

# Particle spectrograph

## Wave operator and propagator

$\sigma_{1+}^{\#1} + \alpha\beta$	$\sigma_{1+}^{\#2} + \alpha\beta$	$\tau_{1+}^{\#1} + \alpha\beta$	$\sigma_{1-}^{\#1} - \alpha$	$\sigma_{1-}^{\#2} - \alpha$	$\tau_{1-}^{\#1} - \alpha$	$\tau_{1-}^{\#2} - \alpha$
$\sigma_{1+}^{\#1} + \alpha\beta$	$0$	$-\frac{\sqrt{2}}{t_1+k^2t_1}$	$0$	$0$	$0$	$0$
$\sigma_{1+}^{\#2} + \alpha\beta$	$-\frac{\sqrt{2}}{t_1+k^2t_1}$	$\frac{1}{(1+k^2)^2t_1}$	$0$	$0$	$0$	$0$
$\tau_{1+}^{\#1} + \alpha\beta$	$\frac{i\sqrt{2}k}{t_1+k^2t_1}$	$-\frac{ik}{(1+k^2)^2t_1}$	$0$	$0$	$0$	$0$
$\sigma_{1-}^{\#1} + \alpha$	$0$	$0$	$\frac{6}{(3+4k^2)^2t_1}$	$\frac{6\sqrt{2}}{(3+4k^2)^2t_1}$	$0$	$\frac{12ik}{(3+4k^2)^2t_1}$
$\sigma_{1-}^{\#2} + \alpha$	$0$	$0$	$\frac{6\sqrt{2}}{(3+4k^2)^2t_1}$	$\frac{12}{(3+4k^2)^2t_1}$	$0$	$\frac{12i\sqrt{2}k}{(3+4k^2)^2t_1}$
$\tau_{1-}^{\#1} + \alpha$	$0$	$0$	$0$	$0$	$0$	$0$
$\tau_{1-}^{\#2} + \alpha$	$0$	$0$	$-\frac{12ik}{(3+4k^2)^2t_1}$	$-\frac{12i\sqrt{2}k}{(3+4k^2)^2t_1}$	$0$	$\frac{24k^2}{(3+4k^2)^2t_1}$

Quadratic (free) Lagrangian density

$$-\frac{1}{3}t_1\omega_{\lambda'}^{\alpha'}\omega_{\kappa\alpha}^{\kappa}-t_1\omega_{\kappa\lambda}^{\kappa}\omega_{\lambda'}^{\lambda'}+f^{\alpha\beta}\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}+\frac{2}{3}r_2\partial^\beta\omega^{\theta\alpha}_{\kappa}\partial_\theta\omega_{\alpha\beta}^{\kappa}-$$
$$\frac{1}{3}r_2\partial_\theta\omega_{\alpha\beta}^{\kappa}\partial_\kappa\omega^{\alpha\beta\theta}-\frac{2}{3}r_2\partial_\theta\omega_{\alpha\beta}^{\kappa}\partial_\kappa\omega^{\theta\alpha\beta}+4r_3\partial_\alpha\omega_{\lambda}^{\alpha}\partial_\kappa\omega^{\theta\kappa\lambda}-$$
$$4r_3\partial_\theta\omega_{\lambda}^{\alpha}\partial_\kappa\omega^{\theta\kappa\lambda}-\frac{1}{2}t_1\partial^\alpha f_{\theta\kappa}\partial^\kappa f_{\alpha}^{\theta}-\frac{1}{2}t_1\partial^\alpha f_{\kappa\theta}\partial^\theta f_{\alpha}^{\kappa}-\frac{1}{2}t_1\partial^\alpha f_{\lambda}^{\kappa}\partial^\kappa f_{\alpha}^{\lambda}+$$
$$\frac{1}{3}t_1\omega_{\kappa\alpha}^{\alpha}\partial^\kappa f_{\lambda'}^{\lambda'}+\frac{1}{3}t_1\omega_{\kappa\lambda}^{\lambda}\partial^\kappa f_{\lambda'}^{\lambda'}+\frac{2}{3}t_1\partial^\alpha f_{\kappa\alpha}\partial^\kappa f_{\lambda'}^{\lambda'}-\frac{1}{3}t_1\partial_\kappa f_{\lambda}^{\lambda}\partial^\kappa f_{\lambda'}^{\lambda'}+$$
$$2t_1\omega_{\lambda\kappa\theta}\partial^\kappa f_{\lambda'}^{\theta}-\frac{1}{3}t_1\omega_{\lambda\alpha}\partial^\alpha f_{\kappa}^{\lambda'}-\frac{1}{3}t_1\omega_{\lambda\lambda}^{\lambda}\partial^\kappa f_{\kappa}^{\lambda'}+\frac{1}{2}t_1\partial^\alpha f_{\lambda}^{\kappa}\partial^\kappa f_{\lambda\alpha}^{\lambda}+$$
$$\frac{1}{2}t_1\partial_\kappa f_{\theta}^{\lambda}\partial^\kappa f_{\lambda}^{\theta}+\frac{1}{2}t_1\partial_\kappa f_{\theta}^{\lambda}\partial^\kappa f_{\lambda}^{\theta}-\frac{1}{3}t_1\partial^\alpha f_{\lambda}^{\kappa}\partial^\kappa f_{\alpha}^{\lambda}+\frac{1}{3}r_2\partial_\kappa\omega^{\alpha\beta\theta}\partial^\theta\omega_{\alpha\beta}^{\kappa}+$$
$$\frac{2}{3}r_2\partial_\kappa\omega^{\theta\alpha\beta}\partial^\kappa\omega_{\alpha\beta\theta}-\frac{2}{3}r_2\partial^\beta\omega_{\lambda'}^{\alpha\lambda}\partial_\lambda\omega_{\alpha\beta}^{\lambda'}+\frac{2}{3}r_2\partial^\beta\omega_{\lambda'}^{\lambda\alpha}\partial_\lambda\omega_{\alpha\beta}^{\lambda'}-$$
$$4r_3\partial^\beta\omega_{\lambda'}^{\lambda\alpha}\partial_\lambda\omega_{\alpha\beta}^{\lambda'}-4r_3\partial_\alpha\omega_{\lambda}^{\alpha}\partial_\theta\omega_{\lambda}^{\theta\kappa}+\frac{1}{\kappa}+4r_3\partial_\theta\omega_{\lambda}^{\alpha}\partial_\alpha\omega_{\lambda}^{\lambda\theta\kappa}$$

$\omega_{1+}^{\#1} + \alpha\beta$	$\omega_{1+}^{\#2} + \alpha\beta$	$f_{1+}^{\#1} + \alpha\beta$	$\omega_{1-}^{\#1} - \alpha$	$\omega_{1-}^{\#2} - \alpha$	$f_{1-}^{\#1} - \alpha$	$f_{1-}^{\#2} - \alpha$
$\omega_{1+}^{\#1} + \alpha\beta$	$-\frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$0$	$0$	$0$	$0$
$\omega_{1+}^{\#2} + \alpha\beta$	$-\frac{t_1}{\sqrt{2}}$	$0$	$0$	$0$	$0$	$0$
$f_{1+}^{\#1} + \alpha\beta$	$\frac{ikt_1}{\sqrt{2}}$	$0$	$0$	$0$	$0$	$0$
$\omega_{1-}^{\#1} + \alpha$	$0$	$0$	$\frac{t_1}{6}$	$\frac{t_1}{3\sqrt{2}}$	$0$	$\frac{ikt_1}{3}$
$\omega_{1-}^{\#2} + \alpha$	$0$	$0$	$\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	$0$	$\frac{1}{3}i\sqrt{2}kt_1$
$f_{1-}^{\#1} + \alpha$	$0$	$0$	$0$	$0$	$0$	$0$
$f_{1-}^{\#2} + \alpha$	$0$	$0$	$-\frac{1}{3}ik t_1$	$-\frac{1}{3}i\sqrt{2}kt_1$	$0$	$\frac{2k^2t_1}{3}$

$\sigma_{0+}^{\#1} +$	$\tau_{0+}^{\#1} +$	$\tau_{0+}^{\#2} +$	$\sigma_{0-}^{\#1} -$
$\sigma_{0+}^{\#1} +$	$\frac{1}{6k^2r_3}$	$0$	$0$
$\tau_{0+}^{\#1} +$	$0$	$0$	$0$
$\tau_{0+}^{\#2} +$	$0$	$0$	$0$
$\sigma_{0-}^{\#1} -$	$0$	$0$	$\frac{1}{k^2r_2-t_1}$

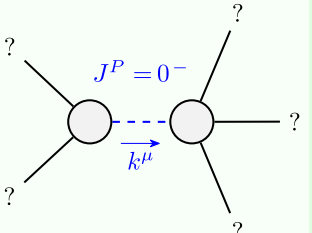
$\omega_{2+}^{\#1} + \alpha\beta$	$f_{2+}^{\#1} + \alpha\beta$	$\omega_{2-}^{\#1} - \alpha\beta\chi$	
$\omega_{2+}^{\#1} + \alpha\beta$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2+}^{\#1} + \alpha\beta$	$\frac{ikt_1}{\sqrt{2}}$	$k^2t_1$	0
$\omega_{2-}^{\#1} + \alpha\beta\chi$	0	0	$\frac{t_1}{2}$

$\omega_{0+}^{\#1} +$	$f_{0+}^{\#1} +$	$f_{0+}^{\#2} +$	$\omega_{0-}^{\#1} -$
$\omega_{0+}^{\#1} +$	$6k^2r_3$	$0$	$0$
$f_{0+}^{\#1} +$	$0$	$0$	$0$
$f_{0+}^{\#2} +$	$0$	$0$	$0$
$\omega_{0-}^{\#1} -$	$0$	$0$	$k^2r_2-t_1$

Source constraints/gauge generators	Multiplicities
SO(3) irreps	
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#1\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\sigma_{1-}^{\#1\alpha} == \sigma_{1-}^{\#2\alpha}$	3
$\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	19

$\sigma_+^{\#1} + \alpha\beta$	$\tau_+^{\#1} + \alpha\beta$	$\sigma_-^{\#1} - \alpha\beta\chi$	
$\sigma_+^{\#1} + \alpha\beta$	$\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
$\tau_+^{\#1} + \alpha\beta$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\sigma_-^{\#1} - \alpha\beta\chi$	0	0	$\frac{2}{t_1}$

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$\frac{t_1}{r_2} > 0$
Spin:	0
Parity:	Odd

No massless particles (sepicles on)

## Unitarity conditions

$r_2 < 0 \&\& t_1 < 0$