

Particle spectrograph

Wave operator and propagator

| $\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{1+}^{\#2} \alpha\beta$ | $\tau_{1+}^{\#1} \alpha\beta$                   | $\sigma_{1-}^{\#1} \alpha$                       | $\sigma_{1-}^{\#2} \alpha$                   | $\tau_{1-}^{\#1} \alpha$ | $\tau_{1-}^{\#2} \alpha$                             |
|---|---------------------------------|---|--|--|--------------------------|--|
| $\sigma_{1+}^{\#1} \dagger^{\alpha\beta}$ | 0                               | $-\frac{\sqrt{2}}{t_1+k^2}t_1$                  | $-\frac{i\sqrt{2}k}{t_1+k^2}t_1$                 | 0  | 0                        | 0  |
| $\sigma_{1+}^{\#2} \dagger^{\alpha\beta}$ | $-\frac{\sqrt{2}}{t_1+k^2}t_1$  | $\frac{-2k^2(2r_3+r_5)+t_1}{(1+k^2)^2}t_1$      | $\frac{-2ik^3(2r_3+r_5)+ik t_1}{(1+k^2)^2}t_1^2$ | 0  | 0                        | 0  |
| $\tau_{1+}^{\#1} \dagger^{\alpha\beta}$   | $\frac{i\sqrt{2}k}{t_1+k^2}t_1$ | $\frac{i(2k^3(2r_3+r_5)-kt_1)}{(1+k^2)^2}t_1^2$ | $\frac{-2k^4(2r_3+r_5)+k^2t_1}{(1+k^2)^2}t_1^2$  | 0  | 0                        | 0  |
| $\sigma_{1-}^{\#1} \dagger^{\alpha}$      | 0                               | 0   | 0  | $\frac{1}{k^2(-r_1+2r_3+r_5)}$               | 0                        | $\frac{i}{k(1+2k^2)(r_1-2r_3-r_5)}$                  |
| $\sigma_{1-}^{\#2} \dagger^{\alpha}$      | 0                               | 0   | 0  | $\frac{1}{\sqrt{2}(k^2+2k^4)(r_1-2r_3-r_5)}$ | 0                        | $\frac{1}{\sqrt{2}k(1+2k^2)^2(r_1-2r_3-r_5)}t_1$     |
| $\tau_{1-}^{\#1} \dagger^{\alpha}$        | 0                               | 0   | 0  | 0  | 0                        | 0  |
| $\tau_{1-}^{\#2} \dagger^{\alpha}$        | 0                               | 0   | 0  | $\frac{i}{k(1+2k^2)(-r_1+2r_3+r_5)}$         | 0                        | $\frac{1}{-r_1+2r_3+r_5}+\frac{6k^2}{(1+2k^2)^2}t_1$ |

Quadratic (free) action

$$S=$$
$$\iiint(f^{\alpha\beta} \tau_{\alpha\beta} + \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} + \frac{1}{6}t_1(2\omega^{\alpha i}_{\alpha'}\omega^{\theta}_{\theta'}\omega^{\theta}_{\theta'}\partial_{\theta'}f^{\alpha i} + 4\omega^{\theta}_{\theta'}\partial_{\theta'}f^{\alpha}_{\alpha'} - 2\partial_{\theta'}f^{\theta}_{\theta'}\partial_{\theta'}f^{\alpha}_{\alpha'} - 2\partial_{\theta'}f^{\alpha i}_{\alpha'}\partial_{\theta'}f^{\theta}_{\theta'} + 4\partial_{\theta'}f^{\alpha}_{\alpha'}\partial_{\theta'}f^{\theta}_{\theta'} - 6\partial_{\alpha'}f_{\theta}\partial^{\theta}f^{\alpha i} - 3\partial_{\alpha'}f_{\theta i}\partial^{\theta}f^{\alpha i} + 3\partial_{\theta'}f_{\alpha\theta}\partial^{\theta}f^{\alpha i} + 3\partial_{\theta'}f_{\alpha i}\partial^{\theta}f^{\alpha i} + 3\partial_{\theta'}f_{\alpha}\partial^{\theta}f^{\alpha i} + 6\omega_{\alpha\theta i}(\omega^{\alpha i\theta} + 2\partial^{\theta}f^{\alpha i})) - 2r_3(\partial_{\beta}\omega^{\theta}_{\theta'}\partial_{\theta'}\omega^{\alpha\beta}_{\alpha'} + \partial_{\theta'}\omega^{\theta}_{\beta}\partial_{\theta'}\omega^{\alpha\beta}_{\alpha'} + \partial_{\alpha'}\omega^{\alpha\beta i}\partial_{\theta}\omega^{\theta}_{\beta'} - 2\partial_{\theta'}\omega^{\alpha\beta}_{\alpha'}\partial_{\theta}\omega^{\theta}_{\beta'} + \partial_{\alpha'}\omega^{\alpha\beta i}\partial_{\theta}\omega^{\theta}_{\beta'} - 2\partial_{\theta'}\omega^{\alpha\beta}_{\alpha'}\partial_{\theta}\omega^{\theta}_{\beta'} + 2\partial_{\beta}\omega_{\theta\alpha}\partial^{\theta}\omega^{\alpha\beta i}) + \frac{2}{3}r_1(3\partial_{\beta}\omega^{\theta}_{\theta'}\partial_{\theta'}\omega^{\alpha\beta}_{\alpha'} + 3\partial_{\theta'}\omega^{\theta}_{\beta}\partial_{\theta'}\omega^{\alpha\beta}_{\alpha'} + 3\partial_{\alpha'}\omega^{\alpha\beta i}\partial_{\theta}\omega^{\theta}_{\beta'} - 6\partial_{\theta'}\omega^{\alpha\beta i}\partial_{\theta}\omega^{\theta}_{\beta'} + 3\partial_{\alpha'}\omega^{\alpha\beta i}\partial_{\theta}\omega^{\theta}_{\beta'} - 6\partial_{\theta'}\omega^{\alpha\beta}_{\alpha'}\partial_{\theta}\omega^{\theta}_{\beta'} - 2\partial_{\beta}\omega_{\alpha i\theta}\partial^{\theta}\omega^{\alpha\beta i} + \partial_{\beta}\omega_{\alpha\theta i}\partial^{\theta}\omega^{\alpha\beta i} + 2\partial_{\beta}\omega_{\theta\alpha}\partial^{\theta}\omega^{\alpha\beta i}) + \partial_{\theta}\omega_{\theta\alpha}\partial^{\theta}\omega^{\alpha\beta i} - \partial_{\theta'}\omega_{\alpha\beta\theta}\partial^{\theta}\omega^{\alpha\beta i} + \partial_{\theta}\omega_{\alpha\beta i}\partial^{\theta}\omega^{\alpha\beta i} + \partial_{\theta}\omega_{\alpha i\beta}\partial^{\theta}\omega^{\alpha\beta i}) + r_5(\partial_{\theta'}\omega^{\kappa}_{\kappa'}\partial_{\theta'}\omega^{\alpha i}_{\alpha'} - \partial_{\theta}\omega^{\kappa}_{\kappa'}\partial_{\theta'}\omega^{\alpha i}_{\alpha'} - (\partial_{\alpha'}\omega^{\alpha i\theta} - 2\partial^{\theta}\omega^{\alpha i}_{\alpha'}) (\partial_{\kappa'}\omega^{\kappa}_{\theta'} - \partial_{\kappa}\omega^{\kappa}_{\theta'}))) [t, x, y, z] dz dy dx dt$$

|   | $\omega_{1+}^{\#1} \alpha\beta$ | $\omega_{1+}^{\#2} \alpha\beta$ | $f_{1+}^{\#1} \alpha\beta$ | $\omega_{1-}^{\#1} \alpha$         | $\omega_{1-}^{\#2} \alpha$   | $f_{1-}^{\#1} \alpha$ | $f_{1-}^{\#2} \alpha$       |
|---|---------------------------------|---------------------------------|----------------------------|------------------------------------|------------------------------|-----------------------|-----------------------------|
| $\omega_{1+}^{\#1} \dagger^{\alpha\beta}$ | $k^2(2r_3+r_5)-\frac{t_1}{2}$   | $-\frac{t_1}{\sqrt{2}}$         | $-\frac{ik t_1}{\sqrt{2}}$ | 0                                  | 0                            | 0                     | 0                           |
| $\omega_{1+}^{\#2} \dagger^{\alpha\beta}$ | $-\frac{t_1}{\sqrt{2}}$         | 0                               | 0                          | 0                                  | 0                            | 0                     | 0                           |
| $f_{1+}^{\#1} \dagger^{\alpha\beta}$      | $\frac{ik t_1}{\sqrt{2}}$       | 0                               | 0                          | 0                                  | 0                            | 0                     | 0                           |
| $\omega_{1-}^{\#1} \dagger^{\alpha}$      | 0                               | 0                               | 0                          | $k^2(-r_1+2r_3+r_5)+\frac{t_1}{6}$ | $\frac{t_1}{3\sqrt{2}}$      | 0                     | $\frac{ik t_1}{3}$          |
| $\omega_{1-}^{\#2} \dagger^{\alpha}$      | 0                               | 0                               | 0                          | $\frac{t_1}{3\sqrt{2}}$            | $\frac{t_1}{3}$              | 0                     | $\frac{1}{3}i\sqrt{2}k t_1$ |
| $f_{1-}^{\#1} \dagger^{\alpha}$           | 0                               | 0                               | 0                          | 0                                  | 0                            | 0                     | 0                           |
| $f_{1-}^{\#2} \dagger^{\alpha}$           | 0                               | 0                               | 0                          | $-\frac{1}{3}ik t_1$               | $-\frac{1}{3}i\sqrt{2}k t_1$ | 0                     | $\frac{2k^2t_1}{3}$         |

| $\sigma_0^{\#1} \dagger$ | $\tau_0^{\#1} \dagger$     | $\tau_0^{\#2} \dagger$ | $\sigma_0^{\#1} \dagger$ |
|--------------------------|----------------------------|------------------------|--------------------------|
| $\sigma_0^{\#1} \dagger$ | $\frac{1}{6k^2(-r_1+r_3)}$ | 0                      | 0                        |
| $\tau_0^{\#1} \dagger$   | 0                          | 0                      | 0                        |
| $\tau_0^{\#2} \dagger$   | 0                          | 0                      | 0                        |
| $\sigma_0^{\#1} \dagger$ | 0                          | 0                      | $-\frac{1}{t_1}$         |

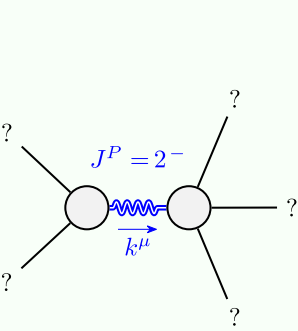
| $\sigma_{2+}^{\#1} \dagger^{\alpha\beta}$     | $\tau_{2+}^{\#1} \dagger^{\alpha\beta}$ | $\sigma_{2-}^{\#1} \alpha\beta\chi$ |
|---|---|-------------------------------------|
| $\frac{2}{(1+2k^2)^2}t_1$                     | $-\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$    | 0                                   |
| $\tau_{2+}^{\#1} \dagger^{\alpha\beta}$       | $\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$     | 0                                   |
| $\sigma_{2-}^{\#1} \dagger^{\alpha\beta\chi}$ | 0                                       | $\frac{2}{2k^2r_1+t_1}$             |

| $\omega_{2+}^{\#1} \dagger^{\alpha\beta}$     | $f_{2+}^{\#1} \alpha\beta$ | $\omega_{2-}^{\#1} \alpha\beta\chi$ |
|---|----------------------------|-------------------------------------|
| $\frac{t_1}{2}$                               | $-\frac{ikt_1}{\sqrt{2}}$  | 0                                   |
| $f_{2+}^{\#1} \dagger^{\alpha\beta}$          | $\frac{ikt_1}{\sqrt{2}}$   | 0                                   |
| $\omega_{2-}^{\#1} \dagger^{\alpha\beta\chi}$ | 0                          | $k^2r_1+\frac{t_1}{2}$              |

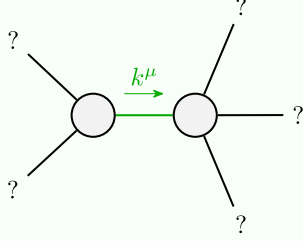
| $\omega_0^{\#1} \dagger$ | $f_0^{\#1} \dagger$ | $\omega_0^{\#2} \dagger$ | $\omega_0^{\#1} \dagger$ |
|--------------------------|---------------------|--------------------------|--------------------------|
| $\omega_0^{\#1} \dagger$ | $6k^2(-r_1+r_3)$    | 0                        | 0                        |
| $f_0^{\#1} \dagger$      | 0                   | 0                        | 0                        |
| $f_0^{\#2} \dagger$      | 0                   | 0                        | 0                        |
| $\omega_0^{\#1} \dagger$ | 0                   | 0                        | $-t_1$                   |

| Source constraints/gauge generators                                  |                |
|--|----------------|
| SO(3) irreps   | Multiplicities |
| $\tau_{0+}^{\#2} == 0$   | 1              |
| $\tau_{0+}^{\#1} == 0$   | 1              |
| $\tau_{1-}^{\#2\alpha} + 2ik \sigma_{1-}^{\#2\alpha} == 0$           | 3              |
| $\tau_{1-}^{\#1\alpha} == 0$   | 3              |
| $\tau_{1+}^{\#1\alpha\beta} + ik \sigma_{1+}^{\#2\alpha\beta} == 0$  | 3              |
| $\tau_{2+}^{\#1\alpha\beta} - 2ik \sigma_{2+}^{\#1\alpha\beta} == 0$ | 5              |
| Total constraints:   | 16             |

Massive and massless spectra



| Massive particle |                         |
|------------------|-------------------------|
| Pole residue:    | $-\frac{1}{r_1} > 0$    |
| Polarisations:   | 5                       |
| Square mass:     | $-\frac{t_1}{2r_1} > 0$ |
| Spin:            | 2                       |
| Parity:          | Odd                     |



| Quadratic pole |                                     |
|----------------|-------------------------------------|
| Pole residue:  | $\frac{1}{(r_1-2r_3-r_5)t_1^2} > 0$ |
| Polarisations: | 2                                   |

Unitarity conditions

$r_1 < 0 \ \&\& \ r_5 < r_1 - 2r_3 \ \&\& \ t_1 > 0$