

## Wave operator and propagator

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} == 0$	1
$\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\sigma_{1-}^{\#2\alpha} == 0$	3
$\sigma_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + i k \sigma_{1+}^{\#1\alpha\beta} == 0$	3
$\sigma_{1+}^{\#1\alpha\beta} == \sigma_{1+}^{\#2\alpha\beta}$	3
$\sigma_{2-}^{\#1\alpha\beta\chi} == 0$	5
$\tau_{2+}^{\#1\alpha\beta} == 0$	5
$\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	36

## Quadratic (free) action

$$\begin{aligned} \delta_F = & \iiint \left( \frac{1}{6} (4t_2 \omega_{\kappa\lambda}' \omega_{\kappa\lambda}' + 2t_2 \omega_{\kappa\lambda}' \omega_{\kappa\lambda}' + 6f^{\alpha\beta} \tau_{\alpha\beta} + 6\omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} + \right. \\ & 4r_2 \partial^\beta \omega_{\kappa}^{\theta\alpha} \partial_\theta \omega_{\alpha\beta}^{\kappa} - 2r_2 \partial_\theta \omega_{\alpha\beta}^{\kappa} \partial_\kappa \omega^{\alpha\beta\theta} - 4r_2 \partial_\theta \omega_{\alpha\beta}^{\kappa} \partial_\kappa \omega^{\theta\alpha\beta} + \\ & t_2 \partial^\alpha f_{\theta\kappa} \partial_\kappa f_{\alpha}^{\theta} - t_2 \partial^\alpha f_{\kappa\theta} \partial_\theta f_{\alpha}^{\theta} + t_2 \partial^\alpha f_{\lambda}^{\theta} \partial_\kappa f_{\alpha\lambda}^{\theta} + 2t_2 \omega_{\theta\kappa} \partial_\kappa f^{\lambda\theta} - \\ & 4t_2 \omega_{\kappa\theta} \partial^\kappa f^{\lambda\theta} - 2t_2 \omega_{\theta\kappa} \partial^\kappa f^{\lambda\theta} + 4t_2 \omega_{\theta\kappa\lambda} \partial^\kappa f^{\lambda\theta} - t_2 \partial^\alpha f_{\lambda}^{\theta} \partial_\kappa f_{\alpha}^{\theta} - \\ & t_2 \partial_\kappa f_{\theta}^{\lambda} \partial^\kappa f_{\lambda}^{\theta} + t_2 \partial_\kappa f_{\theta}^{\lambda} \partial^\kappa f_{\lambda}^{\theta} + 2r_2 \partial_\kappa \omega^{\alpha\beta\theta} \partial_\kappa \omega_{\alpha\beta\theta} + 4r_2 \partial_\kappa \omega^{\theta\alpha\beta} \partial_\kappa \omega_{\alpha\beta\theta} - \\ & \left. 4r_2 \partial^\beta \omega_{\alpha\lambda}^{\theta} \partial_\lambda \omega_{\alpha\beta}^{\theta} + 4r_2 \partial^\beta \omega_{\alpha\lambda}^{\theta} \partial_\lambda \omega_{\alpha\beta}^{\theta} \right) [t, x, y, z] dz dy dx dt \end{aligned}$$

$\omega_1^{\#1} + \alpha\beta$	$\frac{2t_2}{3}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{1}{3}i\sqrt{2}kt_2$	$\omega_1^{\#1-\alpha}$	$\omega_1^{\#2-\alpha}$	$f_1^{\#1-\alpha}$	$f_1^{\#2-\alpha}$
$\omega_1^{\#2} + \alpha\beta$	$\frac{\sqrt{2}t_2}{3}$	$\frac{t_2}{3}$	$\frac{ikt_2}{3}$	0	0	0	0
$f_1^{\#1} + \alpha\beta$	$-\frac{1}{3}i\sqrt{2}kt_2$	$-\frac{1}{3}ikt_2$	$\frac{k^2t_2}{3}$	0	0	0	0
$\omega_1^{\#1} + \alpha$	0	0	0	0	0	0	0
$\omega_1^{\#2} + \alpha$	0	0	0	0	0	0	0
$f_1^{\#1} + \alpha$	0	0	0	0	0	0	0
$f_1^{\#2} + \alpha$	0	0	0	0	0	0	0

The diagram illustrates the decomposition of the tensor product of two representations of the Lie algebra  $\mathfrak{su}(2)$  into irreducible representations. The central grid shows the decomposition of the tensor product of two representations into a direct sum of irreducible representations. The side grids show the decomposition of the tensor product of two representations into a direct sum of irreducible representations.

**Central Grid:**

	$\omega_0^{\#1} \uparrow$	$f_0^{\#1} \uparrow$	$f_0^{\#2} \uparrow$	$\omega_0^{\#1} \uparrow$
$\omega_0^{\#1} \uparrow$	0	0	0	$k^2 r_2 + t_2$
$f_0^{\#1} \uparrow$	0	0	0	0
$f_0^{\#2} \uparrow$	0	0	0	0
$\omega_0^{\#1} \uparrow$	0	0	0	0

**Side Grids:**

**Left Side Grid:**

	$\omega_0^{\#1} \uparrow$	$f_0^{\#1} \uparrow$	$f_0^{\#2} \uparrow$	$\omega_0^{\#1} \uparrow$
$\omega_0^{\#1} \uparrow$	0	0	0	$\frac{1}{k^2 r_2 + t_2}$
$f_0^{\#1} \uparrow$	0	0	0	0
$f_0^{\#2} \uparrow$	0	0	0	0
$\omega_0^{\#1} \uparrow$	0	0	0	0

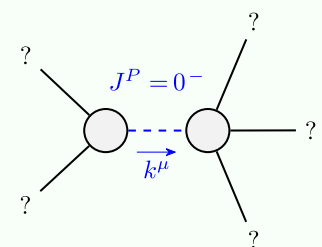
**Right Side Grid:**

	$\omega_2^{\#1} \uparrow \alpha \beta$	$f_2^{\#1} \uparrow \alpha \beta$	$\omega_2^{\#1} \uparrow \alpha \beta \chi$
$\omega_2^{\#1} \uparrow \alpha \beta$	0	0	0
$f_2^{\#1} \uparrow \alpha \beta$	0	0	0
$\omega_2^{\#1} \uparrow \alpha \beta \chi$	0	0	0

**Bottom Grid:**

	$\sigma_{1^+}^{\#1} \alpha \beta$	$\sigma_{1^+}^{\#2} \alpha \beta$	$\tau_{1^+}^{\#1} \alpha \beta$	$\sigma_{1^-}^{\#1} \alpha$	$\sigma_{1^-}^{\#2} \alpha$	$\tau_{1^-}^{\#1} \alpha$	$\tau_{1^-}^{\#2} \alpha$
$\sigma_{1^+}^{\#1} \alpha \beta$	$\frac{6}{(3+k^2)^2 t_2}$	$\frac{3 \sqrt{2}}{(3+k^2)^2 t_2}$	$\frac{3 i \sqrt{2} k}{(3+k^2)^2 t_2}$	0	0	0	0
$\sigma_{1^+}^{\#2} \alpha \beta$	$\frac{3 \sqrt{2}}{(3+k^2)^2 t_2}$	$\frac{3}{(3+k^2)^2 t_2}$	$\frac{3 i k}{(3+k^2)^2 t_2}$	0	0	0	0
$\tau_{1^+}^{\#1} \alpha \beta$	$-\frac{3 i \sqrt{2} k}{(3+k^2)^2 t_2}$	$-\frac{3 i k}{(3+k^2)^2 t_2}$	$\frac{3 k^2}{(3+k^2)^2 t_2}$	0	0	0	0
$\sigma_{1^-}^{\#1} \alpha$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#2} \alpha$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#1} \alpha$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#2} \alpha$	0	0	0	0	0	0	0

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

## Unitarity conditions

$$r_2 < 0 \ \&\& \ t_2 > 0$$