

Particle spectrograph

Wave operator and propagator

| Source constraints | | | |
|--|--|----------------|--|
| SO(3) irreps | Fundamental fields | Multiplicities | |
| $\sigma_0^{\#1} == 0$ | $\epsilon \eta_{\alpha\chi\delta} \sigma^\delta \sigma^{\alpha\beta\chi} == 0$ | 1 | |
| $\tau_0^{\#2} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$ | 1 | |
| $\tau_0^{\#1} - 2 i k \sigma_0^{\#1} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha_\alpha + 2 \partial_\chi \partial^{\chi} \partial_\beta \sigma^{\alpha\beta}_\alpha$ | 1 | |
| $\tau_1^{2\alpha} + 2 i k \sigma_1^{2\alpha} == 0$ | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\alpha \partial_\beta \tau^{\alpha\beta} + 2 \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$ | 3 | |
| $\tau_1^{1\alpha} == 0$ | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\alpha \partial_\beta \tau^{\beta\alpha}$ | 3 | |
| $\tau_1^{\#1\alpha} == 0$ | $\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^\alpha_\chi + \partial_\chi \partial^\chi \tau^{\alpha\beta} == \partial_\chi \partial^\alpha \tau^\beta_\chi + \partial_\chi \partial^\beta \tau^\alpha_\chi + \partial_\chi \partial^\chi \tau^{\beta\alpha}$ | 3 | |
| $\sigma_1^{2\alpha\beta} == 0$ | $\partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$ | 3 | |
| $\sigma_2^{\#1\alpha\beta\chi} == 0$ | $3 \partial_\epsilon \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\delta\epsilon} + 3 \partial_\epsilon \partial^\epsilon \partial_\chi \partial^\alpha \sigma^{\beta\delta}_\delta +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\chi\delta} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\delta\chi} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\chi\delta\alpha} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\beta\delta} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\alpha\delta\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\delta\alpha} +$ $4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\beta\chi} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\chi\beta} +$ $3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\beta \sigma^{\delta\epsilon}_\delta +$ $3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\beta\delta\epsilon} +$ $3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\epsilon \sigma^{\alpha\delta}_\delta ==$ $3 \partial_\epsilon \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 3 \partial_\epsilon \partial^\epsilon \partial_\chi \partial^\beta \sigma^{\alpha\delta}_\delta +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\chi\delta} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\delta\chi} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\chi\delta\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\delta\alpha} +$ $4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\beta\chi} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\chi\beta} +$ $3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\beta \sigma^{\delta\epsilon}_\delta +$ $3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\alpha\delta\epsilon} +$ $3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\epsilon \sigma^{\beta\delta}_\delta$ | 5 | |
| $\tau_2^{\#1\alpha\beta} == 0$ | $4 \partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2 \partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi_\chi +$ $3 \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3 \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} +$ $2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} == 3 \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\chi} +$ $3 \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^\chi_\beta + 3 \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^\alpha_\chi +$ $3 \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^\chi_\alpha + 2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \tau^\chi_\chi$ | 5 | |
| Total constraints/gauge generators: | | 25 | |

| $\sigma_1^{\#1} \dagger^{\alpha\beta}$ | $\sigma_1^{\#2} \dagger^{\alpha\beta}$ | $\tau_1^{\#1} \dagger^{\alpha\beta}$ | $\sigma_1^{\#1} \dagger^\alpha$ | $\sigma_1^{\#2} \dagger^\alpha$ | $\tau_1^{\#1} \dagger^\alpha$ | $\tau_1^{\#2} \dagger^\alpha$ |
|--|--|--------------------------------------|--|---|-------------------------------|--|
| $\frac{1}{k^2 (2 r_3 + r_5)}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $\frac{2}{k^2 (r_3 + 2 r_5)}$ | $\frac{2 \sqrt{2}}{k^2 (1 + 2 k^2) (r_3 + 2 r_5)}$ | 0 | $\frac{4 i}{k (1 + 2 k^2) (r_3 + 2 r_5)}$ |
| 0 | 0 | 0 | $\frac{2 \sqrt{2}}{k^2 (1 + 2 k^2) (r_3 + 2 r_5)}$ | $\frac{3 k^2 (r_3 + 2 r_5) + 4 t_3}{(k + 2 k^3)^2 (r_3 + 2 r_5) t_3}$ | 0 | $\frac{i \sqrt{2} (3 k^2 (r_3 + 2 r_5) + 4 t_3)}{k (1 + 2 k^2)^2 (r_3 + 2 r_5) t_3}$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $-\frac{4 i}{k (1 + 2 k^2) (r_3 + 2 r_5)}$ | $-\frac{i \sqrt{2} (3 k^2 (r_3 + 2 r_5) + 4 t_3)}{k (1 + 2 k^2)^2 (r_3 + 2 r_5) t_3}$ | 0 | $-\frac{6 k^2 (r_3 + 2 r_5) + 8 t_3}{(1 + 2 k^2)^2 (r_3 + 2 r_5) t_3}$ |
| $\tau_1^{\#2} \dagger^\alpha$ | 0 | 0 | 0 | 0 | 0 | 0 |

| $\omega_1^{\#1} \dagger^{\alpha\beta}$ | $\omega_1^{\#2} \dagger^{\alpha\beta}$ | $f_1^{\#1} \dagger^{\alpha\beta}$ | $\omega_1^{\#1} \dagger^\alpha$ | $\omega_1^{\#2} \dagger^\alpha$ | $f_1^{\#1} \dagger^\alpha$ | $f_1^{\#2} \dagger^\alpha$ |
|--|--|-----------------------------------|--|---------------------------------|----------------------------|--------------------------------|
| $k^2 (2 r_3 + r_5)$ | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $k^2 (\frac{\sqrt{3}}{2} + r_5) + \frac{2 t_3}{3}$ | $-\frac{\sqrt{2} t_3}{3}$ | 0 | $-\frac{2}{3} i k t_3$ |
| 0 | 0 | 0 | $-\frac{\sqrt{2} t_3}{3}$ | $\frac{t_3}{3}$ | 0 | $\frac{1}{3} i \sqrt{2} k t_3$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $\frac{2 i k t_3}{3}$ | $-\frac{1}{3} i \sqrt{2} k t_3$ | 0 | $\frac{2 k^2 t_3}{3}$ |

| $\sigma_2^{\#1} \dagger^{\alpha\beta}$ | $\tau_2^{\#1} \dagger^{\alpha\beta}$ | $\sigma_2^{\#1} \dagger^{\alpha\beta\chi}$ |
|--|--------------------------------------|--|
| $-\frac{2}{3 k^2 r_3}$ | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

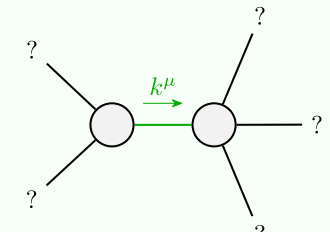
| $\sigma_0^{\#1} \dagger$ | $\tau_0^{\#2} \dagger$ | $\sigma_0^{\#1} \dagger^\alpha$ |
|--|------------------------|---------------------------------|
| $\frac{1}{(1 + 2 k^2)^2 t_3}$ | 0 | 0 |
| $\frac{i \sqrt{2} k}{(1 + 2 k^2)^2 t_3}$ | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

| $\omega_0^{\#1} \dagger$ | $f_0^{\#1} \dagger$ | $\omega_0^{\#2} \dagger$ |
|--------------------------|---------------------|--------------------------|
| t_3 | $-i \sqrt{2} k t_3$ | 0 |
| $i \sqrt{2} k t_3$ | $2 k^2 t_3$ | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

Quadratic (free) action

$$\begin{aligned} S = & \iiint \iiint (f^{\alpha\beta} \tau_{\alpha\beta} + \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - \frac{1}{2} r_3 (\partial_\beta \omega_{ \theta}^\theta \partial' \omega^{\alpha\beta}_\alpha + \partial_\gamma \omega_{ \theta}^\theta \partial' \omega^{\alpha\beta}_\alpha + \\ & \partial_\alpha \omega^{\alpha\beta\gamma} \partial_\theta \omega_{ \beta}^\theta - 2 \partial' \omega^{\alpha\beta}_\alpha \partial_\theta \omega_{ \beta}^\theta + \partial_\alpha \omega^{\alpha\beta\gamma} \partial_\theta \omega_{ \beta}^\theta - \\ & 2 \partial' \omega^{\alpha\beta}_\alpha \partial_\theta \omega_{ \beta}^\theta + 8 \partial_\beta \omega_{ \theta}^\theta \partial^\theta \omega^{\alpha\beta\gamma}) - \\ & \frac{2}{3} t_3 (\omega^{\alpha\gamma}_\alpha \omega_{ \kappa}^\kappa - 2 \omega_{ \kappa}^\kappa \partial_\gamma f^{\alpha\gamma} + 2 \omega_{ \kappa}^\kappa \partial' f^\alpha_\alpha - \\ & \partial_\gamma f^\kappa_\kappa \partial' f^\alpha_\alpha - \partial_\gamma f^{\alpha\gamma} \partial_\kappa f^\kappa_\alpha + 2 \partial' f^\alpha_\alpha \partial_\kappa f^\kappa_\gamma) + \\ & r_5 (\partial_\gamma \omega_{ \kappa}^\kappa \partial^\theta \omega^{\alpha\gamma}_\alpha - \partial_\theta \omega_{ \kappa}^\kappa \partial^\theta \omega^{\alpha\gamma}_\alpha - (\partial_\alpha \omega^{\alpha\gamma\theta} - 2 \partial^\theta \omega^{\alpha\gamma}_\alpha) \\ & (\partial_\kappa \omega_{ \theta}^\kappa - \partial_\kappa \omega_{ \theta}^\kappa)) [t, x, y, z] dz dy dx dt \end{aligned}$$

Massive and massless spectra



Quadratic pole

| | |
|----------------|--|
| Pole residue: | $-\frac{1}{r_3 (2 r_3 + r_5) (r_3 + 2 r_5) p^2} > 0$ |
| Polarisations: | 2 |

(No massive particles)

Unitarity conditions

$$r_3 < 0 \&\& (r_5 < -\frac{r_3}{2} \parallel r_5 > -2 r_3) \parallel r_3 > 0 \&\& -2 r_3 < r_5 < -\frac{r_3}{2}$$