

# Particle spectrograph

## Wave operator and propagator

$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$	$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{1-}^{\#1} \dagger^{\alpha}$	$\sigma_{1-}^{\#2} \dagger^{\alpha}$	$\tau_{1-}^{\#1} \dagger^{\alpha}$	$\tau_{1-}^{\#2} \dagger^{\alpha}$
$\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$	0	$-\frac{i\sqrt{2}k}{t_1+k^2t_1}$	0	0	0	0
$\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{\sqrt{2}}{t_1+k^2t_1}$	$\frac{-2k^2(2r_1+r_5)+t_1}{(1+k^2)^2t_1^2}$	0	0	0	0
$\tau_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{i\sqrt{2}k}{t_1+k^2t_1}$	$\frac{-2k^4(2r_1+r_5)+k^2t_1}{(1+k^2)^2t_1^2}$	0	0	0	0
$\sigma_{1-}^{\#1} \dagger^{\alpha}$	0	0	$\frac{1}{k^2(r_1+r_5)}$	$-\frac{1}{\sqrt{2}(k^2+2k^4)(r_1+r_5)}$	0	$-\frac{i}{k(1+2k^2)(r_1+r_5)}$
$\sigma_{1-}^{\#2} \dagger^{\alpha}$	0	0	$-\frac{1}{\sqrt{2}(k^2+2k^4)(r_1+r_5)}$	$\frac{6k^2(r_1+r_5)+t_1}{2(k+2k^3)^2(r_1+r_5)t_1}$	0	$\frac{i(6k^2(r_1+r_5)+t_1)}{\sqrt{2}k(1+2k^2)^2(r_1+r_5)t_1}$
$\tau_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0
$\tau_{1-}^{\#2} \dagger^{\alpha}$	0	0	$\frac{i}{k(1+2k^2)(r_1+r_5)}$	$-\frac{i(6k^2(r_1+r_5)+t_1)}{\sqrt{2}k(1+2k^2)^2(r_1+r_5)t_1}$	0	$\frac{6k^2(r_1+r_5)+t_1}{(1+2k^2)^2(r_1+r_5)t_1}$

	$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
$\tau_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{2k^2r_1+t_1}$

	$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$
$\omega_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	$k^2t_1$	0
$\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$	0	0	$k^2r_1+\frac{t_1}{2}$

$\sigma_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#2} \dagger$	$\sigma_{0-}^{\#1} \dagger$
$\sigma_{0+}^{\#1} \dagger$	0	0	0
$\tau_{0+}^{\#1} \dagger$	0	0	0
$\tau_{0+}^{\#2} \dagger$	0	0	0
$\sigma_{0-}^{\#1} \dagger$	0	0	$-\frac{1}{t_1}$

$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\omega_{1-}^{\#1} \dagger^{\alpha}$	$\omega_{1-}^{\#2} \dagger^{\alpha}$	$f_{1-}^{\#1} \dagger^{\alpha}$	$f_{1-}^{\#2} \dagger^{\alpha}$
$\omega_{1+}^{\#1} \dagger^{\alpha\beta}$	$k^2(2r_1+r_5)-\frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1+}^{\#2} \dagger^{\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0
$f_{1+}^{\#1} \dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0
$\omega_{1-}^{\#1} \dagger^{\alpha}$	0	0	$k^2(r_1+r_5)+\frac{t_1}{6}$	$\frac{t_1}{3\sqrt{2}}$	0	$\frac{ikt_1}{3}$
$\omega_{1-}^{\#2} \dagger^{\alpha}$	0	0	$\frac{t_1}{3\sqrt{2}}$	$\frac{t_1}{3}$	0	$\frac{1}{3}i\sqrt{2}kt_1$
$f_{1-}^{\#1} \dagger^{\alpha}$	0	0	0	0	0	0
$f_{1-}^{\#2} \dagger^{\alpha}$	0	0	$-\frac{1}{3}ikt_1$	$-\frac{1}{3}i\sqrt{2}kt_1$	0	$\frac{2k^2t_1}{3}$

Quadratic (free) Lagrangian density

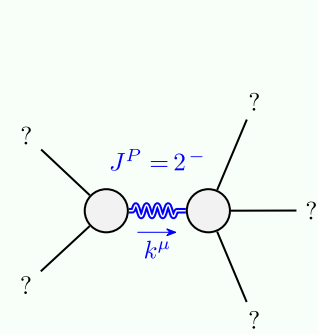
$$\begin{aligned} &-\frac{1}{3}t_1\omega_{\kappa\alpha}^{\alpha'}\omega_{\kappa\alpha}^{\kappa}-t_1\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\kappa\lambda}^{\lambda'}+f^{\alpha\beta}\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}-r_5\partial_{\lambda'}\omega_{\kappa}^{\kappa\lambda}\partial^{\lambda}\omega_{\lambda}^{\alpha}- \\ &\frac{2}{3}r_1\partial^{\beta}\omega_{\kappa}^{\theta\alpha}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}-\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\theta}+\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\theta} \\ &r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\theta}^{\theta\kappa\lambda}+r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\alpha}^{\theta\kappa\lambda}-r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\kappa\theta}+2r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\alpha}^{\kappa\lambda\theta}- \\ &\frac{1}{2}t_1\partial^{\alpha}f_{\theta\kappa}^{\kappa}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{2}t_1\partial^{\alpha}f_{\kappa\theta}^{\theta}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{2}t_1\partial^{\alpha}f_{\kappa}^{\theta}\partial^{\kappa}f_{\alpha}^{\lambda}-\frac{1}{2}t_1\partial_{\kappa'}^{\lambda}\partial^{\kappa}f_{\alpha\lambda}^{\kappa}+\frac{1}{2}t_1\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\lambda}^{\kappa}+ \\ &\frac{1}{3}t_1\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{2}{3}t_1\partial^{\alpha}f_{\kappa\alpha}^{\theta}\partial^{\kappa}f_{\lambda}^{\theta}-\frac{1}{3}t_1\partial_{\kappa'}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+2t_1\omega_{\kappa\theta}^{\theta}\partial^{\kappa}f_{\lambda}^{\theta}- \\ &\frac{1}{3}t_1\omega_{\lambda\alpha}^{\alpha}\partial^{\kappa}f_{\kappa}^{\theta}-\frac{1}{3}t_1\omega_{\lambda}^{\lambda}\partial^{\kappa}f_{\kappa}^{\lambda}+\frac{1}{2}t_1\partial^{\alpha}f_{\kappa}^{\lambda}\partial^{\kappa}f_{\lambda\alpha}^{\kappa}+\frac{1}{2}t_1\partial_{\kappa'}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+ \\ &\frac{1}{2}t_1\partial_{\kappa'}^{\lambda}\partial^{\kappa}f_{\theta}^{\theta}\partial^{\kappa}f_{\lambda}^{\theta}-\frac{1}{3}t_1\partial^{\alpha}f_{\alpha}^{\lambda}\partial^{\kappa}f_{\lambda\kappa}^{\kappa}+\frac{2}{3}r_1\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}-\frac{2}{3}r_1\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}+ \\ &\frac{2}{3}r_1\partial^{\beta}\omega_{\lambda}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda'}-\frac{8}{3}r_1\partial_1^{\beta}\omega_{\lambda}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\lambda'}+r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\theta}^{\theta\kappa}-r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\kappa}^{\kappa} \end{aligned}$$

$\omega_{0+}^{\#1} \dagger$	$f_{0+}^{\#1} \dagger$	$f_{0+}^{\#2} \dagger$	$\omega_{0-}^{\#1} \dagger$
$\omega_{0+}^{\#1} \dagger$	0	0	0
$f_{0+}^{\#1} \dagger$	0	0	0
$f_{0+}^{\#2} \dagger$	0	0	0
$\omega_{0-}^{\#1} \dagger$	0	0	$-t_1$

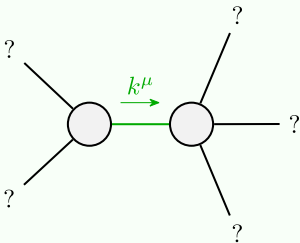
Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\sigma_{0+}^{\#1} == 0$	1
$\tau_{0+}^{\#1} == 0$	1
$\tau_{0+}^{\#2} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	17

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd



Quadratic pole	
Pole residue:	$-\frac{1}{(r_1+r_5)t_1^2} > 0$
Polarisations:	2

## Unitarity conditions

$$r_1 < 0 \&\& r_5 < -r_1 \&\& t_1 > 0$$