Particle spectrograph

Wave operator and propagator

| $	au_1^{\#2}$ | 0 | 0 | 0 | $\frac{4i}{k(1+2k^2)(r_3+2r_5)}$ | $\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$ | 0 | $\frac{6k^2(r_3+2r_5)+8t_3}{(1+2k^2)^2(r_3+2r_5)t_3}$ |
|--|--|--|---|---|---|-----------------------------|--|
| $\tau_{1}^{\#1}{}_{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\sigma_{1}^{\#2}{}_{lpha}$ | 0 | 0 | 0 | $\frac{2\sqrt{2}}{k^2(1+2k^2)(r_3+2r_5)}$ | $\frac{3 k^2 (r_3 + 2 r_5) + 4 t_3}{(k + 2 k^3)^2 (r_3 + 2 r_5) t_3}$ | 0 | $-\frac{i\sqrt{2}(3k^2(r_3+2r_5)+4t_3)}{k(1+2k^2)^2(r_3+2r_5)t_3}$ |
| $\sigma_{1}^{\#1}{}_{\alpha}$ | 0 | 0 | 0 | $\frac{2}{k^2 (r_3 + 2 r_5)}$ | $\frac{2\sqrt{2}}{k^2(1+2k^2)(r_3+2r_5)}$ | 0 | $-\frac{4i}{k(1+2k^2)(r_3+2r_5)}$ |
| $\tau_1^{\#1}{}_{\!$ | $-\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$ | $\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$ | $\frac{3k^2(2r_3+r_5)+2t_2}{(1+k^2)^2(2r_3+r_5)t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1}^{\#2}{}_{\alpha\beta}$ | $-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$ | $\frac{3k^2(2r_3+r_5)+2t_2}{(k+k^3)^2(2r_3+r_5)t_2}$ | $-\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1}^{\#1}{}_{\alpha\beta}$ | | $-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$ | $\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$ | 0 | 0 | 0 | 0 |
| | $\sigma_{1}^{\#1} + \alpha^{eta}$ | $\sigma_{1}^{#2} + \alpha^{\beta}$ | $\tau_{1}^{\#1} + \alpha \beta$ | $\sigma_{1}^{\#1} +^{\alpha}$ | $\sigma_{1}^{\#2} +^{\alpha}$ | $\tau_{1}^{\#1} +^{\alpha}$ | $\tau_1^{\#2} + ^{\alpha}$ |

| _ | $\sigma_{0^+}^{\sharp 1}$ | $\tau_{0}^{\#1}$ | $	au_{0}^{\#2}$ | $\sigma_0^{\#1}$ |
|----------------------|--------------------------------------|---------------------------------------|-----------------|------------------|
| $\sigma_{0}^{\#1}$ † | $\frac{1}{(1+2k^2)^2t_3}$ | $-\frac{i\sqrt{2} k}{(1+2k^2)^2 t_3}$ | 0 | 0 |
| $\tau_{0}^{\#1}$ † | $\frac{i\sqrt{2} k}{(1+2k^2)^2 t_3}$ | $\frac{2k^2}{(1+2k^2)^2t_3}$ | 0 | 0 |
| $\tau_{0}^{\#2}$ † | 0 | 0 | 0 | 0 |
| $\sigma_0^{\#1}$ † | 0 | 0 | 0 | $\frac{1}{t_2}$ |

| | $\omega_{0}^{\sharp 1}$ | $f_{0+}^{\#1}$ | $f_{0}^{#2}$ | $\omega_0^{\#1}$ |
|-------------------------|-------------------------|--------------------|--------------|------------------|
| $\omega_0^{\sharp 1}$ † | t_3 | $-i \sqrt{2} kt_3$ | 0 | 0 |
| $f_{0^{+}}^{#1}\dagger$ | $i \sqrt{2} kt_3$ | $2k^2t_3$ | 0 | 0 |
| $f_{0}^{#2}$ † | 0 | 0 | 0 | 0 |
| $\omega_{0}^{\#1}$ † | 0 | 0 | 0 | t_2 |

| | $\omega_{2^{+}\alpha\beta}^{\#1}$ | $f_{2}^{\#1}{}_{\alpha\beta}$ | $\omega_{2}^{\#1}{}_{\alpha\beta\chi}$ |
|--|-----------------------------------|-------------------------------|--|
| $\omega_{2^{+}}^{\sharp 1}\dagger^{lphaeta}$ | $-\frac{3k^2r_3}{2}$ | 0 | 0 |
| $f_2^{#1} \dagger^{\alpha\beta}$ | 0 | 0 | 0 |
| $\omega_2^{\#1}$ † $^{lphaeta\chi}$ | 0 | 0 | 0 |

| $f_{1}^{\#2}$ | 0 | 0 | 0 | $-\frac{2}{3}$ ikt ₃ | $\frac{1}{3}\bar{l}\sqrt{2}kt_3$ | 0 | $\frac{2k^2t_3}{3}$ |
|----------------------------------|----------------------------------|----------------------------------|--|---|----------------------------------|------------------------------|-----------------------------------|
| $f_{1^-}^{\#1}\alpha$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1}^{\#2}{}_{\alpha}$ | 0 | 0 | 0 | $-\frac{\sqrt{2}t_3}{3}$ | <u>t3</u> 3 | 0 | $-\frac{1}{3}\bar{l}\sqrt{2}kt_3$ |
| $\omega_{1^{-}\alpha}^{\#1}$ | 0 | 0 | 0 | $\lambda^2 \left(\frac{r_3}{2} + r_5 \right) + \frac{2t_3}{3}$ | $-\frac{\sqrt{2}t_3}{3}$ | 0 | <u>2 i k t 3</u> 3 |
| $f_1^{\#1}_{\alpha\beta}$ | $\frac{1}{3}\bar{l}\sqrt{2}kt_2$ | <i>ikt</i> 2 3 | $\frac{k^2 t_2}{3}$ | 0 | 0 | 0 | 0 |
| $\omega_1^{\#2}{}_+\alpha\beta$ | $\frac{\sqrt{2} t_2}{3}$ | t 2 3 | $-rac{1}{3}ec{l}kt_2$ | 0 | 0 | 0 | 0 |
| $\omega_1^{\#1}{}_+\alpha_\beta$ | 2 (2 | $\frac{\sqrt{2} t_2}{3}$ | $-\frac{1}{3}$ \vec{l} $\sqrt{2}$ kt_2 | 0 | 0 | 0 | 0 |
| | $\omega_1^{\#1} + \alpha^{eta}$ | $\omega_1^{#2} + \alpha^{\beta}$ | $f_{1}^{#1} + \alpha \beta$ | $\omega_{1^{\bar{-}}}^{\#1} +^{\alpha}$ | $\omega_1^{\#2} +^{\alpha}$ | $f_{1^{-}}^{\#1} +^{\alpha}$ | $f_1^{#2} + \alpha$ |

| auge generators | Multiplicities | 1 | 1 | 3 | 3 | 3 | 2 | 2 | 21 |
|-------------------------------------|----------------|--------------------|---|--|----------------------------------|---|---|-----------------------------------|--------------------|
| Source constraints/gauge generators | SO(3) irreps | $\tau_0^{#2} == 0$ | $\tau_{0+}^{\#1} - 2\bar{l}k\sigma_{0+}^{\#1} == 0$ | $\tau_{1}^{\#2}{}^{\alpha} + 2ik \sigma_{1}^{\#2}{}^{\alpha} == 0$ | $\tau_{1}^{\#1}{}^{\alpha} == 0$ | $\tau_{1+}^{\#1}\alpha\beta + ik \ \sigma_{1+}^{\#2}\alpha\beta == 0 \ \boxed{3}$ | $\sigma_{2^{-1}}^{\#1}\alpha\beta\chi := 0$ | $\tau_{2+}^{\#1\alpha\beta} == 0$ | Total constraints: |

| | $\sigma_{2^{+}\alpha\beta}^{\#1}$ | $	au_2^{\#1}_{lphaeta}$ | $\sigma_{2}^{\#1}{}_{\alpha\beta\chi}$ |
|---|-----------------------------------|-------------------------|--|
| $\sigma_{2}^{\#1} \dagger^{\alpha\beta}$ | $-\frac{2}{3k^2r_3}$ | 0 | 0 |
| $	au_2^{\#1} \dagger^{lphaeta}$ | 0 | 0 | 0 |
| $\sigma_2^{\sharp 1} \dagger^{\alpha\beta\chi}$ | 0 | 0 | 0 |

Quadratic (free) Lagrangian density

 $\frac{2}{3} t_3 \ \omega_i^{\alpha i} \ \omega_{\kappa \alpha}^{\kappa \kappa} + \frac{2}{3} t_2 \ \omega_i^{\kappa \lambda} \ \omega_{\kappa \lambda}^{i} + \frac{1}{3} t_2 \ \omega_{\kappa \lambda}^{i} \ \omega_{\kappa \lambda}^{i} + f^{\alpha \beta} \ \tau_{\alpha \beta} + \\ \omega^{\alpha \beta \chi} \ \sigma_{\alpha \beta \chi}^{} - \frac{1}{2} r_3 \partial_i \omega^{\kappa \lambda}_{\kappa} \partial^i \omega_{\lambda}^{\alpha}_{\alpha} - r_5 \partial_i \omega^{\kappa \lambda}_{\kappa} \partial^i \omega_{\lambda}^{\alpha}_{\alpha} + \frac{1}{2} r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha}_{\beta} \partial_{\kappa} \omega^{\theta \kappa \lambda} - \\ r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha}_{\beta} \partial_{\kappa} \omega^{\theta \kappa \lambda} - \frac{1}{2} r_3 \partial_{\theta} \omega_{\lambda}^{\alpha}_{\alpha} \partial_{\kappa} \omega^{\theta \kappa \lambda} + r_5 \partial_{\theta} \omega_{\lambda}^{\alpha}_{\alpha} \partial_{\kappa} \omega^{\theta \kappa \lambda} - \frac{1}{2} r_3 \partial_{\alpha} \omega_{\lambda}^{\alpha}_{\beta} \partial_{\kappa} \omega^{\kappa \lambda \theta} - \\ r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha}_{\beta} \partial_{\kappa} \omega^{\kappa \lambda \theta} + r_3 \partial_{\theta} \omega_{\lambda}^{\alpha}_{\alpha} \partial_{\kappa} \omega^{\kappa \lambda \theta} + 2 r_5 \partial_{\theta} \omega_{\lambda}^{\alpha}_{\alpha} \partial_{\kappa} \omega^{\kappa \lambda \theta} + \\ \frac{1}{6} t_2 \partial^{\alpha} f_{\theta \kappa} \partial^{\kappa} f_{\alpha}^{\theta} - \frac{1}{6} t_2 \partial^{\alpha} f_{\kappa \theta} \partial^{\kappa} f_{\alpha}^{\theta} + \frac{1}{6} t_2 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\alpha \lambda}^{-\frac{2}{3}} t_3 \omega_{\kappa \alpha}^{\alpha} \partial^{\kappa} f_i^{i} - \\ \frac{2}{3} t_3 \omega_{\kappa \lambda}^{\lambda} \partial^{\kappa} f_i^{i} - \frac{4}{3} t_3 \partial^{\alpha} f_{\kappa \alpha} \partial^{\kappa} f_i^{i} + \frac{2}{3} t_3 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_i^{i} + \frac{1}{3} t_2 \omega_{i \theta \kappa} \partial^{\kappa} f_i^{i\theta} - \\ \frac{2}{3} t_2 \omega_{i \kappa \theta} \partial^{\kappa} f_i^{i\theta} - \frac{1}{3} t_2 \omega_{\theta i \kappa} \partial^{\kappa} f_{\lambda \alpha}^{i\theta} + \frac{2}{3} t_2 \omega_{\theta \kappa i} \partial^{\kappa} f_i^{i\theta} + \frac{2}{3} t_3 \omega_{\kappa}^{\alpha} \partial^{\kappa} f_i^{\kappa} + \\ \frac{2}{3} t_3 \partial_{\kappa}^{\lambda} \partial^{\kappa} f_{\kappa}^{i} - \frac{1}{6} t_2 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\lambda \alpha}^{i} - \frac{1}{6} t_2 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{i\theta} + \frac{1}{6} t_2 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{i\theta} + \\ \frac{2}{3} t_3 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\kappa}^{i} - \frac{1}{6} t_2 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\lambda \alpha}^{i} - \frac{1}{6} t_2 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{i} + \frac{1}{6} t_2 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{i} + \\ \frac{2}{3} t_3 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{i} - 4 r_3 \partial^{\beta} \omega_{i}^{\lambda} \partial_{\lambda} \omega_{\alpha}^{i} - \frac{1}{2} r_3 \partial_{\alpha} \omega_{\lambda}^{\lambda} \partial^{\lambda} \omega_{\lambda}^{i} \partial^{\lambda} \omega_{\kappa}^{i} + \\ \frac{2}{3} t_3 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\lambda} \omega_{\kappa}^{i} + \frac{1}{2} r_3 \partial_{\theta} \omega_{\lambda}^{\lambda} \partial^{\lambda} \omega_{\kappa}^{i} - r_5 \partial_{\theta} \omega_{\lambda}^{\lambda} \partial^{\lambda} \omega_{\kappa}^{i} \partial^{\lambda} \omega_{\kappa}^{i} + \\ \frac{2}{3} t_3 \partial_{\alpha} \omega_{\lambda}^{\lambda} \partial^{\lambda} \omega_{\kappa}^{i} + \frac{1}{2} r_3 \partial_{\theta} \omega_{\lambda}^{\lambda} \partial^{\lambda} \omega_{\kappa}^{i} - r_5 \partial_{\theta} \omega_{\lambda}^{\lambda} \partial^{\lambda} \omega_{\kappa}^{i} \partial^{\lambda} \omega_{\kappa}^{i} + \\ \frac{2}{3} t_3 \partial_{\alpha}^{\lambda} \omega_{\lambda}^{\lambda} \partial^{\lambda} \omega_{\kappa}^{i} + \frac{1}{2} r_3 \partial_{\theta} \omega_{\lambda}^{\lambda} \partial^{\lambda} \omega_{\kappa}^{i}$

Massive and massless spectra

Quadratic pole

Pole residue:
$$-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2} > 0$$

Polarisations: 2

(No massive particles)

Unitarity conditions

$$r_3 < 0 \&\& (r_5 < -\frac{r_3}{2} || r_5 > -2 r_3) || r_3 > 0 \&\& -2 r_3 < r_5 < -\frac{r_3}{2}$$