Particle spectrograph

Wave operator and propagator

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0^+}^{\#2} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$	1
$\tau_{0^+}^{\#1} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha$	1
$\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$	3
$\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$	$\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} + 2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$	3
$\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$	$-i\,(4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} + 4\,i\,k^\chi\,\partial_\epsilon\partial_\chi\partial^\beta\partial^\alpha\sigma^{\delta\epsilon}{}_\delta - 6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\delta\epsilon} - 6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} + 6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\alpha\delta\beta} + 6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\beta\delta\alpha} - 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi - 4\,i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\chi\sigma^{\delta\epsilon}{}_\delta) == 0$	5
Total constraints/gauge generators:		16

$\tau_{1^+}^{\#1\,\alpha\beta}$

$-\frac{i\sqrt{2}\,k}{t_1+k^2\,t_1}$	$\frac{-2\,i\,k^3\,(2\,r_3+r_5)+i\,k\,t_1}{(1+k^2)^2\,t_1^2}$
$\frac{-2\,i\,k^3\,(2\,r_3+r_5)+k^2\,t_1}{(1+k^2)^2\,t_1^2}$	0
0	0
0	0

$\sigma_{1^+}^{\#2\,\alpha\beta}$

$-\frac{\sqrt{2}}{t_1+k^2\,t_1}$	$\frac{-2\,k^2\,(2\,r_3+r_5)+t_1}{(1+k^2)^2\,t_1^2}$
$\frac{i\,(2\,k^3\,(2\,r_3+r_5)-k\,t_1)}{(1+k^2)^2\,t_1^2}$	0
0	0
0	0

$\sigma_{1^+}^{\#1\,\alpha\beta}$

0	$-\frac{\sqrt{2}}{t_1+k^2\,t_1}$
$\frac{i\,\sqrt{2}\,k}{t_1+k^2\,t_1}$	0
0	0
0	0

$\sigma_{1^-}^{\#1\,\alpha}$

$\frac{1}{k^2\,(2\,r_3+r_5)}$	$-\frac{1}{\sqrt{2}\,(k^2+2\,k^4)\,(2\,r_3+r_5)}$
$-\frac{1}{\sqrt{2}\,(k^2+2\,k^4)\,(2\,r_3+r_5)}$	$\frac{6\,k^2\,(2\,r_3+r_5)+t_1}{2\,(k+2\,k^3)^2\,(2\,r_3+r_5)\,t_1}$
0	0
0	$-\frac{i\,(6\,k^2\,(2\,r_3+r_5)+t_1)}{\sqrt{2}\,k\,(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$

$\sigma_{1^-}^{\#2\,\alpha}$

0	$-\frac{i}{k\,(1+2\,k^2)\,(2\,r_3+r_5)}$
0	$\frac{i\,(6\,k^2\,(2\,r_3+r_5)+t_1)}{\sqrt{2}\,k\,(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$
0	0
0	$\frac{6\,k^2\,(2\,r_3+r_5)+t_1}{(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$

$\tau_{1^-}^{\#1\,\alpha}$

0	$-\frac{i}{k\,(1+2\,k^2)\,(2\,r_3+r_5)}$
0	$\frac{i\,(6\,k^2\,(2\,r_3+r_5)+t_1)}{\sqrt{2}\,k\,(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$
0	0
0	$\frac{6\,k^2\,(2\,r_3+r_5)+t_1}{(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$

$\tau_{1^-}^{\#2\,\alpha}$

0	$-\frac{i}{k\,(1+2\,k^2)\,(2\,r_3+r_5)}$
0	$\frac{i\,(6\,k^2\,(2\,r_3+r_5)+t_1)}{\sqrt{2}\,k\,(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$
0	0
0	$\frac{6\,k^2\,(2\,r_3+r_5)+t_1}{(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$

$\sigma_{1^+}^{\#1\,\dagger\,\alpha\beta}$ $\sigma_{1^+}^{\#2\,\dagger\,\alpha\beta}$ $\tau_{1^+}^{\#1\,\dagger\,\alpha\beta}$ $\sigma_{1^-}^{\#1\,\dagger\,\alpha}$ $\sigma_{1^-}^{\#2\,\dagger\,\alpha}$ $\tau_{1^-}^{\#1\,\dagger\,\alpha}$ $\tau_{1^-}^{\#2\,\dagger\,\alpha}$

Quadratic (free) action

$$S == \int\int\int\int f^{\alpha\beta}\,\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + \frac{1}{6}\,t_1\,(2\,\omega^{\alpha\prime}{}_\alpha{}^\theta{}_{,\theta}\,\omega^{\alpha\prime}{}_\theta{}^\theta{}_{,\iota} - 4\,\omega_\alpha{}^\theta{}_{,\theta}\,\partial_\iota f^{\alpha\prime}{}_\theta + 4\,\omega_\iota{}^\theta{}_{,\theta}\,\partial^\prime f^\alpha{}_\alpha - 2\,\partial_\iota f^{\alpha\prime}{}_\theta\,\partial_\theta f^\alpha{}_\alpha + 4\,\partial^\prime f^\alpha{}_\alpha\,\partial_\theta f^\theta{}_\iota - 6\,\partial_\alpha f_{\iota\theta}\,\partial^\theta f^{\alpha\prime} - 2\,\partial_\iota f^{\alpha\prime}{}_\theta\,\partial_\theta f^\alpha{}_\alpha + 3\,\partial_\iota f^{\alpha\theta}\,\partial^\theta f^{\alpha\prime} + 3\,\partial_\theta f^{\alpha\iota}\,\partial^\theta f^{\alpha\prime} - 6\,\omega_{\alpha\theta\iota}\,(\omega^{\alpha\iota\theta} + 2\,\partial^\theta f^{\alpha\prime}) + \frac{1}{3}\,r_2\,(4\,\partial_\beta\omega_{\alpha\iota\theta} - 2\,\partial_\beta\omega_{\iota\theta\alpha} - \partial_\iota\omega_{\alpha\beta\theta} + \partial_\theta\omega_{\alpha\beta\iota} - 2\,\partial_\theta\omega_{\alpha\iota\beta})\,\partial^\theta\omega^{\alpha\beta\iota} + 2\,r_3\,(\partial_\beta\omega_\iota{}^\theta{}_\theta\,\partial^\prime\omega^{\alpha\beta}{}_\alpha + \partial_\iota\omega_\beta{}^\theta{}_\theta\,\partial^\prime\omega^{\alpha\beta}{}_\alpha + \partial_\alpha\omega^{\alpha\beta\iota}\,\partial_\theta\omega_\beta{}^\theta{}_\iota - \partial_\alpha\omega^{\alpha\beta}{}_\beta\,\partial_\theta\omega_\iota{}^\theta{}_\beta - \partial_\alpha\omega^{\alpha\beta}{}_\iota\,\partial_\theta\omega_\beta{}^\theta{}_\beta) + 2\,\partial^\prime\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_{\iota\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\iota} + 2\,\partial^\prime\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_\iota{}^\theta{}_\beta\,\partial^\theta\omega^{\alpha\beta\iota}) + r_5\,(\partial_\iota\omega_\theta{}^\kappa{}_\kappa\,\partial^\theta\omega^{\alpha\iota}{}_\alpha - \partial_\theta\omega_\iota{}^\kappa{}_\kappa\,\partial^\theta\omega^{\alpha\iota}{}_\alpha - (\partial_\alpha\omega^{\alpha\iota\theta} - 2\,\partial^\theta\omega^{\alpha\iota}{}_\alpha)\,(\partial_\kappa\omega_\iota{}^\kappa{}_\theta - \partial_\kappa\omega_\theta{}^\kappa{}_\iota))\,[t,\,x,\,y,\,z]\,dt\,dx\,dy\,dz$$

$\sigma_{2^+}^{\#1\,\alpha\beta}$ $\tau_{2^+}^{\#1\,\alpha\beta}$ $\sigma_{2^-}^{\#1\,\alpha\beta\chi}$

$\frac{2}{(1+2\,k^2)^2\,t_1}$	$-\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	0
$\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	$\frac{4\,k^2}{(1+2\,k^2)^2\,t_1}$	0
0	0	$\frac{2}{t_1}$

$\sigma_{2^+}^{\#1\,\dagger\,\alpha\beta}$ $\tau_{2^+}^{\#1\,\dagger\,\alpha\beta}$ $\sigma_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$

$\sigma_{0^+}^{\#1}$

0	0	0	$\frac{1}{k^2\,r_2\,t_1}$
0	0	0	0
0	0	0	0
$\frac{1}{6\,k^2\,r_3}$	0	0	0

$\sigma_{0^+}^{\#1\,\dagger}$ $\tau_{0^+}^{\#1\,\dagger}$ $\tau_{0^+}^{\#2\,\dagger}$ $\sigma_{0^-}^{\#1\,\dagger}$

$\omega_{2^+}^{\#1\,\alpha\beta}$

0	$-\frac{i\,k\,t_1}{\sqrt{2}}$	$\frac{t_1}{2}$
$\frac{i\,k\,t_1}{\sqrt{2}}$	$k^2\,t_1$	0
0	0	$\frac{t_1}{2}$

$f_{2^+}^{\#1\,\alpha\beta}$

$\frac{i\,k\,t_1}{\sqrt{2}}$	0	0
0	0	0
0	0	0

$\omega_{2^+}^{\#2\,\alpha\beta}$

0	$-\frac{i\,k\,t_1}{\sqrt{2}}$	$\frac{t_1}{2}$
$\frac{i\,k\,t_1}{\sqrt{2}}$	$k^2\,t_1$	0
0	0	$\frac{t_1}{2}$

$\omega_{2^+}^{\#1\,\dagger\,\alpha\beta}$ $f_{2^+}^{\#1\,\dagger\,\alpha\beta}$ $\omega_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$

$\omega_{0^+}^{\#1}$

0	0	0	0
0	0	0	0
0	0	0	0
$6\,k^2\,r_3$	0	0	$k^2\,r_2 - t_1$

$\omega_{0^+}^{\#1\,\dagger}$ $f_{0^+}^{\#1\,\dagger}$ $f_{0^+}^{\#2\,\dagger}$ $\omega_{0^-}^{\#1\,\dagger}$

$\omega_{1^+}^{\#1\,\alpha\beta}$

$k^2\,(2\,r_3 + r_5) + \frac{t_1}{6}$	$\frac{t_1}{3\,\sqrt{2}}$	0	$-\frac{1}{3}\,i\,k\,t_1$
$\frac{t_1}{3\,\sqrt{2}}$	0	0	0
0	0	0	$-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_1$
$-\frac{1}{3}\,i\,k\,t_1$	$-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_1$	0	$\frac{2\,k^2\,t_1}{3}$

$\omega_{1^+}^{\#2\,\alpha\beta}$

$\frac{i\,k\,t_1}{\sqrt{2}}$	0	0	$-\frac{i\,k\,t_1}{\sqrt{2}}$
0	0	0	0
0	0	0	0
$-\frac{i\,k\,t_1}{\sqrt{2}}$	0	0	$\frac{i\,k\,t_1}{\sqrt{2}}$

$f_{1^+}^{\#1\,\alpha\beta}$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$\omega_{1^-}^{\#1\,\alpha}$

0	0	0	$\frac{i\,k\,t_1}{3}$
0	0	0	$\frac{1}{3}\,i\,\sqrt{2}\,k\,t_1$
0	0	0	0
$\frac{t_1}{3\,\sqrt{2}}$	$\frac{t_1}{3}$	0	$\frac{2\,k^2\,t_1}{3}$

$\omega_{1^-}^{\#2\,\alpha}$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$f_{1^-}^{\#1\,\alpha}$

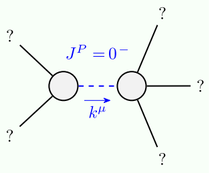
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$f_{1^-}^{\#2\,\alpha}$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

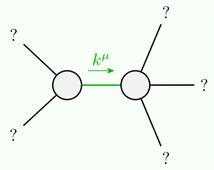
$\omega_{1^+}^{\#1\,\dagger\,\alpha\beta}$ $\omega_{1^+}^{\#2\,\dagger\,\alpha\beta}$ $f_{1^+}^{\#1\,\dagger\,\alpha\beta}$ $\omega_{1^-}^{\#1\,\dagger\,\alpha}$ $\omega_{1^-}^{\#2\,\dagger\,\alpha}$ $f_{1^-}^{\#1\,\dagger\,\alpha}$ $f_{1^-}^{\#2\,\dagger\,\alpha}$

Massive and massless spectra



Massive particle

Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$\frac{t_1}{r_2} > 0$
Spin:	0
Parity:	Odd



Quadratic pole

Pole residue:	$-\frac{1}{(2\,r_3+r_5)\,{t_1}^2} > 0$
Polarisations:	2

Unitarity conditions

$r_2 < 0$ && $r_5 < -2\,r_3$ && $t_1 < 0$