# Particle spectrograph

## Wave operator and propagator

### Source constraints

| SO(3) irreps | Fundamental fields | Multiplicities |
|---|---|---|
| $\tau_{0^+}^{\#2} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$ | 1 |
| $\tau_{0^+}^{\#1} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha$ | 1 |
| $\sigma_{0^+}^{\#1} == 0$ | $\partial_\beta\sigma^{\alpha\beta}{}_\alpha == 0$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#1\,\alpha} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} + 2\,(\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi}{}_\beta - \partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi} + \partial_\delta\partial^\delta\partial_\chi\partial^\chi\sigma^{\alpha\beta}{}_\beta) == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta}$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$ | 3 |
| $\sigma_{1^-}^{\#1\,\alpha} == \sigma_{1^-}^{\#2\,\alpha}$ | $\partial_\chi\partial^\alpha\sigma^{\beta\chi}{}_\beta + \partial_\chi\partial^\chi\sigma^{\alpha\beta}{}_\beta == 0$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$ | $\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} + 2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$ | 3 |
| $\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$ | $-i\,(4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} + 4\,i\,k^\chi\,\partial_\epsilon\partial_\chi\partial^\beta\partial^\alpha\sigma^{\delta\epsilon}{}_\delta - 6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\delta\epsilon} - 6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} + 6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\alpha\delta\beta} + 6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\beta\delta\alpha} - 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi - 4\,i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\chi\sigma^{\delta\epsilon}{}_\delta) == 0$ | 5 |
| Total constraints/gauge generators: | | 20 |

### Quadratic (free) action

$$\begin{aligned}\mathcal{S} == \iiiint\Big(&\tfrac{1}{6}\,(2\,t_1\,\omega^{\alpha\imath}{}_\alpha\,\omega_\imath{}^\theta{}_\theta + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 4\,t_1\,\omega_\alpha{}^\theta{}_\theta\,\partial_\imath f^{\alpha\imath} + \\ &4\,t_1\,\omega_\imath{}^\theta{}_\theta\,\partial^\imath f^\alpha{}_\alpha - 2\,t_1\,\partial_\imath f^\theta{}_\theta\,\partial^\imath f^\alpha{}_\alpha - 2\,t_1\,\partial_\imath f^{\alpha\imath}\,\partial_\theta f_\alpha{}^\theta + \\ &4\,t_1\,\partial^\imath f^\alpha{}_\alpha\,\partial_\theta f_\imath{}^\theta + 4\,t_1\,\omega_{\imath\theta\alpha}\,\partial^\theta f^{\alpha\imath} + 4\,t_2\,\omega_{\imath\theta\alpha}\,\partial^\theta f^{\alpha\imath} - \\ &4\,t_1\,\partial_\alpha f_{\imath\theta}\,\partial^\theta f^{\alpha\imath} + 2\,t_2\,\partial_\alpha f_{\imath\theta}\,\partial^\theta f^{\alpha\imath} - 4\,t_1\,\partial_\alpha f_{\theta\imath}\,\partial^\theta f^{\alpha\imath} - \\ &t_2\,\partial_\alpha f_{\theta\imath}\,\partial^\theta f^{\alpha\imath} + 2\,t_1\,\partial_\imath f_{\alpha\theta}\,\partial^\theta f^{\alpha\imath} - t_2\,\partial_\imath f_{\alpha\theta}\,\partial^\theta f^{\alpha\imath} + \\ &4\,t_1\,\partial_\theta f_{\alpha\imath}\,\partial^\theta f^{\alpha\imath} + t_2\,\partial_\theta f_{\alpha\imath}\,\partial^\theta f^{\alpha\imath} + 2\,t_1\,\partial_\theta f_{\imath\alpha}\,\partial^\theta f^{\alpha\imath} - \\ &t_2\,\partial_\theta f_{\imath\alpha}\,\partial^\theta f^{\alpha\imath} + 2\,(t_1+t_2)\,\omega_{\alpha\imath\theta}\,(\omega^{\alpha\imath\theta} + 2\,\partial^\theta f^{\alpha\imath}) + \\ &2\,\omega_{\alpha\theta\imath}\,((t_1-2\,t_2)\,\omega^{\alpha\imath\theta} + 2\,(2\,t_1-t_2)\,\partial^\theta f^{\alpha\imath}) + \\ &8\,r_2\,\partial_\beta\omega_{\alpha\imath\theta}\,\partial^\theta\omega^{\alpha\beta\imath} - 4\,r_2\,\partial_\beta\omega_{\alpha\theta\imath}\,\partial^\theta\omega^{\alpha\beta\imath} + 4\,r_2\,\partial_\beta\omega_{\imath\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\imath} - \\ &2\,r_2\,\partial_\imath\omega_{\alpha\beta\theta}\,\partial^\theta\omega^{\alpha\beta\imath} + 2\,r_2\,\partial_\theta\omega_{\alpha\beta\imath}\,\partial^\theta\omega^{\alpha\beta\imath} - \\ &4\,r_2\,\partial_\theta\omega_{\alpha\imath\beta}\,\partial^\theta\omega^{\alpha\beta\imath}\Big)\Big)[t,\,x,\,y,\,z]\,dt\,dx\,dy\,dz\end{aligned}$$

| | $\sigma_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^+}^{\#2\,\alpha\beta}$ | $\tau_{1^+}^{\#1\,\alpha\beta}$ | $\sigma_{1^-}^{\#1\,\alpha}$ | $\sigma_{1^-}^{\#2\,\alpha}$ | $\tau_{1^-}^{\#1\,\alpha}$ | $\tau_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{2\,(t_1+t_2)}{3\,t_1\,t_2}$ | $\frac{\sqrt{2}\,(t_1-2\,t_2)}{3\,(1+k^2)\,t_1\,t_2}$ | $\frac{i\,\sqrt{2}\,k\,(t_1-2\,t_2)}{3\,(1+k^2)\,t_1\,t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2}\dagger^{\alpha\beta}$ | $\frac{\sqrt{2}\,(t_1-2\,t_2)}{3\,(1+k^2)\,t_1\,t_2}$ | $\frac{t_1+4\,t_2}{3\,(1+k^2)^2\,t_1\,t_2}$ | $\frac{i\,k\,(t_1+4\,t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $-\frac{i\,\sqrt{2}\,k\,(t_1-2\,t_2)}{3\,(1+k^2)\,t_1\,t_2}$ | $-\frac{i\,k\,(t_1+4\,t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$ | $\frac{k^2\,(t_1+4\,t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{6}{(3+4\,k^2)^2\,t_1}$ | $\frac{6\,\sqrt{2}}{(3+4\,k^2)^2\,t_1}$ | 0 | $\frac{12\,i\,k}{(3+4\,k^2)^2\,t_1}$ |
| $\sigma_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{6\,\sqrt{2}}{(3+4\,k^2)^2\,t_1}$ | $\frac{12}{(3+4\,k^2)^2\,t_1}$ | 0 | $\frac{12\,i\,\sqrt{2}\,k}{(3+4\,k^2)^2\,t_1}$ |
| $\tau_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | $-\frac{12\,i\,k}{(3+4\,k^2)^2\,t_1}$ | $-\frac{12\,i\,\sqrt{2}\,k}{(3+4\,k^2)^2\,t_1}$ | 0 | $\frac{24\,k^2}{(3+4\,k^2)^2\,t_1}$ |

| | $\omega_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^+}^{\#2\,\alpha\beta}$ | $f_{1^+}^{\#1\,\alpha\beta}$ | $\omega_{1^-}^{\#1\,\alpha}$ | $\omega_{1^-}^{\#2\,\alpha}$ | $f_{1^-}^{\#1\,\alpha}$ | $f_{1^-}^{\#2\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{1}{6}\,(t_1+4\,t_2)$ | $-\frac{t_1-2\,t_2}{3\,\sqrt{2}}$ | $-\frac{i\,k\,(t_1-2\,t_2)}{3\,\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2}\dagger^{\alpha\beta}$ | $-\frac{t_1-2\,t_2}{3\,\sqrt{2}}$ | $\frac{t_1+t_2}{3}$ | $\frac{1}{3}\,i\,k\,(t_1+t_2)$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{i\,k\,(t_1-2\,t_2)}{3\,\sqrt{2}}$ | $-\frac{1}{3}\,i\,k\,(t_1+t_2)$ | $\frac{1}{3}\,k^2\,(t_1+t_2)$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{6}$ | $\frac{t_1}{3\,\sqrt{2}}$ | 0 | $\frac{i\,k\,t_1}{3}$ |
| $\omega_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{3\,\sqrt{2}}$ | $\frac{t_1}{3}$ | 0 | $\frac{1}{3}\,i\,\sqrt{2}\,k\,t_1$ |
| $f_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | $-\frac{1}{3}\,i\,k\,t_1$ | $-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_1$ | 0 | $\frac{2\,k^2\,t_1}{3}$ |

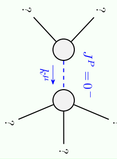
| | $\sigma_{2^+}^{\#1\,\alpha\beta\chi}$ | $\tau_{2^+}^{\#1\,\alpha\beta}$ | $\sigma_{2^+}^{\#1\,\alpha\beta}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1}\dagger^{\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{t_1}$ |
| $\tau_{2^+}^{\#1}\dagger^{\alpha\beta}$ | 0 | $\frac{4\,k^2}{(1+2\,k^2)^2\,t_1}$ | $-\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ |
| $\sigma_{2^+}^{\#1}\dagger^{\alpha\beta}$ | 0 | $\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ | $\frac{2}{(1+2\,k^2)^2\,t_1}$ |

| | $\omega_{2^+}^{\#1\,\alpha\beta\chi}$ | $f_{2^+}^{\#1\,\alpha\beta}$ | $\omega_{2^+}^{\#1\,\alpha\beta}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1}\dagger^{\alpha\beta\chi}$ | 0 | $\frac{i\,k\,t_1}{\sqrt{2}}$ | $\frac{t_1}{2}$ |
| $f_{2^+}^{\#1}\dagger^{\alpha\beta}$ | 0 | $k^2\,t_1$ | $-\frac{i\,k\,t_1}{\sqrt{2}}$ |
| $\omega_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{t_1}{2}$ | 0 | 0 |

| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | 0 |
| $f_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | 0 |
| $f_{0^+}^{\#2}\dagger$ | 0 | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1}\dagger$ | 0 | 0 | 0 | $k^2\,r_2 + t_2$ |

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^+}^{\#2}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#2}\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | $\frac{1}{k^2\,r_2+t_2}$ |

## Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $-\frac{t_2}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

(No massless particles)

## Unitarity conditions

$r_2 < 0\ \&\&\ t_2 > 0$