# Particle spectrograph

## Wave operator and propagator

Source constraints

| SO(3) irreps | Fundamental fields | Multiplicities |
|---|---|---|
| $\tau_{0^+}^{\#2} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$ | 1 |
| $\tau_{0^+}^{\#1} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha{}_\alpha$ | 1 |
| $\sigma_{0^+}^{\#1} == 0$ | $\partial_\beta \sigma^{\alpha\beta}{}_\alpha == 0$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$ | $\partial_\chi \partial_\theta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta} + 2\,\partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$ | $\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\chi \tau^{\alpha\beta} + 2\,\partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 2\,\partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} + 2\,\partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi}$ | 3 |
| $\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$ | $-i\,(4\,\partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2\,\partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi{}_\chi - 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\beta} - 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi} - 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} + 4\,i\,k^\chi\,\partial_\epsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta\epsilon}{}_\delta - 6\,i\,k^\chi\,\partial_\epsilon \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\delta\epsilon} - 6\,i\,k^\chi\,\partial_\epsilon \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} + 6\,i\,k^\chi\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\alpha\delta\beta} + 6\,i\,k^\chi\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\beta\delta\alpha} - 2\,\eta^{\alpha\beta}\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \tau^\chi{}_\chi - 4\,i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi \partial^\phi \partial_\epsilon \partial_\chi \sigma^{\delta\epsilon}{}_\delta) == 0$ | 5 |
| Total constraints/gauge generators: | | 17 |

Quadratic (free) action

$$\mathcal{S} == \iiiint \tfrac{1}{6} (2\,t_1\,\omega^{\alpha\iota}{}_\alpha\,\omega_\iota{}^\theta{}_\theta + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 4\,t_1\,\omega^\theta{}_\alpha{}^\iota\,\partial_\iota f^{\alpha\iota}{}_\theta + 4\,t_1\,\omega_\iota{}^\theta{}_\theta\,\partial^\iota f^\alpha{}_\alpha - 2\,t_1\,\partial_\iota f^{\alpha\iota}{}_\theta\,\partial^\theta f^\alpha{}_\alpha - 2\,t_1\,\partial_\iota f^{\alpha\iota}{}_\theta\,\partial^\theta f^\alpha{}_\alpha + 4\,t_1\,\partial^\iota f^\alpha{}_\alpha\,\partial_\theta f_\iota{}^\theta - 6\,t_1\,\partial_\alpha f_{\iota\theta}\,\partial^\theta f^{\alpha\iota} - 3\,t_1\,\partial_\alpha f_{\theta\iota}\,\partial^\theta f^{\alpha\iota} + 3\,t_1\,\partial_\iota f_{\alpha\theta}\,\partial^\theta f^{\alpha\iota} + 3\,t_1\,\partial_\theta f_{\alpha\iota}\,\partial^\theta f^{\alpha\iota} + 6\,t_1\,\omega_{\alpha\theta\iota}\,(\omega^{\alpha\iota\theta} + 2\,\partial^\theta f^{\alpha\iota}) + 8\,r_2\,\partial_\beta \omega_{\alpha\iota\theta}\,\partial^\theta \omega^{\alpha\beta\iota} - 4\,r_2\,\partial_\beta \omega_{\alpha\theta\iota}\,\partial^\theta \omega^{\alpha\beta\iota} + 4\,r_2\,\partial_\beta \omega_{\iota\theta\alpha}\,\partial^\theta \omega^{\alpha\beta\iota} - 2\,r_2\,\partial_\iota \omega_{\alpha\beta\theta}\,\partial^\theta \omega^{\alpha\beta\iota} + 2\,r_2\,\partial_\theta \omega_{\alpha\beta\iota}\,\partial^\theta \omega^{\alpha\beta\iota} - 4\,r_2\,\partial_\theta \omega_{\alpha\iota\beta}\,\partial^\theta \omega^{\alpha\beta\iota} + 6\,r_5\,\partial_\theta \omega^{\alpha\beta\iota}{}_\kappa\,\partial^\theta \omega^{\alpha\iota}{}_\alpha - 6\,r_5\,\partial_\theta \omega_\iota{}^\kappa{}_\theta\,\partial^\theta \omega^{\alpha\iota}{}_\alpha + 6\,r_5\,\partial_\alpha \omega^{\alpha\iota\theta}\,\partial_\kappa \omega_\iota{}^\kappa{}_\theta + 12\,r_5\,\partial^\theta \omega^{\alpha\iota}{}_\alpha\,\partial_\kappa \omega_\iota{}^\kappa{}_\theta + 6\,r_5\,\partial_\alpha \omega^{\alpha\iota\theta}\,\partial_\kappa \omega_\theta{}^\kappa{}_\iota - 12\,r_5\,\partial^\theta \omega^{\alpha\iota}{}_\alpha\,\partial_\kappa \omega_\theta{}^\kappa{}_\iota)\,[t, x, y, z]\,dz\,dy\,dx\,dt$$

| | $\sigma_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^+}^{\#2\,\alpha\beta}$ | $\tau_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^-}^{\#1\,\alpha}$ | $\sigma_{1^-}^{\#2\,\alpha}$ | $\tau_{1^-}^{\#1\,\alpha}$ | $\tau_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | 0 | $-\frac{\sqrt{2}}{t_1 + k^2\,t_1}$ | $-\frac{i\,\sqrt{2}\,k}{t_1 + k^2\,t_1}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $-\frac{\sqrt{2}}{t_1 + k^2\,t_1}$ | $-\frac{2\,k^2\,r_5 + t_1}{(1 + k^2)^2\,t_1^2}$ | $-\frac{i\,(2\,k^3\,r_5 + t_1)}{(1 + k^2)^2\,t_1^2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $\frac{i\,\sqrt{2}\,k}{t_1 + k^2\,t_1}$ | $\frac{i\,(2\,k^3\,r_5 + t_1)}{(1 + k^2)^2\,t_1^2}$ | $\frac{-2\,k^4\,r_5 + k^2\,t_1}{(1 + k^2)^2\,t_1^2}$ | 0 | $-\frac{1}{\sqrt{2}\,(k^2\,r_5 + 2\,k^4\,r_5)}$ | 0 | $-\frac{i}{k\,r_5 + 2\,k^3\,r_5}$ |
| $\sigma_{1^-}^{\#1\,\alpha}\,\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{1}{k^2\,r_5}$ | $-\frac{1}{\sqrt{2}\,(k^2\,r_5 + 2\,k^4\,r_5)}$ | 0 | $-\frac{i\,(6\,k^2\,r_5 + t_1)}{\sqrt{2}\,k\,(1 + 2\,k^2)^2\,r_5\,t_1}$ |
| $\sigma_{1^-}^{\#2\,\alpha}\,\dagger^{\alpha}$ | 0 | 0 | 0 | $-\frac{1}{\sqrt{2}\,(k^2\,r_5 + 2\,k^4\,r_5)}$ | $\frac{6\,k^2\,r_5 + t_1}{2\,(k + 2\,k^3)^2\,r_5\,t_1}$ | 0 | $\frac{i\,(6\,k^2\,r_5 + t_1)}{\sqrt{2}\,k\,(1 + 2\,k^2)^2\,r_5\,t_1}$ |
| $\tau_{1^-}^{\#1\,\alpha}\,\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2\,\alpha}\,\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{i}{k\,r_5 + 2\,k^3\,r_5}$ | $-\frac{i\,(6\,k^2\,r_5 + t_1)}{\sqrt{2}\,k\,(1 + 2\,k^2)^2\,r_5\,t_1}$ | 0 | $\frac{6\,k^2\,r_5 + t_1}{(1 + 2\,k^2)^2\,r_5\,t_1}$ |

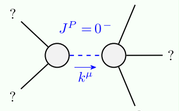
| | $\sigma_{2^+}^{\#1\,\alpha\beta\chi}$ | $\tau_{2^+}^{\#1\,\alpha\beta}$ | $\sigma_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $\frac{2}{(1 + 2\,k^2)^2\,t_1}$ | $-\frac{2\,i\,\sqrt{2}\,k}{(1 + 2\,k^2)^2\,t_1}$ | 0 |
| $\tau_{2^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $\frac{2\,i\,\sqrt{2}\,k}{(1 + 2\,k^2)^2\,t_1}$ | $\frac{4\,k^2}{(1 + 2\,k^2)^2\,t_1}$ | 0 |
| $\sigma_{2^-}^{\#1\,\alpha\beta\chi}\,\dagger^{\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{t_1}$ |

| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1}\,\dagger$ | 0 | 0 | 0 | $k^2\,r_2 - t_1$ |
| $f_{0^+}^{\#1}\,\dagger$ | 0 | 0 | 0 | 0 |
| $f_{0^+}^{\#2}\,\dagger$ | 0 | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1}\,\dagger$ | $k^2\,r_2 - t_1$ | 0 | 0 | $\frac{1}{k^2\,r_2 - t_1}$ |

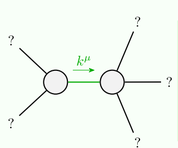
| | $\omega_{2^+}^{\#1\,\alpha\beta}$ | $f_{2^+}^{\#1\,\alpha\beta}$ | $\omega_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1}\,\dagger^{\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{i\,k\,t_1}{\sqrt{2}}$ | 0 |
| $f_{2^+}^{\#1}\,\dagger^{\alpha\beta}$ | $\frac{i\,k\,t_1}{\sqrt{2}}$ | $k^2\,t_1$ | 0 |
| $\omega_{2^-}^{\#1}\,\dagger^{\alpha\beta\chi}$ | 0 | 0 | $\frac{t_1}{2}$ |

| | $\omega_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^+}^{\#2\,\alpha\beta}$ | $f_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^-}^{\#1\,\alpha}$ | $\omega_{1^-}^{\#2\,\alpha}$ | $f_{1^-}^{\#1\,\alpha}$ | $f_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $k^2\,r_5 - \frac{t_1}{2}$ | $-\frac{t_1}{\sqrt{2}}$ | $-\frac{i\,k\,t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $-\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $\frac{i\,k\,t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1\,\alpha}\,\dagger^{\alpha}$ | 0 | 0 | 0 | $k^2\,r_5 + \frac{t_1}{6}$ | $\frac{t_1}{3\,\sqrt{2}}$ | 0 | $\frac{i\,k\,t_1}{3}$ |
| $\omega_{1^-}^{\#2\,\alpha}\,\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{3\,\sqrt{2}}$ | $\frac{t_1}{3}$ | 0 | $\frac{1}{3}\,i\,\sqrt{2}\,k\,t_1$ |
| $f_{1^-}^{\#1\,\alpha}\,\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2\,\alpha}\,\dagger^{\alpha}$ | 0 | 0 | 0 | $-\frac{1}{3}\,i\,k\,t_1$ | $-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_1$ | 0 | $\frac{2\,k^2\,t_1}{3}$ |

## Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $\frac{t_1}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |




| Quadratic pole | |
|---|---|
| Pole residue: | $-\frac{1}{r_5\,{t_1}^2} > 0$ |
| Polarisations: | 2 |

## Unitarity conditions

$r_2 < 0$ && $r_5 < 0$ && $t_1 < 0$