

Wave operator and propagator

Source constraints		Fundamental fields	Multiplicities
$\tau_{0+}^{\#2} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$		1
$\sigma_{0+}^{\#1} == 0$	$\partial_\beta \sigma^{\alpha\beta}{}_\alpha == 0$		1
$\tau_{1-}^{\#2\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta}$		3
$\tau_{1-}^{\#1\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$		3
$\sigma_{1-}^{\#2\alpha} == 0$	$\partial_\chi \partial_\beta \sigma^{\alpha\beta\chi} == 0$		3
$\sigma_{1-}^{\#1\alpha} == 0$	$\partial_\chi \partial^\alpha \sigma^{\beta\chi}{}_\beta + \partial_\chi \partial^\chi \sigma^{\alpha\beta}{}_\beta == \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$		3
$\tau_{1+}^{\#1\alpha\beta} == 0$	$\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\chi\alpha} + \partial_\chi \partial^\chi \tau^{\alpha\beta} == \partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} + \partial_\chi \partial^\chi \tau^{\beta\alpha}$		3
$\sigma_{1+}^{\#2\alpha\beta} == 0$	$\partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$		3
$\sigma_{1+}^{\#1\alpha\beta} == 0$	$\partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\chi\beta} == \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\beta\chi\alpha}$		3
$\sigma_{2+}^{\#1\alpha\beta} == 0$	$2 \partial_\delta \partial^\beta \partial^\alpha \sigma^{\chi\delta}{}_\chi + 3 (\partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\chi\beta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\beta\chi\alpha}) == 3 \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 3 \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta} + 2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\delta \sigma^{\chi\delta}{}_\chi$		5
$\sigma_{2-}^{\#1\alpha\beta\chi} == 0$	$3 \partial_\epsilon \partial_\delta \partial^\chi \partial^\alpha \sigma^{\beta\delta\epsilon} + 3 \partial_\epsilon \partial^\epsilon \partial^\chi \partial^\alpha \sigma^{\beta\delta}{}_\delta + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\chi\delta} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\delta\chi} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\chi\delta\alpha} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\alpha\beta\delta} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\alpha\delta\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\beta\chi\alpha} + 3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\alpha \sigma^{\delta\epsilon}{}_\delta + 3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\beta\delta\epsilon} + 3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\alpha \sigma^{\delta\epsilon}{}_\delta + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\alpha\delta}{}_\delta + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\chi\delta} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\delta\chi} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\chi\delta\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\beta\delta\alpha} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\beta\chi} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\chi\beta} + 3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\delta\epsilon}{}_\delta + 3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\alpha\delta\epsilon} + 3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\delta\epsilon}{}_\delta$		33
Total constraints/gauge generators:			33

Quadratic (free) action

$$S = \int \int \int \int (f^{\alpha\beta} \tau_{\alpha\beta} + \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - 4 \omega_{\alpha}^{\chi} \omega_{\chi}^{\alpha} \partial_{\beta} f^{\alpha\beta} + 4 \partial_{\beta} \omega^{\alpha\beta} + 4 \omega_{\beta}^{\chi} \omega_{\chi}^{\beta} f^{\alpha} - 2 \partial_{\beta} f^{\chi} \partial_{\chi} f^{\alpha} - 2 \partial_{\beta} f^{\alpha\beta} \partial_{\chi} f_{\alpha}^{\chi} + 4 \partial_{\beta} f^{\alpha} \partial_{\chi} f_{\beta}^{\chi} - 4 f^{\alpha\beta} (\partial_{\beta} \omega_{\alpha}^{\chi} \omega_{\chi}^{\alpha} - \partial_{\chi} \omega_{\alpha}^{\chi}) - 4 f_{\alpha}^{\alpha} \partial_{\chi} \omega^{\beta\chi} + 4 \omega_{\alpha\chi\beta} \partial^{\chi} f^{\alpha\beta} - 2 \partial_{\alpha} f_{\beta\chi} \partial^{\chi} f^{\alpha\beta} - \partial_{\alpha} f_{\chi\beta} \partial^{\chi} f^{\alpha\beta} + \partial_{\beta} f_{\alpha\chi} \partial^{\chi} f^{\alpha\beta} + \partial_{\chi} f_{\alpha\beta} \partial^{\chi} f^{\alpha\beta} + \partial_{\chi} f_{\beta\alpha} \partial^{\chi} f^{\alpha\beta}) + \frac{1}{3} \alpha_3 (4 \partial_{\beta} \omega_{\alpha\chi\delta} - 2 \partial_{\beta} \omega_{\alpha\delta\chi} + 2 \partial_{\beta} \omega_{\chi\delta\alpha} - \partial_{\chi} \omega_{\alpha\beta\delta} + \partial_{\delta} \omega_{\alpha\beta\chi} - 2 \partial_{\delta} \omega_{\alpha\chi\beta}) \partial^{\delta} \omega^{\alpha\beta\chi}) [t, x, y, z] dz dy dx dt$$

Figure 1 displays the Clebsch-Gordan coefficients for the addition of angular momenta, organized into several tables. The tables are color-coded: pink for non-zero coefficients and light blue for zero coefficients.

Table 1 (Top Left): Coefficients for the addition of two spin-1/2 particles (labeled with σ and τ).

	$\sigma_0^{#1} \uparrow$	$\tau_0^{#2} \uparrow$	$\tau_0^{#1} \uparrow$	$\sigma_0^{#1} \uparrow$
$\sigma_0^{#1} \uparrow$	0	0	0	0
$\tau_0^{#1} \uparrow$	0	0	$-\frac{1}{4\beta_1 k^2}$	0
$\tau_0^{#2} \uparrow$	0	0	0	0
$\sigma_0^{#1} \uparrow$	0	0	0	$\frac{1}{a_3 k^2}$

Table 2 (Top Right): Coefficients for the addition of two spin-1 particles (labeled with ω , f , and ω).

	$\omega_{0+}^{#1}$	$f_{0+}^{#1}$	$f_{0+}^{#2}$	$\omega_{0+}^{#1}$
$\omega_{0+}^{#1} \uparrow$	0	0	0	0
$f_{0+}^{#1} \uparrow$	0	$-4\beta_1 k^2$	0	0
$f_{0+}^{#2} \uparrow$	0	0	0	0
$\omega_{0+}^{#1} \uparrow$	0	0	0	$\alpha_3 k^2$

Table 3 (Middle Right): Coefficients for the addition of two spin-1 particles (labeled with ω , f , and ω).

	$\omega_{2+}^{#1} \uparrow \alpha \beta$	$f_{2+}^{#1} \uparrow \alpha \beta$	$\omega_{2+}^{#1} \uparrow \alpha \beta \chi$
$\omega_{2+}^{#1} \uparrow \alpha \beta$	0	0	0
$f_{2+}^{#1} \uparrow \alpha \beta$	0	$2\beta_1 k^2$	0
$\omega_{2+}^{#1} \uparrow \alpha \beta \chi$	0	0	0

Table 4 (Bottom Left): Coefficients for the addition of two spin-1/2 particles (labeled with σ and τ).

	$\omega_{1+}^{#1} \uparrow \alpha \beta$	$\omega_{1+}^{#2} \uparrow \alpha \beta$	$f_{1+}^{#1} \uparrow \alpha \beta$	$\omega_{1-}^{#1} \uparrow \alpha$	$\omega_{1-}^{#2} \uparrow \alpha$	$f_{1-}^{#1} \uparrow \alpha$	$f_{1-}^{#2} \uparrow \alpha$
$\omega_{1+}^{#1} \uparrow \alpha \beta$	0	0	0	0	0	0	0
$\omega_{1+}^{#2} \uparrow \alpha \beta$	0	0	0	0	0	0	0
$f_{1+}^{#1} \uparrow \alpha \beta$	0	0	0	0	0	0	0
$\omega_{1-}^{#1} \uparrow \alpha$	0	0	0	0	0	0	0
$\omega_{1-}^{#2} \uparrow \alpha$	0	0	0	0	0	0	0
$f_{1-}^{#1} \uparrow \alpha$	0	0	0	0	0	0	0
$f_{1-}^{#2} \uparrow \alpha$	0	0	0	0	0	0	0

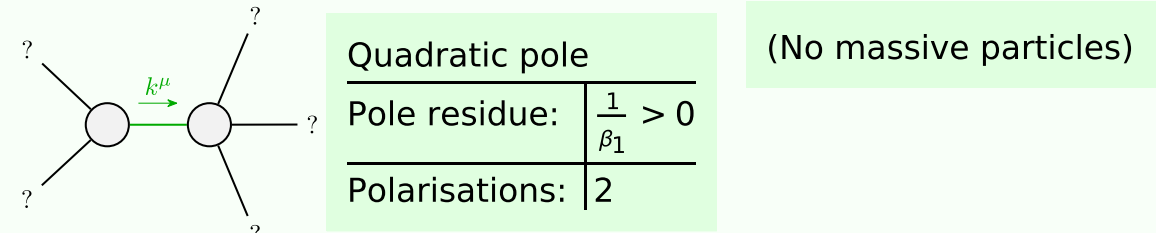
Table 5 (Bottom Right): Coefficients for the addition of two spin-1/2 particles (labeled with σ and τ).

	$\sigma_{1+}^{#1} \uparrow \alpha \beta$	$\sigma_{1+}^{#2} \uparrow \alpha \beta$	$\tau_{1+}^{#1} \uparrow \alpha \beta$	$\sigma_{1-}^{#1} \uparrow \alpha$	$\sigma_{1-}^{#2} \uparrow \alpha$	$\tau_{1-}^{#1} \uparrow \alpha$	$\tau_{1-}^{#2} \uparrow \alpha$
$\sigma_{1+}^{#1} \uparrow \alpha \beta$	0	0	0	0	0	0	0
$\sigma_{1+}^{#2} \uparrow \alpha \beta$	0	0	0	0	0	0	0
$\tau_{1+}^{#1} \uparrow \alpha \beta$	0	0	0	0	0	0	0
$\sigma_{1-}^{#1} \uparrow \alpha$	0	0	0	0	0	0	0
$\sigma_{1-}^{#2} \uparrow \alpha$	0	0	0	0	0	0	0
$\tau_{1-}^{#1} \uparrow \alpha$	0	0	0	0	0	0	0
$\tau_{1-}^{#2} \uparrow \alpha$	0	0	0	0	0	0	0

Table 6 (Bottom Right): Coefficients for the addition of two spin-1/2 particles (labeled with σ and τ).

	$\sigma_{2+}^{#1} \uparrow \alpha \beta$	$\tau_{2+}^{#1} \uparrow \alpha \beta$	$\sigma_{2-}^{#1} \uparrow \alpha \beta \chi$
$\sigma_{2+}^{#1} \uparrow \alpha \beta$	0	0	0

Massive and massless spectra



Unitarity conditions

$$\beta_1 > 0$$