

# Particle spectrograph

## Wave operator and propagator

	$\sigma_{1^+}^{\#1} \uparrow \alpha\beta$	$\sigma_{1^+}^{\#2} \alpha\beta$	$\tau_{1^+}^{\#1} \alpha\beta$	$\sigma_{1^+}^{\#1} \alpha$	$\sigma_{1^+}^{\#2} \alpha$	$\tau_{1^+}^{\#1} \alpha$	$\tau_{1^+}^{\#2} \alpha$
$\sigma_{1^+}^{\#1} \uparrow \alpha\beta$	$\frac{1}{k^2(2r_3+r_5)}$	$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$	$-\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$	0	0	0	0
$\sigma_{1^+}^{\#2} \uparrow \alpha\beta$	$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$	$\frac{3k^2(2r_3+r_5)+2t_2}{(k+k^3)^2(2r_3+r_5)t_2}$	$\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$	0	0	0	0
$\tau_{1^+}^{\#1} \uparrow \alpha\beta$	$\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$	$-\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$	$\frac{3k^2(2r_3+r_5)+2t_2}{(1+k^2)^2(2r_3+r_5)t_2}$	0	0	0	0
$\sigma_{1^+}^{\#1} \uparrow \alpha$	0	0	0	$\frac{2}{k^2(r_3+2r_5)}$	$\frac{2\sqrt{2}}{k^2(1+2k^2)(r_3+2r_5)}$	$\frac{4i}{k(1+2k^2)(r_3+2r_5)}$	0
$\sigma_{1^+}^{\#2} \uparrow \alpha$	0	0	0	$\frac{2\sqrt{2}}{k^2(1+2k^2)(r_3+2r_5)}$	$\frac{3k^2(r_3+2r_5)+4t_3}{(k+2k^3)^2(r_3+2r_5)t_3}$	$\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$	0
$\tau_{1^+}^{\#1} \uparrow \alpha$	0	0	0	0	0	0	0
$\tau_{1^+}^{\#2} \uparrow \alpha$	0	0	0	$-\frac{4i}{k(1+2k^2)(r_3+2r_5)}$	$-\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$	$\frac{6k^2(r_3+2r_5)+8t_3}{(1+2k^2)^2(r_3+2r_5)t_3}$	0

	$\omega_{1^+}^{\#1} \alpha\beta$	$\omega_{1^+}^{\#2} \alpha\beta$	$f_{1^+}^{\#1} \alpha\beta$	$\omega_{1^+}^{\#1} \alpha$	$\omega_{1^+}^{\#2} \alpha$	$f_{1^+}^{\#1} \alpha$	$f_{1^+}^{\#2} \alpha$
$\omega_{1^+}^{\#1} \uparrow \alpha\beta$	$k^2(2r_3+r_5)+\frac{2t_2}{3}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{1}{3}i\sqrt{2}kt_2$	0	0	0	0
$\omega_{1^+}^{\#2} \uparrow \alpha\beta$	$\frac{\sqrt{2}t_2}{3}$	$\frac{t_2}{3}$	$\frac{ikt_2}{3}$	0	0	0	0
$f_{1^+}^{\#1} \uparrow \alpha\beta$	$-\frac{1}{3}i\sqrt{2}kt_2$	$-\frac{1}{3}ikt_2$	$\frac{k^2t_2}{3}$	0	0	0	0
$\omega_{1^+}^{\#1} \uparrow \alpha$	0	0	0	$k^2(\frac{r_3}{2}+r_5)+\frac{2t_3}{3}$	$-\frac{\sqrt{2}t_3}{3}$	0	$-\frac{2}{3}i\sqrt{2}kt_3$
$\omega_{1^+}^{\#2} \uparrow \alpha$	0	0	0	$-\frac{\sqrt{2}t_3}{3}$	$\frac{t_3}{3}$	0	$\frac{1}{3}i\sqrt{2}kt_3$
$f_{1^+}^{\#1} \uparrow \alpha$	0	0	0	0	0	0	0
$f_{1^+}^{\#2} \uparrow \alpha$	0	0	0	$\frac{2ikt_3}{3}$	$-\frac{1}{3}i\sqrt{2}kt_3$	0	$\frac{2k^2t_3}{3}$

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\omega_{0^+}^{\#1}$
$\omega_{0^+}^{\#1} \uparrow$	$t_3$	$-i\sqrt{2}kt_3$	0	0
$f_{0^+}^{\#1} \uparrow$	$i\sqrt{2}kt_3$	$2k^2t_3$	0	0
$f_{0^+}^{\#2} \uparrow$	0	0	0	0
$\omega_{0^+}^{\#1} \uparrow$	0	0	0	$k^2r_2+t_2$

Quadratic (free) action
$\begin{aligned} S_F = & \iiint \left( \frac{1}{6} (4t_3 \omega_{\lambda}^{\alpha'} \omega_{\kappa\alpha}^{\kappa} + 4t_2 \omega_{\lambda}^{\kappa\lambda} \omega_{\kappa\lambda}^{\lambda} + 2t_2 \omega_{\kappa\lambda}^{\lambda} \omega_{\lambda}^{\kappa\lambda} + 6f^{\alpha\beta} \tau_{\alpha\beta} + 6\omega^{\alpha\beta\chi} \right. \\ & \sigma_{\alpha\beta\chi} - 3r_3 \partial_{\lambda} \omega_{\kappa}^{\kappa\lambda} \partial^{\lambda} \omega_{\lambda}^{\alpha} - 6r_5 \partial_{\lambda} \omega_{\kappa}^{\kappa\lambda} \partial^{\lambda} \omega_{\lambda}^{\alpha} + 4r_2 \partial_{\lambda} \omega_{\kappa}^{\theta\alpha} \partial^{\theta} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\alpha\beta}^{\kappa} - \\ & 2r_2 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega_{\alpha\beta\theta} - 4r_2 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega_{\alpha\beta}^{\theta\alpha} + 3r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\theta\kappa\lambda} - \\ & 6r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\theta\kappa\lambda} - 3r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\alpha}^{\theta\kappa\lambda} + 6r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\alpha}^{\theta\kappa\lambda} - \\ & 3r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\kappa\lambda\theta} - 6r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\kappa\lambda\theta} + 6r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\alpha}^{\kappa\lambda\theta} + \\ & 12r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\alpha}^{\kappa\lambda\theta} + t_2 \partial^{\alpha} f_{\theta\kappa} \partial^{\kappa} f_{\alpha}^{\theta} - t_2 \partial^{\alpha} f_{\kappa\theta} \partial^{\kappa} f_{\alpha}^{\theta} + t_2 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\alpha\lambda}^{\kappa} - \\ & 4t_3 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} - 4t_3 \omega_{\kappa\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} - 8t_3 \partial_{\kappa}^{\alpha} f_{\alpha}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + 4t_3 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + \\ & 2t_2 \omega_{\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} - 4t_2 \omega_{\lambda\theta} \partial^{\kappa} f_{\lambda}^{\lambda} - 2t_2 \omega_{\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} + 4t_2 \omega_{\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} + \\ & 4t_3 \omega_{\lambda\alpha}^{\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} + 4t_3 \omega_{\lambda\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} - t_2 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\alpha}^{\lambda} - t_2 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\alpha}^{\lambda} + \\ & t_2 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\alpha}^{\lambda} + 4t_3 \partial^{\alpha} f_{\alpha}^{\lambda} \partial^{\kappa} f_{\lambda\kappa}^{\lambda} + 2r_2 \partial_{\kappa} \omega^{\alpha\beta\theta} \partial^{\kappa} \omega_{\alpha\beta\theta} + \\ & 4r_2 \partial_{\kappa} \omega^{\theta\alpha\beta} \partial^{\kappa} \omega_{\alpha\beta\theta} - 4r_2 \partial_{\lambda}^{\beta} \omega_{\lambda}^{\alpha\lambda} \partial_{\lambda} \omega_{\alpha\beta}^{\lambda} + 4r_2 \partial_{\lambda}^{\beta} \omega_{\lambda}^{\lambda\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\lambda} - \\ & 24r_3 \partial^{\beta} \omega_{\lambda}^{\lambda\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\lambda} - 3r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} + 6r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} + \\ & 3r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} - 6r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} ) [t, x, y, z] dz dy dx dt \end{aligned}$

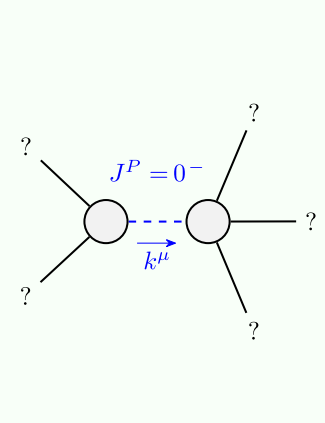
	$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^+}^{\#1}$
$\sigma_{0^+}^{\#1} \uparrow$	$\frac{1}{(1+2k^2)^2 t_3}$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2 t_3}$	0	0
$\tau_{0^+}^{\#1} \uparrow$	$\frac{i\sqrt{2}k}{(1+2k^2)^2 t_3}$	$\frac{2k^2}{(1+2k^2)^2 t_3}$	0	0
$\tau_{0^+}^{\#2} \uparrow$	0	0	0	0
$\sigma_{0^+}^{\#1} \uparrow$	0	0	0	$\frac{1}{k^2 r_2 + t_2}$

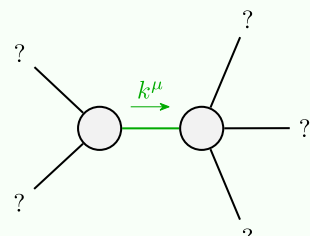
	$\omega_{2^+}^{\#1} \alpha\beta$	$f_{2^+}^{\#1} \alpha\beta$	$\omega_{2^+}^{\#1} \alpha\beta\chi$
$\omega_{2^+}^{\#1} \uparrow \alpha\beta$	$-\frac{3k^2 r_3}{2}$	0	0
$f_{2^+}^{\#1} \uparrow \alpha\beta$	0	0	0
$\omega_{2^+}^{\#1} \uparrow \alpha\beta\chi$	0	0	0

Source constraints/gauge generators	Multiplicities
$\sigma_{0^+}^{\#2} == 0$	1
$\tau_{0^+}^{\#1} - 2ik\sigma_{0^+}^{\#1} == 0$	1
$\tau_{1^+}^{\#2\alpha} + 2ik\sigma_{1^+}^{\#2\alpha} == 0$	3
$\tau_{1^+}^{\#1\alpha} == 0$	3
$\tau_{1^+}^{\#1\alpha\beta} + ik\sigma_{1^+}^{\#2\alpha\beta} == 0$	3
$\sigma_{2^+}^{\#1\alpha\beta\chi} == 0$	5
$\tau_{2^+}^{\#1\alpha\beta} == 0$	5
Total constraints:	21

	$\sigma_{2^+}^{\#1} \alpha\beta$	$\tau_{2^+}^{\#1} \alpha\beta$	$\sigma_{2^+}^{\#1} \alpha\beta\chi$
$\sigma_{2^+}^{\#1} \uparrow \alpha\beta$	$-\frac{2}{3k^2 r_3}$	0	0
$\tau_{2^+}^{\#1} \uparrow \alpha\beta$	0	0	0
$\sigma_{2^+}^{\#1} \uparrow \alpha\beta\chi$	0	0	0

## Massive and massless spectra

	<table><tr><td colspan="2">Massive particle</td></tr><tr><td>Pole residue:</td><td><math>-\frac{1}{r_2} &gt; 0</math></td></tr><tr><td>Polarisations:</td><td>1</td></tr><tr><td>Square mass:</td><td><math>-\frac{t_2}{r_2} &gt; 0</math></td></tr><tr><td>Spin:</td><td>0</td></tr><tr><td>Parity:</td><td>Odd</td></tr></table>	Massive particle		Pole residue:	$-\frac{1}{r_2} > 0$	Polarisations:	1	Square mass:	$-\frac{t_2}{r_2} > 0$	Spin:	0	Parity:	Odd
Massive particle													
Pole residue:	$-\frac{1}{r_2} > 0$												
Polarisations:	1												
Square mass:	$-\frac{t_2}{r_2} > 0$												
Spin:	0												
Parity:	Odd												

	<table><tr><td colspan="2">Quadratic pole</td></tr><tr><td>Pole residue:</td><td><math>-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2} &gt; 0</math></td></tr><tr><td>Polarisations:</td><td>2</td></tr></table>	Quadratic pole		Pole residue:	$-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2} > 0$	Polarisations:	2
Quadratic pole							
Pole residue:	$-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2} > 0$						
Polarisations:	2						

## Unitarity conditions

$$r_2 < 0 \&\& r_3 < 0 \&\& r_5 < -\frac{r_3}{2} \&\& t_2 > 0 \parallel r_2 < 0 \&\& r_3 < 0 \&\& r_5 > -2r_3 \&\& t_2 > 0 \parallel r_2 < 0 \&\& r_3 > 0 \&\& -2r_3 < r_5 < -\frac{r_3}{2} \&\& t_2 > 0$$