically the function called ParticleSpectrum) is not in green, thus wherever we are using PSALTer the output should be quite distinctive.

The first step is to load the PSALTer package.

```
------------------------------------------------------------
Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}
CopyRight (C) 2003-2020, Jose M. Martin-Garcia, under the General Public License.
Connecting to external linux executable...
Connection established.
------------------------------------------------------------
Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}
CopyRight (C) 2002-2021, Jose M. Martin-Garcia, under the General Public License.
------------------------------------------------------------
Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}
CopyRight (C) 2011-2021, Thomas Bäckdahl, under the General Public License.
------------------------------------------------------------
Package xAct`xPert`  version 1.0.6, {2018, 2, 28}
CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia
  and Guillermo A. Mena Marugan, under the General Public License.
** Variable $CovDFormat changed from Prefix to Postfix
** Option AllowUpperDerivatives of ContractMetric changed from False to True
** Option MetricOn of MakeRule changed from None to All
** Option ContractMetrics of MakeRule changed from False to True
------------------------------------------------------------
Package xAct`Invar`  version 2.0.5, {2013, 7, 1}
CopyRight (C) 2006-2020, J. M. Martin-Garcia,
  D. Yllanes and R. Portugal, under the General Public License.
** DefConstantSymbol: Defining constant symbol sigma.
** DefConstantSymbol: Defining constant symbol dim.
** Option CurvatureRelations of DefCovD changed from True to False
```

```
** Variable $CommuteCovDsOnScalars changed from True to False
------------------------------------------------------------
Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}
CopyRight (C) 2005-2021, David Yllanes and
  Jose M. Martin-Garcia, under the General Public License.
------------------------------------------------------------
Package xAct`xTras`  version 1.4.2, {2014, 10, 30}
CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.
** Variable $CovDFormat changed from Postfix to Prefix
** Option CurvatureRelations of DefCovD changed from False to True
------------------------------------------------------------
Package xAct`PSALTer` version 1.0.0-developer, {2023, 4, 17}
CopyRight © 2022, Will E. V. Barker, Zhiyuan Wei, Stephanie Buttigieg,
  Claire Rigouzzo and Cillian Rew, under the General Public License.
------------------------------------------------------------
These packages come with ABSOLUTELY NO WARRANTY; for details type
  Disclaimer[]. This is free software, and you are welcome to redistribute
  it under certain conditions. See the General Public License for details.
------------------------------------------------------------
```

Great, so PSALTer is now loaded and we can start to do some science.

Metric affine gauge theory

Field strength tensors

In this section we will try our analysis of the metric affine gauge theory (MAGT). Our attempt closely follows the very wonderful paper arXiv:1912.01023 which was first brought to my attention by Claire Rigouzzo. The current MAGT implementation in PSALTer follows (to the letter) the conventions established in this paper. We will attempt to recover some key results in this paper, but we will also later look at arXiv:2110.14788, which was brought to my attention by Sebastian Zell.

First we write out rules which define the field strength tensors.

We want to define the curvature in Equation (2.1) on page 4 of arXiv:1912.01023.

```
** DefTensor: Defining tensor MetricAffineCurvature[-m, -n, r, -s].
```

$$\mathcal{F}_{mn}{}^{r}{}_{s} \tag{1}$$

$$\mathcal{A}_{m}{}^{r}{}_{q}\,\mathcal{A}_{n}{}^{q}{}_{s} - \mathcal{A}_{m}{}^{q}{}_{s}\,\mathcal{A}_{n}{}^{r}{}_{q} + \partial_{m}\mathcal{A}_{n}{}^{r}{}_{s} - \partial_{n}\mathcal{A}_{m}{}^{r}{}_{s} \tag{2}$$

Next want to define the torsion in Equation (2.2) on page 5 of arXiv:1912.01023.

```
** DefTensor: Defining tensor MetricAffineTorsion[-m, a, -n].
```

$$\mathcal{T}_{m}{}^{a}{}_{n} \tag{3}$$

$$\mathcal{A}_{m}{}^{a}{}_{n} - \mathcal{A}_{n}{}^{a}{}_{m} \tag{4}$$

And finally the non-metricity in Equation (2.3) on page 5 of arXiv:1912.01023. Watch out for the trivial misprint in the trace valence. Also, since the non-metricity only appears via quadratic invariants we don't need to bother about perturbing the metric here.

```
** DefTensor: Defining tensor MetricAffineNonMetricity[-l, -m, -n].
```

$$\mathcal{Q}_{lmn} \tag{5}$$

$$-\mathcal{A}_{lmn} + \mathcal{A}_{lnm} - \partial_{l} h_{mn} \tag{6}$$

Now we move on to computing the seven contractions defined in Equation (2.5) on page 5 of arXiv:1912.01023. Most of these contractions only appear in quadratic invariants, so we only need these formulae to be accurate to first order in small quantities.

First comes the torsion contraction.

```
** DefTensor: Defining tensor MetricAffineTorsionContraction[-m].
```

$$\mathcal{T}_{m} \tag{7}$$

$$\mathcal{T}_{a}{}^{a}{}_{m} \tag{8}$$

Next the (standard) non-metricity contraction.

```
** DefTensor: Defining tensor MetricAffineNonMetricityContraction[-m].
```

$$\mathcal{Q}_{m} \tag{9}$$

$$\mathcal{Q}_{ma}{}^{a} \tag{10}$$

Next the (tilde) non-metricity contraction.

```
** DefTensor: Defining tensor MetricAffineNonMetricityContractionTilde[-m].
```

$$\tilde{\mathcal{Q}}_{m} \tag{11}$$

$$\mathcal{Q}_{a}{}^{a}{}_{m} \tag{12}$$

Next the (conventional) Ricci tensor.

```
** DefTensor: Defining tensor MetricAffineRicciTensor[-m, -n].
```

$$\mathcal{F}_{mn} \tag{13}$$

$$\mathcal{F}_{mna}{}^{a} \tag{14}$$

Next the first of the pseudo-Ricci tensors.

```
** DefTensor: Defining tensor MetricAffineRicciTensor14[-m, -n].
```

$$\mathcal{F}^{(14)}{}_{mn} \tag{15}$$

$$\mathcal{F}_{amn}{}^{a} \tag{16}$$

Next the second of the pseudo-Ricci tensors.

```
** DefTensor: Defining tensor MetricAffineRicciTensor13[-m, -n].
```

$$\mathcal{F}^{(13)}{}_{mn} \tag{17}$$

$$\mathcal{F}_{am}{}^{a}{}_{n} \tag{18}$$

Now we move on to computing the (conventional) Ricci scalar. This time we need to be careful to retain contributions up to second order in smallness, since this is the only invariant which appears on its own.

```
** DefTensor: Defining tensor MetricAffineRicciScalar[].
```

$$\mathcal{F} \tag{19}$$

$$\left(\left(\eta^{ab}-h^{ab}\right)\mathcal{F}_{ca}{}^{c}{}_{b}\right) \tag{20}$$

The general parity-preserving Lagrangian

Now all the generally-covariant contractions of the field strength tensors have been defined, so we construct the general, parity-preserving Lagrangian proposed in Equation (2.4) on page 5 of arXiv:1912.01023. This is just an exercise in data entry.

$$\begin{aligned}\frac{1}{2}\Big(&-\mathcal{F}^{mnrs}\left(c_{1}\,\mathcal{F}_{mnrs}+c_{2}\,\mathcal{F}_{mnsr}+c_{4}\,\mathcal{F}_{mrns}+c_{5}\,\mathcal{F}_{msnr}+c_{6}\,\mathcal{F}_{msrn}+c_{3}\,\mathcal{F}_{rsmn}\right)-\\&\left(a_{5}\,Q_{nmr}+a_{4}\,Q_{rmn}\right)Q^{rmn}-a_{6}\,Q_{m}\,Q^{m}-a_{8}\,Q_{m}\,\tilde{Q}^{m}-a_{7}\,\tilde{Q}_{m}\,\tilde{Q}^{m}+a_{0}\,\mathcal{F}+c_{16}\,\mathcal{F}^{2}-\\&\mathcal{F}^{(13)mn}\left(c_{7}\,\mathcal{F}^{(13)}{}_{mn}+c_{8}\,\mathcal{F}^{(13)}{}_{nm}\right)-\mathcal{F}^{mn}\left(c_{13}\,\mathcal{F}_{mn}+c_{14}\,\mathcal{F}^{(13)}{}_{mn}+c_{15}\,\mathcal{F}^{(14)}{}_{mn}\right)-\\&\left(c_{11}\,\mathcal{F}^{(13)}{}_{mn}+c_{12}\,\mathcal{F}^{(13)}{}_{nm}\right)\mathcal{F}^{(14)mn}-\mathcal{F}^{(14)mn}\left(c_{9}\,\mathcal{F}^{(14)}{}_{mn}+c_{10}\,\mathcal{F}^{(14)}{}_{nm}\right)-\end{aligned} \tag{21}$$

$$a_{9}\, Q_{mrn}\, \mathcal{T}^{mrn} - \left(a_{2}\, \mathcal{T}_{mnr} + a_{1}\, \mathcal{T}_{mrn}\right) \mathcal{T}^{mrn} - \left(a_{10}\, Q_{m} + a_{11}\, \tilde{Q}_{m}\right) \mathcal{T}^{m} - a_{3}\, \mathcal{T}_{m}\, \mathcal{T}^{m}\Big)$$

This general Lagrangian is something that we must linearize. First, we need the linearized measure, otherwise the Einstein--Hilbert term (which has first-order perturbed contributions) won't have the right linearization.

$$1 + \frac{h^{a}{}_{a}}{2} \tag{22}$$

```
** DefConstantSymbol: Defining constant symbol PerturbativeParameter.
```

Now we attempt the linearization.

$$\begin{aligned}
&\frac{1}{2}\left(-2 a_{1} - 2 a_{4} + a_{9}\right) \mathcal{A}_{abc}\, \mathcal{A}^{abc} + \left(-\frac{a_{2}}{2} + a_{4} - \frac{a_{9}}{2}\right) \mathcal{A}_{acb}\, \mathcal{A}^{abc} + \frac{1}{2}\left(-a_{2} - a_{5}\right) \mathcal{A}^{abc}\, \mathcal{A}_{bac} + \\
&\left(-\frac{a_{0}}{2} + a_{2} + a_{5} - \frac{a_{9}}{2}\right) \mathcal{A}^{abc}\, \mathcal{A}_{bca} + \frac{1}{2}\left(2 a_{1} - a_{5} + a_{9}\right) \mathcal{A}^{abc}\, \mathcal{A}_{cba} + \frac{1}{2} a_{0}\, \mathcal{A}^{a}{}_{a}{}^{b}\, \mathcal{A}^{c}{}_{bc} + \\
&\frac{1}{2} a_{9}\, \mathcal{A}^{abc}\, \partial_{a} h_{bc} - \frac{1}{4} a_{0}\, h^{c}{}_{c}\, \partial_{b} \mathcal{A}^{a}{}_{a}{}^{b} + \frac{1}{4} a_{0}\, h^{c}{}_{c}\, \partial_{b} \mathcal{A}^{ab}{}_{a} - \frac{1}{2} a_{0}\, h_{ac}\, \partial_{b} \mathcal{A}^{abc} - a_{5}\, \mathcal{A}^{abc}\, \partial_{b} h_{ac} + \\
&\left(a_{5} - \frac{a_{9}}{2}\right) \mathcal{A}^{abc}\, \partial_{c} h_{ab} + \frac{1}{2} a_{0}\, h_{bc}\, \partial^{c} \mathcal{A}^{a}{}_{a}{}^{b} - \frac{1}{2} a_{5}\, \partial_{b} h_{ac}\, \partial^{c} h^{ab} - \frac{1}{2} a_{4}\, \partial_{c} h_{ab}\, \partial^{c} h^{ab} + \\
&\frac{1}{2} c_{16}\, \partial_{b} \mathcal{A}^{a}{}_{a}{}^{b}\, \partial_{d} \mathcal{A}^{c}{}_{c}{}^{d} - c_{16}\, \partial_{b} \mathcal{A}^{a}{}_{a}{}^{b}\, \partial_{d} \mathcal{A}^{cd}{}_{c} + \frac{1}{2} c_{16}\, \partial_{b} \mathcal{A}^{ab}{}_{a}\, \partial_{d} \mathcal{A}^{cd}{}_{c} + \frac{1}{2} c_{5}\, \partial_{a} \mathcal{A}_{bcd}\, \partial^{d} \mathcal{A}^{abc} + \\
&c_{4}\, \partial_{a} \mathcal{A}_{bdc}\, \partial^{d} \mathcal{A}^{abc} + c_{6}\, \partial_{a} \mathcal{A}_{cbd}\, \partial^{d} \mathcal{A}^{abc} + \frac{1}{2} c_{5}\, \partial_{a} \mathcal{A}_{cdb}\, \partial^{d} \mathcal{A}^{abc} + c_{1}\, \partial_{a} \mathcal{A}_{dbc}\, \partial^{d} \mathcal{A}^{abc} + \\
&c_{2}\, \partial_{a} \mathcal{A}_{dcb}\, \partial^{d} \mathcal{A}^{abc} - \frac{1}{2} c_{5}\, \partial_{b} \mathcal{A}_{acd}\, \partial^{d} \mathcal{A}^{abc} - \frac{1}{2} c_{4}\, \partial_{b} \mathcal{A}_{adc}\, \partial^{d} \mathcal{A}^{abc} - \frac{1}{2} c_{3}\, \partial_{b} \mathcal{A}_{cda}\, \partial^{d} \mathcal{A}^{abc} - \\
&\frac{1}{2} c_{6}\, \partial_{c} \mathcal{A}_{abd}\, \partial^{d} \mathcal{A}^{abc} - \frac{1}{2} c_{3}\, \partial_{c} \mathcal{A}_{bad}\, \partial^{d} \mathcal{A}^{abc} + c_{3}\, \partial_{c} \mathcal{A}_{bda}\, \partial^{d} \mathcal{A}^{abc} - c_{1}\, \partial_{d} \mathcal{A}_{abc}\, \partial^{d} \mathcal{A}^{abc} - \\
&c_{2}\, \partial_{d} \mathcal{A}_{acb}\, \partial^{d} \mathcal{A}^{abc} - \frac{1}{2} c_{4}\, \partial_{d} \mathcal{A}_{bac}\, \partial^{d} \mathcal{A}^{abc} - \frac{1}{2} c_{5}\, \partial_{d} \mathcal{A}_{bca}\, \partial^{d} \mathcal{A}^{abc} - \frac{1}{2} c_{6}\, \partial_{d} \mathcal{A}_{cba}\, \partial^{d} \mathcal{A}^{abc}
\end{aligned} \tag{23}$$

We see that there is not a great degree of degeneracy among the coupling constants, but bear in mind that we may only see such patterns when surface terms are used to extract the wave operator.

Now we are basically ready to try some spectral analysis, but first we will present the matrices of particle interactions stored in the current MAGT implementation in PSALTer. These are just for reference: when the ParticleSpectrum function is used, much of the output is in the form of matrices, and it can be useful to know which element corresponds to which interaction. The symbols used for the different spin-parity modes are currently a bit cryptic (except for the metric perturbation, which is fairly clear), but the logic is basically as follows. The general connection A is decomposed into parts which are symmetric (denoted Q) and antisymmetric (denoted A again, confusingly) in the second and third indices. These parts are further decomposed into invariant subspaces under the action of the group of spatial rotations. These parts are given with reduced numbers of indices where convenient, and are labelled by spin and parity but also by a cryptic series of superscripts denoting from which Young tableau the mode descends. Zhiyuan is really the expert on decoding these (since he constructed them), but all the modes appear schematically in Table 2, on page 11 of arXiv:1912.01023. So, here are the matrices.

The spin-0 sector. It is pretty big.

$$\begin{pmatrix}
{}^{0^+}h^{\perp}\,{}^{0^+}h^{\perp}\dagger & {}^{0^+}h^{\|}\,{}^{0^+}h^{\perp}\dagger & {}^{0^+}\mathcal{A}^{\|}\,{}^{0^+}h^{\perp}\dagger & {}^{0^+}h^{\perp}\dagger\,{}^{0^+}Q^{\perp t} & {}^{0^+}h^{\perp}\dagger\,{}^{0^+}Q^{\|} & {}^{0^+}h^{\perp}\dagger\,{}^{0^+}Q^{\perp h} & {}^{0^-}\mathcal{A}^{\|}\,{}^{0^+}h^{\perp}\dagger \\
{}^{0^+}h^{\|}\dagger\,{}^{0^+}h^{\perp} & {}^{0^+}h^{\|}\,{}^{0^+}h^{\|}\dagger & {}^{0^+}\mathcal{A}^{\|}\,{}^{0^+}h^{\|}\dagger & {}^{0^+}h^{\|}\dagger\,{}^{0^+}Q^{\perp t} & {}^{0^+}h^{\|}\dagger\,{}^{0^+}Q^{\|} & {}^{0^+}h^{\|}\dagger\,{}^{0^+}Q^{\perp h} & {}^{0^-}\mathcal{A}^{\|}\,{}^{0^+}h^{\|}\dagger \\
{}^{0^+}\mathcal{A}^{\|}\dagger\,{}^{0^+}h^{\perp} & {}^{0^+}\mathcal{A}^{\|}\dagger\,{}^{0^+}h^{\|} & {}^{0^+}\mathcal{A}^{\|}\,{}^{0^+}\mathcal{A}^{\|}\dagger & {}^{0^+}\mathcal{A}^{\|}\dagger\,{}^{0^+}Q^{\perp t} & {}^{0^+}\mathcal{A}^{\|}\dagger\,{}^{0^+}Q^{\|} & {}^{0^+}\mathcal{A}^{\|}\dagger\,{}^{0^+}Q^{\perp h} & {}^{0^-}\mathcal{A}^{\|}\,{}^{0^+}\mathcal{A}^{\|}\dagger \\
{}^{0^+}h^{\perp}\,{}^{0^+}Q^{\perp t}\dagger & {}^{0^+}h^{\|}\,{}^{0^+}Q^{\perp t}\dagger & {}^{0^+}\mathcal{A}^{\|}\,{}^{0^+}Q^{\perp t}\dagger & {}^{0^+}Q^{\perp t}\,{}^{0^+}Q^{\perp t}\dagger & {}^{0^+}Q^{\|}\,{}^{0^+}Q^{\perp t}\dagger & {}^{0^+}Q^{\perp h}\,{}^{0^+}Q^{\perp t}\dagger & {}^{0^-}\mathcal{A}^{\|}\,{}^{0^+}Q^{\perp t}\dagger \\
{}^{0^+}h^{\perp}\,{}^{0^+}Q^{\|}\dagger & {}^{0^+}h^{\|}\,{}^{0^+}Q^{\|}\dagger & {}^{0^+}\mathcal{A}^{\|}\,{}^{0^+}Q^{\|}\dagger & {}^{0^+}Q^{\|}\dagger\,{}^{0^+}Q^{\perp t} & {}^{0^+}Q^{\|}\,{}^{0^+}Q^{\|}\dagger & {}^{0^+}Q^{\|}\dagger\,{}^{0^+}Q^{\perp h} & {}^{0^-}\mathcal{A}^{\|}\,{}^{0^+}Q^{\|}\dagger \\
{}^{0^+}h^{\perp}\,{}^{0^+}Q^{\perp h}\dagger & {}^{0^+}h^{\|}\,{}^{0^+}Q^{\perp h}\dagger & {}^{0^+}\mathcal{A}^{\|}\,{}^{0^+}Q^{\perp h}\dagger & {}^{0^+}Q^{\perp h}\dagger\,{}^{0^+}Q^{\perp t} & {}^{0^+}Q^{\|}\,{}^{0^+}Q^{\perp h}\dagger & {}^{0^+}Q^{\perp h}\,{}^{0^+}Q^{\perp h}\dagger & {}^{0^-}\mathcal{A}^{\|}\,{}^{0^+}Q^{\perp h}\dagger \\
{}^{0^-}\mathcal{A}^{\|}\dagger\,{}^{0^+}h^{\perp} & {}^{0^-}\mathcal{A}^{\|}\dagger\,{}^{0^+}h^{\|} & {}^{0^-}\mathcal{A}^{\|}\dagger\,{}^{0^+}\mathcal{A}^{\|} & {}^{0^-}\mathcal{A}^{\|}\dagger\,{}^{0^+}Q^{\perp t} & {}^{0^-}\mathcal{A}^{\|}\dagger\,{}^{0^+}Q^{\|} & {}^{0^-}\mathcal{A}^{\|}\dagger\,{}^{0^+}Q^{\perp h} & {}^{0^-}\mathcal{A}^{\|}\,{}^{0^-}\mathcal{A}^{\|}\dagger
\end{pmatrix} \tag{24}$$

The spin-1 sector. It is vast.

$$\left(\begin{matrix}
{}^{1^+}\mathcal{A}^{\|ab}\,{}^{1^+}\mathcal{A}^{\|}\dagger_{ab} & {}^{1^+}\mathcal{A}^{\|}\dagger^{ab}\,{}^{1^+}\mathcal{A}^{\perp}{}_{ab} & {}^{1^+}\mathcal{A}^{\|}\dagger^{ab}\,{}^{1^+}Q^{\perp}{}_{ab} & {}^{1^+}\mathcal{A}^{\|}\dagger^{bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}h^{\perp a} & {}^{1^-}\mathcal{A}^{\|a}\,{}^{1^+} \\
{}^{1^+}\mathcal{A}^{\|ab}\,{}^{1^+}\mathcal{A}^{\perp}\dagger_{ab} & {}^{1^+}\mathcal{A}^{\perp ab}\,{}^{1^+}\mathcal{A}^{\perp}\dagger_{ab} & {}^{1^+}\mathcal{A}^{\perp}\dagger^{ab}\,{}^{1^+}Q^{\perp}{}_{ab} & {}^{1^+}\mathcal{A}^{\perp}\dagger^{bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}h^{\perp a} & {}^{1^-}\mathcal{A}^{\|a}\,{}^{1^+} \\
{}^{1^+}\mathcal{A}^{\|ab}\,{}^{1^+}Q^{\perp}\dagger_{ab} & {}^{1^+}\mathcal{A}^{\perp ab}\,{}^{1^+}Q^{\perp}\dagger_{ab} & {}^{1^+}Q^{\perp ab}\,{}^{1^+}Q^{\perp}\dagger_{ab} & \epsilon^{\|}{}_{abc}\,{}^{1^-}h^{\perp a}\,{}^{1^+}Q^{\perp}\dagger^{bc} & {}^{1^-}\mathcal{A}^{\|a}\,\epsilon^{\|} \\
{}^{1^+}\mathcal{A}^{\|bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}h^{\perp}\dagger^{a} & {}^{1^+}\mathcal{A}^{\perp bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}h^{\perp}\dagger^{a} & \epsilon^{\|}{}_{abc}\,{}^{1^-}h^{\perp}\dagger^{a}\,{}^{1^+}Q^{\perp bc} & {}^{1^-}h^{\perp a}\,{}^{1^-}h^{\perp}\dagger_{a} & {}^{1^-}\mathcal{A}^{\|a} \\
{}^{1^-}\mathcal{A}^{\|}\dagger^{a}\,{}^{1^+}\mathcal{A}^{\|bc}\,\epsilon^{\|}{}_{abc} & {}^{1^-}\mathcal{A}^{\|}\dagger^{a}\,{}^{1^+}\mathcal{A}^{\perp bc}\,\epsilon^{\|}{}_{abc} & {}^{1^-}\mathcal{A}^{\|}\dagger^{a}\,\epsilon^{\|}{}_{abc}\,{}^{1^+}Q^{\perp bc} & {}^{1^-}\mathcal{A}^{\|}\dagger^{a}\,{}^{1^-}h^{\perp}{}_{a} & {}^{1^-}\mathcal{A}^{\|a} \\
{}^{1^+}\mathcal{A}^{\|bc}\,{}^{1^-}\mathcal{A}^{\perp}\dagger^{a}\,\epsilon^{\|}{}_{abc} & {}^{1^-}\mathcal{A}^{\perp}\dagger^{a}\,{}^{1^+}\mathcal{A}^{\perp bc}\,\epsilon^{\|}{}_{abc} & {}^{1^-}\mathcal{A}^{\perp}\dagger^{a}\,\epsilon^{\|}{}_{abc}\,{}^{1^+}Q^{\perp bc} & {}^{1^-}\mathcal{A}^{\perp}\dagger^{a}\,{}^{1^-}h^{\perp}{}_{a} & {}^{1^-}\mathcal{A}^{\|a} \\
{}^{1^+}\mathcal{A}^{\|bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}Q^{\perp t}\dagger^{a} & {}^{1^+}\mathcal{A}^{\perp bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}Q^{\perp t}\dagger^{a} & \epsilon^{\|}{}_{abc}\,{}^{1^+}Q^{\perp bc}\,{}^{1^-}Q^{\perp t}\dagger^{a} & {}^{1^-}h^{\perp a}\,{}^{1^-}Q^{\perp t}\dagger_{a} & {}^{1^-}\mathcal{A}^{\|a} \\
{}^{1^+}\mathcal{A}^{\|bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}Q^{\| t}\dagger^{a} & {}^{1^+}\mathcal{A}^{\perp bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}Q^{\| t}\dagger^{a} & \epsilon^{\|}{}_{abc}\,{}^{1^+}Q^{\perp bc}\,{}^{1^-}Q^{\| t}\dagger^{a} & {}^{1^-}h^{\perp a}\,{}^{1^-}Q^{\| t}\dagger_{a} & {}^{1^-}\mathcal{A}^{\|a} \\
{}^{1^+}\mathcal{A}^{\|bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}Q^{\perp h}\dagger^{a} & {}^{1^+}\mathcal{A}^{\perp bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}Q^{\perp h}\dagger^{a} & \epsilon^{\|}{}_{abc}\,{}^{1^+}Q^{\perp bc}\,{}^{1^-}Q^{\perp h}\dagger^{a} & {}^{1^-}h^{\perp a}\,{}^{1^-}Q^{\perp h}\dagger_{a} & {}^{1^-}\mathcal{A}^{\|a} \\
{}^{1^+}\mathcal{A}^{\|bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}Q^{\| h}\dagger^{a} & {}^{1^+}\mathcal{A}^{\perp bc}\,\epsilon^{\|}{}_{abc}\,{}^{1^-}Q^{\| h}\dagger^{a} & \epsilon^{\|}{}_{abc}\,{}^{1^+}Q^{\perp bc}\,{}^{1^-}Q^{\| h}\dagger^{a} & {}^{1^-}h^{\perp a}\,{}^{1^-}Q^{\| h}\dagger_{a} & {}^{1^-}\mathcal{A}^{\|a}
\end{matrix}\right. \tag{25}$$

The spin-2 sector. It is not too bad.

$$\left(\begin{array}{ccccc}
{}^{2^+}h^{\parallel ab}\ {}^{2^+}h^{\parallel}\dagger_{ab} & {}^{2^+}\mathcal{A}^{\parallel ab}\ {}^{2^+}h^{\parallel}\dagger_{ab} & {}^{2^+}h^{\parallel}\dagger^{ab}\ {}^{2^+}Q^{\parallel}{}_{ab} & {}^{2^+}h^{\parallel}\dagger^{ab}\ {}^{2^+}Q^{\perp}{}_{ab} & {}^{2^-}\mathcal{A}^{\parallel c} \\
{}^{2^+}\mathcal{A}^{\parallel}\dagger^{ab}\ {}^{2^+}h^{\parallel}{}_{ab} & {}^{2^+}\mathcal{A}^{\parallel ab}\ {}^{2^+}\mathcal{A}^{\parallel}\dagger_{ab} & {}^{2^+}\mathcal{A}^{\parallel}\dagger^{ab}\ {}^{2^+}Q^{\parallel}{}_{ab} & {}^{2^+}\mathcal{A}^{\parallel}\dagger^{ab}\ {}^{2^+}Q^{\perp}{}_{ab} & {}^{2^-}\mathcal{A}^{\parallel a} \\
{}^{2^+}h^{\parallel ab}\ {}^{2^+}Q^{\parallel}\dagger_{ab} & {}^{2^+}\mathcal{A}^{\parallel ab}\ {}^{2^+}Q^{\parallel}\dagger_{ab} & {}^{2^+}Q^{\parallel ab}\ {}^{2^+}Q^{\parallel}\dagger_{ab} & {}^{2^+}Q^{\parallel}\dagger^{ab}\ {}^{2^+}Q^{\perp}{}_{ab} & {}^{2^-}\mathcal{A}^{\parallel a} \\
{}^{2^+}h^{\parallel ab}\ {}^{2^+}Q^{\perp}\dagger_{ab} & {}^{2^+}\mathcal{A}^{\parallel ab}\ {}^{2^+}Q^{\perp}\dagger_{ab} & {}^{2^+}Q^{\parallel ab}\ {}^{2^+}Q^{\perp}\dagger_{ab} & {}^{2^+}Q^{\perp ab}\ {}^{2^+}Q^{\perp}\dagger_{ab} & {}^{2^-}\mathcal{A}^{\parallel a} \\
{}^{2^-}\mathcal{A}^{\parallel}\dagger^{abc}\ \epsilon^{\parallel}{}_{bcd}\ {}^{2^+}h^{\parallel}{}_{a}{}^{d} & {}^{2^-}\mathcal{A}^{\parallel}\dagger^{abc}\ {}^{2^+}\mathcal{A}^{\parallel}{}_{c}{}^{d}\ \epsilon^{\parallel}{}_{abd} & {}^{2^-}\mathcal{A}^{\parallel}\dagger^{abc}\ \epsilon^{\parallel}{}_{bcd}\ {}^{2^+}Q^{\parallel}{}_{a}{}^{d} & {}^{2^-}\mathcal{A}^{\parallel}\dagger^{abc}\ \epsilon^{\parallel}{}_{bcd}\ {}^{2^+}Q^{\perp}{}_{a}{}^{d} & {}^{2^-}\mathcal{F} \\
\epsilon^{\parallel}{}_{bcd}\ {}^{2^+}h^{\parallel ab}\ {}^{2^-}Q^{\parallel}\dagger_{a}{}^{cd} & {}^{2^+}\mathcal{A}^{\parallel ab}\ \epsilon^{\parallel}{}_{bcd}\ {}^{2^-}Q^{\parallel}\dagger_{a}{}^{cd} & \epsilon^{\parallel}{}_{bcd}\ {}^{2^-}Q^{\parallel}\dagger^{abc}\ {}^{2^+}Q^{\parallel}{}_{a}{}^{d} & \epsilon^{\parallel}{}_{bcd}\ {}^{2^-}Q^{\parallel}\dagger^{abc}\ {}^{2^+}Q^{\perp}{}_{a}{}^{d} & {}^{2^-}\mathcal{F}
\end{array}\right.$$

The spin-3 sector. Yes, despite well-known theorems by Weinberg, Witten etc., there is a higher-spin sector at play! I suppose one must be careful that it does not propagate?

$$\left(\left(\ {}^{3^-}Q^{\parallel abc}\ \ {}^{3^-}Q^{\parallel}\dagger_{abc}\ \right)\right) \tag{27}$$

That deals with all the preliminaries. We can now transition to some spectral analyses.

Einstein-Hilbert theory

The first theory we will look at is the simple Einstein-Hilbert case.

$$\frac{1}{2}\underset{0}{a}.\left(1+\frac{h^{a}{}_{a}}{2}\right)\mathcal{F} \tag{28}$$

Now we linearize it.

$$-\frac{1}{2}\underset{0}{a}.\ \mathcal{A}^{abc}\ \mathcal{A}_{bca}+\frac{1}{2}\underset{0}{a}.\ \mathcal{A}^{a}{}_{a}{}^{b}\ \mathcal{A}^{c}{}_{bc}-\frac{1}{4}\underset{0}{a}.\ h^{c}{}_{c}\ \partial_{b}\mathcal{A}^{a}{}_{a}{}^{b}+ \\ \frac{1}{4}\underset{0}{a}.\ h^{c}{}_{c}\ \partial_{b}\mathcal{A}^{ab}{}_{a}-\frac{1}{2}\underset{0}{a}.\ h_{ac}\ \partial_{b}\mathcal{A}^{abc}+\frac{1}{2}\underset{0}{a}.\ h_{bc}\ \partial^{c}\mathcal{A}^{a}{}_{a}{}^{b} \tag{29}$$

Now we feed the linearized Lagrangian into PSALTer.

```
The (possibly singular) a-matrices associated with
  the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & \frac{i\,a3_{0}\,k}{?} & -\frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 \\ 0 & 0 & \frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 & -\frac{i\,a3_{0}\,k}{?\sqrt{?}} & \frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 \\ 0 & -\frac{i\,a3_{0}\,k}{?\sqrt{?}} & -\frac{a3_{0}}{?} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{a3_{0}}{?} & -\frac{a3_{0}}{?\sqrt{?}} & 0 \\ -\frac{?}{?}\,i\,a_{?}\,k & \frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 & \frac{a3_{0}}{?} & 0 & -\frac{a3_{0}}{?\sqrt{?}} & 0 \\ \frac{i\,a3_{0}\,k}{?\sqrt{?}} & -\frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 & -\frac{a3_{0}}{?\sqrt{?}} & -\frac{a3_{0}}{?\sqrt{?}} & \frac{a3_{0}}{?} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{a3_{0}}{?} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\frac{a3_{0}}{?} & -\frac{a3_{0}}{?\sqrt{?}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{a3_{0}}{?\sqrt{?}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{a3_{0}}{?} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 & -\frac{i\,a3_{0}\,k}{?\sqrt{?}} & \frac{?}{?}\,i\sqrt{\frac{?}{?}}\,a_{?}\,k & -\frac{i\,a3_{0}\,k}{?\sqrt{?}} & -\frac{i\,a3_{0}\,k}{?\sqrt{?}} \\ 0 & 0 & 0 & -\frac{i\,a3_{0}\,k}{?\sqrt{?}} & -\frac{a3_{0}}{?} & \frac{a3_{0}}{?\sqrt{?}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{a3_{0}}{?\sqrt{?}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 & 0 & -\frac{a3_{0}}{?} & \frac{\sqrt{?}\,a3_{0}}{?} & -\frac{a3_{0}}{?\sqrt{?}} & -\frac{a3_{0}}{?} \\ 0 & 0 & 0 & -\frac{?}{?}\,i\sqrt{\frac{?}{?}}\,a_{?}\,k & 0 & 0 & \frac{\sqrt{?}\,a3_{0}}{?} & \frac{a3_{0}}{?} & -\frac{?}{?}\sqrt{\frac{?}{?}}\,a_{?} & -\frac{\sqrt{?}\,a3_{0}}{?} \\ 0 & 0 & 0 & \frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 & 0 & -\frac{a3_{0}}{?\sqrt{?}} & -\frac{?}{?}\sqrt{\frac{?}{?}}\,a_{?} & \frac{a3_{0}}{?} & \frac{a3_{0}}{?\sqrt{?}} \\ 0 & 0 & 0 & \frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 & 0 & -\frac{a3_{0}}{?} & -\frac{\sqrt{?}\,a3_{0}}{?} & \frac{a3_{0}}{?\sqrt{?}} & \frac{?\,a3_{0}}{?} \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & -\frac{i\,a3_{0}\,k}{?\sqrt{?}} & -\frac{i\,a3_{0}\,k}{?\sqrt{?}} & \frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 & 0 \\ \frac{i\,a3_{0}\,k}{?\sqrt{?}} & \frac{a3_{0}}{?} & 0 & 0 & 0 & 0 \\ \frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 & -\frac{a3_{0}}{?} & 0 & 0 & 0 \\ -\frac{i\,a3_{0}\,k}{?\sqrt{?}} & 0 & 0 & \frac{a3_{0}}{?} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{a3_{0}}{?} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-3 sector:

$$\left(-\frac{a_0}{}\right)$$

Gauge constraints on source currents:

$$-6\,i\,\dot{\mathcal{T}}^{\perp}+k\,\dot{\mathcal{Z}}^{\parallel}+2\,k\,\dot{\mathcal{Z}}^{\perp h}==0$$

$$2\,i\,\dot{\mathcal{T}}^{\perp}+k\,\dot{\mathcal{Z}}^{\perp t}==0$$

$$12\,i\,\dot{\mathcal{T}}^{\perp a}+k\left(2\,\dot{\mathcal{Z}}^{\parallel h\,a}+\dot{\mathcal{Z}}^{\parallel t\,a}+3\,\dot{\mathcal{Z}}^{\perp t\,a}+6\,\dot{\mathcal{Y}}^{\perp a}\right)==0$$

$$-6\,i\,\dot{\mathcal{T}}^{\perp a}+k\,\dot{\mathcal{Z}}^{\perp h\,a}==k\left(\dot{\mathcal{Z}}^{\perp t\,a}+3\,\dot{\mathcal{Y}}^{\perp a}\right)$$

$$\dot{\mathcal{Z}}^{\parallel\,abc}==0$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix}
-\frac{k^2}{a_0(\;+k^2)^2} & \frac{\sqrt{\;}}{a_0+a_0k^2} & \frac{i\sqrt{\;}\,k}{a_0+a_0k^2} & -\frac{i\,k}{a_0(\;+k^2)^2} & \frac{i\,k(\;+k^2)}{a_0(\;+k^2)^2} & -\frac{i\sqrt{\;}\,k(\;+k^2)}{a_0(\;+k^2)^2} & 0 \\
\frac{\sqrt{\;}}{a_0+a_0k^2} & \frac{}{a_0k^2} & \frac{i\sqrt{\;}}{a_0k} & \frac{i\sqrt{\;}}{a_0k+a_0k^3} & -\frac{i}{\sqrt{\;}(a_0k+a_0k^3)} & -\frac{i\sqrt{\frac{2}{3}}}{a_0k+a_0k^3} & 0 \\
-\frac{i\sqrt{\;}\,k}{a_0+a_0k^2} & -\frac{i\sqrt{\;}}{a_0k} & 0 & \frac{\sqrt{\;}}{a_0+a_0k^2} & -\frac{\sqrt{\frac{2}{3}}}{a_0+a_0k^2} & -\frac{}{\sqrt{\;}(a_0+a_0k^2)} & 0 \\
\frac{i\,k}{a_0(\;+k^2)^2} & -\frac{i\sqrt{\;}}{a_0k+a_0k^3} & \frac{\sqrt{\;}}{a_0+a_0k^2} & -\frac{}{a_0(\;+k^2)^2} & \frac{(\;+k^2)}{a_0(\;+k^2)^2} & -\frac{\sqrt{\;}(\;+k^2)}{a_0(\;+k^2)^2} & 0 \\
-\frac{i\,k(\;+k^2)}{a_0(\;+k^2)^2} & \frac{i}{\sqrt{\;}(a_0k+a_0k^3)} & -\frac{\sqrt{\frac{2}{3}}}{a_0+a_0k^2} & \frac{(\;+k^2)}{a_0(\;+k^2)^2} & -\frac{(\;+k^2)}{a_0(\;+k^2)^2} & -\frac{\sqrt{\;}(\;+k^2)}{a_0(\;+k^2)^2} & 0 \\
\frac{i\sqrt{\;}\,k(\;+k^2)}{a_0(\;+k^2)^2} & \frac{i\sqrt{\frac{2}{3}}}{a_0k+a_0k^3} & -\frac{}{\sqrt{\;}(\;a3_0+\;a3_0k^2)} & -\frac{\sqrt{\;}(\;+\;k^2)}{a3_0(\;+\;k^2)^2} & -\frac{\sqrt{\;}(\;+\;k^2)}{a3_0(\;+\;k^2)^2} & \frac{(\;+\;k^2)}{a3_0(\;+\;k^2)^2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{}{a3_0}
\end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
0 & -\frac{2\sqrt{2}}{a_{0}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{2\sqrt{2}}{a_{0}} & \frac{2}{a_{0}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{4}{a_{0}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2 k^2}{a_{0} (2+k^2)^2} & \frac{2 i \sqrt{2} k}{a_{0} (2+k^2)} & \frac{i k (4+k^2)}{a_{0} (2+k^2)^2} & -\frac{i k \left(6+5 k^2\right)}{\sqrt{6} a_{0} (2+k^2)^2} & \frac{i \sqrt{\frac{5}{6}} k}{a_{0} (2+k^2)} & -\frac{2 i k (3+k^2)}{\sqrt{3} a_{0} (2+k^2)^2} & \frac{i \sqrt{\frac{2}{3}} k}{a_{0} (2+k^2)} \\
0 & 0 & 0 & -\frac{2 i \sqrt{2} k}{a_{0} (2+k^2)} & 0 & \frac{\sqrt{2} (4+k^2)}{a_{0} (2+k^2)} & -\frac{2 k^2}{\sqrt{3} a_{0} (2+k^2)} & 0 & \frac{\sqrt{\frac{2}{3}} k^2}{a_{0} (2+k^2)} & 0 \\
0 & 0 & 0 & -\frac{i k (4+k^2)}{a_{0} (2+k^2)^2} & \frac{\sqrt{2} (4+k^2)}{a_{0} (2+k^2)} & \frac{(4+k^2)^2}{2 a_{0} (2+k^2)^2} & \frac{k^2 (-2+k^2)}{2 \sqrt{6} a_{0} (2+k^2)^2} & -\frac{\sqrt{\frac{5}{6}} k^2}{4 a_{0}+2 a_{0} k^2} & \frac{k^2 \left(5+2 k^2\right)}{\sqrt{3} a_{0} (2+k^2)^2} & -\frac{k^2}{\sqrt{6} \left(2 a_{0}+a_{0} k^2\right)} \\
0 & 0 & 0 & \frac{i k \left(6+5 k^2\right)}{\sqrt{6} a_{0} (2+k^2)^2} & -\frac{2 k^2}{\sqrt{3} a_{0} (2+k^2)} & \frac{k^2 (-2+k^2)}{2 \sqrt{6} a_{0} (2+k^2)^2} & -\frac{76+52 k^2+3 k^4}{12 a_{0} (2+k^2)^2} & \frac{\sqrt{5} \left(10+3 k^2\right)}{12 a_{0} (2+k^2)} & \frac{-2+k^2}{3 \sqrt{2} a_{0} (2+k^2)^2} & \frac{1}{-2 a_{0}-\frac{8 a_{0}}{2+3 k^2}} \\
0 & 0 & 0 & -\frac{i \sqrt{\frac{5}{6}} k}{2 a_{0}+a_{0} k^2} & 0 & -\frac{\sqrt{\frac{5}{6}} k^2}{4 a_{0}+2 a_{0} k^2} & \frac{\sqrt{5} \left(10+3 k^2\right)}{12 a_{0} (2+k^2)} & \frac{1}{12 a_{0}} & -\frac{\sqrt{\frac{5}{2}}}{6 a_{0}+3 a_{0} k^2} & -\frac{\sqrt{5}}{6 a_{0}} \\
0 & 0 & 0 & \frac{2 i k (3+k^2)}{\sqrt{3} a_{0} (2+k^2)^2} & \frac{\sqrt{\frac{2}{3}} k^2}{a_{0} (2+k^2)} & \frac{k^2 \left(5+2 k^2\right)}{\sqrt{3} a_{0} (2+k^2)^2} & \frac{-2+k^2}{3 \sqrt{2} a_{0} (2+k^2)^2} & -\frac{\sqrt{\frac{5}{2}}}{6 a_{0}+3 a_{0} k^2} & \frac{2 \left(17+14 k^2+3 k^4\right)}{3 a_{0} (2+k^2)^2} & -\frac{\sqrt{2} \left(7+3 k^2\right)}{3 a_{0} (2+k^2)} \\
0 & 0 & 0 & -\frac{i \sqrt{\frac{2}{3}} k}{2 a_{0}+a_{0} k^2} & 0 & -\frac{k^2}{\sqrt{6} \left(2 a_{0}+a_{0} k^2\right)} & \frac{1}{-2 a_{0}-\frac{8 a_{0}}{2+3 k^2}} & -\frac{\sqrt{5}}{6 a_{0}} & -\frac{\sqrt{2} \left(7+3 k^2\right)}{3 a_{0} (2+k^2)} & \frac{5}{3 a_{0}}
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix}
-\frac{8}{a_{0} k^2} & -\frac{4 i \sqrt{2}}{a_{0} k} & \frac{4 i}{\sqrt{3} a_{0} k} & \frac{4 i \sqrt{\frac{2}{3}}}{a_{0} k} & 0 & 0 \\
\frac{4 i \sqrt{2}}{a_{0} k} & 0 & \frac{2 \sqrt{\frac{2}{3}}}{a_{0}} & \frac{4}{\sqrt{3} a_{0}} & 0 & 0 \\
-\frac{4 i}{\sqrt{3} a_{0} k} & \frac{2 \sqrt{\frac{2}{3}}}{a_{0}} & -\frac{8}{3 a_{0}} & -\frac{2 \sqrt{2}}{3 a_{0}} & 0 & 0 \\
-\frac{4 i \sqrt{\frac{2}{3}}}{a_{0} k} & \frac{4}{\sqrt{3} a_{0}} & -\frac{2 \sqrt{2}}{3 a_{0}} & \frac{8}{3 a_{0}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{4}{a_{0}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Matrix for spin-3 sector:

$$\left(-\frac{2}{a_{0}}\right)$$

Square masses:

{{}, {}, {}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{28\, p}{a.}, -\frac{18\, p}{a.}\right\}$$

Overall unitarity conditions:

$a. < 0 \,\&\&\, (p < 0 \,||\, p > 0)$

This completes the spectral analysis. We find that there are no massive poles, and hence no massive gravitons. There are however two massless degrees of freedom which we take to be the graviton polarisations. The unitarity conditions of these polarisations just make sure that the Einstein--Hilbert coupling carries the right sign, i.e. that the square of the Planck mass is positive.

We will show this later, but the source constraints can be decoded as a consequence of the diffeomorphism and projective gauge symmetries of the theory.

Let's pause the calculations here.

Throw: Uncaught Throw[Pause calculation please!] returned to top level.

Out[1]= Hold[Throw[Pause calculation please!]]