# Particle spectrograph

## Wave operator and propagator

Source constraints

| SO(3) irreps | Fundamental fields | Multiplicities |
|---|---|---|
| $\tau_{0^+}^{\#2} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$ | 1 |
| $\tau_{0^+}^{\#1} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$ | $\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} + 2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$ | 3 |
| $\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$ | $-i\,(4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} + 4\,i\,k^\chi\,\partial_\epsilon\partial_\chi\partial^\beta\partial^\alpha\sigma^{\delta\epsilon}{}_\delta - 6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\delta\epsilon} - 6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} + 6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\alpha\delta\beta} + 6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\beta\delta\alpha} - 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi - 4\,i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\chi\sigma^{\delta\epsilon}{}_\delta) == 0$ | 5 |
| Total constraints/gauge generators: | | 16 |

$\sigma_{1^+}^{\#1\,\alpha\beta}$ †

| | $\sigma_{1^+}^{\#1\,\alpha\beta}$ † | $\sigma_{1^+}^{\#2\,\alpha\beta}$ † | $\tau_{1^+}^{\#1\,\alpha\beta}$ † | $\sigma_{1^-}^{\#1\,\alpha}$ † | $\sigma_{1^-}^{\#2\,\alpha}$ † | $\tau_{1^-}^{\#1\,\alpha}$ † | $\tau_{1^-}^{\#2\,\alpha}$ † |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1\,\alpha\beta}$ | 0 | $-\frac{\sqrt{2}}{t_1+k^2\,t_1}$ | $-\frac{i\,\sqrt{2}\,k}{t_1+k^2\,t_1}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2\,\alpha\beta}$ | $-\frac{\sqrt{2}}{t_1+k^2\,t_1}$ | $-\frac{2\,k^2\,(2\,r_3+r_5)+i\,k\,t_1}{(1+k^2)^2\,t_1^2}$ | $-\frac{2\,i\,k^3\,(2\,r_3+r_5)+i\,k^2\,t_1}{(1+k^2)^2\,t_1^2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1\,\alpha\beta}$ | $\frac{i\,\sqrt{2}\,k}{t_1+k^2\,t_1}$ | $-\frac{2\,i\,k^3\,(2\,r_3+r_5)+k^2\,t_1}{(1+k^2)^2\,t_1^2}$ | $-\frac{2\,k^4\,(2\,r_3+r_5)+k^2\,t_1}{(1+k^2)^2\,t_1^2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1\,\alpha}$ | 0 | 0 | 0 | $-\frac{1}{\sqrt{2}\,(k^2+2\,k^4)\,(2\,r_3+r_5)}$ | $\frac{6\,k^2\,(2\,r_3+r_5)+t_1}{2\,(k+2\,k^3)^2\,(2\,r_3+r_5)\,t_1}$ | 0 | $-\frac{i\,(6\,k^2\,(2\,r_3+r_5)+t_1)}{\sqrt{2}\,k\,(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$ |
| $\sigma_{1^-}^{\#2\,\alpha}$ | 0 | 0 | 0 | $\frac{6\,k^2\,(2\,r_3+r_5)+t_1}{2\,(k+2\,k^3)^2\,(2\,r_3+r_5)\,t_1}$ | $-\frac{1}{\sqrt{2}\,(k^2+2\,k^4)\,(2\,r_3+r_5)}$ | 0 | $\frac{i\,(6\,k^2\,(2\,r_3+r_5)+t_1)}{\sqrt{2}\,k\,(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$ |
| $\tau_{1^-}^{\#1\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | $\frac{1}{k^2\,(2\,r_3+r_5)}$ | $-\frac{i}{k\,(1+2\,k^2)\,(2\,r_3+r_5)}$ |
| $\tau_{1^-}^{\#2\,\alpha}$ | 0 | 0 | 0 | $-\frac{i}{k\,(1+2\,k^2)\,(2\,r_3+r_5)}$ | $\frac{i\,(6\,k^2\,(2\,r_3+r_5)+t_1)}{\sqrt{2}\,k\,(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$ | 0 | $\frac{6\,k^2\,(2\,r_3+r_5)+t_1}{(1+2\,k^2)^2\,(2\,r_3+r_5)\,t_1}$ |

$\sigma_{2^+}^{\#1\,\alpha\beta}$ †

| | $\sigma_{2^+}^{\#1\,\alpha\beta}$ † | $\tau_{2^+}^{\#1\,\alpha\beta}$ † | $\sigma_{2^-}^{\#1\,\alpha\beta\chi}$ † |
|---|---|---|---|
| $\sigma_{2^+}^{\#1\,\alpha\beta}$ | $\frac{2}{(1+2\,k^2)^2\,t_1}$ | $-\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ | 0 |
| $\tau_{2^+}^{\#1\,\alpha\beta}$ | $\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$ | $\frac{4\,k^2}{(1+2\,k^2)^2\,t_1}$ | 0 |
| $\sigma_{2^-}^{\#1\,\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{t_1}$ |

$\omega_{2^+}^{\#1\,\alpha\beta}$ †

| | $\omega_{2^+}^{\#1\,\alpha\beta}$ † | $f_{2^+}^{\#1\,\alpha\beta}$ † | $\omega_{2^-}^{\#1\,\alpha\beta\chi}$ † |
|---|---|---|---|
| $\omega_{2^+}^{\#1\,\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{i\,k\,t_1}{\sqrt{2}}$ | 0 |
| $f_{2^+}^{\#1\,\alpha\beta}$ | $\frac{i\,k\,t_1}{\sqrt{2}}$ | $k^2\,t_1$ | 0 |
| $\omega_{2^-}^{\#1\,\alpha\beta\chi}$ | 0 | 0 | $\frac{t_1}{2}$ |

$\sigma_{0^+}^{\#1}$ †

| | $\sigma_{0^+}^{\#1}$ † | $\tau_{0^+}^{\#1}$ † | $\tau_{0^+}^{\#2}$ † | $\sigma_{0^-}^{\#1}$ † |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}$ | $\frac{1}{6\,k^2\,r_3}$ | 0 | 0 | 0 |
| $\tau_{0^+}^{\#1}$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#2}$ | 0 | 0 | 0 | $-\frac{1}{t_1}$ |
| $\sigma_{0^-}^{\#1}$ | 0 | 0 | $-\frac{1}{t_1}$ | 0 |

$\omega_{0^+}^{\#1}$ †

| | $\omega_{0^+}^{\#1}$ † | $f_{0^+}^{\#1}$ † | $f_{0^+}^{\#2}$ † | $\omega_{0^-}^{\#1}$ † |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1}$ | $6\,k^2\,r_3$ | 0 | 0 | 0 |
| $f_{0^+}^{\#1}$ | 0 | 0 | 0 | 0 |
| $f_{0^+}^{\#2}$ | 0 | 0 | 0 | $-t_1$ |
| $\omega_{0^-}^{\#1}$ | 0 | 0 | $-t_1$ | 0 |

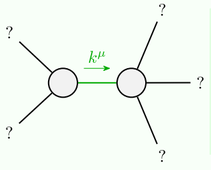
Quadratic (free) action

$$S == \int\int\int\int (f^{\alpha\beta}\,\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + \frac{1}{6}\,t_1\,(2\,\omega^{\alpha\iota}{}_\alpha\,\omega_\iota{}^\theta{}_\theta - 4\,\omega_\alpha{}^\theta{}_\theta\,\partial_\iota f^{\alpha\iota} + 4\,\omega_\iota{}^\theta{}_\theta\,\partial_\iota f^\alpha{}_\alpha - 2\,\partial_\iota f^\alpha{}_\theta\,\partial^\theta f^{\alpha\iota} + 4\,\partial^\iota f^\alpha{}_\alpha\,\partial_\theta f_\iota{}^\theta - 6\,\partial_\alpha f_\iota{}^\theta\,\partial_\theta f^{\alpha\iota} - 3\,\partial_\theta f_{\alpha\iota}\,\partial^\theta f^{\alpha\iota} + 2\,\partial^\theta f^{\alpha\iota}) - 2\,r_3\,(\partial_\beta\omega_\iota{}^\theta{}_\theta\,\partial^\iota\omega^{\alpha\beta}{}_\alpha + \partial_\iota\omega_\beta{}^\theta{}_\theta\,\partial^\iota\omega^{\alpha\beta}{}_\alpha + 2\,\partial^\iota\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_\iota{}^\theta{}_\beta + 2\,\partial^\iota\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_\beta{}^\theta{}_\iota + 2\,\partial_\theta\omega_\alpha{}^\theta{}_\beta\,\partial^\iota\omega^{\alpha\beta}{}_\iota + 2\,\partial_\theta\omega_\alpha{}^\theta{}_\iota\,\partial^\iota\omega^{\alpha\beta}{}_\beta + 2\,\partial_\theta\omega_\iota{}^\theta{}_\beta\,\partial^\iota\omega^{\alpha\beta}{}_\alpha + 2\,\partial_\beta\omega_{\iota\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\iota}) + r_5\,(\partial_\iota\omega_\theta{}^\kappa\,\partial^\theta\omega^{\alpha\iota}{}_\alpha\,\partial_\theta\omega_\iota{}^\kappa{}_\alpha - (\partial_\alpha\omega^{\alpha\iota\theta} - 2\,\partial^\theta\omega^{\alpha\iota}{}_\alpha)\,(\partial_\kappa\omega_\iota{}^\kappa{}_\theta - \partial_\kappa\omega^\kappa{}_{\theta\,\iota}))))[t, x, y, z]\,dz\,dy\,dx\,dt$$

$\omega_{1^+}^{\#1\,\alpha\beta}$ †

| | $\omega_{1^+}^{\#1\,\alpha\beta}$ † | $\omega_{1^+}^{\#2\,\alpha\beta}$ † | $f_{1^+}^{\#1\,\alpha\beta}$ † | $\omega_{1^-}^{\#1\,\alpha}$ † | $\omega_{1^-}^{\#2\,\alpha}$ † | $f_{1^-}^{\#1\,\alpha}$ † | $f_{1^-}^{\#2\,\alpha}$ † |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1\,\alpha\beta}$ | $k^2\,(2\,r_3+r_5)-\frac{t_1}{2}$ | $-\frac{t_1}{\sqrt{2}}$ | $-\frac{i\,k\,t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2\,\alpha\beta}$ | $-\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1\,\alpha\beta}$ | $\frac{i\,k\,t_1}{\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1\,\alpha}$ | 0 | 0 | 0 | $k^2\,(2\,r_3+r_5)+\frac{t_1}{6}$ | $\frac{t_1}{3\,\sqrt{2}}$ | $\frac{i\,k\,t_1}{3}$ | 0 |
| $\omega_{1^-}^{\#2\,\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{3\,\sqrt{2}}$ | $\frac{t_1}{3}$ | $\frac{1}{3}\,i\,\sqrt{2}\,k\,t_1$ | 0 |
| $f_{1^-}^{\#1\,\alpha}$ | 0 | 0 | 0 | 0 | $\frac{1}{3}\,i\,\sqrt{2}\,k\,t_1$ | 0 | $\frac{2\,k^2\,t_1}{3}$ |
| $f_{1^-}^{\#2\,\alpha}$ | 0 | 0 | 0 | 0 | 0 | $\frac{2\,k^2\,t_1}{3}$ | $-\frac{1}{3}\,i\,k\,t_1$ |

## Massive and massless spectra




Quadratic pole

| | |
|---|---|
| Pole residue: | $-\frac{1}{(2\,r_3+r_5)\,t_1^2} > 0$ |
| Polarisations: | 2 |

(No massive particles)

## Unitarity conditions

$r_5 < -2\,r_3\;\&\&\;t_1 < 0\,||\,t_1 > 0$