

# Particle spectrograph

## Wave operator and propagator

Quadratic (free) Lagrangian density

$$\begin{aligned} &\frac{2}{3}t_2\omega_{\lambda'}^{\kappa\lambda}\omega_{\kappa\lambda'}'+\frac{1}{3}t_2\omega_{\kappa\lambda}^{\lambda'}\omega_{\kappa\lambda}^{\lambda'}+\omega_{\alpha\beta}^{\alpha\beta}\tau_{\alpha\beta}+\omega_{\alpha\beta\chi}^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}-r_5\partial_{\lambda}^{\kappa\lambda}\partial_{\kappa}^{\lambda}\omega_{\lambda}^{\alpha}+ \\ &\frac{2}{3}r_2\partial_{\theta}^{\beta}\omega_{\kappa}^{\theta\alpha}\partial_{\theta}^{\alpha}\omega_{\alpha\beta}^{\kappa}-\frac{1}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\omega_{\alpha\beta}^{\kappa}-\frac{2}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\theta}-\frac{2}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\theta}- \\ &r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\theta\kappa\lambda}^{\alpha}+r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\alpha}\omega_{\lambda}^{\theta\kappa\lambda}-r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\kappa\lambda\theta}+2r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\kappa\lambda\theta}+ \\ &\frac{1}{6}t_2\partial_{\kappa}^{\alpha}f_{\theta\kappa}\partial_{\kappa}^{\kappa}f_{\alpha}^{\theta}-\frac{1}{6}t_2\partial_2^{\alpha}f_{\kappa\theta}\partial_{\kappa}^{\kappa}f_{\alpha}^{\theta}+\frac{1}{6}t_2\partial_{\alpha}f_{\kappa}^{\lambda}\partial_{\kappa}^{\kappa}f_{\alpha}^{\lambda}+\frac{1}{3}t_2\omega_{\theta\kappa}\partial_{\kappa}^{\kappa}f_{\alpha}^{\lambda}-\frac{1}{6}t_2\partial_{\alpha}f_{\kappa}^{\lambda}\partial_{\kappa}^{\kappa}f_{\lambda}^{\alpha}- \\ &\frac{2}{3}t_2\omega_{\kappa\theta}\partial_{\kappa}^{\kappa}f_{\theta}^{\lambda}-\frac{1}{3}t_2\omega_{\theta\kappa}\partial_{\kappa}^{\kappa}f_{\theta}^{\lambda}+\frac{2}{3}t_2\omega_{\theta\kappa}\partial_{\kappa}^{\kappa}f_{\theta}^{\lambda}-\frac{1}{6}t_2\partial_{\alpha}f_{\kappa}^{\lambda}\partial_{\kappa}^{\kappa}f_{\lambda}^{\alpha}- \\ &\frac{1}{6}t_2\partial_{\kappa}f_{\theta}^{\lambda}\partial_{\kappa}^{\kappa}f_{\lambda}^{\theta}+\frac{1}{6}t_2\partial_{\kappa}f_{\theta}^{\lambda}\partial_{\kappa}^{\kappa}f_{\lambda}^{\theta}+\frac{1}{3}r_2\partial_{\kappa}\omega_{\alpha\beta\theta}^{\alpha}\partial_{\kappa}^{\kappa}\omega_{\alpha\beta\theta}+\frac{2}{3}r_2\partial_{\kappa}\omega_{\alpha\beta\theta}^{\alpha}\partial_{\kappa}^{\kappa}\omega_{\alpha\beta\theta}- \\ &\frac{2}{3}r_2\partial_{\theta}\omega_{\lambda'}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda'}+\frac{2}{3}r_2\partial_{\theta}^{\beta}\omega_{\alpha\beta}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda'}+r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}^{\lambda}\omega_{\lambda}^{\theta\kappa}-r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}^{\lambda}\omega_{\lambda}^{\theta\kappa} \end{aligned}$$

$\sigma_{1+}^{\#1}+\alpha\beta$	$\sigma_{1+}^{\#2}+\alpha\beta$	$\tau_{1+}^{\#1}+\alpha\beta$	$\sigma_{1-}^{\#1}+\alpha$	$\sigma_{1-}^{\#2}+\alpha$	$\tau_{1-}^{\#1}+\alpha$	$\tau_{1-}^{\#2}+\alpha$
$\frac{1}{k^2r_5}$	$-\frac{\sqrt{2}}{k^2r_5+k^4r_5}$	$-\frac{i\sqrt{2}}{kr_5+k^3r_5}$	0	0	0	0
$-\frac{\sqrt{2}}{k^2r_5+k^4r_5}$	$\frac{3k^2r_5+2t_2}{(k+k^3)^2r_5t_2}$	$\frac{i(3k^2r_5+2t_2)}{k(1+k^2)^2r_5t_2}$	0	0	0	0
$-\frac{i\sqrt{2}}{kr_5+k^3r_5}$	$-\frac{i(3k^2r_5+2t_2)}{k(1+k^2)^2r_5t_2}$	$\frac{3k^2r_5+2t_2}{(1+k^2)^2r_5t_2}$	0	0	0	0
0	0	0	$\frac{1}{k^2r_5}$	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$\omega_{1+}^{\#1}+\alpha\beta$	$\omega_{1+}^{\#2}+\alpha\beta$	$f_{1+}^{\#1}+\alpha\beta$	$\omega_{1-}^{\#1}+\alpha$	$\omega_{1-}^{\#2}+\alpha$	$f_{1-}^{\#1}+\alpha$	$f_{1-}^{\#2}+\alpha$
$k^2r_5+\frac{2t_2}{3}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{1}{3}i\sqrt{2}kt_2$	0	0	0	0
$\frac{\sqrt{2}t_2}{3}$	$\frac{t_2}{3}$	$\frac{ikt_2}{3}$	0	0	0	0
$-\frac{1}{3}i\sqrt{2}kt_2$	$-\frac{1}{3}ikt_2$	$\frac{k^2t_2}{3}$	0	0	0	0
0	0	0	$k^2r_5$	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

	$\omega_{0+}^{\#1}$	$f_{0+}^{\#1}$	$f_{0+}^{\#2}$	$\omega_{0-}^{\#1}$
$\omega_{0+}^{\#1}+$	0	0	0	0
$f_{0+}^{\#1}+$	0	0	0	0
$f_{0+}^{\#2}+$	0	0	0	0
$\omega_{0-}^{\#1}+$	0	0	0	$k^2r_2+t_2$

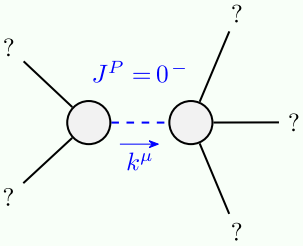
	$\omega_{2+}^{\#1}+\alpha\beta$	$f_{2+}^{\#1}+\alpha\beta$	$\omega_{2-}^{\#1}+\alpha\beta\chi$
$\omega_{2+}^{\#1}+\alpha\beta$	0	0	0
$f_{2+}^{\#1}+\alpha\beta$	0	0	0
$\omega_{2-}^{\#1}+\alpha\beta\chi$	0	0	0

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2}==0$	1
$\tau_{0+}^{\#1}==0$	1
$\sigma_{0+}^{\#1}==0$	1
$\tau_{1-}^{\#2\alpha}==0$	3
$\tau_{1-}^{\#1\alpha}==0$	3
$\sigma_{1-}^{\#2\alpha}==0$	3
$\tau_{1+}^{\#1\alpha\beta}+ik\sigma_{1+}^{\#2\alpha\beta}==0$	3
$\sigma_{2-}^{\#1\alpha\beta\chi}==0$	5
$\tau_{2+}^{\#1\alpha\beta}==0$	5
$\sigma_{2+}^{\#1\alpha\beta}==0$	5
Total constraints:	30

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
$\sigma_{0+}^{\#1}+$	0	0	0	0
$\tau_{0+}^{\#1}+$	0	0	0	0
$\tau_{0+}^{\#2}+$	0	0	0	0
$\sigma_{0-}^{\#1}+$	0	0	0	$\frac{1}{k^2r_2+t_2}$

	$\sigma_{2+}^{\#1}+\alpha\beta$	$\tau_{2+}^{\#1}+\alpha\beta$	$\sigma_{2-}^{\#1}+\alpha\beta\chi$
$\sigma_{2+}^{\#1}+\alpha\beta$	0	0	0
$\tau_{2+}^{\#1}+\alpha\beta$	0	0	0
$\sigma_{2-}^{\#1}+\alpha\beta\chi$	0	0	0

## Massive and massless spectra



Massive particle

Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

## Unitarity conditions

$r_2 < 0 \&\& t_2 > 0$