



Unitarity conditions  
 $\alpha_0 > 0 \ \&\& \ \alpha_3 < 0 \ \&\& \ \beta_1 < \frac{\alpha_0}{4}$

	$\sigma_{1+}^{\#1} \alpha\beta$	$\sigma_{1+}^{\#2} \alpha\beta$	$\tau_{1+}^{\#1} \alpha\beta$	$\sigma_{1-}^{\#1} \alpha$	$\sigma_{1-}^{\#2} \alpha$	$\tau_{1-}^{\#1} \alpha$	$\tau_{1-}^{\#2} \alpha$
$\sigma_{1+}^{\#1} + \alpha\beta$	0	$\frac{2\sqrt{2}}{(\alpha_0 - 4\beta_1)(1+k^2)}$	$\frac{2i\sqrt{2}k}{(\alpha_0 - 4\beta_1)(1+k^2)}$	0	0	0	0
$\sigma_{1+}^{\#2} + \alpha\beta$	$\frac{2\sqrt{2}}{(\alpha_0 - 4\beta_1)(1+k^2)}$	$-\frac{2}{(\alpha_0 - 4\beta_1)(1+k^2)^2}$	$-\frac{2ik}{(\alpha_0 - 4\beta_1)(1+k^2)^2}$	0	0	0	0
$\tau_{1+}^{\#1} + \alpha\beta$	$-\frac{2i\sqrt{2}k}{(\alpha_0 - 4\beta_1)(1+k^2)}$	$\frac{2ik}{(\alpha_0 - 4\beta_1)(1+k^2)^2}$	$-\frac{2k^2}{(\alpha_0 - 4\beta_1)(1+k^2)^2}$	0	0	0	0
$\sigma_{1-}^{\#1} + \alpha$	0	0	0	0	$-\frac{2\sqrt{2}}{(\alpha_0 - 4\beta_1)(1+2k^2)}$	0	$-\frac{4ik}{(\alpha_0 - 4\beta_1)(1+2k^2)}$
$\sigma_{1-}^{\#2} + \alpha$	0	0	0	$-\frac{2\sqrt{2}}{(\alpha_0 - 4\beta_1)(1+2k^2)}$	$-\frac{2}{(\alpha_0 - 4\beta_1)(1+2k^2)^2}$	0	$-\frac{2i\sqrt{2}k}{(\alpha_0 - 4\beta_1)(1+2k^2)^2}$
$\tau_{1-}^{\#1} + \alpha$	0	0	0	0	0	0	0
$\tau_{1-}^{\#2} + \alpha$	0	0	0	$\frac{4ik}{(\alpha_0 - 4\beta_1)(1+2k^2)}$	$\frac{2i\sqrt{2}k}{(\alpha_0 - 4\beta_1)(1+2k^2)^2}$	0	$-\frac{4k^2}{(\alpha_0 - 4\beta_1)(1+2k^2)^2}$

	$\omega_{2^+}^{\#1} \alpha \beta$	$f_{2^+}^{\#1} \alpha \beta$	$\omega_{2^+}^{\#1} \alpha \beta \chi$
$\omega_{2^+}^{\#1} \dagger \alpha \beta$	$-\frac{\alpha_0}{4} + \beta_1$	$\frac{i(\alpha_0 - 4\beta_1)k}{2\sqrt{2}}$	0
$f_{2^+}^{\#1} \dagger \alpha \beta$	$-\frac{i(\alpha_0 - 4\beta_1)k}{2\sqrt{2}}$	$2\beta_1 k^2$	0
$\omega_{2^+}^{\#1} \dagger \alpha \beta \chi$	0	0	$-\frac{\alpha_0}{4} + \beta_1$

	$\omega_1^{\#1} + \alpha\beta$	$\omega_1^{\#2} + \alpha\beta$	$f_1^{\#1} + \alpha\beta$	$\omega_1^{\#1} - \alpha$	$\omega_1^{\#2} - \alpha$	$f_1^{\#1} - \alpha$	$f_1^{\#2} - \alpha$
$\omega_1^{\#1} + \alpha\beta$	$\frac{1}{4}(\alpha_0 - 4\beta_1)$	$\frac{\alpha_0 - 4\beta_1}{2\sqrt{2}}$	$\frac{i(\alpha_0 - 4\beta_1)k}{2\sqrt{2}}$	0	0	0	0
$\omega_1^{\#2} + \alpha\beta$	$\frac{\alpha_0 - 4\beta_1}{2\sqrt{2}}$	0	0	0	0	0	0
$f_1^{\#1} + \alpha\beta$	$-\frac{i(\alpha_0 - 4\beta_1)k}{2\sqrt{2}}$	0	0	0	0	0	0
$\omega_1^{\#1} - \alpha$	0	0	0	$\frac{1}{4}(\alpha_0 - 4\beta_1)$	$-\frac{\alpha_0 - 4\beta_1}{2\sqrt{2}}$	0	$-\frac{1}{2}i(\alpha_0 - 4\beta_1)k$
$\omega_1^{\#2} - \alpha$	0	0	0	$-\frac{\alpha_0 - 4\beta_1}{2\sqrt{2}}$	0	0	0
$f_1^{\#1} - \alpha$	0	0	0	0	0	0	0
$f_1^{\#2} - \alpha$	0	0	0	$\frac{1}{2}i(\alpha_0 - 4\beta_1)k$	0	0	0

	$\sigma_{2+}^{\#1} \alpha \beta$	$\tau_{2+}^{\#1} \alpha \beta$	$\sigma_{2-}^{\#1} \alpha \beta \chi$
$\sigma_{2+}^{\#1} \dagger \alpha \beta$	$-\frac{16 \beta_1}{\alpha_0^2 - 4 \alpha_0 \beta_1}$	$\frac{2 i \sqrt{2}}{\alpha_0 k}$	0
$\tau_{2+}^{\#1} \dagger \alpha \beta$	$-\frac{2 i \sqrt{2}}{\alpha_0 k}$	$\frac{2}{\alpha_0 k^2}$	0
$\sigma_{2-}^{\#1} \dagger \alpha \beta \chi$	0	0	$\frac{1}{-\frac{\alpha_0}{4} + \beta_1}$

Source constraints	
SO(3) irreps	#
$\tau_{0+}^{\#2} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2 i k \sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + i k \sigma_{1+}^{\#2\alpha\beta} == 0$	3
Total #:	10

$\omega_0^{\#1} +$	$\frac{1}{2} (\alpha_0 - 4 \beta_1)$	$-\frac{i (\alpha_0 - 4 \beta_1) k}{\sqrt{2}}$	$0$	$\omega_0^{\#1}$
$f_0^{\#1} +$	$\frac{i (\alpha_0 - 4 \beta_1) k}{\sqrt{2}}$	$-4 \beta_1 k^2$	$0$	$0$
$f_0^{\#2} +$	$0$	$0$	$0$	$0$
$\omega_0^{\#1} +$	$0$	$0$	$0$	$\frac{\alpha_0}{2} - 2 \beta_1 + \alpha_3 k^2$

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	$\frac{8\beta_1}{\alpha_0^2 - 4\alpha_0\beta_1}$	$-\frac{i\sqrt{2}}{\alpha_0 k}$	0	0
$\tau_{0+}^{\#1} \dagger$	$\frac{i\sqrt{2}}{\alpha_0 k}$	$-\frac{1}{\alpha_0 k^2}$	0	0
$\tau_{0+}^{\#2} \dagger$	0	0	0	0
$\sigma_{0-}^{\#1} \dagger$	0	0	0	$\frac{2}{\alpha_0 - 4\beta_1 + 2\alpha_3 k^2}$

$$\begin{aligned}
& \text{Lagrangian density} \\
& -\frac{1}{2}\alpha_0\omega_{\alpha\chi\beta}\omega^{\alpha\beta\chi}-\frac{1}{2}\alpha_0\omega^{\alpha\beta}_\alpha\omega^\chi_\beta\chi+2\beta_1\omega^{\alpha\beta}_\alpha\omega^\chi_\beta\chi-2\beta_1\omega^\chi_\alpha\omega^{\chi\delta}_\alpha\omega^\alpha_{\chi\delta}+ \\
& f^{\alpha\beta}\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}-2\beta_1\omega^\chi_\alpha\chi\partial_\beta f^{\alpha\beta}-2\beta_1\omega^\delta_\alpha\partial_\beta f^{\alpha\beta}-\alpha_0f^{\alpha\beta}\partial_\beta\omega^\chi_\alpha\chi+ \\
& \alpha_0\partial_\beta\omega^{\alpha\beta}_\alpha+\frac{2}{3}\alpha_3\partial^\alpha\omega^{\beta\zeta}_\chi\partial_\beta\omega^\chi_{\zeta\alpha}+2\beta_1\omega^\chi_\beta\chi\partial^\beta f^\alpha_\alpha+2\beta_1\omega^\delta_\beta\omega^\delta_\delta\partial^\beta f^\alpha_\alpha- \\
& 2\beta_1\partial_\beta f^\chi_\chi\partial^\beta f^\alpha_\alpha+\alpha_0f^{\alpha\beta}\partial_\chi\omega^\chi_\alpha\omega^\chi_\beta-f^\alpha_\alpha\partial_\chi\omega^{\beta\chi}_\beta-\frac{2}{3}\alpha_3\partial_\beta\omega^\chi_{\zeta\alpha}\partial_\chi\omega^{\beta\zeta}_\alpha- \\
& \frac{1}{3}\alpha_3\partial_\beta\omega^\chi_{\zeta\alpha}\partial_\chi\omega^{\zeta\alpha\beta}+4\beta_1\omega_{\alpha\chi\beta}\omega^{\chi\alpha\beta}+\beta_1\partial_\chi f^\delta_\beta\partial^\chi f^\beta_\delta+\beta_1\partial_\chi f^\delta_\beta\partial^\chi f^\beta_\delta+ \\
& \frac{2}{3}\alpha_3\partial_\chi\omega^{\beta\zeta}_\alpha\partial^\chi\omega_{\zeta\alpha\beta}+\frac{1}{3}\alpha_3\partial_\chi\omega^{\zeta\alpha\beta}\partial^\chi\omega_{\zeta\alpha\beta}+4\beta_1\partial^\beta f^\alpha_\alpha\partial_\delta f^\delta_\beta- \\
& 2\beta_1\partial_\beta f^\beta_\chi\partial_\delta f^{\chi\delta}+\frac{2}{3}\alpha_3\partial^\beta\omega^\delta_\alpha\partial_\delta\omega^\alpha_{\zeta\beta}-\frac{2}{3}\alpha_3\partial^\beta\omega^\zeta_\alpha\partial_\delta\omega^\delta_{\zeta\beta}- \\
& \beta_1\partial^\chi f^\beta_\zeta\partial^\zeta f^\beta_{\beta\chi}-\beta_1\partial^\chi f^\beta_\zeta\partial^\zeta f^\beta_{\chi\beta}+\beta_1\partial^\chi f_{\delta\zeta}\partial^\zeta f^\delta_\chi-\beta_1\partial^\chi f_{\zeta\delta}\partial^\zeta f^\delta_\chi
\end{aligned}$$

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	$\frac{8\beta_1}{\alpha_0^2 - 4\alpha_0\beta_1}$	$-\frac{i\sqrt{2}}{\alpha_0 k}$	0	0
$\tau_{0+}^{\#1} \dagger$	$\frac{i\sqrt{2}}{\alpha_0 k}$	$-\frac{1}{\alpha_0 k^2}$	0	0
$\tau_{0+}^{\#2} \dagger$	0	0	0	0
$\sigma_{0-}^{\#1} \dagger$	0	0	0	$\frac{2}{\alpha_0 - 4\beta_1 + 2\alpha_3 k^2}$