Lagrangian density

$$-2\beta_1^2\,\omega_{\alpha\chi\beta}\,\omega^{\alpha\beta\chi} - 2\beta_1^2\,\omega_\alpha{}^{\chi\delta}\,\omega_{\chi\delta}{}^\alpha - 2\beta_1^2\,\omega_\alpha{}^\chi{}_\chi\,\partial_\beta f^{\alpha\beta} -$$
$$2\beta_1^2\,\omega_\alpha{}^\delta{}_\delta\,\partial_\beta f^{\alpha\beta} - 4\beta_1^2\,f^{\alpha\beta}\partial_\beta\omega_\alpha{}^\chi{}_\chi + 4\beta_1^2\,\partial_\beta\omega^{\alpha\beta}{}_\alpha +$$
$$\tfrac{2}{3}\partial^\alpha\omega^{\beta\zeta}{}_\chi\,\partial_\beta\omega_{\zeta\alpha}{}^\chi + 2\beta_1^2\,\omega_\beta{}^\chi{}_\chi\,\partial^\beta f^\alpha{}_\alpha -$$
$$2\beta_1^2\,\partial_\beta f^\chi{}_\chi\,\partial^\beta f^\alpha{}_\alpha + 4\beta_1^2\,\partial_\chi\omega_\alpha{}^\chi{}_\beta - 4\beta_1^2\,f^\alpha{}_\alpha\,\partial_\chi\omega^{\beta\chi}{}_\beta -$$
$$\tfrac{2}{3}\partial_\beta\omega_\alpha{}^\chi{}_\zeta\,\partial_\chi\omega^{\zeta\alpha}{}_\beta - \tfrac{1}{3}\partial_\beta\omega_{\zeta\alpha}{}^\chi\,\partial_\chi\omega^{\zeta\alpha\beta} + 4\beta_1^2\,\omega_{\alpha\chi\beta}\,\partial^\chi f^{\alpha\beta} +$$
$$\beta_1^2\,\partial_\chi f_\beta{}^\delta\,\partial^\chi f^\alpha{}_\delta + \beta_1^2\,\partial_\chi f_\beta{}^\delta\,\partial^\chi f^\beta{}_\delta + \tfrac{2}{3}\partial_\chi\omega^{\beta\zeta\alpha}\partial^\chi\omega_{\zeta\alpha\beta} +$$
$$\tfrac{1}{3}\partial_\chi\omega^{\zeta\alpha\beta}\partial^\chi\omega_{\zeta\alpha\beta} + 4\beta_1^2\,\partial^\beta f^\alpha{}_\alpha\,\partial_\delta f_\beta{}^\delta - 2\beta_1^2\,\partial_\beta f^\alpha{}_\delta\,\partial_\delta f^{\chi\delta} +$$
$$\tfrac{2}{3}\partial^\beta\omega_\alpha{}^{\delta\zeta}\partial_\delta\omega_{\zeta\beta}{}^\alpha - \tfrac{2}{3}\partial^\beta\omega_\alpha{}^{\zeta\delta}\partial_\delta\omega_{\zeta\beta}{}^\alpha - \beta_1^2\,\partial^\chi f_\zeta{}^\delta\partial^\zeta f_{\beta\chi} -$$
$$\beta_1^2\,\partial^\chi f_\zeta{}^\beta\,\partial^\zeta f_{\chi\beta} + \beta_1^2\,\partial^\chi f^\alpha{}_\zeta\,\partial^\zeta f_{\delta\zeta}\partial^\delta f^\chi{}_\chi - \beta_1^2\,\partial^\chi f^\alpha{}_\chi\partial^\delta f^\delta{}_\chi$$

Added source term: $\big|\; f^{\alpha\beta}\,\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}$

Top matrix (7×7):

	$\sigma_{1^+}^{\#1}\!\dagger^{\alpha\beta}$	$\sigma_{1^+}^{\#2}\!\dagger^{\alpha\beta}$	$\tau_{1^+}^{\#1}\!\dagger^{\alpha\beta}$	$\sigma_{1^-}^{\#1}\!\dagger^{\alpha}$	$\sigma_{1^-}^{\#2}\!\dagger^{\alpha}$	$\tau_{1^-}^{\#1}\!\dagger^{\alpha}$	$\tau_{1^-}^{\#2}\!\dagger^{\alpha}$
$\tau_{1^-}^{\#2}{}_\alpha$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#1}{}_\alpha$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#2}{}_\alpha$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#1}{}_\alpha$	0	0	0	0	0	0	0
$\tau_{1^+}^{\#1}{}_{\alpha\beta}$	0	0	0	0	0	0	0
$\sigma_{1^+}^{\#2}{}_{\alpha\beta}$	0	0	0	0	0	0	0
$\sigma_{1^+}^{\#1}{}_{\alpha\beta}$	0	0	0	0	0	0	0

3×3 matrix:

	$\sigma_{2^+}^{\#1}{}_{\alpha\beta}$	$\tau_{2^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\sigma_{2^+}^{\#1}\!\dagger^{\alpha\beta}$	0	0	0
$\tau_{2^+}^{\#1}\!\dagger^{\alpha\beta}$	0	$\dfrac{1}{2\beta_1^2 k^2}$	0
$\sigma_{2^-}^{\#1}\!\dagger^{\alpha\beta\chi}$	0	0	0

4×4 matrix:

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\omega_{0^-}^{\#1}$
$\omega_{0^+}^{\#1}\!\dagger$	0	0	0	0
$f_{0^+}^{\#1}\!\dagger$	0	$-4\beta_1^2 k^2$	0	0
$f_{0^+}^{\#2}\!\dagger$	0	0	0	0
$\omega_{0^-}^{\#1}\!\dagger$	0	0	0	k^2

Middle matrix (7×7):

	$\omega_{1^+}^{\#1}\!\dagger^{\alpha\beta}$	$\omega_{1^+}^{\#2}\!\dagger^{\alpha\beta}$	$f_{1^+}^{\#1}\!\dagger^{\alpha\beta}$	$\omega_{1^-}^{\#1}\!\dagger^{\alpha}$	$\omega_{1^-}^{\#2}\!\dagger^{\alpha}$	$f_{1^-}^{\#1}\!\dagger^{\alpha}$	$f_{1^-}^{\#2}\!\dagger^{\alpha}$
$f_{1^-}^{\#2}{}_\alpha$	0	0	0	0	0	0	0
$f_{1^-}^{\#1}{}_\alpha$	0	0	0	0	0	0	0
$\omega_{1^-}^{\#2}{}_\alpha$	0	0	0	0	0	0	0
$\omega_{1^-}^{\#1}{}_\alpha$	0	0	0	0	0	0	0
$f_{1^+}^{\#1}{}_{\alpha\beta}$	0	0	0	0	0	0	0
$\omega_{1^+}^{\#2}{}_{\alpha\beta}$	0	0	0	0	0	0	0
$\omega_{1^+}^{\#1}{}_{\alpha\beta}$	0	0	0	0	0	0	0

3×3 matrix:

	$\omega_{2^+}^{\#1}{}_{\alpha\beta}$	$f_{2^+}^{\#1}{}_{\alpha\beta}$	$\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\omega_{2^+}^{\#1}\!\dagger^{\alpha\beta}$	0	0	0
$f_{2^+}^{\#1}\!\dagger^{\alpha\beta}$	0	$2\beta_1^2 k^2$	0
$\omega_{2^-}^{\#1}\!\dagger^{\alpha\beta\chi}$	0	0	0

4×4 matrix:

	$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^-}^{\#1}$
$\sigma_{0^+}^{\#1}\!\dagger$	0	0	0	0
$\tau_{0^+}^{\#1}\!\dagger$	0	$-\dfrac{1}{4\beta_1^2 k^2}$	0	0
$\tau_{0^+}^{\#2}\!\dagger$	0	0	0	0
$\sigma_{0^-}^{\#1}\!\dagger$	0	0	0	$\dfrac{1}{k^2}$

(No massive particles)

Quadratic pole	
Pole residue:	$\dfrac{1}{\beta_1^2} > 0$
Polarisations:	2

Unitarity conditions: $\beta_1 < 0 \,\|\, \beta_1 > 0$