Particle spectrograph

Wave operator and propagator

Left matrix — spin-1 sector (rows top→bottom: $\tau_{1^-}^{\#2\,\alpha}$, $\tau_{1^-}^{\#1\,\alpha}$, $\sigma_{1^-}^{\#2\,\alpha}$, $\sigma_{1^-}^{\#1\,\alpha}$, $\tau_{1^+}^{\#1\,\alpha\beta}$, $\sigma_{1^+}^{\#2\,\alpha\beta}$, $\sigma_{1^+}^{\#1\,\alpha\beta}$):

	$\sigma_{1^+}^{\#1}{}^{\dagger\,\alpha\beta}$	$\sigma_{1^+}^{\#2}{}^{\dagger\,\alpha\beta}$	$\tau_{1^+}^{\#1}{}^{\dagger\,\alpha\beta}$	$\sigma_{1^-}^{\#1}{}^{\dagger\,\alpha}$	$\sigma_{1^-}^{\#2}{}^{\dagger\,\alpha}$	$\tau_{1^-}^{\#1}{}^{\dagger\,\alpha}$	$\tau_{1^-}^{\#2}{}^{\dagger\,\alpha}$
$\tau_{1^-}^{\#2\,\alpha}$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#1\,\alpha}$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#2\,\alpha}$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#1\,\alpha}$	0	0	0	0	0	0	0
$\tau_{1^+}^{\#1\,\alpha\beta}$	$-\frac{i\sqrt{2}}{k(1+k^2)(2r_3-r_4)}$	$\frac{i(k^2(6r_3-3r_4)+2t_2)}{k(1+k^2)^2(2r_3-r_4)t_2}$	$\frac{\frac{1}{r_3-\frac{r_4}{2}}+\frac{3k^2}{t_2}}{(1+k^2)^2}$	0	0	0	0
$\sigma_{1^+}^{\#2\,\alpha\beta}$	$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3-r_4)}$	$\frac{k^2(6r_3-3r_4)+2t_2}{(k+k^3)^2(2r_3-r_4)t_2}$	$-\frac{i(k^2(6r_3-3r_4)+2t_2)}{k(1+k^2)^2(2r_3-r_4)t_2}$	0	0	0	0
$\sigma_{1^+}^{\#1\,\alpha\beta}$	$\frac{1}{k^2(2r_3-r_4)}$	$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3-r_4)}$	$\frac{i\sqrt{2}}{k(1+k^2)(2r_3-r_4)}$	0	0	0	0

Quadratic (free) action

$$
\begin{aligned}
S_{\mathrm{F}} == \iiiint \Big[&\tfrac{1}{6}\big(4t_2\,\omega_i{}^{\kappa\lambda}\omega_{\kappa\lambda}{}^i + 2t_2\,\omega_{\kappa\lambda}{}^i\,\omega_i{}^{\kappa\lambda} + \\
&6 f^{\alpha\beta}\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi} + 4r_2\,\partial^\beta\omega_\theta{}^{\theta\alpha}\partial_\kappa\omega_{\alpha\beta}{}^\kappa - 2r_2\,\partial_\theta\omega_\alpha{}^\kappa\partial_\kappa\omega_{\alpha\beta\theta} - \\
&4r_2\,\partial_\theta\omega_{\alpha\beta}{}^\kappa\partial_\kappa\omega_\lambda{}^\alpha{}_\theta\partial_\kappa\omega^{\theta\kappa\lambda} + 12r_4\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\partial_\kappa\omega^{\theta\kappa\lambda} - 12r_4\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\partial_\kappa\omega^{\theta\kappa\lambda} + \\
&t_2\,\partial^\alpha f_{\theta\kappa}\partial^\kappa f_\alpha{}^\theta - t_2\,\partial^\alpha f_{\kappa\theta}\partial^\kappa f_\alpha{}^\theta + t_2\,\partial^\alpha f_\kappa{}^\lambda\partial^\kappa f_{\alpha\lambda} + 2t_2\,\omega_{i\theta\kappa}\partial^\kappa f^{i\theta} - \\
&4t_2\,\omega_{i\kappa\theta}\partial^\kappa f^{i\theta} - 2t_2\,\omega_{\theta i\kappa}\partial^\kappa f^{i\theta} + 4t_2\,\omega_{\theta\kappa i}\partial^\kappa f_\lambda{}^\theta - t_2\,\partial^\alpha f_\kappa{}^\lambda\partial^\kappa f_{\lambda\alpha} - \\
&t_2\,\partial_\kappa f_\lambda{}^\theta\partial^\kappa f_\lambda{}^\theta + t_2\,\partial_\kappa f_\lambda{}^\theta + 2r_2\,\partial_\kappa\omega^{\alpha\beta\theta}\partial^\kappa\omega_{\alpha\beta\theta} + 4r_2\,\partial_\kappa\omega^{\theta\alpha\beta}\partial^\kappa\omega_{\alpha\beta\theta} - \\
&4r_2\,\partial^\beta\omega_i{}^{\alpha\lambda}\partial_\lambda\omega_{\alpha\beta}{}^i + 4r_2\,\partial^\beta\omega_i{}^{\lambda\alpha}\partial_\lambda\omega_{\alpha\beta}{}^i - 24r_3\,\partial^\beta\omega_i{}^{\lambda\alpha}\partial_\lambda\omega_{\alpha\beta}{}^i - \\
&12r_4\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\partial^\lambda\omega^{\theta\kappa}{}_\kappa + 12r_4\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\partial^\lambda\omega^{\theta\kappa}{}_\kappa\big)\Big][t,x,y,z]\,dz\,dy\,dx\,dt
\end{aligned}
$$

Middle matrix — spin-1 sector (rows top→bottom: $f_{1^-}^{\#2\,\alpha}$, $f_{1^-}^{\#1\,\alpha}$, $\omega_{1^-}^{\#2\,\alpha}$, $\omega_{1^-}^{\#1\,\alpha}$, $f_{1^+}^{\#1\,\alpha\beta}$, $\omega_{1^+}^{\#2\,\alpha\beta}$, $\omega_{1^+}^{\#1\,\alpha\beta}$):

	$\omega_{1^+}^{\#1}{}^{\dagger\,\alpha\beta}$	$\omega_{1^+}^{\#2}{}^{\dagger\,\alpha\beta}$	$f_{1^+}^{\#1}{}^{\dagger\,\alpha\beta}$	$\omega_{1^-}^{\#1}{}^{\dagger\,\alpha}$	$\omega_{1^-}^{\#2}{}^{\dagger\,\alpha}$	$f_{1^-}^{\#1}{}^{\dagger\,\alpha}$	$f_{1^-}^{\#2}{}^{\dagger\,\alpha}$
$f_{1^-}^{\#2\,\alpha}$	0	0	0	0	0	0	0
$f_{1^-}^{\#1\,\alpha}$	0	0	0	0	0	0	0
$\omega_{1^-}^{\#2\,\alpha}$	0	0	0	0	0	0	0
$\omega_{1^-}^{\#1\,\alpha}$	0	0	0	0	0	0	0
$f_{1^+}^{\#1\,\alpha\beta}$	$\frac{1}{3}i\sqrt{2}\,k t_2$	$\frac{i k t_2}{3}$	$\frac{k^2 t_2}{3}$	0	0	0	0
$\omega_{1^+}^{\#2\,\alpha\beta}$	$\frac{\sqrt{2}\,t_2}{3}$	$\frac{t_2}{3}$	$-\frac{1}{3}i k t_2$	0	0	0	0
$\omega_{1^+}^{\#1\,\alpha\beta}$	$k^2(2r_3-r_4)+\frac{2t_2}{3}$	$\frac{\sqrt{2}\,t_2}{3}$	$-\frac{1}{3}i\sqrt{2}\,k t_2$	0	0	0	0

Spin-0 propagator (σ, τ):

	$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^-}^{\#1}$
$\sigma_{0^+}^{\#1}\dagger$	$\frac{1}{-2k^2r_3+4k^2r_4}$	0	0	0
$\tau_{0^+}^{\#1}\dagger$	0	0	0	0
$\tau_{0^+}^{\#2}\dagger$	0	0	0	0
$\sigma_{0^-}^{\#1}\dagger$	0	0	0	$\frac{1}{k^2 r_2+t_2}$

Spin-0 (ω, f):

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\omega_{0^-}^{\#1}$
$\omega_{0^+}^{\#1}\dagger$	$-2k^2(r_3-2r_4)$	0	0	0
$f_{0^+}^{\#1}\dagger$	0	0	0	0
$f_{0^+}^{\#2}\dagger$	0	0	0	0
$\omega_{0^-}^{\#1}\dagger$	0	0	0	$k^2 r_2+t_2$

Source constraints/gauge generators
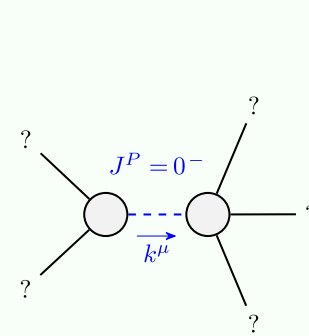
SO(3) irreps	Multiplicities
$\tau_{0^+}^{\#2}==0$	1
$\tau_{0^+}^{\#1}==0$	1
$\tau_{1^-}^{\#2\,\alpha}==0$	3
$\tau_{1^-}^{\#1\,\alpha}==0$	3
$\sigma_{1^-}^{\#2\,\alpha}==0$	3
$\sigma_{1^-}^{\#1\,\alpha}==0$	3
$\tau_{1^+}^{\#1\,\alpha\beta}+i k\,\sigma_{1^+}^{\#2\,\alpha\beta}==0$	3
$\sigma_{2^-}^{\#1\,\alpha\beta\chi}==0$	5
$\tau_{2^+}^{\#1\,\alpha\beta}==0$	5
Total constraints:	27

Spin-2 (σ, τ) matrix (rows top→bottom: $\sigma_{2^-}^{\#1\,\alpha\beta\chi}$, $\tau_{2^+}^{\#1\,\alpha\beta}$, $\sigma_{2^+}^{\#1\,\alpha\beta}$):

	$\sigma_{2^+}^{\#1}{}^{\dagger\,\alpha\beta}$	$\tau_{2^+}^{\#1}{}^{\dagger\,\alpha\beta}$	$\sigma_{2^-}^{\#1}{}^{\dagger\,\alpha\beta\chi}$
$\sigma_{2^-}^{\#1\,\alpha\beta\chi}$	0	0	0
$\tau_{2^+}^{\#1\,\alpha\beta}$	0	0	0
$\sigma_{2^+}^{\#1\,\alpha\beta}$	$\frac{1}{k^2(-2r_3+r_4)}$	0	0

Spin-2 (ω, f) matrix:

	$\omega_{2^+}^{\#1}{}_{\alpha\beta}$	$f_{2^+}^{\#1}{}_{\alpha\beta}$	$\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\omega_{2^+}^{\#1}{}^{\dagger\,\alpha\beta}$	$k^2(-2r_3+r_4)$	0	0
$f_{2^+}^{\#1}{}^{\dagger\,\alpha\beta}$	0	0	0
$\omega_{2^-}^{\#1}{}^{\dagger\,\alpha\beta\chi}$	0	0	0

Massive and massless spectra

$J^P = 0^-$

Massive particle	
Pole residue:	$-\frac{1}{r_2}>0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2}>0$
Spin:	0
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_2<0 \;\&\&\; t_2>0$