Particle spectrograph

Wave operator and propagator

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\tau_0^{\#2}$ == 0	$\partial_\beta\partial_\alpha\tau^{\alpha\beta}$ == 0	1
$\tau_0^{\#1}$ == 0	$\partial_\beta\partial_\alpha\tau^{\alpha\beta}$ == $\partial_\beta\partial^\beta\tau^\alpha{}_\alpha$	1
$\sigma_{0^+}^{\#1}$ == 0	$\partial_\beta\sigma^{\alpha\beta}{}_\alpha$ == 0	1
$\tau_{1^-}^{\#2\,\alpha}$ == 0	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi}$ == $\partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta}$	3
$\tau_{1^-}^{\#1\,\alpha}$ == 0	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi}$ == $\partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$	3
$\sigma_{1^-}^{\#2\,\alpha}$ == 0	$\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$ == 0	3
$\sigma_{1^-}^{\#1\,\alpha}$ == 0	$\partial_\chi\partial^\alpha\sigma^{\beta\chi}{}_\beta + \partial_\chi\partial^\chi\sigma^{\alpha\beta}{}_\beta$ == $\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$	3
$\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#1\,\alpha\beta}$ == 0	$\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} +$ $\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha}$ == $\partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} +$ $\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta}$	3
$\sigma_{1^+}^{\#1\,\alpha\beta}$ == $\sigma_{1^+}^{\#2\,\alpha\beta}$	$3\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} +$ $2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta}$ == $3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha}$	3
$\sigma_{2^-}^{\#1\,\alpha\beta\chi}$ == 0	$3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\alpha\sigma^{\beta\delta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\alpha\sigma^{\beta\delta}{}_\delta +$ $2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\chi\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\delta\chi} +$ $2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\chi\delta\alpha} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\beta\delta} +$ $2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\delta\beta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\beta\chi\alpha} +$ $3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\alpha\sigma^{\delta\epsilon}{}_\delta +$ $3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\beta\delta\epsilon} +$ $3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\alpha\delta}{}_\delta$ == $3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\beta\sigma^{\alpha\delta}{}_\delta +$ $2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\chi\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\delta\chi} +$ $2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\chi\delta\beta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\beta\delta\alpha} +$ $4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\beta\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\chi\beta} +$ $3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\beta\sigma^{\delta\epsilon}{}_\delta +$ $3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\alpha\delta\epsilon} +$ $3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\beta\delta}{}_\delta$	5
$\tau_{2^+}^{\#1\,\alpha\beta}$ == 0	$4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi +$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} +$ $2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta}$ == $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} +$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} +$ $2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi$	5
$\sigma_{2^+}^{\#1\,\alpha\beta}$ == 0	$3\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta} +$ $2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\sigma^{\chi\delta}{}_\chi$ == $2\,\partial_\delta\partial^\beta\partial^\alpha\sigma^{\chi\delta}{}_\chi +$ $3\,(\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\chi\beta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\chi\alpha})$	5
Total constraints/gauge generators:		36

Quadratic (free) action

$$\begin{aligned}\mathcal{S} == \iiint\!\!\int (\tfrac{1}{6}\,(6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} + 4\,t_2\,\omega_{\iota\theta\alpha}\,\partial^\theta f^{\alpha\iota} + 2\,t_2\,\partial_\alpha f_{\iota\theta}\,\partial^\theta f^{\alpha\iota} -\\ t_2\,\partial_\alpha f_{\theta\iota}\,\partial^\theta f^{\alpha\iota} - t_2\,\partial_\iota f_{\alpha\theta}\,\partial^\theta f^{\alpha\iota} + t_2\,\partial_\theta f_{\alpha\iota}\,\partial^\theta f^{\alpha\iota} -\\ t_2\,\partial_\theta f_{\iota\alpha}\,\partial^\theta f^{\alpha\iota} - 4\,t_2\,\omega_{\alpha\theta\iota}\,(\omega^{\alpha\iota\theta} + \partial^\theta f^{\alpha\iota}) +\\ 2\,t_2\,\omega_{\alpha\iota\theta}\,(\omega^{\alpha\iota\theta} + 2\,\partial^\theta f^{\alpha\iota}) + 8\,r_2\,\partial_\beta\omega_{\alpha\iota\theta}\,\partial^\theta\omega^{\alpha\beta\iota} -\\ 4\,r_2\,\partial_\beta\omega_{\alpha\theta\iota}\,\partial^\theta\omega^{\alpha\beta\iota} + 4\,r_2\,\partial_\beta\omega_{\iota\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\iota} -\\ 2\,r_2\,\partial_\iota\omega_{\alpha\beta\theta}\,\partial^\theta\omega^{\alpha\beta\iota} + 2\,r_2\,\partial_\theta\omega_{\alpha\beta\iota}\,\partial^\theta\omega^{\alpha\beta\iota} -\\ 4\,r_2\,\partial_\theta\omega_{\alpha\iota\beta}\,\partial^\theta\omega^{\alpha\beta\iota}))[t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt\end{aligned}$$

	$\omega_{1^+}^{\#1\,\alpha\beta}$	$\omega_{1^+}^{\#2\,\alpha\beta}$	$f_{1^+}^{\#1\,\alpha\beta}$	$\omega_{1^-}^{\#1\,\alpha}$	$\omega_{1^-}^{\#2\,\alpha}$	$f_{1^-}^{\#1\,\alpha}$	$f_{1^-}^{\#2\,\alpha}$
$\omega_{1^+}^{\#1\,\dagger\,\alpha\beta}$	$\frac{2\,t_2}{3}$	$\frac{\sqrt{2}\,t_2}{3}$	$\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2$	0	0	0	0
$\omega_{1^+}^{\#2\,\dagger\,\alpha\beta}$	$\frac{\sqrt{2}\,t_2}{3}$	$\frac{t_2}{3}$	$\frac{i\,k\,t_2}{3}$	0	0	0	0
$f_{1^+}^{\#1\,\dagger\,\alpha\beta}$	$-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_2$	$-\frac{1}{3}\,i\,k\,t_2$	$\frac{k^2\,t_2}{3}$	0	0	0	0
$\omega_{1^-}^{\#1\,\dagger\,\alpha}$	0	0	0	0	0	0	0
$\omega_{1^-}^{\#2\,\dagger\,\alpha}$	0	0	0	0	0	0	0
$f_{1^-}^{\#1\,\dagger\,\alpha}$	0	0	0	0	0	0	0
$f_{1^-}^{\#2\,\dagger\,\alpha}$	0	0	0	0	0	0	0

	$\sigma_{1^+}^{\#1\,\alpha\beta}$	$\sigma_{1^+}^{\#2\,\alpha\beta}$	$\tau_{1^+}^{\#1\,\alpha\beta}$	$\sigma_{1^-}^{\#1\,\alpha}$	$\sigma_{1^-}^{\#2\,\alpha}$	$\tau_{1^-}^{\#1\,\alpha}$	$\tau_{1^-}^{\#2\,\alpha}$
$\sigma_{1^+}^{\#1\,\dagger\,\alpha\beta}$	$\frac{6}{(3+k^2)^2\,t_2}$	$\frac{3\,\sqrt{2}}{(3+k^2)^2\,t_2}$	$\frac{3\,i\,\sqrt{2}\,k}{(3+k^2)^2\,t_2}$	0	0	0	0
$\sigma_{1^+}^{\#2\,\dagger\,\alpha\beta}$	$\frac{3\,\sqrt{2}}{(3+k^2)^2\,t_2}$	$\frac{3}{(3+k^2)^2\,t_2}$	$\frac{3\,i\,k}{(3+k^2)^2\,t_2}$	0	0	0	0
$\tau_{1^+}^{\#1\,\dagger\,\alpha\beta}$	$-\frac{3\,i\,\sqrt{2}\,k}{(3+k^2)^2\,t_2}$	$-\frac{3\,i\,k}{(3+k^2)^2\,t_2}$	$\frac{3\,k^2}{(3+k^2)^2\,t_2}$	0	0	0	0
$\sigma_{1^-}^{\#1\,\dagger\,\alpha}$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#2\,\dagger\,\alpha}$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#1\,\dagger\,\alpha}$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#2\,\dagger\,\alpha}$	0	0	0	0	0	0	0

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$f_{0^+}^{\#1}$	$\omega_{0^+}^{\#1}$
$\omega_{0^+}^{\#1\,\dagger}$	0	0	0	0
$f_{0^+}^{\#2\,\dagger}$	0	0	0	0
$f_{0^+}^{\#1\,\dagger}$	0	0	0	0
$\omega_{0^+}^{\#1\,\dagger}$	$k^2\,r_2 + t_2$	0	0	0

	$\omega_{2^+}^{\#1\,\alpha\beta}$	$f_{2^+}^{\#1\,\alpha\beta}$	$\omega_{2^+}^{\#1\,\alpha\beta}$
$\omega_{2^+}^{\#1\,\dagger\,\alpha\beta}$	0	0	0
$f_{2^+}^{\#1\,\dagger\,\alpha\beta}$	0	0	0
$\omega_{2^+}^{\#1\,\dagger\,\alpha\beta\chi}$	0	0	0

	$\sigma_{2^+}^{\#1\,\alpha\beta}$	$\tau_{2^+}^{\#1\,\alpha\beta}$	$\sigma_{2^-}^{\#1\,\alpha\beta\chi}$
$\sigma_{2^+}^{\#1\,\dagger\,\alpha\beta}$	0	0	0
$\tau_{2^+}^{\#1\,\dagger\,\alpha\beta}$	0	0	0
$\sigma_{2^-}^{\#1\,\dagger\,\alpha\beta\chi}$	0	0	0

	$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\tau_{0^+}^{\#1}$	$\sigma_{0^+}^{\#1}$
$\sigma_{0^+}^{\#1\,\dagger}$	0	0	0	0
$\tau_{0^+}^{\#2\,\dagger}$	0	0	0	0
$\tau_{0^+}^{\#1\,\dagger}$	0	0	0	0
$\sigma_{0^+}^{\#1\,\dagger}$	$k^2\,r_2 + t_2$	$\frac{1}{3}$	0	0

Massive and massless spectra

Massive particle			
Pole residue:	$-\frac{1}{r_2} > 0$		
Square mass:	$-\frac{t_2}{r_2} > 0$		
Polarisations:	1	3	5
Spin:	0	1	2
Parity:	Odd	Even	Odd

(No massless particles)

Unitarity conditions

$r_2 < 0$ && $t_2 > 0$