# Particle spectrograph

### Wave operator and propagator

					lo		lo:
$\tau_{1}^{\#2}{}_{\alpha}$	0	0	0	$-\frac{4 i k}{(\alpha_0 - 4 \beta_1) (1 + 2 k^2)}$	$-\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+2k^2)^2}$	0	$-\frac{4k^2}{(\alpha_0-4\beta_1)(1+2k^2)^2}$
$\tau_{1}^{\#1}{}_{\alpha}$	0	0	0	0	0	0	0
$\sigma_{1}^{\#2}$	0	0	0	$-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+2k^2)}$	$-\frac{2}{(\alpha_0-4\beta_1)(1+2k^2)^2}$	0	$\frac{2 i \sqrt{2} k}{(\alpha_0 - 4 \beta_1) (1 + 2 k^2)^2}$
$\sigma_{1^{-}\alpha}^{\#1}$	0	0	0	0	$-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+2k^2)}$	0	$\frac{4ik}{(\alpha_0-4\beta_1)(1+2k^2)}$
$\tau_{1}^{\#1}{}_{\alpha\beta}$	$\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+k^2)}$	$-\frac{2ik}{(\alpha_0-4\beta_1)(1+k^2)^2}$	$-\frac{2k^2}{(\alpha_0-4\beta_1)(1+k^2)^2}$	0	0	0	0
$\sigma_{1}^{\#2}\alpha_{\beta}$	$\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+k^2)}$	$-\frac{2}{(\alpha_0-4\beta_1)(1+k^2)^2}$	$\frac{2ik}{(\alpha_0-4\beta_1)(1+k^2)^2}$	0	0	0	0
$\sigma_{1}^{\#1}{}_{\alpha\beta}$	0	$\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+k^2)}$	$-\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+k^2)}$	0	0	0	0
	$\sigma_{1}^{\#1} + \alpha \beta$	$\sigma_{1}^{#2} + \alpha \beta$	$\tau_1^{\#1} + \alpha \beta$	$\sigma_{1}^{\#1} +^{lpha}$	$\sigma_{1}^{\#2} +^{lpha}$	$\tau_{1}^{\#_{1}} +^{\alpha}$	$\tau_1^{\#2} + \alpha$

	$\omega_{2^{+}\alpha\beta}^{\#1}$	$f_{2^{+}\alpha\beta}^{\#1}$	$\omega_{2}^{\#1}{}_{\alpha\beta\chi}$
$\omega_{2}^{\#1} \dagger^{\alpha\beta}$	$-\frac{\alpha_0}{4}+\beta_1$	$\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$	0
$f_{2+}^{\#1}\dagger^{\alpha\beta}$	$-\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$	$2 \beta_1 k^2$	0
$\omega_{2}^{\#1}\dagger^{lphaeta\chi}$	0	0	$-\frac{\alpha_0}{4}+\beta_1$

	$\sigma_{2^{+}\alpha\beta}^{\#1}$	$\tau_{2^{+}\alpha\beta}^{\#1}$	$\sigma_{2}^{\#1}{}_{\alpha\beta\chi}$
$\sigma_{2}^{\#1}\dagger^{lphaeta}$	$-\frac{16\beta_1}{\alpha_0^2-4\alpha_0\beta_1}$	$\frac{2i\sqrt{2}}{\alpha_0 k}$	0
$\tau_{2^+}^{\#1}\dagger^{\alpha\beta}$	$-\frac{2i\sqrt{2}}{\alpha_0 k}$	$\frac{2}{\alpha_0 k^2}$	0
$\sigma_2^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{1}{-\frac{\alpha_0}{4} + \beta_1}$

	$\sigma_{0}^{\#1}$	$\tau_{0}^{\#1}$	$ au_{0}^{\#2}$	$\sigma_0^{\sharp 1}$
$\sigma_{0}^{\#1}$ †	$\frac{8\beta_1}{\alpha_0^2 - 4\alpha_0\beta_1}$	$-\frac{i\sqrt{2}}{\alpha_0 k}$	0	0
$\tau_{0}^{\#1}$ †	$\frac{i\sqrt{2}}{\alpha_0 k}$	$-\frac{1}{\alpha_0 k^2}$	0	0
$\tau_{0^{+}}^{\#2}$ †	0	0	0	0
$\sigma_0^{\!\#\!1}\dagger$	0	0	0	$\frac{2}{\alpha_0-4\beta_1+2\alpha_3k^2}$

Quadratic (free) Lagrangian density

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$f_{1^-}^{\#2}{}_{\alpha}$	0	0	0	$-\frac{1}{2}\bar{I}(\alpha_0-4\beta_1)k$	0	0	0
$f_{1^{}}^{\#1}{}_{\alpha}$	0	0	0	0	0	0	0
$\omega_{1^{\bar{-}}\alpha}^{\#2}$	0	0	0	$-\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$	0	0	0
$\omega_{1^{-}}^{\#1}{}_{\alpha}$	0	0	0	$\frac{1}{4} \left( \alpha_0 - 4  \beta_1 \right)$	$-\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$	0	$\bar{i}(\alpha_0-4\beta_1)k$
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$f_{1}^{\#1}{}_{\alpha\beta}$	$\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$	0	0	0	0	0	$0 \qquad \frac{1}{2}  \vec{l}$
$\omega_{1}^{\#2}_{+}$ $\alpha_{\beta}^{\#1}_{+}$	$\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$	0 0	0 0	0 0	0 0	0 0	0 0 $\frac{1}{2}$
		$+^{\alpha\beta}$ $\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$ 0 0	$+^{\alpha\beta} \left[ -\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}} \right]  0  0$	0 0 0	$\omega_{1}^{\#2} + \alpha$ 0 0 0	0 0 0 φ	$+^{\alpha}$ 0 0 0 $\frac{1}{2}$

Source constraints/gauge generators					
SO(3) irreps	Multiplicities				
$\tau_{0^{+}}^{\#2} == 0$	1				
$\tau_1^{\#2\alpha} + 2 i k \sigma_1^{\#2\alpha} == 0$	3				
$\tau_{1}^{\#1\alpha} == 0$	3				
$\tau_{1+}^{\#1}{}^{\alpha\beta} + i k \sigma_{1+}^{\#2}{}^{\alpha\beta} == 0$	3				
Total constraints:	10				

$\omega_{0}^{\#1}$	0	0	0	$\frac{\alpha_0}{2} - 2 \beta_1 + \alpha_3 k^2$
$f_{0}^{\#2}$	0	0	0	0
$f_0^{\#1}$	$-\frac{i(\alpha_0-4\beta_1)k}{\sqrt{2}}$	-4 $\beta_1 k^2$	0	0
$\omega_0^{\#_1}$	$\frac{1}{2} \left( \alpha_0 - 4  \beta_1 \right)$	$\frac{i(\alpha_0-4\beta_1)k}{\sqrt{2}}$	0	0
	$\omega_{0}^{\#1}$ †	$f_{0}^{\#1}$ $\dagger$	$f_0^{#2} \uparrow$	$\omega_{0^-}^{\#1}  \dagger$

# $\begin{array}{l} -\frac{1}{2} \alpha_{0} \ \omega_{\alpha\chi\beta} \ \omega^{\alpha\beta\chi} - \frac{1}{2} \alpha_{0} \ \omega^{\alpha\beta}_{\alpha} \ \omega_{\beta}^{\chi} + 2 \beta_{1} \ \omega^{\alpha\beta}_{\alpha} \ \omega_{\beta}^{\chi} - 2 \beta_{1} \ \omega_{\alpha}^{\chi\delta} \ \omega_{\chi\delta}^{\alpha} + \\ f^{\alpha\beta} \ \tau_{\alpha\beta} + \omega^{\alpha\beta\chi} \ \sigma_{\alpha\beta\chi} - 2 \beta_{1} \ \omega_{\alpha}^{\chi} \ \partial_{\beta} f^{\alpha\beta} - 2 \beta_{1} \ \omega_{\alpha}^{\delta} \ \partial_{\beta} f^{\alpha\beta} - \alpha_{0} \ f^{\alpha\beta} \ \partial_{\beta} \omega_{\alpha}^{\chi} + \\ \alpha_{0} \ \partial_{\beta} \omega^{\alpha\beta}_{\alpha} + \frac{2}{3} \ \alpha_{3} \ \partial^{\alpha} \omega^{\beta\zeta}_{\chi} \ \partial_{\beta} \omega_{\zeta\alpha}^{\chi} + 2 \beta_{1} \ \omega_{\beta}^{\chi} \ \partial^{\beta} f^{\alpha}_{\alpha} + 2 \beta_{1} \ \omega_{\beta}^{\delta} \ \partial^{\beta} f^{\alpha}_{\alpha} - \\ 2 \beta_{1} \ \partial_{\beta} f^{\chi}_{\chi} \ \partial^{\beta} f^{\alpha}_{\alpha} + \alpha_{0} \ f^{\alpha\beta} \ \partial_{\chi} \omega_{\alpha}^{\chi}_{\beta} - \alpha_{0} \ f^{\alpha}_{\alpha} \ \partial_{\chi} \omega^{\beta\chi}_{\beta} - \frac{2}{3} \ \alpha_{3} \ \partial_{\beta} \omega_{\zeta\alpha}^{\chi} \ \partial_{\chi} \omega^{\beta\zeta\alpha} - \\ \frac{1}{3} \ \alpha_{3} \ \partial_{\beta} \omega_{\zeta\alpha}^{\chi} \ \partial_{\chi} \omega^{\zeta\alpha\beta} + 4 \beta_{1} \ \omega_{\alpha\chi\beta} \ \partial^{\chi} f^{\alpha\beta} + \beta_{1} \ \partial_{\chi} f^{\delta}_{\beta} \ \partial^{\chi} f^{\delta}_{\beta} + \beta_{1} \ \partial_{\chi} f^{\delta}_{\beta} \ \partial^{\chi} f^{\delta}_{\beta} + \\ \frac{2}{3} \ \alpha_{3} \ \partial_{\chi} \omega^{\beta\zeta\alpha} \ \partial^{\chi} \omega_{\zeta\alpha\beta} + \frac{1}{3} \ \alpha_{3} \ \partial_{\chi} \omega^{\zeta\alpha\beta} \ \partial^{\chi} \omega_{\zeta\alpha\beta} + 4 \beta_{1} \ \partial^{\beta} f^{\alpha}_{\alpha} \ \partial_{\delta} f^{\delta}_{\beta} - \\ 2 \ \beta_{1} \ \partial_{\beta} f^{\beta}_{\chi} \ \partial_{\delta} f^{\chi\delta} + \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\delta\zeta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\zeta\delta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \\ 2 \ \beta_{1} \ \partial_{\beta} f^{\beta}_{\chi} \ \partial_{\delta} f^{\chi\delta} + \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\delta\zeta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\zeta\delta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \\ 2 \ \beta_{1} \ \partial_{\beta} f^{\beta}_{\chi} \ \partial_{\delta} f^{\chi\delta} + \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\delta\zeta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\zeta\delta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \\ 2 \ \beta_{1} \ \partial_{\beta} f^{\chi\delta}_{\chi} \ \partial_{\delta} f^{\chi\delta} + \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\delta\zeta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\zeta\delta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \\ 2 \ \beta_{1} \ \partial_{\beta} f^{\chi\delta}_{\chi} \ \partial_{\delta} f^{\chi\delta} + \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\delta\zeta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\zeta\delta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \\ 2 \ \beta_{1} \ \partial_{\beta} f^{\chi\delta}_{\chi} \ \partial_{\gamma} f^{\chi\delta}_{\chi} + \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\zeta\delta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \frac{2}{3} \ \alpha_{3} \ \partial^{\beta} \omega_{\alpha}^{\zeta\delta} \ \partial_{\delta} \omega_{\zeta\beta}^{\alpha} - \\ 2 \ \beta_{1} \ \partial_{\gamma} f^{\chi\delta}_{\chi} \ \partial_{\gamma} f^{\chi\delta}_{\chi} \ \partial_{\gamma} f^{\chi\delta}_{\chi} \ \partial_{\gamma} f^{\chi\delta}_{\chi} + \frac{2}{3} \ \partial_{\gamma} \phi^{\chi$

 $\beta_1 \partial^{\chi} f_{\zeta}^{\beta} \partial^{\zeta} f_{\beta \chi} - \beta_1 \partial^{\chi} f_{\zeta}^{\beta} \partial^{\zeta} f_{\chi \beta} + \beta_1 \partial^{\chi} f_{\delta \zeta} \partial^{\zeta} f_{\chi}^{\delta} - \beta_1 \partial^{\chi} f_{\zeta \delta} \partial^{\zeta} f_{\chi}^{\delta}$ 

## Massive and massless spectra

Massive particle
Pole residue: 
$$-\frac{1}{\alpha_3} > 0$$

Polarisations: 1

Square mass:  $-\frac{\alpha_0 - 4\beta_1}{2\alpha_3} > 0$ 

Spin: 0

Parity: Odd

? Quadratic pole Pole residue: 
$$\frac{1}{\alpha_0} > 0$$
 Polarisations: 2

## Unitarity conditions

$$\alpha_0 > 0 \&\& \alpha_3 < 0 \&\& \beta_1 < \frac{\alpha_0}{4}$$