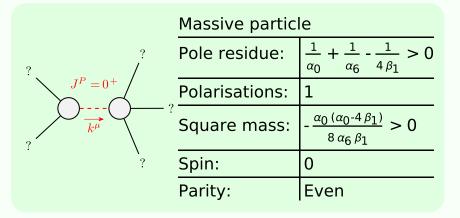
| | $\sigma_{1}^{\#1}{}_{\alpha\beta}$ | $\sigma_1^{\#_2^2}$ | $\tau_{1}^{\#1}_{\alpha\beta}$ | $\sigma_{1^{-}\alpha}^{\#1}$ | $\sigma_{1}^{\#2}{}_{\alpha}$ | $\tau_{1^{-}}^{\#1}\alpha$ | $\tau_{1}^{\#2}{}_{\alpha}$ |
|-------------------------------|--|--|--|--|---|----------------------------|---|
| $r_1^{#1} + \alpha \beta$ | 0 | $\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+k^2)}$ | $\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+k^2)}$ | 0 | 0 | 0 | 0 |
| $ +^{\alpha \beta} $ | $\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+k^2)}$ | $-\frac{2}{(\alpha_0-4\beta_1)(1+k^2)^2}$ | $-\frac{2ik}{(\alpha_0-4\beta_1)(1+k^2)^2}$ | 0 | 0 | 0 | 0 |
| $\tau_1^{\#1} + \alpha \beta$ | | $\frac{2ik}{(\alpha_0-4\beta_1)(1+k^2)^2}$ | $-\frac{2k^2}{(\alpha_0-4\beta_1)(1+k^2)^2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1}^{\#1} +^{lpha}$ | 0 | 0 | 0 | 0 | $-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+2k^2)}$ | 0 | $-\frac{4ik}{(\alpha_0-4\beta_1)(1+2k^2)}$ |
| $\sigma_1^{\#2} +^{\alpha}$ | 0 | 0 | 0 | $-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+2k^2)}$ | $-\frac{2}{(\alpha_0-4\beta_1)(1+2k^2)^2}$ | 0 | $-\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+2k^2)^2}$ |
| $\tau_{1}^{\#_{1}} + \alpha$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1}^{\#2} + ^{\alpha}$ | 0 | 0 | 0 | $\frac{4ik}{(\alpha_0-4\beta_1)(1+2k^2)}$ | $\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+2k^2)^2}$ | 0 | $-\frac{4k^2}{(\alpha_0\!-\!4\beta_1)(1\!+\!2k^2)^2}$ |
| | | | | | | | |

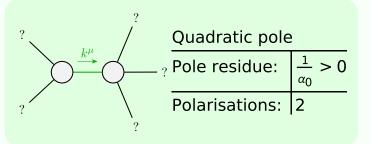
| | $\omega_{2^{+}lphaeta}^{\sharp1}$ | $f_{2}^{\#1}{}_{\alpha\beta}$ | $\omega_{2^{-}\alpha\beta\chi}^{\#1}$ |
|--|--|---|---------------------------------------|
| $\omega_{2}^{\#1}\dagger^{\alpha\beta}$ | $-\frac{\alpha_0}{4}+\beta_1$ | $\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$ | 0 |
| $f_{2}^{#1} \dagger^{\alpha\beta}$ | $-\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$ | $2 \beta_1 k^2$ | 0 |
| $\omega_2^{\#1} \dagger^{\alpha\beta\chi}$ | 0 | 0 | $-\frac{\alpha_0}{4}+\beta_1$ |

| | $\sigma_{2^{+}\alpha\beta}^{\#1}$ | $	au_2^{\#1}{}_{lphaeta}$ | $\sigma_{2}^{\#1}_{\alpha\beta\lambda}$ |
|--|--|---------------------------------|---|
| $\sigma_{2}^{\#1} \dagger^{\alpha\beta}$ | $-\frac{16\beta_1}{\alpha_0^2-4\alpha_0\beta_1}$ | $\frac{2i\sqrt{2}}{\alpha_0 k}$ | 0 |
| $\tau_{2}^{\#1} \dagger^{\alpha\beta}$ | $-\frac{2i\sqrt{2}}{\alpha_0 k}$ | $\frac{2}{\alpha_0 k^2}$ | 0 |
| $\sigma_{2}^{\#1}\dagger^{lphaeta\chi}$ | 0 | 0 | $\frac{1}{-\frac{\alpha_0}{4} + \beta_1}$ |
| | | | |

| $f_{1^-}^{\#2} lpha$ | 0 | 0 | 0 | $-\frac{1}{2}\tilde{I}\left(\alpha_{0}-4\beta_{1}\right)k$ | 0 | 0 | 0 |
|--------------------------------------|---|---|--|--|--|--|--|
| $f_{1^-}^{\#1} \alpha$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1^{-}\alpha}^{\#2}$ | 0 | 0 | 0 | $-\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$ | 0 | 0 | 0 |
| $\omega_{1^{-}}^{\#1}{}_{\alpha}$ | 0 | 0 | 0 | $\frac{1}{4} \left(\alpha_0 - 4 \beta_1 \right)$ | $-\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$ | 0 | $\frac{1}{2}\bar{l}(\alpha_0-4\beta_1)k$ |
| $f_{1}^{\#1}{}_{lphaeta}$ | $\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_1^{\#_2}$ | $\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_1^{\#1}_{+}{}_{\alpha\beta}$ | $\frac{1}{1}(\alpha_0-4\beta_1)$ | $\frac{\alpha_0 - 4 \beta_1}{2 \sqrt{2}}$ | $-\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| | $\omega_1^{#1} + \alpha^{\beta}$ | $\omega_1^{#2} + \alpha \beta$ | $f_{1}^{\#1} + \alpha \beta$ | $\omega_{1}^{\#1} +^{\alpha}$ | $\omega_1^{\#2} +^{lpha}$ | $f_{1^{\bar{-}}}^{\#1} \dagger^{\alpha}$ | $f_1^{#2} + \alpha$ |

| $f_{1^-}^{\#2} lpha$ | 0 | 0 | 0 | $-\frac{1}{2}\bar{l}(\alpha_0-4eta)$ | 0 | 0 | 0 |
|---|--|---|---|--|--|-------|---|
| $f_{1^{}}^{\#1}\alpha$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1^{\bar{-}}\alpha}^{\#2} f_{1^{\bar{-}}\alpha}^{\#1}$ | 0 | 0 | 0 | $-\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$ | 0 | 0 | 0 |
| $\omega_{1^{^{-}}\alpha}^{\#1}$ | 0 | 0 | 0 | $\frac{1}{4} \left(\alpha_0 - 4 \beta_1 \right)$ | $-\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$ | 0 | $\frac{1}{2}\tilde{l}\left(\alpha_0-4eta_1\right)k$ |
| | | | | | | | |
| $f_1^{\#1}$ | $\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1}^{\#2}{}_{\alpha\beta}$ $f_{1}^{\#1}{}_{\alpha\beta}$ | <u> </u> | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 | 0 0 |
| $\omega_{1+lphaeta}^{\#1}$ $\omega_{1+lphaeta}^{\#2}$ $f_{1+lphaeta}^{\#1}$ | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$ | $\frac{\alpha_0-4\beta_1}{2\sqrt{2}}\qquad \qquad 0\qquad \qquad 0$ | $-\frac{i(\alpha_0-4\beta_1)^k}{2\sqrt{2}} \qquad 0 \qquad 0$ | 0 0 0 | 0 0 0 | 0 0 0 | 0 |





Unitarity conditions $\alpha_0 > 0 \&\& \alpha_6 > 0 \&\& \beta_1 < 0 \mid |\beta_1 > \frac{\alpha_0}{4}$

| Lagrangian density | $-\frac{1}{2}\alpha_0\ \omega_{\alpha\chi\beta}\ \omega^{\alpha\beta\chi} - \frac{1}{2}\alpha_0\ \omega^{\alpha\beta}\ \omega^{\chi}_{\beta\ \chi} + 2\beta_1\ \omega^{\alpha\beta}_{\alpha}\ \omega^{\chi}_{\beta\ \chi} - 2\beta_1\ \omega^{\chi\delta}_{\alpha}\ \omega^{\chi\delta}_{\chi\delta} +$ | $f^{\alpha\beta} \tau_{\alpha\beta} + \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - 2\beta_1 \omega_{\alpha\chi}^{\chi} \partial_{\beta} f^{\alpha\beta} - 2\beta_1 \omega_{\alpha\delta}^{\delta} \partial_{\beta} f^{\alpha\beta} - \alpha_0 f^{\alpha\beta} \partial_{\beta} \omega_{\alpha\chi}^{\chi} +$ | $\alpha_0 \partial_\beta \omega^{\alpha\beta}_{\ \alpha} + 2 \beta_1 \omega^{ \chi}_{\beta \chi} \partial^\beta f^\alpha_{\ \alpha} + 2 \beta_1 \omega^{ \delta}_{\beta \delta} \partial^\beta f^\alpha_{\ \alpha} - 2 \beta_1 \partial_\beta f^\chi_{\ \chi} \partial^\beta f^\alpha_{\ \alpha} +$ | $\alpha_0 \ f^{\alpha\beta} \ \partial_\chi \omega_{\alpha \ \beta}^{\ \chi} - \alpha_0 \ f^{\alpha}_{\ \alpha} \ \partial_\chi \omega^{\beta\chi}_{\ \beta} + 4 \ \beta_1 \ \omega_{\alpha\chi\beta} \ \partial^\chi f^{\alpha\beta} + \beta_1 \ \partial_\chi f_{\ \beta}^{\ \delta} + \beta_{\delta} + \beta_{\delta}$ | $\beta_1 \partial_\chi f^\delta_{\ \beta} \partial^\chi f^\beta_{\ \delta} + 4 \beta_1 \partial^\beta f^\alpha_{\ \alpha} \partial_\delta f^\beta_{\ \beta} - 2 \beta_1 \partial_\beta f^\beta_{\ \chi} \partial_\delta f^{\chi\delta} + \tfrac{2}{3} \alpha_6 \partial_\beta \omega^{\alpha\beta}_{\ \alpha} \partial_\delta \omega^{\chi\delta}_{\ \chi} -$ | $\beta_1 \partial^\chi f_\zeta^{\ \beta} \partial^\zeta f_{\beta\chi} - \beta_1 \partial^\chi f_\zeta^{\ \beta} \partial^\zeta f_{\chi\beta} + \beta_1 \partial^\chi f_{\delta\zeta} \partial^\zeta f^\delta_{\ \chi} - \beta_1 \partial^\chi f_{\zeta\delta} \partial^\zeta f^\delta_{\ \chi}$ | |
|--------------------|---|---|---|--|---|---|--|
|--------------------|---|---|---|--|---|---|--|

| | $\sigma^{\sharp 1}_{0^+}$ | $	au_{0}^{\#1}$ | $	au_0^{\#2}$ | $\sigma_{0}^{\#1}$ |
|-------------------------|--|---|---------------|----------------------------------|
| $\sigma_{0}^{\#1}$ † | $\frac{8 \beta_1}{\alpha_0^2 - 4 \alpha_0 \beta_1 + 8 \alpha_6 \beta_1 k^2}$ | $-\frac{i\sqrt{2} (\alpha_0-4\beta_1)}{\alpha_0 (\alpha_0-4\beta_1)k+8\alpha_6\beta_1 k^3}$ | 0 | 0 |
| $\tau_{0}^{\#1}$ † | $\frac{i \sqrt{2} (\alpha_0 - 4 \beta_1)}{\alpha_0 (\alpha_0 - 4 \beta_1) k + 8 \alpha_6 \beta_1 k^3}$ | $-\frac{\alpha_0 - 4 \beta_1 + 2 \alpha_6 k^2}{k^2 (\alpha_0^2 - 4 \alpha_0 \beta_1 + 8 \alpha_6 \beta_1 k^2)}$ | 0 | 0 |
| $	au_{0}^{\#2} \dagger$ | 0 | 0 | 0 | 0 |
| $\sigma_{0}^{\#1}$ † | 0 | 0 | 0 | $\frac{2}{\alpha_0 - 4 \beta_1}$ |

| | | # | 1 | \mathcal{C} | 3 | \mathcal{C} | 1 |
|--|--------------------|--------------|--------------------|--|----------------------------------|---|----------|
| $\beta_1 \ \omega_{\alpha\chi\beta} \ \partial^{\chi} f^{\alpha\beta} + \beta_1 \ \partial_{\chi} f_{\beta}^{\ \delta} \partial^{\chi} f_{\beta}^{\ \beta} +$ $(-2)^{2} \beta_1 \partial_{\beta} f_{\chi}^{\ \beta} \partial_{\delta} f^{\chi\delta} + \frac{2}{3} \alpha_6 \partial_{\beta} \omega^{\alpha\beta}_{\alpha} \partial_{\delta} \omega^{\chi\delta}_{\chi} -$ $(-2)^{3} \beta_{\chi} f_{\delta\zeta} \partial^{\zeta} f^{\delta}_{\chi} - \beta_1 \partial^{\chi} f_{\zeta\delta} \partial^{\zeta} f^{\delta}_{\chi}$ | Source constraints | SO(3) irreps | $\tau_0^{#2} == 0$ | $\tau_{1}^{\#2}{}^{\alpha} + 2ik \ \sigma_{1}^{\#2}{}^{\alpha} == 0 \ 3$ | $\tau_{1}^{\#1}{}^{\alpha} == 0$ | $\tau_{1}^{\#1}\alpha\beta + ik \ \sigma_{1}^{\#2}\alpha\beta == 0 \ 3$ | Total #: |
| $\beta_1 \ \omega_{\alpha\chi\beta} \ \partial^{\chi} f^{\alpha\beta} + \beta_1 \ \partial_{\chi} f_{\beta}^{\ \delta} $ $(5)^{2} \beta_1 \ \partial_{\beta} f_{\chi}^{\ \beta} \ \partial_{\delta} f^{\chi\delta} + \frac{2}{3} \ \alpha_6 \ \partial_{\beta} u$ $(5)^{3} f_{\delta\zeta} \partial^{\zeta} f^{\delta} - \beta_1 \ \partial^{\chi} f_{\zeta\delta} \partial^{\zeta} f^{\delta}$ | $\omega_{0}^{\#1}$ | , (| 0 | 0 | 0 | 0 $\frac{1}{2} (\alpha_0 - 4 \beta_1)$ | |
| β_1 ($\frac{2}{2}\beta_1$ | #5 0+ | | 0 | 0 | 0 | 0 | |

-4 $\beta_1 k^2$

0

 $\omega_{0}^{#1} + f_{0}^{#1} + f_{0}^{#1} + f_{0}^{#2} + f_{0}^{#2} + \omega_{0}^{#1} + f_{0}^{#1} + f_{$

 $\frac{\alpha_0}{2} - 2\beta_1 + \alpha_6 k^2$

 $\omega_{0}^{\#1}$