Particle spectrograph

Wave operator and propagator

$$\begin{array}{|c|c|c|}\hline \frac{t_1}{2} & -\frac{i k t_1}{\sqrt{2}} & 0 \\\hline \frac{i k t_1}{\sqrt{2}} & k^2 t_1 & 0 \\\hline 0 & 0 & k^2 r_1 + \frac{t_1}{2} \\\hline\end{array}$$

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0^+}^{\#2} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$	1
$\tau_{0^+}^{\#1} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha{}_\alpha$	1
$\tau_{1^-}^{\#2\alpha} + 2 i k\, \sigma_{1^-}^{\#2\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta} + 2 \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$	3
$\tau_{1^-}^{\#1\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$	3
$\tau_{1^+}^{\#1\alpha\beta} + i k\, \sigma_{1^+}^{\#2\alpha\beta} == 0$	$\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\chi\alpha} + \partial_\chi \partial^\chi \tau^{\alpha\beta} + 2 \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 2 \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} + \partial_\chi \partial^\chi \tau^{\beta\alpha} + 2 \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$	3
$\tau_{2^+}^{\#1\alpha\beta} - 2 i k\, \sigma_{2^+}^{\#1\alpha\beta} == 0$	$-i\,(4 \partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2 \partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi{}_\chi - 3 \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\chi} - 3 \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\beta} - 3 \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi} - 3 \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\chi\alpha} + 3 \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3 \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} + 4 i k^\chi \partial_\epsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta\epsilon}{}_\delta - 6 i k^\chi \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\delta\epsilon} - 6 i k^\chi \partial_\epsilon \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 2 \eta^{\alpha\beta} \partial_\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} + 6 i k^\chi \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\alpha\delta\beta} + 6 i k^\chi \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\beta\delta\alpha} - 2 \eta^{\alpha\beta} \partial_\epsilon \partial_\delta \partial^\delta \tau^\chi{}_\chi - 4 i \eta^{\alpha\beta} k^\chi \partial_\phi \partial^\phi \partial_\epsilon \partial_\chi \sigma^{\delta\epsilon}{}_\delta) == 0$	5
Total constraints/gauge generators:		16

Central propagator matrix (columns: $\sigma_{1^+}^{\#1}{}^{\dagger\alpha\beta}$, $\sigma_{1^+}^{\#2}{}^{\dagger\alpha\beta}$, $\tau_{1^+}^{\#1}{}^{\dagger\alpha\beta}$, $\sigma_{1^-}^{\#1}{}^{\dagger\alpha}$, $\sigma_{1^-}^{\#2}{}^{\dagger\alpha}$, $\tau_{1^-}^{\#1}{}^{\dagger\alpha}$, $\tau_{1^-}^{\#2}{}^{\dagger\alpha}$):

	$\sigma_{1^+}^{\#1}{}^{\dagger}$	$\sigma_{1^+}^{\#2}{}^{\dagger}$	$\tau_{1^+}^{\#1}{}^{\dagger}$	$\sigma_{1^-}^{\#1}{}^{\dagger}$	$\sigma_{1^-}^{\#2}{}^{\dagger}$	$\tau_{1^-}^{\#1}{}^{\dagger}$	$\tau_{1^-}^{\#2}{}^{\dagger}$
$\tau_{1^-}^{\#2\alpha}$	0	0	0	$\frac{i}{k(1+2k^2)(r_1-2r_3-r_5)}$	$\frac{i(6k^2(r_1-2r_3-r_5)+t_1)}{\sqrt{2}\,k(1+2k^2)^2(r_1-2r_3-r_5)t_1}$	0	$\frac{\frac{1}{-r_1+2r_3+r_5}+\frac{6k^2}{t_1}}{(1+2k^2)^2}$
$\tau_{1^-}^{\#1\alpha}$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#2\alpha}$	0	0	0	$\frac{1}{\sqrt{2}(k^2+2k^4)(r_1-2r_3-r_5)}$	$\frac{\frac{1}{-r_1+2r_3+r_5}+\frac{6k^2}{t_1}}{2(k+2k^3)^2}$	0	$\frac{i(6k^2(r_1-2r_3-r_5)+t_1)}{\sqrt{2}\,k(1+2k^2)^2(r_1-2r_3-r_5)t_1}$
$\sigma_{1^-}^{\#1\alpha}$	0	0	0	$\frac{1}{k^2(-r_1+2r_3+r_5)}$	$\frac{1}{\sqrt{2}(k^2+2k^4)(r_1-2r_3-r_5)}$	0	$\frac{i}{k(1+2k^2)(-r_1+2r_3+r_5)}$
$\tau_{1^+}^{\#1\alpha\beta}$	$-\frac{i\sqrt{2}\,k}{t_1+k^2 t_1}$	$\frac{-2ik^3(2r_3+r_5)+ikt_1}{(1+k^2)^2 t_1^2}$	$\frac{-2k^4(2r_3+r_5)+k^2 t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\sigma_{1^+}^{\#2\alpha\beta}$	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$-\frac{2k^2(2r_3+r_5)+t_1}{(1+k^2)^2 t_1^2}$	$\frac{i(2k^3(2r_3+r_5)+kt_1)}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\sigma_{1^+}^{\#1\alpha\beta}$	0	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$\frac{i\sqrt{2}\,k}{t_1+k^2 t_1}$	0	0	0	0

Upper-right wave operator matrix:

	$\omega_{1^+}^{\#1}{}_{\alpha\beta}$	$\omega_{1^+}^{\#2}{}_{\alpha\beta}$	$f_{1^+}^{\#1}{}_{\alpha\beta}$	$\omega_{1^-}^{\#1}{}_\alpha$	$\omega_{1^-}^{\#2}{}_\alpha$	$f_{1^-}^{\#1}{}_\alpha$	$f_{1^-}^{\#2}{}_\alpha$
$\omega_{1^+}^{\#1}{}^{\dagger\alpha\beta}$	$k^2(2r_3+r_5)-\frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1^+}^{\#2}{}^{\dagger\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_{1^-}^{\#1}{}^{\dagger\alpha}$	0	0	0	$k^2(-r_1+2r_3+r_5)+\frac{t_1}{6}$	$\frac{t_1}{3\sqrt{2}}$	0	$\frac{ikt_1}{3}$
$\omega_{1^-}^{\#2}{}^{\dagger\alpha}$	0	0	0	$\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	0	$\frac{1}{3}i\sqrt{2}\,kt_1$
$f_{1^-}^{\#1}{}^{\dagger\alpha}$	0	0	0	0	0	0	0
$f_{1^-}^{\#2}{}^{\dagger\alpha}$	0	0	0	$-\frac{1}{3}ikt_1$	$-\frac{1}{3}i\sqrt{2}\,kt_1$	0	$\frac{2k^2 t_1}{3}$

Quadratic (free) action

$$S = \int\int\int\int (f^{\alpha\beta}\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi} + \tfrac{1}{6}t_1(2\,\omega^{\alpha\iota}{}_{\iota}\,\omega_{\iota}{}^{\theta}{}_{\theta} - 4\,\omega_\alpha{}^\theta{}_\theta\,\partial_\iota f^{\alpha\iota} + 4\,\omega_\iota{}^\theta{}_\theta\,\partial_\iota f^{\alpha\iota} - 2\,\partial_\iota f^\theta{}_\theta + \dots)\,)[t,x,y,z]\,dz\,dy\,dx\,dt$$

σ_{0^+} block:

	$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^-}^{\#1}$
$\sigma_{0^+}^{\#1}{}^{\dagger}$	$\frac{1}{6k^2(-r_1+r_3)}$	0	1	0
$\tau_{0^+}^{\#1}{}^{\dagger}$	0	0	0	0
$\tau_{0^+}^{\#2}{}^{\dagger}$	1	0	0	0
$\sigma_{0^-}^{\#1}{}^{\dagger}$	0	0	0	$-\frac{1}{t_1}$

σ_{2^+} block:

	$\sigma_{2^+}^{\#1}{}_{\alpha\beta}$	$\tau_{2^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\sigma_{2^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{2}{(1+2k^2)^2 t_1}$	$-\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$	0
$\tau_{2^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$	$\frac{4k^2}{(1+2k^2)^2 t_1}$	0
$\sigma_{2^-}^{\#1}{}^{\dagger\alpha\beta\chi}$	0	0	$\frac{2}{2k^2 r_1 + t_1}$

ω_{0^+} block:

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\omega_{0^-}^{\#1}$
$\omega_{0^+}^{\#1}{}^{\dagger}$	0	0	0	$-t_1$
$f_{0^+}^{\#1}{}^{\dagger}$	0	$6k^2(-r_1+r_3)$	0	0
$f_{0^+}^{\#2}{}^{\dagger}$	0	0	0	0
$\omega_{0^-}^{\#1}{}^{\dagger}$	0	0	0	0

Massive and massless spectra

Massive particle ($J^P = 2^-$)

Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

Quadratic pole

Pole residue:	$\frac{1}{(r_1-2r_3-r_5)t_1^2} > 0$
Polarisations:	2

Unitarity conditions

$r_1 < 0\ \&\&\ r_5 < r_1 - 2r_3\ \&\&\ t_1 > 0$