

Wave operator and propagator

$$\begin{aligned} & \int \int \int \int \left(\frac{1}{6} \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - 3(r_3 + 2r_5) \partial_\lambda \omega_\alpha^{\kappa\lambda} \partial'_\kappa \omega_\lambda^\alpha + 4r_2 \partial^\beta \omega_\kappa^{\theta\alpha} \partial_\theta \omega_{\alpha\beta}^\kappa - 2r_2 \right. \\ & \quad \partial_\theta \omega_{\alpha\beta}^\kappa \partial_\kappa \omega^{\alpha\beta\theta} - 4r_2 \partial_\theta \omega_{\alpha\beta}^\kappa \partial_\kappa \omega^{\theta\alpha\beta} + 3r_3 \partial_\alpha \omega_\lambda^\alpha \partial_\theta \omega^{\theta\kappa\lambda} - 6r_5 \partial_\alpha \omega_\lambda^\alpha \partial_\theta \omega^{\theta\kappa\lambda} - \\ & \quad 3r_3 \partial_\theta \omega_\lambda^\alpha \partial_\alpha \omega^{\theta\kappa\lambda} + 6r_5 \partial_\theta \omega_\lambda^\alpha \partial_\alpha \omega^{\theta\kappa\lambda} - 3r_3 \partial_\alpha \omega_\lambda^\alpha \partial_\kappa \omega^{\kappa\lambda\theta} - \\ & \quad 6r_5 \partial_\alpha \omega_\lambda^\alpha \partial_\theta \omega^{\kappa\lambda\theta} + 6r_3 \partial_\theta \omega_\lambda^\alpha \partial_\alpha \omega^{\kappa\lambda\theta} + 12r_5 \partial_\theta \omega_\lambda^\alpha \partial_\alpha \omega^{\kappa\lambda\theta} + \\ & \quad 2r_2 \partial_\kappa \omega^{\alpha\beta\theta} \partial^\kappa \omega_{\alpha\beta\theta} + 4r_2 \partial_\kappa \omega^{\theta\alpha\beta} \partial^\kappa \omega_{\alpha\beta\theta} - 4r_2 \partial^\beta \omega_{\alpha\beta\theta}^{\alpha\lambda} \partial_\lambda \omega_{\alpha\beta}^{\prime} + 4r_2 \partial^\beta \omega_{\alpha\beta}^{\lambda\alpha} \\ & \quad \partial_\lambda \omega_{\alpha\beta}^{\prime} - 24r_3 \partial^\beta \omega_{\alpha\beta}^{\lambda\alpha} \partial_\lambda \omega_{\alpha\beta}^{\prime} - 3r_3 \partial_\alpha \omega_\lambda^\alpha \partial^\lambda \omega_\theta^{\theta\kappa} + 6r_5 \partial_\alpha \omega_\lambda^\alpha \partial_\theta \omega^{\theta\kappa} + \\ & \quad \left. 3r_3 \partial_\theta \omega_\lambda^\alpha \partial^\lambda \omega_\alpha^{\theta\kappa} - 6r_5 \partial_\theta \omega_\lambda^\alpha \partial^\lambda \omega_\alpha^{\theta\kappa} \right) [t, x, y, z] dz dy dx dt \end{aligned}$$

The diagram illustrates the decomposition of the tensor product of two $SO(3)$ irreps into irreps of the diagonal $SO(3)$ and the resulting irreps of the full $SO(6)$.

Top Row (Input Irreps):

- $\sigma_{2+}^{\#1} \alpha\beta$ (Red box): 0
- $\sigma_{2-}^{\#1} \alpha\beta\chi$ (Blue box): 0

Middle Row (Diagonal $SO(3)$ Irreps):

- $\sigma_{0+}^{\#1} \dagger$ (Red box): 0
- $\sigma_{0-}^{\#1} \dagger$ (Blue box): 0
- $\sigma_{0+}^{\#1} \dagger$ (Red box): 0
- $\sigma_{0-}^{\#1} \dagger$ (Blue box): $\frac{1}{k^2 r_2}$

Bottom Row (Full $SO(6)$ Irreps):

- $\omega_{0+}^{\#1} \dagger$ (Red box): 0
- $\omega_{0-}^{\#1} \dagger$ (Blue box): $k^2 r_2$

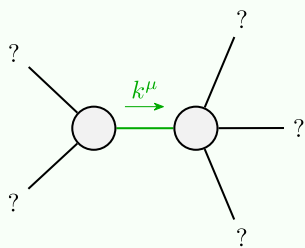
Source constraints/gauge generators (Green box):

$SO(3)$ irreps	Multiplicities
$\sigma_{0+}^{\#1} == 0$	1
$\sigma_{1-}^{\#2\alpha} == 0$	3
$\sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\sigma_{2-}^{\#1\alpha\beta\chi} == 0$	5
Total constraints:	12

Resulting Irreps (Right Side):

- $\omega_{2+}^{\#1} \alpha\beta$ (Red box): $-\frac{3k^2 r_3}{2}$
- $\omega_{2-}^{\#1} \alpha\beta\chi$ (Blue box): 0
- $\omega_{2+}^{\#1} \dagger \alpha\beta\chi$ (Red box): 0
- $\omega_{2-}^{\#1} \dagger \alpha\beta\chi$ (Blue box): 0
- $\omega_{1-}^{\#2} \alpha$ (Blue box): 0
- $\omega_{1-}^{\#1} \alpha$ (Blue box): 0
- $\omega_{1+}^{\#2} \alpha\beta$ (Red box): 0
- $\omega_{1+}^{\#1} \alpha\beta$ (Red box): $k^2 (2r_3 + r_5)$
- $\omega_{1+}^{\#1} \dagger \alpha\beta$ (Red box): 0
- $\omega_{1+}^{\#2} \dagger \alpha\beta$ (Red box): 0
- $\omega_{1-}^{\#1} \dagger \alpha$ (Blue box): $\frac{1}{2} k^2 (r_3 + 2r_5)$
- $\omega_{1-}^{\#2} \dagger \alpha$ (Blue box): 0

Massive and massless spectra



Quadratic pole	
Pole residue:	$-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)} > 0$
Polarisations:	2

Unitarity conditions

$$r_3 < 0 \&\& (r_5 < -\frac{r_3}{2} \parallel r_5 > -2r_3) \parallel r_3 > 0 \&\& -2r_3 < r_5 < -\frac{r_3}{2}$$