# Particle spectrograph

## Wave operator and propagator

| Source constraints | | |
|---|---|---|
| SO(3) irreps | Fundamental fields | Multiplicities |
| $\tau_{0^+}^{\#2} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$ | 1 |
| $\tau_{0^+}^{\#1} - 2\,i\,k\,\sigma_{0^+}^{\#1} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha + 2\,\partial_\chi \partial^\chi \partial_\beta \sigma^{\alpha\beta}{}_\alpha$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$ | $\partial_\chi \partial_\beta \sigma^{\alpha\beta} \tau^{\#2}{}_{\alpha\beta} == \partial_\chi \partial^\chi \partial_\beta \tau^{\#2}{}_\alpha{}^\beta + 2\,\partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | $\partial_\chi \partial_\beta \sigma^{\alpha\beta} \tau^{\#2}{}_{\alpha\beta} == \partial_\chi \partial^\chi \partial_\beta \tau^{\#2}{}_\alpha{}^\beta$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$ | $\partial_\chi \partial^\alpha \tau^{\#2}{}_\alpha{}^\beta{}_\chi + \partial_\chi \partial^\chi \tau^{\#2}{}_\alpha{}^{\alpha\beta} + 2\,\partial_\delta \partial_\chi \partial^\alpha \sigma^{\#2}{}_\alpha{}^{\beta\chi\delta} + 2\,\partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\chi \partial^\alpha \tau^{\#2}{}_\alpha{}^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\#2}{}_\alpha{}^{\alpha\chi} + \partial_\chi \partial^\chi \tau^{\#2}{}_\alpha{}^{\beta\alpha} + 2\,\partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$ | 3 |
| $\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$ | $-i\,(4\,\partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2\,\partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi{}_\chi - 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\chi} - 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi} - 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} + 4\,i\,k^\chi\,\partial_\epsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta\epsilon}{}_\delta - 6\,i\,k^\chi\,\partial_\epsilon \partial_\chi \partial^\alpha \sigma^{\beta\delta\epsilon} - 6\,i\,k^\chi\,\partial_\epsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta\epsilon} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} + 6\,i\,k^\chi\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\alpha\delta\beta} + 6\,i\,k^\chi\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\beta\delta\alpha} - 2\,\eta^{\alpha\beta}\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \tau^\chi{}_\chi + 4\,i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi \partial^\phi \partial_\epsilon \partial_\chi \sigma^{\delta\epsilon}{}_\delta) == 0$ | 5 |
| Total constraints/gauge generators: | | 16 |

Quadratic (free) action

$$S == \iiint\int (f^{\alpha\beta}\,\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + \frac{1}{2}\,t_1\,(2\,\omega^{\alpha\prime}{}_\alpha{}^\theta{}_{,\theta}\,\omega^{\alpha\prime}{}_\alpha{}^\theta{}_{,\theta} - 4\,\omega^\theta{}_{,\theta}\,\partial_\prime f^{\alpha\prime} + 4\,\omega^\theta{}_{,\theta}\,\partial^\prime f^\alpha{}_\alpha{}^\theta - 2\,\partial_\prime f^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha{}^\theta - 2\,\partial_\theta f^{\alpha\prime}{}_\alpha{}^\theta\,\partial_\theta f^\prime{}_\alpha{}^\theta + 4\,\partial^\prime f^\alpha{}_\alpha{}^\theta\,\partial_\theta f^\prime{}_\theta - 2\,\partial_\alpha f^\theta{}_{\iota\theta}\,\partial^\theta f^{\alpha\prime}{}_\iota{}^\theta - \partial_\alpha f_{\theta\iota}\,\partial^\theta f^{\alpha\prime\iota} - \partial_\theta f^{\alpha\prime}{}_\alpha{}^\theta + 2\,\partial^\theta f^{\alpha\prime\prime})) - \frac{1}{3}\,r_1\,(3\,\partial_\beta \omega^\prime{}_\alpha{}^\theta{}_\theta\,\partial^\prime \omega^{\alpha\beta}{}_\alpha{}^\theta - 3\,\partial_\prime \omega^\theta{}_\alpha{}^{\alpha\beta}\,\partial_\theta \omega^\prime{}_\beta{}^\theta + 6\,\partial^\prime \omega^{\alpha\beta}{}_\alpha{}^\theta\,\partial_\theta \omega^\prime{}_\beta{}^\theta + 3\,\partial_\alpha \omega^{\alpha\beta\prime}{}_\beta{}^\theta\,\partial_\theta \omega^\prime{}_\alpha{}^\theta - 6\,\partial^\prime \omega^{\alpha\beta}{}_\alpha{}^\theta\,\partial_\theta \omega^\prime{}_{\beta\,,\theta} + 2\,\partial_\beta \omega^\theta{}_{,\theta}\,\partial^\theta \omega^{\alpha\beta\prime} - 2\,\partial_\beta \omega_{\alpha\theta\prime}\,\partial^\theta \omega^{\alpha\beta\prime} + 8\,\partial_\beta \omega_{\prime\theta\alpha}\,\partial^\theta \omega^{\alpha\beta\prime} + 2\,\partial_\prime \omega_{\alpha\beta\theta}\,\partial^\theta \omega^{\alpha\beta\prime} - 2\,\partial_\theta \omega_{\alpha\beta\prime}\,\partial^\theta \omega^{\alpha\beta\prime} - 2\,\partial_\theta \omega_{\alpha\prime\beta}\,\partial^\theta \omega^{\alpha\beta\prime}))[t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt$$

$\sigma_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^+}^{\#2\,\alpha\beta}$ | $\tau_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^-}^{\#1\,\alpha}$ | $\sigma_{1^-}^{\#2\,\alpha}$ | $\tau_{1^-}^{\#1\,\alpha}$ | $\tau_{1^-}^{\#2\,\alpha}$

| $\sigma_{1^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | 0 | $-\frac{\sqrt{2}}{t_1+k^2\,t_1}$ | $-\frac{i\sqrt{2}\,k}{t_1+k^2\,t_1}$ | 0 | 0 | 0 | 0 |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#2\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $-\frac{\sqrt{2}}{t_1+k^2\,t_1}$ | $-\frac{2\,k^2\,r_1+t_1}{(1+k^2)^2\,t_1^2}$ | $-\frac{i\,(2\,k^3\,r_1+k\,t_1)}{(1+k^2)^2\,t_1^2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $\frac{i\sqrt{2}\,k}{t_1+k^2\,t_1}$ | $\frac{i\,(2\,k^3\,r_1+k\,t_1)}{(1+k^2)^2\,t_1^2}$ | $\frac{-2\,k^4\,r_1+k^2\,t_1}{(1+k^2)^2\,t_1^2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1\,\alpha}\,\dagger^\alpha$ | 0 | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2\,k^2\,t_1}$ | 0 | $\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$ |
| $\sigma_{1^-}^{\#2\,\alpha}\,\dagger^\alpha$ | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2\,k^2\,t_1}$ | $\frac{1}{(1+2\,k^2)^2\,t_1}$ | 0 | $\frac{i\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ |
| $\tau_{1^-}^{\#1\,\alpha}\,\dagger^\alpha$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2\,\alpha}\,\dagger^\alpha$ | 0 | 0 | 0 | $-\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$ | $-\frac{i\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ | 0 | $\frac{2\,k^2}{(1+2\,k^2)^2\,t_1}$ |

| $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|
| $-\frac{1}{(1+2\,k^2)^2\,t_1}$ | $\frac{i\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ | 0 | 0 |
| $-\frac{i\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ | $-\frac{2\,k^2}{(1+2\,k^2)^2\,t_1}$ | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $-\frac{1}{t_1}$ |

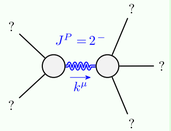
| $\sigma_{2^+}^{\#1\,\alpha\beta}$ | $\tau_{2^+}^{\#1\,\alpha\beta}$ | $\sigma_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|
| $\frac{2}{(1+2\,k^2)^2\,t_1}$ | $-\frac{2\,i\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ | 0 |
| $\frac{2\,i\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ | $\frac{4\,k^2}{(1+2\,k^2)^2\,t_1}$ | 0 |
| 0 | 0 | $\frac{2}{2\,k^2\,r_1+t_1}$ |

| $\omega_{1^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $k^2\,r_1 - \frac{t_1}{2}$ | $-\frac{t_1}{\sqrt{2}}$ | $\frac{i\,k\,t_1}{\sqrt{2}}$ | 0 | $-\frac{t_1}{2}$ | $\frac{t_1}{\sqrt{2}}$ | 0 | $-i\,k\,t_1$ |
|---|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#2\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $-\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 | $-\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 |
| $f_{1^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $\frac{i\,k\,t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1\,\alpha}\,\dagger^\alpha$ | 0 | 0 | 0 | 0 | $i\,k\,t_1$ | 0 | 0 | 0 |
| $\omega_{1^-}^{\#2\,\alpha}\,\dagger^\alpha$ | 0 | 0 | 0 | $\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#1\,\alpha}\,\dagger^\alpha$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2\,\alpha}\,\dagger^\alpha$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| $\omega_{0^+}^{\#1}\,\dagger$ | $-t_1$ | $i\sqrt{2}\,k\,t_1$ | 0 | 0 |
|---|---|---|---|---|
| $f_{0^+}^{\#1}\,\dagger$ | $-i\sqrt{2}\,k\,t_1$ | $-2\,k^2\,t_1$ | 0 | 0 |
| $f_{0^+}^{\#2}\,\dagger$ | 0 | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1}\,\dagger$ | 0 | 0 | 0 | $-t_1$ |

| $\omega_{2^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{i\,k\,t_1}{\sqrt{2}}$ | 0 |
|---|---|---|---|
| $f_{2^+}^{\#1\,\alpha\beta}\,\dagger^{\alpha\beta}$ | $\frac{i\,k\,t_1}{\sqrt{2}}$ | $k^2\,t_1$ | 0 |
| $\omega_{2^-}^{\#1\,\alpha\beta\chi}\,\dagger^{\alpha\beta\chi}$ | 0 | 0 | $k^2\,r_1 + \frac{t_1}{2}$ |

## Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_1} > 0$ |
| Polarisations: | 5 |
| Square mass: | $-\frac{t_1}{2\,r_1} > 0$ |
| Spin: | 2 |
| Parity: | Odd |

(No massless particles)

## Unitarity conditions

$r_1 < 0$ && $t_1 > 0$