

# Particle spectrograph

## Wave operator and propagator

	$\sigma_{1^+ \alpha \beta}^{\#1}$	$\sigma_{1^+ \alpha \beta}^{\#2}$	$\tau_{1^+ \alpha \beta}^{\#1}$	$\sigma_{1^+ \alpha}^{\#1}$	$\sigma_{1^+ \alpha}^{\#2}$	$\tau_{1^+ \alpha}^{\#1}$	$\tau_{1^+ \alpha}^{\#2}$
$\sigma_{1^+ \dagger}^{\#1} \dagger \alpha \beta$	$\frac{2(t_1+t_2)}{3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)}$	$\frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$	$\frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$	0	0	0	0
$\sigma_{1^+ \dagger}^{\#2} \dagger \alpha \beta$	$\frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$	$\frac{6k^2(2r_1+r_5)+t_1+4t_2}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$	$\frac{ik(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$	0	0	0	0
$\tau_{1^+ \dagger}^{\#1} \dagger \alpha \beta$	$-\frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$	$-\frac{ik(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$	$-\frac{k^2(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$	0	0	0	0
$\sigma_{1^+ \dagger}^{\#1} \dagger \alpha$	0	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	0	$\frac{2ik}{t_1+2k^2t_1}$
$\sigma_{1^+ \dagger}^{\#2} \dagger \alpha$	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	$\frac{-2k^2(r_1+r_5)+t_1}{(t_1+2k^2t_1)^2}$	0	$-\frac{i\sqrt{2}k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2t_1)^2}$
$\tau_{1^+ \dagger}^{\#1} \dagger \alpha$	0	0	0	0	0	0	0
$\tau_{1^+ \dagger}^{\#2} \dagger \alpha$	0	0	0	$-\frac{2ik}{t_1+2k^2t_1}$	$\frac{i\sqrt{2}k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2t_1)^2}$	0	$\frac{-4k^4(r_1+r_5)+2k^2t_1}{(t_1+2k^2t_1)^2}$

$\omega_2^{\#1} + \alpha\beta$	$\frac{t_1}{2}$	$-\frac{ik t_1}{\sqrt{2}}$	$0$	$\omega_2^{\#1} + \alpha\beta$
$\omega_2^{\#1} + \alpha\beta$	$\frac{ik t_1}{\sqrt{2}}$	$k^2 t_1$	$0$	$\omega_2^{\#1} + \alpha\beta$
$\omega_2^{\#1} + \alpha\beta$	$0$	$0$	$k^2 r_1 + \frac{t_1}{2}$	$\omega_2^{\#1} + \alpha\beta$

Source constraints		
SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0+}^{\#2} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == 0$	1
$\tau_{0+}^{\#1} - 2 \, i \, k \, \sigma_{0+}^{\#1} == 0$	$\partial_\beta \partial_\alpha \tau^{\alpha\beta} == \partial_\beta \partial^\beta \tau^\alpha{}_\alpha + 2 \, \partial_\chi \partial^\chi \partial_\beta \sigma^{\alpha\beta}{}_\alpha$	1
$\tau_1^{2\alpha} + 2 \, i \, k \, \sigma_1^{2\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\alpha\beta} + 2 \, \partial_\delta \partial^\delta \partial_\chi \partial_\beta \sigma^{\alpha\beta\chi}$	3
$\tau_1^{\#1\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha \tau^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta \tau^{\beta\alpha}$	3
$\tau_{1+}^{\#1\alpha\beta} + i \, k \, \sigma_{1+}^{\#2\alpha\beta} == 0$	$\partial_\chi \partial^\alpha \tau^{\beta\chi} + \partial_\chi \partial^\beta \tau^{\chi\alpha} + \partial_\chi \partial^\chi \tau^{\alpha\beta} +$ $2 \, \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\chi\delta} + 2 \, \partial_\delta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} ==$ $\partial_\chi \partial^\alpha \tau^{\chi\beta} + \partial_\chi \partial^\beta \tau^{\alpha\chi} +$ $\partial_\chi \partial^\chi \tau^{\beta\alpha} + 2 \, \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\chi\delta}$	3
$\tau_{2+}^{\#1\alpha\beta} - 2 \, i \, k \, \sigma_{2+}^{\#1\alpha\beta} == 0$	$-i \, (4 \, \partial_\delta \partial_\chi \partial^\beta \partial^\alpha \tau^{\chi\delta} + 2 \, \partial_\delta \partial^\delta \partial^\beta \partial^\alpha \tau^\chi{}_\chi -$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\beta\chi} - 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\alpha \tau^{\chi\beta} -$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\alpha\chi} - 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\beta \tau^{\chi\alpha} +$ $3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\alpha\beta} + 3 \, \partial_\delta \partial^\delta \partial_\chi \partial^\chi \tau^{\beta\alpha} +$ $4 \, i \, k^\chi \, \partial_\epsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta\epsilon}{}_\delta -$ $6 \, i \, k^\chi \, \partial_\epsilon \partial_\delta \partial_\chi \partial^\alpha \sigma^{\beta\delta\epsilon}{}_\epsilon -$ $6 \, i \, k^\chi \, \partial_\epsilon \partial_\delta \partial_\chi \partial^\beta \sigma^{\alpha\delta\epsilon}{}_\epsilon +$ $2 \, \eta^{\alpha\beta} \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \tau^{\chi\delta} +$ $6 \, i \, k^\chi \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\alpha\delta\beta} +$ $6 \, i \, k^\chi \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial_\chi \sigma^{\beta\delta\alpha} -$ $2 \, \eta^{\alpha\beta} \, \partial_\epsilon \partial^\epsilon \partial_\delta \partial^\delta \tau^\chi{}_\chi -$ $4 \, i \, \eta^{\alpha\beta} \, k^\chi \, \partial_\phi \partial^\phi \partial_\epsilon \partial_\chi \sigma^{\delta\epsilon}{}_\delta) == 0$	5
Total constraints/gauge generators:		16

$\omega_1^{\#1} \dagger \alpha \beta$	$\frac{1}{6} (6 k^2 (2 r_1 + r_5) + t_1 + 4 t_2)$	$\omega_1^{\#2} \dagger \alpha \beta$	$-\frac{t_1 - 2 t_2}{3 \sqrt{2}}$	$f_1^{\#1} \dagger \alpha \beta$	$-\frac{i k (t_1 - 2 t_2)}{3 \sqrt{2}}$	$\omega_1^{\#1} \dagger \alpha$	0	$\omega_1^{\#2} \dagger \alpha$	0	$f_1^{\#1} \dagger \alpha$	0	$f_1^{\#2} \dagger \alpha$	0
$\omega_1^{\#2} \dagger \alpha \beta$	$-\frac{t_1 - 2 t_2}{3 \sqrt{2}}$	$\omega_1^{\#1} \dagger \alpha \beta$	$\frac{t_1 + t_2}{3}$	$f_1^{\#2} \dagger \alpha \beta$	$\frac{1}{3} i k (t_1 + t_2)$	$\omega_1^{\#1} \dagger \alpha$	0	$\omega_1^{\#2} \dagger \alpha$	0	$f_1^{\#1} \dagger \alpha$	0	$f_1^{\#2} \dagger \alpha$	0
$f_1^{\#1} \dagger \alpha \beta$	$\frac{i k (t_1 - 2 t_2)}{3 \sqrt{2}}$	$f_1^{\#2} \dagger \alpha \beta$	$-\frac{1}{3} i k (t_1 + t_2)$	$\omega_1^{\#1} \dagger \alpha \beta$	$\frac{1}{3} k^2 (t_1 + t_2)$	$\omega_1^{\#1} \dagger \alpha$	0	$\omega_1^{\#2} \dagger \alpha$	0	$f_1^{\#1} \dagger \alpha$	0	$f_1^{\#2} \dagger \alpha$	0
$\omega_1^{\#1} \dagger \alpha$	0	$\omega_1^{\#2} \dagger \alpha$	0	$f_1^{\#1} \dagger \alpha$	0	$\omega_1^{\#1} \dagger \alpha$	$k^2 (r_1 + r_5) - \frac{t_1}{2}$	$\omega_1^{\#2} \dagger \alpha$	$\frac{t_1}{\sqrt{2}}$	$f_1^{\#1} \dagger \alpha$	0	$f_1^{\#2} \dagger \alpha$	$i k t_1$
$\omega_1^{\#2} \dagger \alpha$	0	$\omega_1^{\#1} \dagger \alpha$	0	$f_1^{\#2} \dagger \alpha$	0	$\omega_1^{\#1} \dagger \alpha$	$\frac{t_1}{\sqrt{2}}$	$\omega_1^{\#2} \dagger \alpha$	0	$f_1^{\#1} \dagger \alpha$	0	$f_1^{\#2} \dagger \alpha$	0
$f_1^{\#1} \dagger \alpha$	0	$f_1^{\#2} \dagger \alpha$	0	$\omega_1^{\#1} \dagger \alpha$	0	$\omega_1^{\#1} \dagger \alpha$	0	$\omega_1^{\#2} \dagger \alpha$	0	$f_1^{\#1} \dagger \alpha$	0	$f_1^{\#2} \dagger \alpha$	0
$f_1^{\#2} \dagger \alpha$	0	$f_1^{\#1} \dagger \alpha$	0	$\omega_1^{\#1} \dagger \alpha$	0	$\omega_1^{\#1} \dagger \alpha$	$-i k t_1$	$\omega_1^{\#2} \dagger \alpha$	0	$f_1^{\#1} \dagger \alpha$	0	$f_1^{\#2} \dagger \alpha$	0

	$\omega_0^{\#1} \dagger$	$f_0^{\#1}$	$f_0^{\#2}$	$\omega_0^{\#1}$
$\omega_0^{\#1} \dagger$	$-t_1$	$i \sqrt{2} k t_1$	0	0
$f_0^{\#1} \dagger$	$-i \sqrt{2} k t_1$	$-2 k^2 t_1$	0	0
$f_0^{\#2} \dagger$	0	0	0	0
$\omega_0^{\#1} \dagger$	0	0	0	$t_2$

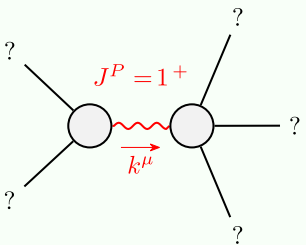
	$\sigma_2^{\#1} + \alpha \beta$	$\tau_2^{\#1} + \alpha \beta$	$\sigma_2^{\#1} - \alpha \beta$
$\sigma_2^{\#1} + \alpha \beta$	$\frac{2}{(1+2k^2)^2 t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	0
$\tau_2^{\#1} + \alpha \beta$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	$\frac{4k^2}{(1+2k^2)^2 t_1}$	0
$\sigma_2^{\#1} + \alpha \beta$	0	0	$\frac{2}{2k^2 t_1 + t_2}$

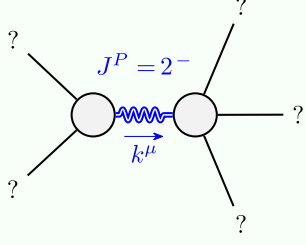
	$\sigma_0^{\#1}$	$\tau_0^{\#1}$	$\tau_0^{\#2}$	$\sigma_0^{\#1}$
$\sigma_0^{\#1} \dagger$	$-\frac{1}{(1+2k^2)^2 t_1}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2 t_1}$	0	0
$\tau_0^{\#1} \dagger$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2 t_1}$	$-\frac{2k^2}{(1+2k^2)^2 t_1}$	0	0
$\tau_0^{\#2} \dagger$	0	0	0	0
$\sigma_0^{\#1} \dagger$	0	0	0	$\frac{1}{t_2}$

$$\begin{aligned}
S = & \iiint \left( \frac{1}{6} (6 t_1 \omega^{\alpha i}_{\alpha} \omega^{\theta}_{i\theta} + 6 f^{\alpha\beta} \tau_{\alpha\beta} + 6 \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - 12 t_1 \omega^{\theta}_{\alpha\theta} \partial_i f^{\alpha i} + 12 t_1 \right. \\
& \omega^{\theta}_{i\theta} \partial' f^{\alpha}_{\alpha} - 6 t_1 \partial_i f^{\theta}_{\theta} \partial' f^{\alpha}_{\alpha} - 6 t_1 \partial_i f^{\alpha i} \partial_{\theta} f^{\theta}_{\alpha} + \\
& 12 t_1 \partial' f^{\alpha}_{\alpha} \partial_{\theta} f^{\theta}_{i\theta} + 4 t_1 \omega_{i\theta\alpha} \partial^{\theta} f^{\alpha i} + 4 t_2 \omega_{i\theta\alpha} \partial^{\theta} f^{\alpha i} - \\
& 4 t_1 \partial_{\alpha} f_{i\theta} \partial^{\theta} f^{\alpha i} + 2 t_2 \partial_{\alpha} f_{i\theta} \partial^{\theta} f^{\alpha i} - 4 t_1 \partial_{\alpha} f_{\theta i} \partial^{\theta} f^{\alpha i} - \\
& t_2 \partial_{\alpha} f_{\theta i} \partial^{\theta} f^{\alpha i} + 2 t_1 \partial_i f_{\alpha\theta} \partial^{\theta} f^{\alpha i} - t_2 \partial_i f_{\alpha\theta} \partial^{\theta} f^{\alpha i} + \\
& 4 t_1 \partial_{\theta} f_{\alpha i} \partial^{\theta} f^{\alpha i} + t_2 \partial_{\theta} f_{\alpha i} \partial^{\theta} f^{\alpha i} + 2 t_1 \partial_{\theta} f_{i\alpha} \partial^{\theta} f^{\alpha i} - \\
& t_2 \partial_{\theta} f_{i\alpha} \partial^{\theta} f^{\alpha i} + 2 (t_1 + t_2) \omega_{\alpha i\theta} (\omega^{\alpha i\theta} + 2 \partial^{\theta} f^{\alpha i}) + \\
& 2 \omega_{\alpha\theta i} ((t_1 - 2 t_2) \omega^{\alpha i\theta} + 2 (2 t_1 - t_2) \partial^{\theta} f^{\alpha i}) - \\
& 8 r_1 \partial_{\beta} \omega_{\alpha i\theta} \partial^{\theta} \omega^{\alpha\beta i} + 4 r_1 \partial_{\beta} \omega_{\alpha\theta i} \partial^{\theta} \omega^{\alpha\beta i} - \\
& 16 r_1 \partial_{\beta} \omega_{i\theta\alpha} \partial^{\theta} \omega^{\alpha\beta i} - 4 r_1 \partial_i \omega_{\alpha\beta\theta} \partial^{\theta} \omega^{\alpha\beta i} + \\
& 4 r_1 \partial_{\theta} \omega_{\alpha\beta i} \partial^{\theta} \omega^{\alpha\beta i} + 4 r_1 \partial_{\theta} \omega_{\alpha i\beta} \partial^{\theta} \omega^{\alpha\beta i} + \\
& 6 r_5 \partial_i \omega_{\theta}^{\kappa} \partial^{\theta} \omega^{\alpha i}_{\alpha} - 6 r_5 \partial_{\theta} \omega_i^{\kappa} \partial^{\theta} \omega^{\alpha i}_{\alpha} - 6 r_5 \partial_{\alpha} \omega^{\alpha i\theta} \\
& \partial_{\kappa} \omega_i^{\kappa}_{\theta} + 12 r_5 \partial^{\theta} \omega^{\alpha i}_{\alpha} \partial_{\kappa} \omega_i^{\kappa}_{\theta} + 6 r_5 \partial_{\alpha} \omega^{\alpha i\theta} \partial_{\kappa} \omega_{\theta}^{\kappa}_{i} - \\
& \left. 12 r_5 \partial^{\theta} \omega^{\alpha i}_{\alpha} \partial_{\kappa} \omega_{\theta}^{\kappa}_{i} \right) [t, x, y, z] dz dy dx dt
\end{aligned}$$

## Massive and massless spectra



Massive particle	
Pole residue:	$\frac{-3t_1t_2(t_1+t_2)+6r_1(t_1^2+2t_2^2)+3r_5(t_1^2+2t_2^2)}{(2r_1+r_5)(t_1+t_2)(-3t_1t_2+4r_1(t_1+t_2)+2r_5(t_1+t_2))} > 0$
Polarisations:	3
Square mass:	$-\frac{3t_1t_2}{2(2r_1+r_5)(t_1+t_2)} > 0$
Spin:	1
Parity:	Even



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

## Unitarity conditions

$$r_1 < 0 \ \&\& \ r_5 > -2r_1 \ \&\& \ t_1 > 0 \ \&\& \ -t_1 < t_2 < 0$$