Particle spectrograph

Wave operator and propagator

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\sigma_{0^+}^{\#1} == 0$	$\partial_\beta \sigma^{\alpha\beta}{}_\alpha == 0$	1
$\tau_{0^+}^{\#1} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha{}_\alpha$	1
$\tau_{0^+}^{\#2} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$	1
$\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#1\,\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} + 2\,(\partial_\delta \partial^\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi}{}_\beta - \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi} + \partial_\delta \partial^\delta \partial_\chi \partial^\chi \sigma^{\alpha\beta}{}_\beta) == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta}$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$	3
$\sigma_{1^-}^{\#1\,\alpha} == \sigma_{1^-}^{\#2\,\alpha}$	$\partial_\chi \partial^\alpha \sigma^{\beta\chi}{}_\beta + \partial_\chi \partial^\chi \sigma^{\alpha\beta}{}_\beta == 0$	3
$\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$	$\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\chi\alpha} + \partial_\chi \partial^\chi \tau^{\alpha\beta} + 2\,\partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 2\,\partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} + \partial_\chi \partial^\chi \tau^{\beta\alpha} + 2\,\partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$	3
$\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$	$-i\,(4\,\partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2\,\partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi{}_\chi - 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\beta} - 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi} - 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} + 4\,i\,k^\chi\,\partial_\epsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta\epsilon}{}_\delta - 6\,i\,k^\chi\,\partial_\epsilon \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\delta\epsilon} - 6\,i\,k^\chi\,\partial_\epsilon \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} + 6\,i\,k^\chi\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\alpha\delta\beta} + 6\,i\,k^\chi\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\beta\delta\alpha} - 2\,\eta^{\alpha\beta}\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \tau^\chi{}_\chi - 4\,i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi \partial^\phi \partial_\epsilon \partial_\chi \sigma^{\delta\epsilon}{}_\delta) == 0$	5
Total constraints/gauge generators:		20

Quadratic (free) action

$$\begin{aligned}
\mathcal{S} == \iiiint \Big(&\tfrac{1}{6}\,(2\,t_1\,\omega^{\alpha\prime}{}_\alpha\,\omega_\prime{}^\theta{}_\theta + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 4\,t_1\,\omega_\alpha{}^\theta{}_\theta\,\partial_\prime f^{\alpha\prime} + \\
&4\,t_1\,\omega_\prime{}^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha - 2\,t_1\,\partial_\prime f^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha - 6\,r_1\,\partial_\beta \omega_\prime{}^\theta{}_\theta\,\partial^\prime \omega^{\alpha\beta}{}_\alpha + \\
&6\,r_1\,\partial_\prime \omega_\beta{}^\theta{}_\theta\,\partial^\prime \omega^{\alpha\beta}{}_\alpha - 2\,t_1\,\partial_\prime f^{\alpha\prime}\,\partial_\theta f_\alpha{}^\theta + 4\,t_1\,\partial^\prime f^\alpha{}_\alpha\,\partial_\theta f_\prime{}^\theta + \\
&6\,r_1\,\partial_\alpha \omega^{\alpha\beta\prime}\,\partial_\theta \omega_\beta{}^\theta{}_\prime - 12\,r_1\,\partial^\prime \omega^{\alpha\beta}{}_\alpha\,\partial_\theta \omega_\beta{}^\theta{}_\prime - \\
&6\,r_1\,\partial_\alpha \omega^{\alpha\beta\prime}\,\partial_\theta \omega_\prime{}^\theta{}_\beta + 12\,r_1\,\partial^\prime \omega^{\alpha\beta}{}_\alpha\,\partial_\theta \omega_\prime{}^\theta{}_\beta - \\
&6\,t_1\,\partial_\alpha f_{\prime\theta}\,\partial^\theta f^{\alpha\prime} - 3\,t_1\,\partial_\alpha f_{\theta\prime}\,\partial^\theta f^{\alpha\prime} + 3\,t_1\,\partial_\prime f_{\alpha\theta}\,\partial^\theta f^{\alpha\prime} + \\
&3\,t_1\,\partial_\theta f_{\alpha\prime}\,\partial^\theta f^{\alpha\prime} + 3\,t_1\,\partial_\theta f_{\prime\alpha}\,\partial^\theta f^{\alpha\prime} + \\
&6\,t_1\,\omega_{\alpha\theta\prime}\,(\omega^{\alpha\prime\theta} + 2\,\partial^\theta f^{\alpha\prime}) - 8\,r_1\,\partial_\beta \omega_{\alpha\prime\theta}\,\partial^\theta \omega^{\alpha\beta\prime} + \\
&4\,r_1\,\partial_\beta \omega_{\alpha\theta\prime}\,\partial^\theta \omega^{\alpha\beta\prime} - 16\,r_1\,\partial_\beta \omega_{\prime\theta\alpha}\,\partial^\theta \omega^{\alpha\beta\prime} - \\
&4\,r_1\,\partial_\prime \omega_{\alpha\beta\theta}\,\partial^\theta \omega^{\alpha\beta\prime} + 4\,r_1\,\partial_\theta \omega_{\alpha\beta\prime}\,\partial^\theta \omega^{\alpha\beta\prime} + \\
&4\,r_1\,\partial_\theta \omega_{\alpha\prime\beta}\,\partial^\theta \omega^{\alpha\beta\prime})\Big)[t, x, y, z]\,dt\,dz\,dy\,dx\,dt
\end{aligned}$$

	$\sigma_{1^+}^{\#1\,\alpha\beta}$	$\sigma_{1^+}^{\#2\,\alpha\beta}$	$\tau_{1^+}^{\#1\,\alpha\beta}$	$\sigma_{1^-}^{\#1\,\alpha}$	$\sigma_{1^-}^{\#2\,\alpha}$	$\tau_{1^-}^{\#1\,\alpha}$	$\tau_{1^-}^{\#2\,\alpha}$
$\sigma_{1^+}^{\#1\,\alpha\beta\,\dagger}$	0	$-\frac{\sqrt{2}}{t_1 + k^2 t_1}$	$-\frac{i\sqrt{2}\,k}{t_1 + k^2 t_1}$	0	0	0	0
$\sigma_{1^+}^{\#2\,\alpha\beta\,\dagger}$	$-\frac{\sqrt{2}}{t_1 + k^2 t_1}$	$\frac{-2k^2 r_1 + t_1}{(1+k^2)^2 t_1^2}$	$-\frac{i(2k^3 r_1 - k t_1)}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\tau_{1^+}^{\#1\,\alpha\beta\,\dagger}$	$\frac{i\sqrt{2}\,k}{t_1 + k^2 t_1}$	$\frac{i(2k^3 r_1 - k t_1)}{(1+k^2)^2 t_1^2}$	$\frac{-2k^4 r_1 + k^2 t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\sigma_{1^-}^{\#1\,\alpha\,\dagger}$	0	0	0	$\frac{6}{(3+4k^2)^2 t_1}$	$\frac{6\sqrt{2}}{(3+4k^2)^2 t_1}$	0	$\frac{12ik}{(3+4k^2)^2 t_1}$
$\sigma_{1^-}^{\#2\,\alpha\,\dagger}$	0	0	0	$\frac{6\sqrt{2}}{(3+4k^2)^2 t_1}$	$\frac{12}{(3+4k^2)^2 t_1}$	0	$\frac{12i\sqrt{2}\,k}{(3+4k^2)^2 t_1}$
$\tau_{1^-}^{\#1\,\alpha\,\dagger}$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#2\,\alpha\,\dagger}$	0	0	0	$-\frac{12ik}{(3+4k^2)^2 t_1}$	$-\frac{12i\sqrt{2}\,k}{(3+4k^2)^2 t_1}$	0	$\frac{24k^2}{(3+4k^2)^2 t_1}$

	$\omega_{1^+}^{\#1\,\alpha\beta}$	$\omega_{1^+}^{\#2\,\alpha\beta}$	$f_{1^+}^{\#1\,\alpha\beta}$	$\omega_{1^-}^{\#1\,\alpha}$	$\omega_{1^-}^{\#2\,\alpha}$	$f_{1^-}^{\#1\,\alpha}$	$f_{1^-}^{\#2\,\alpha}$
$\omega_{1^+}^{\#1\,\alpha\beta\,\dagger}$	$k^2 r_1 - \frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1^+}^{\#2\,\alpha\beta\,\dagger}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1^+}^{\#1\,\alpha\beta\,\dagger}$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_{1^-}^{\#1\,\alpha\,\dagger}$	0	0	0	$\frac{t_1}{6}$	$\frac{t_1}{3\sqrt{2}}$	0	$\frac{ikt_1}{3}$
$\omega_{1^-}^{\#2\,\alpha\,\dagger}$	0	0	0	$\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	0	$\frac{1}{3}i\sqrt{2}\,kt_1$
$f_{1^-}^{\#1\,\alpha\,\dagger}$	0	0	0	0	0	0	0
$f_{1^-}^{\#2\,\alpha\,\dagger}$	0	0	0	$-\frac{1}{3}ikt_1$	$-\frac{1}{3}i\sqrt{2}\,kt_1$	0	$\frac{2k^2 t_1}{3}$

	$\sigma_{2^+}^{\#1\,\alpha\beta}$	$\tau_{2^+}^{\#1\,\alpha\beta}$	$\sigma_{2^+}^{\#2\,\alpha\beta}$
$\sigma_{2^+}^{\#1\,\alpha\beta\,\dagger}$	$\frac{2}{2k^2 r_1 + t_1}$	0	0
$\tau_{2^+}^{\#1\,\alpha\beta\,\dagger}$	0	$\frac{4k^2}{(1+2k^2)^2 t_1}$	$\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$
$\sigma_{2^+}^{\#2\,\alpha\beta\,\dagger}$	0	$\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$	$\frac{2}{(1+2k^2)^2 t_1}$

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$f_{0^+}^{\#1}$	$\omega_{0^+}^{\#2}$
$\omega_{0^+}^{\#1\,\dagger}$	0	0	0	0
$f_{0^+}^{\#1\,\dagger}$	0	0	0	0
$f_{0^+}^{\#2\,\dagger}$	0	0	0	0
$\omega_{0^+}^{\#2\,\dagger}$	0	0	0	$-t_1$

	$\tau_{2^+}^{\#1\,\alpha\beta}$	$\sigma_{2^+}^{\#2\,\alpha\beta}$	$\sigma_2^{\#1\,\alpha\beta\chi}$
$\tau_{2^+}^{\#1\,\alpha\beta\,\dagger}$	$\frac{4k^2}{(1+2k^2)^2 t_1}$	$\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$	0
$\sigma_{2^+}^{\#2\,\alpha\beta\,\dagger}$	$\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$	$\frac{2}{(1+2k^2)^2 t_1}$	0
$\sigma_2^{\#1\,\alpha\beta\chi\,\dagger}$	0	0	$\frac{2}{2k^2 r_1 + t_1}$

	$\omega_{0^+}^{\#1}$	$\tau_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^+}^{\#1}$
$\omega_{0^+}^{\#1\,\dagger}$	0	0	0	0
$\tau_{0^+}^{\#1\,\dagger}$	0	0	0	0
$\tau_{0^+}^{\#2\,\dagger}$	0	0	0	0
$\sigma_{0^+}^{\#1\,\dagger}$	0	0	0	$-\frac{1}{t_1}$

Massive and massless spectra

Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2 r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0 \;\&\&\; t_1 > 0$