

## Wave operator and propagator

## Quadratic (free) action

$$S = - \int \int \int (\omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - 2 r_3 (\partial_\beta \omega_{\gamma\theta}^\theta \partial' \omega_{\alpha}^{\alpha\beta} + \partial_\gamma \omega_{\beta\theta}^\theta \partial' \omega_{\alpha}^{\alpha\beta} + \partial_\alpha \omega_{\beta\gamma}^{\alpha\beta\gamma} - 2 \partial' \omega_{\beta\gamma}^{\alpha\beta} \partial_\alpha \omega_{\beta\gamma}^\theta + \partial_\alpha \omega_{\beta\gamma}^{\alpha\beta\gamma} \partial_\theta \omega_{\beta\gamma}^\theta - 2 \partial' \omega_{\alpha}^{\alpha\beta} \partial_\beta \omega_{\gamma\theta}^\theta + 2 \partial_\beta \omega_{\gamma\theta}^\theta \partial^\theta \omega_{\alpha}^{\alpha\beta\gamma}) + \frac{2}{3} r_1 (3 \partial_\beta \omega_{\gamma\theta}^\theta \partial' \omega_{\alpha}^{\alpha\beta} + 3 \partial_\gamma \omega_{\beta\theta}^\theta \partial' \omega_{\alpha}^{\alpha\beta} + 3 \partial_\alpha \omega_{\beta\gamma}^{\alpha\beta\gamma} \partial_\theta \omega_{\beta\gamma}^\theta - 6 \partial' \omega_{\alpha}^{\alpha\beta} \partial_\beta \omega_{\gamma\theta}^\theta + 3 \partial_\alpha \omega_{\beta\gamma}^{\alpha\beta\gamma} \partial_\theta \omega_{\beta\gamma}^\theta - 6 \partial' \omega_{\alpha}^{\alpha\beta} \partial_\beta \omega_{\gamma\theta}^\theta + 2 \partial_\beta \omega_{\gamma\theta}^\theta \partial^\theta \omega_{\alpha}^{\alpha\beta\gamma} - \partial_\gamma \omega_{\alpha\beta\theta} \partial^\theta \omega_{\alpha}^{\alpha\beta\gamma} + \partial_\theta \omega_{\alpha\beta\gamma} \partial^\theta \omega_{\alpha}^{\alpha\beta\gamma} + \partial_\theta \omega_{\alpha\beta\gamma} \partial^\theta \omega_{\alpha}^{\alpha\beta\gamma}) + r_5 (\partial_\gamma \omega_{\theta\kappa}^\kappa \partial^\theta \omega_{\alpha}^{\alpha\gamma} - \partial_\theta \omega_{\gamma\kappa}^\kappa \partial^\theta \omega_{\alpha}^{\alpha\gamma} - (\partial_\alpha \omega_{\gamma\theta}^{\alpha\gamma} - 2 \partial^\theta \omega_{\alpha}^{\alpha\gamma})) [t, x, y, z] dz dy dx dt$$

## Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\sigma_0^{\#1} == 0$	$\epsilon \eta_{\alpha\beta\chi\delta} \partial^\delta \sigma^{\alpha\beta\chi} == 0$	1
$\sigma_1^{\#2\alpha} == 0$	$\partial_\chi \partial_\beta \sigma^{\alpha\beta\chi} == 0$	3
$\sigma_{1+}^{\#2\alpha\beta} == 0$	$\partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$	3
$\sigma_{2+}^{\#1\alpha\beta} == 0$	$3 \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 3 \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta} + 2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\delta \sigma^{\chi\delta}$ $2 \partial_\delta \partial^\beta \partial^\alpha \sigma^{\chi\delta} + 3 (\partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\chi\beta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\beta\chi\alpha})$	5
Total constraints/gauge generators:		12

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The diagram illustrates the decomposition of the tensor product of two representations of the Lie algebra  $\mathfrak{so}(3,1)$ . The representations are labeled by their highest weights and the corresponding Dynkin diagram. The decomposition is shown as a sequence of steps, starting from the top left and moving downwards and to the right.

**Top Row:**

- $\omega_{2+}^{\#1} \alpha \beta$  (Pink box)
- $\omega_{2-}^{\#1} \alpha \beta \chi$  (Light blue box)
- $\sigma_{2+}^{\#1} \alpha \beta$  (Light orange box)
- $\sigma_{2-}^{\#1} \alpha \beta \chi$  (Light orange box)
- $\sigma_{0+}^{\#1}$  (Light orange box)
- $\sigma_{0-}^{\#1}$  (Light orange box)

**Second Row:**

- $\omega_{1+}^{\#1} \alpha \beta$  (Pink box)
- $\omega_{1-}^{\#1} \alpha$  (Light blue box)
- $\omega_{0+}^{\#1}$  (Light orange box)
- $\omega_{0-}^{\#1}$  (Light orange box)

**Third Row:**

- $\omega_{1+}^{\#2} \alpha \beta$  (Pink box)
- $\omega_{1-}^{\#2} \alpha$  (Light blue box)
- $\omega_{0+}^{\#2}$  (Light orange box)
- $\omega_{0-}^{\#2}$  (Light orange box)

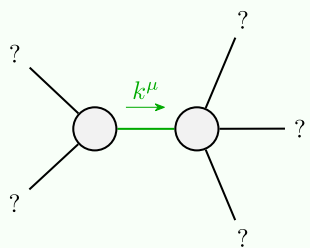
**Bottom Row:**

- $\omega_{1+}^{\#1} \alpha \beta$  (Pink box)
- $\omega_{1-}^{\#1} \alpha$  (Light blue box)
- $\omega_{0+}^{\#1}$  (Light orange box)
- $\omega_{0-}^{\#1}$  (Light orange box)

**Arrows and Labels:**

- Arrows indicate the decomposition of the tensor product of two representations into a direct sum of other representations.
- Labels on the arrows include  $\alpha$ ,  $\beta$ , and  $\chi$ , which represent the simple roots of the Lie algebra.

## Massive and massless spectra



Quadratic pole	
Pole residue:	$\frac{1}{r_1(r_1-2r_3-r_5)(2r_3+r_5)} > 0$
Polarisations:	2

(No massive particles)

## Unitarity conditions

$$r_1 < 0 \&\& (r_5 < r_1 - 2r_3 \parallel r_5 > -2r_3) \parallel r_1 > 0 \&\& -2r_3 < r_5 < r_1 - 2r_3$$