# Particle spectrograph

## Wave operator and propagator

| $\tau_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^+}^{\#2}{}_{\alpha\beta}$ | $\tau_{1^+}^{\#2}{}_{\alpha}$ | $\sigma_{1^-}^{\#1}{}_{\alpha}$ | $\sigma_{1^-}^{\#2}{}_{\alpha}$ | $\tau_{1^-}^{\#1}{}_{\alpha}$ | $\tau_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|
| $-\frac{i\sqrt{2}\,k}{t_1+k^2 t_1}$ | $-\frac{\sqrt{2}}{t_1+k^2 t_1}$ | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{i\sqrt{2}\,k}{t_1+k^2 t_1}$ | $\frac{i(2k^3 r_5-k t_1)}{(1+k^2)^2 t_1^2}$ | $\frac{-2k^4 r_5+k^2 t_1}{(1+k^2)^2 t_1^2}$ | $\frac{i}{k r_5+2k^3 r_5}$ | $\frac{i(6k^2 r_5+t_1)}{\sqrt{2}\,k(1+2k^2)^2 r_5 t_1}$ | 0 | $\frac{6k^2 r_5+t_1}{(1+2k^2)^2 r_5 t_1}$ |
| 0 | 0 | 0 | $-\frac{1}{\sqrt{2}\,(k^2 r_5+2k^4 r_5)}$ | $\frac{6k^2 r_5+t_1}{2(k+2k^3)^2 r_5 t_1}$ | 0 | $-\frac{i}{k r_5+2k^3 r_5}$ |
| 0 | 0 | 0 | $\frac{1}{k^2 r_5}$ | $-\frac{1}{\sqrt{2}\,(k^2 r_5+2k^4 r_5)}$ | $-\frac{i}{k r_5+2k^3 r_5}$ | 0 |

Quadratic (free) Lagrangian density

$$
\begin{aligned}
&-\tfrac{1}{3} t_1\, \omega_{\prime}^{\alpha\prime}{}_{\kappa\alpha}\, \omega_{\prime}^{\kappa}{}_{\kappa\lambda} - t_1\, \omega_{\prime}^{\kappa\lambda}{}_{\kappa}\, \omega_{\kappa\lambda}{}^{\prime} + f^{\alpha\beta}\,\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - r_5\,\partial_{\prime}\omega^{\kappa\lambda}{}_{\kappa}\,\partial^{\prime}\omega^{\alpha}{}_{\lambda\alpha} + \\
&\tfrac{2}{3} r_2\,\partial^{\beta}\omega^{\theta\alpha}{}_{\kappa}\,\partial_{\theta}\omega^{\kappa}{}_{\alpha\beta} - \tfrac{2}{3} r_2\,\partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\,\partial_{\kappa}\omega^{\theta\alpha\beta} - \\
&r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial_{\kappa}\omega^{\theta\kappa\lambda} - r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial_{\kappa}\omega^{\kappa\lambda\theta} + 2\,r_5\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial_{\kappa}\omega^{\kappa\lambda\theta} - \\
&\tfrac{1}{2} t_1\,\partial^{\alpha} f_{\theta\kappa}\,\partial^{\kappa} f_{\alpha}{}^{\theta} - \tfrac{1}{2} t_1\,\partial^{\alpha} f_{\kappa\theta}\,\partial^{\kappa} f_{\alpha}{}^{\theta} + t_1\,\omega_{\kappa\lambda}{}^{\lambda}\,\partial^{\kappa} f_{\prime}^{\prime} + \tfrac{2}{3} t_1\,\partial^{\alpha} f_{\kappa\alpha}\,\partial^{\kappa} f_{\prime}^{\prime} - \\
&\tfrac{1}{3} t_1\,\omega_{\kappa\lambda}{}^{\lambda}\,\partial^{\kappa} f_{\prime}^{\prime} + \tfrac{2}{3} t_1\,\partial^{\alpha} f_{\kappa\alpha}\,\partial^{\kappa} f_{\prime}^{\prime} - \tfrac{1}{3} t_1\,\omega_{\iota\alpha}{}^{\alpha}\,\partial^{\kappa} f_{\kappa}^{\prime} - \tfrac{1}{3} t_1\,\omega_{\iota\lambda}{}^{\lambda}\,\partial^{\kappa} f_{\kappa}^{\prime} + \\
&\tfrac{1}{2} t_1\,\partial_{\kappa} f^{\lambda}{}_{\theta}\,\partial^{\kappa} f_{\lambda}{}^{\theta} - \tfrac{1}{3} t_1\,\partial^{\alpha} f_{\lambda}{}^{\lambda}\,\partial^{\kappa} f_{\alpha\kappa} + \tfrac{2}{3} r_2\,\partial_{\kappa}\omega^{\alpha\beta\theta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} + \tfrac{2}{3} r_2\,\partial_{\kappa}\omega^{\theta\alpha\beta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} - \\
&\tfrac{2}{3} r_2\,\partial^{\beta}\omega_{\prime}^{\alpha\lambda}\,\partial_{\lambda}\omega_{\alpha\beta}{}^{\prime} + r_5\,\partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa} - r_5\,\partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}
\end{aligned}
$$

| $\omega_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^+}^{\#2}{}_{\alpha\beta}$ | $f_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^-}^{\#1}{}_{\alpha}$ | $\omega_{1^-}^{\#2}{}_{\alpha}$ | $f_{1^-}^{\#1}{}_{\alpha}$ | $f_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|
| $k^2 r_5 - \frac{t_1}{2}$ | $-\frac{t_1}{\sqrt{2}}$ | $\frac{i k t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $-\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\frac{i k t_1}{\sqrt{2}}$ | 0 | 0 | $-\frac{1}{3} i k t_1$ | $-\frac{t_1}{3\sqrt{2}}$ | $\frac{1}{3} i\sqrt{2}\,k t_1$ | $\frac{2k^2 t_1}{3}$ |
| 0 | 0 | 0 | $k^2 r_5 + \frac{t_1}{6}$ | $\frac{t_1}{3\sqrt{2}}$ | 0 | $\frac{i k t_1}{3}$ |
| 0 | 0 | 0 | $\frac{t_1}{3\sqrt{2}}$ | $\frac{t_1}{3}$ | 0 | $\frac{1}{3} i\sqrt{2}\,k t_1$ |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $-\frac{1}{3} i k t_1$ | $-\frac{1}{3} i\sqrt{2}\,k t_1$ | 0 | $\frac{2k^2 t_1}{3}$ |

| $\sigma_{2^+}^{\#1}{}_{\alpha\beta}$ | $\tau_{2^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|
| $\frac{2}{(1+2k^2)^2 t_1}$ | $-\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | 0 |
| $\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | $\frac{4k^2}{(1+2k^2)^2 t_1}$ | 0 |
| 0 | 0 | $\frac{2}{t_1}$ |

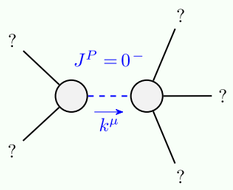
| $\omega_{2^+}^{\#1}{}_{\alpha\beta}$ | $f_{2^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|
| $\frac{t_1}{2}$ | $-\frac{i k t_1}{\sqrt{2}}$ | 0 |
| $\frac{i k t_1}{\sqrt{2}}$ | $k^2 t_1$ | 0 |
| 0 | 0 | $\frac{t_1}{2}$ |

| $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $\frac{1}{k^2 r_2 - t_1}$ |

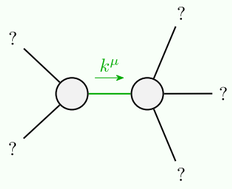
| $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | $k^2 r_2 - t_1$ |

| Source constraints/gauge generators | |
|---|---|
| SO(3) irreps | Multiplicities |
| $\tau_{0^+}^{\#2} == 0$ | 1 |
| $\tau_{0^+}^{\#1} == 0$ | 1 |
| $\sigma_{0^+}^{\#1} == 0$ | 1 |
| $\tau_{1^-}^{\#2\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\alpha} == 0$ | 3 |
| $\tau_{1^-}^{\#1\alpha} == 0$ | 3 |
| $\tau_{1^+}^{\#1\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$ | 3 |
| $\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$ | 5 |
| Total constraints: | 17 |

## Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $\frac{t_1}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |




| Quadratic pole | |
|---|---|
| Pole residue: | $-\frac{1}{r_5 {t_1}^2} > 0$ |
| Polarisations: | 2 |

## Unitarity conditions

$r_2 < 0 \;\&\&\; r_5 < 0 \;\&\&\; t_1 < 0$