Particle spectrograph

Wave operator and propagator

Quadratic (free) action

$$\begin{aligned}S_F == \iiint\Big(&\tfrac{1}{6}\,(-6t_1\,\omega_{\iota}{}^{\alpha\iota}\,\omega_{\kappa\alpha}{}^{\kappa} - 2\,(t_1-2t_2)\,\omega_{\iota}{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}' + 2t_1\,\omega_{\kappa\lambda}{}'\,\omega^{\kappa\lambda}{}_{\iota} + 2t_2\,\omega_{\kappa\lambda}{}'\,\omega^{\kappa\lambda}{}_{\iota} + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + 12r_1\,\partial_\iota\omega^{\kappa\lambda}{}_{\kappa}\,\partial'\omega_{\lambda}{}^{\alpha} - 4r_1\,\partial^\beta\omega^{\theta\alpha}{}_{\kappa}\,\partial_\theta\omega_{\alpha\beta}{}^{\kappa} - \\ &4r_1\,\partial_\theta\omega_{\alpha\beta}{}^{\kappa}\,\partial_\kappa\omega^{\alpha\beta\theta} + 4r_1\,\partial_\theta\omega_{\alpha\beta}{}^{\kappa}\,\partial_\kappa\omega^{\theta\lambda}{}_{\lambda} + 12r_1\,\partial_\theta\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_\kappa\omega^{\kappa\lambda\theta} - 24r_1\,\partial_\theta\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_\kappa\omega^{\kappa\lambda\theta} - \\ &12r_1\,\partial_\theta\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_\kappa\omega^{\theta\kappa\lambda} + 12r_1\,\partial_\alpha\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial_\kappa\omega^{\kappa\lambda\theta} - \\ &2t_1\,\partial^\alpha f_{\theta\kappa}\,\partial^\kappa f_{\alpha}{}^{\theta} + t_2\,\partial^\alpha f_{\theta\kappa}\,\partial^\kappa f_{\alpha}{}^{\theta} - 4t_1\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f_{\alpha}{}^{\theta} - t_2\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f_{\alpha}{}^{\theta} - \\ &2t_1\,\partial^\alpha f^{\lambda}{}_{\kappa}\,\partial^\kappa f_{\alpha\lambda} + t_2\,\partial^\alpha f^{\lambda}{}_{\kappa}\,\partial^\kappa f_{\alpha\lambda} + 6t_1\,\partial^\kappa f_{\alpha\lambda}\,\partial^\alpha f^{\lambda}{}_{\kappa} + 6t_1\,\omega_{\kappa\alpha}{}^{\alpha}\,\partial^\kappa f'_{\iota} + 6t_1\,\omega_{\kappa\lambda}{}^{\lambda}\,\partial^\kappa f'_{\iota} + \\ &12t_1\,\partial^\alpha f_{\kappa\alpha}\,\partial^\kappa f'_{\iota} - 6t_1\,\partial_\kappa f^{\lambda}{}_{\lambda}\,\partial^\kappa f'_{\iota} + 2t_1\,\omega_{\iota\theta\kappa}\,\partial^\kappa f'^{\iota\theta} + 2t_2\,\omega_{\iota\theta\kappa}\,\partial^\kappa f'^{\iota\theta} + \\ &8t_1\,\omega_{\iota\kappa\theta}\,\partial^\kappa f'^{\iota\theta} - 4t_2\,\omega_{\iota\kappa\theta}\,\partial^\kappa f'^{\iota\theta} - 2t_2\,\omega_{\theta\iota\kappa}\,\partial^\kappa f'^{\iota\theta} + \\ &4t_1\,\omega_{\theta\kappa\iota}\,\partial^\kappa f'^{\iota\theta} + 4t_2\,\omega_{\theta\kappa\iota}\,\partial^\kappa f'^{\iota\theta} - 6t_1\,\omega_{\iota\alpha}{}^{\alpha}\,\partial^\kappa f'_{\kappa} - 6t_1\,\omega_{\iota\lambda}{}^{\lambda}\,\partial^\kappa f'_{\kappa} + \\ &2t_1\,\partial^\alpha f^{\lambda}{}_{\kappa}\,\partial^\kappa f_{\lambda\alpha} - t_2\,\partial^\alpha f^{\lambda}{}_{\kappa}\,\partial^\kappa f_{\lambda\alpha} + 2t_1\,\partial_\kappa f^{\lambda}{}_{\theta}\,\partial^\kappa f_{\lambda}{}^{\theta} - 2t_2\,\partial_\kappa f^{\lambda}{}_{\theta}\,\partial^\kappa f_{\lambda}{}^{\theta} + \\ &4t_1\,\partial_\kappa f^{\lambda}{}_{\theta}\,\partial^\kappa f_{\lambda}{}^{\theta} + t_2\,\partial_\kappa f^{\lambda}{}_{\theta}\,\partial^\kappa f_{\lambda}{}^{\theta} - 6t_1\,\partial^\alpha f^{\lambda}{}_{\alpha}\,\partial^\kappa f_{\lambda\kappa} + 4r_1\,\partial_\kappa\omega^{\alpha\beta\theta}\,\partial^\kappa\omega_{\alpha\beta\theta} - \\ &4r_1\,\partial_\kappa\omega^{\theta\alpha\beta}\,\partial^\kappa\omega_{\alpha\beta\theta} + 4r_1\,\partial^\beta\omega_{\iota}{}^{\alpha\lambda}\,\partial_\lambda\omega_{\alpha\beta}{}' - 16r_1\,\partial^\beta\omega_{\iota}{}^{\lambda\alpha}\,\partial_\lambda\omega_{\alpha\beta}{}' - \\ &12r_1\,\partial_\alpha\omega_{\lambda}{}^{\alpha}\,\partial^\lambda\omega^{\theta\kappa}{}_{\kappa} + 12r_1\,\partial_\theta\omega_{\lambda}{}^{\alpha}\,\partial^\lambda\omega^{\theta\kappa}{}_{\kappa}\Big)[t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt\end{aligned}$$

	$\tau_{1^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{1^+}^{\#1}{}_{\alpha}$	$\sigma_{1^+}^{\#2}{}_{\alpha}$	$\tau_{1^-}^{\#1}{}_{\alpha}$	$\tau_{1^-}^{\#2}{}_{\alpha}$
$\sigma_{1^+}^{\#1\,\dagger}{}^{\alpha\beta}$	$\frac{i\sqrt{2}\,k\,(t_1-2t_2)}{3\,(1+k^2)\,t_1\,t_2}$	0	0	0	0
$\sigma_{1^+}^{\#2\,\dagger}{}^{\alpha\beta}$	$\frac{i\,k\,(t_1+4t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$	0	0	0	0
$\tau_{1^+}^{\#1\,\dagger}{}^{\alpha\beta}$	$\frac{k^2\,(t_1+4t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$	0	0	0	0
$\sigma_{1^-}^{\#1\,\dagger}{}^{\alpha}$	0	0	$\frac{\sqrt{2}}{t_1+2k^2\,t_1}$	0	$\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$
$\sigma_{1^-}^{\#2\,\dagger}{}^{\alpha}$	0	$\frac{\sqrt{2}}{t_1+2k^2\,t_1}$	$\frac{2\,k^2\,r_1+t_1}{(t_1+2\,k^2\,t_1)^2}$	0	$\frac{i\,\sqrt{2}\,k\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$
$\tau_{1^-}^{\#1\,\dagger}{}^{\alpha}$	0	0	0	0	0
$\tau_{1^-}^{\#2\,\dagger}{}^{\alpha}$	0	$-\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$	$-\frac{i\,\sqrt{2}\,k\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$	0	$\frac{2\,k^2\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$

	$f_{1^+}^{\#1}{}_{\alpha\beta}$	$\omega_{1^-}^{\#1}{}_{\alpha}$	$\omega_{1^-}^{\#2}{}_{\alpha}$	$f_{1^-}^{\#1}{}_{\alpha}$	$f_{1^-}^{\#2}{}_{\alpha}$
$\omega_{1^+}^{\#1\,\dagger}{}^{\alpha\beta}$	$-\frac{i\,k\,(t_1-2t_2)}{3\,\sqrt{2}}$	0	0	0	0
$\omega_{1^+}^{\#2\,\dagger}{}^{\alpha\beta}$	$\frac{1}{3}\,i\,k\,(t_1+t_2)$	0	0	0	0
$f_{1^+}^{\#1\,\dagger}{}^{\alpha\beta}$	$\frac{1}{3}\,k^2\,(t_1+t_2)$	0	0	0	0
$\omega_{1^-}^{\#1\,\dagger}{}^{\alpha}$	0	$-k^2\,r_1-\frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$i\,k\,t_1$
$\omega_{1^-}^{\#2\,\dagger}{}^{\alpha}$	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
$f_{1^-}^{\#1\,\dagger}{}^{\alpha}$	0	0	0	0	0
$f_{1^-}^{\#2\,\dagger}{}^{\alpha}$	0	$-i\,k\,t_1$	0	0	0

	$\omega_{1^+}^{\#1}{}_{\alpha\beta}$	$\omega_{1^+}^{\#2}{}_{\alpha\beta}$
$\omega_{1^+}^{\#1\,\dagger}{}^{\alpha\beta}$	$\frac{1}{6}\,(t_1+4\,t_2)$	$-\frac{t_1-2t_2}{3\,\sqrt{2}}$
$\omega_{1^+}^{\#2\,\dagger}{}^{\alpha\beta}$	$-\frac{t_1-2t_2}{3\,\sqrt{2}}$	$\frac{t_1+t_2}{3}$
$f_{1^+}^{\#1\,\dagger}{}^{\alpha\beta}$	$\frac{i\,k\,(t_1-2t_2)}{3\,\sqrt{2}}$	$-\frac{1}{3}\,i\,k\,(t_1+t_2)$
$\omega_{1^-}^{\#1\,\dagger}{}^{\alpha}$	0	0
$\omega_{1^-}^{\#2\,\dagger}{}^{\alpha}$	0	0
$f_{1^-}^{\#1\,\dagger}{}^{\alpha}$	0	0
$f_{1^-}^{\#2\,\dagger}{}^{\alpha}$	0	0

	$\sigma_{2^+}^{\#1}{}_{\alpha\beta}$	$\tau_{2^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\sigma_{2^+}^{\#1\,\dagger}{}^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2\,t_1}$	$-\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	0
$\tau_{2^+}^{\#1\,\dagger}{}^{\alpha\beta}$	$\frac{2\,i\,\sqrt{2}\,k}{(1+2k^2)^2\,t_1}$	$\frac{4\,k^2}{(1+2\,k^2)^2\,t_1}$	0
$\sigma_{2^-}^{\#1\,\dagger}{}^{\alpha\beta\chi}$	0	0	$\frac{2}{2\,k^2\,r_1+t_1}$

	$\omega_{2^+}^{\#1}{}_{\alpha\beta}$	$f_{2^+}^{\#1}{}_{\alpha\beta}$	$\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\omega_{2^+}^{\#1\,\dagger}{}^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{i\,k\,t_1}{\sqrt{2}}$	0
$f_{2^+}^{\#1\,\dagger}{}^{\alpha\beta}$	$\frac{i\,k\,t_1}{\sqrt{2}}$	$k^2\,t_1$	0
$\omega_{2^-}^{\#1\,\dagger}{}^{\alpha\beta\chi}$	0	0	$k^2\,r_1+\frac{t_1}{2}$

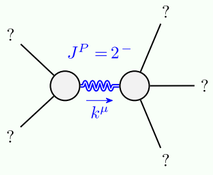
	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\omega_{0^-}^{\#1}$
$\omega_{0^+}^{\#1\,\dagger}$	$-t_1$	$i\,\sqrt{2}\,k\,t_1$	0	0
$f_{0^+}^{\#1\,\dagger}$	$-i\,\sqrt{2}\,k\,t_1$	$-2\,k^2\,t_1$	0	0
$f_{0^+}^{\#2\,\dagger}$	0	0	0	0
$\omega_{0^-}^{\#1\,\dagger}$	0	0	0	t_2

	$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^-}^{\#1}$
$\sigma_{0^+}^{\#1\,\dagger}$	$-\frac{1}{(1+2k^2)^2\,t_1}$	$\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	0	0
$\tau_{0^+}^{\#1\,\dagger}$	$\frac{i\,\sqrt{2}\,k}{(1+2k^2)^2\,t_1}$	$-\frac{2\,k^2}{(1+2\,k^2)^2\,t_1}$	0	0
$\tau_{0^+}^{\#2\,\dagger}$	0	0	0	0
$\sigma_{0^-}^{\#1\,\dagger}$	0	0	0	$\frac{1}{t_2}$

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau_{0^+}^{\#2} == 0$	1
$\tau_{0^+}^{\#1} - 2\,i\,k\,\sigma_{0^+}^{\#1} == 0$	1
$\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	3
$\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$	3
$\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$	5
Total constraints:	16

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2\,r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0$ && $t_1 > 0$