

# Wave operator and propagator

Source constraints		
SO(3) irreps	Fundamental fields	Multiplicities
$\sigma_0^{\#1} == 0$	$\epsilon \eta_{\alpha\beta\chi\delta} \partial^\delta \sigma^{\alpha\beta\chi} == 0$	1
$\tau_{0+}^{\#2} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$	1
$\tau_{0+}^{\#1} - 2 i k \sigma_{0+}^{\#1} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha_\alpha + 2 \partial_\chi \partial^\chi \partial_\beta \sigma^{\alpha\beta}_\alpha$	1
$\tau_1^{\#2\alpha} + 2 i k \sigma_1^{\#2\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta} + 2 \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$	3
$\tau_1^{\#1\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$	3
$\tau_{1+}^{\#1\alpha\beta} == 0$	$\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\chi\alpha} + \partial_\chi \partial^\chi \tau^{\alpha\beta} ==$ $\partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} + \partial_\chi \partial^\chi \tau^{\beta\alpha}$	3
$\sigma_{1+}^{\#2\alpha\beta} == 0$	$\partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$	3
$\tau_{2+}^{\#1\alpha\beta} == 0$	$4 \partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2 \partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi_\chi +$ $3 \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3 \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} +$ $2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} == 3 \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\chi} +$ $3 \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\beta} + 3 \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi} +$ $3 \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\chi\alpha} + 2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \tau^\chi_\chi$	5
$\sigma_{2+}^{\#1\alpha\beta} == 0$	$3 \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 3 \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta} +$ $2 \eta^{\alpha\beta} \partial_\epsilon \partial^\epsilon \partial_\delta \sigma^{\chi\delta}_\chi == 2 \partial_\delta \partial^\beta \partial^\alpha \sigma^{\chi\delta}_\chi +$ $3 (\partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\chi\beta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\beta\chi\alpha})$	5
Total constraints/gauge generators:		25

$\sigma_1^{\#1} \dagger \alpha\beta$	$\frac{1}{k^2(2r_1+r_5)}$	$\sigma_{1+\alpha\beta}^{\#2}$	$\tau_{1+\alpha\beta}^{\#1}$	$\sigma_{1-\alpha}^{\#1}$	$\sigma_{1-\alpha}^{\#2}$	$\tau_{1-\alpha}^{\#1}$	$\tau_{1-\alpha}^{\#2}$
$\sigma_1^{\#1} \dagger \alpha\beta$	0	0	0	0	0	0	0
$\sigma_1^{\#2} \dagger \alpha\beta$	0	0	0	0	0	0	0
$\tau_1^{\#1} \dagger \alpha\beta$	0	0	0	0	0	0	0
$\sigma_1^{\#1} \dagger \alpha$	0	0	0	$\frac{1}{k^2(r_1+r_5)}$	$\frac{\sqrt{2}}{k^2(1+2k^2)(r_1+r_5)}$	0	$\frac{2i}{k(1+2k^2)(r_1+r_5)}$
$\sigma_1^{\#2} \dagger \alpha$	0	0	0	$\frac{\sqrt{2}}{k^2(1+2k^2)(r_1+r_5)}$	$\frac{3k^2(r_1+r_5)+2t_3}{(k+2k^3)^2(r_1+r_5)t_3}$	0	$\frac{i\sqrt{2}(3k^2(r_1+r_5)+2t_3)}{k(1+2k^2)^2(r_1+r_5)t_3}$
$\tau_1^{\#1} \dagger \alpha$	0	0	0	0	0	0	0
$\tau_1^{\#2} \dagger \alpha$	0	0	0	$-\frac{2i}{k(1+2k^2)(r_1+r_5)}$	$-\frac{i\sqrt{2}(3k^2(r_1+r_5)+2t_3)}{k(1+2k^2)^2(r_1+r_5)t_3}$	0	$\frac{6k^2(r_1+r_5)+4t_3}{(1+2k^2)^2(r_1+r_5)t_3}$

## Quadratic (free) action

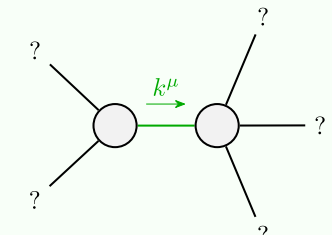
$$\begin{aligned}
S = & \iiint \left( \frac{1}{3} (-2t_3 \omega_{\alpha}^{\omega'} (-2t_3 \omega_{\alpha}^{\omega'} \omega_{\kappa}^{\kappa} + 3 f^{\alpha\beta} \tau_{\alpha\beta}^{\alpha\beta} + 3 \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi}^{\alpha\beta\chi} + 4 t_3 \omega_{\alpha}^{\kappa} \partial_{\kappa} f^{\alpha\iota} - \right. \\
& 4 t_3 \omega_{\iota}^{\kappa} \partial_{\kappa} f^{\alpha} - 2 t_3 \partial_{\iota} f^{\kappa} \partial' f^{\alpha} - 4 r_1 \partial_{\beta} \omega_{\alpha\iota\theta}^{\alpha\beta\iota} \partial^{\theta} \omega^{\alpha\beta\iota} + \\
& 2 r_1 \partial_{\beta} \omega_{\alpha\theta\iota}^{\alpha\beta\iota} \partial^{\theta} \omega_{\iota\theta\alpha}^{\alpha\beta\iota} - 8 r_1 \partial_{\beta} \omega_{\iota\theta\alpha}^{\alpha\beta\iota} \partial^{\theta} \omega^{\alpha\beta\iota} - 2 r_1 \partial_{\iota} \omega_{\alpha\beta\theta}^{\alpha\beta\iota} \\
& \partial^{\theta} \omega^{\alpha\beta\iota} + 2 r_1 \partial_{\theta} \omega_{\alpha\beta\iota}^{\alpha\beta\iota} \partial^{\theta} \omega^{\alpha\beta\iota} + 2 r_1 \partial_{\theta} \omega_{\alpha\iota\beta}^{\alpha\beta\iota} \partial^{\theta} \omega^{\alpha\beta\iota} + \\
& 3 r_5 \partial_{\iota} \omega_{\theta}^{\omega'} \partial^{\theta} \omega_{\alpha}^{\omega'} - 3 r_5 \partial_{\theta} \omega_{\alpha}^{\omega'} \partial^{\theta} \omega_{\iota}^{\omega'} + \\
& 2 t_3 \partial_{\iota} f^{\alpha\iota} \partial_{\kappa} f^{\kappa} - 4 t_3 \partial_{\iota} f^{\alpha} \partial_{\kappa} f^{\kappa} - 3 r_5 \partial_{\alpha} \omega^{\alpha\iota\theta} \partial_{\kappa} \omega_{\iota}^{\kappa} + \\
& 6 r_5 \partial^{\theta} \omega_{\alpha}^{\omega'} \partial_{\kappa} \omega_{\iota}^{\kappa} + 3 r_5 \partial_{\alpha} \omega^{\alpha\iota\theta} \partial_{\kappa} \omega_{\theta}^{\kappa} - \\
& \left. 6 r_5 \partial^{\theta} \omega_{\alpha}^{\omega'} \partial_{\kappa} \omega_{\theta}^{\kappa} \right) [t, x, y, z] dz dy dx dt
\end{aligned}$$

	$\omega_1^{\#1} + \alpha\beta$	$\omega_1^{\#2} + \alpha\beta$	$f_1^{\#1} + \alpha\beta$	$\omega_1^{\#1} - \alpha$	$\omega_1^{\#2} - \alpha$	$f_1^{\#1} - \alpha$	$f_1^{\#2} - \alpha$
$\omega_1^{\#1} + \alpha\beta$	$k^2(2r_1 + r_5)$	0	0	0	0	0	0
$\omega_1^{\#2} + \alpha\beta$	0	0	0	0	0	0	0
$f_1^{\#1} + \alpha\beta$	0	0	0	0	0	0	0
$\omega_1^{\#1} + \alpha$	0	0	0	$k^2(r_1 + r_5) + \frac{2t_3}{3}$	$-\frac{\sqrt{2}t_3}{3}$	0	$-\frac{2}{3}i k t_3$
$\omega_1^{\#2} + \alpha$	0	0	0	$-\frac{\sqrt{2}t_3}{3}$	$\frac{t_3}{3}$	0	$\frac{1}{3}i\sqrt{2} k t_3$
$f_1^{\#1} + \alpha$	0	0	0	0	0	0	0
$f_1^{\#2} + \alpha$	0	0	0	$\frac{2i k t_3}{3}$	$-\frac{1}{3}i\sqrt{2} k t_3$	0	$\frac{2k^2 t_3}{3}$

$$\begin{array}{c}
\begin{array}{ccc}
\omega_{2+}^{\#1} & f_{2+}^{\#1} & \omega_{2-}^{\#1} \\
\downarrow \uparrow^{\alpha\beta} & \downarrow \uparrow^{\alpha\beta} & \downarrow \uparrow^{\alpha\beta\chi}
\end{array}
\begin{array}{|c|c|c|}
\hline
0 & 0 & 0 \\
\hline
0 & 0 & 0 \\
\hline
0 & 0 & k^2 r_1 \\
\hline
\end{array}
\end{array}
\qquad
\begin{array}{c}
\begin{array}{ccc}
\sigma_{2+}^{\#1} & \tau_{2+}^{\#1} & \sigma_{2-}^{\#1} \\
\downarrow \uparrow^{\alpha\beta} & \downarrow \uparrow^{\alpha\beta} & \downarrow \uparrow^{\alpha\beta\chi}
\end{array}
\begin{array}{|c|c|c|}
\hline
0 & 0 & 0 \\
\hline
0 & 0 & 0 \\
\hline
0 & 0 & \frac{1}{k^2 r_1} \\
\hline
\end{array}
\end{array}$$
  

$$\begin{array}{c}
\begin{array}{ccc}
\sigma_{0+}^{\#1} & \tau_{0+}^{\#1} & \tau_{0+}^{\#2} & \sigma_{0-}^{\#1} \\
\downarrow \uparrow^{\alpha\beta} & \downarrow \uparrow^{\alpha\beta} & \downarrow \uparrow^{\alpha\beta} & \downarrow \uparrow^{\alpha\beta\chi}
\end{array}
\begin{array}{|c|c|c|c|}
\hline
\frac{1}{(1+2k^2)^2 t_3} & -\frac{i\sqrt{2}k}{(1+2k^2)^2 t_3} & 0 & 0 \\
\hline
\frac{i\sqrt{2}k}{(1+2k^2)^2 t_3} & \frac{2k^2}{(1+2k^2)^2 t_3} & 0 & 0 \\
\hline
0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 \\
\hline
\end{array}
\end{array}
\qquad
\begin{array}{c}
\begin{array}{ccc}
f_{0+}^{\#2} & \omega_{0-}^{\#1} & f_{0+}^{\#1} \\
\downarrow \uparrow^{\alpha\beta\chi} & \downarrow \uparrow^{\alpha\beta\chi} & \downarrow \uparrow^{\alpha\beta}
\end{array}
\begin{array}{|c|c|c|c|}
\hline
0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 \\
\hline
-i\sqrt{2}kt_3 & 2k^2 t_3 & 0 & 0 \\
\hline
t_3 & i\sqrt{2}kt_3 & 0 & 0 \\
\hline
\end{array}
\end{array}$$

Quadratic pole	
Pole residue:	$-\frac{1}{r_1(r_1+r_5)(2r_1+r_5)p^2} > 0$
Polarisations:	2



Quadratic pole	
Pole residue:	$-\frac{1}{r_1(r_1+r_5)(2r_1+r_5)p^2} > 0$
Polarisations:	2

(No massive particles)

$$r_1 < 0 \&\& (r_5 < -r_1 \parallel r_5 > -2r_1) \parallel r_1 > 0 \&\& -2r_1 < r_5 < -r_1$$

$$r_1 < 0 \&\& (r_5 < -r_1 \parallel r_5 > -2r_1) \parallel r_1 > 0 \&\& -2r_1 < r_5 < -r_1$$