Particle spectrograph

Wave operator and propagator

Quadratic (free) action

$$S_F == \int\int\int\int \frac{1}{6} \left(\begin{array}{l} (-6\,t_1\,\omega_{\iota}^{\alpha\prime}\,\omega_{\kappa\alpha}^{\ \ \kappa} - 2\,(t_1 - 2\,t_2)\,\omega_{\iota}^{\kappa\lambda}\,\omega_{\kappa\lambda}^{\ \ \prime} + 2\,t_1\,\omega_{\kappa\lambda}^{\ \ \prime}\,\omega^{\kappa\lambda}{}_{\iota}^{\prime} + 2\,t_2\,\omega_{\kappa\lambda}^{\ \ \prime}\,\omega^{\kappa\lambda}{}_{\iota}^{\prime} + \\ 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + 12\,r_1\,\partial_{\iota}\omega^{\kappa\lambda}\,\partial^{\prime}\omega_{\lambda\ \alpha}^{\ \alpha} - 4\,r_1\,\partial^{\beta}\omega^{\theta\alpha}\,\partial_{\theta}\omega_{\alpha\beta}^{\ \ \kappa} + 4\,r_2 \\ \partial^{\beta}\omega^{\theta\alpha}\,\partial_{\theta}\omega_{\alpha\beta}^{\ \ \kappa} - 4\,r_1\,\partial_{\theta}\omega_{\alpha\beta}^{\ \ \kappa}\,\partial_{\kappa}\omega^{\alpha\beta\theta} - 2\,r_2\,\partial_{\theta}\omega_{\alpha\beta}^{\ \ \kappa}\,\partial_{\kappa}\omega^{\alpha\beta\theta} + 4\,r_1\,\partial_{\theta}\omega_{\alpha\beta}^{\ \ \kappa}\,\partial_{\kappa}\omega^{\theta\alpha\beta} - \\ 4\,r_2\,\partial_{\theta}\omega_{\alpha\beta}^{\ \ \kappa}\,\partial_{\kappa}\omega^{\theta\alpha\beta} + 12\,r_1\,\partial_{\alpha}\omega_{\lambda\ \theta}^{\ \alpha}\,\partial_{\kappa}\omega^{\theta\kappa\lambda} - 12\,r_1\,\partial_{\theta}\omega_{\lambda\ \alpha}^{\ \alpha}\,\partial_{\kappa}\omega^{\theta\kappa\lambda} + \\ 12\,r_1\,\partial_{\alpha}\omega_{\lambda\ \theta}^{\ \alpha}\,\partial_{\kappa}\omega^{\kappa\lambda\theta} - 24\,r_1\,\partial_{\theta}\omega_{\lambda\ \alpha}^{\ \alpha}\,\partial_{\kappa}\omega^{\kappa\lambda\theta} - 2\,t_1\,\partial^{\alpha}f_{\theta\kappa}\,\partial^{\kappa}f_{\ \alpha}^{\theta} + \\ t_2\,\partial^{\alpha}f_{\theta\kappa}\,\partial^{\kappa}f_{\ \alpha}^{\theta} - 4\,t_1\,\partial^{\alpha}f_{\kappa\theta}\,\partial^{\kappa}f_{\ \alpha}^{\theta} - 2\,t_2\,\partial^{\alpha}f_{\kappa\theta}\,\partial^{\kappa}f_{\ \alpha}^{\theta} + 2\,t_1\,\partial^{\alpha}f_{\ \kappa}^{\lambda}\,\partial^{\kappa}f_{\alpha\lambda}^{\prime} + \\ t_2\,\partial^{\alpha}f_{\ \kappa}^{\lambda}\,\partial^{\kappa}f_{\alpha\lambda}^{\prime} + 6\,t_1\,\omega_{\kappa\alpha}^{\ \ \alpha}\,\partial^{\kappa}f_{\ \prime}^{\prime} + 6\,t_1\,\omega_{\kappa\lambda}^{\ \ \lambda}\,\partial^{\kappa}f_{\ \prime}^{\prime} + 12\,t_1\,\partial^{\alpha}f_{\kappa\alpha}\,\partial^{\kappa}f_{\ \prime}^{\prime} - \\ 6\,t_1\,\partial_{\kappa}f_{\ \lambda}^{\lambda}\,\partial^{\kappa}f_{\ \prime}^{\prime} + 2\,t_1\,\omega_{\prime\theta\kappa}\,\partial^{\kappa}f^{\prime\theta} + 2\,t_2\,\omega_{\prime\theta\kappa}\,\partial^{\kappa}f^{\prime\theta} + 8\,t_1\,\omega_{\prime\kappa\theta}\,\partial^{\kappa}f^{\prime\theta} - \\ 4\,t_2\,\omega_{\prime\kappa\theta}\,\partial^{\kappa}f^{\prime\theta} - 2\,t_2\,\omega_{\theta\prime\kappa}\,\partial^{\kappa}f^{\prime\theta} + 4\,t_1\,\omega_{\theta\kappa\prime}\,\partial^{\kappa}f^{\prime\theta} + \\ 4\,t_2\,\omega_{\theta\kappa\prime}\,\partial^{\kappa}f^{\prime\theta} - 6\,t_1\,\omega_{\prime\alpha}^{\ \ \alpha}\,\partial^{\kappa}f_{\ \kappa}^{\prime} - 6\,t_1\,\omega_{\prime\lambda}^{\ \ \lambda}\,\partial^{\kappa}f_{\ \kappa}^{\prime} + 2\,t_1\,\partial^{\alpha}f_{\ \kappa}^{\lambda}\,\partial^{\kappa}f_{\lambda\alpha}^{\prime} - \\ t_2\,\partial^{\alpha}f_{\ \kappa}^{\lambda}\,\partial^{\kappa}f_{\lambda\alpha}^{\prime} + 2\,t_1\,\partial_{\kappa}f_{\ \theta}^{\lambda}\,\partial^{\kappa}f_{\ \lambda}^{\theta} - 2\,t_2\,\partial_{\kappa}f_{\ \theta}^{\lambda}\,\partial^{\kappa}f_{\ \lambda}^{\theta} + 4\,t_1\,\partial_{\kappa}f_{\ \theta}^{\lambda}\,\partial^{\kappa}f_{\ \lambda}^{\theta} + \\ t_2\,\partial_{\kappa}f_{\ \theta}^{\lambda}\,\partial^{\kappa}f_{\ \lambda}^{\theta} - 6\,t_1\,\partial^{\alpha}f_{\ \lambda}^{\lambda}\,\partial^{\kappa}f_{\ \kappa}^{\prime} + 4\,r_1\,\partial_{\kappa}\omega^{\alpha\beta\theta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} + 2\,r_2\,\partial_{\kappa}\omega^{\alpha\beta\theta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} - \\ 4\,r_1\,\partial_{\kappa}\omega^{\theta\alpha\beta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} + 4\,r_2\,\partial_{\kappa}\omega^{\theta\alpha\beta}\,\partial^{\kappa}\omega_{\alpha\beta\theta} + 4\,r_1\,\partial^{\beta}\omega_{\ \ \ \iota}^{\alpha\lambda}\,\partial_{\lambda}\omega_{\alpha\beta}^{\ \ \prime} - \\ 4\,r_2\,\partial^{\beta}\omega_{\ \ \ \iota}^{\alpha\lambda}\,\partial_{\lambda}\omega_{\alpha\beta}^{\ \ \prime} - 16\,r_1\,\partial^{\beta}\omega_{\ \ \ \iota}^{\lambda\alpha}\,\partial_{\lambda}\omega_{\alpha\beta}^{\ \ \prime} + 4\,r_2\,\partial^{\beta}\omega_{\ \ \ \iota}^{\lambda\alpha}\,\partial_{\lambda}\omega_{\alpha\beta}^{\ \ \prime} - \\ 12\,r_1\,\partial_{\alpha}\omega_{\lambda\ \theta}^{\ \alpha}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}^{\prime} + 12\,r_1\,\partial_{\theta}\omega_{\lambda\ \alpha}^{\ \alpha}\,\partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}^{\prime} \end{array} \right) [t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt$$

$\sigma_{1^+}^{\#1}{}^{\alpha\beta}\,\sigma_{1^+}^{\#2}{}^{\alpha\beta}\,\tau_{1^+}^{\#1}{}^{\alpha\beta}\,\sigma_{1^-}^{\#1}{}^{\alpha}\,\sigma_{1^-}^{\#2}{}^{\alpha}\,\tau_{1^-}^{\#1}{}^{\alpha}\,\tau_{1^-}^{\#2}{}^{\alpha}$

$\sigma_{1^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{2\,(t_1+t_2)}{3\,t_1\,t_2}$	$\frac{\sqrt{2}\,(t_1-2\,t_2)}{3\,(1+k^2)\,t_1\,t_2}$	$-\frac{i\,\sqrt{2}\,k\,(t_1-2\,t_2)}{3\,(1+k^2)\,t_1\,t_2}$	0	0	0	0
$\sigma_{1^+}^{\#2}{}^{\dagger\alpha\beta}$	$\frac{\sqrt{2}\,(t_1-2\,t_2)}{3\,(1+k^2)\,t_1\,t_2}$	$\frac{t_1+4\,t_2}{3\,(1+k^2)^2\,t_1\,t_2}$	$-\frac{i\,k\,(t_1+4\,t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$	0	0	0	0
$\tau_{1^+}^{\#1}{}^{\dagger\alpha\beta}$	$-\frac{i\,\sqrt{2}\,k\,(t_1-2\,t_2)}{3\,(1+k^2)\,t_1\,t_2}$	$-\frac{i\,k\,(t_1+4\,t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$	$\frac{k^2\,(t_1+4\,t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$	0	0	0	0
$\sigma_{1^-}^{\#1}{}^{\dagger\alpha}$	0	0	0	0	$\frac{\sqrt{2}}{t_1+2\,k^2\,t_1}$	0	$\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$
$\sigma_{1^-}^{\#2}{}^{\dagger\alpha}$	0	0	0	$\frac{\sqrt{2}}{t_1+2\,k^2\,t_1}$	$\frac{2\,k^2\,r_1+t_1}{(t_1+2\,k^2\,t_1)^2}$	0	$\frac{i\,\sqrt{2}\,k\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$
$\tau_{1^-}^{\#1}{}^{\dagger\alpha}$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#2}{}^{\dagger\alpha}$	0	0	0	$-\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$	$-\frac{i\,\sqrt{2}\,k\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$	0	$\frac{2\,k^2\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$

$\omega_{1^+}^{\#1}{}^{\alpha\beta}\,\omega_{1^+}^{\#2}{}^{\alpha\beta}\,f_{1^+}^{\#1}{}^{\alpha\beta}\,\omega_{1^-}^{\#1}{}^{\alpha}\,\omega_{1^-}^{\#2}{}^{\alpha}\,f_{1^-}^{\#1}{}^{\alpha}\,f_{1^-}^{\#2}{}^{\alpha}$

$\omega_{1^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{1}{6}\,(t_1+4\,t_2)$	$-\frac{t_1-2\,t_2}{3\,\sqrt{2}}$	$\frac{i\,k\,(t_1-2\,t_2)}{3\,\sqrt{2}}$	0	0	0	$i\,k\,t_1$
$\omega_{1^+}^{\#2}{}^{\dagger\alpha\beta}$	$-\frac{t_1-2\,t_2}{3\,\sqrt{2}}$	$\frac{t_1+t_2}{3}$	$-\frac{1}{3}\,i\,k\,(t_1+t_2)$	0	0	0	0
$f_{1^+}^{\#1}{}^{\dagger\alpha\beta}$	$-\frac{i\,k\,(t_1-2\,t_2)}{3\,\sqrt{2}}$	$\frac{1}{3}\,i\,k\,(t_1+t_2)$	$\frac{1}{3}\,k^2\,(t_1+t_2)$	0	0	0	0
$\omega_{1^-}^{\#1}{}^{\dagger\alpha}$	0	0	0	$-k^2\,r_1-\frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$-i\,k\,t_1$
$\omega_{1^-}^{\#2}{}^{\dagger\alpha}$	0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
$f_{1^-}^{\#1}{}^{\dagger\alpha}$	0	0	0	0	0	0	0
$f_{1^-}^{\#2}{}^{\dagger\alpha}$	0	0	0	$i\,k\,t_1$	0	0	0

$\omega_{2^+}^{\#1}{}^{\alpha\beta}\,f_{2^+}^{\#1}{}^{\alpha\beta}\,\omega_{2^-}^{\#1}{}^{\alpha\beta\chi}$

$\omega_{2^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{i\,k\,t_1}{\sqrt{2}}$	0
$f_{2^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{i\,k\,t_1}{\sqrt{2}}$	$k^2\,t_1$	0
$\omega_{2^-}^{\#1}{}^{\dagger\alpha\beta\chi}$	0	0	$k^2\,r_1+\frac{t_1}{2}$

$\sigma_{2^+}^{\#1}{}^{\alpha\beta}\,\tau_{2^+}^{\#1}{}^{\alpha\beta}\,\sigma_{2^-}^{\#1}{}^{\alpha\beta\chi}$

$\sigma_{2^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{2}{(1+2\,k^2)^2\,t_1}$	$-\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	0
$\tau_{2^+}^{\#1}{}^{\dagger\alpha\beta}$	$\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	$\frac{4\,k^2}{(1+2\,k^2)^2\,t_1}$	0
$\sigma_{2^-}^{\#1}{}^{\dagger\alpha\beta\chi}$	0	0	$\frac{2}{2\,k^2\,r_1+t_1}$

$\omega_{0^+}^{\#1}{}^{\dagger}\,f_{0^+}^{\#1}{}^{\dagger}\,f_{0^+}^{\#2}{}^{\dagger}\,\omega_{0^-}^{\#1}{}^{\dagger}$

$\omega_{0^+}^{\#1}{}^{\dagger}$	$-t_1$	$i\,\sqrt{2}\,k\,t_1$	0	0
$f_{0^+}^{\#1}{}^{\dagger}$	$-i\,\sqrt{2}\,k\,t_1$	$-2\,k^2\,t_1$	0	0
$f_{0^+}^{\#2}{}^{\dagger}$	0	0	0	$k^2\,r_2+t_2$
$\omega_{0^-}^{\#1}{}^{\dagger}$	0	0	$k^2\,r_2+t_2$	0

$\sigma_{0^+}^{\#1}\,\tau_{0^+}^{\#2}\,\sigma_{0^-}^{\#1}$

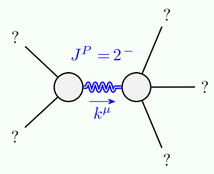
$\tau_{0^+}^{\#1}$	$\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	0	0
$\tau_{0^+}^{\#2}$	$\frac{2\,k^2}{(1+2\,k^2)^2\,t_1}$	0	0
$\sigma_{0^-}^{\#1}$	0	0	$\frac{1}{k^2\,r_2+t_2}$

$\sigma_{0^+}^{\#1}{}^{\dagger}\,\tau_{0^+}^{\#1}{}^{\dagger}\,\tau_{0^+}^{\#2}{}^{\dagger}\,\sigma_{0^-}^{\#1}{}^{\dagger}$

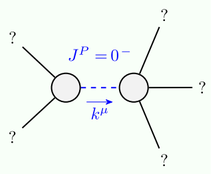
$\sigma_{0^+}^{\#1}{}^{\dagger}$	$-\frac{1}{(1+2\,k^2)^2\,t_1}$	$-\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	0
$\tau_{0^+}^{\#1}{}^{\dagger}$	$-\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	$-\frac{2\,k^2}{(1+2\,k^2)^2\,t_1}$	0
$\tau_{0^+}^{\#2}{}^{\dagger}$	0	0	0
$\sigma_{0^-}^{\#1}{}^{\dagger}$	0	0	$\frac{1}{k^2\,r_2+t_2}$

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau_{0^+}^{\#2} == 0$	1
$\tau_{0^+}^{\#1} - 2\,i\,k\,\sigma_{0^+}^{\#1} == 0$	1
$\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	3
$\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$	3
$\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$	5
Total constraints:	16

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2\,r_1} > 0$
Spin:	2
Parity:	Odd



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0 \;\&\&\; r_2 < 0 \;\&\&\; t_1 > 0 \;\&\&\; t_2 > 0$