

# Particle spectrograph

## Wave operator and propagator

	$\omega_{1+}^{\#1}{}_{\alpha\beta}$	$\omega_{1+}^{\#2}{}_{\alpha\beta}$	$f_{1+}^{\#1}{}_{\alpha\beta}$	$\omega_{1-}^{\#1}{}_{\alpha}$	$\omega_{1-}^{\#2}{}_{\alpha}$	$f_{1-}^{\#1}{}_{\alpha}$	$f_{1-}^{\#2}{}_{\alpha}$
$\omega_{1+}^{\#1}{}_{\dagger\alpha\beta}$	$k^2(2r_3+r_5)+\frac{2t_2}{3}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{1}{3}i\sqrt{2}kt_2$	0	0	0	0
$\omega_{1+}^{\#2}{}_{\dagger\alpha\beta}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{t_2}{3}$	$\frac{ikt_2}{3}$	0	0	0	0
$f_{1+}^{\#1}{}_{\dagger\alpha\beta}$	$-\frac{1}{3}i\sqrt{2}kt_2$	$-\frac{1}{3}ikt_2$	$\frac{k^2t_2}{3}$	0	0	0	0
$\omega_{1-}^{\#1}{}_{\dagger\alpha}$	0	0	0	$k^2(\frac{r_3}{2}+r_5)+\frac{2t_3}{3}$	$-\frac{\sqrt{2}t_3}{3}$	0	$-\frac{2}{3}ik t_3$
$\omega_{1-}^{\#2}{}_{\dagger\alpha}$	0	0	0	$-\frac{\sqrt{2}t_3}{3}$	$\frac{t_3}{3}$	0	$\frac{1}{3}i\sqrt{2}kt_3$
$f_{1-}^{\#1}{}_{\dagger\alpha}$	0	0	0	0	0	0	0
$f_{1-}^{\#2}{}_{\dagger\alpha}$	0	0	0	$\frac{2ikt_3}{3}$	$-\frac{1}{3}i\sqrt{2}kt_3$	0	$\frac{2k^2t_3}{3}$

$\sigma_0^{\#1}{}_{\dagger}$	$\tau_0^{\#1}{}_{\dagger}$	$\tau_0^{\#2}{}_{\dagger}$	$\sigma_0^{\#1}{}_{-}$
$\frac{1}{(1+2k^2)^2}t_3$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_3$	0	0
$\frac{i\sqrt{2}k}{(1+2k^2)^2}t_3$	$\frac{2k^2}{(1+2k^2)^2}t_3$	0	0
0	0	0	0
0	0	0	$\frac{1}{t_2}$

$\omega_0^{\#1}{}_{\dagger}$	$f_0^{\#1}{}_{\dagger}$	$\omega_0^{\#2}{}_{\dagger}$	$\omega_0^{\#1}{}_{-}$
$t_3$	$-i\sqrt{2}kt_3$	0	0
$i\sqrt{2}kt_3$	$2k^2t_3$	0	0
0	0	0	0
0	0	0	$t_2$

$\omega_2^{\#1}{}_{\dagger\alpha\beta}$	$f_2^{\#1}{}_{\dagger\alpha\beta}$	$\omega_2^{\#1}{}_{-\alpha\beta\chi}$
$-\frac{3k^2r_3}{2}$	0	0
0	0	0
0	0	0

Quadratic (free) action

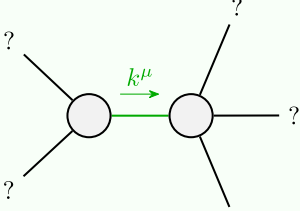
$$S= \iiint\int (\frac{1}{6}(-4t_3\omega^{\alpha i}{}_{\alpha}\omega_{i\kappa}^{\kappa}+6f^{\alpha\beta}\tau_{\alpha\beta}+6\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}+8t_3\omega_{\alpha\kappa}^{\kappa}\partial_{i}f^{\alpha i}-8t_3\omega_{i\kappa}^{\kappa}\partial'f^{\alpha}{}_{\alpha}+4t_3\partial_{i}f_{\kappa}^{\kappa}\partial'f^{\alpha}{}_{\alpha}-3r_3\partial_{\beta}\omega_{i\theta}^{\theta}\partial'\omega^{\alpha\beta}{}_{\alpha}-3r_3\partial_{i}\omega_{\beta\theta}^{\theta}\partial'\omega^{\alpha\beta}{}_{\alpha}-3r_3\partial_{\alpha}\omega^{\alpha\beta i}\partial_{\theta}\omega_{\beta i}^{\theta}+6r_3\partial'\omega^{\alpha\beta}{}_{\alpha}\partial_{\theta}\omega_{\beta i}^{\theta}-3r_3\partial_{\alpha}\omega^{\alpha\beta i}\partial_{\theta}\omega_{i\beta}^{\theta}+6r_3\partial'\omega^{\alpha\beta}{}_{\alpha}\partial_{\theta}\omega_{i\beta}^{\theta}+4t_2\omega_{i\theta\alpha}\partial^{\theta}f^{\alpha i}+2t_2\partial_{\alpha}f_{i\theta}\partial^{\theta}f^{\alpha i}-t_2\partial_{\alpha}f_{\theta i}\partial^{\theta}f^{\alpha i}-t_2\partial_{i}f_{\alpha\theta}\partial^{\theta}f^{\alpha i}+t_2\partial_{\theta}f_{\alpha i}\partial^{\theta}f^{\alpha i}-t_2\partial_{\theta}f_{i\alpha}\partial^{\theta}f^{\alpha i}-4t_2\omega_{\alpha\theta i}(\omega^{\alpha i\theta}+\partial^{\theta}f^{\alpha i})+2t_2\omega_{\alpha i\theta}(\omega^{\alpha i\theta}+2\partial^{\theta}f^{\alpha i})-24r_3\partial_{\beta}\omega_{i\theta\alpha}\partial^{\theta}\omega^{\alpha\beta i}+6r_5\partial_{i}\omega_{\theta\kappa}^{\kappa}\partial^{\theta}\omega^{\alpha i}{}_{\alpha}-6r_5\partial_{\theta}\omega_{i\kappa}^{\kappa}\partial^{\theta}\omega^{\alpha i}{}_{\alpha}+4t_3\partial_{i}f^{\alpha i}\partial_{\kappa}f_{\alpha}^{\kappa}-8t_3\partial'f^{\alpha}{}_{\alpha}\partial_{\kappa}f_{i\kappa}^{\kappa}-6r_5\partial_{\alpha}\omega^{\alpha i\theta}\partial_{\kappa}\omega_{i\theta}^{\kappa}+12r_5\partial^{\theta}\omega^{\alpha i}{}_{\alpha}\partial_{\kappa}\omega_{i\theta}^{\kappa}+6r_5\partial_{\alpha}\omega^{\alpha i\theta}\partial_{\kappa}\omega_{\theta i}^{\kappa}-12r_5\partial^{\theta}\omega^{\alpha i}{}_{\alpha}\partial_{\kappa}\omega_{\theta i}^{\kappa})) [t, x, y, z] dz dy dx dt$$

$\sigma_2^{\#1}{}_{\dagger\alpha\beta}$	$\tau_2^{\#1}{}_{\dagger\alpha\beta}$	$\sigma_2^{\#1}{}_{-\alpha\beta\chi}$
$-\frac{2}{3k^2}r_3$	0	0
0	0	0
0	0	0

Source constraints	Fundamental fields	Multiplicities
SO(3) irreps		
$\tau_0^{\#2}{}_{\dagger}==0$	$\partial_{\beta}\partial_{\alpha}\tau^{\alpha\beta}==0$	1
$\tau_0^{\#1}{}_{\dagger}-2iik\sigma_0^{\#1}{}_{\dagger}==0$	$\partial_{\beta}\partial_{\alpha}\tau^{\alpha\beta}==\partial_{\beta}\partial^{\beta}\tau^{\alpha}{}_{\alpha}+2\partial_{\chi}\partial^{\chi}\partial_{\beta}\sigma^{\alpha\beta}{}_{\alpha}$	1
$\tau_1^{\#2\alpha}{}_{\dagger}+2ik\sigma_1^{\#2\alpha}{}_{\dagger}==0$	$\partial_{\chi}\partial_{\beta}\partial^{\alpha}t^{\beta\chi}==\partial_{\chi}\partial^{\chi}\partial_{\beta}t^{\alpha\beta}+2\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial_{\beta}\sigma^{\alpha\beta\chi}$	3
$\tau_1^{\#1\alpha}{}_{\dagger}==0$	$\partial_{\chi}\partial_{\beta}\partial^{\alpha}t^{\beta\chi}==\partial_{\chi}\partial^{\chi}\partial_{\beta}t^{\beta\alpha}$	3
$\tau_1^{\#1\alpha\beta}{}_{\dagger}+ik\sigma_1^{\#2\alpha\beta}{}_{\dagger}==0$	$\partial_{\chi}\partial^{\alpha}t^{\beta\chi}+\partial_{\chi}\partial^{\beta}t^{\chi\alpha}+\partial_{\chi}\partial^{\chi}t^{\alpha\beta}+2\partial_{\delta}\partial_{\chi}\partial^{\alpha}\sigma^{\beta\chi\delta}+2\partial_{\delta}\partial_{\chi}\partial^{\delta}\partial_{\chi}\sigma^{\alpha\beta\chi}==\partial_{\chi}\partial^{\chi}t^{\beta\alpha}+\partial_{\chi}\partial^{\alpha}t^{\chi\beta}+\partial_{\chi}\partial^{\beta}t^{\chi\alpha}+\partial_{\chi}\partial^{\chi}t^{\beta\alpha}+2\partial_{\delta}\partial_{\chi}\partial^{\beta}\sigma^{\alpha\chi\delta}+2\partial_{\delta}\partial_{\chi}\partial^{\delta}\partial_{\chi}\sigma^{\alpha\chi\delta}$	3
$\sigma_2^{\#1\alpha\beta\chi}==0$	$3\partial_{\epsilon}\partial_{\delta}\partial^{\chi}\partial^{\alpha}\sigma^{\beta\delta\epsilon}+3\partial_{\epsilon}\partial^{\epsilon}\partial^{\chi}\partial_{\alpha}\sigma^{\beta\delta}{}_{\delta}+2\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\alpha\chi\delta}+4\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\alpha\delta\chi}+2\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\chi\delta\alpha}+4\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\alpha\delta\alpha}+2\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\alpha\delta\beta}+2\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\alpha\beta\chi}+2\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\alpha\beta\delta}+4\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\alpha\beta\chi}+2\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\beta}\sigma^{\alpha\chi\beta}+3\eta^{\alpha\chi}\partial_{\phi}\partial^{\phi}\partial_{\epsilon}\partial^{\beta}\sigma^{\delta\epsilon}{}_{\delta}+3\eta^{\beta\chi}\partial_{\phi}\partial^{\phi}\partial_{\epsilon}\partial_{\delta}\sigma^{\alpha\delta\epsilon}+3\eta^{\alpha\chi}\partial_{\phi}\partial^{\phi}\partial_{\epsilon}\partial^{\delta}\sigma^{\beta\delta}{}_{\delta}$	5
$\tau_2^{\#1\alpha\beta}{}_{\dagger}==0$	$4\partial_{\delta}\partial_{\chi}\partial^{\beta}\partial^{\alpha}\chi^{\delta}+2\partial_{\delta}\partial^{\delta}\partial^{\beta}\partial^{\alpha}\tau^{\chi}{}_{\chi}+3\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\chi}\tau^{\alpha\beta}+3\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\beta}\tau^{\alpha\chi}+2\eta^{\alpha\beta}\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial_{\chi}\tau^{\chi\delta}==3\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\alpha}t^{\beta\chi}+3\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\beta}\tau^{\alpha\chi}+3\partial_{\delta}\partial^{\delta}\partial_{\chi}\partial^{\beta}\tau^{\chi\alpha}+2\eta^{\alpha\beta}\partial_{\epsilon}\partial^{\epsilon}\partial_{\delta}\partial^{\delta}\tau^{\chi}{}_{\chi}$	5
Total constraints/gauge generators:		21

	$\sigma_{1+}^{\#1}{}_{\alpha\beta}$	$\sigma_{1+}^{\#2}{}_{\alpha\beta}$	$\tau_{1+}^{\#1}{}_{\alpha\beta}$	$\sigma_{1-}^{\#1}{}_{\alpha}$	$\sigma_{1-}^{\#2}{}_{\alpha}$	$\tau_{1-}^{\#1}{}_{\alpha}$	$\tau_{1-}^{\#2}{}_{\alpha}$
$\sigma_{1+}^{\#1}{}_{\dagger\alpha\beta}$	$\frac{1}{k^2(2r_3+r_5)}$	$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$	$-\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$	0	0	0	0
$\sigma_{1+}^{\#2}{}_{\dagger\alpha\beta}$	$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$	$\frac{3k^2(2r_3+r_5)+2t_2}{(k+k^3)^2(2r_3+r_5)t_2}$	$\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$	0	0	0	0
$\tau_{1+}^{\#1}{}_{\dagger\alpha\beta}$	$\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$	$-\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$	$\frac{3k^2(2r_3+r_5)+2t_2}{(1+k^2)^2(2r_3+r_5)t_2}$	0	0	0	0
$\sigma_{1-}^{\#1}{}_{\dagger\alpha}$	0	0	0	$\frac{2}{k^2(r_3+2r_5)}$	$\frac{2\sqrt{2}}{k^2(1+2k^2)(r_3+2r_5)}$	0	$\frac{4i}{k(1+2k^2)(r_3+2r_5)}$
$\sigma_{1-}^{\#2}{}_{\dagger\alpha}$	0	0	0	$\frac{2\sqrt{2}}{k^2(1+2k^2)(r_3+2r_5)}$	$\frac{3k^2(r_3+2r_5)+4t_3}{(k+2k^3)^2(r_3+2r_5)t_3}$	0	$\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$
$\tau_{1-}^{\#1}{}_{\dagger\alpha}$	0	0	0	0	0	0	0
$\tau_{1-}^{\#2}{}_{\dagger\alpha}$	0	0	0	$-\frac{4i}{k(1+2k^2)(r_3+2r_5)}$	$-\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$	0	$\frac{6k^2(r_3+2r_5)+8t_3}{(1+2k^2)^2(r_3+2r_5)t_3}$

## Massive and massless spectra



Quadratic pole

Pole residue:  $-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2} > 0$

Polarisations: 2

(No massive particles)

## Unitarity conditions

$$r_3 < 0 \&\& (r_5 < -\frac{r_3}{2} \parallel r_5 > -2r_3) \parallel r_3 > 0 \&\& -2r_3 < r_5 < -\frac{r_3}{2}$$