

# Particle spectrograph

## Wave operator and propagator

	$\sigma_{1^+}^{\#1} \alpha\beta$	$\sigma_{1^+}^{\#2} \alpha\beta$	$\tau_{1^+}^{\#1} \alpha\beta$	$\sigma_{1^+}^{\#1} \alpha$	$\sigma_{1^+}^{\#2} \alpha$	$\tau_{1^+}^{\#1} \alpha$	$\tau_{1^+}^{\#2} \alpha$
$\sigma_{1^+}^{\#1} \dagger^{\alpha\beta}$	0	$-\frac{\sqrt{2}}{t_1+k^2}t_1$	$-\frac{i\sqrt{2}k}{t_1+k^2}t_1$	0	0	0	0
$\sigma_{1^+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{\sqrt{2}}{t_1+k^2}t_1$	$\frac{-2k^2r_5+t_1}{(1+k^2)^2}t_1^2$	$-\frac{i(2k^3r_5-kt_1)}{(1+k^2)^2}t_1^2$	0	0	0	0
$\tau_{1^+}^{\#1} \dagger^{\alpha\beta}$	$\frac{i\sqrt{2}k}{t_1+k^2}t_1$	$\frac{i(2k^3r_5-kt_1)}{(1+k^2)^2}t_1^2$	$\frac{-2k^4r_5+k^2t_1}{(1+k^2)^2}t_1^2$	0	0	0	0
$\sigma_{1^+}^{\#1} \dagger^\alpha$	0	0	0	$-\frac{2(t_1+t_3)}{3t_1t_3+2k^2r_5(t_1+t_3)}$	$-\frac{\sqrt{2}(t_1-2t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{2ik(t_1-2t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{2ik(t_1-2t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$
$\sigma_{1^+}^{\#2} \dagger^\alpha$	0	0	0	$-\frac{\sqrt{2}(t_1-2t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{6k^2r_5+t_1+4t_3}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{i\sqrt{2}k(6k^2r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{i\sqrt{2}k(6k^2r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$
$\tau_{1^+}^{\#1} \dagger^\alpha$	0	0	0	0	0	0	0
$\tau_{1^+}^{\#2} \dagger^\alpha$	0	0	0	$-\frac{2ik(t_1-2t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{i\sqrt{2}k(6k^2r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{2k^2(6k^2r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{2k^2(6k^2r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$

	$\omega_{1^+}^{\#1} \alpha\beta$	$\omega_{1^+}^{\#2} \alpha\beta$	$f_{1^+}^{\#1} \alpha\beta$	$\omega_{1^+}^{\#1} \alpha$	$\omega_{1^+}^{\#2} \alpha$	$f_{1^+}^{\#1} \alpha$	$f_{1^+}^{\#2} \alpha$
$\omega_{1^+}^{\#1} \dagger^{\alpha\beta}$	$k^2r_5-\frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1^+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1^+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_{1^+}^{\#1} \dagger^\alpha$	0	0	0	$\frac{1}{6}(6k^2r_5+t_1+4t_3)$	$\frac{t_1-2t_3}{3\sqrt{2}}$	0	$\frac{1}{3}ik(t_1-2t_3)$
$\omega_{1^+}^{\#2} \dagger^\alpha$	0	0	0	$\frac{t_1-2t_3}{3\sqrt{2}}$	$\frac{t_1+t_3}{3}$	0	$\frac{1}{3}i\sqrt{2}k(t_1+t_3)$
$f_{1^+}^{\#1} \dagger^\alpha$	0	0	0	0	0	0	0
$f_{1^+}^{\#2} \dagger^\alpha$	0	0	0	$-\frac{1}{3}ik(t_1-2t_3)$	$-\frac{1}{3}i\sqrt{2}k(t_1+t_3)$	0	$\frac{2}{3}k^2(t_1+t_3)$

Quadratic (free) Lagrangian density	
$-\frac{1}{3}t_1\omega_{\lambda'}^{\alpha'}\omega_{\kappa\alpha}^{\kappa}+\frac{2}{3}t_3\omega_{\lambda'}^{\alpha'}\omega_{\kappa\alpha}^{\kappa}-t_1\omega_{\lambda'}^{\kappa\lambda}\omega_{\kappa\lambda}^{\lambda}+f^{\alpha\beta}\tau_{\alpha\beta}+$ $\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}-r_5\partial_{\lambda'}\omega_{\kappa}^{\kappa\lambda}\partial'_{\lambda}\omega_{\lambda}^{\alpha\alpha}+\frac{2}{3}r_2\partial^{\beta}\omega_{\alpha}^{\theta\alpha}+\frac{2}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}-\frac{1}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta}-$ $\frac{2}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta}-r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\lambda}^{\theta\kappa\lambda}+r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\alpha}^{\theta\kappa\lambda}-r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\lambda}^{\theta\kappa\lambda}+$ $2r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\alpha}^{\kappa\lambda\theta}-\frac{1}{2}t_1\partial_{\lambda'}\omega_{\theta\kappa}^{\alpha}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{2}t_1\partial_{\lambda'}\omega_{\theta\kappa}^{\alpha}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{2}t_1\partial_{\lambda'}\omega_{\theta\kappa}^{\alpha}\partial^{\kappa}f_{\alpha}^{\theta}+$ $\frac{1}{3}t_1\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\lambda'}^{\lambda}-\frac{2}{3}t_3\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\lambda'}^{\lambda}+\frac{1}{3}t_1\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\lambda'}^{\lambda}-\frac{2}{3}t_3\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\lambda'}^{\lambda}+$ $\frac{2}{3}t_1\partial^{\alpha}f_{\kappa\alpha}\partial^{\kappa}f_{\lambda'}^{\lambda}-\frac{4}{3}t_3\partial^{\alpha}f_{\kappa\alpha}\partial^{\kappa}f_{\lambda'}^{\lambda}-\frac{1}{3}t_1\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda'}^{\lambda}+\frac{2}{3}t_3\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda'}^{\lambda}+$ $2t_1\omega_{\lambda\kappa\theta}\partial^{\kappa}f_{\lambda'}^{\theta}-\frac{1}{3}t_1\omega_{\lambda\alpha}^{\alpha}\partial^{\kappa}f_{\lambda'}^{\lambda}+\frac{2}{3}t_3\omega_{\lambda\alpha}^{\alpha}\partial^{\kappa}f_{\lambda'}^{\lambda}-\frac{1}{3}t_1\omega_{\lambda\alpha}^{\lambda}\partial^{\kappa}f_{\lambda'}^{\lambda}+$ $\frac{2}{3}t_3\omega_{\lambda\alpha}^{\lambda}\partial^{\kappa}f_{\lambda'}^{\lambda}+\frac{1}{2}t_1\partial^{\alpha}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda\alpha}^{\alpha}+\frac{1}{2}t_1\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{1}{2}t_1\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}-$ $\frac{1}{3}t_1\partial^{\alpha}f_{\lambda}^{\alpha}\partial^{\kappa}f_{\lambda\kappa}^{\kappa}+\frac{2}{3}t_3\partial^{\alpha}f_{\lambda}^{\alpha}\partial^{\kappa}f_{\lambda\kappa}^{\kappa}+\frac{1}{3}r_2\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\theta}\omega_{\alpha\beta\theta}^{\kappa}+\frac{2}{3}r_2\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\theta}\omega_{\alpha\beta\theta}^{\kappa}-$ $\frac{2}{3}r_2\partial^{\beta}\omega_{\lambda'}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda\alpha}+\frac{2}{3}r_2\partial^{\beta}\omega_{\lambda'}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda\alpha}+r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\kappa}^{\theta\kappa}-r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\alpha}^{\theta\kappa}$	

	$\sigma_{2^+}^{\#1} \alpha\beta$	$\tau_{2^+}^{\#1} \alpha\beta$	$\sigma_{2^+}^{\#1} \alpha\beta\chi$
$\sigma_{2^+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
$\tau_{2^+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\sigma_{2^+}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{t_1}$

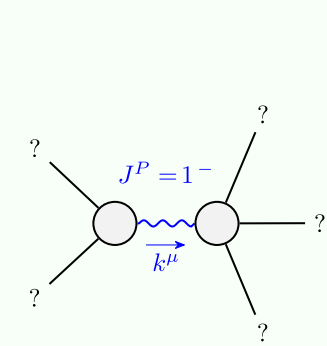
Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau_{0^+}^{\#2} == 0$	1
$\tau_{0^+}^{\#1} - 2ik\sigma_{0^+}^{\#1} == 0$	1
$\tau_{1^+}^{\#2\alpha} + 2ik\sigma_{1^+}^{\#2\alpha} == 0$	3
$\tau_{1^+}^{\#1\alpha} == 0$	3
$\tau_{1^+}^{\#1\alpha\beta} + ik\sigma_{1^+}^{\#2\alpha\beta} == 0$	3
$\tau_{2^+}^{\#1\alpha\beta} - 2ik\sigma_{2^+}^{\#1\alpha\beta} == 0$	5
Total constraints:	16

$\omega_{0^+}^{\#1}$	0	0	0	$k^2r_2-t_1$
$f_{0^+}^{\#2}$	0	0	0	0
$f_{0^+}^{\#1}$	$-i\sqrt{2}kt_3$	$2k^2t_3$	0	0
$\omega_{0^+}^{\#1}$	$t_3$	$i\sqrt{2}kt_3$	0	0

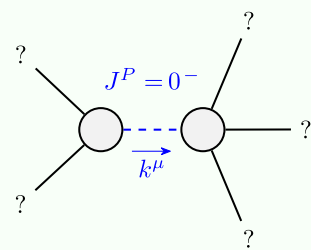
	$\omega_{2^+}^{\#1} \alpha\beta$	$f_{2^+}^{\#1} \alpha\beta$	$\omega_{2^+}^{\#1} \alpha\beta\chi$
$\omega_{2^+}^{\#1} \dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2^+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	$k^2t_1$	0
$\omega_{2^+}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{t_1}{2}$

$\sigma_{0^+}^{\#1}$	0	0	0	$\frac{1}{k^2r_2-t_1}$
$\tau_{0^+}^{\#2}$	0	0	0	0
$\tau_{0^+}^{\#1}$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$	$\frac{2k^2}{(1+2k^2)^2t_3}$	0	0
$\sigma_{0^+}^{\#1}$	$\frac{1}{(1+2k^2)^2t_3}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$	0	0

## Massive and massless spectra



Massive particle	
Pole residue:	$\frac{6t_1t_3(t_1+t_3)-3r_5(t_1^2+2t_3^2)}{2r_5(t_1+t_3)(-3t_1t_3+r_5(t_1+t_3))} > 0$
Polarisations:	3
Square mass:	$-\frac{3t_1t_3}{2r_5t_1+2r_5t_3} > 0$
Spin:	1
Parity:	Odd



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$\frac{t_1}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

## Unitarity conditions

$r_2 < 0 \&\& r_5 < 0 \&\& t_1 < 0 \&\& 0 < t_3 < -t_1$