

# Particle spectrograph

## Wave operator and propagator

	$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2} \alpha\beta$	$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1-}^{\#1} \alpha$	$\sigma_{1-}^{\#2} \alpha$	$\tau_{1-}^{\#1} \alpha$	$\tau_{1-}^{\#2} \alpha$
$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	0	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$-\frac{i\sqrt{2}k}{t_1+k^2 t_1}$	0	0	0	0
$\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$\frac{-2k^2(2r_3+r_5)+t_1}{(1+k^2)^2 t_1^2}$	$\frac{-2ik^3(2r_3+r_5)+ikt_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{i\sqrt{2}k}{t_1+k^2 t_1}$	$\frac{i(2k^3(2r_3+r_5)-kt_1)}{(1+k^2)^2 t_1^2}$	$\frac{-2k^4(2r_3+r_5)+k^2 t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\sigma_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	$\frac{1}{k^2(2r_3+r_5)}$	$-\frac{1}{\sqrt{2}(k^2+2k^4)(2r_3+r_5)}$	0	$-\frac{i}{k(1+2k^2)(2r_3+r_5)}$
$\sigma_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$-\frac{1}{\sqrt{2}(k^2+2k^4)(2r_3+r_5)}$	$\frac{6k^2(2r_3+r_5)+t_1}{2(k+2k^3)^2(2r_3+r_5)t_1}$	0	$\frac{i(6k^2(2r_3+r_5)+t_1)}{\sqrt{2}k(1+2k^2)^2(2r_3+r_5)t_1}$
$\tau_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$\tau_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$\frac{i}{k(1+2k^2)(2r_3+r_5)}$	$-\frac{i(6k^2(2r_3+r_5)+t_1)}{\sqrt{2}k(1+2k^2)^2(2r_3+r_5)t_1}$	0	$\frac{6k^2(2r_3+r_5)+t_1}{(1+2k^2)^2(2r_3+r_5)t_1}$

	$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{2+}^{\#1} \dagger^{\alpha\beta\chi}$
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2 t_1}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	0
$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	$\frac{4k^2}{(1+2k^2)^2 t_1}$	0
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{t_1}$

	$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1+}^{\#2} \alpha\beta$	$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1-}^{\#1} \alpha$	$\omega_{1-}^{\#2} \alpha$	$f_{1-}^{\#1} \alpha$	$f_{1-}^{\#2} \alpha$
$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$k^2(2r_3+r_5)-\frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	$k^2(2r_3+r_5)+\frac{t_1}{6}$	$\frac{t_1}{3\sqrt{2}}$	0	$\frac{ikt_1}{3}$
$\omega_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	0	$\frac{1}{3}i\sqrt{2}kt_1$
$f_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$f_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$-\frac{1}{3}i\sqrt{2}kt_1$	$-\frac{1}{3}i\sqrt{2}kt_1$	0	$\frac{2k^2 t_1}{3}$

Quadratic (free) Lagrangian density

$$\begin{aligned}
 &-\frac{1}{3}t_1\omega_{\lambda}^{\alpha\lambda}\omega_{\kappa\alpha}^{\kappa}-t_1\omega_{\kappa\alpha}^{\kappa}\omega_{\lambda}^{\lambda}-t_1\omega_{\lambda}^{\lambda}\omega_{\kappa\alpha}^{\kappa}+f_{\alpha\beta}^{\alpha\beta}\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}-2r_3\partial_{\lambda}\omega_{\kappa}^{\kappa\lambda}\partial^{\lambda}\omega_{\lambda}^{\alpha}- \\
 &r_5\partial_{\lambda}\omega_{\kappa}^{\kappa\lambda}\partial^{\lambda}\omega_{\lambda}^{\alpha}+2r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\theta}^{\theta\kappa\lambda}-r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\theta}^{\theta\kappa\lambda}-2r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}+ \\
 &r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}-2r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\theta}^{\theta\kappa\lambda}-r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\theta}^{\theta\kappa\lambda}+4r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}+ \\
 &2r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\alpha}^{\kappa\lambda\theta}-\frac{1}{2}t_1\partial^{\alpha}f_{\theta\kappa}^{\kappa}f_{\alpha}^{\theta}-\frac{1}{2}t_1\partial^{\alpha}f_{\kappa\theta}^{\theta}f_{\alpha}^{\kappa}-\frac{1}{2}t_1\partial^{\alpha}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\alpha}^{\kappa}+\frac{1}{3}t_1\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\lambda}^{\lambda}+\frac{2}{3}t_1\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\lambda}-\frac{1}{3}t_1\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\lambda}+ \\
 &2t_1\omega_{\lambda\kappa\theta}\partial^{\kappa}f_{\theta}^{\theta\lambda}-\frac{1}{3}t_1\omega_{\lambda\alpha}^{\alpha}\partial^{\kappa}f_{\lambda}^{\lambda}-\frac{1}{3}t_1\omega_{\lambda\alpha}^{\alpha}\partial^{\kappa}f_{\lambda}^{\lambda}+\frac{1}{2}t_1\omega_{\lambda\alpha}^{\alpha}\partial^{\kappa}f_{\lambda}^{\lambda}+\frac{1}{2}t_1\omega_{\lambda\alpha}^{\alpha}\partial^{\kappa}f_{\lambda}^{\lambda}-\frac{1}{2}t_1\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\lambda}-4r_3\partial^{\beta}\omega_{\lambda}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda}- \\
 &2r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\theta}^{\theta\kappa}+r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\theta}^{\theta\kappa}+2r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\kappa}^{\theta\kappa}-r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\kappa}^{\theta\kappa}
 \end{aligned}$$

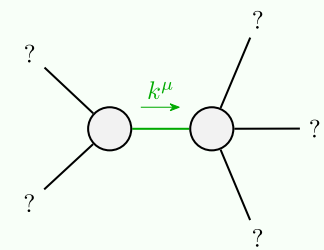
	$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{2-}^{\#1} \alpha\beta\chi$
$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	$k^2 t_1$	0
$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{t_1}{2}$

Source constraints/gauge generators	Multiplicities
$\text{SO}(3)$ irreps	
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	16

	$\omega_{0+}^{\#1}$	$f_{0+}^{\#1}$	$f_{0+}^{\#2}$	$\omega_{0-}^{\#1}$
$\omega_{0+}^{\#1} \dagger$	$6k^2 r_3$	0	0	0
$f_{0+}^{\#1} \dagger$	0	0	0	0
$f_{0+}^{\#2} \dagger$	0	0	0	0
$\omega_{0-}^{\#1} \dagger$	0	0	0	$-t_1$

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	$\frac{1}{6k^2 r_3}$	0	0	0
$\tau_{0+}^{\#1} \dagger$	0	0	0	0
$\tau_{0+}^{\#2} \dagger$	0	0	0	0
$\sigma_{0-}^{\#1} \dagger$	0	0	0	$-\frac{1}{t_1}$

## Massive and massless spectra



Quadratic pole	
Pole residue:	$-\frac{1}{(2r_3+r_5)t_1^2} > 0$
Polarisations:	2

(No massive particles)

## Unitarity conditions

$$r_5 < -2r_3 \ \&\& \ t_1 < 0 \ || \ t_1 > 0$$