

Particle spectrograph

Wave operator and propagator

| Source constraints | | |
|---|--|----------------|
| SO(3) irreps | Fundamental fields | Multiplicities |
| $\tau_{0+}^{\#2} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$ | 1 |
| $\tau_{0+}^{\#1} - 2 \, i \, k \, \sigma_{0+}^{\#1} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha_\alpha + 2 \, \partial_\chi \partial^\chi \partial_\beta \sigma^{\alpha\beta}_\alpha$ | 1 |
| $\tau_1^{\#2\,\alpha} + 2 \, i \, k \, \, \sigma_1^{\#2\,\alpha} == 0$ | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta} + 2 \, \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$ | 3 |
| $\tau_1^{\#1\,\alpha} == 0$ | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$ | 3 |
| $\tau_1^{\#1\,\alpha\beta} + i \, k \, \sigma_1^{\#2\,\alpha\beta} == 0$ | $\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\chi\alpha} + \partial_\chi \partial^\chi \tau^{\alpha\beta} +$ $2 \, \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 2 \, \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} ==$ $\partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} +$ $\partial_\chi \partial^\chi \tau^{\beta\alpha} + 2 \, \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$ | 3 |
| $\tau_2^{\#1\,\alpha\beta} - 2 \, i \, k \, \sigma_2^{\#1\,\alpha\beta} == 0$ | $-i \, (4 \, \partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2 \, \partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi_\chi -$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\chi} - 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\beta} -$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi} - 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\chi\alpha} +$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} +$ $4 \, i \, k^\chi \, \partial_\epsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta\epsilon}_\delta -$ $6 \, i \, k^\chi \, \partial_\epsilon \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\delta\epsilon} -$ $6 \, i \, k^\chi \, \partial_\epsilon \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\delta\epsilon} +$ $2 \, \eta^{\alpha\beta} \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} +$ $6 \, i \, k^\chi \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\alpha\delta\beta} +$ $6 \, i \, k^\chi \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\beta\delta\alpha} -$ $2 \, \eta^{\alpha\beta} \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \tau^\chi_\chi -$ $4 \, i \, \eta^{\alpha\beta} \, k^\chi \, \partial_\phi \partial^\phi \partial_\epsilon \partial_\chi \sigma^{\delta\epsilon}_\delta) == 0$ | 5 |
| Total constraints/gauge generators: | | 16 |

Quadratic (free) action

$$S == \int \int \int \int (\frac{1}{6} (2 \, t_1 \, \omega^\alpha_\alpha \, \omega^\theta_{\theta'} - 4 \, t_3 \, \omega^\alpha_{\alpha'} \, \omega^\kappa_{\kappa'} +$$

$$6 \, f^{\alpha\beta} \, \tau_{\alpha\beta} + 6 \, \omega^{\alpha\beta\chi} \, \sigma_{\alpha\beta\chi} - 4 \, t_1 \, \omega^\theta_{\alpha} \, \omega^\theta_{\beta'} \, \partial_\beta f^{\alpha\alpha'} +$$

$$8 \, t_3 \, \omega^\kappa_{\alpha} \, \omega^\kappa_{\beta'} \, \partial_\beta f^{\alpha\alpha'} + 4 \, t_1 \, \omega^\theta_{\beta'} \, \partial_\beta f^\alpha_{\alpha'} - 8 \, t_3 \, \omega^\kappa_{\beta'} \, \partial_\beta f^\alpha_{\alpha'} -$$

$$2 \, t_1 \, \partial_\beta f^\theta_{\theta'} \, \partial_\beta f^\alpha_{\alpha'} + 4 \, t_3 \, \partial_\beta f^\kappa_{\kappa'} \, \partial_\beta f^\alpha_{\alpha'} - 2 \, t_1 \, \partial_\beta f^{\alpha\alpha'} \, \partial_\beta f^\theta_{\theta'} +$$

$$4 \, t_1 \, \partial_\beta f^\alpha_{\alpha'} \, \partial_\beta f^\theta_{\theta'} + 4 \, t_1 \, \omega_{\theta\alpha} \, \partial^\theta f^{\alpha\alpha'} + 4 \, t_2 \, \omega_{\theta\alpha} \, \partial^\theta f^{\alpha\alpha'} -$$

$$4 \, t_1 \, \partial_\alpha f_{\beta'} \partial^\theta f^{\alpha\beta'} + 2 \, t_2 \, \partial_\alpha f_{\beta'} \partial^\theta f^{\alpha\beta'} - 4 \, t_1 \, \partial_\alpha f_{\theta'} \partial^\theta f^{\alpha\theta'} -$$

$$t_2 \, \partial_\alpha f_{\theta'} \partial^\theta f^{\alpha\theta'} + 2 \, t_1 \, \partial_\beta f_{\alpha\theta'} \partial^\theta f^{\alpha\theta'} - t_2 \, \partial_\beta f_{\alpha\theta'} \partial^\theta f^{\alpha\theta'} +$$

$$4 \, t_1 \, \partial_\theta f_{\alpha'} \partial^\theta f^{\alpha\alpha'} + t_2 \, \partial_\theta f_{\alpha'} \partial^\theta f^{\alpha\alpha'} + 2 \, t_1 \, \partial_\theta f_{\beta'} \partial^\theta f^{\alpha\beta'} -$$

$$t_2 \, \partial_\theta f_{\beta'} \partial^\theta f^{\alpha\beta'} + 2 \, (t_1 + t_2) \, \omega_{\alpha\theta} \, (\omega^{\alpha\theta} + 2 \, \partial^\theta f^{\alpha\theta'}) +$$

$$2 \, \omega_{\alpha\theta'} \, ((t_1 - 2 \, t_2) \, \omega^{\alpha\theta} + 2 \, (2 \, t_1 - t_2) \, \partial^\theta f^{\alpha\theta'}) +$$

$$8 \, r_2 \, \partial_\beta \omega_{\alpha\theta'} \partial^\theta \omega^{\alpha\beta\theta'} - 4 \, r_2 \, \partial_\beta \omega_{\alpha\theta'} \partial^\theta \omega^{\alpha\beta\theta'} +$$

$$4 \, r_2 \, \partial_\beta \omega_{\theta\alpha} \partial^\theta \omega^{\alpha\beta\theta'} - 2 \, r_2 \, \partial_\beta \omega_{\alpha\theta'} \partial^\theta \omega^{\alpha\beta\theta'} +$$

$$2 \, r_2 \, \partial_\theta \omega_{\alpha\beta'} \partial^\theta \omega^{\alpha\beta\theta'} - 4 \, r_2 \, \partial_\theta \omega_{\alpha\beta'} \partial^\theta \omega^{\alpha\beta\theta'} +$$

$$4 \, t_3 \, \partial_\beta f^{\alpha\alpha'} \partial_\alpha f^\kappa_{\kappa'} - 8 \, t_3 \, \partial_\beta f^\alpha_{\alpha'} \, \partial_\alpha f^\kappa_{\kappa'}) [t, \, x, \, y, \, z] \, dz \, dy \, dx \, dt$$

| $\sigma_{0+}^{\#1}$ | $\tau_{0+}^{\#1}$ | $\tau_{0+}^{\#2}$ | $\sigma_0^{\#1}$ |
|--|---|-------------------|--------------------------|
| $\frac{1}{(1+2\,k^2)^2\,t_3}$ | $-\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_3}$ | 0 | 0 |
| $\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_3}$ | $\frac{2\,k^2}{(1+2\,k^2)^2\,t_3}$ | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $\frac{1}{k^2\,r_2+t_2}$ |

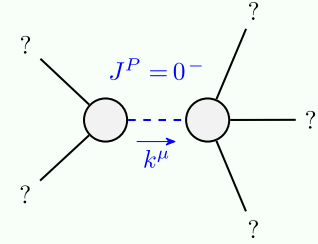
| $\omega_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{1+}^{\#2} \dagger^{\alpha\beta}$ | $f_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{1-}^{\#1} \dagger_\alpha$ | $\omega_{1-}^{\#2} \dagger_\alpha$ | $f_{1-}^{\#1} \dagger_\alpha$ | $f_{1-}^{\#2} \dagger_\alpha$ |
|--|---|---|--|---|-------------------------------|--|
| $\frac{1}{6} \, (t_1 + 4 \, t_2)$ | $-\frac{t_1-2\,t_2}{3 \, \sqrt{2}}$ | $-\frac{i \, k \, (t_1-2\,t_2)}{3 \, \sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $\frac{-\,t_1-2\,t_2}{3 \, \sqrt{2}}$ | $\frac{t_1+t_2}{3}$ | $\frac{1}{3} \, i \, k \, (t_1 + t_2)$ | 0 | 0 | 0 | 0 |
| $\frac{i \, k \, (t_1-2\,t_2)}{3 \, \sqrt{2}}$ | $-\frac{1}{3} \, i \, k \, (t_1 + t_2)$ | $\frac{1}{3} \, k^2 \, (t_1 + t_2)$ | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $\frac{1}{6} \, (t_1 + 4 \, t_3)$ | $\frac{t_1-2\,t_3}{3 \, \sqrt{2}}$ | 0 | $\frac{1}{3} \, i \, k \, (t_1 - 2 \, t_3)$ |
| 0 | 0 | 0 | $\frac{t_1-2\,t_3}{3 \, \sqrt{2}}$ | $\frac{t_1+t_3}{3}$ | 0 | $\frac{1}{3} \, i \, \sqrt{2} \, k \, (t_1 + t_3)$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $-\frac{1}{3} \, i \, k \, (t_1 - 2 \, t_3)$ | $-\frac{1}{3} \, i \, \sqrt{2} \, k \, (t_1 + t_3)$ | 0 | $\frac{2}{3} \, k^2 \, (t_1 + t_3)$ |

| $\omega_{2+}^{\#1} \dagger^{\alpha\beta}$ | $f_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$ |
|---|--------------------------------------|---|
| $\frac{t_1}{2}$ | $-\frac{i \, k \, t_1}{\sqrt{2}}$ | $\frac{t_1}{2}$ |
| $\frac{i \, k \, t_1}{\sqrt{2}}$ | $k^2 \, t_1$ | 0 |
| 0 | 0 | 0 |

| $\omega_{0+}^{\#1}$ | $f_{0+}^{\#1}$ | $f_{0+}^{\#2}$ | $\omega_0^{\#1}$ |
|-----------------------------|------------------------------|----------------|--------------------|
| t_3 | $-i \, \sqrt{2} \, k \, t_3$ | 0 | 0 |
| $i \, \sqrt{2} \, k \, t_3$ | $2 \, k^2 \, t_3$ | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $k^2 \, r_2 + t_2$ |

| $\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\tau_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$ |
|---|--|---|
| $\frac{2}{(1+2\,k^2)^2\,t_1}$ | $-\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ | 0 |
| $\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ | $\frac{4\,k^2}{(1+2\,k^2)^2\,t_1}$ | 0 |
| 0 | 0 | $\frac{2}{t_1}$ |

Massive and massless spectra



| Massive particle | |
|------------------|------------------------|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $-\frac{t_2}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

(No massless particles)

Unitarity conditions

$r_2 < 0 \,\&\& \, t_2 > 0$