# Particle spectrograph

## Wave operator and propagator

| | $\sigma_{1^-\,\alpha}^{\#1}$ | $\sigma_{1^-\,\alpha}^{\#2}$ | $\tau_{1^-\,\alpha}^{\#1}$ | $\tau_{1^-\,\alpha}^{\#2}$ |
|---|---|---|---|---|
| $\sigma_{1^+\,\alpha\beta}^{\#1\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+\,\alpha\beta}^{\#2\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+\,\alpha\beta}^{\#1\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1\,\dagger\,\alpha}$ | $\frac{2}{k^2\,(2\,r_3+r_5)}$ | $\frac{2\,\sqrt{2}}{k^2\,(1+2\,k^2)\,(r_3+2\,r_5)}$ | 0 | $\frac{4\,i}{k\,(1+2\,k^2)\,(r_3+2\,r_5)}$ |
| $\sigma_{1^-}^{\#2\,\dagger\,\alpha}$ | $\frac{2\,\sqrt{2}}{k^2\,(1+2\,k^2)\,(r_3+2\,r_5)}$ | $\frac{3\,k^2\,(r_3+2\,r_5)+4\,t_3}{(k+2\,k^3)^2\,(r_3+2\,r_5)\,t_3}$ | 0 | $\frac{i\,\sqrt{2}\,(3\,k^2\,(r_3+2\,r_5)+4\,t_3)}{k\,(1+2\,k^2)^2\,(r_3+2\,r_5)\,t_3}$ |
| $\tau_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2\,\dagger\,\alpha}$ | $\frac{4\,i}{k\,(1+2\,k^2)\,(r_3+2\,r_5)}$ | $-\frac{i\,\sqrt{2}\,(3\,k^2\,(r_3+2\,r_5)+4\,t_3)}{k\,(1+2\,k^2)^2\,(r_3+2\,r_5)\,t_3}$ | 0 | $\frac{6\,k^2\,(r_3+2\,r_5)+8\,t_3}{(1+2\,k^2)^2\,(r_3+2\,r_5)\,t_3}$ |

| | $\sigma_{1^+\,\alpha\beta}^{\#1}$ | $\sigma_{1^+\,\alpha\beta}^{\#2}$ | $\tau_{1^+\,\alpha\beta}^{\#1}$ |
|---|---|---|---|
| $\sigma_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $\frac{1}{k^2\,(2\,r_3+r_5)}$ | 0 | 0 |
| $\sigma_{1^+}^{\#2\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 |
| $\tau_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 |

| | $\omega_{1^-\,\alpha}^{\#1}$ | $\omega_{1^-\,\alpha}^{\#2}$ | $f_{1^-\,\alpha}^{\#1}$ | $f_{1^-\,\alpha}^{\#2}$ |
|---|---|---|---|---|
| $\omega_{1^+\,\alpha\beta}^{\#1\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+\,\alpha\beta}^{\#2\,\dagger}$ | 0 | 0 | 0 | 0 |
| $f_{1^+\,\alpha\beta}^{\#1\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1\,\dagger\,\alpha}$ | $k^2\,(\frac{r_3}{2}+r_5)+\frac{2\,t_3}{3}$ | $-\frac{\sqrt{2}\,t_3}{3}$ | 0 | $-\frac{2}{3}\,i\,k\,t_3$ |
| $\omega_{1^-}^{\#2\,\dagger\,\alpha}$ | $-\frac{\sqrt{2}\,t_3}{3}$ | $\frac{t_3}{3}$ | 0 | $\frac{1}{3}\,i\,\sqrt{2}\,k\,t_3$ |
| $f_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2\,\dagger\,\alpha}$ | $\frac{2\,i\,k\,t_3}{3}$ | $-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_3$ | 0 | $\frac{2\,k^2\,t_3}{3}$ |

| | $\omega_{1^+\,\alpha\beta}^{\#1}$ | $\omega_{1^+\,\alpha\beta}^{\#2}$ | $f_{1^+\,\alpha\beta}^{\#1}$ |
|---|---|---|---|
| $\omega_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $k^2\,(2\,r_3+r_5)$ | 0 | 0 |
| $\omega_{1^+}^{\#2\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $f_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 |
| $\omega_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 |
| $f_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 |
| $f_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 |

Quadratic (free) Lagrangian density

$$\frac{2}{3}\,t_3\,\omega_{\prime}^{\alpha\prime}\,\omega_{\kappa\alpha}{}^{\kappa}+f^{\alpha\beta}\,\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}-\frac{1}{2}\,r_3\,\partial_{\prime}\omega^{\kappa\lambda}{}_{\kappa}\,\partial^{\prime}\omega_{\lambda}{}^{\alpha}{}_{\alpha}-r_5\,\partial_{\prime}\omega^{\kappa\lambda}{}_{\kappa}\,\partial^{\prime}\omega_{\lambda}{}^{\alpha}{}_{\alpha}-\frac{1}{2}\,r_3\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial_{\kappa}\omega^{\theta\kappa\lambda}-r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial_{\kappa}\omega^{\theta\kappa\lambda}-\frac{1}{2}\,r_3\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_{\kappa}\omega^{\kappa\lambda\theta}-r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial_{\kappa}\omega^{\kappa\lambda\theta}-\frac{1}{2}\,r_3\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_{\kappa}\omega^{\kappa\lambda\theta}+2\,r_5\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_{\kappa}\omega^{\kappa\lambda\theta}+\frac{2}{3}\,t_3\,\omega_{\kappa\alpha}\,\partial^{\kappa}f^{\prime}{}_{\prime}+\frac{2}{3}\,t_3\,\partial_{\kappa}f^{\lambda}{}_{\lambda}\,\partial^{\kappa}f^{\prime}{}_{\prime}+\frac{2}{3}\,t_3\,\omega_{\kappa\alpha}\,\partial^{\kappa}f^{\prime}{}_{\prime}-\frac{2}{3}\,t_3\,\omega_{\kappa\lambda}{}^{\lambda}\,\partial^{\kappa}f^{\prime}{}_{\prime}-\frac{4}{3}\,t_3\,\partial^{\alpha}f_{\kappa\alpha}\,\partial^{\kappa}f^{\prime}{}_{\prime}+\frac{2}{3}\,t_3\,\omega_{\prime\alpha}{}^{\alpha}\,\partial^{\kappa}f^{\prime}{}_{\kappa}+\frac{2}{3}\,t_3\,\omega_{\prime\lambda}{}^{\lambda}\,\partial^{\kappa}f^{\prime}{}_{\kappa}+\frac{2}{3}\,t_3\,\partial^{\alpha}f^{\lambda}{}_{\alpha}\,\partial^{\kappa}f_{\lambda\kappa}-4\,r_3\,\partial^{\beta}\omega_{\prime}{}^{\lambda\alpha}\,\partial_{\lambda}\omega_{\alpha\beta}{}^{\prime}-\frac{1}{2}\,r_3\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}-r_5\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}+\frac{1}{2}\,r_3\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}+r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}$$

| | $\sigma_{2^+\,\alpha\beta}^{\#1}$ | $\tau_{2^+\,\alpha\beta}^{\#1}$ | $\sigma_{2^-\,\alpha\beta\chi}^{\#1}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | $-\frac{2}{3\,k^2\,r_3}$ | 0 | 0 |
| $\tau_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $\sigma_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$ | 0 | 0 | 0 |

| | $\omega_{2^+\,\alpha\beta}^{\#1}$ | $f_{2^+\,\alpha\beta}^{\#1}$ | $\omega_{2^-\,\alpha\beta\chi}^{\#1}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | $-\frac{3\,k^2\,r_3}{2}$ | 0 | 0 |
| $f_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $\omega_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$ | 0 | 0 | 0 |

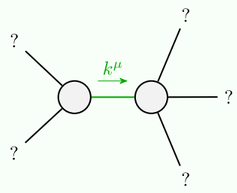
| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1\,\dagger}$ | $t_3$ | $-i\,\sqrt{2}\,k\,t_3$ | 0 | 0 |
| $f_{0^+}^{\#1\,\dagger}$ | $i\,\sqrt{2}\,k\,t_3$ | $2\,k^2\,t_3$ | 0 | 0 |
| $f_{0^+}^{\#2\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1\,\dagger}$ | 0 | 0 | 0 | 0 |

Source constraints/gauge generators

| SO(3) irreps | Multiplicities |
|---|---|
| $\sigma_{0^-}^{\#1} == 0$ | 1 |
| $\tau_{0^+}^{\#2} == 0$ | 1 |
| $\tau_{0^+}^{\#1} - 2\,i\,k\,\sigma_{0^+}^{\#1} == 0$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} == 0$ | 3 |
| $\sigma_{1^+}^{\#2\,\alpha\beta} == 0$ | 3 |
| $\sigma_{2^-}^{\#1\,\alpha\beta\chi} == 0$ | 5 |
| $\tau_{2^+}^{\#1\,\alpha\beta} == 0$ | 5 |
| Total constraints: | 25 |

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1\,\dagger}$ | $\frac{1}{(1+2\,k^2)^2\,t_3}$ | $-\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_3}$ | 0 | 0 |
| $\tau_{0^+}^{\#1\,\dagger}$ | $\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_3}$ | $\frac{2\,k^2}{(1+2\,k^2)^2\,t_3}$ | 0 | 0 |
| $\tau_{0^+}^{\#2\,\dagger}$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1\,\dagger}$ | 0 | 0 | 0 | 0 |

## Massive and massless spectra




Quadratic pole

| | |
|---|---|
| Pole residue: | $-\frac{1}{r_3\,(2\,r_3+r_5)\,(r_3+2\,r_5)\,p^2} > 0$ |
| Polarisations: | 2 |

(No massive particles)

## Unitarity conditions

$$r_3 < 0 \,\&\&\, (r_5 < -\frac{r_3}{2} \,||\, r_5 > -2\,r_3) \,||\, r_3 > 0 \,\&\&\, -2\,r_3 < r_5 < -\frac{r_3}{2}$$