Particle spectrograph

Wave operator and propagator

	$\omega_{1^+}^{\#1}{}_{\alpha\beta}$	$\omega_{1^+}^{\#2}{}_{\alpha\beta}$	$f_{1^+}^{\#1}{}_{\alpha\beta}$	$\omega_{1^-}^{\#1}{}_{\alpha}$	$\omega_{1^-}^{\#2}{}_{\alpha}$	$f_{1^-}^{\#1}{}_{\alpha}$	$f_{1^-}^{\#2}{}_{\alpha}$
$\omega_{1^+}^{\#1}\dagger^{\alpha\beta}$	$k^2 r_5 - \frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$\frac{i k t_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1^+}^{\#2}\dagger^{\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{i k t_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_{1^-}^{\#1}\dagger^{\alpha}$	0	0	0	$\frac{1}{6}(6 k^2 r_5 + t_1 + 4 t_3)$	$\frac{t_1 - 2 t_3}{3\sqrt{2}}$	0	$\frac{1}{3} i k (t_1 - 2 t_3)$
$\omega_{1^-}^{\#2}\dagger^{\alpha}$	0	0	0	$\frac{t_1 - 2 t_3}{3\sqrt{2}}$	$\frac{t_1 + t_3}{3}$	0	$\frac{1}{3} i \sqrt{2}\, k (t_1 + t_3)$
$f_{1^-}^{\#1}\dagger^{\alpha}$	0	0	0	0	0	0	0
$f_{1^-}^{\#2}\dagger^{\alpha}$	0	0	0	$-\frac{1}{3} i k (t_1 - 2 t_3)$	$-\frac{1}{3} i \sqrt{2}\, k (t_1 + t_3)$	0	$\frac{2}{3} k^2 (t_1 + t_3)$

Left propagator matrix:

	$\sigma_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\sigma_{1^+}^{\#2}\dagger^{\alpha\beta}$	$\tau_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\sigma_{1^-}^{\#1}\dagger^{\alpha}$	$\sigma_{1^-}^{\#2}\dagger^{\alpha}$	$\tau_{1^-}^{\#1}\dagger^{\alpha}$	$\tau_{1^-}^{\#2}\dagger^{\alpha}$
$\tau_{1^-}^{\#2}{}_{\alpha}$	0	0	0	$-\frac{2 i k (t_1 - 2 t_3)}{(1+2 k^2)(3 t_1 t_3 + 2 k^2 r_5 (t_1+t_3))}$	$\frac{i \sqrt{2}\, k (6 k^2 r_5 + t_1 + 4 t_3)}{(1+2 k^2)^2 (3 t_1 t_3 + 2 k^2 r_5 (t_1+t_3))}$	0	$\frac{2 k^2 (6 k^2 r_5 + t_1 + 4 t_3)}{(1+2 k^2)^2 (3 t_1 t_3 + 2 k^2 r_5 (t_1+t_3))}$
$\tau_{1^-}^{\#1}{}_{\alpha}$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#2}{}_{\alpha}$	0	0	0	$-\frac{\sqrt{2}(t_1 - 2 t_3)}{(1+2 k^2)(3 t_1 t_3 + 2 k^2 r_5 (t_1+t_3))}$	$\frac{6 k^2 r_5 + t_1 + 4 t_3}{(1+2 k^2)^2 (3 t_1 t_3 + 2 k^2 r_5 (t_1+t_3))}$	0	$-\frac{i \sqrt{2}\, k (6 k^2 r_5 + t_1 + 4 t_3)}{(1+2 k^2)^2 (3 t_1 t_3 + 2 k^2 r_5 (t_1+t_3))}$
$\sigma_{1^-}^{\#1}{}_{\alpha}$	0	0	0	$\frac{2(t_1 + t_3)}{3 t_1 t_3 + 2 k^2 r_5 (t_1+t_3)}$	$-\frac{\sqrt{2}(t_1 - 2 t_3)}{(1+2 k^2)(3 t_1 t_3 + 2 k^2 r_5 (t_1+t_3))}$	0	$\frac{2 i k (t_1 - 2 t_3)}{(1+2 k^2)(3 t_1 t_3 + 2 k^2 r_5 (t_1+t_3))}$
$\tau_{1^+}^{\#1}{}_{\alpha\beta}$	$-\frac{i \sqrt{2}\, k}{t_1 + k^2 t_1}$	$-\frac{i(2 k^3 r_5 - k t_1)}{(1+k^2)^2 t_1^2}$	$-\frac{2 k^4 r_5 + k^2 t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\sigma_{1^+}^{\#2}{}_{\alpha\beta}$	$-\frac{\sqrt{2}}{t_1 + k^2 t_1}$	$-\frac{2 k^2 r_5 + t_1}{(1+k^2)^2 t_1^2}$	$\frac{i(2 k^3 r_5 - k t_1)}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\sigma_{1^+}^{\#1}{}_{\alpha\beta}$	0	$-\frac{\sqrt{2}}{t_1 + k^2 t_1}$	$\frac{i \sqrt{2}\, k}{t_1 + k^2 t_1}$	0	0	0	0

ω_0 block:

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\omega_{0^-}^{\#1}$
$\omega_{0^+}^{\#1}\dagger$	t_3	$-i\sqrt{2}\, k t_3$	0	0
$f_{0^+}^{\#1}\dagger$	$i\sqrt{2}\, k t_3$	$2 k^2 t_3$	0	0
$f_{0^+}^{\#2}\dagger$	0	0	0	0
$\omega_{0^-}^{\#1}\dagger$	0	0	0	$k^2 r_2 - t_1$

ω_2 block:

	$\omega_{2^+}^{\#1}{}_{\alpha\beta}$	$f_{2^+}^{\#1}{}_{\alpha\beta}$	$\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\omega_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$	0	0	$\frac{t_1}{2}$
$f_{2^+}^{\#1}\dagger^{\alpha\beta}$	$-\frac{i k t_1}{\sqrt{2}}$	$k^2 t_1$	0
$\omega_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$\frac{i k t_1}{\sqrt{2}}$	0

σ_2 / τ_2 block:

	$\sigma_{2^+}^{\#1}{}_{\alpha\beta}$	$\tau_{2^+}^{\#1}{}_{\alpha\beta}$	$\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$
$\sigma_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{2}{(1+2 k^2)^2 t_1}$	$-\frac{2 i \sqrt{2}\, k}{(1+2 k^2)^2 t_1}$	0
$\tau_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{2 i \sqrt{2}\, k}{(1+2 k^2)^2 t_1}$	$\frac{4 k^2}{(1+2 k^2)^2 t_1}$	0
$\sigma_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{t_1}$

σ_0 block:

	$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^-}^{\#1}$
$\sigma_{0^-}^{\#1}\dagger$	0	0	0	$\frac{1}{k^2 r_2 - t_1}$
$\tau_{0^+}^{\#2}\dagger$	0	0	0	0
$\tau_{0^+}^{\#1}\dagger$	$-\frac{i\sqrt{2}\, k}{(1+2 k^2)^2 t_3}$	$\frac{2 k^2}{(1+2 k^2)^2 t_3}$	0	0
$\sigma_{0^+}^{\#1}\dagger$	$\frac{1}{(1+2 k^2)^2 t_3}$	$\frac{i\sqrt{2}\, k}{(1+2 k^2)^2 t_3}$	0	0

Quadratic (free) action

$$S_F = \iiiint \Big(\tfrac{1}{6}\big(-2(t_1 - 2 t_3)\,\omega_i{}^{\alpha l}\,\omega_{i\,\kappa\alpha}{}^{\kappa} - 6 t_1\,\omega_i{}^{\kappa\lambda}\,\omega_{i\,\kappa\lambda}{}^{l} + 6 f^{\alpha\beta}\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - $$
$$6 r_5\,\partial_i\omega^{\kappa\lambda}{}_\kappa\,\partial_\kappa\omega_{\lambda}{}^{\alpha}{}_{\alpha\beta} + 4 r_2\,\partial_\theta\omega_{\alpha\beta}{}^{\kappa}{}_{\alpha}\,\partial_\kappa\omega^{\theta\alpha\beta} - 2 r_2\,\partial_\theta\omega_{\alpha\beta}{}^{\kappa}\,\partial_\kappa\omega^{\theta\kappa\lambda} - $$
$$6 r_5\,\partial_\alpha\omega_{\lambda}{}^{\alpha}{}_\theta\,\partial_\kappa\omega^{\kappa\lambda\theta} + 12 r_5\,\partial_\theta\omega_{\lambda}{}^{\alpha}{}_\alpha\,\partial_\kappa\omega^{\kappa\lambda\theta} - 3 t_1\,\partial^\alpha f^{\kappa}{}_{\theta\kappa}\,\partial^\theta f_{\alpha} - 3 t_1\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f_{\alpha}{}^{\theta} - $$
$$3 t_1\,\partial^\alpha f^{\lambda}{}_{\kappa}\,\partial^\kappa f_{\alpha\lambda} + 2 t_1\,\omega_{\kappa\alpha}{}^{\alpha}\,\partial^\kappa f^{l}{}_{l} - 4 t_3\,\omega_{\kappa\alpha}{}^{\alpha}\,\partial^\kappa f^{l}{}_{l} + $$
$$4 t_3\,\partial_\kappa f^{\lambda}{}_{l}\,\partial^\kappa f^{l}{}_{l} + 12 t_1\,\omega_{l\kappa\theta}\,\partial^{l\theta} f - 2 t_1\,\omega_{l\kappa\theta}{}^{\alpha}\,\partial^\kappa f^{l}{}_{\kappa} + 4 t_3\,\omega_{l\alpha}{}^{\alpha}\,\partial^\kappa f^{l}{}_{\kappa} - $$
$$2 t_1\,\omega_{l\lambda}{}^{\lambda}\,\partial^\kappa f^{l}{}_{\kappa} + 4 t_3\,\omega_{l\lambda}{}^{\lambda}\,\partial^\kappa f^{l}{}_{\kappa} + 3 t_1\,\partial_\kappa f^{\lambda}{}_{\theta}\,\partial^\kappa f_{\lambda}{}^{\theta} + 3 t_1\,\partial_\kappa f_{\theta}{}^{\lambda}\,\partial^\kappa f_{\lambda}{}^{\theta}{}_{\kappa} + $$
$$4 r_2\,\partial_\kappa\omega^{\theta\alpha\beta}\,\partial_\kappa\omega_{\theta\alpha\beta} - 4 r_2\,\partial^\theta\omega^{\alpha\beta}{}_\theta\,\partial_\kappa\omega_{\lambda\alpha}{}^{l} + 4 r_2\,\partial^\beta\omega^{\lambda\alpha}{}_\beta\,\partial_\lambda\omega_{\alpha}{}^{l} + $$
$$6 r_5\,\partial_\alpha\omega_{\lambda}{}^{\alpha}{}_\theta\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa - 6 r_5\,\partial_\theta\omega_{\lambda}{}^{\alpha}{}_\alpha\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa \big) \Big)[t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt$$

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau_{0^+}^{\#2} == 0$	1
$\tau_{0^+}^{\#1} - 2 i k\,\sigma_{0^+}^{\#1} == 0$	1
$\tau_{1^-}^{\#2\,\alpha} + 2 i k\,\sigma_{1^-}^{\#2\,\alpha} == 0$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	3
$\tau_{1^+}^{\#1\,\alpha\beta} + i k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$	3
$\tau_{2^+}^{\#1\,\alpha\beta} - 2 i k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$	5
Total constraints:	16

Massive and massless spectra
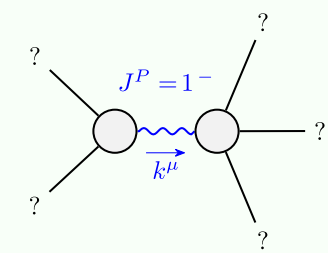
Massive particle ($J^P = 1^-$, k^μ)

Pole residue:	$\frac{6 t_1 t_3 (t_1+t_3) - 3 r_5 (t_1^2 + 2 t_3^2)}{2 r_5 (t_1+t_3)(-3 t_1 t_3 + r_5 (t_1+t_3))} > 0$
Polarisations:	3
Square mass:	$-\frac{3 t_1 t_3}{2 r_5 t_1 + 2 r_5 t_3} > 0$
Spin:	1
Parity:	Odd
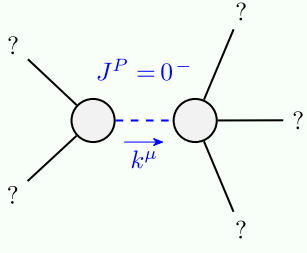
Massive particle ($J^P = 0^-$, k^μ)

Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$\frac{t_1}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_2 < 0 \;\&\&\; r_5 < 0 \;\&\&\; t_1 < 0 \;\&\&\; 0 < t_3 < -t_1$