Particle spectrograph

Wave operator and propagator

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\sigma_{0^-}^{\#1} == 0$	$\epsilon \eta_{\alpha\beta\chi\delta}\, \partial^{\delta} \sigma^{\alpha\beta\chi} == 0$	1
$\tau_{0^+}^{\#2} == 0$	$\partial_{\beta}\partial_{\alpha}\tau^{\alpha\beta} == 0$	1
$\tau_{0^+}^{\#1} - 2\,i\,k\,\sigma_{0^+}^{\#1} == 0$	$\partial_{\beta}\partial_{\alpha}\tau^{\alpha\beta} == \partial_{\beta}\partial^{\beta}\tau^{\alpha}{}_{\alpha} + 2\,\partial_{\chi}\partial^{\chi}\partial_{\beta}\sigma^{\alpha\beta}{}_{\alpha}$	1
$\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$	$\partial_{\chi}\partial_{\beta}\partial^{\alpha}\tau^{\beta\chi} == \partial_{\chi}\partial^{\chi}\partial_{\beta}\tau^{\alpha\beta} + 2\,\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial_{\beta}\sigma^{\alpha\beta\chi}$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	$\partial_{\chi}\partial_{\beta}\partial^{\alpha}\tau^{\beta\chi} == \partial_{\chi}\partial^{\chi}\partial_{\beta}\tau^{\beta\alpha}$	3
$\tau_{1^+}^{\#1\,\alpha\beta} == 0$	$\partial_{\chi}\partial^{\alpha}\tau^{\beta\chi} + \partial_{\chi}\partial^{\beta}\tau^{\chi\alpha} + \partial_{\chi}\partial^{\chi}\tau^{\alpha\beta} == \partial_{\chi}\partial^{\alpha}\tau^{\chi\beta} + \partial_{\chi}\partial^{\beta}\tau^{\alpha\chi} + \partial_{\chi}\partial^{\chi}\tau^{\beta\alpha}$	3
$\sigma_{1^+}^{\#2\,\alpha\beta} == 0$	$\partial_{\delta}\partial_{\chi}\partial^{\alpha}\sigma^{\beta\chi\delta} + \partial_{\delta}\partial^{\delta}\partial_{\chi}\sigma^{\alpha\beta\chi} == \partial_{\delta}\partial_{\chi}\partial^{\beta}\sigma^{\alpha\chi\delta}$	3
$\tau_{2^+}^{\#1\,\alpha\beta} == 0$	$4\,\partial_{\delta}\partial_{\chi}\partial^{\beta}\partial^{\alpha}\tau^{\chi\delta} + 2\,\partial_{\delta}\partial^{\delta}\partial^{\beta}\partial^{\alpha}\tau^{\chi}{}_{\chi} + 3\,\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\chi}\tau^{\alpha\beta} + 3\,\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\chi}\tau^{\beta\alpha} + 2\,\eta^{\alpha\beta}\,\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial_{\chi}\tau^{\chi\delta} == 3\,\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\alpha}\tau^{\beta\chi} + 3\,\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\beta}\tau^{\alpha\chi} + 3\,\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\beta}\tau^{\chi\alpha} + 2\,\eta^{\alpha\beta}\,\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\chi}\tau^{\chi}{}_{\chi}$	5
$\sigma_{2^+}^{\#1\,\alpha\beta} == 0$	$3\,\partial_{\delta}\partial_{\chi}\partial^{\alpha}\sigma^{\beta\chi\delta} + 3\,\partial_{\delta}\partial_{\chi}\partial^{\beta}\sigma^{\alpha\chi\delta} + 2\,\eta^{\alpha\beta}\,\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\sigma^{\chi\delta}{}_{\chi} == 2\,\partial_{\delta}\partial^{\beta}\partial^{\alpha}\sigma^{\chi\delta}{}_{\chi} + 3\,(\partial_{\delta}\partial^{\delta}\partial_{\chi}\sigma^{\alpha\chi\beta} + \partial_{\delta}\partial^{\delta}\partial_{\chi}\sigma^{\beta\chi\alpha})$	5
Total constraints/gauge generators:		25

$$
\begin{array}{c|ccccccc}
 & \sigma_{1^+}^{\#1\,\alpha\beta} & \sigma_{1^+}^{\#2\,\alpha\beta} & \tau_{1^+}^{\#1\,\alpha\beta} & \sigma_{1^-}^{\#1\,\alpha} & \sigma_{1^-}^{\#2\,\alpha} & \tau_{1^-}^{\#1\,\alpha} & \tau_{1^-}^{\#2\,\alpha} \\
\sigma_{1^+}^{\#1\,\dagger\,\alpha\beta} & \frac{1}{k^2\,(2\,r_1+r_5)} & 0 & 0 & 0 & 0 & 0 & 0 \\
\sigma_{1^+}^{\#2\,\dagger\,\alpha\beta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tau_{1^+}^{\#1\,\dagger\,\alpha\beta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sigma_{1^-}^{\#1\,\dagger\,\alpha} & 0 & 0 & 0 & \frac{1}{k^2\,(r_1+r_5)} & \frac{\sqrt{2}}{k^2\,(1+2\,k^2)\,(r_1+r_5)} & 0 & \frac{2\,i}{k\,(1+2\,k^2)\,(r_1+r_5)} \\
\sigma_{1^-}^{\#2\,\dagger\,\alpha} & 0 & 0 & 0 & \frac{\sqrt{2}}{k^2\,(1+2\,k^2)\,(r_1+r_5)} & \frac{3\,k^2\,(r_1+r_5)+2\,t_3}{(k+2\,k^3)^2\,(r_1+r_5)\,t_3} & 0 & \frac{i\,\sqrt{2}\,(3\,k^2\,(r_1+r_5)+2\,t_3)}{k\,(1+2\,k^2)^2\,(r_1+r_5)\,t_3} \\
\tau_{1^-}^{\#1\,\dagger\,\alpha} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tau_{1^-}^{\#2\,\dagger\,\alpha} & 0 & 0 & 0 & -\frac{i\,\sqrt{2}\,(3\,k^2\,(r_1+r_5)+2\,t_3)}{k\,(1+2\,k^2)^2\,(r_1+r_5)\,t_3} & 0 & 0 & \frac{6\,k^2\,(r_1+r_5)+4\,t_3}{(1+2\,k^2)^2\,(r_1+r_5)\,t_3}
\end{array}
$$

Quadratic (free) action

$$
\begin{aligned}
S == \int\int\int\int \Big(&-2\,t_3\,\mathcal{A}^{\alpha\prime}{}_{\alpha}\,\mathcal{A}_{\prime}{}^{\theta}{}_{\theta} + 3\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 3\,\mathcal{A}^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + 4\,t_3\,\mathcal{A}_{\alpha}{}^{\theta}\,\partial_{\prime}f^{\alpha\prime} - 4\,t_3\,\mathcal{A}_{\prime}{}^{\theta}\,\partial^{\prime}f^{\alpha}{}_{\alpha} + 2\,t_3\,\partial_{\prime}f^{\alpha\prime}{}_{\theta}\,\partial^{\theta}f^{\alpha}{}_{\alpha} - \\
&4\,t_3\,\partial^{\prime}f^{\alpha}{}_{\alpha}\,\partial_{\theta}f_{\prime}{}^{\theta} - 4\,r_1\,\partial_{\beta}\mathcal{A}_{\alpha\prime\theta}\,\partial^{\theta}\mathcal{A}^{\alpha\beta\prime} + 2\,r_1\,\partial_{\beta}\mathcal{A}_{\prime\theta\alpha}\,\partial^{\theta}\mathcal{A}^{\alpha\beta\prime} - 8\,r_1\,\partial_{\beta}\mathcal{A}_{\prime\theta\alpha}\,\partial^{\theta}\mathcal{A}^{\alpha\beta\prime} - 2\,r_1\,\partial_{\prime}\mathcal{A}_{\alpha\beta\theta}\,\partial^{\theta}\mathcal{A}^{\alpha\beta\prime} + \\
&2\,r_1\,\partial_{\theta}\mathcal{A}_{\alpha\beta\prime}\,\partial^{\theta}\mathcal{A}^{\alpha\beta\prime} + 2\,r_1\,\partial_{\theta}\mathcal{A}_{\alpha\prime\beta}\,\partial^{\theta}\mathcal{A}^{\alpha\beta\prime} + \\
&3\,r_5\,\partial_{\mathcal{A}}{}_{\theta}{}^{\kappa}\,\partial^{\theta}\mathcal{A}^{\alpha\prime}{}_{\alpha} - 3\,r_5\,\partial_{\theta}\mathcal{A}_{\prime}{}^{\kappa}\,\partial^{\theta}\mathcal{A}^{\alpha\prime}{}_{\alpha} - 3\,r_5\,\partial_{\alpha}\mathcal{A}^{\alpha\prime\theta} \\
&\partial_{\kappa}\mathcal{A}_{\prime}{}^{\kappa}{}_{\theta} + 6\,r_5\,\partial^{\theta}\mathcal{A}^{\alpha\prime}{}_{\alpha}\,\partial_{\kappa}\mathcal{A}_{\prime}{}^{\kappa}{}_{\theta} + 3\,r_5\,\partial_{\alpha}\mathcal{A}^{\alpha\prime\theta}\,\partial_{\kappa}\mathcal{A}_{\theta}{}^{\kappa}{}_{\prime} - \\
&6\,r_5\,\partial^{\theta}\mathcal{A}^{\alpha\prime}{}_{\alpha}\,\partial_{\kappa}\mathcal{A}_{\theta}{}^{\kappa}{}_{\prime}\Big)[t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt
\end{aligned}
$$

$$
\begin{array}{c|ccccccc}
 & \mathcal{A}_{1^+}^{\#1\,\alpha\beta} & \mathcal{A}_{1^+}^{\#2\,\alpha\beta} & f_{1^+}^{\#1\,\alpha\beta} & \mathcal{A}_{1^-}^{\#1\,\alpha} & \mathcal{A}_{1^-}^{\#2\,\alpha} & f_{1^-}^{\#1\,\alpha} & f_{1^-}^{\#2\,\alpha} \\
\mathcal{A}_{1^+}^{\#1\,\dagger\,\alpha\beta} & k^2\,(2\,r_1+r_5) & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathcal{A}_{1^+}^{\#2\,\dagger\,\alpha\beta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
f_{1^+}^{\#1\,\dagger\,\alpha\beta} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathcal{A}_{1^-}^{\#1\,\dagger\,\alpha} & 0 & 0 & 0 & k^2\,(r_1+r_5)+\frac{2\,t_3}{3} & -\frac{\sqrt{2}\,t_3}{3} & 0 & -\frac{2}{3}\,i\,k\,t_3 \\
\mathcal{A}_{1^-}^{\#2\,\dagger\,\alpha} & 0 & 0 & 0 & -\frac{\sqrt{2}\,t_3}{3} & \frac{t_3}{3} & 0 & \frac{1}{3}\,i\,\sqrt{2}\,k\,t_3 \\
f_{1^-}^{\#1\,\dagger\,\alpha} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
f_{1^-}^{\#2\,\dagger\,\alpha} & 0 & 0 & 0 & \frac{2\,i\,k\,t_3}{3} & -\frac{1}{3}\,i\,\sqrt{2}\,k\,t_3 & 0 & \frac{2\,k^2\,t_3}{3}
\end{array}
$$

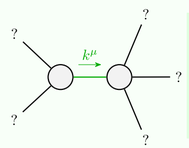
$$
\begin{array}{c|ccc}
 & \mathcal{A}_{2^+}^{\#1\,\alpha\beta} & f_{2^+}^{\#1\,\alpha\beta} & \mathcal{A}_{2^-}^{\#1\,\alpha\beta\chi} \\
\mathcal{A}_{2^+}^{\#1\,\dagger\,\alpha\beta} & 0 & 0 & 0 \\
f_{2^+}^{\#1\,\dagger\,\alpha\beta} & 0 & 0 & 0 \\
\mathcal{A}_{2^-}^{\#1\,\dagger\,\alpha\beta\chi} & 0 & 0 & k^2\,r_1
\end{array}
$$

$$
\begin{array}{c|cccc}
 & \sigma_{0^+}^{\#1} & \tau_{0^+}^{\#1} & \tau_{0^+}^{\#2} & \sigma_{0^-}^{\#1} \\
\sigma_{0^+}^{\#1\,\dagger} & \frac{1}{(1+2\,k^2)^2\,t_3} & -\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_3} & 0 & 0 \\
\tau_{0^+}^{\#1\,\dagger} & \frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_3} & \frac{2\,k^2}{(1+2\,k^2)^2\,t_3} & 0 & 0 \\
\tau_{0^+}^{\#2\,\dagger} & 0 & 0 & 0 & 0 \\
\sigma_{0^-}^{\#1\,\dagger} & 0 & 0 & 0 & 0
\end{array}
$$

$$
\begin{array}{c|ccc}
 & \sigma_{2^+}^{\#1\,\alpha\beta} & \tau_{2^+}^{\#1\,\alpha\beta} & \sigma_{2^-}^{\#1\,\alpha\beta\chi} \\
\sigma_{2^+}^{\#1\,\dagger\,\alpha\beta} & 0 & 0 & \frac{1}{k^2\,r_1} \\
\tau_{2^+}^{\#1\,\dagger\,\alpha\beta} & 0 & 0 & 0 \\
\sigma_{2^-}^{\#1\,\dagger\,\alpha\beta\chi} & \frac{1}{k^2\,r_1} & 0 & 0
\end{array}
$$

$$
\begin{array}{c|cccc}
 & \mathcal{A}_{0^+}^{\#1} & f_{0^+}^{\#1} & f_{0^+}^{\#2} & \mathcal{A}_{0^-}^{\#1} \\
\mathcal{A}_{0^+}^{\#1\,\dagger} & t_3 & -i\,\sqrt{2}\,k\,t_3 & 2\,k^2\,t_3 & 0 \\
f_{0^+}^{\#1\,\dagger} & -i\,\sqrt{2}\,k\,t_3 & 0 & 0 & 0 \\
f_{0^+}^{\#2\,\dagger} & 2\,k^2\,t_3 & 0 & 0 & 0 \\
\mathcal{A}_{0^-}^{\#1\,\dagger} & 0 & 0 & 0 & 0
\end{array}
$$

Massive and massless spectra



Quadratic pole

Pole residue:	$-\frac{1}{r_1\,(r_1+r_5)\,(2\,r_1+r_5)\,p^2} > 0$
Polarisations:	2

(No massive particles)

Unitarity conditions

$r_1 < 0 \,\&\&\, (r_5 < -r_1 \,||\, r_5 > -2\,r_1) \,||\, r_1 > 0 \,\&\&\, -2\,r_1 < r_5 < -r_1$