# Particle spectrograph

Lagrangian density

$$-t_1\,\omega_{\iota}{}^{\alpha\iota}\,\omega_{\kappa\alpha}{}^{\kappa}-\frac{1}{3}t_1\,\omega_{\iota}{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}'+\frac{2}{3}t_2\,\omega_{\iota}{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}'\,\omega^{\kappa\lambda}{}_{\iota}+\frac{1}{3}t_1\,\omega_{\kappa\lambda}{}'\,\omega^{\kappa\lambda}{}_{\iota}+\frac{1}{3}t_2\,\omega_{\kappa\lambda}{}'\,\omega^{\kappa\lambda}{}_{\iota}+2r_1\partial_{\iota}\omega^{\kappa\lambda}{}_{\kappa}\partial'\omega_{\lambda}{}^{\alpha}{}_{\alpha}-\frac{2}{3}r_1\partial^{\beta}\omega^{\theta\alpha}{}_{\kappa}\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}+$$
$$\frac{2}{3}r_2\partial^{\beta}\omega^{\theta\alpha}{}_{\kappa}\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}-\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta}-\frac{1}{3}r_2\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta}+\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta}-\frac{2}{3}r_2\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta}+2r_1\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\partial_{\kappa}\omega^{\theta\kappa\lambda}-$$
$$2r_1\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}+2r_1\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\partial_{\kappa}\omega^{\kappa\lambda\theta}-4r_1\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}-\frac{1}{3}t_1\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}{}^{\theta}+\frac{1}{6}t_2\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}{}^{\theta}-\frac{2}{3}t_1\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}{}^{\theta}-\frac{1}{6}t_2\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}{}^{\theta}-$$
$$\frac{1}{3}t_1\partial^{\alpha}f^{\lambda}{}_{\kappa}\partial^{\kappa}f_{\alpha\lambda}+\frac{1}{6}t_2\partial^{\alpha}f^{\lambda}{}_{\kappa}\partial^{\kappa}f_{\alpha\lambda}+t_1\,\omega_{\kappa\alpha}{}^{\alpha}\,\partial^{\kappa}f'_{\iota}+t_1\,\omega_{\kappa\lambda}{}^{\lambda}\,\partial^{\kappa}f'_{\iota}+2t_1\partial^{\alpha}f_{\kappa\alpha}\partial^{\kappa}f'_{\iota}-t_1\partial_{\kappa}f^{\lambda}{}_{\lambda}\partial^{\kappa}f'_{\iota}+\frac{1}{3}t_1\,\omega_{\iota\theta\kappa}\,\partial^{\kappa}f'^{\iota\theta}+$$
$$\frac{1}{3}t_2\,\omega_{\iota\theta\kappa}\,\partial^{\kappa}f'^{\iota\theta}+\frac{4}{3}t_1\,\omega_{\iota\kappa\theta}\,\partial^{\kappa}f'^{\iota\theta}-\frac{2}{3}t_2\,\omega_{\iota\kappa\theta}\,\partial^{\kappa}f'^{\iota\theta}-\frac{1}{3}t_1\,\omega_{\theta\iota\kappa}\,\partial^{\kappa}f'^{\iota\theta}-\frac{1}{3}t_2\,\omega_{\theta\iota\kappa}\,\partial^{\kappa}f'^{\iota\theta}+\frac{2}{3}t_1\,\omega_{\theta\kappa\iota}\,\partial^{\kappa}f'^{\iota\theta}+\frac{2}{3}t_2\,\omega_{\theta\kappa\iota}\,\partial^{\kappa}f'^{\iota\theta}-$$
$$t_1\,\omega_{\iota\alpha}{}^{\alpha}\,\partial^{\kappa}f'_{\kappa}-t_1\,\omega_{\iota\lambda}{}^{\lambda}\,\partial^{\kappa}f'_{\kappa}+\frac{1}{3}t_1\partial^{\alpha}f^{\lambda}{}_{\kappa}\partial^{\kappa}f_{\lambda\alpha}-\frac{1}{6}t_2\partial^{\alpha}f^{\lambda}{}_{\kappa}\partial^{\kappa}f_{\lambda\alpha}+\frac{1}{3}t_1\partial_{\kappa}f_{\theta}{}^{\lambda}\partial^{\kappa}f_{\lambda}{}^{\theta}-\frac{1}{6}t_2\partial_{\kappa}f_{\theta}{}^{\lambda}\partial^{\kappa}f_{\lambda}{}^{\theta}+\frac{2}{3}t_1\partial_{\kappa}f^{\lambda}{}_{\theta}\partial^{\kappa}f_{\lambda}{}^{\theta}+$$
$$\frac{1}{6}t_2\partial_{\kappa}f^{\lambda}{}_{\theta}\partial^{\kappa}f_{\lambda}{}^{\theta}-t_1\partial^{\alpha}f^{\lambda}{}_{\alpha}\partial^{\kappa}f_{\lambda\kappa}+\frac{2}{3}r_1\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+\frac{1}{3}r_2\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}-\frac{2}{3}r_1\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}+\frac{2}{3}r_2\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}+$$
$$\frac{2}{3}r_1\partial^{\beta}\omega_{\iota}{}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}{}'-\frac{2}{3}r_2\partial^{\beta}\omega_{\iota}{}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}{}'-\frac{8}{3}r_1\partial^{\beta}\omega_{\iota}{}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}{}'+\frac{2}{3}r_2\partial^{\beta}\omega_{\iota}{}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}{}'-2r_1\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}+2r_1\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}$$

Added source term: $f^{\alpha\beta}\,\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}$

# Wave operator

| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1}\dagger$ | $-t_1$ | $i\sqrt{2}\,k\,t_1$ | 0 | 0 |
| $f_{0^+}^{\#1}\dagger$ | $-i\sqrt{2}\,k\,t_1$ | $-2k^2\,t_1$ | 0 | 0 |
| $f_{0^+}^{\#2}\dagger$ | 0 | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1}\dagger$ | 0 | 0 | 0 | $k^2\,r_2+t_2$ |

| | $\omega_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^+}^{\#2}{}_{\alpha\beta}$ | $f_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^-}^{\#1}{}_{\alpha}$ | $\omega_{1^-}^{\#2}{}_{\alpha}$ | $f_{1^-}^{\#1}{}_{\alpha}$ | $f_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{1}{6}(t_1+4t_2)$ | $-\frac{t_1-2t_2}{3\sqrt{2}}$ | $-\frac{ik(t_1-2t_2)}{3\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2}\dagger^{\alpha\beta}$ | $-\frac{t_1-2t_2}{3\sqrt{2}}$ | $\frac{t_1+t_2}{3}$ | $\frac{1}{3}ik(t_1+t_2)$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{ik(t_1-2t_2)}{3\sqrt{2}}$ | $-\frac{1}{3}ik(t_1+t_2)$ | $\frac{1}{3}k^2(t_1+t_2)$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | $-k^2r_1-\frac{t_1}{2}$ | $\frac{t_1}{\sqrt{2}}$ | 0 | $ikt_1$ |
| $\omega_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 |
| $f_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | $-ikt_1$ | 0 | 0 | 0 |

| | $\omega_{2^+}^{\#1}{}_{\alpha\beta}$ | $f_{2^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{ikt_1}{\sqrt{2}}$ | 0 |
| $f_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{ikt_1}{\sqrt{2}}$ | $k^2\,t_1$ | 0 |
| $\omega_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$ | 0 | 0 | $k^2r_1+\frac{t_1}{2}$ |

# Saturated propagator

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}\dagger$ | $-\frac{1}{(1+2k^2)^2t_1}$ | $\frac{i\sqrt{2}\,k}{(1+2k^2)^2t_1}$ | 0 | 0 |
| $\tau_{0^+}^{\#1}\dagger$ | $-\frac{i\sqrt{2}\,k}{(1+2k^2)^2t_1}$ | $-\frac{2k^2}{(1+2k^2)^2t_1}$ | 0 | 0 |
| $\tau_{0^+}^{\#2}\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1}\dagger$ | 0 | 0 | 0 | $\frac{1}{k^2r_2+t_2}$ |

| | $\sigma_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^+}^{\#2}{}_{\alpha\beta}$ | $\tau_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^-}^{\#1}{}_{\alpha}$ | $\sigma_{1^-}^{\#2}{}_{\alpha}$ | $\tau_{1^-}^{\#1}{}_{\alpha}$ | $\tau_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{2(t_1+t_2)}{3t_1t_2}$ | $\frac{\sqrt{2}\,(t_1-2t_2)}{3(1+k^2)t_1t_2}$ | $\frac{i\sqrt{2}\,k(t_1-2t_2)}{3(1+k^2)t_1t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2}\dagger^{\alpha\beta}$ | $\frac{\sqrt{2}\,(t_1-2t_2)}{3(1+k^2)t_1t_2}$ | $\frac{t_1+4t_2}{3(1+k^2)^2t_1t_2}$ | $\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $-\frac{i\sqrt{2}\,k(t_1-2t_2)}{3(1+k^2)t_1t_2}$ | $-\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$ | $\frac{k^2(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2t_1}$ | 0 | $\frac{2ik}{t_1+2k^2t_1}$ |
| $\sigma_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2t_1}$ | $\frac{2k^2r_1+t_1}{(t_1+2k^2t_1)^2}$ | 0 | $\frac{i\sqrt{2}\,k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$ |
| $\tau_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | $-\frac{2ik}{t_1+2k^2t_1}$ | $-\frac{i\sqrt{2}\,k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$ | 0 | $\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$ |

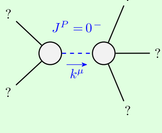
| | $\sigma_{2^+}^{\#1}{}_{\alpha\beta}$ | $\tau_{2^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{2}{(1+2k^2)^2t_1}$ | $-\frac{2i\sqrt{2}\,k}{(1+2k^2)^2t_1}$ | 0 |
| $\tau_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{2i\sqrt{2}\,k}{(1+2k^2)^2t_1}$ | $\frac{4k^2}{(1+2k^2)^2t_1}$ | 0 |
| $\sigma_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{2k^2r_1+t_1}$ |

# Source constraints

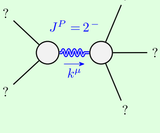
Source constraints

| SO(3) irreps | # |
|---|---|
| $\tau_{0^+}^{\#2}==0$ | 1 |
| $\tau_{0^+}^{\#1}-2ik\,\sigma_{0^+}^{\#1}==0$ | 1 |
| $\tau_{1^-}^{\#2\alpha}+2ik\,\sigma_{1^-}^{\#2\alpha}==0$ | 3 |
| $\tau_{1^-}^{\#1\alpha}==0$ | 3 |
| $\tau_{1^+}^{\#1\alpha\beta}+ik\,\sigma_{1^+}^{\#2\alpha\beta}==0$ | 3 |
| $\tau_{2^+}^{\#1\alpha\beta}-2ik\,\sigma_{2^+}^{\#1\alpha\beta}==0$ | 5 |
| Total #: | 16 |

# Massive spectrum




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_2}>0$ |
| Polarisations: | 1 |
| Square mass: | $-\frac{t_2}{r_2}>0$ |
| Spin: | 0 |
| Parity: | Odd |




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_1}>0$ |
| Polarisations: | 5 |
| Square mass: | $-\frac{t_1}{2r_1}>0$ |
| Spin: | 2 |
| Parity: | Odd |

# Massless spectrum

(No massless particles)

# Unitarity conditions

Unitarity conditions

$$r_1<0\;\&\&\;r_2<0\;\&\&\;t_1>0\;\&\&\;t_2>0$$