

# Particle spectrograph

## Wave operator and propagator

### Quadratic (free) Lagrangian density

$$\begin{aligned} &\frac{2}{3}t_2\omega_{\lambda'}^{\kappa\lambda}\omega_{\kappa\lambda}'+\frac{1}{3}t_2\omega_{\kappa\lambda}'\omega_{\lambda'}^{\kappa\lambda}+f_{\lambda'}^{\alpha\beta}\omega_{\alpha\beta}'+\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}+\frac{3}{2}r_3\partial_{\lambda'}\omega_{\lambda}^{\kappa\lambda}\partial_{\kappa}\omega_{\lambda}^{\alpha}+\\ &\frac{2}{3}r_2\partial^{\beta}\omega_{\kappa}^{\theta\alpha}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}-\frac{1}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta}-\frac{2}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta}+\\ &\frac{5}{2}r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\theta\kappa\lambda}-\frac{5}{2}r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}+\frac{3}{2}r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}-\\ &3r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}+\frac{1}{6}t_2\partial^{\alpha}f_{\alpha}^{\kappa}\partial_{\kappa}f_{\alpha}^{\theta}-\frac{1}{6}t_2\partial_{\kappa}f_{\alpha}^{\theta}\partial^{\alpha}f_{\kappa}^{\theta}+\frac{1}{6}t_2\partial^{\alpha}f_{\lambda}^{\lambda}\partial_{\kappa}^{\kappa}f_{\alpha\lambda}+\\ &\frac{1}{3}t_2\omega_{\theta\kappa}\partial^{\kappa}f_{\lambda}^{\theta}-\frac{2}{3}t_2\omega_{\lambda\theta}\partial^{\kappa}f_{\theta}^{\lambda}-\frac{1}{3}t_2\omega_{\theta\kappa}\partial^{\kappa}f_{\theta}^{\lambda}+\frac{2}{3}t_2\omega_{\theta\kappa\lambda}\partial^{\kappa}f_{\lambda}^{\theta}-\\ &\frac{1}{6}t_2\partial^{\alpha}f_{\kappa}^{\lambda}\partial_{\lambda}^{\kappa}f_{\alpha}^{\theta}-\frac{1}{6}t_2\partial_2\partial_{\kappa}f_{\lambda}^{\theta}+\frac{1}{6}t_2\partial_{\kappa}f_{\lambda}^{\theta}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{1}{3}r_2\partial_{\alpha}\omega_{\lambda}^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+\\ &\frac{2}{3}r_2\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}-\frac{2}{3}r_2\partial_{\alpha}\omega_{\lambda}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}'+\frac{2}{3}r_2\partial^{\beta}\omega_{\lambda}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}'+\\ &4r_3\partial^{\beta}\omega_{\lambda}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}'+\frac{5}{2}r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\theta\kappa}+\frac{5}{2}r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\alpha}^{\theta\kappa}\end{aligned}$$

|  | $\omega_{1+}^{\#1}\dagger^{\alpha\beta}$ | $\omega_{1+}^{\#2}\dagger^{\alpha\beta}$ | $f_{1+}^{\#1}\dagger^{\alpha\beta}$ | $\omega_{1-}^{\#1}\dagger^{\alpha}$ | $\omega_{1-}^{\#2}\dagger^{\alpha}$ | $f_{1-}^{\#1}\dagger^{\alpha}$ | $f_{1-}^{\#2}\dagger^{\alpha}$ |
|--|--|--|-------------------------------------|-------------------------------------|-------------------------------------|--------------------------------|--------------------------------|
| $\omega_{1+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{2t_2}{3}$                         | $\frac{\sqrt{2}t_2}{3}$                  | $\frac{1}{3}i\sqrt{2}kt_2$          | 0                                   | 0                                   | 0                              | 0                              |
| $\omega_{1+}^{\#2}\dagger^{\alpha\beta}$ | $\frac{\sqrt{2}t_2}{3}$                  | $\frac{t_2}{3}$                          | $\frac{ikt_2}{3}$                   | 0                                   | 0                                   | 0                              | 0                              |
| $f_{1+}^{\#1}\dagger^{\alpha\beta}$      | $-\frac{1}{3}i\sqrt{2}kt_2$              | $-\frac{1}{3}ikt_2$                      | $\frac{k^2t_2}{3}$                  | 0                                   | 0                                   | 0                              | 0                              |
| $\omega_{1-}^{\#1}\dagger^{\alpha}$      | 0  | 0  | 0                                   | $-\frac{3k^2r_3}{2}$                | 0                                   | 0                              | 0                              |
| $\omega_{1-}^{\#2}\dagger^{\alpha}$      | 0  | 0  | 0                                   | 0                                   | 0                                   | 0                              | 0                              |
| $f_{1-}^{\#1}\dagger^{\alpha}$           | 0  | 0  | 0                                   | 0                                   | 0                                   | 0                              | 0                              |
| $f_{1-}^{\#2}\dagger^{\alpha}$           | 0  | 0  | 0                                   | 0                                   | 0                                   | 0                              | 0                              |

|  | $\sigma_{1+}^{\#1}\dagger^{\alpha\beta}$ | $\sigma_{1+}^{\#2}\dagger^{\alpha\beta}$ | $\tau_{1+}^{\#1}\dagger^{\alpha\beta}$ | $\sigma_{1-}^{\#1}\dagger^{\alpha}$ | $\sigma_{1-}^{\#2}\dagger^{\alpha}$ | $\tau_{1-}^{\#1}\dagger^{\alpha}$ | $\tau_{1-}^{\#2}\dagger^{\alpha}$ |
|--|--|--|--|-------------------------------------|-------------------------------------|-----------------------------------|-----------------------------------|
| $\sigma_{1+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{6}{(3+k^2)^2t_2}$                 | $\frac{3\sqrt{2}}{(3+k^2)^2t_2}$         | $\frac{3i\sqrt{2}k}{(3+k^2)^2t_2}$     | 0                                   | 0                                   | 0                                 | 0                                 |
| $\sigma_{1+}^{\#2}\dagger^{\alpha\beta}$ | $\frac{3\sqrt{2}}{(3+k^2)^2t_2}$         | $\frac{3}{(3+k^2)^2t_2}$                 | $\frac{3ik}{(3+k^2)^2t_2}$             | 0                                   | 0                                   | 0                                 | 0                                 |
| $\tau_{1+}^{\#1}\dagger^{\alpha\beta}$   | $-\frac{3i\sqrt{2}k}{(3+k^2)^2t_2}$      | $-\frac{3ik}{(3+k^2)^2t_2}$              | $\frac{3k^2}{(3+k^2)^2t_2}$            | 0                                   | 0                                   | 0                                 | 0                                 |
| $\sigma_{1-}^{\#1}\dagger^{\alpha}$      | 0  | 0  | 0                                      | $-\frac{2}{3k^2r_3}$                | 0                                   | 0                                 | 0                                 |
| $\sigma_{1-}^{\#2}\dagger^{\alpha}$      | 0  | 0  | 0                                      | 0                                   | 0                                   | 0                                 | 0                                 |
| $\tau_{1-}^{\#1}\dagger^{\alpha}$        | 0  | 0  | 0                                      | 0                                   | 0                                   | 0                                 | 0                                 |
| $\tau_{1-}^{\#2}\dagger^{\alpha}$        | 0  | 0  | 0                                      | 0                                   | 0                                   | 0                                 | 0                                 |

|  | $\omega_{2+}^{\#1}\dagger^{\alpha\beta}$ | $f_{2+}^{\#1}\dagger^{\alpha\beta}$ | $\omega_{2-}^{\#1}\dagger^{\alpha\beta\chi}$ |
|--|--|-------------------------------------|--|
| $\omega_{2+}^{\#1}\dagger^{\alpha\beta}$     | $-\frac{3k^2r_3}{2}$                     | 0                                   | 0  |
| $f_{2+}^{\#1}\dagger^{\alpha\beta}$          | 0  | 0                                   | 0  |
| $\omega_{2-}^{\#1}\dagger^{\alpha\beta\chi}$ | 0  | 0                                   | 0  |

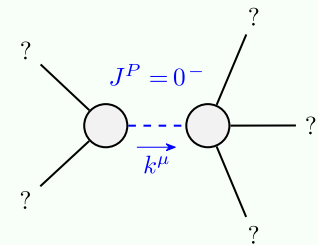
|                            | $\omega_{0+}^{\#1}\dagger$ | $f_{0+}^{\#1}\dagger$ | $f_{0+}^{\#2}\dagger$ | $\omega_{0-}^{\#1}\dagger$ |
|----------------------------|----------------------------|-----------------------|-----------------------|----------------------------|
| $\omega_{0+}^{\#1}\dagger$ | 0                          | 0                     | 0                     | 0                          |
| $f_{0+}^{\#1}\dagger$      | 0                          | 0                     | 0                     | 0                          |
| $f_{0+}^{\#2}\dagger$      | 0                          | 0                     | 0                     | 0                          |
| $\omega_{0-}^{\#1}\dagger$ | 0                          | 0                     | 0                     | $k^2r_2+t_2$               |

| Source constraints/gauge generators                                |                |
|--|----------------|
| SO(3) irreps   | Multiplicities |
| $\tau_{0+}^{\#2} == 0$   | 1              |
| $\tau_{0+}^{\#1} == 0$   | 1              |
| $\sigma_{0+}^{\#1} == 0$   | 1              |
| $\tau_{1-}^{\#2\alpha} == 0$                                       | 3              |
| $\tau_{1-}^{\#1\alpha} == 0$                                       | 3              |
| $\sigma_{1-}^{\#2\alpha} == 0$                                     | 3              |
| $\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#1\alpha\beta} == 0$ | 3              |
| $\sigma_{1+}^{\#1\alpha\beta} == \sigma_{1+}^{\#2\alpha\beta}$     | 3              |
| $\sigma_{2-}^{\#1\alpha\beta\chi} == 0$                            | 5              |
| $\tau_{2+}^{\#1\alpha\beta} == 0$                                  | 5              |
| Total constraints:   | 28             |

|  | $\sigma_{2+}^{\#1}\dagger^{\alpha\beta}$ | $\tau_{2+}^{\#1}\dagger^{\alpha\beta}$ | $\sigma_{2-}^{\#1}\dagger^{\alpha\beta\chi}$ |
|--|--|--|--|
| $\sigma_{2+}^{\#1}\dagger^{\alpha\beta}$     | $-\frac{2}{3k^2r_3}$                     | 0                                      | 0  |
| $\tau_{2+}^{\#1}\dagger^{\alpha\beta}$       | 0  | 0                                      | 0  |
| $\sigma_{2-}^{\#1}\dagger^{\alpha\beta\chi}$ | 0  | 0                                      | 0  |

|                            | $\sigma_{0+}^{\#1}\dagger$ | $\tau_{0+}^{\#1}\dagger$ | $\tau_{0+}^{\#2}\dagger$ | $\sigma_{0-}^{\#1}\dagger$ |
|----------------------------|----------------------------|--------------------------|--------------------------|----------------------------|
| $\sigma_{0+}^{\#1}\dagger$ | 0                          | 0                        | 0                        | 0                          |
| $\tau_{0+}^{\#1}\dagger$   | 0                          | 0                        | 0                        | 0                          |
| $\tau_{0+}^{\#2}\dagger$   | 0                          | 0                        | 0                        | 0                          |
| $\sigma_{0-}^{\#1}\dagger$ | 0                          | 0                        | 0                        | $\frac{1}{k^2r_2+t_2}$     |

## Massive and massless spectra



| Massive particle |                        |
|------------------|------------------------|
| Pole residue:    | $-\frac{1}{r_2} > 0$   |
| Polarisations:   | 1                      |
| Square mass:     | $-\frac{t_2}{r_2} > 0$ |
| Spin:            | 0                      |
| Parity:          | Odd                    |

(No massless particles)

## Unitarity conditions

$$r_2 < 0 \ \&\& \ t_2 > 0$$