Particle spectrograph

Wave operator and propagator

Quadratic (free) action

$S_F ==$

$\iiint [\tfrac{1}{3}(-3t_1\,\omega_i{}^{\alpha i}\,\omega_{\kappa\alpha}{}^{\kappa} - t_1\,\omega_i{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}^{i} + t_1\,\omega_{\kappa\lambda}{}^{i}\,\omega^{\kappa\lambda}{}_{i} + 3f^{\alpha\beta}\tau_{\alpha\beta} + 3\,\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}$
$+ 6r_1\,\partial_i\omega_{\lambda}{}^{\alpha}\,\partial_\kappa\omega^{\theta\kappa\lambda}{}_{\alpha} - 2r_1\,\partial^\beta\omega^{\theta\alpha}{}_{\kappa}\partial_\theta\omega_{\alpha\beta}{}^{\kappa} - 2r_1\,\partial_\theta\omega_{\alpha\beta}{}^{\kappa}\partial_\kappa\omega^{\theta\kappa\lambda}$
$+ 2r_1\,\partial_\theta\omega_{\alpha\beta}{}^{\kappa}\partial_\kappa\omega^{\theta\alpha\beta} + 6r_1\,\partial_\alpha\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial_\kappa\omega^{\theta\kappa\lambda} - 6r_1\,\partial_\theta\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_\kappa\omega^{\kappa\lambda\theta}$
$- 12r_1\,\partial_\theta\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_\kappa\omega^{\kappa\lambda\theta} - t_1\,\partial^\alpha f_{\theta\kappa}\,\partial^\kappa f_{\alpha}{}^{\theta} - 2t_1\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f_{\alpha}{}^{\theta}$
$+ t_1\,\partial^\kappa f^{\lambda}{}_{\kappa}\,\partial^\kappa f_{\alpha\lambda} + 3t_1\,\omega_{\kappa}{}^{\lambda}\,\partial^\kappa f^{i}{}_{i} + 3t_1\,\omega_{\kappa\alpha}{}^{\kappa}\,\partial^\alpha f^{i}{}_{i} + 6t_1\,\partial^\alpha f_{\kappa\alpha}\,\partial^\kappa f^{i}{}_{i}$
$+ 3t_1\,\partial_{\kappa}f^{\lambda}{}_{\lambda}\,\partial^\kappa f^{i}{}_{i} + t_1\,\omega_{i\theta\kappa}\,\partial^\kappa f^{i\theta} + 4t_1\,\omega_{i\kappa\theta}\,\partial^\kappa f^{i\theta} - t_1\,\omega_{\theta i\kappa}\,\partial^\kappa f^{i\theta} +$
$2t_1\,\omega_{\theta\kappa i}\,\partial^\kappa f^{i\theta} - 3t_1\,\omega_{i\alpha}{}^{\alpha}\,\partial^\kappa f^{i}{}_{\kappa} - 3t_1\,\omega_{i\lambda}{}^{\lambda}\,\partial^\kappa f_{\kappa}{}^{i} + t_1\,\partial^\kappa f^{\lambda}{}_{\alpha}\,\partial^\kappa f_{\lambda\alpha} +$
$t_1\,\partial_\kappa f_{\theta}{}^{\lambda}\,\partial^\kappa f^{\theta}{}_{\lambda} + 2t_1\,\partial_\kappa f^{\lambda}{}_{\alpha}\,\partial^\kappa f_{\lambda\kappa} + 2r_1\,\partial_\kappa\omega^{\alpha\beta\theta}\partial^\kappa\omega_{\alpha\beta\theta} - 3t_1\,\partial_\kappa\omega_{\theta i}{}^{\lambda\alpha}\partial_\lambda\omega^{i}{}_{\alpha\beta}$
$- 8r_1\,\partial_\kappa\omega^{\alpha\beta\theta}\partial^\lambda\omega_{\alpha}{}^{\kappa}{}_{\beta} + 2r_1\,\partial_\theta\omega^{\theta\alpha\beta}\partial_\kappa\omega^{\kappa}{}_{\alpha\beta} - r_1\,\partial_\lambda\omega^{\alpha\lambda}{}_{\theta}\,\partial_i\omega^{\theta}{}_{\alpha\beta}$
$+ 2r_1\,\partial_\kappa\omega^{\theta\alpha\beta}\partial^\kappa\omega_{\theta\alpha\beta} + 6r_1\,\partial_\theta\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial^\lambda\omega^{\theta\kappa}{}_{\kappa})]\,(t,x,y,z)\,dz\,dy\,dx\,dt$

First operator matrix (spin-1):

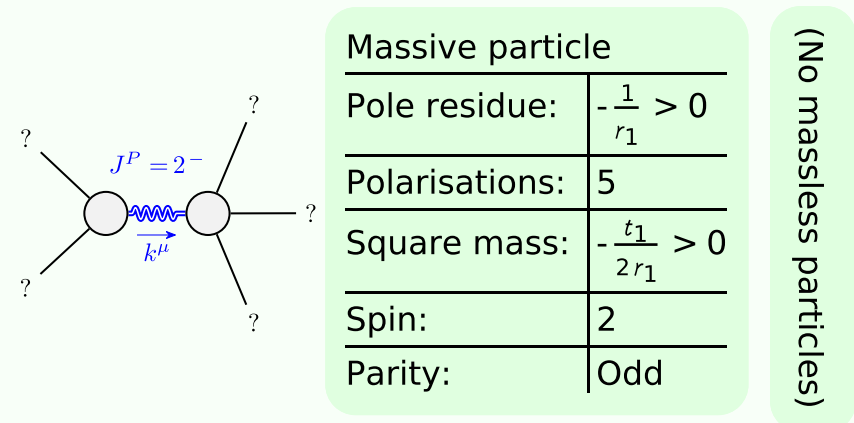|	$\sigma_{1+}^{\#1}\dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2}\dagger^{\alpha\beta}$	$\tau_{1+}^{\#1}\dagger^{\alpha\beta}$	$\sigma_{1-}^{\#1}\dagger^{\alpha}$	$\sigma_{1-}^{\#2}\dagger^{\alpha}$	$\tau_{1-}^{\#1}\dagger^{\alpha}$	$\tau_{1-}^{\#2}\dagger^{\alpha}$
$\tau_{1-}^{\#2}{}_{\alpha}$	0	0	0	$\frac{2ik}{t_1+2k^2t_1}$	$\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$	0	$\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
$\tau_{1-}^{\#1}{}_{\alpha}$	0	0	0	0	0	0	0
$\sigma_{1-}^{\#2}{}_{\alpha}$	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	$\frac{2k^2r_1+t_1}{(t_1+2k^2t_1)^2}$	0	$-\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
$\sigma_{1-}^{\#1}{}_{\alpha}$	0	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	0	$-\frac{2ik}{t_1+2k^2t_1}$
$\tau_{1+}^{\#1}{}_{\alpha\beta}$	$\frac{6i\sqrt{2}k}{(3+2k^2)^2t_1}$	$\frac{12ik}{(3+2k^2)^2t_1}$	$\frac{12k^2}{(3+2k^2)^2t_1}$	0	0	0	0
$\sigma_{1+}^{\#2}{}_{\alpha\beta}$	$-\frac{6\sqrt{2}}{(3+2k^2)^2t_1}$	$\frac{12}{(3+2k^2)^2t_1}$	$\frac{12ik}{(3+2k^2)^2t_1}$	0	0	0	0
$\sigma_{1+}^{\#1}{}_{\alpha\beta}$	$\frac{6}{(3+2k^2)^2t_1}$	$\frac{6\sqrt{2}}{(3+2k^2)^2t_1}$	$\frac{6i\sqrt{2}k}{(3+2k^2)^2t_1}$	0	0	0	0

Spin-0 matrix:

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
$\sigma_{0+}^{\#1}\dagger$	$-\frac{1}{(1+2k^2)^2t_1}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	0	0
$\tau_{0+}^{\#1}\dagger$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0	0
$\tau_{0+}^{\#2}\dagger$	0	0	0	0
$\sigma_{0-}^{\#1}\dagger$	0	0	0	0

Spin-2 matrix:

	$\sigma_{2+}^{\#1}{}_{\alpha\beta}$	$\tau_{2+}^{\#1}{}_{\alpha\beta}$	$\sigma_{2-}^{\#1}{}_{\alpha\beta\chi}$
$\sigma_{2+}^{\#1}\dagger^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
$\tau_{2+}^{\#1}\dagger^{\alpha\beta}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\sigma_{2-}^{\#1}\dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{2k^2r_1+t_1}$

Spin-0 ω/f matrix:

	$\omega_{0+}^{\#1}\dagger$	$f_{0+}^{\#1}\dagger$	$f_{0+}^{\#2}\dagger$	$\omega_{0-}^{\#1}\dagger$
$\omega_{0-}^{\#1}\dagger$	0	0	0	0
$f_{0+}^{\#2}\dagger$	0	0	0	0
$f_{0+}^{\#1}\dagger$	$i\sqrt{2}kt_1$	$-2k^2t_1$	0	0
$\omega_{0+}^{\#1}\dagger$	$-t_1$	$-i\sqrt{2}kt_1$	0	0

Spin-1 ω/f matrix:

	$\omega_{1+}^{\#1}\dagger^{\alpha\beta}$	$\omega_{1+}^{\#2}\dagger^{\alpha\beta}$	$f_{1+}^{\#1}\dagger^{\alpha\beta}$	$\omega_{1-}^{\#1}\dagger^{\alpha}$	$\omega_{1-}^{\#2}\dagger^{\alpha}$	$f_{1-}^{\#1}\dagger^{\alpha}$	$f_{1-}^{\#2}\dagger^{\alpha}$
$f_{1-}^{\#2}{}_{\alpha}$	0	0	0	ikt_1	0	0	0
$f_{1-}^{\#1}{}_{\alpha}$	0	0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0
$\omega_{1-}^{\#2}{}_{\alpha}$	0	0	0	$-k^2r_1-\frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$-ikt_1$
$\omega_{1-}^{\#1}{}_{\alpha}$	0	0	0				
$f_{1+}^{\#1}{}_{\alpha\beta}$	$\frac{ikt_1}{3\sqrt{2}}$	$\frac{ikt_1}{3}$	$\frac{k^2t_1}{3}$	0	0	0	0
$\omega_{1+}^{\#2}{}_{\alpha\beta}$	$-\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	$-\frac{1}{3}ikt_1$	0	0	0	0
$\omega_{1+}^{\#1}{}_{\alpha\beta}$	$\frac{t_1}{6}$	$-\frac{t_1}{3\sqrt{2}}$	$\frac{ikt_1}{3\sqrt{2}}$	0	0	0	0

Spin-2 ω/f matrix:

	$\omega_{2+}^{\#1}\dagger^{\alpha\beta}$	$f_{2+}^{\#1}\dagger^{\alpha\beta}$	$\omega_{2-}^{\#1}\dagger^{\alpha\beta\chi}$
$\omega_{2-}^{\#1}\dagger^{\alpha\beta\chi}$	0	0	$k^2r_1+\frac{t_1}{2}$
$f_{2+}^{\#1}\dagger^{\alpha\beta}$	$-\frac{ikt_1}{\sqrt{2}}$	k^2t_1	0
$\omega_{2+}^{\#1}\dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$\frac{ikt_1}{\sqrt{2}}$	0

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\sigma_{0-}^{\#1} == 0$	1
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} - 2ik\,\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\,\alpha} + 2ik\,\sigma_{1-}^{\#2\,\alpha} == 0$	3
$\tau_{1-}^{\#1\,\alpha} == 0$	3
$\tau_{1+}^{\#1\,\alpha\beta} - 2ik\,\sigma_{1+}^{\#1\,\alpha\beta} == 0$	3
$2\,\sigma_{1+}^{\#1\,\alpha\beta} + \sigma_{1+}^{\#2\,\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\,\alpha\beta} - 2ik\,\sigma_{2+}^{\#1\,\alpha\beta} == 0$	5
Total constraints:	20

Massive and massless spectra

$J^P = 2^-$, $\vec{k}^\mu$

Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0\ \&\&\ t_1 > 0$