

Particle spectrograph

Wave operator and propagator

	$\sigma_{1^+ \alpha \beta}^{\#1}$	$\sigma_{1^+ \alpha \beta}^{\#2}$	$\tau_{1^+ \alpha \beta}^{\#1}$	$\sigma_{1^+ \alpha}^{\#1}$	$\sigma_{1^+ \alpha}^{\#2}$	$\tau_{1^+ \alpha}^{\#1}$	$\tau_{1^+ \alpha}^{\#2}$
$\sigma_{1^+ \dagger \alpha \beta}^{\#1}$	0	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$-\frac{i \sqrt{2} k}{t_1+k^2 t_1}$	0	0	0	0
$\sigma_{1^+ \dagger \alpha \beta}^{\#2}$	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$\frac{-2 k^2 (2 r_1+r_5)+t_1}{(1+k^2)^2 t_1^2}$	$\frac{-2 i k^3 (2 r_1+r_5)+i k t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\tau_{1^+ \dagger \alpha \beta}^{\#1}$	$\frac{i \sqrt{2} k}{t_1+k^2 t_1}$	$\frac{i (2 k^3 (2 r_1+r_5)-k t_1)}{(1+k^2)^2 t_1^2}$	$\frac{-2 k^4 (2 r_1+r_5)+k^2 t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\sigma_{1^+ \dagger \alpha}^{\#1}$	0	0	0	$\frac{2 (t_1+t_3)}{3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3)}$	$-\frac{\sqrt{2} (t_1-2 t_3)}{(1+2 k^2) (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$	0	$-\frac{2 i k (t_1-2 t_3)}{(1+2 k^2) (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$
$\sigma_{1^+ \dagger \alpha}^{\#2}$	0	0	0	$-\frac{\sqrt{2} (t_1-2 t_3)}{(1+2 k^2) (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$	$\frac{6 k^2 (r_1+r_5)+t_1+4 t_3}{(1+2 k^2)^2 (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$	0	$\frac{i \sqrt{2} k (6 k^2 (r_1+r_5)+t_1+4 t_3)}{(1+2 k^2)^2 (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$
$\tau_{1^+ \dagger \alpha}^{\#1}$	0	0	0	0	0	0	0
$\tau_{1^+ \dagger \alpha}^{\#2}$	0	0	0	$\frac{2 i k (t_1-2 t_3)}{(1+2 k^2) (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$	$-\frac{i \sqrt{2} k (6 k^2 (r_1+r_5)+t_1+4 t_3)}{(1+2 k^2)^2 (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$	0	$\frac{2 k^2 (6 k^2 (r_1+r_5)+t_1+4 t_3)}{(1+2 k^2)^2 (3 t_1 t_3+2 k^2 (r_1+r_5) (t_1+t_3))}$

Source constraints		
SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0^+}^{\#2} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha \beta} == 0$	1
$\tau_{0^+}^{\#1} - 2 \, i \, k \, \sigma_{0^+}^{\#1} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha \beta} == \partial_\beta \partial^\beta \tau^\alpha_\alpha + 2 \, \partial_\chi \partial^X \partial_\beta \sigma^{\alpha \beta}_\alpha$	1
$\tau_1^{\#2 \alpha} + 2 \, i \, k \, \sigma_1^{\#2 \alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta \chi} == \partial_\chi \partial^X \partial_\beta \tau^{\alpha \beta} + 2 \, \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha \beta \chi}$	3
$\tau_1^{\#1 \alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta \chi} == \partial_\chi \partial^X \partial_\beta \tau^{\beta \alpha}$	3
$\tau_{1^+}^{\#1 \alpha \beta} + i \, k \, \sigma_{1^+}^{\#2 \alpha \beta} == 0$	$\partial_\chi \partial^\alpha \tau^{\beta \chi} + \partial_\chi \partial^\beta \tau^{\chi \alpha} + \partial_\chi \partial^\chi \tau^{\alpha \beta} +$ $2 \, \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta \chi \delta} + 2 \, \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha \beta \chi} ==$ $\partial_\chi \partial^\alpha \tau^{\chi \beta} + \partial_\chi \partial^\beta \tau^{\alpha \chi} +$ $\partial_\chi \partial^\chi \tau^{\beta \alpha} + 2 \, \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha \chi \delta}$	3
$\tau_{2^+}^{\#1 \alpha \beta} - 2 \, i \, k \, \sigma_{2^+}^{\#1 \alpha \beta} == 0$	$-i \, (4 \, \partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi \delta} + 2 \, \partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^{\chi}_\chi -$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta \chi} - 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi \beta} -$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha \chi} - 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\chi \alpha} +$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^X \tau^{\alpha \beta} + 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^X \tau^{\beta \alpha} +$ $4 \, i \, k^X \, \partial_\epsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta \epsilon}_\delta -$ $6 \, i \, k^X \, \partial_\epsilon \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta \delta \epsilon} -$ $6 \, i \, k^X \, \partial_\epsilon \partial_\delta \partial_\chi \partial^\alpha \sigma^{\alpha \delta \epsilon} +$ $2 \, \eta^{\alpha \beta} \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{\chi \delta} +$ $6 \, i \, k^X \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\alpha \delta \beta} +$ $6 \, i \, k^X \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\beta \delta \alpha} -$ $2 \, \eta^{\alpha \beta} \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \tau^{\chi}_\chi -$ $4 \, i \, \eta^{\alpha \beta} \, k^X \, \partial_\phi \partial^\phi \partial_\epsilon \partial_\chi \sigma^{\delta \epsilon}_\delta) == 0$	5
Total constraints/gauge generators:		16

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\omega_{0^+}^{\#1}$		$\omega_{2^+ \alpha \beta}^{\#1}$	$f_{2^+ \alpha \beta}^{\#1}$	$\omega_{2^+ \alpha \beta \chi}^{\#1}$
$\omega_{0^+ \dagger}^{\#1}$	t_3	$-i \sqrt{2} k t_3$	0	0	$\omega_{2^+ \dagger \alpha \beta}^{\#1}$	$\frac{t_1}{2}$	$-\frac{i k t_1}{\sqrt{2}}$	0
$f_{0^+ \dagger}^{\#1}$	$i \sqrt{2} k t_3$	$2 k^2 t_3$	0	0	$f_{2^+ \dagger \alpha \beta}^{\#1}$	$\frac{i k t_1}{\sqrt{2}}$	$k^2 t_1$	0
$f_{0^+ \dagger}^{\#2}$	0	0	0	0	$\omega_{2^+ \dagger \alpha \beta \chi}^{\#1}$	0	0	$k^2 r_1 + \frac{t_1}{2}$
$\omega_{0^+}^{\#1}$	0	0	0	$-t_1$				
	$\omega_{1^+ \alpha \beta}^{\#1}$	$\omega_{1^+ \alpha \beta}^{\#2}$	$f_{1^+ \alpha \beta}^{\#1}$		$\omega_{1^+ \alpha}^{\#1}$	$\omega_{1^+ \alpha}^{\#2}$	$f_{1^+ \alpha}^{\#1}$	$f_{1^+ \alpha}^{\#2}$
$\omega_{1^+ \dagger \alpha \beta}^{\#1}$	$k^2 (2 r_1 + r_5) - \frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{i k t_1}{\sqrt{2}}$		0	0	0	0
$\omega_{1^+ \dagger \alpha \beta}^{\#2}$	$-\frac{t_1}{\sqrt{2}}$	0	0		0	0	0	0
$f_{1^+ \dagger \alpha \beta}^{\#1}$	$\frac{i k t_1}{\sqrt{2}}$	0	0		0	0	0	0
$\omega_{1^+ \dagger \alpha}^{\#1}$	0	0	0		$\frac{1}{6} (6 k^2 (r_1 + r_5) + t_1 + 4 t_3)$	$\frac{t_1-2 t_3}{3 \sqrt{2}}$	0	$\frac{1}{3} i k (t_1 - 2 t_3)$
$\omega_{1^+ \dagger \alpha}^{\#2}$	0	0	0		$\frac{t_1-2 t_3}{3 \sqrt{2}}$	$\frac{t_1+t_3}{3}$	0	$\frac{1}{3} i \sqrt{2} k (t_1 + t_3)$
$f_{1^+ \dagger \alpha}^{\#1}$	0	0	0		0	0	0	0
$f_{1^+ \dagger \alpha}^{\#2}$	0	0	0		$-\frac{1}{3} i k (t_1 - 2 t_3)$	$-\frac{1}{3} i \sqrt{2} k (t_1 + t_3)$	0	$\frac{2}{3} k^2 (t_1 + t_3)$

Quadratic (free) action	
$S == \iiint \iiint (\frac{1}{6} (2 \, \omega^{\alpha i}_\alpha (t_1 \, \omega_{\mid \theta}^\theta - 2 \, t_3 \, \omega_{\mid \kappa}^\kappa) + 6 \, f^{\alpha \beta} \, \tau_{\alpha \beta} + 6 \, \omega^{\alpha \beta \chi} \, \sigma_{\alpha \beta \chi} -$ $4 \, t_1 \, \omega_\alpha^\theta \partial_{\mid f}^{\alpha i} + 8 \, t_3 \, \omega_\alpha^\kappa \partial_{\mid f}^{\alpha i} + 4 \, t_1 \, \omega_{\mid \theta}^\theta \partial' f^\alpha_\alpha -$ $8 \, t_3 \, \omega_{\mid \kappa}^\kappa \partial' f^\alpha_\alpha - 2 \, t_1 \partial_{\mid f} \omega_{\theta}^\theta \partial' f^\alpha_\alpha + 4 \, t_3 \partial_{\mid f} \omega_{\kappa}^\kappa \partial' f^\alpha_\alpha -$ $2 \, t_1 \partial_{\mid f}^{\alpha i} \partial_\theta f^\alpha_\theta + 4 \, t_1 \partial' f^\alpha_\alpha \partial_\theta f_{\mid \theta}^\theta - 6 \, t_1 \partial_\alpha f_{\mid \theta} \partial^\theta f^{\alpha i} -$ $3 \, t_1 \partial_\alpha f_{\theta \mid} \partial^\theta f^{\alpha i} + 3 \, t_1 \partial_{\mid f} \omega_{\alpha \theta} \partial^\theta f^{\alpha i} + 3 \, t_1 \partial_\theta f_{\alpha \mid} \partial^\theta f^{\alpha i} +$ $3 \, t_1 \partial_\theta f_{\mid \alpha} \partial^\theta f^{\alpha i} + 6 \, t_1 \, \omega_{\alpha \theta \mid} (\, \omega^{\alpha i \theta} + 2 \, \partial^\theta f^{\alpha i}) -$ $8 \, r_1 \, \partial_\beta \omega_{\alpha \mid \theta} \partial^\theta \omega^{\alpha \beta \mid} + 4 \, r_1 \, \partial_\beta \omega_{\alpha \theta \mid} \partial^\theta \omega^{\alpha \beta \mid} -$ $16 \, r_1 \, \partial_\beta \omega_{\mid \theta \alpha} \partial^\theta \omega^{\alpha \beta \mid} - 4 \, r_1 \, \partial_{\mid} \omega_{\alpha \beta \theta} \partial^\theta \omega^{\alpha \beta \mid} +$ $4 \, r_1 \, \partial_\theta \omega_{\alpha \beta \mid} \partial^\theta \omega^{\alpha \beta \mid} + 4 \, r_1 \, \partial_\theta \omega_{\alpha \mid \beta} \partial^\theta \omega^{\alpha \beta \mid} +$ $6 \, r_5 \, \partial_{\mid} \omega_{\theta \kappa}^\kappa \partial^\theta \omega^{\alpha i}_\alpha - 6 \, r_5 \, \partial_\theta \omega_{\mid \kappa}^\kappa \partial^\theta \omega^{\alpha i}_\alpha +$ $4 \, t_3 \partial_{\mid f}^{\alpha i} \partial_\kappa f^\alpha_\kappa - 8 \, t_3 \partial' f^\alpha_\alpha \partial_\kappa f_{\mid \kappa}^\kappa - 6 \, r_5 \, \partial_\alpha \omega^{\alpha i \theta} \partial_\kappa \omega_{\mid \theta}^\kappa +$ $12 \, r_5 \, \partial^\theta \omega^{\alpha i}_\alpha \partial_\kappa \omega_{\mid \theta}^\kappa + 6 \, r_5 \, \partial_\alpha \omega^{\alpha i \theta} \partial_\kappa \omega_{\theta \mid \kappa}^\kappa -$ $12 \, r_5 \, \partial^\theta \omega^{\alpha i}_\alpha \partial_\kappa \omega_{\theta \mid \kappa}^\kappa)) [t, \, x, \, y, \, z] \, d z \, d y \, d x \, d t$	

$\sigma_{2^+ \alpha \beta}^{\#1}$	$\tau_{2^+ \alpha \beta}^{\#1}$	$\sigma_{2^+ \alpha \beta \chi}^{\#1}$
0	0	$\frac{2}{2 k^2 r_1+t_1}$

$\sigma_{0^+ \dagger}^{\#1}$	$\tau_{0^+ \dagger}^{\#2}$	$\sigma_{0^+ \dagger}^{\#1}$
0	0	$-\frac{1}{t_1}$
0	0	0
0	0	0

Massive and massless spectra

Massive particle	
Pole residue:	$-\frac{3 \, (-2 \, t_1 \, t_3 \, (t_1+t_3)+r_1 \, (t_1^2+2 \, t_3^2))+r_5 \, (t_1^2+2 \, t_3^2))}{2 \, (r_1+r_5) \, (t_1+t_3) \, (-3 \, t_1 \, t_3+r_1 \, (t_1+t_3)+r_5 \, (t_1+t_3))} > 0$
Polarisations:	3
Square mass:	$-\frac{3 \, t_1 \, t_3}{2 \, (r_1+r_5) \, (t_1+t_3)} > 0$
Spin:	1
Parity:	Odd

Massive particle		(No massless particles)
Pole residue:	$-\frac{1}{r_1} > 0$	
Polarisations:	5	
Square mass:	$-\frac{t_1}{2 \, r_1} > 0$	
Spin:	2	
Parity:	Odd	

Unitarity conditions

$r_1 < 0 \, \&\& \, r_5 < -r_1 \, \&\& \, t_1 > 0 \, \&\& \, t_3 < -t_1 \, || \, t_3 > 0$