

Particle spectrograph

Wave operator and propagator

Quadratic (free) action

$$\begin{aligned} S = & \iiint (\frac{1}{6} (6 t_1 \omega_{\alpha}^{\alpha'} \omega_{\beta}^{\theta} + 6 f^{\alpha \beta} \tau_{\alpha \beta} + 6 \omega^{\alpha \beta \chi} \sigma_{\alpha \beta \chi} - 12 t_1 \omega_{\alpha}^{\theta} \partial_{\beta} f^{\alpha} + 12 t_1 \omega_{\beta}^{\theta} \partial_{\alpha} f^{\alpha} - \\ & \omega_{\beta}^{\theta} \partial_{\alpha} f^{\alpha} - 6 t_1 \partial_{\beta} \omega_{\alpha}^{\theta} \partial_{\beta} f^{\alpha} - 12 r_1 \partial_{\beta} \omega_{\beta}^{\theta} \partial_{\beta} \omega_{\alpha}^{\alpha \beta} + 12 r_1 \partial_{\beta} \omega_{\beta}^{\theta} \partial_{\beta} \omega_{\alpha}^{\alpha \beta} - \\ & 6 t_1 \partial_{\beta} \omega_{\alpha}^{\alpha'} \partial_{\beta} \omega_{\alpha}^{\theta} + 12 t_1 \partial_{\beta} \omega_{\alpha}^{\alpha} \partial_{\beta} \omega_{\alpha}^{\theta} + 12 r_1 \partial_{\alpha} \omega_{\beta}^{\alpha \beta} \partial_{\theta} \omega_{\beta}^{\theta} - 24 r_1 \partial_{\beta} \omega_{\alpha}^{\alpha \beta} \partial_{\theta} \omega_{\beta}^{\theta} - \\ & 12 r_1 \partial_{\alpha} \omega_{\beta}^{\alpha \beta} \partial_{\theta} \omega_{\beta}^{\theta} + 24 r_1 \partial_{\beta} \omega_{\alpha}^{\alpha \beta} \partial_{\theta} \omega_{\beta}^{\theta} + 4 t_1 \omega_{\beta}^{\theta} \partial_{\beta} f^{\alpha} + 4 t_2 \omega_{\beta}^{\theta} \partial_{\theta} f^{\alpha} - \\ & 4 t_1 \partial_{\alpha} f_{\beta}^{\alpha'} \partial_{\theta} f^{\alpha} + 2 t_2 \partial_{\alpha} f_{\beta}^{\alpha'} \partial_{\theta} f^{\alpha} - 4 t_1 \partial_{\alpha} f_{\beta}^{\alpha'} \partial_{\theta} f^{\alpha} - t_2 \partial_{\alpha} f_{\beta}^{\alpha'} \partial_{\theta} f^{\alpha} + \\ & 2 t_1 \partial_{\beta} f_{\alpha}^{\alpha'} \partial_{\theta} f^{\alpha} - t_2 \partial_{\beta} f_{\alpha}^{\alpha'} \partial_{\theta} f^{\alpha} + 4 t_1 \partial_{\beta} f_{\alpha}^{\alpha'} \partial_{\theta} f^{\alpha} + t_2 \partial_{\beta} f_{\alpha}^{\alpha'} \partial_{\theta} f^{\alpha} + \\ & 2 t_1 \partial_{\beta} f_{\alpha}^{\alpha'} \partial_{\theta} f^{\alpha} - t_2 \partial_{\beta} f_{\alpha}^{\alpha'} \partial_{\theta} f^{\alpha} + 2 (t_1 + t_2) \omega_{\alpha \theta} (\omega^{\alpha \theta} + 2 \partial^{\theta} f^{\alpha'}) + \\ & 2 \omega_{\alpha \theta} ((t_1 - 2 t_2) \omega^{\alpha \theta} + 2 (2 t_1 - t_2) \partial^{\theta} f^{\alpha'}) - 8 r_1 \partial_{\beta} \omega_{\alpha \theta} \partial^{\theta} \omega^{\alpha \beta} + \\ & 8 r_2 \partial_{\beta} \omega_{\alpha \theta} \partial^{\theta} \omega^{\alpha \beta} + 4 r_1 \partial_{\beta} \omega_{\alpha \theta} \partial^{\theta} \omega^{\alpha \beta} - 4 r_2 \partial_{\beta} \omega_{\alpha \theta} \partial^{\theta} \omega^{\alpha \beta} - \\ & 16 r_1 \partial_{\beta} \omega_{\theta \alpha} \partial^{\theta} \omega_{\alpha \beta} + 4 r_2 \partial_{\beta} \omega_{\theta \alpha} \partial^{\theta} \omega_{\alpha \beta} - 4 r_1 \partial_{\beta} \omega_{\alpha \theta} \partial^{\theta} \omega_{\alpha \beta} - \\ & 2 r_2 \partial_{\beta} \omega_{\alpha \theta} \partial^{\theta} \omega_{\alpha \beta} + 4 r_1 \partial_{\theta} \omega_{\alpha \beta} \partial^{\theta} \omega_{\alpha \beta} + 2 r_2 \partial_{\theta} \omega_{\alpha \beta} \partial^{\theta} \omega_{\alpha \beta} + \\ & 4 r_1 \partial_{\theta} \omega_{\alpha \beta} \partial^{\theta} \omega_{\alpha \beta} - 4 r_2 \partial_{\theta} \omega_{\alpha \beta} \partial^{\theta} \omega_{\alpha \beta})) [t, x, y, z] d x d y d z d t \end{aligned}$$

$\sigma_{1+}^{\#1} \dagger^{\alpha \beta}$	$\sigma_{1+}^{\#2} \dagger^{\alpha \beta}$	$\tau_{1+}^{\#1} \dagger^{\alpha \beta}$	$\sigma_{1-}^{\#1} \dagger^{\alpha}$	$\sigma_{1-}^{\#2} \dagger^{\alpha}$	$\tau_{1-}^{\#1} \dagger^{\alpha}$	$\tau_{1-}^{\#2} \dagger^{\alpha}$
$\frac{2(t_1+t_2)}{3t_1t_2}$	$\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	0	0	0	0
$\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{t_1+4t_2}{3(1+k^2)^2t_1t_2}$	$\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0	0
$-\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$-\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	$\frac{k^2(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0	0
0	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	0	$\frac{2ik}{t_1+2k^2t_1}$
0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	$\frac{2k^2r_1+t_1}{(t_1+2k^2t_1)^2}$	0	$\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
0	0	0	0	0	0	0
0	0	0	$-\frac{2ik}{t_1+2k^2t_1}$	$-\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$	0	$\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$

$\omega_{1+}^{\#1} \dagger^{\alpha \beta}$	$\omega_{1+}^{\#2} \dagger^{\alpha \beta}$	$f_{1+}^{\#1} \dagger^{\alpha \beta}$	$\omega_{1-}^{\#1} \dagger^{\alpha}$	$\omega_{1-}^{\#2} \dagger^{\alpha}$	$f_{1-}^{\#1} \dagger^{\alpha}$	$f_{1-}^{\#2} \dagger^{\alpha}$
$\frac{1}{6}(t_1+4t_2)$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$-\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	0	0	0	0
$-\frac{t_1-2t_2}{3\sqrt{2}}$	$\frac{t_1+t_2}{3}$	$\frac{1}{3}ik(t_1+t_2)$	0	0	0	0
$\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	$-\frac{1}{3}ik(t_1+t_2)$	$\frac{1}{3}k^2(t_1+t_2)$	0	0	0	0
0	0	0	$-k^2r_1-\frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$ik t_1$
0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
0	0	0	0	0	0	0
0	0	0	$-ik t_1$	0	0	0

$\omega_{2+}^{\#1} \dagger^{\alpha \beta}$	$f_{2+}^{\#1} \dagger^{\alpha \beta}$	$\omega_{2-}^{\#1} \dagger^{\alpha \beta \chi}$
$\frac{t_1}{2}$	$-\frac{ik t_1}{\sqrt{2}}$	0
$\frac{ik t_1}{\sqrt{2}}$	$k^2 t_1$	0
0	0	$k^2 r_1 + \frac{t_1}{2}$

$\sigma_{2+}^{\#1} \dagger^{\alpha \beta}$	$\tau_{2+}^{\#1} \dagger^{\alpha \beta}$	$\sigma_{2-}^{\#1} \dagger^{\alpha \beta \chi}$
$\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
0	0	$\frac{2}{2k^2r_1+t_1}$

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	16

$\sigma_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#2} \dagger$	$\sigma_{0-}^{\#1} \dagger$
$-\frac{1}{(1+2k^2)^2t_1}$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	0	0
$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0	0
0	0	0	0
0	0	0	$\frac{1}{k^2r_2+t_2}$

$\omega_{0+}^{\#1} \dagger$	$f_{0+}^{\#1} \dagger$	$f_{0+}^{\#2} \dagger$	$\omega_{0-}^{\#1} \dagger$
$-t_1$	$i\sqrt{2}k t_1$	0	0
$-i\sqrt{2}k t_1$	$-2k^2 t_1$	0	0
0	0	0	0
0	0	0	$k^2 r_2 + t_2$

Massive and massless spectra

Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

Unitarity conditions

$$r_1 < 0 \ \&\& \ r_2 < 0 \ \&\& \ t_1 > 0 \ \&\& \ t_2 > 0$$