## Large matrix (1⁺ / 1⁻ sector)

| | $\sigma_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^+}^{\#2}{}_{\alpha\beta}$ | $\tau_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^-}^{\#1}{}_{\alpha}$ | $\sigma_{1^-}^{\#2}{}_{\alpha}$ | $\tau_{1^-}^{\#1}{}_{\alpha}$ | $\tau_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\tau_{1^-}^{\#2}{}_{\alpha}$ | 0 | 0 | 0 | $\frac{2ik\,t_1}{1+2k^2 t_1}$ | $-\frac{i\sqrt2\,k(2k^2 r_5-t_1)}{(1+2k^2 t_1)^2}$ | 0 | $\frac{-4k^4 r_5+2k^2 t_1}{(t_1+2k^2 t_1)^2}$ |
| $\tau_{1^-}^{\#1}{}_{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#2}{}_{\alpha}$ | 0 | 0 | 0 | $\frac{\sqrt2}{t_1+2k^2 t_1}$ | $\frac{-2k^2 r_5+t_1}{(t_1+2k^2 t_1)^2}$ | 0 | $\frac{i\sqrt2\,k(2k^2 r_5-t_1)}{(t_1+2k^2 t_1)^2}$ |
| $\sigma_{1^-}^{\#1}{}_{\alpha}$ | 0 | 0 | 0 | 0 | $\frac{\sqrt2}{t_1+2k^2 t_1}$ | 0 | $-\frac{2ik}{1+2k^2 t_1}$ |
| $\tau_{1^+}^{\#1}{}_{\alpha\beta}$ | $\frac{i\sqrt2\,k}{t_1+k^2 t_1}$ | $-\frac{i(2k^3 r_5-k t_1)}{(1+k^2)^2 t_1^2}$ | $-\frac{2k^4 r_5+k^2 t_1}{(1+k^2)^2 t_1^2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2}{}_{\alpha\beta}$ | $-\frac{\sqrt2}{t_1+k^2 t_1}$ | $-\frac{2k^2 r_5+t_1}{(1+k^2)^2 t_1^2}$ | $\frac{i(2k^3 r_5-k t_1)}{(1+k^2)^2 t_1^2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#1}{}_{\alpha\beta}$ | 0 | $-\frac{\sqrt2}{t_1+k^2 t_1}$ | $\frac{i\sqrt2\,k}{t_1+k^2 t_1}$ | 0 | 0 | 0 | 0 |

## Lagrangian density

$$-t_1\,\omega_{\iota}{}^{\alpha\iota}\,\omega_{\kappa\alpha}{}^{\kappa}-t_1\,\omega_{\iota}{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}^{\iota}-r_5\,\partial_{\iota}\omega_{\kappa\lambda}{}^{\iota}\,\partial_{\theta}\omega^{\kappa\lambda\theta}{}_{\kappa}+\tfrac{2}{3}r_2\,\partial_{\beta}\omega_{\theta\theta}{}^{\theta}\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}-$$
$$\tfrac{1}{3}r_2\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\,\partial_{\kappa}\omega^{\alpha\beta\theta}-\tfrac{2}{3}r_2\,\partial_{\theta}\omega_{\alpha\beta\theta}\,\partial_{\kappa}\omega^{\kappa\alpha\beta}+\omega_{\gamma\kappa\theta}\,\partial_{\kappa}\omega^{\theta}{}_{\alpha\beta}{}^{\kappa}-$$
$$r_5\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_{\kappa}\omega^{\theta\kappa\lambda}-r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial_{\kappa}\omega^{\kappa\lambda\theta}+2r_5\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_{\kappa}\omega^{\kappa\lambda\theta}-$$
$$\tfrac{1}{2}t_1\,\partial^{\alpha}f_{\theta\kappa}\,\partial^{\kappa}f_{\alpha}{}^{\theta}-\tfrac{1}{2}t_1\,\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}{}^{\theta}-\tfrac{1}{2}t_1\,\partial^{\alpha}f_{\kappa}{}^{\lambda}\,\partial^{\kappa}f_{\alpha\lambda}+$$
$$t_1\,\omega_{\kappa\alpha}{}^{\alpha}\,\partial^{\kappa}f^{\iota}{}_{\iota}+t_1\,\omega_{\kappa\lambda}{}^{\lambda}\,\partial^{\kappa}f^{\iota}{}_{\iota}+2t_1\,\partial^{\alpha}f_{\kappa\alpha}\,\partial^{\kappa}f^{\iota}{}_{\iota}-t_1\,\partial_{\kappa}f^{\lambda}{}_{\lambda}\,\partial^{\kappa}f^{\iota}{}_{\iota}+$$
$$2t_1\,\omega_{\iota\kappa\theta}\,\partial^{\kappa}f^{\iota\theta}-t_1\,\omega_{\iota\alpha}{}^{\alpha}\,\partial^{\kappa}f^{\iota}{}_{\kappa}-t_1\,\omega_{\iota\lambda}{}^{\lambda}\,\partial^{\kappa}f^{\iota}{}_{\kappa}-t_1\,\partial^{\alpha}f_{\kappa}{}^{\theta}\,\partial^{\kappa}f_{\lambda\alpha}+$$
$$\tfrac{1}{2}t_1\,\partial_{\kappa}f_{\theta}{}^{\lambda}\,\partial^{\kappa}f_{\lambda}{}^{\theta}+\tfrac{1}{2}t_1\,\partial_{\kappa}f^{\lambda}{}_{\theta}\,\partial^{\kappa}f_{\lambda}{}^{\theta}-t_1\,\partial^{\alpha}f^{\lambda}{}_{\alpha}\,\partial^{\kappa}f_{\lambda\kappa}+$$
$$\tfrac{1}{3}r_2\,\partial_{\kappa}\omega^{\alpha\beta\theta}\,\partial^{\kappa}\omega_{\alpha\beta\theta}+\tfrac{2}{3}r_2\,\partial_{\kappa}\omega^{\theta\alpha\beta}\,\partial^{\kappa}\omega_{\alpha\beta\theta}-\tfrac{2}{3}r_2\,\partial^{\beta}\omega_{\iota}{}^{\alpha\lambda}\,\partial_{\lambda}\omega_{\alpha\beta}{}^{\iota}+$$
$$\tfrac{2}{3}r_2\,\partial^{\beta}\omega_{\iota}{}^{\lambda\alpha}\,\partial_{\lambda}\omega_{\alpha\beta}{}^{\iota}+r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}-r_5\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}$$

**Added source term:** $\big|\,f^{\alpha\beta}\,\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}$

## 0⁺ / 0⁻ sector (σ, τ)

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}\dagger$ | $-\frac{1}{(1+2k^2)^2 t_1}$ | $\frac{i\sqrt2\,k}{(1+2k^2)^2 t_1}$ | 0 | 0 |
| $\tau_{0^+}^{\#1}\dagger$ | $\frac{i\sqrt2\,k}{(1+2k^2)^2 t_1}$ | $-\frac{2k^2}{(1+2k^2)^2 t_1}$ | 0 | 0 |
| $\tau_{0^+}^{\#2}\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1}\dagger$ | 0 | 0 | 0 | $\frac{1}{k^2 r_2-t_1}$ |

## Source constraints

| SO(3) irreps | # |
|---|---|
| $\tau_{0^+}^{\#2}==0$ | 1 |
| $\tau_{0^+}^{\#1}-2ik\,\sigma_{0^+}^{\#1}==0$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha}+2ik\,\sigma_{1^-}^{\#2\,\alpha}==0$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha}==0$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta}+ik\,\sigma_{1^+}^{\#2\,\alpha\beta}==0$ | 3 |
| $\tau_{2^+}^{\#1\,\alpha\beta}-2ik\,\sigma_{2^+}^{\#1\,\alpha\beta}==0$ | 5 |
| Total #: | 16 |

## 0⁺ / 0⁻ sector (ω, f)

| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1}\dagger$ | 0 | $\bar i\sqrt2\,k t_1$ | 0 | 0 |
| $f_{0^+}^{\#1}\dagger$ | $-t_1$ | $-2k^2 t_1$ | 0 | 0 |
| $f_{0^+}^{\#2}\dagger$ | $-\bar i\sqrt2\,k t_1$ | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1}\dagger$ | 0 | 0 | 0 | $k^2 r_2-t_1$ |

## 1⁺ / 1⁻ sector (ω, f)

| | $\omega_{1^+}^{\#1}{}^{\alpha\beta}$ | $\omega_{1^+}^{\#2}{}^{\alpha\beta}$ | $f_{1^+}^{\#1}{}^{\alpha\beta}$ | $\omega_{1^-}^{\#1}{}^{\alpha}$ | $\omega_{1^-}^{\#2}{}^{\alpha}$ | $f_{1^-}^{\#1}{}^{\alpha}$ | $f_{1^-}^{\#2}{}^{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^-}^{\#2}{}_{\alpha}$ | 0 | 0 | 0 | $ik t_1$ | 0 | 0 | 0 |
| $f_{1^-}^{\#2}{}_{\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{\sqrt2}$ | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1}{}_{\alpha}$ | 0 | 0 | 0 | $k^2 r_5-\frac{t_1}{2}$ | $\frac{t_1}{\sqrt2}$ | 0 | $-ik t_1$ |
| $\omega_{1^+}^{\#1}{}_{\alpha\beta}$ | $k^2 r_5-\frac{t_1}{2}$ | $-\frac{t_1}{\sqrt2}$ | $\frac{ik t_1}{\sqrt2}$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1}{}_{\alpha\beta}$ | $-\frac{ik t_1}{\sqrt2}$ | $\frac{t_1}{\sqrt2}$ | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2}{}_{\alpha\beta}$ | $-\frac{t_1}{\sqrt2}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#1}{}_{\alpha\beta}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## 2⁺ / 2⁻ sector (ω, f)

| | $\omega_{2^+}^{\#1}{}_{\alpha\beta}$ | $f_{2^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{ik t_1}{\sqrt2}$ | 0 |
| $f_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{ik t_1}{\sqrt2}$ | $k^2 t_1$ | 0 |
| $\omega_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$ | 0 | 0 | $\frac{t_1}{2}$ |

## 2⁺ / 2⁻ sector (σ, τ)

| | $\sigma_{2^+}^{\#1}{}_{\alpha\beta}$ | $\tau_{2^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{t_1}$ |
| $\tau_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{2i\sqrt2\,k}{(1+2k^2)^2 t_1}$ | $\frac{4k^2}{(1+2k^2)^2 t_1}$ | 0 |
| $\sigma_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{2}{(1+2k^2)^2 t_1}$ | $\frac{2i\sqrt2\,k}{(1+2k^2)^2 t_1}$ | 0 |

## Massive particle



$J^P = 0^-$,  $\vec{k}^{\mu}$

| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_2}>0$ |
| Polarisations: | 1 |
| Square mass: | $\frac{t_1}{r_2}>0$ |
| Spin: | 0 |
| Parity: | Odd |

**Unitary conditions** $r_2<0\ \&\&\ t_1<0$

(No massless particles)