In[1]:=
```
Get@FileNameJoin@{NotebookDirectory[], "Calibration.m"};
```

First we import some formatting...

...okay, that's better, from now on any commentary written inside this Calibration.m wrapper will present as blue text (i.e. this text is not part of PSALTer, it is just a use-case). Next we load the PSALTer package:

```
-------------------------------------------------------------

Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}

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Connecting to external linux executable...

Connection established.

-------------------------------------------------------------

Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}

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Package xAct`xPert`  version 1.0.6, {2018, 2, 28}

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  and Guillermo A. Mena Marugan, under the General Public License.

** Variable $PrePrint assigned value ScreenDollarIndices

** Variable $CovDFormat changed from Prefix to Postfix

** Option AllowUpperDerivatives of ContractMetric changed from False to True

** Option MetricOn of MakeRule changed from None to All

** Option ContractMetrics of MakeRule changed from False to True

-------------------------------------------------------------

Package xAct`Invar`  version 2.0.5, {2013, 7, 1}

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  D. Yllanes and R. Portugal, under the General Public License.

** DefConstantSymbol: Defining constant symbol sigma.

** DefConstantSymbol: Defining constant symbol dim.

** Option CurvatureRelations of DefCovD changed from True to False

** Variable $CommuteCovDsOnScalars changed from True to False

-------------------------------------------------------------

Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}

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  Jose M. Martin-Garcia, under the General Public License.
```

Now we set up the general Lagrangian:

$$-\underset{\cdot}{\lambda}\,\mathcal{R}^{ij}{}_{ij}+\left(\frac{\underset{1}{r}}{3}+\frac{\underset{2}{r}}{6}\right)\mathcal{R}_{ijhl}\;\mathcal{R}^{ijhl}+\left(\frac{2\,\underset{1}{r}}{3}-\frac{2\,\underset{2}{r}}{3}\right)\mathcal{R}_{ihjl}\;\mathcal{R}^{ijhl}+$$

$$\left(\underset{4}{r}+\underset{5}{r}\right)\mathcal{R}_{i\ jl}^{\ l}\;\mathcal{R}^{ihj}{}_{h}+\left(\underset{4}{r}-\underset{5}{r}\right)\mathcal{R}^{ihj}{}_{h}\;\mathcal{R}_{j\ il}^{\ l}+\left(\frac{\underset{1}{r}}{3}+\frac{\underset{2}{r}}{6}-\underset{3}{r}\right)\mathcal{R}^{ijhl}\;\mathcal{R}_{hlij}+$$

$$\left(\frac{\underset{\cdot}{\lambda}}{4}+\frac{\underset{1}{t}}{3}+\frac{\underset{2}{t}}{12}\right)\mathcal{T}_{ijh}\;\mathcal{T}^{ijh}+\left(-\frac{\underset{\cdot}{\lambda}}{2}-\frac{\underset{1}{t}}{3}+\frac{\underset{2}{t}}{6}\right)\mathcal{T}^{ijh}\;\mathcal{T}_{jhi}+\left(-\underset{\cdot}{\lambda}-\frac{\underset{1}{t}}{3}+\frac{2\,\underset{3}{t}}{3}\right)\mathcal{T}_{i}^{\ ji}\;\mathcal{T}_{hj}^{\ \ h}$$

We also knock up some simple tools to linearise the Lagrangian:

** DefConstantSymbol: Defining constant symbol PerturbativeParameter.

Now we would like to check the basic
  Einstein-Cartan theory. Here is the full nonlinear Lagrangian:

$$\underset{1}{t}\,\mathcal{R}^{ij}{}_{ij}$$

To use PSALTer, you have to first linearise
  this Lagrangian to second order around the desired vacuum:

$$t_{\cdot 1}\, \mathcal{A}_{q\,j\,i}\ \mathcal{A}^{q\,i\,j} + t_{\cdot 1}\, \mathcal{A}^{q\,i}{}_{q}\ \mathcal{A}_{i}{}^{j}{}_{j} + 2\, t_{\cdot 1}\, f^{q\,i}\ \partial_{i}\mathcal{A}_{q}{}^{j}{}_{j} - 2\, t_{\cdot 1}\, \partial_{i}\mathcal{A}^{q\,i}{}_{q} - 2\, t_{\cdot 1}\, f^{q\,i}\ \partial_{j}\mathcal{A}_{q}{}^{j}{}_{i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

** ParticleSpectrum..

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -t_{\cdot 1} & -\frac{i\,k\,t_{\cdot 1}}{\sqrt{2}} & -i\sqrt{\frac{3}{2}}\,k\,t_{\cdot 1} & 0 \\ \frac{i\,k\,t_{\cdot 1}}{\sqrt{2}} & 0 & 0 & 0 \\ i\sqrt{\frac{3}{2}}\,k\,t_{\cdot 1} & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_{\cdot 1} \end{pmatrix}, \begin{pmatrix} -\frac{t_{\cdot 1}}{2} & -\frac{t_{\cdot 1}}{\sqrt{2}} & -\frac{i\,k\,t_{\cdot 1}}{\sqrt{2}} & 0 & 0 & 0 & 0 \\ -\frac{t_{\cdot 1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{i\,k\,t_{\cdot 1}}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{t_{\cdot 1}}{2} & \frac{t_{\cdot 1}}{\sqrt{2}} & 0 & i\,k\,t_{\cdot 1} \\ 0 & 0 & 0 & \frac{t_{\cdot 1}}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i\,k\,t_{\cdot 1} & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{t_{\cdot 1}}{2} & -\frac{i\,k\,t_{\cdot 1}}{\sqrt{2}} & 0 \\ \frac{i\,k\,t_{\cdot 1}}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & \frac{t_{\cdot 1}}{2} \end{pmatrix}\right\}$$

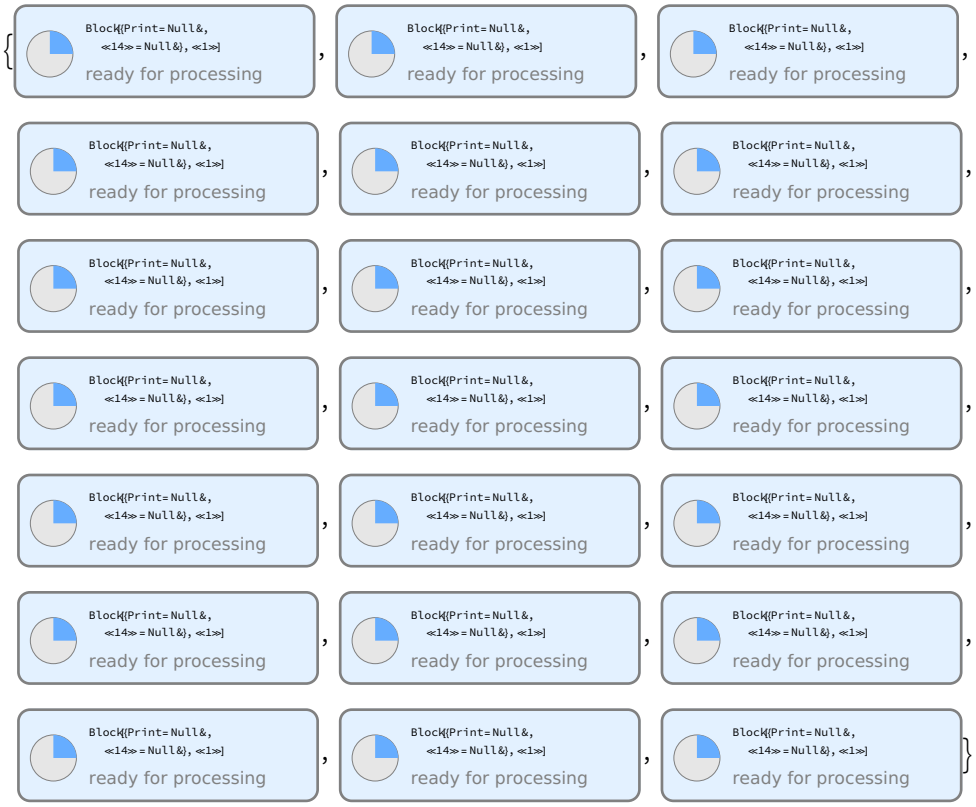
Gauge constraints on source currents:

$$\left\{ {}^{0^+}_{\cdot}\tau^{\parallel} == {}^{0^+}_{\cdot}\tau^{\perp},\ -2\,i\,k\ {}^{1^-}_{\cdot}\sigma^{\perp\,q} == {}^{1^-}_{\cdot}\tau^{\perp\,q},\ {}^{1^-}_{\cdot}\tau^{\parallel\,q} == 0,\ -i\,k\ {}^{1^+}_{\cdot}\sigma^{\perp\,q\,b} == {}^{1^+}_{\cdot}\tau^{\parallel\,q\,b} \right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} 0 & -\frac{i}{2\sqrt{2}\,k\,t_{\cdot 1}} & -\frac{i\sqrt{\frac{3}{2}}}{2\,k\,t_{\cdot 1}} & 0 \\ \frac{i}{2\sqrt{2}\,k\,t_{\cdot 1}} & \frac{1}{8k^2\,t_{\cdot 1}} & \frac{\sqrt{3}}{8k^2\,t_{\cdot 1}} & 0 \\ \frac{i\sqrt{\frac{3}{2}}}{2\,k\,t_{\cdot 1}} & \frac{\sqrt{3}}{8k^2\,t_{\cdot 1}} & \frac{3}{8k^2\,t_{\cdot 1}} & 0 \\ 0 & 0 & 0 & -\frac{1}{t_{\cdot 1}} \end{pmatrix},$$

$$\begin{pmatrix} 0 & -\frac{\sqrt{2}}{t_{\cdot 1}+k^2\,t_{\cdot 1}} & -\frac{i\sqrt{2}\,k}{t_{\cdot 1}+k^2\,t_{\cdot 1}} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{2}}{t_{\cdot 1}+k^2\,t_{\cdot 1}} & \frac{1}{(1+k^2)\,t_{\cdot 1}} & \frac{i\,k}{(1+k^2)\,t_{\cdot 1}} & 0 & 0 & 0 & 0 \\ \frac{i\sqrt{2}\,k}{t_{\cdot 1}+k^2\,t_{\cdot 1}} & -\frac{i\,k}{(1+k^2)\,t_{\cdot 1}} & \frac{k^2}{(1+k^2)\,t_{\cdot 1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\sqrt{2}}{t_{\cdot 1}+2k^2\,t_{\cdot 1}} & 0 & \frac{2\,i\,k}{t_{\cdot 1}+2k^2\,t_{\cdot 1}} \\ 0 & 0 & 0 & \frac{\sqrt{2}}{t_{\cdot 1}+2k^2\,t_{\cdot 1}} & \frac{1}{(1+2k^2)\,t_{\cdot 1}} & 0 & \frac{i\sqrt{2}\,k}{(1+2k^2)\,t_{\cdot 1}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2\,i\,k}{t_{\cdot 1}+2k^2\,t_{\cdot 1}} & -\frac{i\sqrt{2}\,k}{(1+2k^2)\,t_{\cdot 1}} & 0 & \frac{2k^2}{(1+2k^2)\,t_{\cdot 1}} \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i\sqrt{2}}{k\,t_{\cdot 1}} & 0 \\ \frac{i\sqrt{2}}{k\,t_{\cdot 1}} & -\frac{1}{k^2\,t_{\cdot 1}} & 0 \\ 0 & 0 & \frac{2}{t_{\cdot 1}} \end{pmatrix}\right\}$$



During evaluation of In[1]:=

Launching Parallel Subkernels

CurrentGroup	1 of 1: Local kernel (8 kernels)
Launched/Collected/Expected	8/8/8

LinkObject: Unable to communicate with closed link LinkObject[/usr/local/Wolfram/Mathematica/13.2/Executables/wolfram –noinit –subkernel –wstp, 813, 15].

LinkObject: Unable to communicate with closed link LinkObject[/usr/local/Wolfram/Mathematica/13.2/Executables/wolfram –noinit –subkernel –wstp, 815, 17].

Parallel`Developer`ConnectKernel: 2 of 8 kernels failed to initialize.