## Particle spectrograph

## Wave operator and propagator

	$\sigma_{2^{+}lphaeta}^{\!\#1}$	$ au_2^{\#1}_{lphaeta}$	$\sigma_{2^{-}\alpha\beta\chi}^{\#1}$
$\sigma_{2^{+}}^{\sharp 1}\dagger^{lphaeta}$	$\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
$\tau_{2}^{\#1} \dagger^{\alpha\beta}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\sigma_2^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{t_1}$

_	${\mathcal R}_0^{\sharp 1}$	$f_{0}^{#1}$	$f_{0}^{#2}$	$\mathcal{A}_0^{\#1}$
$\mathcal{A}_{0}^{\#1}\dagger$	$t_3$	$-i \sqrt{2} kt_3$	0	0
$f_{0}^{\#1}\dagger$	$i\sqrt{2} kt_3$	$2k^2t_3$	0	0
$f_{0}^{#2} \dagger$	0	0	0	0
$\mathcal{A}_{0}^{\sharp 1}$ †	0	0	0	$-t_1$
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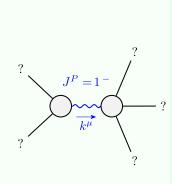
	$\mathcal{A}_{2}^{\#1}{}_{lphaeta}$	$f_{2^{+}\alpha\beta}^{\#1}$	$\mathcal{A}_{2^{-}lphaeta\chi}^{\#1}$
$\mathcal{A}_{2}^{\sharp 1}\dagger^{lphaeta}$	<u>t</u> 1 2	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2}^{#1} \dagger^{\alpha\beta}$	$\frac{i k t_1}{\sqrt{2}}$	$k^2 t_1$	0
$\mathcal{A}_{2}^{\sharp_{1}}\!\dagger^{lphaeta\chi}$	0	0	<u>t</u> 1 2

Source constraints	T			
SO(3) irreps	Fundamental fields	Multiplicities		
$\tau_0^{\#2} == 0$	$\partial_{\beta}\partial_{\alpha}\tau^{\alpha\beta} == 0$	1		
$\tau_{0+}^{\#1} - 2  \bar{\imath}  k  \sigma_{0+}^{\#1} == 0$	$\partial_{\beta}\partial_{\alpha}\tau^{\alpha\beta} = \partial_{\beta}\partial^{\beta}\tau^{\alpha}_{\alpha} + 2\partial_{\chi}\partial^{\chi}\partial_{\beta}\sigma^{\alpha\beta}_{\alpha}$	1		
$\tau_{1^{-}}^{\#2\alpha} + 2  \bar{\imath}  k  \sigma_{1^{-}}^{\#2\alpha} == 0$	$\partial_{\chi}\partial_{\beta}\partial^{\alpha}\tau^{\beta\chi} = \partial_{\chi}\partial^{\chi}\partial_{\beta}\tau^{\alpha\beta} + 2\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial_{\beta}\sigma^{\alpha\beta\chi}$	3		
$\tau_{1}^{\#1}{}^{\alpha} == 0$	$\partial_{\chi}\partial_{\beta}\partial^{\alpha}\tau^{\beta\chi} == \partial_{\chi}\partial^{\chi}\partial_{\beta}\tau^{\beta\alpha}$	3		
$\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$	$\partial_{\chi}\partial^{\alpha}\tau^{\beta\chi} + \partial_{\chi}\partial^{\beta}\tau^{\chi\alpha} + \partial_{\chi}\partial^{\chi}\tau^{\alpha\beta} +$	3		
	$2 \partial_{\delta} \partial_{\chi} \partial^{\alpha} \sigma^{\beta \chi \delta} + 2 \partial_{\delta} \partial^{\delta} \partial_{\chi} \sigma^{\alpha \beta \chi} = =$			
	$\partial_{\chi}\partial^{\alpha}\tau^{\chi\beta} + \partial_{\chi}\partial^{\beta}\tau^{\alpha\chi} +$			
	$\partial_{\chi}\partial^{\chi}\tau^{\beta\alpha} + 2\partial_{\delta}\partial_{\chi}\partial^{\beta}\sigma^{\alpha\chi\delta}$			
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	$-i \left(4 \partial_{\delta} \partial_{\chi} \partial^{\beta} \partial^{\alpha} \tau^{\chi \delta} + 2 \partial_{\delta} \partial^{\delta} \partial^{\beta} \partial^{\alpha} \tau^{\chi}_{\chi} - \right)$	5		
	$3 \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\alpha} \tau^{\beta \chi} - 3 \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\alpha} \tau^{\chi \beta} -$			
	$3 \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\beta} \tau^{\alpha \chi} - 3 \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\beta} \tau^{\chi \alpha} +$			
	$3 \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\chi} \tau^{\alpha\beta} + 3 \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\chi} \tau^{\beta\alpha} +$			
	$4 i k^{\chi} \partial_{\epsilon} \partial_{\chi} \partial^{\beta} \partial^{\alpha} \sigma^{\delta \epsilon}_{\delta} -$			
	$6 i k^{\chi} \partial_{\epsilon} \partial_{\delta} \partial_{\chi} \partial^{\alpha} \sigma^{\beta \delta \epsilon} -$			
	$6 i k^{\chi} \partial_{\epsilon} \partial_{\delta} \partial_{\chi} \partial^{\beta} \sigma^{\alpha \delta \epsilon} +$			
	$2 \eta^{\alpha\beta} \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial_{\chi} \tau^{\chi\delta} +$			
	$6 i k^{\chi} \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial_{\chi} \sigma^{\alpha \delta \beta} +$			
	$6 i k^{\chi} \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial_{\chi} \sigma^{\beta \delta \alpha} -$			
	$2 \eta^{\alpha\beta} \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial^{\delta} \tau^{X}_{X} -$			
	$4 i \eta^{\alpha\beta} k^{X} \partial_{\phi} \partial^{\phi} \partial_{\epsilon} \partial_{X} \sigma^{\delta\epsilon} \delta = 0$			
Total constraints/gauge generators: 16				

	$\mathcal{A}_{1}^{\#1}{}_{lphaeta}$	$\mathcal{A}_{1}^{\#2}{}_{lphaeta}$	$f_{1}^{\#1}$	$_eta$ F	${rac{1}{1}}^{rac{1}{lpha}}$	${\mathcal R}_{1^- \; lpha}^{ ext{#2}}$	$f_{1-\alpha}^{\#1}$	$f_{1-\alpha}^{\#2}$	_				
$\mathcal{A}_{1}^{\sharp 1}\dagger^{lphaeta}$	$k^2 r_5 - \frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$		0	0	0	0	$\sigma_{0}^{\#1}$	0	0	0	$-\frac{1}{t_1}$
$\mathcal{A}_{1}^{\#2}\dagger^{lphaeta}$	$-\frac{t_1}{\sqrt{2}}$	0	0		0	0	0	0	$\tau_{0}^{\#2}$	0	0	0	0
$f_{1}^{\#1}\dagger^{\alpha\beta}$	$\frac{i k t_1}{\sqrt{2}}$	0	0		0	0	0	0	$\tau_{0}^{\#1}$	$\frac{t_0+}{i\sqrt{2}k}$	$\frac{2k^2}{(1+2k^2)^2 t_3}$	0	0
${\cal R}_1^{\sharp 1}\dagger^lpha$	0	0	0	$\frac{1}{6}$ (6 $k^2 r_5$	$+t_1+4t_3$ )	$\frac{t_1-2t_3}{3\sqrt{2}}$	0	$\frac{1}{3} \bar{i} k (t_1 - 2 t_3)$					
$\mathcal{A}_{1}^{\#2}\dagger^{lpha}$	0	0	0	<u>t</u> 1	$-2t_3$	<u>t1+t3</u> 3	0	$\frac{1}{3}\bar{l}\sqrt{2}k(t_1+t_3)$	$\sigma_{0}^{\#1}$	$\frac{1}{(1+2k^2)^2t_3}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$	0	0
$f_{1}^{#1} \dagger^{\alpha}$	0	0	0		0	0	0	0	ρ				
$f_{1}^{#2} \dagger^{\alpha}$	0	0	0	$-\frac{1}{3}ik$	$(t_1 - 2t_3)$	$-\frac{1}{3}i\sqrt{2}k(t_1+t_2)$	t <sub>3</sub> ) 0	$\frac{2}{3}k^2(t_1+t_3)$		$\sigma_{0}^{\#1}$ †	$ au_0^{\#1}\dagger$	$\tau_{0}^{\#2} \uparrow$	$\sigma_{0}^{\#1}$ $\dagger$
	$\sigma_{1}^{\#1}{}_{lphaeta}$	$\sigma_{1}^{\#2}$	ιβ	$ au_{1}^{\#1}{}_{lphaeta}$	(	$\sigma_{1^{-}lpha}^{\sharp1}$		$\sigma_{1-lpha}^{\#2}$	$ au_1^{\#1}{}_{lpha}$		τ	#2 1 α	
$\sigma_{1}^{\sharp 1} \dagger^{lphaeta}$	0	$-\frac{\sqrt{2}}{t_1+k^2}$	$\frac{}{t_1}$	$-\frac{i\sqrt{2}k}{t_1+k^2t_1}$		0	0		0	0			
$\sigma_{1}^{\#2} \dagger^{\alpha\beta}$	$-\frac{\sqrt{2}}{t_1+k^2t_1}$	$\frac{-2k^2r_5-(1+k^2)^2}{(1+k^2)^2}$	$\frac{+t_1}{t_1^2}$	$\frac{i(2k^3r_5-kt_1)}{(1+k^2)^2t_1^2}$		0		0	0			0	
$\tau_{1}^{\#1} + \alpha\beta$	$\frac{i \sqrt{2} k}{t_1 + k^2 t_1}$	$\frac{i(2k^3r_{5-1})^2}{(1+k^2)^2}$		$\frac{-2 k^4 r_5 + k^2 t_1}{(1+k^2)^2 t_1^2}$		0	0		0	0 0			
$\sigma_{1}^{\sharp_{1}}\dagger^{lpha}$	0	0		0	$\frac{2(t_1+t_3)}{3t_1t_3+2k^2r_5(t_1+t_3)}$		$-\frac{\sqrt{2} (t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2r_5(t_1+t_3))}$		0	$-\frac{2 i k (t_1-2 t_3)}{(1+2 k^2) (3 t_1 t_3+2 k^2 r_5 (t_1+t_3))}$		<sub>3</sub> ) <sup>2</sup> <sub>r5</sub> (t <sub>1</sub> +t <sub>3</sub> ))	
$\sigma_1^{\#2} \dagger^{\alpha}$	0	0		0	$-\frac{\sqrt{2} (t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2r_5(t_1+t_3))}$		$\frac{6 k^2 r_5 + t_1 + 4 t_3}{(1 + 2 k^2)^2 (3 t_1 t_3 + 2 k^2 r_5 (t_1 + t_3))}$		0	$\frac{i \sqrt{2} k (6 k^2 r_5 + t_1 + 4 t_3)}{(1 + 2 k^2)^2 (3 t_1 t_3 + 2 k^2 r_5 (t_1 + t_3))}$		$\frac{1+4t_3}{2}r_5(t_1+t_3)$	
$\tau_1^{\#1} + \alpha$	0	0		0		0	0		0	0			
$\tau_1^{\#2} \uparrow^{\alpha}$	0	0		0		$t(t_1-2t_3)$ $t_3+2k^2r_5(t_1+t_3)$		$\frac{1}{2} k (6k^2 r_5 + t_1 + 4t_3)$ $\frac{1}{2} (3t_1 t_3 + 2k^2 r_5 (t_1 + t_3))$	- O	$\frac{2}{(1+2k^2)^2}$	$k^2 (6 k^2)^2 (3 t_1 t_2)^2$	$r_5 + t_1$ $3 + 2k$	+4 <i>t</i> <sub>3</sub> ) <sup>2</sup> r <sub>5</sub> (t <sub>1</sub> +t <sub>3</sub> ))

Quadratic (free) action
S== S
$\iiint (\frac{1}{6}\left(2\left(t_{1}-2t_{3}\right)\mathcal{A}^{\alpha\prime}_{\alpha}\mathcal{A}^{\theta}_{i}+6f^{\alpha\beta}\tau_{\alpha\beta}+6\mathcal{A}^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}-4t_{1}\mathcal{A}^{\theta}_{\alpha}\partial_{i}f^{\alpha\prime}+$
$8t_3\mathcal{R}_{\alpha}^{\ \ \theta}\partial_{\scriptscriptstyle{l}}\!f^{\alpha\prime} + 4t_1\mathcal{R}_{\scriptscriptstyle{l}}^{\ \ \theta}\partial^{\prime}\!f^{\alpha}_{\ \alpha} - 8t_3\mathcal{R}_{\scriptscriptstyle{l}}^{\ \ \theta}\partial^{\prime}\!f^{\alpha}_{\ \alpha} -$
$2t_1\partial_{\scriptscriptstyle j} f^{\theta}_{}\partial^{\prime} f^{\alpha}_{} + 4t_3\partial_{\scriptscriptstyle j} f^{\theta}_{}\partial^{\prime} f^{\alpha}_{} - 2t_1\partial_{\scriptscriptstyle j} f^{\alpha\prime}\partial_{\theta} f_{}^{} +$
$4t_3\partial_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{$
$6t_1\partial_\alpha f_{,\theta}\partial^\theta f^{\alpha\prime} - 3t_1\partial_\alpha f_{\theta\prime}\partial^\theta f^{\alpha\prime} + 3t_1\partial_\prime f_{\alpha\theta}\partial^\theta f^{\alpha\prime} +$
$3t_1\partial_{\theta}f_{\alpha\prime}\partial^{\theta}f^{\alpha\prime}+3t_1\partial_{\theta}f_{\prime\alpha}\partial^{\theta}f^{\alpha\prime}+$
$6t_1\mathcal{A}_{lpha heta_l}(\mathcal{A}^{lpha l  heta} + 2\partial^ heta f^{lpha l}) + 6r_5\partial_l\mathcal{A}_ heta^{\kappa}\partial^ heta\mathcal{A}_lpha^{\kappa}$ -
$6r_5\partial_ heta \mathcal{R}_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_$
$12r_5\partial^ heta \mathcal{A}^{lpha\prime}_{}\partial_{\kappa} \mathcal{A}^{\prime}_{}+6r_5\partial_lpha \mathcal{A}^{lpha\prime}_{}\partial_\kappa \mathcal{A}^{}_{'}-$
$12r_5\partial^{ heta}\mathcal{A}^{lpha'}_{}\partial_{\kappa}\mathcal{A}^{\prime}_{}))[t,  ext{$lpha$},  ext{$eta$},  ext{$eta$},  ext{$eta$},  ext{$eta$}$

## Massive and massless spectra



Massive particle					
Pole residue:	$\frac{6t_1t_3(t_1+t_3)-3r_5(t_1^2+2t_3^2)}{2r_5(t_1+t_3)(-3t_1t_3+r_5(t_1+t_3))} > 0$				
Polarisations:	3				
Square mass:	$-\frac{3t_1t_3}{2r_5t_1+2r_5t_3} > 0$				
Spin:	1				
Parity:	Odd				

(No massless particles)	

## Unitarity conditions

 $r_5 < 0 \&\& (t_1 < 0 \&\& 0 < t_3 < -t_1) || (t_1 > 0 \&\& (t_3 < -t_1 || t_3 > 0))$