

# Particle spectrograph

## Wave operator and propagator

$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2} \alpha\beta$	$\tau_{1+}^{\#1} \alpha\beta$	$\sigma_{1-}^{\#1} \alpha$	$\sigma_{1-}^{\#2} \alpha$	$\tau_{1-}^{\#1} \alpha$	$\tau_{1-}^{\#2} \alpha$
$\dfrac{2(t_1+t_2)}{3t_1t_2+2k^2r_5(t_1+t_2)}$	$\dfrac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2r_5(t_1+t_2))}$	$\dfrac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2r_5(t_1+t_2))}$	0	0	0	0
$\dfrac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2r_5(t_1+t_2))}$	$\dfrac{6k^2r_5+t_1+4t_2}{(1+k^2)^2(3t_1t_2+2k^2r_5(t_1+t_2))}$	$\dfrac{ik(6k^2r_5+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2r_5(t_1+t_2))}$	0	0	0	0
$-\dfrac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2r_5(t_1+t_2))}$	$-\dfrac{ik(6k^2r_5+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2r_5(t_1+t_2))}$	$\dfrac{k^2(6k^2r_5+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2r_5(t_1+t_2))}$	0	0	0	0
0	0	0	0	$\dfrac{\sqrt{2}}{t_1+2k^2t_1}$	0	$\dfrac{2ik}{t_1+2k^2t_1}$
0	0	0	0	$\dfrac{-2k^2r_5+t_1}{(t_1+2k^2t_1)^2}$	0	$-\dfrac{i\sqrt{2}k(2k^2r_5-t_1)}{(t_1+2k^2t_1)^2}$
0	0	0	0	0	0	0
0	0	0	$-\dfrac{2ik}{t_1+2k^2t_1}$	$\dfrac{i\sqrt{2}k(2k^2r_5-t_1)}{(t_1+2k^2t_1)^2}$	0	$\dfrac{-4k^4r_5+2k^2t_1}{(t_1+2k^2t_1)^2}$

Quadratic (free) action

$$S_F = \iiint (\frac{1}{6} (-6t_1\omega_{\kappa}^{\alpha'}\omega_{\kappa\alpha}^{\kappa}-2(t_1-2t_2)\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\kappa\lambda}^{\kappa\lambda}+2t_1\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\kappa\lambda}^{\kappa\lambda}+2t_2\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\kappa\lambda}^{\kappa\lambda}+6f^{\alpha\beta}\tau_{\alpha\beta}+6\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}-6r_5\partial_{\lambda}\omega_{\lambda}^{\kappa\lambda}\partial'_{\kappa}\omega_{\lambda}^{\alpha}+4r_2\partial^{\beta}\omega_{\kappa}^{\theta\alpha}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}-2r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\theta\kappa\lambda}-4r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta}-6r_5\partial_5\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}+6r_5\partial_5\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}-6r_5\partial_5\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}+12r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}-2t_1\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}^{\theta}+t_2\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}^{\theta}-4t_1\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}^{\theta}-t_2\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}^{\theta}-2t_1\partial^{\alpha}f_{\kappa}^{\lambda}\partial^{\kappa}f_{\alpha\lambda}+t_2\partial^{\alpha}f_{\kappa}^{\lambda}\partial^{\kappa}f_{\alpha\lambda}+6t_1\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\lambda}^{\kappa\lambda}+6t_1\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\kappa\lambda}+12t_1\partial^{\alpha}f_{\kappa\alpha}\partial^{\kappa}f_{\lambda}^{\lambda}-6t_1\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\lambda}+2t_1\omega_{\theta\kappa}\partial^{\kappa}f^{\theta\lambda}+2t_2\omega_{\theta\kappa}\partial^{\kappa}f^{\theta\lambda}+8t_1\omega_{\lambda\theta}\partial^{\kappa}f^{\theta\lambda}-4t_2\omega_{\lambda\theta}\partial^{\kappa}f^{\theta\lambda}-2t_1\omega_{\theta\lambda}\partial^{\kappa}f^{\theta\lambda}-2t_2\omega_{\theta\lambda}\partial^{\kappa}f^{\theta\lambda}+4t_1\omega_{\theta\kappa}\partial^{\kappa}f^{\theta\lambda}+4t_2\omega_{\theta\kappa}\partial^{\kappa}f^{\theta\lambda}-6t_1\omega_{\lambda\alpha}^{\alpha}\partial^{\kappa}f_{\kappa}^{\lambda}-6t_1\omega_{\lambda\lambda}^{\lambda}\partial^{\kappa}f_{\kappa}^{\lambda}+2t_1\partial^{\alpha}f_{\kappa}^{\lambda}\partial^{\kappa}f_{\lambda\alpha}-t_2\partial^{\alpha}f_{\kappa}^{\lambda}\partial^{\kappa}f_{\lambda\alpha}+2t_1\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\lambda}-t_2\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\lambda}+4t_1\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\lambda}+t_2\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\lambda}-6t_1\partial^{\alpha}f_{\alpha}^{\lambda}\partial^{\kappa}f_{\lambda\kappa}+2r_2\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+4r_2\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}-4r_2\partial_2\partial^{\beta}\omega_{\lambda}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\kappa\lambda}+4r_2\partial_2\partial^{\beta}\omega_{\lambda}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\kappa\lambda}+6r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\theta}^{\theta\kappa}-6r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\kappa}^{\theta\kappa})[t,x,y,z]dzdydxdt$$

	$\omega_{1+}^{\#1} \alpha\beta$	$\omega_{1+}^{\#2} \alpha\beta$	$f_{1+}^{\#1} \alpha\beta$	$\omega_{1-}^{\#1} \alpha$	$\omega_{1-}^{\#2} \alpha$	$f_{1-}^{\#1} \alpha$	$f_{1-}^{\#2} \alpha$
$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{1}{6}(6k^2r_5+t_1+4t_2)$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$-\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	0	0	0	0
$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$\frac{t_1+t_2}{3}$	$\frac{1}{3}ik(t_1+t_2)$	0	0	0	0
$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	$-\frac{1}{3}ik(t_1+t_2)$	$\frac{1}{3}k^2(t_1+t_2)$	0	0	0	0
$\omega_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	$k^2r_5-\frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$ik t_1$
$\omega_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
$f_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$f_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$-ik t_1$	0	0	0

	$\sigma_{2+}^{\#1} \alpha\beta$	$\tau_{2+}^{\#1} \alpha\beta$	$\sigma_{2-}^{\#1} \alpha\beta\chi$
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{t_1}$

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	$-\frac{1}{(1+2k^2)^2t_1}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	0	0
$\tau_{0+}^{\#1} \dagger$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0	0
$\tau_{0+}^{\#2} \dagger$	0	0	0	0
$\sigma_{0-}^{\#1} \dagger$	0	0	0	$\frac{1}{k^2r_2+t_2}$

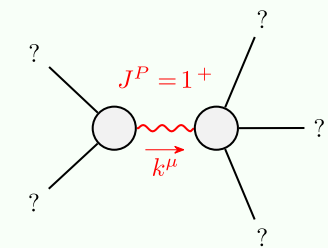
	$\omega_{0+}^{\#1}$	$f_{0+}^{\#1}$	$f_{0+}^{\#2}$	$\omega_{0-}^{\#1}$
$\omega_{0+}^{\#1} \dagger$	$-t_1$	$i\sqrt{2}kt_1$	0	0
$f_{0+}^{\#1} \dagger$	$-i\sqrt{2}kt_1$	$-2k^2t_1$	0	0
$f_{0+}^{\#2} \dagger$	0	0	0	0
$\omega_{0-}^{\#1} \dagger$	0	0	0	$k^2r_2+t_2$

### Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	16

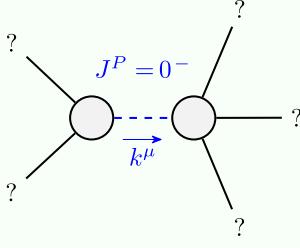
	$\omega_{2+}^{\#1} \alpha\beta$	$f_{2+}^{\#1} \alpha\beta$	$\omega_{2-}^{\#1} \alpha\beta\chi$
$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	$k^2t_1$	0
$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{t_1}{2}$

## Massive and massless spectra



### Massive particle

Pole residue:	$\frac{-3t_1t_2(t_1+t_2)+3r_5(t_1^2+2t_2^2)}{r_5(t_1+t_2)(-3t_1t_2+2r_5(t_1+t_2))} > 0$
Polarisations:	3
Square mass:	$-\frac{3t_1t_2}{2r_5t_1+2r_5t_2} > 0$
Spin:	1
Parity:	Even



### Massive particle

Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

## Unitarity conditions

$$r_2 < 0 \&\& r_5 > 0 \&\& t_1 < 0 \&\& t_2 > -t_1$$