Lagrangian density

$$
\begin{aligned}
\mathcal{L} =\;& -t_1\,\omega_i{}^{\alpha I}\,\omega_{\kappa\alpha}{}^\kappa - \tfrac{1}{3}t_1\,\omega_i{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}^I + \tfrac{1}{3}t_1\,\omega_{\kappa\lambda}{}^I\,\omega^{\kappa\lambda}{}_I + f^{\alpha\beta}\,\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + \\
& 2\,r_1\,\partial_\iota\omega^{\kappa\lambda}{}_\kappa\,\partial^\iota\omega_{\lambda}{}^\alpha{}_\alpha - \tfrac{2}{3}r_1\,\partial^\beta\omega^{\theta\alpha}{}_\kappa\,\partial_\theta\omega_{\alpha\beta}{}^\kappa - \\
& \tfrac{2}{3}r_1\,\partial_\theta\omega_{\alpha\beta}{}^\kappa\,\partial_\kappa\omega^{\alpha\beta\theta} - \tfrac{1}{3}r_2\,\partial_\theta\omega_\kappa{}^{\kappa\theta}\,\partial_\kappa\omega^{\theta\alpha\beta}{}_\alpha - \\
& \tfrac{2}{3}r_2\,\partial_\theta\omega_{\alpha\beta}{}^\kappa\,\partial_\kappa\omega^{\theta\alpha\beta} - 2\,r_1\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial_\kappa\omega^{\theta\kappa\lambda} + \\
& 2\,r_1\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial_\kappa\omega^{\kappa\lambda\theta} - 4\,r_1\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial_\kappa\omega^{\kappa\lambda\theta} - \tfrac{1}{3}t_1\,\partial^\alpha f_{\theta\kappa}\,\partial^\theta f_\alpha{}^\theta - \\
& \tfrac{2}{3}t_1\,\partial^\alpha f_{\theta\kappa}\,\partial^\theta f_\alpha{}^\kappa - \tfrac{2}{3}t_1\,\partial^\alpha f_{\kappa\alpha}\,\partial^\kappa f^\iota{}_\iota - \\
& \tfrac{1}{3}t_1\,\partial^\alpha f_\kappa{}^\lambda\,\partial^\kappa f_{\alpha\lambda} + t_1\,\omega_{\kappa\alpha}{}^\alpha\,\partial^\kappa f^\iota{}_\iota + t_1\,\omega_{\kappa\lambda}{}^\lambda\,\partial^\kappa f_\iota{}^\iota - \\
& t_1\,\partial_\kappa f^\lambda{}_\lambda\,\partial^\kappa f^\iota{}_\iota + \tfrac{1}{3}t_1\,\omega_{\iota\theta\kappa}\,\partial^\kappa f^{\iota\theta} + \tfrac{4}{3}t_1\,\omega_{\iota\theta\kappa}\,\partial^\kappa f^{\iota\theta} + \\
& \tfrac{2}{3}t_1\,\omega_{\theta\kappa\iota}\,\partial^\kappa f^\iota{}_\kappa - t_1\,\omega_{\iota\alpha}{}^\lambda\,\partial^\kappa f^\iota{}_\kappa + \tfrac{1}{3}t_1\,\omega_{\iota\lambda}{}^\lambda\,\partial^\kappa f_{\lambda\kappa} + \\
& \tfrac{1}{3}t_1\,\partial_\kappa f_\lambda{}^\theta\,\partial^\kappa f_\lambda{}^\theta + \tfrac{2}{3}t_1\,\partial_\kappa f^\lambda{}_\theta\,\partial^\kappa f_\lambda{}^\theta - t_1\,\partial^\alpha f_\alpha{}^\kappa\,\partial^\kappa f_{\lambda\alpha} + \\
& \tfrac{1}{3}r_2\,\partial_\kappa\omega^{\alpha\beta\theta}\,\partial^\kappa\omega_{\alpha\beta\theta} - \tfrac{2}{3}r_1\,\partial_\kappa\omega^{\theta\alpha\beta}\,\partial^\kappa\omega_{\alpha\beta\theta} + \\
& \tfrac{1}{3}r_2\,\partial_\beta\omega^{\alpha\beta}{}_\theta\,\partial^\kappa\omega_{\alpha\beta}{}^\theta + \tfrac{2}{3}r_1\,\partial_\kappa\omega^{\theta\alpha\beta}\,\partial^\kappa\omega_{\alpha\beta\theta} + \\
& \tfrac{2}{3}r_1\,\partial_\beta\omega^{\alpha\lambda}\partial_\lambda\omega_\iota{}^{\alpha\beta}{}' - \tfrac{1}{3}r_1\,\partial^\beta\omega_\iota{}^{\lambda\alpha}\partial_\lambda\omega_{\alpha\beta}{}' + \\
& \tfrac{2}{3}r_2\,\partial^\beta\omega_\iota{}^{\lambda\alpha}\partial_\lambda\omega_{\alpha\beta}{}' - 2\,r_1\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa + 2\,r_1\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa
\end{aligned}
$$

Matrix (1^+ / 1^- sector)

	$\sigma^{\#1}_{1^+}{}^{\dagger\alpha\beta}$	$\sigma^{\#2}_{1^+}{}^{\dagger\alpha\beta}$	$\tau^{\#1}_{1^+}{}^{\dagger\alpha\beta}$	$\sigma^{\#1}_{1^-}{}^{\dagger\alpha}$	$\sigma^{\#2}_{1^-}{}^{\dagger\alpha}$	$\tau^{\#1}_{1^-}{}^{\dagger\alpha}$	$\tau^{\#2}_{1^-}{}^{\dagger\alpha}$
$\tau^{\#2}_{1^-}{}^\alpha$	0	0	0	$\frac{2ik}{t_1+2k^2t_1}$	$\frac{i\sqrt2\,k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$	0	$\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
$\tau^{\#1}_{1^-}{}^\alpha$	0	0	0	0	0	0	0
$\sigma^{\#2}_{1^-}{}^\alpha$	0	0	0	$\frac{\sqrt2}{t_1+2k^2t_1}$	$\frac{2k^2r_1+t_1}{(t_1+2k^2t_1)^2}$	0	$-\frac{i\sqrt2\,k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
$\sigma^{\#1}_{1^-}{}^\alpha$	0	0	0	0	$\frac{\sqrt2}{t_1+2k^2t_1}$	0	$-\frac{2ik}{t_1+2k^2t_1}$
$\tau^{\#1}_{1^+\alpha\beta}$	$-\frac{6i\sqrt2\,k}{(3+2k^2)^2t_1}$	$\frac{12ik}{(3+2k^2)^2t_1}$	$\frac{12k^2}{(3+2k^2)^2t_1}$	0	0	0	0
$\sigma^{\#2}_{1^+\alpha\beta}$	$\frac{6\sqrt2}{(3+2k^2)^2t_1}$	$-\frac{12}{(3+2k^2)^2t_1}$	$-\frac{12ik}{(3+2k^2)^2t_1}$	0	0	0	0
$\sigma^{\#1}_{1^+\alpha\beta}$	$\frac{6}{(3+2k^2)^2t_1}$	$\frac{6\sqrt2}{(3+2k^2)^2t_1}$	$\frac{6i\sqrt2\,k}{(3+2k^2)^2t_1}$	0	0	0	0

Source constraints

SO(3) irreps	#
$\tau^{\#2}_{0^+} == 0$	1
$\tau^{\#1}_{0^+} - 2ik\,\sigma^{\#1}_{0^+} == 0$	1
$\tau^{\#2\alpha}_{1^-} + 2ik\,\sigma^{\#2\alpha}_{1^-} == 0$	3
$\tau^{\#1\alpha}_{1^-} == 0$	3
$\tau^{\#1\alpha\beta}_{1^+} - 2ik\,\sigma^{\#1\alpha\beta}_{1^+} == 0$	3
$2\,\sigma^{\#1\alpha\beta}_{1^+} + \sigma^{\#2\alpha\beta}_{1^+} == 0$	3
$\tau^{\#1\alpha\beta}_{2^+} - 2ik\,\sigma^{\#1\alpha\beta}_{2^+} == 0$	5
Total #:	19

Matrix (1^+ / 1^-, ω–f sector)

	$\omega^{\#1}_{1^+}{}^{\dagger\alpha\beta}$	$\omega^{\#2}_{1^+}{}^{\dagger\alpha\beta}$	$f^{\#1}_{1^+}{}^{\dagger\alpha\beta}$	$\omega^{\#1}_{1^-}{}^{\dagger\alpha}$	$\omega^{\#2}_{1^-}{}^{\dagger\alpha}$	$f^{\#1}_{1^-}{}^{\dagger\alpha}$	$f^{\#2}_{1^-}{}^{\dagger\alpha}$
$f^{\#2}_{1^-}{}^\alpha$	0	0	0	ikt_1	0	0	0
$f^{\#1}_{1^-}{}^\alpha$	0	0	0	$\frac{t_1}{\sqrt2}$	0	0	0
$\omega^{\#2}_{1^-}{}^\alpha$	0	0	0	$-k^2r_1-\frac{t_1}{2}$	$\frac{t_1}{\sqrt2}$	0	$-ikt_1$
$\omega^{\#1}_{1^-}{}^\alpha$	0	0	0	$\frac{t_1}{\sqrt2}$	0	0	0
$f^{\#1}_{1^+\alpha\beta}$	$-\frac{ikt_1}{3\sqrt2}$	$\frac{ikt_1}{3}$	$\frac{k^2t_1}{3}$	0	0	0	0
$\omega^{\#2}_{1^+\alpha\beta}$	$-\frac{t_1}{3\sqrt2}$	$\frac{t_1}{3}$	$-\frac{1}{3}ikt_1$	0	0	0	0
$\omega^{\#1}_{1^+\alpha\beta}$	$\frac{t_1}{6}$	$-\frac{t_1}{3\sqrt2}$	$\frac{ikt_1}{3\sqrt2}$	0	0	0	0

Matrix (2^+ / 2^- sector, saturated)

	$\sigma^{\#1}_{2^+\alpha\beta}$	$\tau^{\#1}_{2^+\alpha\beta}$	$\sigma^{\#1}_{2^-\alpha\beta\chi}$
$\sigma^{\#1}_{2^-\alpha\beta\chi}$	0	0	$\frac{2}{2k^2r_1+t_1}$
$\tau^{\#1}_{2^+\alpha\beta}$	$-\frac{2i\sqrt2\,k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\sigma^{\#1}_{2^+\alpha\beta}$	$\frac{2}{(1+2k^2)^2t_1}$	$\frac{2i\sqrt2\,k}{(1+2k^2)^2t_1}$	0

Row labels (daggers): $\sigma^{\#1}_{2^+}{}^{\dagger\alpha\beta}$, $\tau^{\#1}_{2^+}{}^{\dagger\alpha\beta}$, $\sigma^{\#1}_{2^-}{}^{\dagger\alpha\beta\chi}$

Matrix (2^+ / 2^-)

	$\omega^{\#1}_{2^+\alpha\beta}$	$f^{\#1}_{2^+\alpha\beta}$	$\omega^{\#1}_{2^-\alpha\beta\chi}$
$\omega^{\#1}_{2^+}{}^{\dagger\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt2}$	0
$f^{\#1}_{2^+}{}^{\dagger\alpha\beta}$	$\frac{ikt_1}{\sqrt2}$	k^2t_1	0
$\omega^{\#1}_{2^-}{}^{\dagger\alpha\beta\chi}$	0	0	$k^2r_1+\frac{t_1}{2}$

Matrix (0^+ / 0^- sector, saturated)

	$\sigma^{\#1}_{0^+}$	$\tau^{\#1}_{0^+}$	$\tau^{\#2}_{0^+}$	$\sigma^{\#1}_{0^-}$
$\tau^{\#2}_{0^+}$	0	0	0	$\frac{1}{k^2r_2}$
$\tau^{\#1}_{0^+}$	$\frac{i\sqrt2\,k}{(1+2k^2)^2t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0	0
$\sigma^{\#1}_{0^+}$	$-\frac{1}{(1+2k^2)^2t_1}$	$-\frac{i\sqrt2\,k}{(1+2k^2)^2t_1}$	0	0

Row labels (daggers): $\sigma^{\#1}_{0^+}{}^\dagger$, $\tau^{\#1}_{0^+}{}^\dagger$, $\tau^{\#2}_{0^+}{}^\dagger$, $\sigma^{\#1}_{0^-}{}^\dagger$

Matrix (0^+ / 0^-)

	$\omega^{\#1}_{0^+}$	$f^{\#1}_{0^+}$	$f^{\#2}_{0^+}$	$\omega^{\#1}_{0^-}$
$\omega^{\#1}_{0^+}{}^\dagger$	$-t_1$	$i\sqrt2\,kt_1$	0	0
$f^{\#1}_{0^+}{}^\dagger$	$-i\sqrt2\,kt_1$	$-2k^2t_1$	0	0
$f^{\#2}_{0^+}{}^\dagger$	0	0	0	0
$\omega^{\#1}_{0^-}{}^\dagger$	0	0	0	k^2r_2

Massive particle

$J^P = 2^-$, k^μ

Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

$r_1 < 0$ && $t_1 > 0$