

# Particle spectrograph

## Wave operator and propagator

	$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$	$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1-}^{\#1} \dagger^{\alpha}$	$\sigma_{1-}^{\#2} \dagger^{\alpha}$	$\tau_{1-}^{\#1} \dagger^{\alpha}$	$\tau_{1-}^{\#2} \dagger^{\alpha}$
$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	0	$-\frac{\sqrt{2}}{t_1+k^2}t_1$	$-\frac{i\sqrt{2}k}{t_1+k^2}t_1$	0	0	0	0
$\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{\sqrt{2}}{t_1+k^2}t_1$	$\frac{1}{(1+k^2)^2}t_1$	$\frac{ik}{(1+k^2)^2}t_1$	0	0	0	0
$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{i\sqrt{2}k}{t_1+k^2}t_1$	$-\frac{ik}{(1+k^2)^2}t_1$	$\frac{k^2}{(1+k^2)^2}t_1$	0	0	0	0
$\sigma_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2}t_1$	0	$\frac{2ik}{t_1+2k^2}t_1$
$\sigma_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2}t_1$	$\frac{2k^2r_1+t_1}{(t_1+2k^2t_1)^2}$	0	$\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
$\tau_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$\tau_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$-\frac{2ik}{t_1+2k^2}t_1$	$-\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$	0	$\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$

### Quadratic (free) Lagrangian density

$$\begin{aligned}
 &-t_1\omega_{\lambda'}^{\alpha'}\omega_{\kappa\alpha}^{\kappa}-t_1\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\lambda'}^{\lambda}+f^{\alpha\beta}\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}+2r_1\partial_{\lambda}\omega^{\kappa\lambda}_{\kappa}\partial'_{\kappa}\omega_{\lambda}^{\alpha}- \\
 &\frac{2}{3}r_1\partial^{\beta}\omega^{\theta\alpha}_{\kappa}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}-\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta}+\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta}+ \\
 &2r_1\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega^{\theta\kappa\lambda}-2r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}+2r_1\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}- \\
 &4r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}-\frac{1}{2}t_1\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{2}t_1\partial^{\alpha}f_{\kappa\theta}\partial^{\theta}f_{\alpha}^{\kappa}-\frac{1}{2}t_1\partial^{\alpha}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\alpha}^{\lambda}\partial^{\kappa}f_{\alpha\lambda}+ \\
 &t_1\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f'_{\lambda}+t_1\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f'_{\lambda}+2t_1\partial^{\alpha}f_{\kappa\alpha}\partial^{\kappa}f'_{\lambda}-t_1\partial_{\kappa}f'_{\lambda}\partial^{\kappa}f'_{\lambda}+2t_1\omega_{\lambda\kappa\theta}\partial^{\kappa}f'^{\theta}- \\
 &t_1\omega_{\lambda\alpha}^{\alpha}\partial^{\kappa}f'_{\kappa}-t_1\omega_{\lambda\lambda}^{\lambda}\partial^{\kappa}f'_{\kappa}+\frac{1}{2}t_1\partial^{\alpha}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda\alpha}^{\kappa}+\frac{1}{2}t_1\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\theta}^{\lambda}+ \\
 &\frac{1}{2}t_1\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}-t_1\partial^{\alpha}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda\kappa}^{\kappa}+\frac{2}{3}r_1\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}-\frac{2}{3}r_1\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}+ \\
 &\frac{2}{3}r_1\partial^{\beta}\omega_{\lambda}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda}-\frac{8}{3}r_1\partial^{\beta}\omega_{\lambda}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda}-2r_1\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\lambda}^{\theta\kappa}+2r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\lambda}^{\theta\kappa}_{\kappa}
 \end{aligned}$$

	$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1-}^{\#1} \dagger^{\alpha}$	$\omega_{1-}^{\#2} \dagger^{\alpha}$	$f_{1-}^{\#1} \dagger^{\alpha}$	$f_{1-}^{\#2} \dagger^{\alpha}$
$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$-\frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	$-k^2r_1-\frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$i\tilde{k}t_1$
$\omega_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
$f_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0	0
$f_{1-}^{\#2} \dagger^{\alpha}$	0	0	0	$-i\tilde{k}t_1$	0	0	0

	$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2}t_1$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$	0
$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$	$\frac{4k^2}{(1+2k^2)^2}t_1$	0
$\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{2k^2r_1+t_1}$

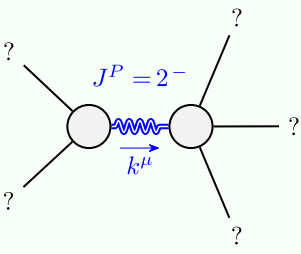
	$\sigma_0^{\#1} \dagger$	$\tau_0^{\#1} \dagger$	$\tau_0^{\#2} \dagger$	$\sigma_0^{\#1} \dagger$
$\sigma_0^{\#1} \dagger$	$-\frac{1}{(1+2k^2)^2}t_1$	$\frac{i\sqrt{2}k}{(1+2k^2)^2}t_1$	0	0
$\tau_0^{\#1} \dagger$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_1$	$-\frac{2k^2}{(1+2k^2)^2}t_1$	0	0
$\tau_0^{\#2} \dagger$	0	0	0	0
$\sigma_0^{\#1} \dagger$	0	0	0	$-\frac{1}{t_1}$

	$\omega_0^{\#1} \dagger$	$f_0^{\#1} \dagger$	$f_0^{\#2} \dagger$	$\omega_0^{\#1} \dagger$
$\omega_0^{\#1} \dagger$	$-t_1$	$i\sqrt{2}kt_1$	0	0
$f_0^{\#1} \dagger$	$-i\sqrt{2}kt_1$	$-2k^2t_1$	0	0
$f_0^{\#2} \dagger$	0	0	0	0
$\omega_0^{\#1} \dagger$	0	0	0	$-t_1$

	$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	$k^2t_1$	0
$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$k^2r_1+\frac{t_1}{2}$

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} - 2i\tilde{k}\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2i\tilde{k}\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + i\tilde{k}\sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2i\tilde{k}\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	16

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

## Unitarity conditions

$$r_1 < 0 \&\& t_1 > 0$$