

# Particle spectrograph

# Wave operator and propagator

$\Delta_{1+\alpha\beta}^{\#1}$	0	$-\frac{2\sqrt{2}}{a_0}$	0	$\Delta_{1+\alpha\beta}^{\#3}$	0	$\Delta_{1-\alpha}^{\#1}$	0	$\Delta_{1-\alpha}^{\#2}$	0	$\Delta_{1-\alpha}^{\#3}$	0	$\Delta_{1-\alpha}^{\#4}$	0	$\Delta_{1-\alpha}^{\#5}$	0	$\Delta_{1-\alpha}^{\#6}$	0	$\mathcal{I}_{1-\alpha}^{\#1}$	0
$\Delta_{1+\alpha\beta}^{\#2}$	$-\frac{2\sqrt{2}}{a_0}$	$\frac{2}{a_0}$	0	0	0	$\Delta_{1-\alpha}^{\#1}$	0	$\Delta_{1-\alpha}^{\#2}$	0	$\Delta_{1-\alpha}^{\#3}$	0	$\Delta_{1-\alpha}^{\#4}$	0	$\Delta_{1-\alpha}^{\#5}$	0	$\Delta_{1-\alpha}^{\#6}$	0	$\mathcal{I}_{1-\alpha}^{\#2}$	0
$\Delta_{1+\alpha\beta}^{\#3}$	0	$\frac{4}{a_0}$	0	0	0	$\Delta_{1-\alpha}^{\#1}$	0	$\Delta_{1-\alpha}^{\#2}$	0	$\Delta_{1-\alpha}^{\#3}$	0	$\Delta_{1-\alpha}^{\#4}$	0	$\Delta_{1-\alpha}^{\#5}$	0	$\Delta_{1-\alpha}^{\#6}$	0	$\mathcal{I}_{1-\alpha}^{\#3}$	0
$\Delta_{1-}^{\#1}$	0	0	0	0	$\frac{2\sqrt{2}}{a_0}$	$\Delta_{1-}^{\#1}$	0	$\Delta_{1-}^{\#2}$	$\frac{2\sqrt{2}}{a_0}$	$\Delta_{1-}^{\#3}$	0	$\Delta_{1-}^{\#4}$	0	$\Delta_{1-}^{\#5}$	0	$\Delta_{1-}^{\#6}$	0	$\mathcal{I}_{1-}^{\#1}$	0
$\Delta_{1-}^{\#2}$	0	0	0	0	$\frac{2\sqrt{2}}{a_0}$	$\Delta_{1-}^{\#1}$	$\frac{2\sqrt{2}}{a_0}$	$\Delta_{1-}^{\#2}$	0	$\Delta_{1-}^{\#3}$	0	$\Delta_{1-}^{\#4}$	0	$\Delta_{1-}^{\#5}$	0	$\Delta_{1-}^{\#6}$	0	$\mathcal{I}_{1-}^{\#2}$	0
$\Delta_{1-}^{\#3}$	0	0	0	0	0	$\Delta_{1-}^{\#1}$	0	$\Delta_{1-}^{\#2}$	0	$\Delta_{1-}^{\#3}$	$-\frac{19}{12a_0}$	$\Delta_{1-}^{\#4}$	$\frac{5\sqrt{5}}{12a_0}$	$\Delta_{1-}^{\#5}$	$-\frac{1}{6\sqrt{2}a_0}$	$\Delta_{1-}^{\#6}$	$-\frac{1}{6a_0}$	$\mathcal{I}_{1-}^{\#3}$	0
$\Delta_{1-}^{\#4}$	0	0	0	0	0	$\Delta_{1-}^{\#1}$	0	$\Delta_{1-}^{\#2}$	0	$\Delta_{1-}^{\#3}$	$\frac{5\sqrt{5}}{12a_0}$	$\Delta_{1-}^{\#4}$	$\frac{1}{12a_0}$	$\Delta_{1-}^{\#5}$	$-\frac{\sqrt{5}}{6a_0}$	$\Delta_{1-}^{\#6}$	$-\frac{\sqrt{5}}{6a_0}$	$\mathcal{I}_{1-}^{\#4}$	0
$\Delta_{1-}^{\#5}$	0	0	0	0	0	$\Delta_{1-}^{\#1}$	0	$\Delta_{1-}^{\#2}$	0	$\Delta_{1-}^{\#3}$	$-\frac{1}{6\sqrt{2}a_0}$	$\Delta_{1-}^{\#4}$	$\frac{\sqrt{2}}{6a_0}$	$\Delta_{1-}^{\#5}$	$\frac{17}{6a_0}$	$\Delta_{1-}^{\#6}$	$-\frac{7}{3\sqrt{2}a_0}$	$\mathcal{I}_{1-}^{\#5}$	0
$\Delta_{1-}^{\#6}$	0	0	0	0	0	$\Delta_{1-}^{\#1}$	0	$\Delta_{1-}^{\#2}$	0	$\Delta_{1-}^{\#3}$	$-\frac{1}{6a_0}$	$\Delta_{1-}^{\#4}$	$-\frac{\sqrt{5}}{6a_0}$	$\Delta_{1-}^{\#5}$	$-\frac{7}{3\sqrt{2}a_0}$	$\Delta_{1-}^{\#6}$	$\frac{5}{3a_0}$	$\mathcal{I}_{1-}^{\#6}$	0
$\mathcal{I}_{1-}^{\#1}$	0	0	0	0	0	$\Delta_{1-}^{\#1}$	0	$\Delta_{1-}^{\#2}$	0	$\Delta_{1-}^{\#3}$	0	$\Delta_{1-}^{\#4}$	0	$\Delta_{1-}^{\#5}$	0	$\Delta_{1-}^{\#6}$	0	$\mathcal{I}_{1-}^{\#7}$	0

$\Gamma_{1+\alpha\beta}^{\#1}$	$\Gamma_{1+\alpha\beta}^{\#2}$	$\Gamma_{1+\alpha\beta}^{\#3}$	$\Gamma_{1-\alpha}^{\#1}$	$\Gamma_{1-\alpha}^{\#2}$	$\Gamma_{1-\alpha}^{\#3}$	$\Gamma_{1-\alpha}^{\#4}$	$\Gamma_{1-\alpha}^{\#5}$	$\Gamma_{1-\alpha}^{\#6}$	$\mu_{1-\alpha}^{\#1}$
$\#1 \uparrow_{1+}$	$-\frac{a_0}{4}$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0	0
$\#2 \uparrow_{1+}$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0	0	0
$\#3 \uparrow_{1+}$	0	$\frac{a_0}{4}$	0	0	0	0	0	0	0
$\Gamma_{1-}^{\#1 \uparrow + \alpha}$	0	0	$-\frac{a_0}{4}$	$\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0
$\Gamma_{1-}^{\#2 \uparrow + \alpha}$	0	0	$\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0
$\Gamma_{1-}^{\#3 \uparrow + \alpha}$	0	0	0	0	$-\frac{a_0}{3}$	$\frac{\sqrt{5}a_0}{6}$	$-\frac{a_0}{6\sqrt{2}}$	$-\frac{a_0}{6}$	0
$\Gamma_{1-}^{\#4 \uparrow + \alpha}$	0	0	0	0	$\frac{\sqrt{5}a_0}{6}$	$\frac{a_0}{3}$	$-\frac{1}{6}\sqrt{\frac{5}{2}}a_0$	$-\frac{\sqrt{5}a_0}{6}$	0
$\Gamma_{1-}^{\#5 \uparrow + \alpha}$	0	0	0	0	$-\frac{a_0}{6\sqrt{2}}$	$-\frac{1}{6}\sqrt{\frac{5}{2}}a_0$	$\frac{a_0}{3}$	$\frac{a_0}{6\sqrt{2}}$	0
$\Gamma_{1-}^{\#6 \uparrow + \alpha}$	0	0	0	0	$-\frac{a_0}{6}$	$-\frac{\sqrt{5}a_0}{6}$	$\frac{a_0}{6\sqrt{2}}$	$\frac{5a_0}{12}$	0
$\mu_{1-}^{\#1 \uparrow + \alpha}$	0	0	0	0	0	0	0	0	0

## Quadratic (free) action

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$$\begin{aligned} & \int \int \int \int \left( \frac{1}{8} h^{\alpha\beta} \mathcal{T}_{\alpha\beta} - 4 \Gamma^{\alpha\beta\chi} (a_0 \Gamma_{\beta\chi\alpha} - 2 \Delta_{\alpha\beta\chi} + a_0 \partial_\beta h_{\alpha\chi}) + 2 a_0 \Gamma^{\alpha\beta} \Gamma^{\alpha\beta} h^{\chi}{}_{\chi} - \right. \\ & 2 a_0 h^{\chi}{}_{\chi} \partial_\beta \Gamma^{\alpha}{}_{\beta} h^{\alpha}{}_{\chi} + 2 a_0 h^{\chi}{}_{\chi} \partial_\beta \Gamma^{\alpha\beta}{}_{\alpha} - 4 a_0 h_{\alpha\chi} \partial_\beta \Gamma^{\alpha\beta\chi} + \\ & 4 a_0 h^{\alpha\beta} \partial_\beta \partial_\alpha h^{\chi}{}_{\chi} - a_0 \partial_\beta h^{\chi}{}_{\chi} \partial^\beta h^{\alpha}{}_{\alpha} - 4 a_0 \partial_\alpha h^{\alpha\beta} \partial_\chi h^{\chi}{}_{\beta} + \\ & 4 a_0 \partial^\beta h^{\alpha}{}_{\alpha} \partial_\chi h^{\chi}{}_{\beta} + 2 a_0 \Gamma^{\alpha}{}_{\beta} \Gamma^{\alpha}{}_{\beta} (2 \Gamma^{\chi}{}_{\beta\chi} - \partial_\beta h^{\chi}{}_{\chi} + 2 \partial_\chi h^{\chi}{}_{\beta}) - \\ & 8 a_0 h^{\alpha\beta} \partial_\chi \partial_\beta h^{\chi}{}_{\alpha} + 2 a_0 h^{\alpha}{}_{\alpha} \partial_\chi \partial_\alpha h^{\beta\chi} + \\ & 4 a_0 h^{\alpha\beta} \partial_\chi \partial_\alpha h^{\chi}{}_{\beta} - 2 a_0 h^{\alpha}{}_{\alpha} \partial_\chi \partial^\chi h^{\beta}{}_{\beta} - \\ & 2 a_0 \partial_\beta h_{\alpha\chi} \partial^\chi h^{\alpha\beta} + 3 a_0 \partial_\chi h_{\alpha\beta} \partial^\chi h^{\alpha\beta} + \\ & \left. 4 a_0 h_{\beta\gamma} \partial^\chi \Gamma^{\alpha}{}_{\alpha}{}^{\beta\gamma} \right) [t, x, y, z] dz dy dx dt \end{aligned}$$

## Source constraints

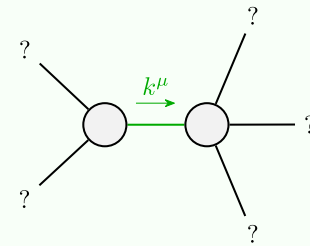
SO(3) irreps	Fundamental fields	Multiplicities
$\mathcal{T}_{0+}^{\#2} = 0$	$\partial_\beta \partial_\alpha \mathcal{T}^{\alpha\beta} = 0$	1
$\Delta_{0+}^{\#3} + 2\Delta_{0+}^{\#4} + 3\Delta_{0+}^{\#7} = 0$	$\partial_\alpha \Delta^{\alpha\beta}_\beta = 0$	1
$\mathcal{T}_{1-}^{\#1\alpha} = 0$	$\partial_\chi \partial_\beta \partial^\alpha \mathcal{T}^{\beta\chi} = \partial_\chi \partial^\chi \partial_\beta \mathcal{T}^{\alpha\beta}$	3
$2\Delta_{1-}^{\#6\alpha} + \Delta_{1-}^{\#4\alpha} + 2\Delta_{1-}^{\#5\alpha} + \Delta_{1-}^{\#3\alpha} = 0$	$\partial_\beta \partial^\alpha \Delta^{\beta\chi}_\chi = \partial_\chi \partial^\chi \Delta^{\alpha\beta}_\beta$	3
Total constraints/gauge generators:		8

$\Delta_0^{\#1} +$	$\Delta_0^{\#2} +$	$\Delta_0^{\#3} +$	$\Delta_0^{\#4} +$	$\mathcal{T}_0^{\#1} +$	$\mathcal{T}_0^{\#2} +$	$\Delta_0^{\#1} +$
$-\frac{2}{a_0}$	0	0	0	0	0	0
0	$-\frac{3}{4a_0}$	$\frac{5}{4a_0}$	$-\frac{1}{2\sqrt{2}a_0}$	0	0	0

$\Delta_2^{\#1} \dagger \alpha\beta$	$\frac{4}{a_0}$	0	0	0	0	0	0	0	0
$\Delta_2^{\#2} \dagger \alpha\beta$	0	$-\frac{2}{a_0}$	0	0	0	0	0	0	0
$\Delta_2^{\#3} \dagger \alpha\beta$	0	0	$\frac{4}{a_0}$	0	0	0	0	0	0
$\mathcal{T}_2^{\#1} \dagger \alpha\beta$	0	0	0	$-\frac{8}{a_0 k^2}$	0	0	0	0	0
$\Delta_2^{\#1} \dagger \alpha\beta_X$	0	0	0	0	0	$\frac{4}{a_0}$	0	0	0
$\Delta_2^{\#2} \dagger \alpha\beta_X$	0	0	0	0	0	0	0	$\frac{4}{a_0}$	0

$\Gamma_0^{\#1} \vdash$	$\Gamma_0^{\#2} \vdash$	$\Gamma_0^{\#3} \vdash$	$\Gamma_0^{\#4} \vdash$	$h_0^{\#1} \vdash$	$h_0^{\#2} \vdash$	$\Gamma_0^{\#1}$
$-\frac{a_0}{2}$	0	0	0	0	0	$-\frac{a_0}{2}$
0	0	$\frac{a_0}{2}$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0
0	$\frac{a_0}{2}$	0	$-\frac{a_0}{2\sqrt{2}}$	0	0	0
0	$-\frac{a_0}{2\sqrt{2}}$	$-\frac{a_0}{2\sqrt{2}}$	$\frac{a_0}{2}$	0	$\frac{a_0 k^2}{4}$	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
$\Gamma_0^{\#1} \vdash$	$\Gamma_0^{\#2} \vdash$	$\Gamma_0^{\#3} \vdash$	$\Gamma_0^{\#4} \vdash$	$h_0^{\#1} \vdash$	$h_0^{\#2} \vdash$	$\Gamma_0^{\#1} \vdash$
$\Gamma_{2^+}^{\#1} \vdash \alpha\beta$	$\Gamma_{2^+}^{\#2} \vdash \alpha\beta$	$\Gamma_{2^+}^{\#3} \vdash \alpha\beta$	$h_{2^+}^{\#1} \vdash \alpha\beta$	$\Gamma_{2^+}^{\#1} \vdash \alpha\beta\chi$	$\Gamma_{2^+}^{\#2} \vdash \alpha\beta\chi$	
$\frac{a_0}{4}$	0	0	0	0	0	
0	$-\frac{a_0}{2}$	0	0	0	0	
0	0	$\frac{a_0}{4}$	0	0	0	
$h_{2^+}^{\#1} \vdash \alpha\beta$	0	0	0	$-\frac{a_0 k^2}{8}$	0	
$\Gamma_{2^+}^{\#1} \vdash \alpha\beta\chi$	0	0	0	0	$\frac{a_0}{4}$	0
$\Gamma_{2^+}^{\#2} \vdash \alpha\beta\chi$	0	0	0	0	0	$\frac{a_0}{4}$

# Massive and massless spectra



Quadratic pole	
Pole residue:	$-\frac{1}{a_0} > 0$
Polarisations:	2

(No massive particles)

## Unitarity conditions

$$a_0 < 0$$