

Particle spectrograph

Wave operator and propagator

| Source constraints | | |
|---|--|----------------|
| SO(3) irreps | Fundamental fields | Multiplicities |
| $\tau_{0+}^{\#2} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$ | 1 |
| $\tau_{0+}^{\#1} - 2 i k \sigma_{0+}^{\#1} == 0$ | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha_\alpha + 2 \partial_\chi \partial^\chi \partial_\beta \sigma^{\alpha\beta}_\alpha$ | 1 |
| $\tau_{1+}^{\#2\alpha} + 2 i k \sigma_{1+}^{\#2\alpha} == 0$ | $\partial_\chi \partial_\beta \partial_\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta} + 2 \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$ | 3 |
| $\tau_{1+}^{\#1\alpha} == 0$ | $\partial_\chi \partial_\beta \partial_\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$ | 3 |
| $\tau_{1+}^{\alpha\beta} + i k \sigma_{1+}^{\#1\alpha\beta} == 0$ | $\partial_\chi \partial_\beta \tau^{\alpha\beta\chi} + \partial_\chi \partial^\beta \tau^\alpha_\chi + \partial_\chi \partial^\chi \tau^{\alpha\beta} + \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\beta\chi\alpha} ==$ $\partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\beta\chi\alpha} ==$ $\partial_\chi \partial^\alpha \tau^\beta_\chi + \partial_\chi \partial^\beta \tau^\alpha_\chi + \partial_\chi \partial^\chi \tau^{\beta\alpha} + \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\chi\beta}$ | 3 |
| $\sigma_{1+}^{\#1\alpha\beta} == \sigma_{1+}^{\#2\alpha\beta}$ | $3 \partial_\delta \partial_\chi \partial_\alpha \sigma^{\beta\chi\delta} +$ $2 \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\chi\beta} ==$ $3 \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\beta\chi\alpha}$ | 3 |
| $\sigma_{2+}^{\#1\alpha\beta\chi} == 0$ | $3 \partial_\epsilon \partial_\delta \partial_\chi \partial_\alpha \sigma^{\beta\delta\epsilon} + 3 \partial_\epsilon \partial^\epsilon \partial_\chi \partial_\alpha \sigma^{\beta\delta}_\delta +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\alpha\chi\delta} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\alpha\delta\chi} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\chi\delta\alpha} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\alpha\beta\delta} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\alpha\delta\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\beta\chi\alpha} +$ $3 \eta^{\beta\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial_\alpha \sigma^{\delta\epsilon}_\delta +$ $3 \eta^{\alpha\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial_\delta \sigma^{\beta\delta\epsilon} +$ $3 \eta^{\beta\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial_\alpha \sigma^{\alpha\delta}_\delta ==$ $3 \partial_\epsilon \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 3 \partial_\epsilon \partial^\epsilon \partial_\chi \partial_\beta \sigma^{\alpha\delta}_\delta +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\beta\chi\delta} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\alpha\delta\chi} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\alpha\chi\delta\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\beta\delta\alpha} +$ $4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\alpha\beta\chi} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\beta \sigma^{\alpha\chi\beta} +$ $3 \eta^{\alpha\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial_\beta \sigma^{\delta\epsilon}_\delta +$ $3 \eta^{\beta\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial_\alpha \sigma^{\alpha\delta\epsilon} +$ $3 \eta^{\alpha\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial_\beta \sigma^{\beta\delta}_\delta$ | 5 |
| $\tau_{2+}^{\#1\alpha\beta} == 0$ | $4 \partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2 \partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi_\chi +$ $3 \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\alpha\beta} + 3 \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\alpha} +$ $2 \eta^{\alpha\beta} \partial_\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} ==$ $3 \partial_\delta \partial^\delta \partial_\chi \partial_\alpha \tau^{\beta\chi} + 3 \partial_\delta \partial^\delta \partial_\chi \partial_\alpha \tau^{\chi\alpha} +$ $3 \partial_\delta \partial^\delta \partial_\chi \partial_\beta \tau^{\alpha\chi} + 3 \partial_\delta \partial^\delta \partial_\chi \partial_\beta \tau^{\chi\alpha} +$ $2 \eta^{\alpha\beta} \partial_\epsilon \partial_\delta \partial_\alpha \tau^\chi_\chi$ | 5 |
| Total constraints/gauge generators: | | 24 |

Quadratic (free) action

$$S = \int \int \int \int \Big(\frac{1}{6} (-4 t_3 \omega_{\alpha}^{\alpha\iota} \omega_{\iota\kappa}^{\kappa} + 6 f^{\alpha\beta} \tau_{\alpha\beta} + 6 \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} + 8 t_3 \omega_{\alpha\kappa}^{\kappa} \partial_{\iota} f^{\alpha\iota} - 8 t_3 \omega_{\iota\kappa}^{\kappa} \partial_{\iota} f^{\alpha}_{\alpha} + 4 t_3 \partial_{\iota} f^{\kappa}_{\kappa} \partial_{\iota} f^{\alpha}_{\alpha} - 15 r_3 \partial_{\beta} \omega_{\iota\beta}^{\beta} \partial_{\iota} \omega^{\alpha\beta}_{\alpha} + 9 r_3 \partial_{\iota} \omega_{\beta\beta}^{\beta} \partial_{\iota} \omega^{\alpha\beta}_{\alpha} + 9 r_3 \partial_{\alpha} \omega^{\alpha\beta\iota} \partial_{\theta} \omega_{\beta\iota}^{\theta} - 18 r_3 \partial_{\iota} \omega_{\alpha\beta}^{\alpha\beta} \partial_{\theta} \omega_{\beta\iota}^{\theta} - 15 r_3 \partial_{\alpha} \omega^{\alpha\beta\iota} \partial_{\theta} \omega_{\iota\beta}^{\theta} + 30 r_3 \partial_{\iota} \omega_{\alpha\beta}^{\alpha\beta} \partial_{\theta} \omega_{\iota\beta}^{\theta} + 4 t_2 \omega_{\theta\alpha} \partial^{\theta} f^{\alpha\iota} + 2 t_2 \partial_{\alpha} f_{\theta} \partial^{\theta} f^{\alpha\iota} - t_2 \partial_{\alpha} f_{\theta\iota} \partial^{\theta} f^{\alpha\iota} - t_2 \partial_{\iota} f_{\alpha\theta} \partial^{\theta} f^{\alpha\iota} + t_2 \partial_{\theta} f_{\alpha\iota} \partial^{\theta} f^{\alpha\iota} - t_2 \partial_{\theta} f_{\iota\alpha} \partial^{\theta} f^{\alpha\iota} - 4 t_2 \omega_{\alpha\theta\iota} (\omega^{\alpha\iota\theta} + \partial^{\theta} f^{\alpha\iota}) + 2 t_2 \omega_{\alpha\iota\theta} (\omega^{\alpha\iota\theta} + 2 \partial^{\theta} f^{\alpha\iota}) + 8 r_2 \partial_{\beta} \omega_{\alpha\iota\theta} \partial^{\theta} \omega^{\alpha\beta\iota} - 4 r_2 \partial_{\beta} \omega_{\alpha\theta\iota} \partial^{\theta} \omega^{\alpha\beta\iota} + 4 r_2 \partial_{\beta} \omega_{\iota\theta\alpha} \partial^{\theta} \omega^{\alpha\beta\iota} - 2 r_2 \partial_{\theta} \omega_{\alpha\beta\theta} \partial^{\theta} \omega^{\alpha\beta\iota} + 24 r_3 \partial_{\beta} \omega_{\iota\theta\alpha} \partial^{\theta} \omega^{\alpha\beta\iota} - 2 r_2 \partial_{\theta} \omega_{\alpha\beta\theta} \partial^{\theta} \omega^{\alpha\beta\iota} + 2 r_2 \partial_{\theta} \omega_{\alpha\beta\theta} \partial^{\theta} \omega^{\alpha\beta\iota} - 4 r_2 \partial_{\theta} \omega_{\alpha\iota\beta} \partial^{\theta} \omega^{\alpha\beta\iota} + 4 t_3 \partial_{\iota} f^{\alpha\iota} \partial_{\alpha} \omega_{\iota\kappa}^{\kappa} - 8 t_3 \partial_{\iota} f^{\alpha}_{\alpha} \partial_{\iota} \omega_{\kappa}^{\kappa}) [t, x, y, z] dz dy dx dt$$

| | | | | | | |
|--|-----------------------------------|---|---|---|-------------------|--|
| $\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{1+}^{\#2}$ | $\tau_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{1+}^{\#1}$ | $\sigma_{1+}^{\#2}$ | $\tau_{1+}^{\#1}$ | $\tau_{1+}^{\#2}$ |
| $\frac{6}{(3+k^2)^2} t_2$ | $\frac{3\sqrt{2}}{(3+k^2)^2} t_2$ | $\frac{3 i \sqrt{2} k}{(3+k^2)^2} t_2$ | 0 | 0 | 0 | 0 |
| $\frac{3\sqrt{2}}{(3+k^2)^2} t_2$ | $\frac{3}{(3+k^2)^2} t_2$ | $\frac{3 i k}{(3+k^2)^2} t_2$ | 0 | 0 | 0 | 0 |
| $\tau_{1+}^{\#1} \dagger^{\alpha\beta} - \frac{3 i \sqrt{2} k}{(3+k^2)^2} t_2$ | $-\frac{3 i k}{(3+k^2)^2} t_2$ | $\frac{3 k^2}{(3+k^2)^2} t_2$ | 0 | 0 | 0 | 0 |
| $\sigma_{1+}^{\#1} \dagger^{\alpha}$ | 0 | 0 | $-\frac{2}{3 k^2 r_3}$ | $-\frac{2 \sqrt{2}}{3 k^2 r_3 + 6 k^4 r_3}$ | 0 | $-\frac{4 i}{3 k r_3 + 6 k^3 r_3}$ |
| $\sigma_{1+}^{\#2} \dagger^{\alpha}$ | 0 | 0 | $-\frac{2 \sqrt{2}}{3 k^2 r_3 + 6 k^4 r_3}$ | $\frac{9 k^2 r_3 - 4 t_3}{3 (k + 2 k^2)^2 r_3 t_3}$ | 0 | $\frac{i \sqrt{2} (9 k^2 r_3 - 4 t_3)}{3 k (1 + 2 k^2)^2 r_3 t_3}$ |
| $\tau_{1+}^{\#1} \dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1+}^{\#2} \dagger^{\alpha}$ | 0 | 0 | $\frac{4 i}{3 k r_3 + 6 k^3 r_3}$ | $-\frac{i \sqrt{2} (9 k^2 r_3 - 4 t_3)}{3 k (1 + 2 k^2)^2 r_3 t_3}$ | 0 | $\frac{2 (9 k^2 r_3 - 4 t_3)}{3 (1 + 2 k^2)^2 r_3 t_3}$ |

| | | | | | | |
|---|--|--------------------------------|------------------------------------|---------------------------------|----------------|--------------------------------|
| $\omega_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{1+}^{\#2}$ | $f_{1+}^{\#1}$ | $\omega_{1+}^{\#1}$ | $\omega_{1+}^{\#2}$ | $f_{1+}^{\#1}$ | $f_{1+}^{\#2}$ |
| $\frac{2 t_2}{3}$ | $\frac{\sqrt{2} t_2}{3}$ | $\frac{1}{3} i \sqrt{2} k t_2$ | 0 | 0 | 0 | 0 |
| $\omega_{1+}^{\#2} \dagger^{\alpha\beta}$ | $\frac{t_2}{3}$ | $\frac{i k t_2}{3}$ | 0 | 0 | 0 | 0 |
| $f_{1+}^{\#1} \dagger^{\alpha\beta} - \frac{1}{3} i \sqrt{2} k t_2$ | $\frac{1}{3} i \sqrt{2} k t_2 - \frac{1}{3} i k t_2$ | $\frac{k^2 t_2}{3}$ | 0 | 0 | 0 | 0 |
| $\omega_{1+}^{\#1} \dagger^{\alpha}$ | 0 | 0 | $\frac{1}{6} (-9 k^2 r_3 + 4 t_3)$ | $-\frac{\sqrt{2} t_3}{3}$ | 0 | $-\frac{2}{3} i k t_3$ |
| $\omega_{1+}^{\#2} \dagger^{\alpha}$ | 0 | 0 | $-\frac{\sqrt{2} t_3}{3}$ | $\frac{t_3}{3}$ | 0 | $\frac{1}{3} i \sqrt{2} k t_3$ |
| $f_{1+}^{\#1} \dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1+}^{\#2} \dagger^{\alpha}$ | 0 | 0 | $\frac{2 i k t_3}{3}$ | $-\frac{1}{3} i \sqrt{2} k t_3$ | 0 | $\frac{2 k^2 t_3}{3}$ |

$\omega_{0+}^{\#1} \dagger$
 $f_{0+}^{\#1} \dagger$
 $f_{0+}^{\#2} \dagger$
 $\omega_{0+}^{\#1} \dagger$

$\omega_{0+}^{\#1} \dagger$
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$\sigma_{0+}^{\#1} \dagger$
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 $\sigma_{2+}^{\#1} \dagger^{\alpha\beta\chi}$

Massive and massless spectra

| Massive particle | |
|------------------|------------------------|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $-\frac{t_2}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

(No massless particles)
(separable particles)

Unitarity conditions

$r_2 < 0 \ \&\& \ t_2 > 0$