

# Particle spectrograph

## Wave operator and propagator

Quadratic (free) action

$S_F ==$

$$\begin{aligned} & \iiint (\frac{1}{6} (-2(t_1-2t_3) \omega_{\alpha'}^{\alpha'} \omega_{\kappa}^{\kappa} - 6t_1 \omega_{\kappa\lambda}^{\kappa\lambda} \omega_{\lambda'}^{\lambda'} + 6f^{\alpha\beta} \tau_{\alpha\beta} + 6\omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} + \\ & 4r_2 \partial^\beta \omega_{\kappa}^{\theta\alpha} \partial_\theta \omega_{\alpha\beta}^{\kappa} - 2r_2 \partial_\theta \omega_{\alpha\beta}^{\kappa} \partial_\kappa \omega^{\alpha\beta\theta} - 4r_2 \partial_\theta \omega_{\alpha\beta}^{\kappa} \partial_\kappa \omega^{\theta\alpha\beta} - \\ & 3t_1 \partial^\alpha f_{\theta\kappa} \partial_\kappa f_{\alpha}^{\theta} - 3t_1 \partial^\alpha f_{\kappa\theta} \partial_\theta f_{\alpha}^{\kappa} - 3t_1 \partial^\alpha f_{\alpha}^{\theta} \partial_\kappa f_{\lambda}^{\theta} + 2t_1 \omega_{\kappa\alpha}^{\alpha} \partial_\kappa f_{\lambda}^{\theta} - \\ & 4t_3 \omega_{\kappa\alpha}^{\alpha} \partial_\kappa f_{\lambda}^{\theta} + 2t_1 \omega_{\kappa\lambda}^{\lambda} \partial_\kappa f_{\lambda}^{\theta} - 4t_3 \omega_{\kappa\lambda}^{\lambda} \partial_\kappa f_{\lambda}^{\theta} + 4t_1 \partial^\alpha f_{\kappa\alpha} \partial_\kappa f_{\lambda}^{\theta} - \\ & 8t_3 \partial^\alpha f_{\kappa\alpha} \partial_\kappa f_{\lambda}^{\theta} - 2t_1 \partial_\kappa f_{\lambda}^{\theta} \partial_\lambda f_{\lambda}^{\theta} + 4t_3 \partial_\kappa f_{\lambda}^{\theta} \partial_\lambda f_{\lambda}^{\theta} + 12t_1 \omega_{\kappa\theta} \partial_\kappa f_{\lambda}^{\theta} - \\ & 2t_1 \omega_{\alpha\kappa}^{\alpha} \partial_\kappa f_{\lambda}^{\theta} + 4t_3 \omega_{\alpha\kappa}^{\alpha} \partial_\kappa f_{\lambda}^{\theta} - 2t_1 \omega_{\lambda\alpha}^{\lambda} \partial_\kappa f_{\lambda}^{\theta} + 4t_3 \omega_{\lambda\alpha}^{\lambda} \partial_\kappa f_{\lambda}^{\theta} + \\ & 3t_1 \partial^\alpha f_{\kappa}^{\lambda} \partial_\kappa f_{\lambda\alpha}^{\theta} + 3t_1 \partial_\kappa f_{\lambda}^{\theta} \partial_\lambda f_{\lambda}^{\theta} + 3t_1 \partial_\kappa f_{\theta}^{\lambda} \partial_\lambda f_{\lambda}^{\theta} - 2t_1 \partial^\alpha f_{\lambda}^{\theta} \partial_\alpha f_{\lambda\kappa}^{\theta} + \\ & 4t_3 \partial^\alpha f_{\lambda}^{\theta} \partial_\alpha f_{\lambda\kappa}^{\theta} + 2r_2 \partial_\kappa \omega^{\alpha\beta\theta} \partial_\kappa \omega_{\alpha\beta\theta} + 4r_2 \partial_\kappa \omega^{\theta\alpha\beta} \partial_\kappa \omega_{\alpha\beta\theta} - \\ & 4r_2 \partial^\beta \omega_{\lambda}^{\alpha\lambda} \partial_\lambda \omega_{\alpha\beta}^{\lambda'} + 4r_2 \partial^\beta \omega_{\lambda}^{\lambda\alpha} \partial_\lambda \omega_{\alpha\beta}^{\lambda'}) [t, x, y, z] d^4x dt \end{aligned}$$

| $\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$ | $\tau_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{1-}^{\#1} \dagger^{\alpha}$       | $\sigma_{1-}^{\#2} \dagger^{\alpha}$              | $\tau_{1-}^{\#1} \dagger^{\alpha}$ | $\tau_{1-}^{\#2} \dagger^{\alpha}$               |
|---|---|---|--|---|------------------------------------|--|
| 0   | $-\frac{\sqrt{2}}{t_1+k^2} t_1$           | $-\frac{i\sqrt{2}k}{t_1+k^2} t_1$       | 0  | 0   | 0                                  | 0  |
| $-\frac{\sqrt{2}}{t_1+k^2} t_1$           | $\frac{1}{(1+k^2)^2} t_1$                 | $\frac{ik}{(1+k^2)^2} t_1$              | 0  | 0   | 0                                  | 0  |
| $\frac{i\sqrt{2}k}{t_1+k^2} t_1$          | $-\frac{ik}{(1+k^2)^2} t_1$               | $\frac{k^2}{(1+k^2)^2} t_1$             | 0  | 0   | 0                                  | 0  |
| 0   | 0   | 0                                       | $\frac{2(t_1+t_3)}{3t_1t_3}$               | $-\frac{\sqrt{2}(t_1-2t_3)}{3(1+2k^2)t_1t_3}$     | 0                                  | $-\frac{2ikt_1-4ikt_3}{3t_1t_3+6k^2t_1t_3}$      |
| 0   | 0   | 0                                       | 0  | $\frac{t_1+4t_3}{3(1+2k^2)^2t_1t_3}$              | 0                                  | $\frac{i\sqrt{2}k(t_1+4t_3)}{3(1+2k^2)^2t_1t_3}$ |
| 0   | 0   | 0                                       | 0  | 0   | 0                                  | 0  |
| 0   | 0   | 0                                       | $\frac{2ikt_1-4ikt_3}{3t_1t_3+6k^2t_1t_3}$ | $-\frac{i\sqrt{2}k(t_1+4t_3)}{3(1+2k^2)^2t_1t_3}$ | 0                                  | $\frac{2k^2(t_1+4t_3)}{3(1+2k^2)^2t_1t_3}$       |

| $\omega_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{1+}^{\#2} \dagger^{\alpha\beta}$ | $f_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{1-}^{\#1} \dagger^{\alpha}$ | $\omega_{1-}^{\#2} \dagger^{\alpha}$ | $f_{1-}^{\#1} \dagger^{\alpha}$ | $f_{1-}^{\#2} \dagger^{\alpha}$  |
|---|---|--------------------------------------|--------------------------------------|--------------------------------------|---------------------------------|----------------------------------|
| $-\frac{t_1}{2}$                          | $-\frac{t_1}{\sqrt{2}}$                   | $-\frac{ikt_1}{\sqrt{2}}$            | 0                                    | 0                                    | 0                               | 0                                |
| $-\frac{t_1}{\sqrt{2}}$                   | 0   | 0                                    | 0                                    | 0                                    | 0                               | 0                                |
| $\frac{ikt_1}{\sqrt{2}}$                  | 0   | 0                                    | 0                                    | 0                                    | 0                               | 0                                |
| 0   | 0   | 0                                    | $\frac{1}{6}(t_1+4t_3)$              | $\frac{t_1-2t_3}{3\sqrt{2}}$         | 0                               | $\frac{1}{3}ik(t_1-2t_3)$        |
| 0   | 0   | 0                                    | $\frac{t_1-2t_3}{3\sqrt{2}}$         | $\frac{t_1+t_3}{3}$                  | 0                               | $\frac{1}{3}i\sqrt{2}k(t_1+t_3)$ |
| 0   | 0   | 0                                    | 0                                    | 0                                    | 0                               | 0                                |
| 0   | 0   | 0                                    | $-\frac{1}{3}ik(t_1-2t_3)$           | $-\frac{1}{3}i\sqrt{2}k(t_1+t_3)$    | 0                               | $\frac{2}{3}k^2(t_1+t_3)$        |

| Source constraints/gauge generators                                 |                |
|---|----------------|
| SO(3) irreps  | Multiplicities |
| $\tau_{0+}^{\#2} == 0$  | 1              |
| $\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$                       | 1              |
| $\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$           | 3              |
| $\tau_{1-}^{\#1\alpha} == 0$  | 3              |
| $\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$  | 3              |
| $\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$ | 5              |
| Total constraints:  | 16             |

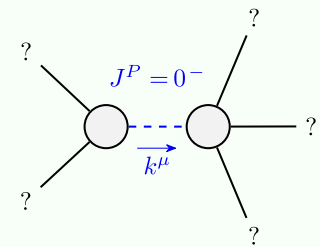
| $\omega_{2+}^{\#1} \dagger^{\alpha\beta}$ | $f_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$ |
|---|--------------------------------------|---|
| $\frac{t_1}{2}$                           | $-\frac{ikt_1}{\sqrt{2}}$            | 0   |
| $\frac{ikt_1}{\sqrt{2}}$                  | $k^2t_1$                             | 0   |
| 0   | 0                                    | $\frac{t_1}{2}$                               |

| $\sigma_{0+}^{\#1} \dagger$        | $\tau_{0+}^{\#1} \dagger$           | $\tau_{0+}^{\#2} \dagger$ | $\sigma_{0-}^{\#1} \dagger$ |
|------------------------------------|-------------------------------------|---------------------------|-----------------------------|
| $\frac{1}{(1+2k^2)^2t_3}$          | $-\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$ | 0                         | 0                           |
| $\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$ | $\frac{2k^2}{(1+2k^2)^2t_3}$        | 0                         | 0                           |
| 0                                  | 0                                   | 0                         | 0                           |
| 0                                  | 0                                   | 0                         | $\frac{1}{k^2r_2-t_1}$      |

| $\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\tau_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$ |
|---|---|---|
| $-\frac{2}{(1+2k^2)^2t_1}$                | $-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$    | 0   |
| $\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$       | $\frac{4k^2}{(1+2k^2)^2t_1}$            | 0   |
| 0   | 0                                       | $\frac{2}{t_1}$                               |

| $\omega_{0+}^{\#1} \dagger$ | $f_{0+}^{\#1} \dagger$ | $\omega_{0+}^{\#2} \dagger$ | $\omega_{0-}^{\#1} \dagger$ |
|-----------------------------|------------------------|-----------------------------|-----------------------------|
| $t_3$                       | $-i\sqrt{2}kt_3$       | 0                           | 0                           |
| $i\sqrt{2}kt_3$             | $2k^2t_3$              | 0                           | 0                           |
| 0                           | 0                      | 0                           | 0                           |
| 0                           | 0                      | 0                           | $k^2r_2-t_1$                |

## Massive and massless spectra



Massive particle

|                |                       |
|----------------|-----------------------|
| Pole residue:  | $-\frac{1}{r_2} > 0$  |
| Polarisations: | 1                     |
| Square mass:   | $\frac{t_1}{r_2} > 0$ |
| Spin:          | 0                     |
| Parity:        | Odd                   |

(No massless particles)

## Unitarity conditions

$$r_2 < 0 \&\& t_1 < 0$$