In[ ]:=
```
Get@FileNameJoin@{NotebookDirectory[], "Calibration.m"};
```

```
------------------------------------------------------------

Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}

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Connecting to external linux executable...

Connection established.

------------------------------------------------------------

Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}

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------------------------------------------------------------

Package xAct`xPlain` version 1.0.0-developer, {2023, 6, 10}

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------------------------------------------------------------

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  type Disclaimer[]. This is free software, and you are welcome to redistribute
  it under certain conditions. See the General Public License for details.

------------------------------------------------------------
```

PSALTer Calibration

Key observation: During the calibration run, we need to write some commentary, which will appear in this green text, or as numbered equations/expressions with a green background. The output of the PSALTer package (specifically the function called ParticleSpectrum) is not in green, thus wherever we are using PSALTer the output should be quite distinctive.

The first step is to load the PSALTer package.

```
------------------------------------------------------------

Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}

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------------------------------------------------------------

Package xAct`xPert`  version 1.0.6, {2018, 2, 28}

CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia
  and Guillermo A. Mena Marugan, under the General Public License.

** Variable $CovDFormat changed from Prefix to Postfix

** Option AllowUpperDerivatives of ContractMetric changed from False to True

** Option MetricOn of MakeRule changed from None to All

** Option ContractMetrics of MakeRule changed from False to True

------------------------------------------------------------

Package xAct`Invar`  version 2.0.5, {2013, 7, 1}

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  D. Yllanes and R. Portugal, under the General Public License.

** DefConstantSymbol: Defining constant symbol sigma.
```

```
** DefConstantSymbol: Defining constant symbol dim.

** Option CurvatureRelations of DefCovD changed from True to False

** Variable $CommuteCovDsOnScalars changed from True to False

------------------------------------------------------------

Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}
```



```
------------------------------------------------------------

Package xAct`xTras`  version 1.4.2, {2014, 10, 30}
```



```
** Variable $CovDFormat changed from Postfix to Prefix

** Option CurvatureRelations of DefCovD changed from False to True

------------------------------------------------------------

Package xAct`PSALTer` version 1.0.0-developer, {2023, 6, 17}
```



```
------------------------------------------------------------
```



```
------------------------------------------------------------
```

Great, so PSALTer is now loaded and we can start to do some science.

Scalar field theory

Key observation: We will test the ScalarTheory module.

Massless scalar (shift-symmetric field)

Let's begin by looking at a massless scalar field theory.

$$\underset{1}{\alpha.}\, \partial_a\varphi\, \partial^a\varphi \tag{1}$$

Now we shove the Lagrangian into PSALTer.

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \underset{1}{\alpha.}\, k^2 \end{pmatrix}$$

Gauge constraints on source currents:

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{\underset{1}{\alpha.}\, k^2} \end{pmatrix}$$

Square masses:

{{}, {}}

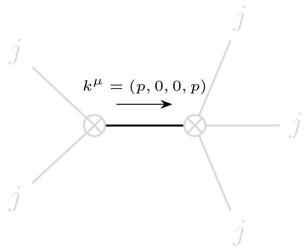
Massive pole residues:

{{}, {}}

Massless eigenvalues:

$\left\{\frac{1}{\alpha_{1}}\right\}$

Overall particle spectrum:



Massless particle

Pole residue:	$\frac{s}{\alpha^{P}_{1}} > 0$
Polarisations:	1

Overall unitarity conditions:

$\alpha_{1} > 0$

The result is much as you would expect. There is one massless polarisation, supported by a no-ghost condition which bounds the kinetic part of the Hamiltonian from below.

Massive scalar (Higgs field, pions)

Now for the massive case.

$$-\alpha_{2}\,\varphi^{2} + \alpha_{1}\,\partial_{a}\varphi\,\partial^{a}\varphi \qquad (2)$$

We apply PSALTer again.

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$\left(-\alpha_{2} + \alpha_{1}\,k^{2}\right)$

Gauge constraints on source currents:

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$\left(\frac{1}{-\alpha_{2} + \alpha_{1}\,k^{2}}\right)$

Square masses:

$\left\{\left\{\frac{\alpha_{2}}{\alpha_{1}}\right\}, \{\}\right\}$

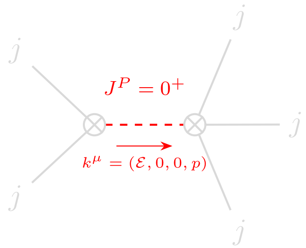
Massive pole residues:

$$\left\{\left\{\frac{1}{\underset{1}{\alpha.}}\right\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$\frac{1}{\underset{1}{\alpha.}} > 0$
Square mass:	$\frac{\underset{2}{\alpha.}}{\underset{1}{\alpha.}} > 0$
Spin:	0
Parity:	Even

Overall unitarity conditions:

$\underset{1}{\alpha.} > 0 \;\&\&\; \underset{2}{\alpha.} > 0$

We find that the massless eigenvalue has disappeared, but the propagator develops a massive pole whose no-ghost condition is equivalent. There is an additional no-tachyon condition on the Klein-Gordon mass.

Vector field theory

Key observation: We will test the VectorTheory module.

Maxwell field (quantum electrodynamics)

The first pure 1-form theory we might think to try is due to Maxwell. We know from kindergarten that if we contract the square of the Maxwell tensor, we get a viable kinetic term which propagates the two massless photon polarisations. Let's try this out.

$$\alpha. \left(\partial_a \mathcal{B}_b - \partial_b \mathcal{B}_a\right)\left(\partial^a \mathcal{B}^b - \partial^b \mathcal{B}^a\right) \tag{3}$$

We need to expand the brackets before passing to PSALTer.

$$-2\,\alpha.\,\partial_a \mathcal{B}_b\,\partial^b \mathcal{B}^a + 2\,\alpha.\,\partial_b \mathcal{B}_a\,\partial^b \mathcal{B}^a \tag{4}$$

Now we shove the Lagrangian into PSALTer.

The (possibly singular) *a*-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

(0)

Matrix for spin-1 sector:

$\left(2\,\underset{1}{\alpha\cdot}\,k^2\right)$

Gauge constraints on source currents:

${}^{0^+}_{\cdot}\mathcal{J} == 0$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

(0)

Matrix for spin-1 sector:

$\left(\frac{1}{2\,\underset{1}{\alpha\cdot}\,k^2}\right)$

Square masses:

{{}, {}, {}, {}}

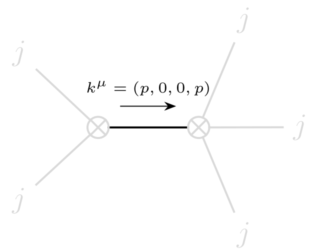
Massive pole residues:

{{}, {}, {}, {}}

Massless eigenvalues:

$\left\{-\frac{1}{2\,\underset{1}{\alpha\cdot}},\ -\frac{1}{2\,\underset{1}{\alpha\cdot}}\right\}$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{1}{\underset{1}{\alpha\cdot}} > 0$
Polarisations:	2

Overall unitarity conditions:

$\underset{1}{\alpha\cdot} < 0$

The output above makes sense. There are no mass terms in our Lagrangian, and hence no massive poles in the propagator. Instead, there are two massless eigenvalues which suggest that the vector part of the theory propagates two massless polarisations. The no-ghost condition of this massless vector simply demands that our kinetic coupling be negative: this is why in school we are told to put a -1/4 factor in front of the QED Lagrangian. What about the gauge constraints on the source currents? There is only one such constraint, which tells us that the positive-parity scalar part of the QED current (think the chiral current, or some such four-vector source) must vanish. Reverse-engineering this condition from momentum to position space, we see that the four-divergence of the source must vanish. Of course it must: this is just charge conservation. The conservation law is intimately connected to the gauge symmetries of the theory, according to Noether: these symmetries are manifest as singularities (zeroes) in the matrix form of the Lagrangian operator, though there are no spin-parity degeneracies in the 1-

form and so all these matrices are just single elements.

Proca field (electroweak bosons)

Having investigated the massless theory, we keep the same kinetic setup but just add a mass term. This is of course the Proca theory, which finds a place higher up in the standard model.

$$\alpha_{3}\, \mathcal{B}_a\, \mathcal{B}^a + \alpha_{1} \left(\partial_a \mathcal{B}_b - \partial_b \mathcal{B}_a\right)\left(\partial^a \mathcal{B}^b - \partial^b \mathcal{B}^a\right) \quad (5)$$

Again we just need to expand those brackets before passing to PSALTer.

$$\alpha_{3}\, \mathcal{B}_a\, \mathcal{B}^a - 2\,\alpha_{1}\, \partial_a \mathcal{B}_b\, \partial^b \mathcal{B}^a + 2\,\alpha_{1}\, \partial_b \mathcal{B}_a\, \partial^b \mathcal{B}^a \quad (6)$$

Now we shove the Lagrangian into PSALTer.

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \alpha_{3} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \alpha_{3} + 2\,\alpha_{1}\, k^2 \end{pmatrix}$$

Gauge constraints on source currents:

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{\alpha_{3}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{\alpha_{3} + 2\,\alpha_{1}\, k^2} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \left\{-\frac{\alpha_{3}}{2\,\alpha_{1}}\right\}\right\}$$

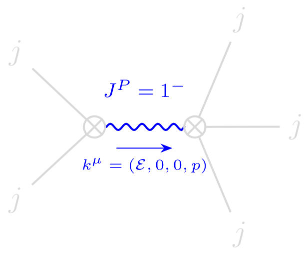
Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \left\{-\frac{1}{2\,\alpha_{1}}\right\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{2\alpha_{\cdot 1}} > 0$
Square mass:	$-\frac{\alpha_{\cdot 3}}{2\alpha_{\cdot 1}} > 0$
Spin:	1
Parity:	Odd

Overall unitarity conditions:

$\alpha_{\cdot 1} < 0 \,\&\&\, \alpha_{\cdot 3} > 0$

Once again, the result makes sense. If you write out the Proca equation of motion and take the divergence, you see that the presence of the mass term restricts the 1-form to be divergence-free, which is another way of saying that the helicity-0 mode vanishes on shell. This is not a gauge condition (evidenced by the fact that the Lagrangian operator matrices are non-singular), but it does mean that in common with Maxwell's theory, we are stuck with the parity-odd vector mode. What is this mode doing? The theory is now massive, and so there is a massive pole in the propagator. There are now two unitarity conditions: the original no-ghost condition of QED and a new no-tachyon condition which protects the Proca mass from becoming imaginary.

Sickly quantum electrodynamics

Now let's try something a bit more ambitious. What if we didn't have the QED Lagrangian as inspiration, but we wanted to construct a general (and not necessarily gauge-invariant) 1-form theory? In the first instance, we'll take the case without any masses. Up to surface terms, there are two kinetic terms we could try which are consistent with the basic requirement of Lorentz invariance.

$$\alpha_{\cdot}\, \partial_a \mathcal{B}_b\, \partial^a \mathcal{B}^b + \alpha_{\cdot}\, \partial_a \mathcal{B}^a\, \partial_b \mathcal{B}^b \qquad (7)$$

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$\left(\left(\alpha_{\cdot 1} + \alpha_{\cdot 2}\right) k^2 \right)$

Matrix for spin-1 sector:

$\left(\alpha_{\cdot 1} k^2 \right)$

Gauge constraints on source currents:

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$\left(\frac{1}{\left(\alpha_{\cdot 1} + \alpha_{\cdot 2}\right) k^2} \right)$

Matrix for spin-1 sector:

$$\left(\frac{}{\alpha_{1} k^2} \right)$$

Square masses:

{{}, {}, {}, {}}

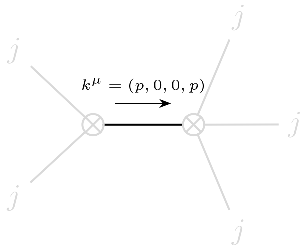
Massive pole residues:

{{}, {}, {}, {}}

Massless eigenvalues:

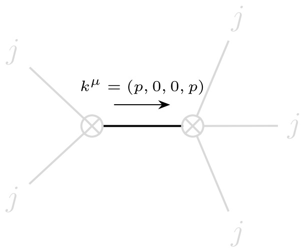
$$\left\{ \frac{-2\,\alpha_{\cdot} - \alpha_{\cdot}}{2\,\alpha_{\cdot}\left(\alpha_{\cdot} + \alpha_{\cdot}\right)}, -\frac{1}{\alpha_{\cdot}}, -\frac{1}{\alpha_{\cdot}}, \frac{2\,\alpha_{\cdot} + \alpha_{\cdot}}{2\,\alpha_{\cdot}\left(\alpha_{\cdot} + \alpha_{\cdot}\right)} \right\}$$

Overall particle spectrum:



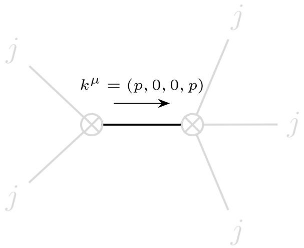
Massless particle

Pole residue:	$-\frac{S}{\alpha P_1} - \frac{S}{\alpha P M_1\, \alpha P_2} > 0$
Polarisations:	1



Massless particle

Pole residue:	$-\frac{S}{\alpha P_1} > 0$
Polarisations:	2



Massless particle

Pole residue:	$\frac{S}{\alpha P_1} + \frac{S}{\alpha P M_1\, \alpha P_2} > 0$
Polarisations:	1

Overall unitarity conditions:

False

Notice the suspicious appearance of two extra massless eigenvalues, alongside the familiar photon polarisations. These carry different signs, and thus cannot be positive-definite: the theory is immutably sick, and the no-ghost condition is simply `False'. What has happened here is a result of the Ostrogradsky theorem. Our kinetic structure has destroyed the gauge-invariance of the theory, and so the helicity-0 part of the field (the divergence of some scalar superpotential) has begun to move. Because the helicity-0 part contains an implicit divergence, that part of the theory now contains four implicit derivatives, and is a sickly higher-derivative model. The Ostrogradsky theorem says that derivative decoupling will bifurcate the helicity-0 mode into two modes, one of which is always a ghost. How to get rid of the ghost? We clearly can't do it at the level of the eigenvalues, so we look a few lines above to the Lagrangian matrix structure. The Scalar sector can be killed off entirely, spawning a singular one-element matrix and thus a new gauge symmetry, only by imposing the QED condition. This is of course just what we expect to find.

Sickly Proca field

For completeness, it behoves us to look at the general massive case.

$$\alpha_{\cdot}\, \mathcal{B}_{a}\, \mathcal{B}^{a} + \alpha_{\cdot}\, \partial_{a}\mathcal{B}_{b}\, \partial^{a}\mathcal{B}^{b} + \alpha_{\cdot}\, \partial_{a}\mathcal{B}^{a}\, \partial_{b}\mathcal{B}^{b} \tag{8}$$

We enter this into PSALTer.

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$\left(\alpha_{3} + \left(\alpha_{1} + \alpha_{2}\right) k^2\right)$

Matrix for spin-1 sector:

$\left(\alpha_{3} + \alpha_{1} k^2\right)$

Gauge constraints on source currents:

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$\left(\frac{1}{\alpha_{3} + \left(\alpha_{1} + \alpha_{2}\right) k^2}\right)$

Matrix for spin-1 sector:

$\left(\frac{}{\alpha_{3} + \alpha_{1} k^2}\right)$

Square masses:

$\left\{\left\{-\frac{\alpha_{\cdot}}{\alpha_{\cdot} + \alpha_{\cdot}}\right\}, \{\}, \{\}, \left\{-\frac{\alpha_{\cdot}}{\alpha_{\cdot}}\right\}\right\}$

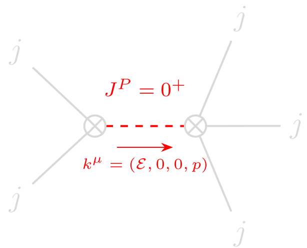
Massive pole residues:

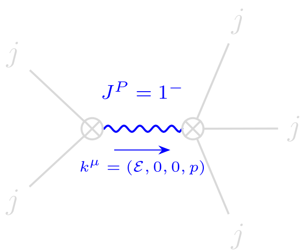
$\left\{\left\{\frac{1}{\alpha_{\cdot} + \alpha_{\cdot}}\right\}, \{\}, \{\}, \left\{-\frac{1}{\alpha_{\cdot}}\right\}\right\}$

Massless eigenvalues:

{}

Overall particle spectrum:





Massive particle

Pole residue:	$\frac{S}{\alpha P_{1} \mathrm{PM}\, \alpha P_{2}} > 0$
Square mass:	$-\frac{\alpha P_{3}}{\alpha P_{1} \mathrm{PM}\, \alpha P_{2}} > 0$
Spin:	0
Parity:	Even

Massive particle

Pole residue:	$-\frac{S}{\alpha P_{1}} > 0$
Square mass:	$-\frac{\alpha P_{3}}{\alpha P_{1}} > 0$
Spin:	1
Parity:	Odd

Overall unitarity conditions:

False

Once again, the theory is sick in the helicity-0 sector. In case the massive parity-odd vector is unitary, then the helicity-0 mode must either be a ghost or a tachyon.

Tensor field theory

Key observation: We will test the TensorTheory module.

Fierz-Pauli (linear gravity)

The natural theory to check will be the Fierz-Pauli theory.

$$\alpha_{\cdot}\left(-\partial^{a}h_{ab}\,\partial^{b}h^{c}{}_{c}+\frac{1}{2}\,\partial_{b}h^{a}{}_{a}\,\partial^{b}h^{c}{}_{c}-\frac{1}{2}\,\partial_{c}h^{ab}\,\partial^{c}h_{ab}+\partial_{b}h^{ab}\,\partial^{c}h_{ac}\right) \quad (9)$$

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 \\ 0 & \alpha_{\cdot 1}\, k^2 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{\alpha_{\cdot 1}\, k^2}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\cdot}\mathcal{T}^{\perp} == 0$$

$${}^{1^-}_{\cdot}\mathcal{T}^{\perp a} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{\alpha_{\cdot 1}\, k^2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{\alpha_{\cdot 1}\, k^2} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

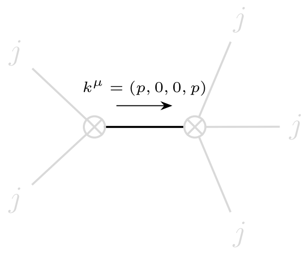
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{4\,p^2}{\alpha_{\cdot 1}},\ -\frac{2\,p^2}{\alpha_{\cdot 1}}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{p^2}{\alpha_{1}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(p < 0 \;\&\&\; \alpha_{1} < 0\right) \,||\, \left(p > 0 \;\&\&\; \alpha_{1} < 0\right)$$

The Fierz-Pauli theory thus propagates two massless polarisations, and the no-ghost condition is consistent with a positive Einstein or Newton-Cavendish constant, or a positive square Planck mass. The diffeomorphism invariance of the theory is manifest as a gauge symmetry, whose constraints on the source currents are commensurate with the conservation of the matter stress-energy tensor.

Massive gravity

We now include the unique mass term which corresponds to massive gravity, i.e. `Fierz-Pauli tuning'.

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & -\sqrt{3}\,\alpha_{2} \\ -\sqrt{3}\,\alpha_{2} & -2\,\alpha_{2} + \alpha_{1}\,k^2 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \alpha_{2} \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \alpha_{2} - \frac{\alpha_{1}\,k^2}{2} \end{pmatrix}$$

Gauge constraints on source currents:

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{2\,\alpha_{2} - \alpha_{1}\,k^2}{3\,\alpha_{2}^2} & -\frac{1}{\sqrt{3}\,\alpha_{2}} \\ -\frac{1}{\sqrt{3}\,\alpha_{2}} & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{\alpha_{2}} \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{1}{\alpha_{2} - \frac{\alpha_{1}\,k^2}{2}} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \left\{\frac{2\,\alpha._{2}}{\alpha._{1}}\right\}, \{\}\right\}$$

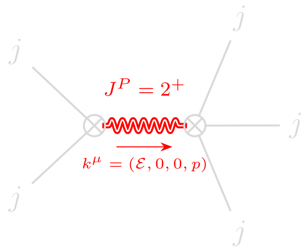
Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \left\{-\frac{2}{\alpha._{1}}\right\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:



Massive particle	
Pole residue:	$-\frac{2}{\alpha._{1}} > 0$
Square mass:	$\frac{2\,\alpha._{2}}{\alpha._{1}} > 0$
Spin:	2
Parity:	Even

Overall unitarity conditions:

$\alpha._{1} < 0$ && $\alpha._{2} < 0$

There is no massless sector. The propagator develops a massive pole in the positive-parity tensor sector. The no-ghost condition is as before, but now a no-tachyon condition protects the graviton mess.

Sick Fierz-Pauli (first variation)

Returning to the case without any mass terms, we should check that deviations to the Fierz-Pauli action are unacceptable. Let's vary the fourth term to some degree.

$$\alpha.\left(-\partial^{a}h_{ab}\,\partial^{b}h^{c}{}_{c} + \frac{1}{2}\,\partial_{b}h^{a}{}_{a}\,\partial^{b}h^{c}{}_{c} - \frac{1}{2}\,\partial_{c}h^{ab}\,\partial^{c}h_{ab}\right) + \alpha.\,\partial_{b}h^{ab}\,\partial^{c}h_{ac} \tag{10}$$

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} (-\alpha. + \alpha.)\,k & 0 \\ 0 & \alpha.\,k \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -(-\alpha. + \alpha.)\,k \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{\alpha_1 k^2}{} \end{pmatrix}$$

Gauge constraints on source currents:

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{}{\left(-\alpha_1+\alpha_2\right) k^2} & 0 \\ 0 & \frac{}{\alpha_1 k^2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\frac{}{\left(\alpha_1-\alpha_2\right) k^2} \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{\alpha_{1} k^2} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

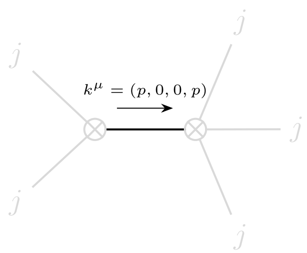
Massive pole residues:

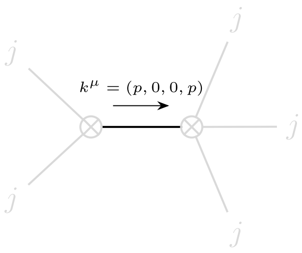
{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{4 p^2}{\alpha_1}, -\frac{2 p^2}{\alpha_1}, -\frac{2\left(2\alpha_1-\alpha_2\right)p^2}{\alpha_1\left(\alpha_1-\alpha_2\right)}, -\frac{2\left(2\alpha_1-\alpha_2\right)p^2}{\alpha_1\left(\alpha_1-\alpha_2\right)}, \frac{2\left(2\alpha_1-\alpha_2\right)p^2}{\alpha_1\left(\alpha_1-\alpha_2\right)}, \frac{2\left(2\alpha_1-\alpha_2\right)p^2}{\alpha_1\left(\alpha_1-\alpha_2\right)}, -\frac{\left(6\alpha_1-\alpha_2\right)p^2}{4\alpha_1\left(\alpha_1-\alpha_2\right)},\right.$$
$$\left.\frac{\left(6\alpha_1-\alpha_2\right)p^2}{2\alpha_1\left(\alpha_1-\alpha_2\right)}, \frac{\left(-2\alpha_1+\alpha_2-\sqrt{20\alpha_1^2-36\alpha_1\alpha_2+17\alpha_2^2}\right)p^2}{4\alpha_1\left(\alpha_1-\alpha_2\right)}, \frac{\left(-2\alpha_1+\alpha_2+\sqrt{20\alpha_1^2-36\alpha_1\alpha_2+17\alpha_2^2}\right)p^2}{4\alpha_1\left(\alpha_1-\alpha_2\right)}\right\}$$

Overall particle spectrum:



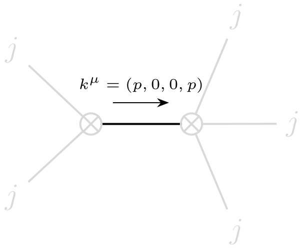


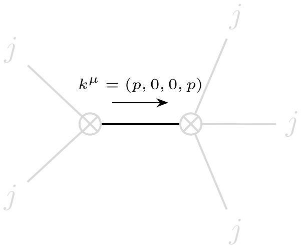
Massless particle

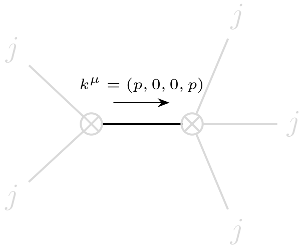
Pole residue:	$-\frac{p^2}{\alpha_1\text{P}} > 0$
Polarisations:	2

Massless particle

Pole residue:	$\frac{\text{JJT }\alpha_1\text{PM }\alpha_2\text{PK}p^2}{\alpha_1\text{PJ}\alpha_1\text{PO}\alpha_2\text{PK}} > 0$
Polarisations:	2







Massless particle

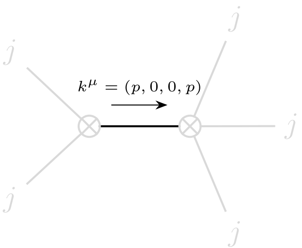
Pole residue:	$\frac{\text{JT }\alpha_1\text{PO}\alpha_2\text{PK}p^2}{\alpha_1\text{PJ}\alpha_1\text{PO}\alpha_2\text{PK}} > 0$
Polarisations:	2

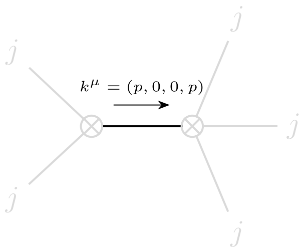
Massless particle

Pole residue:	$\frac{\text{JJX }\alpha_1\text{PM }\alpha_2\text{PK}p^2}{\alpha_1\text{PJ}\alpha_1\text{PO}\alpha_2\text{PK}} > 0$
Polarisations:	1

Massless particle

Pole residue:	$\frac{\text{JX }\alpha_1\text{PO}\alpha_2\text{PK}p^2}{\alpha_1\text{PJ}\alpha_1\text{PO}\alpha_2\text{PK}} > 0$
Polarisations:	1





Massless particle

Pole residue:	$\frac{\text{JJT }\alpha_1\text{PM }\alpha_2\text{PO}\sqrt{\text{TR }\alpha_1\text{P}^2\text{OUX }\alpha_1\text{P}\alpha_2\text{PM SY }\alpha_2\text{P}^2}\text{ K}p^2}{\alpha_1\text{PJ}\alpha_1\text{PO}\alpha_2\text{PK}} > 0$
Polarisations:	1

Massless particle

Pole residue:	$\frac{\text{JJT }\alpha_1\text{PM }\alpha_2\text{PM }\sqrt{\text{TR }\alpha_1\text{P}^2\text{OUX }\alpha_1\text{P}\alpha_2\text{PM SY }\alpha_2\text{P}^2}\text{ K}p^2}{\alpha_1\text{PJ}\alpha_1\text{PO}\alpha_2\text{PK}} > 0$
Polarisations:	1

Overall unitarity conditions:

False

So this variation has no gauge symmetries, too many propagating species and no hope of unitarity.

Sick Fierz-Pauli (second variation)

This time let's wiggle the third term.

$$-\frac{1}{2}\alpha_{2}\,\partial_{c}h^{ab}\,\partial^{c}h_{ab}+\alpha_{1}\left(-\partial^{a}h_{ab}\,\partial^{b}h^{c}{}_{c}+\frac{1}{2}\,\partial_{b}h^{a}{}_{a}\,\partial^{b}h^{c}{}_{c}+\partial_{b}h^{ab}\,\partial^{c}h_{ac}\right) \tag{11}$$

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{2}\left(\alpha_{1}-\alpha_{2}\right)k^2 & 0 \\ 0 & \frac{1}{2}\left(3\,\alpha_{1}-\alpha_{2}\right)k^2 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{2}\left(\alpha_{1}-\alpha_{2}\right)k^2 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{\alpha_{2}\,k^2}{2} \end{pmatrix}$$

Gauge constraints on source currents:

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{2}{\left(\alpha_{1}-\alpha_{2}\right)k^2} & 0 \\ 0 & \frac{2}{\left(3\,\alpha_{1}-\alpha_{2}\right)k^2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{2}{\left(\alpha_{1}-\alpha_{2}\right)k^2} \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{\alpha_{2}\,k^2} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

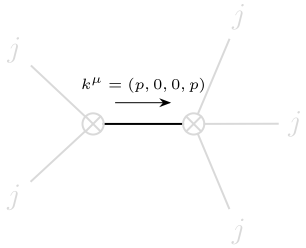
Massive pole residues:

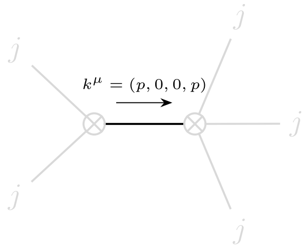
{{}, {}, {}, {}, {}, {}}

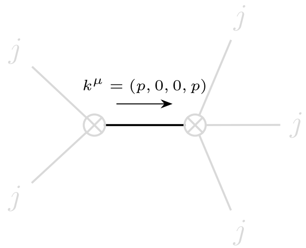
Massless eigenvalues:

$$\left\{-\frac{2\left(\alpha_{1}-2\,\alpha_{2}\right)p^2}{\left(\alpha_{1}-\alpha_{2}\right)\alpha_{2}},\ -\frac{2\left(\alpha_{1}-2\,\alpha_{2}\right)p^2}{\left(\alpha_{1}-\alpha_{2}\right)\alpha_{2}},\ \frac{2\left(\alpha_{1}-2\,\alpha_{2}\right)p^2}{\left(\alpha_{1}-\alpha_{2}\right)\alpha_{2}},\ \frac{2\left(\alpha_{1}-2\,\alpha_{2}\right)p^2}{\left(\alpha_{1}-\alpha_{2}\right)\alpha_{2}},\ -\frac{4\,p^2}{\alpha_{2}},\ -\frac{2\left(3\,\alpha_{1}^2-4\,\alpha_{1}\,\alpha_{2}+\alpha_{2}^2\right)p^2}{\left(\alpha_{1}-\alpha_{2}\right)\left(3\,\alpha_{1}-\alpha_{2}\right)\alpha_{2}},\right.$$
$$-\frac{\left(\alpha_{1}^2-6\,\alpha_{1}\,\alpha_{2}+2\,\alpha_{2}^2\right)p^2}{\left(\alpha_{1}-\alpha_{2}\right)\left(3\,\alpha_{1}-\alpha_{2}\right)\alpha_{2}},\ \frac{2\left(\alpha_{1}^2-6\,\alpha_{1}\,\alpha_{2}+2\,\alpha_{2}^2\right)p^2}{\left(\alpha_{1}-\alpha_{2}\right)\left(3\,\alpha_{1}-\alpha_{2}\right)\alpha_{2}},\ \frac{\left(-2\,\alpha_{1}^2+5\,\alpha_{1}\,\alpha_{2}-2\,\alpha_{2}^2-\sqrt{4\,\alpha_{1}^4-8\,\alpha_{1}^3\,\alpha_{2}+5\,\alpha_{1}^2\,\alpha_{2}^2}\right)p^2}{\left(\alpha_{1}-\alpha_{2}\right)\left(3\,\alpha_{1}-\alpha_{2}\right)\alpha_{2}},$$
$$\left.\frac{\left(-2\,\alpha_{1}^2+5\,\alpha_{1}\,\alpha_{2}-2\,\alpha_{2}^2+\sqrt{4\,\alpha_{1}^4-8\,\alpha_{1}^3\,\alpha_{2}+5\,\alpha_{1}^2\,\alpha_{2}^2}\right)p^2}{\left(\alpha_{1}-\alpha_{2}\right)\left(3\,\alpha_{1}-\alpha_{2}\right)\alpha_{2}}\right\}$$

Overall particle spectrum:







Massless particle

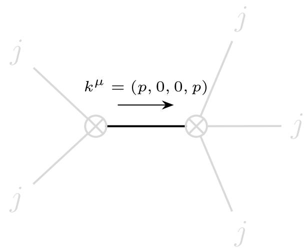
Pole residue:	$-\frac{(\alpha_1-2\alpha_2)\,p^2}{(\alpha_1-\alpha_2)\,\alpha_2} > 0$
Polarisations:	2

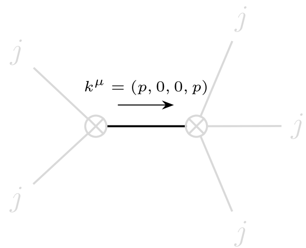
Massless particle

Pole residue:	$\frac{(\alpha_1-2\alpha_2)\,p^2}{(\alpha_1-\alpha_2)\,\alpha_2} > 0$
Polarisations:	2

Massless particle

Pole residue:	$-\frac{p^2}{\alpha_2} > 0$
Polarisations:	2



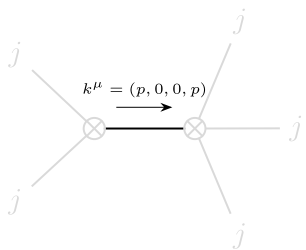


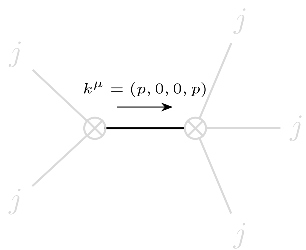
Massless particle

Pole residue:	$-\frac{(\alpha_1^2-6\,\alpha_1\,\alpha_2+2\,\alpha_2^2)\,p^2}{(\alpha_1-\alpha_2)\,(3\,\alpha_1-\alpha_2)\,\alpha_2} > 0$
Polarisations:	1

Massless particle

Pole residue:	$\frac{(\alpha_1^2-6\,\alpha_1\,\alpha_2+2\,\alpha_2^2)\,p^2}{(\alpha_1-\alpha_2)\,(3\,\alpha_1-\alpha_2)\,\alpha_2} > 0$
Polarisations:	1





Massless particle

Pole residue:	$-\frac{\left(2\,\alpha_1^2-5\,\alpha_1\,\alpha_2+2\,\alpha_2^2+\sqrt{\alpha_1^2\,(4\,\alpha_1^2-8\,\alpha_1\,\alpha_2+5\,\alpha_2^2)}\right)p^2}{(\alpha_1-\alpha_2)\,(3\,\alpha_1-\alpha_2)\,\alpha_2} > 0$
Polarisations:	1

Massless particle

Pole residue:	$\frac{\left(-2\,\alpha_1^2+5\,\alpha_1\,\alpha_2-2\,\alpha_2^2+\sqrt{\alpha_1^2\,(4\,\alpha_1^2-8\,\alpha_1\,\alpha_2+5\,\alpha_2^2)}\right)p^2}{(\alpha_1-\alpha_2)\,(3\,\alpha_1-\alpha_2)\,\alpha_2} > 0$
Polarisations:	1

Overall unitarity conditions:

False

Again this variation has no gauge symmetries, too many propagating species and no hope of unitarity.

Sick Fierz-Pauli (third variation)

This time let's wiggle the second term.

$$-\alpha_2\,\partial^a h_{ab}\,\partial^b h^c{}_c + \alpha_1\left(\frac{1}{2}\,\partial_b h^a{}_a\,\partial^b h^c{}_c - \frac{1}{2}\,\partial_c h^{ab}\,\partial^c h_{ab} + \partial_b h^{ab}\,\partial^c h_{ac}\right) \qquad (12)$$

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} (\alpha_{.} - \alpha_{.})k & -\sqrt{3}\,(\alpha_{.} - \alpha_{.})k \\ -\sqrt{3}\,(\alpha_{.} - \alpha_{.})k & \alpha_{.}\,k \end{pmatrix}$$

Matrix for spin-1 sector:

(0)

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{\alpha_{1}\,k^2}{\;} \end{pmatrix}$$

Gauge constraints on source currents:

$$.\bar{\mathcal{T}}^{\perp^{a}} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{\alpha_{1}}{(\alpha_{1}-\alpha_{2})(\alpha_{1}+\ \alpha_{2})k^2} & -\frac{\sqrt{\ }}{(\alpha_{1}+\ \alpha_{2})k^2} \\ -\frac{\sqrt{\ }}{(\alpha_{1}+\ \alpha_{2})k^2} & \frac{\;}{(\alpha_{1}+\ \alpha_{2})k^2} \end{pmatrix}$$

Matrix for spin-1 sector:

(0)

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{\alpha_{.1}\,k^2} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

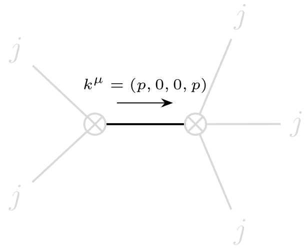
Massive pole residues:

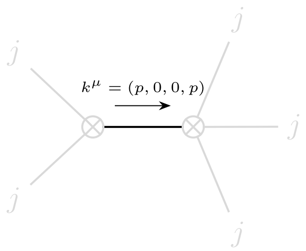
{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{4\,p^2}{\alpha_{.1}},\ \frac{2\left(-\alpha_{.1}{}^2 - 2\,\alpha_{.1}\,\alpha_{.2} + 3\,\alpha_{.2}{}^2\right)p^2}{\alpha_{.1}\left(\alpha_{.1} - \alpha_{.2}\right)\left(\alpha_{.1} + 3\,\alpha_{.2}\right)},\ \frac{2\left(\alpha_{.1}{}^2 - 2\,\alpha_{.1}\,\alpha_{.2} + 5\,\alpha_{.2}{}^2\right)p^2}{\alpha_{.1}\left(\alpha_{.1} - \alpha_{.2}\right)\left(\alpha_{.1} + 3\,\alpha_{.2}\right)}\right\}$$

Overall particle spectrum:





Massless particle

Pole residue:	$-\frac{p^2}{\alpha P_{1}} > 0$
Polarisations:	2

Massless particle

Pole residue:	$\frac{\mathrm{J}\alpha P_{1}^2 \mathrm{OT}\, \alpha P_{1} \alpha P_{2} \mathrm{M\,W}\, \alpha P_{2}^2 \mathrm{K} p^2}{\alpha P_{1} \mathrm{J} \alpha P_{1} \mathrm{O} \alpha P_{2} \mathrm{KJ} \alpha P_{1} \mathrm{M\,U}\, \alpha P_{2} \mathrm{K}} > 0$
Polarisations:	1

Overall unitarity conditions:

$$\left(p < 0 \;\&\&\; \alpha._{1} < 0 \;\&\&\; \left(\alpha._{2} < \alpha._{1} \;\|\; \alpha._{2} > -\frac{\alpha._{1}}{3}\right)\right) \;\|\; \left(p > 0 \;\&\&\; \alpha._{1} < 0 \;\&\&\; \left(\alpha._{2} < \alpha._{1} \;\|\; \alpha._{2} > -\frac{\alpha._{1}}{3}\right)\right)$$

This time we have what looks to be a viable theory with an extra massless scalar. However the diffeomorphism gauge symmetry has been lost, and the stress-energy tensor is not conserved.

Sick Fierz-Pauli (fourth variation)

This time let's wiggle the first term.

$$\frac{1}{2}\,\alpha._{2}\,\partial_b h^a{}_a\,\partial^b h^c{}_c + \alpha._{1}\left(-\partial^a h_{ab}\,\partial^b h^c{}_c - \frac{1}{2}\,\partial_c h^{ab}\,\partial^c h_{ab} + \partial_b h^{ab}\,\partial^c h_{ac}\right) \qquad (13)$$

The (possibly singular) *a*-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{2}\left(-\alpha._{1} + \alpha._{2}\right) k^2 & \frac{1}{2}\sqrt{3}\left(-\alpha._{1} + \alpha._{2}\right) k^2 \\ \frac{1}{2}\sqrt{3}\left(-\alpha._{1} + \alpha._{2}\right) k^2 & -\frac{1}{2}\left(\alpha._{1} - 3\,\alpha._{2}\right) k^2 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{\alpha._{1}\, k^2}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$$\overset{1^-}{\cdot}\mathcal{T}^{\perp a} == 0$$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally
analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{\alpha._{1} - 3\,\alpha._{2}}{\alpha._{1}\left(\alpha._{1} - \alpha._{2}\right) k^2} & -\frac{\sqrt{3}}{\alpha._{1}\, k^2} \\ -\frac{\sqrt{3}}{\alpha._{1}\, k^2} & \frac{1}{\alpha._{1}\, k^2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{\alpha_{1} k^2} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

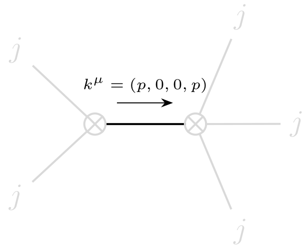
Massive pole residues:

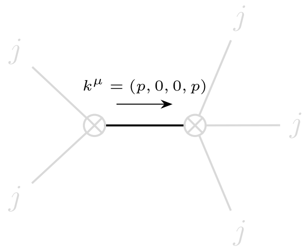
{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{4\,p^2}{\alpha_{1} - \alpha_{2}},\ -\frac{4\,p^2}{\alpha_{1}},\ -\frac{2\,p^2}{\alpha_{1}}\right\}$$

Overall particle spectrum:





Massless particle

Pole residue:	$\frac{p^2}{\text{O}\alpha\text{P}_{1}\text{M}\ \alpha\text{P}_{2}} > 0$
Polarisations:	1

Massless particle

Pole residue:	$-\frac{p^2}{\alpha\text{P}_{1}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(p < 0\ \&\&\ \alpha_{1} < 0\ \&\&\ \alpha_{2} > \alpha_{1}\right) \,||\, \left(p > 0\ \&\&\ \alpha_{1} < 0\ \&\&\ \alpha_{2} > \alpha_{1}\right)$$

Another case with a partial gauge symmetry and an extra scalar mode.

Sick massive gravity

Finally, let's break the `Fierz-Pauli tuning'.

$$\alpha_{2}\, h_{ab}\, h^{ab} - \alpha_{3}\, h^{a}{}_{a}\, h^{b}{}_{b} + \alpha_{1}\left(-\partial^{a} h_{ab}\, \partial^{b} h^{c}{}_{c} + \frac{1}{2}\, \partial_{b} h^{a}{}_{a}\, \partial^{b} h^{c}{}_{c} - \frac{1}{2}\, \partial_{c} h^{ab}\, \partial^{c} h_{ab} + \partial_{b} h^{ab}\, \partial^{c} h_{ac}\right) \quad (14)$$

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \alpha_{2} - \alpha_{3} & -\sqrt{3}\, \alpha_{3} \\ -\sqrt{3}\, \alpha_{3} & \alpha_{2} - 3\, \alpha_{3} + \alpha_{1} k^2 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \alpha_{2} \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \alpha_{2} - \frac{\alpha_{1} k^2}{2} \end{pmatrix}$$

Gauge constraints on source currents:

```
The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
   analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:
```

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{}{\alpha_2+\alpha_3\left(- \ -\frac{3\,\alpha._3}{\alpha._2-3\,\alpha._3+\alpha._1\,k^2}\right)} & \frac{\sqrt{\ }\ \alpha_3}{\alpha_2\left(\alpha_2-\ \alpha_3\right)+\alpha_1\left(\alpha_2-\alpha_3\right)k^2} \\ \frac{\sqrt{\ }\ \alpha_3}{\alpha_2\left(\alpha_2-\ \alpha_3\right)+\alpha_1\left(\alpha_2-\alpha_3\right)k^2} & \frac{}{\frac{\alpha._2\left(\alpha._2-4\,\alpha._3\right)}{\alpha._2-\alpha._3}+\alpha_1\ k^2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{}{\alpha_2} \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{}{\alpha_2-\frac{\alpha._1\,k^2}{2}} \end{pmatrix}$$

Square masses:

$$\left\{\left\{-\frac{\alpha.\left(\alpha.-4\,\alpha.\right)}{\alpha.\left(\alpha.-\alpha.\right)}\right\},\{\},\{\},\{\},\left\{\frac{2\,\alpha.}{\alpha.}\right\},\{\}\right\}$$

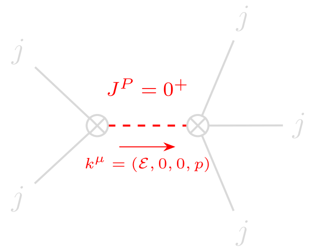
Massive pole residues:

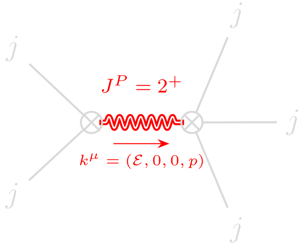
$$\left\{\left\{\frac{\alpha.\ \ -2\,\alpha.\,\alpha.+4\,\alpha.}{\alpha.\left(\alpha.-\alpha.\right)}\right\},\{\},\{\},\{\},\left\{-\frac{2}{\alpha.}\right\},\{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:





Massive particle

Pole residue:	$\frac{\alpha P_2^2\,OT\ \alpha P_2\,\alpha P_3 M\ V\ \alpha P_3^2}{\alpha P_1 J \alpha P_2 O \alpha P_3 K^2} > 0$
Square mass:	$-\frac{\alpha P_2 J \alpha P_2 O V\ \alpha P_3 K}{\alpha P_1 J \alpha P_2 O \alpha P_3 K} > 0$
Spin:	0
Parity:	Even

Massive particle

Pole residue:	$-\frac{T}{\alpha P_1} > 0$
Square mass:	$\frac{T\ \alpha P_2}{\alpha P_1} > 0$
Spin:	2
Parity:	Even

Overall unitarity conditions:

False

The consequence is seen in the positive-parity scalar sector, which develops a massive pole. This is the Boulware-Deser ghost, which always spoils the unitarity of the theory.

Poincaré gauge theory (PGT)

Key observation: We will test the PoincareGaugeTheory module.

Now we set up the general Lagrangian:

$$-\lambda_{\cdot} \mathcal{R}^{ij}{}_{ij} + \left(\frac{r_{\underset{1}{\cdot}}}{3} + \frac{r_{\underset{2}{\cdot}}}{6}\right) \mathcal{R}_{ijhl} \mathcal{R}^{ijhl} + \left(\frac{2 r_{\underset{1}{\cdot}}}{3} - \frac{2 r_{\underset{2}{\cdot}}}{3}\right) \mathcal{R}_{ihjl} \mathcal{R}^{ijhl} +$$
$$\left(r_{\underset{4}{\cdot}} + r_{\underset{5}{\cdot}}\right) \mathcal{R}_{i}{}^{l}{}_{jl} \mathcal{R}^{ihj}{}_{h} + \left(r_{\underset{4}{\cdot}} - r_{\underset{5}{\cdot}}\right) \mathcal{R}^{ihj}{}_{h} \mathcal{R}_{j}{}^{l}{}_{il} + \left(\frac{r_{\underset{1}{\cdot}}}{3} + \frac{r_{\underset{2}{\cdot}}}{6} - r_{\underset{3}{\cdot}}\right) \mathcal{R}^{ijhl} \mathcal{R}_{hlij} + \tag{15}$$
$$\left(\frac{\lambda_{\cdot}}{4} + \frac{t_{\underset{1}{\cdot}}}{3} + \frac{t_{\underset{2}{\cdot}}}{12}\right) \mathcal{T}_{ijh} \mathcal{T}^{ijh} + \left(-\frac{\lambda_{\cdot}}{2} - \frac{t_{\underset{1}{\cdot}}}{3} + \frac{t_{\underset{2}{\cdot}}}{6}\right) \mathcal{T}^{ijh} \mathcal{T}_{jhi} + \left(-\lambda_{\cdot} - \frac{t_{\underset{1}{\cdot}}}{3} + \frac{2 t_{\underset{3}{\cdot}}}{3}\right) \mathcal{T}_{i}{}^{ji} \mathcal{T}_{hj}{}^{h}$$

```
** DefConstantSymbol: Defining constant symbol PerturbativeParameter.
```

Einstein-Cartan theory (ECT)

Now we would like to check the basic Einstein-Cartan theory. Here is the full nonlinear Lagrangian:

$$t_{\underset{1}{\cdot}} \mathcal{R}^{ij}{}_{ij} \tag{16}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{\underset{1}{\cdot}} \mathcal{A}_{\alpha ji} \mathcal{A}^{\alpha ij} + t_{\underset{1}{\cdot}} \mathcal{A}^{\alpha i}{}_{\alpha} \mathcal{A}_{i}{}^{j}{}_{j} + 2 t_{\underset{1}{\cdot}} f^{\alpha i} \partial_i \mathcal{A}_{\alpha}{}^{j}{}_{j} - 2 t_{\underset{1}{\cdot}} \partial_i \mathcal{A}^{\alpha i}{}_{\alpha} - 2 t_{\underset{1}{\cdot}} f^{\alpha i} \partial_j \mathcal{A}_{\alpha}{}^{j}{}_{i} \tag{17}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} -t_{\underset{1}{\cdot}} & -\frac{i k t_{\underset{1}{\cdot}}}{\sqrt{2}} & -i\sqrt{\frac{3}{2}}\, k t_{\underset{1}{\cdot}} & 0 \\ \frac{i k t_{\underset{1}{\cdot}}}{\sqrt{2}} & 0 & 0 & 0 \\ i\sqrt{\frac{3}{2}}\, k t_{\underset{1}{\cdot}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_{\underset{1}{\cdot}} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} -\frac{t_{1}}{2} & -\frac{t_{1}}{\sqrt{2}} & -\frac{i k t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{t_{1}}{2} & \frac{t_{1}}{\sqrt{2}} & 0 & i k t_{1} \\ 0 & 0 & 0 & \frac{t_{1}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i k t_{1} & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{2} & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{t_{1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\cdot}\tau^{\parallel} == {}^{0^+}_{\cdot}\tau^{\perp}$$

$$2 i k \, {}^{1^-}_{\cdot}\sigma^{\perp a} + {}^{1^-}_{\cdot}\tau^{\perp a} == 0$$

$${}^{1^-}_{\cdot}\tau^{\parallel a} == 0$$

$$i k \, {}^{1^+}_{\cdot}\sigma^{\perp ab} + {}^{1^+}_{\cdot}\tau^{\parallel ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & -\frac{i}{2\sqrt{2}\, k t_{1}} & -\frac{i\sqrt{\frac{3}{2}}}{2 k t_{1}} & 0 \\ \frac{i}{2\sqrt{2}\, k t_{1}} & \frac{1}{8 k^2 t_{1}} & \frac{\sqrt{3}}{8 k^2 t_{1}} & 0 \\ \frac{i\sqrt{\frac{3}{2}}}{2 k t_{1}} & \frac{\sqrt{3}}{8 k^2 t_{1}} & \frac{3}{8 k^2 t_{1}} & 0 \\ 0 & 0 & 0 & -\frac{1}{t_{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & -\frac{i\sqrt{2}\, k}{t_{1}+k^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{1}{(1+k^2)^2 t_{1}} & \frac{i k}{(1+k^2)^2 t_{1}} & 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\, k}{t_{1}+k^2 t_{1}} & -\frac{i k}{(1+k^2)^2 t_{1}} & \frac{k^2}{(1+k^2)^2 t_{1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & 0 & \frac{2 i k}{t_{1}+2k^2 t_{1}} \\ 0 & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & \frac{1}{(1+2k^2)^2 t_{1}} & 0 & \frac{i\sqrt{2}\, k}{(1+2k^2)^2 t_{1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2 i k}{t_{1}+2k^2 t_{1}} & -\frac{i\sqrt{2}\, k}{(1+2k^2)^2 t_{1}} & 0 & \frac{2k^2}{(1+2k^2)^2 t_{1}} \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & -\frac{i\sqrt{2}}{k\, t_{\dot{1}}} & 0 \\ \frac{i\sqrt{2}}{k\, t_{\dot{1}}} & -\frac{1}{k^2\, t_{\dot{1}}} & 0 \\ 0 & 0 & \frac{2}{t_{\dot{1}}} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

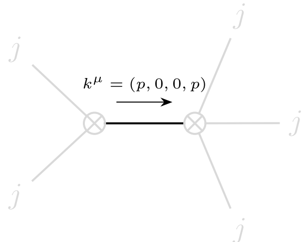
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{9\, p^2}{t_{\dot{1}}}, -\frac{9\, p^2}{t_{\dot{1}}}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{p^2}{t^{P}_{\dot{1}}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(p < 0 \,\&\&\, t_{\dot{1}} < 0\right) \,||\, \left(p > 0 \,\&\&\, t_{\dot{1}} < 0\right)$$

Okay, so that is the end of the PSALTer output for Einstein-Cartan gravity. What we find are no propagating massive modes, but instead two degrees of freedom in the massive sector. The no-ghost conditions on these massless d.o.f restrict the sign in front of the Einstein-Hilbert term to be negative (which is what we expect for our conventions).

General relativity (GR)

Using Karananas' coefficients, it is particularly easy to also look at GR, instead of Einstein-Cartan theory. The difference here is that the quadratic torsion coefficients are manually removed. Here is the nonlinear Lagrangian:

$$-\lambda_{\dot{}}\, \mathcal{R}^{ij}{}_{ij} + \frac{1}{4}\lambda_{\dot{}}\, \mathcal{T}_{ij\hbar}\, \mathcal{T}^{ij\hbar} + \frac{1}{2}\lambda_{\dot{}}\, \mathcal{T}^{ij\hbar}\, \mathcal{T}_{ji\hbar} + \lambda_{\dot{}}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{\hbar}{}_{j\hbar} \tag{18}$$

Here is the linearised theory:

$$\begin{aligned} &-2\lambda_{\dot{}}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} - 2\lambda_{\dot{}}\, f^{ab}\, \partial_{b}\mathcal{A}_{a}{}^{i}{}_{i} + 2\lambda_{\dot{}}\, \partial_{b}\mathcal{A}^{ab}{}_{a} + 2\lambda_{\dot{}}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - \\ &\lambda_{\dot{}}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + 2\lambda_{\dot{}}\, f^{ab}\, \partial_{i}\mathcal{A}_{a}{}^{i}{}_{b} - \lambda_{\dot{}}\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} + 2\lambda_{\dot{}}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} + 2\lambda_{\dot{}}\, \mathcal{A}_{bia}\, \partial^{i} f^{ab} - \\ &\lambda_{\dot{}}\, \partial_{a} f_{bi}\, \partial^{i} f^{ab} + \frac{1}{2}\lambda_{\dot{}}\, \partial_{a} f_{ib}\, \partial^{i} f^{ab} - \frac{1}{2}\lambda_{\dot{}}\, \partial_{b} f_{ai}\, \partial^{i} f^{ab} + \frac{1}{2}\lambda_{\dot{}}\, \partial_{i} f_{ab}\, \partial^{i} f^{ab} + \frac{1}{2}\lambda_{\dot{}}\, \partial_{i} f_{ba}\, \partial^{i} f^{ab} \end{aligned} \tag{19}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & \frac{3 i k \lambda_{:}}{\sqrt{2}} & i \sqrt{\frac{3}{2}}\, k \lambda_{:} & 0 \\ -\frac{3 i k \lambda_{:}}{\sqrt{2}} & -2 k^2 \lambda_{:} & 0 & 0 \\ -i \sqrt{\frac{3}{2}}\, k \lambda_{:} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & 0 & i \sqrt{2}\, k \lambda_{:} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -i \sqrt{2}\, k \lambda_{:} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & i k \lambda_{:} & -i k \lambda_{:} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i k \lambda_{:} & 0 & 0 & 0 \\ 0 & 0 & 0 & i k \lambda_{:} & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & k^2 \lambda_{:} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^-}_{\bullet}\sigma^{\parallel} == 0$$

$${}^{1^-}_{\bullet}\tau^{\parallel}{}^{a} + {}^{1^-}_{\bullet}\tau^{\perp}{}^{a} == 0$$

$${}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} == 0$$

$${}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab} == 0$$

$${}^{2^-}_{\bullet}\sigma^{\parallel}{}^{abc} == 0$$

$${}^{2^+}_{\bullet}\sigma^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & \frac{i \sqrt{\frac{2}{3}}}{k \lambda_{:}} & 0 \\ 0 & -\frac{1}{2 k^2 \lambda_{:}} & \frac{\sqrt{3}}{2 k^2 \lambda_{:}} & 0 \\ -\frac{i \sqrt{\frac{2}{3}}}{k \lambda_{:}} & \frac{\sqrt{3}}{2 k^2 \lambda_{:}} & -\frac{3}{2 k^2 \lambda_{:}} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & 0 & \frac{i}{\sqrt{2}\,k\,\lambda_{:}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i}{\sqrt{2}\,k\,\lambda_{:}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{i}{2\,k\,\lambda_{:}} & -\frac{i}{2\,k\,\lambda_{:}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{i}{2\,k\,\lambda_{:}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{i}{2\,k\,\lambda_{:}} & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{k^2\,\lambda_{:}} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

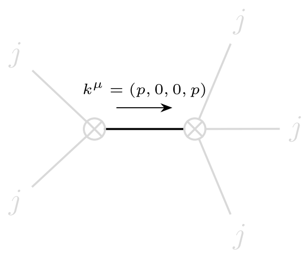
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{p^2}{\lambda_{:}}, \frac{p^2}{\lambda_{:}}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$\frac{p^2}{\lambda_{:}} > 0$
Polarisations:	2

Overall unitarity conditions:

$(p < 0 \;\&\&\; \lambda_{:} > 0) \;||\; (p > 0 \;\&\&\; \lambda_{:} > 0)$

Thus, the conclusions are the same, as expected.

Performing the grand PGT survey

We are now ready to check that PSALTer is getting the physics right by running it on the 58 theories in arXiv:1910.14197.

Case 1

Now for a new theory. Here is the full nonlinear Lagrangian for Case 1 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2}\, \mathcal{R}_{ij\hbar l}\, \mathcal{R}^{ij\hbar l} - \frac{2}{3} r_{2}\, \mathcal{R}_{i\hbar jl}\, \mathcal{R}^{ij\hbar l} + \left(\frac{r_{3}}{2} + r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{\hbar}\, \mathcal{R}_{j}{}^{l}{}_{\hbar l} + \quad (20)$$

$$\frac{1}{6}\left(r_{\dot 2}-6\,r_{\dot 3}\right)\mathcal{R}^{ijkl}\,\mathcal{R}_{klij}+\frac{1}{2}\left(r_{\dot 3}-2\,r_{\dot 5}\right)\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl}+\frac{1}{12}\,t_{\dot 2}\,\mathcal{T}_{ijk}\,\mathcal{T}^{ijk}-\frac{1}{6}\,t_{\dot 2}\,\mathcal{T}^{ijk}\,\mathcal{T}_{jik}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3}\,t_{\dot 2}\,\mathcal{A}_{abi}\,\mathcal{A}^{abi}-\frac{2}{3}\,t_{\dot 2}\,\mathcal{A}_{aib}\,\mathcal{A}^{abi}+\left(-\frac{r_{\dot 3}}{2}+r_{\dot 5}\right)\partial_b\mathcal{A}_i{}^j{}_j\,\partial^i\mathcal{A}^{ab}{}_a+\\
&\left(-\frac{r_{\dot 3}}{2}-r_{\dot 5}\right)\partial_i\mathcal{A}_b{}^j{}_j\,\partial^i\mathcal{A}^{ab}{}_a-\frac{2}{3}\,t_{\dot 2}\,\mathcal{A}_{abi}\,\partial^i f^{ab}+\frac{2}{3}\,t_{\dot 2}\,\mathcal{A}_{aib}\,\partial^i f^{ab}-\frac{2}{3}\,t_{\dot 2}\,\mathcal{A}_{bia}\,\partial^i f^{ab}+\\
&\frac{1}{3}\,t_{\dot 2}\,\partial_a f_{bi}\,\partial^i f^{ab}-\frac{1}{6}\,t_{\dot 2}\,\partial_a f_{ib}\,\partial^i f^{ab}-\frac{1}{6}\,t_{\dot 2}\,\partial_b f_{ai}\,\partial^i f^{ab}+\frac{1}{6}\,t_{\dot 2}\,\partial_i f_{ab}\,\partial^i f^{ab}-\frac{1}{6}\,t_{\dot 2}\,\partial_i f_{ba}\,\partial^i f^{ab}+\\
&\left(-\frac{r_{\dot 3}}{2}-r_{\dot 5}\right)\partial_a\mathcal{A}^{abi}\,\partial_j\mathcal{A}_b{}^j{}_i+\left(r_{\dot 3}+2\,r_{\dot 5}\right)\partial^i\mathcal{A}^{ab}{}_a\,\partial_j\mathcal{A}_b{}^j{}_i+\left(-\frac{r_{\dot 3}}{2}+r_{\dot 5}\right)\partial_a\mathcal{A}^{abi}\,\partial_j\mathcal{A}_i{}^j{}_b+\\
&\left(r_{\dot 3}-2\,r_{\dot 5}\right)\partial^i\mathcal{A}^{ab}{}_a\,\partial_j\mathcal{A}_i{}^j{}_b+\frac{4}{3}\,r_{\dot 2}\,\partial_b\mathcal{A}_{aij}\,\partial^j\mathcal{A}^{abi}-\frac{2}{3}\,r_{\dot 2}\,\partial_b\mathcal{A}_{aji}\,\partial^j\mathcal{A}^{abi}+\\
&\frac{2}{3}\left(r_{\dot 2}-6\,r_{\dot 3}\right)\partial_b\mathcal{A}_{ija}\,\partial^j\mathcal{A}^{abi}-\frac{1}{3}\,r_{\dot 2}\,\partial_i\mathcal{A}_{abj}\,\partial^j\mathcal{A}^{abi}+\frac{1}{3}\,r_{\dot 2}\,\partial_j\mathcal{A}_{abi}\,\partial^j\mathcal{A}^{abi}-\frac{2}{3}\,r_{\dot 2}\,\partial_j\mathcal{A}_{aib}\,\partial^j\mathcal{A}^{abi}
\end{aligned}\tag{21}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\,r_{\dot 2}+t_{\dot 2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
k^2\left(2\,r_{\dot 3}+r_{\dot 5}\right)+\frac{2\,t_{\dot 2}}{3} & \frac{\sqrt{2}\,t_{\dot 2}}{3} & -\frac{1}{3}\,i\,\sqrt{2}\,k\,t_{\dot 2} & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}\,t_{\dot 2}}{3} & \frac{t_{\dot 2}}{3} & -\frac{1}{3}\,i\,k\,t_{\dot 2} & 0 & 0 & 0 & 0 \\
\frac{1}{3}\,i\,\sqrt{2}\,k\,t_{\dot 2} & \frac{i\,k\,t_{\dot 2}}{3} & \frac{k^2\,t_{\dot 2}}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2}\,k^2\left(r_{\dot 3}+2\,r_{\dot 5}\right) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{3\,k^2\,r_{\dot 3}}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^+}\tau^{\perp} == 0$

${}^{0^+}\tau^{\parallel} == 0$

${}^{0^+}\sigma^{\parallel} == 0$

${}^{1^-}\tau^{\perp a} == 0$

${}^{1^-}\tau^{\parallel a} == 0$

$${}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} == 0$$

$$-i\,k\;{}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab} + {}^{1^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

$${}^{2^-}_{\bullet}\sigma^{\parallel}{}^{abc} == 0$$

$${}^{2^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{\underset{2}{\bullet}} + t_{\underset{2}{\bullet}}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right)} & -\frac{\sqrt{2}}{k^2 (1+k^2)\left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right)} & \frac{i\sqrt{2}}{k (1+k^2)\left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right)} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{k^2 (1+k^2)\left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right)} & \frac{3 k^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right) + 2 t_{\underset{2}{\bullet}}}{(k+k^3)^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right) t_{\underset{2}{\bullet}}} & -\frac{i\left(3 k^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right) + 2 t_{\underset{2}{\bullet}}\right)}{k (1+k^2)^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right) t_{\underset{2}{\bullet}}} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}}{k (1+k^2)\left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right)} & \frac{i\left(3 k^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right) + 2 t_{\underset{2}{\bullet}}\right)}{k (1+k^2)^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right) t_{\underset{2}{\bullet}}} & \frac{3 k^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right) + 2 t_{\underset{2}{\bullet}}}{(1+k^2)^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right) t_{\underset{2}{\bullet}}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{k^2 \left(r_{\underset{3}{\bullet}} + 2 r_{\underset{5}{\bullet}}\right)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{}{k^2 r_{3}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\},\left\{-\frac{t_{\bullet}}{r_{\bullet}}\right\},\{\},\{\},\{\},\{\}\right\}$$

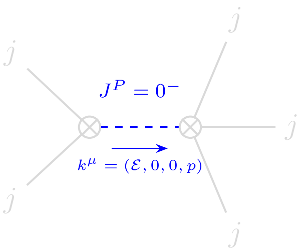
Massive pole residues:

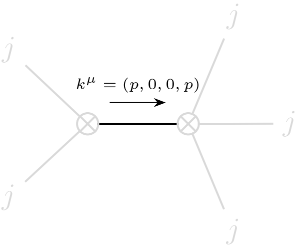
$$\left\{\{\},\left\{-\frac{1}{r_{\bullet}}\right\},\{\},\{\},\{\},\{\}\right\}$$

Massless eigenvalues:

$$\left\{-\frac{45\,r_{\bullet}\;+20\,r_{\bullet}\,r_{\bullet}+4\,r_{\bullet}}{r_{\bullet}\left(2\,r_{\bullet}+r_{\bullet}\right)\left(r_{\bullet}+2\,r_{\bullet}\right)},-\frac{45\,r_{\bullet}\;+20\,r_{\bullet}\,r_{\bullet}+4\,r_{\bullet}}{r_{\bullet}\left(2\,r_{\bullet}+r_{\bullet}\right)\left(r_{\bullet}+2\,r_{\bullet}\right)}\right\}$$

Overall particle spectrum:





Massive particle

Pole residue:	$-\frac{S}{r_{P_2}} > 0$
Square mass:	$-\frac{t_{P_2}}{r_{P_2}} > 0$
Spin:	0
Parity:	Odd

Massless particle

Pole residue:	$-\frac{T}{r_{P_3}} + \frac{Y}{T r_{P_3} M r_{P_5}} - \frac{TV}{r_{P_3} M T r_{P_5}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(r_{\cdot 2} < 0 \,\&\&\, r_{\cdot 3} < 0 \,\&\&\, r_{\cdot 5} < -\frac{r_{\cdot 3}}{2} \,\&\&\, t_{\cdot 2} > 0\right) \,||$$

$$\left(r_{\cdot 2} < 0 \,\&\&\, r_{\cdot 3} < 0 \,\&\&\, r_{\cdot 5} > -2\, r_{\cdot 3} \,\&\&\, t_{\cdot 2} > 0\right) \,||\, \left(r_{\cdot 2} < 0 \,\&\&\, r_{\cdot 3} > 0 \,\&\&\, -2\, r_{\cdot 3} < r_{\cdot 5} < -\frac{r_{\cdot 3}}{2} \,\&\&\, t_{\cdot 2} > 0\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\begin{gathered}\left(r_{\cdot} < 0 \,\&\&\, r_{\cdot} < 0 \,\&\&\, r_{\cdot} < -\frac{r_{\cdot}}{2} \,\&\&\, t_{\cdot} > 0\right) \,|| \\ \left(r_{\cdot} < 0 \,\&\&\, r_{\cdot} < 0 \,\&\&\, r_{\cdot} > -2\, r_{\cdot} \,\&\&\, t_{\cdot} > 0\right) \,||\, \left(r_{\cdot} < 0 \,\&\&\, r_{\cdot} > 0 \,\&\&\, -2\, r_{\cdot} < r_{\cdot} < -\frac{r_{\cdot}}{2} \,\&\&\, t_{\cdot} > 0\right)\end{gathered} \tag{22}$$

Okay, that concludes the analysis of this theory.

Case 2

Now for a new theory. Here is the full nonlinear Lagrangian for Case 2 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{gathered}\frac{1}{6} r_{\cdot}\, \mathcal{R}_{ij\mathfrak{k}l}\, \mathcal{R}^{ij\mathfrak{k}l} - \frac{2}{3} r_{\cdot}\, \mathcal{R}_{i\mathfrak{k}jl}\, \mathcal{R}^{ij\mathfrak{k}l} + \left(\frac{r_{\cdot}}{2} + r_{\cdot}\right) \mathcal{R}^{ij\mathfrak{k}}{}_{i}\, \mathcal{R}_{j}{}^{l}{}_{\mathfrak{k}l} + \frac{1}{6}\left(r_{\cdot} - 6\, r_{\cdot}\right) \mathcal{R}^{ij\mathfrak{k}l}\, \mathcal{R}_{\mathfrak{k}lij} + \\ \frac{1}{2}\left(r_{\cdot} - 2\, r_{\cdot}\right) \mathcal{R}^{ij\mathfrak{k}}{}_{i}\, \mathcal{R}_{\mathfrak{k}}{}^{l}{}_{jl} + \frac{1}{12} t_{\cdot}\, \mathcal{T}_{ij\mathfrak{k}}\, \mathcal{T}^{ij\mathfrak{k}} - \frac{1}{6} t_{\cdot}\, \mathcal{T}^{ij\mathfrak{k}}\, \mathcal{T}_{ji\mathfrak{k}} - \frac{2}{3} t_{\cdot}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{\mathfrak{k}}{}_{j\mathfrak{k}}\end{gathered} \tag{23}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{gathered}\frac{1}{3} t_{\cdot}\, \mathcal{A}_{abi}\, \mathcal{A}^{abi} - \frac{2}{3} t_{\cdot}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} - \frac{2}{3} t_{\cdot}\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} + \frac{4}{3} t_{\cdot}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} - \frac{4}{3} t_{\cdot}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + \\ \frac{2}{3} t_{\cdot}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{\cdot}\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} - \frac{4}{3} t_{\cdot}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} + \left(-\frac{r_{\cdot}}{2} + r_{\cdot}\right) \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} +\end{gathered}$$

$$\left(-\frac{r.}{2}-r.\right)\partial_i\mathcal{A}_b{}^j{}_j\;\partial^i\mathcal{A}^{ab}{}_a-\frac{2}{3}t.\;\mathcal{A}_{abi}\;\partial^i f^{ab}+\frac{2}{3}t.\;\mathcal{A}_{aib}\;\partial^i f^{ab}-\frac{2}{3}t.\;\mathcal{A}_{bia}\;\partial^i f^{ab}+$$
$$\frac{1}{3}t.\;\partial_a f_{bi}\;\partial^i f^{ab}-\frac{1}{6}t.\;\partial_a f_{ib}\;\partial^i f^{ab}-\frac{1}{6}t.\;\partial_b f_{ai}\;\partial^i f^{ab}+\frac{1}{6}t.\;\partial_i f_{ab}\;\partial^i f^{ab}-\frac{1}{6}t.\;\partial_i f_{ba}\;\partial^i f^{ab}+$$
$$\left(-\frac{r.}{2}-r.\right)\partial_a\mathcal{A}^{abi}\;\partial_j\mathcal{A}_b{}^j{}_i+\left(r.+2r.\right)\partial^i\mathcal{A}^{ab}{}_a\;\partial_j\mathcal{A}_b{}^j{}_i+\left(-\frac{r.}{2}+r.\right)\partial_a\mathcal{A}^{abi}\;\partial_j\mathcal{A}_i{}^j{}_b+$$
$$\left(r.-2r.\right)\partial^i\mathcal{A}^{ab}{}_a\;\partial_j\mathcal{A}_i{}^j{}_b+\frac{4}{3}r.\;\partial_b\mathcal{A}_{aij}\;\partial^j\mathcal{A}^{abi}-\frac{2}{3}r.\;\partial_b\mathcal{A}_{aji}\;\partial^j\mathcal{A}^{abi}+$$
$$\frac{2}{3}\left(r.-6r.\right)\partial_b\mathcal{A}_{ija}\;\partial^j\mathcal{A}^{abi}-\frac{1}{3}r.\;\partial_i\mathcal{A}_{abj}\;\partial^j\mathcal{A}^{abi}+\frac{1}{3}r.\;\partial_j\mathcal{A}_{abi}\;\partial^j\mathcal{A}^{abi}-\frac{2}{3}r.\;\partial_j\mathcal{A}_{aib}\;\partial^j\mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} t. & -i\sqrt{2}\,k\,t. & 0 & 0 \\ i\sqrt{2}\,k\,t. & 2k\;t. & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\;\;r.+t. \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k\left(2r.+r.\right)+\frac{t_2}{} & \frac{\sqrt{\ }\;t_2}{} & --i\sqrt{2}\,k\,t. & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\ }\;t_2}{} & \frac{t_2}{} & --i\,k\,t. & 0 & 0 & 0 & 0 \\ -i\sqrt{2}\,k\,t. & \frac{i\,k\,t_2}{} & \frac{k^2\,t_2}{} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\left(\frac{r_3}{}+r.\right)+\frac{t_3}{} & -\frac{\sqrt{\ }\;t_3}{} & --i\,k\,t. & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{\ }\;t_3}{} & \frac{t_3}{} & -i\sqrt{2}\,k\,t. & 0 \\ 0 & 0 & 0 & \frac{i\,k\,t_3}{} & --i\sqrt{2}\,k\,t. & \frac{k^2\,t_3}{} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{k^2\,r_3}{} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{+}_{\cdot}\tau^{\perp} == 0$$

$$-2\,i\,k\;{}^{+}_{\cdot}\sigma^{\parallel}+{}^{+}_{\cdot}\tau^{\parallel} == 0$$

$${}^{-}_{\cdot}\tau^{\perp}{}^{a} == 0$$

$$2\,i\,k\;{}^{-}_{\cdot}\sigma^{\perp}{}^{a}+{}^{-}_{\cdot}\tau^{\parallel}{}^{a} == 0$$

$$-i\,k\;{}^{+}_{\cdot}\sigma^{\perp}{}^{ab}+{}^{+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

$${}^{-}_{\cdot}\sigma^{\parallel}{}^{abc} == 0$$

$${}^{+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{}{(+k^2)^2 t_3} & -\frac{i\sqrt{\ } k}{(+k^2)^2 t_3} & 0 & 0 \\ \frac{i\sqrt{\ } k}{(+k^2)^2 t_3} & \frac{k^2}{(+k^2)^2 t_3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_2 + t_2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{}{k^2(r_3+r_5)} & -\frac{\sqrt{\ }}{k^2(+k^2)(r_3+r_5)} & \frac{i\sqrt{\ }}{k(+k^2)(r_3+r_5)} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{\ }}{k^2(+k^2)(r_3+r_5)} & \frac{k^2(r_3+r_5)+t_2}{(k+k^3)^2(r_3+r_5)t_2} & -\frac{i(k^2(r_3+r_5)+t_2)}{k(+k^2)^2(r_3+r_5)t_2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{\ }}{k(+k^2)(r_3+r_5)} & \frac{i(k^2(r_3+r_5)+t_2)}{k(+k^2)^2(r_3+r_5)t_2} & \frac{k^2(r_3+r_5)+t_2}{(+k^2)^2(r_3+r_5)t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2(r_3+r_5)} & \frac{\sqrt{\ }}{k^2(+k^2)(r_3+r_5)} & \frac{i}{k(+k^2)(r_3+r_5)} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{\ }}{k^2(+k^2)(r_3+r_5)} & \frac{k^2(r_3+r_5)+t_3}{(k+k^3)^2(r_3+r_5)t_3} & \frac{i\sqrt{\ }(k^2(r_3+r_5)+t_3)}{k(+k^2)^2(r_3+r_5)t_3} & 0 \\ 0 & 0 & 0 & -\frac{i}{k(+k^2)(r_3+r_5)} & -\frac{i\sqrt{\ }(k^2(r_3+r_5)+t_3)}{k(+k^2)^2(r_3+r_5)t_3} & \frac{k^2(r_3+r_5)+t_3}{(+k^2)^2(r_3+r_5)t_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{}{k^2 r_3} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{.}}{r_{.}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

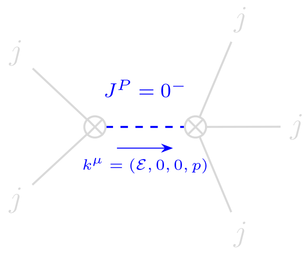
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{.}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

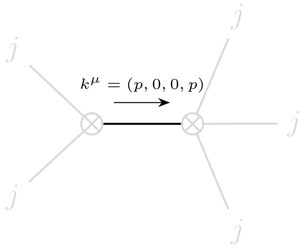
$$\left\{-\frac{403 r_{.}\ + 172 r_{.} r_{.} + 28 r_{.}}{6 r_{.} (2 r_{.} + r_{.})(r_{.} + 2 r_{.})}, -\frac{403 r_{.}\ + 172 r_{.} r_{.} + 28 r_{.}}{6 r_{.} (2 r_{.} + r_{.})(r_{.} + 2 r_{.})}\right\}$$

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{r_{2}} > 0$
Square mass:	$-\frac{t_{2}}{r_{2}} > 0$
Spin:	0
Parity:	Odd



Massless particle

Pole residue:	$-\frac{SV}{r_{3}} + \frac{WY}{T r_{3} M r_{5}} - \frac{TSX}{r_{3} M T r_{5}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(r_{2} < 0 \;\&\&\; r_{3} < 0 \;\&\&\; r_{5} < -\frac{r_{3}}{2} \;\&\&\; t_{2} > 0\right) \,||$$

$$\left(r_{2} < 0 \;\&\&\; r_{3} < 0 \;\&\&\; r_{5} > -2\, r_{3} \;\&\&\; t_{2} > 0\right) || \left(r_{2} < 0 \;\&\&\; r_{3} > 0 \;\&\&\; -2\, r_{3} < r_{5} < -\frac{r_{3}}{2} \;\&\&\; t_{2} > 0\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r. < 0 \;\&\&\; r. < 0 \;\&\&\; r. < -\frac{r.}{2} \;\&\&\; t. > 0\right) || \left(r. < 0 \;\&\&\; r. < 0 \;\&\&\; r. > -2\, r. \;\&\&\; t. > 0\right) || \left(r. < 0 \;\&\&\; r. > 0 \;\&\&\; -2\, r. < r. < -\frac{r.}{2} \;\&\&\; t. > 0\right) \tag{25}$$

Okay, that concludes the analysis of this theory.

Case 3

Now for a new theory. Here is the full nonlinear Lagrangian for Case 3 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r.\, \mathcal{R}_{ijhl}\, \mathcal{R}^{ijhl} - \frac{2}{3} r.\, \mathcal{R}_{ihjl}\, \mathcal{R}^{ijhl} + r.\, \mathcal{R}^{ij}{}_{i}{}^{h}\, \mathcal{R}_{j}{}^{l}{}_{hl} + \frac{1}{6} r.\, \mathcal{R}^{ijhl}\, \mathcal{R}_{hlij} - r.\, \mathcal{R}^{ij}{}_{i}{}^{h}\, \mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{4} t.\, \mathcal{T}_{ijh}\, \mathcal{T}^{ijh} + \frac{1}{2} t.\, \mathcal{T}^{ijh}\, \mathcal{T}_{jih} + \frac{1}{3} t.\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{h}{}_{jh} \tag{26}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t.\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} + \frac{1}{3} t.\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3} t.\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} + \frac{2}{3} t.\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - \frac{1}{3} t.\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - \frac{1}{3} t.\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} + \frac{2}{3} t.\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} + r.\, \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} - r.\, \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} + 2 t.\, \mathcal{A}_{bia}\, \partial^{i} f^{ab} - t.\, \partial_{a} f_{bi}\, \partial^{i} f^{ab} + \frac{1}{2} t.\, \partial_{a} f_{ib}\, \partial^{i} f^{ab} - \frac{1}{2} t.\, \partial_{b} f_{ai}\, \partial^{i} f^{ab} +$$

(27)

$$\frac{1}{2} t.\, \partial_i f_{ab}\, \partial^i f^{ab} + \frac{1}{2} t.\, \partial_i f_{ba}\, \partial^i f^{ab} - r.\, \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_b{}^j{}_i + 2\, r.\, \partial^i \mathcal{A}^{ab}{}_a\, \partial_j \mathcal{A}_b{}^j{}_i +$$
$$r.\, \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_i{}^j{}_b - 2\, r.\, \partial^i \mathcal{A}^{ab}{}_a\, \partial_j \mathcal{A}_i{}^j{}_b + \frac{4}{3} r.\, \partial_b \mathcal{A}_{aij}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r.\, \partial_b \mathcal{A}_{aji}\, \partial^j \mathcal{A}^{abi} +$$
$$\frac{2}{3} r.\, \partial_b \mathcal{A}_{ija}\, \partial^j \mathcal{A}^{abi} - \frac{1}{3} r.\, \partial_i \mathcal{A}_{abj}\, \partial^j \mathcal{A}^{abi} + \frac{1}{3} r.\, \partial_j \mathcal{A}_{abi}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r.\, \partial_j \mathcal{A}_{aib}\, \partial^j \mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\ r. - t. \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k\ r. - \frac{t_1}{} & -\frac{t_1}{\sqrt{}} & \frac{i k t_1}{\sqrt{}} & 0 & 0 & 0 & 0 \\ -\frac{t_1}{\sqrt{}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_1}{\sqrt{}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\ r. + \frac{t_1}{} & \frac{t_1}{\sqrt{}} & \frac{i k t_1}{} & 0 \\ 0 & 0 & 0 & \frac{t_1}{\sqrt{}} & \frac{t_1}{} & -i\sqrt{2}\, k t. & 0 \\ 0 & 0 & 0 & --i k t. & --i\sqrt{2}\, k t. & \frac{k^2 t_1}{} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_1}{} & -\frac{i k t_1}{\sqrt{}} & 0 \\ \frac{i k t_1}{\sqrt{}} & k\ t. & 0 \\ 0 & 0 & \frac{t_1}{} \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{+}{.}\tau^{\perp} == 0$

$\overset{+}{.}\tau^{\parallel} == 0$

$\overset{+}{.}\sigma^{\parallel} == 0$

$\overset{-}{.}\tau^{\perp}{}^{a} == 0$

$2 i k\ \overset{-}{.}\sigma^{\perp}{}^{a} + \overset{-}{.}\tau^{\parallel}{}^{a} == 0$

$-i k\ \overset{+}{.}\sigma^{\perp}{}^{ab} + \overset{+}{.}\tau^{\parallel}{}^{ab} == 0$

$-2 i k\ \overset{+}{.}\sigma^{\parallel}{}^{ab} + \overset{+}{.}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally
analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_2 - t_1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{}}{t_1 + k^2 t_1} & \frac{i\sqrt{}\,k}{t_1 + k^2 t_1} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{}}{t_1 + k^2 t_1} & \frac{-k^2 r_5 + t_1}{(+k^2)^2 t_1^2} & \frac{i(k^3 r_5 - k t_1)}{(+k^2)^2 t_1^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{}\,k}{t_1 + k^2 t_1} & -\frac{i(k^3 r_5 - k t_1)}{(+k^2)^2 t_1^2} & \frac{-k^4 r_5 + k^2 t_1}{(+k^2)^2 t_1^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_5} & -\frac{}{\sqrt{}\left(k^2 r_5 + k^4 r_5\right)} & -\frac{i}{k r_5 + k^3 r_5} & 0 \\ 0 & 0 & 0 & -\frac{}{\sqrt{}\left(k^2 r_5 + k^4 r_5\right)} & \frac{k^2 r_5 + t_1}{(k + k^3)^2 r_5 t_1} & \frac{i(k^2 r_5 + t_1)}{\sqrt{}\,k(+k^2)^2 r_5 t_1} & 0 \\ 0 & 0 & 0 & \frac{i}{k r_5 + k^3 r_5} & -\frac{i(k^2 r_5 + t_1)}{\sqrt{}\,k(+k^2)^2 r_5 t_1} & \frac{k^2 r_5 + t_1}{(+k^2)^2 r_5 t_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{}{(+k^2)^2 t_1} & -\frac{i\sqrt{}\,k}{(+k^2)^2 t_1} & 0 \\ \frac{i\sqrt{}\,k}{(+k^2)^2 t_1} & \frac{k^2}{(+k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t.}{r.}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

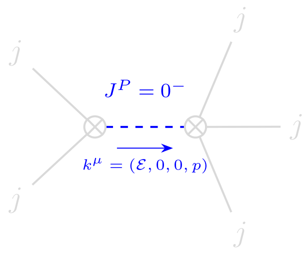
Massive pole residues:

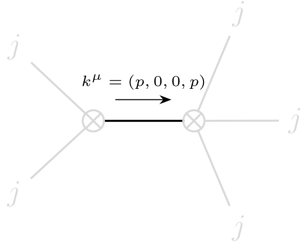
$$\left\{\{\}, \left\{-\frac{1}{r.}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\left\{-\frac{7t. + 2r.t.p + 4r.\ p}{2r.t.}, -\frac{7t. + 2r.t.p + 4r.\ p}{2r.t.}\right\}$$

Overall particle spectrum:





Massive particle

Pole residue:	$-\frac{S}{r\underset{2}{P}} > 0$
Square mass:	$\frac{t\underset{1}{P}}{r\underset{2}{P}} > 0$
Spin:	0
Parity:	Odd

Massless particle

Pole residue:	$-\frac{Y}{r\underset{5}{P}} - \frac{T p^2}{t\underset{1}{P}} - \frac{V r\underset{5}{P} p^4}{t\underset{1}{P}^2} > 0$
Polarisations:	2

Overall unitarity conditions:

$p \in \mathbb{R} \,\&\&\, r. < 0 \,\&\&\, r. < 0 \,\&\&\, t. < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\underset{2}{r.} < 0 \,\&\&\, \underset{5}{r.} < 0 \,\&\&\, \underset{1}{t.} < 0 \tag{28}$$

Okay, that concludes the analysis of this theory.

Case 4

Now for a new theory. Here is the full nonlinear Lagrangian for Case 4 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}\underset{1}{r.}\,\mathcal{R}_{ij\hbar l}\,\mathcal{R}^{ij\hbar l} + \frac{2}{3}\underset{1}{r.}\,\mathcal{R}_{i\hbar jl}\,\mathcal{R}^{ij\hbar l} + \underset{5}{r.}\,\mathcal{R}^{ij}{}_{i}{}^{\hbar}\,\mathcal{R}_{j}{}^{l}{}_{\hbar l} - \frac{2}{3}\underset{1}{r.}\,\mathcal{R}^{ij\hbar l}\,\mathcal{R}_{\hbar lij} - \underset{5}{r.}\,\mathcal{R}^{ij}{}_{i}{}^{\hbar}\,\mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{4}\underset{1}{t.}\,\mathcal{T}_{ij\hbar}\,\mathcal{T}^{ij\hbar} + \frac{1}{2}\underset{1}{t.}\,\mathcal{T}^{ij\hbar}\,\mathcal{T}_{ji\hbar} + \frac{1}{3}\underset{1}{t.}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{\hbar}{}_{j\hbar} \tag{29}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\underset{1}{t.}\,\mathcal{A}_{aib}\,\mathcal{A}^{abi} + \frac{1}{3}\underset{1}{t.}\,\mathcal{A}^{ab}{}_{a}\,\mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3}\underset{1}{t.}\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial_a f^{ab} + \frac{2}{3}\underset{1}{t.}\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial^b f^{a}{}_{a} - \\
&\frac{1}{3}\underset{1}{t.}\,\partial_b f^{i}{}_{i}\,\partial^b f^{a}{}_{a} - \frac{1}{3}\underset{1}{t.}\,\partial_a f^{ab}\,\partial_i f^{i}{}_{b} + \frac{2}{3}\underset{1}{t.}\,\partial^b f^{a}{}_{a}\,\partial_i f^{i}{}_{b} + \underset{5}{r.}\,\partial_b\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^i\mathcal{A}^{ab}{}_{a} - \\
&\underset{5}{r.}\,\partial_i\mathcal{A}_{b}{}^{j}{}_{j}\,\partial^i\mathcal{A}^{ab}{}_{a} + 2\underset{1}{t.}\,\mathcal{A}_{bia}\,\partial^i f^{ab} - \underset{1}{t.}\,\partial_a f_{bi}\,\partial^i f^{ab} + \frac{1}{2}\underset{1}{t.}\,\partial_a f_{ib}\,\partial^i f^{ab} - \frac{1}{2}\underset{1}{t.}\,\partial_b f_{ai}\,\partial^i f^{ab} + \\
&\frac{1}{2}\underset{1}{t.}\,\partial_i f_{ab}\,\partial^i f^{ab} + \frac{1}{2}\underset{1}{t.}\,\partial_i f_{ba}\,\partial^i f^{ab} - \underset{5}{r.}\,\partial_a\mathcal{A}^{abi}\,\partial_j\mathcal{A}_{b}{}^{j}{}_{i} + 2\underset{5}{r.}\,\partial^i\mathcal{A}^{ab}{}_{a}\,\partial_j\mathcal{A}_{b}{}^{j}{}_{i} + \\
&\underset{5}{r.}\,\partial_a\mathcal{A}^{abi}\,\partial_j\mathcal{A}_{i}{}^{j}{}_{b} - 2\underset{5}{r.}\,\partial^i\mathcal{A}^{ab}{}_{a}\,\partial_j\mathcal{A}_{i}{}^{j}{}_{b} - \frac{4}{3}\underset{1}{r.}\,\partial_b\mathcal{A}_{aij}\,\partial^j\mathcal{A}^{abi} + \frac{2}{3}\underset{1}{r.}\,\partial_b\mathcal{A}_{aji}\,\partial^j\mathcal{A}^{abi} - \\
&\frac{8}{3}\underset{1}{r.}\,\partial_b\mathcal{A}_{ija}\,\partial^j\mathcal{A}^{abi} - \frac{2}{3}\underset{1}{r.}\,\partial_i\mathcal{A}_{abj}\,\partial^j\mathcal{A}^{abi} + \frac{2}{3}\underset{1}{r.}\,\partial_j\mathcal{A}_{abi}\,\partial^j\mathcal{A}^{abi} + \frac{2}{3}\underset{1}{r.}\,\partial_j\mathcal{A}_{aib}\,\partial^j\mathcal{A}^{abi}
\end{aligned} \tag{30}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_{\dot{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^2\left(2 r_{\dot{1}} + r_{\dot{5}}\right) - \frac{t_{\dot{1}}}{2} & -\frac{t_{\dot{1}}}{\sqrt{2}} & \frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\left(r_{\dot{1}} + r_{\dot{5}}\right) + \frac{t_{\dot{1}}}{6} & \frac{t_{\dot{1}}}{3\sqrt{2}} & \frac{i k t_{\dot{1}}}{3} & 0 \\ 0 & 0 & 0 & \frac{t_{\dot{1}}}{3\sqrt{2}} & \frac{t_{\dot{1}}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{\dot{1}} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} i k t_{\dot{1}} & -\frac{1}{3} i \sqrt{2}\, k t_{\dot{1}} & \frac{2 k^2 t_{\dot{1}}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{\dot{1}}}{2} & -\frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 \\ \frac{i k t_{\dot{1}}}{\sqrt{2}} & k^2 t_{\dot{1}} & 0 \\ 0 & 0 & k^2 r_{\dot{1}} + \frac{t_{\dot{1}}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{0^+}{\cdot}\sigma^{\parallel} == 0$

$\overset{0^+}{\cdot}\tau^{\parallel} == 0$

$\overset{0^+}{\cdot}\tau^{\perp} == 0$

$\overset{1^-}{\cdot}\tau^{\perp\,a} == 0$

$2 i k\, \overset{1^-}{\cdot}\sigma^{\perp\,a} + \overset{1^-}{\cdot}\tau^{\parallel\,a} == 0$

$-i k\, \overset{1^+}{\cdot}\sigma^{\perp\,ab} + \overset{1^+}{\cdot}\tau^{\parallel\,ab} == 0$

$-2 i k\, \overset{2^+}{\cdot}\sigma^{\parallel\,ab} + \overset{2^+}{\cdot}\tau^{\parallel\,ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_{\dot{1}}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2k^2\left(2r_{1}+r_{5}\right)+t_{1}}{(1+k^2)^2 t_{1}^2} & \frac{i\left(2k^3\left(2r_{1}+r_{5}\right)-k\,t_{1}\right)}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & \frac{-2ik^3\left(2r_{1}+r_{5}\right)+i\,k\,t_{1}}{(1+k^2)^2 t_{1}^2} & \frac{-2k^4\left(2r_{1}+r_{5}\right)+k^2 t_{1}}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2\left(r_{1}+r_{5}\right)} & -\frac{1}{\sqrt{2}\left(k^2+2k^4\right)\left(r_{1}+r_{5}\right)} & -\frac{i}{k\left(1+2k^2\right)\left(r_{1}+r_{5}\right)} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}\left(k^2+2k^4\right)\left(r_{1}+r_{5}\right)} & \frac{6k^2\left(r_{1}+r_{5}\right)+t_{1}}{2\left(k+2k^3\right)^2\left(r_{1}+r_{5}\right)t_{1}} & \frac{i\left(6k^2\left(r_{1}+r_{5}\right)+t_{1}\right)}{\sqrt{2}\,k\left(1+2k^2\right)^2\left(r_{1}+r_{5}\right)t_{1}} & 0 \\ 0 & 0 & 0 & \frac{i}{k\left(1+2k^2\right)\left(r_{1}+r_{5}\right)} & -\frac{i\left(6k^2\left(r_{1}+r_{5}\right)+t_{1}\right)}{\sqrt{2}\,k\left(1+2k^2\right)^2\left(r_{1}+r_{5}\right)t_{1}} & \frac{6k^2\left(r_{1}+r_{5}\right)+t_{1}}{\left(1+2k^2\right)^2\left(r_{1}+r_{5}\right)t_{1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{\left(1+2k^2\right)^2 t_{1}} & -\frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} & 0 \\ \frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} & \frac{4k^2}{\left(1+2k^2\right)^2 t_{1}} & 0 \\ 0 & 0 & \frac{2}{2k^2 r_{1}+t_{1}} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{1}}{2r_{1}}\right\}\right\}$$

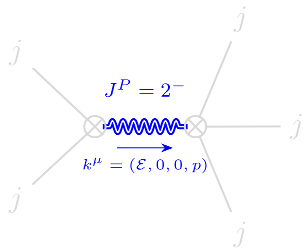
Massive pole residues:

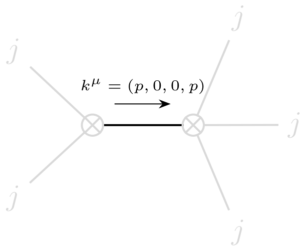
$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{1}}\right\}\right\}$$

Massless eigenvalues:

$$\left\{-\frac{7t_{1}^2+2r_{1}t_{1}p^2+2r_{5}t_{1}p^2+4r_{1}^2p^4+8r_{1}r_{5}p^4+4r_{5}^2p^4}{2\left(r_{1}+r_{5}\right)t_{1}^2}, -\frac{7t_{1}^2+2r_{1}t_{1}p^2+2r_{5}t_{1}p^2+4r_{1}^2p^4+8r_{1}r_{5}p^4+4r_{5}^2p^4}{2\left(r_{1}+r_{5}\right)t_{1}^2}\right\}$$

Overall particle spectrum:





Massive particle

Pole residue:	$-\frac{1}{r_{1}} > 0$
Square mass:	$-\frac{t_{1}}{2r_{1}} > 0$
Spin:	2
Parity:	Odd

Massless particle

Pole residue:	$-\frac{7}{r_{1}+r_{5}} + \frac{-2t_{1}p^2-4\left(r_{1}+r_{5}\right)p^4}{t_{1}^2} > 0$
Polarisations:	2

Overall unitarity conditions:

$p \in \mathbb{R} \,\&\&\, r_{1} < 0 \,\&\&\, r_{5} < -r_{1} \,\&\&\, t_{1} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\underset{1}{r} < 0 \,\&\&\, \underset{5}{r} < -\underset{1}{r} \,\&\&\, \underset{1}{t} > 0 \tag{31}$$

Okay, that concludes the analysis of this theory.

Case 5

Now for a new theory. Here is the full nonlinear Lagrangian for Case 5 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}\underset{1}{r}\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl}+\frac{2}{3}\underset{1}{r}\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl}+\underset{5}{r}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl}-\frac{2}{3}\underset{1}{r}\,\mathcal{R}^{ijkl}\,\mathcal{R}_{klij}-\underset{5}{r}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl}+\frac{1}{3}\underset{1}{t}\,\mathcal{T}_{ijk}\,\mathcal{T}^{ijk}+\frac{1}{3}\underset{1}{t}\,\mathcal{T}^{ijk}\,\mathcal{T}_{jik}+\underset{1}{t}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk} \tag{32}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\frac{1}{3}\underset{1}{t}\,\mathcal{A}_{abi}\,\mathcal{A}^{abi}+\frac{1}{3}\underset{1}{t}\,\mathcal{A}_{aib}\,\mathcal{A}^{abi}+\underset{1}{t}\,\mathcal{A}^{ab}{}_{a}\,\mathcal{A}_{b}{}^{i}{}_{i}-2\underset{1}{t}\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial_{a}f^{ab}+2\underset{1}{t}\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial^{b}f^{a}{}_{a}-\\&\underset{1}{t}\,\partial_{b}f^{i}{}_{i}\,\partial^{b}f^{a}{}_{a}-\underset{1}{t}\,\partial_{a}f^{ab}\,\partial_{i}f^{i}{}_{b}+2\underset{1}{t}\,\partial^{b}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{b}+\underset{5}{r}\,\partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}-\underset{5}{r}\,\partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}-\\&\frac{2}{3}\underset{1}{t}\,\mathcal{A}_{abi}\,\partial^{i}f^{ab}+\frac{2}{3}\underset{1}{t}\,\mathcal{A}_{aib}\,\partial^{i}f^{ab}+\frac{4}{3}\underset{1}{t}\,\mathcal{A}_{bia}\,\partial^{i}f^{ab}-\frac{2}{3}\underset{1}{t}\,\partial_{a}f_{bi}\,\partial^{i}f^{ab}+\frac{1}{3}\underset{1}{t}\,\partial_{a}f_{ib}\,\partial^{i}f^{ab}-\\&\frac{2}{3}\underset{1}{t}\,\partial_{b}f_{ai}\,\partial^{i}f^{ab}+\frac{2}{3}\underset{1}{t}\,\partial_{i}f_{ab}\,\partial^{i}f^{ab}+\frac{1}{3}\underset{1}{t}\,\partial_{i}f_{ba}\,\partial^{i}f^{ab}-\underset{5}{r}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}+2\underset{5}{r}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}+\\&\underset{5}{r}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}-2\underset{5}{r}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}-\frac{4}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{abi}-\\&\frac{8}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{abi}-\frac{2}{3}\underset{1}{r}\,\partial_{i}\mathcal{A}_{abj}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{abi}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{aib}\,\partial^{j}\mathcal{A}^{abi}\end{aligned} \tag{33}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix}-\underset{1}{t} & i\sqrt{2}\,k\,\underset{1}{t} & 0 & 0\\ -i\sqrt{2}\,k\,\underset{1}{t} & -2k^{2}\,\underset{1}{t} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} k^2\left(2 r_{\dot{1}} + r_{\dot{5}}\right) + \frac{t_{\dot{1}}}{6} & -\frac{t_{\dot{1}}}{3\sqrt{2}} & \frac{i k t_{\dot{1}}}{3\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{\dot{1}}}{3\sqrt{2}} & \frac{t_{\dot{1}}}{3} & -\frac{1}{3} i k t_{\dot{1}} & 0 & 0 & 0 & 0 \\ -\frac{i k t_{\dot{1}}}{3\sqrt{2}} & \frac{i k t_{\dot{1}}}{3} & \frac{k^2 t_{\dot{1}}}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\left(r_{\dot{1}} + r_{\dot{5}}\right) - \frac{t_{\dot{1}}}{2} & \frac{t_{\dot{1}}}{\sqrt{2}} & i k t_{\dot{1}} & 0 \\ 0 & 0 & 0 & \frac{t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i k t_{\dot{1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{\dot{1}}}{2} & -\frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 \\ \frac{i k t_{\dot{1}}}{\sqrt{2}} & k^2 t_{\dot{1}} & 0 \\ 0 & 0 & k^2 r_{\dot{1}} + \frac{t_{\dot{1}}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^-}_{\bullet}\sigma^{\parallel} == 0$$

$${}^{0^+}_{\bullet}\tau^{\perp} == 0$$

$$-2 i k\, {}^{0^+}_{\bullet}\sigma^{\parallel} + {}^{0^+}_{\bullet}\tau^{\parallel} == 0$$

$${}^{1^-}_{\bullet}\tau^{\perp\,a} == 0$$

$$2 i k\, {}^{1^-}_{\bullet}\sigma^{\perp\,a} + {}^{1^-}_{\bullet}\tau^{\parallel\,a} == 0$$

$$-i k\, {}^{1^+}_{\bullet}\sigma^{\perp\,ab} + {}^{1^+}_{\bullet}\tau^{\parallel\,ab} == 0$$

$$-2 i k\, {}^{2^+}_{\bullet}\sigma^{\parallel\,ab} + {}^{2^+}_{\bullet}\tau^{\parallel\,ab} == 0$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{\left(1+2k^2\right)^2 t_{\dot{1}}} & \frac{i\sqrt{2}\, k}{\left(1+2k^2\right)^2 t_{\dot{1}}} & 0 & 0 \\ -\frac{i\sqrt{2}\, k}{\left(1+2k^2\right)^2 t_{\dot{1}}} & -\frac{2k^2}{\left(1+2k^2\right)^2 t_{\dot{1}}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2 (2 r_{1}+r_{5})} & \frac{1}{\sqrt{2} (k^2+k^4) (2 r_{1}+r_{5})} & -\frac{i}{\sqrt{2} (k+k^3) (2 r_{1}+r_{5})} & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2} (k^2+k^4) (2 r_{1}+r_{5})} & \frac{6 k^2 (2 r_{1}+r_{5})+t_{1}}{2 (k+k^3)^2 (2 r_{1}+r_{5}) t_{1}} & -\frac{i (6 k^2 (2 r_{1}+r_{5})+t_{1})}{2 k (1+k^2)^2 (2 r_{1}+r_{5}) t_{1}} & 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{2} (k+k^3) (2 r_{1}+r_{5})} & \frac{i (6 k^2 (2 r_{1}+r_{5})+t_{1})}{2 k (1+k^2)^2 (2 r_{1}+r_{5}) t_{1}} & \frac{6 k^2 (2 r_{1}+r_{5})+t_{1}}{2 (1+k^2)^2 (2 r_{1}+r_{5}) t_{1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2 k^2 t_{1}} & \frac{2 i k}{t_{1}+2 k^2 t_{1}} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2 k^2 t_{1}} & \frac{-2 k^2 (r_{1}+r_{5})+t_{1}}{(t_{1}+2 k^2 t_{1})^2} & -\frac{i \sqrt{2} k (2 k^2 (r_{1}+r_{5})-t_{1})}{(t_{1}+2 k^2 t_{1})^2} & 0 \\ 0 & 0 & 0 & -\frac{2 i k}{t_{1}+2 k^2 t_{1}} & \frac{i \sqrt{2} k (2 k^2 (r_{1}+r_{5})-t_{1})}{(t_{1}+2 k^2 t_{1})^2} & \frac{-4 k^4 (r_{1}+r_{5})+2 k^2 t_{1}}{(t_{1}+2 k^2 t_{1})^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2 k^2)^2 t_{1}} & -\frac{2 i \sqrt{2} k}{(1+2 k^2)^2 t_{1}} & 0 \\ \frac{2 i \sqrt{2} k}{(1+2 k^2)^2 t_{1}} & \frac{4 k^2}{(1+2 k^2)^2 t_{1}} & 0 \\ 0 & 0 & \frac{2}{2 k^2 r_{1}+t_{1}} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{1}}{2 r_{1}}\right\}\right\}$$

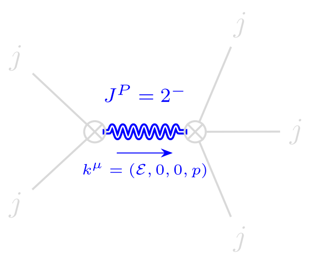
Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{1}}\right\}\right\}$$

Massless eigenvalues:

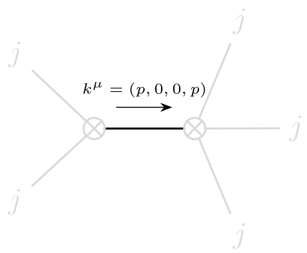
$$\left\{\frac{9 t_{1}^2+4 r_{1} t_{1} p^2+2 r_{5} t_{1} p^2+8 r_{1}^2 p^4+8 r_{1} r_{5} p^4+2 r_{5}^2 p^4}{(2 r_{1}+r_{5}) t_{1}^2}, \frac{9 t_{1}^2+4 r_{1} t_{1} p^2+2 r_{5} t_{1} p^2+8 r_{1}^2 p^4+8 r_{1} r_{5} p^4+2 r_{5}^2 p^4}{(2 r_{1}+r_{5}) t_{1}^2}\right\}$$

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{1}} > 0$
Square mass:	$-\frac{t_{1}}{2 r_{1}} > 0$
Spin:	2
Parity:	Odd



Massless particle

Pole residue:	$\frac{9}{2 r_{1}+r_{5}} + \frac{2 p^2 (t_{1}+(2 r_{1}+r_{5}) p^2)}{t_{1}^2} > 0$
Polarisations:	2

Overall unitarity conditions:

$p \in \mathbb{R}$ && $r_{1} < 0$ && $r_{5} > -2 r_{1}$ && $t_{1} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{1} < 0 \;\&\&\; r_{5} > -2 r_{1} \;\&\&\; t_{1} > 0 \tag{34}$$

Okay, that concludes the analysis of this theory.

Case 6

Now for a new theory. Here is the full nonlinear Lagrangian for Case 6 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ij\hbar l} \mathcal{R}^{ij\hbar l} - \frac{2}{3} r_{2} \mathcal{R}_{i\hbar jl} \mathcal{R}^{ij\hbar l} + \left(2 r_{3} + r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{j}{}^{l}{}_{\hbar l} + \frac{1}{6}\left(r_{2} - 6 r_{3}\right) \mathcal{R}^{ij\hbar l} \mathcal{R}_{\hbar lij} + \left(2 r_{3} - r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{4} t_{1} \mathcal{T}_{ij\hbar} \mathcal{T}^{ij\hbar} + \frac{1}{2} t_{1} \mathcal{T}^{ij\hbar} \mathcal{T}_{ji\hbar} + \frac{1}{3} t_{1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{\hbar}{}_{j\hbar} \tag{35}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned} & t_{1} \mathcal{A}_{aib} \mathcal{A}^{abi} + \frac{1}{3} t_{1} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3} t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} + \frac{2}{3} t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} - \\ & \frac{1}{3} t_{1} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} - \frac{1}{3} t_{1} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} + \frac{2}{3} t_{1} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} + \left(-2 r_{3} + r_{5}\right) \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + \\ & \left(-2 r_{3} - r_{5}\right) \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + 2 t_{1} \mathcal{A}_{bia} \partial^{i} f^{ab} - t_{1} \partial_{a} f_{bi} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{a} f_{ib} \partial^{i} f^{ab} - \\ & \frac{1}{2} t_{1} \partial_{b} f_{ai} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{i} f_{ab} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{i} f_{ba} \partial^{i} f^{ab} + \left(-2 r_{3} - r_{5}\right) \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \\ & 2\left(2 r_{3} + r_{5}\right) \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \left(-2 r_{3} + r_{5}\right) \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + \left(4 r_{3} - 2 r_{5}\right) \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + \\ & \frac{4}{3} r_{2} \partial_{b} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{2} \partial_{b} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3}\left(r_{2} - 6 r_{3}\right) \partial_{b} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{abi} - \\ & \frac{1}{3} r_{2} \partial_{i} \mathcal{A}_{abj} \partial^{j} \mathcal{A}^{abi} + \frac{1}{3} r_{2} \partial_{j} \mathcal{A}_{abi} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{2} \partial_{j} \mathcal{A}_{aib} \partial^{j} \mathcal{A}^{abi} \end{aligned} \tag{36}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} 6 k^2 r_{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{2} - t_{1} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} k^2\left(2 r_{3}+r_{5}\right)-\frac{t_{1}}{2} & -\frac{t_{1}}{\sqrt{2}} & \frac{i k t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\left(2 r_{3}+r_{5}\right)+\frac{t_{1}}{6} & \frac{t_{1}}{3\sqrt{2}} & \frac{i k t_{1}}{3} & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{3\sqrt{2}} & \frac{t_{1}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{1} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} i k t_{1} & -\frac{1}{3} i \sqrt{2}\, k t_{1} & \frac{2 k^2 t_{1}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{2} & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & k^2 t_{1} & 0 \\ 0 & 0 & \frac{t_{1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\bullet}\tau^{\perp} == 0$$

$${}^{0^+}_{\bullet}\tau^{\parallel} == 0$$

$${}^{1^-}_{\bullet}\tau^{\perp\,a} == 0$$

$$2 i k\, {}^{1^-}_{\bullet}\sigma^{\perp\,a} + {}^{1^-}_{\bullet}\tau^{\parallel\,a} == 0$$

$$-i k\, {}^{1^+}_{\bullet}\sigma^{\perp\,ab} + {}^{1^+}_{\bullet}\tau^{\parallel\,ab} == 0$$

$$-2 i k\, {}^{2^+}_{\bullet}\sigma^{\parallel\,ab} + {}^{2^+}_{\bullet}\tau^{\parallel\,ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{6 k^2 r_{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{2}-t_{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{i\sqrt{2}\, k}{t_{1}+k^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2 k^2\left(2 r_{3}+r_{5}\right)+t_{1}}{(1+k^2)^2 t_{1}^2} & \frac{i\left(2 k^3\left(2 r_{3}+r_{5}\right)-k t_{1}\right)}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\, k}{t_{1}+k^2 t_{1}} & \frac{-2 i k^3\left(2 r_{3}+r_{5}\right)+i k t_{1}}{(1+k^2)^2 t_{1}^2} & \frac{-2 k^4\left(2 r_{3}+r_{5}\right)+k^2 t_{1}}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2\left(2 r_{3}+r_{5}\right)} & -\frac{1}{\sqrt{2}\left(k^2+2 k^4\right)\left(2 r_{3}+r_{5}\right)} & -\frac{i}{k\left(1+2 k^2\right)\left(2 r_{3}+r_{5}\right)} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}\left(k^2+2 k^4\right)\left(2 r_{3}+r_{5}\right)} & \frac{6 k^2\left(2 r_{3}+r_{5}\right)+t_{1}}{2\left(k+2 k^3\right)^2\left(2 r_{3}+r_{5}\right) t_{1}} & \frac{i\left(6 k^2\left(2 r_{3}+r_{5}\right)+t_{1}\right)}{\sqrt{2}\, k\left(1+2 k^2\right)^2\left(2 r_{3}+r_{5}\right) t_{1}} & 0 \\ 0 & 0 & 0 & \frac{i}{k\left(1+2 k^2\right)\left(2 r_{3}+r_{5}\right)} & -\frac{i\left(6 k^2\left(2 r_{3}+r_{5}\right)+t_{1}\right)}{\sqrt{2}\, k\left(1+2 k^2\right)^2\left(2 r_{3}+r_{5}\right) t_{1}} & \frac{6 k^2\left(2 r_{3}+r_{5}\right)+t_{1}}{\left(1+2 k^2\right)^2\left(2 r_{3}+r_{5}\right) t_{1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_1} & -\frac{2 i \sqrt{2} k}{(1+2k^2)^2 t_1} & 0 \\ \frac{2 i \sqrt{2} k}{(1+2k^2)^2 t_1} & \frac{4k^2}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

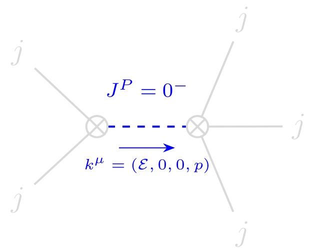
Massive pole residues:

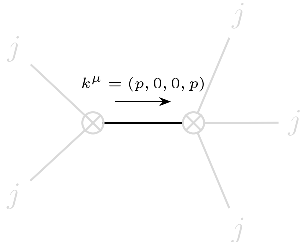
$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\left\{-\frac{7 t_1^2 + 4 r_3 t_1 p^2 + 2 r_5 t_1 p^2 + 16 r_3^2 p^4 + 16 r_3 r_5 p^4 + 4 r_5^2 p^4}{2 (2 r_3 + r_5) t_1^2}, -\frac{7 t_1^2 + 4 r_3 t_1 p^2 + 2 r_5 t_1 p^2 + 16 r_3^2 p^4 + 16 r_3 r_5 p^4 + 4 r_5^2 p^4}{2 (2 r_3 + r_5) t_1^2}\right\}$$

Overall particle spectrum:





Massive particle

Pole residue:	$-\frac{1}{r_2} > 0$
Square mass:	$\frac{t_1}{r_2} > 0$
Spin:	0
Parity:	Odd

Massless particle

Pole residue:	$-\frac{7}{2 r_3 + r_5} + \frac{-2 t_1 p^2 - 4 (2 r_3 + r_5) p^4}{t_1^2} > 0$
Polarisations:	2

Overall unitarity conditions:

$$(p \mid r.) \in \mathbb{R} \;\&\&\; r. < 0 \;\&\&\; r. < -2 r. \;\&\&\; t. < 0$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_3 \in \mathbb{R} \;\&\&\; r_2 < 0 \;\&\&\; r_5 < -2 r_3 \;\&\&\; t_1 < 0 \qquad (37)$$

Okay, that concludes the analysis of this theory.

Case 7

Now for a new theory. Here is the full nonlinear Lagrangian for Case 7 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{1} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} + \frac{2}{3} r_{1} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \left(-2 r_{1} + 2 r_{3} + r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{3} \left(r_{1} - 3 r_{3}\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij} + \left(-2 r_{1} + 2 r_{3} - r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4} t_{1} \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{2} t_{1} \mathcal{T}^{ijk} \mathcal{T}_{jik} + \frac{1}{3} t_{1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk} \tag{38}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
& t_{1} \mathcal{A}_{aib} \mathcal{A}^{abi} + \frac{1}{3} t_{1} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3} t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} + \frac{2}{3} t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} - \\
& \frac{1}{3} t_{1} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} - \frac{1}{3} t_{1} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} + \frac{2}{3} t_{1} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} + \left(2 r_{1} - 2 r_{3} + r_{5}\right) \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + \\
& \left(2 r_{1} - 2 r_{3} - r_{5}\right) \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + 2 t_{1} \mathcal{A}_{bia} \partial^{i} f^{ab} - t_{1} \partial_{a} f_{bi} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{a} f_{ib} \partial^{i} f^{ab} - \\
& \frac{1}{2} t_{1} \partial_{b} f_{ai} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{i} f_{ab} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{i} f_{ba} \partial^{i} f^{ab} + \left(2 r_{1} - 2 r_{3} - r_{5}\right) \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \\
& \left(-4 r_{1} + 4 r_{3} + 2 r_{5}\right) \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \left(2 r_{1} - 2 r_{3} + r_{5}\right) \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - \\
& 2 \left(2 r_{1} - 2 r_{3} + r_{5}\right) \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - \frac{4}{3} r_{1} \partial_{b} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{b} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{abi} + \\
& \frac{4}{3} \left(r_{1} - 3 r_{3}\right) \partial_{b} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{1} \partial_{i} \mathcal{A}_{abj} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{abi} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{aib} \partial^{j} \mathcal{A}^{abi}
\end{aligned} \tag{39}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 6 k^2 \left(-r_{1} + r_{3}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_{1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
k^2 \left(2 r_{3} + r_{5}\right) - \frac{t_{1}}{2} & -\frac{t_{1}}{\sqrt{2}} & \frac{i k t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
-\frac{t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{i k t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k^2 \left(-r_{1} + 2 r_{3} + r_{5}\right) + \frac{t_{1}}{6} & \frac{t_{1}}{3 \sqrt{2}} & \frac{i k t_{1}}{3} & 0 \\
0 & 0 & 0 & \frac{t_{1}}{3 \sqrt{2}} & \frac{t_{1}}{3} & \frac{1}{3} i \sqrt{2} k t_{1} & 0 \\
0 & 0 & 0 & -\frac{1}{3} i k t_{1} & -\frac{1}{3} i \sqrt{2} k t_{1} & \frac{2 k^2 t_{1}}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{2} & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & k^2 t_{1} & 0 \\ 0 & 0 & k^2 r_{1} + \frac{t_{1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\bullet}\tau^{\perp} == 0$$

$${}^{0^+}_{\bullet}\tau^{\parallel} == 0$$

$${}^{1^-}_{\bullet}\tau^{\perp}{}^{a} == 0$$

$$2\,i\,k\;{}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} + {}^{1^-}_{\bullet}\tau^{\parallel}{}^{a} == 0$$

$$-i\,k\;{}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab} + {}^{1^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

$$-2\,i\,k\;{}^{2^+}_{\bullet}\sigma^{\parallel}{}^{ab} + {}^{2^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{6k^2(-r_1+r_3)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{-2k^2(2r_3+r_5)+t_1}{(1+k^2)^2 t_1^2} & \frac{i\left(2k^3(2r_3+r_5)-k t_1\right)}{(1+k^2)^2 t_1^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & \frac{-2ik^3(2r_3+r_5)+i k t_1}{(1+k^2)^2 t_1^2} & \frac{-2k^4(2r_3+r_5)+k^2 t_1}{(1+k^2)^2 t_1^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2(-r_1+2r_3+r_5)} & \frac{1}{\sqrt{2}(k^2+2k^4)(r_1-2r_3-r_5)} & \frac{i}{k(1+2k^2)(r_1-2r_3-r_5)} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}(k^2+2k^4)(r_1-2r_3-r_5)} & \frac{\frac{1}{-r_1+2r_3+r_5}+\frac{6k^2}{t_1}}{2(k+2k^3)^2} & \frac{i\left(6k^2(r_1-2r_3-r_5)-t_1\right)}{\sqrt{2}\,k(1+2k^2)^2(r_1-2r_3-r_5)t_1} & 0 \\ 0 & 0 & 0 & \frac{i}{k(1+2k^2)(-r_1+2r_3+r_5)} & -\frac{i\left(6k^2(r_1-2r_3-r_5)-t_1\right)}{\sqrt{2}\,k(1+2k^2)^2(r_1-2r_3-r_5)t_1} & \frac{\frac{1}{-r_1+2r_3+r_5}+\frac{6k^2}{t_1}}{(1+2k^2)^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_1} & -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & 0 \\ \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & \frac{4k^2}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{2k^2 r_1+t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\},\{\},\{\},\{\},\{\},\left\{-\frac{t_1}{2r_1}\right\}\right\}$$

Massive pole residues:

$$\left\{\{\},\{\},\{\},\{\},\{\},\left\{-\frac{1}{r_1}\right\}\right\}$$

Massless eigenvalues:

$$\left\{\frac{7 t_{1}^{2}-2 r_{1} t_{1} p^{2}+4 r_{3} t_{1} p^{2}+2 r_{5} t_{1} p^{2}+4 r_{1}^{2} p^{4}-16 r_{1} r_{3} p^{4}+16 r_{3}^{2} p^{4}-8 r_{1} r_{5} p^{4}+16 r_{3} r_{5} p^{4}+4 r_{5}^{2} p^{4}}{2\left(r_{1}-2 r_{3}-r_{5}\right) t_{1}^{2}},\right.$$
$$\left.\frac{7 t_{1}^{2}-2 r_{1} t_{1} p^{2}+4 r_{3} t_{1} p^{2}+2 r_{5} t_{1} p^{2}+4 r_{1}^{2} p^{4}-16 r_{1} r_{3} p^{4}+16 r_{3}^{2} p^{4}-8 r_{1} r_{5} p^{4}+16 r_{3} r_{5} p^{4}+4 r_{5}^{2} p^{4}}{2\left(r_{1}-2 r_{3}-r_{5}\right) t_{1}^{2}}\right\}$$

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Overall particle spectrum:
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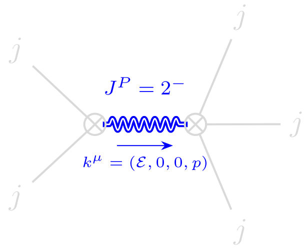
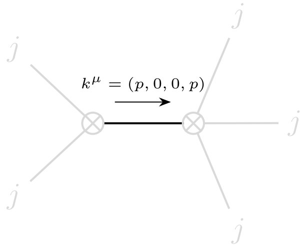




Massive particle

Pole residue:	$-\frac{1}{r_{1}} > 0$
Square mass:	$-\frac{t_{1}}{2 r_{1}} > 0$
Spin:	2
Parity:	Odd

Massless particle

Pole residue:	$\frac{7}{r_{1}-2 r_{3}-r_{5}} + \frac{-2 t_{1} p^{2}+4\left(r_{1}-2 r_{3}-r_{5}\right) p^{4}}{t_{1}^{2}} > 0$
Polarisations:	2

```
Overall unitarity conditions:
```

$\left(p \mid r_{\cdot}\right) \in \mathbb{R}\ \&\&\ r_{\cdot} < 0\ \&\&\ r_{\cdot} < r_{\cdot} - 2 r_{\cdot}\ \&\&\ t_{\cdot} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{3} \in \mathbb{R}\ \&\&\ r_{1} < 0\ \&\&\ r_{5} < r_{1} - 2 r_{3}\ \&\&\ t_{1} > 0 \tag{40}$$

Okay, that concludes the analysis of this theory.

Case 8

Now for a new theory. Here is the full nonlinear Lagrangian for Case 8 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{1} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} + \frac{2}{3} r_{1} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + r_{5} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} - \frac{2}{3} r_{1} \mathcal{R}^{ijkl} \mathcal{R}_{klij} - r_{5} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} - \frac{2}{3} t_{3} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk} \tag{41}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$-\frac{2}{3} t_{3} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} + \frac{4}{3} t_{3} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} - \frac{4}{3} t_{3} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{3} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{3} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} - \frac{4}{3} t_{3} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} + r_{5} \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} - r_{5} \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} - r_{5} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + 2 r_{5} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + r_{5} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - 2 r_{5} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - \frac{4}{3} r_{1} \partial_{b} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{b} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{abi} - \tag{42}$$

$$\frac{8}{3} r_{1}\, \partial_b \mathcal{A}_{ija}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{1}\, \partial_i \mathcal{A}_{abj}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_{1}\, \partial_j \mathcal{A}_{abi}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_{1}\, \partial_j \mathcal{A}_{aib}\, \partial^j \mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} t_{3} & -i\sqrt{2}\, k\, t_{3} & 0 & 0 \\ i\sqrt{2}\, k\, t_{3} & 2k^2 t_{3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^2\left(2 r_{1} + r_{5}\right) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\left(r_{1} + r_{5}\right) + \frac{2 t_{3}}{3} & -\frac{\sqrt{2}\, t_{3}}{3} & -\frac{2}{3} i\, k\, t_{3} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}\, t_{3}}{3} & \frac{t_{3}}{3} & \frac{1}{3} i\sqrt{2}\, k\, t_{3} & 0 \\ 0 & 0 & 0 & \frac{2 i k t_{3}}{3} & -\frac{1}{3} i\sqrt{2}\, k\, t_{3} & \frac{2 k^2 t_{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k^2 r_{1} \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^-}\sigma^{\parallel} == 0$

${}^{0^+}\tau^{\perp} == 0$

$-2 i k\, {}^{0^+}\sigma^{\parallel} + {}^{0^+}\tau^{\parallel} == 0$

${}^{1^-}\tau^{\perp\, a} == 0$

$2 i k\, {}^{1^-}\sigma^{\perp\, a} + {}^{1^-}\tau^{\parallel\, a} == 0$

${}^{1^+}\tau^{\parallel\, ab} == 0$

${}^{1^+}\sigma^{\perp\, ab} == 0$

${}^{2^+}\tau^{\parallel\, ab} == 0$

${}^{2^+}\sigma^{\parallel\, ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{\left(1+2k^2\right)^2 t_{3}} & -\frac{i\sqrt{2}\, k}{\left(1+2k^2\right)^2 t_{3}} & 0 & 0 \\ \frac{i\sqrt{2}\, k}{\left(1+2k^2\right)^2 t_{3}} & \frac{2k^2}{\left(1+2k^2\right)^2 t_{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2 (2 r_1 + r_5)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 (r_1 + r_5)} & \frac{\sqrt{2}}{k^2 (1+2k^2)(r_1 + r_5)} & \frac{2 i}{k (1+2k^2)(r_1 + r_5)} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{k^2 (1+2k^2)(r_1 + r_5)} & \frac{3 k^2 (r_1 + r_5) + 2 t_3}{(k + 2k^3)^2 (r_1 + r_5) t_3} & \frac{i \sqrt{2} (3 k^2 (r_1 + r_5) + 2 t_3)}{k (1+2k^2)^2 (r_1 + r_5) t_3} & 0 \\ 0 & 0 & 0 & -\frac{2 i}{k (1+2k^2)(r_1 + r_5)} & -\frac{i \sqrt{2} (3 k^2 (r_1 + r_5) + 2 t_3)}{k (1+2k^2)^2 (r_1 + r_5) t_3} & \frac{6 k^2 (r_1 + r_5) + 4 t_3}{(1+2k^2)^2 (r_1 + r_5) t_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{k^2 r_1} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

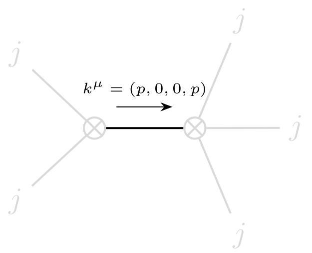
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{ \frac{-5 r_1^2 - 4 r_1 r_5 - 3 r_5^2}{r_1 (r_1 + r_5)(2 r_1 + r_5)}, \frac{-5 r_1^2 - 4 r_1 r_5 - 3 r_5^2}{r_1 (r_1 + r_5)(2 r_1 + r_5)} \right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{3}{r_1} - \frac{4}{r_1 + r_5} + \frac{9}{2 r_1 + r_5} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(r_1 < 0 \,\&\&\, \left(r_5 < -r_1 \,\|\, r_5 > -2 r_1\right)\right) \,\|\, \left(r_1 > 0 \,\&\&\, -2 r_1 < r_5 < -r_1\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_1 < 0 \,\&\&\, \left(r_5 < -r_1 \,\|\, r_5 > -2 r_1\right)\right) \,\|\, \left(r_1 > 0 \,\&\&\, -2 r_1 < r_5 < -r_1\right) \tag{43}$$

Okay, that concludes the analysis of this theory.

Case 9

Now for a new theory. Here is the full nonlinear Lagrangian for Case 9 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{1} \mathcal{R}_{ij\hbar l} \mathcal{R}^{ij\hbar l} + \frac{2}{3} r_{1} \mathcal{R}_{i\hbar jl} \mathcal{R}^{ij\hbar l} + r_{5} \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{j}{}^{l}{}_{\hbar l} - \frac{2}{3} r_{1} \mathcal{R}^{ij\hbar l} \mathcal{R}_{\hbar lij} - r_{5} \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{\hbar}{}^{l}{}_{jl} \tag{44}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned} & r_{5} \partial_{b}\mathcal{A}_{i}{}^{j}{}_{j} \partial^{i}\mathcal{A}^{ab}{}_{a} - r_{5} \partial_{i}\mathcal{A}_{b}{}^{j}{}_{j} \partial^{i}\mathcal{A}^{ab}{}_{a} - r_{5} \partial_{a}\mathcal{A}^{abi} \partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} + 2 r_{5} \partial^{i}\mathcal{A}^{ab}{}_{a} \partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} + \\ & r_{5} \partial_{a}\mathcal{A}^{abi} \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} - 2 r_{5} \partial^{i}\mathcal{A}^{ab}{}_{a} \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} - \frac{4}{3} r_{1} \partial_{b}\mathcal{A}_{aij} \partial^{j}\mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{b}\mathcal{A}_{aji} \partial^{j}\mathcal{A}^{abi} - \\ & \frac{8}{3} r_{1} \partial_{b}\mathcal{A}_{ija} \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{1} \partial_{i}\mathcal{A}_{abj} \partial^{j}\mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{j}\mathcal{A}_{abi} \partial^{j}\mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{j}\mathcal{A}_{aib} \partial^{j}\mathcal{A}^{abi} \end{aligned} \tag{45}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^2 (2 r_{1} + r_{5}) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 (r_{1} + r_{5}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k^2 r_{1} \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^-}_{\cdot}\sigma^{\parallel} == 0$

${}^{0^+}_{\cdot}\tau^{\perp} == 0$

${}^{0^+}_{\cdot}\tau^{\parallel} == 0$

${}^{0^+}_{\cdot}\sigma^{\parallel} == 0$

${}^{1^-}_{\cdot}\tau^{\perp a} == 0$

${}^{1^-}_{\cdot}\tau^{\parallel a} == 0$

${}^{1^-}_{\cdot}\sigma^{\perp a} == 0$

${}^{1^+}_{\cdot}\tau^{\parallel ab} == 0$

${}^{1^+}_{\cdot}\sigma^{\perp ab} == 0$

$${}^{2^+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

$${}^{2^+}_{\cdot}\sigma^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2\left(2 r_1 + r_5\right)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2\left(r_1 + r_5\right)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{k^2 r_1} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

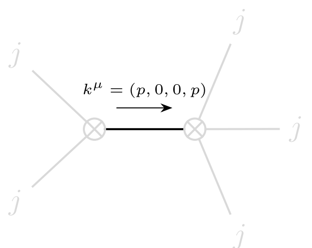
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{-4 r_1^2 - 4 r_1 r_5 - 3 r_5^2}{r_1\left(r_1 + r_5\right)\left(2 r_1 + r_5\right)}, \frac{-4 r_1^2 - 4 r_1 r_5 - 3 r_5^2}{r_1\left(r_1 + r_5\right)\left(2 r_1 + r_5\right)}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{3}{r_1} - \frac{3}{r_1 + r_5} + \frac{8}{2 r_1 + r_5} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(r_1 < 0 \,\&\&\, \left(r_5 < -r_1 \,\|\, r_5 > -2 r_1\right)\right) \,\|\, \left(r_1 > 0 \,\&\&\, -2 r_1 < r_5 < -r_1\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{1} < 0 \,\&\&\, \left(r_{5} < -r_{1} \,||\, r_{5} > -2\,r_{1}\right)\right) \,||\, \left(r_{1} > 0 \,\&\&\, -2\,r_{1} < r_{5} < -r_{1}\right) \tag{46}$$

Okay, that concludes the analysis of this theory.

Case 10

Now for a new theory. Here is the full nonlinear Lagrangian for Case 10 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\left(\frac{r_{3}}{2} + r_{5}\right) \mathcal{R}^{ij\hbar}{}_{i}\, \mathcal{R}_{j}{}^{l}{}_{\hbar l} - r_{3}\, \mathcal{R}^{ij\hbar l}\, \mathcal{R}_{\hbar lij} + \frac{1}{2}\left(r_{3} - 2\,r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{\hbar}\, \mathcal{R}_{\hbar}{}^{l}{}_{jl} \tag{47}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\left(-\frac{r_{3}}{2} + r_{5}\right) \partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{ab}{}_{a} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{ab}{}_{a} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_{a}\mathcal{A}^{abi}\, \partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} + \\ &\left(r_{3} + 2\,r_{5}\right) \partial^{i}\mathcal{A}^{ab}{}_{a}\, \partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} + \left(-\frac{r_{3}}{2} + r_{5}\right) \partial_{a}\mathcal{A}^{abi}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + \left(r_{3} - 2\,r_{5}\right) \partial^{i}\mathcal{A}^{ab}{}_{a}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} - 4\,r_{3}\, \partial_{b}\mathcal{A}_{ija}\, \partial^{j}\mathcal{A}^{abi}\end{aligned} \tag{48}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^2\left(2\,r_{3} + r_{5}\right) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} k^2\left(r_{3} + 2\,r_{5}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{3\,k^2\, r_{3}}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{0^-}{\cdot}\sigma^{\parallel} == 0$

$\overset{0^+}{\cdot}\tau^{\perp} == 0$

$\overset{0^+}{\cdot}\tau^{\parallel} == 0$

$\overset{0^+}{\cdot}\sigma^{\parallel} == 0$

$\overset{1^-}{\cdot}\tau^{\perp\,a} == 0$

${}^{1^-}_{\bullet}\tau^{\parallel}{}^{a} == 0$

${}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} == 0$

${}^{1^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$

${}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab} == 0$

${}^{2^-}_{\bullet}\sigma^{\parallel}{}^{abc} == 0$

${}^{2^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2\left(2 r_{\underset{3}{\cdot}}+r_{\underset{5}{\cdot}}\right)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{k^2\left(r_{\underset{3}{\cdot}}+2 r_{\underset{5}{\cdot}}\right)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{3 k^2 r_{\underset{3}{\cdot}}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

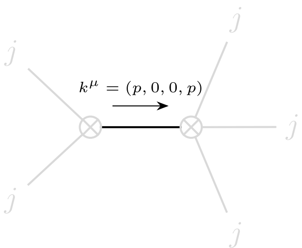
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{33 r_{\underset{3}{\cdot}}^2+20 r_{\underset{3}{\cdot}} r_{\underset{5}{\cdot}}+4 r_{\underset{5}{\cdot}}^2}{r_{\underset{3}{\cdot}}\left(2 r_{\underset{3}{\cdot}}+r_{\underset{5}{\cdot}}\right)\left(r_{\underset{3}{\cdot}}+2 r_{\underset{5}{\cdot}}\right)}, -\frac{33 r_{\underset{3}{\cdot}}^2+20 r_{\underset{3}{\cdot}} r_{\underset{5}{\cdot}}+4 r_{\underset{5}{\cdot}}^2}{r_{\underset{3}{\cdot}}\left(2 r_{\underset{3}{\cdot}}+r_{\underset{5}{\cdot}}\right)\left(r_{\underset{3}{\cdot}}+2 r_{\underset{5}{\cdot}}\right)}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{T}{rP_{3}} + \frac{U}{T\, rP_{3} M\, rP_{5}} - \frac{SX}{rP_{3} M\, T\, rP_{5}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(r_{3} < 0 \,\&\&\, \left(r_{5} < -\frac{r_{3}}{2} \,\|\, r_{5} > -2\, r_{3}\right)\right) \| \left(r_{3} > 0 \,\&\&\, -2\, r_{3} < r_{5} < -\frac{r_{3}}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{3} < 0 \,\&\&\, \left(r_{5} < -\frac{r_{3}}{2} \,\|\, r_{5} > -2\, r_{3}\right)\right) \| \left(r_{3} > 0 \,\&\&\, -2\, r_{3} < r_{5} < -\frac{r_{3}}{2}\right) \tag{49}$$

Okay, that concludes the analysis of this theory.

Case 11

Now for a new theory. Here is the full nonlinear Lagrangian for Case 11 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{.} \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} - \frac{2}{3} r_{.} \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} + \left(\frac{r_{3}}{2} + r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6}\left(r_{.} - 6\, r_{.}\right) \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + \frac{1}{2}\left(r_{.} - 2\, r_{.}\right) \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl} \tag{50}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\left(-\frac{r_{3}}{2} + r_{5}\right) \partial_b \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^i \mathcal{A}^{ab}{}_{a} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_i \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^i \mathcal{A}^{ab}{}_{a} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_{b}{}^{j}{}_{i} + \\
&\left(r_{3} + 2\, r_{5}\right) \partial^i \mathcal{A}^{ab}{}_{a}\, \partial_j \mathcal{A}_{b}{}^{j}{}_{i} + \left(-\frac{r_{3}}{2} + r_{5}\right) \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{b} + \left(r_{3} - 2\, r_{5}\right) \partial^i \mathcal{A}^{ab}{}_{a}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{b} + \\
&\frac{4}{3} r_{2}\, \partial_b \mathcal{A}_{aij}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{2}\, \partial_b \mathcal{A}_{aji}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3}\left(r_{2} - 6\, r_{3}\right) \partial_b \mathcal{A}_{ija}\, \partial^j \mathcal{A}^{abi} - \\
&\frac{1}{3} r_{2}\, \partial_i \mathcal{A}_{abj}\, \partial^j \mathcal{A}^{abi} + \frac{1}{3} r_{2}\, \partial_j \mathcal{A}_{abi}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{2}\, \partial_j \mathcal{A}_{aib}\, \partial^j \mathcal{A}^{abi}
\end{aligned} \tag{51}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{\underset{2}{\bullet}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} k^2 \left(r_{\underset{3}{\bullet}} + 2 r_{\underset{5}{\bullet}}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{3 k^2 r_{\underset{3}{\bullet}}}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^+}_{\bullet}\tau^{\perp} == 0$

${}^{0^+}_{\bullet}\tau^{\parallel} == 0$

${}^{0^+}_{\bullet}\sigma^{\parallel} == 0$

${}^{1^-}_{\bullet}\tau^{\perp}{}^{a} == 0$

${}^{1^-}_{\bullet}\tau^{\parallel}{}^{a} == 0$

${}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} == 0$

${}^{1^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$

${}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab} == 0$

${}^{2^-}_{\bullet}\sigma^{\parallel}{}^{abc} == 0$

${}^{2^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{\underset{2}{\bullet}}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2 \left(2 r_{\underset{3}{\bullet}} + r_{\underset{5}{\bullet}}\right)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{k^2 \left(r_{\underset{3}{\bullet}} + 2 r_{\underset{5}{\bullet}}\right)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{3k^2 r_{\dot{3}}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

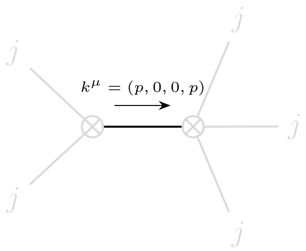
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{33 r_{\dot{3}}^2 + 20 r_{\dot{3}} r_{\dot{5}} + 4 r_{\dot{5}}^2}{r_{\dot{3}}\left(2 r_{\dot{3}} + r_{\dot{5}}\right)\left(r_{\dot{3}} + 2 r_{\dot{5}}\right)}, -\frac{33 r_{\dot{3}}^2 + 20 r_{\dot{3}} r_{\dot{5}} + 4 r_{\dot{5}}^2}{r_{\dot{3}}\left(2 r_{\dot{3}} + r_{\dot{5}}\right)\left(r_{\dot{3}} + 2 r_{\dot{5}}\right)}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{T}{rP_3} + \frac{U}{T\, rP_3 M\, rP_5} - \frac{SX}{rP_3 M\, T\, rP_5} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(r_{\dot{3}} < 0 \,\&\&\, \left(r_{\dot{5}} < -\frac{r_{\dot{3}}}{2} \,\|\, r_{\dot{5}} > -2 r_{\dot{3}}\right)\right) \| \left(r_{\dot{3}} > 0 \,\&\&\, -2 r_{\dot{3}} < r_{\dot{5}} < -\frac{r_{\dot{3}}}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{\dot{3}} < 0 \,\&\&\, \left(r_{\dot{5}} < -\frac{r_{\dot{3}}}{2} \,\|\, r_{\dot{5}} > -2 r_{\dot{3}}\right)\right) \| \left(r_{\dot{3}} > 0 \,\&\&\, -2 r_{\dot{3}} < r_{\dot{5}} < -\frac{r_{\dot{3}}}{2}\right) \tag{52}$$

Okay, that concludes the analysis of this theory.

Case 12

Now for a new theory. Here is the full nonlinear Lagrangian for Case 12 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\left(\frac{r_{\dot{3}}}{2} + r_{\dot{5}}\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} - r_{\dot{3}} \mathcal{R}^{ijkl} \mathcal{R}_{klij} + \frac{1}{2}\left(r_{\dot{3}} - 2 r_{\dot{5}}\right) \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12} t_{\dot{2}} \mathcal{T}_{ijk} \mathcal{T}^{ijk} - \frac{1}{6} t_{\dot{2}} \mathcal{T}^{ijk} \mathcal{T}_{jik} \tag{53}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\frac{1}{3}\dot{t}_{2}\,\mathcal{A}_{abi}\,\mathcal{A}^{abi}-\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{aib}\,\mathcal{A}^{abi}+\left(-\frac{\dot{r}_{3}}{2}+\dot{r}_{5}\right)\partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}+\left(-\frac{\dot{r}_{3}}{2}-\dot{r}_{5}\right)\partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}-\\&\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{abi}\,\partial^{i}f^{ab}+\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{aib}\,\partial^{i}f^{ab}-\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{bia}\,\partial^{i}f^{ab}+\frac{1}{3}\dot{t}_{2}\,\partial_{a}f_{bi}\,\partial^{i}f^{ab}-\frac{1}{6}\dot{t}_{2}\,\partial_{a}f_{ib}\,\partial^{i}f^{ab}-\\&\frac{1}{6}\dot{t}_{2}\,\partial_{b}f_{ai}\,\partial^{i}f^{ab}+\frac{1}{6}\dot{t}_{2}\,\partial_{i}f_{ab}\,\partial^{i}f^{ab}-\frac{1}{6}\dot{t}_{2}\,\partial_{i}f_{ba}\,\partial^{i}f^{ab}+\left(-\frac{\dot{r}_{3}}{2}-\dot{r}_{5}\right)\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}+\\&\left(\dot{r}_{3}+2\dot{r}_{5}\right)\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}+\left(-\frac{\dot{r}_{3}}{2}+\dot{r}_{5}\right)\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}+\left(\dot{r}_{3}-2\dot{r}_{5}\right)\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}-4\dot{r}_{3}\,\partial_{b}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{abi}\end{aligned}\tag{54}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&\dot{t}_{2}\end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}k^{2}\left(2\dot{r}_{3}+\dot{r}_{5}\right)+\frac{2\dot{t}_{2}}{3}&\frac{\sqrt{2}\,\dot{t}_{2}}{3}&-\frac{1}{3}i\sqrt{2}\,k\dot{t}_{2}&0&0&0&0\\\frac{\sqrt{2}\,\dot{t}_{2}}{3}&\frac{\dot{t}_{2}}{3}&-\frac{1}{3}i\,k\dot{t}_{2}&0&0&0&0\\\frac{1}{3}i\sqrt{2}\,k\dot{t}_{2}&\frac{i k\dot{t}_{2}}{3}&\frac{k^{2}\dot{t}_{2}}{3}&0&0&0&0\\0&0&0&\frac{1}{2}k^{2}\left(\dot{r}_{3}+2\dot{r}_{5}\right)&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\\0&0&0&0&0&0&0\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix}-\frac{3k^{2}\dot{r}_{3}}{2}&0&0\\0&0&0\\0&0&0\end{pmatrix}$$

Gauge constraints on source currents:

$\overset{0^+}{\cdot}\sigma^{\parallel} == 0$

$\overset{0^+}{\cdot}\tau^{\parallel} == 0$

$\overset{0^+}{\cdot}\tau^{\perp} == 0$

$\overset{1^-}{\cdot}\tau^{\perp}{}^{a} == 0$

$\overset{1^-}{\cdot}\tau^{\parallel}{}^{a} == 0$

$\overset{1^-}{\cdot}\sigma^{\perp}{}^{a} == 0$

$-i\,k\,\overset{1^+}{\cdot}\sigma^{\perp}{}^{ab} + \overset{1^+}{\cdot}\tau^{\parallel}{}^{ab} == 0$

$\overset{2^-}{\cdot}\sigma^{\parallel}{}^{abc} == 0$

$\overset{2^+}{\cdot}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2 (2 r_{3}+r_{5})} & -\frac{\sqrt{2}}{k^2 (1+k^2)(2 r_{3}+r_{5})} & \frac{i \sqrt{2}}{k (1+k^2)(2 r_{3}+r_{5})} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{k^2 (1+k^2)(2 r_{3}+r_{5})} & \frac{3 k^2 (2 r_{3}+r_{5})+2 t_{2}}{(k+k^3)^2 (2 r_{3}+r_{5}) t_{2}} & -\frac{i (3 k^2 (2 r_{3}+r_{5})+2 t_{2})}{k (1+k^2)^2 (2 r_{3}+r_{5}) t_{2}} & 0 & 0 & 0 & 0 \\ -\frac{i \sqrt{2}}{k (1+k^2)(2 r_{3}+r_{5})} & \frac{i (3 k^2 (2 r_{3}+r_{5})+2 t_{2})}{k (1+k^2)^2 (2 r_{3}+r_{5}) t_{2}} & \frac{3 k^2 (2 r_{3}+r_{5})+2 t_{2}}{(1+k^2)^2 (2 r_{3}+r_{5}) t_{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{k^2 (r_{3}+2 r_{5})} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{3 k^2 r_{3}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

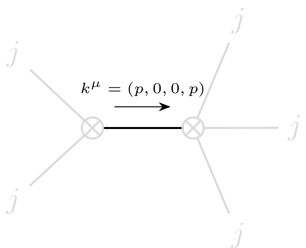
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{45 r_{3}^2+20 r_{3} r_{5}+4 r_{5}^2}{r_{3} (2 r_{3}+r_{5})(r_{3}+2 r_{5})}, -\frac{45 r_{3}^2+20 r_{3} r_{5}+4 r_{5}^2}{r_{3} (2 r_{3}+r_{5})(r_{3}+2 r_{5})}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{2}{r_{3}} + \frac{7}{2 r_{3}+r_{5}} - \frac{24}{r_{3}+2 r_{5}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(r_{3} < 0 \,\&\&\, \left(r_{5} < -\frac{r_{3}}{2} \,\|\, r_{5} > -2 r_{3}\right)\right) \,\|\, \left(r_{3} > 0 \,\&\&\, -2 r_{3} < r_{5} < -\frac{r_{3}}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{\dot{3}} < 0 \,\&\&\, \left(r_{\dot{5}} < -\frac{r_{\dot{3}}}{2} \,\|\, r_{\dot{5}} > -2\, r_{\dot{3}}\right)\right) \,\|\, \left(r_{\dot{3}} > 0 \,\&\&\, -2\, r_{\dot{3}} < r_{\dot{5}} < -\frac{r_{\dot{3}}}{2}\right) \tag{55}$$

Okay, that concludes the analysis of this theory.

Case 13

Now for a new theory. Here is the full nonlinear Lagrangian for Case 13 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{3} r_{\dot{1}}\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} + \frac{2}{3} r_{\dot{1}}\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} + \left(-2\, r_{\dot{1}} + 2\, r_{\dot{3}} + r_{\dot{5}}\right) \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{j}{}^{l}{}_{kl} + \\ &\frac{1}{3}\left(r_{\dot{1}} - 3\, r_{\dot{3}}\right) \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + \left(-2\, r_{\dot{1}} + 2\, r_{\dot{3}} - r_{\dot{5}}\right) \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl}\end{aligned} \tag{56}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\left(2\, r_{\dot{1}} - 2\, r_{\dot{3}} + r_{\dot{5}}\right) \partial_b \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^i \mathcal{A}^{ab}{}_{a} + \left(2\, r_{\dot{1}} - 2\, r_{\dot{3}} - r_{\dot{5}}\right) \partial_i \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^i \mathcal{A}^{ab}{}_{a} + \left(2\, r_{\dot{1}} - 2\, r_{\dot{3}} - r_{\dot{5}}\right) \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_{b}{}^{j}{}_{i} + \\ &\left(-4\, r_{\dot{1}} + 4\, r_{\dot{3}} + 2\, r_{\dot{5}}\right) \partial^i \mathcal{A}^{ab}{}_{a}\, \partial_j \mathcal{A}_{b}{}^{j}{}_{i} + \left(2\, r_{\dot{1}} - 2\, r_{\dot{3}} + r_{\dot{5}}\right) \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{b} - \\ &2\left(2\, r_{\dot{1}} - 2\, r_{\dot{3}} + r_{\dot{5}}\right) \partial^i \mathcal{A}^{ab}{}_{a}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{b} - \frac{4}{3} r_{\dot{1}}\, \partial_b \mathcal{A}_{aij}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_{\dot{1}}\, \partial_b \mathcal{A}_{aji}\, \partial^j \mathcal{A}^{abi} + \\ &\frac{4}{3}\left(r_{\dot{1}} - 3\, r_{\dot{3}}\right) \partial_b \mathcal{A}_{ija}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{\dot{1}}\, \partial_i \mathcal{A}_{abj}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_{\dot{1}}\, \partial_j \mathcal{A}_{abi}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_{\dot{1}}\, \partial_j \mathcal{A}_{aib}\, \partial^j \mathcal{A}^{abi}\end{aligned} \tag{57}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} 6\, k^2 \left(-r_{\dot{1}} + r_{\dot{3}}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} k^2 \left(2\, r_{\dot{3}} + r_{\dot{5}}\right) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 \left(-r_{\dot{1}} + 2\, r_{\dot{3}} + r_{\dot{5}}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k^2\, r_{\dot{1}} \end{pmatrix}$$

```
Gauge constraints on source currents:
```

$\overset{0^-}{\bullet}\sigma^{\parallel} == 0$

$\overset{0^+}{\bullet}\tau^{\perp} == 0$

$\overset{0^+}{\bullet}\tau^{\parallel} == 0$

$\overset{1^-}{\bullet}\tau^{\perp}{}^{a} == 0$

$\overset{1^-}{\bullet}\tau^{\parallel}{}^{a} == 0$

$\overset{1^-}{\bullet}\sigma^{\perp}{}^{a} == 0$

$\overset{1^+}{\bullet}\tau^{\parallel}{}^{ab} == 0$

$\overset{1^+}{\bullet}\sigma^{\perp}{}^{ab} == 0$

$\overset{2^+}{\bullet}\tau^{\parallel}{}^{ab} == 0$

$\overset{2^+}{\bullet}\sigma^{\parallel}{}^{ab} == 0$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{6k^2\left(-r_{\underset{1}{\cdot}}+r_{\underset{3}{\cdot}}\right)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2\left(2r_{\underset{3}{\cdot}}+r_{\underset{5}{\cdot}}\right)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2\left(-r_{\underset{1}{\cdot}}+2r_{\underset{3}{\cdot}}+r_{\underset{5}{\cdot}}\right)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{k^2 r_{\underset{1}{\cdot}}} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

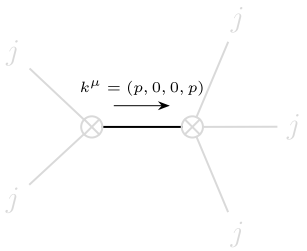
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{8r_{\underset{1}{\cdot}}^2-16r_{\underset{1}{\cdot}}r_{\underset{3}{\cdot}}+12r_{\underset{3}{\cdot}}^2-8r_{\underset{1}{\cdot}}r_{\underset{5}{\cdot}}+12r_{\underset{3}{\cdot}}r_{\underset{5}{\cdot}}+3r_{\underset{5}{\cdot}}^2}{r_{\underset{1}{\cdot}}\left(r_{\underset{1}{\cdot}}-2r_{\underset{3}{\cdot}}-r_{\underset{5}{\cdot}}\right)\left(2r_{\underset{3}{\cdot}}+r_{\underset{5}{\cdot}}\right)}, \frac{8r_{\underset{1}{\cdot}}^2-16r_{\underset{1}{\cdot}}r_{\underset{3}{\cdot}}+12r_{\underset{3}{\cdot}}^2-8r_{\underset{1}{\cdot}}r_{\underset{5}{\cdot}}+12r_{\underset{3}{\cdot}}r_{\underset{5}{\cdot}}+3r_{\underset{5}{\cdot}}^2}{r_{\underset{1}{\cdot}}\left(r_{\underset{1}{\cdot}}-2r_{\underset{3}{\cdot}}-r_{\underset{5}{\cdot}}\right)\left(2r_{\underset{3}{\cdot}}+r_{\underset{5}{\cdot}}\right)}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{U}{r P_{1}} + \frac{U}{r P_{1} O T r P_{3} O r P_{5}} + \frac{Z}{T r P_{3} M r P_{5}} > 0$
Polarisations:	2

Overall unitarity conditions:

$r_{3} \in \mathbb{R} \,\&\&\, \left(\left(r_{1} < 0 \,\&\&\, \left(r_{5} < r_{1} - 2 r_{3} \,\|\, r_{5} > -2 r_{3}\right)\right) \| \left(r_{1} > 0 \,\&\&\, -2 r_{3} < r_{5} < r_{1} - 2 r_{3}\right)\right)$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{3} \in \mathbb{R} \,\&\&\, \left(\left(r_{1} < 0 \,\&\&\, \left(r_{5} < r_{1} - 2 r_{3} \,\|\, r_{5} > -2 r_{3}\right)\right) \| \left(r_{1} > 0 \,\&\&\, -2 r_{3} < r_{5} < r_{1} - 2 r_{3}\right)\right) \tag{58}$$

Okay, that concludes the analysis of this theory.

Case 14

Now for a new theory. Here is the full nonlinear Lagrangian for Case 14 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\left(\frac{r_{3}}{2} + r_{5}\right) \mathcal{R}^{ij\hbar}{}_{i}\, \mathcal{R}_{j}{}^{l}{}_{\hbar l} - r_{3}\, \mathcal{R}^{ij\hbar l}\, \mathcal{R}_{\hbar l ij} + \frac{1}{2}\left(r_{3} - 2 r_{5}\right) \mathcal{R}^{ij\hbar}{}_{i}\, \mathcal{R}_{\hbar}{}^{l}{}_{jl} - \frac{2}{3} t_{3}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{\hbar}{}_{j\hbar} \tag{59}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$-\frac{2}{3} t_{3}\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} + \frac{4}{3} t_{3}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} - \frac{4}{3} t_{3}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{3}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{3}\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} - \frac{4}{3} t_{3}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} + \left(-\frac{r_{3}}{2} + r_{5}\right) \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \left(r_{3} + 2 r_{5}\right) \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \left(-\frac{r_{3}}{2} + r_{5}\right) \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + \left(r_{3} - 2 r_{5}\right) \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - 4 r_{3}\, \partial_{b} \mathcal{A}_{ija}\, \partial^{j} \mathcal{A}^{abi} \tag{60}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} t_{3} & -i \sqrt{2}\, k\, t_{3} & 0 & 0 \\ i \sqrt{2}\, k\, t_{3} & 2 k^{2} t_{3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^2\left(2\dot{r}_3+\dot{r}_5\right) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\left(\frac{\dot{r}_3}{2}+\dot{r}_5\right)+\frac{2\dot{t}_3}{3} & -\frac{\sqrt{2}\,\dot{t}_3}{3} & -\frac{2}{3}\, i\, k\, \dot{t}_3 & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}\,\dot{t}_3}{3} & \frac{\dot{t}_3}{3} & \frac{1}{3}\, i\, \sqrt{2}\, k\, \dot{t}_3 & 0 \\ 0 & 0 & 0 & \frac{2\, i\, k\, \dot{t}_3}{3} & -\frac{1}{3}\, i\, \sqrt{2}\, k\, \dot{t}_3 & \frac{2 k^2 \dot{t}_3}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{3 k^2 \dot{r}_3}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{0^-}{\bullet}\sigma^{\parallel} == 0$

$\overset{0^+}{\bullet}\tau^{\perp} == 0$

$-2\, i\, k\, \overset{0^+}{\bullet}\sigma^{\parallel} + \overset{0^+}{\bullet}\tau^{\parallel} == 0$

$\overset{1^-}{\bullet}{\tau^{\perp}}^{a} == 0$

$2\, i\, k\, \overset{1^-}{\bullet}{\sigma^{\perp}}^{a} + \overset{1^-}{\bullet}{\tau^{\parallel}}^{a} == 0$

$\overset{1^+}{\bullet}{\tau^{\parallel}}^{ab} == 0$

$\overset{1^+}{\bullet}{\sigma^{\perp}}^{ab} == 0$

$\overset{2^-}{\bullet}{\sigma^{\parallel}}^{abc} == 0$

$\overset{2^+}{\bullet}{\tau^{\parallel}}^{ab} == 0$

The Drazin (Moore–Penrose) inverses of these a–matrices, which are functionally analogous to the inverse b–matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{\left(1+2k^2\right)^2 \dot{t}_3} & -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 \dot{t}_3} & 0 & 0 \\ \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 \dot{t}_3} & \frac{2k^2}{\left(1+2k^2\right)^2 \dot{t}_3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2\left(2\dot{r}_3+\dot{r}_5\right)} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{k^2\left(\dot{r}_3+2\dot{r}_5\right)} & \frac{2\sqrt{2}}{k^2\left(1+2k^2\right)\left(\dot{r}_3+2\dot{r}_5\right)} & \frac{4 i}{k\left(1+2k^2\right)\left(\dot{r}_3+2\dot{r}_5\right)} & 0 \\ 0 & 0 & 0 & \frac{2\sqrt{2}}{k^2\left(1+2k^2\right)\left(\dot{r}_3+2\dot{r}_5\right)} & \frac{3k^2\left(\dot{r}_3+2\dot{r}_5\right)+4\dot{t}_3}{\left(k+2k^3\right)^2\left(\dot{r}_3+2\dot{r}_5\right)\dot{t}_3} & \frac{i\sqrt{2}\left(3k^2\left(\dot{r}_3+2\dot{r}_5\right)+4\dot{t}_3\right)}{k\left(1+2k^2\right)^2\left(\dot{r}_3+2\dot{r}_5\right)\dot{t}_3} & 0 \\ 0 & 0 & 0 & -\frac{4 i}{k\left(1+2k^2\right)\left(\dot{r}_3+2\dot{r}_5\right)} & -\frac{i\sqrt{2}\left(3k^2\left(\dot{r}_3+2\dot{r}_5\right)+4\dot{t}_3\right)}{k\left(1+2k^2\right)^2\left(\dot{r}_3+2\dot{r}_5\right)\dot{t}_3} & \frac{6k^2\left(\dot{r}_3+2\dot{r}_5\right)+8\dot{t}_3}{\left(1+2k^2\right)^2\left(\dot{r}_3+2\dot{r}_5\right)\dot{t}_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{3k^2 r_3} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

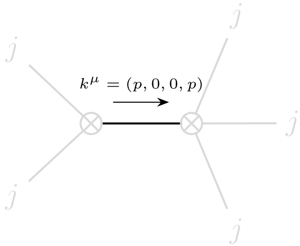
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{-445 r_3^2 - 268 r_3 r_5 - 52 r_5^2}{12 r_3 (2 r_3 + r_5)(r_3 + 2 r_5)}, \frac{-445 r_3^2 - 268 r_3 r_5 - 52 r_5^2}{12 r_3 (2 r_3 + r_5)(r_3 + 2 r_5)}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{TX}{r_3 P} + \frac{Ua}{T r_3 PM r_5 P} - \frac{TSX}{r_3 PM T r_5 P} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(r_3 < 0 \;\&\&\; \left(r_5 < -\frac{r_3}{2} \;\|\; r_5 > -2 r_3\right)\right) \| \left(r_3 > 0 \;\&\&\; -2 r_3 < r_5 < -\frac{r_3}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_3 < 0 \;\&\&\; \left(r_5 < -\frac{r_3}{2} \;\|\; r_5 > -2 r_3\right)\right) \| \left(r_3 > 0 \;\&\&\; -2 r_3 < r_5 < -\frac{r_3}{2}\right) \tag{61}$$

Okay, that concludes the analysis of this theory.

Case 15

Now for a new theory. Here is the full nonlinear Lagrangian for Case 15 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\left(\frac{r_3}{2} + r_5\right) \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{j}{}^{l}{}_{\hbar l} - r_3 \mathcal{R}^{ij\hbar l} \mathcal{R}_{\hbar lij} + \frac{1}{2}\left(r_3 - 2 r_5\right) \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{12} t_2 \mathcal{T}_{ij\hbar} \mathcal{T}^{ij\hbar} - \frac{1}{6} t_2 \mathcal{T}^{ij\hbar} \mathcal{T}_{ji\hbar} - \frac{2}{3} t_3 \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{\hbar}{}_{j\hbar} \tag{62}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{2} \mathcal{A}_{abi} \mathcal{A}^{abi} - \frac{2}{3} t_{2} \mathcal{A}_{aib} \mathcal{A}^{abi} - \frac{2}{3} t_{3} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} + \frac{4}{3} t_{3} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} - \frac{4}{3} t_{3} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{3} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{3} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} - \frac{4}{3} t_{3} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} + \left(-\frac{r_{3}}{2} + r_{5}\right) \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} - \frac{2}{3} t_{2} \mathcal{A}_{abi} \partial^{i} f^{ab} + \frac{2}{3} t_{2} \mathcal{A}_{aib} \partial^{i} f^{ab} - \frac{2}{3} t_{2} \mathcal{A}_{bia} \partial^{i} f^{ab} + \frac{1}{3} t_{2} \partial_{a} f_{bi} \partial^{i} f^{ab} - \frac{1}{6} t_{2} \partial_{a} f_{ib} \partial^{i} f^{ab} - \frac{1}{6} t_{2} \partial_{b} f_{ai} \partial^{i} f^{ab} + \frac{1}{6} t_{2} \partial_{i} f_{ab} \partial^{i} f^{ab} - \frac{1}{6} t_{2} \partial_{i} f_{ba} \partial^{i} f^{ab} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \left(r_{3} + 2 r_{5}\right) \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \left(-\frac{r_{3}}{2} + r_{5}\right) \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + \left(r_{3} - 2 r_{5}\right) \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - 4 r_{3} \partial_{b} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{abi} \tag{63}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} t_{3} & -i\sqrt{2}\, k t_{3} & 0 & 0 \\ i\sqrt{2}\, k t_{3} & 2k^{2} t_{3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_{2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^{2}\left(2 r_{3} + r_{5}\right) + \frac{2 t_{2}}{3} & \frac{\sqrt{2}\, t_{2}}{3} & -\frac{1}{3} i \sqrt{2}\, k t_{2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}\, t_{2}}{3} & \frac{t_{2}}{3} & -\frac{1}{3} i k t_{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{2} & \frac{i k t_{2}}{3} & \frac{k^{2} t_{2}}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^{2}\left(\frac{r_{3}}{2} + r_{5}\right) + \frac{2 t_{3}}{3} & -\frac{\sqrt{2}\, t_{3}}{3} & -\frac{2}{3} i k t_{3} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}\, t_{3}}{3} & \frac{t_{3}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{3} & 0 \\ 0 & 0 & 0 & \frac{2 i k t_{3}}{3} & -\frac{1}{3} i \sqrt{2}\, k t_{3} & \frac{2 k^{2} t_{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{3 k^{2} r_{3}}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^+}\tau^{\perp} == 0$

$-2 i k\, {}^{0^+}\sigma^{\parallel} + {}^{0^+}\tau^{\parallel} == 0$

${}^{1^-}\tau^{\perp a} == 0$

$2 i k\, {}^{1^-}\sigma^{\perp a} + {}^{1^-}\tau^{\parallel a} == 0$

$-i\, k\, \overset{1^{+}}{\cdot}\sigma^{\perp}{}^{ab} + \overset{1^{+}}{\cdot}\tau^{\parallel}{}^{ab} == 0$

$\overset{2^{-}}{\cdot}\sigma^{\parallel}{}^{abc} == 0$

$\overset{2^{+}}{\cdot}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{(1+2k^2)^2 \dot{t}_3} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 \dot{t}_3} & 0 & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 \dot{t}_3} & \frac{2k^2}{(1+2k^2)^2 \dot{t}_3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\dot{t}_2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2\left(2\dot{r}_3+\dot{r}_5\right)} & -\frac{\sqrt{2}}{k^2(1+k^2)\left(2\dot{r}_3+\dot{r}_5\right)} & \frac{i\sqrt{2}}{k(1+k^2)\left(2\dot{r}_3+\dot{r}_5\right)} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{k^2(1+k^2)\left(2\dot{r}_3+\dot{r}_5\right)} & \frac{3k^2\left(2\dot{r}_3+\dot{r}_5\right)+2\dot{t}_2}{(k+k^3)^2\left(2\dot{r}_3+\dot{r}_5\right)\dot{t}_2} & -\frac{i\left(3k^2\left(2\dot{r}_3+\dot{r}_5\right)+2\dot{t}_2\right)}{k(1+k^2)^2\left(2\dot{r}_3+\dot{r}_5\right)\dot{t}_2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}}{k(1+k^2)\left(2\dot{r}_3+\dot{r}_5\right)} & \frac{i\left(3k^2\left(2\dot{r}_3+\dot{r}_5\right)+2\dot{t}_2\right)}{k(1+k^2)^2\left(2\dot{r}_3+\dot{r}_5\right)\dot{t}_2} & \frac{3k^2\left(2\dot{r}_3+\dot{r}_5\right)+2\dot{t}_2}{(1+k^2)^2\left(2\dot{r}_3+\dot{r}_5\right)\dot{t}_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{k^2\left(\dot{r}_3+2\dot{r}_5\right)} & \frac{2\sqrt{2}}{k^2(1+2k^2)\left(\dot{r}_3+2\dot{r}_5\right)} & \frac{4i}{k(1+2k^2)\left(\dot{r}_3+2\dot{r}_5\right)} & 0 \\ 0 & 0 & 0 & \frac{2\sqrt{2}}{k^2(1+2k^2)\left(\dot{r}_3+2\dot{r}_5\right)} & \frac{3k^2\left(\dot{r}_3+2\dot{r}_5\right)+4\dot{t}_3}{(k+2k^3)^2\left(\dot{r}_3+2\dot{r}_5\right)\dot{t}_3} & \frac{i\sqrt{2}\left(3k^2\left(\dot{r}_3+2\dot{r}_5\right)+4\dot{t}_3\right)}{k(1+2k^2)^2\left(\dot{r}_3+2\dot{r}_5\right)\dot{t}_3} & 0 \\ 0 & 0 & 0 & -\frac{4i}{k(1+2k^2)\left(\dot{r}_3+2\dot{r}_5\right)} & -\frac{i\sqrt{2}\left(3k^2\left(\dot{r}_3+2\dot{r}_5\right)+4\dot{t}_3\right)}{k(1+2k^2)^2\left(\dot{r}_3+2\dot{r}_5\right)\dot{t}_3} & \frac{6k^2\left(\dot{r}_3+2\dot{r}_5\right)+8\dot{t}_3}{(1+2k^2)^2\left(\dot{r}_3+2\dot{r}_5\right)\dot{t}_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{3k^2\dot{r}_3} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

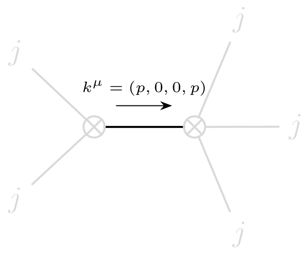
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{403\dot{r}_3^2+172\dot{r}_3\dot{r}_5+28\dot{r}_5^2}{6\dot{r}_3\left(2\dot{r}_3+\dot{r}_5\right)\left(\dot{r}_3+2\dot{r}_5\right)},\ -\frac{403\dot{r}_3^2+172\dot{r}_3\dot{r}_5+28\dot{r}_5^2}{6\dot{r}_3\left(2\dot{r}_3+\dot{r}_5\right)\left(\dot{r}_3+2\dot{r}_5\right)}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{14}{r_{3}} + \frac{57}{2r_{3}+r_{5}} - \frac{216}{r_{3}+2r_{5}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(r_{\cdot} < 0 \; \&\& \left(r_{\cdot} < -\frac{r_{\cdot}}{2} \;||\; r_{\cdot} > -2\, r_{\cdot}\right)\right) \;||\; \left(r_{\cdot} > 0 \; \&\& \; -2\, r_{\cdot} < r_{\cdot} < -\frac{r_{\cdot}}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{3} < 0 \; \&\& \left(r_{5} < -\frac{r_{3}}{2} \;||\; r_{5} > -2\, r_{3}\right)\right) \;||\; \left(r_{3} > 0 \; \&\& \; -2\, r_{3} < r_{5} < -\frac{r_{3}}{2}\right) \tag{64}$$

Okay, that concludes the analysis of this theory.

Case 16

Now for a new theory. Here is the full nonlinear Lagrangian for Case 16 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{6} r_{2}\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} - \frac{2}{3} r_{2}\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} + \left(\frac{r_{3}}{2} + r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{j}{}^{l}{}_{kl} + \\ &\frac{1}{6}\left(r_{2} - 6\, r_{3}\right) \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + \frac{1}{2}\left(r_{3} - 2\, r_{5}\right) \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl} - \frac{2}{3} t_{3}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{k}{}_{jk}\end{aligned} \tag{65}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&-\frac{2}{3} t_{3}\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} + \frac{4}{3} t_{3}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} - \frac{4}{3} t_{3}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{3}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + \\ &\frac{2}{3} t_{3}\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} - \frac{4}{3} t_{3}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} + \left(-\frac{r_{3}}{2} + r_{5}\right) \partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{ab}{}_{a} + \left(-\frac{r_{3}}{2} - r_{5}\right) \partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{ab}{}_{a} + \\ &\left(-\frac{r_{3}}{2} - r_{5}\right) \partial_{a}\mathcal{A}^{abi}\, \partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} + \left(r_{3} + 2\, r_{5}\right) \partial^{i}\mathcal{A}^{ab}{}_{a}\, \partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} + \left(-\frac{r_{3}}{2} + r_{5}\right) \partial_{a}\mathcal{A}^{abi}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + \\ &\left(r_{3} - 2\, r_{5}\right) \partial^{i}\mathcal{A}^{ab}{}_{a}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3} r_{2}\, \partial_{b}\mathcal{A}_{aij}\, \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{2}\, \partial_{b}\mathcal{A}_{aji}\, \partial^{j}\mathcal{A}^{abi} + \\ &\frac{2}{3}\left(r_{2} - 6\, r_{3}\right) \partial_{b}\mathcal{A}_{ija}\, \partial^{j}\mathcal{A}^{abi} - \frac{1}{3} r_{2}\, \partial_{a}\mathcal{A}_{abj}\, \partial^{j}\mathcal{A}^{abi} + \frac{1}{3} r_{2}\, \partial_{j}\mathcal{A}_{abi}\, \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{2}\, \partial_{j}\mathcal{A}_{aib}\, \partial^{j}\mathcal{A}^{abi}\end{aligned} \tag{66}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} t_{3} & -i\sqrt{2}\,k\,t_{3} & 0 & 0 \\ i\sqrt{2}\,k\,t_{3} & 2k^2 t_{3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^2\left(2r_{3}+r_{5}\right) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\left(\frac{r_{3}}{2}+r_{5}\right)+\frac{2t_{3}}{3} & -\frac{\sqrt{2}\,t_{3}}{3} & -\frac{2}{3}\,i\,k\,t_{3} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}\,t_{3}}{3} & \frac{t_{3}}{3} & \frac{1}{3}\,i\sqrt{2}\,k\,t_{3} & 0 \\ 0 & 0 & 0 & \frac{2\,i\,k\,t_{3}}{3} & -\frac{1}{3}\,i\sqrt{2}\,k\,t_{3} & \frac{2k^2 t_{3}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{3k^2 r_{3}}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{0^+}{\cdot}\tau^{\perp} == 0$

$-2\,i\,k\,\overset{0^+}{\cdot}\sigma^{\parallel} + \overset{0^+}{\cdot}\tau^{\parallel} == 0$

$\overset{1^-}{\cdot}\tau^{\perp\,a} == 0$

$2\,i\,k\,\overset{1^-}{\cdot}\sigma^{\perp\,a} + \overset{1^-}{\cdot}\tau^{\parallel\,a} == 0$

$\overset{1^+}{\cdot}\tau^{\parallel\,ab} == 0$

$\overset{1^+}{\cdot}\sigma^{\perp\,ab} == 0$

$\overset{2^-}{\cdot}\sigma^{\parallel\,abc} == 0$

$\overset{2^+}{\cdot}\tau^{\parallel\,ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{\left(1+2k^2\right)^2 t_{3}} & -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{3}} & 0 & 0 \\ \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{3}} & \frac{2k^2}{\left(1+2k^2\right)^2 t_{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2 (2 r_{\cdot 3} + r_{\cdot 5})} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2}{k^2 (r_{\cdot 3} + 2 r_{\cdot 5})} & \frac{2\sqrt{2}}{k^2 (1+2k^2)(r_{\cdot 3} + 2 r_{\cdot 5})} & \frac{4 i}{k (1+2k^2)(r_{\cdot 3} + 2 r_{\cdot 5})} & 0 \\ 0 & 0 & 0 & \frac{2\sqrt{2}}{k^2 (1+2k^2)(r_{\cdot 3} + 2 r_{\cdot 5})} & \frac{3k^2 (r_{\cdot 3} + 2 r_{\cdot 5}) + 4 t_{\cdot 3}}{(k+2k^3)^2 (r_{\cdot 3} + 2 r_{\cdot 5}) t_{\cdot 3}} & \frac{i \sqrt{2} (3k^2 (r_{\cdot 3} + 2 r_{\cdot 5}) + 4 t_{\cdot 3})}{k (1+2k^2)^2 (r_{\cdot 3} + 2 r_{\cdot 5}) t_{\cdot 3}} & 0 \\ 0 & 0 & 0 & -\frac{4 i}{k (1+2k^2)(r_{\cdot 3} + 2 r_{\cdot 5})} & -\frac{i \sqrt{2} (3k^2 (r_{\cdot 3} + 2 r_{\cdot 5}) + 4 t_{\cdot 3})}{k (1+2k^2)^2 (r_{\cdot 3} + 2 r_{\cdot 5}) t_{\cdot 3}} & \frac{6k^2 (r_{\cdot 3} + 2 r_{\cdot 5}) + 8 t_{\cdot 3}}{(1+2k^2)^2 (r_{\cdot 3} + 2 r_{\cdot 5}) t_{\cdot 3}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{3k^2 r_{\cdot 3}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

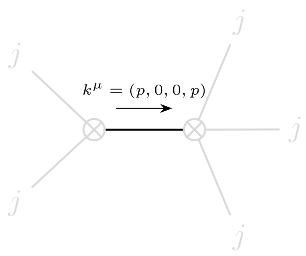
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{ \frac{-445 r_{\cdot 3}^2 - 268 r_{\cdot 3} r_{\cdot 5} - 52 r_{\cdot 5}^2}{12 r_{\cdot 3} (2 r_{\cdot 3} + r_{\cdot 5})(r_{\cdot 3} + 2 r_{\cdot 5})}, \frac{-445 r_{\cdot 3}^2 - 268 r_{\cdot 3} r_{\cdot 5} - 52 r_{\cdot 5}^2}{12 r_{\cdot 3} (2 r_{\cdot 3} + r_{\cdot 5})(r_{\cdot 3} + 2 r_{\cdot 5})} \right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{26}{r_{\cdot 3}} + \frac{39}{2 r_{\cdot 3} + r_{\cdot 5}} - \frac{216}{r_{\cdot 3} + 2 r_{\cdot 5}} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(r_{\cdot} < 0 \,\&\&\, \left(r_{\cdot} < -\frac{r_{\cdot}}{2} \,\|\, r_{\cdot} > -2 r_{\cdot}\right)\right) \| \left(r_{\cdot} > 0 \,\&\&\, -2 r_{\cdot} < r_{\cdot} < -\frac{r_{\cdot}}{2}\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{\cdot 3} < 0 \,\&\&\, \left(r_{\cdot 5} < -\frac{r_{\cdot 3}}{2} \,\|\, r_{\cdot 5} > -2 r_{\cdot 3}\right)\right) \| \left(r_{\cdot 3} > 0 \,\&\&\, -2 r_{\cdot 3} < r_{\cdot 5} < -\frac{r_{\cdot 3}}{2}\right) \quad (67)$$

Okay, that concludes the analysis of this theory.

Case 17

Now for a new theory. Here is the full nonlinear Lagrangian for Case 17 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\dot{r}_{5}\, \mathcal{R}^{ij\hbar}{}_{i}\, \mathcal{R}_{j}{}^{l}{}_{\hbar l} - \dot{r}_{5}\, \mathcal{R}^{ij\hbar}{}_{i}\, \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{4} \dot{t}_{1}\, \mathcal{T}_{ij\hbar}\, \mathcal{T}^{ij\hbar} + \frac{1}{2} \dot{t}_{1}\, \mathcal{T}^{ij\hbar}\, \mathcal{T}_{ji\hbar} + \frac{1}{3} \dot{t}_{1}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{\hbar}{}_{j\hbar} \tag{68}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\dot{t}_{1}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} + \frac{1}{3} \dot{t}_{1}\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3} \dot{t}_{1}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} + \frac{2}{3} \dot{t}_{1}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - \frac{1}{3} \dot{t}_{1}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - \\
&\frac{1}{3} \dot{t}_{1}\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} + \frac{2}{3} \dot{t}_{1}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} + \dot{r}_{5}\, \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} - \dot{r}_{5}\, \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} + 2 \dot{t}_{1}\, \mathcal{A}_{bia}\, \partial^{i} f^{ab} - \\
&\dot{t}_{1}\, \partial_{a} f_{bi}\, \partial^{i} f^{ab} + \frac{1}{2} \dot{t}_{1}\, \partial_{a} f_{ib}\, \partial^{i} f^{ab} - \frac{1}{2} \dot{t}_{1}\, \partial_{b} f_{ai}\, \partial^{i} f^{ab} + \frac{1}{2} \dot{t}_{1}\, \partial_{i} f_{ab}\, \partial^{i} f^{ab} + \frac{1}{2} \dot{t}_{1}\, \partial_{i} f_{ba}\, \partial^{i} f^{ab} - \\
&\dot{r}_{5}\, \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + 2 \dot{r}_{5}\, \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \dot{r}_{5}\, \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - 2 \dot{r}_{5}\, \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b}
\end{aligned} \tag{69}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\dot{t}_{1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
k^2 \dot{r}_{5} - \frac{\dot{t}_{1}}{2} & -\frac{\dot{t}_{1}}{\sqrt{2}} & \frac{i k \dot{t}_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\
-\frac{\dot{t}_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{i k \dot{t}_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & k^2 \dot{r}_{5} + \frac{\dot{t}_{1}}{6} & \frac{\dot{t}_{1}}{3\sqrt{2}} & \frac{i k \dot{t}_{1}}{3} & 0 \\
0 & 0 & 0 & \frac{\dot{t}_{1}}{3\sqrt{2}} & \frac{\dot{t}_{1}}{3} & \frac{1}{3} i \sqrt{2}\, k \dot{t}_{1} & 0 \\
0 & 0 & 0 & -\frac{1}{3} i k \dot{t}_{1} & -\frac{1}{3} i \sqrt{2}\, k \dot{t}_{1} & \frac{2 k^2 \dot{t}_{1}}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix}
\frac{\dot{t}_{1}}{2} & -\frac{i k \dot{t}_{1}}{\sqrt{2}} & 0 \\
\frac{i k \dot{t}_{1}}{\sqrt{2}} & k^2 \dot{t}_{1} & 0 \\
0 & 0 & \frac{\dot{t}_{1}}{2}
\end{pmatrix}$$

Gauge constraints on source currents:

$\overset{0^+}{\cdot}\sigma^{\parallel} == 0$

$\overset{0^+}{\cdot}\tau^{\parallel} == 0$

$\overset{0^+}{\cdot}\tau^{\perp} == 0$

$\overset{1^-}{\cdot}\tau^{\perp a} == 0$

$2\, i\, k\ {}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} + {}^{1^-}_{\bullet}\tau^{\parallel}{}^{a} == 0$

$-i\, k\ {}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab} + {}^{1^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$

$-2\, i\, k\ {}^{2^+}_{\bullet}\sigma^{\parallel}{}^{ab} + {}^{2^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{-2k^2 r_5+t_1}{(1+k^2)^2 t_1^2} & \frac{i(2k^3 r_5-k t_1)}{(1+k^2)^2 t_1^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & -\frac{i(2k^3 r_5-k t_1)}{(1+k^2)^2 t_1^2} & \frac{-2k^4 r_5+k^2 t_1}{(1+k^2)^2 t_1^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_5} & -\frac{1}{\sqrt{2}\,(k^2 r_5+2k^4 r_5)} & -\frac{i}{k r_5+2k^3 r_5} & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}\,(k^2 r_5+2k^4 r_5)} & \frac{6k^2 r_5+t_1}{2(k+2k^3)^2 r_5 t_1} & \frac{i(6k^2 r_5+t_1)}{\sqrt{2}\,k(1+2k^2)^2 r_5 t_1} & 0 \\ 0 & 0 & 0 & \frac{i}{k r_5+2k^3 r_5} & -\frac{i(6k^2 r_5+t_1)}{\sqrt{2}\,k(1+2k^2)^2 r_5 t_1} & \frac{6k^2 r_5+t_1}{(1+2k^2)^2 r_5 t_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_1} & -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & 0 \\ \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & \frac{4k^2}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{t_1} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

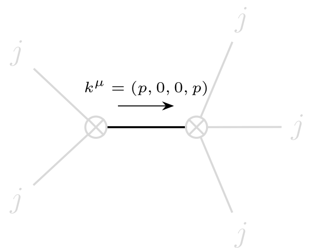
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{7 t_1^2+2 r_5 t_1 p^2+4 r_5^2 p^4}{2 r_5 t_1^2},\ -\frac{7 t_1^2+2 r_5 t_1 p^2+4 r_5^2 p^4}{2 r_5 t_1^2}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{7}{r_{\cdot 5}} - \frac{2p^2}{t_{\cdot 1}} - \frac{4 r_{\cdot 5}\, p^4}{t_{\cdot 1}^2} > 0$
Polarisations:	2

Overall unitarity conditions:

$p \in \mathbb{R}\ \&\&\ r_{\cdot 5} < 0\ \&\&\ \left(t_{\cdot 1} < 0 \,\|\, t_{\cdot 1} > 0\right)$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$t_{\cdot} \neq 0\ \&\&\ r_{\cdot} < 0 \tag{70}$$

Okay, that concludes the analysis of this theory.

Case 18

Now for a new theory. Here is the full nonlinear Lagrangian for Case 18 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$r_{\cdot}\, \mathcal{R}^{ijh}{}_{i}\, \mathcal{R}_{j}{}^{l}{}_{hl} - r_{\cdot}\, \mathcal{R}^{ijh}{}_{i}\, \mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{3} t_{\cdot}\, \mathcal{T}_{ijh}\, \mathcal{T}^{ijh} + \frac{1}{3} t_{\cdot}\, \mathcal{T}^{ijh}\, \mathcal{T}_{jih} + t_{\cdot}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{h}{}_{jh} \tag{71}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3} t_{\cdot}\, \mathcal{A}_{abi}\, \mathcal{A}^{abi} + \frac{1}{3} t_{\cdot}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} + t_{\cdot}\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} - 2 t_{\cdot}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} + \\
&2 t_{\cdot}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - t_{\cdot}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - t_{\cdot}\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} + 2 t_{\cdot}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} + r_{\cdot}\, \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} - \\
&r_{\cdot}\, \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} - \frac{2}{3} t_{\cdot}\, \mathcal{A}_{abi}\, \partial^{i} f^{ab} + \frac{2}{3} t_{\cdot}\, \mathcal{A}_{aib}\, \partial^{i} f^{ab} + \frac{4}{3} t_{\cdot}\, \mathcal{A}_{bia}\, \partial^{i} f^{ab} - \\
&\frac{2}{3} t_{\cdot}\, \partial_{a} f_{bi}\, \partial^{i} f^{ab} + \frac{1}{3} t_{\cdot}\, \partial_{a} f_{ib}\, \partial^{i} f^{ab} - \frac{2}{3} t_{\cdot}\, \partial_{b} f_{ai}\, \partial^{i} f^{ab} + \frac{2}{3} t_{\cdot}\, \partial_{i} f_{ab}\, \partial^{i} f^{ab} + \frac{1}{3} t_{\cdot}\, \partial_{i} f_{ba}\, \partial^{i} f^{ab} - \\
&r_{\cdot}\, \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + 2 r_{\cdot}\, \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + r_{\cdot}\, \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - 2 r_{\cdot}\, \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b}
\end{aligned} \tag{72}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -t_{\bullet} & i\sqrt{2}\,k\,t_{\bullet} & 0 & 0 \\ -i\sqrt{2}\,k\,t_{\bullet} & -2\,k\,t_{\bullet} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k\,r_{\bullet}+\frac{t_{1}}{} & -\frac{t_{1}}{\sqrt{}} & \frac{i\,k\,t_{1}}{\sqrt{}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1}}{\sqrt{}} & \frac{t_{1}}{} & --\,i\,k\,t_{\bullet} & 0 & 0 & 0 & 0 \\ -\frac{i\,k\,t_{1}}{\sqrt{}} & \frac{i\,k\,t_{1}}{} & \frac{k^{2}\,t_{1}}{} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\,r_{\bullet}-\frac{t_{1}}{} & \frac{t_{1}}{\sqrt{}} & i\,k\,t_{\bullet} & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{\sqrt{}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\,k\,t_{\bullet} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{} & -\frac{i\,k\,t_{1}}{\sqrt{}} & 0 \\ \frac{i\,k\,t_{1}}{\sqrt{}} & k\,t_{\bullet} & 0 \\ 0 & 0 & \frac{t_{1}}{} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{-}_{\bullet}\sigma^{\parallel} == 0$$

$${}^{+}_{\bullet}\tau^{\perp} == 0$$

$$-2\,i\,k\;{}^{+}_{\bullet}\sigma^{\parallel} + {}^{+}_{\bullet}\tau^{\parallel} == 0$$

$${}^{-}_{\bullet}\tau^{\perp}{}^{a} == 0$$

$$2\,i\,k\;{}^{-}_{\bullet}\sigma^{\perp}{}^{a} + {}^{-}_{\bullet}\tau^{\parallel}{}^{a} == 0$$

$$-i\,k\;{}^{+}_{\bullet}\sigma^{\perp}{}^{ab} + {}^{+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

$$-2\,i\,k\;{}^{+}_{\bullet}\sigma^{\parallel}{}^{ab} + {}^{+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{}{\left(+\,k^{2}\right)^{2}t_{1}} & \frac{i\sqrt{}\,k}{\left(+\,k^{2}\right)^{2}t_{1}} & 0 & 0 \\ -\frac{i\sqrt{}\,k}{\left(+\,k^{2}\right)^{2}t_{1}} & -\frac{k^{2}}{\left(+\,k^{2}\right)^{2}t_{1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{}{k^2 r_5} & \frac{}{\sqrt{\ }\left(k^2 r_5 + k^4 r_5\right)} & -\frac{i}{\sqrt{\ }\left(k r_5 + k^3 r_5\right)} & 0 & 0 & 0 & 0 \\ \frac{}{\sqrt{\ }\left(k^2 r_5 + k^4 r_5\right)} & \frac{k^2 r_5 + t_1}{(k+k^3)^2 r_5 t_1} & -\frac{i\left(k^2 r_5 + t_1\right)}{k\left(+ k^2\right)^2 r_5 t_1} & 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{\ }\left(k r_5 + k^3 r_5\right)} & \frac{i\left(k^2 r_5 + t_1\right)}{k\left(+ k^2\right)^2 r_5 t_1} & \frac{k^2 r_5 + t_1}{\left(+ k^2\right)^2 r_5 t_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{\ }}{t_1 + k^2 t_1} & \frac{i k}{t_1 + k^2 t_1} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{\ }}{t_1 + k^2 t_1} & \frac{- k^2 r_5 + t_1}{\left(t_1 + k^2 t_1\right)^2} & -\frac{i \sqrt{\ } k\left(k^2 r_5 - t_1\right)}{\left(t_1 + k^2 t_1\right)^2} & 0 \\ 0 & 0 & 0 & -\frac{i k}{t_1 + k^2 t_1} & \frac{i \sqrt{\ } k\left(k^2 r_5 - t_1\right)}{\left(t_1 + k^2 t_1\right)^2} & \frac{- k^4 r_5 + k^2 t_1}{\left(t_1 + k^2 t_1\right)^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_1} & -\frac{2 i \sqrt{2} k}{(1+2k^2)^2 t_1} & 0 \\ \frac{2 i \sqrt{2} k}{(1+2k^2)^2 t_1} & \frac{4 k^2}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{t_1} \end{pmatrix}$$

```
Square masses:
```

{{}, {}, {}, {}, {}, {}}

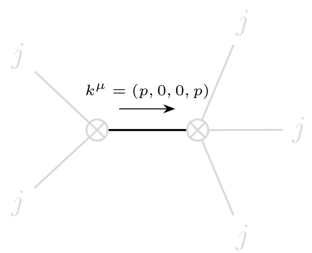
```
Massive pole residues:
```

{{}, {}, {}, {}, {}, {}}

```
Massless eigenvalues:
```

$$\left\{\frac{9 t_1^2 + 2 r_5 t_1 p^2 + 2 r_5^2 p^4}{r_5 t_1^2}, \frac{9 t_1^2 + 2 r_5 t_1 p^2 + 2 r_5^2 p^4}{r_5 t_1^2}\right\}$$

```
Overall particle spectrum:
```




```
Massless particle
```

Pole residue:	$\frac{9}{r_5} + \frac{2p^2}{t_1} + \frac{2 r_5 p^4}{t_1^2} > 0$
Polarisations:	2

```
Overall unitarity conditions:
```

$p \in \mathbb{R} \,\&\&\, r_5 > 0 \,\&\&\, \left(t_1 < 0 \,\|\, t_1 > 0\right)$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$t_{\cdot} \neq 0 \,\&\&\, r_{\cdot} > 0 \tag{73}$$

Okay, that concludes the analysis of this theory.

Case 19

Now for a new theory. Here is the full nonlinear Lagrangian for Case 19 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\left(2 r_{\cdot} + r_{\cdot}\right) \mathcal{R}^{ij h}{}_{i}\, \mathcal{R}_{j}{}^{l}{}_{h l} - r_{\cdot}\, \mathcal{R}^{i j h l}\, \mathcal{R}_{h l i j} + \left(2 r_{\cdot} - r_{\cdot}\right) \mathcal{R}^{i j h}{}_{i}\, \mathcal{R}_{h}{}^{l}{}_{j l} + \frac{1}{4} t_{\cdot}\, \mathcal{T}_{i j h}\, \mathcal{T}^{i j h} + \frac{1}{2} t_{\cdot}\, \mathcal{T}^{i j h}\, \mathcal{T}_{j i h} + \frac{1}{3} t_{\cdot}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{h}{}_{j h} \tag{74}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_{\cdot}\, \mathcal{A}_{a i b}\, \mathcal{A}^{a b i} + \frac{1}{3} t_{\cdot}\, \mathcal{A}^{a b}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3} t_{\cdot}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{a b} + \frac{2}{3} t_{\cdot}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - \frac{1}{3} t_{\cdot}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - \frac{1}{3} t_{\cdot}\, \partial_{a} f^{a b}\, \partial_{i} f^{i}{}_{b} + \frac{2}{3} t_{\cdot}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} + \left(-2 r_{\cdot} + r_{\cdot}\right) \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{a b}{}_{a} + \left(-2 r_{\cdot} - r_{\cdot}\right) \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{a b}{}_{a} + 2 t_{\cdot}\, \mathcal{A}_{b i a}\, \partial^{i} f^{a b} - t_{\cdot}\, \partial_{a} f_{b i}\, \partial^{i} f^{a b} + \frac{1}{2} t_{\cdot}\, \partial_{a} f_{i b}\, \partial^{i} f^{a b} - \frac{1}{2} t_{\cdot}\, \partial_{b} f_{a i}\, \partial^{i} f^{a b} + \frac{1}{2} t_{\cdot}\, \partial_{i} f_{a b}\, \partial^{i} f^{a b} + \frac{1}{2} t_{\cdot}\, \partial_{i} f_{b a}\, \partial^{i} f^{a b} + \left(-2 r_{\cdot} - r_{\cdot}\right) \partial_{a} \mathcal{A}^{a b i}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + 2 \left(2 r_{\cdot} + r_{\cdot}\right) \partial^{i} \mathcal{A}^{a b}{}_{a}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \left(-2 r_{\cdot} + r_{\cdot}\right) \partial_{a} \mathcal{A}^{a b i}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + \left(4 r_{\cdot} - 2 r_{\cdot}\right) \partial^{i} \mathcal{A}^{a b}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - 4 r_{\cdot}\, \partial_{b} \mathcal{A}_{i j a}\, \partial^{j} \mathcal{A}^{a b i} \tag{75}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} 6 k\, r_{\cdot} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_{\cdot} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} k \left(2 r_{\cdot} + r_{\cdot}\right) - \frac{t_{1}}{} & -\frac{t_{1}}{\sqrt{}} & \frac{i k t_{1}}{\sqrt{}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1}}{\sqrt{}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_{1}}{\sqrt{}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k \left(2 r_{\cdot} + r_{\cdot}\right) + \frac{t_{1}}{} & \frac{t_{1}}{\sqrt{}} & \frac{i k t_{1}}{} & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{\sqrt{}} & \frac{t_{1}}{} & - i \sqrt{2}\, k t_{\cdot} & 0 \\ 0 & 0 & 0 & -- i k t_{\cdot} & -- i \sqrt{2}\, k t_{\cdot} & \frac{k^{2} t_{1}}{} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} \frac{t_1}{} & -\frac{i\,k\,t_1}{\sqrt{}} & 0 \\ \frac{i\,k\,t_1}{\sqrt{}} & k\;t. & 0 \\ 0 & 0 & \frac{t_1}{} \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{+}{.}\tau^{\perp} == 0$

$\overset{+}{.}\tau^{\parallel} == 0$

$\overset{-}{.}\tau^{\perp}{}^{a} == 0$

$2\,i\,k\;\overset{-}{.}\sigma^{\perp}{}^{a} + \overset{-}{.}\tau^{\parallel}{}^{a} == 0$

$-i\,k\;\overset{+}{.}\sigma^{\perp}{}^{ab} + \overset{+}{.}\tau^{\parallel}{}^{ab} == 0$

$-2\,i\,k\;\overset{+}{.}\sigma^{\parallel}{}^{ab} + \overset{+}{.}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{}{k^2\,r_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{}{t_1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{}}{t_1+k^2\,t_1} & \frac{i\sqrt{}\;k}{t_1+k^2\,t_1} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{}}{t_1+k^2\,t_1} & \frac{-\;k^2\left(r_3+r_5\right)+t_1}{\left(+k^2\right)^2 t_1{}^2} & \frac{i\left(k^3\left(r_3+r_5\right)-k\,t_1\right)}{\left(+k^2\right)^2 t_1{}^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{}\;k}{t_1+k^2\,t_1} & \frac{-\;i\,k^3\left(r_3+r_5\right)+i\,k\,t_1}{\left(+k^2\right)^2 t_1{}^2} & \frac{-\;k^4\left(r_3+r_5\right)+k^2\,t_1}{\left(+k^2\right)^2 t_1{}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2\left(r_3+r_5\right)} & -\frac{}{\sqrt{}\left(k^2+\;k^4\right)\left(r_3+r_5\right)} & -\frac{i}{k\left(+\;k^2\right)\left(r_3+r_5\right)} & 0 \\ 0 & 0 & 0 & -\frac{}{\sqrt{}\left(k^2+\;k^4\right)\left(r_3+r_5\right)} & \frac{k^2\left(r_3+r_5\right)+t_1}{\left(k+\;k^3\right)^2\left(r_3+r_5\right)t_1} & \frac{i\left(k^2\left(r_3+r_5\right)+t_1\right)}{\sqrt{}\;k\left(+\;k^2\right)^2\left(r_3+r_5\right)t_1} & 0 \\ 0 & 0 & 0 & \frac{i}{k\left(+\;k^2\right)\left(r_3+r_5\right)} & -\frac{i\left(k^2\left(r_3+r_5\right)+t_1\right)}{\sqrt{}\;k\left(+\;k^2\right)^2\left(r_3+r_5\right)t_1} & \frac{k^2\left(r_3+r_5\right)+t_1}{\left(+\;k^2\right)^2\left(r_3+r_5\right)t_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{}{\left(+\;k^2\right)^2 t_1} & -\frac{i\sqrt{}\;k}{\left(+\;k^2\right)^2 t_1} & 0 \\ \frac{i\sqrt{}\;k}{\left(+\;k^2\right)^2 t_1} & \frac{k^2}{\left(+\;k^2\right)^2 t_1} & 0 \\ 0 & 0 & \frac{}{t_1} \end{pmatrix}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

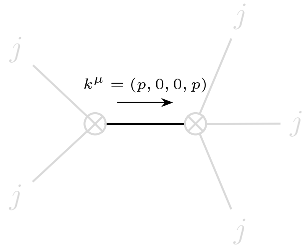
Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{7t_{.}\ +4r_{.}t_{.}p+2r_{.}t_{.}p+16r_{.}\ p+16r_{.}r_{.}p+4r_{.}\ p}{2\left(2r_{.}+r_{.}\right)t_{.}},-\frac{7t_{.}\ +4r_{.}t_{.}p+2r_{.}t_{.}p+16r_{.}\ p+16r_{.}r_{.}p+4r_{.}\ p}{2\left(2r_{.}+r_{.}\right)t_{.}}\right\}$$

Overall particle spectrum:



Massless particle

Pole residue:	$-\frac{Y}{T r_3 P M r_5 P} + \frac{ОТ t_1 P p^2 ОV JT r_3 PM r_5 PK p^4}{t_1 P^2} > 0$
Polarisations:	2

Overall unitarity conditions:

$$\left(p \mid r_{\underset{3}{.}}\right) \in \mathbb{R}\ \&\&\ r_{\underset{5}{.}} < -2\, r_{\underset{3}{.}}\ \&\&\ \left(t_{\underset{1}{.}} < 0\ \|\ t_{\underset{1}{.}} > 0\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{.} \in \mathbb{R}\ \&\&\ t_{.} \neq 0\ \&\&\ r_{.} < -2\, r_{.} \tag{76}$$

Okay, that concludes the analysis of this theory.

Case 20

Now for a new theory. Here is the full nonlinear Lagrangian for Case 20 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{6} r_{.}\, \mathcal{R}_{ijkl}\ \mathcal{R}^{ijkl} - \frac{2}{3} r_{.}\, \mathcal{R}_{ikjl}\ \mathcal{R}^{ijkl} + \frac{1}{6} r_{.}\, \mathcal{R}^{ijkl}\ \mathcal{R}_{klij} +\\ &\frac{1}{12}\left(4t_{.}+t_{.}\right)\mathcal{T}_{ijk}\ \mathcal{T}^{ijk} + \frac{1}{6}\left(2t_{.}-t_{.}\right)\mathcal{T}^{ijk}\ \mathcal{T}_{jik} + \frac{1}{3}\left(t_{.}-2t_{.}\right)\mathcal{T}^{i}{}_{i}{}^{j}\ \mathcal{T}^{k}{}_{jk}\end{aligned} \tag{77}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\frac{1}{3}\left(t_{.}+t_{.}\right)\mathcal{A}_{abi}\ \mathcal{A}^{abi} + \frac{1}{3}\left(t_{.}-2t_{.}\right)\mathcal{A}_{aib}\ \mathcal{A}^{abi} + \frac{1}{3}\left(t_{.}-2t_{.}\right)\mathcal{A}^{ab}{}_{a}\ \mathcal{A}_{b}{}^{i}{}_{i} -\\ &\frac{2}{3}\left(t_{.}-2t_{.}\right)\mathcal{A}_{b}{}^{i}{}_{i}\ \partial_{a}f^{ab} + \frac{2}{3}\left(t_{.}-2t_{.}\right)\mathcal{A}_{b}{}^{i}{}_{i}\ \partial^{b}f^{a}{}_{a} + \frac{1}{3}\left(-t_{.}+2t_{.}\right)\partial_{b}f^{i}{}_{i}\ \partial^{b}f^{a}{}_{a} +\\ &\frac{1}{3}\left(-t_{.}+2t_{.}\right)\partial_{a}f^{ab}\ \partial_{i}f^{i}{}_{b} + \frac{2}{3}\left(t_{.}-2t_{.}\right)\partial^{b}f^{a}{}_{a}\ \partial_{i}f^{i}{}_{b} - \frac{2}{3}\left(t_{.}+t_{.}\right)\mathcal{A}_{abi}\ \partial^{i}f^{ab} +\\ &\frac{2}{3}\left(t_{.}+t_{.}\right)\mathcal{A}_{aib}\ \partial^{i}f^{ab} + \frac{2}{3}\left(2t_{.}-t_{.}\right)\mathcal{A}_{bia}\ \partial^{i}f^{ab} + \frac{1}{3}\left(-2t_{.}+t_{.}\right)\partial_{a}f_{bi}\ \partial^{i}f^{ab} +\\ &\frac{1}{6}\left(2t_{.}-t_{.}\right)\partial_{a}f_{ib}\ \partial^{i}f^{ab} + \frac{1}{6}\left(-4t_{.}-t_{.}\right)\partial_{b}f_{ai}\ \partial^{i}f^{ab} + \frac{1}{6}\left(4t_{.}+t_{.}\right)\partial_{i}f_{ab}\ \partial^{i}f^{ab} +\\ &\frac{1}{6}\left(2t_{.}-t_{.}\right)\partial_{i}f_{ba}\ \partial^{i}f^{ab} + \frac{4}{3} r_{.}\, \partial_{b}\mathcal{A}_{aij}\ \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{.}\, \partial_{b}\mathcal{A}_{aji}\ \partial^{j}\mathcal{A}^{abi} +\end{aligned} \tag{78}$$

$$\frac{2}{3} r_{\cdot}\, \partial_{b}\mathcal{A}_{ija}\, \partial^{j}\mathcal{A}^{abi} - \frac{1}{3} r_{\cdot}\, \partial_{i}\mathcal{A}_{abj}\, \partial^{j}\mathcal{A}^{abi} + \frac{1}{3} r_{\cdot}\, \partial_{j}\mathcal{A}_{abi}\, \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{\cdot}\, \partial_{j}\mathcal{A}_{aib}\, \partial^{j}\mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} t_{\cdot} & -i\sqrt{2}\, k t_{\cdot} & 0 & 0 \\ i\sqrt{2}\, k t_{\cdot} & 2 k\ t_{\cdot} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\ r_{\cdot} + t_{\cdot} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\left(t_{\cdot} + 4 t_{\cdot}\right) & -\frac{t_1 - t_2}{\sqrt{}} & \frac{i k\left(t_1 - t_2\right)}{\sqrt{}} & 0 & 0 & 0 & 0 \\ -\frac{t_1 - t_2}{\sqrt{}} & \frac{t_1 + t_2}{} & --\, i k\left(t_{\cdot} + t_{\cdot}\right) & 0 & 0 & 0 & 0 \\ -\frac{i k\left(t_1 - t_2\right)}{\sqrt{}} & -\, i k\left(t_{\cdot} + t_{\cdot}\right) & -\, k\ \left(t_{\cdot} + t_{\cdot}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\left(t_{\cdot} + 4 t_{\cdot}\right) & \frac{t_1 - t_3}{\sqrt{}} & -\, i k\left(t_{\cdot} - 2 t_{\cdot}\right) & 0 \\ 0 & 0 & 0 & \frac{t_1 - t_3}{\sqrt{}} & \frac{t_1 + t_3}{} & -\, i\sqrt{2}\ k\left(t_{\cdot} + t_{\cdot}\right) & 0 \\ 0 & 0 & 0 & --\, i k\left(t_{\cdot} - 2 t_{\cdot}\right) & --\, i\sqrt{2}\ k\left(t_{\cdot} + t_{\cdot}\right) & -\, k\ \left(t_{\cdot} + t_{\cdot}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_1}{} & -\frac{i k t_1}{\sqrt{}} & 0 \\ \frac{i k t_1}{\sqrt{}} & k\ t_{\cdot} & 0 \\ 0 & 0 & \frac{t_1}{} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{+}_{\cdot}\tau^{\perp} == 0$$

$$-2 i k\, {}^{+}_{\cdot}\sigma^{\parallel} + {}^{+}_{\cdot}\tau^{\parallel} == 0$$

$${}^{-}_{\cdot}\tau^{\perp\, a} == 0$$

$$2 i k\, {}^{-}_{\cdot}\sigma^{\perp\, a} + {}^{-}_{\cdot}\tau^{\parallel\, a} == 0$$

$$-i k\, {}^{+}_{\cdot}\sigma^{\perp\, ab} + {}^{+}_{\cdot}\tau^{\parallel\, ab} == 0$$

$$-2 i k\, {}^{+}_{\cdot}\sigma^{\parallel\, ab} + {}^{+}_{\cdot}\tau^{\parallel\, ab} == 0$$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{}{(\ +\ k^2)^2 t_3} & -\frac{i \sqrt{\ }\ k}{(\ +\ k^2)^2 t_3} & 0 & 0 \\ \frac{i \sqrt{\ }\ k}{(\ +\ k^2)^2 t_3} & \frac{k^2}{(\ +\ k^2)^2 t_3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_2 + t_2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{(t_1+t_2)}{t_1 t_2} & \frac{\sqrt{\ }\,(t_1 - t_2)}{(\ +k^2) t_1 t_2} & -\frac{i \sqrt{\ }\ k (t_1 - t_2)}{(\ +k^2) t_1 t_2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\ }\,(t_1 - t_2)}{(\ +k^2) t_1 t_2} & \frac{t_1 + t_2}{(\ +k^2)^2 t_1 t_2} & -\frac{i k (t_1 + t_2)}{(\ +k^2)^2 t_1 t_2} & 0 & 0 & 0 & 0 \\ \frac{i \sqrt{\ }\ k (t_1 - t_2)}{(\ +k^2) t_1 t_2} & \frac{i k (t_1 + t_2)}{(\ +k^2)^2 t_1 t_2} & \frac{k^2 (t_1 + t_2)}{(\ +k^2)^2 t_1 t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(t_1+t_3)}{t_1 t_3} & -\frac{\sqrt{\ }\,(t_1 - t_3)}{(\ +\ k^2) t_1 t_3} & -\frac{i k t_1 - i k t_3}{t_1 t_3 + k^2 t_1 t_3} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{\ }\,(t_1 - t_3)}{(\ +\ k^2) t_1 t_3} & \frac{t_1 + t_3}{(\ +\ k^2)^2 t_1 t_3} & \frac{i \sqrt{\ }\ k (t_1 + t_3)}{(\ +\ k^2)^2 t_1 t_3} & 0 \\ 0 & 0 & 0 & \frac{i k t_1 - i k t_3}{t_1 t_3 + k^2 t_1 t_3} & -\frac{i \sqrt{\ }\ k (t_1 + t_3)}{(\ +\ k^2)^2 t_1 t_3} & \frac{k^2 (t_1 + t_3)}{(\ +\ k^2)^2 t_1 t_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{}{(\ +\ k^2)^2 t_1} & -\frac{i \sqrt{\ }\ k}{(\ +\ k^2)^2 t_1} & 0 \\ \frac{i \sqrt{\ }\ k}{(\ +\ k^2)^2 t_1} & \frac{k^2}{(\ +\ k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{.}}{r_{.}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

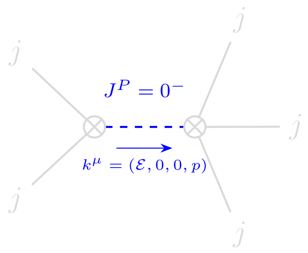
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{.}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{r^{P}_{2}} > 0$
Square mass:	$-\frac{t^{P}_{2}}{r^{P}_{2}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r. < 0$ && $t. > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\underset{2}{r.} < 0 \;\&\&\; \underset{2}{t.} > 0 \tag{79}$$

Okay, that concludes the analysis of this theory.

Case 21

Now for a new theory. Here is the full nonlinear Lagrangian for Case 21 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\underset{2}{r.}\,\mathcal{R}_{\mathfrak{i}\mathfrak{j}\mathfrak{k}\mathfrak{l}}\,\mathcal{R}^{\mathfrak{i}\mathfrak{j}\mathfrak{k}\mathfrak{l}} - \frac{2}{3}\underset{2}{r.}\,\mathcal{R}_{\mathfrak{i}\mathfrak{k}\mathfrak{j}\mathfrak{l}}\,\mathcal{R}^{\mathfrak{i}\mathfrak{j}\mathfrak{k}\mathfrak{l}} + \frac{1}{6}\underset{2}{r.}\,\mathcal{R}^{\mathfrak{i}\mathfrak{j}\mathfrak{k}\mathfrak{l}}\,\mathcal{R}_{\mathfrak{k}\mathfrak{l}\mathfrak{i}\mathfrak{j}} + \frac{1}{4}\underset{1}{t.}\,\mathcal{T}_{\mathfrak{i}\mathfrak{j}\mathfrak{k}}\,\mathcal{T}^{\mathfrak{i}\mathfrak{j}\mathfrak{k}} + \frac{1}{2}\underset{1}{t.}\,\mathcal{T}^{\mathfrak{i}\mathfrak{j}\mathfrak{k}}\,\mathcal{T}_{\mathfrak{j}\mathfrak{i}\mathfrak{k}} + \frac{1}{3}\left(\underset{1}{t.} - 2\underset{3}{t.}\right)\mathcal{T}^{\mathfrak{i}}{}_{\mathfrak{i}}{}^{\mathfrak{j}}\,\mathcal{T}^{\mathfrak{k}}{}_{\mathfrak{j}\mathfrak{k}} \tag{80}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\underset{1}{t.}\,\mathcal{A}_{\mathfrak{a}\mathfrak{i}\mathfrak{b}}\,\mathcal{A}^{\mathfrak{a}\mathfrak{b}\mathfrak{i}} + \frac{1}{3}\left(\underset{1}{t.} - 2\underset{3}{t.}\right)\mathcal{A}^{\mathfrak{a}\mathfrak{b}}{}_{\mathfrak{a}}\,\mathcal{A}_{\mathfrak{b}}{}^{\mathfrak{i}}{}_{\mathfrak{i}} - \frac{2}{3}\left(\underset{1}{t.} - 2\underset{3}{t.}\right)\mathcal{A}_{\mathfrak{b}}{}^{\mathfrak{i}}{}_{\mathfrak{i}}\,\partial_{\mathfrak{a}}f^{\mathfrak{a}\mathfrak{b}} + \\
&\frac{2}{3}\left(\underset{1}{t.} - 2\underset{3}{t.}\right)\mathcal{A}_{\mathfrak{b}}{}^{\mathfrak{i}}{}_{\mathfrak{i}}\,\partial^{\mathfrak{b}}f^{\mathfrak{a}}{}_{\mathfrak{a}} + \frac{1}{3}\left(-\underset{1}{t.} + 2\underset{3}{t.}\right)\partial_{\mathfrak{b}}f^{\mathfrak{i}}{}_{\mathfrak{i}}\,\partial^{\mathfrak{b}}f^{\mathfrak{a}}{}_{\mathfrak{a}} + \frac{1}{3}\left(-\underset{1}{t.} + 2\underset{3}{t.}\right)\partial_{\mathfrak{a}}f^{\mathfrak{a}\mathfrak{b}}\,\partial_{\mathfrak{i}}f^{\mathfrak{i}}{}_{\mathfrak{b}} + \\
&\frac{2}{3}\left(\underset{1}{t.} - 2\underset{3}{t.}\right)\partial^{\mathfrak{b}}f^{\mathfrak{a}}{}_{\mathfrak{a}}\,\partial_{\mathfrak{i}}f^{\mathfrak{i}}{}_{\mathfrak{b}} + 2\underset{1}{t.}\,\mathcal{A}_{\mathfrak{b}\mathfrak{i}\mathfrak{a}}\,\partial^{\mathfrak{i}}f^{\mathfrak{a}\mathfrak{b}} - \underset{1}{t.}\,\partial_{\mathfrak{a}}f_{\mathfrak{b}\mathfrak{i}}\,\partial^{\mathfrak{i}}f^{\mathfrak{a}\mathfrak{b}} + \frac{1}{2}\underset{1}{t.}\,\partial_{\mathfrak{a}}f_{\mathfrak{i}\mathfrak{b}}\,\partial^{\mathfrak{i}}f^{\mathfrak{a}\mathfrak{b}} - \frac{1}{2}\underset{1}{t.}\,\partial_{\mathfrak{b}}f_{\mathfrak{a}\mathfrak{i}}\,\partial^{\mathfrak{i}}f^{\mathfrak{a}\mathfrak{b}} + \\
&\frac{1}{2}\underset{1}{t.}\,\partial_{\mathfrak{i}}f_{\mathfrak{a}\mathfrak{b}}\,\partial^{\mathfrak{i}}f^{\mathfrak{a}\mathfrak{b}} + \frac{1}{2}\underset{1}{t.}\,\partial_{\mathfrak{i}}f_{\mathfrak{b}\mathfrak{a}}\,\partial^{\mathfrak{i}}f^{\mathfrak{a}\mathfrak{b}} + \frac{4}{3}\underset{2}{r.}\,\partial_{\mathfrak{b}}\mathcal{A}_{\mathfrak{a}\mathfrak{i}\mathfrak{j}}\,\partial^{\mathfrak{j}}\mathcal{A}^{\mathfrak{a}\mathfrak{b}\mathfrak{i}} - \frac{2}{3}\underset{2}{r.}\,\partial_{\mathfrak{b}}\mathcal{A}_{\mathfrak{a}\mathfrak{j}\mathfrak{i}}\,\partial^{\mathfrak{j}}\mathcal{A}^{\mathfrak{a}\mathfrak{b}\mathfrak{i}} + \\
&\frac{2}{3}\underset{2}{r.}\,\partial_{\mathfrak{b}}\mathcal{A}_{\mathfrak{i}\mathfrak{j}\mathfrak{a}}\,\partial^{\mathfrak{j}}\mathcal{A}^{\mathfrak{a}\mathfrak{b}\mathfrak{i}} - \frac{1}{3}\underset{2}{r.}\,\partial_{\mathfrak{i}}\mathcal{A}_{\mathfrak{a}\mathfrak{b}\mathfrak{j}}\,\partial^{\mathfrak{j}}\mathcal{A}^{\mathfrak{a}\mathfrak{b}\mathfrak{i}} + \frac{1}{3}\underset{2}{r.}\,\partial_{\mathfrak{j}}\mathcal{A}_{\mathfrak{a}\mathfrak{b}\mathfrak{i}}\,\partial^{\mathfrak{j}}\mathcal{A}^{\mathfrak{a}\mathfrak{b}\mathfrak{i}} - \frac{2}{3}\underset{2}{r.}\,\partial_{\mathfrak{j}}\mathcal{A}_{\mathfrak{a}\mathfrak{i}\mathfrak{b}}\,\partial^{\mathfrak{j}}\mathcal{A}^{\mathfrak{a}\mathfrak{b}\mathfrak{i}}
\end{aligned} \tag{81}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} t_{3} & -i\sqrt{2}\,k\,t_{3} & 0 & 0 \\ i\sqrt{2}\,k\,t_{3} & 2k^2\,t_{3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{2} - t_{1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\frac{t_{1}}{} & -\frac{t_{1}}{\sqrt{}} & \frac{i k t_{1}}{\sqrt{}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1}}{\sqrt{}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_{1}}{\sqrt{}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\left(t. + 4t.\right) & \frac{t_{1} - t_{3}}{\sqrt{}} & -ik\left(t. - 2t.\right) & 0 \\ 0 & 0 & 0 & \frac{t_{1} - t_{3}}{\sqrt{}} & \frac{t_{1} + t_{3}}{} & -i\sqrt{2}\,k\left(t. + t.\right) & 0 \\ 0 & 0 & 0 & --ik\left(t. - 2t.\right) & --i\sqrt{2}\,k\left(t. + t.\right) & -k\left(t. + t.\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{} & -\frac{i k t_{1}}{\sqrt{}} & 0 \\ \frac{i k t_{1}}{\sqrt{}} & k\,t. & 0 \\ 0 & 0 & \frac{t_{1}}{} \end{pmatrix}$$

Gauge constraints on source currents:

$\dot{\tau}^{\perp} == 0$

$-2ik\,\dot{\sigma}^{\parallel} + \dot{\tau}^{\parallel} == 0$

$\dot{\tau}^{\perp\,a} == 0$

$2ik\,\dot{\sigma}^{\perp\,a} + \dot{\tau}^{\parallel\,a} == 0$

$-ik\,\dot{\sigma}^{\perp\,ab} + \dot{\tau}^{\parallel\,ab} == 0$

$-2ik\,\dot{\sigma}^{\parallel\,ab} + \dot{\tau}^{\parallel\,ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{}{(+k^2)^2 t_{3}} & -\frac{i\sqrt{}\,k}{(+k^2)^2 t_{3}} & 0 & 0 \\ \frac{i\sqrt{}\,k}{(+k^2)^2 t_{3}} & \frac{k^2}{(+k^2)^2 t_{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_{2} - t_{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{\ }}{t_1 + k^2 t_1} & \frac{i \sqrt{\ } k}{t_1 + k^2 t_1} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{\ }}{t_1 + k^2 t_1} & \frac{}{(+ k^2)^2 t_1} & -\frac{i k}{(+ k^2)^2 t_1} & 0 & 0 & 0 & 0 \\ -\frac{i \sqrt{\ } k}{t_1 + k^2 t_1} & \frac{i k}{(+ k^2)^2 t_1} & \frac{k^2}{(+ k^2)^2 t_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(t_1 + t_3)}{t_1 t_3} & -\frac{\sqrt{\ } (t_1 - t_3)}{(+ k^2) t_1 t_3} & -\frac{i k t_1 - i k t_3}{t_1 t_3 + k^2 t_1 t_3} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{\ } (t_1 - t_3)}{(+ k^2) t_1 t_3} & \frac{t_1 + t_3}{(+ k^2)^2 t_1 t_3} & \frac{i \sqrt{\ } k (t_1 + t_3)}{(+ k^2)^2 t_1 t_3} & 0 \\ 0 & 0 & 0 & \frac{i k t_1 - i k t_3}{t_1 t_3 + k^2 t_1 t_3} & -\frac{i \sqrt{\ } k (t_1 + t_3)}{(+ k^2)^2 t_1 t_3} & \frac{k^2 (t_1 + t_3)}{(+ k^2)^2 t_1 t_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{}{(+ k^2)^2 t_1} & -\frac{i \sqrt{\ } k}{(+ k^2)^2 t_1} & 0 \\ \frac{i \sqrt{\ } k}{(+ k^2)^2 t_1} & \frac{k^2}{(+ k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t.}{r.}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

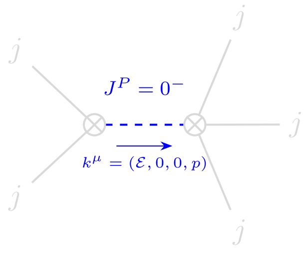
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r.}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r._2} > 0$
Square mass:	$\frac{t._1}{r._2} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r. < 0 \,\&\&\, t. < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r. < 0 \,\&\&\, t. < 0 \tag{82}$$

Okay, that concludes the analysis of this theory.

Case 22

Now for a new theory. Here is the full nonlinear Lagrangian for Case 22 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r. \, \mathcal{R}_{ijkl} \, \mathcal{R}^{ijkl} - \frac{2}{3} r. \, \mathcal{R}_{ikjl} \, \mathcal{R}^{ijkl} + \frac{1}{6} r. \, \mathcal{R}^{ijkl} \, \mathcal{R}_{klij} + \frac{1}{12} \left(4 t. + t.\right) \mathcal{T}_{ijk} \, \mathcal{T}^{ijk} + \frac{1}{6} \left(2 t. - t.\right) \mathcal{T}^{ijk} \, \mathcal{T}_{jik} + t. \, \mathcal{T}^{i}{}_{i}{}^{j} \, \mathcal{T}^{k}{}_{jk} \tag{83}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned} &\frac{1}{3} \left(t. + t.\right) \mathcal{A}_{abi} \, \mathcal{A}^{abi} + \frac{1}{3} \left(t. - 2 t.\right) \mathcal{A}_{aib} \, \mathcal{A}^{abi} + t. \, \mathcal{A}^{ab}{}_{a} \, \mathcal{A}_{b}{}^{i}{}_{i} - \\ &2 t. \, \mathcal{A}_{b}{}^{i}{}_{i} \, \partial_{a} f^{ab} + 2 t. \, \mathcal{A}_{b}{}^{i}{}_{i} \, \partial^{b} f^{a}{}_{a} - t. \, \partial_{b} f^{i}{}_{i} \, \partial^{b} f^{a}{}_{a} - t. \, \partial_{a} f^{ab} \, \partial_{i} f^{i}{}_{b} + 2 t. \, \partial^{b} f^{a}{}_{a} \, \partial_{i} f^{i}{}_{b} - \\ &\frac{2}{3} \left(t. + t.\right) \mathcal{A}_{abi} \, \partial^{i} f^{ab} + \frac{2}{3} \left(t. + t.\right) \mathcal{A}_{aib} \, \partial^{i} f^{ab} + \frac{2}{3} \left(2 t. - t.\right) \mathcal{A}_{bia} \, \partial^{i} f^{ab} + \\ &\frac{1}{3} \left(-2 t. + t.\right) \partial_{a} f_{bi} \, \partial^{i} f^{ab} + \frac{1}{6} \left(2 t. - t.\right) \partial_{a} f_{ib} \, \partial^{i} f^{ab} + \frac{1}{6} \left(-4 t. - t.\right) \partial_{b} f_{ai} \, \partial^{i} f^{ab} + \\ &\frac{1}{6} \left(4 t. + t.\right) \partial_{i} f_{ab} \, \partial^{i} f^{ab} + \frac{1}{6} \left(2 t. - t.\right) \partial_{i} f_{ba} \, \partial^{i} f^{ab} + \frac{4}{3} r. \, \partial_{b} \mathcal{A}_{aij} \, \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r. \, \partial_{b} \mathcal{A}_{aji} \, \partial^{j} \mathcal{A}^{abi} + \\ &\frac{2}{3} r. \, \partial_{b} \mathcal{A}_{ija} \, \partial^{j} \mathcal{A}^{abi} - \frac{1}{3} r. \, \partial_{i} \mathcal{A}_{abj} \, \partial^{j} \mathcal{A}^{abi} + \frac{1}{3} r. \, \partial_{j} \mathcal{A}_{abi} \, \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r. \, \partial_{j} \mathcal{A}_{aib} \, \partial^{j} \mathcal{A}^{abi} \end{aligned} \tag{84}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

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The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} -t. & i \sqrt{2} \, k t. & 0 & 0 \\ -i \sqrt{2} \, k t. & -2 k \, t. & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k \, r. + t. \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} \frac{1}{6} \left(t._{1} + 4 t._{2}\right) & -\frac{t._{1} - 2 t._{2}}{3 \sqrt{2}} & \frac{i k \left(t._{1} - 2 t._{2}\right)}{3 \sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t._{1} - 2 t._{2}}{3 \sqrt{2}} & \frac{t._{1} + t._{2}}{3} & -\frac{1}{3} i k \left(t._{1} + t._{2}\right) & 0 & 0 & 0 & 0 \\ -\frac{i k \left(t._{1} - 2 t._{2}\right)}{3 \sqrt{2}} & \frac{1}{3} i k \left(t._{1} + t._{2}\right) & \frac{1}{3} k^{2} \left(t._{1} + t._{2}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{t._{1}}{2} & \frac{t._{1}}{\sqrt{2}} & i k t._{1} & 0 \\ 0 & 0 & 0 & \frac{t._{1}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i k t._{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_1}{} & -\frac{i k t_1}{\sqrt{}} & 0 \\ \frac{i k t_1}{\sqrt{}} & k\, t_{\cdot} & 0 \\ 0 & 0 & \frac{t_1}{} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{+}_{\cdot}\tau^{\perp} == 0$$

$$-2 i k\ {}^{+}_{\cdot}\sigma^{\parallel} + {}^{+}_{\cdot}\tau^{\parallel} == 0$$

$${}^{-}_{\cdot}\tau^{\perp}{}^{a} == 0$$

$$2 i k\ {}^{-}_{\cdot}\sigma^{\perp}{}^{a} + {}^{-}_{\cdot}\tau^{\parallel}{}^{a} == 0$$

$$-i k\ {}^{+}_{\cdot}\sigma^{\perp}{}^{ab} + {}^{+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

$$-2 i k\ {}^{+}_{\cdot}\sigma^{\parallel}{}^{ab} + {}^{+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{}{(+k^2)^2 t_1} & \frac{i\sqrt{}\,k}{(+k^2)^2 t_1} & 0 & 0 \\ -\frac{i\sqrt{}\,k}{(+k^2)^2 t_1} & -\frac{k^2}{(+k^2)^2 t_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_2 + t_2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{(t_1 + t_2)}{t_1 t_2} & \frac{\sqrt{}\,(t_1 - t_2)}{(+k^2) t_1 t_2} & -\frac{i\sqrt{}\,k(t_1 - t_2)}{(+k^2) t_1 t_2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{}\,(t_1 - t_2)}{(+k^2) t_1 t_2} & \frac{t_1 + t_2}{(+k^2)^2 t_1 t_2} & -\frac{i k (t_1 + t_2)}{(+k^2)^2 t_1 t_2} & 0 & 0 & 0 & 0 \\ \frac{i\sqrt{}\,k(t_1 - t_2)}{(+k^2) t_1 t_2} & \frac{i k (t_1 + t_2)}{(+k^2)^2 t_1 t_2} & \frac{k^2 (t_1 + t_2)}{(+k^2)^2 t_1 t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{}}{t_1 + k^2 t_1} & \frac{i k}{t_1 + k^2 t_1} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{}}{t_1 + k^2 t_1} & \frac{}{(+k^2)^2 t_1} & \frac{i\sqrt{}\,k}{(+k^2)^2 t_1} & 0 \\ 0 & 0 & 0 & -\frac{i k}{t_1 + k^2 t_1} & -\frac{i\sqrt{}\,k}{(+k^2)^2 t_1} & \frac{k^2}{(+k^2)^2 t_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{}{(+k^2)^2 t_1} & -\frac{i\sqrt{}\,k}{(+k^2)^2 t_1} & 0 \\ \frac{i\sqrt{}\,k}{(+k^2)^2 t_1} & \frac{k^2}{(+k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\},\ \left\{-\frac{t_{\cdot 2}}{r_{\cdot 2}}\right\},\ \{\},\ \{\},\ \{\},\ \{\}\right\}$$

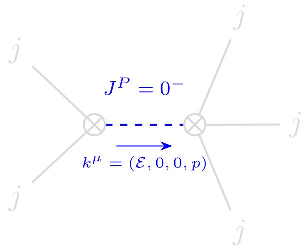
Massive pole residues:

$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{r^P_2} > 0$
Square mass:	$-\frac{t^P_2}{r^P_2} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_2 < 0 \,\&\&\, t_2 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_2 < 0 \,\&\&\, t_2 > 0 \tag{85}$$

Okay, that concludes the analysis of this theory.

Case 23

Now for a new theory. Here is the full nonlinear Lagrangian for Case 23 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_2 \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_2 \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \frac{1}{6} r_2 \mathcal{R}^{ijkl} \mathcal{R}_{klij} + \frac{1}{4} t_1 \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{2} t_1 \mathcal{T}^{ijk} \mathcal{T}_{jik} + t_1 \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk} \tag{86}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned} & t_1 \mathcal{A}_{aib} \mathcal{A}^{abi} + t_1 \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} - 2 t_1 \mathcal{A}_{b}{}^{i}{}_{i} \partial_a f^{ab} + 2 t_1 \mathcal{A}_{b}{}^{i}{}_{i} \partial^b f^{a}{}_{a} - t_1 \partial_b f^{i}{}_{i} \partial^b f^{a}{}_{a} - \\ & t_1 \partial_a f^{ab} \partial_i f^{i}{}_{b} + 2 t_1 \partial^b f^{a}{}_{a} \partial_i f^{i}{}_{b} + 2 t_1 \mathcal{A}_{bia} \partial^i f^{ab} - t_1 \partial_a f_{bi} \partial^i f^{ab} + \frac{1}{2} t_1 \partial_a f_{ib} \partial^i f^{ab} - \\ & \frac{1}{2} t_1 \partial_b f_{ai} \partial^i f^{ab} + \frac{1}{2} t_1 \partial_i f_{ab} \partial^i f^{ab} + \frac{1}{2} t_1 \partial_i f_{ba} \partial^i f^{ab} + \frac{4}{3} r_2 \partial_b \mathcal{A}_{aij} \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_2 \partial_b \mathcal{A}_{aji} \partial^j \mathcal{A}^{abi} + \end{aligned} \tag{87}$$

$$\frac{2}{3} r_{\dot 2}\, \partial_b \mathcal{A}_{ija}\, \partial^j \mathcal{A}^{abi} - \frac{1}{3} r_{\dot 2}\, \partial_i \mathcal{A}_{abj}\, \partial^j \mathcal{A}^{abi} + \frac{1}{3} r_{\dot 2}\, \partial_j \mathcal{A}_{abi}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{\dot 2}\, \partial_j \mathcal{A}_{aib}\, \partial^j \mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -t_{\dot 1} & i\sqrt{2}\, k\, t_{\dot 1} & 0 & 0 \\ -i\sqrt{2}\, k\, t_{\dot 1} & -2k^2 t_{\dot 1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{\dot 2} - t_{\dot 1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\frac{t_{\dot 1}}{2} & -\frac{t_{\dot 1}}{\sqrt{2}} & \frac{i k t_{\dot 1}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{\dot 1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_{\dot 1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{t_{\dot 1}}{2} & \frac{t_{\dot 1}}{\sqrt{2}} & i k t_{\dot 1} & 0 \\ 0 & 0 & 0 & \frac{t_{\dot 1}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i k t_{\dot 1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{\dot 1}}{2} & -\frac{i k t_{\dot 1}}{\sqrt{2}} & 0 \\ \frac{i k t_{\dot 1}}{\sqrt{2}} & k^2 t_{\dot 1} & 0 \\ 0 & 0 & \frac{t_{\dot 1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\cdot}\tau^{\perp} == 0$$

$$-2 i k\, {}^{0^+}_{\cdot}\sigma^{\parallel} + {}^{0^+}_{\cdot}\tau^{\parallel} == 0$$

$${}^{1^-}_{\cdot}\tau^{\perp a} == 0$$

$$2 i k\, {}^{1^-}_{\cdot}\sigma^{\perp a} + {}^{1^-}_{\cdot}\tau^{\parallel a} == 0$$

$$-i k\, {}^{1^+}_{\cdot}\sigma^{\perp ab} + {}^{1^+}_{\cdot}\tau^{\parallel ab} == 0$$

$$-2 i k\, {}^{2^+}_{\cdot}\sigma^{\parallel ab} + {}^{2^+}_{\cdot}\tau^{\parallel ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{(1+2k^2)^2 t_{\dot 1}} & \frac{i\sqrt{2}\, k}{(1+2k^2)^2 t_{\dot 1}} & 0 & 0 \\ -\frac{i\sqrt{2}\, k}{(1+2k^2)^2 t_{\dot 1}} & -\frac{2k^2}{(1+2k^2)^2 t_{\dot 1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{\dot 2} - t_{\dot 1}} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{\dot{t}_1+k^2\dot{t}_1} & \frac{i\sqrt{2}k}{\dot{t}_1+k^2\dot{t}_1} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{\dot{t}_1+k^2\dot{t}_1} & \frac{1}{(1+k^2)^2\dot{t}_1} & -\frac{ik}{(1+k^2)^2\dot{t}_1} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}k}{\dot{t}_1+k^2\dot{t}_1} & \frac{ik}{(1+k^2)^2\dot{t}_1} & \frac{k^2}{(1+k^2)^2\dot{t}_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{\dot{t}_1+2k^2\dot{t}_1} & \frac{2ik}{\dot{t}_1+2k^2\dot{t}_1} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{\dot{t}_1+2k^2\dot{t}_1} & \frac{1}{(1+2k^2)^2\dot{t}_1} & \frac{i\sqrt{2}k}{(1+2k^2)^2\dot{t}_1} & 0 \\ 0 & 0 & 0 & -\frac{2ik}{\dot{t}_1+2k^2\dot{t}_1} & -\frac{i\sqrt{2}k}{(1+2k^2)^2\dot{t}_1} & \frac{2k^2}{(1+2k^2)^2\dot{t}_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2\dot{t}_1} & -\frac{2i\sqrt{2}k}{(1+2k^2)^2\dot{t}_1} & 0 \\ \frac{2i\sqrt{2}k}{(1+2k^2)^2\dot{t}_1} & \frac{4k^2}{(1+2k^2)^2\dot{t}_1} & 0 \\ 0 & 0 & \frac{2}{\dot{t}_1} \end{pmatrix}$$

```
Square masses:
```

$$\left\{\{\}, \left\{\frac{\dot{t}_1}{\dot{r}_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

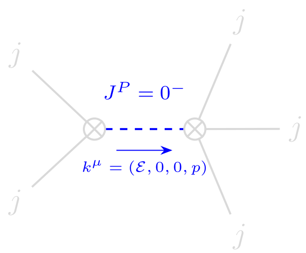
```
Massive pole residues:
```

$$\left\{\{\}, \left\{-\frac{1}{\dot{r}_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

```
Massless eigenvalues:
```

{}

```
Overall particle spectrum:
```



Massive particle

Pole residue:	$-\frac{1}{\dot{r}_2} > 0$
Square mass:	$\frac{\dot{t}_1}{\dot{r}_2} > 0$
Spin:	0
Parity:	Odd

```
Overall unitarity conditions:
```

$\dot{r}_2 < 0$ && $\dot{t}_1 < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\underset{2}{r.} < 0 \;\&\&\; \underset{1}{t.} < 0 \tag{88}$$

Okay, that concludes the analysis of this theory.

Case 24

Now for a new theory. Here is the full nonlinear Lagrangian for Case 24 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\underset{2}{r.}\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl} - \frac{2}{3}\underset{2}{r.}\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl} + \underset{5}{r.}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6}\underset{2}{r.}\,\mathcal{R}^{ijkl}\,\mathcal{R}_{klij} - \underset{5}{r.}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12}\underset{2}{t.}\,\mathcal{T}_{ijk}\,\mathcal{T}^{ijk} - \frac{1}{6}\underset{2}{t.}\,\mathcal{T}^{ijk}\,\mathcal{T}_{jik} - \frac{2}{3}\underset{3}{t.}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk} \tag{89}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3}\underset{2}{t.}\,\mathcal{A}_{abi}\,\mathcal{A}^{abi} - \frac{2}{3}\underset{2}{t.}\,\mathcal{A}_{aib}\,\mathcal{A}^{abi} - \frac{2}{3}\underset{3}{t.}\,\mathcal{A}^{ab}{}_{a}\,\mathcal{A}_{b}{}^{i}{}_{i} + \frac{4}{3}\underset{3}{t.}\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial_{a}f^{ab} - \frac{4}{3}\underset{3}{t.}\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial^{b}f^{a}{}_{a} + \\
&\frac{2}{3}\underset{3}{t.}\,\partial_{b}f^{i}{}_{i}\,\partial^{b}f^{a}{}_{a} + \frac{2}{3}\underset{3}{t.}\,\partial_{a}f^{ab}\,\partial_{i}f^{i}{}_{b} - \frac{4}{3}\underset{3}{t.}\,\partial^{b}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{b} + \underset{5}{r.}\,\partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a} - \underset{5}{r.}\,\partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a} - \\
&\frac{2}{3}\underset{2}{t.}\,\mathcal{A}_{abi}\,\partial^{i}f^{ab} + \frac{2}{3}\underset{2}{t.}\,\mathcal{A}_{aib}\,\partial^{i}f^{ab} - \frac{2}{3}\underset{2}{t.}\,\mathcal{A}_{bia}\,\partial^{i}f^{ab} + \frac{1}{3}\underset{2}{t.}\,\partial_{a}f_{bi}\,\partial^{i}f^{ab} - \frac{1}{6}\underset{2}{t.}\,\partial_{a}f_{ib}\,\partial^{i}f^{ab} - \\
&\frac{1}{6}\underset{2}{t.}\,\partial_{b}f_{ai}\,\partial^{i}f^{ab} + \frac{1}{6}\underset{2}{t.}\,\partial_{i}f_{ab}\,\partial^{i}f^{ab} - \frac{1}{6}\underset{2}{t.}\,\partial_{i}f_{ba}\,\partial^{i}f^{ab} - \underset{5}{r.}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} + 2\underset{5}{r.}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} + \\
&\underset{5}{r.}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} - 2\underset{5}{r.}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3}\underset{2}{r.}\,\partial_{b}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{abi} - \frac{2}{3}\underset{2}{r.}\,\partial_{b}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{abi} + \\
&\frac{2}{3}\underset{2}{r.}\,\partial_{b}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{abi} - \frac{1}{3}\underset{2}{r.}\,\partial_{i}\mathcal{A}_{abj}\,\partial^{j}\mathcal{A}^{abi} + \frac{1}{3}\underset{2}{r.}\,\partial_{j}\mathcal{A}_{abi}\,\partial^{j}\mathcal{A}^{abi} - \frac{2}{3}\underset{2}{r.}\,\partial_{j}\mathcal{A}_{aib}\,\partial^{j}\mathcal{A}^{abi}
\end{aligned} \tag{90}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:
```

$$\begin{pmatrix} \underset{3}{t.} & -i\sqrt{2}\,k\,\underset{3}{t.} & 0 & 0 \\ i\sqrt{2}\,k\,\underset{3}{t.} & 2k^{2}\,\underset{3}{t.} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^{2}\,\underset{2}{r.} + \underset{2}{t.} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} k\ r_{\cdot} + \frac{t_{2}}{} & \frac{\sqrt{\ }\ t_{2}}{} & --i\sqrt{2}\ k t_{\cdot} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\ }\ t_{2}}{} & \frac{t_{2}}{} & --i\, k t_{\cdot} & 0 & 0 & 0 & 0 \\ -i\sqrt{2}\ k t_{\cdot} & \frac{i\, k\, t_{2}}{} & \frac{k^{2}\ t_{2}}{} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\ r_{\cdot} + \frac{t_{3}}{} & -\frac{\sqrt{\ }\ t_{3}}{} & --i\, k t_{\cdot} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{\ }\ t_{3}}{} & \frac{t_{3}}{} & -i\sqrt{2}\ k t_{\cdot} & 0 \\ 0 & 0 & 0 & \frac{i\, k\, t_{3}}{} & --i\sqrt{2}\ k t_{\cdot} & \frac{k^{2}\ t_{3}}{} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$\cdot\overset{+}{\tau}^{\perp} == 0$

$-2\, i\, k\ \cdot\overset{+}{\sigma}^{\parallel} + \cdot\overset{+}{\tau}^{\parallel} == 0$

$\cdot\overset{-}{\tau}^{\perp\, a} == 0$

$2\, i\, k\ \cdot\overset{-}{\sigma}^{\perp\, a} + \cdot\overset{-}{\tau}^{\parallel\, a} == 0$

$-i\, k\ \cdot\overset{+}{\sigma}^{\perp\, ab} + \cdot\overset{+}{\tau}^{\parallel\, ab} == 0$

$\cdot\overset{-}{\sigma}^{\parallel\, abc} == 0$

$\cdot\overset{+}{\tau}^{\parallel\, ab} == 0$

$\cdot\overset{+}{\sigma}^{\parallel\, ab} == 0$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{}{(\ +\ k^{2})^{2}\, t_{3}} & -\frac{i\sqrt{\ }\ k}{(\ +\ k^{2})^{2}\, t_{3}} & 0 & 0 \\ \frac{i\sqrt{\ }\ k}{(\ +\ k^{2})^{2}\, t_{3}} & \frac{k^{2}}{(\ +\ k^{2})^{2}\, t_{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^{2}\ r_{2} + t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2 r_5} & -\frac{\sqrt{2}}{k^2 r_5+k^4 r_5} & \frac{i\sqrt{2}}{k r_5+k^3 r_5} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{k^2 r_5+k^4 r_5} & \frac{3k^2 r_5+2t_2}{(k+k^3)^2 r_5 t_2} & -\frac{i(3k^2 r_5+2t_2)}{k(1+k^2)^2 r_5 t_2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}}{k r_5+k^3 r_5} & \frac{i(3k^2 r_5+2t_2)}{k(1+k^2)^2 r_5 t_2} & \frac{3k^2 r_5+2t_2}{(1+k^2)^2 r_5 t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_5} & \frac{\sqrt{2}}{k^2 r_5+2k^4 r_5} & \frac{2i}{k r_5+2k^3 r_5} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{k^2 r_5+2k^4 r_5} & \frac{3k^2 r_5+2t_3}{(k+2k^3)^2 r_5 t_3} & \frac{i\sqrt{2}(3k^2 r_5+2t_3)}{k(1+2k^2)^2 r_5 t_3} & 0 \\ 0 & 0 & 0 & -\frac{2i}{k r_5+2k^3 r_5} & -\frac{i\sqrt{2}(3k^2 r_5+2t_3)}{k(1+2k^2)^2 r_5 t_3} & \frac{6k^2 r_5+4t_3}{(1+2k^2)^2 r_5 t_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_2}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

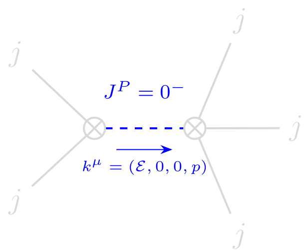
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_2} > 0$
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_2 < 0$ && $t_2 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\underset{2}{\bullet}} < 0 \,\&\&\, t_{\underset{2}{\bullet}} > 0 \tag{91}$$

Okay, that concludes the analysis of this theory.

Case 25

Now for a new theory. Here is the full nonlinear Lagrangian for Case 25 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\underset{2}{\bullet}} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{\underset{2}{\bullet}} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \frac{1}{6} r_{\underset{2}{\bullet}} \mathcal{R}^{ijkl} \mathcal{R}_{klij} + \frac{1}{12} t_{\underset{2}{\bullet}} \mathcal{T}_{ijk} \mathcal{T}^{ijk} - \frac{1}{6} t_{\underset{2}{\bullet}} \mathcal{T}^{ijk} \mathcal{T}_{jik} - \frac{2}{3} t_{\underset{3}{\bullet}} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk} \tag{92}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3} t_{\underset{2}{\bullet}} \mathcal{A}_{abi} \mathcal{A}^{abi} - \frac{2}{3} t_{\underset{2}{\bullet}} \mathcal{A}_{aib} \mathcal{A}^{abi} - \frac{2}{3} t_{\underset{3}{\bullet}} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} + \frac{4}{3} t_{\underset{3}{\bullet}} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} - \\
&\frac{4}{3} t_{\underset{3}{\bullet}} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{\underset{3}{\bullet}} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{\underset{3}{\bullet}} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} - \frac{4}{3} t_{\underset{3}{\bullet}} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} - \frac{2}{3} t_{\underset{2}{\bullet}} \mathcal{A}_{abi} \partial^{i} f^{ab} + \\
&\frac{2}{3} t_{\underset{2}{\bullet}} \mathcal{A}_{aib} \partial^{i} f^{ab} - \frac{2}{3} t_{\underset{2}{\bullet}} \mathcal{A}_{bia} \partial^{i} f^{ab} + \frac{1}{3} t_{\underset{2}{\bullet}} \partial_{a} f_{bi} \partial^{i} f^{ab} - \frac{1}{6} t_{\underset{2}{\bullet}} \partial_{a} f_{ib} \partial^{i} f^{ab} - \frac{1}{6} t_{\underset{2}{\bullet}} \partial_{b} f_{ai} \partial^{i} f^{ab} + \\
&\frac{1}{6} t_{\underset{2}{\bullet}} \partial_{i} f_{ab} \partial^{i} f^{ab} - \frac{1}{6} t_{\underset{2}{\bullet}} \partial_{i} f_{ba} \partial^{i} f^{ab} + \frac{4}{3} r_{\underset{2}{\bullet}} \partial_{b} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{\underset{2}{\bullet}} \partial_{b} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{abi} + \\
&\frac{2}{3} r_{\underset{2}{\bullet}} \partial_{b} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{abi} - \frac{1}{3} r_{\underset{2}{\bullet}} \partial_{i} \mathcal{A}_{abj} \partial^{j} \mathcal{A}^{abi} + \frac{1}{3} r_{\underset{2}{\bullet}} \partial_{j} \mathcal{A}_{abi} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{\underset{2}{\bullet}} \partial_{j} \mathcal{A}_{aib} \partial^{j} \mathcal{A}^{abi}
\end{aligned} \tag{93}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix}
t_{\underset{3}{\bullet}} & -i\sqrt{2}\, k\, t_{\underset{3}{\bullet}} & 0 & 0 \\
i\sqrt{2}\, k\, t_{\underset{3}{\bullet}} & 2k^2 t_{\underset{3}{\bullet}} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & k^2 r_{\underset{2}{\bullet}} + t_{\underset{2}{\bullet}}
\end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
\frac{2t_{\underset{2}{\bullet}}}{3} & \frac{\sqrt{2}\, t_{\underset{2}{\bullet}}}{3} & -\frac{1}{3} i\sqrt{2}\, k\, t_{\underset{2}{\bullet}} & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}\, t_{\underset{2}{\bullet}}}{3} & \frac{t_{\underset{2}{\bullet}}}{3} & -\frac{1}{3} i\, k\, t_{\underset{2}{\bullet}} & 0 & 0 & 0 & 0 \\
\frac{1}{3} i\sqrt{2}\, k\, t_{\underset{2}{\bullet}} & \frac{i k t_{\underset{2}{\bullet}}}{3} & \frac{k^2 t_{\underset{2}{\bullet}}}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2t_{\underset{3}{\bullet}}}{3} & -\frac{\sqrt{2}\, t_{\underset{3}{\bullet}}}{3} & -\frac{2}{3} i\, k\, t_{\underset{3}{\bullet}} & 0 \\
0 & 0 & 0 & -\frac{\sqrt{2}\, t_{\underset{3}{\bullet}}}{3} & \frac{t_{\underset{3}{\bullet}}}{3} & \frac{1}{3} i\sqrt{2}\, k\, t_{\underset{3}{\bullet}} & 0 \\
0 & 0 & 0 & \frac{2 i k t_{\underset{3}{\bullet}}}{3} & -\frac{1}{3} i\sqrt{2}\, k\, t_{\underset{3}{\bullet}} & \frac{2k^2 t_{\underset{3}{\bullet}}}{3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\cdot}\tau^{\perp} == 0$$

$$-2\, i\, k\, {}^{0^+}_{\cdot}\sigma^{\parallel} + {}^{0^+}_{\cdot}\tau^{\parallel} == 0$$

$${}^{1^-}_{\cdot}\tau^{\perp}{}^{a} == 0$$

$$-i\, k\, {}^{1^-}_{\cdot}\sigma^{\parallel}{}^{a} + {}^{1^-}_{\cdot}\tau^{\parallel}{}^{a} == 0$$

$${}^{1^-}_{\cdot}\sigma^{\parallel}{}^{a} + 2\, {}^{1^-}_{\cdot}\sigma^{\perp}{}^{a} == 0$$

$$-i\, k\, {}^{1^+}_{\cdot}\sigma^{\parallel}{}^{ab} + {}^{1^+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

$${}^{1^+}_{\cdot}\sigma^{\parallel}{}^{ab} == {}^{1^+}_{\cdot}\sigma^{\perp}{}^{ab}$$

$${}^{2^-}_{\cdot}\sigma^{\parallel}{}^{abc} == 0$$

$${}^{2^+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

$${}^{2^+}_{\cdot}\sigma^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{(1+2k^2)^2 t_{3}} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & 0 & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & \frac{2k^2}{(1+2k^2)^2 t_{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{2} + t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{6}{(3+k^2)^2 t_{2}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{2}} & -\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{2}} & 0 & 0 & 0 & 0 \\ \frac{3\sqrt{2}}{(3+k^2)^2 t_{2}} & \frac{3}{(3+k^2)^2 t_{2}} & -\frac{3ik}{(3+k^2)^2 t_{2}} & 0 & 0 & 0 & 0 \\ \frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{2}} & \frac{3ik}{(3+k^2)^2 t_{2}} & \frac{3k^2}{(3+k^2)^2 t_{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{(3+2k^2)^2 t_{3}} & -\frac{3\sqrt{2}}{(3+2k^2)^2 t_{3}} & -\frac{6ik}{(3+2k^2)^2 t_{3}} & 0 \\ 0 & 0 & 0 & -\frac{3\sqrt{2}}{(3+2k^2)^2 t_{3}} & \frac{3}{(3+2k^2)^2 t_{3}} & \frac{3i\sqrt{2}\,k}{(3+2k^2)^2 t_{3}} & 0 \\ 0 & 0 & 0 & \frac{6ik}{(3+2k^2)^2 t_{3}} & -\frac{3i\sqrt{2}\,k}{(3+2k^2)^2 t_{3}} & \frac{6k^2}{(3+2k^2)^2 t_{3}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

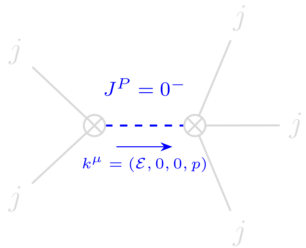
Massive pole residues:

$\left\{\{\}, \left\{-\frac{1}{r_{\cdot 2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{\cdot 2}} > 0$
Square mass:	$-\frac{t_{\cdot 2}}{r_{\cdot 2}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{\cdot 2} < 0 \;\&\&\; t_{\cdot 2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\cdot 2} < 0 \;\&\&\; t_{\cdot 2} > 0 \tag{94}$$

Okay, that concludes the analysis of this theory.

Case 26

Now for a new theory. Here is the full nonlinear Lagrangian for Case 26 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\cdot 2}\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} - \frac{2}{3} r_{\cdot 2}\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} + \frac{1}{6} r_{\cdot 2}\, \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + \frac{1}{12} t_{\cdot 2}\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} - \frac{1}{6} t_{\cdot 2}\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik} \tag{95}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\frac{1}{3} t_{\cdot 2}\, \mathcal{A}_{abi}\, \mathcal{A}^{abi} - \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} - \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{abi}\, \partial^i f^{ab} + \frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{aib}\, \partial^i f^{ab} - \\ &\frac{2}{3} t_{\cdot 2}\, \mathcal{A}_{bia}\, \partial^i f^{ab} + \frac{1}{3} t_{\cdot 2}\, \partial_a f_{bi}\, \partial^i f^{ab} - \frac{1}{6} t_{\cdot 2}\, \partial_a f_{ib}\, \partial^i f^{ab} - \frac{1}{6} t_{\cdot 2}\, \partial_b f_{ai}\, \partial^i f^{ab} + \\ &\frac{1}{6} t_{\cdot 2}\, \partial_i f_{ab}\, \partial^i f^{ab} - \frac{1}{6} t_{\cdot 2}\, \partial_i f_{ba}\, \partial^i f^{ab} + \frac{4}{3} r_{\cdot 2}\, \partial_b \mathcal{A}_{aij}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{\cdot 2}\, \partial_b \mathcal{A}_{aji}\, \partial^j \mathcal{A}^{abi} + \\ &\frac{2}{3} r_{\cdot 2}\, \partial_b \mathcal{A}_{ija}\, \partial^j \mathcal{A}^{abi} - \frac{1}{3} r_{\cdot 2}\, \partial_i \mathcal{A}_{abj}\, \partial^j \mathcal{A}^{abi} + \frac{1}{3} r_{\cdot 2}\, \partial_j \mathcal{A}_{abi}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{\cdot 2}\, \partial_j \mathcal{A}_{aib}\, \partial^j \mathcal{A}^{abi}\end{aligned} \tag{96}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{\underset{2}{\cdot}} + t_{\underset{2}{\cdot}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{2 t_{\underset{2}{\cdot}}}{3} & \frac{\sqrt{2}\, t_{\underset{2}{\cdot}}}{3} & -\frac{1}{3} i \sqrt{2}\, k t_{\underset{2}{\cdot}} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}\, t_{\underset{2}{\cdot}}}{3} & \frac{t_{\underset{2}{\cdot}}}{3} & -\frac{1}{3} i k t_{\underset{2}{\cdot}} & 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{\underset{2}{\cdot}} & \frac{i k t_{\underset{2}{\cdot}}}{3} & \frac{k^2 t_{\underset{2}{\cdot}}}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^+}_{\cdot}\tau^{\perp} == 0$

${}^{0^+}_{\cdot}\tau^{\parallel} == 0$

${}^{0^+}_{\cdot}\sigma^{\parallel} == 0$

${}^{1^-}_{\cdot}\tau^{\perp\,a} == 0$

${}^{1^-}_{\cdot}\tau^{\parallel\,a} == 0$

${}^{1^-}_{\cdot}\sigma^{\perp\,a} == 0$

${}^{1^-}_{\cdot}\sigma^{\parallel\,a} == 0$

$-i k\, {}^{1^+}_{\cdot}\sigma^{\parallel\,ab} + {}^{1^+}_{\cdot}\tau^{\parallel\,ab} == 0$

${}^{1^+}_{\cdot}\sigma^{\parallel\,ab} == {}^{1^+}_{\cdot}\sigma^{\perp\,ab}$

${}^{2^-}_{\cdot}\sigma^{\parallel\,abc} == 0$

${}^{2^+}_{\cdot}\tau^{\parallel\,ab} == 0$

${}^{2^+}_{\cdot}\sigma^{\parallel\,ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{\underset{2}{\cdot}} + t_{\underset{2}{\cdot}}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{6}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & -\frac{3 i \sqrt{2} k}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & 0 & 0 & 0 & 0 \\ \frac{3\sqrt{2}}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & \frac{3}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & -\frac{3 i k}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & 0 & 0 & 0 & 0 \\ \frac{3 i \sqrt{2} k}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & \frac{3 i k}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & \frac{3 k^2}{(3+k^2)^2 t_{\underset{2}{\cdot}}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\underset{2}{\cdot}}}{r_{\underset{2}{\cdot}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

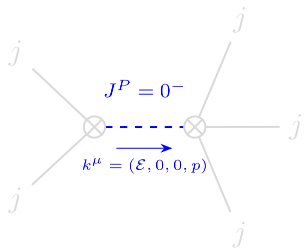
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\underset{2}{\cdot}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{\underset{2}{\cdot}}} > 0$
Square mass:	$-\frac{t_{\underset{2}{\cdot}}}{r_{\underset{2}{\cdot}}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{\underset{2}{\cdot}} < 0 \,\&\&\, t_{\underset{2}{\cdot}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\cdot} < 0 \,\&\&\, t_{\cdot} > 0 \tag{97}$$

Okay, that concludes the analysis of this theory.

Case 27

Now for a new theory. Here is the full nonlinear Lagrangian for Case 27 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\bullet}\, \mathcal{R}_{ijhl}\, \mathcal{R}^{ijhl} - \frac{2}{3} r_{\bullet}\, \mathcal{R}_{ihjl}\, \mathcal{R}^{ijhl} + \frac{1}{6}\left(r_{\bullet} - 6\, r_{\bullet}\right) \mathcal{R}^{ijhl}\, \mathcal{R}_{hlij} + r_{\bullet}\, \mathcal{R}^{ij\,h}{}_{i}\, \mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{12} t_{\bullet}\, \mathcal{T}_{ijh}\, \mathcal{T}^{ijh} - \frac{1}{6} t_{\bullet}\, \mathcal{T}^{ijh}\, \mathcal{T}_{jih} \tag{98}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{\bullet}\, \mathcal{A}_{abi}\, \mathcal{A}^{abi} - \frac{2}{3} t_{\bullet}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} - r_{\bullet}\, \partial_b \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^i \mathcal{A}^{ab}{}_{a} - \frac{2}{3} t_{\bullet}\, \mathcal{A}_{abi}\, \partial^i f^{ab} + \frac{2}{3} t_{\bullet}\, \mathcal{A}_{aib}\, \partial^i f^{ab} - \frac{2}{3} t_{\bullet}\, \mathcal{A}_{bia}\, \partial^i f^{ab} + \frac{1}{3} t_{\bullet}\, \partial_a f_{bi}\, \partial^i f^{ab} - \frac{1}{6} t_{\bullet}\, \partial_a f_{ib}\, \partial^i f^{ab} - \frac{1}{6} t_{\bullet}\, \partial_b f_{ai}\, \partial^i f^{ab} + \frac{1}{6} t_{\bullet}\, \partial_i f_{ab}\, \partial^i f^{ab} - \frac{1}{6} t_{\bullet}\, \partial_i f_{ba}\, \partial^i f^{ab} - r_{\bullet}\, \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{b} + 2\, r_{\bullet}\, \partial^i \mathcal{A}^{ab}{}_{a}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3} r_{\bullet}\, \partial_b \mathcal{A}_{aij}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{\bullet}\, \partial_b \mathcal{A}_{aji}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3}\left(r_{\bullet} - 6\, r_{\bullet}\right) \partial_b \mathcal{A}_{ija}\, \partial^j \mathcal{A}^{abi} - \frac{1}{3} r_{\bullet}\, \partial_i \mathcal{A}_{abj}\, \partial^j \mathcal{A}^{abi} + \frac{1}{3} r_{\bullet}\, \partial_j \mathcal{A}_{abi}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{\bullet}\, \partial_j \mathcal{A}_{aib}\, \partial^j \mathcal{A}^{abi} \tag{99}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\; r_{\bullet} + t_{\bullet} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} -\left(9 k\; r_{\bullet} + 4 t_{\bullet}\right) & \frac{\sqrt{\ }\; t_{\bullet 2}}{} & --\, i \sqrt{2}\, k t_{\bullet} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\ }\; t_{\bullet 2}}{} & \frac{t_{\bullet 2}}{} & --\, i\, k t_{\bullet} & 0 & 0 & 0 & 0 \\ -\, i \sqrt{2}\, k t_{\bullet} & \frac{i\, k\, t_{\bullet 2}}{} & \frac{k^2\, t_{\bullet 2}}{} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} -\frac{3 k^2\, r_{\bullet 3}}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

```
Gauge constraints on source currents:
```

${}^{0^+}_{\bullet}\tau^{\perp} == 0$

${}^{0^+}_{\bullet}\tau^{\parallel} == 0$

${}^{0^+}_{\bullet}\sigma^{\parallel} == 0$

${}^{1^-}_{\bullet}\tau^{\perp a} == 0$

$${}^{1^-}_{\bullet}\tau^{\parallel\,\mathfrak{a}} == 0$$

$${}^{1^-}_{\bullet}\sigma^{\perp\,\mathfrak{a}} == 0$$

$${}^{1^-}_{\bullet}\sigma^{\parallel\,\mathfrak{a}} == 0$$

$$-i\,k\ {}^{1^+}_{\bullet}\sigma^{\perp\,\mathfrak{a}\mathfrak{b}} + {}^{1^+}_{\bullet}\tau^{\parallel\,\mathfrak{a}\mathfrak{b}} == 0$$

$${}^{2^-}_{\bullet}\sigma^{\parallel\,\mathfrak{a}\mathfrak{b}\mathfrak{c}} == 0$$

$${}^{2^+}_{\bullet}\tau^{\parallel\,\mathfrak{a}\mathfrak{b}} == 0$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{\bullet 2}+t_{\bullet 2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{2}{3k^2 r_{\bullet 3}} & -\frac{2\sqrt{2}}{3k^2 r_{\bullet 3}+3k^4 r_{\bullet 3}} & \frac{2i\sqrt{2}}{3k\,r_{\bullet 3}+3k^3 r_{\bullet 3}} & 0 & 0 & 0 & 0 \\ -\frac{2\sqrt{2}}{3k^2 r_{\bullet 3}+3k^4 r_{\bullet 3}} & \frac{9k^2 r_{\bullet 3}+4t_{\bullet 2}}{3(k+k^3)^2 r_{\bullet 3} t_{\bullet 2}} & -\frac{i(9k^2 r_{\bullet 3}+4t_{\bullet 2})}{3k(1+k^2)^2 r_{\bullet 3} t_{\bullet 2}} & 0 & 0 & 0 & 0 \\ -\frac{2i\sqrt{2}}{3k\,r_{\bullet 3}+3k^3 r_{\bullet 3}} & \frac{i(9k^2 r_{\bullet 3}+4t_{\bullet 2})}{3k(1+k^2)^2 r_{\bullet 3} t_{\bullet 2}} & \frac{9k^2 r_{\bullet 3}+4t_{\bullet 2}}{3(1+k^2)^2 r_{\bullet 3} t_{\bullet 2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{3k^2 r_{\bullet 3}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\bullet 2}}{r_{\bullet 2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

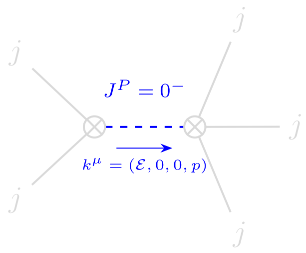
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\bullet 2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{\cdot 2}} > 0$
Square mass:	$-\frac{t_{\cdot 2}}{r_{\cdot 2}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{\cdot 2} < 0 \;\&\&\; t_{\cdot 2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\cdot} < 0 \;\&\&\; t_{\cdot} > 0 \tag{100}$$

Okay, that concludes the analysis of this theory.

Case 28

Now for a new theory. Here is the full nonlinear Lagrangian for Case 28 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\cdot} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{\cdot} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + r_{\cdot} \mathcal{R}^{ij\ k}_{\ \ i} \mathcal{R}_{j\ kl}^{\ l} + \frac{1}{6} r_{\cdot} \mathcal{R}^{ijkl} \mathcal{R}_{klij} - r_{\cdot} \mathcal{R}^{ij\ k}_{\ \ i} \mathcal{R}_{k\ jl}^{\ l} + \frac{1}{12} t_{\cdot} \mathcal{T}_{ijk} \mathcal{T}^{ijk} - \frac{1}{6} t_{\cdot} \mathcal{T}^{ijk} \mathcal{T}_{jik} \tag{101}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\frac{1}{3} t_{\cdot} \mathcal{A}_{abi} \mathcal{A}^{abi} - \frac{2}{3} t_{\cdot} \mathcal{A}_{aib} \mathcal{A}^{abi} + r_{\cdot} \partial_b \mathcal{A}_{i\ j}^{\ j} \partial^i \mathcal{A}^{ab}_{\ \ a} - r_{\cdot} \partial_i \mathcal{A}_{b\ j}^{\ j} \partial^i \mathcal{A}^{ab}_{\ \ a} - \frac{2}{3} t_{\cdot} \mathcal{A}_{abi} \partial^i f^{ab} + \\ &\frac{2}{3} t_{\cdot} \mathcal{A}_{aib} \partial^i f^{ab} - \frac{2}{3} t_{\cdot} \mathcal{A}_{bia} \partial^i f^{ab} + \frac{1}{3} t_{\cdot} \partial_a f_{bi} \partial^i f^{ab} - \frac{1}{6} t_{\cdot} \partial_a f_{ib} \partial^i f^{ab} - \frac{1}{6} t_{\cdot} \partial_b f_{ai} \partial^i f^{ab} + \\ &\frac{1}{6} t_{\cdot} \partial_i f_{ab} \partial^i f^{ab} - \frac{1}{6} t_{\cdot} \partial_i f_{ba} \partial^i f^{ab} - r_{\cdot} \partial_a \mathcal{A}^{abi} \partial_j \mathcal{A}_{b\ i}^{\ j} + 2 r_{\cdot} \partial^i \mathcal{A}^{ab}_{\ \ a} \partial_j \mathcal{A}_{b\ i}^{\ j} + \\ &r_{\cdot} \partial_a \mathcal{A}^{abi} \partial_j \mathcal{A}_{i\ b}^{\ j} - 2 r_{\cdot} \partial^i \mathcal{A}^{ab}_{\ \ a} \partial_j \mathcal{A}_{i\ b}^{\ j} + \frac{4}{3} r_{\cdot} \partial_b \mathcal{A}_{aij} \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{\cdot} \partial_b \mathcal{A}_{aji} \partial^j \mathcal{A}^{abi} + \\ &\frac{2}{3} r_{\cdot} \partial_b \mathcal{A}_{ija} \partial^j \mathcal{A}^{abi} - \frac{1}{3} r_{\cdot} \partial_i \mathcal{A}_{abj} \partial^j \mathcal{A}^{abi} + \frac{1}{3} r_{\cdot} \partial_j \mathcal{A}_{abi} \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{\cdot} \partial_j \mathcal{A}_{aib} \partial^j \mathcal{A}^{abi}\end{aligned} \tag{102}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\, r_{\cdot} + t_{\cdot} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k\, r_{\cdot} + \frac{t_{2}}{} & \frac{\sqrt{\ }\, t_{2}}{} & --i\sqrt{2}\, k t_{\cdot} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\ }\, t_{2}}{} & \frac{t_{2}}{} & --i k t_{\cdot} & 0 & 0 & 0 & 0 \\ -i\sqrt{2}\, k t_{\cdot} & \frac{i k t_{2}}{} & \frac{k^2 t_{2}}{} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\, r_{\cdot} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

${}^{+}_{\cdot}\tau^{\perp} == 0$

${}^{+}_{\cdot}\tau^{\parallel} == 0$

${}^{+}_{\cdot}\sigma^{\parallel} == 0$

${}^{-}_{\cdot}\tau^{\perp\, a} == 0$

${}^{-}_{\cdot}\tau^{\parallel\, a} == 0$

${}^{-}_{\cdot}\sigma^{\perp\, a} == 0$

$-i k\ {}^{+}_{\cdot}\sigma^{\perp\, ab} + {}^{+}_{\cdot}\tau^{\parallel\, ab} == 0$

${}^{-}_{\cdot}\sigma^{\parallel\, abc} == 0$

${}^{+}_{\cdot}\tau^{\parallel\, ab} == 0$

${}^{+}_{\cdot}\sigma^{\parallel\, ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_{2} + t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2 r_5} & -\frac{\sqrt{2}}{k^2 r_5+k^4 r_5} & \frac{i \sqrt{2}}{k r_5+k^3 r_5} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{k^2 r_5+k^4 r_5} & \frac{3 k^2 r_5+2 t_2}{(k+k^3)^2 r_5 t_2} & -\frac{i\left(3 k^2 r_5+2 t_2\right)}{k (1+k^2)^2 r_5 t_2} & 0 & 0 & 0 & 0 \\ -\frac{i \sqrt{2}}{k r_5+k^3 r_5} & \frac{i\left(3 k^2 r_5+2 t_2\right)}{k (1+k^2)^2 r_5 t_2} & \frac{3 k^2 r_5+2 t_2}{(1+k^2)^2 r_5 t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_5} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_2}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

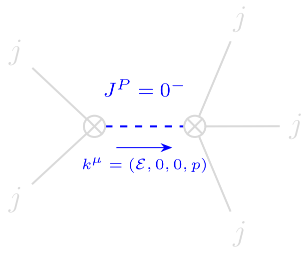
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{r\mathrm{P}_2} > 0$
Square mass:	$-\frac{t\mathrm{P}_2}{r\mathrm{P}_2} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_2 < 0 \;\&\&\; t_2 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_2 < 0 \;\&\&\; t_2 > 0 \tag{103}$$

Okay, that concludes the analysis of this theory.

Case 29

Now for a new theory. Here is the full nonlinear Lagrangian for Case 29 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{6}\left(2\dot{r}_{1}+\dot{r}_{2}\right)\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl}+\frac{2}{3}\left(\dot{r}_{1}-\dot{r}_{2}\right)\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl}-\\&2\dot{r}_{1}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl}+\frac{1}{6}\left(-4\dot{r}_{1}+\dot{r}_{2}\right)\mathcal{R}^{ijkl}\,\mathcal{R}_{klij}+2\dot{r}_{1}\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl}+\\&\frac{1}{12}\dot{t}_{2}\,\mathcal{T}_{ijk}\,\mathcal{T}^{ijk}-\frac{1}{6}\dot{t}_{2}\,\mathcal{T}^{ijk}\,\mathcal{T}_{jik}-\frac{2}{3}\dot{t}_{3}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk}\end{aligned}\tag{104}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\frac{1}{3}\dot{t}_{2}\,\mathcal{A}_{abi}\,\mathcal{A}^{abi}-\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{aib}\,\mathcal{A}^{abi}-\frac{2}{3}\dot{t}_{3}\,\mathcal{A}^{ab}{}_{a}\,\mathcal{A}_{b}{}^{i}{}_{i}+\frac{4}{3}\dot{t}_{3}\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial_{a}f^{ab}-\frac{4}{3}\dot{t}_{3}\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial^{b}f^{a}{}_{a}+\\&\frac{2}{3}\dot{t}_{3}\,\partial_{b}f^{i}{}_{i}\,\partial^{b}f^{a}{}_{a}+\frac{2}{3}\dot{t}_{3}\,\partial_{a}f^{ab}\,\partial_{i}f^{i}{}_{b}-\frac{4}{3}\dot{t}_{3}\,\partial^{b}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{b}-2\dot{r}_{1}\,\partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}+\\&2\dot{r}_{1}\,\partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}-\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{abi}\,\partial^{i}f^{ab}+\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{aib}\,\partial^{i}f^{ab}-\frac{2}{3}\dot{t}_{2}\,\mathcal{A}_{bia}\,\partial^{i}f^{ab}+\\&\frac{1}{3}\dot{t}_{2}\,\partial_{a}f_{bi}\,\partial^{i}f^{ab}-\frac{1}{6}\dot{t}_{2}\,\partial_{a}f_{ib}\,\partial^{i}f^{ab}-\frac{1}{6}\dot{t}_{2}\,\partial_{b}f_{ai}\,\partial^{i}f^{ab}+\frac{1}{6}\dot{t}_{2}\,\partial_{i}f_{ab}\,\partial^{i}f^{ab}-\frac{1}{6}\dot{t}_{2}\,\partial_{i}f_{ba}\,\partial^{i}f^{ab}+\\&2\dot{r}_{1}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}-4\dot{r}_{1}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}-2\dot{r}_{1}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}+4\dot{r}_{1}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}-\\&\frac{4}{3}\left(\dot{r}_{1}-\dot{r}_{2}\right)\partial_{b}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\left(\dot{r}_{1}-\dot{r}_{2}\right)\partial_{b}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\left(-4\dot{r}_{1}+\dot{r}_{2}\right)\partial_{b}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{abi}+\\&\frac{1}{3}\left(-2\dot{r}_{1}-\dot{r}_{2}\right)\partial_{i}\mathcal{A}_{abj}\,\partial^{j}\mathcal{A}^{abi}+\frac{1}{3}\left(2\dot{r}_{1}+\dot{r}_{2}\right)\partial_{j}\mathcal{A}_{abi}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\left(\dot{r}_{1}-\dot{r}_{2}\right)\partial_{j}\mathcal{A}_{aib}\,\partial^{j}\mathcal{A}^{abi}\end{aligned}\tag{105}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix}\dot{t}_{3} & -i\sqrt{2}\,k\,\dot{t}_{3} & 0 & 0\\ i\sqrt{2}\,k\,\dot{t}_{3} & 2k^{2}\,\dot{t}_{3} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & k^{2}\,\dot{r}_{2}+\dot{t}_{2}\end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix}\frac{2\dot{t}_{2}}{3} & \frac{\sqrt{2}\,\dot{t}_{2}}{3} & -\frac{1}{3}i\sqrt{2}\,k\,\dot{t}_{2} & 0 & 0 & 0 & 0\\ \frac{\sqrt{2}\,\dot{t}_{2}}{3} & \frac{\dot{t}_{2}}{3} & -\frac{1}{3}i\,k\,\dot{t}_{2} & 0 & 0 & 0 & 0\\ \frac{1}{3}i\sqrt{2}\,k\,\dot{t}_{2} & \frac{i\,k\,\dot{t}_{2}}{3} & \frac{k^{2}\,\dot{t}_{2}}{3} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -k^{2}\,\dot{r}_{1}+\frac{2\dot{t}_{3}}{3} & -\frac{\sqrt{2}\,\dot{t}_{3}}{3} & -\frac{2}{3}i\,k\,\dot{t}_{3} & 0\\ 0 & 0 & 0 & -\frac{\sqrt{2}\,\dot{t}_{3}}{3} & \frac{\dot{t}_{3}}{3} & \frac{1}{3}i\sqrt{2}\,k\,\dot{t}_{3} & 0\\ 0 & 0 & 0 & \frac{2i\,k\,\dot{t}_{3}}{3} & -\frac{1}{3}i\sqrt{2}\,k\,\dot{t}_{3} & \frac{2k^{2}\,\dot{t}_{3}}{3} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k^2 r_1 \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}\tau^{\perp} == 0$$

$$-2\,i\,k\,{}^{0^+}\sigma^{\parallel} + {}^{0^+}\tau^{\parallel} == 0$$

$${}^{1^-}\tau^{\perp\,a} == 0$$

$$2\,i\,k\,{}^{1^-}\sigma^{\perp\,a} + {}^{1^-}\tau^{\parallel\,a} == 0$$

$$-i\,k\,{}^{1^+}\sigma^{\parallel\,ab} + {}^{1^+}\tau^{\parallel\,ab} == 0$$

$${}^{1^+}\sigma^{\parallel\,ab} == {}^{1^+}\sigma^{\perp\,ab}$$

$${}^{2^+}\tau^{\parallel\,ab} == 0$$

$${}^{2^+}\sigma^{\parallel\,ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{(1+2k^2)^2 t_3} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3} & 0 & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3} & \frac{2k^2}{(1+2k^2)^2 t_3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_2 + t_2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{6}{(3+k^2)^2 t_2} & \frac{3\sqrt{2}}{(3+k^2)^2 t_2} & -\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_2} & 0 & 0 & 0 & 0 \\ \frac{3\sqrt{2}}{(3+k^2)^2 t_2} & \frac{3}{(3+k^2)^2 t_2} & -\frac{3ik}{(3+k^2)^2 t_2} & 0 & 0 & 0 & 0 \\ \frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_2} & \frac{3ik}{(3+k^2)^2 t_2} & \frac{3k^2}{(3+k^2)^2 t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{k^2 r_1} & -\frac{\sqrt{2}}{k^2 r_1 + 2k^4 r_1} & -\frac{2i}{k r_1 + 2k^3 r_1} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}}{k^2 r_1 + 2k^4 r_1} & \frac{3k^2 r_1 - 2t_3}{(k+2k^3)^2 r_1 t_3} & \frac{i\sqrt{2}\,(3k^2 r_1 - 2t_3)}{k(1+2k^2)^2 r_1 t_3} & 0 \\ 0 & 0 & 0 & \frac{2i}{k r_1 + 2k^3 r_1} & -\frac{i\sqrt{2}\,(3k^2 r_1 - 2t_3)}{k(1+2k^2)^2 r_1 t_3} & \frac{6k^2 r_1 - 4t_3}{(1+2k^2)^2 r_1 t_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{k^2 r_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_2}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

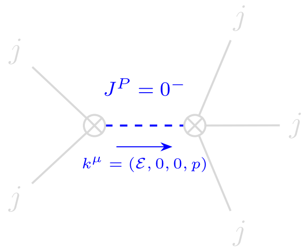
Massive pole residues:

$\{\{\}, \{-\frac{1}{r_{\overset{.}{2}}}\}, \{\}, \{\}, \{\}, \{\}\}$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{\overset{.}{2}}} > 0$
Square mass:	$-\frac{t_{\overset{.}{2}}}{r_{\overset{.}{2}}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{\overset{.}{2}} < 0 \,\&\&\, t_{\overset{.}{2}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r. < 0 \,\&\&\, t. > 0 \tag{106}$$

Okay, that concludes the analysis of this theory.

Case 30

Now for a new theory. Here is the full nonlinear Lagrangian for Case 30 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\left(2\,r. + r.\right) \mathcal{R}_{ijkl}\ \mathcal{R}^{ijkl} + \frac{2}{3}\left(r. - r.\right) \mathcal{R}_{ikjl}\ \mathcal{R}^{ijkl} - 2\,r.\ \mathcal{R}^{ij}{}_{i}{}^{k}\ \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6}\left(-4\,r. + r.\right) \mathcal{R}^{ijkl}\ \mathcal{R}_{klij} + 2\,r.\ \mathcal{R}^{ij}{}_{i}{}^{k}\ \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12}\,t.\ \mathcal{T}_{ijk}\ \mathcal{T}^{ijk} - \frac{1}{6}\,t.\ \mathcal{T}^{ijk}\ \mathcal{T}_{jik} \tag{107}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\,t.\ \mathcal{A}_{abi}\ \mathcal{A}^{abi} - \frac{2}{3}\,t.\ \mathcal{A}_{aib}\ \mathcal{A}^{abi} - 2\,r.\ \partial_b\mathcal{A}_{i}{}^{j}{}_{j}\ \partial^i\mathcal{A}^{ab}{}_{a} + 2\,r.\ \partial_i\mathcal{A}_{b}{}^{j}{}_{j}\ \partial^i\mathcal{A}^{ab}{}_{a} - \frac{2}{3}\,t.\ \mathcal{A}_{abi}\ \partial^i f^{ab} + \frac{2}{3}\,t.\ \mathcal{A}_{aib}\ \partial^i f^{ab} - \frac{2}{3}\,t.\ \mathcal{A}_{bia}\ \partial^i f^{ab} + \frac{1}{3}\,t.\ \partial_a f_{bi}\ \partial^i f^{ab} - \frac{1}{6}\,t.\ \partial_a f_{ib}\ \partial^i f^{ab} - \frac{1}{6}\,t.\ \partial_b f_{ai}\ \partial^i f^{ab} + \frac{1}{6}\,t.\ \partial_i f_{ab}\ \partial^i f^{ab} - \frac{1}{6}\,t.\ \partial_i f_{ba}\ \partial^i f^{ab} + 2\,r.\ \partial_a\mathcal{A}^{abi}\ \partial_j\mathcal{A}_{b}{}^{j}{}_{i} - 4\,r.\ \partial^i\mathcal{A}^{ab}{}_{a}\ \partial_j\mathcal{A}_{b}{}^{j}{}_{i} - 2\,r.\ \partial_a\mathcal{A}^{abi}\ \partial_j\mathcal{A}_{i}{}^{j}{}_{b} + 4\,r.\ \partial^i\mathcal{A}^{ab}{}_{a}\ \partial_j\mathcal{A}_{i}{}^{j}{}_{b} - \tag{108}$$

$$\frac{4}{3}\left(r_{\cdot}-r_{\cdot}\right)\partial_{b}\mathcal{A}_{a\,i\,j}\,\partial^{j}\mathcal{A}^{a\,b\,i}+\frac{2}{3}\left(r_{\cdot}-r_{\cdot}\right)\partial_{b}\mathcal{A}_{a\,j\,i}\,\partial^{j}\mathcal{A}^{a\,b\,i}+\frac{2}{3}\left(-4\,r_{\cdot}+r_{\cdot}\right)\partial_{b}\mathcal{A}_{i\,j\,a}\,\partial^{j}\mathcal{A}^{a\,b\,i}+$$
$$\frac{1}{3}\left(-2\,r_{\cdot}-r_{\cdot}\right)\partial_{i}\mathcal{A}_{a\,b\,j}\,\partial^{j}\mathcal{A}^{a\,b\,i}+\frac{1}{3}\left(2\,r_{\cdot}+r_{\cdot}\right)\partial_{j}\mathcal{A}_{a\,b\,i}\,\partial^{j}\mathcal{A}^{a\,b\,i}+\frac{2}{3}\left(r_{\cdot}-r_{\cdot}\right)\partial_{j}\mathcal{A}_{a\,i\,b}\,\partial^{j}\mathcal{A}^{a\,b\,i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\;r_{\cdot}+t_{\cdot} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{t_{2}}{\,} & \frac{\sqrt{\;}\;t_{2}}{\,} & --i\sqrt{2}\,k\,t_{\cdot} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\;}\;t_{2}}{\,} & \frac{t_{2}}{\,} & --i\,k\,t_{\cdot} & 0 & 0 & 0 & 0 \\ -i\sqrt{2}\,k\,t_{\cdot} & \frac{i\,k\,t_{2}}{\,} & \frac{k^{2}\,t_{2}}{\,} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k\;r_{\cdot} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k\;r_{\cdot} \end{pmatrix}$$

Gauge constraints on source currents:

${}^{+}_{\cdot}\tau^{\perp} == 0$

${}^{+}_{\cdot}\tau^{\parallel} == 0$

${}^{+}_{\cdot}\sigma^{\parallel} == 0$

${}^{-}_{\cdot}\tau^{\perp}{}^{a} == 0$

${}^{-}_{\cdot}\tau^{\parallel}{}^{a} == 0$

${}^{-}_{\cdot}\sigma^{\perp}{}^{a} == 0$

$-i\,k\;{}^{+}_{\cdot}\sigma^{\parallel}{}^{a\,b}+{}^{+}_{\cdot}\tau^{\parallel}{}^{a\,b} == 0$

${}^{+}_{\cdot}\sigma^{\parallel}{}^{a\,b} == {}^{+}_{\cdot}\sigma^{\perp}{}^{a\,b}$

${}^{+}_{\cdot}\tau^{\parallel}{}^{a\,b} == 0$

${}^{+}_{\cdot}\sigma^{\parallel}{}^{a\,b} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_2 + t_2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{}{(\ +k^2)^2 t_2} & \frac{\sqrt{\ }}{(\ +k^2)^2 t_2} & -\frac{i \sqrt{\ } k}{(\ +k^2)^2 t_2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\ }}{(\ +k^2)^2 t_2} & \frac{}{(\ +k^2)^2 t_2} & -\frac{i k}{(\ +k^2)^2 t_2} & 0 & 0 & 0 & 0 \\ \frac{i \sqrt{\ } k}{(\ +k^2)^2 t_2} & \frac{i k}{(\ +k^2)^2 t_2} & \frac{k^2}{(\ +k^2)^2 t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{}{k^2 r_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{}{k^2 r_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{.}}{r_{.}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

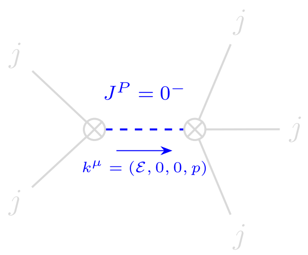
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{.}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{.2}} > 0$
Square mass:	$-\frac{t_{.2}}{r_{.2}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{.2} < 0 \;\&\&\; t_{.2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\cdot} < 0 \,\&\&\, t_{\cdot} > 0 \tag{109}$$

Okay, that concludes the analysis of this theory.

Case 31

Now for a new theory. Here is the full nonlinear Lagrangian for Case 31 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\cdot}\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} - \frac{2}{3} r_{\cdot}\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} + \frac{1}{6}\left(r_{\cdot} - 6 r_{\cdot}\right) \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + 4 r_{\cdot}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12} t_{\cdot}\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} - \frac{1}{6} t_{\cdot}\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik} \tag{110}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3} t_{\cdot}\, \mathcal{A}_{abi}\, \mathcal{A}^{abi} - \frac{2}{3} t_{\cdot}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} - 4 r_{\cdot}\, \partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{ab}{}_{a} - \\
&\frac{2}{3} t_{\cdot}\, \mathcal{A}_{abi}\, \partial^{i} f^{ab} + \frac{2}{3} t_{\cdot}\, \mathcal{A}_{aib}\, \partial^{i} f^{ab} - \frac{2}{3} t_{\cdot}\, \mathcal{A}_{bia}\, \partial^{i} f^{ab} + \frac{1}{3} t_{\cdot}\, \partial_{a} f_{bi}\, \partial^{i} f^{ab} - \\
&\frac{1}{6} t_{\cdot}\, \partial_{a} f_{ib}\, \partial^{i} f^{ab} - \frac{1}{6} t_{\cdot}\, \partial_{b} f_{ai}\, \partial^{i} f^{ab} + \frac{1}{6} t_{\cdot}\, \partial_{i} f_{ab}\, \partial^{i} f^{ab} - \frac{1}{6} t_{\cdot}\, \partial_{i} f_{ba}\, \partial^{i} f^{ab} - \\
&4 r_{\cdot}\, \partial_{a}\mathcal{A}^{abi}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + 8 r_{\cdot}\, \partial^{i}\mathcal{A}^{ab}{}_{a}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3} r_{\cdot}\, \partial_{b}\mathcal{A}_{aij}\, \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{\cdot}\, \partial_{b}\mathcal{A}_{aji}\, \partial^{j}\mathcal{A}^{abi} + \\
&\frac{2}{3}\left(r_{\cdot} - 6 r_{\cdot}\right) \partial_{b}\mathcal{A}_{ija}\, \partial^{j}\mathcal{A}^{abi} - \frac{1}{3} r_{\cdot}\, \partial_{i}\mathcal{A}_{abj}\, \partial^{j}\mathcal{A}^{abi} + \frac{1}{3} r_{\cdot}\, \partial_{j}\mathcal{A}_{abi}\, \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{\cdot}\, \partial_{j}\mathcal{A}_{aib}\, \partial^{j}\mathcal{A}^{abi}
\end{aligned} \tag{111}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 6 k\, r_{\cdot} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\, r_{\cdot} + t_{\cdot} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{2 {t_{\cdot}}_{2}}{3} & \frac{\sqrt{2}\, {t_{\cdot}}_{2}}{3} & -\frac{1}{3} i \sqrt{2}\, k {t_{\cdot}}_{2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}\, {t_{\cdot}}_{2}}{3} & \frac{{t_{\cdot}}_{2}}{3} & -\frac{1}{3} i k {t_{\cdot}}_{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k {t_{\cdot}}_{2} & \frac{i k {t_{\cdot}}_{2}}{3} & \frac{k^{2} {t_{\cdot}}_{2}}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$\underset{\bullet}{\overset{0^+}{}}\tau^{\perp} == 0$

$\underset{\bullet}{\overset{0^+}{}}\tau^{\parallel} == 0$

$\underset{\bullet}{\overset{1^-}{}}\tau^{\perp}{}^{a} == 0$

$\underset{\bullet}{\overset{1^-}{}}\tau^{\parallel}{}^{a} == 0$

$\underset{\bullet}{\overset{1^-}{}}\sigma^{\perp}{}^{a} == 0$

$\underset{\bullet}{\overset{1^-}{}}\sigma^{\parallel}{}^{a} == 0$

$-i\,k\ \underset{\bullet}{\overset{1^+}{}}\sigma^{\parallel}{}^{ab} + \underset{\bullet}{\overset{1^+}{}}\tau^{\parallel}{}^{ab} == 0$

$\underset{\bullet}{\overset{1^+}{}}\sigma^{\parallel}{}^{ab} == \underset{\bullet}{\overset{1^+}{}}\sigma^{\perp}{}^{ab}$

$\underset{\bullet}{\overset{2^-}{}}\sigma^{\parallel}{}^{abc} == 0$

$\underset{\bullet}{\overset{2^+}{}}\tau^{\parallel}{}^{ab} == 0$

$\underset{\bullet}{\overset{2^+}{}}\sigma^{\parallel}{}^{ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{6k^2 r_{\underset{3}{\bullet}}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{\underset{2}{\bullet}} + t_{\underset{2}{\bullet}}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{6}{(3+k^2)^2 t_{\underset{2}{\bullet}}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{\underset{2}{\bullet}}} & -\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{\underset{2}{\bullet}}} & 0 & 0 & 0 & 0 \\ \frac{3\sqrt{2}}{(3+k^2)^2 t_{\underset{2}{\bullet}}} & \frac{3}{(3+k^2)^2 t_{\underset{2}{\bullet}}} & -\frac{3ik}{(3+k^2)^2 t_{\underset{2}{\bullet}}} & 0 & 0 & 0 & 0 \\ \frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{\underset{2}{\bullet}}} & \frac{3ik}{(3+k^2)^2 t_{\underset{2}{\bullet}}} & \frac{3k^2}{(3+k^2)^2 t_{\underset{2}{\bullet}}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\underset{2}{\bullet}}}{r_{\underset{2}{\bullet}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

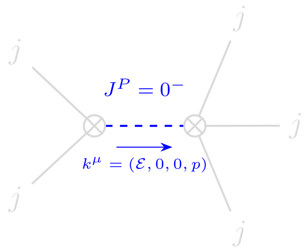
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\underset{2}{\bullet}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{\cdot 2}} > 0$
Square mass:	$-\frac{t_{\cdot 2}}{r_{\cdot 2}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{\cdot 2} < 0 \,\&\&\, t_{\cdot 2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\cdot} < 0 \,\&\&\, t_{\cdot} > 0 \tag{112}$$

Okay, that concludes the analysis of this theory.

Case 32

Now for a new theory. Here is the full nonlinear Lagrangian for Case 32 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\cdot}\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} - \frac{2}{3} r_{\cdot}\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} + \frac{1}{6} r_{\cdot}\, \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + \frac{1}{12}\left(4 t_{\cdot} + t_{\cdot}\right) \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} + \frac{1}{6}\left(2 t_{\cdot} - t_{\cdot}\right) \mathcal{T}^{ijk}\, \mathcal{T}_{jik} + \frac{1}{3} t_{\cdot}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{k}{}_{jk} \tag{113}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3}\left(t_{\cdot} + t_{\cdot}\right) \mathcal{A}_{abi}\, \mathcal{A}^{abi} + \frac{1}{3}\left(t_{\cdot} - 2 t_{\cdot}\right) \mathcal{A}_{aib}\, \mathcal{A}^{abi} + \frac{1}{3} t_{\cdot}\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3} t_{\cdot}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} + \\
&\frac{2}{3} t_{\cdot}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - \frac{1}{3} t_{\cdot}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - \frac{1}{3} t_{\cdot}\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} + \frac{2}{3} t_{\cdot}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} - \\
&\frac{2}{3}\left(t_{\cdot} + t_{\cdot}\right) \mathcal{A}_{abi}\, \partial^{i} f^{ab} + \frac{2}{3}\left(t_{\cdot} + t_{\cdot}\right) \mathcal{A}_{aib}\, \partial^{i} f^{ab} + \frac{2}{3}\left(2 t_{\cdot} - t_{\cdot}\right) \mathcal{A}_{bia}\, \partial^{i} f^{ab} + \\
&\frac{1}{3}\left(-2 t_{\cdot} + t_{\cdot}\right) \partial_{a} f_{bi}\, \partial^{i} f^{ab} + \frac{1}{6}\left(2 t_{\cdot} - t_{\cdot}\right) \partial_{a} f_{ib}\, \partial^{i} f^{ab} + \frac{1}{6}\left(-4 t_{\cdot} - t_{\cdot}\right) \partial_{b} f_{ai}\, \partial^{i} f^{ab} + \\
&\frac{1}{6}\left(4 t_{\cdot} + t_{\cdot}\right) \partial_{i} f_{ab}\, \partial^{i} f^{ab} + \frac{1}{6}\left(2 t_{\cdot} - t_{\cdot}\right) \partial_{i} f_{ba}\, \partial^{i} f^{ab} + \frac{4}{3} r_{\cdot}\, \partial_{b} \mathcal{A}_{aij}\, \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{\cdot}\, \partial_{b} \mathcal{A}_{aji}\, \partial^{j} \mathcal{A}^{abi} + \\
&\frac{2}{3} r_{\cdot}\, \partial_{b} \mathcal{A}_{ija}\, \partial^{j} \mathcal{A}^{abi} - \frac{1}{3} r_{\cdot}\, \partial_{i} \mathcal{A}_{abj}\, \partial^{j} \mathcal{A}^{abi} + \frac{1}{3} r_{\cdot}\, \partial_{j} \mathcal{A}_{abi}\, \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{\cdot}\, \partial_{j} \mathcal{A}_{aib}\, \partial^{j} \mathcal{A}^{abi}
\end{aligned} \tag{114}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\; r_{\cdot} + t_{\cdot} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\left(t_{\cdot} + 4\, t_{\cdot}\right) & -\frac{t_{1} - t_{2}}{\sqrt{\;}} & \frac{i\, k\left(t_{1} - t_{2}\right)}{\sqrt{\;}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1} - t_{2}}{\sqrt{\;}} & \frac{t_{1} + t_{2}}{} & --\, i\, k\left(t_{\cdot} + t_{\cdot}\right) & 0 & 0 & 0 & 0 \\ -\frac{i\, k\left(t_{1} - t_{2}\right)}{\sqrt{\;}} & -\, i\, k\left(t_{\cdot} + t_{\cdot}\right) & -\, k\;\left(t_{\cdot} + t_{\cdot}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{} & \frac{t_{1}}{\sqrt{\;}} & \frac{i\, k\, t_{1}}{} & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{\sqrt{\;}} & \frac{t_{1}}{} & -\, i\,\sqrt{2}\; k\, t_{\cdot} & 0 \\ 0 & 0 & 0 & --\, i\, k\, t_{\cdot} & --\, i\,\sqrt{2}\; k\, t_{\cdot} & \frac{k^{2}\, t_{1}}{} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{} & -\frac{i\, k\, t_{1}}{\sqrt{\;}} & 0 \\ \frac{i\, k\, t_{1}}{\sqrt{\;}} & k\;\, t_{\cdot} & 0 \\ 0 & 0 & \frac{t_{1}}{} \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{+}{\cdot}\tau^{\perp} == 0$

$\overset{+}{\cdot}\tau^{\parallel} == 0$

$\overset{+}{\cdot}\sigma^{\parallel} == 0$

$\overset{-}{\cdot}\tau^{\perp}{}^{a} == 0$

$2\, i\, k\;\; \overset{-}{\cdot}\sigma^{\parallel}{}^{a} + \overset{-}{\cdot}\tau^{\parallel}{}^{a} == 0$

$\overset{-}{\cdot}\sigma^{\parallel}{}^{a} == \overset{-}{\cdot}\sigma^{\perp}{}^{a}$

$-i\, k\;\; \overset{+}{\cdot}\sigma^{\perp}{}^{ab} + \overset{+}{\cdot}\tau^{\parallel}{}^{ab} == 0$

$-2\, i\, k\;\; \overset{+}{\cdot}\sigma^{\parallel}{}^{ab} + \overset{+}{\cdot}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally
analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^{2}\, r_{\cdot 2} + t_{\cdot 2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{2(t_1+t_2)}{3t_1 t_2} & \frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1 t_2} & -\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1 t_2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1 t_2} & \frac{t_1+4t_2}{3(1+k^2)^2 t_1 t_2} & -\frac{ik(t_1+4t_2)}{3(1+k^2)^2 t_1 t_2} & 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1 t_2} & \frac{ik(t_1+4t_2)}{3(1+k^2)^2 t_1 t_2} & \frac{k^2(t_1+4t_2)}{3(1+k^2)^2 t_1 t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{(3+4k^2)^2 t_1} & \frac{6\sqrt{2}}{(3+4k^2)^2 t_1} & \frac{12ik}{(3+4k^2)^2 t_1} & 0 \\ 0 & 0 & 0 & \frac{6\sqrt{2}}{(3+4k^2)^2 t_1} & \frac{12}{(3+4k^2)^2 t_1} & \frac{12i\sqrt{2}k}{(3+4k^2)^2 t_1} & 0 \\ 0 & 0 & 0 & -\frac{12ik}{(3+4k^2)^2 t_1} & -\frac{12i\sqrt{2}k}{(3+4k^2)^2 t_1} & \frac{24k^2}{(3+4k^2)^2 t_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_1} & -\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1} & 0 \\ \frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1} & \frac{4k^2}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_2}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

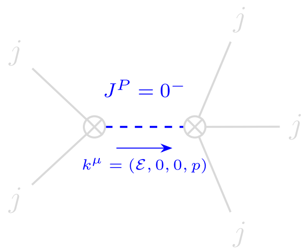
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_2} > 0$
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_2 < 0$ && $t_2 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\dot{r}_2 < 0 \,\&\&\, \dot{t}_2 > 0 \tag{115}$$

Okay, that concludes the analysis of this theory.

Case 33

Now for a new theory. Here is the full nonlinear Lagrangian for Case 33 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{6}\dot{r}_2\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl}-\frac{2}{3}\dot{r}_2\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl}+\frac{1}{6}\dot{r}_2\,\mathcal{R}^{ijkl}\,\mathcal{R}_{klij}+\\&\frac{1}{4}\dot{t}_1\,\mathcal{T}_{ijk}\,\mathcal{T}^{ijk}+\frac{1}{2}\dot{t}_1\,\mathcal{T}^{ijk}\,\mathcal{T}_{jik}+\frac{1}{3}\dot{t}_1\,\mathcal{T}^{i\ \ j}_{\ i}\,\mathcal{T}^{k}_{\ jk}\end{aligned} \tag{116}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\dot{t}_1\,\mathcal{A}_{aib}\,\mathcal{A}^{abi}+\frac{1}{3}\dot{t}_1\,\mathcal{A}^{ab}_{\ \ a}\,\mathcal{A}_{b\ i}^{\ i}-\frac{2}{3}\dot{t}_1\,\mathcal{A}_{b\ i}^{\ i}\,\partial_a f^{ab}+\frac{2}{3}\dot{t}_1\,\mathcal{A}_{b\ i}^{\ i}\,\partial^b f^a_{\ a}-\frac{1}{3}\dot{t}_1\,\partial_b f^i_{\ i}\,\partial^b f^a_{\ a}-\\&\frac{1}{3}\dot{t}_1\,\partial_a f^{ab}\,\partial_i f^i_{\ b}+\frac{2}{3}\dot{t}_1\,\partial^b f^a_{\ a}\,\partial_i f^i_{\ b}+2\dot{t}_1\,\mathcal{A}_{bia}\,\partial^i f^{ab}-\dot{t}_1\,\partial_a f_{bi}\,\partial^i f^{ab}+\frac{1}{2}\dot{t}_1\,\partial_a f_{ib}\,\partial^i f^{ab}-\\&\frac{1}{2}\dot{t}_1\,\partial_b f_{ai}\,\partial^i f^{ab}+\frac{1}{2}\dot{t}_1\,\partial_i f_{ab}\,\partial^i f^{ab}+\frac{1}{2}\dot{t}_1\,\partial_i f_{ba}\,\partial^i f^{ab}+\frac{4}{3}\dot{r}_2\,\partial_b\mathcal{A}_{aij}\,\partial^j\mathcal{A}^{abi}-\frac{2}{3}\dot{r}_2\,\partial_b\mathcal{A}_{aji}\,\partial^j\mathcal{A}^{abi}+\\&\frac{2}{3}\dot{r}_2\,\partial_b\mathcal{A}_{ija}\,\partial^j\mathcal{A}^{abi}-\frac{1}{3}\dot{r}_2\,\partial_i\mathcal{A}_{abj}\,\partial^j\mathcal{A}^{abi}+\frac{1}{3}\dot{r}_2\,\partial_j\mathcal{A}_{abi}\,\partial^j\mathcal{A}^{abi}-\frac{2}{3}\dot{r}_2\,\partial_j\mathcal{A}_{aib}\,\partial^j\mathcal{A}^{abi}\end{aligned} \tag{117}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&k^2\dot{r}_2-\dot{t}_1\end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix}-\frac{\dot{t}_1}{2}&-\frac{\dot{t}_1}{\sqrt{2}}&\frac{i k\dot{t}_1}{\sqrt{2}}&0&0&0&0\\-\frac{\dot{t}_1}{\sqrt{2}}&0&0&0&0&0&0\\-\frac{i k\dot{t}_1}{\sqrt{2}}&0&0&0&0&0&0\\0&0&0&\frac{\dot{t}_1}{6}&\frac{\dot{t}_1}{3\sqrt{2}}&\frac{i k\dot{t}_1}{3}&0\\0&0&0&\frac{\dot{t}_1}{3\sqrt{2}}&\frac{\dot{t}_1}{3}&\frac{1}{3}i\sqrt{2}\,k\dot{t}_1&0\\0&0&0&-\frac{1}{3}i k\dot{t}_1&-\frac{1}{3}i\sqrt{2}\,k\dot{t}_1&\frac{2k^2\dot{t}_1}{3}&0\\0&0&0&0&0&0&0\end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} \frac{t_{1}}{2} & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & k^2 t_{1} & 0 \\ 0 & 0 & \frac{t_{1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}\tau^{\perp} == 0$$

$${}^{0^+}\tau^{\parallel} == 0$$

$${}^{0^+}\sigma^{\parallel} == 0$$

$${}^{1^-}\tau^{\perp}{}^{a} == 0$$

$$2 i k \, {}^{1^-}\sigma^{\parallel}{}^{a} + {}^{1^-}\tau^{\parallel}{}^{a} == 0$$

$${}^{1^-}\sigma^{\parallel}{}^{a} == {}^{1^-}\sigma^{\perp}{}^{a}$$

$$-i k \, {}^{1^+}\sigma^{\perp}{}^{ab} + {}^{1^+}\tau^{\parallel}{}^{ab} == 0$$

$$-2 i k \, {}^{2^+}\sigma^{\parallel}{}^{ab} + {}^{2^+}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{2} - t_{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{1}{(1+k^2)^2 t_{1}} & -\frac{i k}{(1+k^2)^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & \frac{i k}{(1+k^2)^2 t_{1}} & \frac{k^2}{(1+k^2)^2 t_{1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{(3+4k^2)^2 t_{1}} & \frac{6\sqrt{2}}{(3+4k^2)^2 t_{1}} & \frac{12 i k}{(3+4k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 & \frac{6\sqrt{2}}{(3+4k^2)^2 t_{1}} & \frac{12}{(3+4k^2)^2 t_{1}} & \frac{12 i\sqrt{2}\,k}{(3+4k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 & -\frac{12 i k}{(3+4k^2)^2 t_{1}} & -\frac{12 i\sqrt{2}\,k}{(3+4k^2)^2 t_{1}} & \frac{24 k^2}{(3+4k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_{1}} & -\frac{2 i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & 0 \\ \frac{2 i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & \frac{4k^2}{(1+2k^2)^2 t_{1}} & 0 \\ 0 & 0 & \frac{2}{t_{1}} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_{1}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

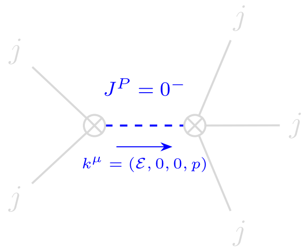
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{.2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

```
Massless eigenvalues:
```

$\{\}$

```
Overall particle spectrum:
```




```
Massive particle
```

Pole residue:	$-\frac{1}{r_{.2}} > 0$
Square mass:	$\frac{t_{.1}}{r_{.2}} > 0$
Spin:	0
Parity:	Odd

```
Overall unitarity conditions:
```

$r. < 0 \,\&\&\, t. < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r. < 0 \,\&\&\, t. < 0 \tag{118}$$

Okay, that concludes the analysis of this theory.

Case 34

Now for a new theory. Here is the full nonlinear Lagrangian for Case 34 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r. \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r. \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + r. \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6}\left(r. - 6 r.\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij} + 3 r. \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12} t. \mathcal{T}_{ijk} \mathcal{T}^{ijk} - \frac{1}{6} t. \mathcal{T}^{ijk} \mathcal{T}_{jik} \tag{119}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t. \mathcal{A}_{abi} \mathcal{A}^{abi} - \frac{2}{3} t. \mathcal{A}_{aib} \mathcal{A}^{abi} - 3 r. \partial_b \mathcal{A}_{i}{}^{j}{}_{j} \partial^i \mathcal{A}^{ab}{}_{a} - r. \partial_i \mathcal{A}_{b}{}^{j}{}_{j} \partial^i \mathcal{A}^{ab}{}_{a} - \frac{2}{3} t. \mathcal{A}_{abi} \partial^i f^{ab} + \frac{2}{3} t. \mathcal{A}_{aib} \partial^i f^{ab} - \frac{2}{3} t. \mathcal{A}_{bia} \partial^i f^{ab} + \frac{1}{3} t. \partial_a f_{bi} \partial^i f^{ab} - \frac{1}{6} t. \partial_a f_{ib} \partial^i f^{ab} - \frac{1}{6} t. \partial_b f_{ai} \partial^i f^{ab} + \frac{1}{6} t. \partial_i f_{ab} \partial^i f^{ab} - \frac{1}{6} t. \partial_i f_{ba} \partial^i f^{ab} - r. \partial_a \mathcal{A}^{abi} \partial_j \mathcal{A}_{b}{}^{j}{}_{i} + 2 r. \partial^i \mathcal{A}^{ab}{}_{a} \partial_j \mathcal{A}_{b}{}^{j}{}_{i} - \tag{120}$$

$$3\,r_{\bullet}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{i\ b}^{\ j}+6\,r_{\bullet}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{i\ b}^{\ j}+\frac{4}{3}\,r_{\bullet}\,\partial_{b}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{abi}-\frac{2}{3}\,r_{\bullet}\,\partial_{b}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{abi}+$$
$$\frac{2}{3}\left(r_{\bullet}-6\,r_{\bullet}\right)\partial_{b}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{abi}-\frac{1}{3}\,r_{\bullet}\,\partial_{i}\mathcal{A}_{abj}\,\partial^{j}\mathcal{A}^{abi}+\frac{1}{3}\,r_{\bullet}\,\partial_{j}\mathcal{A}_{abi}\,\partial^{j}\mathcal{A}^{abi}-\frac{2}{3}\,r_{\bullet}\,\partial_{j}\mathcal{A}_{aib}\,\partial^{j}\mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 6\,k\,r_{\bullet} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\,r_{\bullet}+t_{\bullet} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k\,r_{\bullet}+\frac{t_{2}}{} & \frac{\sqrt{\ }\,t_{2}}{} & --i\sqrt{2}\,k\,t_{\bullet} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\ }\,t_{2}}{} & \frac{t_{2}}{} & --i\,k\,t_{\bullet} & 0 & 0 & 0 & 0 \\ -i\sqrt{2}\,k\,t_{\bullet} & \frac{i\,k\,t_{2}}{} & \frac{k^{2}\,t_{2}}{} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\,r_{\bullet} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{+}{\cdot}\tau^{\perp} == 0$

$\overset{+}{\cdot}\tau^{\parallel} == 0$

$\overset{-}{\cdot}\tau^{\perp}{}^{a} == 0$

$\overset{-}{\cdot}\tau^{\parallel}{}^{a} == 0$

$\overset{-}{\cdot}\sigma^{\perp}{}^{a} == 0$

$-i\,k\;\overset{+}{\cdot}\sigma^{\perp}{}^{ab}+\overset{+}{\cdot}\tau^{\parallel}{}^{ab} == 0$

$\overset{-}{\cdot}\sigma^{\parallel}{}^{abc} == 0$

$\overset{+}{\cdot}\tau^{\parallel}{}^{ab} == 0$

$\overset{+}{\cdot}\sigma^{\parallel}{}^{ab} == 0$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{}{k^2 r_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_2 + t_2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{}{k^2 r_3} & -\frac{\sqrt{}}{k^2 r_3 + k^4 r_3} & \frac{i\sqrt{}}{k r_3 + k^3 r_3} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{}}{k^2 r_3 + k^4 r_3} & \frac{k^2 r_3 + t_2}{(k+k^3)^2 r_3 t_2} & -\frac{i(k^2 r_3 + t_2)}{k(+k^2)^2 r_3 t_2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{}}{k r_3 + k^3 r_3} & \frac{i(k^2 r_3 + t_2)}{k(+k^2)^2 r_3 t_2} & \frac{k^2 r_3 + t_2}{(+k^2)^2 r_3 t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\cdot}}{r_{\cdot}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

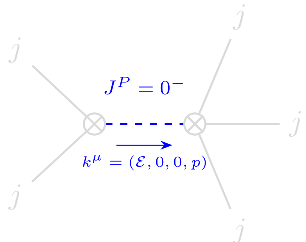
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\cdot}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{rP_2} > 0$
Square mass:	$-\frac{tP_2}{rP_2} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{\cdot_2} < 0 \,\&\&\, t_{\cdot_2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{2} < 0 \,\&\&\, t_{2} > 0 \tag{121}$$

Okay, that concludes the analysis of this theory.

Case 35

Now for a new theory. Here is the full nonlinear Lagrangian for Case 35 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{6} r_{2}\, \mathcal{R}_{ijhl}\, \mathcal{R}^{ijhl} - \frac{2}{3} r_{2}\, \mathcal{R}_{ihjl}\, \mathcal{R}^{ijhl} - \frac{3}{2} r_{3}\, \mathcal{R}^{ij}{}_{i}{}^{h}\, \mathcal{R}_{j}{}^{l}{}_{hl} + \\ &\frac{1}{6}\left(r_{2} - 6 r_{3}\right) \mathcal{R}^{ijhl}\, \mathcal{R}_{hlij} + \frac{5}{2} r_{3}\, \mathcal{R}^{ij}{}_{i}{}^{h}\, \mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{12} t_{2}\, \mathcal{T}_{ijh}\, \mathcal{T}^{ijh} - \frac{1}{6} t_{2}\, \mathcal{T}^{ijh}\, \mathcal{T}_{jih}\end{aligned} \tag{122}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\frac{1}{3} t_{2}\, \mathcal{A}_{abi}\, \mathcal{A}^{abi} - \frac{2}{3} t_{2}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} - \frac{5}{2} r_{3}\, \partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{ab}{}_{a} + \frac{3}{2} r_{3}\, \partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{ab}{}_{a} - \\ &\frac{2}{3} t_{2}\, \mathcal{A}_{abi}\, \partial^{i}f^{ab} + \frac{2}{3} t_{2}\, \mathcal{A}_{aib}\, \partial^{i}f^{ab} - \frac{2}{3} t_{2}\, \mathcal{A}_{bia}\, \partial^{i}f^{ab} + \frac{1}{3} t_{2}\, \partial_{a}f_{bi}\, \partial^{i}f^{ab} - \frac{1}{6} t_{2}\, \partial_{a}f_{ib}\, \partial^{i}f^{ab} - \\ &\frac{1}{6} t_{2}\, \partial_{b}f_{ai}\, \partial^{i}f^{ab} + \frac{1}{6} t_{2}\, \partial_{i}f_{ab}\, \partial^{i}f^{ab} - \frac{1}{6} t_{2}\, \partial_{i}f_{ba}\, \partial^{i}f^{ab} + \frac{3}{2} r_{3}\, \partial_{a}\mathcal{A}^{abi}\, \partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} - 3 r_{3}\, \partial^{i}\mathcal{A}^{ab}{}_{a}\, \partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} - \\ &\frac{5}{2} r_{3}\, \partial_{a}\mathcal{A}^{abi}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + 5 r_{3}\, \partial^{i}\mathcal{A}^{ab}{}_{a}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3} r_{2}\, \partial_{b}\mathcal{A}_{aij}\, \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{2}\, \partial_{b}\mathcal{A}_{aji}\, \partial^{j}\mathcal{A}^{abi} + \\ &\frac{2}{3}\left(r_{2} - 6 r_{3}\right) \partial_{b}\mathcal{A}_{ija}\, \partial^{j}\mathcal{A}^{abi} - \frac{1}{3} r_{2}\, \partial_{i}\mathcal{A}_{abj}\, \partial^{j}\mathcal{A}^{abi} + \frac{1}{3} r_{2}\, \partial_{j}\mathcal{A}_{abi}\, \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{2}\, \partial_{j}\mathcal{A}_{aib}\, \partial^{j}\mathcal{A}^{abi}\end{aligned} \tag{123}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{2} + t_{2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{2 t_{2}}{3} & \frac{\sqrt{2}\, t_{2}}{3} & -\frac{1}{3} i \sqrt{2}\, k\, t_{2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}\, t_{2}}{3} & \frac{t_{2}}{3} & -\frac{1}{3} i\, k\, t_{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k\, t_{2} & \frac{i k t_{2}}{3} & \frac{k^2 t_{2}}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{3 k^2 r_{3}}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{3 k^2 r_{\overset{\bullet}{3}}}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\bullet}\tau^{\perp} == 0$$

$${}^{0^+}_{\bullet}\tau^{\parallel} == 0$$

$${}^{0^+}_{\bullet}\sigma^{\parallel} == 0$$

$${}^{1^-}_{\bullet}\tau^{\perp}{}^{a} == 0$$

$${}^{1^-}_{\bullet}\tau^{\parallel}{}^{a} == 0$$

$${}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} == 0$$

$$-i\, k\, {}^{1^+}_{\bullet}\sigma^{\parallel}{}^{ab} + {}^{1^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

$${}^{1^+}_{\bullet}\sigma^{\parallel}{}^{ab} == {}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab}$$

$${}^{2^-}_{\bullet}\sigma^{\parallel}{}^{abc} == 0$$

$${}^{2^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 \left(r_{\overset{\bullet}{2}} + t_{\overset{\bullet}{2}}\right)} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{6}{(3+k^2)^2 t_{\overset{\bullet}{2}}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{\overset{\bullet}{2}}} & -\frac{3 i \sqrt{2} k}{(3+k^2)^2 t_{\overset{\bullet}{2}}} & 0 & 0 & 0 & 0 \\ \frac{3\sqrt{2}}{(3+k^2)^2 t_{\overset{\bullet}{2}}} & \frac{3}{(3+k^2)^2 t_{\overset{\bullet}{2}}} & -\frac{3 i k}{(3+k^2)^2 t_{\overset{\bullet}{2}}} & 0 & 0 & 0 & 0 \\ \frac{3 i \sqrt{2} k}{(3+k^2)^2 t_{\overset{\bullet}{2}}} & \frac{3 i k}{(3+k^2)^2 t_{\overset{\bullet}{2}}} & \frac{3 k^2}{(3+k^2)^2 t_{\overset{\bullet}{2}}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{3 k^2 r_{\overset{\bullet}{3}}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{3 k^2 r_{\overset{\bullet}{3}}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{\overset{\bullet}{2}}}{r_{\overset{\bullet}{2}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

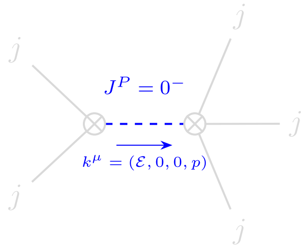
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{\overset{\bullet}{2}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{r^P_{2}} > 0$
Square mass:	$-\frac{t^P_{2}}{r^P_{2}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{\underset{2}{\cdot}} < 0 \,\&\&\, t_{\underset{2}{\cdot}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\underset{2}{\cdot}} < 0 \,\&\&\, t_{\underset{2}{\cdot}} > 0 \tag{124}$$

Okay, that concludes the analysis of this theory.

Case 36

Now for a new theory. Here is the full nonlinear Lagrangian for Case 36 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{gathered}\frac{1}{6}\left(2 r_{\underset{1}{\cdot}}+r_{\underset{2}{\cdot}}\right) \mathcal{R}_{ij\mathfrak{k}l}\, \mathcal{R}^{ij\mathfrak{k}l}+\frac{2}{3}\left(r_{\underset{1}{\cdot}}-r_{\underset{2}{\cdot}}\right) \mathcal{R}_{i\mathfrak{k}jl}\, \mathcal{R}^{ij\mathfrak{k}l}-2 r_{\underset{1}{\cdot}}\, \mathcal{R}^{ij}{}_{i}{}^{\mathfrak{k}}\, \mathcal{R}_{j}{}^{l}{}_{\mathfrak{k}l}+ \\ \frac{1}{6}\left(2 r_{\underset{1}{\cdot}}+r_{\underset{2}{\cdot}}-6 r_{\underset{3}{\cdot}}\right) \mathcal{R}^{ij\mathfrak{k}l}\, \mathcal{R}_{\mathfrak{k}lij}+\left(-2 r_{\underset{1}{\cdot}}+4 r_{\underset{3}{\cdot}}\right) \mathcal{R}^{ij}{}_{i}{}^{\mathfrak{k}}\, \mathcal{R}_{\mathfrak{k}}{}^{l}{}_{jl}+\frac{1}{12} t_{\underset{2}{\cdot}}\, \mathcal{T}_{ij\mathfrak{k}}\, \mathcal{T}^{ij\mathfrak{k}}-\frac{1}{6} t_{\underset{2}{\cdot}}\, \mathcal{T}^{ij\mathfrak{k}}\, \mathcal{T}_{ji\mathfrak{k}}\end{gathered} \tag{125}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{gathered}\frac{1}{3} t_{\underset{2}{\cdot}}\, \mathcal{A}_{abi}\, \mathcal{A}^{abi}-\frac{2}{3} t_{\underset{2}{\cdot}}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi}+2\left(r_{\underset{1}{\cdot}}-2 r_{\underset{3}{\cdot}}\right) \partial_b \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^i \mathcal{A}^{ab}{}_{a}+2 r_{\underset{1}{\cdot}}\, \partial_i \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^i \mathcal{A}^{ab}{}_{a}- \\ \frac{2}{3} t_{\underset{2}{\cdot}}\, \mathcal{A}_{abi}\, \partial^i f^{ab}+\frac{2}{3} t_{\underset{2}{\cdot}}\, \mathcal{A}_{aib}\, \partial^i f^{ab}-\frac{2}{3} t_{\underset{2}{\cdot}}\, \mathcal{A}_{bia}\, \partial^i f^{ab}+\frac{1}{3} t_{\underset{2}{\cdot}}\, \partial_a f_{bi}\, \partial^i f^{ab}-\frac{1}{6} t_{\underset{2}{\cdot}}\, \partial_a f_{ib}\, \partial^i f^{ab}- \\ \frac{1}{6} t_{\underset{2}{\cdot}}\, \partial_b f_{ai}\, \partial^i f^{ab}+\frac{1}{6} t_{\underset{2}{\cdot}}\, \partial_i f_{ab}\, \partial^i f^{ab}-\frac{1}{6} t_{\underset{2}{\cdot}}\, \partial_i f_{ba}\, \partial^i f^{ab}+2 r_{\underset{1}{\cdot}}\, \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_{b}{}^{j}{}_{i}- \\ 4 r_{\underset{1}{\cdot}}\, \partial^i \mathcal{A}^{ab}{}_{a}\, \partial_j \mathcal{A}_{b}{}^{j}{}_{i}+2\left(r_{\underset{1}{\cdot}}-2 r_{\underset{3}{\cdot}}\right) \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{b}+\left(-4 r_{\underset{1}{\cdot}}+8 r_{\underset{3}{\cdot}}\right) \partial^i \mathcal{A}^{ab}{}_{a}\, \partial_j \mathcal{A}_{i}{}^{j}{}_{b}- \\ \frac{4}{3}\left(r_{\underset{1}{\cdot}}-r_{\underset{2}{\cdot}}\right) \partial_b \mathcal{A}_{aij}\, \partial^j \mathcal{A}^{abi}+\frac{2}{3}\left(r_{\underset{1}{\cdot}}-r_{\underset{2}{\cdot}}\right) \partial_b \mathcal{A}_{aji}\, \partial^j \mathcal{A}^{abi}+\frac{2}{3}\left(2 r_{\underset{1}{\cdot}}+r_{\underset{2}{\cdot}}-6 r_{\underset{3}{\cdot}}\right) \partial_b \mathcal{A}_{ija}\, \partial^j \mathcal{A}^{abi}+\end{gathered} \tag{126}$$

$$\frac{1}{3}\left(-2\, r_{1}-r_{2}\right) \partial_i \mathcal{A}_{abj}\, \partial^j \mathcal{A}^{abi}+\frac{1}{3}\left(2\, r_{1}+r_{2}\right) \partial_j \mathcal{A}_{abi}\, \partial^j \mathcal{A}^{abi}+\frac{2}{3}\left(r_{1}-r_{2}\right) \partial_j \mathcal{A}_{aib}\, \partial^j \mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 6\, k^2 \left(-r_{1}+r_{3}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\, r_{2}+t_{2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{2 t_{2}}{3} & \frac{\sqrt{2}\, t_{2}}{3} & -\frac{1}{3} i \sqrt{2}\, k\, t_{2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}\, t_{2}}{3} & \frac{t_{2}}{3} & -\frac{1}{3} i\, k\, t_{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k\, t_{2} & \frac{i\, k\, t_{2}}{3} & \frac{k^2 t_{2}}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k^2\, r_{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & k^2\, r_{1} \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^+}\tau^{\perp} == 0$

${}^{0^+}\tau^{\parallel} == 0$

${}^{1^-}\tau^{\perp\, a} == 0$

${}^{1^-}\tau^{\parallel\, a} == 0$

${}^{1^-}\sigma^{\perp\, a} == 0$

$-i\, k\, {}^{1^+}\sigma^{\parallel\, ab} + {}^{1^+}\tau^{\parallel\, ab} == 0$

${}^{1^+}\sigma^{\parallel\, ab} == {}^{1^+}\sigma^{\perp\, ab}$

${}^{2^+}\tau^{\parallel\, ab} == 0$

${}^{2^+}\sigma^{\parallel\, ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{6\, k^2 \left(-r_{1}+r_{3}\right)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2\, r_{2}+t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{6}{(3+k^2)^2 t_{2}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{2}} & -\frac{3 i \sqrt{2} k}{(3+k^2)^2 t_{2}} & 0 & 0 & 0 & 0 \\ \frac{3\sqrt{2}}{(3+k^2)^2 t_{2}} & \frac{3}{(3+k^2)^2 t_{2}} & -\frac{3 i k}{(3+k^2)^2 t_{2}} & 0 & 0 & 0 & 0 \\ \frac{3 i \sqrt{2} k}{(3+k^2)^2 t_{2}} & \frac{3 i k}{(3+k^2)^2 t_{2}} & \frac{3 k^2}{(3+k^2)^2 t_{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{k^2 r_{1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{k^2 r_{1}} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

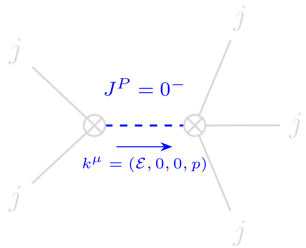
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{r^{P}_{2}} > 0$
Square mass:	$-\frac{t^{P}_{2}}{r^{P}_{2}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{2} < 0 \;\&\&\; t_{2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\dot{2}} < 0 \,\&\&\, t_{\dot{2}} > 0 \tag{127}$$

Okay, that concludes the analysis of this theory.

Case 37

Now for a new theory. Here is the full nonlinear Lagrangian for Case 37 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}
&\frac{1}{6} r_{\dot{2}} \mathcal{R}_{ijhl} \mathcal{R}^{ijhl} - \frac{2}{3} r_{\dot{2}} \mathcal{R}_{ihjl} \mathcal{R}^{ijhl} - \frac{3}{2} r_{\dot{3}} \mathcal{R}^{ij}{}_{i}{}^{h} \mathcal{R}_{j}{}^{l}{}_{hl} + \frac{1}{6} \left(r_{\dot{2}} - 6 r_{\dot{3}}\right) \mathcal{R}^{ijhl} \mathcal{R}_{hlij} + \\
&\frac{5}{2} r_{\dot{3}} \mathcal{R}^{ij}{}_{i}{}^{h} \mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{12} t_{\dot{2}} \mathcal{T}_{ijh} \mathcal{T}^{ijh} - \frac{1}{6} t_{\dot{2}} \mathcal{T}^{ijh} \mathcal{T}_{jih} - \frac{2}{3} t_{\dot{3}} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{h}{}_{jh}
\end{aligned} \tag{128}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3} t_{\dot{2}} \mathcal{A}_{abi} \mathcal{A}^{abi} - \frac{2}{3} t_{\dot{2}} \mathcal{A}_{aib} \mathcal{A}^{abi} - \frac{2}{3} t_{\dot{3}} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} + \frac{4}{3} t_{\dot{3}} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} - \frac{4}{3} t_{\dot{3}} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} + \\
&\frac{2}{3} t_{\dot{3}} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{\dot{3}} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} - \frac{4}{3} t_{\dot{3}} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} - \frac{5}{2} r_{\dot{3}} \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + \frac{3}{2} r_{\dot{3}} \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} - \\
&\frac{2}{3} t_{\dot{2}} \mathcal{A}_{abi} \partial^{i} f^{ab} + \frac{2}{3} t_{\dot{2}} \mathcal{A}_{aib} \partial^{i} f^{ab} - \frac{2}{3} t_{\dot{2}} \mathcal{A}_{bia} \partial^{i} f^{ab} + \frac{1}{3} t_{\dot{2}} \partial_{a} f_{bi} \partial^{i} f^{ab} - \frac{1}{6} t_{\dot{2}} \partial_{a} f_{ib} \partial^{i} f^{ab} - \\
&\frac{1}{6} t_{\dot{2}} \partial_{b} f_{ai} \partial^{i} f^{ab} + \frac{1}{6} t_{\dot{2}} \partial_{i} f_{ab} \partial^{i} f^{ab} - \frac{1}{6} t_{\dot{2}} \partial_{i} f_{ba} \partial^{i} f^{ab} + \frac{3}{2} r_{\dot{3}} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} - 3 r_{\dot{3}} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} - \\
&\frac{5}{2} r_{\dot{3}} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + 5 r_{\dot{3}} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3} r_{\dot{2}} \partial_{b} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{\dot{2}} \partial_{b} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{abi} + \\
&\frac{2}{3} \left(r_{\dot{2}} - 6 r_{\dot{3}}\right) \partial_{b} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{abi} - \frac{1}{3} r_{\dot{2}} \partial_{i} \mathcal{A}_{abj} \partial^{j} \mathcal{A}^{abi} + \frac{1}{3} r_{\dot{2}} \partial_{j} \mathcal{A}_{abi} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{\dot{2}} \partial_{j} \mathcal{A}_{aib} \partial^{j} \mathcal{A}^{abi}
\end{aligned} \tag{129}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} t_{\dot{3}} & -i\sqrt{2}\,k\,t_{\dot{3}} & 0 & 0 \\ i\sqrt{2}\,k\,t_{\dot{3}} & 2k^2 t_{\dot{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{\dot{2}} + t_{\dot{2}} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} \frac{2t_{\dot{2}}}{3} & \frac{\sqrt{2}\,t_{\dot{2}}}{3} & -\frac{1}{3} i\sqrt{2}\,k\,t_{\dot{2}} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}\,t_{\dot{2}}}{3} & \frac{t_{\dot{2}}}{3} & -\frac{1}{3} i\,k\,t_{\dot{2}} & 0 & 0 & 0 & 0 \\ \frac{1}{3} i\sqrt{2}\,k\,t_{\dot{2}} & \frac{i\,k\,t_{\dot{2}}}{3} & \frac{k^2 t_{\dot{2}}}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6}\left(-9k^2 r_{\dot{3}} + 4t_{\dot{3}}\right) & -\frac{\sqrt{2}\,t_{\dot{3}}}{3} & -\frac{2}{3} i\,k\,t_{\dot{3}} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}\,t_{\dot{3}}}{3} & \frac{t_{\dot{3}}}{3} & \frac{1}{3} i\sqrt{2}\,k\,t_{\dot{3}} & 0 \\ 0 & 0 & 0 & \frac{2i\,k\,t_{\dot{3}}}{3} & -\frac{1}{3} i\sqrt{2}\,k\,t_{\dot{3}} & \frac{2k^2 t_{\dot{3}}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} -\frac{3k^2 r_{3}}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\bullet}\tau^{\perp} == 0$$

$$-2\,i\,k\,{}^{0^+}_{\bullet}\sigma^{\parallel} + {}^{0^+}_{\bullet}\tau^{\parallel} == 0$$

$${}^{1^-}_{\bullet}\tau^{\perp\,a} == 0$$

$$2\,i\,k\,{}^{1^-}_{\bullet}\sigma^{\perp\,a} + {}^{1^-}_{\bullet}\tau^{\parallel\,a} == 0$$

$$-i\,k\,{}^{1^+}_{\bullet}\sigma^{\parallel\,ab} + {}^{1^+}_{\bullet}\tau^{\parallel\,ab} == 0$$

$${}^{1^+}_{\bullet}\sigma^{\parallel\,ab} == {}^{1^+}_{\bullet}\sigma^{\perp\,ab}$$

$${}^{2^-}_{\bullet}\sigma^{\parallel\,abc} == 0$$

$${}^{2^+}_{\bullet}\tau^{\parallel\,ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{(1+2k^2)^2 t_{3}} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & 0 & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{3}} & \frac{2k^2}{(1+2k^2)^2 t_{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{2}+t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{6}{(3+k^2)^2 t_{2}} & \frac{3\sqrt{2}}{(3+k^2)^2 t_{2}} & -\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{2}} & 0 & 0 & 0 & 0 \\ \frac{3\sqrt{2}}{(3+k^2)^2 t_{2}} & \frac{3}{(3+k^2)^2 t_{2}} & -\frac{3ik}{(3+k^2)^2 t_{2}} & 0 & 0 & 0 & 0 \\ \frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_{2}} & \frac{3ik}{(3+k^2)^2 t_{2}} & \frac{3k^2}{(3+k^2)^2 t_{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{3k^2 r_{3}} & -\frac{2\sqrt{2}}{3k^2 r_{3}+6k^4 r_{3}} & -\frac{4i}{3k r_{3}+6k^3 r_{3}} & 0 \\ 0 & 0 & 0 & -\frac{2\sqrt{2}}{3k^2 r_{3}+6k^4 r_{3}} & \frac{9k^2 r_{3}-4t_{3}}{3(k+2k^3)^2 r_{3} t_{3}} & \frac{i\sqrt{2}\left(9k^2 r_{3}-4t_{3}\right)}{3k(1+2k^2)^2 r_{3} t_{3}} & 0 \\ 0 & 0 & 0 & \frac{4i}{3k r_{3}+6k^3 r_{3}} & -\frac{i\sqrt{2}\left(9k^2 r_{3}-4t_{3}\right)}{3k(1+2k^2)^2 r_{3} t_{3}} & \frac{2\left(9k^2 r_{3}-4t_{3}\right)}{3(1+2k^2)^2 r_{3} t_{3}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{2}{3k^2 r_{3}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\},\left\{-\frac{t_{2}}{r_{2}}\right\},\{\},\{\},\{\},\{\}\right\}$$

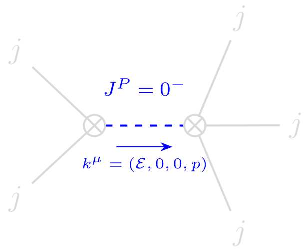
Massive pole residues:

$\left\{\{\}, \left\{-\frac{1}{r_{\underset{2}{.}}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{\underset{2}{.}}} > 0$
Square mass:	$-\frac{t_{\underset{2}{.}}}{r_{\underset{2}{.}}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{\underset{2}{.}} < 0 \&\& t_{\underset{2}{.}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{.} < 0 \&\& t_{.} > 0 \tag{130}$$

Okay, that concludes the analysis of this theory.

Case 38

Now for a new theory. Here is the full nonlinear Lagrangian for Case 38 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{.} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{.} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \frac{1}{6}\left(r_{.} - 6 r_{.}\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij} + 4 r_{.} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12}\left(4 t_{.} + t_{.}\right) \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{6}\left(2 t_{.} - t_{.}\right) \mathcal{T}^{ijk} \mathcal{T}_{jik} + \frac{1}{3} t_{.} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk} \tag{131}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\left(t_{.} + t_{.}\right) \mathcal{A}_{abi} \mathcal{A}^{abi} + \frac{1}{3}\left(t_{.} - 2 t_{.}\right) \mathcal{A}_{aib} \mathcal{A}^{abi} + \frac{1}{3} t_{.} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3} t_{.} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} + \frac{2}{3} t_{.} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} - \frac{1}{3} t_{.} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} - \frac{1}{3} t_{.} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} + \frac{2}{3} t_{.} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} - 4 r_{.} \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} - \frac{2}{3}\left(t_{.} + t_{.}\right) \mathcal{A}_{abi} \partial^{i} f^{ab} + \frac{2}{3}\left(t_{.} + t_{.}\right) \mathcal{A}_{aib} \partial^{i} f^{ab} +$$

$$\frac{2}{3}\left(2\,t_{\cdot}-t_{\cdot}\right)\mathcal{A}_{bia}\,\partial^{i}f^{ab}+\frac{1}{3}\left(-2\,t_{\cdot}+t_{\cdot}\right)\partial_{a}f_{bi}\,\partial^{i}f^{ab}+\frac{1}{6}\left(2\,t_{\cdot}-t_{\cdot}\right)\partial_{a}f_{ib}\,\partial^{i}f^{ab}+$$
$$\frac{1}{6}\left(-4\,t_{\cdot}-t_{\cdot}\right)\partial_{b}f_{ai}\,\partial^{i}f^{ab}+\frac{1}{6}\left(4\,t_{\cdot}+t_{\cdot}\right)\partial_{i}f_{ab}\,\partial^{i}f^{ab}+\frac{1}{6}\left(2\,t_{\cdot}-t_{\cdot}\right)\partial_{i}f_{ba}\,\partial^{i}f^{ab}-$$
$$4\,r_{\cdot}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{i\ \,b}^{\ j}+8\,r_{\cdot}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{i\ \,b}^{\ j}+\frac{4}{3}\,r_{\cdot}\,\partial_{b}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{abi}-\frac{2}{3}\,r_{\cdot}\,\partial_{b}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{abi}+$$
$$\frac{2}{3}\left(r_{\cdot}-6\,r_{\cdot}\right)\partial_{b}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{abi}-\frac{1}{3}\,r_{\cdot}\,\partial_{i}\mathcal{A}_{abj}\,\partial^{j}\mathcal{A}^{abi}+\frac{1}{3}\,r_{\cdot}\,\partial_{j}\mathcal{A}_{abi}\,\partial^{j}\mathcal{A}^{abi}-\frac{2}{3}\,r_{\cdot}\,\partial_{j}\mathcal{A}_{aib}\,\partial^{j}\mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 6\,k\,r_{\cdot} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\,r_{\cdot}+t_{\cdot} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\left(t_{\cdot}+4\,t_{\cdot}\right) & -\frac{t_1-t_2}{\sqrt{}} & \frac{i\,k\left(t_1-t_2\right)}{\sqrt{}} & 0 & 0 & 0 & 0 \\ -\frac{t_1-t_2}{\sqrt{}} & \frac{t_1+t_2}{} & --i\,k\left(t_{\cdot}+t_{\cdot}\right) & 0 & 0 & 0 & 0 \\ -\frac{i\,k\left(t_1-t_2\right)}{\sqrt{}} & -i\,k\left(t_{\cdot}+t_{\cdot}\right) & -k\left(t_{\cdot}+t_{\cdot}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{t_1}{} & \frac{t_1}{\sqrt{}} & \frac{i\,k\,t_1}{} & 0 \\ 0 & 0 & 0 & \frac{t_1}{\sqrt{}} & \frac{t_1}{} & -i\sqrt{2}\,k\,t_{\cdot} & 0 \\ 0 & 0 & 0 & --i\,k\,t_{\cdot} & --i\sqrt{2}\,k\,t_{\cdot} & \frac{k^2\,t_1}{} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{\cdot 1}}{2} & -\frac{i\,k\,t_{\cdot 1}}{\sqrt{2}} & 0 \\ \frac{i\,k\,t_{\cdot 1}}{\sqrt{2}} & k^2\,t_{\cdot 1} & 0 \\ 0 & 0 & \frac{t_{\cdot 1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\cdot}\tau^{\perp} == 0$$

$${}^{0^+}_{\cdot}\tau^{\parallel} == 0$$

$${}^{1^-}_{\cdot}\tau^{\perp\,a} == 0$$

$$2\,i\,k\,{}^{1^-}_{\cdot}\sigma^{\parallel\,a}+{}^{1^-}_{\cdot}\tau^{\parallel\,a} == 0$$

$${}^{1^-}_{\cdot}\sigma^{\parallel\,a} == {}^{1^-}_{\cdot}\sigma^{\perp\,a}$$

$$-i\,k\,{}^{1^+}_{\cdot}\sigma^{\perp\,ab}+{}^{1^+}_{\cdot}\tau^{\parallel\,ab} == 0$$

$$-2\,i\,k\,{}^{2^+}_{\cdot}\sigma^{\parallel\,ab}+{}^{2^+}_{\cdot}\tau^{\parallel\,ab} == 0$$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally
analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{6 k^2 r_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_2 + t_2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{2 (t_1 + t_2)}{3 t_1 t_2} & \frac{\sqrt{2} (t_1 - 2 t_2)}{3 (1+k^2) t_1 t_2} & -\frac{i \sqrt{2} k (t_1 - 2 t_2)}{3 (1+k^2) t_1 t_2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2} (t_1 - 2 t_2)}{3 (1+k^2) t_1 t_2} & \frac{t_1 + 4 t_2}{3 (1+k^2)^2 t_1 t_2} & -\frac{i k (t_1 + 4 t_2)}{3 (1+k^2)^2 t_1 t_2} & 0 & 0 & 0 & 0 \\ \frac{i \sqrt{2} k (t_1 - 2 t_2)}{3 (1+k^2) t_1 t_2} & \frac{i k (t_1 + 4 t_2)}{3 (1+k^2)^2 t_1 t_2} & \frac{k^2 (t_1 + 4 t_2)}{3 (1+k^2)^2 t_1 t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{(3+4 k^2)^2 t_1} & \frac{6 \sqrt{2}}{(3+4 k^2)^2 t_1} & \frac{12 i k}{(3+4 k^2)^2 t_1} & 0 \\ 0 & 0 & 0 & \frac{6 \sqrt{2}}{(3+4 k^2)^2 t_1} & \frac{12}{(3+4 k^2)^2 t_1} & \frac{12 i \sqrt{2} k}{(3+4 k^2)^2 t_1} & 0 \\ 0 & 0 & 0 & -\frac{12 i k}{(3+4 k^2)^2 t_1} & -\frac{12 i \sqrt{2} k}{(3+4 k^2)^2 t_1} & \frac{24 k^2}{(3+4 k^2)^2 t_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2 k^2)^2 t_1} & -\frac{2 i \sqrt{2} k}{(1+2 k^2)^2 t_1} & 0 \\ \frac{2 i \sqrt{2} k}{(1+2 k^2)^2 t_1} & \frac{4 k^2}{(1+2 k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_2}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

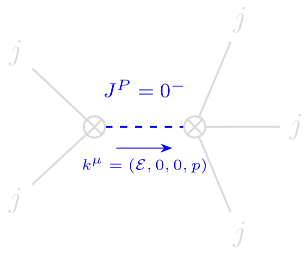
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{s}{r^P_{2}} > 0$
Square mass:	$-\frac{t^P_{2}}{r^P_{2}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{2} < 0 \,\&\&\, t_{2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{2} < 0 \,\&\&\, t_{2} > 0 \tag{133}$$

Okay, that concludes the analysis of this theory.

Case 39

Now for a new theory. Here is the full nonlinear Lagrangian for Case 39 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{2} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \frac{1}{6}\left(r_{2} - 6 r_{3}\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij} + 4 r_{3} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4} t_{1} \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{2} t_{1} \mathcal{T}^{ijk} \mathcal{T}_{jik} + \frac{1}{3} t_{1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk} \tag{134}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned} & t_{1} \mathcal{A}_{aib} \mathcal{A}^{abi} + \frac{1}{3} t_{1} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3} t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} + \frac{2}{3} t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} - \frac{1}{3} t_{1} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} - \\ & \frac{1}{3} t_{1} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} + \frac{2}{3} t_{1} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} - 4 r_{3} \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + 2 t_{1} \mathcal{A}_{bia} \partial^{i} f^{ab} - \\ & t_{1} \partial_{a} f_{bi} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{a} f_{ib} \partial^{i} f^{ab} - \frac{1}{2} t_{1} \partial_{b} f_{ai} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{i} f_{ab} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{i} f_{ba} \partial^{i} f^{ab} - \\ & 4 r_{3} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + 8 r_{3} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3} r_{2} \partial_{b} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{2} \partial_{b} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{abi} + \\ & \frac{2}{3}\left(r_{2} - 6 r_{3}\right) \partial_{b} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{abi} - \frac{1}{3} r_{2} \partial_{i} \mathcal{A}_{abj} \partial^{j} \mathcal{A}^{abi} + \frac{1}{3} r_{2} \partial_{j} \mathcal{A}_{abi} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{2} \partial_{j} \mathcal{A}_{aib} \partial^{j} \mathcal{A}^{abi} \end{aligned} \tag{135}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 6k^2 r_{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{2} - t_{1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\frac{t_{1}}{2} & -\frac{t_{1}}{\sqrt{2}} & \frac{i k t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{6} & \frac{t_{1}}{3\sqrt{2}} & \frac{i k t_{1}}{3} & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{3\sqrt{2}} & \frac{t_{1}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{1} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} i k t_{1} & -\frac{1}{3} i \sqrt{2}\, k t_{1} & \frac{2k^2 t_{1}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{2} & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & k^2 t_{1} & 0 \\ 0 & 0 & \frac{t_{1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\cdot}\tau^{\perp} == 0$$

$${}^{0^+}_{\cdot}\tau^{\parallel} == 0$$

$${}^{1^-}_{\cdot}\tau^{\perp}{}^{a} == 0$$

$$2 i k\, {}^{1^-}_{\cdot}\sigma^{\parallel}{}^{a} + {}^{1^-}_{\cdot}\tau^{\parallel}{}^{a} == 0$$

$${}^{1^-}_{\cdot}\sigma^{\parallel}{}^{a} == {}^{1^-}_{\cdot}\sigma^{\perp}{}^{a}$$

$$-i k\, {}^{1^+}_{\cdot}\sigma^{\perp}{}^{ab} + {}^{1^+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

$$-2 i k\, {}^{2^+}_{\cdot}\sigma^{\parallel}{}^{ab} + {}^{2^+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{6k^2 r_{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{2} - t_{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{\dot{t}_1+k^2\dot{t}_1} & \frac{i\sqrt{2}k}{\dot{t}_1+k^2\dot{t}_1} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{\dot{t}_1+k^2\dot{t}_1} & \frac{1}{(1+k^2)^2\dot{t}_1} & -\frac{ik}{(1+k^2)^2\dot{t}_1} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}k}{\dot{t}_1+k^2\dot{t}_1} & \frac{ik}{(1+k^2)^2\dot{t}_1} & \frac{k^2}{(1+k^2)^2\dot{t}_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{(3+4k^2)^2\dot{t}_1} & \frac{6\sqrt{2}}{(3+4k^2)^2\dot{t}_1} & \frac{12ik}{(3+4k^2)^2\dot{t}_1} & 0 \\ 0 & 0 & 0 & \frac{6\sqrt{2}}{(3+4k^2)^2\dot{t}_1} & \frac{12}{(3+4k^2)^2\dot{t}_1} & \frac{12i\sqrt{2}k}{(3+4k^2)^2\dot{t}_1} & 0 \\ 0 & 0 & 0 & -\frac{12ik}{(3+4k^2)^2\dot{t}_1} & -\frac{12i\sqrt{2}k}{(3+4k^2)^2\dot{t}_1} & \frac{24k^2}{(3+4k^2)^2\dot{t}_1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2\dot{t}_1} & -\frac{2i\sqrt{2}k}{(1+2k^2)^2\dot{t}_1} & 0 \\ \frac{2i\sqrt{2}k}{(1+2k^2)^2\dot{t}_1} & \frac{4k^2}{(1+2k^2)^2\dot{t}_1} & 0 \\ 0 & 0 & \frac{2}{\dot{t}_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{\frac{\dot{t}_1}{\dot{r}_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

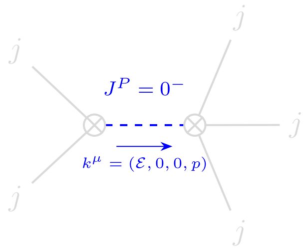
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{\dot{r}_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{\dot{r}_2} > 0$
Square mass:	$\frac{\dot{t}_1}{\dot{r}_2} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$\dot{r}_2 < 0 \; \&\& \; \dot{t}_1 < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{2} < 0 \,\&\&\, t_{1} < 0 \tag{136}$$

Okay, that concludes the analysis of this theory.

Case 40

Now for a new theory. Here is the full nonlinear Lagrangian for Case 40 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{2} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{2} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + \frac{1}{6}\left(r_{2} - 6 r_{3}\right) \mathcal{R}^{ijkl} \mathcal{R}_{klij} + 2 r_{4} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12} t_{2} \mathcal{T}_{ijk} \mathcal{T}^{ijk} - \frac{1}{6} t_{2} \mathcal{T}^{ijk} \mathcal{T}_{jik} \tag{137}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3} t_{2} \mathcal{A}_{abi} \mathcal{A}^{abi} - \frac{2}{3} t_{2} \mathcal{A}_{aib} \mathcal{A}^{abi} - 2 r_{4} \partial_{b}\mathcal{A}_{i}{}^{j}{}_{j} \partial^{i}\mathcal{A}^{ab}{}_{a} - \\
&\frac{2}{3} t_{2} \mathcal{A}_{abi} \partial^{i} f^{ab} + \frac{2}{3} t_{2} \mathcal{A}_{aib} \partial^{i} f^{ab} - \frac{2}{3} t_{2} \mathcal{A}_{bia} \partial^{i} f^{ab} + \frac{1}{3} t_{2} \partial_{a} f_{bi} \partial^{i} f^{ab} - \\
&\frac{1}{6} t_{2} \partial_{a} f_{ib} \partial^{i} f^{ab} - \frac{1}{6} t_{2} \partial_{b} f_{ai} \partial^{i} f^{ab} + \frac{1}{6} t_{2} \partial_{i} f_{ab} \partial^{i} f^{ab} - \frac{1}{6} t_{2} \partial_{i} f_{ba} \partial^{i} f^{ab} - \\
&2 r_{4} \partial_{a}\mathcal{A}^{abi} \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + 4 r_{4} \partial^{i}\mathcal{A}^{ab}{}_{a} \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3} r_{2} \partial_{b}\mathcal{A}_{aij} \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{2} \partial_{b}\mathcal{A}_{aji} \partial^{j}\mathcal{A}^{abi} + \\
&\frac{2}{3}\left(r_{2} - 6 r_{3}\right) \partial_{b}\mathcal{A}_{ija} \partial^{j}\mathcal{A}^{abi} - \frac{1}{3} r_{2} \partial_{i}\mathcal{A}_{abj} \partial^{j}\mathcal{A}^{abi} + \frac{1}{3} r_{2} \partial_{j}\mathcal{A}_{abi} \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{2} \partial_{j}\mathcal{A}_{aib} \partial^{j}\mathcal{A}^{abi}
\end{aligned} \tag{138}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

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The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} -2k^2\left(r_{3} - 2 r_{4}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{2} + t_{2} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} k^2\left(2 r_{3} - r_{4}\right) + \frac{2 t_{2}}{3} & \frac{\sqrt{2}\, t_{2}}{3} & -\frac{1}{3} i \sqrt{2}\, k t_{2} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}\, t_{2}}{3} & \frac{t_{2}}{3} & -\frac{1}{3} i k t_{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} i \sqrt{2}\, k t_{2} & \frac{i k t_{2}}{3} & \frac{k^2 t_{2}}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} k^2\left(-2 r_{3} + r_{4}\right) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\bullet}\tau^{\perp} == 0$$

$${}^{0^+}_{\bullet}\tau^{\parallel} == 0$$

$${}^{1^-}_{\bullet}\tau^{\perp}{}^{a} == 0$$

$${}^{1^-}_{\bullet}\tau^{\parallel}{}^{a} == 0$$

$${}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} == 0$$

$${}^{1^-}_{\bullet}\sigma^{\parallel}{}^{a} == 0$$

$$-i\,k\;{}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab} + {}^{1^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

$${}^{2^-}_{\bullet}\sigma^{\parallel}{}^{abc} == 0$$

$${}^{2^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{-2k^2 r_{3}+4k^2 r_{4}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{2}+t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{1}{k^2(2r_{3}-r_{4})} & -\frac{\sqrt{2}}{k^2(1+k^2)(2r_{3}-r_{4})} & \frac{i\sqrt{2}}{k(1+k^2)(2r_{3}-r_{4})} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{k^2(1+k^2)(2r_{3}-r_{4})} & \frac{k^2(6r_{3}-3r_{4})+2t_{2}}{(k+k^3)^2(2r_{3}-r_{4})t_{2}} & -\frac{i(k^2(6r_{3}-3r_{4})+2t_{2})}{k(1+k^2)^2(2r_{3}-r_{4})t_{2}} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}}{k(1+k^2)(2r_{3}-r_{4})} & \frac{i(k^2(6r_{3}-3r_{4})+2t_{2})}{k(1+k^2)^2(2r_{3}-r_{4})t_{2}} & \frac{\frac{1}{r_{3}-\frac{r_{4}}{2}}+\frac{3k^2}{t_{2}}}{(1+k^2)^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{1}{k^2(-2r_{3}+r_{4})} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_{2}}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

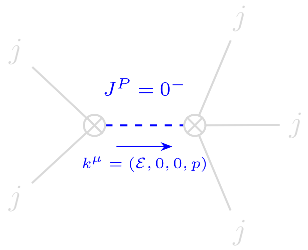
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_{2}}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

$$\{\}$$

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{\cdot_2}} > 0$
Square mass:	$-\frac{t_{\cdot_2}}{r_{\cdot_2}} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_{\cdot_2} < 0 \;\&\&\; t_{\cdot_2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\cdot} < 0 \;\&\&\; t_{\cdot} > 0 \tag{139}$$

Okay, that concludes the analysis of this theory.

Case 41

Now for a new theory. Here is the full nonlinear Lagrangian for Case 41 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6} r_{\cdot}\, \mathcal{R}_{ij\hbar l}\, \mathcal{R}^{ij\hbar l} - \frac{2}{3} r_{\cdot}\, \mathcal{R}_{i\hbar jl}\, \mathcal{R}^{ij\hbar l} + \frac{1}{6}\left(r_{\cdot} - 6 r_{\cdot}\right) \mathcal{R}^{ij\hbar l}\, \mathcal{R}_{\hbar lij} + r_{\cdot}\, \mathcal{R}^{ij}{}_{i}{}^{\hbar}\, \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{12} t_{\cdot}\, \mathcal{T}_{ij\hbar}\, \mathcal{T}^{ij\hbar} - \frac{1}{6} t_{\cdot}\, \mathcal{T}^{ij\hbar}\, \mathcal{T}_{ji\hbar} - \frac{2}{3} t_{\cdot}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{\hbar}{}_{j\hbar} \tag{140}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3} t_{\cdot}\, \mathcal{A}_{abi}\, \mathcal{A}^{abi} - \frac{2}{3} t_{\cdot}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} - \frac{2}{3} t_{\cdot}\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} + \frac{4}{3} t_{\cdot}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} - \\
&\frac{4}{3} t_{\cdot}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{\cdot}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + \frac{2}{3} t_{\cdot}\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} - \frac{4}{3} t_{\cdot}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} - \\
&r_{\cdot}\, \partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{ab}{}_{a} - \frac{2}{3} t_{\cdot}\, \mathcal{A}_{abi}\, \partial^{i} f^{ab} + \frac{2}{3} t_{\cdot}\, \mathcal{A}_{aib}\, \partial^{i} f^{ab} - \frac{2}{3} t_{\cdot}\, \mathcal{A}_{bia}\, \partial^{i} f^{ab} + \\
&\frac{1}{3} t_{\cdot}\, \partial_{a} f_{bi}\, \partial^{i} f^{ab} - \frac{1}{6} t_{\cdot}\, \partial_{a} f_{ib}\, \partial^{i} f^{ab} - \frac{1}{6} t_{\cdot}\, \partial_{b} f_{ai}\, \partial^{i} f^{ab} + \frac{1}{6} t_{\cdot}\, \partial_{i} f_{ab}\, \partial^{i} f^{ab} - \frac{1}{6} t_{\cdot}\, \partial_{i} f_{ba}\, \partial^{i} f^{ab} - \\
&r_{\cdot}\, \partial_{a}\mathcal{A}^{abi}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + 2 r_{\cdot}\, \partial^{i}\mathcal{A}^{ab}{}_{a}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3} r_{\cdot}\, \partial_{b}\mathcal{A}_{aij}\, \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{\cdot}\, \partial_{b}\mathcal{A}_{aji}\, \partial^{j}\mathcal{A}^{abi} + \\
&\frac{2}{3}\left(r_{\cdot} - 6 r_{\cdot}\right) \partial_{b}\mathcal{A}_{ija}\, \partial^{j}\mathcal{A}^{abi} - \frac{1}{3} r_{\cdot}\, \partial_{i}\mathcal{A}_{abj}\, \partial^{j}\mathcal{A}^{abi} + \frac{1}{3} r_{\cdot}\, \partial_{j}\mathcal{A}_{abi}\, \partial^{j}\mathcal{A}^{abi} - \frac{2}{3} r_{\cdot}\, \partial_{j}\mathcal{A}_{aib}\, \partial^{j}\mathcal{A}^{abi}
\end{aligned} \tag{141}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} t. & -i\sqrt{2}\,k\,t. & 0 & 0 \\ i\sqrt{2}\,k\,t. & 2\,k\,t. & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\,r.+t. \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\left(9\,k\,r.+4\,t.\right) & \frac{\sqrt{\ }\,t_{2}}{} & --i\sqrt{2}\,k\,t. & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\ }\,t_{2}}{} & \frac{t_{2}}{} & --i\,k\,t. & 0 & 0 & 0 & 0 \\ -i\sqrt{2}\,k\,t. & \frac{i\,k\,t_{2}}{} & \frac{k^{2}\,t_{2}}{} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{t_{3}}{} & -\frac{\sqrt{\ }\,t_{3}}{} & --i\,k\,t. & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{\ }\,t_{3}}{} & \frac{t_{3}}{} & -i\sqrt{2}\,k\,t. & 0 \\ 0 & 0 & 0 & \frac{i\,k\,t_{3}}{} & --i\sqrt{2}\,k\,t. & \frac{k^{2}\,t_{3}}{} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{k^{2}\,r_{3}}{} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauge constraints on source currents:

$\dot{\tau}^{\perp} == 0$

$-2\,i\,k\,\dot{\sigma}^{\parallel} + \dot{\tau}^{\parallel} == 0$

$\dot{\tau}^{\perp\,a} == 0$

$-i\,k\,\dot{\sigma}^{\parallel\,a} + \dot{\tau}^{\parallel\,a} == 0$

$\dot{\sigma}^{\parallel\,a} + 2\,\dot{\sigma}^{\perp\,a} == 0$

$-i\,k\,\dot{\sigma}^{\perp\,ab} + \dot{\tau}^{\parallel\,ab} == 0$

$\dot{\sigma}^{\parallel\,abc} == 0$

$\dot{\tau}^{\parallel\,ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{}{\left(+k^{2}\right)^{2}\,t_{3}} & -\frac{i\sqrt{\ }\,k}{\left(+k^{2}\right)^{2}\,t_{3}} & 0 & 0 \\ \frac{i\sqrt{\ }\,k}{\left(+k^{2}\right)^{2}\,t_{3}} & \frac{k^{2}}{\left(+k^{2}\right)^{2}\,t_{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^{2}\,r_{2}+t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{}{k^2 r_3} & -\frac{\sqrt{}}{k^2 r_3 + k^4 r_3} & \frac{i\sqrt{}}{k r_3 + k^3 r_3} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{}}{k^2 r_3 + k^4 r_3} & \frac{k^2 r_3 + t_2}{(k+k^3)^2 r_3 t_2} & -\frac{i\left(k^2 r_3 + t_2\right)}{k(+k^2)^2 r_3 t_2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{}}{k r_3 + k^3 r_3} & \frac{i\left(k^2 r_3 + t_2\right)}{k(+k^2)^2 r_3 t_2} & \frac{k^2 r_3 + t_2}{(+k^2)^2 r_3 t_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{(+k^2)^2 t_3} & -\frac{\sqrt{}}{(+k^2)^2 t_3} & -\frac{i k}{(+k^2)^2 t_3} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{}}{(+k^2)^2 t_3} & \frac{}{(+k^2)^2 t_3} & \frac{i\sqrt{}\,k}{(+k^2)^2 t_3} & 0 \\ 0 & 0 & 0 & \frac{i k}{(+k^2)^2 t_3} & -\frac{i\sqrt{}\,k}{(+k^2)^2 t_3} & \frac{k^2}{(+k^2)^2 t_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} -\frac{}{k^2 r_3} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{-\frac{t_.}{r_.}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

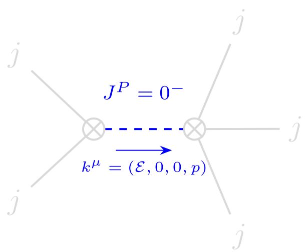
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_.}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_2} > 0$
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_2 < 0 \,\&\&\, t_2 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{2} < 0 \&\& t_{2} > 0 \tag{142}$$

Okay, that concludes the analysis of this theory.

Case 42

Now for a new theory. Here is the full nonlinear Lagrangian for Case 42 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{6} r_{2}\, \mathcal{R}_{ijhl}\, \mathcal{R}^{ijhl} - \frac{2}{3} r_{2}\, \mathcal{R}_{ihjl}\, \mathcal{R}^{ijhl} + r_{5}\, \mathcal{R}^{ij h}{}_{i}\, \mathcal{R}_{j}{}^{l}{}_{hl} + \frac{1}{6} r_{2}\, \mathcal{R}^{ijhl}\, \mathcal{R}_{hlij} - \\ &r_{5}\, \mathcal{R}^{ijh}{}_{i}\, \mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{4} t_{1}\, \mathcal{T}_{ijh}\, \mathcal{T}^{ijh} + \frac{1}{2} t_{1}\, \mathcal{T}^{ijh}\, \mathcal{T}_{jih} + \frac{1}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{h}{}_{jh}\end{aligned} \tag{143}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&t_{1}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} + \frac{1}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} + \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + \\ &\frac{1}{3}\left(-t_{1} + 2 t_{3}\right) \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} + \frac{1}{3}\left(-t_{1} + 2 t_{3}\right) \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} + \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} + \\ &r_{5}\, \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} - r_{5}\, \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} + 2 t_{1}\, \mathcal{A}_{bia}\, \partial^{i} f^{ab} - t_{1}\, \partial_{a} f_{bi}\, \partial^{i} f^{ab} + \frac{1}{2} t_{1}\, \partial_{a} f_{ib}\, \partial^{i} f^{ab} - \\ &\frac{1}{2} t_{1}\, \partial_{b} f_{ai}\, \partial^{i} f^{ab} + \frac{1}{2} t_{1}\, \partial_{i} f_{ab}\, \partial^{i} f^{ab} + \frac{1}{2} t_{1}\, \partial_{i} f_{ba}\, \partial^{i} f^{ab} - r_{5}\, \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + 2 r_{5}\, \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \\ &r_{5}\, \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - 2 r_{5}\, \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3} r_{2}\, \partial_{b} \mathcal{A}_{aij}\, \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{2}\, \partial_{b} \mathcal{A}_{aji}\, \partial^{j} \mathcal{A}^{abi} + \\ &\frac{2}{3} r_{2}\, \partial_{b} \mathcal{A}_{ija}\, \partial^{j} \mathcal{A}^{abi} - \frac{1}{3} r_{2}\, \partial_{i} \mathcal{A}_{abj}\, \partial^{j} \mathcal{A}^{abi} + \frac{1}{3} r_{2}\, \partial_{j} \mathcal{A}_{abi}\, \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{2}\, \partial_{j} \mathcal{A}_{aib}\, \partial^{j} \mathcal{A}^{abi}\end{aligned} \tag{144}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} t_{3} & -i \sqrt{2}\, k\, t_{3} & 0 & 0 \\ i \sqrt{2}\, k\, t_{3} & 2 k^{2} t_{3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^{2} r_{2} - t_{1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^{2} r_{5} - \frac{t_{1}}{2} & -\frac{t_{1}}{\sqrt{2}} & \frac{i k t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6}\left(6 k^{2} r_{5} + t_{1} + 4 t_{3}\right) & \frac{t_{1} - 2 t_{3}}{3 \sqrt{2}} & \frac{1}{3} i k \left(t_{1} - 2 t_{3}\right) & 0 \\ 0 & 0 & 0 & \frac{t_{1} - 2 t_{3}}{3 \sqrt{2}} & \frac{t_{1} + t_{3}}{3} & \frac{1}{3} i \sqrt{2}\, k \left(t_{1} + t_{3}\right) & 0 \\ 0 & 0 & 0 & -\frac{1}{3} i k \left(t_{1} - 2 t_{3}\right) & -\frac{1}{3} i \sqrt{2}\, k \left(t_{1} + t_{3}\right) & \frac{2}{3} k^{2} \left(t_{1} + t_{3}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_1}{} & -\frac{i k t_1}{\sqrt{}} & 0 \\ \frac{i k t_1}{\sqrt{}} & k\ t. & 0 \\ 0 & 0 & \frac{t_1}{} \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{+}{.}\tau^{\perp} == 0$

$-2\, i\, k\ \overset{+}{.}\sigma^{\parallel} + \overset{+}{.}\tau^{\parallel} == 0$

$\overset{-}{.}\tau^{\perp\, a} == 0$

$2\, i\, k\ \overset{-}{.}\sigma^{\perp\, a} + \overset{-}{.}\tau^{\parallel\, a} == 0$

$-i\, k\ \overset{+}{.}\sigma^{\perp\, ab} + \overset{+}{.}\tau^{\parallel\, ab} == 0$

$-2\, i\, k\ \overset{+}{.}\sigma^{\parallel\, ab} + \overset{+}{.}\tau^{\parallel\, ab} == 0$

The Drazin (Moore–Penrose) inverses of these a–matrices, which are functionally analogous to the inverse b–matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{}{(\ +k^2)^2 t_3} & -\frac{i\sqrt{}\ k}{(\ +k^2)^2 t_3} & 0 & 0 \\ \frac{i\sqrt{}\ k}{(\ +k^2)^2 t_3} & \frac{k^2}{(\ +k^2)^2 t_3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_2 - t_1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{}}{t_1 + k^2 t_1} & \frac{i\sqrt{}\ k}{t_1 + k^2 t_1} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{}}{t_1 + k^2 t_1} & \frac{-\ k^2 r_5 + t_1}{(\ +k^2)^2 t_1{}^2} & \frac{i(\ k^3 r_5 - k t_1)}{(\ +k^2)^2 t_1{}^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{}\ k}{t_1 + k^2 t_1} & -\frac{i(\ k^3 r_5 - k t_1)}{(\ +k^2)^2 t_1{}^2} & \frac{-\ k^4 r_5 + k^2 t_1}{(\ +k^2)^2 t_1{}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(t_1 + t_3)}{t_1 t_3 + \ k^2 r_5 (t_1 + t_3)} & -\frac{\sqrt{}\ (t_1 - \ t_3)}{(\ +\ k^2)(\ t_1 t_3 + \ k^2 r_5 (t_1 + t_3))} & -\frac{i k (t_1 - \ t_3)}{(\ +\ k^2)(\ t_1 t_3 + \ k^2 r_5 (t_1 + t_3))} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}\ (t._1 - 2 t._3)}{(1+2k^2)(3 t._1 t._3 + 2 k^2 r._5 (t._1 + t._3))} & \frac{6 k^2 r._5 + t._1 + 4 t._3}{(1+2k^2)^2 (3 t._1 t._3 + 2 k^2 r._5 (t._1 + t._3))} & \frac{i\sqrt{2}\ k (6 k^2 r._5 + t._1 + 4 t._3)}{(1+2k^2)^2 (3 t._1 t._3 + 2 k^2 r._5 (t._1 + t._3))} & 0 \\ 0 & 0 & 0 & \frac{2 i k (t._1 - 2 t._3)}{(1+2k^2)(3 t._1 t._3 + 2 k^2 r._5 (t._1 + t._3))} & -\frac{i\sqrt{2}\ k (6 k^2 r._5 + t._1 + 4 t._3)}{(1+2k^2)^2 (3 t._1 t._3 + 2 k^2 r._5 (t._1 + t._3))} & \frac{2 k^2 (6 k^2 r._5 + t._1 + 4 t._3)}{(1+2k^2)^2 (3 t._1 t._3 + 2 k^2 r._5 (t._1 + t._3))} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t._1} & -\frac{2 i\sqrt{2}\ k}{(1+2k^2)^2 t._1} & 0 \\ \frac{2 i\sqrt{2}\ k}{(1+2k^2)^2 t._1} & \frac{4 k^2}{(1+2k^2)^2 t._1} & 0 \\ 0 & 0 & \frac{2}{t._1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t.}{r.}\right\}, \{\}, \left\{-\frac{3\,t.\,t.}{2\,r.\,t. + 2\,r.\,t.}\right\}, \{\}, \{\}\right\}$$

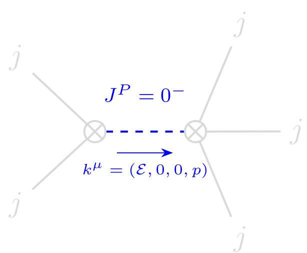
Massive pole residues:

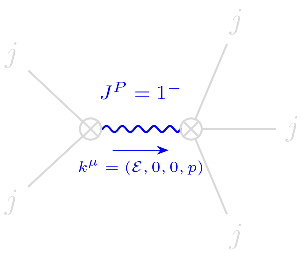
$$\left\{\{\}, \left\{-\frac{1}{r.}\right\}, \{\}, \left\{\frac{6\,t.\,t.\left(t. + t.\right) - 3\,r.\left(t. \;\; + 2\,t. \;\;\right)}{2\,r.\left(t. + t.\right)\left(-3\,t.\,t. + r.\left(t. + t.\right)\right)}\right\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:





Massive particle

Pole residue:	$-\frac{1}{r._2} > 0$
Square mass:	$\frac{t._1}{r._2} > 0$
Spin:	0
Parity:	Odd

Massive particle

Pole residue:	$\frac{6\,t._1\,t._3\,(t._1+t._3)-3\,r._5\,(t._1^2+2\,t._3^2)}{2\,r._5\,(t._1+t._3)\,(-3\,t._1\,t._3+r._5\,(t._1+t._3))} > 0$
Square mass:	$-\frac{3\,t._1\,t._3}{2\,r._5\,t._1+2\,r._5\,t._3} > 0$
Spin:	1
Parity:	Odd

Overall unitarity conditions:

$r. < 0 \,\&\&\, r. < 0 \,\&\&\, t. < 0 \,\&\&\, 0 < t. < -t.$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r. < 0 \,\&\&\, r. < 0 \,\&\&\, t. < 0 \,\&\&\, 0 < t. < -t. \tag{145}$$

Okay, that concludes the analysis of this theory.

Case 43

Now for a new theory. Here is the full nonlinear Lagrangian for Case 43 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{6}\,r.\,\mathcal{R}_{ijkl}\,\mathcal{R}^{ijkl} - \frac{2}{3}\,r.\,\mathcal{R}_{ikjl}\,\mathcal{R}^{ijkl} + r.\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6}\,r.\,\mathcal{R}^{ijkl}\,\mathcal{R}_{klij} - \\ r.\,\mathcal{R}^{ij}{}_{i}{}^{k}\,\mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12}\left(4\,t. + t.\right)\mathcal{T}_{ijk}\,\mathcal{T}^{ijk} + \frac{1}{6}\left(2\,t. - t.\right)\mathcal{T}^{ijk}\,\mathcal{T}_{jik} + t.\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{k}{}_{jk} \tag{146}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\left(t. + t.\right)\mathcal{A}_{abi}\,\mathcal{A}^{abi} + \frac{1}{3}\left(t. - 2\,t.\right)\mathcal{A}_{aib}\,\mathcal{A}^{abi} + t.\,\mathcal{A}^{ab}{}_{a}\,\mathcal{A}_{b}{}^{i}{}_{i} - 2\,t.\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial_{a}f^{ab} +$$

$$2\,t_{\cdot}\,\mathcal{A}_{b\ \ i}^{\ i}\ \partial^{b}f^{a}{}_{a} - t_{\cdot}\,\partial_{b}f^{i}{}_{i}\ \partial^{b}f^{a}{}_{a} - t_{\cdot}\,\partial_{a}f^{ab}\ \partial_{i}f^{i}{}_{b} + 2\,t_{\cdot}\,\partial^{b}f^{a}{}_{a}\ \partial_{i}f^{i}{}_{b} + r_{\cdot}\,\partial_{b}\mathcal{A}_{i\ \ j}^{\ j}\ \partial^{i}\mathcal{A}^{ab}{}_{a} -$$
$$r_{\cdot}\,\partial_{i}\mathcal{A}_{b\ \ j}^{\ j}\ \partial^{i}\mathcal{A}^{ab}{}_{a} - \frac{2}{3}\left(t_{\cdot} + t_{\cdot}\right)\mathcal{A}_{abi}\ \partial^{i}f^{ab} + \frac{2}{3}\left(t_{\cdot} + t_{\cdot}\right)\mathcal{A}_{aib}\ \partial^{i}f^{ab} + \frac{2}{3}\left(2\,t_{\cdot} - t_{\cdot}\right)\mathcal{A}_{bia}\ \partial^{i}f^{ab} +$$
$$\frac{1}{3}\left(-2\,t_{\cdot} + t_{\cdot}\right)\partial_{a}f_{bi}\ \partial^{i}f^{ab} + \frac{1}{6}\left(2\,t_{\cdot} - t_{\cdot}\right)\partial_{a}f_{ib}\ \partial^{i}f^{ab} + \frac{1}{6}\left(-4\,t_{\cdot} - t_{\cdot}\right)\partial_{b}f_{ai}\ \partial^{i}f^{ab} +$$
$$\frac{1}{6}\left(4\,t_{\cdot} + t_{\cdot}\right)\partial_{i}f_{ab}\ \partial^{i}f^{ab} + \frac{1}{6}\left(2\,t_{\cdot} - t_{\cdot}\right)\partial_{i}f_{ba}\ \partial^{i}f^{ab} - r_{\cdot}\,\partial_{a}\mathcal{A}^{abi}\ \partial_{j}\mathcal{A}_{b\ \ i}^{\ j} + 2\,r_{\cdot}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\ \partial_{j}\mathcal{A}_{b\ \ i}^{\ j} +$$
$$r_{\cdot}\,\partial_{a}\mathcal{A}^{abi}\ \partial_{j}\mathcal{A}_{i\ \ b}^{\ j} - 2\,r_{\cdot}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\ \partial_{j}\mathcal{A}_{i\ \ b}^{\ j} + \frac{4}{3}\,r_{\cdot}\,\partial_{b}\mathcal{A}_{aij}\ \partial^{j}\mathcal{A}^{abi} - \frac{2}{3}\,r_{\cdot}\,\partial_{b}\mathcal{A}_{aji}\ \partial^{j}\mathcal{A}^{abi} +$$
$$\frac{2}{3}\,r_{\cdot}\,\partial_{b}\mathcal{A}_{ija}\ \partial^{j}\mathcal{A}^{abi} - \frac{1}{3}\,r_{\cdot}\,\partial_{i}\mathcal{A}_{abj}\ \partial^{j}\mathcal{A}^{abi} + \frac{1}{3}\,r_{\cdot}\,\partial_{j}\mathcal{A}_{abi}\ \partial^{j}\mathcal{A}^{abi} - \frac{2}{3}\,r_{\cdot}\,\partial_{j}\mathcal{A}_{aib}\ \partial^{j}\mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -t_{\cdot} & i\sqrt{2}\,k\,t_{\cdot} & 0 & 0 \\ -i\sqrt{2}\,k\,t_{\cdot} & -2\,k\,t_{\cdot} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\,r_{\cdot} + t_{\cdot} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\left(6\,k\,r_{\cdot} + t_{\cdot} + 4\,t_{\cdot}\right) & -\frac{t_{1} - t_{2}}{\sqrt{}} & \frac{i\,k\left(t_{1} - t_{2}\right)}{\sqrt{}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1} - t_{2}}{\sqrt{}} & \frac{t_{1} + t_{2}}{} & --\,i\,k\left(t_{\cdot} + t_{\cdot}\right) & 0 & 0 & 0 & 0 \\ -\frac{i\,k\left(t_{1} - t_{2}\right)}{\sqrt{}} & -\,i\,k\left(t_{\cdot} + t_{\cdot}\right) & -\,k\left(t_{\cdot} + t_{\cdot}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\,r_{\cdot} - \frac{t_{1}}{} & \frac{t_{1}}{\sqrt{}} & i\,k\,t_{\cdot} & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{\sqrt{}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\,k\,t_{\cdot} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{} & -\frac{i\,k\,t_{1}}{\sqrt{}} & 0 \\ \frac{i\,k\,t_{1}}{\sqrt{}} & k\,t_{\cdot} & 0 \\ 0 & 0 & \frac{t_{1}}{} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{+}_{\cdot}\tau^{\perp} == 0$$

$$-2\,i\,k\ {}^{+}_{\cdot}\sigma^{\parallel} + {}^{+}_{\cdot}\tau^{\parallel} == 0$$

$${}^{-}_{\cdot}\tau^{\perp}{}^{a} == 0$$

$$2\,i\,k\ {}^{-}_{\cdot}\sigma^{\perp}{}^{a} + {}^{-}_{\cdot}\tau^{\parallel}{}^{a} == 0$$

$$-i\,k\ {}^{+}_{\cdot}\sigma^{\perp}{}^{ab} + {}^{+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

$$-2\,i\,k\ {}^{+}_{\cdot}\sigma^{\parallel}{}^{ab} + {}^{+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{}{(+k^2)^2 t_1} & \frac{i\sqrt{\ }\,k}{(+k^2)^2 t_1} & 0 & 0 \\ -\frac{i\sqrt{\ }\,k}{(+k^2)^2 t_1} & -\frac{k^2}{(+k^2)^2 t_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{}{k^2 r_2 + t_2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{(t_1+t_2)}{t_1 t_2 + k^2 r_5 (t_1+t_2)} & \frac{\sqrt{\ }\,(t_1 - t_2)}{(+k^2)(t_1 t_2 + k^2 r_5 (t_1+t_2))} & -\frac{i\sqrt{\ }\,k(t_1 - t_2)}{(+k^2)(t_1 t_2 + k^2 r_5 (t_1+t_2))} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\ }\,(t_1 - t_2)}{(+k^2)(t_1 t_2 + k^2 r_5 (t_1+t_2))} & \frac{k^2 r_5 + t_1 + t_2}{(+k^2)^2 (t_1 t_2 + k^2 r_5 (t_1+t_2))} & -\frac{i k (k^2 r_5 + t_1 + t_2)}{(+k^2)^2 (t_1 t_2 + k^2 r_5 (t_1+t_2))} & 0 & 0 & 0 & 0 \\ \frac{i\sqrt{\ }\,k(t_1 - t_2)}{(+k^2)(t_1 t_2 + k^2 r_5 (t_1+t_2))} & \frac{i k (k^2 r_5 + t_1 + t_2)}{(+k^2)^2 (t_1 t_2 + k^2 r_5 (t_1+t_2))} & \frac{k^2 (k^2 r_5 + t_1 + t_2)}{(+k^2)^2 (t_1 t_2 + k^2 r_5 (t_1+t_2))} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{\ }}{t_1 + k^2 t_1} & \frac{i k}{t_1 + k^2 t_1} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{\ }}{t_1 + k^2 t_1} & \frac{- k^2 r_5 + t_1}{(t_1 + k^2 t_1)^2} & -\frac{i\sqrt{\ }\,k(k^2 r_5 - t_1)}{(t_1 + k^2 t_1)^2} & 0 \\ 0 & 0 & 0 & -\frac{i k}{t_1 + k^2 t_1} & \frac{i\sqrt{\ }\,k(k^2 r_5 - t_1)}{(t_1 + k^2 t_1)^2} & \frac{- k^4 r_5 + k^2 t_1}{(t_1 + k^2 t_1)^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_1} & -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & 0 \\ \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & \frac{4k^2}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\},\left\{-\frac{t_2}{r_2}\right\},\left\{-\frac{3 t_1 t_2}{2 r_5 t_1 + 2 r_5 t_2}\right\},\{\},\{\},\{\}\right\}$$

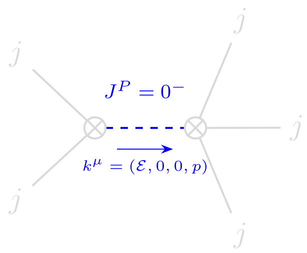
Massive pole residues:

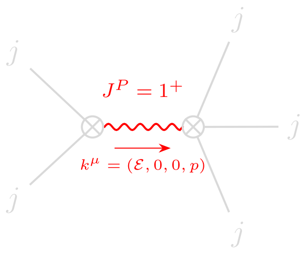
$$\left\{\{\},\left\{-\frac{1}{r_2}\right\},\left\{\frac{-3 t_1 t_2 (t_1 + t_2) + 3 r_5 (t_1^2 + 2 t_2^2)}{r_5 (t_1 + t_2)(-3 t_1 t_2 + 2 r_5 (t_1 + t_2))}\right\},\{\},\{\},\{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:





Massive particle

Pole residue:	$-\frac{S}{r_2 P} > 0$
Square mass:	$-\frac{t_2 P}{r_2 P} > 0$
Spin:	0
Parity:	Odd

Massive particle

Pole residue:	$\frac{ΩU\, t_1 P t_2 P J t_1 P M t_2 P K M U\, r_5 P J t_1 P^2 M T t_2 P^2 K}{r_5 P J t_1 P M t_2 P K J ΩU\, t_1 P t_2 P M T\, r_5 P J t_1 P M t_2 P K K} > 0$
Square mass:	$-\frac{U\, t_1 P t_2 P}{T\, r_5 P t_1 P M T\, r_5 P t_2 P} > 0$
Spin:	1
Parity:	Even

Overall unitarity conditions:

$r. < 0\ \&\&\ r. > 0\ \&\&\ t. < 0\ \&\&\ t. > -t.$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{2} < 0\ \&\&\ r_{5} > 0\ \&\&\ t_{1} < 0\ \&\&\ t_{2} > -t_{1} \tag{148}$$

Okay, that concludes the analysis of this theory.

Case 44

Now for a new theory. Here is the full nonlinear Lagrangian for Case 44 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{3} r_{1} \mathcal{R}_{ijhl} \mathcal{R}^{ijhl} + \frac{2}{3} r_{1} \mathcal{R}_{ihjl} \mathcal{R}^{ijhl} + r_{5} \mathcal{R}^{ij}{}_{i}{}^{h} \mathcal{R}_{j}{}^{l}{}_{hl} - \frac{2}{3} r_{1} \mathcal{R}^{ijhl} \mathcal{R}_{hlij} - \\ &r_{5} \mathcal{R}^{ij}{}_{i}{}^{h} \mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{4} t_{1} \mathcal{T}_{ijh} \mathcal{T}^{ijh} + \frac{1}{2} t_{1} \mathcal{T}^{ijh} \mathcal{T}_{jih} + \frac{1}{3} \left(t_{1} - 2 t_{3}\right) \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{h}{}_{jh}\end{aligned} \tag{149}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&t_{1} \mathcal{A}_{aib} \mathcal{A}^{abi} + \frac{1}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} + \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} + \\ &\frac{1}{3}\left(-t_{1} + 2 t_{3}\right) \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} + \frac{1}{3}\left(-t_{1} + 2 t_{3}\right) \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} + \frac{2}{3}\left(t_{1} - 2 t_{3}\right) \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} + \\ &r_{5} \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} - r_{5} \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + 2 t_{1} \mathcal{A}_{bia} \partial^{i} f^{ab} - t_{1} \partial_{a} f_{bi} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{a} f_{ib} \partial^{i} f^{ab} - \\ &\frac{1}{2} t_{1} \partial_{b} f_{ai} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{i} f_{ab} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{i} f_{ba} \partial^{i} f^{ab} - r_{5} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + 2 r_{5} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \\ &r_{5} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - 2 r_{5} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - \frac{4}{3} r_{1} \partial_{b} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{b} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{abi} - \\ &\frac{8}{3} r_{1} \partial_{b} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{1} \partial_{i} \mathcal{A}_{abj} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{abi} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{aib} \partial^{j} \mathcal{A}^{abi}\end{aligned} \tag{150}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \underset{3}{t} & -i\sqrt{2}\,k\,\underset{3}{t} & 0 & 0 \\ i\sqrt{2}\,k\,\underset{3}{t} & 2k^2\,\underset{3}{t} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\underset{1}{t} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^2\left(2\underset{1}{r}+\underset{5}{r}\right)-\frac{\underset{1}{t}}{2} & -\frac{\underset{1}{t}}{\sqrt{2}} & \frac{i k \underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{\underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k \underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6}\left(6k^2\left(\underset{1}{r}+\underset{5}{r}\right)+\underset{1}{t}+4\underset{3}{t}\right) & \frac{\underset{1}{t}-2\underset{3}{t}}{3\sqrt{2}} & \frac{1}{3}i k\left(\underset{1}{t}-2\underset{3}{t}\right) & 0 \\ 0 & 0 & 0 & \frac{\underset{1}{t}-2\underset{3}{t}}{3\sqrt{2}} & \frac{\underset{1}{t}+\underset{3}{t}}{3} & \frac{1}{3}i\sqrt{2}\,k\left(\underset{1}{t}+\underset{3}{t}\right) & 0 \\ 0 & 0 & 0 & -\frac{1}{3}i k\left(\underset{1}{t}-2\underset{3}{t}\right) & -\frac{1}{3}i\sqrt{2}\,k\left(\underset{1}{t}+\underset{3}{t}\right) & \frac{2}{3}k^2\left(\underset{1}{t}+\underset{3}{t}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{\underset{1}{t}}{2} & -\frac{i k \underset{1}{t}}{\sqrt{2}} & 0 \\ \frac{i k \underset{1}{t}}{\sqrt{2}} & k^2\,\underset{1}{t} & 0 \\ 0 & 0 & k^2\,\underset{1}{r}+\frac{\underset{1}{t}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\bullet}\tau^{\perp} == 0$$

$$-2 i k\, {}^{0^+}_{\bullet}\sigma^{\parallel} + {}^{0^+}_{\bullet}\tau^{\parallel} == 0$$

$${}^{1^-}_{\bullet}\tau^{\perp}{}^{a} == 0$$

$$2 i k\, {}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} + {}^{1^-}_{\bullet}\tau^{\parallel}{}^{a} == 0$$

$$-i k\, {}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab} + {}^{1^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

$$-2 i k\, {}^{2^+}_{\bullet}\sigma^{\parallel}{}^{ab} + {}^{2^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{\left(1+2k^2\right)^2\underset{3}{t}} & -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2\underset{3}{t}} & 0 & 0 \\ \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2\underset{3}{t}} & \frac{2k^2}{\left(1+2k^2\right)^2\underset{3}{t}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\underset{1}{t}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & 0 & 0 & 0 & 0 \\
-\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2k^2\left(2r_{1}+r_{5}\right)+t_{1}}{(1+k^2)^2 t_{1}^2} & \frac{i\left(2k^3\left(2r_{1}+r_{5}\right)-k\,t_{1}\right)}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\
-\frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & \frac{-2ik^3\left(2r_{1}+r_{5}\right)+ik\,t_{1}}{(1+k^2)^2 t_{1}^2} & \frac{-2k^4\left(2r_{1}+r_{5}\right)+k^2 t_{1}}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{2\left(t_{1}+t_{3}\right)}{3t_{1}t_{3}+2k^2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)} & -\frac{\sqrt{2}\left(t_{1}-2t_{3}\right)}{(1+2k^2)\left(3t_{1}t_{3}+2k^2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)\right)} & -\frac{2ik\left(t_{1}-2t_{3}\right)}{(1+2k^2)\left(3t_{1}t_{3}+2k^2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)\right)} & 0 \\
0 & 0 & 0 & -\frac{\sqrt{2}\left(t_{1}-2t_{3}\right)}{(1+2k^2)\left(3t_{1}t_{3}+2k^2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)\right)} & \frac{6k^2\left(r_{1}+r_{5}\right)+t_{1}+4t_{3}}{(1+2k^2)^2\left(3t_{1}t_{3}+2k^2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)\right)} & \frac{i\sqrt{2}\,k\left(6k^2\left(r_{1}+r_{5}\right)+t_{1}+4t_{3}\right)}{(1+2k^2)^2\left(3t_{1}t_{3}+2k^2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)\right)} & 0 \\
0 & 0 & 0 & \frac{2ik\left(t_{1}-2t_{3}\right)}{(1+2k^2)\left(3t_{1}t_{3}+2k^2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)\right)} & -\frac{i\sqrt{2}\,k\left(6k^2\left(r_{1}+r_{5}\right)+t_{1}+4t_{3}\right)}{(1+2k^2)^2\left(3t_{1}t_{3}+2k^2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)\right)} & \frac{k^2\left(k^2\left(r_{1}+r_{5}\right)+t_{1}+4t_{3}\right)}{(1+2k^2)^2\left(3t_{1}t_{3}+2k^2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)\right)} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix}
\frac{2}{(1+2k^2)^2 t_{1}} & -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & 0 \\
\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & \frac{4k^2}{(1+2k^2)^2 t_{1}} & 0 \\
0 & 0 & \frac{2}{2k^2 r_{1}+t_{1}}
\end{pmatrix}$$

Square masses:

$$\left\{\{\},\{\},\{\},\left\{-\frac{3t_{1}t_{3}}{2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)}\right\},\{\},\left\{-\frac{t_{1}}{2r_{1}}\right\}\right\}$$

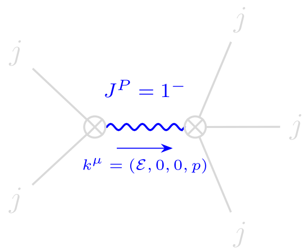
Massive pole residues:

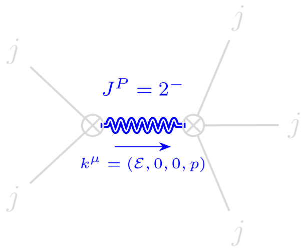
$$\left\{\{\},\{\},\{\},\left\{-\frac{3\left(-2t_{1}t_{3}\left(t_{1}+t_{3}\right)+r_{1}\left(t_{1}^2+2t_{3}^2\right)+r_{5}\left(t_{1}^2+2t_{3}^2\right)\right)}{2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)\left(-3t_{1}t_{3}+r_{1}\left(t_{1}+t_{3}\right)+r_{5}\left(t_{1}+t_{3}\right)\right)}\right\},\{\},\left\{-\frac{1}{r_{1}}\right\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:





Massive particle

Pole residue:	$-\frac{3\left(-2t_{1}t_{3}\left(t_{1}+t_{3}\right)+r_{1}\left(t_{1}^2+2t_{3}^2\right)+r_{5}\left(t_{1}^2+2t_{3}^2\right)\right)}{2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)\left(-3t_{1}t_{3}+r_{1}\left(t_{1}+t_{3}\right)+r_{5}\left(t_{1}+t_{3}\right)\right)} > 0$
Square mass:	$-\frac{3t_{1}t_{3}}{2\left(r_{1}+r_{5}\right)\left(t_{1}+t_{3}\right)} > 0$
Spin:	1
Parity:	Odd

Massive particle

Pole residue:	$-\frac{1}{r_{1}} > 0$
Square mass:	$-\frac{t_{1}}{2r_{1}} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r_{1} < 0 \,\&\&\, r_{5} < -r_{1} \,\&\&\, t_{1} > 0 \,\&\&\, \left(t_{3} < -t_{1} \,\|\, t_{3} > 0\right)$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r_{1} < 0 \,\&\&\, r_{5} < -r_{1} \,\&\&\, t_{1} > 0 \,\&\&\, t_{3} < -t_{1}\right) \| \left(r_{1} < 0 \,\&\&\, r_{5} < -r_{1} \,\&\&\, t_{1} > 0 \,\&\&\, t_{3} > 0\right) \tag{151}$$

Okay, that concludes the analysis of this theory.

Case 45

Now for a new theory. Here is the full nonlinear Lagrangian for Case 45 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{1} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} + \frac{2}{3} r_{1} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + r_{5} \mathcal{R}^{ij\,k}{}_{i} \mathcal{R}_{j}{}^{l}{}_{kl} - \frac{2}{3} r_{1} \mathcal{R}^{ijkl} \mathcal{R}_{klij} - r_{5} \mathcal{R}^{ij\,k}{}_{i} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{12}\left(4 t_{1} + t_{2}\right) \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{6}\left(2 t_{1} - t_{2}\right) \mathcal{T}^{ijk} \mathcal{T}_{jik} + t_{1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk} \tag{152}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3}\left(t_{1} + t_{2}\right) \mathcal{A}_{abi} \mathcal{A}^{abi} + \frac{1}{3}\left(t_{1} - 2 t_{2}\right) \mathcal{A}_{aib} \mathcal{A}^{abi} + t_{1} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} - 2 t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} + \\
&2 t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} - t_{1} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} - t_{1} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} + 2 t_{1} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} + r_{5} \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} - \\
&r_{5} \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} - \frac{2}{3}\left(t_{1} + t_{2}\right) \mathcal{A}_{abi} \partial^{i} f^{ab} + \frac{2}{3}\left(t_{1} + t_{2}\right) \mathcal{A}_{aib} \partial^{i} f^{ab} + \frac{2}{3}\left(2 t_{1} - t_{2}\right) \mathcal{A}_{bia} \partial^{i} f^{ab} + \\
&\frac{1}{3}\left(-2 t_{1} + t_{2}\right) \partial_{a} f_{bi} \partial^{i} f^{ab} + \frac{1}{6}\left(2 t_{1} - t_{2}\right) \partial_{a} f_{ib} \partial^{i} f^{ab} + \frac{1}{6}\left(-4 t_{1} - t_{2}\right) \partial_{b} f_{ai} \partial^{i} f^{ab} + \\
&\frac{1}{6}\left(4 t_{1} + t_{2}\right) \partial_{i} f_{ab} \partial^{i} f^{ab} + \frac{1}{6}\left(2 t_{1} - t_{2}\right) \partial_{i} f_{ba} \partial^{i} f^{ab} - r_{5} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + 2 r_{5} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \\
&r_{5} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - 2 r_{5} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - \frac{4}{3} r_{1} \partial_{b} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{b} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{abi} - \\
&\frac{8}{3} r_{1} \partial_{b} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{1} \partial_{i} \mathcal{A}_{abj} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{abi} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{aib} \partial^{j} \mathcal{A}^{abi}
\end{aligned} \tag{153}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} -t_{1} & i\sqrt{2}\, k\, t_{1} & 0 & 0 \\ -i\sqrt{2}\, k\, t_{1} & -2 k^{2} t_{1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_{2} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} \frac{1}{6}\left(6k^2\left(2r_{1}+r_{5}\right)+t_{1}+4t_{2}\right) & -\frac{t_{1}-2t_{2}}{3\sqrt{2}} & \frac{i k\left(t_{1}-2t_{2}\right)}{3\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1}-2t_{2}}{3\sqrt{2}} & \frac{t_{1}+t_{2}}{3} & -\frac{1}{3} i k\left(t_{1}+t_{2}\right) & 0 & 0 & 0 & 0 \\ -\frac{i k\left(t_{1}-2t_{2}\right)}{3\sqrt{2}} & \frac{1}{3} i k\left(t_{1}+t_{2}\right) & \frac{1}{3}k^2\left(t_{1}+t_{2}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\left(r_{1}+r_{5}\right)-\frac{t_{1}}{2} & \frac{t_{1}}{\sqrt{2}} & i k t_{1} & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i k t_{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{2} & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & k^2 t_{1} & 0 \\ 0 & 0 & k^2 r_{1}+\frac{t_{1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^+}\tau^{\perp} == 0$

$-2 i k\, {}^{0^+}\sigma^{\parallel} + {}^{0^+}\tau^{\parallel} == 0$

${}^{1^-}\tau^{\perp a} == 0$

$2 i k\, {}^{1^-}\sigma^{\perp a} + {}^{1^-}\tau^{\parallel a} == 0$

$-i k\, {}^{1^+}\sigma^{\perp ab} + {}^{1^+}\tau^{\parallel ab} == 0$

$-2 i k\, {}^{2^+}\sigma^{\parallel ab} + {}^{2^+}\tau^{\parallel ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{\left(1+2k^2\right)^2 t_{1}} & \frac{i\sqrt{2}k}{\left(1+2k^2\right)^2 t_{1}} & 0 & 0 \\ -\frac{i\sqrt{2}k}{\left(1+2k^2\right)^2 t_{1}} & -\frac{2k^2}{\left(1+2k^2\right)^2 t_{1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
\frac{2(t_1+t_2)}{3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)} & \frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))} & -\frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))} & 0 & 0 & 0 \\
\frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))} & \frac{6k^2(2r_1+r_5)+t_1+4t_2}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))} & -\frac{ik(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))} & 0 & 0 & 0 \\
\frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))} & \frac{ik(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))} & \frac{k^2(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t_1+2k^2t_1} & \frac{2ik}{t_1+2k^2t_1} \\
0 & 0 & 0 & \frac{\sqrt{2}}{t_1+2k^2t_1} & \frac{-2k^2(r_1+r_5)+t_1}{(t_1+2k^2t_1)^2} & -\frac{i\sqrt{2}k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2t_1)^2} \\
0 & 0 & 0 & -\frac{2ik}{t_1+2k^2t_1} & \frac{i\sqrt{2}k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2t_1)^2} & \frac{-4k^4(r_1+r_5)+2k^2t_1}{(t_1+2k^2t_1)^2} \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix}
\frac{2}{(1+2k^2)^2t_1} & -\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1} & 0 \\
\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1} & \frac{4k^2}{(1+2k^2)^2t_1} & 0 \\
0 & 0 & \frac{2}{2k^2r_1+t_1}
\end{pmatrix}$$

Square masses:

$$\left\{\{\},\{\},\left\{-\frac{3t_1t_2}{2(2r_1+r_5)(t_1+t_2)}\right\},\{\},\{\},\left\{-\frac{t_1}{2r_1}\right\}\right\}$$

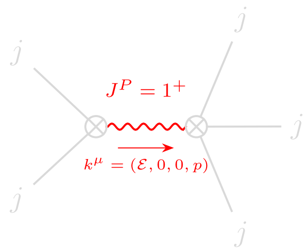
Massive pole residues:

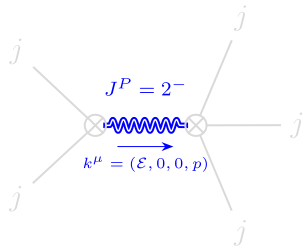
$$\left\{\{\},\{\},\left\{\frac{-3t_1t_2(t_1+t_2)+6r_1(t_1^2+2t_2^2)+3r_5(t_1^2+2t_2^2)}{(2r_1+r_5)(t_1+t_2)(-3t_1t_2+4r_1(t_1+t_2)+2r_5(t_1+t_2))}\right\},\{\},\{\},\left\{-\frac{1}{r_1}\right\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:





Massive particle

Pole residue:	$\frac{-3t_1t_2(t_1+t_2)+6r_1(t_1^2+2t_2^2)+3r_5(t_1^2+2t_2^2)}{(2r_1+r_5)(t_1+t_2)(-3t_1t_2+4r_1(t_1+t_2)+2r_5(t_1+t_2))} > 0$
Square mass:	$-\frac{3t_1t_2}{2(2r_1+r_5)(t_1+t_2)} > 0$
Spin:	1
Parity:	Even

Massive particle

Pole residue:	$-\frac{1}{r_1} > 0$
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r_{1} < 0 \;\&\&\; r_{5} > -2\, r_{1} \;\&\&\; t_{1} > 0 \;\&\&\; -t_{1} < t_{2} < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{1} < 0 \;\&\&\; r_{5} > -2\, r_{1} \;\&\&\; t_{1} > 0 \;\&\&\; -t_{1} < t_{2} < 0 \tag{154}$$

Okay, that concludes the analysis of this theory.

Case 46

Now for a new theory. Here is the full nonlinear Lagrangian for Case 46 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}
&\frac{1}{6}\left(2 r_{1}+r_{2}\right) \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl}+\frac{2}{3}\left(r_{1}-r_{2}\right) \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl}- \\
&2 r_{1}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{j}{}^{l}{}_{kl}+\frac{1}{6}\left(-4 r_{1}+r_{2}\right) \mathcal{R}^{ijkl}\, \mathcal{R}_{klij}+2 r_{1}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl}+ \\
&\frac{1}{12}\left(4 t_{1}+t_{2}\right) \mathcal{T}_{ijk}\, \mathcal{T}^{ijk}+\frac{1}{6}\left(2 t_{1}-t_{2}\right) \mathcal{T}^{ijk}\, \mathcal{T}_{jik}+t_{1}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{k}{}_{jk}
\end{aligned} \tag{155}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3}\left(t_{1}+t_{2}\right) \mathcal{A}_{abi}\, \mathcal{A}^{abi}+\frac{1}{3}\left(t_{1}-2 t_{2}\right) \mathcal{A}_{aib}\, \mathcal{A}^{abi}+t_{1}\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i}- \\
&2 t_{1}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab}+2 t_{1}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a}-t_{1}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a}-t_{1}\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b}+2 t_{1}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b}- \\
&2 r_{1}\, \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a}+2 r_{1}\, \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a}-\frac{2}{3}\left(t_{1}+t_{2}\right) \mathcal{A}_{abi}\, \partial^{i} f^{ab}+\frac{2}{3}\left(t_{1}+t_{2}\right) \mathcal{A}_{aib}\, \partial^{i} f^{ab}+ \\
&\frac{2}{3}\left(2 t_{1}-t_{2}\right) \mathcal{A}_{bia}\, \partial^{i} f^{ab}+\frac{1}{3}\left(-2 t_{1}+t_{2}\right) \partial_{a} f_{bi}\, \partial^{i} f^{ab}+\frac{1}{6}\left(2 t_{1}-t_{2}\right) \partial_{a} f_{ib}\, \partial^{i} f^{ab}+ \\
&\frac{1}{6}\left(-4 t_{1}-t_{2}\right) \partial_{b} f_{ai}\, \partial^{i} f^{ab}+\frac{1}{6}\left(4 t_{1}+t_{2}\right) \partial_{i} f_{ab}\, \partial^{i} f^{ab}+\frac{1}{6}\left(2 t_{1}-t_{2}\right) \partial_{i} f_{ba}\, \partial^{i} f^{ab}+ \\
&2 r_{1}\, \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i}-4 r_{1}\, \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i}-2 r_{1}\, \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b}+4 r_{1}\, \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b}- \\
&\frac{4}{3}\left(r_{1}-r_{2}\right) \partial_{b} \mathcal{A}_{aij}\, \partial^{j} \mathcal{A}^{abi}+\frac{2}{3}\left(r_{1}-r_{2}\right) \partial_{b} \mathcal{A}_{aji}\, \partial^{j} \mathcal{A}^{abi}+\frac{2}{3}\left(-4 r_{1}+r_{2}\right) \partial_{b} \mathcal{A}_{ija}\, \partial^{j} \mathcal{A}^{abi}+ \\
&\frac{1}{3}\left(-2 r_{1}-r_{2}\right) \partial_{i} \mathcal{A}_{abj}\, \partial^{j} \mathcal{A}^{abi}+\frac{1}{3}\left(2 r_{1}+r_{2}\right) \partial_{j} \mathcal{A}_{abi}\, \partial^{j} \mathcal{A}^{abi}+\frac{2}{3}\left(r_{1}-r_{2}\right) \partial_{j} \mathcal{A}_{aib}\, \partial^{j} \mathcal{A}^{abi}
\end{aligned} \tag{156}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} -t_{1} & i\sqrt{2}\, k\, t_{1} & 0 & 0 \\ -i\sqrt{2}\, k\, t_{1} & -2 k^{2} t_{1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^{2} r_{2}+t_{2} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} \frac{1}{6}\left(t_{1}+4\,t_{2}\right) & -\frac{t_{1}-2\,t_{2}}{3\sqrt{2}} & \frac{i\,k\left(t_{1}-2\,t_{2}\right)}{3\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1}-2\,t_{2}}{3\sqrt{2}} & \frac{t_{1}+t_{2}}{3} & -\frac{1}{3}\,i\,k\left(t_{1}+t_{2}\right) & 0 & 0 & 0 & 0 \\ -\frac{i\,k\left(t_{1}-2\,t_{2}\right)}{3\sqrt{2}} & \frac{1}{3}\,i\,k\left(t_{1}+t_{2}\right) & \frac{1}{3}\,k^{2}\left(t_{1}+t_{2}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k^{2}\,r_{1}-\frac{t_{1}}{2} & \frac{t_{1}}{\sqrt{2}} & i\,k\,t_{1} & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\,k\,t_{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{2} & -\frac{i\,k\,t_{1}}{\sqrt{2}} & 0 \\ \frac{i\,k\,t_{1}}{\sqrt{2}} & k^{2}\,t_{1} & 0 \\ 0 & 0 & k^{2}\,r_{1}+\frac{t_{1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^+}\tau^{\perp} == 0$

$-2\,i\,k\,{}^{0^+}\sigma^{\parallel} + {}^{0^+}\tau^{\parallel} == 0$

${}^{1^-}\tau^{\perp a} == 0$

$2\,i\,k\,{}^{1^-}\sigma^{\perp a} + {}^{1^-}\tau^{\parallel a} == 0$

$-i\,k\,{}^{1^+}\sigma^{\perp ab} + {}^{1^+}\tau^{\parallel ab} == 0$

$-2\,i\,k\,{}^{2^+}\sigma^{\parallel ab} + {}^{2^+}\tau^{\parallel ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{\left(1+2\,k^{2}\right)^{2}\,t_{1}} & \frac{i\,\sqrt{2}\,k}{\left(1+2\,k^{2}\right)^{2}\,t_{1}} & 0 & 0 \\ -\frac{i\,\sqrt{2}\,k}{\left(1+2\,k^{2}\right)^{2}\,t_{1}} & -\frac{2\,k^{2}}{\left(1+2\,k^{2}\right)^{2}\,t_{1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^{2}\,r_{2}+t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{2\left(t^{\bullet}_{1}+t^{\bullet}_{2}\right)}{3\,t^{\bullet}_{1}\,t^{\bullet}_{2}} & \frac{\sqrt{2}\left(t^{\bullet}_{1}-2\,t^{\bullet}_{2}\right)}{3\left(1+k^{2}\right)t^{\bullet}_{1}\,t^{\bullet}_{2}} & -\frac{\mathrm{i}\sqrt{2}\,k\left(t^{\bullet}_{1}-2\,t^{\bullet}_{2}\right)}{3\left(1+k^{2}\right)t^{\bullet}_{1}\,t^{\bullet}_{2}} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}\left(t^{\bullet}_{1}-2\,t^{\bullet}_{2}\right)}{3\left(1+k^{2}\right)t^{\bullet}_{1}\,t^{\bullet}_{2}} & \frac{t^{\bullet}_{1}+4\,t^{\bullet}_{2}}{3\left(1+k^{2}\right)^{2}t^{\bullet}_{1}\,t^{\bullet}_{2}} & -\frac{\mathrm{i}\,k\left(t^{\bullet}_{1}+4\,t^{\bullet}_{2}\right)}{3\left(1+k^{2}\right)^{2}t^{\bullet}_{1}\,t^{\bullet}_{2}} & 0 & 0 & 0 & 0 \\ \frac{\mathrm{i}\sqrt{2}\,k\left(t^{\bullet}_{1}-2\,t^{\bullet}_{2}\right)}{3\left(1+k^{2}\right)t^{\bullet}_{1}\,t^{\bullet}_{2}} & \frac{\mathrm{i}\,k\left(t^{\bullet}_{1}+4\,t^{\bullet}_{2}\right)}{3\left(1+k^{2}\right)^{2}t^{\bullet}_{1}\,t^{\bullet}_{2}} & \frac{k^{2}\left(t^{\bullet}_{1}+4\,t^{\bullet}_{2}\right)}{3\left(1+k^{2}\right)^{2}t^{\bullet}_{1}\,t^{\bullet}_{2}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t^{\bullet}_{1}+2\,k^{2}\,t^{\bullet}_{1}} & \frac{2\,\mathrm{i}\,k}{t^{\bullet}_{1}+2\,k^{2}\,t^{\bullet}_{1}} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{t^{\bullet}_{1}+2\,k^{2}\,t^{\bullet}_{1}} & \frac{2\,k^{2}\,r^{\bullet}_{1}+t^{\bullet}_{1}}{\left(t^{\bullet}_{1}+2\,k^{2}\,t^{\bullet}_{1}\right)^{2}} & \frac{\mathrm{i}\sqrt{2}\,k\left(2\,k^{2}\,r^{\bullet}_{1}+t^{\bullet}_{1}\right)}{\left(t^{\bullet}_{1}+2\,k^{2}\,t^{\bullet}_{1}\right)^{2}} & 0 \\ 0 & 0 & 0 & -\frac{2\,\mathrm{i}\,k}{t^{\bullet}_{1}+2\,k^{2}\,t^{\bullet}_{1}} & -\frac{\mathrm{i}\sqrt{2}\,k\left(2\,k^{2}\,r^{\bullet}_{1}+t^{\bullet}_{1}\right)}{\left(t^{\bullet}_{1}+2\,k^{2}\,t^{\bullet}_{1}\right)^{2}} & \frac{2\,k^{2}\left(2\,k^{2}\,r^{\bullet}_{1}+t^{\bullet}_{1}\right)}{\left(t^{\bullet}_{1}+2\,k^{2}\,t^{\bullet}_{1}\right)^{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{\left(1+2\,k^{2}\right)^{2}t^{\bullet}_{1}} & -\frac{2\,\mathrm{i}\sqrt{2}\,k}{\left(1+2\,k^{2}\right)^{2}t^{\bullet}_{1}} & 0 \\ \frac{2\,\mathrm{i}\sqrt{2}\,k}{\left(1+2\,k^{2}\right)^{2}t^{\bullet}_{1}} & \frac{4\,k^{2}}{\left(1+2\,k^{2}\right)^{2}t^{\bullet}_{1}} & 0 \\ 0 & 0 & \frac{2}{2\,k^{2}\,r^{\bullet}_{1}+t^{\bullet}_{1}} \end{pmatrix}$$

Square masses:

$$\left\{\{\},\left\{-\frac{t^{\bullet}_{2}}{r^{\bullet}_{2}}\right\},\{\},\{\},\{\},\left\{-\frac{t^{\bullet}_{1}}{2\,r^{\bullet}_{1}}\right\}\right\}$$

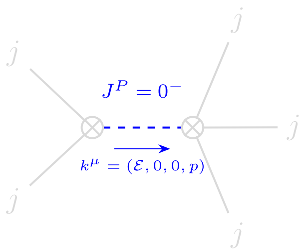
Massive pole residues:

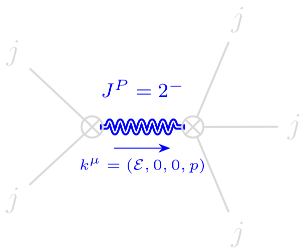
$$\left\{\{\},\left\{-\frac{1}{r^{\bullet}_{2}}\right\},\{\},\{\},\{\},\left\{-\frac{1}{r^{\bullet}_{1}}\right\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:





Massive particle

Pole residue:	$-\frac{S}{rP_{2}} > 0$
Square mass:	$-\frac{tP_{2}}{rP_{2}} > 0$
Spin:	0
Parity:	Odd

Massive particle

Pole residue:	$-\frac{S}{rP_{1}} > 0$
Square mass:	$-\frac{tP_{1}}{T\,rP_{1}} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r^{\bullet}_{1} < 0 \,\&\&\, r^{\bullet}_{2} < 0 \,\&\&\, t^{\bullet}_{1} > 0 \,\&\&\, t^{\bullet}_{2} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{1} < 0 \,\&\&\, r_{2} < 0 \,\&\&\, t_{1} > 0 \,\&\&\, t_{2} > 0 \tag{157}$$

Okay, that concludes the analysis of this theory.

Case 47

Now for a new theory. Here is the full nonlinear Lagrangian for Case 47 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$r_{5}\,\mathcal{R}^{ij\hbar}{}_{i}\,\mathcal{R}_{j}{}^{l}{}_{\hbar l}-r_{5}\,\mathcal{R}^{ij\hbar}{}_{i}\,\mathcal{R}_{\hbar}{}^{l}{}_{jl}+\frac{1}{4}t_{1}\,\mathcal{T}_{ij\hbar}\,\mathcal{T}^{ij\hbar}+\frac{1}{2}t_{1}\,\mathcal{T}^{ij\hbar}\,\mathcal{T}_{ji\hbar}+\frac{1}{3}\left(t_{1}-2t_{3}\right)\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{\hbar}{}_{j\hbar} \tag{158}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&t_{1}\,\mathcal{A}_{aib}\,\mathcal{A}^{abi}+\frac{1}{3}\left(t_{1}-2t_{3}\right)\mathcal{A}^{ab}{}_{a}\,\mathcal{A}_{b}{}^{i}{}_{i}-\frac{2}{3}\left(t_{1}-2t_{3}\right)\mathcal{A}_{b}{}^{i}{}_{i}\,\partial_{a}f^{ab}+\\&\frac{2}{3}\left(t_{1}-2t_{3}\right)\mathcal{A}_{b}{}^{i}{}_{i}\,\partial^{b}f^{a}{}_{a}+\frac{1}{3}\left(-t_{1}+2t_{3}\right)\partial_{b}f^{i}{}_{i}\,\partial^{b}f^{a}{}_{a}+\frac{1}{3}\left(-t_{1}+2t_{3}\right)\partial_{a}f^{ab}\,\partial_{i}f^{i}{}_{b}+\\&\frac{2}{3}\left(t_{1}-2t_{3}\right)\partial^{b}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{b}+r_{5}\,\partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}-r_{5}\,\partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}+2t_{1}\,\mathcal{A}_{bia}\,\partial^{i}f^{ab}-\\&t_{1}\,\partial_{a}f_{bi}\,\partial^{i}f^{ab}+\frac{1}{2}t_{1}\,\partial_{a}f_{ib}\,\partial^{i}f^{ab}-\frac{1}{2}t_{1}\,\partial_{b}f_{ai}\,\partial^{i}f^{ab}+\frac{1}{2}t_{1}\,\partial_{i}f_{ab}\,\partial^{i}f^{ab}+\frac{1}{2}t_{1}\,\partial_{i}f_{ba}\,\partial^{i}f^{ab}-\\&r_{5}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}+2r_{5}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}+r_{5}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}-2r_{5}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}\end{aligned} \tag{159}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} t_{3} & -i\sqrt{2}\,k\,t_{3} & 0 & 0\\ i\sqrt{2}\,k\,t_{3} & 2k^{2}t_{3} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -t_{1}\end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} k^{2}r_{5}-\frac{t_{1}}{2} & -\frac{t_{1}}{\sqrt{2}} & \frac{i\,k\,t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0\\ -\frac{t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0\\ -\frac{i\,k\,t_{1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{6}\left(6k^{2}r_{5}+t_{1}+4t_{3}\right) & \frac{t_{1}-2t_{3}}{3\sqrt{2}} & \frac{1}{3}\,i\,k\left(t_{1}-2t_{3}\right) & 0\\ 0 & 0 & 0 & \frac{t_{1}-2t_{3}}{3\sqrt{2}} & \frac{t_{1}+t_{3}}{3} & \frac{1}{3}\,i\sqrt{2}\,k\left(t_{1}+t_{3}\right) & 0\\ 0 & 0 & 0 & -\frac{1}{3}\,i\,k\left(t_{1}-2t_{3}\right) & -\frac{1}{3}\,i\sqrt{2}\,k\left(t_{1}+t_{3}\right) & \frac{2}{3}k^{2}\left(t_{1}+t_{3}\right) & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} \frac{t_1}{2} & -\frac{i k t_1}{\sqrt{2}} & 0 \\ \frac{i k t_1}{\sqrt{2}} & k^2 t_1 & 0 \\ 0 & 0 & \frac{t_1}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}\tau^{\perp} == 0$$

$$-2\, i\, k\, {}^{0^+}\sigma^{\parallel} + {}^{0^+}\tau^{\parallel} == 0$$

$${}^{1^-}\tau^{\perp}{}^{a} == 0$$

$$2\, i\, k\, {}^{1^-}\sigma^{\perp}{}^{a} + {}^{1^-}\tau^{\parallel}{}^{a} == 0$$

$$-i\, k\, {}^{1^+}\sigma^{\perp}{}^{ab} + {}^{1^+}\tau^{\parallel}{}^{ab} == 0$$

$$-2\, i\, k\, {}^{2^+}\sigma^{\parallel}{}^{ab} + {}^{2^+}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{(1+2k^2)^2 t_3} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3} & 0 & 0 \\ \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3} & \frac{2k^2}{(1+2k^2)^2 t_3} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_1} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{-2k^2 r_5+t_1}{(1+k^2)^2 t_1^2} & \frac{i(2k^3 r_5-k t_1)}{(1+k^2)^2 t_1^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{t_1+k^2 t_1} & -\frac{i(2k^3 r_5-k t_1)}{(1+k^2)^2 t_1^2} & \frac{-2k^4 r_5+k^2 t_1}{(1+k^2)^2 t_1^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(t_1+t_3)}{3 t_1 t_3+2k^2 r_5 (t_1+t_3)} & -\frac{\sqrt{2}\,(t_1-2t_3)}{(1+2k^2)(3 t_1 t_3+2k^2 r_5 (t_1+t_3))} & -\frac{2 i k (t_1-2t_3)}{(1+2k^2)(3 t_1 t_3+2k^2 r_5 (t_1+t_3))} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}\,(t_1-2t_3)}{(1+2k^2)(3 t_1 t_3+2k^2 r_5 (t_1+t_3))} & \frac{6k^2 r_5+t_1+4t_3}{(1+2k^2)^2 (3 t_1 t_3+2k^2 r_5 (t_1+t_3))} & \frac{i\sqrt{2}\,k(6k^2 r_5+t_1+4t_3)}{(1+2k^2)^2 (3 t_1 t_3+2k^2 r_5 (t_1+t_3))} & 0 \\ 0 & 0 & 0 & \frac{2 i k (t_1-2t_3)}{(1+2k^2)(3 t_1 t_3+2k^2 r_5 (t_1+t_3))} & -\frac{i\sqrt{2}\,k(6k^2 r_5+t_1+4t_3)}{(1+2k^2)^2 (3 t_1 t_3+2k^2 r_5 (t_1+t_3))} & \frac{2k^2(6k^2 r_5+t_1+4t_3)}{(1+2k^2)^2 (3 t_1 t_3+2k^2 r_5 (t_1+t_3))} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_1} & -\frac{2 i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & 0 \\ \frac{2 i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & \frac{4k^2}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\},\{\},\{\},\left\{-\frac{3 t_1 t_3}{2 r_5 t_1+2 r_5 t_3}\right\},\{\},\{\}\right\}$$

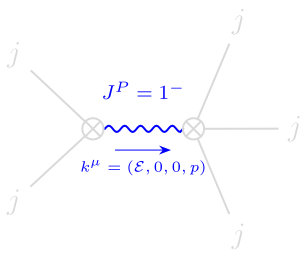
Massive pole residues:

$$\left\{\{\},\{\},\{\},\left\{\frac{6\,t_{1}\,t_{3}\left(t_{1}+t_{3}\right)-3\,r_{5}\left(t_{1}^{2}+2\,t_{3}^{2}\right)}{2\,r_{5}\left(t_{1}+t_{3}\right)\left(-3\,t_{1}\,t_{3}+r_{5}\left(t_{1}+t_{3}\right)\right)}\right\},\{\},\{\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:



Massive particle

Pole residue:	$\frac{6t_{1}t_{3}(t_{1}+t_{3})-3r_{5}(t_{1}^{2}+2t_{3}^{2})}{2r_{5}(t_{1}+t_{3})(-3t_{1}t_{3}+r_{5}(t_{1}+t_{3}))} > 0$
Square mass:	$-\frac{3t_{1}t_{3}}{2r_{5}t_{1}+2r_{5}t_{3}} > 0$
Spin:	1
Parity:	Odd

Overall unitarity conditions:

$$r_{5} < 0 \,\&\&\, \left(\left(t_{1} < 0 \,\&\&\, 0 < t_{3} < -t_{1}\right) \,||\, \left(t_{1} > 0 \,\&\&\, \left(t_{3} < -t_{1} \,||\, t_{3} > 0\right)\right)\right)$$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\left(r. < 0 \,\&\&\, t. < 0 \,\&\&\, 0 < t. < -t.\right) \,||\, \left(r. < 0 \,\&\&\, t. > 0 \,\&\&\, t. < -t.\right) \,||\, \left(r. < 0 \,\&\&\, t. > 0 \,\&\&\, t. > 0\right) \qquad (160)$$

Okay, that concludes the analysis of this theory.

Case 48

Now for a new theory. Here is the full nonlinear Lagrangian for Case 48 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$r.\,\mathcal{R}^{ij\;\;k}_{\;\;i}\,\mathcal{R}_{j\;\;kl}^{\;\;l}-r.\,\mathcal{R}^{ij\;\;k}_{\;\;i}\,\mathcal{R}_{k\;\;jl}^{\;\;l}+\frac{1}{12}\left(4\,t.+t.\right)\mathcal{T}_{ijk}\,\mathcal{T}^{ijk}+\frac{1}{6}\left(2\,t.-t.\right)\mathcal{T}^{ijk}\,\mathcal{T}_{jik}+t.\,\mathcal{T}^{i\;\;j}_{\;\;i}\,\mathcal{T}^{k}_{\;\;jk} \qquad (161)$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3}\left(t.+t.\right)\mathcal{A}_{abi}\,\mathcal{A}^{abi}+\frac{1}{3}\left(t.-2\,t.\right)\mathcal{A}_{aib}\,\mathcal{A}^{abi}+t.\,\mathcal{A}^{ab}_{\;\;\;a}\,\mathcal{A}_{b\;\;i}^{\;\;i}-2\,t.\,\mathcal{A}_{b\;\;i}^{\;\;i}\,\partial_{a}f^{ab}+2\,t.\,\mathcal{A}_{b\;\;i}^{\;\;i}\,\partial^{b}f^{a}_{\;\;a}-t.\,\partial_{b}f^{i}_{\;\;i}\,\partial^{b}f^{a}_{\;\;a}-t.\,\partial_{a}f^{ab}\,\partial_{i}f^{i}_{\;\;b}+2\,t.\,\partial^{b}f^{a}_{\;\;a}\,\partial_{i}f^{i}_{\;\;b}+$$

$$r_{.}\,\partial_b\mathcal{A}_i{}^j{}_j\,\partial^i\mathcal{A}^{ab}{}_a - r_{.}\,\partial_i\mathcal{A}_b{}^j{}_j\,\partial^i\mathcal{A}^{ab}{}_a - \frac{2}{3}\left(t_{.} + t_{.}\right)\mathcal{A}_{abi}\,\partial^i f^{ab} + \frac{2}{3}\left(t_{.} + t_{.}\right)\mathcal{A}_{aib}\,\partial^i f^{ab} +$$
$$\frac{2}{3}\left(2\,t_{.} - t_{.}\right)\mathcal{A}_{bia}\,\partial^i f^{ab} + \frac{1}{3}\left(-2\,t_{.} + t_{.}\right)\partial_a f_{bi}\,\partial^i f^{ab} + \frac{1}{6}\left(2\,t_{.} - t_{.}\right)\partial_a f_{ib}\,\partial^i f^{ab} +$$
$$\frac{1}{6}\left(-4\,t_{.} - t_{.}\right)\partial_b f_{ai}\,\partial^i f^{ab} + \frac{1}{6}\left(4\,t_{.} + t_{.}\right)\partial_i f_{ab}\,\partial^i f^{ab} + \frac{1}{6}\left(2\,t_{.} - t_{.}\right)\partial_i f_{ba}\,\partial^i f^{ab} -$$
$$r_{.}\,\partial_a\mathcal{A}^{abi}\,\partial_j\mathcal{A}_b{}^j{}_i + 2\,r_{.}\,\partial^i\mathcal{A}^{ab}{}_a\,\partial_j\mathcal{A}_b{}^j{}_i + r_{.}\,\partial_a\mathcal{A}^{abi}\,\partial_j\mathcal{A}_i{}^j{}_b - 2\,r_{.}\,\partial^i\mathcal{A}^{ab}{}_a\,\partial_j\mathcal{A}_i{}^j{}_b$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -t_{.} & i\sqrt{2}\,k\,t_{.} & 0 & 0 \\ -i\sqrt{2}\,k\,t_{.} & -2\,k\;t_{.} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_{.} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\left(6\,k\;r_{.} + t_{.} + 4\,t_{.}\right) & -\frac{t_1 - t_2}{\sqrt{}} & \frac{i\,k\left(t_1 - t_2\right)}{\sqrt{}} & 0 & 0 & 0 & 0 \\ -\frac{t_1 - t_2}{\sqrt{}} & \frac{t_1 + t_2}{} & --\,i\,k\left(t_{.} + t_{.}\right) & 0 & 0 & 0 & 0 \\ -\frac{i\,k\left(t_1 - t_2\right)}{\sqrt{}} & -\,i\,k\left(t_{.} + t_{.}\right) & -\,k\;\left(t_{.} + t_{.}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k\;r_{.} - \frac{t_1}{} & \frac{t_1}{\sqrt{}} & i\,k\,t_{.} & 0 \\ 0 & 0 & 0 & \frac{t_1}{\sqrt{}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\,k\,t_{.} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_1}{} & -\frac{i\,k\,t_1}{\sqrt{}} & 0 \\ \frac{i\,k\,t_1}{\sqrt{}} & k\;t_{.} & 0 \\ 0 & 0 & \frac{t_1}{} \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{+}{.}\tau^{\perp} == 0$

$-2\,i\,k\;\overset{+}{.}\sigma^{\parallel} + \overset{+}{.}\tau^{\parallel} == 0$

$\overset{-}{.}\tau^{\perp}{}^{a} == 0$

$2\,i\,k\;\overset{-}{.}\sigma^{\perp}{}^{a} + \overset{-}{.}\tau^{\parallel}{}^{a} == 0$

$-i\,k\;\overset{+}{.}\sigma^{\perp}{}^{ab} + \overset{+}{.}\tau^{\parallel}{}^{ab} == 0$

$-2\,i\,k\;\overset{+}{.}\sigma^{\parallel}{}^{ab} + \overset{+}{.}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally
analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{}{(+k^2)^2 t_1} & \frac{i\sqrt{\ }\,k}{(+k^2)^2 t_1} & 0 & 0 \\ -\frac{i\sqrt{\ }\,k}{(+k^2)^2 t_1} & -\frac{k^2}{(+k^2)^2 t_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{}{t_2} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{(t_1+t_2)}{t_1 t_2 + k^2 r_5 (t_1+t_2)} & \frac{\sqrt{\ }(t_1 - t_2)}{(+k^2)(t_1 t_2 + k^2 r_5 (t_1+t_2))} & -\frac{i\sqrt{\ }\,k(t_1 - t_2)}{(+k^2)(t_1 t_2 + k^2 r_5 (t_1+t_2))} & 0 & 0 & 0 & 0 \\ \frac{\sqrt{\ }(t_1 - t_2)}{(+k^2)(t_1 t_2 + k^2 r_5 (t_1+t_2))} & \frac{k^2 r_5 + t_1 + t_2}{(+k^2)^2(t_1 t_2 + k^2 r_5 (t_1+t_2))} & -\frac{i k(k^2 r_5 + t_1 + t_2)}{(+k^2)^2(t_1 t_2 + k^2 r_5 (t_1+t_2))} & 0 & 0 & 0 & 0 \\ \frac{i\sqrt{\ }\,k(t_1 - t_2)}{(+k^2)(t_1 t_2 + k^2 r_5 (t_1+t_2))} & \frac{i k(k^2 r_5 + t_1 + t_2)}{(+k^2)^2(t_1 t_2 + k^2 r_5 (t_1+t_2))} & \frac{k^2(k^2 r_5 + t_1 + t_2)}{(+k^2)^2(t_1 t_2 + k^2 r_5 (t_1+t_2))} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{\ }}{t_1 + k^2 t_1} & \frac{i k}{t_1 + k^2 t_1} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{\ }}{t_1 + k^2 t_1} & \frac{- k^2 r_5 + t_1}{(t_1 + k^2 t_1)^2} & -\frac{i\sqrt{\ }\,k(k^2 r_5 - t_1)}{(t_1 + k^2 t_1)^2} & 0 \\ 0 & 0 & 0 & -\frac{i k}{t_1 + k^2 t_1} & \frac{i\sqrt{\ }\,k(k^2 r_5 - t_1)}{(t_1 + k^2 t_1)^2} & \frac{- k^4 r_5 + k^2 t_1}{(t_1 + k^2 t_1)^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_1} & -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & 0 \\ \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & \frac{4k^2}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \left\{-\frac{3 t_1 t_2}{2 r_5 t_1 + 2 r_5 t_2}\right\}, \{\}, \{\}, \{\}\right\}$$

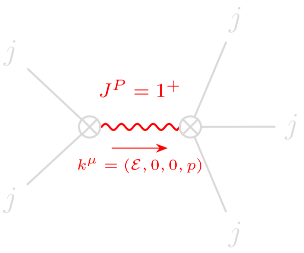
Massive pole residues:

$$\left\{\{\}, \{\}, \left\{\frac{-3 t_1 t_2 (t_1 + t_2) + 3 r_5 (t_1^2 + 2 t_2^2)}{r_5 (t_1 + t_2)(-3 t_1 t_2 + 2 r_5 (t_1 + t_2))}\right\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$\frac{\mathrm{OU}\,t_{1}\mathrm{P}\,t_{2}\mathrm{PJ}\,t_{1}\mathrm{PM}\,t_{2}\mathrm{PKMU}\,r_{5}\mathrm{PJ}\,t_{1}\mathrm{P}^2\mathrm{MT}\,t_{2}\mathrm{P}^2\mathrm{K}}{r_{5}\mathrm{PJ}\,t_{1}\mathrm{PM}\,t_{2}\mathrm{PKJOU}\,t_{1}\mathrm{P}\,t_{2}\mathrm{PMT}\,r_{5}\mathrm{PJ}\,t_{1}\mathrm{PM}\,t_{2}\mathrm{PKK}} > 0$
Square mass:	$-\frac{\mathrm{U}\,t_{1}\mathrm{P}\,t_{2}\mathrm{P}}{\mathrm{T}\,r_{5}\mathrm{P}\,t_{1}\mathrm{PMT}\,r_{5}\mathrm{P}\,t_{2}\mathrm{P}} > 0$
Spin:	1
Parity:	Even

Overall unitarity conditions:

$r_{5} > 0 \,\&\&\, \left(\left(t_{1} < 0 \,\&\&\, \left(t_{2} < 0 \,\|\, t_{2} > -t_{1}\right)\right) \,\|\, \left(t_{1} > 0 \,\&\&\, -t_{1} < t_{2} < 0\right)\right)$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{5} > 0 \,\&\&\, \left(\left(t_{1} < 0 \,\&\&\, \left(t_{2} < 0 \,\|\, t_{2} > -t_{1}\right)\right) \,\|\, \left(t_{1} > 0 \,\&\&\, -t_{1} < t_{2} < 0\right)\right) \tag{163}$$

Okay, that concludes the analysis of this theory.

Case 49

Now for a new theory. Here is the full nonlinear Lagrangian for Case 49 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{6} r_{2} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} - \frac{2}{3} r_{2} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} + r_{5} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} + \frac{1}{6} r_{2} \mathcal{R}^{ijkl} \mathcal{R}_{klij} - \\ &r_{5} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4} t_{1} \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{2} t_{1} \mathcal{T}^{ijk} \mathcal{T}_{jik} + t_{1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}\end{aligned} \tag{164}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&t_{1} \mathcal{A}_{aib} \mathcal{A}^{abi} + t_{1} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} - 2 t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} + 2 t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} - \\ &t_{1} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} - t_{1} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} + 2 t_{1} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} + r_{5} \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} - r_{5} \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + \\ &2 t_{1} \mathcal{A}_{bia} \partial^{i} f^{ab} - t_{1} \partial_{a} f_{bi} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{a} f_{ib} \partial^{i} f^{ab} - \frac{1}{2} t_{1} \partial_{b} f_{ai} \partial^{i} f^{ab} + \\ &\frac{1}{2} t_{1} \partial_{i} f_{ab} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{i} f_{ba} \partial^{i} f^{ab} - r_{5} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + 2 r_{5} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} + \\ &r_{5} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - 2 r_{5} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + \frac{4}{3} r_{2} \partial_{b} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{2} \partial_{b} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{abi} + \\ &\frac{2}{3} r_{2} \partial_{b} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{abi} - \frac{1}{3} r_{2} \partial_{i} \mathcal{A}_{abj} \partial^{j} \mathcal{A}^{abi} + \frac{1}{3} r_{2} \partial_{j} \mathcal{A}_{abi} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{2} \partial_{j} \mathcal{A}_{aib} \partial^{j} \mathcal{A}^{abi}\end{aligned} \tag{165}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\underset{1}{t} & i\sqrt{2}\,k\,\underset{1}{t} & 0 & 0 \\ -i\sqrt{2}\,k\,\underset{1}{t} & -2k^2\,\underset{1}{t} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\,\underset{2}{r}-\underset{1}{t} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^2\,\underset{5}{r}-\frac{\underset{1}{t}}{2} & -\frac{\underset{1}{t}}{\sqrt{2}} & \frac{i\,k\,\underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{\underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i\,k\,\underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2\,\underset{5}{r}-\frac{\underset{1}{t}}{2} & \frac{\underset{1}{t}}{\sqrt{2}} & i\,k\,\underset{1}{t} & 0 \\ 0 & 0 & 0 & \frac{\underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\,k\,\underset{1}{t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{\underset{1}{t}}{2} & -\frac{i\,k\,\underset{1}{t}}{\sqrt{2}} & 0 \\ \frac{i\,k\,\underset{1}{t}}{\sqrt{2}} & k^2\,\underset{1}{t} & 0 \\ 0 & 0 & \frac{\underset{1}{t}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\bullet}\tau^{\perp} == 0$$

$$-2\,i\,k\,{}^{0^+}_{\bullet}\sigma^{\parallel} + {}^{0^+}_{\bullet}\tau^{\parallel} == 0$$

$${}^{1^-}_{\bullet}\tau^{\perp}{}^{a} == 0$$

$$2\,i\,k\,{}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} + {}^{1^-}_{\bullet}\tau^{\parallel}{}^{a} == 0$$

$$-i\,k\,{}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab} + {}^{1^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

$$-2\,i\,k\,{}^{2^+}_{\bullet}\sigma^{\parallel}{}^{ab} + {}^{2^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{(1+2k^2)^2\,\underset{1}{t}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2\,\underset{1}{t}} & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2\,\underset{1}{t}} & -\frac{2k^2}{(1+2k^2)^2\,\underset{1}{t}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2\,\underset{2}{r}-\underset{1}{t}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{i \sqrt{2} k}{t_1+k^2 t_1} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_1+k^2 t_1} & \frac{-2 k^2 r_5+t_1}{(1+k^2)^2 t_1{}^2} & \frac{i\left(2 k^3 r_5-k t_1\right)}{(1+k^2)^2 t_1{}^2} & 0 & 0 & 0 & 0 \\ -\frac{i \sqrt{2} k}{t_1+k^2 t_1} & -\frac{i\left(2 k^3 r_5-k t_1\right)}{(1+k^2)^2 t_1{}^2} & \frac{-2 k^4 r_5+k^2 t_1}{(1+k^2)^2 t_1{}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t_1+2 k^2 t_1} & \frac{2 i k}{t_1+2 k^2 t_1} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{t_1+2 k^2 t_1} & \frac{-2 k^2 r_5+t_1}{\left(t_1+2 k^2 t_1\right)^2} & -\frac{i \sqrt{2} k\left(2 k^2 r_5-t_1\right)}{\left(t_1+2 k^2 t_1\right)^2} & 0 \\ 0 & 0 & 0 & -\frac{2 i k}{t_1+2 k^2 t_1} & \frac{i \sqrt{2} k\left(2 k^2 r_5-t_1\right)}{\left(t_1+2 k^2 t_1\right)^2} & \frac{-4 k^4 r_5+2 k^2 t_1}{\left(t_1+2 k^2 t_1\right)^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2 k^2)^2 t_1} & -\frac{2 i \sqrt{2} k}{(1+2 k^2)^2 t_1} & 0 \\ \frac{2 i \sqrt{2} k}{(1+2 k^2)^2 t_1} & \frac{4 k^2}{(1+2 k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \left\{\frac{t_1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

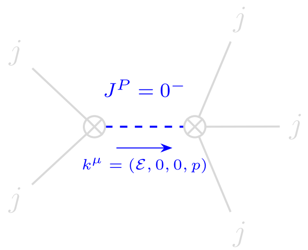
Massive pole residues:

$$\left\{\{\}, \left\{-\frac{1}{r_2}\right\}, \{\}, \{\}, \{\}, \{\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{r^P_2} > 0$
Square mass:	$\frac{t^P_1}{r^P_2} > 0$
Spin:	0
Parity:	Odd

Overall unitarity conditions:

$r_2 < 0 \,\&\&\, t_1 < 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\underset{2}{r} < 0 \,\&\&\, \underset{1}{t} < 0 \tag{166}$$

Okay, that concludes the analysis of this theory.

Case 50

Now for a new theory. Here is the full nonlinear Lagrangian for Case 50 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}\underset{1}{r}\,\mathcal{R}_{ijhl}\,\mathcal{R}^{ijhl}+\frac{2}{3}\underset{1}{r}\,\mathcal{R}_{ihjl}\,\mathcal{R}^{ijhl}-\underset{1}{r}\,\mathcal{R}^{ij}{}_{i}{}^{h}\,\mathcal{R}_{j}{}^{l}{}_{hl}-\frac{2}{3}\underset{1}{r}\,\mathcal{R}^{ijhl}\,\mathcal{R}_{hlij}+\underset{1}{r}\,\mathcal{R}^{ij}{}_{i}{}^{h}\,\mathcal{R}_{h}{}^{l}{}_{jl}+\frac{1}{4}\underset{1}{t}\,\mathcal{T}_{ijh}\,\mathcal{T}^{ijh}+\frac{1}{2}\underset{1}{t}\,\mathcal{T}^{ijh}\,\mathcal{T}_{jih}+\frac{1}{3}\left(\underset{1}{t}-2\underset{3}{t}\right)\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{h}{}_{jh} \tag{167}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\underset{1}{t}\,\mathcal{A}_{aib}\,\mathcal{A}^{abi}+\frac{1}{3}\left(\underset{1}{t}-2\underset{3}{t}\right)\mathcal{A}^{ab}{}_{a}\,\mathcal{A}_{b}{}^{i}{}_{i}-\frac{2}{3}\left(\underset{1}{t}-2\underset{3}{t}\right)\mathcal{A}_{b}{}^{i}{}_{i}\,\partial_{a}f^{ab}+\frac{2}{3}\left(\underset{1}{t}-2\underset{3}{t}\right)\mathcal{A}_{b}{}^{i}{}_{i}\,\partial^{b}f^{a}{}_{a}+\\&\frac{1}{3}\left(-\underset{1}{t}+2\underset{3}{t}\right)\partial_{b}f^{i}{}_{i}\,\partial^{b}f^{a}{}_{a}+\frac{1}{3}\left(-\underset{1}{t}+2\underset{3}{t}\right)\partial_{a}f^{ab}\,\partial_{i}f^{i}{}_{b}+\frac{2}{3}\left(\underset{1}{t}-2\underset{3}{t}\right)\partial^{b}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{b}-\\&\underset{1}{r}\,\partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}+\underset{1}{r}\,\partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}+2\underset{1}{t}\,\mathcal{A}_{bia}\,\partial^{i}f^{ab}-\underset{1}{t}\,\partial_{a}f_{bi}\,\partial^{i}f^{ab}+\frac{1}{2}\underset{1}{t}\,\partial_{a}f_{ib}\,\partial^{i}f^{ab}-\\&\frac{1}{2}\underset{1}{t}\,\partial_{b}f_{ai}\,\partial^{i}f^{ab}+\frac{1}{2}\underset{1}{t}\,\partial_{i}f_{ab}\,\partial^{i}f^{ab}+\frac{1}{2}\underset{1}{t}\,\partial_{i}f_{ba}\,\partial^{i}f^{ab}+\underset{1}{r}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}-2\underset{1}{r}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}-\\&\underset{1}{r}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}+2\underset{1}{r}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}-\frac{4}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{abi}-\\&\frac{8}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{abi}-\frac{2}{3}\underset{1}{r}\,\partial_{i}\mathcal{A}_{abj}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{abi}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{aib}\,\partial^{j}\mathcal{A}^{abi}\end{aligned} \tag{168}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

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The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
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```
Matrix for spin-0 sector:
```

$$\begin{pmatrix}\underset{3}{t} & -i\sqrt{2}\,k\,\underset{3}{t} & 0 & 0\\ i\sqrt{2}\,k\,\underset{3}{t} & 2k^{2}\,\underset{3}{t} & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -\underset{1}{t}\end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} k^2 r_{\underset{1}{\bullet}} - \frac{t_{\underset{1}{\bullet}}}{2} & -\frac{t_{\underset{1}{\bullet}}}{\sqrt{2}} & \frac{i k t_{\underset{1}{\bullet}}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{\underset{1}{\bullet}}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_{\underset{1}{\bullet}}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6}\left(t_{\underset{1}{\bullet}} + 4 t_{\underset{3}{\bullet}}\right) & \frac{t_{\underset{1}{\bullet}} - 2 t_{\underset{3}{\bullet}}}{3\sqrt{2}} & \frac{1}{3} i k \left(t_{\underset{1}{\bullet}} - 2 t_{\underset{3}{\bullet}}\right) & 0 \\ 0 & 0 & 0 & \frac{t_{\underset{1}{\bullet}} - 2 t_{\underset{3}{\bullet}}}{3\sqrt{2}} & \frac{t_{\underset{1}{\bullet}} + t_{\underset{3}{\bullet}}}{3} & \frac{1}{3} i \sqrt{2}\, k \left(t_{\underset{1}{\bullet}} + t_{\underset{3}{\bullet}}\right) & 0 \\ 0 & 0 & 0 & -\frac{1}{3} i k \left(t_{\underset{1}{\bullet}} - 2 t_{\underset{3}{\bullet}}\right) & -\frac{1}{3} i \sqrt{2}\, k \left(t_{\underset{1}{\bullet}} + t_{\underset{3}{\bullet}}\right) & \frac{2}{3} k^2 \left(t_{\underset{1}{\bullet}} + t_{\underset{3}{\bullet}}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{\underset{1}{\bullet}}}{2} & -\frac{i k t_{\underset{1}{\bullet}}}{\sqrt{2}} & 0 \\ \frac{i k t_{\underset{1}{\bullet}}}{\sqrt{2}} & k^2 t_{\underset{1}{\bullet}} & 0 \\ 0 & 0 & k^2 r_{\underset{1}{\bullet}} + \frac{t_{\underset{1}{\bullet}}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\bullet}\tau^{\perp} == 0$$

$$-2 i k\, {}^{0^+}_{\bullet}\sigma^{\parallel} + {}^{0^+}_{\bullet}\tau^{\parallel} == 0$$

$${}^{1^-}_{\bullet}\tau^{\perp}{}^{a} == 0$$

$$2 i k\, {}^{1^-}_{\bullet}\sigma^{\perp}{}^{a} + {}^{1^-}_{\bullet}\tau^{\parallel}{}^{a} == 0$$

$$-i k\, {}^{1^+}_{\bullet}\sigma^{\perp}{}^{ab} + {}^{1^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

$$-2 i k\, {}^{2^+}_{\bullet}\sigma^{\parallel}{}^{ab} + {}^{2^+}_{\bullet}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{\left(1+2k^2\right)^2 t_{\underset{3}{\bullet}}} & -\frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{\underset{3}{\bullet}}} & 0 & 0 \\ \frac{i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{\underset{3}{\bullet}}} & \frac{2k^2}{\left(1+2k^2\right)^2 t_{\underset{3}{\bullet}}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_{\underset{1}{\bullet}}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2k^2 r_{1}+t_{1}}{(1+k^2)^2 t_{1}^2} & \frac{i\left(2k^3 r_{1}-k t_{1}\right)}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & -\frac{i\left(2k^3 r_{1}-k t_{1}\right)}{(1+k^2)^2 t_{1}^2} & \frac{-2k^4 r_{1}+k^2 t_{1}}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2\left(t_{1}+t_{3}\right)}{3 t_{1} t_{3}} & -\frac{\sqrt{2}\left(t_{1}-2t_{3}\right)}{3\left(1+2k^2\right) t_{1} t_{3}} & -\frac{2ik t_{1}-4ik t_{3}}{3 t_{1} t_{3}+6k^2 t_{1} t_{3}} & 0 \\ 0 & 0 & 0 & -\frac{\sqrt{2}\left(t_{1}-2t_{3}\right)}{3\left(1+2k^2\right) t_{1} t_{3}} & \frac{t_{1}+4t_{3}}{3\left(1+2k^2\right)^2 t_{1} t_{3}} & \frac{i\sqrt{2}\,k\left(t_{1}+4t_{3}\right)}{3\left(1+2k^2\right)^2 t_{1} t_{3}} & 0 \\ 0 & 0 & 0 & \frac{2ik t_{1}-4ik t_{3}}{3 t_{1} t_{3}+6k^2 t_{1} t_{3}} & -\frac{i\sqrt{2}\,k\left(t_{1}+4t_{3}\right)}{3\left(1+2k^2\right)^2 t_{1} t_{3}} & \frac{2k^2\left(t_{1}+4t_{3}\right)}{3\left(1+2k^2\right)^2 t_{1} t_{3}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Matrix for spin-2 sector:
```

$$\begin{pmatrix} \frac{2}{\left(1+2k^2\right)^2 t_{1}} & -\frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} & 0 \\ \frac{2i\sqrt{2}\,k}{\left(1+2k^2\right)^2 t_{1}} & \frac{4k^2}{\left(1+2k^2\right)^2 t_{1}} & 0 \\ 0 & 0 & \frac{2}{2k^2 r_{1}+t_{1}} \end{pmatrix}$$

```
Square masses:
```

$$\left\{\{\},\{\},\{\},\{\},\{\},\left\{-\frac{t_{1}}{2 r_{1}}\right\}\right\}$$

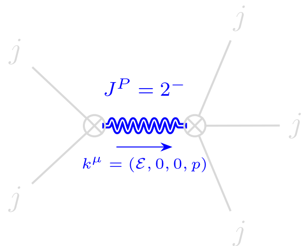
```
Massive pole residues:
```

$$\left\{\{\},\{\},\{\},\{\},\{\},\left\{-\frac{1}{r_{1}}\right\}\right\}$$

```
Massless eigenvalues:
```

{}

```
Overall particle spectrum:
```




```
Massive particle
```

Pole residue:	$-\frac{1}{r_{1}} > 0$
Square mass:	$-\frac{t_{1}}{2 r_{1}} > 0$
Spin:	2
Parity:	Odd

```
Overall unitarity conditions:
```

$r_{1} < 0 \,\&\&\, t_{1} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\dot{1}} < 0 \,\&\&\, t_{\dot{1}} > 0 \tag{169}$$

Okay, that concludes the analysis of this theory.

Case 51

Now for a new theory. Here is the full nonlinear Lagrangian for Case 51 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{\dot{1}}\, \mathcal{R}_{ij\hbar l}\, \mathcal{R}^{ij\hbar l} + \frac{2}{3} r_{\dot{1}}\, \mathcal{R}_{i\hbar jl}\, \mathcal{R}^{ij\hbar l} - 2 r_{\dot{1}}\, \mathcal{R}^{ij\hbar}{}_i\, \mathcal{R}_j{}^l{}_{\hbar l} - \frac{2}{3} r_{\dot{1}}\, \mathcal{R}^{ij\hbar l}\, \mathcal{R}_{\hbar lij} + 2 r_{\dot{1}}\, \mathcal{R}^{ij\hbar}{}_i\, \mathcal{R}_\hbar{}^l{}_{jl} + \frac{1}{12}\left(4 t_{\dot{1}} + t_{\dot{2}}\right) \mathcal{T}_{ij\hbar}\, \mathcal{T}^{ij\hbar} + \frac{1}{6}\left(2 t_{\dot{1}} - t_{\dot{2}}\right) \mathcal{T}^{ij\hbar}\, \mathcal{T}_{ji\hbar} + t_{\dot{1}}\, \mathcal{T}^i{}_i{}^j\, \mathcal{T}^\hbar{}_{j\hbar} \tag{170}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\frac{1}{3}\left(t_{\dot{1}} + t_{\dot{2}}\right) \mathcal{A}_{abi}\, \mathcal{A}^{abi} + \frac{1}{3}\left(t_{\dot{1}} - 2 t_{\dot{2}}\right) \mathcal{A}_{aib}\, \mathcal{A}^{abi} + t_{\dot{1}}\, \mathcal{A}^{ab}{}_a\, \mathcal{A}_b{}^i{}_i - 2 t_{\dot{1}}\, \mathcal{A}_b{}^i{}_i\, \partial_a f^{ab} + \\
&2 t_{\dot{1}}\, \mathcal{A}_b{}^i{}_i\, \partial^b f^a{}_a - t_{\dot{1}}\, \partial_b f^i{}_i\, \partial^b f^a{}_a - t_{\dot{1}}\, \partial_a f^{ab}\, \partial_i f^i{}_b + 2 t_{\dot{1}}\, \partial^b f^a{}_a\, \partial_i f^i{}_b - 2 r_{\dot{1}}\, \partial_b \mathcal{A}_i{}^j{}_j\, \partial^i \mathcal{A}^{ab}{}_a + \\
&2 r_{\dot{1}}\, \partial_i \mathcal{A}_b{}^j{}_j\, \partial^i \mathcal{A}^{ab}{}_a - \frac{2}{3}\left(t_{\dot{1}} + t_{\dot{2}}\right) \mathcal{A}_{abi}\, \partial^i f^{ab} + \frac{2}{3}\left(t_{\dot{1}} + t_{\dot{2}}\right) \mathcal{A}_{aib}\, \partial^i f^{ab} + \frac{2}{3}\left(2 t_{\dot{1}} - t_{\dot{2}}\right) \mathcal{A}_{bia}\, \partial^i f^{ab} + \\
&\frac{1}{3}\left(-2 t_{\dot{1}} + t_{\dot{2}}\right) \partial_a f_{bi}\, \partial^i f^{ab} + \frac{1}{6}\left(2 t_{\dot{1}} - t_{\dot{2}}\right) \partial_a f_{ib}\, \partial^i f^{ab} + \frac{1}{6}\left(-4 t_{\dot{1}} - t_{\dot{2}}\right) \partial_b f_{ai}\, \partial^i f^{ab} + \\
&\frac{1}{6}\left(4 t_{\dot{1}} + t_{\dot{2}}\right) \partial_i f_{ab}\, \partial^i f^{ab} + \frac{1}{6}\left(2 t_{\dot{1}} - t_{\dot{2}}\right) \partial_i f_{ba}\, \partial^i f^{ab} + 2 r_{\dot{1}}\, \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_b{}^j{}_i - 4 r_{\dot{1}}\, \partial^i \mathcal{A}^{ab}{}_a\, \partial_j \mathcal{A}_b{}^j{}_i - \\
&2 r_{\dot{1}}\, \partial_a \mathcal{A}^{abi}\, \partial_j \mathcal{A}_i{}^j{}_b + 4 r_{\dot{1}}\, \partial^i \mathcal{A}^{ab}{}_a\, \partial_j \mathcal{A}_i{}^j{}_b - \frac{4}{3} r_{\dot{1}}\, \partial_b \mathcal{A}_{aij}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_{\dot{1}}\, \partial_b \mathcal{A}_{aji}\, \partial^j \mathcal{A}^{abi} - \\
&\frac{8}{3} r_{\dot{1}}\, \partial_b \mathcal{A}_{ija}\, \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_{\dot{1}}\, \partial_i \mathcal{A}_{abj}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_{\dot{1}}\, \partial_j \mathcal{A}_{abi}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_{\dot{1}}\, \partial_j \mathcal{A}_{aib}\, \partial^j \mathcal{A}^{abi}
\end{aligned} \tag{171}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} -t_{\dot{1}} & i\sqrt{2}\, k\, t_{\dot{1}} & 0 & 0 \\ -i\sqrt{2}\, k\, t_{\dot{1}} & -2 k^2 t_{\dot{1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & t_{\dot{2}} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} \frac{1}{6}\left(t_{1}+4\,t_{2}\right) & -\frac{t_{1}-2\,t_{2}}{3\sqrt{2}} & \frac{i\,k\left(t_{1}-2\,t_{2}\right)}{3\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{1}-2\,t_{2}}{3\sqrt{2}} & \frac{t_{1}+t_{2}}{3} & -\frac{1}{3}\,i\,k\left(t_{1}+t_{2}\right) & 0 & 0 & 0 & 0 \\ -\frac{i\,k\left(t_{1}-2\,t_{2}\right)}{3\sqrt{2}} & \frac{1}{3}\,i\,k\left(t_{1}+t_{2}\right) & \frac{1}{3}\,k^2\left(t_{1}+t_{2}\right) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k^2\,r_{1}-\frac{t_{1}}{2} & \frac{t_{1}}{\sqrt{2}} & i\,k\,t_{1} & 0 \\ 0 & 0 & 0 & \frac{t_{1}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\,k\,t_{1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{2} & -\frac{i\,k\,t_{1}}{\sqrt{2}} & 0 \\ \frac{i\,k\,t_{1}}{\sqrt{2}} & k^2\,t_{1} & 0 \\ 0 & 0 & k^2\,r_{1}+\frac{t_{1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}\tau^{\perp} == 0$$

$$-2\,i\,k\,{}^{0^+}\sigma^{\parallel}+{}^{0^+}\tau^{\parallel} == 0$$

$${}^{1^-}\tau^{\perp}{}^{a} == 0$$

$$2\,i\,k\,{}^{1^-}\sigma^{\perp}{}^{a}+{}^{1^-}\tau^{\parallel}{}^{a} == 0$$

$$-i\,k\,{}^{1^+}\sigma^{\perp}{}^{ab}+{}^{1^+}\tau^{\parallel}{}^{ab} == 0$$

$$-2\,i\,k\,{}^{2^+}\sigma^{\parallel}{}^{ab}+{}^{2^+}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore–Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{\left(1+2\,k^2\right)^2 t_{1}} & \frac{i\sqrt{2}\,k}{\left(1+2\,k^2\right)^2 t_{1}} & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{\left(1+2\,k^2\right)^2 t_{1}} & -\frac{2\,k^2}{\left(1+2\,k^2\right)^2 t_{1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{t_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix}
\frac{2(t_1+t_2)}{3t_1 t_2} & \frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1 t_2} & -\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1 t_2} & 0 & 0 & 0 & 0 \\
\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1 t_2} & \frac{t_1+4t_2}{3(1+k^2)^2 t_1 t_2} & -\frac{ik(t_1+4t_2)}{3(1+k^2)^2 t_1 t_2} & 0 & 0 & 0 & 0 \\
\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1 t_2} & \frac{ik(t_1+4t_2)}{3(1+k^2)^2 t_1 t_2} & \frac{k^2(t_1+4t_2)}{3(1+k^2)^2 t_1 t_2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t_1+2k^2 t_1} & \frac{2ik}{t_1+2k^2 t_1} & 0 \\
0 & 0 & 0 & \frac{\sqrt{2}}{t_1+2k^2 t_1} & \frac{2k^2 r_1+t_1}{(t_1+2k^2 t_1)^2} & \frac{i\sqrt{2}k(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2} & 0 \\
0 & 0 & 0 & -\frac{2ik}{t_1+2k^2 t_1} & -\frac{i\sqrt{2}k(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2} & \frac{2k^2(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix}
\frac{2}{(1+2k^2)^2 t_1} & -\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1} & 0 \\
\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1} & \frac{4k^2}{(1+2k^2)^2 t_1} & 0 \\
0 & 0 & \frac{2}{2k^2 r_1+t_1}
\end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_1}{2r_1}\right\}\right\}$$

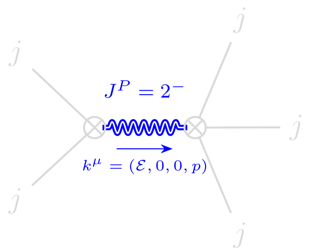
Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_1}\right\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_1} > 0$
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r_1 < 0$ && $t_1 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\underset{1}{r} < 0 \,\&\&\, \underset{1}{t} > 0 \tag{172}$$

Okay, that concludes the analysis of this theory.

Case 52

Now for a new theory. Here is the full nonlinear Lagrangian for Case 52 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}\underset{1}{r}\,\mathcal{R}_{ij\hbar l}\,\mathcal{R}^{ij\hbar l}+\frac{2}{3}\underset{1}{r}\,\mathcal{R}_{i\hbar jl}\,\mathcal{R}^{ij\hbar l}+\underset{5}{r}\,\mathcal{R}^{ij}{}_{i}{}^{\hbar}\,\mathcal{R}_{j}{}^{l}{}_{\hbar l}-\frac{2}{3}\underset{1}{r}\,\mathcal{R}^{ij\hbar l}\,\mathcal{R}_{\hbar lij}- \underset{5}{r}\,\mathcal{R}^{ij}{}_{i}{}^{\hbar}\,\mathcal{R}_{\hbar}{}^{l}{}_{jl}+\frac{1}{4}\underset{1}{t}\,\mathcal{T}_{ij\hbar}\,\mathcal{T}^{ij\hbar}+\frac{1}{2}\underset{1}{t}\,\mathcal{T}^{ij\hbar}\,\mathcal{T}_{ji\hbar}+\underset{1}{t}\,\mathcal{T}^{i}{}_{i}{}^{j}\,\mathcal{T}^{\hbar}{}_{j\hbar} \tag{173}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\underset{1}{t}\,\mathcal{A}_{aib}\,\mathcal{A}^{abi}+\underset{1}{t}\,\mathcal{A}^{ab}{}_{a}\,\mathcal{A}_{b}{}^{i}{}_{i}-2\underset{1}{t}\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial_{a}f^{ab}+2\underset{1}{t}\,\mathcal{A}_{b}{}^{i}{}_{i}\,\partial^{b}f^{a}{}_{a}-\\&\underset{1}{t}\,\partial_{b}f^{i}{}_{i}\,\partial^{b}f^{a}{}_{a}-\underset{1}{t}\,\partial_{a}f^{ab}\,\partial_{i}f^{i}{}_{b}+2\underset{1}{t}\,\partial^{b}f^{a}{}_{a}\,\partial_{i}f^{i}{}_{b}+\underset{5}{r}\,\partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}-\underset{5}{r}\,\partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\,\partial^{i}\mathcal{A}^{ab}{}_{a}+\\&2\underset{1}{t}\,\mathcal{A}_{bia}\,\partial^{i}f^{ab}-\underset{1}{t}\,\partial_{a}f_{bi}\,\partial^{i}f^{ab}+\frac{1}{2}\underset{1}{t}\,\partial_{a}f_{ib}\,\partial^{i}f^{ab}-\frac{1}{2}\underset{1}{t}\,\partial_{b}f_{ai}\,\partial^{i}f^{ab}+\\&\frac{1}{2}\underset{1}{t}\,\partial_{i}f_{ab}\,\partial^{i}f^{ab}+\frac{1}{2}\underset{1}{t}\,\partial_{i}f_{ba}\,\partial^{i}f^{ab}-\underset{5}{r}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}+2\underset{5}{r}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}+\\&\underset{5}{r}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}-2\underset{5}{r}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}-\frac{4}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{abi}-\\&\frac{8}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{abi}-\frac{2}{3}\underset{1}{r}\,\partial_{i}\mathcal{A}_{abj}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{abi}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{aib}\,\partial^{j}\mathcal{A}^{abi}\end{aligned} \tag{174}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} -\underset{1}{t} & i\sqrt{2}\,k\,\underset{1}{t} & 0 & 0 \\ -i\sqrt{2}\,k\,\underset{1}{t} & -2k^{2}\,\underset{1}{t} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\underset{1}{t} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} k^{2}\left(2\underset{1}{r}+\underset{5}{r}\right)-\frac{\underset{1}{t}}{2} & -\frac{\underset{1}{t}}{\sqrt{2}} & \frac{i k \underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{\underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k \underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^{2}\left(\underset{1}{r}+\underset{5}{r}\right)-\frac{\underset{1}{t}}{2} & \frac{\underset{1}{t}}{\sqrt{2}} & i\,k\,\underset{1}{t} & 0 \\ 0 & 0 & 0 & \frac{\underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\,k\,\underset{1}{t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{2} & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & k^2 t_{1} & 0 \\ 0 & 0 & k^2 r_{1} + \frac{t_{1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^+}_{\cdot}\tau^{\perp} == 0$$

$$-2 i k\, {}^{0^+}_{\cdot}\sigma^{\parallel} + {}^{0^+}_{\cdot}\tau^{\parallel} == 0$$

$${}^{1^-}_{\cdot}\tau^{\perp}{}^{a} == 0$$

$$2 i k\, {}^{1^-}_{\cdot}\sigma^{\perp}{}^{a} + {}^{1^-}_{\cdot}\tau^{\parallel}{}^{a} == 0$$

$$-i k\, {}^{1^+}_{\cdot}\sigma^{\perp}{}^{ab} + {}^{1^+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

$$-2 i k\, {}^{2^+}_{\cdot}\sigma^{\parallel}{}^{ab} + {}^{2^+}_{\cdot}\tau^{\parallel}{}^{ab} == 0$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{(1+2k^2)^2 t_{1}} & \frac{i\sqrt{2}k}{(1+2k^2)^2 t_{1}} & 0 & 0 \\ -\frac{i\sqrt{2}k}{(1+2k^2)^2 t_{1}} & -\frac{2k^2}{(1+2k^2)^2 t_{1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{i\sqrt{2}k}{t_{1}+k^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2k^2(2r_{1}+r_{5})+t_{1}}{(1+k^2)^2 t_{1}^2} & \frac{i(2k^3(2r_{1}+r_{5})-k t_{1})}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}k}{t_{1}+k^2 t_{1}} & \frac{-2ik^3(2r_{1}+r_{5})+ikt_{1}}{(1+k^2)^2 t_{1}^2} & \frac{-2k^4(2r_{1}+r_{5})+k^2 t_{1}}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & \frac{2ik}{t_{1}+2k^2 t_{1}} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & \frac{-2k^2(r_{1}+r_{5})+t_{1}}{(t_{1}+2k^2 t_{1})^2} & -\frac{i\sqrt{2}k(2k^2(r_{1}+r_{5})-t_{1})}{(t_{1}+2k^2 t_{1})^2} & 0 \\ 0 & 0 & 0 & -\frac{2ik}{t_{1}+2k^2 t_{1}} & \frac{i\sqrt{2}k(2k^2(r_{1}+r_{5})-t_{1})}{(t_{1}+2k^2 t_{1})^2} & \frac{-4k^4(r_{1}+r_{5})+2k^2 t_{1}}{(t_{1}+2k^2 t_{1})^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_{1}} & -\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_{1}} & 0 \\ \frac{2i\sqrt{2}k}{(1+2k^2)^2 t_{1}} & \frac{4k^2}{(1+2k^2)^2 t_{1}} & 0 \\ 0 & 0 & \frac{2}{2k^2 r_{1}+t_{1}} \end{pmatrix}$$

Square masses:

$$\left\{\{\},\{\},\{\},\{\},\{\},\left\{-\frac{t._{1}}{2\,r._{1}}\right\}\right\}$$

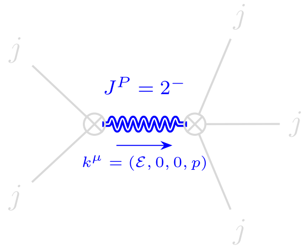
Massive pole residues:

$$\left\{\{\},\{\},\{\},\{\},\{\},\left\{-\frac{1}{r._{1}}\right\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r._{1}} > 0$
Square mass:	$-\frac{t._{1}}{2\,r._{1}} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r._{1} < 0 \;\&\&\; t._{1} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r. < 0 \;\&\&\; t. > 0 \tag{175}$$

Okay, that concludes the analysis of this theory.

Case 53

Now for a new theory. Here is the full nonlinear Lagrangian for Case 53 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r.\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} + \frac{2}{3} r.\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} - r.\, \mathcal{R}^{ij\;k}_{\;\;i}\, \mathcal{R}_{j\;kl}^{\;l} - \frac{2}{3} r.\, \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + r.\, \mathcal{R}^{ij\;k}_{\;\;i}\, \mathcal{R}_{k\;jl}^{\;l} + \frac{1}{4} t.\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} + \frac{1}{2} t.\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik} + t.\, \mathcal{T}^{i\;j}_{\;i}\, \mathcal{T}^{k}_{\;jk} \tag{176}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t.\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} + t.\, \mathcal{A}^{ab}_{\;\;a}\, \mathcal{A}_{b\;i}^{\;i} - 2 t.\, \mathcal{A}_{b\;i}^{\;i}\, \partial_a f^{ab} + 2 t.\, \mathcal{A}_{b\;i}^{\;i}\, \partial^b f^a_{\;a} - t.\, \partial_b f^i_{\;i}\, \partial^b f^a_{\;a} - t.\, \partial_a f^{ab}\, \partial_i f^i_{\;b} + 2 t.\, \partial^b f^a_{\;a}\, \partial_i f^i_{\;b} - r.\, \partial_b \mathcal{A}_{i\;j}^{\;j}\, \partial^i \mathcal{A}^{ab}_{\;\;a} + r.\, \partial_i \mathcal{A}_{b\;j}^{\;j}\, \partial^i \mathcal{A}^{ab}_{\;\;a} +$$

$$2\,t.\;\mathcal{A}_{bia}\;\partial^{i}f^{ab}-t.\;\partial_{a}f_{bi}\;\partial^{i}f^{ab}+\frac{1}{2}\,t.\;\partial_{a}f_{ib}\;\partial^{i}f^{ab}-\frac{1}{2}\,t.\;\partial_{b}f_{ai}\;\partial^{i}f^{ab}+$$
$$\frac{1}{2}\,t.\;\partial_{i}f_{ab}\;\partial^{i}f^{ab}+\frac{1}{2}\,t.\;\partial_{i}f_{ba}\;\partial^{i}f^{ab}+r.\;\partial_{a}\mathcal{A}^{abi}\;\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}-2\,r.\;\partial^{i}\mathcal{A}^{ab}{}_{a}\;\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}-$$
$$r.\;\partial_{a}\mathcal{A}^{abi}\;\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}+2\,r.\;\partial^{i}\mathcal{A}^{ab}{}_{a}\;\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}-\frac{4}{3}\,r.\;\partial_{b}\mathcal{A}_{aij}\;\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\,r.\;\partial_{b}\mathcal{A}_{aji}\;\partial^{j}\mathcal{A}^{abi}-$$
$$\frac{8}{3}\,r.\;\partial_{b}\mathcal{A}_{ija}\;\partial^{j}\mathcal{A}^{abi}-\frac{2}{3}\,r.\;\partial_{i}\mathcal{A}_{abj}\;\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\,r.\;\partial_{j}\mathcal{A}_{abi}\;\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\,r.\;\partial_{j}\mathcal{A}_{aib}\;\partial^{j}\mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -t_{\dot{1}} & i\sqrt{2}\,k\,t_{\dot{1}} & 0 & 0 \\ -i\sqrt{2}\,k\,t_{\dot{1}} & -2k^{2}\,t_{\dot{1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_{\dot{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^{2}\,r_{\dot{1}}-\frac{t_{\dot{1}}}{2} & -\frac{t_{\dot{1}}}{\sqrt{2}} & \frac{i\,k\,t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i\,k\,t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{t_{\dot{1}}}{2} & \frac{t_{\dot{1}}}{\sqrt{2}} & i\,k\,t_{\dot{1}} & 0 \\ 0 & 0 & 0 & \frac{t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\,k\,t_{\dot{1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{\dot{1}}}{2} & -\frac{i\,k\,t_{\dot{1}}}{\sqrt{2}} & 0 \\ \frac{i\,k\,t_{\dot{1}}}{\sqrt{2}} & k^{2}\,t_{\dot{1}} & 0 \\ 0 & 0 & k^{2}\,r_{\dot{1}}+\frac{t_{\dot{1}}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$${}^{0^{+}}_{\cdot}\tau^{\perp} == 0$$

$$-2\,i\,k\,{}^{0^{+}}_{\cdot}\sigma^{\parallel}+{}^{0^{+}}_{\cdot}\tau^{\parallel} == 0$$

$${}^{1^{-}}_{\cdot}\tau^{\perp\,a} == 0$$

$$2\,i\,k\,{}^{1^{-}}_{\cdot}\sigma^{\perp\,a}+{}^{1^{-}}_{\cdot}\tau^{\parallel\,a} == 0$$

$$-i\,k\,{}^{1^{+}}_{\cdot}\sigma^{\perp\,ab}+{}^{1^{+}}_{\cdot}\tau^{\parallel\,ab} == 0$$

$$-2\,i\,k\,{}^{2^{+}}_{\cdot}\sigma^{\parallel\,ab}+{}^{2^{+}}_{\cdot}\tau^{\parallel\,ab} == 0$$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{(1+2k^2)^2 \underset{1}{t}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 \underset{1}{t}} & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 \underset{1}{t}} & -\frac{2k^2}{(1+2k^2)^2 \underset{1}{t}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\underset{1}{t}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{\underset{1}{t}+k^2 \underset{1}{t}} & \frac{i\sqrt{2}\,k}{\underset{1}{t}+k^2 \underset{1}{t}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{\underset{1}{t}+k^2 \underset{1}{t}} & \frac{-2k^2 \underset{1}{r}+\underset{1}{t}}{(1+k^2)^2 \underset{1}{t}^2} & \frac{i\left(2k^3 \underset{1}{r}-k\underset{1}{t}\right)}{(1+k^2)^2 \underset{1}{t}^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{\underset{1}{t}+k^2 \underset{1}{t}} & -\frac{i\left(2k^3 \underset{1}{r}-k\underset{1}{t}\right)}{(1+k^2)^2 \underset{1}{t}^2} & \frac{-2k^4 \underset{1}{r}+k^2 \underset{1}{t}}{(1+k^2)^2 \underset{1}{t}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{\underset{1}{t}+2k^2 \underset{1}{t}} & \frac{2ik}{\underset{1}{t}+2k^2 \underset{1}{t}} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{\underset{1}{t}+2k^2 \underset{1}{t}} & \frac{1}{(1+2k^2)^2 \underset{1}{t}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 \underset{1}{t}} & 0 \\ 0 & 0 & 0 & -\frac{2ik}{\underset{1}{t}+2k^2 \underset{1}{t}} & -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 \underset{1}{t}} & \frac{2k^2}{(1+2k^2)^2 \underset{1}{t}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 \underset{1}{t}} & -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 \underset{1}{t}} & 0 \\ \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 \underset{1}{t}} & \frac{4k^2}{(1+2k^2)^2 \underset{1}{t}} & 0 \\ 0 & 0 & \frac{2}{2k^2 \underset{1}{r}+\underset{1}{t}} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{\underset{1}{t}}{2\underset{1}{r}}\right\}\right\}$$

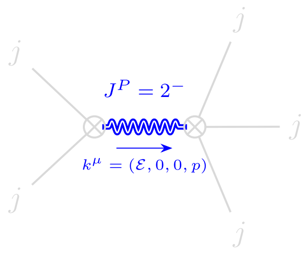
Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{\underset{1}{r}}\right\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{r\mathrm{P}_{1}} > 0$
Square mass:	$-\frac{t\mathrm{P}_{1}}{\mathrm{T}\,r\mathrm{P}_{1}} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r_{1} < 0 \,\&\&\, t_{1} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{1} < 0 \,\&\&\, t_{1} > 0 \tag{178}$$

Okay, that concludes the analysis of this theory.

Case 54

Now for a new theory. Here is the full nonlinear Lagrangian for Case 54 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{3} r_{1} \mathcal{R}_{ijkl} \mathcal{R}^{ijkl} + \frac{2}{3} r_{1} \mathcal{R}_{ikjl} \mathcal{R}^{ijkl} - 2 r_{1} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{j}{}^{l}{}_{kl} - \frac{2}{3} r_{1} \mathcal{R}^{ijkl} \mathcal{R}_{klij} + \\ &2 r_{1} \mathcal{R}^{ij}{}_{i}{}^{k} \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4} t_{1} \mathcal{T}_{ijk} \mathcal{T}^{ijk} + \frac{1}{2} t_{1} \mathcal{T}^{ijk} \mathcal{T}_{jik} + t_{1} \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{k}{}_{jk}\end{aligned} \tag{179}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&t_{1} \mathcal{A}_{aib} \mathcal{A}^{abi} + t_{1} \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} - 2 t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial_{a} f^{ab} + 2 t_{1} \mathcal{A}_{b}{}^{i}{}_{i} \partial^{b} f^{a}{}_{a} - t_{1} \partial_{b} f^{i}{}_{i} \partial^{b} f^{a}{}_{a} - \\ &t_{1} \partial_{a} f^{ab} \partial_{i} f^{i}{}_{b} + 2 t_{1} \partial^{b} f^{a}{}_{a} \partial_{i} f^{i}{}_{b} - 2 r_{1} \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + 2 r_{1} \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j} \partial^{i} \mathcal{A}^{ab}{}_{a} + \\ &2 t_{1} \mathcal{A}_{bia} \partial^{i} f^{ab} - t_{1} \partial_{a} f_{bi} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{a} f_{ib} \partial^{i} f^{ab} - \frac{1}{2} t_{1} \partial_{b} f_{ai} \partial^{i} f^{ab} + \\ &\frac{1}{2} t_{1} \partial_{i} f_{ab} \partial^{i} f^{ab} + \frac{1}{2} t_{1} \partial_{i} f_{ba} \partial^{i} f^{ab} + 2 r_{1} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} - 4 r_{1} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} - \\ &2 r_{1} \partial_{a} \mathcal{A}^{abi} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + 4 r_{1} \partial^{i} \mathcal{A}^{ab}{}_{a} \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - \frac{4}{3} r_{1} \partial_{b} \mathcal{A}_{aij} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{b} \mathcal{A}_{aji} \partial^{j} \mathcal{A}^{abi} - \\ &\frac{8}{3} r_{1} \partial_{b} \mathcal{A}_{ija} \partial^{j} \mathcal{A}^{abi} - \frac{2}{3} r_{1} \partial_{i} \mathcal{A}_{abj} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{abi} \partial^{j} \mathcal{A}^{abi} + \frac{2}{3} r_{1} \partial_{j} \mathcal{A}_{aib} \partial^{j} \mathcal{A}^{abi}\end{aligned} \tag{180}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -t_{\dot{1}} & i\sqrt{2}\,k\,t_{\dot{1}} & 0 & 0 \\ -i\sqrt{2}\,k\,t_{\dot{1}} & -2k^2 t_{\dot{1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_{\dot{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} -\frac{t_{\dot{1}}}{2} & -\frac{t_{\dot{1}}}{\sqrt{2}} & \frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k^2 r_{\dot{1}} - \frac{t_{\dot{1}}}{2} & \frac{t_{\dot{1}}}{\sqrt{2}} & i k t_{\dot{1}} & 0 \\ 0 & 0 & 0 & \frac{t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i k t_{\dot{1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{\dot{1}}}{2} & -\frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 \\ \frac{i k t_{\dot{1}}}{\sqrt{2}} & k^2 t_{\dot{1}} & 0 \\ 0 & 0 & k^2 r_{\dot{1}} + \frac{t_{\dot{1}}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{0^+}{\cdot}\tau^{\perp} == 0$

$-2 i k\, \overset{0^+}{\cdot}\sigma^{\parallel} + \overset{0^+}{\cdot}\tau^{\parallel} == 0$

$\overset{1^-}{\cdot}\tau^{\perp}{}^{a} == 0$

$2 i k\, \overset{1^-}{\cdot}\sigma^{\perp}{}^{a} + \overset{1^-}{\cdot}\tau^{\parallel}{}^{a} == 0$

$-i k\, \overset{1^+}{\cdot}\sigma^{\perp}{}^{ab} + \overset{1^+}{\cdot}\tau^{\parallel}{}^{ab} == 0$

$-2 i k\, \overset{2^+}{\cdot}\sigma^{\parallel}{}^{ab} + \overset{2^+}{\cdot}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{(1+2k^2)^2 t_{\dot{1}}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot{1}}} & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{\dot{1}}} & -\frac{2k^2}{(1+2k^2)^2 t_{\dot{1}}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_{\dot{1}}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{i\sqrt{2}k}{t_{1}+k^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{1}{(1+k^2)^2 t_{1}} & -\frac{i k}{(1+k^2)^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}k}{t_{1}+k^2 t_{1}} & \frac{i k}{(1+k^2)^2 t_{1}} & \frac{k^2}{(1+k^2)^2 t_{1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & \frac{2 i k}{t_{1}+2k^2 t_{1}} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & \frac{2k^2 r_{1}+t_{1}}{(t_{1}+2k^2 t_{1})^2} & \frac{i\sqrt{2}k(2k^2 r_{1}+t_{1})}{(t_{1}+2k^2 t_{1})^2} & 0 \\ 0 & 0 & 0 & -\frac{2 i k}{t_{1}+2k^2 t_{1}} & -\frac{i\sqrt{2}k(2k^2 r_{1}+t_{1})}{(t_{1}+2k^2 t_{1})^2} & \frac{2k^2(2k^2 r_{1}+t_{1})}{(t_{1}+2k^2 t_{1})^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_{1}} & -\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_{1}} & 0 \\ \frac{2i\sqrt{2}k}{(1+2k^2)^2 t_{1}} & \frac{4k^2}{(1+2k^2)^2 t_{1}} & 0 \\ 0 & 0 & \frac{2}{2k^2 r_{1}+t_{1}} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{1}}{2 r_{1}}\right\}\right\}$$

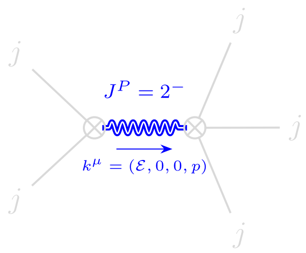
Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{1}}\right\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{1}} > 0$
Square mass:	$-\frac{t_{1}}{2 r_{1}} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r_{1} < 0$ && $t_{1} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$\underset{1}{r} < 0 \,\&\&\, \underset{1}{t} > 0 \tag{181}$$

Okay, that concludes the analysis of this theory.

Case 55

Now for a new theory. Here is the full nonlinear Lagrangian for Case 55 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3}\underset{1}{r}\, \mathcal{R}_{ijkl}\, \mathcal{R}^{ijkl} + \frac{2}{3}\underset{1}{r}\, \mathcal{R}_{ikjl}\, \mathcal{R}^{ijkl} - \underset{1}{r}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{j}{}^{l}{}_{kl} - \frac{2}{3}\underset{1}{r}\, \mathcal{R}^{ijkl}\, \mathcal{R}_{klij} + \underset{1}{r}\, \mathcal{R}^{ij}{}_{i}{}^{k}\, \mathcal{R}_{k}{}^{l}{}_{jl} + \frac{1}{4}\underset{1}{t}\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} + \frac{1}{2}\underset{1}{t}\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik} + \frac{1}{3}\underset{1}{t}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{k}{}_{jk} \tag{182}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}
&\underset{1}{t}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} + \frac{1}{3}\underset{1}{t}\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3}\underset{1}{t}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a}f^{ab} + \frac{2}{3}\underset{1}{t}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b}f^{a}{}_{a} - \\
&\frac{1}{3}\underset{1}{t}\, \partial_{b}f^{i}{}_{i}\, \partial^{b}f^{a}{}_{a} - \frac{1}{3}\underset{1}{t}\, \partial_{a}f^{ab}\, \partial_{i}f^{i}{}_{b} + \frac{2}{3}\underset{1}{t}\, \partial^{b}f^{a}{}_{a}\, \partial_{i}f^{i}{}_{b} - \underset{1}{r}\, \partial_{b}\mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{ab}{}_{a} + \\
&\underset{1}{r}\, \partial_{i}\mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i}\mathcal{A}^{ab}{}_{a} + 2\underset{1}{t}\, \mathcal{A}_{bia}\, \partial^{i}f^{ab} - \underset{1}{t}\, \partial_{a}f_{bi}\, \partial^{i}f^{ab} + \frac{1}{2}\underset{1}{t}\, \partial_{a}f_{ib}\, \partial^{i}f^{ab} - \frac{1}{2}\underset{1}{t}\, \partial_{b}f_{ai}\, \partial^{i}f^{ab} + \\
&\frac{1}{2}\underset{1}{t}\, \partial_{i}f_{ab}\, \partial^{i}f^{ab} + \frac{1}{2}\underset{1}{t}\, \partial_{i}f_{ba}\, \partial^{i}f^{ab} + \underset{1}{r}\, \partial_{a}\mathcal{A}^{abi}\, \partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} - 2\underset{1}{r}\, \partial^{i}\mathcal{A}^{ab}{}_{a}\, \partial_{j}\mathcal{A}_{b}{}^{j}{}_{i} - \\
&\underset{1}{r}\, \partial_{a}\mathcal{A}^{abi}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} + 2\underset{1}{r}\, \partial^{i}\mathcal{A}^{ab}{}_{a}\, \partial_{j}\mathcal{A}_{i}{}^{j}{}_{b} - \frac{4}{3}\underset{1}{r}\, \partial_{b}\mathcal{A}_{aij}\, \partial^{j}\mathcal{A}^{abi} + \frac{2}{3}\underset{1}{r}\, \partial_{b}\mathcal{A}_{aji}\, \partial^{j}\mathcal{A}^{abi} - \\
&\frac{8}{3}\underset{1}{r}\, \partial_{b}\mathcal{A}_{ija}\, \partial^{j}\mathcal{A}^{abi} - \frac{2}{3}\underset{1}{r}\, \partial_{i}\mathcal{A}_{abj}\, \partial^{j}\mathcal{A}^{abi} + \frac{2}{3}\underset{1}{r}\, \partial_{j}\mathcal{A}_{abi}\, \partial^{j}\mathcal{A}^{abi} + \frac{2}{3}\underset{1}{r}\, \partial_{j}\mathcal{A}_{aib}\, \partial^{j}\mathcal{A}^{abi}
\end{aligned} \tag{183}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

```
The (possibly singular) a-matrices associated
  with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:
```

```
Matrix for spin-0 sector:
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\underset{1}{t} \end{pmatrix}$$

```
Matrix for spin-1 sector:
```

$$\begin{pmatrix} k^2 \underset{1}{r} - \frac{\underset{1}{t}}{2} & -\frac{\underset{1}{t}}{\sqrt{2}} & \frac{i k \underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{\underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k \underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\underset{1}{t}}{6} & \frac{\underset{1}{t}}{3\sqrt{2}} & \frac{i k \underset{1}{t}}{3} & 0 \\ 0 & 0 & 0 & \frac{\underset{1}{t}}{3\sqrt{2}} & \frac{\underset{1}{t}}{3} & \frac{1}{3} i \sqrt{2}\, k \underset{1}{t} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} i k \underset{1}{t} & -\frac{1}{3} i \sqrt{2}\, k \underset{1}{t} & \frac{2 k^2 \underset{1}{t}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{1}}{2} & -\frac{i k t_{1}}{\sqrt{2}} & 0 \\ \frac{i k t_{1}}{\sqrt{2}} & k^2 t_{1} & 0 \\ 0 & 0 & k^2 r_{1} + \frac{t_{1}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{0^+}{\cdot}\sigma^{\parallel} == 0$

$\overset{0^+}{\cdot}\tau^{\parallel} == 0$

$\overset{0^+}{\cdot}\tau^{\perp} == 0$

$\overset{1^-}{\cdot}\tau^{\perp\, a} == 0$

$2\, i\, k\, \overset{1^-}{\cdot}\sigma^{\parallel\, a} + \overset{1^-}{\cdot}\tau^{\parallel\, a} == 0$

$\overset{1^-}{\cdot}\sigma^{\parallel\, a} == \overset{1^-}{\cdot}\sigma^{\perp\, a}$

$-i\, k\, \overset{1^+}{\cdot}\sigma^{\perp\, ab} + \overset{1^+}{\cdot}\tau^{\parallel\, ab} == 0$

$-2\, i\, k\, \overset{2^+}{\cdot}\sigma^{\parallel\, ab} + \overset{2^+}{\cdot}\tau^{\parallel\, ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2k^2 r_{1}+t_{1}}{(1+k^2)^2 t_{1}^2} & \frac{i\left(2k^3 r_{1}-k t_{1}\right)}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{t_{1}+k^2 t_{1}} & -\frac{i\left(2k^3 r_{1}-k t_{1}\right)}{(1+k^2)^2 t_{1}^2} & \frac{-2k^4 r_{1}+k^2 t_{1}}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{(3+4k^2)^2 t_{1}} & \frac{6\sqrt{2}}{(3+4k^2)^2 t_{1}} & \frac{12 i k}{(3+4k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 & \frac{6\sqrt{2}}{(3+4k^2)^2 t_{1}} & \frac{12}{(3+4k^2)^2 t_{1}} & \frac{12 i\sqrt{2}\,k}{(3+4k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 & -\frac{12 i k}{(3+4k^2)^2 t_{1}} & -\frac{12 i\sqrt{2}\,k}{(3+4k^2)^2 t_{1}} & \frac{24 k^2}{(3+4k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_{1}} & -\frac{2 i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & 0 \\ \frac{2 i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & \frac{4k^2}{(1+2k^2)^2 t_{1}} & 0 \\ 0 & 0 & \frac{2}{2k^2 r_{1}+t_{1}} \end{pmatrix}$$

Square masses:

$$\left\{\{\},\{\},\{\},\{\},\{\},\left\{-\frac{t_{\dot{1}}}{2r_{\dot{1}}}\right\}\right\}$$

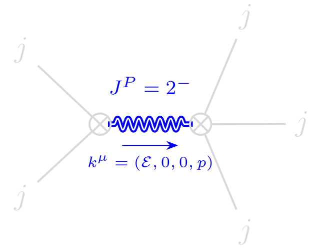
Massive pole residues:

$$\left\{\{\},\{\},\{\},\{\},\{\},\left\{-\frac{1}{r_{\dot{1}}}\right\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{rP_{1}} > 0$
Square mass:	$-\frac{tP_{1}}{T\,rP_{1}} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r_{\dot{1}} < 0 \,\&\&\, t_{\dot{1}} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{\dot{1}} < 0 \,\&\&\, t_{\dot{1}} > 0 \tag{184}$$

Okay, that concludes the analysis of this theory.

Case 56

Now for a new theory. Here is the full nonlinear Lagrangian for Case 56 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_{\dot{1}} \mathcal{R}_{ijhl}\mathcal{R}^{ijhl} + \frac{2}{3} r_{\dot{1}} \mathcal{R}_{ihjl}\mathcal{R}^{ijhl} - 2 r_{\dot{1}} \mathcal{R}^{ij}{}_{i}{}^{h}\mathcal{R}_{j}{}^{l}{}_{hl} - \frac{2}{3} r_{\dot{1}} \mathcal{R}^{ijhl}\mathcal{R}_{hlij} + 2 r_{\dot{1}} \mathcal{R}^{ij}{}_{i}{}^{h}\mathcal{R}_{h}{}^{l}{}_{jl} + \frac{1}{3} t_{\dot{1}} \mathcal{T}_{ijh}\mathcal{T}^{ijh} + \frac{1}{3} t_{\dot{1}} \mathcal{T}^{ijh}\mathcal{T}_{jih} + t_{\dot{1}} \mathcal{T}^{i}{}_{i}{}^{j}\mathcal{T}^{h}{}_{jh} \tag{185}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\frac{1}{3} t_{\dot{1}} \mathcal{A}_{abi}\mathcal{A}^{abi} + \frac{1}{3} t_{\dot{1}} \mathcal{A}_{aib}\mathcal{A}^{abi} + t_{\dot{1}} \mathcal{A}^{ab}{}_{a}\mathcal{A}_{b}{}^{i}{}_{i} - 2 t_{\dot{1}} \mathcal{A}_{b}{}^{i}{}_{i}\partial_a f^{ab} + 2 t_{\dot{1}} \mathcal{A}_{b}{}^{i}{}_{i}\partial^b f^{a}{}_{a} - t_{\dot{1}} \partial_b f^{i}{}_{i}\partial^b f^{a}{}_{a} - t_{\dot{1}} \partial_a f^{ab}\partial_i f^{i}{}_{b} + 2 t_{\dot{1}} \partial^b f^{a}{}_{a}\partial_i f^{i}{}_{b} - 2 r_{\dot{1}} \partial_b \mathcal{A}_{i}{}^{j}{}_{j}\partial^i \mathcal{A}^{ab}{}_{a} + 2 r_{\dot{1}} \partial_i \mathcal{A}_{b}{}^{j}{}_{j}\partial^i \mathcal{A}^{ab}{}_{a} -$$

$$\frac{2}{3}\underset{1}{t}\,\mathcal{A}_{abi}\,\partial^{i}f^{ab}+\frac{2}{3}\underset{1}{t}\,\mathcal{A}_{aib}\,\partial^{i}f^{ab}+\frac{4}{3}\underset{1}{t}\,\mathcal{A}_{bia}\,\partial^{i}f^{ab}-\frac{2}{3}\underset{1}{t}\,\partial_{a}f_{bi}\,\partial^{i}f^{ab}+\frac{1}{3}\underset{1}{t}\,\partial_{a}f_{ib}\,\partial^{i}f^{ab}-$$
$$\frac{2}{3}\underset{1}{t}\,\partial_{b}f_{ai}\,\partial^{i}f^{ab}+\frac{2}{3}\underset{1}{t}\,\partial_{i}f_{ab}\,\partial^{i}f^{ab}+\frac{1}{3}\underset{1}{t}\,\partial_{i}f_{ba}\,\partial^{i}f^{ab}+2\underset{1}{r}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}-4\underset{1}{r}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{b}{}^{j}{}_{i}-$$
$$2\underset{1}{r}\,\partial_{a}\mathcal{A}^{abi}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}+4\underset{1}{r}\,\partial^{i}\mathcal{A}^{ab}{}_{a}\,\partial_{j}\mathcal{A}_{i}{}^{j}{}_{b}-\frac{4}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{aij}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{aji}\,\partial^{j}\mathcal{A}^{abi}-$$
$$\frac{8}{3}\underset{1}{r}\,\partial_{b}\mathcal{A}_{ija}\,\partial^{j}\mathcal{A}^{abi}-\frac{2}{3}\underset{1}{r}\,\partial_{i}\mathcal{A}_{abj}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{abi}\,\partial^{j}\mathcal{A}^{abi}+\frac{2}{3}\underset{1}{r}\,\partial_{j}\mathcal{A}_{aib}\,\partial^{j}\mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\underset{1}{t} & i\sqrt{2}\,k\,\underset{1}{t} & 0 & 0 \\ -i\sqrt{2}\,k\,\underset{1}{t} & -2k^{2}\underset{1}{t} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{\underset{1}{t}}{6} & -\frac{\underset{1}{t}}{3\sqrt{2}} & \frac{i k \underset{1}{t}}{3\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{\underset{1}{t}}{3\sqrt{2}} & \frac{\underset{1}{t}}{3} & -\frac{1}{3} i k \underset{1}{t} & 0 & 0 & 0 & 0 \\ -\frac{i k \underset{1}{t}}{3\sqrt{2}} & \frac{i k \underset{1}{t}}{3} & \frac{k^{2}\underset{1}{t}}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k^{2}\underset{1}{r}-\frac{\underset{1}{t}}{2} & \frac{\underset{1}{t}}{\sqrt{2}} & i k \underset{1}{t} & 0 \\ 0 & 0 & 0 & \frac{\underset{1}{t}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i k \underset{1}{t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{\underset{1}{t}}{2} & -\frac{i k \underset{1}{t}}{\sqrt{2}} & 0 \\ \frac{i k \underset{1}{t}}{\sqrt{2}} & k^{2}\underset{1}{t} & 0 \\ 0 & 0 & k^{2}\underset{1}{r}+\frac{\underset{1}{t}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^-}\sigma^{\parallel} == 0$

${}^{0^+}\tau^{\perp} == 0$

$-2 i k\, {}^{0^+}\sigma^{\parallel} + {}^{0^+}\tau^{\parallel} == 0$

${}^{1^-}\tau^{\perp}{}^{a} == 0$

$2 i k\, {}^{1^-}\sigma^{\perp}{}^{a} + {}^{1^-}\tau^{\parallel}{}^{a} == 0$

$2 i k\, {}^{1^+}\sigma^{\parallel}{}^{ab} + {}^{1^+}\tau^{\parallel}{}^{ab} == 0$

$2\, {}^{1^+}\sigma^{\parallel}{}^{ab} + {}^{1^+}\sigma^{\perp}{}^{ab} == 0$

$-2 i k\, {}^{2^+}\sigma^{\parallel}{}^{ab} + {}^{2^+}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore–Penrose) inverses of these a–matrices, which are functionally analogous to the inverse b–matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{(1+2k^2)^2 t_1} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & -\frac{2k^2}{(1+2k^2)^2 t_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{6}{(3+2k^2)^2 t_1} & -\frac{6\sqrt{2}}{(3+2k^2)^2 t_1} & \frac{6i\sqrt{2}\,k}{(3+2k^2)^2 t_1} & 0 & 0 & 0 & 0 \\ -\frac{6\sqrt{2}}{(3+2k^2)^2 t_1} & \frac{12}{(3+2k^2)^2 t_1} & -\frac{12ik}{(3+2k^2)^2 t_1} & 0 & 0 & 0 & 0 \\ -\frac{6i\sqrt{2}\,k}{(3+2k^2)^2 t_1} & \frac{12ik}{(3+2k^2)^2 t_1} & \frac{12k^2}{(3+2k^2)^2 t_1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t_1+2k^2 t_1} & \frac{2ik}{t_1+2k^2 t_1} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{t_1+2k^2 t_1} & \frac{2k^2 r_1+t_1}{(t_1+2k^2 t_1)^2} & \frac{i\sqrt{2}\,k(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2} & 0 \\ 0 & 0 & 0 & -\frac{2ik}{t_1+2k^2 t_1} & -\frac{i\sqrt{2}\,k(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2} & \frac{2k^2(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_1} & -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & 0 \\ \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1} & \frac{4k^2}{(1+2k^2)^2 t_1} & 0 \\ 0 & 0 & \frac{2}{2k^2 r_1+t_1} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_1}{2 r_1}\right\}\right\}$$

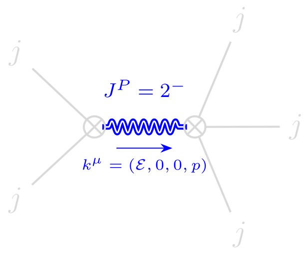
Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_1}\right\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{r_{1}^{P}} > 0$
Square mass:	$-\frac{t_{1}^{P}}{T\, r_{1}^{P}} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r_{1} < 0 \;\&\&\; t_{1} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{1} < 0 \;\&\&\; t_{1} > 0 \tag{187}$$

Okay, that concludes the analysis of this theory.

Case 57

Now for a new theory. Here is the full nonlinear Lagrangian for Case 57 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\begin{aligned}&\frac{1}{6}\left(2 r_{1} + r_{2}\right) \mathcal{R}_{ij\hbar l}\, \mathcal{R}^{ij\hbar l} + \frac{2}{3}\left(r_{1} - r_{2}\right) \mathcal{R}_{i\hbar jl}\, \mathcal{R}^{ij\hbar l} - \\ &2 r_{1}\, \mathcal{R}^{ij}{}_{i}{}^{\hbar}\, \mathcal{R}_{j}{}^{l}{}_{\hbar l} + \frac{1}{6}\left(-4 r_{1} + r_{2}\right) \mathcal{R}^{ij\hbar l}\, \mathcal{R}_{\hbar lij} + 2 r_{1}\, \mathcal{R}^{ij}{}_{i}{}^{\hbar}\, \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \\ &\frac{1}{3} t_{1}\, \mathcal{T}_{ij\hbar}\, \mathcal{T}^{ij\hbar} + \frac{1}{3} t_{1}\, \mathcal{T}^{ij\hbar}\, \mathcal{T}_{ji\hbar} + t_{1}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{\hbar}{}_{j\hbar}\end{aligned} \tag{188}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\begin{aligned}&\frac{1}{3} t_{1}\, \mathcal{A}_{abi}\, \mathcal{A}^{abi} + \frac{1}{3} t_{1}\, \mathcal{A}_{aib}\, \mathcal{A}^{abi} + t_{1}\, \mathcal{A}^{ab}{}_{a}\, \mathcal{A}_{b}{}^{i}{}_{i} - 2 t_{1}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial_{a} f^{ab} + \\ &2 t_{1}\, \mathcal{A}_{b}{}^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - t_{1}\, \partial_{b} f^{i}{}_{i}\, \partial^{b} f^{a}{}_{a} - t_{1}\, \partial_{a} f^{ab}\, \partial_{i} f^{i}{}_{b} + 2 t_{1}\, \partial^{b} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{b} - 2 r_{1}\, \partial_{b} \mathcal{A}_{i}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} + \\ &2 r_{1}\, \partial_{i} \mathcal{A}_{b}{}^{j}{}_{j}\, \partial^{i} \mathcal{A}^{ab}{}_{a} - \frac{2}{3} t_{1}\, \mathcal{A}_{abi}\, \partial^{i} f^{ab} + \frac{2}{3} t_{1}\, \mathcal{A}_{aib}\, \partial^{i} f^{ab} + \frac{4}{3} t_{1}\, \mathcal{A}_{bia}\, \partial^{i} f^{ab} - \\ &\frac{2}{3} t_{1}\, \partial_{a} f_{bi}\, \partial^{i} f^{ab} + \frac{1}{3} t_{1}\, \partial_{a} f_{ib}\, \partial^{i} f^{ab} - \frac{2}{3} t_{1}\, \partial_{b} f_{ai}\, \partial^{i} f^{ab} + \frac{2}{3} t_{1}\, \partial_{i} f_{ab}\, \partial^{i} f^{ab} + \frac{1}{3} t_{1}\, \partial_{i} f_{ba}\, \partial^{i} f^{ab} + \\ &2 r_{1}\, \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} - 4 r_{1}\, \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{b}{}^{j}{}_{i} - 2 r_{1}\, \partial_{a} \mathcal{A}^{abi}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} + 4 r_{1}\, \partial^{i} \mathcal{A}^{ab}{}_{a}\, \partial_{j} \mathcal{A}_{i}{}^{j}{}_{b} - \\ &\frac{4}{3}\left(r_{1} - r_{2}\right) \partial_{b} \mathcal{A}_{aij}\, \partial^{j} \mathcal{A}^{abi} + \frac{2}{3}\left(r_{1} - r_{2}\right) \partial_{b} \mathcal{A}_{aji}\, \partial^{j} \mathcal{A}^{abi} + \frac{2}{3}\left(-4 r_{1} + r_{2}\right) \partial_{b} \mathcal{A}_{ija}\, \partial^{j} \mathcal{A}^{abi} +\end{aligned} \tag{189}$$

$$\frac{1}{3}\left(-2\, r_{\underset{1}{\bullet}} - r_{\underset{2}{\bullet}}\right) \partial_i \mathcal{A}_{abj}\, \partial^j \mathcal{A}^{abi} + \frac{1}{3}\left(2\, r_{\underset{1}{\bullet}} + r_{\underset{2}{\bullet}}\right) \partial_j \mathcal{A}_{abi}\, \partial^j \mathcal{A}^{abi} + \frac{2}{3}\left(r_{\underset{1}{\bullet}} - r_{\underset{2}{\bullet}}\right) \partial_j \mathcal{A}_{aib}\, \partial^j \mathcal{A}^{abi}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated
with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -t_{\underset{1}{\bullet}} & i\sqrt{2}\, k\, t_{\underset{1}{\bullet}} & 0 & 0 \\ -i\sqrt{2}\, k\, t_{\underset{1}{\bullet}} & -2 k^2 t_{\underset{1}{\bullet}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k^2 r_{\underset{2}{\bullet}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{t_{\underset{1}{\bullet}}}{6} & -\frac{t_{\underset{1}{\bullet}}}{3\sqrt{2}} & \frac{i k t_{\underset{1}{\bullet}}}{3\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{\underset{1}{\bullet}}}{3\sqrt{2}} & \frac{t_{\underset{1}{\bullet}}}{3} & -\frac{1}{3} i k t_{\underset{1}{\bullet}} & 0 & 0 & 0 & 0 \\ -\frac{i k t_{\underset{1}{\bullet}}}{3\sqrt{2}} & \frac{i k t_{\underset{1}{\bullet}}}{3} & \frac{k^2 t_{\underset{1}{\bullet}}}{3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k^2 r_{\underset{1}{\bullet}} - \frac{t_{\underset{1}{\bullet}}}{2} & \frac{t_{\underset{1}{\bullet}}}{\sqrt{2}} & i k t_{\underset{1}{\bullet}} & 0 \\ 0 & 0 & 0 & \frac{t_{\underset{1}{\bullet}}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -i k t_{\underset{1}{\bullet}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{\underset{1}{\bullet}}}{2} & -\frac{i k t_{\underset{1}{\bullet}}}{\sqrt{2}} & 0 \\ \frac{i k t_{\underset{1}{\bullet}}}{\sqrt{2}} & k^2 t_{\underset{1}{\bullet}} & 0 \\ 0 & 0 & k^2 r_{\underset{1}{\bullet}} + \frac{t_{\underset{1}{\bullet}}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

${}^{0^+}_{\bullet}\tau^{\perp} == 0$

$-2 i k\, {}^{0^+}_{\bullet}\sigma^{\parallel} + {}^{0^+}_{\bullet}\tau^{\parallel} == 0$

${}^{1^-}_{\bullet}\tau^{\perp\, a} == 0$

$2 i k\, {}^{1^-}_{\bullet}\sigma^{\perp\, a} + {}^{1^-}_{\bullet}\tau^{\parallel\, a} == 0$

$2 i k\, {}^{1^+}_{\bullet}\sigma^{\parallel\, ab} + {}^{1^+}_{\bullet}\tau^{\parallel\, ab} == 0$

$2\, {}^{1^+}_{\bullet}\sigma^{\parallel\, ab} + {}^{1^+}_{\bullet}\sigma^{\perp\, ab} == 0$

$-2 i k\, {}^{2^+}_{\bullet}\sigma^{\parallel\, ab} + {}^{2^+}_{\bullet}\tau^{\parallel\, ab} == 0$

The Drazin (Moore–Penrose) inverses of these a-matrices, which are functionally
analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} -\frac{1}{(1+2k^2)^2 t_{1}} & \frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & 0 & 0 \\ -\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & -\frac{2k^2}{(1+2k^2)^2 t_{1}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k^2 r_{2}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} \frac{6}{(3+2k^2)^2 t_{1}} & -\frac{6\sqrt{2}}{(3+2k^2)^2 t_{1}} & \frac{6i\sqrt{2}\,k}{(3+2k^2)^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{6\sqrt{2}}{(3+2k^2)^2 t_{1}} & \frac{12}{(3+2k^2)^2 t_{1}} & -\frac{12ik}{(3+2k^2)^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{6i\sqrt{2}\,k}{(3+2k^2)^2 t_{1}} & \frac{12ik}{(3+2k^2)^2 t_{1}} & \frac{12k^2}{(3+2k^2)^2 t_{1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & \frac{2ik}{t_{1}+2k^2 t_{1}} & 0 \\ 0 & 0 & 0 & \frac{\sqrt{2}}{t_{1}+2k^2 t_{1}} & \frac{2k^2 r_{1}+t_{1}}{(t_{1}+2k^2 t_{1})^2} & \frac{i\sqrt{2}\,k(2k^2 r_{1}+t_{1})}{(t_{1}+2k^2 t_{1})^2} & 0 \\ 0 & 0 & 0 & -\frac{2ik}{t_{1}+2k^2 t_{1}} & -\frac{i\sqrt{2}\,k(2k^2 r_{1}+t_{1})}{(t_{1}+2k^2 t_{1})^2} & \frac{2k^2(2k^2 r_{1}+t_{1})}{(t_{1}+2k^2 t_{1})^2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2k^2)^2 t_{1}} & -\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & 0 \\ \frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_{1}} & \frac{4k^2}{(1+2k^2)^2 t_{1}} & 0 \\ 0 & 0 & \frac{2}{2k^2 r_{1}+t_{1}} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{1}}{2 r_{1}}\right\}\right\}$$

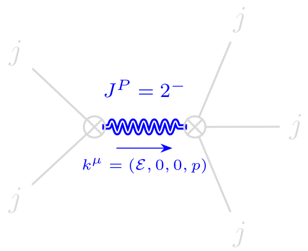
Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{1}}\right\}\right\}$$

Massless eigenvalues:

$\{\}$

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{S}{r^P_1} > 0$
Square mass:	$-\frac{t^P_1}{T\, r^P_1} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r_1 < 0 \,\&\&\, t_1 > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_1 < 0 \,\&\&\, t_1 > 0 \tag{190}$$

Okay, that concludes the analysis of this theory.

Case 58

Now for a new theory. Here is the full nonlinear Lagrangian for Case 58 as defined by the second column of TABLE V. in arXiv:1910.14197:

$$\frac{1}{3} r_1 \mathcal{R}_{ij\hbar l} \mathcal{R}^{ij\hbar l} + \frac{2}{3} r_1 \mathcal{R}_{i\hbar jl} \mathcal{R}^{ij\hbar l} - r_1 \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{j}{}^{l}{}_{\hbar l} + \frac{1}{3} \left(r_1 - 3 r_3\right) \mathcal{R}^{ij\hbar l} \mathcal{R}_{\hbar lij} + \left(-3 r_1 + 4 r_3\right) \mathcal{R}^{ij}{}_{i}{}^{\hbar} \mathcal{R}_{\hbar}{}^{l}{}_{jl} + \frac{1}{4} t_1 \mathcal{T}_{ij\hbar} \mathcal{T}^{ij\hbar} + \frac{1}{2} t_1 \mathcal{T}^{ij\hbar} \mathcal{T}_{ji\hbar} + \frac{1}{3} t_1 \mathcal{T}^{i}{}_{i}{}^{j} \mathcal{T}^{\hbar}{}_{j\hbar} \tag{191}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$t_1 \mathcal{A}_{aib} \mathcal{A}^{abi} + \frac{1}{3} t_1 \mathcal{A}^{ab}{}_{a} \mathcal{A}_{b}{}^{i}{}_{i} - \frac{2}{3} t_1 \mathcal{A}_{b}{}^{i}{}_{i} \partial_a f^{ab} + \frac{2}{3} t_1 \mathcal{A}_{b}{}^{i}{}_{i} \partial^b f^{a}{}_{a} - \frac{1}{3} t_1 \partial_b f^{i}{}_{i} \partial^b f^{a}{}_{a} - \frac{1}{3} t_1 \partial_a f^{ab} \partial_i f^{i}{}_{b} + \frac{2}{3} t_1 \partial^b f^{a}{}_{a} \partial_i f^{i}{}_{b} + \left(3 r_1 - 4 r_3\right) \partial_b \mathcal{A}_{i}{}^{j}{}_{j} \partial^i \mathcal{A}^{ab}{}_{a} + r_1 \partial_i \mathcal{A}_{b}{}^{j}{}_{j} \partial^i \mathcal{A}^{ab}{}_{a} + 2 t_1 \mathcal{A}_{bia} \partial^i f^{ab} - t_1 \partial_a f_{bi} \partial^i f^{ab} + \frac{1}{2} t_1 \partial_a f_{ib} \partial^i f^{ab} - \frac{1}{2} t_1 \partial_b f_{ai} \partial^i f^{ab} + \frac{1}{2} t_1 \partial_i f_{ab} \partial^i f^{ab} + \frac{1}{2} t_1 \partial_i f_{ba} \partial^i f^{ab} + r_1 \partial_a \mathcal{A}^{abi} \partial_j \mathcal{A}_{b}{}^{j}{}_{i} - 2 r_1 \partial^i \mathcal{A}^{ab}{}_{a} \partial_j \mathcal{A}_{b}{}^{j}{}_{i} + \left(3 r_1 - 4 r_3\right) \partial_a \mathcal{A}^{abi} \partial_j \mathcal{A}_{i}{}^{j}{}_{b} + \left(-6 r_1 + 8 r_3\right) \partial^i \mathcal{A}^{ab}{}_{a} \partial_j \mathcal{A}_{i}{}^{j}{}_{b} - \frac{4}{3} r_1 \partial_b \mathcal{A}_{aij} \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_1 \partial_b \mathcal{A}_{aji} \partial^j \mathcal{A}^{abi} + \frac{4}{3} \left(r_1 - 3 r_3\right) \partial_b \mathcal{A}_{ija} \partial^j \mathcal{A}^{abi} - \frac{2}{3} r_1 \partial_i \mathcal{A}_{abj} \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_1 \partial_j \mathcal{A}_{abi} \partial^j \mathcal{A}^{abi} + \frac{2}{3} r_1 \partial_j \mathcal{A}_{aib} \partial^j \mathcal{A}^{abi} \tag{192}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} 6k^2\left(-r_{\dot{1}}+r_{\dot{3}}\right) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_{\dot{1}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} k^2 r_{\dot{1}} - \frac{t_{\dot{1}}}{2} & -\frac{t_{\dot{1}}}{\sqrt{2}} & \frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{t_{\dot{1}}}{6} & \frac{t_{\dot{1}}}{3\sqrt{2}} & \frac{i k t_{\dot{1}}}{3} & 0 \\ 0 & 0 & 0 & \frac{t_{\dot{1}}}{3\sqrt{2}} & \frac{t_{\dot{1}}}{3} & \frac{1}{3} i \sqrt{2}\, k t_{\dot{1}} & 0 \\ 0 & 0 & 0 & -\frac{1}{3} i k t_{\dot{1}} & -\frac{1}{3} i \sqrt{2}\, k t_{\dot{1}} & \frac{2k^2 t_{\dot{1}}}{3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{t_{\dot{1}}}{2} & -\frac{i k t_{\dot{1}}}{\sqrt{2}} & 0 \\ \frac{i k t_{\dot{1}}}{\sqrt{2}} & k^2 t_{\dot{1}} & 0 \\ 0 & 0 & k^2 r_{\dot{1}} + \frac{t_{\dot{1}}}{2} \end{pmatrix}$$

Gauge constraints on source currents:

$\overset{0^+}{\cdot}\tau^{\perp} == 0$

$\overset{0^+}{\cdot}\tau^{\parallel} == 0$

$\overset{1^-}{\cdot}\tau^{\perp}{}^{a} == 0$

$2 i k \overset{1^-}{\cdot}\sigma^{\parallel}{}^{a} + \overset{1^-}{\cdot}\tau^{\parallel}{}^{a} == 0$

$\overset{1^-}{\cdot}\sigma^{\parallel}{}^{a} == \overset{1^-}{\cdot}\sigma^{\perp}{}^{a}$

$-i k \overset{1^+}{\cdot}\sigma^{\perp}{}^{ab} + \overset{1^+}{\cdot}\tau^{\parallel}{}^{ab} == 0$

$-2 i k \overset{2^+}{\cdot}\sigma^{\parallel}{}^{ab} + \overset{2^+}{\cdot}\tau^{\parallel}{}^{ab} == 0$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

Matrix for spin-0 sector:

$$\begin{pmatrix} \frac{1}{6k^2\left(-r_{\dot{1}}+r_{\dot{3}}\right)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{t_{\dot{1}}} \end{pmatrix}$$

Matrix for spin-1 sector:

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{i \sqrt{2} k}{t_{1}+k^2 t_{1}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_{1}+k^2 t_{1}} & \frac{-2 k^2 r_{1}+t_{1}}{(1+k^2)^2 t_{1}^2} & \frac{i (2 k^3 r_{1}-k t_{1})}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ -\frac{i \sqrt{2} k}{t_{1}+k^2 t_{1}} & -\frac{i (2 k^3 r_{1}-k t_{1})}{(1+k^2)^2 t_{1}^2} & \frac{-2 k^4 r_{1}+k^2 t_{1}}{(1+k^2)^2 t_{1}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{6}{(3+4 k^2)^2 t_{1}} & \frac{6 \sqrt{2}}{(3+4 k^2)^2 t_{1}} & \frac{12 i k}{(3+4 k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 & \frac{6 \sqrt{2}}{(3+4 k^2)^2 t_{1}} & \frac{12}{(3+4 k^2)^2 t_{1}} & \frac{12 i \sqrt{2} k}{(3+4 k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 & -\frac{12 i k}{(3+4 k^2)^2 t_{1}} & -\frac{12 i \sqrt{2} k}{(3+4 k^2)^2 t_{1}} & \frac{24 k^2}{(3+4 k^2)^2 t_{1}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Matrix for spin-2 sector:

$$\begin{pmatrix} \frac{2}{(1+2 k^2)^2 t_{1}} & -\frac{2 i \sqrt{2} k}{(1+2 k^2)^2 t_{1}} & 0 \\ \frac{2 i \sqrt{2} k}{(1+2 k^2)^2 t_{1}} & \frac{4 k^2}{(1+2 k^2)^2 t_{1}} & 0 \\ 0 & 0 & \frac{2}{2 k^2 r_{1}+t_{1}} \end{pmatrix}$$

Square masses:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{t_{1}}{2 r_{1}}\right\}\right\}$$

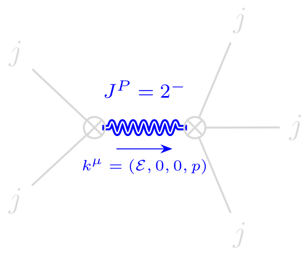
Massive pole residues:

$$\left\{\{\}, \{\}, \{\}, \{\}, \{\}, \left\{-\frac{1}{r_{1}}\right\}\right\}$$

Massless eigenvalues:

{}

Overall particle spectrum:



Massive particle

Pole residue:	$-\frac{1}{r_{1}} > 0$
Square mass:	$-\frac{t_{1}}{2 r_{1}} > 0$
Spin:	2
Parity:	Odd

Overall unitarity conditions:

$r_{1} < 0 \&\& t_{1} > 0$

So, that's the end of the PSALTer output for this theory. You can check the particle content against TABLE IV. in arXiv:1910.14197. If you take the overall unitarity conditions from the final column in TABLE V., and decompose

them using Mathematica's Reduce function, you get the following (to be compared with the PSALTer conditions above):

$$r_{1} < 0 \&\& t_{1} > 0 \tag{193}$$

Okay, that concludes the analysis of this theory.

How long did this take?

Okay, that's all the cases. You can see from the timing below (in seconds) that each theory takes about a minute to process:

```
{{78.9861, Null}, {87.5718, Null}, {86.8373, Null}, {85.4683, Null}, {88.8962, Null},
 {84.4542, Null}, {87.1309, Null}, {76.0278, Null}, {71.432, Null}, {67.6685, Null},
 {70.7438, Null}, {79.3202, Null}, {71.2068, Null}, {76.6342, Null}, {86.7894, Null},
 {82.237, Null}, {71.4899, Null}, {81.4018, Null}, {83.496, Null}, {83.5964, Null},
 {80.0211, Null}, {72.7033, Null}, {69.3987, Null}, {79.4875, Null}, {85.8399, Null},
 {80.6192, Null}, {77.0416, Null}, {82.869, Null}, {79.2548, Null}, {74.8961, Null},
 {76.7133, Null}, {73.9799, Null}, {72.4471, Null}, {84.7398, Null}, {83.926, Null},
 {80.6374, Null}, {86.6551, Null}, {79.0109, Null}, {75.5521, Null}, {79.0092, Null},
 {79.1425, Null}, {79.9517, Null}, {81.8146, Null}, {105.782, Null},
 {106.597, Null}, {91.9829, Null}, {79.8907, Null}, {80.1098, Null},
 {76.4209, Null}, {91.534, Null}, {86.7758, Null}, {77.9385, Null}, {76.4671, Null},
 {76.006, Null}, {90.7328, Null}, {78.7596, Null}, {80.8779, Null}, {91.4459, Null}}
```
(194)

Key observation: We have now reached the end of the PSALTer calibration script.