

# Particle spectrograph

## Wave operator and propagator

### Quadratic (free) Lagrangian density

$$\begin{aligned} &\frac{2}{3}t_2\omega_{\lambda'}^{\kappa\lambda}\omega_{\kappa\lambda}^{'\lambda}+\frac{1}{3}t_2\omega_{\kappa\lambda}^{'\lambda}\omega_{\lambda'}^{\kappa\lambda}+f_{\alpha\beta}^{\alpha\beta}\tau_{\alpha\beta}^{\alpha\beta}+\omega_{\alpha\beta\chi}^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}^{\alpha\beta\chi}+2r_1\partial_{\lambda'}\omega_{\kappa\lambda}^{\kappa\lambda}\partial^{\lambda}\omega_{\lambda}^{\alpha\alpha}- \\ &\frac{2}{3}r_1\partial_{\lambda}^{\beta}\omega_{\kappa}^{\theta\alpha}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}+\frac{2}{3}r_2\partial_{\lambda}^{\beta}\omega_{\kappa}^{\theta\alpha}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}-\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\theta\theta}- \\ &\frac{1}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\theta\theta}+\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\theta\alpha\beta}-\frac{2}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega_{\alpha\beta}^{\theta\alpha\beta}- \\ &2r_1\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\lambda}^{\theta\kappa\lambda}+4r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\theta\kappa\lambda}+2r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\alpha}\partial_{\alpha}\omega_{\lambda}^{\kappa\lambda\theta}+ \\ &4r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\theta\kappa\lambda}+2r_1\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\kappa\lambda\theta}-4r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\alpha}\partial_{\alpha}\omega_{\lambda}^{\kappa\lambda\theta}+ \\ &\frac{1}{6}t_2\partial_{\lambda}^{\alpha}f_{\theta\kappa}^{\alpha}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{6}t_2\partial_{\lambda}^{\alpha}f_{\kappa\theta}^{\alpha}\partial^{\kappa}f_{\alpha}^{\theta}+\frac{1}{6}t_2\partial_{\lambda}^{\alpha}f_{\alpha}^{\theta}\partial^{\kappa}f_{\kappa}^{\lambda}+\frac{1}{6}t_2\partial_{\lambda}^{\alpha}f_{\alpha\lambda}^{\kappa}\partial^{\kappa}f_{\alpha\lambda}^{\lambda\theta}- \\ &\frac{2}{3}t_2\omega_{\lambda\kappa\theta}^{\alpha}\partial^{\kappa}f^{\lambda\theta}-\frac{1}{3}t_2\omega_{\theta\lambda\kappa}^{\alpha}\partial^{\kappa}f^{\lambda\theta}+\frac{2}{3}t_2\omega_{\theta\kappa\lambda}^{\alpha}\partial^{\kappa}f^{\lambda\theta}-\frac{1}{6}t_2\partial_{\lambda}^{\alpha}f_{\kappa}^{\lambda}\partial^{\kappa}f_{\lambda\alpha}^{\lambda}- \\ &\frac{1}{6}t_2\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{1}{6}t_2\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{2}{3}r_1\partial_{\kappa}\omega_{\alpha\beta\theta}^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+\frac{2}{3}r_2\partial_{\kappa}\omega_{\alpha\beta\theta}^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}+ \\ &\frac{1}{3}r_2\partial_{\kappa}\omega_{\alpha\beta\theta}^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}-\frac{2}{3}r_1\partial_{\kappa}\omega_{\alpha\beta\theta}^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}+\frac{2}{3}r_2\partial_{\kappa}\omega_{\alpha\beta\theta}^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}+ \\ &\frac{2}{3}r_1\partial_{\beta}\omega_{\lambda'}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{'\lambda}-\frac{2}{3}r_2\partial_{\beta}\omega_{\lambda'}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{'\lambda}+\frac{4}{3}r_1\partial_{\beta}\omega_{\lambda'}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{'\lambda}+ \\ &\frac{2}{3}r_2\partial_{\beta}\omega_{\lambda'}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{'\lambda}-4r_3\partial_{\beta}\omega_{\lambda'}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{'\lambda}+2r_1\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\lambda}^{\theta\kappa}- \\ &4r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\lambda}^{\theta\kappa}-2r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\lambda}^{\theta\kappa}+4r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\lambda}^{\theta\kappa} \end{aligned}$$

	$\omega_{1+}^{\#1}\alpha\beta$	$\omega_{1+}^{\#2}\alpha\beta$	$f_{1+}^{\#1}\alpha\beta$	$\omega_{1-}^{\#1}\alpha$	$\omega_{1-}^{\#2}\alpha$	$f_{1-}^{\#1}\alpha$	$f_{1-}^{\#2}\alpha$
$\omega_{1+}^{\#1}\alpha\beta$	$\frac{2t_2}{3}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{1}{3}i\sqrt{2}kt_2$	0	0	0	0
$\omega_{1+}^{\#2}\alpha\beta$	$\frac{\sqrt{2}t_2}{3}$	$\frac{t_2}{3}$	$\frac{ikt_2}{3}$	0	0	0	0
$f_{1+}^{\#1}\alpha\beta$	$-\frac{1}{3}i\sqrt{2}kt_2$	$-\frac{1}{3}i\sqrt{2}kt_2$	$\frac{k^2t_2}{3}$	0	0	0	0
$\omega_{1-}^{\#1}\alpha$	0	0	0	$-k^2r_1$	0	0	0
$\omega_{1-}^{\#2}\alpha$	0	0	0	0	0	0	0
$f_{1-}^{\#1}\alpha$	0	0	0	0	0	0	0
$f_{1-}^{\#2}\alpha$	0	0	0	0	0	0	0

	$\sigma_{1+}^{\#1}\alpha\beta$	$\sigma_{1+}^{\#2}\alpha\beta$	$\tau_{1+}^{\#1}\alpha\beta$	$\sigma_{1-}^{\#1}\alpha$	$\sigma_{1-}^{\#2}\alpha$	$\tau_{1-}^{\#1}\alpha$	$\tau_{1-}^{\#2}\alpha$
$\sigma_{1+}^{\#1}\alpha\beta$	$\frac{6}{(3+k^2)^2t_2}$	$\frac{3\sqrt{2}}{(3+k^2)^2t_2}$	$\frac{3i\sqrt{2}k}{(3+k^2)^2t_2}$	0	0	0	0
$\sigma_{1+}^{\#2}\alpha\beta$	$\frac{3\sqrt{2}}{(3+k^2)^2t_2}$	$\frac{3}{(3+k^2)^2t_2}$	$\frac{3ik}{(3+k^2)^2t_2}$	0	0	0	0
$\tau_{1+}^{\#1}\alpha\beta$	$-\frac{3i\sqrt{2}k}{(3+k^2)^2t_2}$	$-\frac{3ik}{(3+k^2)^2t_2}$	$\frac{3k^2}{(3+k^2)^2t_2}$	0	0	0	0
$\sigma_{1-}^{\#1}\alpha$	0	0	0	$-\frac{1}{k^2r_1}$	0	0	0
$\sigma_{1-}^{\#2}\alpha$	0	0	0	0	0	0	0
$\tau_{1-}^{\#1}\alpha$	0	0	0	0	0	0	0
$\tau_{1-}^{\#2}\alpha$	0	0	0	0	0	0	0

	$\omega_{0+}^{\#1}$	$f_{0+}^{\#1}$	$f_{0+}^{\#2}$	$\omega_{0-}^{\#1}$
$\omega_{0+}^{\#1}\dagger$	$6k^2(-r_1+r_3)$	0	0	0
$f_{0+}^{\#1}\dagger$	0	0	0	0
$f_{0+}^{\#2}\dagger$	0	0	0	0
$\omega_{0-}^{\#1}\dagger$	0	0	0	$k^2r_2+t_2$

### Source constraints/gauge generators

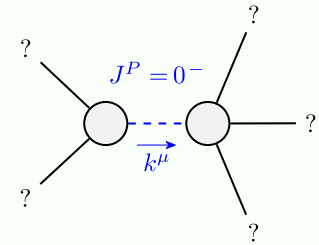
SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2}==0$	1
$\tau_{0+}^{\#1}==0$	1
$\tau_{1-}^{\#2\alpha}==0$	3
$\tau_{1-}^{\#1\alpha}==0$	3
$\sigma_{1-}^{\#2\alpha}==0$	3
$\tau_{1+}^{\#1\alpha\beta}+ik\sigma_{1+}^{\#1\alpha\beta}==0$	3
$\sigma_{1+}^{\#1\alpha\beta}==\sigma_{1+}^{\#2\alpha\beta}$	3
$\tau_{2+}^{\#1\alpha\beta}==0$	5
$\sigma_{2+}^{\#1\alpha\beta}==0$	5
Total constraints:	27

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
$\sigma_{0+}^{\#1}\dagger$	$\frac{1}{6k^2(-r_1+r_3)}$	0	0	0
$\tau_{0+}^{\#1}\dagger$	0	0	0	0
$\tau_{0+}^{\#2}\dagger$	0	0	0	0
$\sigma_{0-}^{\#1}\dagger$	0	0	0	$\frac{1}{k^2r_2+t_2}$

	$\sigma_{2+}^{\#1}\alpha\beta$	$\tau_{2+}^{\#1}\alpha\beta$	$\sigma_{2-}^{\#1}\alpha\beta\chi$
$\sigma_{2+}^{\#1}\dagger^{\alpha\beta}$	0	0	0
$\tau_{2+}^{\#1}\dagger^{\alpha\beta}$	0	0	0
$\sigma_{2-}^{\#1}\dagger^{\alpha\beta\chi}$	0	0	$\frac{1}{k^2r_1}$

	$\omega_{2+}^{\#1}\alpha\beta$	$f_{2+}^{\#1}\alpha\beta$	$\omega_{2-}^{\#1}\alpha\beta\chi$
$\omega_{2+}^{\#1}\dagger^{\alpha\beta}$	0	0	0
$f_{2+}^{\#1}\dagger^{\alpha\beta}$	0	0	0
$\omega_{2-}^{\#1}\dagger^{\alpha\beta\chi}$	0	0	$k^2r_1$

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

## Unitarity conditions

$$r_2 < 0 \ \&\& \ t_2 > 0$$