Particle spectrograph

Wave operator and propagator

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\tau^{\#2}_{0^+} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$	1
$\tau^{\#1}_{0^+} - 2\,i\,k\,\sigma^{\#1}_{0^+} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha + 2\,\partial_\chi\partial^\chi\partial_\beta\sigma^{\alpha\beta}{}_\alpha$	1
$\tau^{\#2}_{1^-}{}^{\alpha} + 2\,i\,k\,\sigma^{\#2}_{1^-}{}^{\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$	3
$\tau^{\#1}_{1^-}{}^{\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$	3
$\tau^{\#1}_{1^+}{}^{\alpha\beta} + i\,k\,\sigma^{\#1}_{1^+}{}^{\alpha\beta} == 0$	$\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} +$ $\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha} ==$ $\partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} +$ $\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta}$	3
$\sigma^{\#1}_{1^+}{}^{\alpha\beta} == \sigma^{\#2}_{1^+}{}^{\alpha\beta}$	$3\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} +$ $2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta} ==$ $3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha}$	3
$\sigma^{\#1}_{2^-}{}^{\alpha\beta\chi} == 0$	$3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\alpha\sigma^{\beta\delta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\alpha\sigma^{\beta\delta}{}_\delta +$ $2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\chi\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\delta\chi} +$ $2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\chi\delta\alpha} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\beta\delta} +$ $2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\delta\beta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\beta\chi\alpha} +$ $3\;\eta^{\beta\chi}\;\partial_\phi\partial^\phi\partial_\epsilon\partial^\alpha\sigma^{\delta\epsilon}{}_\delta +$ $3\;\eta^{\alpha\chi}\;\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\beta\delta\epsilon} +$ $3\;\eta^{\beta\chi}\;\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\alpha\delta}{}_\delta ==$ $3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\beta\sigma^{\alpha\delta}{}_\delta +$ $2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\chi\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\delta\chi} +$ $2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\chi\delta\beta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\beta\delta\alpha} +$ $4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\beta\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\chi\beta} +$ $3\;\eta^{\alpha\chi}\;\partial_\phi\partial^\phi\partial_\epsilon\partial^\beta\sigma^{\delta\epsilon}{}_\delta +$ $3\;\eta^{\beta\chi}\;\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\alpha\delta\epsilon} +$ $3\;\eta^{\alpha\chi}\;\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\beta\delta}{}_\delta$	5
$\tau^{\#1}_{2^+}{}^{\alpha\beta} == 0$	$4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi +$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} +$ $2\;\eta^{\alpha\beta}\;\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} ==$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} +$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} +$ $2\;\eta^{\alpha\beta}\;\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi$	5
Total constraints/gauge generators:		24

Quadratic (free) action

$$
\begin{aligned}
\mathcal{S} == \iiiint\!\Big(\tfrac{1}{6}\,(&-4\,t_3\;\omega^{\alpha\iota}{}_\alpha\;\omega_\iota{}^\kappa{}_\kappa + 6\;f^{\alpha\beta}\;\tau_{\alpha\beta} + 6\;\omega^{\alpha\beta\chi}\;\sigma_{\alpha\beta\chi} + 8\,t_3\;\omega_\alpha{}^\kappa{}_\kappa\,\partial_\iota f^{\alpha\iota} -\\
&8\,t_3\;\omega_\iota{}^\kappa{}_\kappa\,\partial^\iota f^\alpha{}_\alpha + 4\,t_3\,\partial_\iota f^\kappa{}_\kappa\,\partial^\iota f^\alpha{}_\alpha - 15\,r_3\,\partial_\beta\omega_\iota{}^\theta{}_\theta\,\partial^\iota\omega^{\alpha\beta}{}_\alpha +\\
&9\,r_3\,\partial_\iota\omega_\beta{}^\theta{}_\theta\,\partial^\iota\omega^{\alpha\beta}{}_\alpha + 9\,r_3\,\partial_\alpha\omega^{\alpha\beta\iota}\,\partial_\theta\omega_\beta{}^\theta{}_\iota -\\
&18\,r_3\,\partial^\iota\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_\beta{}^\theta{}_\iota - 15\,r_3\,\partial_\alpha\omega^{\alpha\beta\iota}\,\partial_\theta\omega_\iota{}^\theta{}_\beta +\\
&30\,r_3\,\partial^\iota\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_\iota{}^\theta{}_\beta + 4\,t_2\;\omega_{\iota\theta\alpha}\,\partial^\theta f^{\alpha\iota} + 2\,t_2\,\partial_\alpha f_{\iota\theta}\,\partial^\theta f^{\alpha\iota} -\\
&t_2\,\partial_\alpha f_{\theta\iota}\,\partial^\theta f^{\alpha\iota} - t_2\,\partial_\iota f_{\alpha\theta}\,\partial^\theta f^{\alpha\iota} + t_2\,\partial_\theta f_{\alpha\iota}\,\partial^\theta f^{\alpha\iota} -\\
&t_2\,\partial_\theta f_{\iota\alpha}\,\partial^\theta f^{\alpha\iota} - 4\,t_2\;\omega_{\alpha\theta\iota}\,(\omega^{\alpha\iota\theta} + \partial^\theta f^{\alpha\iota}) +\\
&2\,t_2\;\omega_{\alpha\iota\theta}\,(\omega^{\alpha\iota\theta} + 2\,\partial^\theta f^{\alpha\iota}) + 8\,r_2\,\partial_\beta\omega_{\alpha\iota\theta}\,\partial^\theta\omega^{\alpha\beta\iota} -\\
&4\,r_2\,\partial_\beta\omega_{\alpha\theta\iota}\,\partial^\theta\omega^{\alpha\beta\iota} + 4\,r_2\,\partial_\beta\omega_{\iota\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\iota} -\\
&24\,r_3\,\partial_\beta\omega_{\iota\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\iota} - 2\,r_2\,\partial_\iota\omega_{\alpha\beta\theta}\,\partial^\theta\omega^{\alpha\beta\iota} +\\
&2\,r_2\,\partial_\theta\omega_{\alpha\beta\iota}\,\partial^\theta\omega^{\alpha\beta\iota} - 4\,r_2\,\partial_\theta\omega_{\alpha\iota\beta}\,\partial^\theta\omega^{\alpha\beta\iota} +\\
&4\,t_3\,\partial_\iota f^{\alpha\iota}\,\partial_\kappa f_\alpha{}^\kappa - 8\,t_3\,\partial^\iota f^\alpha{}_\alpha\,\partial_\kappa f_\iota{}^\kappa)\Big)[t,x,y,z]\,dz\,dy\,dx\,dt
\end{aligned}
$$

$$
\begin{array}{c|ccc|cccc}
 & \sigma^{\#1}_{1^+}{}_{\alpha\beta} & \sigma^{\#2}_{1^+}{}_{\alpha\beta} & \tau^{\#1}_{1^+}{}_{\alpha\beta} & \sigma^{\#1}_{1^-}{}_\alpha & \sigma^{\#2}_{1^-}{}_\alpha & \tau^{\#1}_{1^-}{}_\alpha & \tau^{\#2}_{1^-}{}_\alpha \\
\hline
\sigma^{\#1}_{1^+}\dagger^{\alpha\beta} & \frac{6}{(3+k^2)^2\,t_2} & \frac{3\sqrt{2}}{(3+k^2)^2\,t_2} & \frac{3\,i\,\sqrt{2}\,k}{(3+k^2)^2\,t_2} & 0 & 0 & 0 & 0 \\
\sigma^{\#2}_{1^+}\dagger^{\alpha\beta} & \frac{3\sqrt{2}}{(3+k^2)^2\,t_2} & \frac{3}{(3+k^2)^2\,t_2} & \frac{3\,i\,k}{(3+k^2)^2\,t_2} & 0 & 0 & 0 & 0 \\
\tau^{\#1}_{1^+}\dagger^{\alpha\beta} & -\frac{3\,i\,\sqrt{2}\,k}{(3+k^2)^2\,t_2} & -\frac{3\,i\,k}{(3+k^2)^2\,t_2} & \frac{3\,k^2}{(3+k^2)^2\,t_2} & 0 & 0 & 0 & 0 \\
\hline
\sigma^{\#1}_{1^-}\dagger^{\alpha} & 0 & 0 & 0 & -\frac{2}{3\,k^2\,r_3} & -\frac{2\sqrt{2}}{3\,k^2\,r_3+6\,k^4\,r_3} & 0 & -\frac{4\,i}{3\,k\,r_3+6\,k^3\,r_3} \\
\sigma^{\#2}_{1^-}\dagger^{\alpha} & 0 & 0 & 0 & -\frac{2\sqrt{2}}{3\,k^2\,r_3+6\,k^4\,r_3} & \frac{9\,k^2\,r_3-4\,t_3}{3\,(k+2\,k^3)^2\,r_3\,t_3} & 0 & \frac{i\,\sqrt{2}\,(9\,k^2\,r_3-4\,t_3)}{3\,k\,(1+2\,k^2)^2\,r_3\,t_3} \\
\tau^{\#1}_{1^-}\dagger^{\alpha} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tau^{\#2}_{1^-}\dagger^{\alpha} & 0 & 0 & 0 & \frac{4\,i}{3\,k\,r_3+6\,k^3\,r_3} & -\frac{i\,\sqrt{2}\,(9\,k^2\,r_3-4\,t_3)}{3\,k\,(1+2\,k^2)^2\,r_3\,t_3} & 0 & \frac{2\,(9\,k^2\,r_3-4\,t_3)}{3\,(1+2\,k^2)^2\,r_3\,t_3}
\end{array}
$$

$$
\begin{array}{c|ccc|cccc}
 & \omega^{\#1}_{1^+}{}_{\alpha\beta} & \omega^{\#2}_{1^+}{}_{\alpha\beta} & f^{\#1}_{1^+}{}_{\alpha\beta} & \omega^{\#1}_{1^-}{}_\alpha & \omega^{\#2}_{1^-}{}_\alpha & f^{\#1}_{1^-}{}_\alpha & f^{\#2}_{1^-}{}_\alpha \\
\hline
\omega^{\#1}_{1^+}\dagger^{\alpha\beta} & \frac{2\,t_2}{3} & \frac{\sqrt{2}\,t_2}{3} & \frac{1}{3}\,i\,\sqrt{2}\,k\,t_2 & 0 & 0 & 0 & 0 \\
\omega^{\#2}_{1^+}\dagger^{\alpha\beta} & \frac{\sqrt{2}\,t_2}{3} & \frac{t_2}{3} & \frac{i\,k\,t_2}{3} & 0 & 0 & 0 & 0 \\
f^{\#1}_{1^+}\dagger^{\alpha\beta} & -\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2 & -\frac{1}{3}\,i\,k\,t_2 & \frac{k^2\,t_2}{3} & 0 & 0 & 0 & 0 \\
\hline
\omega^{\#1}_{1^-}\dagger^{\alpha} & 0 & 0 & 0 & \frac{1}{6}\,(-9\,k^2\,r_3+4\,t_3) & -\frac{\sqrt{2}\,t_3}{3} & 0 & -\frac{2}{3}\,i\,k\,t_3 \\
\omega^{\#2}_{1^-}\dagger^{\alpha} & 0 & 0 & 0 & -\frac{\sqrt{2}\,t_3}{3} & \frac{t_3}{3} & 0 & \frac{1}{3}\,i\,\sqrt{2}\,k\,t_3 \\
f^{\#1}_{1^-}\dagger^{\alpha} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
f^{\#2}_{1^-}\dagger^{\alpha} & 0 & 0 & 0 & \frac{2\,i\,k\,t_3}{3} & -\frac{1}{3}\,i\,\sqrt{2}\,k\,t_3 & 0 & \frac{2\,k^2\,t_3}{3}
\end{array}
$$

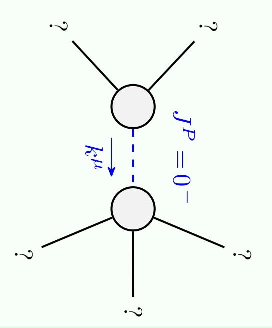
$$
\begin{array}{c|cc|c}
 & \omega^{\#1}_{2^+}{}_{\alpha\beta} & f^{\#1}_{2^+}{}_{\alpha\beta} & \omega^{\#1}_{2^-}{}_{\alpha\beta\chi} \\
\hline
\omega^{\#1}_{2^+}\dagger^{\alpha\beta} & -\frac{3\,k^2\,r_3}{2} & 0 & 0 \\
f^{\#1}_{2^+}\dagger^{\alpha\beta} & 0 & 0 & 0 \\
\hline
\omega^{\#1}_{2^-}\dagger^{\alpha\beta\chi} & 0 & 0 & 0
\end{array}
$$

$$
\begin{array}{c|cc|c}
 & \sigma^{\#1}_{2^+}{}_{\alpha\beta} & \tau^{\#1}_{2^+}{}_{\alpha\beta} & \sigma^{\#1}_{2^-}{}_{\alpha\beta\chi} \\
\hline
\sigma^{\#1}_{2^+}\dagger^{\alpha\beta} & -\frac{2}{3\,k^2\,r_3} & 0 & 0 \\
\tau^{\#1}_{2^+}\dagger^{\alpha\beta} & 0 & 0 & 0 \\
\hline
\sigma^{\#1}_{2^-}\dagger^{\alpha\beta\chi} & 0 & 0 & 0
\end{array}
$$

$$
\begin{array}{c|ccc|c}
 & \omega^{\#1}_{0^+} & f^{\#1}_{0^+} & f^{\#2}_{0^+} & \omega^{\#1}_{0^-} \\
\hline
\omega^{\#1}_{0^+}\dagger & t_3 & -i\,\sqrt{2}\,k\,t_3 & 0 & 0 \\
f^{\#1}_{0^+}\dagger & i\,\sqrt{2}\,k\,t_3 & 2\,k^2\,t_3 & 0 & 0 \\
f^{\#2}_{0^+}\dagger & 0 & 0 & 0 & 0 \\
\hline
\omega^{\#1}_{0^-}\dagger & 0 & 0 & 0 & k^2\,r_2+t_2
\end{array}
$$

$$
\begin{array}{c|ccc|c}
 & \sigma^{\#1}_{0^+} & \tau^{\#1}_{0^+} & \tau^{\#2}_{0^+} & \sigma^{\#1}_{0^-} \\
\hline
\sigma^{\#1}_{0^+}\dagger & \frac{1}{(1+2\,k^2)^2\,t_3} & -\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_3} & 0 & 0 \\
\tau^{\#1}_{0^+}\dagger & \frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_3} & \frac{2\,k^2}{(1+2\,k^2)^2\,t_3} & 0 & 0 \\
\tau^{\#2}_{0^+}\dagger & 0 & 0 & 0 & 0 \\
\hline
\sigma^{\#1}_{0^-}\dagger & 0 & 0 & 0 & \frac{1}{k^2\,r_2+t_2}
\end{array}
$$

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_2 < 0\ \&\&\ t_2 > 0$