

Particle spectrograph

Wave operator and propagator

| | $\omega_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^+}^{\#2}{}_{\alpha\beta}$ | $f_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^-}^{\#1}{}_{\alpha}$ | $\omega_{1^-}^{\#2}{}_{\alpha}$ | $f_{1^-}^{\#1}{}_{\alpha}$ | $f_{1^-}^{\#2}{}_{\alpha}$ |
|--|--------------------------------------|--------------------------------------|---------------------------------|---|---------------------------------|----------------------------|----------------------------|
| $\omega_{1^+}^{\#1}{}_{\dagger}{}^{\alpha\beta}$ | $k^2(2r_3+r_5)+\frac{2t_2}{3}$ | $\frac{\sqrt{2}t_2}{3}$ | $\frac{1}{3}i\sqrt{2}kt_2$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2}{}_{\dagger}{}^{\alpha\beta}$ | $\frac{\sqrt{2}t_2}{3}$ | $\frac{t_2}{3}$ | $\frac{ikt_2}{3}$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1}{}_{\dagger}{}^{\alpha\beta}$ | $-\frac{1}{3}i\sqrt{2}kt_2$ | $-\frac{1}{3}ikt_2$ | $\frac{k^2t_2}{3}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1}{}_{\dagger}{}^{\alpha}$ | 0 | 0 | 0 | $k^2(\frac{r_3}{2}+r_5)+\frac{2t_3}{3}$ | $-\frac{\sqrt{2}t_3}{3}$ | 0 | $-\frac{2}{3}ikt_3$ |
| $\omega_{1^-}^{\#2}{}_{\dagger}{}^{\alpha}$ | 0 | 0 | 0 | $-\frac{\sqrt{2}t_3}{3}$ | $\frac{t_3}{3}$ | 0 | $\frac{1}{3}i\sqrt{2}kt_3$ |
| $f_{1^-}^{\#1}{}_{\dagger}{}^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2}{}_{\dagger}{}^{\alpha}$ | 0 | 0 | 0 | $\frac{2ikt_3}{3}$ | $-\frac{1}{3}i\sqrt{2}kt_3$ | 0 | $\frac{2k^2t_3}{3}$ |

Source constraints/gauge generators

| SO(3) irreps | Multiplicities |
|--|----------------|
| $\tau_{0^+}^{\#2} == 0$ | 1 |
| $\tau_{0^+}^{\#1} - 2ik\sigma_{0^+}^{\#1} == 0$ | 1 |
| $\tau_{1^-}^{\#2\alpha} + 2ik\sigma_{1^-}^{\#2\alpha} == 0$ | 3 |
| $\tau_{1^-}^{\#1\alpha} == 0$ | 3 |
| $\tau_{1^+}^{\#1\alpha\beta} + ik\sigma_{1^+}^{\#2\alpha\beta} == 0$ | 3 |
| $\sigma_{2^-}^{\#1\alpha\beta\chi} == 0$ | 5 |
| $\tau_{2^+}^{\#1\alpha\beta} == 0$ | 5 |
| Total constraints: | 21 |

Quadratic (free) action

$$\begin{aligned} S_F = & \iiint \left(\frac{1}{6} (4t_3\omega_{\kappa\alpha}^{\alpha'}\omega_{\kappa\alpha}^{\kappa} + 4t_2\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\kappa\lambda}^{\lambda} + 2t_2\omega_{\kappa\lambda}^{\lambda}\omega_{\kappa\lambda}^{\kappa\lambda} + 6f^{\alpha\beta}\tau_{\alpha\beta} + 6\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi} - 3r_3\partial_{\lambda}\omega_{\kappa}^{\kappa\lambda}\partial'_{\lambda}\omega_{\alpha}^{\alpha} - 6r_5\partial_{\lambda}\omega_{\kappa}^{\kappa\lambda}\partial'_{\lambda}\omega_{\alpha}^{\alpha} + 4r_2\partial^{\beta}\omega_{\alpha}^{\theta\alpha}\partial_{\theta}\omega_{\alpha\beta}^{\kappa} - \right. \\ & 2r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta} - 4r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta} + 3r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\lambda}^{\theta\kappa\lambda} - \\ & 6r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\lambda}^{\theta\kappa\lambda} - 3r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\lambda}^{\kappa\lambda\theta} + 6r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\kappa\lambda\theta} + \\ & 12r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\kappa\lambda\theta} + t_2\partial^{\alpha}f_{\theta\kappa}\partial_{\kappa}f_{\alpha}^{\theta} - t_2\partial^{\alpha}f_{\kappa\theta}\partial_{\theta}f_{\alpha}^{\kappa} + t_2\partial^{\alpha}f_{\lambda}^{\kappa}\partial_{\kappa}f_{\alpha\lambda}^{\lambda} - \\ & 4t_3\omega_{\kappa\alpha}^{\alpha}\partial_{\kappa}f_{\lambda}^{\lambda} - 4t_3\omega_{\kappa\lambda}^{\lambda}\partial_{\kappa}f_{\lambda}^{\lambda} - 8t_3\partial^{\alpha}f_{\kappa\alpha}\partial_{\kappa}f_{\lambda}^{\lambda} + 4t_3\partial_{\kappa}f_{\lambda}^{\lambda}\partial_{\kappa}f_{\lambda}^{\lambda} + \\ & 2t_2\omega_{\theta\kappa}\partial_{\kappa}f^{\lambda\theta} - 4t_2\omega_{\lambda\theta}\partial_{\kappa}f^{\lambda\theta} - 2t_2\omega_{\theta\kappa}\partial_{\kappa}f^{\lambda\theta} + 4t_2\omega_{\theta\kappa\lambda}\partial_{\kappa}f^{\lambda\theta} + \\ & 4t_3\omega_{\lambda\alpha}^{\alpha}\partial_{\kappa}f_{\lambda}^{\theta} + 4t_3\omega_{\lambda\lambda}^{\lambda}\partial_{\kappa}f_{\lambda}^{\lambda} - t_2\partial^{\alpha}f_{\lambda}^{\lambda}\partial_{\kappa}f_{\lambda\alpha}^{\theta} - t_2\partial_{\kappa}f_{\lambda}^{\theta}\partial_{\lambda}f_{\alpha}^{\kappa} + \\ & t_2\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta} + 4t_3\partial^{\alpha}f_{\lambda}^{\lambda}\partial_{\alpha}^{\kappa}f_{\lambda\kappa}^{\lambda} + 2r_2\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta} + \\ & 4r_2\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta} - 4r_2\partial_{\lambda}^{\beta}\omega_{\lambda}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda} + 4r_2\partial_{\lambda}^{\beta}\omega_{\lambda}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda} - \\ & 24r_3\partial_{\lambda}^{\beta}\omega_{\lambda}^{\lambda\alpha}\partial_{\alpha}\omega_{\alpha\beta}^{\lambda} - 3r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}^{\lambda}\omega_{\lambda}^{\theta\kappa} + 6r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}^{\lambda}\omega_{\lambda}^{\theta\kappa} + \\ & 3r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\alpha}^{\lambda}\omega_{\lambda}^{\theta\kappa} - 6r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\alpha}^{\lambda}\omega_{\lambda}^{\theta\kappa})) [t, x, y, z] dz dy dx dt \end{aligned}$$

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^+}^{\#1}$ |
|----------------------------------|------------------------------------|-------------------------------------|--------------------|------------------------|
| $\sigma_{0^+}^{\#1}{}_{\dagger}$ | $\frac{1}{(1+2k^2)^2t_3}$ | $-\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$ | 0 | 0 |
| $\tau_{0^+}^{\#1}{}_{\dagger}$ | $\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$ | $\frac{2k^2}{(1+2k^2)^2t_3}$ | 0 | 0 |
| $\tau_{0^+}^{\#2}{}_{\dagger}$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^+}^{\#1}{}_{\dagger}$ | 0 | 0 | 0 | $\frac{1}{k^2r_2+t_2}$ |

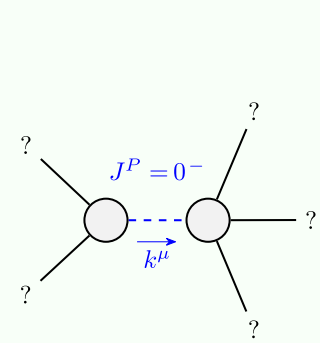
| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^+}^{\#1}$ |
|----------------------------------|----------------------|------------------|-----------------|----------------------|
| $\omega_{0^+}^{\#1}{}_{\dagger}$ | t_3 | $-i\sqrt{2}kt_3$ | 0 | 0 |
| $f_{0^+}^{\#1}{}_{\dagger}$ | $i\sqrt{2}kt_3$ | $2k^2t_3$ | 0 | 0 |
| $f_{0^+}^{\#2}{}_{\dagger}$ | 0 | 0 | 0 | 0 |
| $\omega_{0^+}^{\#1}{}_{\dagger}$ | 0 | 0 | 0 | $k^2r_2+t_2$ |

| | $\sigma_{2^+}^{\#1}{}_{\alpha\beta}$ | $\tau_{2^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{2^+}^{\#1}{}_{\alpha\beta\chi}$ |
|--|--------------------------------------|------------------------------------|--|
| $\sigma_{2^+}^{\#1}{}_{\dagger}{}^{\alpha\beta}$ | $-\frac{2}{3k^2r_3}$ | 0 | 0 |
| $\tau_{2^+}^{\#1}{}_{\dagger}{}^{\alpha\beta}$ | 0 | 0 | 0 |
| $\sigma_{2^+}^{\#1}{}_{\dagger}{}^{\alpha\beta\chi}$ | 0 | 0 | 0 |

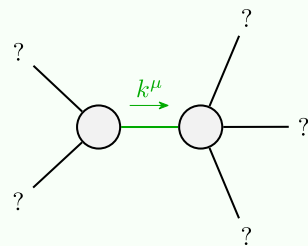
| | $\omega_{2^+}^{\#1}{}_{\alpha\beta}$ | $f_{2^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{2^+}^{\#1}{}_{\alpha\beta\chi}$ |
|--|--------------------------------------|---------------------------------|--|
| $\omega_{2^+}^{\#1}{}_{\dagger}{}^{\alpha\beta}$ | $-\frac{3k^2r_3}{2}$ | 0 | 0 |
| $f_{2^+}^{\#1}{}_{\dagger}{}^{\alpha\beta}$ | 0 | 0 | 0 |
| $\omega_{2^+}^{\#1}{}_{\dagger}{}^{\alpha\beta\chi}$ | 0 | 0 | 0 |

| | $\sigma_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^+}^{\#2}{}_{\alpha\beta}$ | $\tau_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^-}^{\#1}{}_{\alpha}$ | $\sigma_{1^-}^{\#2}{}_{\alpha}$ | $\tau_{1^-}^{\#1}{}_{\alpha}$ | $\tau_{1^-}^{\#2}{}_{\alpha}$ |
|--|--|---|--|---|--|-------------------------------|---|
| $\sigma_{1^+}^{\#1}{}_{\dagger}{}^{\alpha\beta}$ | $\frac{1}{k^2(2r_3+r_5)}$ | $-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$ | $-\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2}{}_{\dagger}{}^{\alpha\beta}$ | $-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$ | $\frac{3k^2(2r_3+r_5)+2t_2}{(k+k^3)^2(2r_3+r_5)t_2}$ | $\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1}{}_{\dagger}{}^{\alpha\beta}$ | $\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$ | $-\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$ | $\frac{3k^2(2r_3+r_5)+2t_2}{(1+k^2)^2(2r_3+r_5)t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1}{}_{\dagger}{}^{\alpha}$ | 0 | 0 | 0 | $\frac{2}{k^2(r_3+2r_5)}$ | $\frac{2\sqrt{2}}{k^2(1+2k^2)(r_3+2r_5)}$ | 0 | $\frac{4i}{k(1+2k^2)(r_3+2r_5)}$ |
| $\sigma_{1^-}^{\#2}{}_{\dagger}{}^{\alpha}$ | 0 | 0 | 0 | $\frac{2\sqrt{2}}{k^2(1+2k^2)(r_3+2r_5)}$ | $\frac{3k^2(r_3+2r_5)+4t_3}{(k+2k^3)^2(r_3+2r_5)t_3}$ | 0 | $\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$ |
| $\tau_{1^-}^{\#1}{}_{\dagger}{}^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2}{}_{\dagger}{}^{\alpha}$ | 0 | 0 | 0 | $-\frac{4i}{k(1+2k^2)(r_3+2r_5)}$ | $-\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$ | 0 | $\frac{6k^2(r_3+2r_5)+8t_3}{(1+2k^2)^2(r_3+2r_5)t_3}$ |

Massive and massless spectra



| Massive particle | |
|------------------|------------------------|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $-\frac{t_2}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |



| Quadratic pole | |
|----------------|---|
| Pole residue: | $-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2} > 0$ |
| Polarisations: | 2 |

Unitarity conditions

$$r_2 < 0 \&\& r_3 < 0 \&\& r_5 < -\frac{r_3}{2} \&\& t_2 > 0 \parallel r_2 < 0 \&\& r_3 < 0 \&\& r_5 > -2r_3 \&\& t_2 > 0 \parallel r_2 < 0 \&\& r_3 > 0 \&\& -2r_3 < r_5 < -\frac{r_3}{2} \&\& t_2 > 0$$