Particle spectrograph

Wave operator and propagator

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\tau^{\#2}_{0^+} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$	1
$\tau^{\#1}_{0^+} - 2 i k\, \sigma^{\#1}_{0^+} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha + 2\,\partial_\chi\partial^\chi\partial_\beta\sigma^{\alpha\beta}{}_\alpha$	1
$\tau^{\#2\alpha}_{1^-} - i k\, \sigma^{\#1\alpha}_{1^-} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} + \partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta} + \partial_\delta\partial^\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi}{}_\beta + \partial_\delta\partial^\delta\partial_\chi\partial^\chi\sigma^{\alpha\beta}{}_\beta$	3
$\tau^{\#1\alpha}_{1^-} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$	3
$\sigma^{\#1\alpha}_{1^-} + 2\,\sigma^{\#2\alpha}_{1^-} == 0$	$\partial_\chi\partial^\alpha\sigma^{\beta\chi}{}_\beta + \partial_\chi\partial^\chi\sigma^{\alpha\beta}{}_\beta == 3\,\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$	3
$\tau^{\#1\alpha\beta}_{1^+} + i k\, \sigma^{\#1\alpha\beta}_{1^+} == 0$	$\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} + \partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} + \partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta}$	3
$\sigma^{\#1\alpha\beta}_{1^+} == \sigma^{\#2\alpha\beta}_{1^+}$	$3\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta} == 3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha}$	3
$\sigma^{\#1\alpha\beta\chi}_{2^-} == 0$	$3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\alpha\sigma^{\beta\delta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\alpha\sigma^{\beta\delta}{}_\delta + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\chi\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\delta\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\chi\delta\alpha} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\beta\delta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\delta\beta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\beta\chi\alpha} + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\alpha\sigma^{\delta\epsilon}{}_\delta + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\beta\delta\epsilon} + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\alpha\delta}{}_\delta == 3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\beta\sigma^{\alpha\delta}{}_\delta + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\chi\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\delta\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\chi\delta\beta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\beta\delta\alpha} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\beta\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\chi\beta} + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\beta\sigma^{\delta\epsilon}{}_\delta + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\alpha\delta\epsilon} + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\beta\delta}{}_\delta$	5
$\tau^{\#1\alpha\beta}_{2^+} == 0$	$4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} == 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi$	5
$\sigma^{\#1\alpha\beta}_{2^+} == 0$	$3\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\sigma^{\chi\delta}{}_\chi == 2\,\partial_\delta\partial^\beta\partial^\alpha\sigma^{\chi\delta}{}_\chi + 3\,(\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha})$	5
Total constraints/gauge generators:		32

	$\sigma^{\#1}_{1^+\alpha\beta}$	$\sigma^{\#2}_{1^+\alpha\beta}$	$\tau^{\#1}_{1^+\alpha\beta}$	$\sigma^{\#1}_{1^-\alpha}$	$\sigma^{\#2}_{1^-\alpha}$	$\tau^{\#1}_{1^-\alpha}$	$\tau^{\#2}_{1^-\alpha}$
$\sigma^{\#1}_{1^+}\dagger^{\alpha\beta}$	$\frac{6}{(3+k^2)^2 t_2}$	$\frac{3\sqrt{2}}{(3+k^2)^2 t_2}$	$\frac{3 i \sqrt{2} k}{(3+k^2)^2 t_2}$	0	0	0	0
$\sigma^{\#2}_{1^+}\dagger^{\alpha\beta}$	$\frac{3\sqrt{2}}{(3+k^2)^2 t_2}$	$\frac{3}{(3+k^2)^2 t_2}$	$\frac{3 i k}{(3+k^2)^2 t_2}$	0	0	0	0
$\tau^{\#1}_{1^+}\dagger^{\alpha\beta}$	$-\frac{3 i \sqrt{2} k}{(3+k^2)^2 t_2}$	$-\frac{3 i k}{(3+k^2)^2 t_2}$	$\frac{3 k^2}{(3+k^2)^2 t_2}$	0	0	0	0
$\sigma^{\#1}_{1^-}\dagger^{\alpha}$	0	0	0	$\frac{6}{(3+2k^2)^2 t_3}$	$-\frac{3\sqrt{2}}{(3+2k^2)^2 t_3}$	0	$-\frac{6 i k}{(3+2k^2)^2 t_3}$
$\sigma^{\#2}_{1^-}\dagger^{\alpha}$	0	0	0	$-\frac{3\sqrt{2}}{(3+2k^2)^2 t_3}$	$\frac{3}{(3+2k^2)^2 t_3}$	0	$\frac{3 i \sqrt{2} k}{(3+2k^2)^2 t_3}$
$\tau^{\#1}_{1^-}\dagger^{\alpha}$	0	0	0	0	0	0	0
$\tau^{\#2}_{1^-}\dagger^{\alpha}$	0	0	0	$\frac{6 i k}{(3+2k^2)^2 t_3}$	$-\frac{3 i \sqrt{2} k}{(3+2k^2)^2 t_3}$	0	$\frac{6 k^2}{(3+2k^2)^2 t_3}$

Quadratic (free) action

$$S == \iiiint\Big(\tfrac{1}{6}\big(-4 t_3\, \omega^{\alpha\iota}{}_\alpha\, \omega^\kappa{}_{\iota\kappa} + 6 f^{\alpha\beta}\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi} + 8 t_3\, \omega_\alpha{}^\kappa{}_\kappa\, \partial_\iota f^{\alpha\iota} - 8 t_3\, \omega_\iota{}^\kappa{}_\kappa\, \partial^\iota f^\alpha{}_\alpha + 4 t_3\, \partial_\iota f^\kappa{}_\kappa\, \partial^\iota f^\alpha{}_\alpha + 4 t_2\, \omega_{\iota\theta\alpha}\, \partial^\theta f^{\alpha\iota} + 2 t_2\, \partial_\alpha f_{\iota\theta}\, \partial^\theta f^{\alpha\iota} - t_2\, \partial_\alpha f_{\theta\iota}\, \partial^\theta f^{\alpha\iota} - t_2\, \partial_\iota f_{\alpha\theta}\, \partial^\theta f^{\alpha\iota} + t_2\, \partial_\theta f_{\alpha\iota}\, \partial^\theta f^{\alpha\iota} - t_2\, \partial_\theta f_{\iota\alpha}\, \partial^\theta f^{\alpha\iota} - 4 t_2\, \omega_{\alpha\theta\iota}\,(\omega^{\alpha\iota\theta} + \partial^\theta f^{\alpha\iota}) + 2 t_2\, \omega_{\alpha\iota\theta}\,(\omega^{\alpha\iota\theta} + 2\,\partial^\theta f^{\alpha\iota}) + 8 r_2\, \partial_\beta\omega_{\alpha\iota\theta}\, \partial^\theta\omega^{\alpha\beta\iota} - 4 r_2\, \partial_\beta\omega_{\alpha\theta\iota}\, \partial^\theta\omega^{\alpha\beta\iota} + 4 r_2\, \partial_\beta\omega_{\iota\theta\alpha}\, \partial^\theta\omega^{\alpha\beta\iota} - 2 r_2\, \partial_\iota\omega_{\alpha\beta\theta}\, \partial^\theta\omega^{\alpha\beta\iota} + 2 r_2\, \partial_\theta\omega_{\alpha\beta\iota}\, \partial^\theta\omega^{\alpha\beta\iota} - 4 r_2\, \partial_\theta\omega_{\alpha\iota\beta}\, \partial^\theta\omega^{\alpha\beta\iota} + 4 t_3\, \partial_\iota f^{\alpha\iota}\, \partial_\kappa f_\alpha{}^\kappa - 8 t_3\, \partial_\iota f^\alpha{}_\alpha\, \partial_\kappa f_\iota{}^\kappa\big)\Big)[t, x, y, z]\, dz\, dy\, dx\, dt$$

	$\omega^{\#1}_{1^+\alpha\beta}$	$\omega^{\#2}_{1^+\alpha\beta}$	$f^{\#1}_{1^+\alpha\beta}$	$\omega^{\#1}_{1^-\alpha}$	$\omega^{\#2}_{1^-\alpha}$	$f^{\#1}_{1^-\alpha}$	$f^{\#2}_{1^-\alpha}$
$\omega^{\#1}_{1^+}\dagger^{\alpha\beta}$	$\frac{2 t_2}{3}$	$\frac{\sqrt{2}\, t_2}{3}$	$\frac{1}{3} i \sqrt{2}\, k t_2$	0	0	0	0
$\omega^{\#2}_{1^+}\dagger^{\alpha\beta}$	$\frac{\sqrt{2}\, t_2}{3}$	$\frac{t_2}{3}$	$\frac{i k t_2}{3}$	0	0	0	0
$f^{\#1}_{1^+}\dagger^{\alpha\beta}$	$-\frac{1}{3} i \sqrt{2}\, k t_2$	$-\frac{1}{3} i k t_2$	$\frac{k^2 t_2}{3}$	0	0	0	0
$\omega^{\#1}_{1^-}\dagger^{\alpha}$	0	0	0	$\frac{2 t_3}{3}$	$-\frac{\sqrt{2}\, t_3}{3}$	0	$-\frac{2}{3} i k t_3$
$\omega^{\#2}_{1^-}\dagger^{\alpha}$	0	0	0	$-\frac{\sqrt{2}\, t_3}{3}$	$\frac{t_3}{3}$	0	$\frac{1}{3} i \sqrt{2}\, k t_3$
$f^{\#1}_{1^-}\dagger^{\alpha}$	0	0	0	0	0	0	0
$f^{\#2}_{1^-}\dagger^{\alpha}$	0	0	0	$\frac{2 i k t_3}{3}$	$-\frac{1}{3} i \sqrt{2}\, k t_3$	0	$\frac{2 k^2 t_3}{3}$

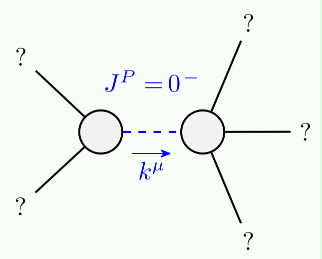
	$\omega^{\#1}_{0^+}$	$f^{\#1}_{0^+}$	$f^{\#2}_{0^+}$	$\omega^{\#1}_{0^-}$
$\omega^{\#1}_{0^+}\dagger$	t_3	$-i\sqrt{2}\, k t_3$	0	0
$f^{\#1}_{0^+}\dagger$	$i\sqrt{2}\, k t_3$	$2 k^2 t_3$	0	0
$f^{\#2}_{0^+}\dagger$	0	0	0	0
$\omega^{\#1}_{0^-}\dagger$	0	0	0	$k^2 r_2 + t_2$

	$\sigma^{\#1}_{0^+}$	$\tau^{\#1}_{0^+}$	$\tau^{\#2}_{0^+}$	$\sigma^{\#1}_{0^-}$
$\sigma^{\#1}_{0^+}\dagger$	$\frac{1}{(1+2k^2)^2 t_3}$	$-\frac{i\sqrt{2}\, k}{(1+2k^2)^2 t_3}$	0	0
$\tau^{\#1}_{0^+}\dagger$	$\frac{i\sqrt{2}\, k}{(1+2k^2)^2 t_3}$	$\frac{2 k^2}{(1+2k^2)^2 t_3}$	0	0
$\tau^{\#2}_{0^+}\dagger$	0	0	0	0
$\sigma^{\#1}_{0^-}\dagger$	0	0	0	$\frac{1}{k^2 r_2 + t_2}$

	$\sigma^{\#1}_{2^+\alpha\beta}$	$\tau^{\#1}_{2^+\alpha\beta}$	$\sigma^{\#1}_{2^-\alpha\beta\chi}$
$\sigma^{\#1}_{2^+}\dagger^{\alpha\beta}$	0	0	0
$\tau^{\#1}_{2^+}\dagger^{\alpha\beta}$	0	0	0
$\sigma^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$	0	0	0

	$\omega^{\#1}_{2^+\alpha\beta}$	$f^{\#1}_{2^+\alpha\beta}$	$\omega^{\#1}_{2^-\alpha\beta\chi}$
$\omega^{\#1}_{2^+}\dagger^{\alpha\beta}$	0	0	0
$f^{\#1}_{2^+}\dagger^{\alpha\beta}$	0	0	0
$\omega^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$	0	0	0

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_2 < 0 \,\&\&\, t_2 > 0$