

Particle spectrograph

Wave operator and propagator

	$\sigma_{1^{+}\alpha\beta}^{\#1}$	$\sigma_{1^{+}\alpha\beta}^{\#2}$	$\tau_{1^{+}\alpha\beta}^{\#1}$	$\sigma_{1^{-}\alpha}^{\#1}$	$\sigma_{1^{-}\alpha}^{\#2}$	$\tau_{1^{-}\alpha}^{\#1}$	$\tau_{1^{-}\alpha}^{\#2}$
$\sigma_{1^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\frac{2\left(t_1+t_2\right)}{3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)}$	$\frac{\sqrt{2}\left(t_1-2t_2\right)}{\left(1+k^2\right)\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	$\frac{i\sqrt{2}k\left(t_1-2t_2\right)}{\left(1+k^2\right)\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	0	0	0	0
$\sigma_{1^{+}}^{\#2}\uparrow^{\alpha\beta}$	$\frac{\sqrt{2}\left(t_1-2t_2\right)}{\left(1+k^2\right)\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	$\frac{6k^2\left(2r_1+r_5\right)+t_1+4t_2}{\left(1+k^2\right)^2\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	$\frac{ik\left(6k^2\left(2r_1+r_5\right)+t_1+4t_2\right)}{\left(1+k^2\right)^2\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	0	0	0	0
$\tau_{1^{+}}^{\#1}\uparrow^{\alpha\beta}$	$-\frac{i\sqrt{2}k\left(t_1-2t_2\right)}{\left(1+k^2\right)\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	$-\frac{ik\left(6k^2\left(2r_1+r_5\right)+t_1+4t_2\right)}{\left(1+k^2\right)^2\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	$\frac{k^2\left(6k^2\left(2r_1+r_5\right)+t_1+4t_2\right)}{\left(1+k^2\right)^2\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	0	0	0	0
$\sigma_{1^{-}}^{\#1}\uparrow^{\alpha}$	0	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	0	$\frac{2ik}{t_1+2k^2t_1}$
$\sigma_{1^{-}}^{\#2}\uparrow^{\alpha}$	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	$\frac{-2k^2\left(r_1+r_5\right)+t_1}{\left(t_1+2k^2t_1\right)^2}$	0	$-\frac{i\sqrt{2}k\left(2k^2\left(r_1+r_5\right)+t_1\right)}{\left(t_1+2k^2t_1\right)^2}$
$\tau_{1^{-}}^{\#1}\uparrow^{\alpha}$	0	0	0	0	0	0	0
$\tau_{1^{-}}^{\#2}\uparrow^{\alpha}$	0	0	0	$-\frac{2ik}{t_1+2k^2t_1}$	$\frac{i\sqrt{2}k\left(2k^2\left(r_1+r_5\right)+t_1\right)}{\left(t_1+2k^2t_1\right)^2}$	0	$\frac{-4k^4\left(r_1+r_5\right)+2k^2t_1}{\left(t_1+2k^2t_1\right)^2}$

$\omega_{2^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\omega_{2^{+}}^{\#1}f_{2^{+}\alpha\beta}$	$\omega_{2^{-}}^{\#1}\uparrow^{\alpha\beta\chi}$
$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0
$\omega_{2^{-}}^{\#1}\uparrow^{\alpha\beta\chi}$	0	$k^2r_1+\frac{t_1}{2}$

Source constraints		
SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0^{+}}^{\#2} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$	1
$\tau_{0^{+}}^{\#1}-2\,i\,k\,\sigma_{0^{+}}^{\#1} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha + 2\,\partial_\chi\partial^\chi\partial_\beta\sigma^{\alpha\beta}{}_\alpha$	1
$\tau_{1^{-}}^{\#2\alpha} + 2\,i\,k\,\sigma_{1^{-}}^{\#2\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$	3
$\tau_{1^{-}}^{\#1\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$	3
$\tau_{1^{+}}^{\#1\alpha\beta} + i\,k\,\sigma_{1^{+}}^{\#2\alpha\beta} == 0$	$\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} +$ $2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} ==$ $\partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} +$ $\partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$	3
$\tau_{2^{+}}^{\#1\alpha\beta} - 2\,i\,k\,\sigma_{2^{+}}^{\#1\alpha\beta} == 0$	$-i\,(4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^{\chi}{}_\chi -$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} -$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} +$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} +$ $4\,i\,k^\chi\,\partial_\epsilon\partial_\chi\partial^\beta\partial^\alpha\sigma^{\delta\epsilon}{}_\delta -$ $6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\delta\epsilon} -$ $6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\delta\epsilon} +$ $2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} +$ $6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\alpha\delta\beta} +$ $6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\beta\delta\alpha} -$ $2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^{\chi}{}_\chi -$ $4\,i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\chi\sigma^{\delta\epsilon}{}_\delta) == 0$	5
Total constraints/gauge generators:		16

$\omega_{1^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\omega_{1^{+}}^{\#2}\uparrow^{\alpha\beta}$	$f_{1^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\omega_{1^{-}\alpha}^{\#1}$	$\omega_{1^{-}\alpha}^{\#2}$	$f_{1^{-}\alpha}^{\#1}$	$f_{1^{-}\alpha}^{\#2}$
$\frac{1}{6}\frac{1}{(1+2k^2)^2}t_1$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$-\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	0	0	0	0
$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_1$	$\frac{t_1+t_2}{3}$	$\frac{1}{3}ik(t_1+t_2)$	0	0	0	0
$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_1$	$-\frac{1}{3}ik(t_1+t_2)$	$\frac{1}{3}k^2(t_1+t_2)$	$k^2(r_1+r_5)-\frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$iik t_1$
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

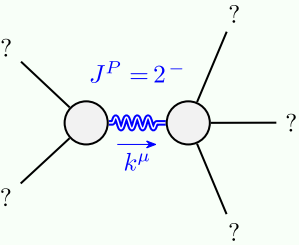
$\sigma_{0^{+}}^{\#1}\uparrow$	$\tau_{0^{+}}^{\#1}\uparrow$	$\sigma_{0^{+}}^{\#2}\uparrow$	$\omega_{0^{-}}^{\#1}\uparrow$
$-\frac{1}{(1+2k^2)^2}t_1$	$\frac{i\sqrt{2}k}{(1+2k^2)^2}t_1$	$\frac{2}{(1+2k^2)^2}t_1$	-t <sub>1</sub>
$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_1$	$-\frac{2k^2}{(1+2k^2)^2}t_1$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$	$i\sqrt{2}kt_1$
0	0	0	-2k <sup>2</sup> t <sub>1</sub>
0	0	0	0
0	0	0	t <sub>2</sub>

$\sigma_{2^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\tau_{2^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\sigma_{2^{-}}^{\#1}\uparrow^{\alpha\beta\chi}$
$\frac{2}{(1+2k^2)^2}t_1$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$	0
0	$\frac{4k^2}{(1+2k^2)^2}t_1$	$\frac{2}{2k^2r_1+t_1}$

Quadratic (free) action
$S = \iiint (\frac{1}{6}(6t_1\omega^{\alpha i}{}_\alpha\omega_{,i}{}^\theta{}_\theta + 6f^{\alpha\beta}\tau_{\alpha\beta} + 6\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi} - 12t_1\omega_\alpha{}^\theta{}_\theta\partial_{,i}f^{\alpha i} + 12t_1\omega_{,i}{}^\theta{}_\theta\partial'f^\alpha{}_\alpha - 6t_1\partial_{,i}f^\theta{}_\theta\partial'f^\alpha{}_\alpha - 6t_1\partial_{,i}f^{\alpha i}\partial_\theta f_\alpha{}^\theta + 12t_1\partial'f^\alpha{}_\alpha\partial_\theta f_{,i}{}^\theta + 4t_1\omega_{,i\theta\alpha}\partial^\theta f^{\alpha i} + 4t_2\omega_{,i\theta\alpha}\partial^\theta f^{\alpha i} - 4t_1\partial_\alpha f_{,i\theta}\partial^\theta f^{\alpha i} + 2t_2\partial_\alpha f_{,i\theta}\partial^\theta f^{\alpha i} - 4t_1\partial_\alpha f_{\theta i}\partial^\theta f^{\alpha i} - t_2\partial_\alpha f_{\theta i}\partial^\theta f^{\alpha i} + 2t_1\partial_{,i}f_{\alpha\theta}\partial^\theta f^{\alpha i} - t_2\partial_{,i}f_{\alpha\theta}\partial^\theta f^{\alpha i} + 4t_1\partial_\theta f_{\alpha i}\partial^\theta f^{\alpha i} + t_2\partial_\theta f_{\alpha i}\partial^\theta f^{\alpha i} + 2t_1\partial_\theta f_{i\alpha}\partial^\theta f^{\alpha i} - t_2\partial_\theta f_{i\alpha}\partial^\theta f^{\alpha i} + 2(t_1+t_2)\omega_{\alpha i\theta}(\omega^{\alpha i\theta} + 2\partial^\theta f^{\alpha i}) + 2\omega_{\alpha\theta i}((t_1-2t_2)\omega^{\alpha i\theta} + 2(2t_1-t_2)\partial^\theta f^{\alpha i}) - 8r_1\partial_\beta\omega_{\alpha i\theta}\partial^\theta\omega^{\alpha\beta i} + 4r_1\partial_\beta\omega_{\alpha\theta i}\partial^\theta\omega^{\alpha\beta i} - 16r_1\partial_\beta\omega_{,i\theta\alpha}\partial^\theta\omega^{\alpha\beta i} - 4r_1\partial_{,i}\omega_{\alpha\beta\theta}\partial^\theta\omega^{\alpha\beta i} + 4r_1\partial_\theta\omega_{\alpha\beta i}\partial^\theta\omega^{\alpha\beta i} + 4r_1\partial_\theta\omega_{\alpha i\beta}\partial^\theta\omega^{\alpha\beta i} + 6r_5\partial_{,i}\omega_\theta{}^\kappa{}_\kappa\partial^\theta\omega^{\alpha i}{}_\alpha - 6r_5\partial_\theta\omega_{,i}{}^\kappa{}_\kappa\partial^\theta\omega^{\alpha i}{}_\alpha - 6r_5\partial_\alpha\omega^{\alpha i\theta}\partial_\kappa\omega_{,i}{}^\kappa{}_\theta + 12r_5\partial^\theta\omega^{\alpha i}{}_\alpha\partial_\kappa\omega_{,i}{}^\kappa{}_\theta + 6r_5\partial_\alpha\omega^{\alpha i\theta}\partial_\kappa\omega_\theta{}^\kappa{}_{,i} - 12r_5\partial^\theta\omega^{\alpha i}{}_\alpha\partial_\kappa\omega_\theta{}^\kappa{}_{,i})) [t, x, y, z] dz dy dx dt$

Massive and massless spectra

Massive particle	
Pole residue:	$\frac{-3t_1t_2(t_1+t_2)+6r_1(t_1^2+2t_2^2)+3r_5(t_1^2+2t_2^2)}{(2r_1+r_5)(t_1+t_2)(-3t_1t_2+4r_1(t_1+t_2)+2r_5(t_1+t_2))} > 0$
Polarisations:	3
Square mass:	$-\frac{3t_1t_2}{2(2r_1+r_5)(t_1+t_2)} > 0$
Spin:	1
Parity:	Even



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0 \ \&\& \ r_5 > -2\,r_1 \ \&\& \ t_1 > 0 \ \&\& \ -t_1 < t_2 < 0$