Particle spectrograph

Wave operator and propagator

Quadratic (free) Lagrangian density

$$-t_1 \omega_\iota{}^{\alpha\iota}\omega_{\kappa\alpha}{}^\kappa - \tfrac13 t_1 \omega_\iota{}^{\kappa\lambda}\omega_{\kappa\lambda}{}^\iota + \tfrac23 t_2 \omega_\iota{}^{\kappa\lambda}\omega_{\kappa\lambda}{}^\iota + \tfrac13 t_1 \omega_{\kappa\lambda}{}^\iota\omega^{\kappa\lambda}{}_\iota +$$
$$\tfrac13 t_2 \omega_{\kappa\lambda}{}^\iota \omega^{\kappa\lambda}{}_\iota + f^{\alpha\beta}\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi} + 2r_1 \partial_\iota\omega^{\kappa\lambda}{}_\kappa \partial^\iota\omega_\lambda{}^\alpha{}_\alpha -$$
$$\tfrac23 r_1 \partial^\beta\omega^{\theta\alpha}{}_\kappa \partial_\theta\omega_{\alpha\beta}{}^\kappa - \tfrac23 r_1 \partial_\theta\omega_{\alpha\beta}{}^\kappa\partial_\kappa\omega^{\alpha\beta\theta} + \tfrac23 r_1 \partial_\theta\omega_{\alpha\beta}{}^\kappa \partial_\kappa\omega^{\theta\alpha\beta} +$$
$$2r_1\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\partial_\kappa\omega^{\theta\kappa\lambda} - 2r_1\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\partial_\kappa\omega^{\theta\kappa\lambda} + 2r_1\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\partial_\kappa\omega^{\kappa\lambda\theta} -$$
$$4r_1\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\partial_\kappa\omega^{\kappa\lambda\theta} - \tfrac13 t_1\partial^\alpha f_{\theta\kappa}\partial^\kappa f_\alpha{}^\theta + \tfrac16 t_2 \partial^\alpha f_{\theta\kappa}\partial^\kappa f_\alpha{}^\theta - \tfrac23 t_1 \partial^\alpha f_{\kappa\theta}\partial^\kappa f_\alpha{}^\theta -$$
$$\tfrac16 t_2\partial^\alpha f_{\kappa\theta}\partial^\kappa f_\alpha{}^\theta - \tfrac13 t_1\partial^\alpha f^\lambda{}_\kappa\partial^\kappa f_{\alpha\lambda} + \tfrac16 t_2\partial^\alpha f^\lambda{}_\kappa \partial^\kappa f_{\alpha\lambda} + t_1\omega_{\kappa\alpha}{}^\alpha\partial^\kappa f^\iota{}_\iota +$$
$$t_1\omega_{\kappa\lambda}{}^\lambda\partial^\kappa f^\iota{}_\iota + 2t_1\partial^\alpha f_{\kappa\alpha}\partial^\kappa f^\iota{}_\iota - t_1\partial_\kappa f^\lambda{}_\lambda\partial^\kappa f^\iota{}_\iota + \tfrac13 t_1\omega_{\iota\theta\kappa}\partial^\kappa f^{\iota\theta} +$$
$$\tfrac13 t_2 \omega_{\iota\theta\kappa}\partial^\kappa f^{\iota\theta} + \tfrac43 t_1\omega_{\iota\kappa\theta}\partial^\kappa f^{\iota\theta} - \tfrac23 t_2\omega_{\iota\kappa\theta}\partial^\kappa f^{\iota\theta} - \tfrac13 t_1\omega_{\theta\iota\kappa}\partial^\kappa f^{\iota\theta} -$$
$$\tfrac13 t_2\omega_{\theta\iota\kappa}\partial^\kappa f^{\iota\theta} + \tfrac23 t_1\omega_{\theta\kappa\iota}\partial^\kappa f^{\iota\theta} + \tfrac23 t_2\omega_{\theta\kappa\iota}\partial^\kappa f^{\iota\theta} - t_1\omega_{\iota\alpha}{}^\alpha\partial^\kappa f^\iota{}_\kappa -$$
$$t_1\omega_{\iota\lambda}{}^\lambda\partial^\kappa f^\iota{}_\kappa + \tfrac13 t_1\partial^\alpha f^\lambda{}_\kappa\partial^\kappa f_{\lambda\alpha} - \tfrac16 t_2\partial^\alpha f^\lambda{}_\kappa\partial^\kappa f_{\lambda\alpha} + \tfrac13 t_1\partial_\kappa f_\theta{}^\lambda\partial^\kappa f_\lambda{}^\theta -$$
$$\tfrac16 t_2\partial_\kappa f_\theta{}^\lambda\partial^\kappa f_\lambda{}^\theta + \tfrac23 t_1\partial_\kappa f^\lambda{}_\theta\partial^\kappa f_\lambda{}^\theta + \tfrac16 t_2\partial_\kappa f^\lambda{}_\theta\partial^\kappa f_\lambda{}^\theta - t_1\partial^\alpha f^\lambda{}_\alpha\partial^\kappa f_{\lambda\kappa} +$$
$$\tfrac23 r_1\partial_\kappa\omega^{\alpha\beta\theta}\partial^\kappa\omega_{\alpha\beta\theta} - \tfrac23 r_1\partial_\kappa\omega^{\theta\alpha\beta}\partial^\kappa\omega_{\alpha\beta\theta} + \tfrac23 r_1\partial^\beta\omega_\iota{}^{\alpha\lambda}\partial_\lambda\omega_{\alpha\beta}{}^\iota -$$
$$\tfrac83 r_1\partial^\beta\omega_\iota{}^{\lambda\alpha}\partial_\lambda\omega_{\alpha\beta}{}^\iota - 2r_1\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\partial^\lambda\omega^{\theta\kappa}{}_\kappa + 2r_1\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\partial^\lambda\omega^{\theta\kappa}{}_\kappa$$

	$\sigma^{\#1}_{1^+}{}_{\alpha\beta}$	$\sigma^{\#2}_{1^+}{}_{\alpha\beta}$	$\tau^{\#1}_{1^+}{}_{\alpha\beta}$	$\sigma^{\#1}_{1^-}{}_\alpha$	$\sigma^{\#2}_{1^-}{}_\alpha$	$\tau^{\#1}_{1^-}{}_\alpha$	$\tau^{\#2}_{1^-}{}_\alpha$
$\sigma^{\#1}_{1^+}\dagger^{\alpha\beta}$	$\frac{2(t_1+t_2)}{3t_1t_2}$	$\frac{\sqrt2(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{i\sqrt2 k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	0	0	0	0
$\sigma^{\#2}_{1^+}\dagger^{\alpha\beta}$	$\frac{\sqrt2(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{t_1+4t_2}{3(1+k^2)^2t_1t_2}$	$\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0	0
$\tau^{\#1}_{1^+}\dagger^{\alpha\beta}$	$-\frac{i\sqrt2 k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$-\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	$\frac{k^2(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0	0
$\sigma^{\#1}_{1^-}\dagger^\alpha$	0	0	0	0	$\frac{\sqrt2}{t_1+2k^2t_1}$	0	$\frac{2ik}{t_1+2k^2t_1}$
$\sigma^{\#2}_{1^-}\dagger^\alpha$	0	0	0	$\frac{\sqrt2}{t_1+2k^2t_1}$	$\frac{2k^2r_1+t_1}{(t_1+2k^2t_1)^2}$	0	$\frac{i\sqrt2 k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
$\tau^{\#1}_{1^-}\dagger^\alpha$	0	0	0	0	0	0	0
$\tau^{\#2}_{1^-}\dagger^\alpha$	0	0	0	$-\frac{2ik}{t_1+2k^2t_1}$	$-\frac{i\sqrt2 k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$	0	$\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$

	$\omega^{\#1}_{1^+}{}_{\alpha\beta}$	$\omega^{\#2}_{1^+}{}_{\alpha\beta}$	$f^{\#1}_{1^+}{}_{\alpha\beta}$	$\omega^{\#1}_{1^-}{}_\alpha$	$\omega^{\#2}_{1^-}{}_\alpha$	$f^{\#1}_{1^-}{}_\alpha$	$f^{\#2}_{1^-}{}_\alpha$
$\omega^{\#1}_{1^+}\dagger^{\alpha\beta}$	$\frac16(t_1+4t_2)$	$-\frac{t_1-2t_2}{3\sqrt2}$	$-\frac{ik(t_1-2t_2)}{3\sqrt2}$	0	0	0	0
$\omega^{\#2}_{1^+}\dagger^{\alpha\beta}$	$-\frac{t_1-2t_2}{3\sqrt2}$	$\frac{t_1+t_2}{3}$	$\frac13 ik(t_1+t_2)$	0	0	0	0
$f^{\#1}_{1^+}\dagger^{\alpha\beta}$	$\frac{ik(t_1-2t_2)}{3\sqrt2}$	$-\frac13 ik(t_1+t_2)$	$\frac13 k^2(t_1+t_2)$	0	0	0	0
$\omega^{\#1}_{1^-}\dagger^\alpha$	0	0	0	$-k^2r_1-\frac{t_1}{2}$	$\frac{t_1}{\sqrt2}$	0	ikt_1
$\omega^{\#2}_{1^-}\dagger^\alpha$	0	0	0	$\frac{t_1}{\sqrt2}$	0	0	0
$f^{\#1}_{1^-}\dagger^\alpha$	0	0	0	0	0	0	0
$f^{\#2}_{1^-}\dagger^\alpha$	0	0	0	$-ikt_1$	0	0	0

	$\omega^{\#1}_{2^+}{}_{\alpha\beta}$	$f^{\#1}_{2^+}{}_{\alpha\beta}$	$\omega^{\#1}_{2^-}{}_{\alpha\beta\chi}$
$\omega^{\#1}_{2^+}\dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt2}$	0
$f^{\#1}_{2^+}\dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt2}$	k^2t_1	0
$\omega^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$	0	0	$k^2r_1+\frac{t_1}{2}$

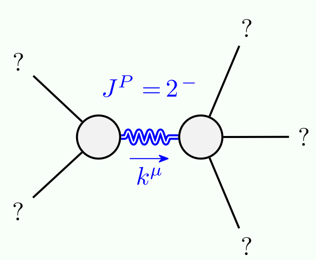
	$\sigma^{\#1}_{0^+}$	$\tau^{\#1}_{0^+}$	$\tau^{\#2}_{0^+}$	$\sigma^{\#1}_{0^-}$
$\sigma^{\#1}_{0^+}\dagger$	$-\frac{1}{(1+2k^2)^2t_1}$	$\frac{i\sqrt2 k}{(1+2k^2)^2t_1}$	0	0
$\tau^{\#1}_{0^+}\dagger$	$-\frac{i\sqrt2 k}{(1+2k^2)^2t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0	0
$\tau^{\#2}_{0^+}\dagger$	0	0	0	0
$\sigma^{\#1}_{0^-}\dagger$	0	0	0	$\frac{1}{t_2}$

	$\omega^{\#1}_{0^+}$	$f^{\#1}_{0^+}$	$f^{\#2}_{0^+}$	$\omega^{\#1}_{0^-}$
$\omega^{\#1}_{0^+}\dagger$	$-t_1$	$i\sqrt2 kt_1$	0	0
$f^{\#1}_{0^+}\dagger$	$-i\sqrt2 kt_1$	$-2k^2t_1$	0	0
$f^{\#2}_{0^+}\dagger$	0	0	0	0
$\omega^{\#1}_{0^-}\dagger$	0	0	0	t_2

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau^{\#2}_{0^+} == 0$	1
$\tau^{\#1}_{0^+} - 2ik\,\sigma^{\#1}_{0^+} == 0$	1
$\tau^{\#2}_{1^-}{}^\alpha + 2ik\,\sigma^{\#2}_{1^-}{}^\alpha == 0$	3
$\tau^{\#1}_{1^-}{}^\alpha == 0$	3
$\tau^{\#1}_{1^+}{}^{\alpha\beta} + ik\,\sigma^{\#2}_{1^+}{}^{\alpha\beta} == 0$	3
$\tau^{\#1}_{2^+}{}^{\alpha\beta} - 2ik\,\sigma^{\#1}_{2^+}{}^{\alpha\beta} == 0$	5
Total constraints:	16

	$\sigma^{\#1}_{2^+}{}_{\alpha\beta}$	$\tau^{\#1}_{2^+}{}_{\alpha\beta}$	$\sigma^{\#1}_{2^-}{}_{\alpha\beta\chi}$
$\sigma^{\#1}_{2^+}\dagger^{\alpha\beta}$	$\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt2 k}{(1+2k^2)^2t_1}$	0
$\tau^{\#1}_{2^+}\dagger^{\alpha\beta}$	$\frac{2i\sqrt2 k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\sigma^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$	0	0	$\frac{2}{2k^2r_1+t_1}$

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0 \,\&\&\, t_1 > 0$