| Lagrangian density   |
|--|
| $-t_1 \; \omega_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{$   |
| $2 r_1 \partial_i \omega^{\kappa \lambda}_{\ \kappa} \partial^i \omega_{\lambda \alpha}^{\ \alpha} - \frac{2}{3} r_1 \partial^\beta \omega^{\theta \alpha}_{\ \kappa} \partial_\theta \omega_{\alpha \beta}^{\ \kappa} + \frac{2}{3} r_2 \partial^\beta \omega^{\theta \alpha}_{\ \kappa} \partial_\theta \omega_{\alpha \beta}^{\ \kappa} -$      |
| $\frac{2}{3}r_{1}\partial_{\theta}\omega_{\alpha\beta}^{ \  \   k}\partial_{\kappa}\omega^{\alpha\beta\theta} - \frac{1}{3}r_{2}\partial_{\theta}\omega_{\alpha\beta}^{ \   \kappa}\partial_{\kappa}\omega^{\alpha\beta\theta} + \frac{2}{3}r_{1}\partial_{\theta}\omega_{\alpha\beta}^{ \   \kappa}\partial_{\kappa}\omega^{\theta\alpha\beta} -$ |
| $\frac{2}{3} r_2  \partial_\theta \omega_{\alpha\beta}^{} + 2 k_1  \partial_\alpha \omega_{\lambda}^{\alpha}  \partial_\kappa \omega^{\theta \kappa \lambda} - 2  r_1  \partial_\theta \omega_{\lambda}^{\alpha}  \partial_\kappa \omega^{\theta \kappa \lambda} +$  |
| $2r_1\partial_\alpha\omega_\lambda^{\ \alpha}_{\ \ \theta}\partial_\kappa\omega^{\kappa\lambda\theta}-4r_1\partial_\theta\omega_\lambda^{\ \alpha}_{\ \ \alpha}\partial_\kappa\omega^{\kappa\lambda\theta}-\frac{1}{3}t_1\partial^\alpha f_{\theta\kappa}\partial^\kappa f_{\alpha}^{\ \theta}-$   |
| $rac{2}{3}t_{1}\partial^{lpha}\!f_{\kappa	heta}\partial^{\kappa}\!f_{lpha}^{}\!-\!rac{1}{3}t_{1}\partial^{lpha}\!f^{\lambda}_{}\partial^{\kappa}\!f_{lpha\lambda}^{}\!+\!t_{1}\;\omega_{\kappalpha}^{lpha}\partial^{\kappa}\!f'_{}^{}\!+$  |
| $t_1 \; \omega_{\kappa\lambda}^{\;\;\lambda} \; \partial^\kappa f'_{\;\; } + 2  t_1  \partial^\alpha f_{\;\; \kappa\alpha}  \partial^\kappa f'_{\;\; } - t_1  \partial_\kappa f^\lambda_{\;\; }  \partial^\kappa f'_{\;\; } + \frac{1}{3}  t_1 \;  \omega_{_{!}\theta\kappa} \; \partial^\kappa f^{_{!}\theta}  +$                                 |
| $rac{4}{3}t_{1}\;\omega_{_{IK}\theta}\;\partial^{\kappa}f^{_{I}\theta}-rac{1}{3}t_{1}\;\omega_{	heta_{IK}}\;\partial^{\kappa}f^{_{I}	heta}+rac{2}{3}t_{1}\;\omega_{	heta_{K}}\;\partial^{\kappa}f^{_{I}	heta}-t_{1}\;\omega_{_{I}lpha}\;\partial^{\kappa}f^{_{I}}_{\;\;\kappa}-$  |
| $t_1\;\omega_{_{I}\lambda}^{\;\;\lambda}\;\partial^{\kappa}f_{_{K}}'+rac{1}{3}t_1\;\partial^{lpha}f_{_{A}}\;\partial^{\kappa}f_{\;\lambdalpha}+rac{1}{3}t_1\;\partial_{\kappa}f_{\;\;\theta}^{\;\;\lambda}\;\partial^{\kappa}f_{\;\;\lambda}^{\;\;	heta}+$   |
| $rac{2}{3}t_1\partial_\kappa f^\lambda_{\theta}\partial^\kappa f_{\lambda}^{\theta}$ - $t_1\partial^\alpha f^\lambda_{\kappa}$ $\partial^\kappa f_{\lambda\kappa}$ + $rac{2}{3}r_1\partial_\kappa \omega^{\alpha\beta\theta}\partial^\kappa \omega_{\alpha\beta\theta}$ +  |
| $\frac{1}{3} r_2  \partial_{\kappa} \omega^{\alpha\beta\theta}  \partial^{\kappa} \omega_{\alpha\beta\theta} - \frac{2}{3} r_1  \partial_{\kappa} \omega^{\theta\alpha\beta}  \partial^{\kappa} \omega_{\alpha\beta\theta} + \frac{2}{3} r_2  \partial_{\kappa} \omega^{\theta\alpha\beta}  \partial^{\kappa} \omega_{\alpha\beta\theta} +$        |
| $rac{2}{3}r_1\partial^{eta}\omega_{\alpha}^{\ lpha\lambda}\partial_{\lambda}\omega_{lphaeta}^{\ \prime}-rac{2}{3}r_2\partial^{eta}\omega_{\alpha}^{\ lpha\lambda}\partial_{\lambda}\omega_{lphaeta}^{\ \prime}-rac{8}{3}r_1\partial^{eta}\omega_{\lambda}^{\ \lambdalpha}\partial_{\lambda}\omega_{lpha}^{\ \prime}+$                           |
| $rac{2}{3}r_2\partial^{eta}\omega_{\lambda}{}^{\lambdalpha}\partial_{\lambda}\omega_{lphaeta}{}^{\prime}$ - $2r_1\partial_{lpha}\omega_{\lambda}{}^{lpha}\partial^{\lambda}\omega^{	heta\kappa}{}_{\kappa}$ + $2r_1\partial_{	heta}\omega_{\lambda}{}^{lpha}\partial^{\lambda}\omega^{	heta\kappa}{}_{\kappa}$                                    |
| Added source term: $f^{\alpha\beta}   	au_{lphaeta} + \omega^{lphaeta\chi}   \sigma_{lphaeta\chi}$   |
|  |

| ${\mathfrak t}_1^{\#2}$            | 0                                    | 0                                  | 0                                   | $\frac{ik}{k^2 t_1}$               | $\frac{k^2 r_1 + t_1)}{k^2 t_1)^2}$   | 0                            | $\frac{2r_1+t_1)}{r^2t_1)^2}$                         |
|------------------------------------|--------------------------------------|------------------------------------|-------------------------------------|------------------------------------|---------------------------------------|------------------------------|---|
|                                    | )                                    | )                                  | )                                   | $\frac{2ik}{t_1 + 2k^2t_1}$        | $\frac{i\sqrt{2}}{(t_1+2k^2t_1)^2}$   | )                            | $\frac{2 k^2 (2 k^2 r_1 + t_1)}{(t_1 + 2 k^2 t_1)^2}$ |
| $\tau_{1}^{\#1}{}_{\alpha}$        | 0                                    | 0                                  | 0                                   | 0                                  | 0                                     | 0                            | 0   |
| $\sigma_{1}^{\#2}{}_{\alpha}$      | 0                                    | 0                                  | 0                                   | $\frac{\sqrt{2}}{t_1 + 2 k^2 t_1}$ | $\frac{2k^2r_1+t_1}{(t_1+2k^2t_1)^2}$ | 0                            | $-\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$    |
| $\sigma_{1}^{\#1}{}_{\alpha}$      | 0                                    | 0                                  | 0                                   | 0                                  | $\frac{\sqrt{2}}{t_1 + 2k^2t_1}$      | 0                            | $-\frac{2ik}{t_1+2k^2t_1}$                            |
| $\tau_1^{\#1}{}_+\alpha\beta$      | $-\frac{6i\sqrt{2}k}{(3+2k^2)^2t_1}$ | $\frac{12ik}{(3+2k^2)^2t_1}$       | $\frac{12k^2}{(3+2k^2)^2t_1}$       | 0                                  | 0                                     | 0                            | 0   |
| $\sigma_{1}^{\#2}{}_{\alpha\beta}$ | $-\frac{6\sqrt{2}}{(3+2k^2)^2t_1}$   | $\frac{12}{(3+2k^2)^2t_1}$         | $-\frac{12ik}{(3+2k^2)^2t_1}$       | 0                                  | 0                                     | 0                            | 0   |
| $\sigma_{1}^{\#1}{}_{\alpha\beta}$ | $\frac{6}{(3+2k^2)^2t_1}$            | $-\frac{6\sqrt{2}}{(3+2k^2)^2t_1}$ | $\frac{6i\sqrt{2}k}{(3+2k^2)^2t_1}$ | 0                                  | 0                                     | 0                            | 0   |
|                                    | $\sigma_{1}^{\#1} + \alpha \beta$    | $\sigma_1^{#2} + \alpha \beta$     | $\tau_1^{\#1} + ^{\alpha\beta}$     | $\sigma_1^{\#1} +^\alpha$          | $\sigma_1^{\#2} +^{\alpha}$           | $\tau_{1}^{\#_{1}} + \alpha$ | $\tau_{1}^{\#2} +^{\alpha}$                           |

|   | #                | _                               |   | 8  | m                           | m   r                         | n   u                    | 19                    |
|---|------------------|---------------------------------|---|--|-----------------------------|-------------------------------|--------------------------|-----------------------|
| Source constraints                                | SO(3) irreps     | $\tau_{0+}^{\#2} == 0$          | $\tau_{0+}^{\#1} - 2  \bar{l}  k  \sigma_{0+}^{\#1} == 0$ | $\tau_{1}^{\#2}{}^{\alpha} + 2ik \sigma_{1}^{\#2}{}^{\alpha} == 0$ | $t_1^{\#1}{}^{\alpha} == 0$ | 0                             | $+ O_1^{\pm} = 0$        | Total #:              |
| f#2   | , 1 α            | 0                               | 0   | 0  | ikt <sub>1</sub>            | 0                             | 0                        | 0                     |
| $f_{1}^{#1}$                                      | , τ α            | 0                               | 0   | 0  | 0                           | 0                             | 0                        | 0                     |
| $\omega_{1}^{*2}$ , $f_{1}^{*1}$ , $f_{1}^{*2}$ . | $\alpha$         | 0                               | 0   | 0  | $\frac{t_1}{\sqrt{2}}$      | . 0                           | 0                        | 0                     |
| $\omega_{1}^{\#1}$                                | -1 α             | 0                               | 0   | 0  | $-k^2 r_1 - \frac{t_1}{2}$  | $\frac{t_1}{\sqrt{2}}$        | 0                        | $-ec{\imath}kt_1$     |
| f#1<br>f + +                                      | 1. αβ            | $-\frac{ikt_1}{3\sqrt{2}}$      | <u>i</u> kt <u>1</u><br>3                                 | $\frac{k^2 t_1}{3}$  | 0                           | 0                             | 0                        | 0                     |
| $\omega_{+}^{*2}$ , $\omega_{+}^{*1}$             | $1 \alpha \beta$ | $-\frac{\epsilon_1}{3\sqrt{2}}$ | £ 3   | $-\frac{1}{3}$ $\bar{l}$ $kt_1$                                    | 0                           | 0                             | 0                        | 0                     |
| $\omega_{1+\tilde{z},o}^{\#1}$                    | 1 αβ             | 6<br>6                          | $-\frac{t_1}{3\sqrt{2}}$                                  | $\frac{ikt_1}{3\sqrt{2}}$  | 0                           | 0                             | 0                        | 0                     |
|   |                  | $\omega_1^{\#1} + \alpha^{eta}$ | $\omega_1^{\#2} + \alpha^{\beta}$                         | $f_1^{#1} + \alpha \beta$  | $\omega_{1}^{\#1} +^{lpha}$ | $\omega_{1}^{\#2} +^{\alpha}$ | $f_{1}^{\#1} +^{\alpha}$ | $f_{1}^{#2} + \alpha$ |

| $\omega_{0^{\text{-}}}^{\#1}$ | 0                         | 0                | 0                   | $k^2 r_2$                   |  |
|-------------------------------|---------------------------|------------------|---------------------|-----------------------------|--|
| $f_{0}^{\#2}$                 | 0                         | 0                | 0                   | 0                           |  |
| $f_0^{\#1}$                   | $i\sqrt{2}kt_1$           | $-2 k^2 t_1$     | 0                   | 0                           |  |
| $\omega_{0}^{\#1}$            | -¢ <sub>1</sub>           | $-i\sqrt{2}kt_1$ | 0                   | 0                           |  |
| ·                             | $\omega_{0}^{\#1}\dagger$ | $f_{0}^{\#1}$ †  | $f_0^{#2} \uparrow$ | $\omega_{0}^{\#1} \dotplus$ |  |

| $\omega_{2^{-}}^{\#1}{}_{\alpha\beta\chi}$                      | 0                             | 0                         | $k^2 r_1 + \frac{t_1}{2}$         |
|---|-------------------------------|---------------------------|-----------------------------------|
| $f_{2}^{\#1}$   | $-\frac{ikt_1}{\sqrt{2}}$     | $k^2 t_1$                 | 0                                 |
| $\omega_{2}^{\#1}{}_{\alpha\beta}\ f_{2}^{\#1}{}_{\alpha\beta}$ | $\frac{t_1}{2}$               | $\frac{ikt_1}{\sqrt{2}}$  | 0                                 |
|   | $\omega_2^{\#1} + ^{lphaeta}$ | $f_2^{#1} + \alpha \beta$ | $\omega_{2}^{#1} +^{lphaeta\chi}$ |

| _  | $\sigma_{2^{+}lphaeta}^{\#1}$       | $	au_2^{\#1}_{lphaeta}$              | $\sigma_{2}^{\#1}{}_{\alpha\beta\chi}$ |
|--|-------------------------------------|--------------------------------------|--|
| $\sigma_{2}^{\#1} \dagger^{\alpha\beta}$ | $\frac{2}{(1+2k^2)^2t_1}$           | $-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$ | 0                                      |
| $\tau_{2+}^{\#1} \dagger^{\alpha\beta}$  | $\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$ | $\frac{4k^2}{(1+2k^2)^2t_1}$         | 0                                      |
| $\sigma_{2}^{\#1} \dagger^{lphaeta\chi}$ | 0                                   | 0                                    | $\frac{2}{2k^2r_1+t_1}$                |

|  | $\sigma_0^{\#1}$                       | $\tau_{0}^{\#1}$                     | $	au_{0}^{\#2}$ | $\sigma_0^{\#1}$    |
|--|--|--------------------------------------|-----------------|---------------------|
| $\sigma_{0^{+}}^{\#1}$ †   | $-\frac{1}{(1+2k^2)^2t_1}$             | $\frac{i\sqrt{2} k}{(1+2k^2)^2 t_1}$ | 0               | 0                   |
| $\tau_{0}^{\#1}$ †   | $-\frac{i \sqrt{2} k}{(1+2k^2)^2 t_1}$ | $-\frac{2k^2}{(1+2k^2)^2t_1}$        | 0               | 0                   |
| $\tau_{0^{+}}^{\#2}$ †   | 0                                      | 0                                    | 0               | 0                   |
| $\sigma_{0}^{\!$ | 0                                      | 0                                    | 0               | $\frac{1}{k^2 r_2}$ |

|                  | Massive particle |                         |  |
|------------------|------------------|-------------------------|--|
| ? $J^P = 2^{-/}$ | Pole residue:    | $-\frac{1}{r_1} > 0$    |  |
| 2                | Polarisations:   | 5                       |  |
| $\vec{k}^{\mu}$  | Square mass:     | $-\frac{t_1}{2r_1} > 0$ |  |
| ?                | Spin:            | 2                       |  |
| ·                | Parity:          | Odd                     |  |

Unitarity conditions  $r_1 < 0 \&\& t_1 > 0$ 

(No massless particles)