Particle spectrograph

Wave operator and propagator

	$\Delta^{\#1}_{1^{+}}{}_{\alpha\beta}$	$\Delta^{\#2}_{1^{+}}{}_{\alpha\beta}$	$\Delta^{\#3}_{1^{+}}{}_{\alpha\beta}$	$\Delta^{\#1}_{1^{-}}{}_{\alpha}$	$\Delta^{\#2}_{1^{-}}{}_{\alpha}$	$\Delta^{\#3}_{1^{-}}{}_{\alpha}$	$\Delta^{\#4}_{1^{-}}{}_{\alpha}$	$\Delta^{\#5}_{1^{-}}{}_{\alpha}$	$\Delta^{\#6}_{1^{-}}{}_{\alpha}$	$\mathcal{T}^{\#1}_{1^{-}}{}_{\alpha}$
$\Delta^{\#1}_{1^{+}}{}^{\dagger\alpha\beta}$	0	$-\frac{2\sqrt{2}}{a_0}$	0	0	0	0	0	0	0	0
$\Delta^{\#2}_{1^{+}}{}^{\dagger\alpha\beta}$	$-\frac{2\sqrt{2}}{a_0}$	$\frac{2}{a_0}$	0	0	0	0	0	0	0	0
$\Delta^{\#3}_{1^{+}}{}^{\dagger\alpha\beta}$	0	0	$\frac{4}{a_0}$	0	0	0	0	0	0	0
$\Delta^{\#1}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	$\frac{2\sqrt{2}}{a_0}$	0	0	0	0	0
$\Delta^{\#2}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	$\frac{2\sqrt{2}}{a_0}$	$\frac{2}{a_0}$	0	0	0	0	0
$\Delta^{\#3}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	0	$-\frac{19}{12a_0}$	$\frac{5\sqrt{5}}{12a_0}$	$-\frac{1}{6\sqrt{2}\,a_0}$	$-\frac{1}{6a_0}$	0
$\Delta^{\#4}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	0	$\frac{5\sqrt{5}}{12a_0}$	$\frac{1}{12a_0}$	$-\frac{\sqrt{\frac{5}{2}}}{6a_0}$	$-\frac{\sqrt{5}}{6a_0}$	0
$\Delta^{\#5}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	0	$-\frac{1}{6\sqrt{2}\,a_0}$	$-\frac{\sqrt{\frac{5}{2}}}{6a_0}$	$\frac{17}{6a_0}$	$-\frac{7}{3\sqrt{2}\,a_0}$	0
$\Delta^{\#6}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	0	$-\frac{1}{6a_0}$	$-\frac{\sqrt{5}}{6a_0}$	$-\frac{7}{3\sqrt{2}\,a_0}$	$\frac{5}{3a_0}$	0
$\mathcal{T}^{\#1}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	0	0	0	0	0	0

	$\Gamma^{\#1}_{1^{+}}{}_{\alpha\beta}$	$\Gamma^{\#2}_{1^{+}}{}_{\alpha\beta}$	$\Gamma^{\#3}_{1^{+}}{}_{\alpha\beta}$	$\Gamma^{\#1}_{1^{-}}{}_{\alpha}$	$\Gamma^{\#2}_{1^{-}}{}_{\alpha}$	$\Gamma^{\#3}_{1^{-}}{}_{\alpha}$	$\Gamma^{\#4}_{1^{-}}{}_{\alpha}$	$\Gamma^{\#5}_{1^{-}}{}_{\alpha}$	$\Gamma^{\#6}_{1^{-}}{}_{\alpha}$	$h^{\#1}_{1^{-}}{}_{\alpha}$
$\Gamma^{\#1}_{1^{+}}{}^{\dagger\alpha\beta}$	$-\frac{a_0}{4}$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0	0	0
$\Gamma^{\#2}_{1^{+}}{}^{\dagger\alpha\beta}$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0	0	0	0
$\Gamma^{\#3}_{1^{+}}{}^{\dagger\alpha\beta}$	0	0	$\frac{a_0}{4}$	0	0	0	0	0	0	0
$\Gamma^{\#1}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	$-\frac{a_0}{4}$	$\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0
$\Gamma^{\#2}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	$\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0
$\Gamma^{\#3}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	0	$-\frac{a_0}{3}$	$\frac{\sqrt{5}\,a_0}{6}$	$-\frac{a_0}{6\sqrt{2}}$	$-\frac{a_0}{6}$	0
$\Gamma^{\#4}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	0	$\frac{\sqrt{5}\,a_0}{6}$	$\frac{a_0}{3}$	$-\frac{1}{6}\sqrt{\frac{5}{2}}\,a_0$	$-\frac{\sqrt{5}\,a_0}{6}$	0
$\Gamma^{\#5}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	0	$-\frac{a_0}{6\sqrt{2}}$	$-\frac{1}{6}\sqrt{\frac{5}{2}}\,a_0$	$\frac{a_0}{3}$	$\frac{a_0}{6\sqrt{2}}$	0
$\Gamma^{\#6}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	0	$-\frac{a_0}{6}$	$-\frac{\sqrt{5}\,a_0}{6}$	$\frac{a_0}{6\sqrt{2}}$	$\frac{5a_0}{12}$	0
$h^{\#1}_{1^{-}}{}^{\dagger\alpha}$	0	0	0	0	0	0	0	0	0	0

Quadratic (free) action

$$
\begin{aligned}
S==&\iiiint\Big(\tfrac{1}{8}\big(8\,h^{\alpha\beta}\,\mathcal{T}_{\alpha\beta}-4\,\Gamma^{\alpha\beta\chi}\,(a_0\,\Gamma_{\beta\chi\alpha}-2\,\Delta_{\alpha\beta\chi}+a_0\,\partial_{\beta}h_{\alpha\chi})+2\,a_0\,\Gamma^{\alpha\beta}{}_{\alpha}\,\partial_{\beta}h^{\chi}{}_{\chi}-\\
&2\,a_0\,h^{\chi}{}_{\chi}\,\partial_{\beta}\Gamma^{\alpha}{}_{\alpha}{}^{\beta}+2\,a_0\,h^{\chi}{}_{\chi}\,\partial_{\beta}\Gamma^{\alpha\beta}{}_{\alpha}-4\,a_0\,h_{\alpha\chi}\,\partial_{\beta}\Gamma^{\alpha\beta\chi}+\\
&4\,a_0\,h^{\alpha\beta}\,\partial_{\beta}\partial_{\alpha}h^{\chi}{}_{\chi}-a_0\,\partial_{\beta}h^{\chi}{}_{\chi}\,\partial^{\beta}h^{\alpha}{}_{\alpha}-4\,a_0\,\partial_{\alpha}h^{\alpha\beta}\,\partial_{\chi}h_{\beta}{}^{\chi}+\\
&4\,a_0\,\partial^{\beta}h^{\alpha}{}_{\alpha}\,\partial_{\chi}h_{\beta}{}^{\chi}+2\,a_0\,\Gamma^{\alpha}{}_{\alpha}{}^{\beta}\,(2\,\Gamma^{\chi}{}_{\beta\chi}-\partial_{\beta}h^{\chi}{}_{\chi}+2\,\partial_{\chi}h_{\beta}{}^{\chi})-\\
&8\,a_0\,h^{\alpha\beta}\,\partial_{\chi}\partial_{\beta}h_{\alpha}{}^{\chi}+2\,a_0\,h^{\alpha}{}_{\alpha}\,\partial_{\chi}\partial_{\beta}h^{\beta\chi}+\\
&4\,a_0\,h^{\alpha\beta}\,\partial_{\chi}\partial^{\chi}h_{\alpha\beta}-2\,a_0\,h^{\alpha}{}_{\alpha}\,\partial_{\chi}\partial^{\chi}h^{\beta}{}_{\beta}-\\
&2\,a_0\,\partial_{\beta}h_{\alpha\chi}\,\partial^{\chi}h^{\alpha\beta}+3\,a_0\,\partial_{\chi}h_{\alpha\beta}\,\partial^{\chi}h^{\alpha\beta}+\\
&4\,a_0\,h_{\beta\chi}\,\partial^{\chi}\Gamma^{\alpha}{}_{\alpha}{}^{\beta}\big)\Big)[t,x,y,z]\,dz\,dy\,dx\,dt
\end{aligned}
$$

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\mathcal{T}^{\#2}_{0^{+}}==0$	$\partial_{\beta}\partial_{\alpha}\mathcal{T}^{\alpha\beta}==0$	1
$\Delta^{\#3}_{0^{+}}+2\,\Delta^{\#4}_{0^{+}}+3\,\Delta^{\#2}_{0^{+}}==0$	$\partial_{\alpha}\Delta^{\alpha\beta}{}_{\beta}==0$	1
$\mathcal{T}^{\#1}_{1^{-}}{}^{\alpha}==0$	$\partial_{\chi}\partial_{\beta}\partial^{\alpha}\mathcal{T}^{\beta\chi}==\partial_{\chi}\partial^{\chi}\partial_{\beta}\mathcal{T}^{\alpha\beta}$	3
$2\,\Delta^{\#6}_{1^{-}}{}^{\alpha}+\Delta^{\#4}_{1^{-}}{}^{\alpha}+2\,\Delta^{\#5}_{1^{-}}{}^{\alpha}+\Delta^{\#3}_{1^{-}}{}^{\alpha}==0$	$\partial_{\beta}\partial^{\alpha}\Delta^{\beta\chi}{}_{\chi}==\partial_{\chi}\partial^{\chi}\Delta^{\alpha\beta}{}_{\beta}$	3
Total constraints/gauge generators:		8

	$\Gamma^{\#1}_{3^{-}}{}_{\alpha\beta\chi}$
$\Gamma^{\#1}_{3^{-}}{}^{\dagger\alpha\beta\chi}$	$-\frac{a_0}{2}$

	$\Delta^{\#1}_{3^{-}}{}_{\alpha\beta\chi}$
$\Delta^{\#1}_{3^{-}}{}^{\dagger\alpha\beta\chi}$	$-\frac{2}{a_0}$

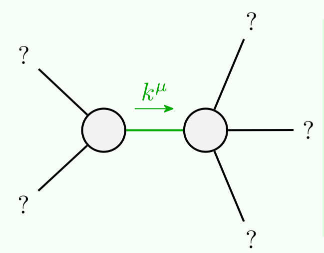
	$\Delta^{\#1}_{0^{+}}$	$\Delta^{\#2}_{0^{+}}$	$\Delta^{\#3}_{0^{+}}$	$\Delta^{\#4}_{0^{+}}$	$\mathcal{T}^{\#1}_{0^{+}}$	$\mathcal{T}^{\#2}_{0^{+}}$	$\Delta^{\#1}_{0^{-}}$
$\Delta^{\#1}_{0^{+}}{}^{\dagger}$	$-\frac{2}{a_0}$	0	0	0	0	0	0
$\Delta^{\#2}_{0^{+}}{}^{\dagger}$	0	$-\frac{3}{4a_0}$	$\frac{5}{4a_0}$	$-\frac{1}{2\sqrt{2}\,a_0}$	0	0	0
$\Delta^{\#3}_{0^{+}}{}^{\dagger}$	0	$\frac{5}{4a_0}$	$-\frac{3}{4a_0}$	$-\frac{1}{2\sqrt{2}\,a_0}$	0	0	0
$\Delta^{\#4}_{0^{+}}{}^{\dagger}$	0	$-\frac{1}{2\sqrt{2}\,a_0}$	$-\frac{1}{2\sqrt{2}\,a_0}$	$\frac{1}{2a_0}$	0	0	0
$\mathcal{T}^{\#1}_{0^{+}}{}^{\dagger}$	0	0	0	0	$\frac{4}{a_0k^2}$	0	0
$\mathcal{T}^{\#2}_{0^{+}}{}^{\dagger}$	0	0	0	0	0	0	0
$\Delta^{\#1}_{0^{-}}{}^{\dagger}$	0	0	0	0	0	0	$-\frac{2}{a_0}$

	$\Delta^{\#1}_{2^{+}}{}_{\alpha\beta}$	$\Delta^{\#2}_{2^{+}}{}_{\alpha\beta}$	$\Delta^{\#3}_{2^{+}}{}_{\alpha\beta}$	$\mathcal{T}^{\#1}_{2^{+}}{}_{\alpha\beta}$	$\Delta^{\#1}_{2^{-}}{}_{\alpha\beta\chi}$	$\Delta^{\#2}_{2^{-}}{}_{\alpha\beta\chi}$
$\Delta^{\#1}_{2^{+}}{}^{\dagger\alpha\beta}$	$\frac{4}{a_0}$	0	0	0	0	0
$\Delta^{\#2}_{2^{+}}{}^{\dagger\alpha\beta}$	0	$-\frac{2}{a_0}$	0	0	0	0
$\Delta^{\#3}_{2^{+}}{}^{\dagger\alpha\beta}$	0	0	$\frac{4}{a_0}$	0	0	0
$\mathcal{T}^{\#1}_{2^{+}}{}^{\dagger\alpha\beta}$	0	0	0	$-\frac{8}{a_0k^2}$	0	0
$\Delta^{\#1}_{2^{-}}{}^{\dagger\alpha\beta\chi}$	0	0	0	0	$\frac{4}{a_0}$	0
$\Delta^{\#2}_{2^{-}}{}^{\dagger\alpha\beta\chi}$	0	0	0	0	0	$\frac{4}{a_0}$

	$\Gamma^{\#1}_{0^{+}}$	$\Gamma^{\#2}_{0^{+}}$	$\Gamma^{\#3}_{0^{+}}$	$\Gamma^{\#4}_{0^{+}}$	$h^{\#1}_{0^{+}}$	$h^{\#2}_{0^{+}}$	$\Gamma^{\#1}_{0^{-}}$
$\Gamma^{\#1}_{0^{+}}{}^{\dagger}$	$-\frac{a_0}{2}$	0	0	0	0	0	0
$\Gamma^{\#2}_{0^{+}}{}^{\dagger}$	0	0	$\frac{a_0}{2}$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0
$\Gamma^{\#3}_{0^{+}}{}^{\dagger}$	0	$\frac{a_0}{2}$	0	$-\frac{a_0}{2\sqrt{2}}$	0	0	0
$\Gamma^{\#4}_{0^{+}}{}^{\dagger}$	0	$-\frac{a_0}{2\sqrt{2}}$	$-\frac{a_0}{2\sqrt{2}}$	$\frac{a_0}{2}$	0	0	0
$h^{\#1}_{0^{+}}{}^{\dagger}$	0	0	0	0	$\frac{a_0k^2}{4}$	0	0
$h^{\#2}_{0^{+}}{}^{\dagger}$	0	0	0	0	0	0	0
$\Gamma^{\#1}_{0^{-}}{}^{\dagger}$	0	0	0	0	0	0	$-\frac{a_0}{2}$

	$\Gamma^{\#1}_{2^{+}}{}_{\alpha\beta}$	$\Gamma^{\#2}_{2^{+}}{}_{\alpha\beta}$	$\Gamma^{\#3}_{2^{+}}{}_{\alpha\beta}$	$h^{\#1}_{2^{+}}{}_{\alpha\beta}$	$\Gamma^{\#1}_{2^{-}}{}_{\alpha\beta\chi}$	$\Gamma^{\#2}_{2^{-}}{}_{\alpha\beta\chi}$
$\Gamma^{\#1}_{2^{+}}{}^{\dagger\alpha\beta}$	$\frac{a_0}{4}$	0	0	0	0	0
$\Gamma^{\#2}_{2^{+}}{}^{\dagger\alpha\beta}$	0	$-\frac{a_0}{2}$	0	0	0	0
$\Gamma^{\#3}_{2^{+}}{}^{\dagger\alpha\beta}$	0	0	$\frac{a_0}{4}$	0	0	0
$h^{\#1}_{2^{+}}{}^{\dagger\alpha\beta}$	0	0	0	$-\frac{a_0k^2}{8}$	0	0
$\Gamma^{\#1}_{2^{-}}{}^{\dagger\alpha\beta\chi}$	0	0	0	0	$\frac{a_0}{4}$	0
$\Gamma^{\#2}_{2^{-}}{}^{\dagger\alpha\beta\chi}$	0	0	0	0	0	$\frac{a_0}{4}$

Massive and massless spectra



Quadratic pole	
Pole residue:	$-\frac{1}{a_0}>0$
Polarisations:	2

(No massive particles)

Unitarity conditions

$a_0 < 0$