## Particle spectrograph

Wave operator and propagator

| $\tau_{1}^{\#2}{}_{\alpha}$   | 0   | 0   | 0   | $\frac{2ik}{t_1 + 2k^2t_1}$          | $-\frac{i\sqrt{2}k(2k^2r_5-t_1)}{(t_1+2k^2t_1)^2}$ | 0                           | $\frac{-4k^4r_5+2k^2t_1}{(t_1+2k^2t_1)^2}$        |  |
|---|---|---|---|--------------------------------------|--|-----------------------------|---|--|
| $\tau_{1}^{\#1}{}_{\alpha}$   | 0   | 0   | 0   | 0                                    | 0  | 0                           | 0   |  |
| $\sigma_{1}^{\#2}{}_{\alpha}$   | 0   | 0   | 0   | $\frac{\sqrt{2}}{t_1 + 2 k^2 t_1}$   | $\frac{-2k^2r_5+t_1}{(t_1+2k^2t_1)^2}$             | 0                           | $\frac{i\sqrt{2}k(2k^2r_5-t_1)}{(t_1+2k^2t_1)^2}$ |  |
| $\sigma_{1}^{\#1}{}_{\alpha}$   | 0   | 0   | 0   | $0$ $\frac{\sqrt{2}}{t_1 + 2k^2t_1}$ |  | 0                           | $-\frac{2ik}{t_1+2k^2t_1}$                        |  |
| ${\mathfrak l}_1^{\#1}{}_{\!$ | $\frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2t_5(t_1+t_2))}$            | $\frac{ik(6k^2r_5+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2r_5(t_1+t_2))}$                | $\frac{k^2 \left(6  k^2  r_5 + t_1 + 4  t_2\right)}{\left(1 + k^2\right)^2 \left(3  t_1  t_2 + 2  k^2  r_5  \left(t_1 + t_2\right)\right)}$ | 0                                    | 0  | 0                           | 0   |  |
| $\sigma_{1}^{\#2}{}_{\alpha\beta}$  | $\frac{\sqrt{2} (t_1 - 2t_2)}{(1 + k^2) (3t_1 t_2 + 2k^2 r_5 (t_1 + t_2))}$ | $\frac{6 k^2 r_5 + t_1 + 4 t_2}{(1 + k^2)^2 (3 t_1 t_2 + 2 k^2 r_5 (t_1 + t_2))}$ | $-\frac{i k (6 k^2 r_5 + t_1 + 4 t_2)}{(1 + k^2)^2 (3 t_1 t_2 + 2 k^2 r_5 (t_1 + t_2))}$  | 0                                    | 0  | 0                           | 0   |  |
| $\sigma_{1}^{\#1}{}_{\alpha\beta}$  | 3t1t;   | $\frac{\sqrt{2} (t_1 - 2t_2)}{(1 + k^2) (3t_1t_2 + 2k^2t_5(t_1 + t_2))}$          | $i \sqrt{2} k(t_1-2t_2) + k^2)(3t_1t_2 + 2k^2r_5(t_1+t_2))$   | 0                                    | 0  | 0                           | 0   |  |
|   | $r_1^{\#1} + \alpha \beta$  | $r_1^{#2} + \alpha \beta$   | $\begin{bmatrix} r_1^{*1} + \alpha \beta \end{bmatrix} - \frac{1}{(1-\alpha)^2}$  | $\sigma_{1}^{\#1} +^{lpha}$          | $\sigma_{1}^{\#2} +^{lpha}$                        | $\tau_{1}^{\#1} +^{\alpha}$ | $t_{1}^{#2} + \alpha$                             |  |

| $\omega_{1+}^{\#1} + \alpha^{\beta} \left[ \frac{1}{6} \left( 6 k^{2} r_{5} + t_{1} + 4 t_{2} \right) \right] - \frac{t_{1} - 2 t_{2}}{3 \sqrt{2}} - \frac{i k (t_{1} - 2 t_{2})}{3 \sqrt{2}} $ |                          |                 |
|---|--------------------------|-----------------|
| $6$ $3\sqrt{2}$ $3\sqrt{2}$   | 0 0                      | 0               |
| $\omega_{1}^{\#2} + \alpha^{\beta}$   | 0 0                      | 0               |
| $f_{1}^{\#1} + \alpha \beta = \frac{i k (t_{1} - 2 t_{2})}{3 \sqrt{2}} - \frac{1}{3} i k (t_{1} + t_{2}) \frac{1}{3} k^{2} (t_{1} + t_{2}) = 0$   | 0 0                      | 0               |
| $\omega_{1}^{\#1} + {}^{\alpha}$ 0 0 $k^2 r_5 - \frac{t_1}{2}$  | $\frac{t_1}{\sqrt{2}}$ C | $\bar{l} k t_1$ |
| $\omega_{1}^{\#2} \uparrow^{\alpha}$ 0 0 $\frac{t_{1}}{\sqrt{2}}$   | 0 0                      | 0               |
| $f_{1}^{#1} \uparrow^{\alpha}$ 0 0 0  | 0 0                      | 0               |
| $f_{1}^{\#2} \uparrow^{\alpha}$ 0 0 -ikt <sub>1</sub>   | 0 0                      | 0               |

 $12r_5\,\partial_\theta\omega_\lambda^{\ \alpha}\partial_\kappa\omega^{\kappa\lambda\theta}-2\,t_1\,\partial^\alpha f_{\,\theta\kappa}\,\partial^\kappa f_{\,\alpha}^{\ \theta}+t_2\,\partial^\alpha f_{\,\theta\kappa}\,\partial^\kappa f_{\,\alpha}^{\ \theta}-4\,t_1\,\partial^\alpha f_{\,\kappa\theta}\,\partial^\kappa f_{\,\alpha}^{\ \theta}-$ 

 $6r_5\partial_\alpha\omega_\lambda^{\phantom{\lambda}\alpha}_{\phantom{\lambda}\theta}\partial_\kappa\omega^{\theta\kappa\lambda} + 6r_5\partial_\theta\omega_\lambda^{\phantom{\lambda}\alpha}_{\phantom{\lambda}\alpha}\partial_\kappa\omega^{\theta\kappa\lambda} - 6r_5\partial_\alpha\omega_\lambda^{\phantom{\lambda}\alpha}_{\phantom{\lambda}\theta}\partial_\kappa\omega^{\kappa\lambda\theta} +$ 

 $S_{F} == \iiint (\frac{1}{6} (-6t_{1} \omega_{\kappa \alpha}^{\alpha \prime} + 2(t_{1} - 2t_{2}) \omega_{\kappa \lambda}^{\kappa \lambda} + 2t_{1} \omega_{\kappa \lambda}^{\prime \prime})$ 

Quadratic (free) action

 $2t_2 \ \omega_{\kappa\lambda}^{\phantom{\kappa\lambda}\prime} \ \omega^{\kappa\lambda}^{\phantom{\kappa\lambda}\prime} + 6 \ f^{\alpha\beta} \ \tau_{\alpha\beta} + 6 \ \omega^{\alpha\beta\chi} \ \sigma_{\alpha\beta\chi} - 6 \ r_5 \ \partial_{\phantom{\kappa\lambda}\kappa}^{\phantom{\kappa\lambda}\prime} \ \partial^{\prime}\omega_{\phantom{\kappa\lambda}\alpha}^{\phantom{\lambda}\alpha} \ .$ 

 $6t_1\ \omega_{\kappa\lambda}^{\ \lambda}\ \partial^{\kappa}f'_{\ \prime} + 12t_1\,\partial^{\alpha}f_{\ \kappa\alpha}\,\partial^{\kappa}f'_{\ \prime} - 6t_1\,\partial_{\kappa}f^{\lambda}_{\ \lambda}\,\partial^{\kappa}f'_{\ \prime} + 2t_1\ \omega_{\ell\theta\kappa}\ \partial^{\kappa}f^{\ell\theta} +$ 

 $2t_2 \, \omega_{\theta_{IK}} \, \partial^{\kappa} f^{I\theta} + 4t_1 \, \omega_{\theta_{KI}} \, \partial^{\kappa} f^{I\theta} + 4t_2 \, \omega_{\theta_{KI}} \, \partial^{\kappa} f^{I\theta} - 6t_1 \, \omega_{I\alpha}^{\quad \alpha} \, \partial^{\kappa} f^{I}_{\kappa}$ 

 $\int_{\kappa} -6r_5 \, \partial_{\theta} \omega_{\lambda}^{\alpha} \, \partial^{\lambda} \omega^{\theta \kappa}_{\kappa}) [t, x, y, z] dz dy dx dt$ 

 $2t_2\;\omega_{_{I}\theta_{K}}\,\partial^{K}f^{^{I}\theta}+8t_1\;\omega_{_{IK}\theta}\;\partial^{K}f^{^{I}\theta}-4t_2\;\omega_{_{IK}\theta}\;\partial^{K}f^{^{I}\theta}-2t_1\;\omega_{\theta_{IK}}\;\partial^{K}f^{^{I}\theta}-$ 

 $t_2 \partial^{\alpha} f_{\kappa\theta} \partial^{\kappa} f_{\alpha}^{\ \theta} - 2 t_1 \partial^{\alpha} f^{\lambda}_{\ \kappa} \partial^{\kappa} f_{\alpha\lambda} + t_2 \partial^{\alpha} f^{\lambda}_{\ \kappa} \partial^{\kappa} f_{\alpha\lambda} + 6 t_1 \omega_{\kappa\alpha}^{\ \alpha} \partial^{\kappa} f'_{\ \gamma} +$ 

(No massless particles)

| $\sigma_{0}^{\#1}$   | 0                                  | 0                                   | 0                  | $\frac{1}{t_2}$        |                    |                         |                   |                 |                              |
|----------------------|------------------------------------|-------------------------------------|--------------------|------------------------|--------------------|-------------------------|-------------------|-----------------|------------------------------|
| •                    |                                    |                                     |                    | 7                      | $\omega_{0}^{\#1}$ | 0                       | 0                 | 0               | $t_2$                        |
| $\tau_{0}^{\#2}$     | 0                                  | 0                                   | 0                  | 0                      | •                  |                         |                   |                 | 1                            |
|                      | - 1                                | <u>t</u> 1                          |                    |                        | $f_{0}^{\#2}$      | 0                       | 0                 | 0               | 0                            |
| $\tau_{0}^{\#1}$     | $\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$ | $-\frac{2k^2}{(1+2k^2)^2t_1}$       | 0                  | 0                      | $f_0^{\#1}$        | $i\sqrt{2}kt_1$         | $-2 k^2 t_1$      | 0               | 0                            |
| $\sigma_{0^+}^{\#1}$ | $-\frac{1}{(1+2k^2)^2t_1}$         | $-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$ | 0                  | 0                      | $\omega_{0}^{\#1}$ | -t <sub>1</sub>         | $-i\sqrt{2} kt_1$ | 0               | 0                            |
|                      | $\sigma_{0}^{\#1}$ †               | $\tau_{0}^{\#1}$ †                  | $\tau_{0}^{\#2}$ † | $\sigma_{0^-}^{\#1}$ † |                    | $\omega_0^{\#1}\dagger$ | $f_{0}^{\#1}$ †   | $f_{0}^{\#2}$ † | $\omega_{0}^{\#1}$ $\dagger$ |

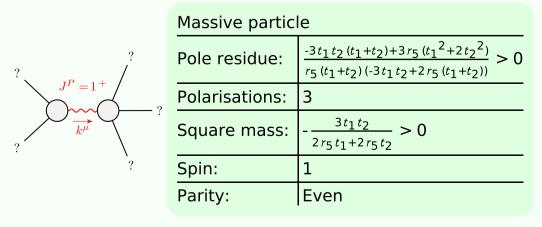
0

 $\tau_2^{\#1} + \alpha\beta$ 

| Source constraints/gauge generators  |                |  |  |  |
|--|----------------|--|--|--|
| SO(3) irreps   | Multiplicities |  |  |  |
| $\tau_{0+}^{\#2} == 0$   | 1              |  |  |  |
| $\tau_{0+}^{\#1} - 2  i  k  \sigma_{0+}^{\#1} == 0$                              | 1              |  |  |  |
| $\tau_{1}^{\#2\alpha} + 2 i k \sigma_{1}^{\#2\alpha} == 0$                       | 3              |  |  |  |
| $\tau_{1}^{\#1\alpha} == 0$  | 3              |  |  |  |
| $\tau_{1+}^{\#1\alpha\beta} + i k \sigma_{1+}^{\#2\alpha\beta} == 0$             | 3              |  |  |  |
| $\tau_{2+}^{\#1}{}^{\alpha\beta} - 2 i k \sigma_{2+}^{\#1}{}^{\alpha\beta} == 0$ | 5              |  |  |  |
| Total constraints:   | 16             |  |  |  |

|  | $\omega_{2^{+}\alpha\beta}^{\#1}$ | $f_{2}^{\#1}{}_{lphaeta}$ | $\omega_2^{\#1}{}_{lphaeta_{\lambda}}$ |
|--|-----------------------------------|---------------------------|--|
| $\omega_2^{\sharp 1} \dagger^{lpha eta}$ | <u>t</u> 1 2                      | $-\frac{ikt_1}{\sqrt{2}}$ | 0                                      |
| $f_2^{#1} \dagger^{\alpha\beta}$         | $\frac{i k t_1}{\sqrt{2}}$        | $k^2 t_1$                 | 0                                      |
| $\nu_2^{\#1} \dagger^{\alpha\beta\chi}$  | 0                                 | 0                         | <u>t</u> 1<br>2                        |

## Massive and massless spectra



## **Unitarity conditions**

 $r_5 > 0 \&\& (t_1 < 0 \&\& (t_2 < 0 || t_2 > -t_1)) || (t_1 > 0 \&\& -t_1 < t_2 < 0)$