

# Particle spectrograph

## Wave operator and propagator

Quadratic (free) action

$S_F ==$

$$\begin{aligned} & \iiint (\frac{1}{6} (4 t_2 \omega_{\kappa\lambda}^{\kappa\lambda} \omega_{\kappa\lambda}^{\prime} + 2 t_2 \omega_{\kappa\lambda}^{\kappa\lambda} \omega_{\kappa\lambda}^{\prime} + 6 f^{\alpha\beta} \tau_{\alpha\beta} + 6 \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - 3 r_3 \partial_{\lambda} \omega_{\kappa}^{\kappa\lambda} \\ & \partial' \omega_{\lambda}^{\alpha} - 6 r_5 \partial_{\lambda} \omega_{\kappa}^{\kappa\lambda} \partial' \omega_{\lambda}^{\alpha} + 4 r_2 \partial^{\beta} \omega_{\kappa}^{\theta\alpha} \partial_{\theta} \omega_{\alpha\beta}^{\kappa} - 2 r_2 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\alpha\beta\theta} - \\ & 4 r_2 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\theta\alpha\beta} + 3 r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\theta\kappa\lambda} - 6 r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega^{\theta\kappa\lambda} - \\ & 3 r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\theta\kappa\lambda} + 6 r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\theta\kappa\lambda} - 3 r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega^{\kappa\lambda\theta} - 6 r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \\ & \partial_{\kappa} \omega^{\kappa\lambda\theta} + 6 r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\kappa\lambda\theta} + 12 r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\kappa\lambda\theta} + t_2 \partial^{\alpha} f_{\theta\kappa} \partial^{\kappa} f_{\alpha}^{\theta} - \\ & t_2 \partial^{\alpha} f_{\kappa\theta} \partial^{\kappa} f_{\alpha}^{\theta} + t_2 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\alpha\lambda} + 2 t_2 \omega_{\theta\kappa} \partial^{\kappa} f^{\theta} - 4 t_2 \omega_{\kappa\theta} \partial^{\kappa} f^{\theta} - \\ & 2 t_2 \omega_{\theta\kappa} \partial^{\kappa} f^{\theta} + 4 t_2 \omega_{\theta\kappa} \partial^{\kappa} f^{\theta} - t_2 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\theta\lambda} - t_2 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} + \\ & t_2 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} + 2 r_2 \partial_{\kappa} \omega^{\alpha\beta\theta} \partial^{\kappa} \omega_{\alpha\beta\theta} + 4 r_2 \partial_{\kappa} \omega^{\theta\alpha\beta} \partial^{\kappa} \omega_{\alpha\beta\theta} - 4 r_2 \partial^{\beta} \omega_{\lambda}^{\alpha\lambda} \partial_{\lambda} \omega_{\alpha\beta}^{\alpha} + \\ & 4 r_2 \partial^{\beta} \omega_{\lambda}^{\lambda\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\prime} - 24 r_3 \partial^{\beta} \omega_{\lambda}^{\lambda\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\prime} - 3 r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega^{\alpha} \partial^{\lambda} \omega_{\kappa}^{\theta\kappa} + 6 r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \\ & \partial^{\lambda} \omega_{\kappa}^{\theta\kappa} + 3 r_3 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\kappa}^{\theta\kappa} - 6 r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\kappa}^{\theta\kappa} )) [t, x, y, z] dz dy dx dt \end{aligned}$$

$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$	$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1-}^{\#1} \dagger^{\alpha}$	$\sigma_{1-}^{\#2} \dagger^{\alpha}$	$\tau_{1-}^{\#1} \dagger^{\alpha}$	$\tau_{1-}^{\#2} \dagger^{\alpha}$
$\frac{1}{k^2 (2r_3+r_5)}$	$-\frac{\sqrt{2}}{k^2 (1+k^2) (2r_3+r_5)}$	$-\frac{i\sqrt{2}}{k (1+k^2) (2r_3+r_5)}$	0	0	0	0
$-\frac{\sqrt{2}}{k^2 (1+k^2) (2r_3+r_5)}$	$\frac{3k^2 (2r_3+r_5)+2t_2}{(k+\lambda^3)^2 (2r_3+r_5) t_2}$	$\frac{i (3k^2 (2r_3+r_5)+2t_2)}{k (1+k^2)^2 (2r_3+r_5) t_2}$	0	0	0	0
$\frac{i\sqrt{2}}{k (1+k^2) (2r_3+r_5)}$	$-\frac{i (3k^2 (2r_3+r_5)+2t_2)}{k (1+k^2)^2 (2r_3+r_5) t_2}$	$\frac{3k^2 (2r_3+r_5)+2t_2}{(1+k^2)^2 (2r_3+r_5) t_2}$	0	0	0	0
0	0	0	$\frac{2}{k^2 (r_3+2r_5)}$	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1-}^{\#1} \dagger^{\alpha}$	$\omega_{1-}^{\#2} \dagger^{\alpha}$	$f_{1-}^{\#1} \dagger^{\alpha}$	$f_{1-}^{\#2} \dagger^{\alpha}$
$k^2 (2r_3+r_5) + \frac{2t_2}{3}$	$\frac{\sqrt{2} t_2}{3}$	$\frac{1}{3} i \sqrt{2} k t_2$	0	0	0	0
$\frac{\sqrt{2} t_2}{3}$	$\frac{t_2}{3}$	$\frac{i k t_2}{3}$	0	0	0	0
$-\frac{1}{3} i \sqrt{2} k t_2$	$-\frac{1}{3} i k t_2$	$\frac{k^2 t_2}{3}$	0	0	0	0
0	0	0	$\frac{1}{2} k^2 (r_3+2r_5)$	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} == 0$	1
$\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + i k \sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\sigma_{2-}^{\#1\alpha\beta\chi} == 0$	5
$\tau_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	25

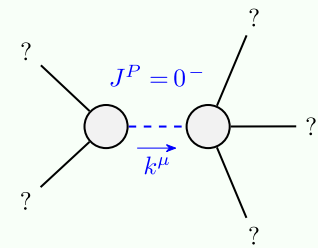
$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$-\frac{3k^2 r_3}{2}$	0	0
0	0	0
0	0	0

$\omega_{0+}^{\#1} \dagger^{\alpha}$	$f_{0+}^{\#1} \dagger^{\alpha}$	$f_{0+}^{\#2} \dagger^{\alpha}$	$\omega_0^{\#1} \dagger^{\alpha}$
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	$k^2 r_2 + t_2$

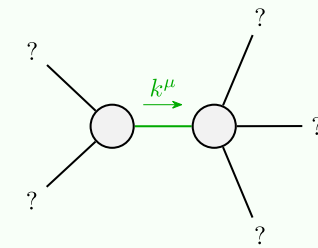
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$-\frac{2}{3k^2 r_3}$	0	0
0	0	0
0	0	0

$\sigma_{0+}^{\#1} \dagger^{\alpha}$	$\tau_{0+}^{\#1} \dagger^{\alpha}$	$\tau_{0+}^{\#2} \dagger^{\alpha}$	$\sigma_{0-}^{\#1} \dagger^{\alpha}$
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	$\frac{1}{k^2 r_2 + t_2}$

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd



Quadratic pole	
Pole residue:	$-\frac{1}{r_3 (2r_3+r_5) (r_3+2r_5) p^2} > 0$
Polarisations:	2

## Unitarity conditions

$$r_2 < 0 \&\& r_3 < 0 \&\& r_5 < -\frac{r_3}{2} \&\& t_2 > 0 \parallel r_2 < 0 \&\& r_3 < 0 \&\& r_5 > -2r_3 \&\& t_2 > 0 \parallel r_2 < 0 \&\& r_3 > 0 \&\& -2r_3 < r_5 < -\frac{r_3}{2} \&\& t_2 > 0$$