Particle spectrograph

Wave operator and propagator

	$\omega_{1^+}^{\#1}{}_{\alpha\beta}$	$\omega_{1^+}^{\#2}{}_{\alpha\beta}$	$f_{1^+}^{\#1}{}_{\alpha\beta}$	$\omega_{1^-}^{\#1}{}_{\alpha}$	$\omega_{1^-}^{\#2}{}_{\alpha}$	$f_{1^-}^{\#1}{}_{\alpha}$	$f_{1^-}^{\#2}{}_{\alpha}$
$\omega_{1^+}^{\#1}{}^{\dagger\alpha\beta}$	$k^2(2r_3+r_5)-\frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1^+}^{\#2}{}^{\dagger\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_{1^-}^{\#1}{}^{\dagger\alpha}$	0	0	0	$k^2(-r_1+2r_3+r_5)+\frac{t_1}{6}$	$\frac{t_1}{3\sqrt{2}}$	0	$\frac{ikt_1}{3}$
$\omega_{1^-}^{\#2}{}^{\dagger\alpha}$	0	0	0	$\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	0	$\frac{1}{3}i\sqrt{2}\,kt_1$
$f_{1^-}^{\#1}{}^{\dagger\alpha}$	0	0	0	0	0	0	0
$f_{1^-}^{\#2}{}^{\dagger\alpha}$	0	0	0	$-\frac{1}{3}ikt_1$	$-\frac{1}{3}i\sqrt{2}\,kt_1$	0	$\frac{2k^2t_1}{3}$

	$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^-}^{\#1}$
$\sigma_{0^+}^{\#1}{}^{\dagger}$	$\frac{1}{6k^2(-r_1+r_3)}$	0	0	0
$\tau_{0^+}^{\#1}{}^{\dagger}$	0	0	0	0
$\tau_{0^+}^{\#2}{}^{\dagger}$	0	0	0	0
$\sigma_{0^-}^{\#1}{}^{\dagger}$	0	0	0	$-\frac{1}{t_1}$

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\omega_{0^-}^{\#1}$
$\omega_{0^+}^{\#1}{}^{\dagger}$	$6k^2(-r_1+r_3)$	0	0	0
$f_{0^+}^{\#1}{}^{\dagger}$	0	0	0	0
$f_{0^+}^{\#2}{}^{\dagger}$	0	0	0	0
$\omega_{0^-}^{\#1}{}^{\dagger}$	0	0	0	$-t_1$

	$\omega_{2^+}^{\#1}{}_{\alpha\beta}$	$f_{2^+}^{\#1}{}_{\alpha\beta}$	$\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\omega_{2^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	k^2t_1	0
$\omega_{2^-}^{\#1}{}^{\dagger\alpha\beta\chi}$	0	0	$k^2r_1+\frac{t_1}{2}$

	$\sigma_{2^+}^{\#1}{}_{\alpha\beta}$	$\tau_{2^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\sigma_{2^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}\,k}{(1+2k^2)^2t_1}$	0
$\tau_{2^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{2i\sqrt{2}\,k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\sigma_{2^-}^{\#1}{}^{\dagger\alpha\beta\chi}$	0	0	$\frac{2}{2k^2r_1+t_1}$

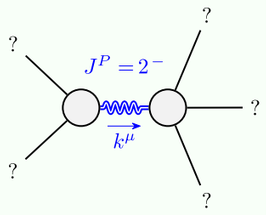
	$\sigma_{1^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{1^+}^{\#2}{}_{\alpha\beta}$	$\tau_{1^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{1^-}^{\#1}{}_{\alpha}$	$\sigma_{1^-}^{\#2}{}_{\alpha}$	$\tau_{1^-}^{\#1}{}_{\alpha}$	$\tau_{1^-}^{\#2}{}_{\alpha}$
$\sigma_{1^+}^{\#1}{}^{\dagger\alpha\beta}$	0	$-\frac{\sqrt{2}}{t_1+k^2t_1}$	$-\frac{i\sqrt{2}\,k}{t_1+k^2t_1}$	0	0	0	0
$\sigma_{1^+}^{\#2}{}^{\dagger\alpha\beta}$	$-\frac{\sqrt{2}}{t_1+k^2t_1}$	$\frac{-2k^2(2r_3+r_5)+t_1}{(1+k^2)^2t_1^2}$	$\frac{-2ik^3(2r_3+r_5)+ikt_1}{(1+k^2)^2t_1^2}$	0	0	0	0
$\tau_{1^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{i\sqrt{2}\,k}{t_1+k^2t_1}$	$\frac{i(2k^3(2r_3+r_5)-kt_1)}{(1+k^2)^2t_1^2}$	$\frac{-2k^4(2r_3+r_5)+k^2t_1}{(1+k^2)^2t_1^2}$	0	0	0	0
$\sigma_{1^-}^{\#1}{}^{\dagger\alpha}$	0	0	0	$\frac{1}{k^2(-r_1+2r_3+r_5)}$	$\frac{1}{\sqrt{2}\,(k^2+2k^4)(r_1-2r_3-r_5)}$	$\frac{i}{k(1+2k^2)(-r_1+2r_3+r_5)}$	$-\frac{i(6k^2(r_1-2r_3-r_5)-t_1)}{\sqrt{2}\,k(1+2k^2)^2(r_1-2r_3-r_5)t_1}$
$\sigma_{1^-}^{\#2}{}^{\dagger\alpha}$	0	0	0	$\frac{1}{\sqrt{2}\,(k^2+2k^4)(r_1-2r_3-r_5)}$	$\frac{\frac{1}{-r_1+2r_3+r_5}+\frac{6k^2}{t_1}}{2(k+2k^3)^2}$	$\frac{i}{k(1+2k^2)(r_1-2r_3-r_5)}$	$\frac{i(6k^2(r_1-2r_3-r_5)-t_1)}{\sqrt{2}\,k(1+2k^2)^2(r_1-2r_3-r_5)t_1}$
$\tau_{1^-}^{\#1}{}^{\dagger\alpha}$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#2}{}^{\dagger\alpha}$	0	0	0	0	0	0	$\frac{1}{-r_1+2r_3+r_5}+\frac{6k^2}{t_1}$ / $(1+2k^2)^2$

Quadratic (free) action

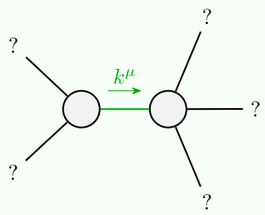
$$
\begin{aligned}
S = = \iiint [& f^{\alpha\beta}\,\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + \tfrac{1}{6}t_1(2\,\omega^{\alpha\prime}{}_{\alpha}\,\omega_{\prime}{}^{\theta}{}_{\theta} - 4\,\omega_{\alpha}{}^{\theta}{}_{\theta}\,\partial_{\prime}f^{\alpha\prime} + 4\,\omega_{\prime}{}^{\theta}{}_{\theta}\,\partial^{\prime}f^{\alpha}{}_{\alpha} - \\
& 2\,\partial_{\prime}f^{\theta}{}_{\theta}\partial^{\prime}f^{\alpha}{}_{\alpha} - 2\,\partial_{\prime}f^{\alpha\prime}\,\partial_{\theta}f^{\alpha}{}_{\alpha}\partial_{\theta}f_{\prime}{}^{\theta} - 6\,\partial_{\alpha}f_{\prime\theta}\,\partial^{\theta}f^{\alpha\prime} - 3\,\partial_{\alpha}f_{\theta\prime}\,\partial^{\theta}f^{\alpha\prime} + \\
& 3\,\partial_{\prime}f_{\alpha\theta}\partial^{\theta}f^{\alpha\prime} + 3\,\partial_{\theta}f_{\alpha\prime}\,\partial^{\theta}f^{\alpha\prime} + 3\,\partial_{\theta}f_{\prime\alpha}\,\partial^{\theta}f^{\alpha\prime} + 6\,\omega_{\alpha\theta\prime}\,(\omega^{\alpha\prime\theta} + 2\,\partial^{\theta}f^{\alpha\prime}) - \\
& 2\,r_3\,(\partial_{\beta}\omega_{\prime}{}^{\theta}{}_{\theta}\,\partial^{\prime}\omega^{\alpha\beta}{}_{\alpha} + \partial_{\prime}\omega_{\beta}{}^{\theta}{}_{\theta}\,\partial^{\prime}\omega^{\alpha\beta}{}_{\alpha} + \partial_{\alpha}\omega^{\alpha\beta\prime}\,\partial_{\theta}\omega_{\prime}{}^{\theta}{}_{\beta} - 2\,\partial^{\prime}\omega^{\alpha\beta}{}_{\alpha}\,\partial_{\theta}\omega_{\beta}{}^{\theta}{}_{\prime} + \\
& \partial_{\alpha}\omega^{\alpha\beta\prime}\,\partial_{\theta}\omega_{\prime}{}^{\theta}{}_{\beta} - 2\,\partial^{\prime}\omega^{\alpha\beta}{}_{\alpha}\,\partial_{\theta}\omega_{\prime}{}^{\theta}{}_{\beta} + 2\,\partial_{\theta}\omega_{\alpha\prime}\,\partial^{\theta}\omega^{\alpha\beta\prime} + 2\,\partial_{\theta}\omega_{\prime\alpha}\,\partial^{\theta}\omega^{\alpha\beta\prime}) + \\
& \tfrac{2}{3}r_1\,(3\,\partial_{\beta}\omega_{\prime}{}^{\theta}{}_{\theta}\,\partial^{\prime}\omega^{\alpha\beta}{}_{\alpha} + 3\,\partial_{\prime}\omega_{\beta}{}^{\theta}{}_{\theta}\,\partial^{\prime}\omega^{\alpha\beta}{}_{\alpha} - 6\,\partial^{\prime}\omega^{\alpha\beta}{}_{\alpha}\,\partial_{\theta}\omega_{\prime}{}^{\theta}{}_{\beta} - 2\,\partial_{\beta}\omega_{\alpha\prime}\,\partial^{\theta}\omega^{\alpha\beta\prime} + \\
& 3\,\partial_{\alpha}\omega^{\alpha\beta\prime}\,\partial_{\theta}\omega_{\prime}{}^{\theta}{}_{\beta} - 6\,\partial^{\prime}\omega^{\alpha\beta}{}_{\alpha}\,\partial_{\theta}\omega_{\prime}{}^{\theta}{}_{\beta} - 2\,\partial_{\beta}\omega_{\alpha\theta}\,\partial^{\theta}\omega^{\alpha\beta\prime} + \partial_{\theta}\omega_{\alpha\beta\prime}\,\partial^{\theta}\omega^{\alpha\beta\prime}) + \\
& 2\,\partial_{\beta}\omega_{\prime\theta\alpha}\,\partial^{\theta}\omega^{\alpha\beta\prime} - \partial_{\prime}\omega_{\alpha\beta\theta}\,\partial^{\theta}\omega^{\alpha\beta\prime} + \partial_{\theta}\omega_{\alpha\beta\prime}\,\partial^{\theta}\omega^{\alpha\beta\prime}) + \\
& r_5\,(\partial_{\prime}\omega_{\theta}{}^{\kappa}{}_{\kappa}\,\partial^{\theta}\omega^{\alpha\prime}{}_{\alpha} - \partial_{\theta}\omega_{\prime}{}^{\kappa}{}_{\kappa}\,\partial^{\theta}\omega^{\alpha\prime}{}_{\alpha} - (\partial_{\alpha}\omega^{\alpha\prime\theta} - 2\,\partial^{\theta}\omega^{\alpha\prime}{}_{\alpha})\,(\partial_{\kappa}\omega_{\prime}{}^{\kappa}{}_{\theta} - \partial_{\kappa}\omega_{\theta\prime}{}^{\kappa}))) [\\
& t, x, y, z]\,dz\,dy\,dx\,dt
\end{aligned}
$$

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau_{0^+}^{\#2} == 0$	1
$\tau_{0^+}^{\#1} == 0$	1
$\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	3
$\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$	3
$\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$	5
Total constraints:	16

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd



Quadratic pole	
Pole residue:	$\frac{1}{(r_1-2r_3-r_5)t_1^2} > 0$
Polarisations:	2

Unitarity conditions

$r_1 < 0 \;\&\&\; r_5 < r_1 - 2r_3 \;\&\&\; t_1 > 0$