

Wave operator and propagator

The diagram shows two vertices (circles) connected by a dashed line representing a particle exchange. The left vertex has two incoming lines (labeled with '?') and two outgoing lines (labeled with '?'). The right vertex has two incoming lines (labeled with '?') and one outgoing line (labeled with '?'). The dashed line is labeled with $J^P = 0^-$ and k^μ with an arrow pointing from left to right.

Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

$$r_2 < 0 \ \&\& \ t_2 > 0$$
$$\begin{aligned}
& -\frac{1}{3}t_1\omega_{\lambda'}^{\alpha\lambda}\omega_{\kappa\alpha}^{\kappa-\frac{1}{3}}t_1\omega_{\lambda'}^{\kappa\lambda}\omega_{\kappa\lambda'}^{\lambda'}+\frac{2}{3}t_2\omega_{\lambda'}^{\kappa\lambda}\omega_{\kappa\lambda'}^{\lambda'}+\frac{1}{3}t_1\omega_{\kappa\lambda}^{\lambda'}\omega^{\kappa\lambda}+ \\
& \frac{1}{3}t_2\omega_{\kappa\lambda}^{\lambda'}\omega_{\kappa\lambda}^{\kappa\lambda}+\omega_{\alpha\beta\chi}^{\alpha\beta}\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}+\frac{2}{3}r_2\partial^\theta\omega_{\kappa}^{\theta\alpha}\partial_\theta\omega_{\alpha\beta}^{\kappa-} \\
& \frac{1}{3}r_2\partial_\theta\omega_{\alpha\beta}^{\kappa}\partial_\kappa\omega^{\alpha\beta\theta}-\frac{2}{3}r_2\partial_\theta\omega_{\alpha\beta}^{\kappa}\partial_\kappa\omega^{\theta\alpha\beta}-\frac{1}{3}t_1\partial^\alpha f_{\theta\kappa}\partial^\kappa f_{\alpha}^{\theta}+ \\
& \frac{1}{6}t_2\partial^\alpha f_{\theta\kappa}\partial^\kappa f_{\alpha}^{\theta}-\frac{2}{3}t_1\partial^\alpha f_{\kappa\theta}\partial^\kappa f_{\alpha}^{\theta}-\frac{1}{6}t_2\partial^\alpha f_{\kappa\theta}\partial^\kappa f_{\alpha}^{\theta}-\frac{1}{3}t_1\partial^\alpha f_{\lambda}^{\kappa}\partial^\kappa f_{\alpha\lambda}^{\kappa}+ \\
& \frac{1}{6}t_2\partial^\alpha f_{\lambda}^{\kappa}\partial^\kappa f_{\alpha\lambda}^{\kappa}+\frac{1}{3}t_1\omega_{\kappa\alpha}^{\alpha}\partial^\kappa f_{\lambda'}^{\lambda}+\frac{1}{3}t_1\omega_{\kappa\lambda}^{\lambda}\partial^\kappa f_{\lambda'}^{\lambda}+\frac{2}{3}t_1\partial^\alpha f_{\kappa\alpha}\partial^\kappa f_{\lambda'}^{\lambda}- \\
& \frac{1}{3}t_1\partial_\kappa f_{\lambda}^{\lambda}\partial^\kappa f_{\lambda}^{\lambda}+\frac{1}{3}t_1\omega_{\lambda\theta\kappa}\partial^\kappa f_{\lambda'}^{\theta}+\frac{1}{3}t_2\omega_{\lambda\theta\kappa}\partial^\kappa f_{\lambda'}^{\theta}+\frac{4}{3}t_1\omega_{\lambda\kappa\theta}\partial^\kappa f_{\lambda'}^{\theta}- \\
& \frac{2}{3}t_2\omega_{\lambda\kappa\theta}\partial^\kappa f_{\lambda'}^{\theta}-\frac{1}{3}t_1\omega_{\lambda\theta\kappa}\partial^\kappa f_{\lambda'}^{\theta}-\frac{1}{3}t_2\omega_{\theta\lambda\kappa}\partial^\kappa f_{\lambda'}^{\theta}+\frac{2}{3}t_1\omega_{\theta\kappa\lambda}\partial^\kappa f_{\lambda'}^{\theta}+ \\
& \frac{2}{3}t_2\omega_{\theta\kappa\lambda}\partial^\kappa f_{\lambda'}^{\theta}-\frac{1}{3}t_1\omega_{\lambda\alpha}^{\alpha}\partial^\kappa f_{\lambda'}^{\lambda}-\frac{1}{3}t_1\omega_{\lambda\lambda}^{\lambda}\partial^\kappa f_{\lambda'}^{\lambda}+\frac{1}{3}t_1\partial^\alpha f_{\lambda}^{\kappa}\partial^\kappa f_{\lambda\alpha}^{\kappa}- \\
& \frac{1}{6}t_2\partial^\alpha f_{\lambda}^{\kappa}\partial^\kappa f_{\lambda\alpha}^{\kappa}+\frac{1}{3}t_1\partial_\kappa f_{\theta}^{\lambda}\partial^\kappa f_{\lambda}^{\theta}-\frac{1}{6}t_2\partial_\kappa f_{\theta}^{\lambda}\partial^\kappa f_{\lambda}^{\theta}+\frac{2}{3}t_1\partial_\kappa f_{\theta}^{\lambda}\partial^\kappa f_{\lambda}^{\theta}+ \\
& \frac{1}{6}t_2\partial_\kappa f_{\theta}^{\lambda}\partial^\kappa f_{\lambda}^{\theta}-\frac{1}{3}t_1\partial^\alpha f_{\lambda}^{\kappa}\partial^\kappa f_{\lambda\kappa}^{\kappa}+\frac{1}{3}r_2\partial_\kappa\omega^{\alpha\beta\theta}\partial^\kappa\omega_{\alpha\beta\theta}+ \\
& \frac{2}{3}r_2\partial_\kappa\omega^{\theta\alpha\beta}\partial^\kappa\omega_{\alpha\beta\theta}-\frac{2}{3}r_2\partial^\beta\omega_{\lambda'}^{\alpha\lambda}\partial_\lambda\omega_{\alpha\beta}^{\lambda'}+\frac{2}{3}r_2\partial^\beta\omega_{\lambda'}^{\lambda\alpha}\partial_\lambda\omega_{\alpha\beta}^{\lambda'}
\end{aligned}$$

	$\sigma_{1+}^{\#1} + \alpha\beta$	$\sigma_{1+}^{\#2} + \alpha\beta$	$\tau_{1+}^{\#1} + \alpha\beta$	$\sigma_{1-}^{\#1} + \alpha$	$\sigma_{1-}^{\#2} + \alpha$	$\tau_{1-}^{\#1} + \alpha$	$\tau_{1-}^{\#2} + \alpha$
$\sigma_{1+}^{\#1} + \alpha\beta$	$\frac{2(t_1+t_2)}{3t_1t_2}$	$\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	0	0	0	0
$\sigma_{1+}^{\#2} + \alpha\beta$	$\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{t_1+4t_2}{3(1+k^2)^2t_1t_2}$	$\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0	0
$\tau_{1+}^{\#1} + \alpha\beta$	$-\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$-\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	$\frac{k^2(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0	0
$\sigma_{1-}^{\#1} + \alpha$	0	0	0	$\frac{6}{(3+4k^2)^2t_1}$	$\frac{6\sqrt{2}}{(3+4k^2)^2t_1}$	0	$\frac{12ik}{(3+4k^2)^2t_1}$
$\sigma_{1-}^{\#2} + \alpha$	0	0	0	$\frac{6\sqrt{2}}{(3+4k^2)^2t_1}$	$\frac{12}{(3+4k^2)^2t_1}$	0	$\frac{12i\sqrt{2}k}{(3+4k^2)^2t_1}$
$\tau_{1-}^{\#1} + \alpha$	0	0	0	0	0	0	0
$\tau_{1-}^{\#2} + \alpha$	0	0	0	$-\frac{12ik}{(3+4k^2)^2t_1}$	$-\frac{12i\sqrt{2}k}{(3+4k^2)^2t_1}$	0	$\frac{24k^2}{(3+4k^2)^2t_1}$

$\omega_{1+}^{\#1} \dagger \alpha \beta$	$\omega_{1+}^{\#2} \dagger \alpha \beta$	$f_{1+}^{\#1} \dagger \alpha \beta$	$\omega_{1-}^{\#1} \alpha$	$\omega_{1-}^{\#2} \alpha$	$f_{1-}^{\#1} \alpha$	$f_{1-}^{\#2} \alpha$
$\frac{1}{6} (t_1 + 4t_2)$	$-\frac{t_1 - 2t_2}{3 \sqrt{2}}$	$-\frac{i k (t_1 - 2t_2)}{3 \sqrt{2}}$	0	0	0	0
$-\frac{t_1 - 2t_2}{3 \sqrt{2}}$	$\frac{t_1 + t_2}{3}$	$\frac{1}{3} i k (t_1 + t_2)$	0	0	0	0
$\frac{i k (t_1 - 2t_2)}{3 \sqrt{2}}$	$-\frac{1}{3} i k (t_1 + t_2)$	$\frac{1}{3} k^2 (t_1 + t_2)$	0	0	0	0
$\omega_{1-}^{\#1} \dagger \alpha$	0	0	$\frac{t_1}{6}$	$\frac{t_1}{3 \sqrt{2}}$	0	$\frac{i k t_1}{3}$
$\omega_{1-}^{\#2} \dagger \alpha$	0	0	$\frac{t_1}{3 \sqrt{2}}$	$\frac{t_1}{3}$	0	$\frac{1}{3} i \sqrt{2} k t_1$
$f_{1-}^{\#1} \dagger \alpha$	0	0	0	0	0	0
$f_{1-}^{\#2} \dagger \alpha$	0	0	$-\frac{1}{3} i k t_1$	$-\frac{1}{3} i \sqrt{2} k t_1$	0	$\frac{2 k^2 t_1}{3}$

Source constraints/gauge generators	SO(3) irreps	Multiplicities
	$\tau_{0+}^{\#2} == 0$	1
	$\tau_{0+}^{\#1} == 0$	1
	$\sigma_{0+}^{\#1} == 0$	1
	$\tau_{1-}^{\#2\alpha} + 2i k \sigma_{1-}^{\#1\alpha} == 0$	3
	$\tau_{1-}^{\#1\alpha} == 0$	3
	$\sigma_{1-}^{\#1\alpha} == \sigma_{1-}^{\#2\alpha}$	3
	$\tau_{1+}^{\#1\alpha\beta} + i k \sigma_{1+}^{\#2\alpha\beta} == 0$	3
	$\tau_{2+}^{\#1\alpha\beta} - 2i k \sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:		20

The diagram illustrates the decomposition of the tensor product of two fundamental representations of $SU(3)$ into irreducible representations. The left part shows the decomposition of $f_2^+ \times f_2^+$ into $f_2^+ + f_0^+ + f_0^-$. The right part shows the decomposition of $f_0^+ \times f_0^+$ into $f_0^+ + f_0^- + f_2^+$.

Left Part: $f_2^+ \times f_2^+$ decomposition

The product representation $f_2^+ \times f_2^+$ is shown as a box containing $\frac{t_1}{2}$ and $-\frac{ik t_1}{\sqrt{2}}$. This decomposes into the sum of three irreducible representations: f_2^+ (containing $\frac{ik t_1}{\sqrt{2}}$ and $k^2 t_1$), f_0^+ (containing 0), and f_0^- (containing $\frac{t_1}{2}$).

Right Part: $f_0^+ \times f_0^+$ decomposition

The product representation $f_0^+ \times f_0^+$ is shown as a box containing 0 and 0. This decomposes into the sum of three irreducible representations: f_0^+ (containing 0), f_0^- (containing 0), and f_2^+ (containing $k^2 r_2 + t_2$).

$\sigma_2^{\#1} \dagger \alpha\beta$	$\frac{2}{(1+2k^2)^2 t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	$\sigma_2^{\#1} \alpha\beta\chi$	0
$\tau_2^{\#1} \dagger \alpha\beta$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	$\frac{4k^2}{(1+2k^2)^2 t_1}$		0
$\sigma_2^{\#1} \dagger \alpha\beta\chi$	0	0		$\frac{2}{t_1}$

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0+}^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	0	0	0	0
$\tau_{0+}^{\#1} \dagger$	0	0	0	0
$\tau_{0+}^{\#2} \dagger$	0	0	0	0
$\sigma_{0+}^{\#1} \dagger$	0	0	0	$\frac{1}{k^2 r_2 + t_2}$