

Particle spectrograph

Wave operator and propagator

|   | $\sigma_{1^+\alpha\beta}^{\#1}$ | $\sigma_{1^+\alpha\beta}^{\#2}$                 | $\tau_{1^+\alpha\beta}^{\#1}$                   | $\sigma_{1^-\alpha}^{\#1}$   | $\sigma_{1^-\alpha}^{\#2}$   | $\tau_{1^-\alpha}^{\#1}$ | $\tau_{1^-\alpha}^{\#2}$  |
|---|---------------------------------|---|---|--|--|--------------------------|---|
| $\sigma_{1^+}^{\#1}\dagger^{\alpha\beta}$ | 0                               | $-\frac{\sqrt{2}}{t_1+k^2t_1}$                  | $-\frac{i\sqrt{2}k}{t_1+k^2t_1}$                | 0  | 0  | 0                        | 0   |
| $\sigma_{1^+}^{\#2}\dagger^{\alpha\beta}$ | $-\frac{\sqrt{2}}{t_1+k^2t_1}$  | $\frac{-2k^2(2r_1+r_5)+t_1}{(1+k^2)^2t_1^2}$    | $\frac{-2ik^3(2r_1+r_5)+ikt_1}{(1+k^2)^2t_1^2}$ | 0  | 0  | 0                        | 0   |
| $\tau_{1^+}^{\#1}\dagger^{\alpha\beta}$   | $\frac{i\sqrt{2}k}{t_1+k^2t_1}$ | $\frac{i(2k^3(2r_1+r_5)-kt_1)}{(1+k^2)^2t_1^2}$ | $\frac{-2k^4(2r_1+r_5)+k^2t_1}{(1+k^2)^2t_1^2}$ | 0  | 0  | 0                        | 0   |
| $\sigma_{1^+}^{\#1}\dagger^\alpha$        | 0                               | 0   | 0   | $\frac{2(t_1+t_3)}{3t_1t_3+2k^2(r_1+r_5)(t_1+t_3)}$                    | $-\frac{\sqrt{2}(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$                   | 0                        | $-\frac{2ik(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$                       |
| $\sigma_{1^+}^{\#2}\dagger^\alpha$        | 0                               | 0   | 0   | $-\frac{\sqrt{2}(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$ | $\frac{6k^2(r_1+r_5)+t_1+4t_3}{(1+2k^2)^2(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$              | 0                        | $\frac{i\sqrt{2}k(6k^2(r_1+r_5)+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$ |
| $\tau_{1^+}^{\#1}\dagger^\alpha$          | 0                               | 0   | 0   | 0  | 0  | 0                        | 0   |
| $\tau_{1^+}^{\#2}\dagger^\alpha$          | 0                               | 0   | 0   | $\frac{2ik(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$       | $-\frac{i\sqrt{2}k(6k^2(r_1+r_5)+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$ | 0                        | $\frac{2k^2(6k^2(r_1+r_5)+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$       |

| Source constraints  |  |                |
|---|--|----------------|
| SO(3) irreps  | Fundamental fields   | Multiplicities |
| $\tau_{0^+}^{\#2}==0$   | $\partial_\beta\partial_\alpha\tau^{\alpha\beta}==0$   | 1              |
| $\tau_{0^+}^{\#1}-2\,i\,k\,\sigma_{0^+}^{\#1}==0$                       | $\partial_\beta\partial_\alpha\tau^{\alpha\beta}==\partial_\beta\partial^\beta\tau^\alpha_\alpha+2\,\partial_\chi\partial^X\partial_\beta\sigma^{\alpha\beta}_\alpha$  | 1              |
| $\tau_1^{\#2\alpha}+2\,i\,k\,\sigma_1^{\#2\alpha}==0$                   | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi}==\partial_\chi\partial^X\partial_\beta\tau^{\alpha\beta}+2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$   | 3              |
| $\tau_1^{\#1\alpha}==0$   | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi}==\partial_\chi\partial^X\partial_\beta\tau^{\beta\alpha}$  | 3              |
| $\tau_{1^+}^{\#1\alpha\beta}+i\,k\,\sigma_{1^+}^{\#2\alpha\beta}==0$    | $\partial_\chi\partial^\alpha\tau^{\beta\chi}+\partial_\chi\partial^\beta\tau^{\chi\alpha}+\partial_\chi\partial^\chi\tau^{\alpha\beta}+$<br>$2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta}+2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi}==$<br>$\partial_\chi\partial^\alpha\tau^{\chi\beta}+\partial_\chi\partial^\beta\tau^{\alpha\chi}+$<br>$\partial_\chi\partial^\chi\tau^{\beta\alpha}+2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$  | 3              |
| $\tau_{2^+}^{\#1\alpha\beta}-2\,i\,k\,\sigma_{2^+}^{\#1\alpha\beta}==0$ | $-i\,(4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta}+2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^{\chi\chi}_\chi-$<br>$3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi}-3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta}-$<br>$3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi}-3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha}+$<br>$3\,\partial_\delta\partial^\delta\partial_\chi\partial^X\tau^{\alpha\beta}+3\,\partial_\delta\partial^\delta\partial_\chi\partial^X\tau^{\beta\alpha}+$<br>$4\,i\,k^X\,\partial_\epsilon\partial_\chi\partial^\beta\partial^\alpha\sigma^{\delta\epsilon}_\delta-$<br>$6\,i\,k^X\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\delta\epsilon}_-$<br>$6\,i\,k^X\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\delta\epsilon}+$<br>$2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta}+$<br>$6\,i\,k^X\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\alpha\delta\beta}+$<br>$6\,i\,k^X\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\beta\delta\alpha}-$<br>$2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^{\chi\chi}_\chi-$<br>$4\,i\,\eta^{\alpha\beta}\,k^X\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\chi\sigma^{\delta\epsilon}_\delta)==0$ | 5              |
| Total constraints/gauge generators:                                     |  | 16             |

|   | $\omega_{0^+}^{\#1}$            | $f_{0^+}^{\#1}$                 | $f_{0^+}^{\#2}$            | $\omega_{0^+}^{\#1}$ |   | $\omega_{2^+}^{\#1}\alpha\beta$   | $f_{2^+}^{\#1}\alpha\beta$ | $\omega_{2^+}^{\#1}\alpha\beta\chi$ |
|---|---------------------------------|---------------------------------|----------------------------|----------------------|---|-----------------------------------|----------------------------|-------------------------------------|
| $\omega_{0^+}^{\#1}\dagger$               | $t_3$                           | $-i\sqrt{2}kt_3$                | 0                          | 0                    | $\omega_{2^+}^{\#1}\dagger^{\alpha\beta}$     | $\frac{t_1}{2}$                   | $-\frac{ikt_1}{\sqrt{2}}$  | 0                                   |
| $f_{0^+}^{\#1}\dagger$                    | $i\sqrt{2}kt_3$                 | $2k^2t_3$                       | 0                          | 0                    | $f_{2^+}^{\#1}\dagger^{\alpha\beta}$          | $\frac{ikt_1}{\sqrt{2}}$          | $k^2t_1$                   | 0                                   |
| $f_{0^+}^{\#2}\dagger$                    | 0                               | 0                               | 0                          | 0                    | $\omega_{2^+}^{\#1}\dagger^{\alpha\beta\chi}$ | 0                                 | 0                          | $k^2r_1+\frac{t_1}{2}$              |
| $\omega_{0^+}^{\#1}\dagger$               | 0                               | 0                               | 0                          | $-t_1$               |   |                                   |                            |                                     |
|   | $\omega_{1^+}^{\#1}\alpha\beta$ | $\omega_{1^+}^{\#2}\alpha\beta$ | $f_{1^+}^{\#1}\alpha\beta$ |                      | $\omega_{1^+}^{\#1}\alpha$                    | $\omega_{1^+}^{\#2}\alpha$        | $f_{1^+}^{\#1}\alpha$      | $f_{1^+}^{\#2}\alpha$               |
| $\omega_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $k^2(2r_1+r_5)-\frac{t_1}{2}$   | $-\frac{t_1}{\sqrt{2}}$         | $-\frac{ikt_1}{\sqrt{2}}$  |                      | 0   | 0                                 | 0                          | 0                                   |
| $\omega_{1^+}^{\#2}\dagger^{\alpha\beta}$ | $-\frac{t_1}{\sqrt{2}}$         | 0                               | 0                          |                      | 0   | 0                                 | 0                          | 0                                   |
| $f_{1^+}^{\#1}\dagger^{\alpha\beta}$      | $\frac{ikt_1}{\sqrt{2}}$        | 0                               | 0                          |                      | 0   | 0                                 | 0                          | 0                                   |
| $\omega_{1^+}^{\#1}\dagger^\alpha$        | 0                               | 0                               | 0                          |                      | $\frac{1}{6}(6k^2(r_1+r_5)+t_1+4t_3)$         | $\frac{t_1-2t_3}{3\sqrt{2}}$      | 0                          | $\frac{1}{3}ik(t_1-2t_3)$           |
| $\omega_{1^+}^{\#2}\dagger^\alpha$        | 0                               | 0                               | 0                          |                      | $\frac{t_1-2t_3}{3\sqrt{2}}$                  | $\frac{t_1+t_3}{3}$               | 0                          | $\frac{1}{3}i\sqrt{2}k(t_1+t_3)$    |
| $f_{1^+}^{\#1}\dagger^\alpha$             | 0                               | 0                               | 0                          |                      | 0   | 0                                 | 0                          | 0                                   |
| $f_{1^+}^{\#2}\dagger^\alpha$             | 0                               | 0                               | 0                          |                      | $-\frac{1}{3}ik(t_1-2t_3)$                    | $-\frac{1}{3}i\sqrt{2}k(t_1+t_3)$ | 0                          | $\frac{2}{3}k^2(t_1+t_3)$           |

| Quadratic (free) action   |
|---|
| $S==\iiint\!\!\!\int(\frac{1}{6}(2\,\omega^{\alpha i}_\alpha(t_1\,\omega_{\,\,\theta}^{\,\,\theta}-2\,t_3\,\omega_{\,\,\kappa}^{\,\,\kappa})+6\,f^{\alpha\beta}\,\tau_{\alpha\beta}+6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}-$<br>$4\,t_1\,\omega_{\alpha\,\,\theta}^{\,\,\theta}\partial_{\,\,\prime}f^{\alpha i}+8\,t_3\,\omega_{\alpha\,\,\kappa}^{\,\,\kappa}\partial_{\,\,\prime}f^{\alpha i}+4\,t_1\,\omega_{\,\,\theta}^{\,\,\theta}\partial_{\,\,\prime}f^\alpha_\alpha-$<br>$8\,t_3\,\omega_{\,\,\kappa}^{\,\,\kappa}\partial_{\,\,\prime}f^\alpha_\alpha-2\,t_1\partial_{\,\,\prime}f_{\,\,\theta}^{\,\,\theta}\partial_{\,\,\prime}f^\alpha_\alpha+4\,t_3\partial_{\,\,\prime}f_{\,\,\kappa}^{\,\,\kappa}\partial_{\,\,\prime}f^\alpha_\alpha-$<br>$2\,t_1\partial_{\,\,\prime}f^{\alpha i}\partial_{\theta}f_{\alpha}^{\,\,\theta}+4\,t_1\partial_{\,\,\prime}f^\alpha_\alpha\partial_{\theta}f_{\,\,\prime}^{\,\,\theta}-6\,t_1\partial_{\alpha}f_{\,\,\theta}^{\,\,\theta}\partial^\theta f^{\alpha i}-$<br>$3\,t_1\partial_{\alpha}f_{\theta\,\,\prime}^{\,\,\theta}\partial^\theta f^{\alpha i}+3\,t_1\partial_{\,\,\prime}f_{\alpha\theta}^{\,\,\theta}\partial^\theta f^{\alpha i}+3\,t_1\partial_{\theta}f_{\alpha i}^{\,\,\theta}\partial^\theta f^{\alpha i}+$<br>$3\,t_1\partial_{\theta}f_{\,\,\prime}^{\,\,\theta}\partial^\theta f^{\alpha i}+6\,t_1\,\omega_{\alpha\theta\,\,\prime}(\,\omega^{\alpha i\theta}+2\,\partial^\theta f^{\alpha i})-$<br>$8\,r_1\,\partial_\beta\omega_{\alpha i\theta}^{\,\,\theta}\partial^\theta\omega^{\alpha\beta i}+4\,r_1\,\partial_\beta\omega_{\alpha\theta\,\,\prime}\partial^\theta\omega^{\alpha\beta i}-$<br>$16\,r_1\,\partial_\beta\omega_{\,\,\theta\alpha}^{\,\,\theta}\partial^\theta\omega^{\alpha\beta i}-4\,r_1\,\partial_{\,\,\prime}\omega_{\alpha\beta\theta}^{\,\,\theta}\partial^\theta\omega^{\alpha\beta i}+$<br>$4\,r_1\,\partial_\theta\omega_{\alpha\beta\,\,\prime}^{\,\,\theta}\partial^\theta\omega^{\alpha\beta i}+4\,r_1\,\partial_\theta\omega_{\alpha\,\,\prime\beta}^{\,\,\theta}\partial^\theta\omega^{\alpha\beta i}+$<br>$6\,r_5\,\partial_{\,\,\prime}\omega_{\theta\,\,\kappa}^{\,\,\kappa}\partial^\theta\omega^{\alpha i}_\alpha-6\,r_5\,\partial_\theta\omega_{\,\,\prime\,\kappa}^{\,\,\kappa}\partial^\theta\omega^{\alpha i}_\alpha+$<br>$4\,t_3\partial_{\,\,\prime}f^{\alpha i}\partial_{\kappa}f_{\alpha}^{\,\,\kappa}-8\,t_3\partial_{\,\,\prime}f^\alpha_\alpha\partial_{\kappa}f_{\,\,\prime}^{\,\,\kappa}-6\,r_5\,\partial_\alpha\omega^{\alpha i\theta}\partial_{\kappa}\omega_{\,\,\theta}^{\,\,\kappa}+$<br>$12\,r_5\,\partial^\theta\omega^{\alpha i}_\alpha\partial_{\kappa}\omega_{\,\,\theta}^{\,\,\kappa}+6\,r_5\,\partial_\alpha\omega^{\alpha i\theta}\partial_{\kappa}\omega_{\theta\,\,\prime}^{\,\,\kappa}-$<br>$12\,r_5\,\partial^\theta\omega^{\alpha i}_\alpha\partial_{\kappa}\omega_{\theta\,\,\prime}^{\,\,\kappa}))[\,t,\,x,\,y,\,z\,]\,dz\,dy\,dx\,dt$ |

| $\sigma_{2^+}^{\#1}\alpha\beta\chi$ | $\sigma_{2^+}^{\#1}\alpha\beta$ | $\sigma_{2^+}^{\#1}\alpha\beta\chi$ |
|-------------------------------------|---------------------------------|-------------------------------------|
| 0                                   | 0                               | $\frac{2}{2k^2r_1+t_1}$             |

| $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^+}^{\#1}$ |
|----------------------|--------------------|----------------------|
| 0                    | 0                  | 0                    |
| $-\frac{1}{t_1}$     | 0                  | 0                    |

Massive and massless spectra

| Massive particle |   |
|------------------|---|
| Pole residue:    | $-\frac{3(-2t_1t_3(t_1+t_3)+r_1(t_1^2+2t_3^2)+r_5(t_1^2+2t_3^2))}{2(r_1+r_5)(t_1+t_3)(-3t_1t_3+r_1(t_1+t_3)+r_5(t_1+t_3))}>0$ |
| Polarisations:   | 3   |
| Square mass:     | $-\frac{3t_1t_3}{2(r_1+r_5)(t_1+t_3)}>0$  |
| Spin:            | 1   |
| Parity:          | Odd   |

| Massive particle |                       | (No massless particles) |
|------------------|-----------------------|-------------------------|
| Pole residue:    | $-\frac{1}{r_1}>0$    |                         |
| Polarisations:   | 5                     |                         |
| Square mass:     | $-\frac{t_1}{2r_1}>0$ |                         |
| Spin:            | 2                     |                         |
| Parity:          | Odd                   |                         |

Unitarity conditions

$r_1 < 0 \ \&\& \ r_5 < -r_1 \ \&\& \ t_1 > 0 \ \&\& \ t_3 < -t_1 \ || \ t_3 > 0$