Particle spectrograph

Wave operator and propagator

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\tau^{\#2}_{0^+} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$	1
$\tau^{\#1}_{0^+} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha{}_\alpha$	1
$\sigma^{\#1}_{0^+} == 0$	$\partial_\beta \sigma^{\alpha\beta}{}_\alpha == 0$	1
$\tau^{\#2\,\alpha}_{1^-} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta}$	3
$\tau^{\#1\,\alpha}_{1^-} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$	3
$\sigma^{\#2\,\alpha}_{1^-} == 0$	$\partial_\chi \partial_\beta \sigma^{\alpha\beta\chi} == 0$	3
$\sigma^{\#1\,\alpha}_{1^-} == 0$	$\partial_\chi \partial^\alpha \sigma^{\beta\chi}{}_\beta + \partial_\chi \partial^\chi \sigma^{\alpha\beta}{}_\beta == \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$	3
$\tau^{\#1\,\alpha\beta}_{1^+} + i\, k\ \sigma^{\#2\,\alpha\beta}_{1^+} == 0$	$\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\chi\alpha} + \partial_\chi \partial^\chi \tau^{\alpha\beta} +$ $2\,\partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 2\,\partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} ==$ $\partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} +$ $\partial_\chi \partial^\chi \tau^{\beta\alpha} + 2\,\partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$	3
$\sigma^{\#1\,\alpha\beta\chi}_{2^-} == 0$	$3\,\partial_\epsilon \partial_\delta \partial^\chi \partial^\alpha \sigma^{\beta\delta\epsilon} + 3\,\partial_\epsilon \partial^\epsilon \partial^\chi \partial^\alpha \sigma^{\beta\delta}{}_\delta +$ $2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\chi\delta} + 4\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\delta\chi} +$ $2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\chi\delta\alpha} + 4\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\alpha\beta\delta} +$ $2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\alpha\delta\beta} + 2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\beta\chi\alpha} +$ $3\ \eta^{\beta\chi}\ \partial_\phi \partial^\phi \partial_\epsilon \partial^\alpha \sigma^{\delta\epsilon}{}_\delta +$ $3\ \eta^{\alpha\chi}\ \partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\beta\delta\epsilon} +$ $3\ \eta^{\beta\chi}\ \partial_\phi \partial^\phi \partial_\epsilon \partial^\epsilon \sigma^{\alpha\delta}{}_\delta ==$ $3\,\partial_\epsilon \partial_\delta \partial^\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 3\,\partial_\epsilon \partial^\epsilon \partial^\chi \partial^\beta \sigma^{\alpha\delta}{}_\delta +$ $2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\chi\delta} + 4\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\delta\chi} +$ $2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\chi\delta\beta} + 2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\beta\delta\alpha} +$ $4\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\beta\chi} + 2\,\partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\chi\beta} +$ $3\ \eta^{\alpha\chi}\ \partial_\phi \partial^\phi \partial_\epsilon \partial^\beta \sigma^{\delta\epsilon}{}_\delta +$ $3\ \eta^{\beta\chi}\ \partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\alpha\delta\epsilon} +$ $3\ \eta^{\alpha\chi}\ \partial_\phi \partial^\phi \partial_\epsilon \partial^\epsilon \sigma^{\beta\delta}{}_\delta$	5
$\tau^{\#1\,\alpha\beta}_{2^+} == 0$	$4\,\partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2\,\partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi{}_\chi +$ $3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} +$ $2\ \eta^{\alpha\beta}\ \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} ==$ $3\,\partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\chi} + 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\beta} +$ $3\,\partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi} + 3\,\partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\chi\alpha} +$ $2\ \eta^{\alpha\beta}\ \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \tau^\chi{}_\chi$	5
Total constraints/gauge generators:		28

Quadratic (free) action

$$
\begin{aligned}
\mathcal{S} == \iiiint \Big(\tfrac{1}{6}\big(6\ f^{\alpha\beta}\ \tau_{\alpha\beta} + 6\ \omega^{\alpha\beta\chi}\ \sigma_{\alpha\beta\chi} - 6\, r_3\, \partial_\beta \omega_\iota{}^\theta{}_\theta \partial^\iota \omega^{\alpha\beta}{}_\alpha - 6\, r_3\, \partial_\alpha \omega^{\alpha\beta\iota} \partial_\theta \omega_\iota{}^\theta{}_\beta + {} \\
12\, r_3\, \partial^\iota \omega^{\alpha\beta}{}_\alpha \partial_\theta \omega_\iota{}^\theta{}_\beta + 4\, t_2\ \omega_{\iota\theta\alpha}\ \partial^\theta f^{\alpha\iota} + 2\, t_2\, \partial_\alpha f_{\iota\theta} \partial^\theta f^{\alpha\iota} - {} \\
t_2\, \partial_\alpha f_{\theta\iota} \partial^\theta f^{\alpha\iota} - t_2\, \partial_\iota f_{\alpha\theta} \partial^\theta f^{\alpha\iota} + t_2\, \partial_\theta f_{\alpha\iota} \partial^\theta f^{\alpha\iota} - {} \\
t_2\, \partial_\theta f_{\iota\alpha} \partial^\theta f^{\alpha\iota} - 4\, t_2\ \omega_{\alpha\theta\iota}\ (\omega^{\alpha\iota\theta} + \partial^\theta f^{\alpha\iota}) + {} \\
2\, t_2\ \omega_{\alpha\iota\theta}\ (\omega^{\alpha\iota\theta} + 2\,\partial^\theta f^{\alpha\iota}) + 8\, r_2\, \partial_\beta \omega_{\alpha\iota\theta} \partial^\theta \omega^{\alpha\beta\iota} - {} \\
4\, r_2\, \partial_\beta \omega_{\alpha\theta\iota} \partial^\theta \omega^{\alpha\beta\iota} + 4\, r_2\, \partial_\beta \omega_{\iota\theta\alpha} \partial^\theta \omega^{\alpha\beta\iota} - 24\, r_3\, \partial_\beta \omega_{\iota\theta\alpha} \\
\partial^\theta \omega^{\alpha\beta\iota} - 2\, r_2\, \partial_\iota \omega_{\alpha\beta\theta} \partial^\theta \omega^{\alpha\beta\iota} + 2\, r_2\, \partial_\theta \omega_{\alpha\beta\iota} \partial^\theta \omega^{\alpha\beta\iota} - {} \\
4\, r_2\, \partial_\theta \omega_{\alpha\iota\beta} \partial^\theta \omega^{\alpha\beta\iota}\big)\Big)[t, x, y, z]\, dz\, dy\, dx\, dt
\end{aligned}
$$

	$\sigma^{\#1}_{1^+\,\alpha\beta}$	$\sigma^{\#2}_{1^+\,\alpha\beta}$	$\tau^{\#1}_{1^+\,\alpha\beta}$	$\sigma^{\#1}_{1^-\,\alpha}$	$\sigma^{\#2}_{1^-\,\alpha}$	$\tau^{\#1}_{1^-\,\alpha}$	$\tau^{\#2}_{1^-\,\alpha}$
$\sigma^{\#1}_{1^+}\dagger^{\alpha\beta}$	$\frac{2}{3\,k^2\,r_3}$	$-\frac{2\sqrt{2}}{3\,k^2\,r_3+3\,k^4\,r_3}$	$-\frac{2\,i\sqrt{2}}{3\,k\,r_3+3\,k^3\,r_3}$	0	0	0	0
$\sigma^{\#2}_{1^+}\dagger^{\alpha\beta}$	$-\frac{2\sqrt{2}}{3\,k^2\,r_3+3\,k^4\,r_3}$	$\frac{9\,k^2\,r_3+4\,t_2}{3\,(k+k^3)^2\,r_3\,t_2}$	$\frac{i\,(9\,k^2\,r_3+4\,t_2)}{3\,k\,(1+k^2)^2\,r_3\,t_2}$	0	0	0	0
$\tau^{\#1}_{1^+}\dagger^{\alpha\beta}$	$\frac{2\,i\sqrt{2}}{3\,k\,r_3+3\,k^3\,r_3}$	$-\frac{i\,(9\,k^2\,r_3+4\,t_2)}{3\,k\,(1+k^2)^2\,r_3\,t_2}$	$\frac{9\,k^2\,r_3+4\,t_2}{3\,(1+k^2)^2\,r_3\,t_2}$	0	0	0	0
$\sigma^{\#1}_{1^-}\dagger^{\alpha}$	0	0	0	0	0	0	0
$\sigma^{\#2}_{1^-}\dagger^{\alpha}$	0	0	0	0	0	0	0
$\tau^{\#1}_{1^-}\dagger^{\alpha}$	0	0	0	0	0	0	0
$\tau^{\#2}_{1^-}\dagger^{\alpha}$	0	0	0	0	0	0	0

	$\omega^{\#1}_{1^+\,\alpha\beta}$	$\omega^{\#2}_{1^+\,\alpha\beta}$	$f^{\#1}_{1^+\,\alpha\beta}$	$\omega^{\#1}_{1^-\,\alpha}$	$\omega^{\#2}_{1^-\,\alpha}$	$f^{\#1}_{1^-\,\alpha}$	$f^{\#2}_{1^-\,\alpha}$
$\omega^{\#1}_{1^+}\dagger^{\alpha\beta}$	$\frac{1}{6}\,(9\,k^2\,r_3+4\,t_2)$	$\frac{\sqrt{2}\,t_2}{3}$	$\frac{1}{3}\,i\sqrt{2}\ k\,t_2$	0	0	0	0
$\omega^{\#2}_{1^+}\dagger^{\alpha\beta}$	$\frac{\sqrt{2}\,t_2}{3}$	$\frac{t_2}{3}$	$\frac{i\,k\,t_2}{3}$	0	0	0	0
$f^{\#1}_{1^+}\dagger^{\alpha\beta}$	$-\frac{1}{3}\,i\sqrt{2}\ k\,t_2$	$-\frac{1}{3}\,i\,k\,t_2$	$\frac{k^2\,t_2}{3}$	0	0	0	0
$\omega^{\#1}_{1^-}\dagger^{\alpha}$	0	0	0	0	0	0	0
$\omega^{\#2}_{1^-}\dagger^{\alpha}$	0	0	0	0	0	0	0
$f^{\#1}_{1^-}\dagger^{\alpha}$	0	0	0	0	0	0	0
$f^{\#2}_{1^-}\dagger^{\alpha}$	0	0	0	0	0	0	0

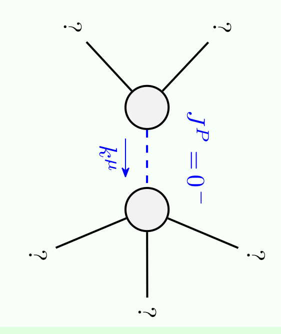
	$\omega^{\#1}_{2^+\,\alpha\beta}$	$f^{\#1}_{2^+\,\alpha\beta}$	$\omega^{\#1}_{2^-\,\alpha\beta\chi}$
$\omega^{\#1}_{2^+}\dagger^{\alpha\beta}$	$-\frac{3\,k^2\,r_3}{2}$	0	0
$f^{\#1}_{2^+}\dagger^{\alpha\beta}$	0	0	0
$\omega^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$	0	0	0

	$\sigma^{\#1}_{0^+}$	$\tau^{\#1}_{0^+}$	$\tau^{\#2}_{0^+}$	$\sigma^{\#1}_{0^-}$
$\sigma^{\#1}_{0^+}\dagger$	0	0	0	0
$\tau^{\#1}_{0^+}\dagger$	0	0	0	0
$\tau^{\#2}_{0^+}\dagger$	0	0	0	0
$\sigma^{\#1}_{0^-}\dagger$	0	0	0	$\frac{1}{k^2\,r_2+t_2}$

	$\omega^{\#1}_{0^+}$	$f^{\#1}_{0^+}$	$f^{\#2}_{0^+}$	$\omega^{\#1}_{0^-}$
$\omega^{\#1}_{0^+}\dagger$	0	0	0	0
$f^{\#1}_{0^+}\dagger$	0	0	0	0
$f^{\#2}_{0^+}\dagger$	0	0	0	0
$\omega^{\#1}_{0^-}\dagger$	0	0	0	$k^2\,r_2+t_2$

	$\sigma^{\#1}_{2^+\,\alpha\beta}$	$\tau^{\#1}_{2^+\,\alpha\beta}$	$\sigma^{\#1}_{2^-\,\alpha\beta\chi}$
$\sigma^{\#1}_{2^+}\dagger^{\alpha\beta}$	$-\frac{2}{3\,k^2\,r_3}$	0	0
$\tau^{\#1}_{2^+}\dagger^{\alpha\beta}$	0	0	0
$\sigma^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$	0	0	0

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_2 < 0\ \&\&\ t_2 > 0$