

# Particle spectrograph

# Wave operator and propagator

$J_1^{\#1} + \alpha\beta$	0	$\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+k^2)}$	$\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+k^2)}$	0	0	0
$J_1^{\#2} + \alpha\beta$	$-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+k^2)}$	$-\frac{2}{(\alpha_0-4\beta_1)(1+k^2)^2}$	$-\frac{2ik}{(\alpha_0-4\beta_1)(1+k^2)^2}$	0	0	0
$\tau_1^{\#1} + \alpha\beta$	$-\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+k^2)}$	$-\frac{2ik}{(\alpha_0-4\beta_1)(1+k^2)^2}$	$-\frac{2k^2}{(\alpha_0-4\beta_1)(1+k^2)^2}$	0	0	0
$\sigma_1^{\#1} + \alpha$	0	0	0	0	$-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+2k^2)}$	$-\frac{4ik}{(\alpha_0-4\beta_1)(1+2k^2)}$
$\sigma_1^{\#2} + \alpha$	0	0	0	$-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+2k^2)}$	$-\frac{2}{(\alpha_0-4\beta_1)(1+2k^2)^2}$	$-\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+2k^2)^2}$
$\tau_1^{\#1} + \alpha$	0	0	0	0	0	0
$\tau_1^{\#2} + \alpha$	0	0	0	$\frac{4ik}{(\alpha_0-4\beta_1)(1+2k^2)}$	$\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+2k^2)^2}$	$-\frac{4k^2}{(\alpha_0-4\beta_1)(1+2k^2)^2}$

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	$\frac{8 \beta_1}{\alpha_0^2 - 4 \alpha_0 \beta_1 + 8 \alpha_6 \beta_1 k^2}$	$-\frac{i \sqrt{2} (\alpha_0 - 4 \beta_1)}{\alpha_0 (\alpha_0 - 4 \beta_1) k + 8 \alpha_6 \beta_1 k^3}$	0	0
$\tau_{0+}^{\#1} \dagger$	$\frac{i \sqrt{2} (\alpha_0 - 4 \beta_1)}{\alpha_0 (\alpha_0 - 4 \beta_1) k + 8 \alpha_6 \beta_1 k^3}$	$-\frac{\alpha_0 - 4 \beta_1 + 2 \alpha_6 k^2}{k^2 (\alpha_0^2 - 4 \alpha_0 \beta_1 + 8 \alpha_6 \beta_1 k^2)}$	0	0
$\tau_{0+}^{\#2} \dagger$	0	0	0	0
$\sigma_{0-}^{\#1} \dagger$	0	0	0	$\frac{2}{\alpha_0 - 4 \beta_1}$

Source constraints			Fundamental fields	Multiplicities
$\text{SO}(3)$ irreps				
$\tau_{0+}^{\#2} == 0$			$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2 \, i \, k \, \sigma_{1-}^{\#2\alpha} == 0$			$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta} + 2 \, \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$	3
$\tau_{1-}^{\#1\alpha} == 0$			$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$	3
$\tau_{1+}^{\#1\alpha\beta} + i \, k \, \sigma_{1+}^{\#2\alpha\beta} == 0$			$\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\chi\alpha} + \partial_\chi \partial^\chi \tau^{\alpha\beta} +$ $2 \, \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 2 \, \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} ==$ $\partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} +$ $\partial_\chi \partial^\chi \tau^{\beta\alpha} + 2 \, \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$	3
Total constraints/gauge generators:				10

## Quadratic (free) action

$$\mathcal{S} =$$

$$\begin{aligned} & \iiint \left( -\frac{1}{2} (\alpha_0 - 4\beta_1) \omega_{\alpha}^{\alpha\beta} \omega_{\beta}^{\chi}{}_{\chi} + f^{\alpha\beta} \tau_{\alpha\beta} + \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - 4\beta_1 \omega_{\alpha}^{\chi}{}_{\chi} \partial_{\beta} f^{\alpha\beta} - \alpha_0 \right. \\ & \quad f^{\alpha\beta} \partial_{\beta} \omega_{\alpha}^{\chi}{}_{\chi} + \alpha_0 \partial_{\beta} \omega_{\alpha}^{\alpha\beta} + 4\beta_1 \omega_{\beta}^{\chi}{}_{\chi} \partial^{\beta} f^{\alpha}{}_{\alpha} - \\ & \quad 2\beta_1 \partial_{\beta} f^{\chi}{}_{\chi} \partial^{\beta} f^{\alpha}{}_{\alpha} - 2\beta_1 \partial_{\beta} f^{\alpha\beta} \partial_{\chi} f^{\chi}{}_{\alpha} + 4\beta_1 \partial^{\beta} f^{\alpha}{}_{\alpha} \partial_{\chi} f^{\chi}{}_{\beta} + \\ & \quad \alpha_0 f^{\alpha\beta} \partial_{\chi} \omega_{\alpha}^{\chi}{}_{\beta} - \alpha_0 f^{\alpha}{}_{\alpha} \partial_{\chi} \omega^{\beta\chi}{}_{\beta} - 2\beta_1 \partial_{\alpha} f_{\beta\chi} \partial^{\chi} f^{\alpha\beta} - \\ & \quad \beta_1 \partial_{\alpha} f_{\chi\beta} \partial^{\chi} f^{\alpha\beta} + \beta_1 \partial_{\beta} f_{\alpha\chi} \partial^{\chi} f^{\alpha\beta} + \beta_1 \partial_{\chi} f_{\alpha\beta} \partial^{\chi} f^{\alpha\beta} + \\ & \quad \beta_1 \partial_{\chi} f_{\beta\alpha} \partial^{\chi} f^{\alpha\beta} - \frac{1}{2} \omega_{\alpha\chi\beta} ((\alpha_0 - 4\beta_1) \omega^{\alpha\beta\chi} - 8\beta_1 \partial^{\chi} f^{\alpha\beta}) + \\ & \quad \left. \frac{2}{3} \alpha_6 \partial_{\beta} \omega_{\alpha}^{\alpha\beta} \partial_{\delta} \omega^{\chi\delta}{}_{\chi} \right) [t, x, y, z] dz dy dx dt \end{aligned}$$

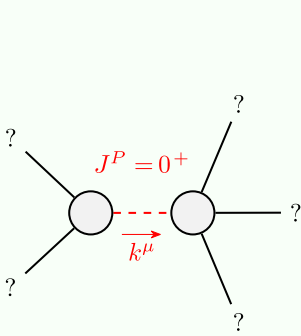
	$\omega_{2^+}^{\#1} \alpha \beta$	$f_{2^+}^{\#1} \alpha \beta$	$\omega_{2^-}^{\#1} \alpha \beta \chi$
$\omega_{2^+}^{\#1} \dagger \alpha \beta$	$-\frac{\alpha_0}{4} + \beta_1$	$\frac{i(\alpha_0 - 4\beta_1)k}{2\sqrt{2}}$	0
$f_{2^+}^{\#1} \dagger \alpha \beta$	$-\frac{i(\alpha_0 - 4\beta_1)k}{2\sqrt{2}}$	$2\beta_1 k^2$	0
$\omega_{2^-}^{\#1} \dagger \alpha \beta \chi$	0	0	$-\frac{\alpha_0}{4} + \beta_1$

	$\omega_{1^+ \alpha \beta}^{\#1}$	$\omega_{1^+ \alpha \beta}^{\#2}$	$f_{1^+ \alpha \beta}^{\#1}$	$\omega_{1^+ \alpha}^{\#1}$	$\omega_{1^+ \alpha}^{\#2}$	$f_{1^+ \alpha}^{\#1}$	$f_{1^+ \alpha}^{\#2}$
$\omega_{1^+}^{\#1} \dagger^{\alpha \beta}$	$\frac{1}{4} (\alpha_0 - 4 \beta_1)$	$\frac{\alpha_0 - 4 \beta_1}{2 \sqrt{2}}$	$\frac{i (\alpha_0 - 4 \beta_1) k}{2 \sqrt{2}}$	0	0	0	0
$\omega_{1^+}^{\#2} \dagger^{\alpha \beta}$	$\frac{\alpha_0 - 4 \beta_1}{2 \sqrt{2}}$	0	0	0	0	0	0
$f_{1^+}^{\#1} \dagger^{\alpha \beta}$	$-\frac{i (\alpha_0 - 4 \beta_1) k}{2 \sqrt{2}}$	0	0	0	0	0	0
$\omega_{1^+}^{\#1} \dagger^{\alpha}$	0	0	0	$\frac{1}{4} (\alpha_0 - 4 \beta_1)$	$-\frac{\alpha_0 - 4 \beta_1}{2 \sqrt{2}}$	0	$-\frac{1}{2} i (\alpha_0 - 4 \beta_1) k$
$\omega_{1^+}^{\#2} \dagger^{\alpha}$	0	0	0	$-\frac{\alpha_0 - 4 \beta_1}{2 \sqrt{2}}$	0	0	0
$f_{1^+}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$f_{1^+}^{\#2} \dagger^{\alpha}$	0	0	0	$\frac{1}{2} i (\alpha_0 - 4 \beta_1) k$	0	0	0

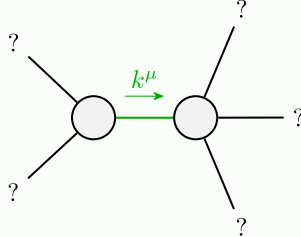
$\omega_0^{\#1} + \omega_0^{\#1+}$	$\omega_0^{\#1}$	$f_0^{\#1+}$	$f_0^{\#1}$	$f_0^{\#2}$	$\omega_0^{\#1}$
$\frac{\alpha_0}{2} - 2\beta_1 + \alpha_6 k^2$	$-\frac{i(\alpha_0 - 4\beta_1)k}{\sqrt{2}}$	0	0	0	0
$\frac{i(\alpha_0 - 4\beta_1)k}{\sqrt{2}}$	$-4\beta_1 k^2$	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	$\frac{1}{2}(\alpha_0 - 4\beta_1)$

$\sigma_{2+}^{\#1} + \alpha\beta$	$-\frac{16\beta_1}{\alpha^2-4\alpha\beta_1}$	$\frac{2i\sqrt{2}}{\alpha_0k}$	$\tau_{2+}^{\#1} + \alpha\beta$	$\sigma_{2-}^{\#1}$	$0$
$\tau_{2+}^{\#1} + \alpha\beta$	$-\frac{2i\sqrt{2}}{\alpha_0k}$	$\frac{2}{\alpha_0k^2}$	$\tau_{2+}^{\#1} + \alpha\beta$	$0$	$0$
$\sigma_{2-}^{\#1} + \alpha\beta\chi$	$0$	$0$	$\sigma_{2-}^{\#1} + \alpha\beta\chi$	$\frac{1}{-\frac{\alpha_0}{4} + \beta_1}$	$0$

# Massive and massless spectra



Massive particle	
Pole residue:	$\frac{1}{\alpha_0} + \frac{1}{\alpha_6} - \frac{1}{4\beta_1} > 0$
Polarisations:	1
Square mass:	$-\frac{\alpha_0(\alpha_0 - 4\beta_1)}{8\alpha_6\beta_1} > 0$
Spin:	0
Parity:	Even



Quadratic pole	
Pole residue:	$\frac{1}{\alpha_0} > 0$
Polarisations:	2

## Unitarity conditions

$$\alpha_0 > 0 \ \&\& \ \alpha_6 > 0 \ \&\& \ \beta_1 < 0 \ || \ \beta_1 > \frac{\alpha_0}{4}$$