# Particle spectrograph

## Wave operator and propagator

|  | $\sigma_{1+}^{\#1}{}^{\alpha\beta}$ | $\sigma_{1+}^{\#2}{}^{\alpha\beta}$ | $\tau_{1+}^{\#1}{}^{\alpha\beta}$ | $\sigma_{1-}^{\#1}{}^{\alpha}$ | $\sigma_{1-}^{\#2}{}^{\alpha}$ | $\tau_{1-}^{\#1}{}^{\alpha}$ | $\tau_{1-}^{\#2}{}^{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1+}^{\#1}{}^{\dagger}{}_{\alpha\beta}$ | $\frac{1}{k^2(2r_3+r_5)}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\sigma_{1+}^{\#2}{}^{\dagger}{}_{\alpha\beta}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1+}^{\#1}{}^{\dagger}{}_{\alpha\beta}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\sigma_{1-}^{\#1}{}^{\dagger}{}_{\alpha}$ | 0 | 0 | 0 | $\frac{2}{k^2(r_3+2r_5)}$ | $\frac{2\sqrt{2}}{k^2(1+2k^2)^2(r_3+2r_5)}$ | 0 | $-\frac{4i}{k(1+2k^2)^2(r_3+2r_5)}$ |
| $\sigma_{1-}^{\#2}{}^{\dagger}{}_{\alpha}$ | 0 | 0 | 0 | $\frac{2\sqrt{2}}{k^2(1+2k^2)^2(r_3+2r_5)}$ | $\frac{3k^2(r_3+2r_5)+4t_3}{(k+2k^3)^2(r_3+2r_5)t_3}$ | 0 | $-\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$ |
| $\tau_{1-}^{\#1}{}^{\dagger}{}_{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1-}^{\#2}{}^{\dagger}{}_{\alpha}$ | 0 | 0 | 0 | $\frac{4i}{k(1+2k^2)(r_3+2r_5)}$ | $\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$ | 0 | $\frac{6k^2(r_3+2r_5)+8t_3}{(1+2k^2)^2(r_3+2r_5)t_3}$ |

### Quadratic (free) Lagrangian density

$$\frac{2}{3}t_3\,\omega_i{}^{\alpha i}\,\omega_{\kappa\alpha}{}^\kappa + f^{\alpha\beta}\,\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - \frac{1}{2}r_3\,\omega_i{}^{\kappa\lambda}\,\partial^l\omega_\lambda{}_\alpha{}^\alpha - r_5\,\partial_l\omega^{\kappa\lambda}{}_\kappa\,\partial^l\omega_\lambda{}_\alpha{}^\alpha +$$
$$\frac{2}{3}r_2\partial_\beta\omega^{\theta\alpha}{}_\kappa\,\partial_\theta\omega_{\alpha\beta}{}^\kappa - \frac{1}{3}r_2\partial\omega_{\alpha\beta}{}^\kappa\partial_\kappa\omega^{\alpha\beta\theta} - \frac{2}{3}r_2\partial_\theta\omega_{\alpha\beta}{}^\kappa\,\partial_\kappa\omega^{\theta\alpha\beta} +$$
...

## Massive and massless spectra
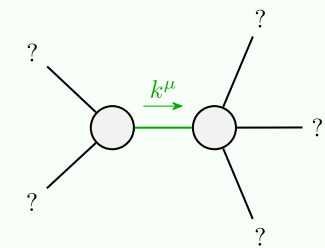


### Quadratic pole

| Pole residue: | $-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2} > 0$ |
|---|---|
| Polarisations: | 2 |

(No massive particles)

## Unitarity conditions

$r_3 < 0\ \&\&\ (r_5 < -\frac{r_3}{2}\ ||\ r_5 > -2r_3)\ ||\ r_3 > 0\ \&\&\ -2r_3 < r_5 < -\frac{r_3}{2}$

|  | $\sigma_{2+}^{\#1}{}_{\alpha\beta}$ | $\tau_{2+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{2-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2+}^{\#1}{}^{\dagger\alpha\beta}$ | $-\frac{2}{3k^2r_3}$ | 0 | 0 |
| $\tau_{2+}^{\#1}{}^{\dagger\alpha\beta}$ | 0 | 0 | 0 |
| $\sigma_{2-}^{\#1}{}^{\dagger\alpha\beta\chi}$ | 0 | 0 | 0 |

|  | $\sigma_{0+}^{\#1}$ | $\tau_{0+}^{\#1}$ | $\tau_{0+}^{\#2}$ | $\sigma_{0-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0+}^{\#1}{}^{\dagger}$ | $\frac{1}{(1+2k^2)^2t_3}$ | $-\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$ | 0 | 0 |
| $\tau_{0+}^{\#1}{}^{\dagger}$ | $\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$ | $\frac{2k^2}{(1+2k^2)^2t_3}$ | 0 | 0 |
| $\tau_{0+}^{\#2}{}^{\dagger}$ | 0 | 0 | 0 | 0 |
| $\sigma_{0-}^{\#1}{}^{\dagger}$ | 0 | 0 | 0 | $\frac{1}{k^2r_2}$ |

|  | $\omega_{2+}^{\#1}{}_{\alpha\beta}$ | $f_{2+}^{\#1}{}_{\alpha\beta}$ | $\omega_{2-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2+}^{\#1}{}^{\dagger}{}_{\alpha\beta}$ | $\frac{3k^2r_3}{2}$ | 0 | 0 |
| $f_{2+}^{\#1}{}^{\dagger}{}_{\alpha\beta}$ | 0 | 0 | 0 |
| $\omega_{2-}^{\#1}{}^{\dagger}{}_{\alpha\beta\chi}$ | 0 | 0 | 0 |

|  | $\omega_{1+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1+}^{\#2}{}_{\alpha\beta}$ | $f_{1+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1-}^{\#1}{}^{\alpha}$ | $\omega_{1-}^{\#2}{}^{\alpha}$ | $f_{1-}^{\#1}{}^{\alpha}$ | $f_{1-}^{\#2}{}^{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1+}^{\#1}{}^{\dagger\alpha\beta}$ | $k^2(2r_3+r_5)$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1+}^{\#2}{}^{\dagger}{}_{\alpha\beta}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1+}^{\#1}{}^{\dagger}{}_{\alpha\beta}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1-}^{\#1}{}^{\alpha}$ | 0 | 0 | 0 | $k^2(\frac{r_3}{2}+r_5)+\frac{2t_3}{3}$ | $-\frac{\sqrt{2}t_3}{3}$ | 0 | $\frac{2ikt_3}{3}$ |
| $\omega_{1-}^{\#2}{}^{\alpha}$ | 0 | 0 | 0 | $-\frac{\sqrt{2}t_3}{3}$ | $\frac{t_3}{3}$ | 0 | $-\frac{1}{3}i\sqrt{2}kt_3$ |
| $f_{1-}^{\#1}{}^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1-}^{\#2}{}^{\alpha}$ | 0 | 0 | 0 | $-\frac{2}{3}ikt_3$ | $\frac{1}{3}i\sqrt{2}kt_3$ | 0 | $\frac{2k^2t_3}{3}$ |

### Source constraints/gauge generators

| SO(3) irreps | Multiplicities |
|---|---|
| $\tau_{0+}^{\#2} == 0$ | 1 |
| $\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$ | 1 |
| $\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$ | 3 |
| $\tau_{1-}^{\#1\alpha} == 0$ | 3 |
| $\tau_{1+}^{\#1\alpha\beta} == 0$ | 3 |
| $\sigma_{1+}^{\#2\alpha\beta} == 0$ | 3 |
| $\sigma_{2-}^{\#1\alpha\beta\chi} == 0$ | 5 |
| $\tau_{2+}^{\#1\alpha\beta} == 0$ | 5 |
| Total constraints: | 24 |

|  | $f_{0+}^{\#2}$ | $\omega_{0-}^{\#1}$ |
|---|---|---|
| ... | 0 | 0 | $k^2r_2$ |
| $f_{0+}^{\#1}$ | 0 | 0 | 0 |
| $\omega_{0+}^{\#1}{}^{\dagger}$ | $t_3$ | $-i\sqrt{2}kt_3$ | 0 | 0 |
| $f_{0+}^{\#1}{}^{\dagger}$ | $i\sqrt{2}kt_3$ | $2k^2t_3$ | 0 | 0 |
| $f_{0+}^{\#2}{}^{\dagger}$ | 0 | 0 | 0 | 0 |
| $\omega_{0-}^{\#1}{}^{\dagger}$ | 0 | 0 | 0 | $k^2r_2$ |