Particle spectrograph

Wave operator and propagator

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0^+}^{\#2} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$	1
$\tau_{0^+}^{\#1} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha$	1
$\tau_{1^-}^{\#2\,\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta}$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$	3
$\sigma_{1^-}^{\#2\,\alpha} == 0$	$\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi} == 0$	3
$\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#1\,\alpha\beta} == 0$	$\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} + \partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\beta\alpha} + \partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta}$	3
$\sigma_{1^+}^{\#1\,\alpha\beta} == \sigma_{1^+}^{\#2\,\alpha\beta}$	$3\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta} + 3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha} == 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta} + 3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha}$	3
$\tau_{2^+}^{\#1\,\alpha\beta} == 0$	$4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} == 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi$	5
$\sigma_{2^+}^{\#1\,\alpha\beta} == 0$	$3\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\sigma^{\chi\delta}{}_\chi == 2\,\partial_\delta\partial^\beta\partial^\alpha\sigma^{\chi\delta}{}_\chi + 3\,(\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha})$	5
Total constraints/gauge generators:		27

Quadratic (free) action

$$\begin{aligned}\mathcal{S} == \iiiint(\tfrac{1}{6}\,(&6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\mathcal{A}^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + 12\,r_1\,\partial_\beta\mathcal{A}_l{}^\theta{}_\theta\,\partial^l\mathcal{A}^{\alpha\beta}{}_\alpha - \\ &24\,r_3\,\partial_\beta\mathcal{A}_l{}^\theta{}_\theta\,\partial^l\mathcal{A}^{\alpha\beta}{}_\alpha + 12\,r_1\,\partial_\iota\mathcal{A}_\beta{}^\theta{}_\theta\,\partial^\iota\mathcal{A}^{\alpha\beta}{}_\alpha + \\ &12\,r_1\,\partial_\alpha\mathcal{A}^{\alpha\beta\iota}\,\partial_\theta\mathcal{A}_\beta{}^\theta{}_\iota - 24\,r_1\,\partial^\iota\mathcal{A}^{\alpha\beta}{}_\alpha\,\partial_\theta\mathcal{A}_\beta{}^\theta{}_\iota + \\ &12\,r_1\,\partial_\alpha\mathcal{A}^{\alpha\beta\iota}\,\partial_\theta\mathcal{A}_l{}^\theta{}_\beta - 24\,r_3\,\partial_\alpha\mathcal{A}^{\alpha\beta\iota}\,\partial_\theta\mathcal{A}_l{}^\theta{}_\beta - \\ &24\,r_1\,\partial^\iota\mathcal{A}^{\alpha\beta}{}_\alpha\,\partial_\theta\mathcal{A}_l{}^\theta{}_\beta + 48\,r_3\,\partial^\iota\mathcal{A}^{\alpha\beta}{}_\alpha\,\partial_\theta\mathcal{A}_l{}^\theta{}_\beta + \\ &4\,t_2\,\mathcal{A}_{\iota\theta\alpha}\,\partial^\theta f^{\alpha\iota} + 2\,t_2\,\partial_\alpha f_{\iota\theta}\,\partial^\theta f^{\alpha\iota} - t_2\,\partial_\alpha f_{\theta\iota}\,\partial^\theta f^{\alpha\iota} - \\ &t_2\,\partial_\iota f_{\alpha\theta}\,\partial^\theta f^{\alpha\iota} + t_2\,\partial_\theta f_{\alpha\iota}\,\partial^\theta f^{\alpha\iota} - t_2\,\partial_\theta f_{\iota\alpha}\,\partial^\theta f^{\alpha\iota} - \\ &4\,t_2\,\mathcal{A}_{\alpha\theta\iota}\,(\mathcal{A}^{\alpha\iota\theta} + \partial^\theta f^{\alpha\iota}) + 2\,t_2\,\mathcal{A}_{\alpha\iota\theta}\,(\mathcal{A}^{\alpha\iota\theta} + 2\,\partial^\theta f^{\alpha\iota}) - \\ &8\,r_1\,\partial_\beta\mathcal{A}_{\alpha\iota\theta}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} + 8\,r_2\,\partial_\beta\mathcal{A}_{\alpha\iota\theta}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} + \\ &4\,r_1\,\partial_\beta\mathcal{A}_{\alpha\theta\iota}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} - 4\,r_2\,\partial_\beta\mathcal{A}_{\alpha\theta\iota}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} + \\ &8\,r_1\,\partial_\beta\mathcal{A}_{\iota\theta\alpha}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} + 4\,r_2\,\partial_\beta\mathcal{A}_{\iota\theta\alpha}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} - \\ &24\,r_3\,\partial_\beta\mathcal{A}_{\iota\theta\alpha}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} - 4\,r_1\,\partial_\iota\mathcal{A}_{\alpha\beta\theta}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} - \\ &2\,r_2\,\partial_\iota\mathcal{A}_{\alpha\beta\theta}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} + 4\,r_1\,\partial_\theta\mathcal{A}_{\alpha\beta\iota}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} + \\ &2\,r_2\,\partial_\theta\mathcal{A}_{\alpha\beta\iota}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} + 4\,r_1\,\partial_\theta\mathcal{A}_{\alpha\iota\beta}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota} - \\ &4\,r_2\,\partial_\theta\mathcal{A}_{\alpha\iota\beta}\,\partial^\theta\mathcal{A}^{\alpha\beta\iota}))[t,\,x,\,y,\,z]\,dt\,dz\,dy\,dx\,dt\end{aligned}$$

	$\mathcal{A}_{1^+}^{\#1\,\alpha\beta}$	$\mathcal{A}_{1^+}^{\#2\,\alpha\beta}$	$f_{1^+}^{\#1\,\alpha\beta}$	$\mathcal{A}_{1^-}^{\#1\,\alpha}$	$\mathcal{A}_{1^-}^{\#2\,\alpha}$	$f_{1^-}^{\#1\,\alpha}$	$f_{1^-}^{\#2\,\alpha}$
$\mathcal{A}_{1^+}^{\#1\,\dagger\,\alpha\beta}$	$\frac{2\,t_2}{3}$	$\frac{\sqrt{2}\,t_2}{3}$	$\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2$	0	0	0	0
$\mathcal{A}_{1^+}^{\#2\,\dagger\,\alpha\beta}$	$\frac{\sqrt{2}\,t_2}{3}$	$\frac{t_2}{3}$	$\frac{i\,k\,t_2}{3}$	0	0	0	0
$f_{1^+}^{\#1\,\dagger\,\alpha\beta}$	$-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2$	$-\frac{1}{3}\,i\,k\,t_2$	$\frac{k^2\,t_2}{3}$	0	0	0	0
$\mathcal{A}_{1^-}^{\#1\,\dagger\,\alpha}$	0	0	0	$-k^2\,r_1$	0	0	0
$\mathcal{A}_{1^-}^{\#2\,\dagger\,\alpha}$	0	0	0	0	0	0	0
$f_{1^-}^{\#1\,\dagger\,\alpha}$	0	0	0	0	0	0	0
$f_{1^-}^{\#2\,\dagger\,\alpha}$	0	0	0	0	0	0	0

	$\sigma_{1^+}^{\#1\,\alpha\beta}$	$\sigma_{1^+}^{\#2\,\alpha\beta}$	$\tau_{1^+}^{\#1\,\alpha\beta}$	$\sigma_{1^-}^{\#1\,\alpha}$	$\sigma_{1^-}^{\#2\,\alpha}$	$\tau_{1^-}^{\#1\,\alpha}$	$\tau_{1^-}^{\#2\,\alpha}$
$\sigma_{1^+}^{\#1\,\dagger\,\alpha\beta}$	$\frac{6}{(3+k^2)^2\,t_2}$	$\frac{3\,\sqrt{2}}{(3+k^2)^2\,t_2}$	$\frac{3\,i\,\sqrt{2}\,k}{(3+k^2)^2\,t_2}$	0	0	0	0
$\sigma_{1^+}^{\#2\,\dagger\,\alpha\beta}$	$\frac{3\,\sqrt{2}}{(3+k^2)^2\,t_2}$	$\frac{3}{(3+k^2)^2\,t_2}$	$\frac{3\,i\,k}{(3+k^2)^2\,t_2}$	0	0	0	0
$\tau_{1^+}^{\#1\,\dagger\,\alpha\beta}$	$-\frac{3\,i\,\sqrt{2}\,k}{(3+k^2)^2\,t_2}$	$-\frac{3\,i\,k}{(3+k^2)^2\,t_2}$	$\frac{3\,k^2}{(3+k^2)^2\,t_2}$	0	0	0	0
$\sigma_{1^-}^{\#1\,\dagger\,\alpha}$	0	0	0	$-\frac{1}{k^2\,r_1}$	0	0	0
$\sigma_{1^-}^{\#2\,\dagger\,\alpha}$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#1\,\dagger\,\alpha}$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#2\,\dagger\,\alpha}$	0	0	0	0	0	0	0

	$\mathcal{A}_{0^+}^{\#1\,\dagger}$	$f_{0^+}^{\#1\,\dagger}$	$f_{0^+}^{\#2\,\dagger}$	$\mathcal{A}_{0^+}^{\#1}$
$\mathcal{A}_{0^+}^{\#1}$	0	0	0	$6\,k^2\,(-r_1+r_3)$
$f_{0^+}^{\#1}$	0	0	0	0
$f_{0^+}^{\#2}$	0	0	0	0
$\mathcal{A}_{0^+}^{\#1}$	$k^2\,r_2+t_2$	0	0	0

	$\sigma_{2^+}^{\#1\,\dagger\,\alpha\beta}$	$\tau_{2^+}^{\#1\,\dagger\,\alpha\beta}$	$\sigma_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$
$\sigma_{2^+}^{\#1\,\alpha\beta}$	0	0	0
$\tau_{2^+}^{\#1\,\alpha\beta}$	0	0	0
$\sigma_{2^-}^{\#1\,\alpha\beta\chi}$	0	0	$\frac{1}{k^2\,r_1}$

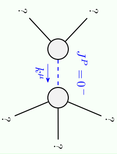
	$\mathcal{A}_{2^+}^{\#1\,\dagger\,\alpha\beta\chi}$	$f_{2^+}^{\#1\,\dagger\,\alpha\beta}$	$\mathcal{A}_{2^+}^{\#1\,\alpha\beta}$
$\mathcal{A}_{2^+}^{\#1\,\alpha\beta}$	0	0	0
$f_{2^+}^{\#1\,\alpha\beta}$	0	0	0
$\mathcal{A}_{2^+}^{\#1\,\alpha\beta\chi}$	$k^2\,r_1$	0	0

	$\sigma_{0^+}^{\#1\,\dagger}$	$\tau_{0^+}^{\#2\,\dagger}$	$\tau_{0^+}^{\#1\,\dagger}$	$\sigma_{0^+}^{\#1}$
$\sigma_{0^+}^{\#1}$	0	0	0	$\frac{1}{6\,k^2\,(-r_1+r_3)}$
$\tau_{0^+}^{\#1}$	0	0	0	0
$\tau_{0^+}^{\#2}$	0	0	0	0
$\sigma_{0^-}^{\#1}$	$\frac{1}{k^2\,r_2+t_2}$	0	0	0

Massive and massless spectra

Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_2 < 0\ \&\&\ t_2 > 0$