

# Particle spectrograph

## Wave operator and propagator

$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2} \alpha\beta$	$\tau_{1+}^{\#1} \alpha\beta$	$\sigma_{1-}^{\#1} \alpha$	$\sigma_{1-}^{\#2} \alpha$	$\tau_{1-}^{\#1} \alpha$	$\tau_{1-}^{\#2} \alpha$
$0$	$-\frac{\sqrt{2}}{t_1+k^2}t_1$	$-\frac{i\sqrt{2}k}{t_1+k^2}t_1$	$0$	$0$	$0$	$0$
$-\frac{\sqrt{2}}{t_1+k^2}t_1$	$-\frac{2k^2(2r_1+r_5)+t_1}{(1+k^2)^2}t_1$	$-\frac{2ik^3(2r_1+r_5)+ik t_1}{(1+k^2)^2}t_1^2$	$0$	$0$	$0$	$0$
$\frac{i\sqrt{2}k}{t_1+k^2}t_1$	$\frac{i(2k^3(2r_1+r_5)-kt_1)}{(1+k^2)^2}t_1^2$	$\frac{2k^4(2r_1+r_5)+k^2t_1}{(1+k^2)^2}t_1^2$	$0$	$0$	$0$	$0$
$0$	$0$	$0$	$-\frac{2(t_1+t_3)}{3t_1t_3+2k^2(r_1+r_5)(t_1+t_3)}$	$-\frac{\sqrt{2}(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$	$0$	$-\frac{2ik(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$
$0$	$0$	$0$	$0$	$0$	$0$	$0$
$0$	$0$	$0$	$0$	$0$	$0$	$0$
$0$	$0$	$0$	$-\frac{2ik(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$	$-\frac{i\sqrt{2}k(6k^2(r_1+r_5)+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$	$0$	$-\frac{2k^2(6k^2(r_1+r_5)+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$

	$\sigma_{2+}^{\#1} \alpha\beta$	$\tau_{2+}^{\#1} \alpha\beta$	$\sigma_{2-}^{\#1} \alpha\beta\chi$
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2}t_1$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$	$0$
$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$	$\frac{4k^2}{(1+2k^2)^2}t_1$	$0$
$\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$	$0$	$0$	$\frac{2}{2k^2r_1+t_1}$

$\omega_{0+}^{\#1} \dagger$	$f_{0+}^{\#1} \dagger$	$\omega_{0+}^{\#2}$	$f_{0+}^{\#2}$	$\omega_0^{\#1}$
$t_3$	$-i\sqrt{2}kt_3$	$0$	$0$	$0$
$f_{0+}^{\#1} \dagger$	$i\sqrt{2}kt_3$	$0$	$0$	$0$
$f_{0+}^{\#2}$	$0$	$0$	$0$	$0$
$\omega_0^{\#1} \dagger$	$0$	$0$	$0$	$-t_1$

$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{2+}^{\#1} \alpha\beta\chi$
$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	$0$
$\frac{ikt_1}{\sqrt{2}}$	$k^2t_1$	$0$
$0$	$0$	$k^2r_1+\frac{t_1}{2}$

	$\omega_{1+}^{\#1} \alpha\beta$	$\omega_{1+}^{\#2} \alpha\beta$	$f_{1+}^{\#1} \alpha\beta$	$\omega_{1-}^{\#1} \alpha$	$\omega_{1-}^{\#2} \alpha$	$f_{1-}^{\#1} \alpha$	$f_{1-}^{\#2} \alpha$
$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$k^2(2r_1+r_5)-\frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	$0$	$0$	$0$	$0$
$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	$0$	$0$	$0$	$0$	$0$	$0$
$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	$0$	$0$	$0$	$0$	$0$	$0$
$\omega_{1-}^{\#1} \dagger^{\alpha}$	$0$	$0$	$0$	$\frac{1}{6}(6k^2(r_1+r_5)+t_1+4t_3)$	$\frac{t_1-2t_3}{3\sqrt{2}}$	$0$	$\frac{1}{3}ik(t_1-2t_3)$
$\omega_{1-}^{\#2} \dagger^{\alpha}$	$0$	$0$	$0$	$\frac{t_1-2t_3}{3\sqrt{2}}$	$\frac{t_1+t_3}{3}$	$0$	$\frac{1}{3}i\sqrt{2}k(t_1+t_3)$
$f_{1-}^{\#1} \dagger^{\alpha}$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$f_{1-}^{\#2} \dagger^{\alpha}$	$0$	$0$	$0$	$-\frac{1}{3}ik(t_1-2t_3)$	$-\frac{1}{3}i\sqrt{2}k(t_1+t_3)$	$0$	$\frac{2}{3}k^2(t_1+t_3)$

### Quadratic (free) action

$S_F ==$

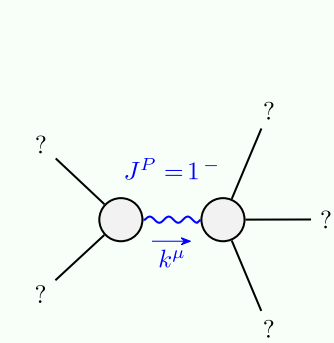
$$\begin{aligned} & \iiint \int (\frac{1}{6} (-2(t_1-2t_3) \omega_{, \alpha}^{\alpha \iota} \omega_{\kappa \alpha}^{\kappa} - 6t_1 \omega_{, \alpha}^{\alpha \iota} \omega_{\kappa \alpha}^{\kappa \lambda} \omega_{\kappa \lambda}^{\iota} + 6f^{\alpha \beta} \tau_{\alpha \beta} + 6\omega^{\alpha \beta \chi} \sigma_{\alpha \beta \chi} - \\ & 6r_5 \partial_{\iota} \omega_{\kappa}^{\kappa \lambda} \partial^{\iota} \omega_{\lambda}^{\alpha} - 4r_1 \partial^{\beta} \omega^{\theta \alpha} \partial_{\theta} \omega_{\alpha \beta}^{\kappa} - 4r_1 \partial_{\theta} \omega_{\alpha \beta}^{\kappa} \partial_{\kappa} \omega^{\alpha \beta \theta} + \\ & 4r_1 \partial_{\theta} \omega_{\alpha \beta}^{\kappa} \partial_{\kappa} \omega^{\theta \alpha \beta} - 6r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\theta \kappa \lambda} + 6r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\theta \kappa \lambda} - \\ & 6r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\kappa \lambda \theta} + 12r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\kappa \lambda \theta} - 3t_1 \partial^{\alpha} f_{\theta \kappa} \partial^{\kappa} f_{\alpha}^{\theta} - 3t_1 \partial^{\alpha} f_{\kappa \theta} \partial^{\kappa} f_{\alpha}^{\theta} - \\ & 3t_1 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\alpha \lambda} + 2t_1 \omega_{\kappa \alpha}^{\alpha} \partial^{\kappa} f_{, \iota}^{\iota} - 4t_3 \omega_{\kappa \alpha}^{\alpha} \partial^{\kappa} f_{, \iota}^{\iota} + 2t_1 \omega_{\kappa \lambda}^{\lambda} \partial^{\kappa} f_{, \iota}^{\iota} - \\ & 4t_3 \omega_{\kappa \lambda}^{\lambda} \partial^{\kappa} f_{, \iota}^{\iota} + 4t_1 \partial^{\alpha} f_{\kappa \alpha} \partial^{\kappa} f_{, \iota}^{\iota} - 8t_3 \partial^{\alpha} f_{\kappa \alpha} \partial^{\kappa} f_{, \iota}^{\iota} - 2t_1 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{, \iota}^{\iota} + \\ & 4t_3 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{, \iota}^{\iota} + 12t_1 \omega_{\iota \kappa \theta} \partial^{\kappa} f^{\iota \theta} - 2t_1 \omega_{\iota \alpha}^{\alpha} \partial^{\kappa} f_{\kappa}^{\iota} + 4t_3 \omega_{\iota \alpha}^{\alpha} \partial^{\kappa} f_{\kappa}^{\iota} - \\ & 2t_1 \omega_{\iota \lambda}^{\lambda} \partial^{\kappa} f_{\kappa}^{\iota} + 4t_3 \omega_{\iota \lambda}^{\lambda} \partial^{\kappa} f_{\kappa}^{\iota} + 3t_1 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\lambda \alpha} + 3t_1 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} + \\ & 3t_1 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} - 2t_1 \partial^{\alpha} f_{\alpha}^{\lambda} \partial^{\kappa} f_{\lambda \kappa} + 4t_3 \partial^{\alpha} f_{\alpha}^{\lambda} \partial^{\kappa} f_{\lambda \kappa} + 4r_1 \partial_{\kappa} \omega^{\alpha \beta \theta} \partial^{\kappa} \omega_{\alpha \beta \theta} - \\ & 4r_1 \partial_{\kappa} \omega^{\theta \alpha \beta} \partial^{\kappa} \omega_{\alpha \beta \theta} + 4r_1 \partial^{\beta} \omega_{, \alpha}^{\alpha \lambda} \partial_{\lambda} \omega_{\alpha \beta}^{\iota} - 16r_1 \partial^{\beta} \omega_{, \alpha}^{\alpha \lambda} \partial_{\lambda} \omega_{\alpha \beta}^{\iota} + \\ & 6r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega^{\theta \kappa} \omega_{\kappa}^{\kappa} - 6r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega^{\theta \kappa} \omega_{\kappa}^{\kappa})) [t, x, y, z] dz dy dx dt \end{aligned}$$

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_0^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	$\frac{1}{(1+2k^2)^2}t_3$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_3$	$0$	$0$
$\tau_{0+}^{\#1} \dagger$	$\frac{i\sqrt{2}k}{(1+2k^2)^2}t_3$	$\frac{2k^2}{(1+2k^2)^2}t_3$	$0$	$0$
$\tau_{0+}^{\#2} \dagger$	$0$	$0$	$0$	$0$
$\sigma_0^{\#1} \dagger$	$0$	$0$	$0$	$-\frac{1}{t_1}$

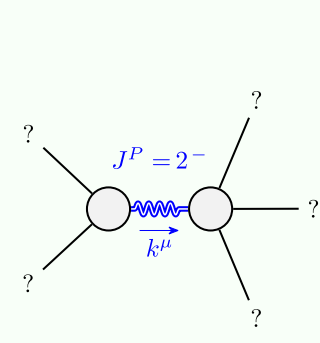
Source constraints/gauge generators

SO(3) irreps	Multiplicities						
$\tau_{0+}^{\#2} == 0$	1						
$\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$	1						
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$	3						
$\tau_{1-}^{\#1\alpha} == 0$	3						
$\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$	3						
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5						
Total constraints:	16						

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{3(-2t_1t_3(t_1+t_3)+r_1(t_1^2+2t_3^2)+r_5(t_1^2+2t_3^2))}{2(r_1+r_5)(t_1+t_3)(-3t_1t_3+r_1(t_1+t_3)+r_5(t_1+t_3))} > 0$
Polarisations:	3
Square mass:	$-\frac{3t_1t_3}{2(r_1+r_5)(t_1+t_3)} > 0$
Spin:	1
Parity:	Odd



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

## Unitarity conditions

$$r_1 < 0 \ \&\& \ r_5 < -r_1 \ \&\& \ t_1 > 0 \ \&\& \ t_3 < -t_1 \ || \ t_3 > 0$$