# Particle spectrograph

## Wave operator and propagator

| | $\sigma_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^+}^{\#2\,\alpha\beta}$ | $\tau_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^-}^{\#1\,\alpha}$ | $\sigma_{1^-}^{\#2\,\alpha}$ | $\tau_{1^-}^{\#1\,\alpha}$ | $\tau_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $\frac{6}{(3+k^2)^2 t_2}$ | $\frac{3\sqrt{2}}{(3+k^2)^2 t_2}$ | $-\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2\,\dagger\,\alpha\beta}$ | $\frac{3\sqrt{2}}{(3+k^2)^2 t_2}$ | $\frac{3}{(3+k^2)^2 t_2}$ | $-\frac{3ik}{(3+k^2)^2 t_2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $-\frac{3i\sqrt{2}\,k}{(3+k^2)^2 t_2}$ | $-\frac{3ik}{(3+k^2)^2 t_2}$ | $\frac{3k^2}{(3+k^2)^2 t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | $\frac{6}{(3+2k^2)^2 t_3}$ | $-\frac{3\sqrt{2}}{(3+2k^2)^2 t_3}$ | 0 | $-\frac{6ik}{(3+2k^2)^2 t_3}$ |
| $\sigma_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 | $-\frac{3\sqrt{2}}{(3+2k^2)^2 t_3}$ | $\frac{3}{(3+2k^2)^2 t_3}$ | 0 | $\frac{3i\sqrt{2}\,k}{(3+2k^2)^2 t_3}$ |
| $\tau_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 | $-\frac{6ik}{(3+2k^2)^2 t_3}$ | $\frac{3i\sqrt{2}\,k}{(3+2k^2)^2 t_3}$ | 0 | $\frac{6k^2}{(3+2k^2)^2 t_3}$ |

Quadratic (free) action

$$S_F == \int\int\int\int (\frac{1}{6}(4\,t_3\,\omega_{\iota}^{\alpha\prime}\,\omega_{\kappa\alpha}^{\kappa\prime} + 4\,t_2\,\omega_{\iota}^{\kappa\lambda}\,\omega_{\kappa\lambda}^{\prime} + 2\,t_2\,\omega_{\iota}^{\kappa\lambda\prime}\,\omega_{\kappa\lambda}^{\prime} + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + 4\,r_2\,\partial^{\beta}\omega^{\theta\alpha}{}_{\kappa}\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa} - 2\,r_2\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\,\partial_{\kappa}\omega^{\alpha\beta\theta} - 4\,r_2\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\,\partial_{\kappa}\omega^{\theta\alpha\beta} + t_2\,\partial^{\alpha}f_{\kappa\theta}\,\partial^{\kappa}f_{\alpha}{}^{\theta} + t_2\,\partial^{\alpha}f^{\kappa}{}_{\theta}\,\partial^{\theta}f_{\kappa\alpha} - 4\,t_3\,\omega_{\kappa\alpha}{}^{\alpha}\,\partial^{\kappa}f^{\prime}{}_{\iota} - 4\,t_3\,\omega_{\kappa\lambda}{}^{\lambda}\,\partial^{\kappa}f^{\prime}{}_{\iota} - 8\,t_3\,\partial^{\alpha}f_{\kappa\alpha}\,\partial^{\kappa}f^{\prime}{}_{\iota} + 4\,t_3\,\partial_{\kappa}f^{\lambda}{}_{\lambda}\,\partial^{\kappa}f^{\prime}{}_{\iota} + 2\,t_2\,\omega_{\iota\theta\kappa}\,\partial^{\kappa}f^{\prime\theta} - 2\,t_2\,\omega_{\iota\kappa\theta}\,\partial^{\kappa}f^{\prime\theta} + 4\,t_2\,\omega_{\theta\kappa\prime}\,\partial^{\kappa}f^{\prime\theta} + 4\,t_3\,\omega_{\iota\alpha}{}^{\alpha}\,\partial^{\kappa}f^{\prime}{}_{\kappa} + 4\,t_2\,\omega_{\iota\kappa\theta}\,\partial^{\kappa}f^{\prime\theta} - 2\,t_2\,\partial^{\alpha}f^{\lambda}{}_{\kappa}\,\partial^{\kappa}f_{\lambda\alpha} - t_2\,\partial_{\kappa}f_{\theta}{}^{\lambda}\,\partial^{\kappa}f_{\lambda}{}^{\theta} + t_2\,\partial_{\kappa}f^{\lambda}{}_{\theta}\,\partial^{\kappa}f_{\lambda}{}^{\theta} + 4\,t_3\,\omega_{\iota\lambda}{}^{\lambda}\,\partial^{\kappa}f^{\prime}{}_{\kappa} - t_2\,\partial^{\alpha}f^{\lambda}{}_{\kappa}\,\partial^{\kappa}f_{\lambda}{}^{\theta} + 2\,r_2\,\partial_{\kappa}\omega^{\alpha\beta\theta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} + 4\,r_2\,\partial_{\kappa}\omega^{\theta\alpha\beta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} + 4\,t_3\,\partial^{\alpha}f^{\lambda}{}_{\alpha}\,\partial^{\kappa}f_{\lambda\kappa} + 2\,r_2\,\partial_{\kappa}\omega^{\theta\alpha\beta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} - 4\,r_2\,\partial^{\beta}\omega_{\iota}^{\alpha\lambda}\,\partial_{\lambda}\omega_{\alpha\beta}^{\prime} + 4\,r_2\,\partial^{\beta}\omega_{\iota}^{\lambda\alpha}\,\partial_{\lambda}\omega_{\alpha\beta}^{\prime}))[t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt$$

| | $\omega_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^+}^{\#2\,\alpha\beta}$ | $f_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^-}^{\#1\,\alpha}$ | $\omega_{1^-}^{\#2\,\alpha}$ | $f_{1^-}^{\#1\,\alpha}$ | $f_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | $\frac{\sqrt{2}\,t_2}{3}$ | $\frac{1}{3}i\sqrt{2}\,k\,t_2$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2\,\dagger\,\alpha\beta}$ | $\frac{\sqrt{2}\,t_2}{3}$ | $\frac{t_2}{3}$ | $\frac{i\,k\,t_2}{3}$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $-\frac{1}{3}i\sqrt{2}\,k\,t_2$ | $-\frac{1}{3}i\,k\,t_2$ | $\frac{k^2\,t_2}{3}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | $\frac{2\,t_3}{3}$ | $-\frac{\sqrt{2}\,t_3}{3}$ | $-\frac{2}{3}i\,k\,t_3$ | $-\frac{2}{3}i\,k\,t_3$ |
| $\omega_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 | $-\frac{\sqrt{2}\,t_3}{3}$ | $\frac{t_3}{3}$ | $\frac{1}{3}i\sqrt{2}\,k\,t_3$ | $\frac{1}{3}i\sqrt{2}\,k\,t_3$ |
| $f_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 | $\frac{2i\,k\,t_3}{3}$ | $-\frac{1}{3}i\sqrt{2}\,k\,t_3$ | 0 | $\frac{2\,k^2\,t_3}{3}$ |

| | $\omega_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^+}^{\#2\,\alpha\beta}$ | $f_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^-}^{\#1\,\alpha}$ | $\omega_{1^-}^{\#2\,\alpha}$ | $f_{1^-}^{\#1\,\alpha}$ | $f_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $\frac{2\,t_2}{3}$ | $\frac{\sqrt{2}\,t_2}{3}$ | $-\frac{1}{3}i\sqrt{2}\,k\,t_2$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2\,\dagger\,\alpha\beta}$ | $\frac{\sqrt{2}\,t_2}{3}$ | $\frac{t_2}{3}$ | $-\frac{1}{3}i\,k\,t_2$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1\,\dagger\,\alpha\beta}$ | $-\frac{1}{3}i\sqrt{2}\,k\,t_2$ | $-\frac{1}{3}i\,k\,t_2$ | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | $-\frac{2}{3}i\,k\,t_3$ | $\frac{\sqrt{2}\,t_3}{3}$ | 0 | $-\frac{2}{3}i\,k\,t_3$ |
| $\omega_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 | $\frac{\sqrt{2}\,t_3}{3}$ | $-\frac{t_3}{3}$ | $\frac{1}{3}i\sqrt{2}\,k\,t_3$ | 0 |
| $f_{1^-}^{\#1\,\dagger\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2\,\dagger\,\alpha}$ | 0 | 0 | 0 | $-\frac{2}{3}i\,k\,t_3$ | $\frac{1}{3}i\sqrt{2}\,k\,t_3$ | 0 | $-\frac{2\,k^2\,t_3}{3}$ |

| | $\omega_{2^+}^{\#1\,\alpha\beta}$ | $f_{2^+}^{\#1\,\alpha\beta}$ | $\omega_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $f_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $\omega_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$ | 0 | 0 | 0 |

| | $\sigma_{2^+}^{\#1\,\alpha\beta}$ | $\tau_{2^+}^{\#1\,\alpha\beta}$ | $\sigma_{2^-}^{\#1\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $\tau_{2^+}^{\#1\,\dagger\,\alpha\beta}$ | 0 | 0 | 0 |
| $\sigma_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$ | 0 | 0 | 0 |

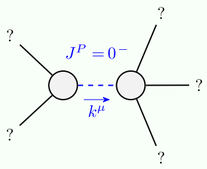
Source constraints/gauge generators

| SO(3) irreps | Multiplicities |
|---|---|
| $\tau_{0^+}^{\#2} == 0$ | 1 |
| $\tau_{0^+}^{\#1} - 2\,i\,k\,\sigma_{0^+}^{\#1} == 0$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} - i\,k\,\sigma_{1^-}^{\#1\,\alpha} == 0$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | 3 |
| $\sigma_{1^-}^{\#1\,\alpha} + 2\,\sigma_{1^-}^{\#2\,\alpha} == 0$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#1\,\alpha\beta} == 0$ | 3 |
| $\sigma_{1^+}^{\#1\,\alpha\beta} == \sigma_{1^+}^{\#2\,\alpha\beta}$ | 3 |
| $\sigma_{2^-}^{\#1\,\alpha\beta\chi} == 0$ | 5 |
| $\tau_{2^+}^{\#1\,\alpha\beta} == 0$ | 5 |
| $\sigma_{2^+}^{\#1\,\alpha\beta} == 0$ | 5 |
| Total constraints: | 32 |

| | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|
| $\omega_{0^+}^{\#1\,\dagger}$ | $t_3$ | $-i\sqrt{2}\,k\,t_3$ | 0 |
| $f_{0^+}^{\#1\,\dagger}$ | $i\sqrt{2}\,k\,t_3$ | $2\,k^2\,t_3$ | 0 |
| $f_{0^+}^{\#2\,\dagger}$ | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1\,\dagger}$ | 0 | 0 | $k^2\,r_2 + t_2$ |

| | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|
| $\sigma_{0^+}^{\#1\,\dagger}$ | $-\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3}$ | 0 | 0 |
| $\tau_{0^+}^{\#1\,\dagger}$ | $\frac{2k^2}{(1+2k^2)^2 t_3}$ | 0 | 0 |
| $\tau_{0^+}^{\#2\,\dagger}$ | 0 | 0 | $\frac{1}{k^2\,r_2+t_2}$ |
| $\sigma_{0^-}^{\#1\,\dagger}$ | 0 | 0 | 0 |

## Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $-\frac{t_2}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

(No massless particles)

## Unitarity conditions

$r_2 < 0 \;\&\&\; t_2 > 0$