## Particle spectrograph

## Wave operator and propagator

SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0}^{\#2} == 0$	$\partial_{\beta}\partial_{\alpha}\tau^{\alpha\beta} == 0$	1
$\tau_0^{\#1} == 0$	$\partial_{\beta}\partial_{\alpha}\tau^{\alpha\beta} == \partial_{\beta}\partial^{\beta}\tau^{\alpha}$	1
$\sigma_{0+}^{\#1} == 0$	$\partial_{\beta}\sigma^{\alpha\beta}_{\alpha} == 0$	1
$\tau_{1}^{\#2}{}^{\alpha} == 0$	$\partial_{\chi}\partial_{\beta}\partial^{\alpha}\tau^{\beta\chi} == \partial_{\chi}\partial^{\chi}\partial_{\beta}\tau^{\alpha\beta}$	æ
$\tau_{1}^{\#1}{}^{\alpha} == 0$	$\partial_{\chi}\partial_{\beta}\partial^{\alpha}\tau^{\beta\chi} == \partial_{\chi}\partial^{\chi}\partial_{\beta}\tau^{\beta\alpha}$	æ
$\sigma_{1}^{#2}\alpha$ == 0	$\partial_{\chi}\partial_{\beta}\sigma^{\alpha\beta\chi} == 0$	8
$\tau_{1+}^{\#1}\alpha\beta + ik \ \sigma_{1+}^{\#2}\alpha\beta == 0$	$\partial_{\chi} \partial^{\alpha} t^{\beta \chi} + \partial_{\chi} \partial^{\beta} t^{\chi \alpha} + \partial_{\chi} \partial^{\chi} t^{\alpha \beta} + \partial_{\chi} \partial^{\alpha} t^{\alpha \beta} $	ю
	$2 \partial_{\delta} \partial_{\chi} \partial^{\alpha} \sigma^{r, \alpha} + 2 \partial_{\delta} \partial_{\gamma} \sigma^{r, \alpha} = =$ $\partial_{\alpha} \partial^{\alpha} r^{\chi \beta} + \partial_{\alpha} \partial^{\beta} r^{\alpha \chi} +$	
	$\partial_{\chi}\partial^{\chi}\tau^{\beta\alpha} + 2\partial_{\delta}\partial_{\chi}\partial^{\beta}\sigma^{\alpha\chi\delta}$	
$\sigma_{2^{-}}^{\#1}\alpha\beta\chi == 0$	$3 \partial_{\epsilon} \partial_{\delta} \partial^{\chi} \partial^{\alpha} \sigma^{\beta \delta \epsilon} + 3 \partial_{\epsilon} \partial^{\epsilon} \partial^{\chi} \partial^{\alpha} \sigma^{\beta \delta} +$	5
	$2\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\alpha\chi\delta} + 4\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\alpha\delta\chi} +$	
	$2 \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial^{\beta} \sigma^{\chi \delta \alpha} + 4 \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial^{\chi} \sigma^{\alpha \beta \delta} +$	
	$2 \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial^{\chi} \sigma^{\alpha \delta \beta} + 2 \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial^{\delta} \sigma^{\beta \chi \alpha} +$	
	$3 \eta^{eta\chi} \partial_{\phi} \partial_{\phi} \partial_{\epsilon} \partial^{\alpha} \sigma^{\delta \epsilon}{}_{\delta} +$	
	$3 \eta^{\alpha\chi} \partial_{\phi} \partial_{\phi} \partial_{\epsilon} \partial_{\delta} \sigma^{\beta\delta\epsilon} +$	
	$3 \eta^{\beta \chi} \partial_{\phi} \partial_{\epsilon} \partial_{\epsilon} \partial^{\epsilon} \sigma^{\alpha \delta}{}_{\delta} = =$	
	$3 \partial_{\epsilon} \partial_{\delta} \partial^{\chi} \partial^{\beta} \sigma^{\alpha \delta \epsilon} + 3 \partial_{\epsilon} \partial^{\epsilon} \partial^{\chi} \partial^{\beta} \sigma^{\alpha \delta} \partial_{\delta} +$	
	$2\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\alpha}\sigma^{\beta\chi\delta} + 4\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\alpha}\sigma^{\beta\delta\chi} +$	
	$2 \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial^{\alpha} \sigma^{\chi \delta \beta} + 2 \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial^{\chi} \sigma^{\beta \delta \alpha} +$	
	$4\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\delta}\sigma^{lphaeta\chi} + 2\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\delta}\sigma^{lpha\chieta} +$	
	$3 \eta^{\alpha\chi} \partial_{\phi} \partial_{\phi} \partial_{\varepsilon} \partial^{\beta} \sigma^{\delta \epsilon}{}_{\delta} +$	
	$3 \eta^{eta\chi}  \partial_{\phi} \partial^{\phi} \partial_{\epsilon} \partial_{\delta} \sigma^{\alpha \delta \epsilon} +$	
	$3~\eta^{lpha\chi}~\partial_{\phi}\partial_{\phi}\partial_{\epsilon}\partial^{\epsilon}\sigma^{eta\delta}{}_{\delta}$	
$\tau_2^{\#1}\alpha\beta == 0$	$4 \partial_{\delta} \partial_{\chi} \partial^{\beta} \partial^{\alpha} \tau^{\chi \delta} + 2 \partial_{\delta} \partial^{\delta} \partial^{\beta} \partial^{\alpha} \tau^{\chi}_{\chi} +$	5
	$3\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\chi}\tau^{\alpha\beta} + 3\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\chi}\tau^{\beta\alpha} +$	
	$2 \eta^{\alpha\beta} \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial_{\chi} \tau^{\chi\delta} ==$	
	$3\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\alpha}\tau^{\beta\chi} + 3\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\alpha}\tau^{\chi\beta} +$	
	$3 \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\beta} \tau^{\alpha \chi} + 3 \partial_{\delta} \partial^{\delta} \partial_{\chi} \partial^{\beta} \tau^{\chi \alpha} +$	
	$2 n^{\alpha\beta} \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \partial^{\delta} \tau_{\chi}^{\chi}$	
$\sigma_{2+}^{\#1}\alpha\beta==0$	$3 \partial_{\delta} \partial_{\chi} \partial^{\alpha} \sigma^{\beta \chi \delta} + 3 \partial_{\delta} \partial_{\chi} \partial^{\beta} \sigma^{\alpha \chi \delta} +$	2
	$2 \eta^{\alpha\beta} \partial_{\epsilon} \partial^{\epsilon} \partial_{\delta} \sigma^{\chi\delta} = 2 \partial_{\delta} \partial^{\beta} \partial^{\alpha} \sigma^{\chi\delta} +$	
	$3 \left( \partial_{\delta} \partial^{\delta} \partial_{\chi} \sigma^{\alpha \chi \beta} + \partial_{\delta} \partial^{\delta} \partial_{\chi} \sigma^{\beta \chi \alpha} \right)$	
Total constraints/gauge generators:	uge generators:	30

Quadratic (free) action
$S == \iiint \left( \frac{1}{6} \left( 6  f^{\alpha \beta}  \tau_{\alpha \beta} + 6  \mathcal{A}^{\alpha \beta \chi}  \sigma_{\alpha \beta \chi} + 4  t_2  \mathcal{A}_{, \theta \alpha}  \partial^{\theta} f^{\alpha \prime} + 2  t_2  \partial_{\alpha} f_{, \theta}  \partial^{\theta} f^{\alpha \prime} - \right) \right)$
$t_2\partial_lpha f_{ heta_I}\partial^ heta f^{lpha_I} - t_2\partial_ u f_{lpha  heta}\partial^ heta f^{lpha_I} + t_2\partial_ heta f_{lpha_I}\partial^ heta f^{lpha_I} -$
$t_2\partial_ heta f_{ lpha}\partial^ heta f^{lpha\prime}$ - $4t_2\mathcal{R}_{lpha heta\prime}$ ( $\mathcal{R}^{lpha\prime}$ $+\partial^ heta f^{lpha\prime}$ ) $+$
$2t_2\mathcal{R}_{lpha^{ec{eta}}}(\mathcal{A}^{lpha^{ec{eta}}}+2\partial^{ heta}f^{lpha^{ec{eta}}})+8r_2\partial_{eta}\mathcal{A}_{lpha^{ec{eta}}}\partial^{ heta}\mathcal{A}_{lpha^{ec{eta}}}.$
$4r_2\partial_eta \mathcal{R}_{lpha heta_l}\partial^ heta \mathcal{R}^{lphaeta_l} + 4r_2\partial_eta \mathcal{H}_{\iota heta_lpha}\partial^ heta \mathcal{R}^{lphaeta_l} -$
$2r_2\partial_{ert}\mathcal{F}_{lphaeta heta}\partial^{artheta}\mathcal{F}^{lphaeta_l}+2r_2\partial_{artheta}\mathcal{F}_{lphaeta_l}\partial^{artheta}\mathcal{F}^{lphaeta_l}$ -
$4r_2\partial_ heta \mathcal{R}_{lphaerteta}\partial^ heta \mathcal{R}^{lphaetaert}+6r_5\partial_ert\mathcal{R}_{eta\kappa}^{\ \ \ }\partial^ heta \mathcal{R}^{lphaert}_{\ \ lpha}$ -
$6r_5\partial_ heta \mathcal{A}_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_{_$
$12r_5\partial^\theta \mathcal{A}^{\alpha\prime}_{\alpha}\partial_\kappa \mathcal{A}^{\kappa}_{\theta} + 6r_5\partial_\alpha \mathcal{A}^{\alpha\prime\theta}\partial_\kappa \mathcal{A}^{\kappa}_{\theta},$
$12r_5\partial^ heta \mathcal{R}^{lpha_\prime}_{lpha}\partial_{\kappa}\mathcal{R}^{}_{ ho}))[t,lpha,eta,z]d\!\!/\!zd\!\!/\!yd\!\!/\!xd\!\!/\!t$
$\sigma_{1}^{\#1}$ $\sigma_{1}^{\#2}$ $\sigma_{1}^{\#2}$ $\sigma_{1}^{\#1}$ $\sigma_{1}^{\#2}$ $\sigma_{1}^{\#2}$ $\sigma_{1}^{\#2}$ $\sigma_{1}^{\#2}$ $\sigma_{1}^{\#2}$ $\sigma_{2}^{\#2}$

)	)	)	)	)	)	)	+ ,					$\alpha\beta$				$\sigma_{0}^{*+} +$	0	0	0	0	
0	0	0	0	0	0	0	$f_{+}^{t1} f_{0}^{#1}$	0	0	0	0	$\mathcal{A}_{2^+}^{\#1}{}_{\alpha\beta}$	0	0	0	$\tau_{0^{+}}^{\#1}$ †	0	0	0	0	
0	0	0	$\frac{1}{k^2 r_5}$	0	0	0	$\mathcal{A}_{0}^{\#1}$	0 +	0	1	0		$\mathcal{A}_2^{\#1} + ^{\alpha\beta}$	$f_2^{\#1} + \alpha \beta$	$\mathcal{A}_{2}^{\#1} +^{lphaeta\chi}$	$\tau_{0^{+}}^{\#2} \dagger$	0	0	0	0	
I 10	t2) t2	2 2 2	۱ ×					$\mathcal{A}_{0}^{\#1}$	$f_{0}^{\#1}$	$f_{0}^{\#2}$	$\mathcal{A}_{0}^{\#1}$ $\dagger$		$\mathcal{A}_{2}^{\#1}$	$f_{2}^{#1}$	$\mathcal{A}_{2}^{\#1}$	$\sigma_{0}^{\#1}$ †	0	0	0	$\frac{1}{k^2 r_2}$	⊢t
$-\frac{\sqrt{k}}{kr_5+k^3r_5}$	$\frac{i(3k^2r_5+2t_2)}{k(1+k^2)^2r_5t_2}$	$\frac{3k^2r_5+2t_2}{(1+k^2)^2r_5t_2}$	0	0	0	0		•		${\cal R}_1^{\#_1^1}$		${\cal R}_1^{\#}$	÷2 + αβ	$f_{1}^{\#}$	1 <sup>+</sup> αβ	${\mathscr R}_{1-lpha}^{\sharp 1}$	$\mathcal{A}_1^{\#2}$	$\frac{2}{\alpha} f$	#1 1 α	$f_{1}^{#2}\alpha$	
- 4	$\frac{i(3)}{k(1)}$						$\mathcal{A}_{\scriptscriptstyle 1}^{\scriptscriptstyle \sharp}$	#1 †°	k	<sup>2</sup> r <sub>5</sub> +	$+\frac{2t_2}{3}$	<u>√2</u>	<u>2</u> t <sub>2</sub>	$\frac{1}{3}\bar{l}\sqrt{2}kt_2$		0	0		0	0	
$-\frac{v^2}{k^2 r_5 + k^4 r_5}$	$\frac{3k^2r_5 + 2t_2}{(k+k^3)^2r_5t_2}$	$\frac{(k+k^3)^2 r_5 t_2}{i (3k^2 r_5 + 2t_2)}$ $\frac{k(1+k^2)^2 r_5 t_2}{0}$ 0		0	0	${\cal A}_1^{\dagger}$	#2 L+ †°	αβ	$\frac{\sqrt{2}}{3}$	t2		<u>2</u> 3		<u>'t2</u> 3	0	0		0	0		
- k <sup>2</sup> r	$\frac{3k^2}{(k+k)^3}$	$-\frac{i(3k^2)}{k(1+i)}$					$f_{1}^{7}$	#1 †°	$-\frac{1}{3}$	$\frac{1}{3}$ i $\sqrt{2}$ kt <sub>2</sub>		$\left  -\frac{1}{3} \right $	ikt <sub>2</sub>	$\frac{k^2 t_2}{3}$		0	0		0	0	
_ 5	75 22 +k <sup>4</sup> r <sub>5</sub> 2 2 2						${\mathscr F}$	7 <sup>#1</sup> †	.α	0	l		0	(	0	$k^2 r_5$	0		0	0	
$\frac{1}{k^2 r_5}$	$\frac{\sqrt{2}}{k^2 r_5 + k^4 r_5}$	$\frac{i\sqrt{2}}{kr_5 + k^3 r_5}$	0	0	0	0	${\mathcal F}$	7 <sup>#2</sup> †	.α	0	l		0		)	0	0		0	0	
αp-	$-\alpha\beta$	+ $+$ $+$ $+$ $+$ $+$ $+$ $+$ $+$ $+$		$\tau_{1}^{\#2} + ^{\alpha}$	l <i>f</i>	6#1 †	.α	0	ı		0	(	0	0	0		0	0			
$\sigma_1^{*}$ † † $\tau^{ap}$	$\sigma_1^{\#2} + \alpha^{\beta}$	$\tau_{1}^{\#1} + \alpha \beta$	$\sigma_{1}^{\#1} +^{\alpha}$	$\sigma_1^{\#^2} +^{\alpha}$	$\tau_{1}^{\#1} +^{\alpha}$	${\mathfrak t}_1^{\#^2}$	f	$f_{1}^{#2} \dagger^{\alpha}$		0			0		)	0	0		0	0	

## Massive and massless spectra

Massive particle

Pole residue: 
$$-\frac{1}{r_2} > 0$$

Polarisations: 1

Square mass:  $-\frac{t_2}{r_2} > 0$ 

Spin: 0

Parity: Odd

Unitarity conditions

 $r_2 < 0 \&\& t_2 > 0$