## Particle spectrograph

## Wave operator and propagator

$\tau_{1}^{\#2}{}_{\alpha}$	0	0	0	$\frac{2ik}{t_1 + 2k^2t_1}$	$-\frac{i\sqrt{2}}{(t_1+2k^2t_1)^2}$	0	$\frac{-4k^4(r_1+r_5)+2k^2t_1}{(t_1+2k^2t_1)^2}$
$\tau_{1^{}}^{\#1}\alpha$	0	0	0	0	0	0	0
$\sigma_{1^{-}\alpha}^{\#2}$	0	0	0	$\frac{\sqrt{2}}{t_1 + 2k^2t_1}$	$\frac{-2 k^2 (r_1 + r_5) + t_1}{(t_1 + 2 k^2 t_1)^2}$	0	$\frac{i\sqrt{2}k(2k^2(r_1+r_5)\cdot t_1)}{(t_1+2k^2t_1)^2}$
$\sigma_{1^{\bar{-}}}^{\#1}{}_{\alpha}$	0	0	0	0	$\frac{\sqrt{2}}{t_1 + 2k^2t_1}$	0	$-\frac{2ik}{t_1+2k^2t_1}$
$\tau_{1}^{\#1}_{\alpha\beta}$	$\frac{i}{\sqrt{2} (k+k^3) (2 r_1 + r_5)}$	$\frac{i(6k^2(2r_1+r_5)+t_1)}{2k(1+k^2)^2(2r_1+r_5)t_1}$	$\frac{6k^2(2r_1+r_5)+t_1}{2(1+k^2)^2(2r_1+r_5)t_1}$	0	0	0	0
$\sigma_{1}^{\#2}{}_{+}\alpha_{\beta}$	$\frac{1}{\sqrt{2} \; (k^2 + k^4)  (2  r_1 + r_5)}$	$\frac{6k^2(2r_1+r_5)+t_1}{2(k+k^3)^2(2r_1+r_5)t_1}$	$-\frac{i(6k^2(2r_1+r_5)+t_1)}{2k(1+k^2)^2(2r_1+r_5)t_1}$	0	0	0	0
$\sigma_{1}^{\#1}{}_{\alpha\beta}$		$\frac{1}{\sqrt{2} \; (k^2 + k^4)  (2  r_1 + r_5)}$	$-\frac{i}{\sqrt{2}\left(k\!+\!k^3\right)\left(2r_1\!+\!r_5\right)}$	0	0	0	0
	$_{1}^{#1}$ $\dagger^{\alpha\beta}$	$_{1}^{#2}+^{\alpha\beta}$	$_{1}^{#1}+^{\alpha\beta}$	$\sigma_{1}^{\#1} + \alpha$	$\sigma_{1}^{\#2} + \alpha$	$\tau_{1}^{\#1} \dagger^{\alpha}$	$t_1^{\#2} +^{\alpha}$

	$\sigma_{0}^{\sharp 1}$	$ au_0^{\#1}$	$ au_{0}^{\#2}$	$\sigma_0^{\#1}$	_	σ#1	<b>-</b> #1
$\sigma_{0}^{\#1}$ †	$-\frac{1}{(1+2k^2)^2t_1}$	$\frac{i \sqrt{2} k}{(1+2k^2)^2 t_1}$	0	0	$\sigma_{\scriptscriptstyle 2}^{\scriptscriptstyle \#1}\dagger^{lphaeta}$	$\sigma_{2+\alpha\beta}^{\#1}$	$\tau_{2+\alpha\beta}^{\#1}$ $2i\sqrt{2}k$
$ au_{0}^{\#1}$ †	$-\frac{i \sqrt{2} k}{(1+2k^2)^2 t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0	0	$\sigma_{2}^{+}$   $\tau_{2}^{\#1}$ † $\alpha\beta$	$\frac{(1+2k^2)^2 t_1}{2i\sqrt{2}k}$	$\frac{(1+2k^2)^2 t_1}{4k^2}$
$\tau_{0}^{\#2}$ †	0	0	0	0		$(1+2k^2)^2t_1$	$(1+2k^2)^2t_1$
$\sigma_0^{\#,1}$ †	0	0	0	0	$\sigma_2^{#1} \dagger^{lphaeta\chi}$	0	0

											$\omega_{0}^{\#1}$	0	0	0	0	>
	$\omega^{\alphaeta\chi}$		a	$f_{\alpha}^{}$ -				_6			$f_{0}^{\#2} \; \omega_{0}^{\#1}$	0	0	0	0	$^{\#1}_{2^-}lphaeta)$
	$\text{SF} == \\ \left\{ \int \int \int \int \left(\frac{1}{3} \left(-3t_1\;\omega_{_{K}\alpha'}^{\;\alpha'}\;\omega_{_{K}\alpha'}^{\;\kappa'} - t_1\;\omega_{_{K}\lambda'}^{\;\kappa\lambda'}\;\omega_{_{K}\lambda'}^{\;\kappa\lambda'} + t_1\;\omega_{_{K}\lambda'}^{\;\kappa\lambda'}\;\omega_{_{K}\lambda'}^{\;\kappa\lambda'} + 3\;f^{\alpha\beta}\;\tau_{_{\alpha\beta}} + 3\;\omega_{^{\alpha\beta\chi}} \right) \right\}$	$_{\kappa}\omega^{\alpha\beta\theta}+$	<u>`</u>	$3r_5\partial_{lpha}\omega_{\lambda}^{a}\partial_{\kappa}\omega^{\kappa\lambda\sigma}+6r_5\partial_{ heta}\omega_{\lambda}^{a}\partial_{\kappa}\omega^{\kappa\lambda\sigma}-t_1\partial^{lpha}f_{$	$\alpha^{\alpha}\partial^{\kappa}f'_{}$ -	+ <sub>θ/</sub>	$^{\kappa}f_{\lambda\alpha}$ +	$t_1 \partial_\kappa f_{\beta}^{\lambda} \partial^\kappa f_{\lambda}^{\theta} + 2  t_1  \partial_\kappa f^{\lambda}_{\theta} \partial^\kappa f_{\theta}^{\theta} - 3  t_1  \partial^\alpha f^{\lambda}_{\alpha} \partial^\kappa f_{\lambda\kappa} + 2  r_1  \partial_\kappa \omega^{\alpha\beta\theta}  \partial^\kappa \omega_{\alpha\beta\theta}^{\theta} - 2  f_1  \partial_\kappa f^{\lambda}_{\alpha\beta\beta}^{\beta\beta} + 2  f_2  f_3  \partial_\kappa f^{\lambda}_{\alpha\beta\beta}^{\beta\beta} + 2  f_3  \partial_\kappa f^{\lambda}_{\alpha\beta\beta}^{\beta\beta} + 2  f_3  \partial_\kappa f^{\lambda}_{\alpha\beta\beta}^{\beta\beta} + 2  f_3  \partial_\kappa f^{\lambda}_{\beta}^{\beta} + 2  f_3  \partial_\kappa f^{$	+	1x dt	$f_0^{\#1}$	$ i\sqrt{2}kt_1 $	$\left  -2 k^2 t_1 \right $	0	0	$\omega_{2}^{\#1}$ $\omega_{2}^{\#1}$ $\alpha_{3}^{\#1}$ $\alpha_{2}^{\#1}$ $\alpha_{2}$
	ر <sup>دک</sup> ر + ع ج	$\sigma_{\alpha\beta\chi} - 3 r_5  \partial_i \omega^{\kappa\lambda}_{\ \kappa}  \partial^i \omega_{\lambda}^{\ \alpha} - 2 r_1  \partial^\beta \omega^{\theta\alpha}_{\ \kappa}  \partial_\theta \omega_{\alpha\beta}^{\ \kappa} - 2 r_1  \partial_\theta \omega_{\alpha\beta}^{\ \kappa}  \partial_\kappa \omega^{\alpha\beta\theta} +$	$2r_1\partial_\theta\omega_{\alpha\beta}^{\kappa}\partial_\kappa\omega^{\theta\alpha\beta}$ - $3r_5\partial_\alpha\omega_\lambda^{\alpha}\partial_\kappa\omega^{\theta\kappa\lambda}$ + $3r_5\partial_\theta\omega_\lambda^{\alpha}\partial_\kappa\omega^{\theta\kappa\lambda}$ -	$\kappa^{\partial^{\kappa} f_{\alpha}^{\ \ \mu}} - 2$	$t_1 \partial^\alpha f^\lambda_{\ \kappa} \partial^\kappa f_{\alpha\lambda} + 3 t_1 \ \omega_{\kappa\alpha}^{\ \alpha} \ \partial^\kappa f'_{\ \prime} + 3 t_1 \ \omega_{\kappa\lambda}^{\ \lambda} \ \partial^\kappa f'_{\ \prime} + 6 t_1 \partial^\alpha f_{\kappa\alpha} \partial^\kappa f'_{\ \prime} -$	$3t_1\partial_{\kappa}f^{\lambda}_{\lambda}\partial^{\kappa}f^{\prime}_{\prime}+t_1\ \omega_{\prime\theta\kappa}\ \partial^{\kappa}f^{\prime\theta}+4t_1\ \omega_{\prime\kappa\theta}\ \partial^{\kappa}f^{\prime\theta}-t_1\ \omega_{\theta\prime\kappa}\ \partial^{\kappa}f^{\prime\theta}+$	$2t_1  \omega_{\theta\kappa\prime}  \partial^\kappa f^{\prime\theta} - 3t_1  \omega_{\prime\alpha}^{}  \partial^\kappa f^\prime_{\kappa} - 3t_1  \omega_{\prime\lambda}^{\lambda}  \partial^\kappa f^\prime_{\kappa} + t_1  \partial^\alpha f^\lambda_{\kappa}  \partial^\kappa f_{\lambda\alpha} +$	$+2r_1\partial_{\kappa}\omega$	$2 r_1 \partial_{\kappa} \omega^{\theta \alpha \beta} \partial^{\kappa} \omega_{\alpha \beta \theta} + 2 r_1 \partial^{\beta} \omega_{\alpha}^{\ \alpha \lambda} \partial_{\lambda} \omega_{\alpha \beta}^{\ \ \prime} - 8 r_1 \partial^{\beta} \omega_{\lambda}^{\ \lambda \alpha} \partial_{\lambda} \omega_{\alpha \beta}^{\ \prime} +$	$3r_5\partial_{lpha}\omega_{\lambda}^{a}\partial^{\lambda}\omega^{\theta\kappa}_{\kappa}-3r_5\partial_{\theta}\omega_{\lambda}^{a}\partial^{\lambda}\omega^{\theta\kappa}_{\kappa}))[t,x,y,z]dzdydxdt$	$\omega_{0}^{\#1}$	-t <sub>1</sub>	$-i\sqrt{2}\ kt_1$	0	0	$\omega_2^{\#1}$
	$_{1}$ $\omega_{\kappa\lambda}^{\prime}$ $_{0}$	υ <sub>αβ</sub> <sup>κ</sup> -2 <i>r</i>	$-3r_5\partial_{\theta}($	$-t_1\partial^lpha f_{ heta}$	$^{\lambda}$ $\partial^{\kappa} f'$	$\theta g^{\kappa} f^{I\theta} - \theta$	$^{\lambda} \partial^{\kappa} f'_{\kappa} +$	$^{\lambda}{}_{\alpha}\partial^{\kappa}f_{\lambda\kappa}$	$8 r_1 \partial^{\beta} u$	[t, x, y,	'	$\omega_{0}^{\#1}\dagger$	$f_{0}^{\#1}$ †	$f_{0}^{\#2}$ †	$\omega_{0}^{\#1}\dagger$	
	$\lambda' + t$	$^{\prime}_{\kappa}\partial_{\theta}\omega$	, 9K7 , 2, 9	S XXA	$t_1 \omega_{_{_{_{_{_{}}}}}}$	$\omega_{_{IK}}$	$\mathcal{E}_{\lambda}$	$_{1}\partial ^{\alpha }f$	υ', ο	$({}^{\theta \kappa}_{\kappa})$				μ	$p_{1}^{\#1}$	3
	$^{\lambda}$ $\omega_{\kappa}$	$^{eta}\omega^{eta_{\mathcal{C}}}$	$^{'}_{\theta}\partial_{\kappa}\omega$	$^{\alpha}_{\alpha}\partial_{\kappa}$	1+3	+ 4 t <sub>1</sub>	$(-3t_1)$	θ-3 <i>t</i>	$^{\chi\gamma}\partial_{\gamma}c$	$\alpha \frac{\partial^{\lambda} u}{\partial \alpha}$	$\omega_1^{\!\scriptscriptstyle\#}$	$^{\ddagger}_{+}$ $^{\alpha}$	$k^2$	(2 r <sub>1</sub>	$+r_{5}$	$(t) + \frac{t}{6}$
_	$-t_1\;\omega_{'}^{\kappa}$	,-2 <i>r</i> 10	$\partial_{lpha} \omega_{\lambda}^{}$	$^{5}\partial_{ heta}\omega_{\lambda}$	$a^{\alpha} \partial^{\kappa} f^{\prime}$	$\partial^{\kappa} f^{I\theta}$	$\alpha \frac{\partial^{k} f'}{\partial \alpha}$	$^{\lambda}_{\theta}\partial^{\kappa}f_{\lambda}$	$_{1}\partial^{eta}\omega_{,}^{'}$	$\partial_{ heta} \omega_{\lambda}^{       $	$\omega_1^{\!\scriptscriptstyle \#}$			-	$\frac{t_1}{3\sqrt{2}}$	
actior	$\omega_{\kappa \alpha}^{ \kappa}$	$\omega_{\lambda}^{\alpha}$	3-3/5	46/	$t_1 \omega_{\kappa}$	<sub>1</sub> ω,θ,	,1 W,c	$f_1 \partial_{\kappa} f$	9+21	,-3 <i>r</i> 5	$f_1^{\sharp}$	$^{\ddagger}$ $^{\dagger}$	β	-	$\frac{i k t_1}{3 \sqrt{2}}$	
ree) a	$\omega_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{$	$\omega^{K\lambda}_{K}$	$^{\kappa}\omega^{\thetalpha_{1}}$	$\kappa e^{\kappa \chi}$	y+3	t' + t'	16- <sub>θ</sub> -3	$\theta$ + 2 $\theta$	$^{\kappa}\omega_{lphaeta}$	$^{\lambda}\omega^{ heta\kappa}$	μ	»# <u> </u>	α		0	
Quadratic (free) action	(-3 <i>t</i> <sub>1</sub>	3 r5 0,1	$\omega_{\alpha\beta}^{\kappa}\hat{o}$	$\omega_{\lambda}^{\ \alpha}_{\ \ \theta}$	$_{\kappa}^{\partial^{\kappa}}f_{\alpha}$	A OK	$_{1KI}$ $\partial^{K}f$	$^{\lambda}\partial^{\kappa}f_{\lambda}$	$0^{etalphaeta}$	$\omega_{\lambda}^{\alpha}{}_{\theta}c$	μ	» <sub>1</sub> -2 †	α		0	
ıadra	$== \iiint \left(\frac{1}{3}\right)$	αβχ - Ξ	$r_1\partial_ heta \iota$	$r_5  \partial_{lpha} \iota$	$\partial^{\alpha} f^{\lambda}$	$t_1  \partial_\kappa f$	$t_1~\omega_{ heta}$	$\partial_{\kappa} f_{\ \theta}^{\ \prime}$	$r_1  \partial_\kappa \iota$	$r_5  \partial_{lpha} \iota$	f	`# <u>1</u> †	α		0	
On	S <sub>F</sub> == []	P	7	m	<i>t</i> <sub>1</sub>	m	7	<i>t</i> <sub>1</sub>	7	m	f	<sup>#2</sup> †	α		0	

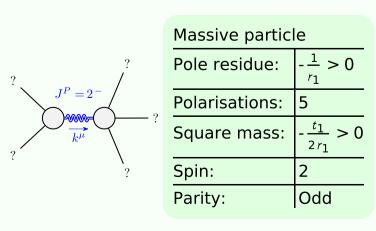
	ors										
	uge generato Multiplicities	. 1	1	4 ,	1	3	3	3	5	17	
	Source constraints/gauge generators SO(3) irreps   Multiplicities	$\sigma_{0}^{#1} == 0$	υ υ	0+ 0 #1 0 =: #1	$\tau_0^{"\ddagger} - 2 i k \sigma_0^{"\ddagger} = 0$	$t_1^{\#2}\alpha + 2ik \sigma_1^{\#2}\alpha = 0$	$\tau_{1}^{\#1}{}^{\alpha} == 0$	$t_1^{\#1}\alpha\beta + ik \ \sigma_1^{\#2}\alpha\beta == 0$	$\tau_{2+}^{\#1}\alpha\beta - 2ik \sigma_{2+}^{\#1}\alpha\beta == 0$	Total constraints:	
$\omega_{2}^{\#1}$ $\omega_{2}^{\#1}$ $\alpha_{2}^{\#1}$ $\alpha_{2}^{\#1}$ $\alpha_{2}$	0	O	>	$k^2 r_1 + \frac{t_1}{2}$	1						
$f_{2}^{\#1}\alpha\beta$	$-\frac{ikt_1}{\sqrt{2}}$	p <sup>2</sup> t.		0							
$\omega_2^{\#1}_{+lphaeta}$	<u>t1</u> 2	$ikt_1$		0							
-	$\omega_2^{\#1} + ^{\alpha \beta}$	$f^{#1} + \alpha \beta$		$\omega_{2^-}^{\#1} +^{lphaeta\chi}$	_						
	$\omega_1^{\#_2}$					$\omega_1^{\!\scriptscriptstyle\#}$	$\frac{1}{\alpha}$	(	$\omega_{1-\alpha}^{\#2}$	$f_{1}^{\#1}\alpha$	$f_{1-\alpha}^{#2}$
$+\frac{t_1}{6}$		<u>1</u> √2				(	)		0	0	0
	tı		īκ	t <sub>1</sub>							

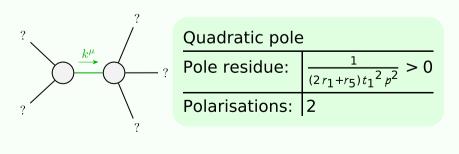
	$\omega_{1^{+}lphaeta}^{\sharp1}$	$\omega_{1^{+}lphaeta}^{\#2}$	$f_{1^{+}\alpha\beta}^{\#1}$	$\omega_{1}^{\sharp 1}{}_{lpha}$	$\omega_{1}^{\#2}{}_{\alpha}$	$f_{1-\alpha}^{\#1}$	$f_{1-\alpha}^{#2}$
$\omega_{\scriptscriptstyle 1}^{\scriptscriptstyle \#1}\dagger^{lphaeta}$	$k^2 (2r_1 + r_5) + \frac{t_1}{6}$	$-\frac{t_1}{3\sqrt{2}}$	$-\frac{ikt_1}{3\sqrt{2}}$	0	0	0	0
$\omega_{1}^{\#2} \dagger^{lphaeta}$	$-\frac{t_1}{3\sqrt{2}}$	<u>t</u> 1 3	<u>i k t 1</u> 3	0	0	0	0
$f_{1}^{\#1}\dagger^{\alpha\beta}$	$\frac{ikt_1}{3\sqrt{2}}$	$-\frac{1}{3}\bar{l}kt_1$	$\frac{k^2t_1}{3}$	0	0	0	0
$\omega_{1}^{\sharp_{1}}$ † $^{lpha}$	0	0	0	$k^2 (r_1 + r_5) - \frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	ikt₁
$\omega_1^{\#2} \dagger^{lpha}$	0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
$f_{1}^{#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$f_{1}^{#2} \dagger^{\alpha}$	0	0	0	$-ar{\it l} \it k t_1$	0	0	0

 $\sigma_{2^-\alpha\beta\chi}^{\#1}$ 

 $\frac{1}{2k^2r_1+t_1}$ 

## Massive and massless spectra





## **Unitarity conditions**

 $r_1 < 0 \&\& r_5 > -2 r_1 \&\& t_1 > 0$