

Particle spectrograph

Wave operator and propagator

| Source constraints  |  |  | Fundamental fields  |  | Multiplicities |  |
|---|--|--|---|--|----------------|--|
| SO(3) irreps  |  |  |   |  |                |  |
| $\tau_0^{\#2} == 0$   |  |  | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$  |  | 1              |  |
| $\tau_{0+}^{\#1} - 2\,i\,k\,\sigma_{0+}^{\#1} == 0$                 |  |  | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha_\alpha + 2\,\partial_\chi \partial^\chi \partial_\beta \sigma^{\alpha\beta}_\alpha$   |  | 1              |  |
| $\tau_{1+}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1+}^{\#2\,\alpha} == 0$ |  |  | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta} + 2\,\partial_\beta \partial^\beta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$   |  | 3              |  |
| $\tau_{1-}^{\#1\,\alpha} == 0$                                      |  |  | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$  |  | 3              |  |
| $\tau_{1+}^{\#1\,\alpha\beta} == 0$                                 |  |  | $\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\alpha\chi} + \partial_\chi \partial^\chi \tau^{\alpha\beta}$  |  | 3              |  |
| $\sigma_{1+}^{\#2\,\alpha\beta} == 0$                               |  |  | $\partial_\theta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + \partial_\theta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\theta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$   |  | 3              |  |
| $\sigma_{2+}^{\#1\,\alpha\beta\chi} == 0$                           |  |  | $3\,\partial_\epsilon \partial_\theta \partial^\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 3\,\partial_\epsilon \partial^\epsilon \partial_\chi \partial^\alpha \sigma^{\beta\delta}_\delta +$<br>$2\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\beta \sigma^{\alpha\chi\delta} + 4\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\beta \sigma^{\alpha\delta\chi} +$<br>$2\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\beta \sigma^{\chi\delta\alpha} + 4\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\chi \sigma^{\alpha\beta\delta} +$<br>$2\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\chi \sigma^{\alpha\delta\beta} + 2\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\delta \sigma^{\beta\chi\alpha} +$<br>$3\,\eta^{\beta\chi}\,\partial_\theta \partial^\theta \partial_\epsilon \partial^\epsilon \sigma^{\delta\epsilon}_\delta +$<br>$3\,\eta^{\alpha\chi}\,\partial_\theta \partial^\theta \partial_\epsilon \partial_\sigma \sigma^{\beta\delta\epsilon} +$<br>$3\,\eta^{\beta\chi}\,\partial_\theta \partial^\theta \partial_\epsilon \partial^\epsilon \sigma^{\alpha\delta}_\delta ==$<br>$3\,\partial_\epsilon \partial_\theta \partial^\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 3\,\partial_\epsilon \partial^\epsilon \partial^\chi \partial^\beta \sigma^{\alpha\delta}_\delta +$<br>$2\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\alpha \sigma^{\beta\chi\delta} + 4\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\beta \sigma^{\alpha\delta\chi} +$<br>$2\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\beta \sigma^{\chi\delta\alpha} + 2\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\chi \sigma^{\alpha\beta\delta} +$<br>$4\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\delta \sigma^{\alpha\beta\chi} + 2\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\delta \sigma^{\alpha\chi\beta} +$<br>$3\,\eta^{\alpha\chi}\,\partial_\theta \partial^\theta \partial_\epsilon \partial^\beta \sigma^{\delta\epsilon}_\delta +$<br>$3\,\eta^{\beta\chi}\,\partial_\theta \partial^\theta \partial_\epsilon \partial_\sigma \sigma^{\alpha\delta\epsilon} +$<br>$3\,\eta^{\alpha\chi}\,\partial_\theta \partial^\theta \partial_\epsilon \partial^\epsilon \sigma^{\beta\delta}_\delta$ |  | 5              |  |
| $\tau_{2+}^{\#1\,\alpha\beta} == 0$                                 |  |  | $4\,\partial_\sigma \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2\,\partial_\sigma \partial^\delta \partial^\beta \partial^\alpha \tau^\chi_\chi +$<br>$3\,\partial_\theta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\alpha\beta} + 3\,\partial_\theta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\alpha} +$<br>$2\,\eta^{\alpha\beta}\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial_\chi \tau^{\chi\delta} == 3\,\partial_\sigma \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\chi} +$<br>$3\,\partial_\theta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\beta} + 3\,\partial_\theta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi} +$<br>$3\,\partial_\theta \partial^\delta \partial_\chi \partial^\beta \tau^{\chi\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon \partial^\epsilon \partial_\theta \partial^\delta \tau^\chi_\chi$  |  | 5              |  |
| Total constraints/gauge generators:                                 |  |  |   |  | 24             |  |

| Quadratic (free) action  |  |
|--|--|
| $S == \iiint (\frac{1}{6} (-4\,t_3\,\omega_{\alpha}^{\alpha\iota}\,\omega_{\iota\,\kappa}^{\kappa} + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + 8\,t_3\,\omega_{\alpha\,\kappa}^{\kappa}\,\partial_{\iota}f^{\alpha\iota} -$<br>$8\,t_3\,\omega_{\iota\,\kappa}^{\kappa}\,\partial_{\iota}f^{\alpha}_{\alpha} + 4\,t_3\,\partial_{\iota}f^{\kappa}_{\kappa}\,\partial_{\iota}f^{\alpha}_{\alpha} - 3\,r_3\,\partial_\beta\omega_{\iota\,\theta}^{\theta}\,\partial_{\iota}\omega^{\alpha\beta}_{\alpha} -$<br>$3\,r_3\,\partial_{\iota}\omega_{\beta\,\theta}^{\theta}\,\partial_{\iota}\omega^{\alpha\beta}_{\alpha} - 3\,r_3\,\partial_\alpha\omega^{\alpha\beta\iota}_{\beta}\,\partial_\theta\omega_{\beta\,\iota}^{\theta} +$<br>$6\,r_3\,\partial_{\iota}\omega^{\alpha\beta}_{\alpha}\,\partial_\theta\omega_{\beta\,\iota}^{\theta} - 3\,r_3\,\partial_\alpha\omega^{\alpha\beta\iota}_{\beta}\,\partial_\theta\omega_{\iota\,\beta}^{\theta} +$<br>$6\,r_3\,\partial_{\iota}\omega^{\alpha\beta}_{\alpha}\,\partial_\theta\omega_{\alpha\theta\iota}^{\theta}\,\partial^{\theta}\omega^{\alpha\beta\iota}_{\beta} + 8\,r_2\,\partial_\beta\omega_{\alpha\iota\theta}^{\theta}\,\partial^{\theta}\omega^{\alpha\beta\iota}_{\iota} -$<br>$4\,r_2\,\partial_\beta\omega_{\alpha\theta\iota}^{\theta}\,\partial^{\theta}\omega^{\alpha\beta\iota}_{\beta} + 4\,r_2\,\partial_\beta\omega_{\iota\theta\alpha}^{\theta}\,\partial^{\theta}\omega^{\alpha\beta\iota}_{\iota} -$<br>$24\,r_3\,\partial_\beta\omega_{\iota\theta\alpha}^{\theta}\,\partial^{\theta}\omega^{\alpha\beta\iota}_{\beta} - 2\,r_2\,\partial_{\iota}\omega_{\alpha\beta\theta}^{\theta}\,\partial^{\theta}\omega^{\alpha\beta\iota}_{\iota} +$<br>$2\,r_2\,\partial_\theta\omega_{\alpha\beta\iota}^{\theta}\,\partial^{\theta}\omega^{\alpha\beta\iota}_{\iota} - 4\,r_2\,\partial_\theta\omega_{\alpha\iota\beta}^{\theta}\,\partial^{\theta}\omega^{\alpha\beta\iota}_{\iota} +$<br>$6\,r_5\,\partial_{\iota}\omega_{\theta\,\kappa}^{\kappa}\,\partial^{\theta}\omega^{\alpha\iota}_{\alpha} - 6\,r_5\,\partial_\theta\omega_{\iota\,\kappa}^{\kappa}\,\partial^{\theta}\omega^{\alpha\iota}_{\alpha} +$<br>$4\,t_3\,\partial_{\iota}f^{\alpha\iota}_{\alpha}\,\partial_{\kappa}f^{\kappa}_{\alpha} - 8\,t_3\,\partial_{\iota}f^{\alpha}_{\alpha}\,\partial_{\kappa}f^{\kappa}_{\iota} - 6\,r_5\,\partial_\alpha\omega^{\alpha\iota\theta}_{\iota}\,\partial_{\kappa}\omega_{\iota\,\theta}^{\kappa} +$<br>$12\,r_5\,\partial^{\theta}\omega^{\alpha\iota}_{\alpha}\,\partial_{\kappa}\omega_{\iota\,\theta}^{\kappa} + 6\,r_5\,\partial_\alpha\omega^{\alpha\iota\theta}_{\iota}\,\partial_{\kappa}\omega_{\theta\,\iota}^{\kappa} -$<br>$12\,r_5\,\partial^{\theta}\omega^{\alpha\iota}_{\alpha}\,\partial_{\kappa}\omega_{\theta\,\iota}^{\kappa})) [t,\,x,\,y,\,z] \,dz\,dy\,dx\,dt$ |  |

| $\sigma_{1+}^{\#1} + ^{\alpha\beta}$ | $\sigma_{1+}^{\#2}$ | $\tau_{1+}^{\#1}$ | $\sigma_{1-}^{\#1}$                       | $\sigma_{1-}^{\#2}$                                   | $\tau_{1-}^{\#1}$ | $\tau_{1-}^{\#2}$   |
|--------------------------------------|---------------------|-------------------|---|---|-------------------|---|
| $\frac{1}{k^2(2r_3+r_5)}$            | 0                   | 0                 | 0   | 0   | 0                 | 0   |
| 0                                    | 0                   | 0                 | 0   | 0   | 0                 | 0   |
| 0                                    | 0                   | 0                 | 0   | 0   | 0                 | 0   |
| 0                                    | 0                   | 0                 | $\frac{2}{k^2(r_3+2r_5)}$                 | $\frac{2\sqrt{2}}{k^2(1+2k^2)(r_3+2r_5)}$             | 0                 | $\frac{4i}{k(1+2k^2)(r_3+2r_5)}$                                  |
| 0                                    | 0                   | 0                 | $\frac{2\sqrt{2}}{k^2(1+2k^2)(r_3+2r_5)}$ | $\frac{3k^2(r_3+2r_5)+4t_3}{(k+2k^2)^2(r_3+2r_5)t_3}$ | 0                 | $\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$ |
| 0                                    | 0                   | 0                 | 0   | 0   | 0                 | 0   |
| 0                                    | 0                   | 0                 | $-\frac{4i}{k(1+2k^2)(r_3+2r_5)}$         | $-\frac{i\sqrt{2}t_3}{k(1+2k^2)^2(r_3+2r_5)t_3}$      | 0                 | $\frac{6k^2(r_3+2r_5)+8t_3}{(1+2k^2)^2(r_3+2r_5)t_3}$             |

| $\omega_{1+}^{\#1} + ^{\alpha\beta}$ | $\omega_{1+}^{\#2}$ | $f_{1+}^{\#1}$ | $\omega_{1-}^{\#1}$                       | $\omega_{1-}^{\#2}$             | $f_{1-}^{\#1}$ | $f_{1-}^{\#2}$                 |
|--------------------------------------|---------------------|----------------|---|---------------------------------|----------------|--------------------------------|
| $k^2(2r_3+r_5)$                      | 0                   | 0              | 0   | 0                               | 0              | 0                              |
| 0                                    | 0                   | 0              | 0   | 0                               | 0              | 0                              |
| 0                                    | 0                   | 0              | 0   | 0                               | 0              | 0                              |
| 0                                    | 0                   | 0              | $k^2(\frac{r_3}{2}+r_5) + \frac{2t_3}{3}$ | $-\frac{\sqrt{2}t_3}{3}$        | 0              | $-\frac{2}{3}i\,k\,t_3$        |
| 0                                    | 0                   | 0              | $-\frac{\sqrt{2}t_3}{3}$                  | $\frac{t_3}{3}$                 | 0              | $\frac{1}{3}i\sqrt{2}\,k\,t_3$ |
| 0                                    | 0                   | 0              | 0   | 0                               | 0              | 0                              |
| 0                                    | 0                   | 0              | $\frac{2ikt_3}{3}$                        | $-\frac{1}{3}i\sqrt{2}\,k\,t_3$ | 0              | $\frac{2k^2t_3}{3}$            |

| $\sigma_{0+}^{\#1} +$     | $\sigma_{0+}^{\#1}$                 | $\tau_{0+}^{\#1}$            | $\sigma_{0+}^{\#2}$ | $\tau_{0+}^{\#2}$ | $\sigma_{0+}^{\#1}$ |
|---------------------------|-------------------------------------|------------------------------|---------------------|-------------------|---------------------|
| $\frac{1}{(1+2k^2)^2t_3}$ | $-\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$ | $\frac{2k^2}{(1+2k^2)^2t_3}$ | 0                   | 0                 | 0                   |
| 0                         | 0                                   | 0                            | 0                   | 0                 | 0                   |
| 0                         | 0                                   | 0                            | 0                   | 0                 | 0                   |
| $\frac{1}{k^2r_2}$        | 0                                   | 0                            | 0                   | 0                 | 0                   |

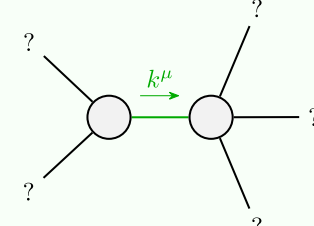
| $\omega_{2+}^{\#1} + ^{\alpha\beta}$ | $f_{2+}^{\#1}$ | $\omega_{2-}^{\#1}$ |
|--------------------------------------|----------------|---------------------|
| $-\frac{3k^2r_3}{2}$                 | 0              | 0                   |
| 0                                    | 0              | 0                   |
| 0                                    | 0              | 0                   |

| $\sigma_{2+}^{\#1} + ^{\alpha\beta}$ | $\tau_{2+}^{\#1}$ | $\sigma_{2-}^{\#1}$ |
|--------------------------------------|-------------------|---------------------|
| $-\frac{2}{3k^2r_3}$                 | 0                 | 0                   |
| 0                                    | 0                 | 0                   |
| 0                                    | 0                 | 0                   |

| $\omega_0^{\#1} +$  | $f_0^{\#1}$          | $\omega_0^{\#2}$ |
|---------------------|----------------------|------------------|
| $t_3$               | $-i\sqrt{2}\,k\,t_3$ | 0                |
| $i\sqrt{2}\,k\,t_3$ | $2k^2t_3$            | 0                |
| 0                   | 0                    | 0                |
| 0                   | 0                    | $k^2r_2$         |

| $\sigma_{2+}^{\#1} + ^{\alpha\beta}$ | $\tau_{2+}^{\#1}$ | $\sigma_{2-}^{\#1}$ |
|--------------------------------------|-------------------|---------------------|
| $-\frac{2}{3k^2r_3}$                 | 0                 | 0                   |
| 0                                    | 0                 | 0                   |
| 0                                    | 0                 | 0                   |

Massive and massless spectra



|                |   |
|----------------|---|
| Quadratic pole |   |
| Pole residue:  | $-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2} > 0$ |
| Polarisations: | 2   |

(No massive particles)

Unitarity conditions

$r_3 < 0 \&\& (r_5 < -\frac{r_3}{2} \parallel r_5 > -2r_3) \parallel r_3 > 0 \&\& -2r_3 < r_5 < -\frac{r_3}{2}$