

# Particle spectrograph

## Wave operator and propagator

$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2}$	$\tau_{1+}^{\#1} \alpha\beta$	$\sigma_{1-}^{\#1} \alpha$	$\sigma_{1-}^{\#2} \alpha$	$\tau_{1-}^{\#1} \alpha$	$\tau_{1-}^{\#2} \alpha$
0	$-\frac{\sqrt{2}}{t_1+k^2}t_1$	$-\frac{i\sqrt{2}k}{t_1+k^2}t_1$	0	0	0	0
$-\frac{\sqrt{2}}{t_1+k^2}t_1$	$\frac{-2k^2r_5+t_1}{(1+k^2)^2}t_1^2$	$-\frac{i(2k^3r_5-kt_1)}{(1+k^2)^2}t_1^2$	0	0	0	0
$\frac{i\sqrt{2}k}{t_1+k^2}t_1$	$\frac{i(2k^3r_5-kt_1)}{(1+k^2)^2}t_1^2$	$\frac{-2k^4r_5+k^2t_1}{(1+k^2)^2}t_1^2$	0	0	0	0
0	0	0	$\frac{1}{k^2r_5}$	$-\frac{1}{\sqrt{2}(k^2r_5+2k^4r_5)}$	0	$-\frac{i}{kr_5+2k^3r_5}$
0	0	0	$-\frac{1}{\sqrt{2}(k^2r_5+2k^4r_5)}$	$\frac{6k^2r_5+t_1}{2(k+2k^3)^2r_5t_1}$	0	$\frac{i(6k^2r_5+t_1)}{\sqrt{2}k(1+2k^2)^2r_5t_1}$
0	0	0	0	0	0	0
0	0	0	$\frac{i}{kr_5+2k^3r_5}$	$-\frac{i(6k^2r_5+t_1)}{\sqrt{2}k(1+2k^2)^2r_5t_1}$	0	$\frac{6k^2r_5+t_1}{(1+2k^2)^2r_5t_1}$

Quadratic (free) action

$$S = \iiint \{ (\frac{1}{6} (2t_1\omega_{\alpha}^{\alpha i}\omega_{\alpha}^{\theta} + 6f^{\alpha\beta}\tau_{\alpha\beta} + 6\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi} - 4t_1\omega_{\alpha}^{\theta}\partial_{\theta}f^{\alpha i} + 4t_1\omega_{\theta}^{\theta}\partial_{\theta}f^{\alpha i} - \partial_{\theta}f^{\alpha}_{\alpha} - 2t_1\partial_{\theta}f^{\theta}_{\theta}\partial_{\theta}f^{\alpha}_{\alpha} - 2t_1\partial_{\theta}f^{\alpha i}\partial_{\theta}f^{\theta}_{\theta} + 4t_1\partial_{\theta}f^{\alpha}_{\alpha}\partial_{\theta}f^{\theta}_{\theta} - 6t_1\partial_{\theta}f_{\theta}\partial_{\theta}f^{\alpha i} - 3t_1\partial_{\alpha}f_{\theta i}\partial_{\theta}f^{\alpha i} + 3t_1\partial_{\theta}f_{\alpha i}\partial_{\theta}f^{\alpha i} + 3t_1\partial_{\theta}f_{\alpha i}\partial_{\theta}f^{\alpha i} + 6t_1\omega_{\alpha\theta i}(\omega^{\alpha i\theta} + 2\partial^{\theta}f^{\alpha i}) + 8r_2\partial_{\beta}\omega_{\alpha i\theta}\partial^{\theta}\omega^{\alpha\beta i} - 4r_2\partial_{\beta}\omega_{\alpha\theta i}\partial^{\theta}\omega^{\alpha\beta i} + 4r_2\partial_{\beta}\omega_{\theta\alpha i}\partial^{\theta}\omega^{\alpha\beta i} - 2r_2\partial_{\theta}\omega_{\alpha\beta\theta}\partial^{\theta}\omega^{\alpha\beta i} + 2r_2\partial_{\theta}\omega_{\alpha\beta i}\partial^{\theta}\omega^{\alpha\beta i} - 4r_2\partial_{\theta}\omega_{\alpha i\beta}\partial^{\theta}\omega^{\alpha\beta i} + 6r_5\partial_{\theta}\omega_{\theta}^{\kappa}\partial^{\theta}\omega_{\kappa}^{\alpha i}\partial^{\theta}\omega_{\alpha}^{\alpha i} - 6r_5\partial_{\theta}\omega_{\alpha}^{\kappa}\partial^{\theta}\omega_{\kappa}^{\alpha i}\partial^{\theta}\omega_{\theta}^{\alpha i} - 6r_5\partial_{\alpha}\omega_{\theta}^{\kappa}\partial^{\theta}\omega_{\kappa}^{\alpha i\theta}\partial_{\kappa}\omega_{\theta}^{\alpha i} + 12r_5\partial^{\theta}\omega_{\alpha}^{\alpha i}\partial_{\kappa}\omega_{\theta}^{\kappa} + 6r_5\partial_{\alpha}\omega^{\alpha i\theta}\partial_{\kappa}\omega_{\theta}^{\kappa} - 12r_5\partial^{\theta}\omega_{\alpha}^{\alpha i}\partial_{\kappa}\omega_{\theta}^{\kappa})) [t, x, y, z] dz dy dx dt$$

	$\omega_{1+}^{\#1} \alpha\beta$	$\omega_{1+}^{\#2}$	$f_{1+}^{\#1} \alpha\beta$	$\omega_{1-}^{\#1} \alpha$	$\omega_{1-}^{\#2}$	$f_{1-}^{\#1} \alpha$	$f_{1-}^{\#2} \alpha$
$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$k^2r_5 - \frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	$k^2r_5 + \frac{t_1}{6}$	$\frac{t_1}{3\sqrt{2}}$	0	$\frac{ikt_1}{3}$
$\omega_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	0	$\frac{1}{3}i\sqrt{2}kt_1$
$f_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$f_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$-\frac{1}{3}i\sqrt{2}kt_1$	$-\frac{1}{3}i\sqrt{2}kt_1$	0	$\frac{2k^2t_1}{3}$

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} == 0$	1
$\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	17

## Massive and massless spectra

Massive particle

Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$\frac{t_1}{r_2} > 0$
Spin:	0
Parity:	Odd

Quadratic pole

Pole residue:	$-\frac{1}{r_5t_1^2} > 0$
Polarisations:	2

## Unitarity conditions

$r_2 < 0 \&\& r_5 < 0 \&\& t_1 < 0$

	$\omega_{2+}^{\#1} \alpha\beta$	$f_{2+}^{\#1} \alpha\beta$	$\omega_{2-}^{\#1} \alpha\beta\chi$	$\sigma_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#2} \dagger$	$\sigma_{0-}^{\#1} \dagger$
$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0
$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	$k^2t_1$	0	0	0	0	0
$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{t_1}{2}$	0	0	0	$\frac{1}{k^2r_2-t_1}$

	$\omega_{0+}^{\#1} \dagger$	$f_{0+}^{\#1} \dagger$	$f_{0+}^{\#2} \dagger$	$\omega_{0-}^{\#1} \dagger$
$\omega_{0+}^{\#1} \dagger$	0	0	0	0
$f_{0+}^{\#1} \dagger$	0	0	0	0
$f_{0+}^{\#2} \dagger$	0	0	0	0
$\omega_{0-}^{\#1} \dagger$	0	0	0	$k^2r_2-t_1$