

# Particle spectrograph

## Wave operator and propagator

$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$	$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1-}^{\#1} \dagger^{\alpha}$	$\sigma_{1-}^{\#2} \dagger^{\alpha}$	$\tau_{1-}^{\#1} \dagger^{\alpha}$	$\tau_{1-}^{\#2} \dagger^{\alpha}$
$\sigma_{1+}^{\#1} \dagger^{\alpha\beta} = \frac{6}{(3+2k^2)^2 t_1}$	$\sigma_{1+}^{\#2} \dagger^{\alpha\beta} = -\frac{6\sqrt{2}}{(3+2k^2)^2 t_1}$	$\tau_{1+}^{\#1} \dagger^{\alpha\beta} = -\frac{6i\sqrt{2}k}{(3+2k^2)^2 t_1}$	$\sigma_{1-}^{\#1} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#2} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#2} \dagger^{\alpha} = 0$
$\sigma_{1+}^{\#2} \dagger^{\alpha\beta} = -\frac{6\sqrt{2}}{(3+2k^2)^2 t_1}$	$\sigma_{1+}^{\#1} \dagger^{\alpha\beta} = \frac{12}{(3+2k^2)^2 t_1}$	$\tau_{1+}^{\#2} \dagger^{\alpha\beta} = \frac{12ik}{(3+2k^2)^2 t_1}$	$\sigma_{1-}^{\#1} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#2} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#2} \dagger^{\alpha} = 0$
$\tau_{1+}^{\#1} \dagger^{\alpha\beta} = \frac{6i\sqrt{2}k}{(3+2k^2)^2 t_1}$	$\tau_{1+}^{\#2} \dagger^{\alpha\beta} = -\frac{12ik}{(3+2k^2)^2 t_1}$	$\tau_{1+}^{\#1} \dagger^{\alpha\beta} = \frac{12k^2}{(3+2k^2)^2 t_1}$	$\sigma_{1-}^{\#1} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#2} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#2} \dagger^{\alpha} = 0$
$\sigma_{1-}^{\#1} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#2} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#1} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#2} \dagger^{\alpha} = \frac{\sqrt{2}}{t_1+2k^2 t_1}$	$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#2} \dagger^{\alpha} = \frac{2ik}{t_1+2k^2 t_1}$
$\sigma_{1-}^{\#2} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#1} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#2} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#1} \dagger^{\alpha} = \frac{\sqrt{2}}{t_1+2k^2 t_1}$	$\sigma_{1-}^{\#2} \dagger^{\alpha} = \frac{2k^2 r_1+t_1}{(t_1+2k^2 t_1)^2}$	$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#2} \dagger^{\alpha} = \frac{i\sqrt{2}k(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2}$
$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#2} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#1} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#2} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#2} \dagger^{\alpha} = 0$
$\tau_{1-}^{\#2} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#2} \dagger^{\alpha} = 0$	$\sigma_{1-}^{\#1} \dagger^{\alpha} = -\frac{2ik}{t_1+2k^2 t_1}$	$\sigma_{1-}^{\#2} \dagger^{\alpha} = -\frac{i\sqrt{2}k(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2}$	$\tau_{1-}^{\#1} \dagger^{\alpha} = 0$	$\tau_{1-}^{\#2} \dagger^{\alpha} = \frac{2k^2(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2}$

### Quadratic (free) Lagrangian density

$$\begin{aligned} & -t_1 \omega_{\lambda'}^{\alpha'} \omega_{\kappa\alpha}^{\kappa} - \frac{1}{3} t_1 \omega_{\kappa\lambda}^{\kappa\lambda} \omega_{\lambda'}^{\lambda'} + \frac{1}{3} t_1 \omega_{\kappa\lambda}^{\lambda'} \omega_{\lambda'}^{\kappa\lambda} + f^{\alpha\beta} \tau_{\alpha\beta} + \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} + \\ & 2r_1 \partial_{\lambda'} \omega_{\kappa}^{\kappa\lambda} \partial' \omega_{\lambda}^{\alpha} - \frac{2}{3} r_1 \partial^{\beta} \omega_{\kappa}^{\theta\alpha} \partial_{\theta} \omega_{\alpha\beta}^{\kappa} - \frac{2}{3} r_1 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\alpha\beta\theta} + \\ & \frac{2}{3} r_1 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\theta\alpha\beta} + 2r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\theta\kappa\lambda} - 2r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\alpha} \omega^{\theta\kappa\lambda} + \\ & 2r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\kappa\lambda\theta} - 4r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\alpha} \omega_{\lambda}^{\theta\kappa\lambda} - \frac{1}{3} t_1 \partial^{\alpha} f_{\theta\kappa} \partial^{\kappa} f_{\alpha}^{\theta} - \\ & \frac{2}{3} t_1 \partial^{\alpha} f_{\kappa\theta} \partial^{\kappa} f_{\alpha}^{\theta} - \frac{1}{3} t_1 \partial_{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\kappa}^{\alpha} + t_1 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f_{\alpha\lambda}^{\lambda} + t_1 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} + t_1 \omega_{\kappa\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + \\ & 2t_1 \partial^{\alpha} f_{\kappa\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} - t_1 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + \frac{1}{3} t_1 \omega_{\lambda\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} + \frac{1}{3} t_1 \omega_{\lambda\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} + \frac{4}{3} t_1 \omega_{\lambda\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} - \\ & \frac{1}{3} t_1 \omega_{\lambda\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} + \frac{2}{3} t_1 \omega_{\theta\kappa\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} - t_1 \omega_{\lambda\alpha}^{\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} - t_1 \omega_{\lambda\alpha}^{\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} + \\ & \frac{1}{3} t_1 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + \frac{1}{3} t_1 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + \frac{2}{3} t_1 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} - t_1 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + \\ & \frac{2}{3} r_1 \partial_{\kappa} \omega^{\alpha\beta\theta} \partial^{\kappa} \omega_{\alpha\beta\theta} - \frac{2}{3} r_1 \partial_{\kappa} \omega^{\theta\alpha\beta} \partial^{\kappa} \omega_{\alpha\beta\theta} + \frac{2}{3} r_1 \partial^{\beta} \omega_{\lambda}^{\alpha\lambda} \partial_{\lambda} \omega_{\alpha\beta}^{\lambda} - \\ & \frac{8}{3} r_1 \partial^{\beta} \omega_{\lambda}^{\lambda\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\lambda} - 2r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} + 2r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\lambda}^{\theta\kappa} \end{aligned}$$

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0+}^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	$-\frac{1}{(1+2k^2)^2 t_1}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2 t_1}$	0	0
$\tau_{0+}^{\#1} \dagger$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2 t_1}$	$-\frac{2k^2}{(1+2k^2)^2 t_1}$	0	0
$\tau_{0+}^{\#2} \dagger$	0	0	0	0
$\sigma_{0+}^{\#1} \dagger$	0	0	0	0

	$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1-}^{\#1} \dagger^{\alpha}$	$\omega_{1-}^{\#2} \dagger^{\alpha}$	$f_{1-}^{\#1} \dagger^{\alpha}$	$f_{1-}^{\#2} \dagger^{\alpha}$
$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{t_1}{6}$	$-\frac{t_1}{3\sqrt{2}}$	$-\frac{ikt_1}{3\sqrt{2}}$	0	0	0	0
$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	$\frac{ikt_1}{3}$	0	0	0	0
$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{3\sqrt{2}}$	$-\frac{1}{3} \frac{ikt_1}{3}$	$\frac{k^2 t_1}{3}$	0	0	0	0
$\omega_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	$-k^2 r_1 - \frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$i k t_1$
$\omega_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
$f_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$f_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$-i k t_1$	0	0	0

	$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{2+}^{\#1} \dagger^{\alpha\beta\chi}$
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2 t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	0
$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	$\frac{4k^2}{(1+2k^2)^2 t_1}$	0
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{2k^2 r_1+t_1}$

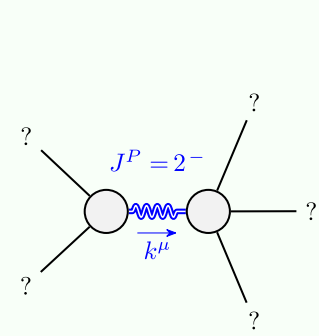
	$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	$k^2 t_1$	0
$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$k^2 r_1 + \frac{t_1}{2}$

	$\omega_{0+}^{\#1}$	$f_{0+}^{\#1}$	$f_{0+}^{\#2}$	$\omega_{0-}^{\#1}$
$\omega_{0+}^{\#1} \dagger$	$-t_1$	$i\sqrt{2} k t_1$	0	0
$f_{0+}^{\#1} \dagger$	$-i\sqrt{2} k t_1$	$-2k^2 t_1$	0	0
$f_{0+}^{\#2} \dagger$	0	0	0	0
$\omega_{0-}^{\#1} \dagger$	0	0	0	0

### Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\sigma_{0-}^{\#1} == 0$	1
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} - 2ik\sigma_{1+}^{\#1\alpha\beta} == 0$	3
$2\sigma_{1+}^{\#1\alpha\beta} + \sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	20

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

## Unitarity conditions

$$r_1 < 0 \text{ \&\& } t_1 > 0$$