

# Particle spectrograph

## Wave operator and propagator

Quadratic (free) action

$$\begin{aligned}
 S = & \int \int \int \int (\frac{1}{3} (3 t_1 \omega_{\alpha}^{\alpha i} \omega_{,\theta}^{\theta} + 3 f^{\alpha \beta} \tau_{\alpha \beta} + 3 \omega^{\alpha \beta \chi} \sigma_{\alpha \beta \chi} - 6 t_1 \omega_{\alpha}^{\theta} \partial_{,f} f^{\alpha i} + 6 t_1 \omega_{,\theta}^{\theta} \partial_{,f}^{\alpha} - \\
 & \partial_{,f}^{\alpha} - 3 t_1 \partial_{,f} f^{\theta} \partial_{,\theta} f^{\alpha} - 6 r_1 \partial_{\beta} \omega_{,\theta}^{\theta} \partial_{,\theta} \omega_{\alpha}^{\alpha \beta} + 6 r_1 \partial_{,\theta} \omega_{\beta}^{\theta} \partial_{,\theta} \omega_{\alpha}^{\alpha \beta} - \\
 & 3 t_1 \partial_{,f} f^{\alpha i} \partial_{\theta} f_{\alpha}^{\theta} + 6 t_1 \partial_{,f} f^{\alpha} \partial_{\theta} f_{\alpha}^{\theta} + 6 r_1 \partial_{\alpha} \omega^{\alpha \beta i} \partial_{\theta} \omega_{\beta}^{\theta} - 12 r_1 \partial_{,\theta} \omega^{\alpha \beta} \partial_{\alpha} \partial_{\theta} \omega_{\beta}^{\theta} - \\
 & 6 r_1 \partial_{\alpha} \omega^{\alpha \beta i} \partial_{\theta} \omega_{,\beta}^{\theta} + 12 r_1 \partial_{,\theta} \omega^{\alpha \beta} \partial_{\theta} \omega_{,\beta}^{\theta} + 2 t_1 \omega_{,\theta \alpha} \partial_{,\theta} f^{\alpha i} - \\
 & 2 t_1 \partial_{\alpha} f_{,\theta} \partial_{,\theta} f^{\alpha i} - 2 t_1 \partial_{\alpha} f_{\theta i} \partial_{,\theta} f^{\alpha i} + t_1 \partial_{,f} f^{\alpha \theta} \partial_{,\theta} f^{\alpha i} + 2 t_1 \partial_{\theta} f_{\alpha i} \partial_{,\theta} f^{\alpha i} + \\
 & t_1 \partial_{\theta} f_{,\alpha} \partial_{,\theta} f^{\alpha i} + t_1 \omega_{\alpha i \theta} (\omega^{\alpha i \theta} + 2 \partial_{,\theta} f^{\alpha i}) + t_1 \omega_{\alpha \theta i} (\omega^{\alpha i \theta} + 4 \partial_{,\theta} f^{\alpha i}) - \\
 & 4 r_1 \partial_{\beta} \omega_{\alpha i \theta} \partial_{,\theta} \omega^{\alpha \beta i} + 4 r_2 \partial_{\beta} \omega_{\alpha i \theta} \partial_{,\theta} \omega^{\alpha \beta i} + 2 r_1 \partial_{\beta} \omega_{\alpha \theta i} \partial_{,\theta} \omega^{\alpha \beta i} - \\
 & 2 r_2 \partial_{\beta} \omega_{\alpha \theta i} \partial_{,\theta} \omega^{\alpha \beta i} - 8 r_1 \partial_{\beta} \omega_{,\theta \alpha} \partial_{,\theta} \omega^{\alpha \beta i} + 2 r_2 \partial_{\beta} \omega_{,\theta \alpha} \partial_{,\theta} \omega^{\alpha \beta i} - \\
 & 2 r_1 \partial_{,\theta} \omega_{\alpha \beta \theta} \partial_{,\theta} \omega^{\alpha \beta i} - r_2 \partial_{,\theta} \omega_{\alpha \beta \theta} \partial_{,\theta} \omega^{\alpha \beta i} + 2 r_1 \partial_{\theta} \omega_{\alpha \beta i} \partial_{,\theta} \omega^{\alpha \beta i} + r_2 \partial_{\theta} \omega_{\alpha \beta i} \partial_{,\theta} \omega^{\alpha \beta i} + \\
 & 2 r_1 \partial_{\theta} \omega_{\alpha i \beta} \partial_{,\theta} \omega^{\alpha \beta i} - 2 r_2 \partial_{\theta} \omega_{\alpha i \beta} \partial_{,\theta} \omega^{\alpha \beta i})) [t, x, y, z] d z d y d x d t
 \end{aligned}$$

| $\sigma_{1+}^{\#1} \dagger^{\alpha \beta}$ | $\sigma_{1+}^{\#2} \dagger^{\alpha \beta}$ | $\tau_{1+}^{\#1} \dagger^{\alpha \beta}$ | $\sigma_{1-}^{\#1} \dagger^{\alpha}$ | $\sigma_{1-}^{\#2} \dagger^{\alpha}$            | $\tau_{1-}^{\#1} \dagger^{\alpha}$ | $\tau_{1-}^{\#2} \dagger^{\alpha}$             |
|--|--|--|--------------------------------------|---|------------------------------------|--|
| $\frac{6}{(3+2k^2)^2} t_1$                 | $-\frac{6\sqrt{2}}{(3+2k^2)^2} t_1$        | $-\frac{6i\sqrt{2}k}{(3+2k^2)^2} t_1$    | 0                                    | 0   | 0                                  | 0  |
| $-\frac{6\sqrt{2}}{(3+2k^2)^2} t_1$        | $\frac{12}{(3+2k^2)^2} t_1$                | $\frac{12ik}{(3+2k^2)^2} t_1$            | 0                                    | 0   | 0                                  | 0  |
| $\frac{6i\sqrt{2}k}{(3+2k^2)^2} t_1$       | $-\frac{12ik}{(3+2k^2)^2} t_1$             | $\frac{12k^2}{(3+2k^2)^2} t_1$           | 0                                    | 0   | 0                                  | 0  |
| 0  | 0  | 0  | 0                                    | $\frac{\sqrt{2}}{t_1+2k^2} t_1$                 | 0                                  | $\frac{2ik}{t_1+2k^2} t_1$                     |
| 0  | 0  | 0  | $\frac{\sqrt{2}}{t_1+2k^2} t_1$      | $\frac{2k^2r_1+t_1}{(t_1+2k^2)^2}$              | 0                                  | $\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2)^2}$ |
| 0  | 0  | 0  | 0                                    | 0   | 0                                  | 0  |
| 0  | 0  | 0  | $-\frac{2ik}{t_1+2k^2} t_1$          | $-\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2)^2}$ | 0                                  | $\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2)^2}$       |

| $\omega_{1+}^{\#1} \dagger^{\alpha \beta}$ | $\omega_{1+}^{\#2} \dagger^{\alpha \beta}$ | $f_{1+}^{\#1} \dagger^{\alpha \beta}$ | $\omega_{1-}^{\#1} \dagger^{\alpha}$ | $\omega_{1-}^{\#2} \dagger^{\alpha}$ | $f_{1-}^{\#1} \dagger^{\alpha}$ | $f_{1-}^{\#2} \dagger^{\alpha}$ |
|--|--|---------------------------------------|--------------------------------------|--------------------------------------|---------------------------------|---------------------------------|
| $\frac{t_1}{6}$                            | $-\frac{t_1}{3\sqrt{2}}$                   | $-\frac{ikt_1}{3\sqrt{2}}$            | 0                                    | 0                                    | 0                               | 0                               |
| $-\frac{t_1}{3\sqrt{2}}$                   | $\frac{t_1}{3}$                            | $\frac{ikt_1}{3}$                     | 0                                    | 0                                    | 0                               | 0                               |
| $\frac{ikt_1}{3\sqrt{2}}$                  | $-\frac{1}{3}ik t_1$                       | $\frac{k^2t_1}{3}$                    | 0                                    | 0                                    | 0                               | 0                               |
| 0  | 0  | 0                                     | $-k^2r_1-\frac{t_1}{2}$              | $\frac{t_1}{\sqrt{2}}$               | 0                               | $i k t_1$                       |
| 0  | 0  | 0                                     | $\frac{t_1}{\sqrt{2}}$               | 0                                    | 0                               | 0                               |
| 0  | 0  | 0                                     | 0                                    | 0                                    | 0                               | 0                               |
| 0  | 0  | 0                                     | $-i k t_1$                           | 0                                    | 0                               | 0                               |

| $\sigma_{0+}^{\#1} \dagger$          | $\tau_{0+}^{\#1} \dagger$           | $\tau_{0+}^{\#2} \dagger$ | $\sigma_{0-}^{\#1} \dagger$ |
|--------------------------------------|-------------------------------------|---------------------------|-----------------------------|
| $-\frac{1}{(1+2k^2)^2} t_1$          | $\frac{i\sqrt{2}k}{(1+2k^2)^2} t_1$ | 0                         | 0                           |
| $-\frac{i\sqrt{2}k}{(1+2k^2)^2} t_1$ | $-\frac{2k^2}{(1+2k^2)^2} t_1$      | 0                         | 0                           |
| 0                                    | 0                                   | 0                         | 0                           |
| 0                                    | 0                                   | 0                         | $\frac{1}{k^2r_2}$          |

| Source constraints/gauge generators                                 |                |
|---|----------------|
| SO(3) irreps  | Multiplicities |
| $\tau_{0+}^{\#2} == 0$  | 1              |
| $\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$                       | 1              |
| $\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$           | 3              |
| $\tau_{1-}^{\#1\alpha} == 0$  | 3              |
| $\tau_{1+}^{\#1\alpha\beta} - 2ik\sigma_{1+}^{\#1\alpha\beta} == 0$ | 3              |
| $2\sigma_{1+}^{\#1\alpha\beta} + \sigma_{1+}^{\#2\alpha\beta} == 0$ | 3              |
| $\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$ | 5              |
| Total constraints:  | 19             |

| $\omega_{0+}^{\#1} \dagger$ | $f_{0+}^{\#1} \dagger$ | $f_{0+}^{\#2} \dagger$ | $\omega_{0-}^{\#1} \dagger$ |
|-----------------------------|------------------------|------------------------|-----------------------------|
| $-t_1$                      | $i\sqrt{2}kt_1$        | 0                      | 0                           |
| $-i\sqrt{2}kt_1$            | $-2k^2t_1$             | 0                      | 0                           |
| 0                           | 0                      | 0                      | 0                           |
| 0                           | 0                      | 0                      | $k^2r_2$                    |

| $\omega_{2+}^{\#1} \dagger^{\alpha \beta}$ | $f_{2+}^{\#1} \dagger^{\alpha \beta}$ | $\omega_{2-}^{\#1} \dagger^{\alpha \beta \chi}$ |
|--|---------------------------------------|---|
| $\frac{t_1}{2}$                            | $-\frac{ikt_1}{\sqrt{2}}$             | 0   |
| $\frac{ikt_1}{\sqrt{2}}$                   | $k^2t_1$                              | 0   |
| 0  | 0                                     | $k^2r_1+\frac{t_1}{2}$                          |

## Massive and massless spectra

| Massive particle |                         |
|------------------|-------------------------|
| Pole residue:    | $-\frac{1}{r_1} > 0$    |
| Polarisations:   | 5                       |
| Square mass:     | $-\frac{t_1}{2r_1} > 0$ |
| Spin:            | 2                       |
| Parity:          | Odd                     |

(No massless particles)

## Unitarity conditions

$$r_1 < 0 \&\& t_1 > 0$$