

Particle spectrograph

Wave operator and propagator

| Source constraints   |   |                |
|--|---|----------------|
| SO(3) irreps   | Fundamental fields  | Multiplicities |
| $\tau_{0+}^{\#2} == 0$   | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$  | 1              |
| $\tau_{0+}^{\#1} == 0$   | $\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha_\alpha$   | 1              |
| $\sigma_{0+}^{\#1} == 0$   | $\partial_\beta \sigma^{\alpha\beta}_\alpha == 0$   | 1              |
| $\tau_{1+}^{\#2\alpha} == 0$   | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta}$  | 3              |
| $\tau_{1+}^{\#1\alpha} == 0$   | $\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$  | 3              |
| $\sigma_{1+}^{\#2\alpha} == 0$                                       | $\partial_\chi \partial_\beta \sigma^{\alpha\beta\chi} == 0$  | 3              |
| $\sigma_{1+}^{\#1\alpha} == 0$                                       | $\partial_\chi \partial^\alpha \sigma^{\beta\chi}_\beta + \partial_\chi \partial^\chi \sigma^{\alpha\beta}_\beta = \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$   | 3              |
| $\tau_{1+}^{\#1\alpha\beta} + i k \sigma_{1+}^{\#2\alpha\beta} == 0$ | $\partial_\chi \partial_\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\chi\alpha} + \partial_\chi \partial^\chi \tau^{\alpha\beta} +$<br>$2 \partial_\phi \partial_\chi \partial^\alpha \sigma^{\beta\chi\phi} + 2 \partial_\phi \partial^\phi \partial_\chi \sigma^{\alpha\beta\chi} ==$<br>$\partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} +$<br>$\partial_\chi \partial^\chi \tau^{\beta\alpha} + 2 \partial_\phi \partial_\chi \partial^\beta \sigma^{\alpha\chi\phi}$  | 3              |
| $\sigma_{2+}^{\#1\alpha\beta\chi} == 0$                              | $3 \partial_\epsilon \partial_\phi \partial^\chi \partial^\alpha \sigma^{\beta\delta\epsilon} + 3 \partial_\epsilon \partial^\epsilon \partial^\chi \partial^\alpha \sigma^{\beta\delta}_\delta +$<br>$2 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\beta \sigma^{\alpha\chi\phi} + 4 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\beta \sigma^{\alpha\delta\chi} +$<br>$2 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\beta \sigma^{\chi\phi\alpha} + 4 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\beta \sigma^{\alpha\beta\delta} +$<br>$2 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\chi \sigma^{\alpha\delta\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\delta \sigma^{\beta\chi\alpha} +$<br>$3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\alpha \sigma^{\delta\epsilon}_\delta +$<br>$3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial_\phi \sigma^{\beta\delta\epsilon} +$<br>$3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\epsilon \sigma^{\alpha\delta}_\delta ==$<br>$3 \partial_\epsilon \partial_\phi \partial^\chi \partial^\beta \sigma^{\alpha\delta\epsilon} + 3 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\chi \sigma^{\alpha\delta}_\delta +$<br>$2 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\alpha \sigma^{\beta\chi\phi} + 4 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\alpha \sigma^{\beta\delta\alpha} +$<br>$2 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\alpha \sigma^{\chi\phi\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\alpha \sigma^{\beta\delta\alpha} +$<br>$4 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\delta \sigma^{\alpha\beta\chi} + 2 \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\delta \sigma^{\alpha\chi\beta} +$<br>$3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\beta \sigma^{\delta\epsilon}_\delta +$<br>$3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial_\phi \sigma^{\alpha\delta\epsilon} +$<br>$3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\epsilon \sigma^{\beta\delta}_\delta$ | 5              |
| $\tau_{2+}^{\#1\alpha\beta} == 0$                                    | $4 \partial_\phi \partial_\chi \partial^\phi \partial^\alpha \tau^{\chi\phi} + 2 \partial_\phi \partial^\phi \partial^\beta \partial^\alpha \tau^{\chi\chi}_\chi +$<br>$3 \partial_\phi \partial^\phi \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3 \partial_\phi \partial^\phi \partial_\chi \partial^\chi \tau^{\beta\alpha} +$<br>$2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\phi \partial_\chi \tau^{\chi\phi} ==$<br>$3 \partial_\phi \partial^\phi \partial_\chi \partial^\alpha \tau^{\beta\chi} + 3 \partial_\phi \partial^\phi \partial_\chi \partial^\alpha \tau^{\chi\beta} +$<br>$3 \partial_\phi \partial^\phi \partial_\chi \partial^\beta \tau^{\alpha\chi} + 3 \partial_\phi \partial^\phi \partial_\chi \partial^\beta \tau^{\chi\alpha} +$<br>$2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\phi \partial^\phi \tau^{\chi\chi}_\chi$  | 5              |
| Total constraints/gauge generators:                                  |   | 28             |

Quadratic (free) action

$$S = \iiint \Big( \frac{1}{6} (6 f^{\alpha\beta} \tau_{\alpha\beta} + 6 \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - 6 r_3 \partial_\beta \omega_{\phantom{\beta}\beta}^\theta \partial^\theta \omega^{\alpha\beta}_\alpha - 6 r_3 \partial_\alpha \omega^{\alpha\beta\theta} \partial_\theta \omega_{\phantom{\beta}\beta}^\theta +$$
$$12 r_3 \partial^\theta \omega^{\alpha\beta}_\alpha \partial_\theta \omega_{\phantom{\beta}\beta}^\theta + 4 t_2 \omega_{\theta\alpha} \partial^\theta f^{\alpha\theta} + t_2 \partial_\theta \omega_{\phantom{\beta}\beta}^\theta \partial^\theta f^{\alpha\theta} + 2 t_2 \partial_\alpha f_{\phantom{\alpha}\alpha}^\theta \partial^\theta f^{\alpha\theta} -$$
$$t_2 \partial_\alpha f_{\phantom{\alpha}\alpha}^\theta \partial^\theta f^{\alpha\theta} - t_2 \partial_\theta f_{\phantom{\alpha}\alpha}^\theta \partial^\theta f^{\alpha\theta} + t_2 \partial_\theta f_{\phantom{\alpha}\alpha}^\theta \partial^\theta f^{\alpha\theta} + t_2 \partial_\theta f_{\phantom{\alpha}\alpha}^\theta \partial^\theta f^{\alpha\theta} +$$
$$t_2 \partial_\theta f_{\phantom{\alpha}\alpha}^\theta \partial^\theta f^{\alpha\theta} - 4 t_2 \omega_{\alpha\theta\theta} (\omega^{\alpha\theta\theta} + 2 \partial^\theta f^{\alpha\theta}) + 8 r_2 \partial_\beta \omega_{\alpha\theta\theta} \partial^\theta \omega^{\alpha\beta\theta} - 24 r_3 \partial_\beta \omega_{\theta\alpha} \partial^\theta \omega^{\alpha\beta\theta} -$$
$$2 t_2 \omega_{\alpha\theta\theta} (\omega^{\alpha\theta\theta} + 2 \partial^\theta f^{\alpha\theta}) + 4 r_2 \partial_\beta \omega_{\alpha\theta\theta} \partial^\theta \omega^{\alpha\beta\theta} + 4 r_2 \partial_\beta \omega_{\theta\alpha} \partial^\theta \omega^{\alpha\beta\theta} - 24 r_3 \partial_\beta \omega_{\theta\alpha} \partial^\theta \omega^{\alpha\beta\theta} -$$
$$4 r_2 \partial_\beta \omega_{\alpha\theta\theta} \partial^\theta \omega^{\alpha\beta\theta} + 2 r_2 \partial_\theta \omega_{\alpha\beta\theta} \partial^\theta \omega^{\alpha\beta\theta} + 2 r_2 \partial_\theta \omega_{\alpha\beta\theta} \partial^\theta \omega^{\alpha\beta\theta} -$$
$$\partial^\theta \omega^{\alpha\beta\theta} - 2 r_2 \partial_\theta \omega_{\alpha\beta\theta} \partial^\theta \omega^{\alpha\beta\theta} ) [t, x, y, z] dz dy dx dt$$

|   |   |   |   |   |   |
|---|---|---|---|---|---|
| $\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$ | $\tau_{1+}^{\#1} \dagger^{\alpha\beta}$             | $\sigma_{1+}^{\#1} \sigma_{1+}^{\#2} \dagger^{\alpha\beta}$ | $\tau_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\tau_{1+}^{\#1} \sigma_{1+}^{\#2} \dagger^{\alpha\beta}$ |
| $\frac{2}{3k^2} r_3$                      | $-\frac{2\sqrt{2}}{3k^2 r_3 + 3k^4 r_3}$  | $-\frac{2i\sqrt{2}}{3kr_3 + 3k^3 r_3}$              | 0   | 0                                       | 0   |
| $\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$ | $-\frac{2\sqrt{2}}{3k^2 r_3 + 3k^4 r_3}$  | $\frac{9k^2 r_3 + 4t_2}{3(k + k^3)^2 r_3 t_2}$      | 0   | 0                                       | 0   |
| $\tau_{1+}^{\#1} \dagger^{\alpha\beta}$   | $\frac{2i\sqrt{2}}{3kr_3 + 3k^3 r_3}$     | $-\frac{i(9k^2 r_3 + 4t_2)}{3k(1 + k^2)^2 r_3 t_2}$ | 0   | 0                                       | 0   |
| $\sigma_{1+}^{\#1} \dagger^\alpha$        | 0   | 0   | 0   | 0                                       | 0   |
| $\sigma_{1+}^{\#2} \dagger^\alpha$        | 0   | 0   | 0   | 0                                       | 0   |
| $\tau_{1+}^{\#1} \dagger^\alpha$          | 0   | 0   | 0   | 0                                       | 0   |
| $\tau_{1+}^{\#2} \dagger^\alpha$          | 0   | 0   | 0   | 0                                       | 0   |

|   |   |                                      |   |                                      |                                      |
|---|---|--------------------------------------|---|--------------------------------------|--------------------------------------|
| $\omega_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{1+}^{\#2} \dagger^{\alpha\beta}$ | $f_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{1+}^{\#1} \omega_{1+}^{\#2} \dagger^{\alpha\beta}$ | $f_{1+}^{\#1} \dagger^{\alpha\beta}$ | $f_{1+}^{\#2} \dagger^{\alpha\beta}$ |
| $\frac{1}{6} (9k^2 r_3 + 4t_2)$           | $\frac{\sqrt{2} t_2}{3}$                  | $\frac{1}{3} i \sqrt{2} k t_2$       | 0   | 0                                    | 0                                    |
| $\omega_{1+}^{\#2} \dagger^{\alpha\beta}$ | $\frac{t_2}{3}$                           | $\frac{ikt_2}{3}$                    | 0   | 0                                    | 0                                    |
| $f_{1+}^{\#1} \dagger^{\alpha\beta}$      | $-\frac{1}{3} i \sqrt{2} k t_2$           | $\frac{k^2 t_2}{3}$                  | 0   | 0                                    | 0                                    |
| $\omega_{1+}^{\#1} \dagger^\alpha$        | 0   | 0                                    | 0   | 0                                    | 0                                    |
| $\omega_{1+}^{\#2} \dagger^\alpha$        | 0   | 0                                    | 0   | 0                                    | 0                                    |
| $f_{1+}^{\#1} \dagger^\alpha$             | 0   | 0                                    | 0   | 0                                    | 0                                    |
| $f_{1+}^{\#2} \dagger^\alpha$             | 0   | 0                                    | 0   | 0                                    | 0                                    |

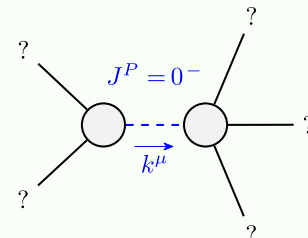
|   |                                      |   |
|---|--------------------------------------|---|
| $\omega_{2+}^{\#1} \dagger^{\alpha\beta}$ | $f_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{2+}^{\#2} \dagger^{\alpha\beta\chi}$ |
| $-\frac{3k^2 r_3}{2}$                     | 0                                    | 0   |
| 0   | 0                                    | 0   |
| 0   | 0                                    | 0   |

|                             |                           |                           |                             |
|-----------------------------|---------------------------|---------------------------|-----------------------------|
| $\sigma_{0+}^{\#1} \dagger$ | $\tau_{0+}^{\#1} \dagger$ | $\tau_{0+}^{\#2} \dagger$ | $\sigma_{0+}^{\#1} \dagger$ |
| 0                           | 0                         | 0                         | 0                           |
| 0                           | 0                         | 0                         | 0                           |
| 0                           | 0                         | 0                         | 0                           |
| $\frac{1}{k^2 r_2 + t_2}$   |                           |                           |                             |

|                          |                     |                          |
|--------------------------|---------------------|--------------------------|
| $\omega_0^{\#1} \dagger$ | $f_0^{\#1} \dagger$ | $\omega_0^{\#1} \dagger$ |
| 0                        | 0                   | 0                        |
| 0                        | 0                   | 0                        |
| 0                        | 0                   | 0                        |
| 0                        | 0                   | 0                        |
| $k^2 r_2 + t_2$          |                     |                          |

|   |   |   |
|---|---|---|
| $\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\tau_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{2+}^{\#1} \dagger^{\alpha\beta\chi}$ |
| $-\frac{2}{3k^2 r_3}$                     | 0                                       | 0   |
| 0   | 0                                       | 0   |
| 0   | 0                                       | 0   |

Massive and massless spectra



|                  |                        |
|------------------|------------------------|
| Massive particle |                        |
| Pole residue:    | $-\frac{1}{r_2} > 0$   |
| Polarisations:   | 1                      |
| Square mass:     | $-\frac{t_2}{r_2} > 0$ |
| Spin:            | 0                      |
| Parity:          | Odd                    |

(see also No massless particles)

Unitarity conditions

$r_2 < 0 \ \&\& \ t_2 > 0$