In[●]:= **Get@FileNameJoin@{NotebookDirectory[], "Calibration.m"};**

```
------------------------------------------------------------

Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}

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Connecting to external linux executable...

Connection established.

------------------------------------------------------------

Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}

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------------------------------------------------------------

Package xAct`xPlain` version 1.0.0-developer, {2023, 5, 24}

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------------------------------------------------------------
```

PSALTer Calibration

Key observation: During the calibration run, we need to write some commentary, which will appear in this green text, or as numbered equations/expressions with a green background. The output of the PSALTer package (specifically the function called ParticleSpectrum) is not in green, thus wherever we are using PSALTer the output should be quite distinctive.

The first step is to load the PSALTer package.

```
------------------------------------------------------------

Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}

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Package xAct`xPert`  version 1.0.6, {2018, 2, 28}

CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia
   and Guillermo A. Mena Marugan, under the General Public License.

** Variable $CovDFormat changed from Prefix to Postfix

** Option AllowUpperDerivatives of ContractMetric changed from False to True

** Option MetricOn of MakeRule changed from None to All

** Option ContractMetrics of MakeRule changed from False to True

------------------------------------------------------------

Package xAct`Invar`  version 2.0.5, {2013, 7, 1}

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   D. Yllanes and R. Portugal, under the General Public License.

** DefConstantSymbol: Defining constant symbol sigma.

** DefConstantSymbol: Defining constant symbol dim.

** Option CurvatureRelations of DefCovD changed from True to False

** Variable $CommuteCovDsOnScalars changed from True to False

------------------------------------------------------------

Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}

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------------------------------------------------------------

Package xAct`xTras`  version 1.4.2, {2014, 10, 30}

CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.

** Variable $CovDFormat changed from Postfix to Prefix

** Option CurvatureRelations of DefCovD changed from False to True

------------------------------------------------------------

Package xAct`PSALTer` version 1.0.0-developer, {2023, 6, 20}

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   Buttigieg, Claire Rigouzzo and Cillian Rew, under the General Public License.

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------------------------------------------------------------
```

Great, so PSALTer is now loaded and we can start to do some science.

Poincaré gauge theory (PGT)

Key observation: We will test the PoincareGaugeTheory module.

Now we set up the general Lagrangian for gravity with curvature and torsion (but kinematically restricted to vanishing nonmetricity)

$$-\lambda_{\bullet}\ \mathcal{R}^{ij}{}_{ij} + \left(\frac{r_{\bullet 1}}{3} + \frac{r_{\bullet 2}}{6}\right)\mathcal{R}_{ijhl}\ \mathcal{R}^{ijhl} + \left(\frac{2\,r_{\bullet 1}}{3} - \frac{2\,r_{\bullet 2}}{3}\right)\mathcal{R}_{ihjl}\ \mathcal{R}^{ijhl} +$$

$$\left(r_{\bullet 4} + r_{\bullet 5}\right)\mathcal{R}_i{}^l{}_{jl}\ \mathcal{R}^{ihj}{}_{h} + \left(r_{\bullet 4} - r_{\bullet 5}\right)\mathcal{R}^{ihj}{}_{h}\ \mathcal{R}_j{}^l{}_{il} + \left(\frac{r_{\bullet 1}}{3} + \frac{r_{\bullet 2}}{6} - r_{\bullet 3}\right)\mathcal{R}^{ijhl}\ \mathcal{R}_{hlij} +$$

$$\left(\frac{\lambda_{\bullet}}{4} + \frac{t_{\bullet 1}}{3} + \frac{t_{\bullet 2}}{12}\right)\mathcal{T}_{ijh}\ \mathcal{T}^{ijh} + \left(-\frac{\lambda_{\bullet}}{2} - \frac{t_{\bullet 1}}{3} + \frac{t_{\bullet 2}}{6}\right)\mathcal{T}^{ijh}\ \mathcal{T}_{jhi} + \left(-\lambda_{\bullet} - \frac{t_{\bullet 1}}{3} + \frac{2\,t_{\bullet 3}}{3}\right)\mathcal{T}_i{}^{ji}\ \mathcal{T}_{hj}{}^{h}$$

(1)

```
** DefConstantSymbol: Defining constant symbol PerturbativeParameter.
```

Antisymmetric Ricci theory (for Jaakko/Syksy)

Here is the full nonlinear Lagrangian:

$$-\lambda_{\bullet}\ \mathcal{R}^{ij}{}_{ij} + r_{\bullet 5}\ \mathcal{R}^{ij}{}_i{}^h\ \mathcal{R}_j{}^l{}_{hl} - r_{\bullet 5}\ \mathcal{R}^{ij}{}_i{}^h\ \mathcal{R}_h{}^l{}_{jl}$$

(2)

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$-\lambda_{\bullet}\ \mathcal{A}_{aji}\ \mathcal{A}^{aij} - \lambda_{\bullet}\ \mathcal{A}^{ai}{}_a\ \mathcal{A}_i{}^j{}_j - 2\,\lambda_{\bullet}\ f^{ai}\ \partial_i\mathcal{A}_a{}^j{}_j +$$

$$2\,\lambda_{\bullet}\ \partial_i\mathcal{A}^{ai}{}_a + 2\,\lambda_{\bullet}\ f^{ai}\ \partial_j\mathcal{A}_a{}^j{}_i + r_{\bullet 5}\ \partial_i\mathcal{A}_j{}^h{}_h\ \partial^j\mathcal{A}^{ai}{}_a - r_{\bullet 5}\ \partial_j\mathcal{A}_i{}^h{}_h\ \partial^j\mathcal{A}^{ai}{}_a -$$

$$r_{\bullet 5}\ \partial_a\mathcal{A}^{aij}\ \partial_h\mathcal{A}_i{}^h{}_j + 2\,r_{\bullet 5}\ \partial^j\mathcal{A}^{ai}{}_a\ \partial_h\mathcal{A}_i{}^h{}_j + r_{\bullet 5}\ \partial_a\mathcal{A}^{aij}\ \partial_h\mathcal{A}_j{}^h{}_i - 2\,r_{\bullet 5}\ \partial^j\mathcal{A}^{ai}{}_a\ \partial_h\mathcal{A}_j{}^h{}_i$$

(3)

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
   with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} \lambda_{\bullet} & \frac{ik\lambda_{\bullet}}{\sqrt{2}} & i\,\sqrt{\frac{3}{2}}\,k\,\lambda_{\bullet} & 0 \\ -\frac{ik\lambda_{\bullet}}{\sqrt{2}} & 0 & 0 & 0 \\ -i\,\sqrt{\frac{3}{2}}\,k\,\lambda_{\bullet} & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{\bullet} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} \frac{\lambda_{\bullet}}{2} + k^2\,r_{\bullet 5} & \frac{\lambda_{\bullet}}{\sqrt{2}} & \frac{ik\lambda_{\bullet}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ \frac{\lambda_{\bullet}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{ik\lambda_{\bullet}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\lambda_{\bullet}}{2} + k^2\,r_{\bullet 5} & -\frac{\lambda_{\bullet}}{\sqrt{2}} & 0 & -i\,k\,\lambda_{\bullet} \\ 0 & 0 & 0 & -\frac{\lambda_{\bullet}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i\,k\,\lambda_{\bullet} & 0 & 0 & 0 \end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} -\dfrac{\lambda_\bullet}{2} & \dfrac{i\,k\,\lambda_\bullet}{\sqrt{2}} & 0 \\[2ex] -\dfrac{i\,k\,\lambda_\bullet}{\sqrt{2}} & 0 & 0 \\[2ex] 0 & 0 & -\dfrac{\lambda_\bullet}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\bullet}\tau^{\parallel} == {}^{0^+}_{\bullet}\tau^{\perp}$$

$$2\,i\,k\,\,{}^{1^-}_{\bullet}\sigma^{\perp\,\mathrm{a}} + {}^{1^-}_{\bullet}\tau^{\perp\,\mathrm{a}} == 0$$

$${}^{1^-}_{\bullet}\tau^{\parallel\,\mathrm{a}} == 0$$

$$i\,k\,\,{}^{1^+}_{\bullet}\sigma^{\perp\,\mathrm{a\,b}} + {}^{1^+}_{\bullet}\tau^{\parallel\,\mathrm{a\,b}} == 0$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & \dfrac{i}{2\sqrt{2}\,k\,\lambda_\bullet} & \dfrac{i\,\sqrt{\frac{3}{2}}}{2\,k\,\lambda_\bullet} & 0 \\[2ex] -\dfrac{i}{2\sqrt{2}\,k\,\lambda_\bullet} & -\dfrac{1}{8\,k^2\,\lambda_\bullet} & -\dfrac{\sqrt{3}}{8\,k^2\,\lambda_\bullet} & 0 \\[2ex] -\dfrac{i\,\sqrt{\frac{3}{2}}}{2\,k\,\lambda_\bullet} & -\dfrac{\sqrt{3}}{8\,k^2\,\lambda_\bullet} & -\dfrac{3}{8\,k^2\,\lambda_\bullet} & 0 \\[2ex] 0 & 0 & 0 & \dfrac{1}{\lambda_\bullet} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & \dfrac{\sqrt{2}}{\lambda_\bullet+k^2\,\lambda_\bullet} & \dfrac{i\,\sqrt{2}\,k}{\lambda_\bullet+k^2\,\lambda_\bullet} & 0 & 0 & 0 & 0 \\[2ex] \dfrac{\sqrt{2}}{\lambda_\bullet+k^2\,\lambda_\bullet} & -\dfrac{\lambda_\bullet+2\,k^2\,r_\bullet}{(1+k^2)^2\,\lambda_\bullet^2} & -\dfrac{i\,k\left(\lambda_\bullet+2\,k^2\,r_\bullet\right)}{(1+k^2)^2\,\lambda_\bullet^2} & 0 & 0 & 0 & 0 \\[2ex] -\dfrac{i\,\sqrt{2}\,k}{\lambda_\bullet+k^2\,\lambda_\bullet} & \dfrac{i\,k\left(\lambda_\bullet+2\,k^2\,r_\bullet\right)}{(1+k^2)^2\,\lambda_\bullet^2} & -\dfrac{k^2\left(\lambda_\bullet+2\,k^2\,r_\bullet\right)}{(1+k^2)^2\,\lambda_\bullet^2} & 0 & 0 & 0 & 0 \\[2ex] 0 & 0 & 0 & 0 & -\dfrac{\sqrt{2}}{\lambda_\bullet+2\,k^2\,\lambda_\bullet} & 0 & -\dfrac{2\,i\,k}{\lambda_\bullet+2\,k^2\,\lambda_\bullet} \\[2ex] 0 & 0 & 0 & -\dfrac{\sqrt{2}}{\lambda_\bullet+2\,k^2\,\lambda_\bullet} & -\dfrac{\lambda_\bullet+2\,k^2\,r_\bullet}{\left(\lambda_\bullet+2\,k^2\,\lambda_\bullet\right)^2} & 0 & -\dfrac{i\,\sqrt{2}\,k\left(\lambda_\bullet+2\,k^2\,r_\bullet\right)}{\left(\lambda_\bullet+2\,k^2\,\lambda_\bullet\right)^2} \\[2ex] 0 & 0 & 0 & 0 & 0 & 0 & 0 \\[2ex] 0 & 0 & 0 & \dfrac{2\,i\,k}{\lambda_\bullet+2\,k^2\,\lambda_\bullet} & \dfrac{i\,\sqrt{2}\,k\left(\lambda_\bullet+2\,k^2\,r_\bullet\right)}{\left(\lambda_\bullet+2\,k^2\,\lambda_\bullet\right)^2} & 0 & -\dfrac{2\,k^2\left(\lambda_\bullet+2\,k^2\,r_\bullet\right)}{\left(\lambda_\bullet+2\,k^2\,\lambda_\bullet\right)^2} \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & \dfrac{i\,\sqrt{2}}{k\,\lambda_\bullet} & 0 \\[2ex] -\dfrac{i\,\sqrt{2}}{k\,\lambda_\bullet} & \dfrac{1}{k^2\,\lambda_\bullet} & 0 \\[2ex] 0 & 0 & -\dfrac{2}{\lambda_\bullet} \end{pmatrix}$$

Square masses:

$$\{\{\}, \{\}, \{\}, \{\}, \{\}, \{\}\}$$

Massive pole residues:

$$\{\{\}, \{\}, \{\}, \{\}, \{\}, \{\}\}$$

Massless eigenvalues:

$$\left\{\dfrac{9\,p^2}{\lambda_\bullet},\ \dfrac{9\,p^2}{\lambda_\bullet}\right\}$$

Overall particle spectrum:
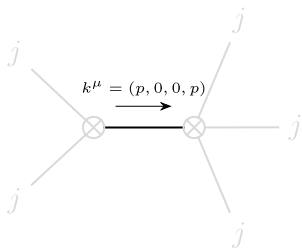
```
Massless particle
```

Pole residue:	$\dfrac{p^2}{\lambda_{\boldsymbol{\cdot}}} > 0$
Polarisations:	2

```
Overall unitarity conditions:
```

$(p < 0 \,\&\&\, \lambda_{\boldsymbol{\cdot}} > 0) \,||\, (p > 0 \,\&\&\, \lambda_{\boldsymbol{\cdot}} > 0)$

Okay, so that is the end of the PSALTer output for antisymmetric Ricci gravity.