

# Particle spectrograph

## Wave operator and propagator

	$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2}$	$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#1} \alpha$	$\sigma_{1+}^{\#2} \alpha$	$\tau_{1+}^{\#1} \alpha$	$\tau_{1+}^{\#2} \alpha$
$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	0	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$-\frac{i\sqrt{2} k}{t_1+k^2 t_1}$	0	0	0	0
$\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$\frac{-2k^2(2r_3+r_5)+t_1}{(1+k^2)^2 t_1^2}$	$\frac{-2ik^3(2r_3+r_5)+ik t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{i\sqrt{2} k}{t_1+k^2 t_1}$	$\frac{i(2k^3(2r_3+r_5)-kt_1)}{(1+k^2)^2 t_1^2}$	$\frac{-2k^4(2r_3+r_5)+k^2 t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\sigma_{1+}^{\#1} \alpha$	0	0	0	$\frac{1}{k^2(-r_1+2r_3+r_5)}$	$\frac{1}{\sqrt{2}(k^2+2k^4)(r_1-2r_3-r_5)}$	0	$\frac{i}{k(1+2k^2)(r_1-2r_3-r_5)}$
$\sigma_{1+}^{\#2} \alpha$	0	0	0	$\frac{1}{\sqrt{2}(k^2+2k^4)(r_1-2r_3-r_5)}$	$\frac{1}{2(k+2k^3)^2} + \frac{6k^2}{-r_1+2r_3+r_5} \frac{t_1}{t_1}$	0	$\frac{i(6k^2(r_1-2r_3-r_5)-t_1)}{\sqrt{2} k(1+2k^2)^2 (r_1-2r_3-r_5) t_1}$
$\tau_{1+}^{\#1} \alpha$	0	0	0	0	0	0	0
$\tau_{1+}^{\#2} \alpha$	0	0	0	$\frac{i}{k(1+2k^2)(-r_1+2r_3+r_5)}$	$-\frac{i(6k^2(r_1-2r_3-r_5)-t_1)}{\sqrt{2} k(1+2k^2)^2 (r_1-2r_3-r_5) t_1}$	0	$\frac{1}{-r_1+2r_3+r_5} + \frac{6k^2}{(1+2k^2)^2} \frac{t_1}{t_1}$

	$\omega_{1+}^{\#1} \alpha\beta$	$\omega_{1+}^{\#2} \alpha\beta$	$f_{1+}^{\#1} \alpha\beta$	$\omega_{1+}^{\#1} \alpha$	$\omega_{1+}^{\#2} \alpha$	$f_{1+}^{\#1} \alpha$	$f_{1+}^{\#2} \alpha$
$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$k^2(2r_3+r_5) - \frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_{1+}^{\#1} \alpha$	0	0	0	$k^2(-r_1+2r_3+r_5) + \frac{t_1}{6}$	$\frac{t_1}{3\sqrt{2}}$	0	$\frac{ikt_1}{3}$
$\omega_{1+}^{\#2} \alpha$	0	0	0	$\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	0	$\frac{1}{3} i \sqrt{2} k t_1$
$f_{1+}^{\#1} \alpha$	0	0	0	0	0	0	0
$f_{1+}^{\#2} \alpha$	0	0	0	$-\frac{1}{3} i k t_1$	$-\frac{1}{3} i \sqrt{2} k t_1$	0	$\frac{2k^2 t_1}{3}$

Quadratic (free) Lagrangian density	
$-\frac{1}{3} t_1 \omega_{\kappa\alpha}^{\alpha'} \omega_{\kappa\alpha}^{\kappa} \omega_{\kappa\alpha}^{\kappa\lambda} \omega_{\kappa\alpha}^{\lambda} \omega_{\kappa\alpha}^{\lambda} + f^{\alpha\beta} \tau_{\alpha\beta} + \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} + 2 r_1 \partial_{\lambda} \omega_{\kappa}^{\kappa\lambda} \partial_{\lambda} \omega_{\kappa}^{\alpha} \alpha -$	
$2 r_3 \partial_{\lambda} \omega_{\kappa}^{\kappa\lambda} \partial_{\lambda} \omega_{\kappa}^{\alpha} \alpha - r_5 \partial_{\lambda} \omega_{\kappa}^{\kappa\lambda} \partial_{\lambda} \omega_{\kappa}^{\alpha} \alpha - \frac{2}{3} r_1 \partial^{\beta} \omega_{\kappa}^{\alpha} \partial_{\theta} \omega_{\alpha\beta}^{\kappa} - \frac{2}{3} r_1 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\alpha\beta\theta} +$	
$\frac{2}{3} r_1 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\alpha\beta\theta} - 2 r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\theta\kappa\lambda} + 2 r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\theta\kappa\lambda} -$	
$r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\theta\kappa\lambda} + 2 r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\theta\kappa\lambda} - 2 r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\theta\kappa\lambda} +$	
$r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\theta\kappa\lambda} + 2 r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\kappa\lambda\theta} - 2 r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\kappa\lambda\theta} +$	
$2 r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\kappa\lambda\theta} - \frac{1}{2} t_1 \partial^{\alpha} f_{\theta\kappa} \partial^{\kappa} f_{\alpha}^{\theta} - \frac{1}{2} t_1 \partial^{\alpha} f_{\kappa\theta} \partial^{\kappa} f_{\alpha}^{\theta} - \frac{1}{2} t_1 \partial^{\alpha} f_{\lambda}^{\kappa} \partial^{\kappa} f_{\alpha}^{\lambda} +$	
$\frac{1}{3} t_1 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} + \frac{1}{3} t_1 \omega_{\kappa\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + \frac{2}{3} t_1 \partial^{\alpha} f_{\kappa\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} - \frac{1}{3} t_1 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} +$	
$2 t_1 \omega_{\kappa\theta} \partial^{\kappa} f_{\lambda}^{\lambda} - \frac{1}{3} t_1 \omega_{\lambda\alpha}^{\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} - \frac{1}{3} t_1 \omega_{\lambda\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + \frac{1}{2} t_1 \partial^{\alpha} f_{\lambda}^{\kappa} \partial^{\kappa} f_{\lambda\alpha} +$	
$\frac{1}{2} t_1 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} + \frac{1}{2} t_1 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\theta}^{\theta} - \frac{1}{3} t_1 \partial^{\alpha} f_{\lambda}^{\alpha} \partial^{\kappa} f_{\lambda\kappa} + \frac{2}{3} r_1 \partial_{\kappa} \omega^{\alpha\beta\theta} \partial^{\kappa} \omega_{\alpha\beta\theta} -$	
$\frac{2}{3} r_1 \partial_{\kappa} \omega^{\theta\alpha\beta} \partial^{\kappa} \omega_{\alpha\beta\theta} + \frac{2}{3} r_1 \partial^{\beta} \omega_{\lambda}^{\alpha\lambda} \partial_{\lambda} \omega_{\alpha\beta}^{\lambda} + \frac{4}{3} r_1 \partial^{\beta} \omega_{\lambda}^{\lambda\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\lambda} -$	
$4 r_3 \partial^{\beta} \omega_{\lambda}^{\lambda\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\lambda} + 2 r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} - 2 r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} +$	
$r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} - 2 r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} + 2 r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} - r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa}$	

	$\sigma_{2+}^{\#1} \alpha\beta$	$\tau_{2+}^{\#1} \alpha\beta$	$\sigma_{2+}^{\#1} \alpha\beta\chi$
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2 t_1}$	$-\frac{2i\sqrt{2} k}{(1+2k^2)^2 t_1}$	0
$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2i\sqrt{2} k}{(1+2k^2)^2 t_1}$	$\frac{4k^2}{(1+2k^2)^2 t_1}$	0
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{2k^2 r_1 + t_1}$

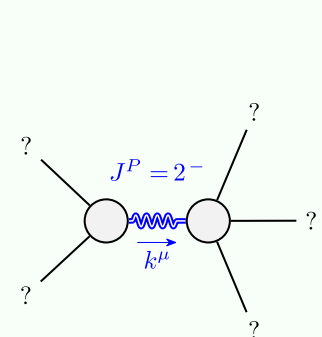
Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} == 0$	1
$\tau_{1+}^{\#2\alpha} + 2ik \sigma_{1+}^{\#2\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + ik \sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik \sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	16

	$\omega_{2+}^{\#1} \alpha\beta\chi$	$f_{2+}^{\#1} \alpha\beta$	$\omega_{2+}^{\#1} \alpha\beta\chi$
$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	0	0	$\frac{t_1}{2}$
$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$-\frac{ikt_1}{\sqrt{2}}$	$k^2 t_1$	0
$\omega_{2+}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$k^2 r_1 + \frac{t_1}{2}$

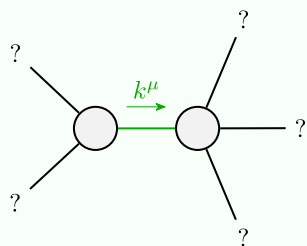
	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0+}^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	$\frac{1}{6k^2(-r_1+r_3)}$	0	0	0
$\tau_{0+}^{\#1} \dagger$	0	0	0	0
$\tau_{0+}^{\#2} \dagger$	0	0	0	0
$\sigma_{0+}^{\#1} \dagger$	0	0	0	$-\frac{1}{t_1}$

	$\omega_0^{\#1}$	$f_0^{\#1}$	$f_0^{\#2}$	$\omega_0^{\#1}$
$\omega_0^{\#1} \dagger$	0	0	0	$-t_1$
$f_0^{\#1} \dagger$	0	0	0	0
$f_0^{\#2} \dagger$	0	0	0	0
$\omega_0^{\#1} \dagger$	$6k^2(-r_1+r_3)$	0	0	0

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd



Quadratic pole	
Pole residue:	$\frac{1}{(r_1-2r_3-r_5)t_1^2} > 0$
Polarisations:	2

## Unitarity conditions

$r_1 < 0 \&\& r_5 < r_1 - 2r_3 \&\& t_1 > 0$