

Particle spectrograph

Wave operator and propagator

	$\omega_{1^+}^{\#1}{}_{\alpha\beta}$	$\omega_{1^+}^{\#2}{}_{\alpha\beta}$	$f_{1^+}^{\#1}{}_{\alpha\beta}$	$\omega_{1^+}^{\#1}{}_{\alpha}$	$\omega_{1^+}^{\#2}{}_{\alpha}$	$f_{1^+}^{\#1}{}_{\alpha}$	$f_{1^+}^{\#2}{}_{\alpha}$
$\omega_{1^+}^{\#1}{}_{\dagger\alpha\beta}$	$k^2(2r_3+r_5)+\frac{2t_2}{3}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{1}{3}i\sqrt{2}kt_2$	0	0	0	0
$\omega_{1^+}^{\#2}{}_{\dagger\alpha\beta}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{t_2}{3}$	$\frac{ikt_2}{3}$	0	0	0	0
$f_{1^+}^{\#1}{}_{\dagger\alpha\beta}$	$-\frac{1}{3}i\sqrt{2}kt_2$	$-\frac{1}{3}ikt_2$	$\frac{k^2t_2}{3}$	0	0	0	0
$\omega_{1^+}^{\#1}{}_{\dagger\alpha}$	0	0	0	$k^2(\frac{r_3}{2}+r_5)+\frac{2t_3}{3}$	$-\frac{\sqrt{2}t_3}{3}$	0	$-\frac{2}{3}ikt_3$
$\omega_{1^+}^{\#2}{}_{\dagger\alpha}$	0	0	0	$-\frac{\sqrt{2}t_3}{3}$	$\frac{t_3}{3}$	0	$\frac{1}{3}i\sqrt{2}kt_3$
$f_{1^+}^{\#1}{}_{\dagger\alpha}$	0	0	0	0	0	0	0
$f_{1^+}^{\#2}{}_{\dagger\alpha}$	0	0	0	$\frac{2ikt_3}{3}$	$-\frac{1}{3}i\sqrt{2}kt_3$	0	$\frac{2k^2t_3}{3}$

$\sigma_0^{\#1}{}_{\dagger}$	$\tau_0^{\#1}{}_{\dagger}$	$\tau_0^{\#2}{}_{\dagger}$	$\sigma_0^{\#1}{}_{\dagger}$
$\frac{1}{(1+2k^2)^2}t_3$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_3$	0	0
$\frac{i\sqrt{2}k}{(1+2k^2)^2}t_3$	$\frac{2k^2}{(1+2k^2)^2}t_3$	0	0
0	0	0	0
0	0	0	$\frac{1}{k^2r_2+t_2}$

$\omega_0^{\#1}{}_{\dagger}$	$f_0^{\#1}{}_{\dagger}$	$f_0^{\#2}{}_{\dagger}$	$\omega_0^{\#1}{}_{\dagger}$
t_3	$-i\sqrt{2}kt_3$	0	0
$i\sqrt{2}kt_3$	$2k^2t_3$	0	0
0	0	0	0
0	0	0	$k^2r_2+t_2$

$\omega_{2^+}^{\#1}{}_{\alpha\beta}f_{2^+}^{\#1}{}_{\alpha\beta}\omega_{2^+}^{\#1}{}_{\alpha\beta\chi}$

$\omega_{2^+}^{\#1}{}_{\dagger\alpha\beta}$	$-\frac{3k^2r_3}{2}$	0	0
$f_{2^+}^{\#1}{}_{\dagger\alpha\beta}$	0	0	0
$\omega_{2^+}^{\#1}{}_{\dagger\alpha\beta\chi}$	0	0	0

$\sigma_{2^+}^{\#1}{}_{\alpha\beta}\tau_{2^+}^{\#1}{}_{\alpha\beta}\sigma_{2^+}^{\#1}{}_{\alpha\beta\chi}$

$\sigma_{2^+}^{\#1}{}_{\dagger\alpha\beta}$	$-\frac{2}{3k^2r_3}$	0	0
$\tau_{2^+}^{\#1}{}_{\dagger\alpha\beta}$	0	0	0
$\sigma_{2^+}^{\#1}{}_{\dagger\alpha\beta\chi}$	0	0	0

Source constraints	Fundamental fields	Multiplicities
SO(3) irreps		
$\tau_{0^+}^{\#2} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$	1
$\tau_{0^+}^{\#1}-2ik\sigma_{0^+}^{\#1} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^{\alpha}_{\alpha} + 2\partial_\chi\partial^{\chi}\partial_\beta\sigma^{\alpha\beta}_{\alpha}$	1
$\tau_{1^+}^{\#2\alpha} + 2ik\sigma_{1^+}^{\#2\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^{\chi}\partial_\beta\tau^{\alpha\beta} + 2\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$	3
$\tau_{1^+}^{\#1\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^{\chi}\partial_\beta\tau^{\beta\alpha}$	3
$\tau_{1^+}^{\#1\alpha\beta} + ik\sigma_{1^+}^{\#2\alpha\beta} == 0$	$\partial_\chi\partial^{\alpha,\beta\chi} + \partial_\chi\partial^{\beta,\chi\alpha} + \partial_\chi\partial^{\chi,\alpha\beta} +$ $2\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} ==$ $\partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\chi\alpha} +$ $\partial_\chi\partial^{\chi,\beta\alpha} + 2\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$	3
$\sigma_{2^+}^{\#1\alpha\beta\chi} == 0$	$3\partial_\epsilon\partial_\delta\partial^{\chi}\partial^\alpha\sigma^{\beta\delta\epsilon} + 3\partial_\epsilon\partial^\epsilon\partial^{\chi}\partial^\alpha\sigma^{\beta\delta}_{\delta} +$ $2\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\chi\delta} + 4\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\delta\chi} +$ $2\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\chi\delta\alpha} + 4\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\chi\alpha\delta} +$ $2\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\delta\beta} + 2\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\delta\beta\chi\alpha} +$ $3\eta^{\beta\chi}\partial_\phi\partial^{\phi}\partial_\epsilon\partial^\epsilon\sigma^{\delta\epsilon}_{\delta} +$ $3\eta^{\alpha\chi}\partial_\phi\partial^{\phi}\partial_\epsilon\partial_\delta\sigma^{\alpha\delta\epsilon} +$ $3\eta^{\beta\chi}\partial_\phi\partial^{\phi}\partial_\epsilon\partial^\epsilon\sigma^{\alpha\delta}_{\delta} ==$ $3\partial_\epsilon\partial_\delta\partial^{\chi}\partial^\beta\sigma^{\alpha\delta\epsilon} + 3\partial_\epsilon\partial^\epsilon\partial^{\chi}\partial^\beta\sigma^{\alpha\delta}_{\delta} +$ $2\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\chi\delta} + 4\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\delta\chi} +$ $2\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\chi\delta\beta} + 2\partial_\epsilon\partial^\epsilon\partial_\delta\partial^{\chi\delta\alpha} +$ $4\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\beta\chi} + 2\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\chi\beta} +$ $3\eta^{\alpha\chi}\partial_\phi\partial^{\phi}\partial_\epsilon\partial^\epsilon\sigma^{\delta\epsilon}_{\delta} +$ $3\eta^{\beta\chi}\partial_\phi\partial^{\phi}\partial_\epsilon\partial_\delta\sigma^{\alpha\delta\epsilon} +$ $3\eta^{\alpha\chi}\partial_\phi\partial^{\phi}\partial_\epsilon\partial^\epsilon\sigma^{\beta\delta}_{\delta}$	5
$\tau_{2^+}^{\#1\alpha\beta} == 0$	$4\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^{\chi}_{\chi} +$ $3\partial_\delta\partial^\delta\partial_\chi\partial^{\chi}\tau^{\alpha\beta} + 3\partial_\delta\partial^\delta\partial_\chi\partial^{\chi}\tau^{\beta\alpha} +$ $2\eta^{\alpha\beta}\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} ==$ $3\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} + 3\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} +$ $2\eta^{\alpha\beta}\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^{\chi}_{\chi}$	5
Total constraints/gauge generators:		21

Quadratic (free) action

$$S = \iiint \Big(\frac{1}{6} (-4t_3\omega_{\alpha}^{\alpha\iota}\omega_{\iota,\kappa}^{\kappa} + 6f^{\alpha\beta}\tau_{\alpha\beta} + 6\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi} + 8t_3\omega_{\alpha,\kappa}^{\kappa}\partial_{\iota}f^{\alpha\iota} -$$

$$8t_3\omega_{\iota,\kappa}^{\kappa}\partial_{\iota}f^{\alpha}_{\alpha} + 4t_3\partial_{\iota}f^{\kappa}_{\kappa}\partial_{\iota}f^{\alpha}_{\alpha} - 3r_3\partial_\beta\omega_{\iota,\theta}^{\theta}\partial^{\iota}\omega^{\alpha\beta}_{\alpha} -$$

$$3r_3\partial_{\iota}\omega_{\beta,\theta}^{\theta}\partial^{\iota}\omega^{\alpha\beta}_{\alpha} - 3r_3\partial_{\alpha}\omega^{\alpha\beta\iota}\partial_\theta\omega_{\beta,\iota}^{\theta} +$$

$$6r_3\partial^{\iota}\omega_{\alpha}^{\alpha\beta}\partial_\theta\omega_{\beta,\iota}^{\theta} - 3r_3\partial_{\alpha}\omega^{\alpha\beta\iota}\partial_\theta\omega_{\iota,\beta}^{\theta} +$$

$$6r_3\partial^{\iota}\omega_{\alpha}^{\alpha\beta}\partial_\theta\omega_{\iota,\beta}^{\theta} + 4t_2\omega_{\iota\theta\alpha}\partial^{\theta}f^{\alpha\iota} + 2t_2\partial_{\alpha}f^{\iota}_{\iota}\partial^{\theta}f^{\alpha\iota} -$$

$$t_2\partial_{\alpha}f_{\theta\iota}\partial^{\theta}f^{\alpha\iota} - t_2\partial_{\iota}f_{\alpha\theta}\partial^{\theta}f^{\alpha\iota} + t_2\partial_{\theta}f_{\alpha\iota}\partial^{\theta}f^{\alpha\iota} -$$

$$t_2\partial_{\theta}f_{\iota\alpha}\partial^{\theta}f^{\alpha\iota} - 4t_2\omega_{\alpha\theta\iota}(\omega^{\alpha\iota\theta} + \partial^{\theta}f^{\alpha\iota}) +$$

$$2t_2\omega_{\alpha\iota\theta}(\omega^{\alpha\iota\theta} + 2\partial^{\theta}f^{\alpha\iota}) + 8r_2\partial_\beta\omega_{\alpha\iota\theta}\partial^{\theta}\omega^{\alpha\beta\iota} -$$

$$4r_2\partial_\beta\omega_{\alpha\theta\iota}\partial^{\theta}\omega^{\alpha\beta\iota} + 4r_2\partial_\beta\omega_{\iota\theta\alpha}\partial^{\theta}\omega^{\alpha\beta\iota} -$$

$$24r_3\partial_\beta\omega_{\iota\theta\alpha}\partial^{\theta}\omega^{\alpha\beta\iota} - 2r_2\partial_{\iota}\omega_{\alpha\beta\theta}\partial^{\theta}\omega^{\alpha\beta\iota} +$$

$$2r_2\partial_\theta\omega_{\alpha\beta\iota}\partial^{\theta}\omega^{\alpha\beta\iota} - 4r_2\partial_\theta\omega_{\alpha\iota\beta}\partial^{\theta}\omega^{\alpha\beta\iota} +$$

$$6r_5\partial_{\iota}\omega_{\theta,\kappa}^{\kappa}\partial^{\theta}\omega_{\alpha}^{\alpha\iota} - 6r_5\partial_\theta\omega_{\iota,\kappa}^{\kappa}\partial^{\theta}\omega_{\alpha}^{\alpha\iota} +$$

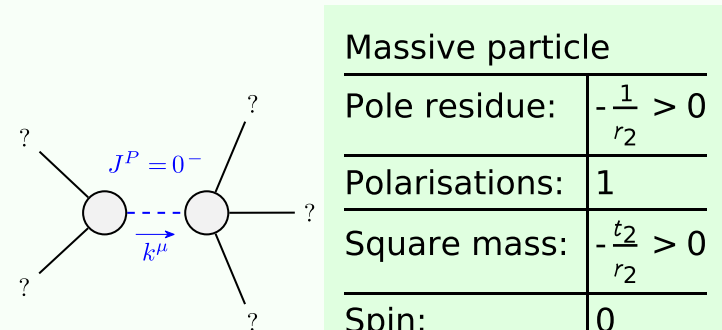
$$4t_3\partial_{\iota}f^{\alpha\iota}\partial_{\kappa}f^{\kappa}_{\alpha} - 8t_3\partial_{\iota}f^{\alpha}_{\alpha}\partial_{\kappa}f^{\kappa}_{\iota} - 6r_5\partial_{\alpha}\omega^{\alpha\iota\theta}\partial_{\kappa}\omega_{\iota,\theta}^{\kappa} +$$

$$12r_5\partial^{\theta}\omega_{\alpha}^{\alpha\iota}\partial_{\kappa}\omega_{\iota,\theta}^{\kappa} + 6r_5\partial_{\alpha}\omega^{\alpha\iota\theta}\partial_{\kappa}\omega_{\theta,\iota}^{\kappa} -$$

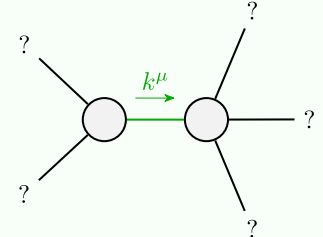
$$12r_5\partial^{\theta}\omega_{\alpha}^{\alpha\iota}\partial_{\kappa}\omega_{\theta,\iota}^{\kappa})[t,x,y,z]dzdydxdt$$

$\sigma_{1^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{1^+}^{\#2}{}_{\alpha\beta}$	$\tau_{1^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{1^+}^{\#1}{}_{\alpha}$	$\tau_{1^+}^{\#1}{}_{\alpha}$	$\sigma_{1^+}^{\#2}{}_{\alpha}$	$\tau_{1^+}^{\#2}{}_{\alpha}$
$\sigma_{1^+}^{\#1}{}_{\dagger\alpha\beta}$	$-\frac{1}{k^2(2r_3+r_5)}$	$-\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$	0	0	0	0
$\sigma_{1^+}^{\#2}{}_{\dagger\alpha\beta}$	$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$	$\frac{i(3k^2(2r_3+r_5)+2t_2)}{(k+k^3)^2(2r_3+r_5)t_2}$	0	0	0	0
$\tau_{1^+}^{\#1}{}_{\dagger\alpha\beta}$	$\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$	$-\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$	0	0	0	0
$\sigma_{1^+}^{\#1}{}_{\dagger\alpha}$	0	0	$\frac{2}{k^2(2r_3+r_5)}$	$\frac{2\sqrt{2}}{k^2(1+k^2)(r_3+2r_5)}$	$\frac{2\sqrt{2}}{k^2(1+k^2)^2(r_3+2r_5)t_3}$	$\frac{4i}{k(1+2k^2)(r_3+2r_5)}$
$\sigma_{1^+}^{\#2}{}_{\dagger\alpha}$	0	0	0	0	0	$\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^3)^2(r_3+2r_5)t_3}$
$\tau_{1^+}^{\#1}{}_{\dagger\alpha}$	0	0	0	0	0	0
$\tau_{1^+}^{\#2}{}_{\dagger\alpha}$	0	0	$-\frac{4i}{k(1+2k^2)(r_3+2r_5)}$	0	$-\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$	$\frac{6k^2(r_3+2r_5)+8t_3}{(1+2k^2)^2(r_3+2r_5)t_3}$

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd



Quadratic pole	
Pole residue:	$-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2} > 0$
Polarisations:	2

Unitarity conditions

$r_2 < 0 \&\& r_3 < 0 \&\& r_5 < -\frac{r_3}{2} \&\& t_2 > 0 \parallel r_2 < 0 \&\& r_3 < 0 \&\& r_5 > -2r_3 \&\& t_2 > 0 \parallel r_2 < 0 \&\& r_3 > 0 \&\& -2r_3 < r_5 < -\frac{r_3}{2} \&\& t_2 > 0$