# Particle spectrograph

## Wave operator and propagator

## Massive and massless spectra

### Source constraints

| SO(3) irreps | Fundamental fields | Multiplicities |
|---|---|---|
| $\sigma_{0^+}^{\#1} == 0$ | $\partial_\beta \sigma^{\alpha\beta}{}_\alpha == 0$ | 1 |
| $\tau_{0^+}^{\#1} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha{}_\alpha$ | 1 |
| $\tau_{0^+}^{\#2} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} == 0$ | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta}$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$ | 3 |
| $\sigma_{1^-}^{\#2\,\alpha} == 0$ | $\partial_\chi \partial_\beta \sigma^{\alpha\beta\chi} == 0$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$ | $\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\chi\alpha} + \partial_\chi \partial^\chi \tau^{\alpha\beta} + 2\,\partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 2\,\partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} + \partial_\chi \partial^\chi \tau^{\beta\alpha} + 2\,\partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$ | 3 |
| $\sigma_{2^-}^{\#1\,\alpha\beta\chi} == 0$ | $3\,\partial_\epsilon \partial_\delta \partial^\chi \partial^\alpha \sigma^{\beta\delta\epsilon} + 3\,\partial_\epsilon \partial^\epsilon \partial^\chi \partial^\alpha \sigma^{\beta\delta}{}_\delta + 2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\chi\delta} + 4\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\chi\delta\alpha} + 2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\alpha\beta\delta} + 2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\alpha\delta\beta} + 2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\beta\chi\alpha} + 3\,\eta^{\beta\chi}\,\partial_\phi \partial^\phi \partial_\epsilon \partial^\alpha \sigma^{\delta\epsilon}{}_\delta + 3\,\eta^{\alpha\chi}\,\partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\beta\delta\epsilon} + 3\,\eta^{\beta\chi}\,\partial_\phi \partial^\phi \partial_\epsilon \partial^\epsilon \sigma^{\alpha\delta}{}_\delta == 3\,\partial_\epsilon \partial_\delta \partial^\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 3\,\partial_\epsilon \partial^\epsilon \partial^\chi \partial^\beta \sigma^{\alpha\delta}{}_\delta + 2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\chi\delta} + 4\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\delta\chi} + 2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\beta\delta\alpha} + 4\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\chi\beta} + 3\,\eta^{\alpha\chi}\,\partial_\phi \partial^\phi \partial_\epsilon \partial^\beta \sigma^{\delta\epsilon}{}_\delta + 3\,\eta^{\beta\chi}\,\partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\alpha\delta\epsilon} + 3\,\eta^{\alpha\chi}\,\partial_\phi \partial^\phi \partial_\epsilon \partial^\epsilon \sigma^{\beta\delta}{}_\delta$ | 5 |
| $\tau_{2^+}^{\#1\,\alpha\beta} == 0$ | $4\,\partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2\,\partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi{}_\chi + 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} == 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\beta} + 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\chi\alpha} + 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \tau^\chi{}_\chi$ | 5 |
| Total constraints/gauge generators: | | 25 |

### Quadratic (free) action

$$\mathcal{S} == \iiiint \left(\frac{1}{6}\,(6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 3\,r_3\,\partial_\beta \omega_\iota{}^\theta{}_\theta\,\partial^\prime \omega^{\alpha\beta}{}_\alpha - 3\,r_3\,\partial_\iota \omega_\beta{}^\theta{}_\theta\,\partial^\prime \omega^{\alpha\beta}{}_\alpha - 3\,r_3\,\partial_\alpha \omega^{\alpha\beta\prime}\,\partial_\theta \omega_\beta{}^\theta{}_\iota + 6\,r_3\,\partial^\prime \omega^{\alpha\beta}{}_\alpha\,\partial_\theta \omega_\beta{}^\theta{}_\iota - 3\,r_3\,\partial_\alpha \omega^{\alpha\beta\prime}\,\partial_\theta \omega_\iota{}^\theta{}_\beta + 6\,r_3\,\partial^\prime \omega^{\alpha\beta}{}_\alpha\,\partial_\theta \omega_\iota{}^\theta{}_\beta + 4\,t_2\,\omega_{\iota\theta\alpha}\,\partial^\theta f^{\alpha\prime} + 2\,t_2\,\partial_\alpha f_{\iota\theta}\,\partial^\theta f^{\alpha\prime} - t_2\,\partial_\alpha f_{\theta\iota}\,\partial^\theta f^{\alpha\prime} - t_2\,\partial_\iota f_{\alpha\theta}\,\partial^\theta f^{\alpha\prime} + t_2\,\partial_\theta f_{\alpha\iota}\,\partial^\theta f^{\alpha\prime} - t_2\,\partial_\theta f_{\iota\alpha}\,\partial^\theta f^{\alpha\prime} + 4\,t_2\,\omega_{\alpha\theta\iota}\,(\omega^{\alpha\iota\theta} + \partial^\theta f^{\alpha\iota}) + 2\,t_2\,\omega_{\alpha\iota\theta}\,(\omega^{\alpha\iota\theta} + 2\,\partial^\theta f^{\alpha\iota}) - 24\,r_3\,\partial_\beta \omega_{\iota\theta\alpha}\,\partial^\theta \omega^{\alpha\beta\iota} + 6\,r_5\,\partial_\iota \omega_\theta{}^\kappa{}_\kappa\,\partial^\theta \omega^{\alpha\iota}{}_\alpha - 6\,r_5\,\partial_\theta \omega_\iota{}^\kappa{}_\kappa\,\partial^\theta \omega^{\alpha\iota}{}_\alpha - 6\,r_5\,\partial_\alpha \omega^{\alpha\iota\theta}\,\partial_\kappa \omega_\iota{}^\kappa{}_\theta + 12\,r_5\,\partial^\theta \omega^{\alpha\iota}{}_\alpha\,\partial_\kappa \omega_\iota{}^\kappa{}_\theta + 6\,r_5\,\partial_\alpha \omega^{\alpha\iota\theta}\,\partial_\kappa \omega_\theta{}^\kappa{}_\iota - 12\,r_5\,\partial^\theta \omega^{\alpha\iota}{}_\alpha\,\partial_\kappa \omega_\theta{}^\kappa{}_\iota)) [t, x, y, z]\,dt\,dx\,dy\,dz$$

| | $\sigma_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^+}^{\#2}{}_{\alpha\beta}$ | $\tau_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^-}^{\#1}{}_{\alpha}$ | $\sigma_{1^-}^{\#2}{}_{\alpha}$ | $\tau_{1^-}^{\#1}{}_{\alpha}$ | $\tau_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{1}{k^2\,(2\,r_3+r_5)}$ | $-\frac{\sqrt{2}}{k^2\,(1+k^2)\,(2\,r_3+r_5)}$ | $-\frac{i\,\sqrt{2}}{k\,(1+k^2)\,(2\,r_3+r_5)}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2}\dagger^{\alpha\beta}$ | $-\frac{\sqrt{2}}{k^2\,(1+k^2)\,(2\,r_3+r_5)}$ | $\frac{3\,k^2\,(2\,r_3+r_5)+2\,t_2}{(k+k^3)^2\,(2\,r_3+r_5)\,t_2}$ | $\frac{i\,(3\,k^2\,(2\,r_3+r_5)+2\,t_2)}{k\,(1+k^2)^2\,(2\,r_3+r_5)\,t_2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{i\,\sqrt{2}}{k\,(1+k^2)\,(2\,r_3+r_5)}$ | $-\frac{i\,(3\,k^2\,(2\,r_3+r_5)+2\,t_2)}{k\,(1+k^2)^2\,(2\,r_3+r_5)\,t_2}$ | $\frac{3\,k^2\,(2\,r_3+r_5)+2\,t_2}{(1+k^2)^2\,(2\,r_3+r_5)\,t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{2}{k^2\,(r_3+2\,r_5)}$ | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | $\omega_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^+}^{\#2}{}_{\alpha\beta}$ | $f_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^-}^{\#1}{}_{\alpha}$ | $\omega_{1^-}^{\#2}{}_{\alpha}$ | $f_{1^-}^{\#1}{}_{\alpha}$ | $f_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $k^2\,(2\,r_3+r_5)+\frac{2\,t_2}{3}$ | $\frac{\sqrt{2}\,t_2}{3}$ | $\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2}\dagger^{\alpha\beta}$ | $\frac{\sqrt{2}\,t_2}{3}$ | $\frac{t_2}{3}$ | $\frac{i\,k\,t_2}{3}$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2$ | $-\frac{1}{3}\,i\,k\,t_2$ | $\frac{k^2\,t_2}{3}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{1}{2}\,k^2\,(r_3+2\,r_5)$ | 0 | 0 | 0 |
| $\omega_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | $\sigma_{2^+}^{\#1}{}_{\alpha\beta}$ | $\tau_{2^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $-\frac{2}{3\,k^2\,r_3}$ | 0 | 0 |
| $\tau_{2^+}^{\#1}\dagger^{\alpha\beta}$ | 0 | 0 | 0 |
| $\sigma_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$ | 0 | 0 | 0 |

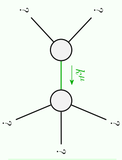
| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\tau_{0^+}^{\#1}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#2}\dagger$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1}\dagger$ | $\frac{1}{t_2}$ | 0 | 0 | 0 |

| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $f_{0^+}^{\#1}$ | $\omega_{0^+}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#2}\dagger$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1}\dagger$ | $t_2$ | 0 | 0 | 0 |

| | 0 | $-\frac{3\,k^2\,r_3}{2}$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |

## Unitarity conditions

$r_3 < 0 \&\& (r_5 < -\frac{r_3}{2} \,||\, r_5 > -2\,r_3) \,||\, r_3 > 0 \&\& -2\,r_3 < r_5 < -\frac{r_3}{2}$




Quadratic pole

| Pole residue: | $-\frac{1}{r_3\,(2\,r_3+r_5)\,(r_3+2\,r_5)\,p^2} > 0$ |
|---|---|
| Polarisations: | 2 |

(No massive particles)