Particle spectrograph

Wave operator and propagator

Quadratic (free) action

$$S_F == \iiint \Big(\tfrac{1}{6}\big(-2t_1\,\omega_I{}^{\alpha I}\,\omega_I\,\omega_{\kappa\alpha}{}^\kappa - 2(t_1-2t_2)\,\omega_I{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}^I + 2t_1\,\omega_{\kappa\lambda}{}^I\,\omega^{\kappa\lambda}{}_I +$$
$$2t_2\,\omega_{\kappa\lambda}{}^I\,\omega^{\kappa\lambda}{}_I + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + 4r_2\,\partial^\beta\omega^{\theta\alpha}{}_\kappa\,\partial_\theta\omega_{\alpha\beta}{}^\kappa -$$
$$2r_2\,\partial_\theta\omega_{\alpha\beta}{}^\kappa\,\partial_\kappa\omega^{\alpha\beta\theta} - 4r_2\,\partial_\kappa\omega_{\alpha}{}^\kappa\,\partial_\theta f_{\theta\kappa}{}^\theta + t_2\,\partial^\alpha f_{\theta\kappa}\,\partial^\kappa f_\alpha{}^\theta -$$
$$4t_1\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f_\alpha{}^\theta - t_2\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f_\alpha{}^\theta - 2t_1\,\partial^\alpha f^\lambda{}_\kappa\,\partial^\kappa f_{\alpha\lambda} + t_2\,\partial^\alpha f^\lambda{}_\kappa\,\partial^\kappa f_{\alpha\lambda} +$$
$$2t_1\,\omega_{\kappa\alpha}{}^\alpha\,\partial^\kappa f^I{}_I + 2t_1\,\omega_{\kappa\lambda}{}^\lambda\,\partial^\kappa f^I{}_I + 4t_1\,\partial^\alpha f_{\kappa\alpha}\,\partial^\kappa f^I{}_I - 2t_1\,\partial_\kappa f^\lambda{}_\lambda\,\partial^\kappa f^I{}_I +$$
$$2t_1\,\omega_{I\theta\kappa}\,\partial^\kappa f^{I\theta} + 2t_2\,\omega_{I\theta\kappa}\,\partial^\kappa f^{I\theta} + 8t_1\,\omega_{I\kappa\theta}\,\partial^\kappa f^{I\theta} - 4t_2\,\omega_{I\kappa\theta}\,\partial^\kappa f^{I\theta} -$$
$$2t_1\,\omega_{\theta I\kappa}\,\partial^\kappa f^{I\theta} - 2t_2\,\omega_{\theta I\kappa}\,\partial^\kappa f^{I\theta} + 4t_1\,\omega_{\theta\kappa I}\,\partial^\kappa f^{I\theta} + 4t_2\,\omega_{\theta\kappa I}\,\partial^\kappa f^{I\theta} -$$
$$2t_1\,\omega_{I\alpha}{}^\alpha\,\partial^\kappa f^I{}_\kappa - 2t_1\,\omega_{I\alpha}{}^\alpha\,\partial^\kappa f^\lambda{}_\lambda - t_2\,\partial_\theta f^\lambda{}_\lambda\,\partial^\kappa f_{\lambda\alpha} - t_2\,\partial^\alpha f^\lambda{}_\theta\,\partial^\kappa f_{\lambda\alpha} +$$
$$2t_1\,\partial^\alpha f^\lambda{}_\theta\,\partial^\kappa f_{\lambda\kappa} - t_2\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f_{\lambda\kappa} + 2r_2\,\partial_\kappa\omega^{\alpha\beta\theta}\,\omega_{\alpha\beta\theta} +$$
$$2t_1\,\partial^\alpha f^\lambda{}_\alpha\,\partial^\kappa f_{\lambda\kappa} + 2r_2\,\partial_\kappa\omega^{\alpha\beta\theta}\,\partial^\kappa\omega_{\alpha\beta\theta} -$$
$$4r_2\,\partial^\beta\omega_I{}^{\alpha\lambda}\,\partial_\lambda\omega_{\alpha\beta}{}^I + 4r_2\,\partial^\beta\omega_I{}^{\lambda\alpha}\,\partial_\lambda\omega_{\alpha\beta}{}^I\big)\big)[t,x,y,z]\,dz\,dy\,dx\,dt$$

First sector (spin-1 mixed):

	$\sigma^{\#1}_{1+}{}^{\dagger\alpha\beta}$	$\sigma^{\#2}_{1+}{}^{\dagger\alpha\beta}$	$\tau^{\#1}_{1+}{}^{\dagger\alpha\beta}$	$\sigma^{\#1}_{1-}{}^{\dagger\alpha}$	$\sigma^{\#2}_{1-}{}^{\dagger\alpha}$	$\tau^{\#1}_{1-}{}^{\dagger\alpha}$	$\tau^{\#2}_{1-}{}^{\dagger\alpha}$
$\tau^{\#2}_{1}{}^{\alpha}$	0	0	0	$\frac{12ik}{(3+4k^2)^2 t_1}$	$\frac{12i\sqrt2 k}{(3+4k^2)^2 t_1}$	0	$\frac{24k^2}{(3+4k^2)^2 t_1}$
$\tau^{\#1}_{1}{}^{\alpha}$	0	0	0	0	0	0	0
$\sigma^{\#2}_{1}{}^{\alpha}$	0	0	0	$\frac{6\sqrt2}{(3+4k^2)^2 t_1}$	$\frac{12}{(3+4k^2)^2 t_1}$	0	$-\frac{12i\sqrt2 k}{(3+4k^2)^2 t_1}$
$\sigma^{\#1}_{1}{}^{\alpha}$	0	0	0	$\frac{6}{(3+4k^2)^2 t_1}$	$\frac{6\sqrt2}{(3+4k^2)^2 t_1}$	0	$-\frac{12ik}{(3+4k^2)^2 t_1}$
$\tau^{\#1}_{1+}{}^{\alpha\beta}$	$\frac{i\sqrt2 k(t_1-2t_2)}{3(1+k^2)t_1 t_2}$	$\frac{ik(t_1+4t_2)}{3(1+k^2)^2 t_1 t_2}$	$\frac{k^2(t_1+4t_2)}{3(1+k^2)^2 t_1 t_2}$	0	0	0	0
$\sigma^{\#2}_{1+}{}^{\alpha\beta}$	$\frac{\sqrt2(t_1-2t_2)}{3(1+k^2)t_1 t_2}$	$\frac{t_1+4t_2}{3(1+k^2)^2 t_1 t_2}$	$-\frac{ik(t_1+4t_2)}{3(1+k^2)^2 t_1 t_2}$	0	0	0	0
$\sigma^{\#1}_{1+}{}^{\alpha\beta}$	$\frac{2(t_1+t_2)}{3t_1 t_2}$	$\frac{\sqrt2(t_1-2t_2)}{3(1+k^2)t_1 t_2}$	$-\frac{i\sqrt2 k(t_1-2t_2)}{3(1+k^2)t_1 t_2}$	0	0	0	0

Second sector:

	$\omega^{\#1}_{1+}{}^{\dagger\alpha\beta}$	$\omega^{\#2}_{1+}{}^{\dagger\alpha\beta}$	$f^{\#1}_{1+}{}^{\dagger\alpha\beta}$	$\omega^{\#1}_{1-}{}^{\dagger\alpha}$	$\omega^{\#2}_{1-}{}^{\dagger\alpha}$	$f^{\#1}_{1-}{}^{\dagger\alpha}$	$f^{\#2}_{1-}{}^{\dagger\alpha}$
$f^{\#1}_{1}{}^{\alpha}$	0	0	0	$\frac{ikt_1}{3}$	$\frac{1}{3}i\sqrt2 kt_1$	0	$\frac{2k^2 t_1}{3}$
$f^{\#1}_{1}{}^{\alpha}$	0	0	0	0	$\frac{1}{3}ikt_1$	0	0
$\omega^{\#2}_{1}{}^{\alpha}$	0	0	0	$\frac{t_1}{3\sqrt2}$	$\frac{t_1}{3}$	0	$-\frac{1}{3}i\sqrt2 kt_1$
$\omega^{\#1}_{1}{}^{\alpha}$	0	0	0	$\frac{t_1}{6}$	$\frac{t_1}{3\sqrt2}$	0	$-\frac{1}{3}ikt_1$
$f^{\#1}_{1+}{}^{\alpha\beta}$	$-\frac{ik(t_1-2t_2)}{3\sqrt2}$	$\frac{1}{3}ik(t_1+t_2)$	$\frac{1}{3}k^2(t_1+t_2)$	0	0	0	0
$\omega^{\#2}_{1+}{}^{\alpha\beta}$	$-\frac{t_1-2t_2}{3\sqrt2}$	$\frac{t_1+t_2}{3}$	$-\frac{1}{3}ik(t_1+t_2)$	0	0	0	0
$\omega^{\#1}_{1+}{}^{\alpha\beta}$	$\frac{1}{6}(t_1+4t_2)$	$-\frac{t_1-2t_2}{3\sqrt2}$	$\frac{ik(t_1-2t_2)}{3\sqrt2}$	0	0	0	0

Spin-2 positive sector:

	$\omega^{\#2}_{2+}{}^{\dagger\alpha\beta}$	$f^{\#1}_{2+}{}^{\dagger\alpha\beta}$	$\omega^{\#1}_{2+}{}^{\dagger\alpha\beta\chi}$
$\omega^{\#2}_{2}{}^{\alpha\beta}$	0	0	$\frac{t_1}{2}$
$f^{\#2}_{2}{}^{\alpha\beta}$	$-\frac{ikt_1}{\sqrt2}$	$k^2 t_1$	0
$\omega^{\#1}_{2}{}^{\alpha\beta\chi}$	$\frac{t_1}{2}$	$\frac{ikt_1}{\sqrt2}$	0

Spin-0 positive sector:

	$\omega^{\#1}_{0+}$	$f^{\#1}_{0+}$	$f^{\#2}_{0+}$	$\omega^{\#1}_{0-}$
$\omega^{\#1}_{0+}{}^{\dagger}$	0	0	0	0
$f^{\#1}_{0+}{}^{\dagger}$	0	0	0	0
$f^{\#2}_{0+}{}^{\dagger}$	0	0	0	0
$\omega^{\#1}_{0-}{}^{\dagger}$	0	0	0	$k^2 r_2 + t_2$

Spin-0 sector (inverse):

	$\sigma^{\#1}_{0+}$	$\tau^{\#1}_{0+}$	$\tau^{\#2}_{0+}$	$\sigma^{\#1}_{0-}$
$\sigma^{\#1}_{0+}{}^{\dagger}$	0	0	0	0
$\tau^{\#1}_{0+}{}^{\dagger}$	0	0	0	0
$\tau^{\#2}_{0+}{}^{\dagger}$	0	0	0	0
$\sigma^{\#1}_{0-}{}^{\dagger}$	0	0	0	$\frac{1}{k^2 r_2 + t_2}$

Spin-2 negative sector:

	$\sigma^{\#1}_{2+}{}^{\alpha\beta}$	$\tau^{\#1}_{2+}{}^{\alpha\beta}$	$\sigma^{\#1}_{2-}{}^{\alpha\beta\chi}$
$\sigma^{\#2}_{2+}{}^{\dagger\alpha\beta}$	$\frac{2}{(1+2k^2)^2 t_1}$	$-\frac{2i\sqrt2 k}{(1+2k^2)^2 t_1}$	0
$\tau^{\#1}_{2+}{}^{\dagger\alpha\beta}$	$\frac{2i\sqrt2 k}{(1+2k^2)^2 t_1}$	$\frac{4k^2}{(1+2k^2)^2 t_1}$	0
$\sigma^{\#1}_{2-}{}^{\dagger\alpha\beta\chi}$	0	0	$\frac{2}{t_1}$

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau^{\#2}_{0+} == 0$	1
$f^{\#1}_{0+} == 0$	1
$\sigma^{\#1}_{0+} == 0$	1
$\tau^{\#2}_{1-}{}^{\alpha} + 2ik\,\sigma^{\#1}_{1-}{}^{\alpha} == 0$	3
$\tau^{\#1}_{1-}{}^{\alpha} == 0$	3
$\sigma^{\#1}_{1}{}^{\alpha} == \sigma^{\#2}_{1}{}^{\alpha}$	3
$\tau^{\#1}_{1+}{}^{\alpha\beta} + ik\,\sigma^{\#2}_{1+}{}^{\alpha\beta} == 0$	3
$f^{\#1}_{2+}{}^{\alpha\beta} - 2ik\,\sigma^{\#1}_{2+}{}^{\alpha\beta} == 0$	5
Total constraints:	20

Massive and massless spectra
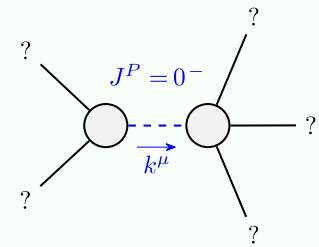
Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

$J^P = 0^-$

$\vec{k^\mu}$

(No massless particles)

Unitarity conditions

$r_2 < 0\ \&\&\ t_2 > 0$