

Particle spectrograph

Wave operator and propagator

Source constraints		Fundamental fields	Multiplicities
SO(3) irreps			
$\tau_{0+}^{\#2} == 0$		$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$	1
$\tau_{0+}^{\#1} - 2 i k \sigma_{0+}^{\#1} == 0$		$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial_\alpha \tau^\alpha_\alpha + 2 \partial_\chi \partial^\chi \partial_\beta \sigma^\alpha\beta_\alpha$	1
$\tau_{1-}^{\#2\alpha} + 2 i k \sigma_{1+}^{\#2\alpha} == 0$		$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial_\beta \partial_\theta \tau^{\epsilon\beta} + 2 \partial_\theta \partial^\theta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$	3
$\tau_{1-}^{\#1\alpha} == 0$		$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$	3
$\tau_{1+}^{\#1\alpha\beta} + i k \sigma_{1+}^{\#2\alpha\beta} == 0$		$\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^\alpha_\alpha + \partial_\chi \partial^\chi \tau^{\alpha\beta}_\alpha + 2 \partial_\theta \partial^\theta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\chi \partial^\alpha \tau^\chi_\beta + \partial_\chi \partial^\beta \tau^{\alpha\chi}_\alpha + \partial_\chi \partial^\chi \tau^{\beta\alpha}_\alpha + 2 \partial_\theta \partial^\theta \partial_\chi \sigma^{\alpha\beta\chi}$	3
$\tau_{2+}^{\#1\alpha\beta} - 2 i k \sigma_{2+}^{\#1\alpha\beta} == 0$		$-i (4 \partial_\theta \partial_\chi \partial^\beta \partial^\alpha \tau^\chi_\delta + 2 \partial_\theta \partial^\theta \partial^\beta \partial^\alpha \tau^\chi_\chi - 3 \partial_\theta \partial^\theta \partial_\chi \partial^\alpha \tau^{\beta\chi}_\alpha - 3 \partial_\theta \partial^\theta \partial_\chi \partial^\beta \tau^{\alpha\chi}_\alpha - 3 \partial_\theta \partial^\theta \partial_\chi \partial^\alpha \tau^{\beta\chi}_\alpha + 3 \partial_\theta \partial^\theta \partial_\chi \partial^\beta \tau^{\alpha\chi}_\alpha + 4 i k^\chi \partial_\epsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta\epsilon}_\delta - 6 i k^\chi \partial_\epsilon \partial_\theta \partial_\chi \partial^\alpha \sigma^{\beta\delta\epsilon}_\epsilon - 6 i k^\chi \partial_\epsilon \partial_\theta \partial_\chi \partial^\beta \sigma^{\alpha\delta\epsilon}_\epsilon + 2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\theta \partial_\chi \tau^\chi_\delta + 6 i k^\chi \partial_\epsilon \partial^\epsilon \partial_\theta \partial_\chi \sigma^{\alpha\delta\beta}_\beta + 6 i k^\chi \partial_\epsilon \partial^\epsilon \partial_\theta \partial_\chi \sigma^{\beta\delta\alpha}_\alpha - 2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\theta \partial^\chi \tau^\chi_\chi - 4 i \eta^{\alpha\beta} k^\chi \partial_\theta \partial^\theta \partial_\epsilon \partial_\chi \sigma^{\delta\epsilon}_\delta) == 0$	5
Total constraints/gauge generators:			16

Quadratic (free) action

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$$\iiint \iiint (\frac{1}{6} (6 t_1 \mathcal{A}^{\alpha i}_\alpha \mathcal{A}^{\theta}_{\theta} + 6 f^{\alpha\beta} \tau_{\alpha\beta} + 6 \mathcal{A}^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - 12 t_1 \mathcal{A}^{\theta}_{\alpha} \partial_{\theta} f^{\alpha i} + 12 t_1 \mathcal{A}^{\theta}_{\theta} \partial_{\theta} f^{\alpha}_{\alpha} - 6 t_1 \partial_{\theta} f^{\alpha}_{\alpha} \partial_{\theta} f^{\alpha}_{\alpha} - 6 t_1 \partial_{\theta} f^{\alpha}_{\theta} \partial_{\theta} f^{\alpha}_{\alpha} + 12 r_1 \partial_{\beta} \mathcal{A}^{\theta}_{\theta} \partial_{\theta} \mathcal{A}^{\alpha\beta}_{\alpha} + 12 r_1 \partial_{\theta} \mathcal{A}^{\theta}_{\beta} \partial_{\theta} \mathcal{A}^{\alpha\beta}_{\alpha} - 6 t_1 \partial_{\theta} f^{\alpha i} \partial_{\theta} f^{\alpha}_{\alpha} + 12 t_1 \partial_{\theta} f^{\alpha}_{\alpha} \partial_{\theta} f^{\theta}_{\theta} + 12 r_1 \partial_{\alpha} \mathcal{A}^{\alpha\beta i} \partial_{\theta} \mathcal{A}^{\theta}_{\beta} - 24 r_1 \partial_{\theta} \mathcal{A}^{\alpha\beta}_{\alpha} \partial_{\theta} \mathcal{A}^{\theta}_{\beta} - 12 r_1 \partial_{\alpha} \mathcal{A}^{\alpha\beta i} \partial_{\theta} \mathcal{A}^{\theta}_{\beta} + 24 r_1 \partial_{\theta} \mathcal{A}^{\alpha\beta}_{\alpha} \partial_{\theta} \mathcal{A}^{\theta}_{\beta} + 4 t_1 \mathcal{A}_{\theta\alpha} \partial^{\theta} f^{\alpha i} + 4 t_2 \mathcal{A}_{\theta\alpha} \partial^{\theta} f^{\alpha i} - 4 t_1 \partial_{\alpha} f_{\theta} \partial^{\theta} f^{\alpha i} + 2 t_2 \partial_{\alpha} f_{\theta} \partial^{\theta} f^{\alpha i} - 4 t_1 \partial_{\alpha} f_{\theta} \partial^{\theta} f^{\alpha i} - t_2 \partial_{\alpha} f_{\theta} \partial^{\theta} f^{\alpha i} + 2 t_1 \partial_{\theta} f_{\alpha\theta} \partial^{\theta} f^{\alpha i} - t_2 \partial_{\theta} f_{\alpha\theta} \partial^{\theta} f^{\alpha i} + 4 t_1 \partial_{\theta} f_{\alpha i} \partial^{\theta} f^{\alpha i} + 2 t_2 \partial_{\theta} f_{\alpha i} \partial^{\theta} f^{\alpha i} - t_2 \partial_{\theta} f_{\alpha i} \partial^{\theta} f^{\alpha i} + 2 (t_1 + t_2) \mathcal{A}_{\alpha i\theta} (\mathcal{A}^{\alpha i\theta} + 2 \partial^{\theta} f^{\alpha i}) + 2 \mathcal{A}_{\alpha\theta i} ((t_1 - 2 t_2) \mathcal{A}^{\alpha i\theta} + 2 (2 t_1 - t_2) \partial^{\theta} f^{\alpha i}) - 8 r_1 \partial_{\beta} \mathcal{A}_{\alpha\theta i} \partial^{\theta} \mathcal{A}^{\alpha\beta i} + 8 r_2 \partial_{\beta} \mathcal{A}_{\alpha\theta i} \partial^{\theta} \mathcal{A}^{\alpha\beta i} + 4 r_1 \partial_{\beta} \mathcal{A}_{\alpha\theta i} \partial^{\theta} \mathcal{A}^{\alpha\beta i} - 4 r_2 \partial_{\beta} \mathcal{A}_{\alpha\theta i} \partial^{\theta} \mathcal{A}^{\alpha\beta i} - 16 r_1 \partial_{\beta} \mathcal{A}_{\theta\alpha} \partial^{\theta} \mathcal{A}^{\alpha\beta i} + 4 r_2 \partial_{\beta} \mathcal{A}_{\theta\alpha} \partial^{\theta} \mathcal{A}^{\alpha\beta i} - 4 r_1 \partial_{\theta} \mathcal{A}_{\alpha\beta\theta} \partial^{\theta} \mathcal{A}^{\alpha\beta i} + 4 r_1 \partial_{\theta} \mathcal{A}_{\alpha\beta i} \partial^{\theta} \mathcal{A}^{\alpha\beta i} + 2 r_2 \partial_{\theta} \mathcal{A}_{\alpha\beta i} \partial^{\theta} \mathcal{A}^{\alpha\beta i} + 4 r_1 \partial_{\theta} \mathcal{A}_{\alpha\beta} \partial^{\theta} \mathcal{A}^{\alpha\beta i} - 4 r_2 \partial_{\theta} \mathcal{A}_{\alpha\beta} \partial^{\theta} \mathcal{A}^{\alpha\beta i})) [t, x, y, z] dz dy dx dt$$

$\sigma_{1+}^{\#1} + \alpha\beta$	$\sigma_{1+}^{\#2} + \alpha\beta$	$\tau_{1+}^{\#1} + \alpha\beta$	$\sigma_{1-}^{\#1} \alpha$	$\sigma_{1-}^{\#2} \alpha$	$\tau_{1-}^{\#1} \alpha$	$\tau_{1-}^{\#2} \alpha$
$\frac{2(t_1+t_2)}{3t_1t_2}$	$\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	0	0	0	0
$\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{t_1+4t_2}{3(1+k^2)^2t_1t_2}$	$\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0	0
$-\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$-\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	$\frac{k^2(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	0	$\frac{2ik}{t_1+2k^2t_1}$
0	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	0	$\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
0	0	0	0	0	0	0
0	0	0	0	$-\frac{2ik}{t_1+2k^2t_1}$	0	$\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$

$\mathcal{A}_{1+}^{\#1} + \alpha\beta$	$\mathcal{A}_{1+}^{\#2} + \alpha\beta$	$f_{1+}^{\#1} + \alpha\beta$	$\mathcal{A}_{1-}^{\#1} \alpha$	$\mathcal{A}_{1-}^{\#2} \alpha$	$f_{1-}^{\#1} \alpha$	$f_{1-}^{\#2} \alpha$
$\frac{1}{6}(t_1+4t_2)$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$-\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	0	0	0	0
$-\frac{t_1-2t_2}{3\sqrt{2}}$	$\frac{t_1+t_2}{3}$	$\frac{1}{3}ik(t_1+t_2)$	0	0	0	0
$\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	$-\frac{1}{3}ik(t_1+t_2)$	$\frac{1}{3}k^2(t_1+t_2)$	0	0	0	0
0	0	0	$-k^2r_1 - \frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$ik t_1$
0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
0	0	0	0	0	0	0
0	0	0	$-ik t_1$	0	0	0

$\sigma_{0+}^{\#1} + \alpha\beta$	$\tau_{0+}^{\#1} + \alpha\beta$	$f_{0+}^{\#2} + \alpha\beta$	$\mathcal{A}_{0+}^{\#1}$
$-\frac{1}{(1+2k^2)^2t_1}$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{i\sqrt{2}kt_1}{-2k^2t_1}$	0
$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0	0
0	0	0	0
0	0	0	$k^2r_2+t_2$

$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0+}^{\#1}$
$-\frac{1}{(1+2k^2)^2t_1}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	0	0
$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0	0
0	0	0	0
0	0	0	$\frac{1}{k^2r_2+t_2}$

$\sigma_{2+}^{\#1} + \alpha\beta$	$\tau_{2+}^{\#1} + \alpha\beta$	$f_{2+}^{\#2} + \alpha\beta$	$\mathcal{A}_{2+}^{\#1} + \alpha\beta$
$-\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{ik t_1}{\sqrt{2}}$	0
$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$-\frac{4k^2}{(1+2k^2)^2t_1}$	$k^2t_1$	0
0	0	0	$k^2r_1 + \frac{t_1}{2}$

$\sigma_{2+}^{\#1}$	$\tau_{2+}^{\#1}$	$\tau_{2+}^{\#2}$	$\sigma_{2+}^{\#1}$
$-\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	0	0
$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$-\frac{4k^2}{(1+2k^2)^2t_1}$	0	0
0	0	0	$\frac{2}{2k^2r_1+t_1}$

Massive and massless spectra

$J^P = 2^-$

Pole residue:  $-\frac{1}{r_1} > 0$

Polarisations: 5

Square mass:  $-\frac{t_1}{2r_1} > 0$

Spin: 2

Parity: Odd

$J^P = 0^-$

Pole residue:  $-\frac{1}{r_2} > 0$

Polarisations: 1

Square mass:  $-\frac{t_2}{r_2} > 0$

Spin: 0

Parity: Odd

(No massless particles)

Unitarity conditions

$r_1 < 0 \ \&\& \ r_2 < 0 \ \&\& \ t_1 > 0 \ \&\& \ t_2 > 0$