

# Particle spectrograph

## Wave operator and propagator

$\Delta_{1+}^{\#1} \uparrow^{\alpha\beta}$	$\Delta_{1+}^{\#2} \uparrow^{\alpha\beta}$	$\Delta_{1+}^{\#3} \uparrow^{\alpha\beta}$	$\Delta_{1+}^{\#1} \uparrow^{\alpha}$	$\Delta_{1+}^{\#2} \uparrow^{\alpha}$	$\Delta_{1+}^{\#3} \uparrow^{\alpha}$	$\Delta_{1+}^{\#4} \uparrow^{\alpha}$	$\Delta_{1+}^{\#5} \uparrow^{\alpha}$	$\Delta_{1+}^{\#6} \uparrow^{\alpha}$	$\mathcal{T}_{1+}^{\#1} \uparrow^{\alpha}$
0	$-\frac{2\sqrt{2}}{a_0}$	0	0	0	0	0	0	0	0
$\Delta_{1+}^{\#2} \uparrow^{\alpha\beta}$	$-\frac{2\sqrt{2}}{a_0}$	0	0	0	0	0	0	0	0
$\Delta_{1+}^{\#3} \uparrow^{\alpha\beta}$	0	$\frac{4}{a_0}$	0	0	0	0	0	0	0
$\Delta_{1+}^{\#1} \uparrow^{\alpha}$	0	0	0	$\frac{2\sqrt{2}}{a_0}$	0	0	0	0	0
$\Delta_{1+}^{\#2} \uparrow^{\alpha}$	0	0	$\frac{2\sqrt{2}}{a_0}$	0	0	0	0	0	0
$\Delta_{1+}^{\#3} \uparrow^{\alpha}$	0	0	0	0	$-\frac{19}{12a_0}$	$\frac{5\sqrt{5}}{12a_0}$	$-\frac{1}{6\sqrt{2}a_0}$	$-\frac{1}{6a_0}$	0
$\Delta_{1+}^{\#4} \uparrow^{\alpha}$	0	0	0	0	$\frac{5\sqrt{5}}{12a_0}$	$\frac{1}{12a_0}$	$-\frac{\sqrt{5}}{6a_0}$	$-\frac{\sqrt{5}}{6a_0}$	0
$\Delta_{1+}^{\#5} \uparrow^{\alpha}$	0	0	0	0	$-\frac{1}{6\sqrt{2}a_0}$	$-\frac{\sqrt{2}}{6a_0}$	$\frac{17}{6a_0}$	$-\frac{7}{3\sqrt{2}a_0}$	0
$\Delta_{1+}^{\#6} \uparrow^{\alpha}$	0	0	0	0	$-\frac{1}{6a_0}$	$-\frac{\sqrt{5}}{6a_0}$	$-\frac{7}{3\sqrt{2}a_0}$	$\frac{5}{3a_0}$	0
$\mathcal{T}_{1+}^{\#1} \uparrow^{\alpha}$	0	0	0	0	0	0	0	0	0

$\Gamma_{1+}^{\#1} \uparrow^{\alpha\beta}$	$\Gamma_{1+}^{\#2} \uparrow^{\alpha\beta}$	$\Gamma_{1+}^{\#3} \uparrow^{\alpha\beta}$	$\Gamma_{1+}^{\#1} \uparrow^{\alpha}$	$\Gamma_{1+}^{\#2} \uparrow^{\alpha}$	$\Gamma_{1+}^{\#3} \uparrow^{\alpha}$	$\Gamma_{1+}^{\#4} \uparrow^{\alpha}$	$\Gamma_{1+}^{\#5} \uparrow^{\alpha}$	$\Gamma_{1+}^{\#6} \uparrow^{\alpha}$	$h_{1+}^{\#1} \uparrow^{\alpha}$
$-\frac{a_0}{4}$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0	0	0
$-\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0	0	0	0
0	0	$\frac{a_0}{4}$	0	0	0	0	0	0	0
0	0	0	$-\frac{a_0}{4}$	$\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0
0	0	0	$\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0
0	0	0	0	0	$-\frac{a_0}{3}$	$\frac{\sqrt{5}a_0}{6}$	$-\frac{a_0}{6\sqrt{2}}$	$-\frac{a_0}{6}$	0
0	0	0	0	0	$\frac{\sqrt{5}a_0}{6}$	$\frac{a_0}{3}$	$-\frac{1}{6}\sqrt{\frac{5}{2}}a_0$	$-\frac{\sqrt{5}a_0}{6}$	0
0	0	0	0	0	$-\frac{a_0}{6\sqrt{2}}$	$-\frac{1}{6}\sqrt{\frac{5}{2}}a_0$	$\frac{a_0}{3}$	$\frac{a_0}{6\sqrt{2}}$	0
0	0	0	0	0	$-\frac{a_0}{6}$	$-\frac{\sqrt{5}a_0}{6}$	$\frac{a_0}{6\sqrt{2}}$	$\frac{5a_0}{12}$	0
0	0	0	0	0	0	0	0	0	0

Quadratic (free) action

$S_F ==$

$$\iiint (\frac{1}{8} (8 h^{\alpha\beta} \mathcal{T}_{\alpha\beta} - 4 \Gamma^{\alpha\beta\chi} (a_0 \Gamma_{\beta\chi\alpha} - 2 \Delta_{\alpha\beta\chi} + a_0 \partial_\beta h_{\alpha\chi}) + 2 a_0 \Gamma^{\alpha\beta}_{\alpha} \partial_\beta h^{\chi}_{\chi} - 2 a_0 h^{\chi}_{\chi} \partial_\beta \Gamma^{\alpha}_{\alpha} \beta + 2 a_0 h^{\chi}_{\chi} \partial_\beta \Gamma^{\alpha\beta}_{\alpha} - 4 a_0 h_{\alpha\chi} \partial_\beta \Gamma^{\alpha\beta\chi} + 4 a_0 h^{\alpha\beta} \partial_\beta \partial_\alpha h^{\chi}_{\chi} - a_0 \partial_\beta h^{\chi}_{\chi} \partial^\beta h^{\alpha}_{\alpha} - 4 a_0 \partial_\alpha h^{\alpha\beta} \partial_\chi h^{\chi}_{\beta} + 4 a_0 \partial^\beta h^{\alpha}_{\alpha} \partial_\chi h^{\chi}_{\beta} + 2 a_0 \Gamma^{\alpha}_{\alpha} \beta (2 \Gamma^{\chi}_{\beta\chi} - \partial_\beta h^{\chi}_{\chi} + 2 \partial_\chi h^{\chi}_{\beta}) - 8 a_0 h^{\alpha\beta} \partial_\chi \partial_\beta h^{\chi}_{\alpha} + 2 a_0 h^{\alpha}_{\alpha} \partial_\chi \partial_\beta h^{\beta\chi} + 4 a_0 h^{\alpha\beta} \partial_\chi \partial^\beta h_{\alpha\beta} - 2 a_0 h^{\alpha}_{\alpha} \partial_\chi \partial^\beta h^{\chi}_{\beta} - 2 a_0 \partial_\beta h_{\alpha\chi} \partial^\chi h^{\alpha\beta} + 3 a_0 \partial_\chi h_{\alpha\beta} \partial^\chi h^{\alpha\beta} + 4 a_0 h_{\beta\chi} \partial^\chi \Gamma^{\alpha}_{\alpha} \beta)) [t, x, y, z] dz dy dx dt$$

$\Delta_{3+}^{\#1} \uparrow^{\alpha\beta\chi}$

$-\frac{2}{a_0}$

$\Gamma_{3+}^{\#1} \uparrow^{\alpha\beta\chi}$

$-\frac{a_0}{2}$

$\Delta_{2+}^{\#1} \uparrow^{\alpha\beta}$	$\Delta_{2+}^{\#2} \uparrow^{\alpha\beta}$	$\Delta_{2+}^{\#3} \uparrow^{\alpha\beta}$	$\mathcal{T}_{2+}^{\#1} \uparrow^{\alpha\beta}$	$\Delta_{2+}^{\#1} \uparrow^{\alpha}$	$\Delta_{2+}^{\#2} \uparrow^{\alpha}$	$\Delta_{2+}^{\#3} \uparrow^{\alpha}$
$\frac{4}{a_0}$	0	0	0	0	0	0
$\Delta_{2+}^{\#1} \uparrow^{\alpha\beta}$	$-\frac{3}{4a_0}$	$\frac{5}{4a_0}$	$-\frac{1}{2\sqrt{2}a_0}$	0	0	0
$\Delta_{2+}^{\#2} \uparrow^{\alpha\beta}$	$\frac{5}{4a_0}$	$-\frac{3}{4a_0}$	$-\frac{1}{2\sqrt{2}a_0}$	0	0	0
$\Delta_{2+}^{\#3} \uparrow^{\alpha\beta}$	$-\frac{1}{2\sqrt{2}a_0}$	$-\frac{1}{2\sqrt{2}a_0}$	$\frac{1}{2a_0}$	0	0	0
$\mathcal{T}_{2+}^{\#1} \uparrow^{\alpha\beta}$	0	0	$-\frac{8}{a_0 k^2}$	0	0	0
$\Delta_{2+}^{\#1} \uparrow^{\alpha\beta\chi}$	0	0	0	$\frac{4}{a_0 k^2}$	0	0
$\Delta_{2+}^{\#2} \uparrow^{\alpha\beta\chi}$	0	0	0	0	$\frac{4}{a_0}$	$\frac{4}{a_0}$

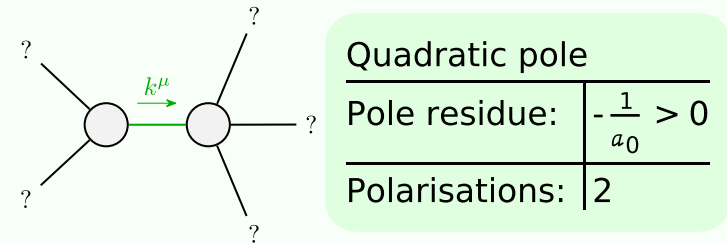
$\Gamma_{2+}^{\#1} \uparrow^{\alpha\beta}$	$\Gamma_{2+}^{\#2} \uparrow^{\alpha\beta}$	$\Gamma_{2+}^{\#3} \uparrow^{\alpha\beta}$	$h_{2+}^{\#1} \uparrow^{\alpha\beta}$	$\Gamma_{2+}^{\#1} \uparrow^{\alpha}$	$\Gamma_{2+}^{\#2} \uparrow^{\alpha}$	$\Gamma_{2+}^{\#3} \uparrow^{\alpha}$
$\frac{a_0}{4}$	0	0	0	0	0	0
$\Gamma_{2+}^{\#1} \uparrow^{\alpha\beta}$	$-\frac{a_0}{2}$	0	0	0	0	0
$\Gamma_{2+}^{\#2} \uparrow^{\alpha\beta}$	0	$\frac{a_0}{4}$	0	0	0	0
$\Gamma_{2+}^{\#3} \uparrow^{\alpha\beta}$	0	0	$-\frac{a_0 k^2}{8}$	0	0	0
$h_{2+}^{\#1} \uparrow^{\alpha\beta}$	0	0	0	$\frac{a_0}{4}$	0	0
$\Gamma_{2+}^{\#1} \uparrow^{\alpha\beta\chi}$	0	0	0	0	$\frac{a_0}{4}$	0
$\Gamma_{2+}^{\#2} \uparrow^{\alpha\beta\chi}$	0	0	0	0	0	$\frac{a_0}{4}$

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\mathcal{T}_{0+}^{\#2} == 0$	1
$\Delta_{0+}^{\#3} + 2 \Delta_{0+}^{\#4} + 3 \Delta_{0+}^{\#2} == 0$	1
$\mathcal{T}_{1+}^{\#1\alpha} == 0$	3
$2 \Delta_{1+}^{\#6\alpha} + \Delta_{1+}^{\#4\alpha} + 2 \Delta_{1+}^{\#5\alpha} + \Delta_{1+}^{\#3\alpha} == 0$	3
Total constraints:	8

	$\Gamma_{0+}^{\#1} \uparrow$	$\Gamma_{0+}^{\#2} \uparrow$	$\Gamma_{0+}^{\#3} \uparrow$	$\Gamma_{0+}^{\#4} \uparrow$	$h_{0+}^{\#1} \uparrow$	$h_{0+}^{\#2} \uparrow$	$\Gamma_{0+}^{\#1} \uparrow$
$\Gamma_{0+}^{\#1} \uparrow$	$-\frac{a_0}{2}$	0	0	0	0	0	0
$\Gamma_{0+}^{\#2} \uparrow$	0	0	$\frac{a_0}{2}$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0
$\Gamma_{0+}^{\#3} \uparrow$	0	$\frac{a_0}{2}$	0	$-\frac{a_0}{2\sqrt{2}}$	0	0	0
$\Gamma_{0+}^{\#4} \uparrow$	0	$-\frac{a_0}{2\sqrt{2}}$	$-\frac{a_0}{2\sqrt{2}}$	$\frac{a_0}{2}$	0	0	0
$h_{0+}^{\#1} \uparrow$	0	0	0	0	$\frac{a_0 k^2}{4}$	0	0
$h_{0+}^{\#2} \uparrow$	0	0	0	0	0	0	0
$\Gamma_{0+}^{\#1} \uparrow$	0	0	0	0	0	0	$-\frac{a_0}{2}$

## Massive and massless spectra



(No massive particles)

## Unitarity conditions

$a_0 < 0$