

# Particle spectrograph

## Wave operator and propagator

Quadratic (free) action

$$S_F == \iiint (\frac{1}{3} (-3 t_1 \omega_{\lambda'}^{\alpha\iota} \omega_{\kappa\alpha}^{-t_1} \omega_{\kappa\lambda}^{\kappa\lambda} \omega_{\kappa\lambda'}^{\lambda'} + t_1 \omega_{\kappa\lambda}^{\lambda'} \omega_{\kappa\lambda}^{\kappa\lambda} + 3 f^{\alpha\beta} \tau_{\alpha\beta} + 3 \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} + 6 r_1 \partial_{\iota} \omega_{\kappa}^{\kappa\lambda} \partial' \omega_{\lambda}^{\alpha} - 2 r_1 \partial^{\beta} \omega_{\kappa}^{\theta\alpha} \partial_{\theta} \omega_{\alpha\beta}^{\kappa} - 2 r_1 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\alpha\beta\theta} + 2 r_1 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\theta\alpha\beta} + 6 r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\theta\kappa\lambda} - 6 r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\theta\kappa\lambda} - 6 r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\kappa\lambda\theta} - 12 r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\kappa\lambda\theta} - t_1 \partial^{\alpha} f_{\theta\kappa} \partial^{\kappa} f_{\alpha}^{\theta} - 2 t_1 \partial^{\alpha} f_{\kappa\theta} \partial^{\kappa} f_{\alpha}^{\theta} - t_1 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\alpha\lambda} + 3 t_1 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f_{\lambda'}^{\iota} + 3 t_1 \omega_{\kappa\lambda}^{\lambda} \partial^{\kappa} f_{\iota}^{\iota} + 6 t_1 \partial^{\alpha} f_{\kappa\alpha} \partial^{\kappa} f_{\lambda'}^{\iota} - 3 t_1 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\iota}^{\iota} + t_1 \omega_{\iota\theta\kappa} \partial^{\kappa} f_{\iota}^{\theta} + 4 t_1 \omega_{\iota\kappa\theta} \partial^{\kappa} f_{\iota}^{\theta} - t_1 \omega_{\theta\iota\kappa} \partial^{\kappa} f_{\iota}^{\theta} + 2 t_1 \omega_{\theta\iota\kappa} \partial^{\kappa} f_{\iota}^{\theta} - 3 t_1 \omega_{\iota\alpha}^{\alpha} \partial^{\kappa} f_{\kappa}^{\lambda} - 3 t_1 \omega_{\iota\lambda}^{\lambda} \partial^{\kappa} f_{\kappa}^{\lambda} + t_1 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\lambda\alpha} + t_1 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} + 2 t_1 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} - 3 t_1 \partial^{\alpha} f_{\lambda}^{\alpha} \partial^{\kappa} f_{\lambda\kappa} + 2 r_1 \partial_{\kappa} \omega^{\alpha\beta\theta} \partial^{\kappa} \omega_{\alpha\beta\theta} - 2 r_1 \partial_{\kappa} \omega^{\theta\alpha\beta} \partial^{\kappa} \omega_{\alpha\beta\theta} + 2 r_1 \partial^{\beta} \omega_{\lambda'}^{\alpha\lambda} \partial_{\lambda} \omega_{\alpha\beta}^{\iota} - 8 r_1 \partial^{\beta} \omega_{\lambda'}^{\lambda\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\iota} - 6 r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\theta\kappa} + 6 r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\alpha}^{\theta\kappa} )) [t, x, y, z] dz dy dx dt$$

$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$	$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1-}^{\#1} \dagger^{\alpha}$	$\sigma_{1-}^{\#2} \dagger^{\alpha}$	$\tau_{1-}^{\#1} \dagger^{\alpha}$	$\tau_{1-}^{\#2} \dagger^{\alpha}$
$\frac{6}{(3+2k^2)^2} t_1$	$-\frac{6\sqrt{2}}{(3+2k^2)^2} t_1$	$-\frac{6i\sqrt{2}k}{(3+2k^2)^2} t_1$	0	0	0	0
$-\frac{6\sqrt{2}}{(3+2k^2)^2} t_1$	$\frac{12}{(3+2k^2)^2} t_1$	$\frac{12ik}{(3+2k^2)^2} t_1$	0	0	0	0
$\frac{6i\sqrt{2}k}{(3+2k^2)^2} t_1$	$-\frac{12ik}{(3+2k^2)^2} t_1$	$\frac{12k^2}{(3+2k^2)^2} t_1$	0	0	0	0
0	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2} t_1$	$\frac{2ik}{t_1+2k^2} t_1$	$\frac{2ik}{t_1+2k^2} t_1$
0	0	0	$\frac{\sqrt{2}}{t_1+2k^2} t_1$	$\frac{2k^2 r_1+t_1}{(t_1+2k^2)^2} t_1$	$\frac{i\sqrt{2}k(2k^2 r_1+t_1)}{(t_1+2k^2)^2} t_1$	$\frac{i\sqrt{2}k(2k^2 r_1+t_1)}{(t_1+2k^2)^2} t_1$
0	0	0	0	0	0	0
0	0	0	$-\frac{2ik}{t_1+2k^2} t_1$	$-\frac{i\sqrt{2}k(2k^2 r_1+t_1)}{(t_1+2k^2)^2} t_1$	0	$\frac{2k^2(2k^2 r_1+t_1)}{(t_1+2k^2)^2} t_1$

$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1-}^{\#1} \dagger^{\alpha}$	$\omega_{1-}^{\#2} \dagger^{\alpha}$	$f_{1-}^{\#1} \dagger^{\alpha}$	$f_{1-}^{\#2} \dagger^{\alpha}$
$\frac{t_1}{6}$	$-\frac{t_1}{3\sqrt{2}}$	$-\frac{ikt_1}{3\sqrt{2}}$	0	0	0	0
$-\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	$\frac{ikt_1}{3}$	0	0	0	0
$\frac{ikt_1}{3\sqrt{2}}$	$-\frac{1}{3} ik t_1$	$\frac{k^2 t_1}{3}$	0	0	0	0
0	0	0	$-k^2 r_1 - \frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	$\frac{t_1}{\sqrt{2}}$	$ik t_1$
0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
0	0	0	0	0	0	0
0	0	0	$-ik t_1$	0	0	0

$\sigma_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#2} \dagger$	$\sigma_{0-}^{\#1} \dagger$
$-\frac{1}{(1+2k^2)^2} t_1$	$\frac{i\sqrt{2}k}{(1+2k^2)^2} t_1$	0	0
$-\frac{i\sqrt{2}k}{(1+2k^2)^2} t_1$	$-\frac{2k^2}{(1+2k^2)^2} t_1$	0	0
0	0	0	0
0	0	0	0

$\omega_{0+}^{\#1} \dagger$	$f_{0+}^{\#1} \dagger$	$f_{0+}^{\#2} \dagger$	$\omega_{0-}^{\#1} \dagger$
-t <sub>1</sub>	$i\sqrt{2}kt_1$	0	0
$-i\sqrt{2}kt_1$	$-2k^2t_1$	0	0
0	0	0	0
0	0	0	0

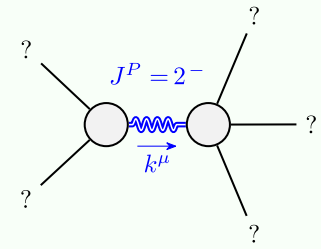
$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$\frac{ikt_1}{\sqrt{2}}$	$k^2 t_1$	0
0	0	$k^2 r_1 + \frac{t_1}{2}$

$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$\frac{2}{(1+2k^2)^2} t_1$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2} t_1$	0
$\frac{2i\sqrt{2}k}{(1+2k^2)^2} t_1$	$\frac{4k^2}{(1+2k^2)^2} t_1$	0
0	0	$\frac{2}{2k^2 r_1+t_1}$

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\sigma_0^{\#1} == 0$	1
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} - 2ik\sigma_{1+}^{\#1\alpha\beta} == 0$	3
$2\sigma_{1+}^{\#1\alpha\beta} + \sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	20

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

## Unitarity conditions

$r_1 < 0 \ \&\& \ t_1 > 0$