# Particle spectrograph

## Wave operator and propagator

| | $\tau_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^+}^{\#2\,\alpha\beta}$ | $\sigma_{1^+}^{\#1\,\alpha\beta}$ | $\tau_{1^-}^{\#1\,\alpha}$ | $\sigma_{1^-}^{\#2\,\alpha}$ | $\sigma_{1^-}^{\#1\,\alpha}$ | $\tau_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\tau_{1^+}^{\#1\,\alpha\beta}$ | $\frac{i\sqrt{2}\,k\,(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | $\frac{ik(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | $\frac{k^2(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2\,\alpha\beta}$ | $\frac{\sqrt{2}\,(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | $\frac{6k^2(2r_1+r_5)+t_1+4t_2}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | $-\frac{ik(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#1\,\alpha\beta}$ | $\frac{2(t_1+t_2)}{3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)}$ | $\frac{\sqrt{2}\,(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | $-\frac{i\sqrt{2}\,k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#1\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#2\,\alpha}$ | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2t_1}$ | $\frac{-2k^2(r_1+r_5)+t_1}{(t_1+2k^2t_1)^2}$ | 0 | $\frac{i\sqrt{2}\,k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2t_1)^2}$ |
| $\sigma_{1^-}^{\#1\,\alpha}$ | 0 | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2t_1}$ | 0 | $-\frac{2ik}{t_1+2k^2t_1}$ |
| $\tau_{1^-}^{\#2\,\alpha}$ | 0 | 0 | 0 | $\frac{2ik}{t_1+2k^2t_1}$ | $-\frac{i\sqrt{2}\,k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2t_1)^2}$ | 0 | $\frac{-4k^4(r_1+r_5)+2k^2t_1}{(t_1+2k^2t_1)^2}$ |

| | $\omega_{2^+}^{\#1\,\alpha\beta}$ | $f_{2^+}^{\#1\,\alpha\beta}$ | $\omega_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{ikt_1}{\sqrt{2}}$ | 0 |
| $f_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | $\frac{ikt_1}{\sqrt{2}}$ | $k^2t_1$ | 0 |
| $\omega_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$ | 0 | 0 | $k^2r_1+\frac{t_1}{2}$ |

| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1\,\dagger}$ | $-t_1$ | $i\sqrt{2}\,kt_1$ | 0 | 0 |
| $f_{0^+}^{\#1\,\dagger}$ | $-i\sqrt{2}\,kt_1$ | $-2k^2t_1$ | 0 | 0 |
| $f_{0^+}^{\#2\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1\,\dagger}$ | 0 | 0 | 0 | $t_2$ |

| | $\sigma_{2^+}^{\#1\,\alpha\beta}$ | $\tau_{2^+}^{\#1\,\alpha\beta}$ | $\sigma_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | $\frac{2}{(1+2k^2)^2t_1}$ | $-\frac{2i\sqrt{2}\,k}{(1+2k^2)^2t_1}$ | 0 |
| $\tau_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | $\frac{2i\sqrt{2}\,k}{(1+2k^2)^2t_1}$ | $\frac{4k^2}{(1+2k^2)^2t_1}$ | 0 |
| $\sigma_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{2k^2r_1+t_1}$ |

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1\,\dagger}$ | $-\frac{1}{(1+2k^2)^2t_1}$ | $\frac{i\sqrt{2}\,k}{(1+2k^2)^2t_1}$ | 0 | 0 |
| $\tau_{0^+}^{\#1\,\dagger}$ | $-\frac{i\sqrt{2}\,k}{(1+2k^2)^2t_1}$ | $-\frac{2k^2}{(1+2k^2)^2t_1}$ | 0 | 0 |
| $\tau_{0^+}^{\#2\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1\,\dagger}$ | 0 | 0 | 0 | $\frac{1}{t_2}$ \| |

| | $f_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^+}^{\#2\,\alpha\beta}$ | $f_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^-}^{\#1\,\alpha}$ | $\omega_{1^-}^{\#2\,\alpha}$ | $f_{1^-}^{\#1\,\alpha}$ | $f_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $f_{1^+}^{\#1\,\alpha\beta}$ | $-\frac{ik(t_1-2t_2)}{3\sqrt{2}}$ | $\frac{1}{3}ik(t_1+t_2)$ | $\frac{1}{3}k^2(t_1+t_2)$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2\,\alpha\beta}$ | $-\frac{t_1-2t_2}{3\sqrt{2}}$ | $\frac{t_1+t_2}{3}$ | $-\frac{1}{3}ik(t_1+t_2)$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1\,\alpha\beta}$ | $\frac{1}{6}(6k^2(2r_1+r_5)+t_1+4t_2)$ | $-\frac{t_1-2t_2}{3\sqrt{2}}$ | $\frac{ik(t_1-2t_2)}{3\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1\,\alpha}$ | 0 | 0 | 0 | $k^2(r_1+r_5)-\frac{t_1}{2}$ | $\frac{t_1}{\sqrt{2}}$ | 0 | $-ikt_1$ |
| $\omega_{1^-}^{\#2\,\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 |
| $f_{1^-}^{\#1\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2\,\alpha}$ | 0 | 0 | 0 | $-ikt_1$ | 0 | 0 | 0 |

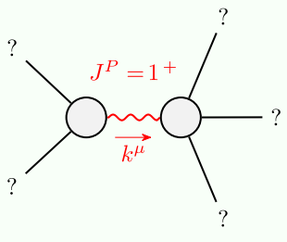
Source constraints/gauge generators

| SO(3) irreps | Multiplicities |
|---|---|
| $\tau_{0^+}^{\#2} == 0$ | 1 |
| $\tau_{0^+}^{\#1} - 2ik\,\sigma_{0^+}^{\#1} == 0$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} + 2ik\,\sigma_{1^-}^{\#2\,\alpha} == 0$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + ik\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$ | 3 |
| $\tau_{2^+}^{\#1\,\alpha\beta} - 2ik\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$ | 5 |
| Total constraints: | 16 |

Quadratic (free) action

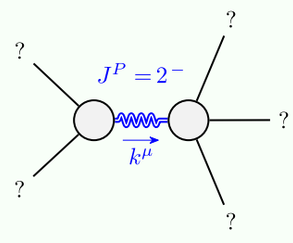
$$S_F == \iiiint\Big(\frac{1}{6}(-6t_1\,\omega_{\iota}{}^{\alpha\iota}\,\omega_{\kappa\alpha}{}^{\kappa} - 2(t_1-2t_2)\,\omega_{\iota}{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}' + 2t_1\,\omega_{\kappa\lambda}{}'\,\omega^{\kappa\lambda}{}_{\iota} + 2t_2\,\omega_{\kappa\lambda}{}'\,\omega^{\kappa\lambda}{}_{\iota} + 6f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 6r_5\,\partial_{\iota}\omega^{\kappa\lambda}{}_{\kappa}\,\partial^{\prime}\omega_{\lambda}{}^{\alpha}{}_{\alpha} - 4r_1\,\partial^{\beta}\omega^{\theta\alpha}{}_{\kappa}\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa} - 4r_1\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\,\partial_{\kappa}\omega^{\alpha\beta\theta} + 4r_1\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\,\partial_{\kappa}\omega^{\theta\alpha\beta} - 6r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial_{\kappa}\omega^{\theta\kappa\lambda} + 6r_5\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_{\kappa}\omega^{\kappa\lambda\theta} - 6r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial_{\kappa}\omega^{\kappa\lambda\theta} + 12r_5\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_{\kappa}\omega^{\kappa\lambda\theta} - 2t_1\,\partial^{\alpha}f_{\theta\kappa}\,\partial^{\kappa}f_{\alpha}{}^{\theta} + t_2\,\partial^{\alpha}f_{\theta\kappa}\,\partial^{\kappa}f_{\alpha}{}^{\theta} - 4t_1\,\partial^{\alpha}f_{\kappa\theta}\,\partial^{\kappa}f_{\alpha}{}^{\theta} - t_2\,\partial^{\alpha}f_{\kappa\theta}\,\partial^{\kappa}f_{\alpha}{}^{\theta} - 2t_1\,\partial^{\alpha}f^{\lambda}{}_{\kappa}\,\partial^{\kappa}f_{\alpha\lambda} + t_2\,\partial^{\alpha}f^{\lambda}{}_{\kappa}\,\partial^{\kappa}f_{\alpha\lambda} + 6t_1\,\omega_{\kappa\alpha}{}^{\alpha}\,\partial^{\kappa}f^{\prime}{}_{\iota} + 6t_1\,\omega_{\kappa\lambda}{}^{\lambda}\,\partial^{\kappa}f^{\prime}{}_{\iota} + 12t_1\,\partial^{\alpha}f_{\kappa\alpha}\,\partial^{\kappa}f^{\prime}{}_{\iota} - 6t_1\,\partial_{\kappa}f^{\lambda}{}_{\lambda}\,\partial^{\kappa}f^{\prime}{}_{\iota} + 2t_1\,\omega_{\iota\theta\kappa}\,\partial^{\kappa}f^{\iota\theta} + 2t_2\,\omega_{\iota\theta\kappa}\,\partial^{\kappa}f^{\iota\theta} + 8t_1\,\omega_{\iota\kappa\theta}\,\partial^{\kappa}f^{\iota\theta} - 4t_2\,\omega_{\iota\kappa\theta}\,\partial^{\kappa}f^{\iota\theta} - 2t_1\,\omega_{\theta\iota\kappa}\,\partial^{\kappa}f^{\iota\theta} - 2t_2\,\omega_{\theta\iota\kappa}\,\partial^{\kappa}f^{\iota\theta} + 4t_1\,\omega_{\theta\kappa\iota}\,\partial^{\kappa}f^{\iota\theta} + 4t_2\,\omega_{\theta\kappa\iota}\,\partial^{\kappa}f^{\iota\theta} - 6t_1\,\omega_{\iota\alpha}{}^{\alpha}\,\partial^{\kappa}f^{\prime}{}_{\kappa} - 6t_1\,\omega_{\iota\lambda}{}^{\lambda}\,\partial^{\kappa}f^{\prime}{}_{\kappa} + 2t_1\,\partial^{\alpha}f^{\lambda}{}_{\kappa}\,\partial^{\kappa}f_{\lambda\alpha} - t_2\,\partial^{\alpha}f^{\lambda}{}_{\kappa}\,\partial^{\kappa}f_{\lambda\alpha} + 2t_1\,\partial_{\kappa}f_{\theta}{}^{\lambda}\,\partial^{\kappa}f_{\lambda}{}^{\theta} - t_2\,\partial_{\kappa}f_{\theta}{}^{\lambda}\,\partial^{\kappa}f_{\lambda}{}^{\theta} + 4t_1\,\partial_{\kappa}f^{\lambda}{}_{\theta}\,\partial^{\kappa}f_{\lambda}{}^{\theta} + t_2\,\partial_{\kappa}f^{\lambda}{}_{\theta}\,\partial^{\kappa}f_{\lambda}{}^{\theta} - 6t_1\,\partial^{\alpha}f^{\lambda}{}_{\alpha}\,\partial^{\kappa}f_{\lambda\kappa} + 4r_1\,\partial_{\kappa}\omega^{\alpha\beta\theta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} - 4r_1\,\partial_{\kappa}\omega^{\theta\alpha\beta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} + 4r_1\,\partial^{\beta}\omega_{\iota}{}^{\alpha\lambda}\,\partial_{\lambda}\omega_{\alpha\beta}{}' - 16r_1\,\partial^{\beta}\omega_{\iota}{}^{\lambda\alpha}\,\partial_{\lambda}\omega_{\alpha\beta}{}' + 6r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa} - 6r_5\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa})\Big)[t, x, y, z]\,dz\,dy\,dx\,dt$$

## Massive and massless spectra




Massive particle

| | |
|---|---|
| Pole residue: | $\frac{-3t_1t_2(t_1+t_2)+6r_1(t_1^2+2t_2^2)+3r_5(t_1^2+2t_2^2)}{(2r_1+r_5)(t_1+t_2)(-3t_1t_2+4r_1(t_1+t_2)+2r_5(t_1+t_2))} > 0$ |
| Polarisations: | 3 |
| Square mass: | $-\frac{3t_1t_2}{2(2r_1+r_5)(t_1+t_2)} > 0$ |
| Spin: | 1 |
| Parity: | Even |




Massive particle

| | |
|---|---|
| Pole residue: | $-\frac{1}{r_1} > 0$ |
| Polarisations: | 5 |
| Square mass: | $-\frac{t_1}{2r_1} > 0$ |
| Spin: | 2 |
| Parity: | Odd |

(No massless particles)

## Unitarity conditions

$r_1 < 0 \;\&\&\; r_5 > -2r_1 \;\&\&\; t_1 > 0 \;\&\&\; -t_1 < t_2 < 0$