In[•]:=
```
Get@FileNameJoin@{NotebookDirectory[], "Calibration.m"};
```

```
-------------------------------------------------------------

Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}

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Connecting to external linux executable...

Connection established.

-------------------------------------------------------------

Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}

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-------------------------------------------------------------

Package xAct`xPlain` version 1.0.0-developer, {2023, 5, 24}

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-------------------------------------------------------------
```

PSALTer Calibration

Key observation: During the calibration run, we need to write some commentary, which will appear in this green text, or as numbered equations/expressions with a green background. The output of the PSALTer package (specifically the function called ParticleSpectrum) is not in green, thus wherever we are using PSALTer the output should be quite distinctive.

The first step is to load the PSALTer package.

```
-------------------------------------------------------------

Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}

CopyRight (C) 2011-2021, Thomas Bäckdahl, under the General Public License.

-------------------------------------------------------------

Package xAct`xPert`  version 1.0.6, {2018, 2, 28}

CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia
  and Guillermo A. Mena Marugan, under the General Public License.

** Variable $CovDFormat changed from Prefix to Postfix

** Option AllowUpperDerivatives of ContractMetric changed from False to True

** Option MetricOn of MakeRule changed from None to All

** Option ContractMetrics of MakeRule changed from False to True

-------------------------------------------------------------

Package xAct`Invar`  version 2.0.5, {2013, 7, 1}

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  D. Yllanes and R. Portugal, under the General Public License.

** DefConstantSymbol: Defining constant symbol sigma.
```

```
** DefConstantSymbol: Defining constant symbol dim.

** Option CurvatureRelations of DefCovD changed from True to False

** Variable $CommuteCovDsOnScalars changed from True to False

-------------------------------------------------------------

Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}

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-------------------------------------------------------------

Package xAct`xTras`  version 1.4.2, {2014, 10, 30}

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** Variable $CovDFormat changed from Postfix to Prefix

** Option CurvatureRelations of DefCovD changed from False to True

-------------------------------------------------------------

Package xAct`PSALTer` version 1.0.0-developer, {2023, 6, 18}

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  Buttigieg, Claire Rigouzzo and Cillian Rew, under the General Public License.

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-------------------------------------------------------------
```

Great, so PSALTer is now loaded and we can start to do some science.

Metric affine gravity (MAG)

Key observation: We will test the MetricAffineGaugeTheory module.

Field strength tensors

Key observation: In this section we will try our analysis of the metric affine gauge theory (MAGT). Our attempt closely follows the very wonderful paper arXiv:1912.01023 which was first brought to my attention by Claire Rigouzzo. The current MAGT implementation in PSALTer follows (to the letter) the conventions established in this paper. We will attempt to recover some key results in this paper, but we will also later look at arXiv:2110.14788, which was brought to my attention by Sebastian Zell.

First we write out rules which define the field strength tensors.

We want to define the curvature in Equation (2.1) on page 4 of arXiv:1912.01023.

```
** DefTensor: Defining tensor MetricAffineCurvature[-m, -n, r, -s].
```

$$\mathcal{F}_{mn\ s}^{\ \ r} \tag{1}$$

$$\mathcal{A}_{m\ q}^{\ \ r}\ \mathcal{A}_{n\ s}^{\ \ q} - \mathcal{A}_{m\ s}^{\ \ q}\ \mathcal{A}_{n\ q}^{\ \ r} + \partial_m \mathcal{A}_{n\ s}^{\ \ r} - \partial_n \mathcal{A}_{m\ s}^{\ \ r} \tag{2}$$

Next want to define the torsion in Equation (2.2) on page 5 of arXiv:1912.01023.

```
** DefTensor: Defining tensor MetricAffineTorsion[-m, a, -n].
```

$$\mathcal{T}_{m\ n}^{\ a} \tag{3}$$

$$\mathcal{A}_{m\ n}^{\ a} - \mathcal{A}_{n\ m}^{\ a} \tag{4}$$

And finally the non-metricity in Equation (2.3) on page 5 of arXiv:1912.01023. Watch out for the trivial misprint in the trace valence. Also, since the non-metricity only appears via quadratic invariants we don't need to bother about perturbing the metric here.

```
** DefTensor: Defining tensor MetricAffineNonMetricity[-l, -m, -n].
```

$$Q_{lmn} \tag{5}$$

$$\mathcal{A}_{lmn} + \mathcal{A}_{lnm} - \partial_l h_{mn} \tag{6}$$

Now we move on to computing the seven contractions defined in Equation (2.5) on page 5 of arXiv:1912.01023. Most of these contractions only appear in quadratic invariants, so we only need these formulae to be accurate to first order in small quantities.

First comes the torsion contraction.

```
** DefTensor: Defining tensor MetricAffineTorsionContraction[-m].
```

$$\mathcal{T}_m \tag{7}$$

$$\mathcal{T}_{a\ m}^{\ a} \tag{8}$$

Next the (standard) non-metricity contraction.

```
** DefTensor: Defining tensor MetricAffineNonMetricityContraction[-m].
```

$$Q_m \tag{9}$$

$$Q_{ma}^{\ \ a} \tag{10}$$

Next the (tilde) non-metricity contraction.

```
** DefTensor: Defining tensor MetricAffineNonMetricityContractionTilde[-m].
```

$$\tilde{Q}_m \tag{11}$$

$$Q_{a\ m}^{\ a} \tag{12}$$

Next the (conventional) Ricci tensor.

```
** DefTensor: Defining tensor MetricAffineRicciTensor[-m, -n].
```

$$\mathcal{F}_{mn} \tag{13}$$

$$\mathcal{F}_{mna}^{\ \ \ a} \tag{14}$$

Next the first of the pseudo-Ricci tensors.

```
** DefTensor: Defining tensor MetricAffineRicciTensor14[-m, -n].
```

$$\mathcal{F}^{(14)}{}_{mn} \tag{15}$$

$$\mathcal{F}_{amn}{}^{a} \tag{16}$$

Next the second of the pseudo-Ricci tensors.

```
** DefTensor: Defining tensor MetricAffineRicciTensor13[-m, -n].
```

$$\mathcal{F}^{(13)}{}_{mn} \tag{17}$$

$$\mathcal{F}_{am\ n}{}^{a} \tag{18}$$

Now we move on to computing the (conventional) Ricci scalar. This time we need to be careful to retain contributions up to second order in smallness, since this is the only invariant which appears on its own.

```
** DefTensor: Defining tensor MetricAffineRicciScalar[].
```

$$\mathcal{F} \tag{19}$$

$$\left(\left(\eta^{ab}-h^{ab}\right)\mathcal{F}_{ca\ b}{}^{c}\right) \tag{20}$$

The general parity-preserving Lagrangian

Now all the generally-covariant contractions of the field strength tensors have been defined, so we construct the general, parity-preserving Lagrangian proposed in Equation (2.4) on page 5 of arXiv:1912.01023. This is just an exercise in data entry.

$$\begin{aligned}
&\frac{1}{2}\Big(-\mathcal{F}^{mnrs}\left(c_{1}.\,\mathcal{F}_{mnrs}+c_{2}.\,\mathcal{F}_{mnsr}+c_{4}.\,\mathcal{F}_{mrns}+c_{5}.\,\mathcal{F}_{msnr}+c_{6}.\,\mathcal{F}_{msrn}+c_{3}.\,\mathcal{F}_{rsmn}\right)-\\
&\quad\left(a_{5}.\,Q_{nmr}+a_{4}.\,Q_{rmn}\right)Q^{rmn}-a_{6}.\,Q_{m}\;Q^{m}-a_{8}.\,Q_{m}\;\tilde{Q}^{m}-a_{7}.\,\tilde{Q}_{m}\;\tilde{Q}^{m}+a_{0}.\mathcal{F}-c_{16}.\mathcal{F}^{2}-\\
&\quad\mathcal{F}^{(13)mn}\left(c_{7}.\,\mathcal{F}^{(13)}{}_{mn}+c_{8}.\,\mathcal{F}^{(13)}{}_{nm}\right)-\mathcal{F}^{mn}\left(c_{13}.\,\mathcal{F}_{mn}+c_{14}.\,\mathcal{F}^{(13)}{}_{mn}+c_{15}.\,\mathcal{F}^{(14)}{}_{mn}\right)-\\
&\quad\left(c_{11}.\,\mathcal{F}^{(13)}{}_{mn}+c_{12}.\,\mathcal{F}^{(13)}{}_{nm}\right)\mathcal{F}^{(14)mn}-\mathcal{F}^{(14)mn}\left(c_{9}.\,\mathcal{F}^{(14)}{}_{mn}+c_{10}.\,\mathcal{F}^{(14)}{}_{nm}\right)-\\
&\quad a_{9}.\,Q_{mrn}\;\mathcal{T}^{mrn}-\left(a_{2}.\,\mathcal{T}_{mnr}+a_{1}.\,\mathcal{T}_{mrn}\right)\mathcal{T}^{mrn}-\left(a_{10}.\,Q_{m}+a_{11}.\,\tilde{Q}_{m}\right)\mathcal{T}^{m}-a_{3}.\,\mathcal{T}_{m}\;\mathcal{T}^{m}\Big)
\end{aligned} \tag{21}$$

This general Lagrangian is something that we must linearize. First, we need the linearized measure, otherwise the Einstein--Hilbert term (which has first-order perturbed contributions) won't have the right linearization.

$$1+\frac{h^{a}{}_{a}}{2} \tag{22}$$

```
** DefConstantSymbol: Defining constant symbol PerturbativeParameter.
```

Now we attempt the linearization.

$$\begin{aligned}
&\left(a_{10}.-\frac{a_{3}.}{2}-2\,a_{6}.\right)\mathcal{A}_{a\ c}{}^{c}\;\mathcal{A}^{ab}{}_{b}+\left(-a_{1}.-a_{4}.-\frac{a_{9}.}{2}\right)\mathcal{A}_{abc}\;\mathcal{A}^{abc}+\frac{1}{2}\left(-a_{2}.-2\,a_{4}.-a_{9}.\right)\mathcal{A}_{acb}\;\mathcal{A}^{abc}+\\
&\frac{1}{2}\left(-a_{2}.-a_{5}.\right)\mathcal{A}^{abc}\;\mathcal{A}_{bac}+\frac{1}{2}\left(-a_{0}.+2\,a_{2}.-2\,a_{5}.+a_{9}.\right)\mathcal{A}^{abc}\;\mathcal{A}_{bca}+\left(-a_{10}.+\frac{a_{11}.}{2}+a_{3}.-a_{8}.\right)\mathcal{A}^{a\ b}_{a}\;\mathcal{A}_{b\ c}{}^{c}+
\end{aligned}$$

$$\frac{1}{2}\left(a_{11}-2a_{8}\right)\mathcal{A}^{ab}{}_{a}\ \mathcal{A}_{b}{}^{c}{}_{c}+\frac{1}{2}\left(2a_{1}-a_{5}+a_{9}\right)\mathcal{A}^{abc}\ \mathcal{A}_{cba}+\frac{1}{2}\left(a_{0}-a_{11}-2a_{7}\right)\mathcal{A}^{a}{}_{a}{}^{b}\ \mathcal{A}^{c}{}_{bc}-$$

$$\frac{1}{2}a_{7}\,\mathcal{A}^{ab}{}_{a}\ \mathcal{A}^{c}{}_{bc}+\frac{1}{2}\left(-a_{11}-a_{3}-a_{7}\right)\mathcal{A}^{a}{}_{a}{}^{b}\ \mathcal{A}^{c}{}_{cb}+\frac{1}{2}\left(4a_{4}+a_{9}\right)\mathcal{A}^{abc}\ \partial_{a}h_{bc}+\left(-\frac{a_{10}}{2}+2a_{6}\right)\mathcal{A}^{ab}{}_{b}\ \partial_{a}h^{c}{}_{c}-$$

$$\frac{1}{4}a_{0}\,h^{c}{}_{c}\ \partial_{b}\mathcal{A}^{a}{}_{a}{}^{b}+\frac{1}{4}a_{0}\,h^{c}{}_{c}\ \partial_{b}\mathcal{A}^{ab}{}_{a}-\frac{1}{2}a_{0}\,h_{ac}\ \partial_{b}\mathcal{A}^{abc}+\frac{1}{2}c_{15}\,\partial^{a}\mathcal{A}^{cd}{}_{d}\ \partial_{b}\mathcal{A}_{ca}{}^{b}+\frac{1}{2}c_{14}\,\partial^{a}\mathcal{A}_{ca}{}^{b}\ \partial_{b}\mathcal{A}^{cd}{}_{d}+$$

$$\frac{1}{2}c_{12}\,\partial^{a}\mathcal{A}^{c}{}_{c}{}^{d}\ \partial_{b}\mathcal{A}_{da}{}^{b}+c_{10}\,\partial^{a}\mathcal{A}^{cd}{}_{c}\ \partial_{b}\mathcal{A}_{da}{}^{b}+a_{5}\,\mathcal{A}^{abc}\ \partial_{b}h_{ac}+\frac{1}{2}\left(a_{10}+a_{8}\right)\mathcal{A}^{a}{}_{a}{}^{b}\ \partial_{b}h^{c}{}_{c}+\frac{1}{2}a_{8}\,\mathcal{A}^{ab}{}_{a}\ \partial_{b}h^{c}{}_{c}-$$

$$\frac{1}{2}a_{6}\,\partial_{b}h^{c}{}_{c}\ \partial^{b}h^{a}{}_{a}+\left(a_{5}-\frac{a_{9}}{2}\right)\mathcal{A}^{abc}\ \partial_{c}h_{ab}+\left(-\frac{a_{11}}{2}+a_{8}\right)\mathcal{A}^{ab}{}_{b}\ \partial_{c}h_{a}{}^{c}+\left(\frac{a_{11}}{2}+a_{7}\right)\mathcal{A}^{a}{}_{a}{}^{b}\ \partial_{c}h_{b}{}^{c}+a_{7}\,\mathcal{A}^{ab}{}_{a}\ \partial_{c}h_{b}{}^{c}-$$

$$\frac{1}{2}a_{7}\,\partial_{a}h^{ab}\ \partial_{c}h_{b}{}^{c}-\frac{1}{2}a_{8}\,\partial^{b}h^{a}{}_{a}\ \partial_{c}h_{b}{}^{c}+\frac{1}{2}a_{0}\,h_{bc}\ \partial^{c}\mathcal{A}^{a}{}_{a}{}^{b}-\frac{1}{2}c_{14}\,\partial_{b}\mathcal{A}_{c}{}^{d}{}_{d}\ \partial^{c}\mathcal{A}^{a}{}_{a}{}^{b}-\frac{1}{2}c_{12}\,\partial_{b}\mathcal{A}^{d}{}_{cd}\ \partial^{c}\mathcal{A}^{a}{}_{a}{}^{b}-$$

$$\frac{1}{2}c_{8}\,\partial_{b}\mathcal{A}^{d}{}_{dc}\ \partial^{c}\mathcal{A}^{a}{}_{a}{}^{b}+\frac{1}{2}c_{14}\,\partial_{c}\mathcal{A}_{b}{}^{d}{}_{d}\ \partial^{c}\mathcal{A}^{a}{}_{a}{}^{b}-\frac{1}{2}c_{11}\,\partial_{c}\mathcal{A}^{d}{}_{bd}\ \partial^{c}\mathcal{A}^{a}{}_{a}{}^{b}-\frac{1}{2}c_{7}\,\partial_{c}\mathcal{A}^{d}{}_{db}\ \partial^{c}\mathcal{A}^{a}{}_{a}{}^{b}-$$

$$\frac{1}{2}c_{15}\,\partial_{b}\mathcal{A}_{c}{}^{d}{}_{d}\ \partial^{c}\mathcal{A}^{ab}{}_{a}-\frac{1}{2}c_{10}\,\partial_{b}\mathcal{A}^{d}{}_{cd}\ \partial^{c}\mathcal{A}^{ab}{}_{a}+\frac{1}{2}c_{15}\,\partial_{c}\mathcal{A}_{b}{}^{d}{}_{d}\ \partial^{c}\mathcal{A}^{ab}{}_{a}-\frac{1}{2}c_{9}\,\partial_{c}\mathcal{A}^{d}{}_{bd}\ \partial^{c}\mathcal{A}^{ab}{}_{a}+$$

$$c_{13}\,\partial_{a}\mathcal{A}_{c}{}^{d}{}_{d}\ \partial^{c}\mathcal{A}^{ab}{}_{b}-c_{13}\,\partial_{c}\mathcal{A}_{a}{}^{d}{}_{d}\ \partial^{c}\mathcal{A}^{ab}{}_{b}-\frac{1}{2}a_{5}\,\partial_{b}h_{ac}\ \partial^{c}h^{ab}-\frac{1}{2}a_{4}\,\partial_{c}h_{ab}\ \partial^{c}h^{ab}-\frac{1}{2}c_{9}\,\partial_{c}\mathcal{A}^{abc}\ \partial_{d}\mathcal{A}_{ab}{}^{d}-$$

$$\frac{1}{2}c_{11}\,\partial_{b}\mathcal{A}^{abc}\ \partial_{d}\mathcal{A}_{ac}{}^{d}-\frac{1}{2}c_{7}\,\partial_{b}\mathcal{A}^{abc}\ \partial_{d}\mathcal{A}_{a}{}^{d}{}_{c}+\frac{1}{2}c_{12}\,\partial^{c}\mathcal{A}_{ba}{}^{b}\ \partial_{d}\mathcal{A}^{ad}{}_{c}-\frac{1}{2}c_{10}\,\partial_{c}\mathcal{A}^{abc}\ \partial_{d}\mathcal{A}_{ba}{}^{d}+$$

$$c_{8}\,\partial^{c}\mathcal{A}^{a}{}_{a}{}^{b}\ \partial_{d}\mathcal{A}_{b}{}^{d}{}_{c}-\frac{1}{2}c_{12}\,\partial_{b}\mathcal{A}^{abc}\ \partial_{d}\mathcal{A}_{ca}{}^{d}+\frac{1}{2}c_{11}\,\partial^{c}\mathcal{A}^{a}{}_{a}{}^{b}\ \partial_{d}\mathcal{A}_{cb}{}^{d}+\frac{1}{2}c_{9}\,\partial^{c}\mathcal{A}^{ab}{}_{a}\ \partial_{d}\mathcal{A}_{cb}{}^{d}-\frac{1}{2}c_{8}\,\partial_{b}\mathcal{A}^{abc}\ \partial_{d}\mathcal{A}_{c}{}^{d}{}_{a}+$$

$$\frac{1}{2}c_{11}\,\partial^{c}\mathcal{A}_{ba}{}^{b}\ \partial_{d}\mathcal{A}_{c}{}^{da}+c_{7}\,\partial^{c}\mathcal{A}^{a}{}_{a}{}^{b}\ \partial_{d}\mathcal{A}_{c}{}^{d}{}_{b}-\frac{1}{2}c_{16}\,\partial_{b}\mathcal{A}^{a}{}_{a}{}^{b}\ \partial_{d}\mathcal{A}^{c}{}_{c}{}^{d}+c_{16}\,\partial_{b}\mathcal{A}^{a}{}_{a}{}^{b}\ \partial_{d}\mathcal{A}^{cd}{}_{c}-\frac{1}{2}c_{16}\,\partial_{b}\mathcal{A}^{ab}{}_{a}\ \partial_{d}\mathcal{A}^{cd}{}_{c}+$$

$$\frac{1}{2}c_{5}\,\partial_{a}\mathcal{A}_{bcd}\ \partial^{d}\mathcal{A}^{abc}+c_{4}\,\partial_{a}\mathcal{A}_{bdc}\ \partial^{d}\mathcal{A}^{abc}+c_{6}\,\partial_{a}\mathcal{A}_{cbd}\ \partial^{d}\mathcal{A}^{abc}+\frac{1}{2}c_{5}\,\partial_{a}\mathcal{A}_{cdb}\ \partial^{d}\mathcal{A}^{abc}+c_{1}\,\partial_{a}\mathcal{A}_{dbc}\ \partial^{d}\mathcal{A}^{abc}+$$

$$c_{2}\,\partial_{a}\mathcal{A}_{dcb}\ \partial^{d}\mathcal{A}^{abc}-\frac{1}{2}c_{5}\,\partial_{b}\mathcal{A}_{acd}\ \partial^{d}\mathcal{A}^{abc}-\frac{1}{2}c_{4}\,\partial_{b}\mathcal{A}_{adc}\ \partial^{d}\mathcal{A}^{abc}-\frac{1}{2}c_{3}\,\partial_{b}\mathcal{A}_{cda}\ \partial^{d}\mathcal{A}^{abc}-\frac{1}{2}c_{6}\,\partial_{c}\mathcal{A}_{abd}\ \partial^{d}\mathcal{A}^{abc}-$$

$$\frac{1}{2}c_{3}\,\partial_{c}\mathcal{A}_{bad}\ \partial^{d}\mathcal{A}^{abc}+c_{3}\,\partial_{c}\mathcal{A}_{bda}\ \partial^{d}\mathcal{A}^{abc}-c_{1}\,\partial_{d}\mathcal{A}_{abc}\ \partial^{d}\mathcal{A}^{abc}-c_{2}\,\partial_{d}\mathcal{A}_{acb}\ \partial^{d}\mathcal{A}^{abc}-\frac{1}{2}c_{4}\,\partial_{d}\mathcal{A}_{bac}\ \partial^{d}\mathcal{A}^{abc}-$$

$$\frac{1}{2}c_{5}\,\partial_{d}\mathcal{A}_{bca}\ \partial^{d}\mathcal{A}^{abc}-\frac{1}{2}c_{6}\,\partial_{d}\mathcal{A}_{cba}\ \partial^{d}\mathcal{A}^{abc}-\frac{1}{2}c_{15}\,\partial_{b}\mathcal{A}_{da}{}^{b}\ \partial^{d}\mathcal{A}^{ac}{}_{c}-\frac{1}{2}c_{14}\,\partial^{a}\mathcal{A}_{da}{}^{b}\ \partial^{d}\mathcal{A}_{b}{}^{c}{}_{c}+\frac{1}{2}c_{9}\,\partial_{b}\mathcal{A}_{da}{}^{b}\ \partial^{d}\mathcal{A}^{ca}{}_{c}$$

We see that there is not a great degree of degeneracy among the coupling constants, but bear in mind that we may only see such patterns when surface terms are used to extract the wave operator.

Key observation: Now we are basically ready to try some spectral analysis, but first we will present the matrices of particle interactions stored in the current MAGT implementation in PSALTer. These are just for reference: when the ParticleSpectrum function is used, much of the output is in the form of matrices, and it can be useful to know which element corresponds to which interaction. The symbols used for the different spin-parity modes are currently a bit cryptic (except for the metric perturbation, which is fairly clear), but the logic is basically as follows. The general connection A is decomposed into parts which are symmetric (denoted Q) and antisymmetric (denoted A again, confusingly) in the second and third indices. These parts are further decomposed into invariant subspaces under the action of the group of spatial rotations. These parts are given with reduced numbers of indices where convenient, and are labelled by spin and parity but also by a cryptic series of superscripts denoting from which Young tableau the mode descends. Zhiyuan is really the expert on decoding these (since he constructed them), but all the modes appear schematically in Table 2, on page 11 of arXiv:1912.01023. So, here are the matrices.

The spin-0 sector. It is pretty big.

$$\begin{pmatrix}
\overset{0}{\cdot}h^{\perp}\,\overset{0}{\cdot}h^{\perp}\dagger & \overset{0}{\cdot}h^{\parallel}\,\overset{0}{\cdot}h^{\perp}\dagger & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}h^{\perp}\dagger & \overset{0}{\cdot}h^{\perp}\dagger\,\overset{0}{\cdot}Q^{\perp t} & \overset{0}{\cdot}h^{\perp}\dagger\,\overset{0}{\cdot}Q^{\parallel} & \overset{0}{\cdot}h^{\perp}\dagger\,\overset{0}{\cdot}Q^{\perp h} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}h^{\perp}\dagger \\
\overset{0}{\cdot}h^{\parallel}\dagger\,\overset{0}{\cdot}h^{\perp} & \overset{0}{\cdot}h^{\parallel}\,\overset{0}{\cdot}h^{\parallel}\dagger & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}h^{\parallel}\dagger & \overset{0}{\cdot}h^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\perp t} & \overset{0}{\cdot}h^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\parallel} & \overset{0}{\cdot}h^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\perp h} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}h^{\parallel}\dagger \\
\overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}h^{\perp} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}h^{\parallel} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger & \overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\perp t} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\parallel} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\perp h} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger \\
\overset{0}{\cdot}h^{\perp}\,\overset{0}{\cdot}Q^{\perp t}\dagger & \overset{0}{\cdot}h^{\parallel}\,\overset{0}{\cdot}Q^{\perp t}\dagger & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}Q^{\perp t}\dagger & \overset{0}{\cdot}Q^{\perp t}\,\overset{0}{\cdot}Q^{\perp t}\dagger & \overset{0}{\cdot}Q^{\parallel}\,\overset{0}{\cdot}Q^{\perp t}\dagger & \overset{0}{\cdot}Q^{\perp h}\,\overset{0}{\cdot}Q^{\perp t}\dagger & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}Q^{\perp t}\dagger \\
\overset{0}{\cdot}h^{\perp}\,\overset{0}{\cdot}Q^{\parallel}\dagger & \overset{0}{\cdot}h^{\parallel}\,\overset{0}{\cdot}Q^{\parallel}\dagger & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}Q^{\parallel}\dagger & \overset{0}{\cdot}Q^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\perp t} & \overset{0}{\cdot}Q^{\parallel}\,\overset{0}{\cdot}Q^{\parallel}\dagger & \overset{0}{\cdot}Q^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\perp h} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}Q^{\parallel}\dagger \\
\overset{0}{\cdot}h^{\perp}\,\overset{0}{\cdot}Q^{\perp h}\dagger & \overset{0}{\cdot}h^{\parallel}\,\overset{0}{\cdot}Q^{\perp h}\dagger & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}Q^{\perp h}\dagger & \overset{0}{\cdot}Q^{\perp h}\dagger\,\overset{0}{\cdot}Q^{\perp t} & \overset{0}{\cdot}Q^{\parallel}\,\overset{0}{\cdot}Q^{\perp h}\dagger & \overset{0}{\cdot}Q^{\perp h}\,\overset{0}{\cdot}Q^{\perp h}\dagger & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}Q^{\perp h}\dagger \\
\overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}h^{\perp} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}h^{\parallel} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}\mathcal{A}^{\parallel} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\perp t} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\parallel} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger\,\overset{0}{\cdot}Q^{\perp h} & \overset{0}{\cdot}\mathcal{A}^{\parallel}\,\overset{0}{\cdot}\mathcal{A}^{\parallel}\dagger
\end{pmatrix} \tag{24}$$

The spin-1 sector. It is vast.

$$\begin{pmatrix}
\overset{1}{\cdot}\mathcal{A}^{\parallel\,ab} & \overset{1}{\cdot}\mathcal{A}^{\parallel}\dagger_{ab} & \overset{1}{\cdot}\mathcal{A}^{\parallel}\dagger^{ab} & \overset{1}{\cdot}\mathcal{A}^{\perp}{}_{ab} & \overset{1}{\cdot}\mathcal{A}^{\parallel}\dagger^{ab} & \overset{1}{\cdot}Q^{\perp}{}_{ab} & \overset{1}{\cdot}\mathcal{A}^{\parallel}\dagger^{bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}h^{\perp\,a} & \overset{1}{\cdot}\mathcal{A}^{\parallel\,a} & \overset{1}{\cdot}\mathcal{A}^{\parallel}\dagger^{bc} & \epsilon^{\parallel}{}_{abc} \\
\overset{1}{\cdot}\mathcal{A}^{\parallel\,ab} & \overset{1}{\cdot}\mathcal{A}^{\perp}\dagger_{ab} & \overset{1}{\cdot}\mathcal{A}^{\perp\,ab} & \overset{1}{\cdot}\mathcal{A}^{\perp}\dagger_{ab} & \overset{1}{\cdot}\mathcal{A}^{\perp}\dagger^{ab} & \overset{1}{\cdot}Q^{\perp}{}_{ab} & \overset{1}{\cdot}\mathcal{A}^{\perp}\dagger^{bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}h^{\perp\,a} & \overset{1}{\cdot}\mathcal{A}^{\perp\,a} & \overset{1}{\cdot}\mathcal{A}^{\perp}\dagger^{bc} & \epsilon^{\parallel}{}_{abc} \\
\overset{1}{\cdot}\mathcal{A}^{\parallel\,ab} & \overset{1}{\cdot}Q^{\perp}\dagger_{ab} & \overset{1}{\cdot}\mathcal{A}^{\perp\,ab} & \overset{1}{\cdot}Q^{\perp}\dagger_{ab} & \overset{1}{\cdot}Q^{\perp\,ab} & \overset{1}{\cdot}Q^{\perp}\dagger_{ab} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}h^{\perp\,a} & \overset{1}{\cdot}Q^{\perp}\dagger^{bc} & \overset{1}{\cdot}\mathcal{A}^{\parallel\,a} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp}\dagger^{bc} & \\
\overset{1}{\cdot}\mathcal{A}^{\parallel\,bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}h^{\perp}\dagger^{a} & \overset{1}{\cdot}\mathcal{A}^{\perp\,bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}h^{\perp}\dagger^{a} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}h^{\perp}\dagger^{a} & \overset{1}{\cdot}Q^{\perp\,bc} & \overset{1}{\cdot}h^{\perp\,a} & \overset{1}{\cdot}h^{\perp}\dagger_{a} & \overset{1}{\cdot}\mathcal{A}^{\parallel\,a} & \overset{1}{\cdot}h^{\perp}\dagger_{a} & \\
\overset{1}{\cdot}\mathcal{A}^{\parallel}\dagger^{a} & \overset{1}{\cdot}\mathcal{A}^{\parallel\,bc} & \epsilon^{\parallel}{}_{abc} & \overset{1}{\cdot}\mathcal{A}^{\parallel}\dagger^{a} & \overset{1}{\cdot}\mathcal{A}^{\perp\,bc} & \epsilon^{\parallel}{}_{abc} & \overset{1}{\cdot}\mathcal{A}^{\parallel}\dagger^{a} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp\,bc} & \overset{1}{\cdot}h^{\perp\,a} & \overset{1}{\cdot}h^{\perp}\dagger_{a} & \overset{1}{\cdot}\mathcal{A}^{\parallel\,a}\,\overset{1}{\cdot}\mathcal{A}^{\parallel}\dagger_{a} \\
\overset{1}{\cdot}\mathcal{A}^{\parallel\,bc} & \overset{1}{\cdot}\mathcal{A}^{\perp}\dagger^{a} & \epsilon^{\parallel}{}_{abc} & \overset{1}{\cdot}\mathcal{A}^{\perp}\dagger^{a} & \overset{1}{\cdot}\mathcal{A}^{\perp\,bc} & \epsilon^{\parallel}{}_{abc} & \overset{1}{\cdot}\mathcal{A}^{\perp}\dagger^{a} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp\,bc} & \overset{1}{\cdot}\mathcal{A}^{\perp}\dagger^{a} & \overset{1}{\cdot}h^{\perp}{}_{a} & \overset{1}{\cdot}\mathcal{A}^{\parallel\,a}\,\overset{1}{\cdot}\mathcal{A}^{\perp}\dagger_{a} \\
\overset{1}{\cdot}\mathcal{A}^{\parallel\,bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp t}\dagger^{a} & \overset{1}{\cdot}\mathcal{A}^{\perp\,bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp t}\dagger^{a} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp\,bc} & \overset{1}{\cdot}Q^{\perp t}\dagger^{a} & \overset{1}{\cdot}h^{\perp\,a} & \overset{1}{\cdot}Q^{\perp t}\dagger_{a} & \overset{1}{\cdot}\mathcal{A}^{\parallel\,a} & \overset{1}{\cdot}Q^{\perp t}\dagger_{a} & \\
\overset{1}{\cdot}\mathcal{A}^{\parallel\,bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\parallel t}\dagger^{a} & \overset{1}{\cdot}\mathcal{A}^{\perp\,bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\parallel t}\dagger^{a} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp\,bc} & \overset{1}{\cdot}Q^{\parallel t}\dagger^{a} & \overset{1}{\cdot}h^{\perp\,a} & \overset{1}{\cdot}Q^{\parallel t}\dagger_{a} & \overset{1}{\cdot}\mathcal{A}^{\parallel\,a} & \overset{1}{\cdot}Q^{\parallel t}\dagger_{a} & \\
\overset{1}{\cdot}\mathcal{A}^{\parallel\,bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp h}\dagger^{a} & \overset{1}{\cdot}\mathcal{A}^{\perp\,bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp h}\dagger^{a} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp\,bc} & \overset{1}{\cdot}Q^{\perp h}\dagger^{a} & \overset{1}{\cdot}h^{\perp\,a} & \overset{1}{\cdot}Q^{\perp h}\dagger_{a} & \overset{1}{\cdot}\mathcal{A}^{\parallel\,a} & \overset{1}{\cdot}Q^{\perp h}\dagger_{a} & \\
\overset{1}{\cdot}\mathcal{A}^{\parallel\,bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\parallel h}\dagger^{a} & \overset{1}{\cdot}\mathcal{A}^{\perp\,bc} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\parallel h}\dagger^{a} & \epsilon^{\parallel}{}_{abc}\,\overset{1}{\cdot}Q^{\perp\,bc} & \overset{1}{\cdot}Q^{\parallel h}\dagger^{a} & \overset{1}{\cdot}h^{\perp\,a} & \overset{1}{\cdot}Q^{\parallel h}\dagger_{a} & \overset{1}{\cdot}\mathcal{A}^{\parallel\,a} & \overset{1}{\cdot}Q^{\parallel h}\dagger_{a} &
\end{pmatrix} \tag{25}$$

The spin-2 sector. It is not too bad.

$$\begin{pmatrix}
\overset{2}{\cdot}h^{\parallel\,ab} & \overset{2}{\cdot}h^{\parallel}\dagger_{ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,ab} & \overset{2}{\cdot}h^{\parallel}\dagger_{ab} & \overset{2}{\cdot}h^{\parallel}\dagger^{ab} & \overset{2}{\cdot}Q^{\parallel}{}_{ab} & \overset{2}{\cdot}h^{\parallel}\dagger^{ab} & \overset{2}{\cdot}Q^{\perp}{}_{ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,abc} & \epsilon^{\parallel}{}_{bcd}\,\overset{2}{\cdot}h^{\parallel} \\
\overset{2}{\cdot}\mathcal{A}^{\parallel}\dagger^{ab} & \overset{2}{\cdot}h^{\parallel}{}_{ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel}\dagger_{ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel}\dagger^{ab} & \overset{2}{\cdot}Q^{\parallel}{}_{ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel}\dagger^{ab} & \overset{2}{\cdot}Q^{\perp}{}_{ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,abc} & \overset{2}{\cdot}\mathcal{A}^{\parallel}\dagger_{c}{}^{d}\ \epsilon \\
\overset{2}{\cdot}h^{\parallel\,ab} & \overset{2}{\cdot}Q^{\parallel}\dagger_{ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,ab} & \overset{2}{\cdot}Q^{\parallel}\dagger_{ab} & \overset{2}{\cdot}Q^{\parallel\,ab} & \overset{2}{\cdot}Q^{\parallel}\dagger_{ab} & \overset{2}{\cdot}Q^{\parallel}\dagger^{ab} & \overset{2}{\cdot}Q^{\perp}{}_{ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,abc} & \epsilon^{\parallel}{}_{bcd}\,\overset{2}{\cdot}Q^{\parallel} \\
\overset{2}{\cdot}h^{\parallel\,ab} & \overset{2}{\cdot}Q^{\perp}\dagger_{ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,ab} & \overset{2}{\cdot}Q^{\perp}\dagger_{ab} & \overset{2}{\cdot}Q^{\parallel\,ab} & \overset{2}{\cdot}Q^{\perp}\dagger_{ab} & \overset{2}{\cdot}Q^{\perp\,ab} & \overset{2}{\cdot}Q^{\perp}\dagger_{ab} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,abc} & \epsilon^{\parallel}{}_{bcd}\,\overset{2}{\cdot}Q^{\perp} \\
\overset{2}{\cdot}\mathcal{A}^{\parallel}\dagger^{abc} & \epsilon^{\parallel}{}_{bcd}\,\overset{2}{\cdot}h^{\parallel}{}_{a}{}^{d} & \overset{2}{\cdot}\mathcal{A}^{\parallel}\dagger^{abc} & \overset{2}{\cdot}\mathcal{A}^{\parallel}{}_{c}{}^{d}\,\epsilon^{\parallel}{}_{abd} & \overset{2}{\cdot}\mathcal{A}^{\parallel}\dagger^{abc} & \epsilon^{\parallel}{}_{bcd}\,\overset{2}{\cdot}Q^{\parallel}{}_{a}{}^{d} & \overset{2}{\cdot}\mathcal{A}^{\parallel}\dagger^{abc} & \epsilon^{\parallel}{}_{bcd}\,\overset{2}{\cdot}Q^{\perp}{}_{a}{}^{d} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,abc} & \overset{2}{\cdot}\mathcal{A}^{\parallel}\dagger_{ab} \\
\epsilon^{\parallel}{}_{bcd}\,\overset{2}{\cdot}h^{\parallel\,ab} & \overset{2}{\cdot}Q^{\parallel}\dagger_{a}{}^{cd} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,ab} & \epsilon^{\parallel}{}_{bcd}\,\overset{2}{\cdot}Q^{\parallel}\dagger_{a}{}^{cd} & \epsilon^{\parallel}{}_{bcd}\,\overset{2}{\cdot}Q^{\parallel}\dagger^{abc} & \overset{2}{\cdot}Q^{\parallel}{}_{a}{}^{d} & \epsilon^{\parallel}{}_{bcd}\,\overset{2}{\cdot}Q^{\parallel}\dagger^{abc} & \overset{2}{\cdot}Q^{\perp}{}_{a}{}^{d} & \overset{2}{\cdot}\mathcal{A}^{\parallel\,abc} & \overset{2}{\cdot}Q^{\parallel}\dagger_{acl}
\end{pmatrix} \tag{26}$$

The spin-3 sector. Yes, despite well-known theorems by Weinberg, Witten etc., there is a higher-spin sector at play! I suppose one must be careful that it does not propagate?

$$\left(\left(\begin{array}{cc} \overset{3}{\cdot}Q^{\parallel\,abc} & \overset{3}{\cdot}Q^{\parallel}\dagger_{abc} \end{array}\right)\right) \tag{27}$$

That deals with all the preliminaries. We can now transition to some spectral analyses.

Einstein-Hilbert theory

The first theory we will look at is the simple Einstein-Hilbert case.

$$\frac{a.\mathcal{F}}{2} \tag{28}$$

Now we linearize it.

$$-\frac{1}{2}a.\,\mathcal{A}^{abc}\;\mathcal{A}_{bca}+\frac{1}{2}a.\,\mathcal{A}^{a\ b}_{\ a}\;\mathcal{A}^{c}_{\ bc}-\frac{1}{4}a.\,h^{c}_{\ c}\;\partial_b\mathcal{A}^{a\ b}_{\ a}+\frac{1}{4}a.\,h^{c}_{\ c}\;\partial_b\mathcal{A}^{ab}_{\ \ a}-\frac{1}{2}a.\,h_{ac}\;\partial_b\mathcal{A}^{abc}+\frac{1}{2}a.\,h_{bc}\;\partial^{c}\mathcal{A}^{a\ b}_{\ a} \tag{29}$$

Now we feed the linearized Lagrangian into PSALTer. This is done with a single call to the ParticleSpectrum function where the linearized Lagrangian above is passed as the input, and it takes 10-15 minutes on my Dutch machine. As usual, the computation of the wave operator matrices and their inverses is basically instantaneous, and almost all of this time is spent in component-value calculations, computing pole residues in the limit of the lightcone.

The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix}
0 & 0 & 0 & 0 & \frac{i a_0 k}{a_0} & -\frac{i a_0 k}{\sqrt{a_0}} & 0 \\
0 & 0 & \frac{i a_0 k}{\sqrt{a_0}} & 0 & -\frac{i a_0 k}{\sqrt{a_0}} & \frac{i a_0 k}{\sqrt{a_0}} & 0 \\
0 & -\frac{i a_0 k}{\sqrt{a_0}} & -\frac{a_0}{a_0} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{a_0}{a_0} & -\frac{a_0}{\sqrt{a_0}} & 0 \\
-i\bar{i}\,a.\,k & \frac{i a_0 k}{\sqrt{a_0}} & 0 & \frac{a_0}{a_0} & 0 & -\frac{a_0}{\sqrt{a_0}} & 0 \\
\frac{i a_0 k}{\sqrt{a_0}} & -\frac{i a_0 k}{\sqrt{a_0}} & 0 & -\frac{a_0}{\sqrt{a_0}} & -\frac{a_0}{\sqrt{a_0}} & \frac{a_0}{a_0} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{a_0}{a_0}
\end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
-\frac{a_0}{a_0} & -\frac{a_0}{\sqrt{a_0}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{a_0}{\sqrt{a_0}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{a_0}{a_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i a_0 k}{\sqrt{a_0}} & 0 & -\frac{i a_0 k}{\sqrt{a_0}} & -i\bar{i}\sqrt{a_0}\,.\,k & -\frac{i a_0 k}{\sqrt{a_0}} & -\frac{i a_0 k}{\sqrt{a_0}} \\
0 & 0 & 0 & -\frac{i a_0 k}{\sqrt{a_0}} & -\frac{a_0}{a_0} & \frac{a_0}{\sqrt{a_0}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{a_0}{\sqrt{a_0}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{i a_0 k}{\sqrt{a_0}} & 0 & 0 & -\frac{a_0}{a_0} & \frac{\sqrt{a_0}}{a_0} & -\frac{a_0}{\sqrt{a_0}} & -\frac{a_0}{a_0} \\
0 & 0 & 0 & -i\bar{i}\sqrt{a_0}\,.\,k & 0 & 0 & \frac{\sqrt{a_0}}{a_0} & \frac{a_0}{a_0} & -\sqrt{a_0}\,.\,\frac{\sqrt{a_0}}{a_0} & \\
0 & 0 & 0 & \frac{i a_0 k}{\sqrt{a_0}} & 0 & 0 & -\frac{a_0}{\sqrt{a_0}} & -\sqrt{a_0}\,.\,\frac{a_0}{a_0} & \frac{a_0}{\sqrt{a_0}} & \\
0 & 0 & 0 & \frac{i a_0 k}{\sqrt{a_0}} & 0 & 0 & -\frac{a_0}{a_0} & \frac{a_0}{\sqrt{a_0}} & \frac{a_0}{a_0} &
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & -\frac{i a_0 k}{\sqrt{\ }} & -\frac{i a_0 k}{\sqrt{\ }} & \frac{i a_0 k}{\sqrt{\ }} & 0 & 0 \\ \frac{i a_0 k}{\sqrt{\ }} & \frac{a_0}{\ } & 0 & 0 & 0 & 0 \\ \frac{i a_0 k}{\sqrt{\ }} & 0 & -\frac{a_0}{\ } & 0 & 0 & 0 \\ -\frac{i a_0 k}{\sqrt{\ }} & 0 & 0 & \frac{a_0}{\ } & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{a_0}{\ } & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{a_0}{\ } \end{pmatrix}$$

Matrix for spin-3 sector:

$$\left(-\frac{a_0}{\ }\right)$$

Gauge constraints on source currents:

$$-6\,i\,\dot{\mathcal{T}}^{\perp} + k\,\dot{\mathcal{Z}}^{\parallel} + 2\,k\,\dot{\mathcal{Z}}^{\perp h} == 0$$

$$2\,i\,\dot{\mathcal{T}}^{\perp} + k\,\dot{\mathcal{Z}}^{\perp t} == 0$$

$$12\,i\,\bar{\mathcal{T}}^{\perp q} + k\left(2\,\bar{\mathcal{Z}}^{\parallel h q} + \bar{\mathcal{Z}}^{\parallel t q} + 3\,\bar{\mathcal{Z}}^{\perp t q} + 6\,\bar{\mathcal{Y}}^{\perp q}\right) == 0$$

$$-6\,i\,\bar{\mathcal{T}}^{\perp q} + k\,\bar{\mathcal{Z}}^{\perp h q} == k\left(\bar{\mathcal{Z}}^{\perp t q} + 3\,\bar{\mathcal{Y}}^{\perp q}\right)$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{k^2}{a_0\left(\ +k^2\right)^2} & \frac{\sqrt{\ }}{a_0+a_0 k^2} & \frac{i\sqrt{\ }k}{a_0+a_0 k^2} & -\frac{i k}{a_0\left(\ +k^2\right)^2} & \frac{i k\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & -\frac{i\sqrt{\ }k\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & 0 \\ \frac{\sqrt{\ }}{a_0+a_0 k^2} & \frac{}{a_0 k^2} & \frac{i\sqrt{\ }}{a_0 k} & \frac{i\sqrt{\ }}{a_0 k+a_0 k^3} & -\frac{i}{\sqrt{\left(\ a_0 k+a_0 k^3\right)}} & -\frac{i\sqrt{\frac{2}{3}}}{a_0 k+a_0 k^3} & 0 \\ -\frac{i\sqrt{\ }k}{a_0+a_0 k^2} & -\frac{i\sqrt{\ }}{a_0 k} & 0 & \frac{\sqrt{\ }}{a_0+a_0 k^2} & -\frac{\sqrt{\frac{2}{3}}}{a_0+a_0 k^2} & -\frac{}{\sqrt{\left(\ a_0+a_0 k^2\right)}} & 0 \\ \frac{i k}{a_0\left(\ +k^2\right)^2} & -\frac{i\sqrt{\ }}{a_0 k+a_0 k^3} & \frac{\sqrt{\ }}{a_0+a_0 k^2} & -\frac{}{a_0\left(\ +k^2\right)^2} & \frac{\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & -\frac{\sqrt{\ }\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & 0 \\ -\frac{i k\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & \frac{i}{\sqrt{\left(\ a_0 k+a_0 k^3\right)}} & -\frac{\sqrt{\frac{2}{3}}}{a_0+a_0 k^2} & \frac{\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & -\frac{\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & -\frac{\sqrt{\ }\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & 0 \\ \frac{i\sqrt{\ }k\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & \frac{i\sqrt{\frac{2}{3}}}{a_0 k+a_0 k^3} & -\frac{}{\sqrt{\left(\ a_0+a_0 k^2\right)}} & -\frac{\sqrt{\ }\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & -\frac{\sqrt{\ }\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & \frac{\left(\ +k^2\right)}{a_0\left(\ +k^2\right)^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{}{a_0} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
0 & -\frac{\sqrt{\ }}{a_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{\sqrt{\ }}{a_0} & \frac{}{a_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{}{a_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{k^2}{a_0 (+k^2)^2} & \frac{i\sqrt{\ }k}{a_0 (+k^2)} & \frac{ik(+k^2)}{a_0 (+k^2)^2} & -\frac{ik(+k^2)}{\sqrt{\ }a_0 (+k^2)^2} & \frac{i\sqrt{\frac{5}{6}}k}{a_0 (+k^2)} & -\frac{ik(+k^2)}{\sqrt{\ }a_0 (+k^2)^2} & \frac{i\sqrt{\frac{2}{3}}k}{a_0 (+k^2)} \\
0 & 0 & 0 & -\frac{i\sqrt{\ }k}{a_0 (+k^2)} & 0 & \frac{\sqrt{\ }(+k^2)}{a_0 (+k^2)} & -\frac{k^2}{\sqrt{\ }a_0 (+k^2)} & 0 & \frac{\sqrt{\frac{2}{3}}k^2}{a_0 (+k^2)} & 0 \\
0 & 0 & 0 & -\frac{ik(+k^2)}{a_0 (+k^2)^2} & \frac{\sqrt{\ }(+k^2)}{a_0 (+k^2)^2} & \frac{(+k^2)^2}{a_0 (+k^2)^2} & \frac{k^2(-+k^2)}{\sqrt{\ }a_0 (+k^2)^2} & -\frac{\sqrt{\frac{5}{6}}k^2}{a_0+a_0k^2} & \frac{k^2(+k^2)}{\sqrt{\ }a_0 (+k^2)^2} & -\frac{k^2}{\sqrt{\ }(a_0+a_0k^2)} \\
0 & 0 & 0 & \frac{ik(+k^2)}{\sqrt{\ }a_0 (+k^2)^2} & -\frac{k^2}{\sqrt{\ }a_0 (+k^2)} & \frac{k^2(-+k^2)}{\sqrt{\ }a_0 (+k^2)^2} & -\frac{+k^2+k^4}{a_0 (+k^2)^2} & \frac{\sqrt{\ }(+k^2)}{a_0 (+k^2)} & \frac{-+k^2}{\sqrt{\ }a_0 (+k^2)^2} & \frac{}{-a_0-\frac{8a_0}{2+3k^2}} \\
0 & 0 & 0 & -\frac{i\sqrt{\frac{5}{6}}k}{a_0+a_0k^2} & 0 & -\frac{\sqrt{\frac{5}{6}}k^2}{a_0+a_0k^2} & \frac{\sqrt{\ }(+k^2)}{a_0 (+k^2)} & \frac{}{a_0} & -\frac{\sqrt{\frac{5}{2}}}{a_0+a_0k^2} & -\frac{\sqrt{\ }}{a_0} \\
0 & 0 & 0 & \frac{ik(+k^2)}{\sqrt{\ }a_0 (+k^2)^2} & \frac{\sqrt{\frac{2}{3}}k^2}{a_0 (+k^2)} & \frac{k^2(+k^2)}{\sqrt{\ }a_0 (+k^2)^2} & \frac{-+k^2}{\sqrt{\ }a_0 (+k^2)^2} & -\frac{\sqrt{\frac{5}{2}}}{a_0+a_0k^2} & \frac{(+k^2+k^4)}{a_0 (+k^2)^2} & -\frac{\sqrt{\ }(+k^2)}{a_0 (+k^2)} \\
0 & 0 & 0 & -\frac{i\sqrt{\frac{2}{3}}k}{a_0+a_0k^2} & 0 & -\frac{k^2}{\sqrt{\ }(a_0+a_0k^2)} & \frac{}{-a_0-\frac{8a_0}{2+3k^2}} & -\frac{\sqrt{\ }}{a_0} & -\frac{\sqrt{\ }(+k^2)}{a_0 (+k^2)} & \frac{}{a_0}
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix}
-\frac{}{a_0 k^2} & -\frac{i\sqrt{\ }}{a_0 k} & \frac{i}{\sqrt{\ }a_0 k} & \frac{i\sqrt{\frac{2}{3}}}{a_0 k} & 0 & 0 \\
\frac{i\sqrt{\ }}{a_0 k} & 0 & \frac{\sqrt{\frac{2}{3}}}{a_0} & \frac{}{\sqrt{\ }a_0} & 0 & 0 \\
-\frac{i}{\sqrt{\ }a_0 k} & \frac{\sqrt{\frac{2}{3}}}{a_0} & -\frac{}{a_0} & -\frac{\sqrt{\ }}{a_0} & 0 & 0 \\
-\frac{i\sqrt{\frac{2}{3}}}{a_0 k} & \frac{}{\sqrt{\ }a_0} & -\frac{\sqrt{\ }}{a_0} & \frac{}{a_0} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{}{a_0} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{}{a_0}
\end{pmatrix}$$

Matrix for spin-3 sector:

$$\begin{pmatrix} -\frac{}{a_0} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}, {}, {}}

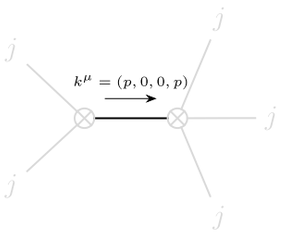
Massive pole residues:

{{}, {}, {}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{28p}{a.}, -\frac{20p}{a.}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{p^2}{a_0 P} > 0$
Polarisations:	2

Overall unitarity conditions:

$a_0 < 0 \&\& (p < 0 \,||\, p > 0)$

This completes the spectral analysis. We find that there are no massive poles, and hence no massive gravitons. There are however two massless degrees of freedom which we take to be the graviton polarisations. The unitarity conditions of these polarisations just make sure that the Einstein--Hilbert coupling carries the right sign, i.e. that the square of the Planck mass is positive. As usual with the PSALTer output, the unitarity conditions also stipulate that the energy (p) of the graviton be a real number, i.e. that its square be positive. This trivial condition typically accompanies the (non-trivial) unitarity condition(s) on the Lagrangian parameters, and is quite natural.

A brief tangent to verify the source constraints

Key observation: What about the source constraints? Stephanie wanted an in-depth discussion of where these came from. These constraints can be decoded as a consequence of the diffeomorphism and projective gauge symmetries of the theory, though it takes a bit of manipulation to see this because in their current form they are expressed in terms of the `cryptic' reduced-index spin-parity modes.

$$-6\,i\,\overset{0^+}{\cdot}\mathcal{T}^{\perp} + k\,\overset{0^+}{\cdot}\mathcal{Z}^{\parallel} + 2\,k\,\overset{0^+}{\cdot}\mathcal{Z}^{\perp h} == 0 \tag{30}$$

$$2\,i\,\overset{0^+}{\cdot}\mathcal{T}^{\perp} + k\,\overset{0^+}{\cdot}\mathcal{Z}^{\perp t} == 0 \tag{31}$$

$$12\,i\,\overset{1^-}{\cdot}\mathcal{T}^{\perp a} + 2\,k\,\overset{1^-}{\cdot}\mathcal{Z}^{\parallel h^a} + k\,\overset{1^-}{\cdot}\mathcal{Z}^{\parallel t^a} + 3\,k\,\overset{1^-}{\cdot}\mathcal{Z}^{\perp t^a} + 6\,k\,\overset{1^-}{\cdot}\mathcal{Y}^{\perp a} == 0 \tag{32}$$

$$-6\,i\,\overset{1^-}{\cdot}\mathcal{T}^{\perp a} + k\,\overset{1^-}{\cdot}\mathcal{Z}^{\perp h^a} - k\,\overset{1^-}{\cdot}\mathcal{Z}^{\perp t^a} - 3\,k\,\overset{1^-}{\cdot}\mathcal{Y}^{\perp a} == 0 \tag{33}$$

Let's expand these in terms of the original source currents which are conjugate to the metric (Einstein stress-energy tensor) and the asymmetric connection (a nameless three-index source which is a combination of the matter spin tensor and the matter dilaton current), both contracted with the momentum in various places.

$$3\,\mathcal{W}^{ab}{}_{b}\,k_a - \frac{6\,i\,\mathcal{T}_{ab}\,k^a\,k^b}{k^2} - \frac{3\,\mathcal{W}^{abc}\,k_a\,k_b\,k_c}{k^2} == 0 \tag{34}$$

$$\frac{2\,i\,\mathcal{T}_{ab}\,k^a\,k^b}{k^2} + \frac{\mathcal{W}^{abc}\,k_a\,k_b\,k_c}{k^2} == 0 \tag{35}$$

$$3\,k\,\mathcal{W}^{ab}{}_{b} - \frac{3\,\mathcal{W}^{bc}{}_{c}\,k^{a}\,k_{b}}{k} + \frac{12\,i\,\mathcal{T}^{a}{}_{b}\,k^{b}}{k} + \frac{6\,\mathcal{W}^{bac}\,k_{b}\,k_{c}}{k} - \frac{12\,i\,\mathcal{T}_{bc}\,k^{a}\,k^{b}\,k^{c}}{k^{3}} - \frac{6\,\mathcal{W}^{bcd}\,k^{a}\,k_{b}\,k_{c}\,k_{d}}{k^{3}} == 0 \tag{36}$$

$$-\frac{6\,i\,\mathcal{T}^{a}{}_{b}\,k^{b}}{k} - \frac{3\,\mathcal{W}^{bac}\,k_{b}\,k_{c}}{k} + \frac{6\,i\,\mathcal{T}_{bc}\,k^{a}\,k^{b}\,k^{c}}{k^{3}} + \frac{3\,\mathcal{W}^{bcd}\,k^{a}\,k_{b}\,k_{c}\,k_{d}}{k^{3}} == 0 \tag{37}$$

That is somewhat clearer. Now add the first and last pairs of constraints.

$$3\,\mathcal{W}^{ab}{}_{b}\,k_{a} == 0 \tag{38}$$

$$3\,k\,\mathcal{W}^{ab}{}_{b} - \frac{3\,\mathcal{W}^{bc}{}_{c}\,k^{a}\,k_{b}}{k} == 0 \tag{39}$$

This leads to a simple covariant expression in the rank-three source alone, which we notice immediately is generated by none other than the projective symmetry in Equation (2.16) on page 7 of arXiv:1912.01023.

$$\mathcal{W}^{ab}{}_{b} == 0 \tag{40}$$

This projective symmetry is to be expected, if we examine the parameter conditions in Equation (2.18) on page 7 of arXiv:1912.01023

Now we add the first and last pairs of constraints. Up to the projective symmetry above, this is a joint constraint on the connection source and the stress-energy tensor. It looks to be generated by precisely the diffeomorphism gauge symmetry Equation (2.14) on page 6 of arXiv:1912.01023, and in fact the constraint itself is written out for us in Equation (4.10) on page 14 of arXiv:1912.01023.

$$-\mathcal{W}^{bc}{}_{c}\,k^{a}\,k_{b} + 2\,i\,\mathcal{T}^{a}{}_{b}\,k^{b} + \mathcal{W}^{bac}\,k_{b}\,k_{c} == 0 \tag{41}$$

Note that the diffeomorphism gauge symmetry is always present in MAGT, so we don't need to carefully check parameter conditions.

Now, we've recovered these (expected) symmetries through their constraints on the conjugate source currents. It would be satisfying if we could go back and show that the linearized Lagrangian itself is symmetric under the relevant transformations.

First we will try the projective symmetry. Here are the infinitesimal transformations suggested by Equation (3.4) on page 8 of arXiv:1912.01023.

```
** DefTensor: Defining tensor DiffPhi[-a].

** DefConstantSymbol: Defining constant symbol Pert.
```

$$\left\{h_{ab}\,,\,\mathcal{A}_{abc}\right\} \tag{42}$$

$$\left\{h_{ab} + \epsilon\,\partial_{a}\xi_{b} + \epsilon\,\partial_{b}\xi_{a}\,,\,\mathcal{A}_{abc} + \epsilon\,\partial_{c}\partial_{a}\xi_{b}\right\} \tag{43}$$

Now let's see how the linearized Lagrangian transforms at first order.

$$\frac{1}{2}\,a_{.}{}_{0}\,\mathcal{A}^{c}{}_{bc}\,\partial^{a}\partial^{b}\xi_{a} - \frac{1}{2}\,a_{.}{}_{0}\,\mathcal{A}_{bca}\,\partial^{a}\partial^{c}\xi^{b} - \frac{1}{2}\,a_{.}{}_{0}\,\partial_{a}\xi_{c}\,\partial_{b}\mathcal{A}^{abc} - \frac{1}{2}\,a_{.}{}_{0}\,\mathcal{A}^{abc}\,\partial_{b}\partial_{a}\xi_{c} +$$
$$\frac{1}{4}\,a_{.}{}_{0}\,h^{c}{}_{c}\,\partial_{b}\partial^{a}\partial_{a}\xi^{b} - \frac{1}{4}\,a_{.}{}_{0}\,h^{c}{}_{c}\,\partial_{b}\partial^{a}\partial^{b}\xi_{a} - \frac{1}{2}\,a_{.}{}_{0}\,h_{ac}\,\partial_{b}\partial^{a}\partial^{c}\xi^{b} - \frac{1}{2}\,a_{.}{}_{0}\,\partial_{b}\mathcal{A}^{abc}\,\partial_{c}\xi_{a} - \tag{44}$$

$$\frac{1}{4}\,a_0\,\partial_b\mathcal{A}^{a\ b}{}_a\,\partial_c\xi^c+\frac{1}{4}\,a_0\,\partial_b\mathcal{A}^{ab}{}_a\,\partial_c\xi^c+\frac{1}{2}\,a_0\,\partial_b\xi_c\,\partial^c\mathcal{A}^{a\ b}{}_a+\frac{1}{2}\,a_0\,\partial_c\xi_b\,\partial^c\mathcal{A}^{a\ b}{}_a-$$
$$\frac{1}{4}\,a_0\,\partial_b\mathcal{A}^{a\ b}{}_a\,\partial^c\xi_c+\frac{1}{4}\,a_0\,\partial_b\mathcal{A}^{ab}{}_a\,\partial^c\xi_c+\frac{1}{2}\,a_0\,h_{bc}\,\partial^c\partial^a\partial^b\xi_a+\frac{1}{2}\,a_0\,\mathcal{A}^{a\ b}{}_a\,\partial^c\partial_c\xi_b$$

This looks non-trivial, but if we take the variational derivative with respect to the vector field which generates the diffeomorphism, we see that the whole thing vanishes.

$$0 \tag{45}$$

Now we would like to try the same thing with the projective symmetry. Again we set up the infinitesimal transformation, this time as suggested by Equation (2.16) on page 7 of arXiv:1912.01023.

```
** DefTensor: Defining tensor ProjPhi[-a].
```

$$\left\{h_{ab}\,,\,\mathcal{A}_{abc}\right\} \tag{46}$$

$$\left\{h_{ab}\,,\,\mathcal{A}_{abc}+\epsilon\,\eta_{bc}\ \lambda_a\right\} \tag{47}$$

Now let's see how the linearized Lagrangian transforms at first order.

$$-\frac{1}{2}\,a_0\,\mathcal{A}_{bca}\ \eta^{cb}\ \lambda^a-\frac{1}{2}\,a_0\,\mathcal{A}^{abc}\ \eta_{ac}\ \lambda_b+\frac{1}{2}\,a_0\,\mathcal{A}^c{}_{bc}\ \lambda^b+\frac{1}{2}\,a_0\,\mathcal{A}^{a\ b}{}_a\ \eta_{cb}\ \lambda^c-\frac{1}{2}\,a_0\,\eta^{cb}\ h_{ac}\ \partial_b\lambda^a+\frac{1}{2}\,a_0\,h_{bc}\ \partial^c\lambda^b \tag{48}$$

Again this looks non-trivial, but if we take the variational derivative with respect to the vector field which generates the projective transformation, we see that the whole thing vanishes.

$$0 \tag{49}$$

These results are quite satisfying, and hopefully illustrate how easy it is to `probe' the Lagrangian for extra symmetries using computer algebra.

Propagating Weyl vector (homothetic correction term)

We've just examined in some detail the very minimal Einstein-Hilbert theory within the metric affine framework. Now we want to move on to something less minimal. We will try adding on to the Einstein-Hilbert term, the quadratic invariant of the homothetic curvature. Of course, this term just acts as the Maxwell invariant for Weyl's vector nonmetricity. We therefore expect to get in addition to the graviton, a massless vector.

Here is the nonlinear Lagrangian.

$$\frac{a_0\,\mathcal{F}}{2}-\frac{1}{2}\,c_{13}\,\mathcal{F}_{mn}\ \mathcal{F}^{mn} \tag{50}$$

Here is the linearised Lagrangian.

$$-\frac{1}{2}\,a_0\,\mathcal{A}^{abc}\ \mathcal{A}_{bca}+\frac{1}{2}\,a_0\,\mathcal{A}^{a\ b}{}_a\ \mathcal{A}^c{}_{bc}-\frac{1}{4}\,a_0\,h^c{}_c\ \partial_b\mathcal{A}^{a\ b}{}_a+\frac{1}{4}\,a_0\,h^c{}_c\ \partial_b\mathcal{A}^{ab}{}_a-$$
$$\frac{1}{2}\,a_0\,h_{ac}\ \partial_b\mathcal{A}^{abc}+\frac{1}{2}\,a_0\,h_{bc}\ \partial^c\mathcal{A}^{a\ b}{}_a+c_{13}\,\partial_a\mathcal{A}_c{}^d{}_d\,\partial^c\mathcal{A}^{ab}{}_b-c_{13}\,\partial_c\mathcal{A}_a{}^d{}_d\,\partial^c\mathcal{A}^{ab}{}_b \tag{51}$$

So we now feed this into PSALTer.

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:
```

$$\begin{pmatrix}
0 & 0 & 0 & 0 & \frac{i\,a3_0\,k}{\cdot} & -\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 \\
0 & 0 & \frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 & -\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & \frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 \\
0 & -\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & -\frac{a3_0}{\cdot} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{a3_0}{\cdot} & -\frac{a3_0}{\cdot\sqrt{\cdot}} & 0 \\
-\frac{\iota}{\cdot}\,i\,a_{\cdot}\,k & \frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 & \frac{a3_0}{\cdot} & 0 & -\frac{a3_0}{\cdot\sqrt{\cdot}} & 0 \\
\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & -\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 & -\frac{a3_0}{\cdot\sqrt{\cdot}} & -\frac{a3_0}{\cdot\sqrt{\cdot}} & \frac{a3_0}{\cdot} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{a3_0}{\cdot}
\end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
-\frac{a3_0}{\cdot} & -\frac{a3_0}{\cdot\sqrt{\cdot}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{a3_0}{\cdot\sqrt{\cdot}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{a3_0}{\cdot} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 & -\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & \frac{\iota}{\cdot}\,i\sqrt{\frac{\cdot}{\cdot}}\,a_{\cdot}\,k & -\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & -\frac{i\,a3_0}{\cdot\sqrt{\cdot}} \\
0 & 0 & 0 & -\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & -\frac{a3_0}{\cdot} & \frac{a3_0}{\cdot\sqrt{\cdot}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{a3_0}{\cdot\sqrt{\cdot}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 & 0 & \frac{\iota}{\cdot}\left(-a_{\cdot}-c_{\iota\cdot}\,k^{\cdot}\right) & \frac{\iota}{\cdot}\sqrt{5}\left(a_{\cdot}-2\,c_{\iota\cdot}\,k^{\cdot}\right) & -\frac{a3_0+{}^{\cdot}c3_{13}\,k^2}{\cdot\sqrt{\cdot}} & \frac{\iota}{\cdot}\left(-a_{\cdot}-4\right. \\
0 & 0 & 0 & -\frac{\iota}{\cdot}\,i\sqrt{\frac{\cdot}{\cdot}}\,a_{\cdot}\,k & 0 & 0 & \frac{\iota}{\cdot}\sqrt{5}\left(a_{\cdot}-2\,c_{\iota\cdot}\,k^{\cdot}\right) & \frac{\iota}{\cdot}\left(a_{\cdot}-5\,c_{\iota\cdot}\,k^{\cdot}\right) & -\frac{\iota}{\cdot}\sqrt{\frac{\cdot}{\cdot}}\left(a_{\cdot}+4\,c_{\iota\cdot}\,k^{\cdot}\right) & -\frac{\iota}{\cdot}\sqrt{5}\left(a_{\cdot}\right. \\
0 & 0 & 0 & \frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 & 0 & -\frac{a3_0+{}^{\cdot}c3_{13}\,k^2}{\cdot\sqrt{\cdot}} & -\frac{\iota}{\cdot}\sqrt{\frac{\cdot}{\cdot}}\left(a_{\cdot}+4\,c_{\iota\cdot}\,k^{\cdot}\right) & \frac{\iota}{\cdot}\left(a_{\cdot}-2\,c_{\iota\cdot}\,k^{\cdot}\right) & \frac{a3_0-{}^{\cdot}c3_{13}}{\cdot\sqrt{\cdot}} \\
0 & 0 & 0 & \frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 & 0 & \frac{\iota}{\cdot}\left(-a_{\cdot}-4\,c_{\iota\cdot}\,k^{\cdot}\right) & -\frac{\iota}{\cdot}\sqrt{5}\left(a_{\cdot}+4\,c_{\iota\cdot}\,k^{\cdot}\right) & \frac{a3_0-{}^{\cdot}c3_{13}\,k^2}{\cdot\sqrt{\cdot}} & \frac{{}^{\cdot}a3_0}{\iota\cdot}--
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix}
0 & -\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & -\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & \frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 & 0 \\
\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & \frac{a3_0}{\cdot} & 0 & 0 & 0 & 0 \\
\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 & -\frac{a3_0}{\cdot} & 0 & 0 & 0 \\
-\frac{i\,a3_0\,k}{\cdot\sqrt{\cdot}} & 0 & 0 & \frac{a3_0}{\cdot} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{a3_0}{\cdot} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{a3_0}{\cdot}
\end{pmatrix}$$

Matrix for spin-3 sector:

$$\left(-\frac{a3_0}{\cdot}\right)$$

Gauge constraints on source currents:

$$-6\,i\,{}^{\cdot}_{\cdot}\mathcal{T}^{\perp}+k\,{}^{\cdot}_{\cdot}\mathcal{Z}^{\parallel}+2\,k\,{}^{\cdot}_{\cdot}\mathcal{Z}^{\perp h}==0$$

$$2\,i\,{}^{\cdot}_{\cdot}\mathcal{T}^{\perp}+k\,{}^{\cdot}_{\cdot}\mathcal{Z}^{\perp t}==0$$

$$-6\,i\,{}^{\iota}_{\cdot}\mathcal{T}^{\perp Q}+k\,{}^{\iota}_{\cdot}\mathcal{Z}^{\perp h Q}==k\left({}^{\iota}_{\cdot}\mathcal{Z}^{\perp t Q}+3\,{}^{\iota}_{\cdot}\mathcal{Y}^{\perp Q}\right)$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix}
-\frac{3k^2}{a_0^3\left(3+3k^2\right)^2} & \frac{\sqrt{3}}{3a_0^3+a_0^3k^2} & \frac{i\sqrt{3}k}{3a_0^3+a_0^3k^2} & -\frac{3ik}{a_0^3\left(3+3k^2\right)^2} & \frac{3ik\left(3+3k^2\right)}{a_0^3\left(3+3k^2\right)^2} & -\frac{i\sqrt{3}k\left(3+3k^2\right)}{a_0^3\left(3+3k^2\right)^2} & 0 \\
\frac{\sqrt{3}}{3a_0^3+a_0^3k^2} & \frac{1}{a_0^3k^2} & \frac{i\sqrt{3}}{a_0^3k} & \frac{3i\sqrt{3}}{3a_0^3k+a_0^3k^3} & -\frac{3i}{\sqrt{3}\left(3a_0^3k+a_0^3k^3\right)} & -\frac{3i\sqrt{\frac{2}{3}}}{3a_0^3k+a_0^3k^3} & 0 \\
-\frac{i\sqrt{3}k}{3a_0^3+a_0^3k^2} & -\frac{i\sqrt{3}}{a_0^3k} & 0 & \frac{\sqrt{3}}{3a_0^3+a_0^3k^2} & -\frac{\sqrt{\frac{2}{3}}}{3a_0^3+a_0^3k^2} & -\frac{3}{\sqrt{3}\left(3a_0^3+a_0^3k^2\right)} & 0 \\
\frac{3ik}{a_0^3\left(3+3k^2\right)^2} & -\frac{3i\sqrt{3}}{3a_0^3k+a_0^3k^3} & \frac{\sqrt{3}}{3a_0^3+a_0^3k^2} & -\frac{3}{a_0^3\left(3+3k^2\right)^2} & \frac{3\left(3+3k^2\right)}{a_0^3\left(3+3k^2\right)^2} & -\frac{3\sqrt{3}\left(3+3k^2\right)}{a_0^3\left(3+3k^2\right)^2} & 0 \\
-\frac{3ik\left(3+3k^2\right)}{a_0^3\left(3+3k^2\right)^2} & \frac{3i}{\sqrt{3}\left(3a_0^3k+a_0^3k^3\right)} & -\frac{\sqrt{\frac{2}{3}}}{3a_0^3+a_0^3k^2} & \frac{3\left(3+3k^2\right)}{a_0^3\left(3+3k^2\right)^2} & -\frac{3\left(3+3k^2\right)}{3a_0^3\left(3+3k^2\right)^2} & -\frac{3\sqrt{3}\left(3+3k^2\right)}{3a_0^3\left(3+3k^2\right)^2} & 0 \\
\frac{i\sqrt{3}k\left(3+3k^2\right)}{a_0^3\left(3+3k^2\right)^2} & \frac{3i\sqrt{\frac{2}{3}}}{3a_0^3k+a_0^3k^3} & -\frac{3}{\sqrt{3}\left(3a_0^3+a_0^3k^2\right)} & -\frac{3\sqrt{3}\left(3+3k^2\right)}{a_0^3\left(3+3k^2\right)^2} & -\frac{3\sqrt{3}\left(3+3k^2\right)}{3a_0^3\left(3+3k^2\right)^2} & \frac{3\left(3+3k^2\right)}{3a_0^3\left(3+3k^2\right)^2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{3}{a_0^3}
\end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
0 & -\frac{3\sqrt{3}}{a_0^3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{3\sqrt{3}}{a_0^3} & \frac{3}{a_0^3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{a_0^3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{3k^2}{a_0^3\left(3+k^2\right)^2} & \frac{3i\sqrt{3}k}{a_0^3\left(3+k^2\right)^2} & \frac{ik\left(3+k^2\right)}{a_0^3\left(3+k^2\right)^2} & -\frac{ik\left(3+k^2\right)}{\sqrt{3}a_0^3\left(3+k^2\right)^2} & \frac{i\sqrt{\frac{5}{6}}k}{a_0^3\left(3+k^2\right)} & -\frac{3ik\left(3+k^2\right)}{\sqrt{3}a_0^3\left(3+k^2\right)^2} \\
0 & 0 & 0 & -\frac{3i\sqrt{3}k}{a_0^3\left(3+k^2\right)^2} & 0 & \frac{\sqrt{3}\left(3+k^2\right)}{a_0^3\left(3+k^2\right)^2} & -\frac{3k^2}{\sqrt{3}a_0^3\left(3+k^2\right)^2} & 0 & \frac{\sqrt{\frac{2}{3}}k^2}{a_0^3\left(3+k^2\right)^2} \\
0 & 0 & 0 & -\frac{ik\left(3+k^2\right)}{a_0^3\left(3+k^2\right)^2} & \frac{\sqrt{3}\left(3+k^2\right)^2}{3a_0^3\left(3+k^2\right)^2} & \frac{\left(3+k^2\right)^2}{3a_0^3\left(3+k^2\right)^2} & \frac{k^2\left(-3+k^2\right)}{3\sqrt{3}a_0^3\left(3+k^2\right)^2} & -\frac{\sqrt{\frac{5}{6}}k^2}{3a_0^3+3a_0^3k^2} & \frac{k^2\left(3+3k^2\right)}{\sqrt{3}a_0^3\left(3+k^2\right)^2} \\
0 & 0 & 0 & \frac{ik\left(3+k^2\right)}{\sqrt{3}a_0^3\left(3+k^2\right)^2} & -\frac{3k^2}{\sqrt{3}a_0^3\left(3+k^2\right)} & \frac{k^2\left(-3+k^2\right)}{3\sqrt{3}a_0^3\left(3+k^2\right)^2} & \frac{i}{3c_{13}^3k^2}\left(-\frac{i}{3c_{13}^3k^2}-\frac{3\left(3+3k^2\right)}{a_0^3\left(3+k^2\right)^2}\right) & \frac{i}{3c_{13}^3k^2}\sqrt{5}\left(-\frac{i}{3c_{13}^3k^2}+\frac{3+3k^2}{3a_0^3+3a_0^3k^2}\right) & -\frac{\frac{1}{c_{13}^3k^2}+\frac{8\left(-2+k^2\right)}{a_0^3\left(2+k^2\right)^2}}{3\sqrt{3}} \\
0 & 0 & 0 & -\frac{i\sqrt{\frac{5}{6}}k}{3a_0^3+3a_0^3k^2} & 0 & -\frac{\sqrt{\frac{5}{6}}k^2}{3a_0^3+3a_0^3k^2} & \frac{i}{3c_{13}^3k^2}\sqrt{5}\left(-\frac{i}{3c_{13}^3k^2}+\frac{3+3k^2}{3a_0^3+3a_0^3k^2}\right) & \frac{i}{3c_{13}^3k^2}\left(\frac{3}{a_0^3}-\frac{3}{c_{13}^3k^2}\right) & \frac{i}{3\sqrt{3}c_{13}^3k^2}\sqrt{\frac{3}{3}}\left(-\frac{i}{c_{13}^3k^2}-\right. \\
0 & 0 & 0 & \frac{3ik\left(3+k^2\right)}{\sqrt{3}a_0^3\left(3+k^2\right)^2} & \frac{\sqrt{\frac{2}{3}}k^2}{a_0^3\left(3+k^2\right)} & \frac{k^2\left(3+3k^2\right)}{\sqrt{3}a_0^3\left(3+k^2\right)^2} & -\frac{\frac{1}{c_{13}^3k^2}+\frac{8\left(-2+k^2\right)}{a_0^3\left(2+k^2\right)^2}}{3\sqrt{3}} & \frac{i}{3c_{13}^3k^2}\sqrt{\frac{3}{3}}\left(-\frac{i}{3c_{13}^3k^2}-\frac{3}{a_0^3\left(3+k^2\right)}\right) & -\frac{i}{3\sqrt{3}c_{13}^3k^2}+\frac{3\left(i+3\right.} \\
0 & 0 & 0 & -\frac{i\sqrt{\frac{2}{3}}k}{3a_0^3+3a_0^3k^2} & 0 & -\frac{k^2}{\sqrt{3}a_0^3\left(3+k^2\right)} & -\frac{i}{3\sqrt{3}c_{13}^3k^2}+\frac{i}{-3a_0^3-\frac{8a_0^3}{2+3k^2}} & -\frac{\sqrt{3}\left(3a_0^3+3c_{13}^3k^2\right)}{3\sqrt{3}a_0^3c_{13}^3k^2} & \frac{\frac{1}{c_{13}^3k^2}-\frac{8\left(7+3k^2\right)}{a_0^3\left(2+k^2\right)}}{i\sqrt{3}}
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{\text{''}}{a3_0\,k^2} & -\frac{\text{'}i\sqrt{\text{'}}}{a3_0\,k} & \frac{\text{'}i}{\sqrt{\text{'}}\,a3_0\,k} & \frac{\text{'}i\sqrt{\frac{2}{3}}}{a3_0\,k} & 0 & 0 \\ \frac{\text{'}i\sqrt{\text{'}}}{a3_0\,k} & 0 & \frac{\text{`}\sqrt{\frac{2}{3}}}{a3_0} & \frac{\text{'}}{\sqrt{\text{'}}\,a3_0} & 0 & 0 \\ -\frac{\text{'}i}{\sqrt{\text{'}}\,a3_0\,k} & \frac{\text{`}\sqrt{\frac{2}{3}}}{a3_0} & -\frac{\text{''}}{\text{`}\,a3_0} & -\frac{\text{`}\sqrt{\text{'}}}{\text{`}\,a3_0} & 0 & 0 \\ -\frac{\text{'}i\sqrt{\frac{2}{3}}}{a3_0\,k} & \frac{\text{'}}{\sqrt{\text{'}}\,a3_0} & -\frac{\text{`}\sqrt{\text{'}}}{\text{`}\,a3_0} & \frac{\text{''}}{\text{`}\,a3_0} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\text{'}}{a3_0} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\text{'}}{a3_0} \end{pmatrix}$$

Matrix for spin-3 sector:

$$\left(-\frac{\text{`}}{a3_0}\right)$$

Square masses:

{{}, {}, {}, {}, {}, {}, {}, {}}

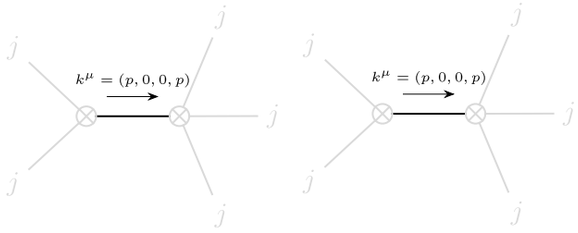
Massive pole residues:

{{}, {}, {}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{1}{4\,c_{\text{`}\,\text{`}}},\ \frac{1}{4\,c_{\text{`}\,\text{`}}},\ -\frac{28\,p^{\text{`}}}{a_{\text{`}}},\ -\frac{20\,p^{\text{`}}}{a_{\text{`}}}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$\frac{S}{c_{P_{13}}} > 0$
Polarisations:	2

Massless particle

Pole residue:	$-\frac{p^2}{a_{P_0}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(p < 0 \;\&\&\; a_{\text{`}\,\text{`}} < 0 \;\&\&\; c_{\text{`}\,\text{`}} > 0\right) \,||\, \left(p > 0 \;\&\&\; a_{\text{`}\,\text{`}} < 0 \;\&\&\; c_{\text{`}\,\text{`}} > 0\right)$$

So these results are consistent with our expectations. We note that an extra higher-spin massless mode is indeed moving, with the expected unitarity constraint. We also see among the source constraints that the projective symmetry is gone, and that there is an extra scalar symmetry (presumably the phase symmetry of the Weyl vector, though the usual dilation interpretation will be lost due to the Einstein-Hilbert term).

Weyl-Proca theory (homothetic and square nonmetric corrections)

The natural extension to the homothetic correction is to add a mass term for the Weyl vector.

Here is the nonlinear Lagrangian.

$$-\frac{1}{2}a_{\cdot}\,Q_{m}\;Q^{m}+\frac{a_{\cdot}\mathcal{F}}{2}-\frac{1}{2}c_{\cdot}\,\mathcal{F}_{mn}\;\mathcal{F}^{mn} \tag{52}$$

Here is the linearised Lagrangian.

$$\begin{aligned}&-2a_{\cdot}\,\mathcal{A}_{q\,c}^{\;\;c}\;\mathcal{A}^{qb}{}_{b}-\frac{1}{2}a_{\cdot}\,\mathcal{A}^{abc}\;\mathcal{A}_{bcq}+\frac{1}{2}a_{\cdot}\,\mathcal{A}^{q\,b}{}_{q}\;\mathcal{A}^{c}{}_{bc}+2a_{\cdot}\,\mathcal{A}^{qb}{}_{b}\;\partial_{q}h^{c}{}_{c}-\frac{1}{4}a_{\cdot}\,h^{c}{}_{c}\;\partial_{b}\mathcal{A}^{q\,b}{}_{q}+\frac{1}{4}a_{\cdot}\,h^{c}{}_{c}\;\partial_{b}\mathcal{A}^{qb}{}_{q}-\\&\frac{1}{2}a_{\cdot}\,h_{qc}\;\partial_{b}\mathcal{A}^{abc}-\frac{1}{2}a_{\cdot}\,\partial_{b}h^{c}{}_{c}\;\partial^{b}h^{q}{}_{q}+\frac{1}{2}a_{\cdot}\,h_{bc}\;\partial^{c}\mathcal{A}^{q\,b}{}_{q}+c_{\cdot}\,\partial_{q}\mathcal{A}_{c\,d}^{\;\;d}\;\partial^{c}\mathcal{A}^{qb}{}_{b}-c_{\cdot}\,\partial_{c}\mathcal{A}_{q\,d}^{\;\;d}\;\partial^{c}\mathcal{A}^{qb}{}_{b}\end{aligned} \tag{53}$$

So we now feed this into PSALTer.

The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix}
-\frac{a_{6}\,k^{2}}{} & --\sqrt{3}\;a_{\cdot}\,k & 0 & -i\,a_{\cdot}\,k & -i\left(a_{\cdot}-4\,a_{\cdot}\right)k & -\frac{i\left(a_{0}+a_{6}\right)k}{\sqrt{}} & 0\\
--\sqrt{3}\;a_{\cdot}\,k & -\frac{a_{6}\,k^{2}}{} & \frac{i\,a_{0}\,k}{\sqrt{}} & -i\sqrt{3}\;a_{\cdot}\,k & -\frac{i\left(a_{0}+a_{6}\right)k}{\sqrt{}} & \frac{i\left(a_{0}-a_{6}\right)k}{\sqrt{}} & 0\\
0 & -\frac{i\,a_{0}\,k}{\sqrt{}} & -\frac{a_{0}}{} & 0 & 0 & 0 & 0\\
i\,a_{\cdot}\,k & i\sqrt{3}\;a_{\cdot}\,k & 0 & -2\,a_{\cdot} & -\left(a_{\cdot}-4\,a_{\cdot}\right) & -\frac{a_{0}+a_{6}}{\sqrt{}} & 0\\
--i\left(a_{\cdot}-4\,a_{\cdot}\right)k & \frac{i\left(a_{0}+a_{6}\right)k}{\sqrt{}} & 0 & -\left(a_{\cdot}-4\,a_{\cdot}\right) & -2\,a_{\cdot} & -\frac{a_{0}+a_{6}}{\sqrt{}} & 0\\
\frac{i\left(a_{0}+a_{6}\right)k}{\sqrt{}} & -\frac{i\left(a_{0}-a_{6}\right)k}{\sqrt{}} & 0 & -\frac{a_{0}+a_{6}}{\sqrt{}} & -\frac{a_{0}+a_{6}}{\sqrt{}} & -\left(a_{\cdot}-8\,a_{\cdot}\right) & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{a_{0}}{}
\end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
-\frac{a_{0}}{} & -\frac{a_{0}}{\sqrt{}} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
-\frac{a_{0}}{\sqrt{}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \frac{a_{0}}{} & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \frac{i\,a_{0}\,k}{\sqrt{}} & 0 & -\frac{i\,a_{0}\,k}{\sqrt{}} & -i\sqrt{-}\;a_{\cdot}\,k & -\frac{i\,a_{0}\,k}{\sqrt{}}\\
0 & 0 & 0 & -\frac{i\,a_{0}\,k}{\sqrt{}} & -\frac{a_{0}}{} & \frac{a_{0}}{\sqrt{}} & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \frac{a_{0}}{\sqrt{}} & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \frac{i\,a_{0}\,k}{\sqrt{}} & 0 & 0 & -\left(-a_{\cdot}-2\,a_{\cdot}-c_{\cdot}\,k\right) & -\sqrt{5}\left(a_{\cdot}-4\,a_{\cdot}-2\,c_{\cdot}\,k\right) & -\frac{a_{0}+a_{6}+c_{13}\,k^{2}}{\sqrt{}}\\
0 & 0 & 0 & --i\sqrt{-}\;a_{\cdot}\,k & 0 & 0 & -\sqrt{5}\left(a_{\cdot}-4\,a_{\cdot}-2\,c_{\cdot}\,k\right) & -\left(a_{\cdot}-5\left(2\,a_{\cdot}+c_{\cdot}\,k\right)\right) & --\sqrt{-}\left(a_{\cdot}+8\,a_{\cdot}+4\,c\right.\\
0 & 0 & 0 & \frac{i\,a_{0}\,k}{\sqrt{}} & 0 & 0 & -\frac{a_{0}+a_{6}+c_{13}\,k^{2}}{\sqrt{}} & --\sqrt{-}\left(a_{\cdot}+8\,a_{\cdot}+4\,c_{\cdot}\,k\right) & -\left(a_{\cdot}-4\,a_{\cdot}-2\,c_{\cdot}\right.\\
0 & 0 & 0 & \frac{i\,a_{0}\,k}{\sqrt{}} & 0 & 0 & -\left(-a_{\cdot}-8\,a_{\cdot}-4\,c_{\cdot}\,k\right) & --\sqrt{5}\left(a_{\cdot}+8\,a_{\cdot}+4\,c_{\cdot}\,k\right) & \frac{a_{0}-\left(a_{6}+c_{13}\,k^{2}\right)}{\sqrt{}}
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & -\frac{i a_0 k}{\sqrt{\ }} & -\frac{i a_0 k}{\sqrt{\ }} & \frac{i a_0 k}{\sqrt{\ }} & 0 & 0 \\ \frac{i a_0 k}{\sqrt{\ }} & \frac{a_0}{\ } & 0 & 0 & 0 & 0 \\ \frac{i a_0 k}{\sqrt{\ }} & 0 & -\frac{a_0}{\ } & 0 & 0 & 0 \\ -\frac{i a_0 k}{\sqrt{\ }} & 0 & 0 & \frac{a_0}{\ } & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{a_0}{\ } & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{a_0}{\ } \end{pmatrix}$$

Matrix for spin-3 sector:

$$\left(-\frac{a_0}{\ } \right)$$

Gauge constraints on source currents:

$$2\,i\,\dot{\mathcal{T}}^{\perp} + k\,\dot{\mathcal{Z}}^{\perp t} == 0$$

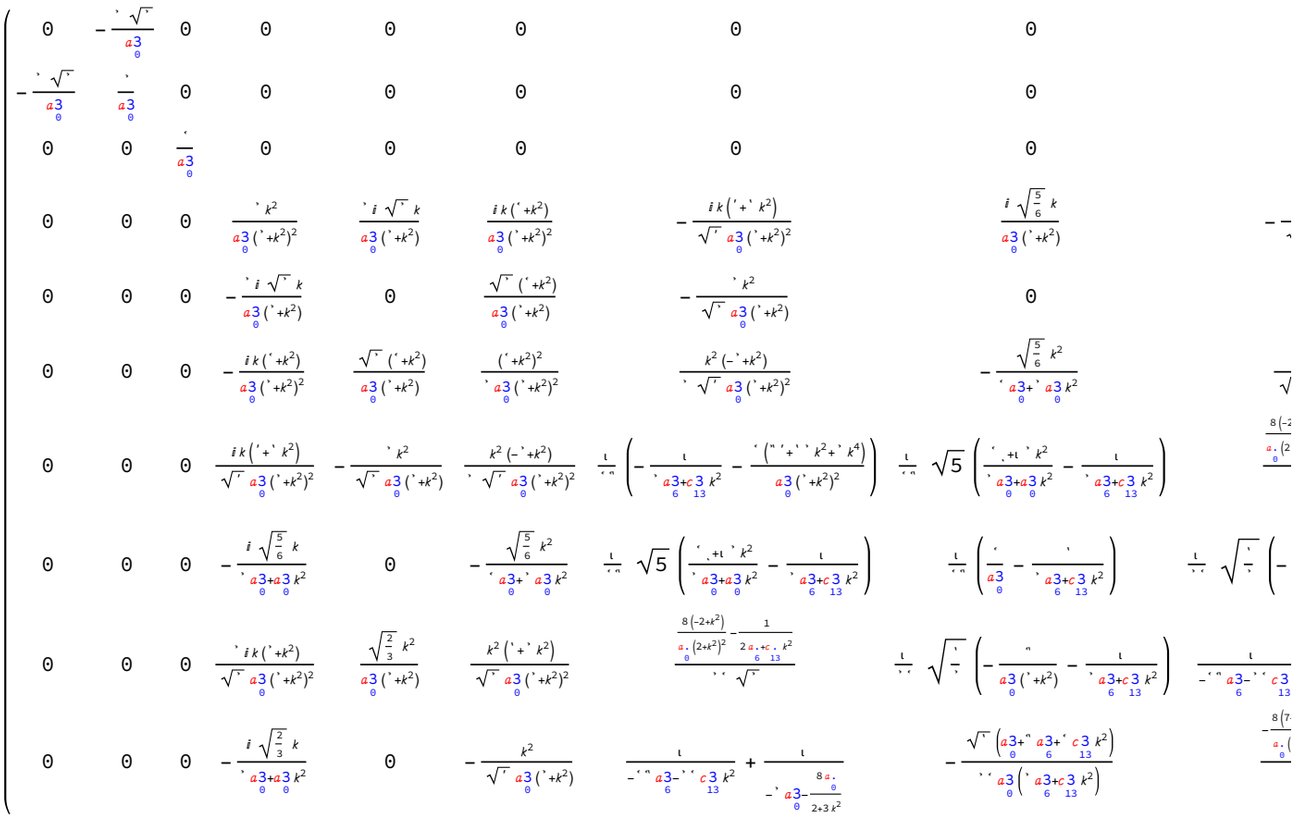
$$-6\,i\,\bar{\mathcal{T}}^{\perp q} + k\,\bar{\mathcal{Z}}^{\perp h q} == k\left(\bar{\mathcal{Z}}^{\perp t q} + 3\,\bar{y}^{\perp q}\right)$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

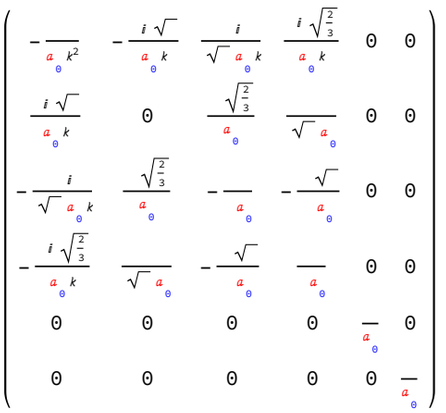
Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{\left(a3_0^{+}\,a3_6\right)k^2}{a3_0\,a3_6\,(+k^2)^2} & 0 & 0 & -\frac{i\left(a3_0^{+}\,a3_6\right)k}{a3_0\,a3_6\,(+k^2)^2} & -\frac{i\left(a3_0^{-}\,a3_6\right)k}{a3_0\,a3_6\,(+k^2)} & -\frac{i\left(a3_0^{+}\,a3_6\right)k}{\sqrt{\ }\,a3_0\,a3_6\,(+k^2)} & 0 \\ 0 & \frac{}{a3_0\,k^2} & \frac{i\sqrt{\ }}{a3_0\,k} & 0 & -\frac{i}{\sqrt{\ }\,a3_0\,k} & -\frac{i\sqrt{\frac{2}{3}}}{a3_0\,k} & 0 \\ 0 & -\frac{i\sqrt{\ }}{a3_0\,k} & 0 & 0 & -\frac{\sqrt{\frac{2}{3}}}{a3_0} & -\frac{}{\sqrt{\ }\,a3_0} & 0 \\ \frac{i\left(a3_0^{+}\,a3_6\right)k}{a3_0\,a3_6\,(+k^2)^2} & 0 & 0 & -\frac{a3_0^{+}\,a3_6}{a3_0\,a3_6\,(+k^2)^2} & -\frac{a3_0^{-}\,a3_6}{a3_0\,a3_6^{+}\,a3_0\,a3_6\,k^2} & -\frac{a3_0^{+}\,a3_6}{\sqrt{\ }\,a3_0\,a3_6\,(+k^2)} & 0 \\ \frac{i\left(a3_0^{-}\,a3_6\right)k}{a3_0\,a3_6\,(+k^2)} & \frac{i}{\sqrt{\ }\,a3_0\,k} & -\frac{\sqrt{\frac{2}{3}}}{a3_0} & -\frac{a3_0^{-}\,a3_6}{a3_0\,a3_6^{+}\,a3_0\,a3_6\,k^2} & -\frac{}{a3_0} - \frac{}{a3_6} & \frac{-a3_0^{+}\,a3_6}{\sqrt{\ }\,a3_0\,a3_6} & 0 \\ \frac{i\left(a3_0^{+}\,a3_6\right)k}{\sqrt{\ }\,a3_0\,a3_6\,(+k^2)} & \frac{i\sqrt{\frac{2}{3}}}{a3_0\,k} & -\frac{}{\sqrt{\ }\,a3_0} & -\frac{a3_0^{+}\,a3_6}{\sqrt{\ }\,a3_0\,a3_6\,(+k^2)} & \frac{-a3_0^{+}\,a3_6}{\sqrt{\ }\,a3_0\,a3_6} & \frac{}{a3_0} - \frac{}{a3_6} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{}{a3_0} \end{pmatrix}$$

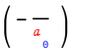
Matrix for spin-1 sector:



Matrix for spin-2 sector:



Matrix for spin-3 sector:



Square masses:

$$\left\{\{\}, \{\}, \{\}, \left\{-\frac{2\,a.}{c.}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

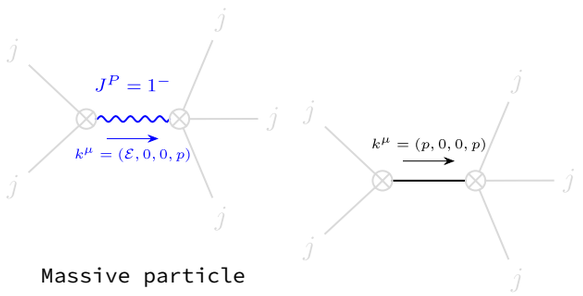
Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \left\{\frac{1}{4\,c.}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\left\{-\frac{28\,p}{a.}, -\frac{20\,p}{a.}\right\}$$

Overall particle spectrum:



Massive particle

Pole residue:	$\frac{S}{V_{cP_{13}}} > 0$
Square mass:	$-\frac{T_{aP_{6}}}{c_{P_{13}}} > 0$
Spin:	1
Parity:	Odd

Massless particle

Pole residue:	$-\frac{p^2}{a_{P_0}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(p < 0 \&\& a_{\_} < 0 \&\& a_{\_} < 0 \&\& c_{\_} > 0\right) \| \left(p > 0 \&\& a_{\_} < 0 \&\& a_{\_} < 0 \&\& c_{\_} > 0\right)$$

These results are again consistent with expectations. We lost the extra scalar source constraint (gauge symmetry) and the massless mode from the last case reveals itself to be a vector when it picks up a mass. The graviton remains as a spectator. Let's stop here for now.

Key observation: We have now reached the end of the PSALTer calibration script.