Particle spectrograph

Wave operator and propagator

$$
\begin{aligned}
\mathcal{S} == \iiiint \big(&\beta\, \mathcal{B}_{\alpha\beta}\, \mathcal{B}^{\alpha\beta} + f^{\alpha\beta}\, \tau_{\alpha\beta} + \mathcal{B}^{\alpha\beta}\, \mathcal{T}_{\alpha\beta} + c_3 (\partial_\beta f^\chi{}_\chi\, \partial^\beta f^\alpha{}_\alpha - 2\, \partial^\beta f^\alpha{}_\alpha\, \partial_\chi f_\beta{}^\chi + \partial_\beta f^{\alpha\beta} (\partial_\chi f_\alpha{}^\chi + 2\, \partial_\chi \mathcal{B}_\alpha{}^\chi) - \\
&\partial_\alpha \mathcal{B}^{\alpha\beta}\, \partial_\chi \mathcal{B}_\beta{}^\chi - 2\, \partial^\beta f^\alpha{}_\alpha\, \partial_\chi \mathcal{B}_\beta{}^\chi) - \tfrac{1}{3}\, \alpha\, (2\, \partial_\beta \mathcal{B}_{\alpha\chi} - \partial_\chi \mathcal{B}_{\alpha\beta})\, \partial^\chi \mathcal{B}^{\alpha\beta} + \\
&c_2 (-2\, \partial_\alpha f_{\beta\chi}\, \partial^\chi f^{\alpha\beta} - 2\, \partial_\alpha \mathcal{B}_{\beta\chi}\, \partial^\chi f^{\alpha\beta} + \partial_\beta f_{\alpha\chi}\, \partial^\chi f^{\alpha\beta} + 4\, \partial_\beta \mathcal{B}_{\alpha\chi}\, \partial^\chi f^{\alpha\beta} + \\
&\partial_\chi f_{\beta\alpha}\, \partial^\chi f^{\alpha\beta} - 2\, \partial_\chi \mathcal{B}_{\alpha\beta}\, \partial^\chi f^{\alpha\beta} + 3\, \partial_\beta \mathcal{B}_{\alpha\chi}\, \partial^\chi \mathcal{B}^{\alpha\beta} - \partial_\chi \mathcal{B}_{\alpha\beta}\, \partial^\chi \mathcal{B}^{\alpha\beta}) + \\
&c_1 (-2\, \partial_\alpha f_{\chi\beta}\, \partial^\chi f^{\alpha\beta} + 4\, \partial_\alpha \mathcal{B}_{\beta\chi}\, \partial^\chi f^{\alpha\beta} + 2\, \partial_\chi f_{\alpha\beta}\, \partial^\chi f^{\alpha\beta} + 4\, \partial_\chi \mathcal{B}_{\alpha\beta}\, \partial^\chi f^{\alpha\beta} - \\
&2\, \partial_\beta \mathcal{B}_{\alpha\chi}\, \partial^\chi \mathcal{B}^{\alpha\beta} + 2\, \partial_\chi \mathcal{B}_{\alpha\beta}\, \partial^\chi \mathcal{B}^{\alpha\beta})\big)[t, x, y, z]\, dz\, dy\, dx\, dt
\end{aligned}
$$

$$\begin{array}{c|c} & \overset{\#1}{2^+} f_{\alpha\beta} \\ \hline \overset{\#1}{2^+} f^\dagger{}^{\alpha\beta} & (2 c_1 + c_2)\, k^2 \end{array}
\qquad
\begin{array}{c|c} & \overset{\#1}{2^+} \tau_{\alpha\beta} \\ \hline \overset{\#1}{2^+} \tau^\dagger{}^{\alpha\beta} & \frac{1}{(2 c_1 + c_2)\, k^2} \end{array}$$

$$\begin{array}{c|cc} & \overset{\#1}{0^+} f & \overset{\#2}{0^+} f \\ \hline \overset{\#1}{0^+} f^\dagger & (2 c_1 + c_2 + 3\, c_3)\, k^2 & 0 \\ \overset{\#2}{0^+} f^\dagger & 0 & 0 \end{array}$$

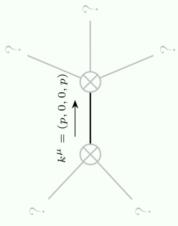
$$\begin{array}{c|ccccc} & \overset{\#1}{1^+} \mathcal{T}_{\alpha\beta} & \overset{\#1}{1^+} \tau_{\alpha\beta} & \overset{\#1}{1^-} \mathcal{T}_\alpha & \overset{\#1}{1^-} \tau_\alpha & \overset{\#2}{1^-} \tau_\alpha \\ \hline
\overset{\#1}{1^+} \mathcal{T}^\dagger{}^{\alpha\beta} & \frac{1}{\frac{k^2 \alpha}{3} + \beta} & -\frac{3}{k^2 \alpha + 3 \beta} & 0 & 0 & 0 \\
\overset{\#1}{1^+} \tau^\dagger{}^{\alpha\beta} & -\frac{3}{k^2 \alpha + 3 \beta} & \frac{1}{(2 c_1 - c_2)\, k^2} + \frac{1}{\frac{k^2 \alpha}{3} + \beta} & 0 & 0 & 0 \\
\overset{\#1}{1^-} \mathcal{T}^\dagger{}^{\alpha} & 0 & 0 & \frac{1}{\beta} & 0 & -\frac{1}{\sqrt{2}\, \beta} \\
\overset{\#1}{1^-} \tau^\dagger{}^{\alpha} & 0 & 0 & 0 & 0 & 0 \\
\overset{\#2}{1^-} \tau^\dagger{}^{\alpha} & 0 & 0 & -\frac{1}{\sqrt{2}\, \beta} & 0 & \frac{1}{(2 c_1 + c_2 + c_3)\, k^2} + \frac{1}{2 \beta}
\end{array}$$

Spin-parity form	Covariant form	Multiplicities
$\overset{\#2}{0^+} \tau == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$	1
$\overset{\#1}{1^-} \tau^\alpha == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$	3
Total expected gauge generators:		4

$$\begin{array}{c|cc} & \overset{\#1}{0^+} \tau & \overset{\#2}{0^+} \tau \\ \hline \overset{\#1}{0^+} \tau^\dagger & \frac{1}{(2 c_1 + c_2 + 3 c_3)\, k^2} & 0 \\ \overset{\#2}{0^+} \tau^\dagger & 0 & 0 \end{array}$$

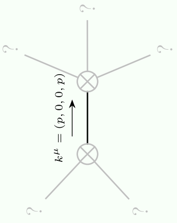
	$\overset{\#1}{1^+} \mathcal{B}_{\alpha\beta}$	$\overset{\#1}{1^+} f_{\alpha\beta}$	$\overset{\#1}{1^-} \mathcal{B}_\alpha$	$\overset{\#1}{1^-} f_\alpha$	$\overset{\#2}{1^-} f_\alpha$
$\overset{\#1}{1^+} \mathcal{B}^\dagger{}^{\alpha\beta}$	$\frac{1}{3} k^2 (6 c_1 - 3 c_2 + \alpha) + \beta$	$(2 c_1 - c_2)\, k^2$	0 0		0
$\overset{\#1}{1^+} f^\dagger{}^{\alpha\beta}$	$(2 c_1 - c_2)\, k^2$	$(2 c_1 - c_2)\, k^2$	0 0		0
$\overset{\#1}{1^-} \mathcal{B}^\dagger{}^{\alpha}$	0 0		$\frac{1}{2} (2 c_1 + c_2 + c_3)\, k^2 + \beta$	0	$\frac{(2 c_1 + c_2 + c_3)\, k^2}{\sqrt{2}}$
$\overset{\#1}{1^-} f^\dagger{}^{\alpha}$	0 0		0 0		0
$\overset{\#2}{1^-} f^\dagger{}^{\alpha}$	0 0		$\frac{(2 c_1 + c_2 + c_3)\, k^2}{\sqrt{2}}$	0	$(2 c_1 + c_2 + c_3)\, k^2$

Massive and massless spectra



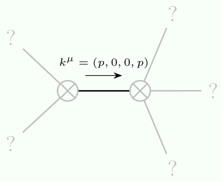
Massless particle

Pole residue:	$-\frac{\sqrt{64 c_1^4 + 32 c_1^3 (c_2 + c_3) + 28 c_1 c_2^2 (c_2 + c_3) + 4 c_1^2 (5 c_2^2 + 2 c_2 c_3 + c_3^2) + c_2^2 (9 c_2^2 + 16 c_2 c_3 + 8 c_3^2)}}{(4 c_1^2 - c_2^2)(2 c_1 + c_2 + c_3)} > 0$
Polarisations:	2



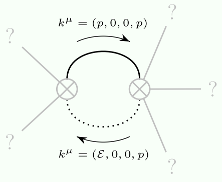
Massless particle

Poleresidue:	$\frac{\sqrt{64 c_1^4 + 32 c_1^3 (c_2 + c_3) + 28 c_1 c_2^2 (c_2 + c_3) + 4 c_1^2 (5 c_2^2 + 2 c_2 c_3 + c_3^2) + c_2^2 (9 c_2^2 + 16 c_2 c_3 + 8 c_3^2)}}{(4 c_1^2 - c_2^2)(2 c_1 + c_2 + c_3)} > 0$
Polarisations:	2



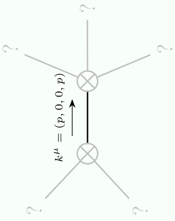
Massless particle

Poleresidue:	$\frac{4 + (2 c_1 + c_2 + c_3)^2}{(2 c_1 + c_2)(2\, c_1 + c_2 + c_3)(2\, c_1 + c_2 + 3 c_3)} > 0$
Polarisations:	1



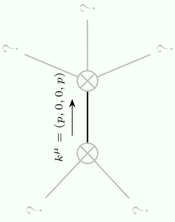
Quartic pole

Pole residue:	$0 < \frac{c_2^2 + 2 c_1 (c_2 + c_3)}{(4 c_1^2 - c_2^2)(2\, c_1 + c_2 + c_3)}\ \&\&\ \frac{c_2^2 + 2 c_1 (c_2 + c_3)}{(4 c_1^2 - c_2^2)(2\, c_1 + c_2 + c_3)} > 0$
Polarisations:	2



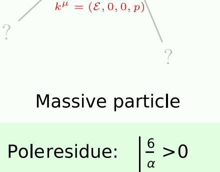
Massless particle

Poleresidue:	$\frac{1}{2 c_1 - c_2} > 0$
Polarisations:	1



Massless particle

Poleresidue:	$\frac{1}{2 c_1 + c_2} > 0$
Polarisations:	2



Massive particle

Poleresidue:	$\frac{6}{\alpha} > 0$
Square mass:	$-\frac{3 \beta}{\alpha} > 0$
Spin:	1
Parity:	Even

Unitarity conditions