

Particle spectrograph

Wave operator and propagator

Source constraints		
SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0+}^{\#2} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$	1
$\tau_{0+}^{\#1} - 2 i k \sigma_{0+}^{\#1} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha_\alpha + 2 \partial_\chi \partial^\chi \partial_\beta \sigma^{\alpha\beta}_\alpha$	1
$\tau_{1+}^{\#2\alpha} + 2 i k \sigma_{1+}^{\#2\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta} + 2 \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$	3
$\tau_{1+}^{\#1\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$	3
$\tau_{1+}^{\alpha\beta} + i k \sigma_{1+}^{\#1\alpha\beta} == 0$	$\partial_\chi \partial_\beta \tau^{\alpha\beta\chi} + \partial_\chi \partial^\beta \tau^\alpha_\chi + \partial_\chi \partial^\chi \tau^{\alpha\beta}_\alpha + \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\beta\chi\alpha} ==$ $\partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\beta\chi\alpha} ==$ $\partial_\chi \partial^\alpha \tau^\beta_\chi + \partial_\chi \partial^\beta \tau^\alpha_\chi + \partial_\chi \partial^\chi \tau^{\beta\alpha}_\alpha + \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\chi\beta}$	3
$\sigma_{1+}^{\#1\alpha\beta} == \sigma_{1+}^{\#2\alpha\beta}$	$3 \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} +$ $2 \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\chi\beta} ==$ $3 \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\beta\chi\alpha}$	3
$\sigma_{2+}^{\#1\alpha\beta\chi} == 0$	$3 \partial_\epsilon \partial_\delta \partial^\chi \partial^\alpha \sigma^{\beta\delta\epsilon} + 3 \partial_\epsilon \partial^\epsilon \partial^\chi \partial^\alpha \sigma^{\beta\delta}_\delta +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\chi\delta} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\delta\chi} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\chi\delta\alpha} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\beta\delta} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\alpha\delta\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\chi\alpha} +$ $3 \eta^{\beta\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial^\alpha \sigma^{\delta\epsilon}_\delta +$ $3 \eta^{\alpha\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial_\delta \sigma^{\beta\delta\epsilon} +$ $3 \eta^{\beta\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial^\alpha \sigma^{\alpha\delta}_\delta ==$ $3 \partial_\epsilon \partial_\delta \partial^\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 3 \partial_\epsilon \partial^\epsilon \partial_\chi \partial^\beta \sigma^{\alpha\delta}_\delta +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\chi\delta} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\delta\chi} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\chi\delta\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\delta\alpha} +$ $4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\alpha\beta\chi} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\alpha\chi\beta} +$ $3 \eta^{\alpha\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial^\beta \sigma^{\delta\epsilon}_\delta +$ $3 \eta^{\beta\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial_\delta \sigma^{\alpha\delta\epsilon} +$ $3 \eta^{\alpha\chi} \partial_\theta \partial^\theta \partial_\epsilon \partial^\beta \sigma^{\beta\delta}_\delta$	5
$\tau_{2+}^{\#1\alpha\beta} == 0$	$4 \partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2 \partial_\delta \partial^\beta \partial^\beta \partial^\alpha \tau^\chi_\chi +$ $3 \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3 \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} +$ $2 \eta^{\alpha\beta} \partial_\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} ==$ $3 \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\chi} + 3 \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\beta} +$ $3 \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi} + 3 \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\chi\alpha} +$ $2 \eta^{\alpha\beta} \partial_\epsilon \partial_\delta \partial^\alpha \tau^\chi_\chi$	5
Total constraints/gauge generators:		24

Quadratic (free) action

$$S = \iiint \left(\frac{1}{6} (-4 t_3 \omega_{\alpha}^{\kappa} \omega_{\kappa}^{\alpha} + 6 f^{\alpha\beta} \tau_{\alpha\beta} + 6 \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} + 8 t_3 \omega_{\alpha}^{\kappa} \omega_{\kappa}^{\alpha} \partial f^{\alpha\chi} - 8 t_3 \omega_{\kappa}^{\alpha} \partial f^{\alpha}_{\kappa} + 4 t_3 \partial f^{\alpha}_{\kappa} \partial f^{\kappa}_{\alpha} - 15 r_3 \partial_\beta \omega_{\gamma}^{\theta} \partial^\gamma \omega^{\alpha\beta}_{\theta} + 9 r_3 \partial_\gamma \omega_{\beta}^{\theta} \partial^\gamma \omega^{\alpha\beta}_{\theta} + 9 r_3 \partial_\alpha \omega^{\alpha\beta\gamma} \partial_\beta \omega_{\gamma}^{\theta} - 18 r_3 \partial^\gamma \omega_{\beta}^{\alpha\beta} \partial_\gamma \omega_{\beta}^{\theta} - 15 r_3 \partial_\alpha \omega^{\alpha\beta\gamma} \partial_\beta \omega_{\gamma}^{\theta} + 30 r_3 \partial^\gamma \omega_{\beta}^{\alpha\beta} \partial_\gamma \omega_{\beta}^{\theta} + 4 t_2 \omega_{\alpha} \partial^\beta f^{\alpha\chi} + 2 t_2 \partial_\alpha f_{\theta} \partial^\beta f^{\alpha\chi} - t_2 \partial_\alpha f_{\theta\gamma} \partial^\beta f^{\alpha\chi} - t_2 \partial f_{\alpha\theta} \partial^\beta f^{\alpha\chi} + t_2 \partial_\theta f_{\alpha\gamma} \partial^\beta f^{\alpha\chi} - t_2 \partial_\theta f_{\gamma\alpha} \partial^\beta f^{\alpha\chi} - 4 t_2 \omega_{\alpha\theta\gamma} (\omega^{\alpha\gamma\theta} + \partial^\theta f^{\alpha\chi}) + 2 t_2 \omega_{\alpha\gamma\theta} (\omega^{\alpha\gamma\theta} + 2 \partial^\theta f^{\alpha\chi}) + 8 r_2 \partial_\beta \omega_{\alpha\gamma\theta} \partial^\theta \omega^{\alpha\beta\gamma} - 4 r_2 \partial_\beta \omega_{\alpha\theta\gamma} \partial^\theta \omega^{\alpha\beta\gamma} + 4 r_2 \partial_\beta \omega_{\gamma\theta\alpha} \partial^\theta \omega^{\alpha\beta\gamma} - 2 r_2 \partial_\theta \omega_{\alpha\beta\gamma} \partial^\theta \omega^{\alpha\beta\gamma} - 4 r_2 \partial_\theta \omega_{\alpha\gamma\beta} \partial^\theta \omega^{\alpha\beta\gamma} + 2 r_2 \partial_\theta \omega_{\alpha\beta\gamma} \partial^\theta \omega^{\alpha\beta\gamma} + 4 t_3 \partial f^{\alpha\chi} \partial_\alpha \omega_{\chi}^{\kappa} - 8 t_3 \partial f^{\alpha}_{\chi} \partial_\alpha \omega^{\kappa\chi}) [t, x, y, z] dz dy dx dt$$

$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2}$	$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#1}$	$\sigma_{1+}^{\#2}$	$\tau_{1+}^{\#1}$	$\tau_{1+}^{\#2}$
$\frac{6}{(3+k^2)^2} t_2$	$\frac{3\sqrt{2}}{(3+k^2)^2} t_2$	$\frac{3 i \sqrt{2} k}{(3+k^2)^2} t_2$	0	0	0	0
$\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$	$\frac{3}{(3+k^2)^2} t_2$	$\frac{3 i k}{(3+k^2)^2} t_2$	0	0	0	0
$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$-\frac{3 i k}{(3+k^2)^2} t_2$	$\frac{3 k^2}{(3+k^2)^2} t_2$	0	0	0	0
$\sigma_{1+}^{\#1} \dagger^\alpha$	0	0	$-\frac{2}{3 k^2 r_3}$	$-\frac{2 \sqrt{2}}{3 k^2 r_3 + 6 k^4 r_3}$	0	$-\frac{4 i}{3 k r_3 + 6 k^3 r_3}$
$\sigma_{1+}^{\#2} \dagger^\alpha$	0	0	$-\frac{2 \sqrt{2}}{3 k^2 r_3 + 6 k^4 r_3}$	$\frac{9 k^2 r_3 - 4 t_3}{3 (k + 2 k^2)^2 r_3 t_3}$	0	$\frac{i \sqrt{2} (9 k^2 r_3 - 4 t_3)}{3 k (1 + 2 k^2)^2 r_3 t_3}$
$\tau_{1+}^{\#1} \dagger^\alpha$	0	0	0	0	0	0
$\tau_{1+}^{\#2} \dagger^\alpha$	0	0	$\frac{4 i}{3 k r_3 + 6 k^3 r_3}$	$-\frac{i \sqrt{2} (9 k^2 r_3 - 4 t_3)}{3 k (1 + 2 k^2)^2 r_3 t_3}$	0	$\frac{2 (9 k^2 r_3 - 4 t_3)}{3 (1 + 2 k^2)^2 r_3 t_3}$

$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1+}^{\#2}$	$f_{1+}^{\#1}$	$\omega_{1+}^{\#1}$	$\omega_{1+}^{\#2}$	$f_{1+}^{\#1}$	$f_{1+}^{\#2}$
$\frac{2 t_2}{3}$	$\frac{\sqrt{2} t_2}{3}$	$\frac{1}{3} i \sqrt{2} k t_2$	0	0	0	0
$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$\frac{t_2}{3}$	$\frac{i k t_2}{3}$	0	0	0	0
$f_{1+}^{\#1} \dagger^{\alpha\beta} - \frac{1}{3} i \sqrt{2} k t_2$	$-\frac{1}{3} i k t_2$	$\frac{k^2 t_2}{3}$	0	0	0	0
$\omega_{1+}^{\#1} \dagger^\alpha$	0	0	$\frac{1}{6} (-9 k^2 r_3 + 4 t_3)$	$-\frac{\sqrt{2} t_3}{3}$	0	$-\frac{2}{3} i k t_3$
$\omega_{1+}^{\#2} \dagger^\alpha$	0	0	$-\frac{\sqrt{2} t_3}{3}$	$\frac{t_3}{3}$	0	$\frac{1}{3} i \sqrt{2} k t_3$
$f_{1+}^{\#1} \dagger^\alpha$	0	0	0	0	0	0
$f_{1+}^{\#2} \dagger^\alpha$	0	0	$\frac{2 i k t_3}{3}$	$-\frac{1}{3} i \sqrt{2} k t_3$	0	$\frac{2 k^2 t_3}{3}$

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$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$
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 $\sigma_{2+}^{\#1} \dagger^{\alpha\beta\chi}$

Massive and massless spectra

Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)
(separable particles)

Unitarity conditions

$r_2 < 0 \ \&\& \ t_2 > 0$