

# Wave operator and propagator

Source constraints		Fundamental fields	Multiplicities
$\sigma_{0+}^{\#1} == 0$	$\partial_\beta \sigma^{\alpha\beta}_\alpha == 0$		1
$\sigma_{1-}^{\#2\alpha} == 0$	$\partial_\chi \partial_\beta \sigma^{\alpha\beta\chi} == 0$		3
$\sigma_{1+}^{\#2\alpha\beta} == 0$	$\partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$		3
$\sigma_{2-}^{\#1\alpha\beta\chi} == 0$	$3 \partial_\epsilon \partial_\delta \partial^\chi \partial^\alpha \sigma^{\beta\delta\epsilon} + 3 \partial_\epsilon \partial^\epsilon \partial^\chi \partial^\alpha \sigma^{\beta\delta}_\delta +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\chi\delta} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\alpha\delta\chi} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\beta \sigma^{\chi\delta\alpha} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\alpha\beta\delta} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\alpha\delta\beta} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\beta\chi\alpha} +$ $3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\alpha \sigma^{\delta\epsilon}_\delta + 3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\beta\delta\epsilon} +$ $3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\epsilon \sigma^{\alpha\delta}_\delta == 3 \partial_\epsilon \partial_\delta \partial^\chi \partial^\beta \sigma^{\alpha\delta\epsilon} +$ $3 \partial_\epsilon \partial^\epsilon \partial^\chi \partial^\beta \sigma^{\alpha\delta}_\delta + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\chi\delta} +$ $4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\beta\delta\chi} + 2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\alpha \sigma^{\chi\delta\beta} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\chi \sigma^{\beta\delta\alpha} + 4 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\beta\chi} +$ $2 \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \sigma^{\alpha\chi\beta} + 3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\beta \sigma^{\delta\epsilon}_\delta +$ $3 \eta^{\beta\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial_\delta \sigma^{\alpha\delta\epsilon} + 3 \eta^{\alpha\chi} \partial_\phi \partial^\phi \partial_\epsilon \partial^\delta \sigma^{\beta\delta}_\delta$	5	
Total constraints/gauge generators:			12

## Quadratic (free) action

$$\begin{aligned}
& + \frac{1}{3} r_2 (4 \partial_\beta \omega_{\alpha\theta} - 2 \partial_\beta \omega_{\alpha\theta_1} + 2 \partial_\beta \omega_{\theta\alpha} - \\
& \quad \partial_1 \omega_{\alpha\beta\theta} + \partial_\theta \omega_{\alpha\beta_1} - 2 \partial_\theta \omega_{\alpha_1\beta}) \partial^\theta \omega^{\alpha\beta_1} - \\
& \frac{1}{2} r_3 (\partial_\beta \omega_{\theta_1\theta}^\theta \partial_1' \omega_{\alpha}^{\alpha\beta} + \partial_1 \omega_{\beta\theta}^\theta \partial_1' \omega_{\alpha}^{\alpha\beta} + \partial_\alpha \omega^{\alpha\beta_1} \partial_\theta \omega_{\beta_1}^\theta - \\
& \quad 2 \partial_1' \omega_{\alpha}^{\alpha\beta} \partial_\theta \omega_{\beta_1}^\theta + \partial_\alpha \omega^{\alpha\beta_1} \partial_\theta \omega_{\theta_1}^\theta - \\
& \quad 2 \partial_1' \omega_{\alpha}^{\alpha\beta} \partial_\theta \omega_{\theta_1}^\theta + 8 \partial_\beta \omega_{\theta\alpha} \partial^\theta \omega^{\alpha\beta_1}) + \\
& r_5 (\partial_1 \omega_{\theta\kappa}^\kappa \partial^\theta \omega_{\alpha}^{\alpha_1} - \partial_\theta \omega_{\kappa\alpha}^\kappa \partial^\theta \omega_{\alpha_1}^{\alpha_1} - (\partial_\alpha \omega^{\alpha_1\theta} - 2 \partial^\theta \omega^{\alpha_1}{}_\alpha) \\
& \quad (\partial_\kappa \omega_{\theta_1}^{\kappa} - \partial_\kappa \omega_{\theta_1'}^\kappa))) [t, x, y, z] dz dy dx dt
\end{aligned}$$

The diagram illustrates the construction of the matrix  $M_{\alpha\beta}^{(1)}$  from various blocks. The blocks are labeled with superscripted indices and colors (pink, light blue, light red).

**Top Left Block:** A 2x2 matrix with labels  $\omega_{2+}^{\#1} \alpha\beta$  and  $\omega_{2-}^{\#1} \alpha\beta\chi$ . The entries are: top-left (pink)  $-\frac{3k^2 r_3}{2}$ , top-right (pink) 0, bottom-left (pink) 0, bottom-right (light blue) 0.

**Top Middle Block:** A 4x4 matrix with labels  $\sigma_{1-}^{\#2} \alpha$ ,  $\sigma_{1-}^{\#1} \alpha$ ,  $\sigma_{1+}^{\#2} \alpha\beta$ , and  $\sigma_{1+}^{\#1} \alpha\beta$ . The entries are: top-left (pink) 0, top-middle (pink) 0, top-right (light blue) 0, top-far-right (light blue) 0; middle-left (pink) 0, middle-middle (pink) 0, middle-right (light blue)  $\frac{2}{k^2 (r_3 + 2r_5)}$ , middle-far-right (light blue) 0; bottom-left (light red) 0, bottom-middle (light red) 0, bottom-right (pink) 0, bottom-far-right (pink) 0; bottom-most-left (light red)  $\frac{1}{k^2 (2r_3 + r_5)}$ , bottom-most-middle (light red) 0, bottom-most-right (pink) 0, bottom-most-far-right (pink) 0.

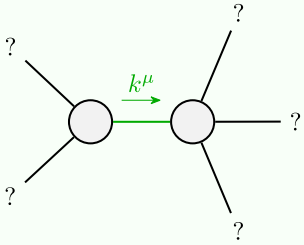
**Top Right Block:** A 2x2 matrix with labels  $\sigma_{0+}^{\#1}$  and  $\sigma_{0-}^{\#1}$ . The entries are: top-left (pink) 0, top-right (light blue)  $\frac{1}{k^2 r_2}$ , bottom-left (light red) 0, bottom-right (pink) 0.

**Middle Right Block:** A 2x2 matrix with labels  $\omega_{0+}^{\#1}$  and  $\omega_{0-}^{\#1}$ . The entries are: top-left (light red) 0, top-right (pink) 0, bottom-left (pink) 0, bottom-right (light blue)  $k^2 r_2$ .

**Bottom Left Block:** A 4x4 matrix with labels  $\omega_{1+}^{\#1} \alpha\beta$ ,  $\omega_{1+}^{\#2} \alpha\beta$ ,  $\omega_{1-}^{\#1} \alpha$ , and  $\omega_{1-}^{\#2} \alpha$ . The entries are: top-left (light red)  $k^2 (2r_3 + r_5)$ , top-middle (light red) 0, top-right (pink) 0, top-far-right (pink) 0; middle-left (light red) 0, middle-middle (light red) 0, middle-right (pink) 0, middle-far-right (pink) 0; bottom-left (pink) 0, bottom-middle (pink) 0, bottom-right (light blue)  $\frac{1}{2} k^2 (r_3 + 2r_5)$ , bottom-far-right (light blue) 0; bottom-most-left (pink) 0, bottom-most-middle (pink) 0, bottom-most-right (light blue) 0, bottom-most-far-right (light blue) 0.

**Bottom Middle Block:** A 2x2 matrix with labels  $\sigma_{2+}^{\#1} \alpha\beta$  and  $\sigma_{2-}^{\#1} \alpha\beta\chi$ . The entries are: top-left (light red)  $-\frac{2}{3k^2 r_3}$ , top-right (pink) 0, bottom-left (pink) 0, bottom-right (light blue) 0.

# Massive and massless spectra



Quadratic pole	
Pole residue:	$-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)} > 0$
Polarisations:	2

(No massive particles)

# Unitarity conditions

$$r_3 < 0 \&\& (r_5 < -\frac{r_3}{2} \parallel r_5 > -2r_3) \parallel r_3 > 0 \&\& -2r_3 < r_5 < -\frac{r_3}{2}$$