In[ ]:=
```
Get@FileNameJoin@{NotebookDirectory[], "Calibration.m"};
```

```
------------------------------------------------------------
Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}
CopyRight (C) 2003-2020, Jose M. Martin-Garcia, under the General Public License.
Connecting to external linux executable...
Connection established.
------------------------------------------------------------
Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}
CopyRight (C) 2002-2021, Jose M. Martin-Garcia, under the General Public License.
------------------------------------------------------------
Package xAct`xPlain` version 1.0.0-developer, {2023, 6, 27}
CopyRight © 2023, Will E. V. Barker and Sebastian Zell, under the General Public License.
------------------------------------------------------------
These packages come with ABSOLUTELY NO WARRANTY; for details type Disclaimer[]. This
  is free software, and you are welcome to redistribute it under certain conditions. See the General Public License for details.
------------------------------------------------------------
```

PSALTer Calibration

Key observation: During the calibration run, we need to write some commentary, which will appear in this green text, or as numbered equations/expressions with a green background. The output of the PSALTer package (specifically the function called ParticleSpectrum) is not in green, thus wherever we are using PSALTer the output should be quite distinctive.

The first step is to load the PSALTer package.

```
------------------------------------------------------------
Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}
CopyRight (C) 2011-2021, Thomas Bäckdahl, under the General Public License.
------------------------------------------------------------
Package xAct`xPert`  version 1.0.6, {2018, 2, 28}
CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia and Guillermo A. Mena Marugan, under the General Public License.
** Variable $CovDFormat changed from Prefix to Postfix
** Option AllowUpperDerivatives of ContractMetric changed from False to True
** Option MetricOn of MakeRule changed from None to All
** Option ContractMetrics of MakeRule changed from False to True
------------------------------------------------------------
Package xAct`Invar`  version 2.0.5, {2013, 7, 1}
CopyRight (C) 2006-2020, J. M. Martin-Garcia, D. Yllanes and R. Portugal, under the General Public License.
** DefConstantSymbol: Defining constant symbol sigma.
** DefConstantSymbol: Defining constant symbol dim.
** Option CurvatureRelations of DefCovD changed from True to False
** Variable $CommuteCovDsOnScalars changed from True to False
------------------------------------------------------------
Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}
CopyRight (C) 2005-2021, David Yllanes and Jose M. Martin-Garcia, under the General Public License.
------------------------------------------------------------
Package xAct`xTras`  version 1.4.2, {2014, 10, 30}
CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.
** Variable $CovDFormat changed from Postfix to Prefix
** Option CurvatureRelations of DefCovD changed from False to True
------------------------------------------------------------
Package xAct`PSALTer` version 1.0.0-developer, {2023, 7, 9}
CopyRight © 2022, Will E. V. Barker, Stephanie Buttigieg, Cillian Rew, Claire Rigouzzo and Zhiyuan Wei, under the General Public License.
------------------------------------------------------------
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  is free software, and you are welcome to redistribute it under certain conditions. See the General Public License for details.
------------------------------------------------------------
```

Great, so PSALTer is now loaded and we can start to do some science.

Scalar field theory

Key observation: We will test the ScalarTheory module.

Massless scalar (shift-symmetric field)

Let's begin by looking at a massless scalar field theory.

$$\alpha_{1}\,\partial_\alpha\varphi\,\partial^\alpha\varphi \tag{1}$$

Now we shove the Lagrangian into PSALTer.

PSALTer results panel

$$S == \iiiint \left(\rho\,\varphi + \alpha_{1}\,\partial_\alpha\varphi\,\partial^\alpha\varphi\right)[t, x, y, z]\, dz\, dy\, dx\, dt$$

Wave operator

$$\begin{array}{c|c} & \overset{0^+}{\cdot}\varphi \\ \hline \overset{0^+}{\cdot}\varphi^\dagger & \alpha_{1}\, k^2 \end{array}$$

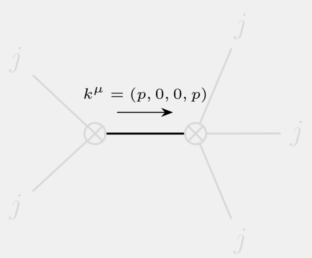
Saturated propagator

$$\begin{array}{c|c} & \overset{0^+}{\cdot}\rho \\ \hline \overset{0^+}{\cdot}\rho^\dagger & \frac{1}{\alpha_{1}\, k^2} \end{array}$$

Source constraints

(None)

Particle spectrum



Massless particle

Pole residue:	$\frac{1}{\alpha_{1}} > 0$
Polarisations:	1

Gauge symmetries

(Not yet implemented)

Unitarity conditions

$\alpha_{1} > 0$

Assumptions

(Not yet implemented)

The result is much as you would expect. There is one massless polarisation, supported by a no-ghost condition which bounds the kinetic part of the Hamiltonian from below.

Massive scalar (Higgs field, pions)

Now for the massive case.

$$-\alpha_{2}\,\varphi^{2}+\alpha_{1}\,\partial_{\alpha}\varphi\,\partial^{\alpha}\varphi \qquad (2)$$

We apply PSALTer again.

PSALTer results panel

$$\mathcal{S} == \iiiint \left(\varphi\left(\rho-\alpha_{2}\,\varphi\right)+\alpha_{1}\,\partial_{\alpha}\varphi\,\partial^{\alpha}\varphi\right)[t,\, x,\, y,\, z]\, dz\, dy\, dx\, dt$$

Wave operator

$$\begin{array}{c|c} & {}^{0^{+}}\varphi \\ \hline {}^{0^{+}}\varphi^{\dagger} & -\alpha_{2}+\alpha_{1}\,k^{2} \end{array}$$

Saturated propagator

$$\begin{array}{c|c} & {}^{0^{+}}\rho \\ \hline {}^{0^{+}}\rho^{\dagger} & \dfrac{1}{-\alpha_{2}+\alpha_{1}\,k^{2}} \end{array}$$

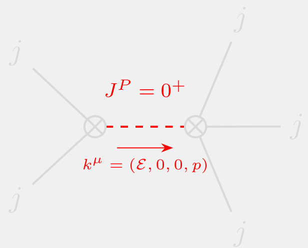
Source constraints

(None)

Gauge symmetries

(Not yet implemented)

Particle spectrum



Massive particle

Pole residue:	$\frac{1}{\alpha_{1}}>0$
Square mass:	$\frac{\alpha_{2}}{\alpha_{1}}>0$
Spin:	0
Parity:	Even

Unitarity conditions

$\alpha_{1}>0\ \&\&\ \alpha_{2}>0$

Assumptions

(Not yet implemented)

We find that the massless eigenvalue has disappeared, but the propagator develops a massive pole whose no-ghost condition is equivalent. There is an additional no-tachyon condition on the Klein-Gordon mass.

Vector field theory

Key observation: We will test the VectorTheory module.

Maxwell field (quantum electrodynamics)

The first pure 1-form theory we might think to try is due to Maxwell. We know from kindergarten that if we contract the square of the Maxwell tensor, we get a viable kinetic term which propagates the two massless photon polarisations. Let's try this out.

$$\underset{1}{\alpha.}\left(\partial_\alpha\mathcal{B}_\beta-\partial_\beta\mathcal{B}_\alpha\right)\left(\partial^\alpha\mathcal{B}^\beta-\partial^\beta\mathcal{B}^\alpha\right) \quad (3)$$

PSALTer results panel

$$\mathcal{S}==\iiiint\left(\mathcal{B}^\alpha\ \mathcal{J}_\alpha+2\,\underset{1}{\alpha.}\left(-\partial_\alpha\mathcal{B}_\beta+\partial_\beta\mathcal{B}_\alpha\right)\partial^\beta\mathcal{B}^\alpha\right)[t,x,y,z]\,dz\,dy\,dx\,dt$$

Wave operator

$$\begin{array}{c|cc} & {}^{0^+}\mathcal{B} & {}^{1^-}\mathcal{B}_\alpha \\ \hline {}^{0^+}\mathcal{B}^\dagger & 0 & \\ {}^{1^-}\mathcal{B}^{\dagger\alpha} & & 2\,\underset{1}{\alpha.}\,k^2 \end{array}$$

Saturated propagator

$$\begin{array}{c|cc} & {}^{0^+}\mathcal{J} & {}^{1^-}\mathcal{J}_\alpha \\ \hline {}^{0^+}\mathcal{J}^\dagger & 0 & \\ {}^{1^-}\mathcal{J}^{\dagger\alpha} & & \frac{1}{2\,\underset{1}{\alpha.}\,k^2} \end{array}$$

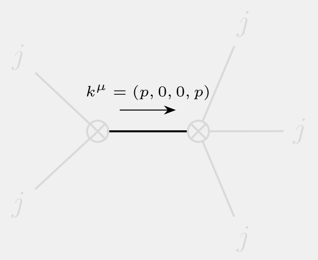
Source constraints

$\boxed{{}^{0^+}\mathcal{J}==0}$

Gauge symmetries

(Not yet implemented)

Particle spectrum



Massless particle

Pole residue:	$-\frac{1}{\underset{1}{\alpha.}}>0$
Polarisations:	2

Unitarity conditions

$\underset{1}{\alpha.}<0$

Assumptions

(Not yet implemented)

The output above makes sense. There are no mass terms in our Lagrangian, and hence no massive poles in the propagator. Instead, there are two massless eigenvalues which suggest that the vector part of the theory propagates two massless polarisations. The no-ghost condition of this massless vector simply demands that our kinetic coupling be negative: this is why in school we are told to put a -1/4 factor in front of the QED Lagrangian. What about the gauge constraints on the source currents? There is only one such constraint, which tells us that the positive-parity scalar part of the QED current (think the chiral current, or some such four-vector source) must vanish. Reverse-engineering this condition from momentum to position space, we see that the four-divergence of the source must vanish. Of course it must: this is just charge conservation. The conservation law is intimately connected to the gauge symmetries of the theory, according to Noether: these symmetries are manifest as singularities (zeroes) in the matrix form of the Lagrangian operator, though there are no spin-parity degeneracies in the 1-form and so all these matrices are just single elements.

Proca field (electroweak bosons)

Having investigated the massless theory, we keep the same kinetic setup but just add a mass term. This is of course the Proca theory, which finds a place higher up in the standard model.

$$\alpha_{\dot{3}}\,\mathcal{B}_{\alpha}\,\mathcal{B}^{\alpha} + \alpha_{\dot{1}}\left(\partial_{\alpha}\mathcal{B}_{\beta} - \partial_{\beta}\mathcal{B}_{\alpha}\right)\left(\partial^{\alpha}\mathcal{B}^{\beta} - \partial^{\beta}\mathcal{B}^{\alpha}\right) \tag{4}$$

PSALTer results panel

$$\mathcal{S} == \iiiint \left(\alpha_{\underset{3}{.}} \mathcal{B}_{\alpha} \mathcal{B}^{\alpha} + \mathcal{B}^{\alpha} \mathcal{J}_{\alpha} + 2 \alpha_{\underset{1}{.}} \left(-\partial_{\alpha}\mathcal{B}_{\beta} + \partial_{\beta}\mathcal{B}_{\alpha} \right) \partial^{\beta}\mathcal{B}^{\alpha} \right)[t, x, y, z] \, dz \, dy \, dx \, dt$$

Wave operator

$$\begin{array}{c|cc} & {}^{0^+}_{\cdot}\mathcal{B} & {}^{1^-}_{\cdot}\mathcal{B}_{\alpha} \\ \hline {}^{0^+}_{\cdot}\mathcal{B}^{\dagger} & \alpha_{\underset{3}{.}} & \\ {}^{1^-}_{\cdot}\mathcal{B}^{\dagger\alpha} & & \alpha_{\underset{3}{.}} + 2 \alpha_{\underset{1}{.}} k^2 \end{array}$$

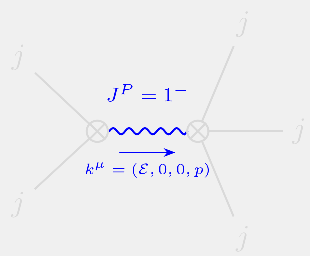
Saturated propagator

$$\begin{array}{c|cc} & {}^{0^+}_{\cdot}\mathcal{J} & {}^{1^-}_{\cdot}\mathcal{J}_{\alpha} \\ \hline {}^{0^+}_{\cdot}\mathcal{J}^{\dagger} & \frac{1}{\alpha_{\underset{3}{.}}} & \\ {}^{1^-}_{\cdot}\mathcal{J}^{\dagger\alpha} & & \frac{1}{\alpha_{\underset{3}{.}} + 2 \alpha_{\underset{1}{.}} k^2} \end{array}$$

Source constraints

(None)

Particle spectrum



Massive particle

Pole residue:	$-\frac{1}{2\alpha_{\underset{1}{.}}} > 0$
Square mass:	$-\frac{\alpha_{\underset{3}{.}}}{2\alpha_{\underset{1}{.}}} > 0$
Spin:	1
Parity:	Odd

Gauge symmetries

(Not yet implemented)

Unitarity conditions

$\alpha_{\underset{1}{.}} < 0 \,\&\&\, \alpha_{\underset{3}{.}} > 0$

Assumptions

(Not yet implemented)

Once again, the result makes sense. If you write out the Proca equation of motion and take the divergence, you see that the presence of the mass term restricts the 1-form to be divergence-free, which is another way of saying that the helicity-0 mode vanishes on shell. This is not a gauge condition (evidenced by the fact that the Lagrangian operator matrices are non-singular), but it does mean that in common with Maxwell's theory, we are stuck with the parity-odd vector mode. What is this mode doing? The theory is now massive, and so there is a massive pole in the propagator. There are now two unitarity conditions: the original no-ghost condition of QED and a new no-tachyon condition which protects the Proca mass from becoming imaginary.

Sickly quantum electrodynamics

Now let's try something a bit more ambitious. What if we didn't have the QED Lagrangian as inspiration, but we wanted to construct a general (and not necessarily gauge-invariant) 1-form theory? In the first instance, we'll take the case without any masses. Up to surface terms, there are two kinetic terms we could try which are consistent with the basic requirement of Lorentz invariance.

$$\underset{1}{\alpha}.\,\partial_\alpha \mathcal{B}_\beta\, \partial^\alpha \mathcal{B}^\beta + \underset{2}{\alpha}.\,\partial_\alpha \mathcal{B}^\alpha\, \partial_\beta \mathcal{B}^\beta \tag{5}$$

PSALTer results panel

$$\mathcal{S} == \iiiint \left(\mathcal{B}^\alpha\, \mathcal{J}_\alpha + \underset{2}{\alpha}.\,\partial_\alpha \mathcal{B}^\alpha\, \partial_\beta \mathcal{B}^\beta + \underset{1}{\alpha}.\,\partial_\beta \mathcal{B}_\alpha\, \partial^\beta \mathcal{B}^\alpha\right)[t, x, y, z]\, dz\, dy\, dx\, dt$$

Wave operator

$$\begin{array}{c|cc} & {}^{0^+}\mathcal{B} & {}^{1^-}\mathcal{B}_\alpha \\ \hline {}^{0^+}\mathcal{B}^\dagger & \left(\underset{1}{\alpha}. + \underset{2}{\alpha}.\right) k^2 & \\ {}^{1^-}\mathcal{B}^{\dagger\alpha} & & \underset{1}{\alpha}.\, k^2 \end{array}$$

Saturated propagator

$$\begin{array}{c|cc} & {}^{0^+}\mathcal{J} & {}^{1^-}\mathcal{J}_\alpha \\ \hline {}^{0^+}\mathcal{J}^\dagger & \frac{1}{\left(\underset{1}{\alpha}. + \underset{2}{\alpha}.\right) k^2} & \\ {}^{1^-}\mathcal{J}^{\dagger\alpha} & & \frac{1}{\underset{1}{\alpha}.\, k^2} \end{array}$$

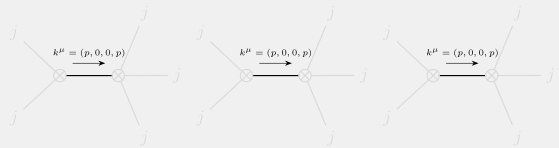
Source constraints

(None)

Gauge symmetries

(Not yet implemented)

Particle spectrum



Massless particle

Pole residue:	$-\frac{1}{\underset{1}{\alpha}.} - \frac{1}{\underset{1}{\alpha}. + \underset{2}{\alpha}.} > 0$
Polarisations	1

Massless particle

Pole residue:	$-\frac{1}{\underset{1}{\alpha}.} > 0$
Polarisations	2

Massless particle

Pole residue:	$\frac{1}{\underset{1}{\alpha}.} + \frac{1}{\underset{1}{\alpha}. + \underset{2}{\alpha}.} > 0$
Polarisations	1

Unitarity conditions

(Impossible)

Assumptions

(Not yet implemented)

Notice the suspicious appearance of two extra massless eigenvalues, alongside the familiar photon polarisations. These carry different signs, and thus cannot be positive-definite: the theory is immutably sick, and the no-ghost condition is simply `False'. What has happened here is a result of the Ostrogradsky theorem. Our kinetic structure has destroyed the gauge-invariance of the theory, and so the helicity-0 part of the field (the divergence of some scalar superpotential) has begun to move. Because the helicity-0 part contains an implicit divergence, that part of the theory now contains four implicit derivatives, and is a sickly higher-derivative model. The Ostrogradsky theorem says that derivative decoupling will bifurcate the helicity-0 mode into two modes, one of which is always a ghost. How to get rid of the ghost? We clearly can't do it at the level of the eigenvalues, so we look a few lines above to the Lagrangian matrix structure. The Scalar sector can be killed off entirely, spawning a singular one-element matrix and thus a new gauge symmetry, only by imposing the QED condition. This is of course just what we expect to find.

Sickly Proca field

For completeness, it behoves us to look at the general massive case.

$$\underset{3}{\alpha}.\,\mathcal{B}_\alpha\, \mathcal{B}^\alpha + \underset{1}{\alpha}.\,\partial_\alpha \mathcal{B}_\beta\, \partial^\alpha \mathcal{B}^\beta + \underset{2}{\alpha}.\,\partial_\alpha \mathcal{B}^\alpha\, \partial_\beta \mathcal{B}^\beta \tag{6}$$

PSALTer results panel

$$\mathcal{S} == \iiiint \left(\alpha_{\cdot 3}\, \mathcal{B}_{\alpha}\, \mathcal{B}^{\alpha} + \mathcal{B}^{\alpha}\, \mathcal{J}_{\alpha} + \alpha_{\cdot 2}\, \partial_{\alpha}\mathcal{B}^{\alpha}\, \partial_{\beta}\mathcal{B}^{\beta} + \alpha_{\cdot 1}\, \partial_{\beta}\mathcal{B}_{\alpha}\, \partial^{\beta}\mathcal{B}^{\alpha} \right)[t, x, y, z]\, dz\, dy\, dx\, dt$$

Wave operator

$$\begin{array}{c|cc} & {}^{0^+}_{\cdot}\mathcal{B} & {}^{1^-}_{\cdot}\mathcal{B}_{\alpha} \\ \hline {}^{0^+}_{\cdot}\mathcal{B}^{\dagger} & \alpha_{\cdot 3} + (\alpha_{\cdot 1} + \alpha_{\cdot 2})\, k^2 & \\ {}^{1^-}_{\cdot}\mathcal{B}^{\dagger\,\alpha} & & \alpha_{\cdot 3} + \alpha_{\cdot 1}\, k^2 \end{array}$$

Saturated propagator

$$\begin{array}{c|cc} & {}^{0^+}_{\cdot}\mathcal{J} & {}^{1^-}_{\cdot}\mathcal{J}_{\alpha} \\ \hline {}^{0^+}_{\cdot}\mathcal{J}^{\dagger} & \frac{1}{\alpha_{\cdot 3} + (\alpha_{\cdot 1} + \alpha_{\cdot 2})\, k^2} & \\ {}^{1^-}_{\cdot}\mathcal{J}^{\dagger\,\alpha} & & \frac{1}{\alpha_{\cdot 3} + \alpha_{\cdot 1}\, k^2} \end{array}$$

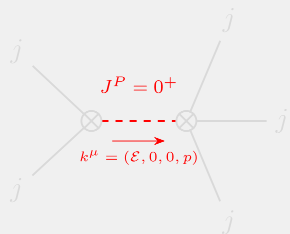
Source constraints

(None)

Gauge symmetries

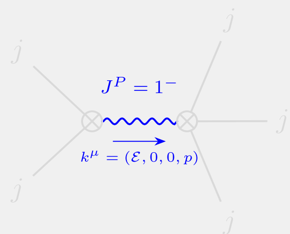
(Not yet implemented)

Particle spectrum



Massive particle

Pole residue:	$\frac{1}{\alpha_{\cdot 1} + \alpha_{\cdot 2}} > 0$
Square mass:	$-\left(\alpha_{\cdot 3} \big/ \left(\alpha_{\cdot 1} + \alpha_{\cdot 2}\right)\right) > 0$
Spin:	0
Parity:	Even



Massive particle

Pole residue:	$-\frac{1}{\alpha_{\cdot 1}} > 0$
Square mass:	$-\frac{\alpha_{\cdot 3}}{\alpha_{\cdot 1}} > 0$
Spin:	1
Parity:	Odd

Unitarity conditions

(Impossible)

Assumptions

(Not yet implemented)

Once again, the theory is sick in the helicity-0 sector. In case the massive parity-odd vector is unitary, then the helicity-0 mode must either be a ghost or a tachyon.

Tensor field theory

Key observation: We will test the TensorTheory module.

Fierz-Pauli (linear gravity)

The natural theory to check will be the Fierz-Pauli theory.

$$\alpha_{1}\left(-\partial^{\alpha}h_{\alpha\beta}\,\partial^{\beta}h^{\chi}{}_{\chi}+\frac{1}{2}\,\partial_{\beta}h^{\alpha}{}_{\alpha}\,\partial^{\beta}h^{\chi}{}_{\chi}-\frac{1}{2}\,\partial_{\chi}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\beta}+\partial_{\beta}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\chi}\right) \tag{7}$$

PSALTer results panel

$$\mathcal{S} == \iiiint\left(h^{\alpha\beta}\ \mathcal{T}_{\alpha\beta}+\tfrac{1}{2}\,\alpha_{1}\left(\partial_{\beta}h^{\chi}{}_{\chi}\,\partial^{\beta}h^{\alpha}{}_{\alpha}+2\,\partial_{\alpha}h^{\alpha\beta}\,\partial_{\chi}h_{\beta}{}^{\chi}-2\,\partial^{\beta}h^{\alpha}{}_{\alpha}\,\partial_{\chi}h_{\beta}{}^{\chi}-\partial_{\chi}h_{\alpha\beta}\,\partial^{\chi}h^{\alpha\beta}\right)\right)[t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt$$

Wave operator

$$\begin{array}{c|cc|c|c}
 & \overset{0^+}{\cdot}h^{\perp} & \overset{0^+}{\cdot}h^{\parallel} & \overset{1^-}{\cdot}h^{\perp}{}_{\alpha} & \overset{2^+}{\cdot}h^{\parallel}{}_{\alpha\beta} \\
\hline
\overset{0^+}{\cdot}h^{\perp}\,\dagger & 0 & 0 & & \\
\overset{0^+}{\cdot}h^{\parallel}\,\dagger & 0 & \alpha_{1}\,k^{2} & & \\
\hline
\overset{1^-}{\cdot}h^{\perp}\,\dagger^{\alpha} & & & 0 & \\
\hline
\overset{2^+}{\cdot}h^{\parallel}\,\dagger^{\alpha\beta} & & & & -\frac{\alpha_{1}\,k^{2}}{2}
\end{array}$$

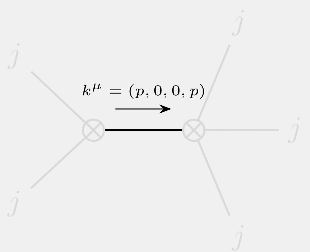
Saturated propagator

$$\begin{array}{c|cc|c|c}
 & \overset{0^+}{\cdot}\mathcal{T}^{\perp} & \overset{0^+}{\cdot}\mathcal{T}^{\parallel} & \overset{1^-}{\cdot}\mathcal{T}^{\perp}{}_{\alpha} & \overset{2^+}{\cdot}\mathcal{T}^{\parallel}{}_{\alpha\beta} \\
\hline
\overset{0^+}{\cdot}\mathcal{T}^{\perp}\,\dagger & 0 & 0 & & \\
\overset{0^+}{\cdot}\mathcal{T}^{\parallel}\,\dagger & 0 & \frac{1}{\alpha_{1}\,k^{2}} & & \\
\hline
\overset{1^-}{\cdot}\mathcal{T}^{\perp}\,\dagger^{\alpha} & & & 0 & \\
\hline
\overset{2^+}{\cdot}\mathcal{T}^{\parallel}\,\dagger^{\alpha\beta} & & & & -\frac{2}{\alpha_{1}\,k^{2}}
\end{array}$$

Source constraints

$\overset{0^+}{\cdot}\mathcal{T}^{\perp} == 0$	$\overset{1^-}{\cdot}\mathcal{T}^{\perp\alpha} == 0$

Particle spectrum



Massless particle

Pole residue:	$-\frac{p^{2}}{\alpha_{1}} > 0$
Polarisations:	2

Gauge symmetries

(Not yet implemented)

Unitarity conditions

$\alpha_{1} < 0$

Assumptions

(Not yet implemented)

The Fierz-Pauli theory thus propagates two massless polarisations, and the no-ghost condition is consistent with a positive Einstein or Newton-Cavendish constant, or a positive square Planck mass. The diffeomorphism invariance of the theory is manifest as a gauge symmetry, whose constraints on the source currents are commensurate with the conservation of the matter stress-energy tensor.

Massive gravity

We now include the unique mass term which corresponds to massive gravity, i.e. `Fierz-Pauli tuning'.

$$\alpha_{2}\left(h_{\alpha\beta}\ h^{\alpha\beta}-h^{\alpha}{}_{\alpha}\ \ h^{\beta}{}_{\beta}\right)+\alpha_{1}\left(-\partial^{\alpha}h_{\alpha\beta}\,\partial^{\beta}h^{\chi}{}_{\chi}+\frac{1}{2}\partial_{\beta}h^{\alpha}{}_{\alpha}\,\partial^{\beta}h^{\chi}{}_{\chi}-\frac{1}{2}\partial_{\chi}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\beta}+\partial_{\beta}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\chi}\right) \quad (8)$$

PSALTer results panel

$$\mathcal{S}==\iiiint\left(\alpha_{2}\left(h_{\alpha\beta}\ h^{\alpha\beta}-h^{\alpha}{}_{\alpha}\ \ h^{\beta}{}_{\beta}\right)+h^{\alpha\beta}\ \mathcal{T}_{\alpha\beta}+\tfrac{1}{2}\alpha_{1}\left(\partial_{\beta}h^{\chi}{}_{\chi}\,\partial^{\beta}h^{\alpha}{}_{\alpha}+2\,\partial_{\alpha}h^{\alpha\beta}\,\partial_{\chi}h_{\beta}{}^{\chi}-2\,\partial^{\beta}h^{\alpha}{}_{\alpha}\,\partial_{\chi}h_{\beta}{}^{\chi}-\partial_{\chi}h_{\alpha\beta}\,\partial^{\chi}h^{\alpha\beta}\right)\right)[t,x,y,z]\,dz\,dy\,dx\,dt$$

Wave operator

$$\begin{array}{c|cc}
 & {}^{0^+}_{\cdot}h^{\perp} & {}^{0^+}_{\cdot}h^{\parallel} \\
\hline
{}^{0^+}_{\cdot}h^{\perp}\dagger & 0 & -\sqrt{3}\,\alpha_{2} \\
{}^{0^+}_{\cdot}h^{\parallel}\dagger & -\sqrt{3}\,\alpha_{2} & -2\,\alpha_{2}+\alpha_{1}\,k^{2}
\end{array}
\qquad
\begin{array}{c|c}
 & {}^{1^-}_{\cdot}h^{\perp}{}_{\alpha} \\
\hline
{}^{1^-}_{\cdot}h^{\perp}\dagger^{\alpha} & \alpha_{2}
\end{array}
\qquad
\begin{array}{c|c}
 & {}^{2^+}_{\cdot}h^{\parallel}{}_{\alpha\beta} \\
\hline
{}^{2^+}_{\cdot}h^{\parallel}\dagger^{\alpha\beta} & \alpha_{2}-\frac{\alpha_{1}\,k^{2}}{2}
\end{array}$$

Saturated propagator

$$\begin{array}{c|cc}
 & {}^{0^+}_{\cdot}\mathcal{T}^{\perp} & {}^{0^+}_{\cdot}\mathcal{T}^{\parallel} \\
\hline
{}^{0^+}_{\cdot}\mathcal{T}^{\perp}\dagger & \frac{2\,\alpha_{2}-\alpha_{1}\,k^{2}}{3\,\alpha_{2}^{2}} & -\frac{1}{\sqrt{3}\,\alpha_{2}} \\
{}^{0^+}_{\cdot}\mathcal{T}^{\parallel}\dagger & -\frac{1}{\sqrt{3}\,\alpha_{2}} & 0
\end{array}
\qquad
\begin{array}{c|c}
 & {}^{1^-}_{\cdot}\mathcal{T}^{\perp}{}_{\alpha} \\
\hline
{}^{1^-}_{\cdot}\mathcal{T}^{\perp}\dagger^{\alpha} & \frac{1}{\alpha_{2}}
\end{array}
\qquad
\begin{array}{c|c}
 & {}^{2^+}_{\cdot}\mathcal{T}^{\parallel}{}_{\alpha\beta} \\
\hline
{}^{2^+}_{\cdot}\mathcal{T}^{\parallel}\dagger^{\alpha\beta} & \frac{1}{\alpha_{2}-\frac{\alpha_{1}\,k^{2}}{2}}
\end{array}$$

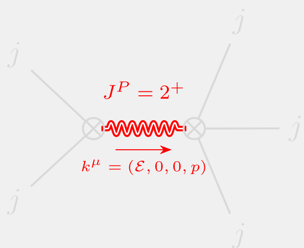
Source constraints

(None)

Gauge symmetries

(Not yet implemented)

Particle spectrum



Massive particle

Pole residue:	$-\frac{2}{\alpha_{1}}>0$
Square mass:	$\frac{2\,\alpha_{2}}{\alpha_{1}}>0$
Spin:	2
Parity:	Even

Unitarity conditions

$\alpha_{1}<0$ && $\alpha_{2}<0$

Assumptions

(Not yet implemented)

There is no massless sector. The propagator develops a massive pole in the positive-parity tensor sector. The no-ghost condition is as before, but now a no-tachyon condition protects the graviton mess.

Sick Fierz-Pauli (first variation)

Returning to the case without any mass terms, we should check that deviations to the Fierz-Pauli action are unacceptable. Let's vary the fourth term to some degree.

$$\alpha_{1}\left(-\partial^{\alpha}h_{\alpha\beta}\,\partial^{\beta}h^{\chi}{}_{\chi}+\frac{1}{2}\,\partial_{\beta}h^{\alpha}{}_{\alpha}\,\partial^{\beta}h^{\chi}{}_{\chi}-\frac{1}{2}\,\partial_{\chi}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\beta}\right)+\alpha_{2}\,\partial_{\beta}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\chi} \tag{9}$$

PSALTer results panel

$$\mathcal{S}==\iiiint\left(h^{\alpha\beta}\,\mathcal{T}_{\alpha\beta}+\alpha_{2}\,\partial_{\alpha}h^{\alpha\beta}\,\partial_{\chi}h_{\beta}{}^{\chi}+\frac{1}{2}\,\alpha_{1}\left(\partial_{\beta}h^{\chi}{}_{\chi}\,\partial^{\beta}h^{\alpha}{}_{\alpha}-2\,\partial^{\beta}h^{\alpha}{}_{\alpha}\,\partial_{\chi}h_{\beta}{}^{\chi}-\partial_{\chi}h_{\alpha\beta}\,\partial^{\chi}h^{\alpha\beta}\right)\right)[t,x,y,z]\,dz\,dy\,dx\,dt$$

Wave operator

$$\begin{array}{c|cc}
 & {}^{0^+}_{\bullet}h^{\perp} & {}^{0^+}_{\bullet}h^{\parallel} \\
\hline
{}^{0^+}_{\bullet}h^{\perp}\dagger & (-\alpha_{1}+\alpha_{2})k^2 & 0 \\
{}^{0^+}_{\bullet}h^{\parallel}\dagger & 0 & \alpha_{1}k^2
\end{array}$$

$$\begin{array}{c|c}
 & {}^{1^-}_{\bullet}h^{\perp}{}_{\alpha} \\
\hline
{}^{1^-}_{\bullet}h^{\perp}\dagger^{\alpha} & \frac{1}{2}(-\alpha_{1}+\alpha_{2})k^2
\end{array}
\qquad
\begin{array}{c|c}
 & {}^{2^+}_{\bullet}h^{\parallel}{}_{\alpha\beta} \\
\hline
{}^{2^+}_{\bullet}h^{\parallel}\dagger^{\alpha\beta} & -\frac{\alpha_{1}k^2}{2}
\end{array}$$

Saturated propagator

$$\begin{array}{c|cc}
 & {}^{0^+}_{\bullet}\mathcal{T}^{\perp} & {}^{0^+}_{\bullet}\mathcal{T}^{\parallel} \\
\hline
{}^{0^+}_{\bullet}\mathcal{T}^{\perp}\dagger & \frac{1}{(-\alpha_{1}+\alpha_{2})k^2} & 0 \\
{}^{0^+}_{\bullet}\mathcal{T}^{\parallel}\dagger & 0 & \frac{1}{\alpha_{1}k^2}
\end{array}$$

$$\begin{array}{c|c}
 & {}^{1^-}_{\bullet}\mathcal{T}^{\perp}{}_{\alpha} \\
\hline
{}^{1^-}_{\bullet}\mathcal{T}^{\perp}\dagger^{\alpha} & \frac{2}{(-\alpha_{1}+\alpha_{2})k^2}
\end{array}
\qquad
\begin{array}{c|c}
 & {}^{2^+}_{\bullet}\mathcal{T}^{\parallel}{}_{\alpha\beta} \\
\hline
{}^{2^+}_{\bullet}\mathcal{T}^{\parallel}\dagger^{\alpha\beta} & -\frac{2}{\alpha_{1}k^2}
\end{array}$$

Source constraints

(None)

Particle spectrum

Massless particle

Pole residue	$-\frac{p^2}{\alpha_{1}}>0$
Polarisations	2

Massless particle

Pole residue	$\frac{(-2\alpha_{1}+\alpha_{2})p^2}{\alpha_{1}(\alpha_{1}-\alpha_{2})}>0$
Polarisations	2

Massless particle

Pole residue	$\frac{(2\alpha_{1}-\alpha_{2})p^2}{\alpha_{1}(\alpha_{1}-\alpha_{2})}>0$
Polarisations	2

Massless particle

Pole residue	$\frac{(-6\alpha_{1}+\alpha_{2})p^2}{\alpha_{1}(\alpha_{1}-\alpha_{2})}>0$
Polarisations	1

Massless particle

Pole residue	$\frac{(6\alpha_{1}-\alpha_{2})p^2}{\alpha_{1}(\alpha_{1}-\alpha_{2})}>0$
Polarisations	1

Massless particle

Pole residue	$\frac{\left(-2\alpha_{1}+\alpha_{2}-\sqrt{20\alpha_{1}^2-36\alpha_{1}\alpha_{2}+17\alpha_{2}^2}\right)p^2}{\alpha_{1}(\alpha_{1}-\alpha_{2})}>0$
Polarisations	1

Massless particle

Pole residue	$\frac{\left(-2\alpha_{1}+\alpha_{2}+\sqrt{20\alpha_{1}^2-36\alpha_{1}\alpha_{2}+17\alpha_{2}^2}\right)p^2}{\alpha_{1}(\alpha_{1}-\alpha_{2})}>0$
Polarisations	1

Gauge symmetries

(Not yet implemented)

Unitarity conditions

(Impossible)

Assumptions

(Not yet implemented)

So this variation has no gauge symmetries, too many propagating species and no hope of unitarity.

Sick Fierz-Pauli (second variation)

This time let's wiggle the third term.

$$-\frac{1}{2}\alpha_{2}\,\partial_{\chi}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\beta}+\alpha_{1}\left(-\partial^{\alpha}h_{\alpha\beta}\,\partial^{\beta}h^{\chi}{}_{\chi}+\frac{1}{2}\partial_{\beta}h^{\alpha}{}_{\alpha}\,\partial^{\beta}h^{\chi}{}_{\chi}+\partial_{\beta}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\chi}\right) \quad (10)$$

PSALTer results panel

$$\mathcal{S}==\iiiint\left(h^{\alpha\beta}\,\mathcal{T}_{\alpha\beta}+\frac{1}{2}\alpha_{1}\,\partial_{\beta}h^{\chi}{}_{\chi}\,\partial^{\beta}h^{\alpha}{}_{\alpha}+\alpha_{1}\left(\partial_{\alpha}h^{\alpha\beta}-\partial^{\beta}h^{\alpha}{}_{\alpha}\right)\partial_{\chi}h_{\beta}{}^{\chi}-\frac{1}{2}\alpha_{2}\,\partial_{\chi}h_{\alpha\beta}\,\partial^{\chi}h^{\alpha\beta}\right)[t,x,y,z]\,dz\,dy\,dx\,dt$$

Wave operator

$$\begin{array}{c|cc} & {}^{0^+}h^{\perp} & {}^{0^+}h^{\parallel} \\ \hline {}^{0^+}h^{\perp}\dagger & \frac{1}{2}(\alpha_{1}-\alpha_{2})k^{2} & 0 \\ {}^{0^+}h^{\parallel}\dagger & 0 & \frac{1}{2}(3\alpha_{1}-\alpha_{2})k^{2} \end{array}\qquad \begin{array}{c|c} & {}^{1^-}h^{\perp}{}_{\alpha} \\ \hline {}^{1^-}h^{\perp}\dagger^{\alpha} & \frac{1}{2}(\alpha_{1}-\alpha_{2})k^{2} \end{array}\qquad \begin{array}{c|c} & {}^{2^+}h^{\parallel}{}_{\alpha\beta} \\ \hline {}^{2^+}h^{\parallel}\dagger^{\alpha\beta} & -\frac{\alpha_{2}k^{2}}{2} \end{array}$$

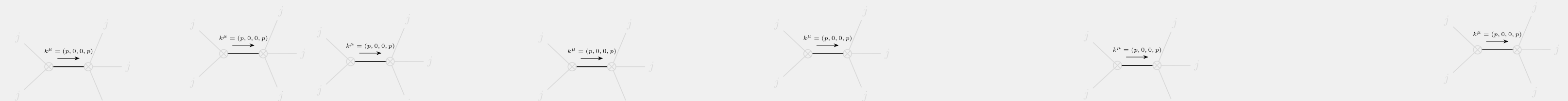
Saturated propagator

$$\begin{array}{c|cc} & {}^{0^+}\mathcal{T}^{\perp} & {}^{0^+}\mathcal{T}^{\parallel} \\ \hline {}^{0^+}\mathcal{T}^{\perp}\dagger & \frac{2}{(\alpha_{1}-\alpha_{2})k^{2}} & 0 \\ {}^{0^+}\mathcal{T}^{\parallel}\dagger & 0 & \frac{2}{(3\alpha_{1}-\alpha_{2})k^{2}} \end{array}\qquad \begin{array}{c|c} & {}^{1^-}\mathcal{T}^{\perp}{}_{\alpha} \\ \hline {}^{1^-}\mathcal{T}^{\perp}\dagger^{\alpha} & \frac{2}{(\alpha_{1}-\alpha_{2})k^{2}} \end{array}\qquad \begin{array}{c|c} & {}^{2^+}\mathcal{T}^{\parallel}{}_{\alpha\beta} \\ \hline {}^{2^+}\mathcal{T}^{\parallel}\dagger^{\alpha\beta} & -\frac{2}{\alpha_{2}k^{2}} \end{array}$$

Source constraints

(None)

Particle spectrum



Massless particle

Pole residue	$-\left(\left((\alpha_{1}-2\alpha_{2})p^{2}\right)/\left((\alpha_{1}-\alpha_{2})\alpha_{2}\right)\right)>0$
Polarisations	2

Massless particle

Pole residue	$\frac{(\alpha_{1}-2\alpha_{2})p^{2}}{(\alpha_{1}-\alpha_{2})\alpha_{2}}>0$
Polarisations	2

Massless particle

Pole residue	$-\frac{p^{2}}{\alpha_{2}}>0$
Polarisations	2

Massless particle

Pole residue	$-\left(\left((\alpha_{1}^{2}-6\alpha_{1}\alpha_{2}+2\alpha_{2}^{2})p^{2}\right)/\left((\alpha_{1}-\alpha_{2})(3\alpha_{1}-\alpha_{2})\alpha_{2}\right)\right)>0$
Polarisations	1

Massless particle

Pole residue	$\frac{(\alpha_{1}^{2}-6\alpha_{1}\alpha_{2}+2\alpha_{2}^{2})p^{2}}{(\alpha_{1}-\alpha_{2})(3\alpha_{1}-\alpha_{2})\alpha_{2}}>0$
Polarisations	1

Massless particle

Pole residue	$-\left(\left(\left(2\alpha_{1}^{2}-5\alpha_{1}\alpha_{2}+2\alpha_{2}^{2}+\sqrt{\alpha_{1}^{2}(4\alpha_{1}^{2}-8\alpha_{1}\alpha_{2}+5\alpha_{2}^{2})}\right)p^{2}\right)/\left((\alpha_{1}-\alpha_{2})(3\alpha_{1}-\alpha_{2})\alpha_{2}\right)\right)>0$
Polarisations	1

Massless particle

Pole residue	$\frac{\left(-2\alpha_{1}^{2}+5\alpha_{1}\alpha_{2}-2\alpha_{2}^{2}+\sqrt{\alpha_{1}^{2}(4\alpha_{1}^{2}-8\alpha_{1}\alpha_{2}+5\alpha_{2}^{2})}\right)p^{2}}{(\alpha_{1}-\alpha_{2})(3\alpha_{1}-\alpha_{2})\alpha_{2}}>0$
Polarisations	1

Gauge symmetries

(Not yet implemented)

Unitarity conditions

(Impossible)

Assumptions

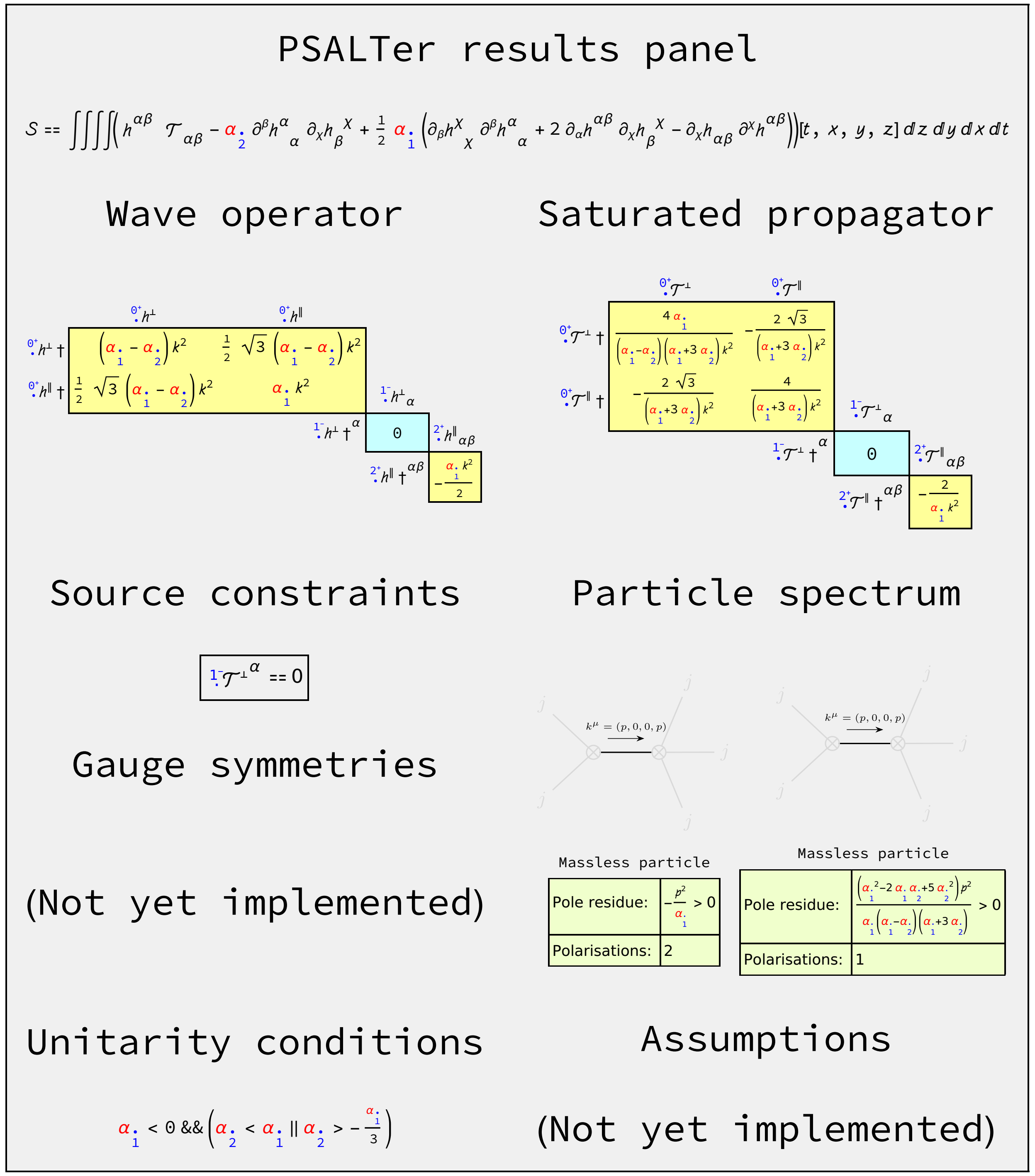
(Not yet implemented)

Again this variation has no gauge symmetries, too many propagating species and no hope of unitarity.

Sick Fierz-Pauli (third variation)

This time let's wiggle the second term.

$$-\alpha_{2}\,\partial^{\alpha}h_{\alpha\beta}\,\partial^{\beta}h^{\chi}{}_{\chi}+\alpha_{1}\left(\frac{1}{2}\partial_{\beta}h^{\alpha}{}_{\alpha}\,\partial^{\beta}h^{\chi}{}_{\chi}-\frac{1}{2}\partial_{\chi}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\beta}+\partial_{\beta}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\chi}\right) \quad (11)$$



This time we have what looks to be a viable theory with an extra massless scalar. However the diffeomorphism gauge symmetry has been lost, and the stress-energy tensor is not conserved.

Sick Fierz-Pauli (fourth variation)

This time let's wiggle the first term.

$$\frac{1}{2}\alpha_{2}\,\partial_{\beta}h^{\alpha}{}_{\alpha}\,\partial^{\beta}h^{\chi}{}_{\chi}+\alpha_{1}\left(-\partial^{\alpha}h_{\alpha\beta}\,\partial^{\beta}h^{\chi}{}_{\chi}-\frac{1}{2}\,\partial_{\chi}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\beta}+\partial_{\beta}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\chi}\right) \tag{12}$$

PSALTer results panel

$$\mathcal{S} == \iiiint \Big(h^{\alpha\beta}\, \mathcal{T}_{\alpha\beta} + \tfrac{1}{2}\,\alpha_{2}\,\partial_{\beta}h^{\chi}{}_{\chi}\,\partial^{\beta}h^{\alpha}{}_{\alpha} + \alpha_{1}\Big(\partial_{\alpha}h^{\alpha\beta}\,\partial_{\chi}h_{\beta}{}^{\chi} - \partial^{\beta}h^{\alpha}{}_{\alpha}\,\partial_{\chi}h_{\beta}{}^{\chi} - \tfrac{1}{2}\,\partial_{\chi}h_{\alpha\beta}\,\partial^{\chi}h^{\alpha\beta}\Big)\Big)[t, x, y, z]\, dz\, dy\, dx\, dt$$

Wave operator

$$\begin{array}{c|cc}
 & {}^{0^+}h^{\perp} & {}^{0^+}h^{\parallel} \\ \hline
{}^{0^+}h^{\perp}\dagger & \frac{1}{2}\left(-\alpha_{1}+\alpha_{2}\right)k^{2} & \frac{1}{2}\sqrt{3}\left(-\alpha_{1}+\alpha_{2}\right)k^{2} \\
{}^{0^+}h^{\parallel}\dagger & \frac{1}{2}\sqrt{3}\left(-\alpha_{1}+\alpha_{2}\right)k^{2} & -\frac{1}{2}\left(\alpha_{1}-3\,\alpha_{2}\right)k^{2}
\end{array}$$

$$\begin{array}{c|c}
 & {}^{1^-}h^{\perp}{}_{\alpha} \\ \hline
{}^{1^-}h^{\perp}\dagger^{\alpha} & 0
\end{array}
\qquad
\begin{array}{c|c}
 & {}^{2^+}h^{\parallel}{}_{\alpha\beta} \\ \hline
{}^{2^+}h^{\parallel}\dagger^{\alpha\beta} & -\frac{\alpha_{1}\,k^{2}}{2}
\end{array}$$

Saturated propagator

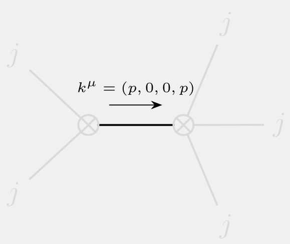
$$\begin{array}{c|cc}
 & {}^{0^+}\mathcal{T}^{\perp} & {}^{0^+}\mathcal{T}^{\parallel} \\ \hline
{}^{0^+}\mathcal{T}^{\perp}\dagger & \frac{\alpha_{1}-3\,\alpha_{2}}{\alpha_{1}\left(\alpha_{1}-\alpha_{2}\right)k^{2}} & -\frac{\sqrt{3}}{\alpha_{1}\,k^{2}} \\
{}^{0^+}\mathcal{T}^{\parallel}\dagger & -\frac{\sqrt{3}}{\alpha_{1}\,k^{2}} & \frac{1}{\alpha_{1}\,k^{2}}
\end{array}$$

$$\begin{array}{c|c}
 & {}^{1^-}\mathcal{T}^{\perp}{}_{\alpha} \\ \hline
{}^{1^-}\mathcal{T}^{\perp}\dagger^{\alpha} & 0
\end{array}
\qquad
\begin{array}{c|c}
 & {}^{2^+}\mathcal{T}^{\parallel}{}_{\alpha\beta} \\ \hline
{}^{2^+}\mathcal{T}^{\parallel}\dagger^{\alpha\beta} & -\frac{2}{\alpha_{1}\,k^{2}}
\end{array}$$

Source constraints

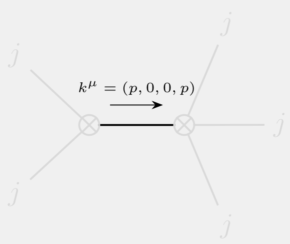
$\boxed{{}^{1^-}\mathcal{T}^{\perp\,\alpha} == 0}$

Particle spectrum



Massless particle

Pole residue:	$\frac{p^{2}}{-\alpha_{1}+\alpha_{2}} > 0$
Polarisations:	1



Massless particle

Pole residue:	$-\frac{p^{2}}{\alpha_{1}} > 0$
Polarisations:	2

Gauge symmetries

(Not yet implemented)

Unitarity conditions

$\alpha_{1} < 0\ \&\&\ \alpha_{2} > \alpha_{1}$

Assumptions

(Not yet implemented)

Another case with a partial gauge symmetry and an extra scalar mode.

Sick massive gravity

Finally, let's break the `Fierz-Pauli tuning'.

$$\alpha_{2}\,h_{\alpha\beta}\,h^{\alpha\beta} - \alpha_{3}\,h^{\alpha}{}_{\alpha}\,h^{\beta}{}_{\beta} + \alpha_{1}\left(-\partial^{\alpha}h_{\alpha\beta}\,\partial^{\beta}h^{\chi}{}_{\chi} + \frac{1}{2}\,\partial_{\beta}h^{\alpha}{}_{\alpha}\,\partial^{\beta}h^{\chi}{}_{\chi} - \frac{1}{2}\,\partial_{\chi}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\beta} + \partial_{\beta}h^{\alpha\beta}\,\partial^{\chi}h_{\alpha\chi}\right) \tag{13}$$

PSALTer results panel

$$\mathcal{S} == \iiiint\left(\alpha_{2}\, h_{\alpha\beta}\, h^{\alpha\beta} - \alpha_{3}\, h^{\alpha}{}_{\alpha}\, h^{\beta}{}_{\beta} + h^{\alpha\beta}\, \mathcal{T}_{\alpha\beta} + \tfrac{1}{2}\alpha_{1}\left(\partial_{\beta}h^{\chi}{}_{\chi}\, \partial^{\beta}h^{\alpha}{}_{\alpha} + 2\, \partial_{\alpha}h^{\alpha\beta}\, \partial_{\chi}h_{\beta}{}^{\chi} - 2\, \partial^{\beta}h^{\alpha}{}_{\alpha}\, \partial_{\chi}h_{\beta}{}^{\chi} - \partial_{\chi}h_{\alpha\beta}\, \partial^{\chi}h^{\alpha\beta}\right)\right)[t, x, y, z]\, dz\, dy\, dx\, dt$$

Wave operator

$$\begin{array}{c|cc|c|c}
 & {}^{0^+}h^{\perp} & {}^{0^+}h^{\parallel} & {}^{1^-}h^{\perp}{}_{\alpha} & {}^{2^+}h^{\parallel}{}_{\alpha\beta} \\
\hline
{}^{0^+}h^{\perp}\dagger & \alpha_{2} - \alpha_{3} & -\sqrt{3}\,\alpha_{3} & & \\
{}^{0^+}h^{\parallel}\dagger & -\sqrt{3}\,\alpha_{3} & \alpha_{2} - 3\,\alpha_{3} + \alpha_{1}\,k^{2} & & \\
\hline
{}^{1^-}h^{\perp}\dagger^{\alpha} & & & \alpha_{2} & \\
\hline
{}^{2^+}h^{\parallel}\dagger^{\alpha\beta} & & & & \alpha_{2} - \frac{\alpha_{1}\,k^{2}}{2}
\end{array}$$

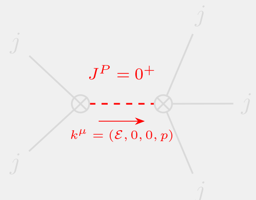
Saturated propagator

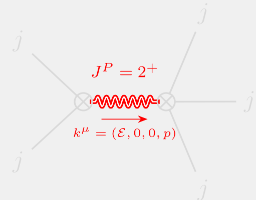
$$\begin{array}{c|cc|c|c}
 & {}^{0^+}\mathcal{T}^{\perp} & {}^{0^+}\mathcal{T}^{\parallel} & {}^{1^-}\mathcal{T}^{\perp}{}_{\alpha} & {}^{2^+}\mathcal{T}^{\parallel}{}_{\alpha\beta} \\
\hline
{}^{0^+}\mathcal{T}^{\perp}\dagger & \frac{1}{\alpha_{2}+\alpha_{3}\left(-1-\frac{3\,\alpha_{3}}{\alpha_{2}-3\,\alpha_{3}+\alpha_{1}\,k^{2}}\right)} & \frac{\sqrt{3}\,\alpha_{3}}{\alpha_{2}\left(\alpha_{2}-4\,\alpha_{3}\right)+\alpha_{1}\left(\alpha_{2}-\alpha_{3}\right)k^{2}} & & \\
{}^{0^+}\mathcal{T}^{\parallel}\dagger & \frac{\sqrt{3}\,\alpha_{3}}{\alpha_{2}\left(\alpha_{2}-4\,\alpha_{3}\right)+\alpha_{1}\left(\alpha_{2}-\alpha_{3}\right)k^{2}} & \frac{1}{\frac{\alpha_{2}\left(\alpha_{2}-4\,\alpha_{3}\right)}{\alpha_{2}-\alpha_{3}}+\alpha_{1}\,k^{2}} & & \\
\hline
{}^{1^-}\mathcal{T}^{\perp}\dagger^{\alpha} & & & \frac{1}{\alpha_{2}} & \\
\hline
{}^{2^+}\mathcal{T}^{\parallel}\dagger^{\alpha\beta} & & & & \frac{1}{\alpha_{2}-\frac{\alpha_{1}\,k^{2}}{2}}
\end{array}$$

Source constraints

(None)

Particle spectrum





Massive particle

Pole residue:	$\frac{\alpha_{2}^{2}-2\,\alpha_{2}\,\alpha_{3}+4\,\alpha_{3}^{2}}{\alpha_{1}\left(\alpha_{2}-\alpha_{3}\right)^{2}} > R$
ÀČḎÃĊẢ Ḇ ÃĎĎb	$-\frac{\alpha_{2}\left(\alpha_{2}-4\,\alpha_{3}\right)}{\alpha_{1}\left(\alpha_{2}-\alpha_{3}\right)} > 0$
Spin:	0
Parity:	Even

Massive particle

Pole residue:	$-\frac{2}{\alpha_{1}} > 0$
Square mass:	$\frac{2\,\alpha_{2}}{\alpha_{1}} > 0$
Spin:	2
Parity:	Even

Gauge symmetries

(Not yet implemented)

Unitarity conditions

(Impossible)

Assumptions

(Not yet implemented)

The consequence is seen in the positive-parity scalar sector, which develops a massive pole. This is the Boulware-Deser ghost, which always spoils the unitarity of the theory.

Poincaré gauge theory (PGT)

Key observation: We will test the PoincareGaugeTheory module.

Here is the inverse translational gauge field, or tetrad.

$$h_{\alpha}{}^{\chi} \tag{14}$$

Here is the translational gauge field, or inverse tetrad.

$$b^{\alpha}{}_{\chi} \tag{15}$$

Here is the Riemann-Cartan tensor.

$$\mathcal{A}^{\alpha\gamma}{}_{\phi}\,\mathcal{A}^{\beta}{}_{\gamma\chi}\,h_{\delta}{}^{\chi}\,h_{\epsilon}{}^{\phi} - \mathcal{A}^{\alpha\gamma}{}_{\chi}\,\mathcal{A}^{\beta}{}_{\gamma\phi}\,h_{\delta}{}^{\chi}\,h_{\epsilon}{}^{\phi} + h_{\delta}{}^{\chi}\,h_{\epsilon}{}^{\phi}\,\partial_{\chi}\mathcal{A}^{\alpha\beta}{}_{\phi} - h_{\delta}{}^{\chi}\,h_{\epsilon}{}^{\phi}\,\partial_{\phi}\mathcal{A}^{\alpha\beta}{}_{\chi} \tag{16}$$

Here is the torsion tensor.

$$\mathcal{A}^{\alpha}{}_{\chi\delta}\,h_{\beta}{}^{\delta} - \mathcal{A}^{\alpha}{}_{\beta\delta}\,h_{\chi}{}^{\delta} + h_{\beta}{}^{\delta}\,h_{\chi}{}^{\epsilon}\,\partial_{\delta}b^{\alpha}{}_{\epsilon} - h_{\beta}{}^{\delta}\,h_{\chi}{}^{\epsilon}\,\partial_{\epsilon}b^{\alpha}{}_{\delta} \tag{17}$$

Now we set up the general Lagrangian. In the first instance we will do this with some coupling constants which are proportional to those used by Hayashi and Shirafuji in Prog. Theor. Phys. 64 (1980) 2222. The normalisations are not absolutely identical, but this should not be a problem.

$$-\frac{1}{2}\alpha_{0}\,\eta^{\alpha\chi}\,\eta^{\beta\delta}\,\mathcal{R}_{\alpha\beta\chi\delta} + \left(\alpha_{1}\,\hat{\mathcal{P}}_{\mathcal{R}}1_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta} + \alpha_{2}\,\hat{\mathcal{P}}_{\mathcal{R}}2_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta} + \alpha_{3}\,\hat{\mathcal{P}}_{\mathcal{R}}3_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta} + \alpha_{4}\,\hat{\mathcal{P}}_{\mathcal{R}}4_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta} + \alpha_{5}\,\hat{\mathcal{P}}_{\mathcal{R}}5_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta} + \alpha_{6}\,\hat{\mathcal{P}}_{\mathcal{R}}6_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta}\right)\mathcal{R}_{\alpha\beta\chi\delta}\,\mathcal{R}^{\iota\theta\gamma\eta} + \left(\beta_{1}\,\hat{\mathcal{P}}_{\mathcal{T}}1_{\iota\gamma\eta}{}^{\alpha\chi\delta} + \beta_{2}\,\hat{\mathcal{P}}_{\mathcal{T}}2_{\iota\gamma\eta}{}^{\alpha\chi\delta} + \beta_{3}\,\hat{\mathcal{P}}_{\mathcal{T}}3_{\iota\gamma\eta}{}^{\alpha\chi\delta}\right)\mathcal{T}_{\alpha\chi\delta}\,\mathcal{T}^{\iota\gamma\eta} \tag{18}$$

In Eq. (18) we are using projectors to extract the Lorentz irreps of the fields. Next we will expand these.

So with the projectors expanded we have the following nonlinear Lagrangian.

$$-\frac{1}{2}\alpha_{0}\,\mathcal{R}^{\alpha\beta}{}_{\alpha\beta} + \frac{1}{6}\left(2\alpha_{1} + 3\alpha_{2} + \alpha_{3}\right)\mathcal{R}_{\alpha\beta\chi\delta}\,\mathcal{R}^{\alpha\beta\chi\delta} + \frac{2}{3}\left(\alpha_{1} - \alpha_{3}\right)\mathcal{R}_{\alpha\chi\beta\delta}\,\mathcal{R}^{\alpha\beta\chi\delta} + \left(-\alpha_{1} - \alpha_{2} + \alpha_{4} + \alpha_{5}\right)\mathcal{R}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{R}_{\beta}{}^{\delta}{}_{\chi\delta} + \frac{1}{6}\left(2\alpha_{1} - 3\alpha_{2} + \alpha_{3}\right)\mathcal{R}^{\alpha\beta\chi\delta}\,\mathcal{R}_{\chi\delta\alpha\beta} + \left(-\alpha_{1} + \alpha_{2} + \alpha_{4} - \alpha_{5}\right)\mathcal{R}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{R}_{\chi}{}^{\delta}{}_{\beta\delta} + \frac{1}{6}\left(2\alpha_{1} - 3\alpha_{4} + \alpha_{6}\right)\mathcal{R}^{\alpha\beta}{}_{\alpha\beta}\,\mathcal{R}^{\chi\delta}{}_{\chi\delta} + \frac{1}{3}\left(2\beta_{1} + \beta_{3}\right)\mathcal{T}_{\alpha\beta\chi}\,\mathcal{T}^{\alpha\beta\chi} + \frac{2}{3}\left(\beta_{1} - \beta_{3}\right)\mathcal{T}^{\alpha\beta\chi}\,\mathcal{T}_{\beta\alpha\chi} + \frac{2}{3}\left(\beta_{1} - \beta_{2}\right)\mathcal{T}^{\alpha}{}_{\alpha}{}^{\beta}\,\mathcal{T}^{\chi}{}_{\beta\chi} \tag{19}$$

We can also use a different set of coupling coefficients, as developed by Karananas.

$$-\lambda_{.}\,\mathcal{R}^{\iota\theta}{}_{\iota\theta} + \left(\frac{r_{.}}{3} + \frac{r_{.}}{6}\right)\mathcal{R}_{\iota\theta\kappa\lambda}\,\mathcal{R}^{\iota\theta\kappa\lambda} + \left(\frac{2r_{x}}{3} - \frac{2r_{y}}{3}\right)\mathcal{R}_{\iota\kappa\theta\lambda}\,\mathcal{R}^{\iota\theta\kappa\lambda} + \left(r_{4} + r_{5}\right)\mathcal{R}_{\iota}{}^{\lambda}{}_{\theta\lambda}\,\mathcal{R}^{\iota\kappa\theta}{}_{\kappa} + \left(r_{4} - r_{5}\right)\mathcal{R}^{\iota\kappa\theta}{}_{\kappa}\,\mathcal{R}_{\theta}{}^{\lambda}{}_{\iota\lambda} + \left(\frac{r_{1}}{3} + \frac{r_{2}}{6} - r_{3}\right)\mathcal{R}^{\iota\theta\kappa\lambda}\,\mathcal{R}_{\kappa\lambda\iota\theta} + \left(\frac{\lambda_{.}}{4} + \frac{t_{1}}{3} + \frac{t_{2}}{12}\right)\mathcal{T}_{\iota\theta\kappa}\,\mathcal{T}^{\iota\theta\kappa} + \left(-\frac{\lambda_{.}}{2} - \frac{t_{1}}{3} + \frac{t_{2}}{6}\right)\mathcal{T}^{\iota\theta\kappa}\,\mathcal{T}_{\theta\kappa\iota} + \left(-\lambda_{4/5} - \frac{t_{x}}{3} + \frac{2t_{z}}{3}\right)\mathcal{T}_{\iota}{}^{\theta\iota}\,\mathcal{T}_{\kappa\theta}{}^{\kappa} \tag{20}$$

Most general PGT

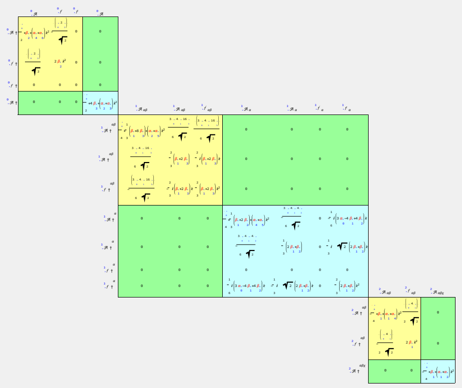
We first want to study the most general PGT. We will do this using the general coupling coefficients defined in Eq. (19).

$$-\frac{1}{2}\alpha_{0}\,\mathcal{R}^{\alpha\beta}{}_{\alpha\beta} + \frac{1}{6}\left(2\alpha_{1} + 3\alpha_{2} + \alpha_{3}\right)\mathcal{R}_{\alpha\beta\chi\delta}\,\mathcal{R}^{\alpha\beta\chi\delta} + \frac{2}{3}\left(\alpha_{1} - \alpha_{3}\right)\mathcal{R}_{\alpha\chi\beta\delta}\,\mathcal{R}^{\alpha\beta\chi\delta} + \left(-\alpha_{1} - \alpha_{2} + \alpha_{4} + \alpha_{5}\right)\mathcal{R}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{R}_{\beta}{}^{\delta}{}_{\chi\delta} + \frac{1}{6}\left(2\alpha_{1} - 3\alpha_{2} + \alpha_{3}\right)\mathcal{R}^{\alpha\beta\chi\delta}\,\mathcal{R}_{\chi\delta\alpha\beta} + \left(-\alpha_{1} + \alpha_{2} + \alpha_{4} - \alpha_{5}\right)\mathcal{R}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{R}_{\chi}{}^{\delta}{}_{\beta\delta} + \frac{1}{6}\left(2\alpha_{1} - 3\alpha_{4} + \alpha_{6}\right)\mathcal{R}^{\alpha\beta}{}_{\alpha\beta}\,\mathcal{R}^{\chi\delta}{}_{\chi\delta} + \frac{1}{3}\left(2\beta_{1} + \beta_{3}\right)\mathcal{T}_{\alpha\beta\chi}\,\mathcal{T}^{\alpha\beta\chi} + \frac{2}{3}\left(\beta_{1} - \beta_{3}\right)\mathcal{T}^{\alpha\beta\chi}\,\mathcal{T}_{\beta\alpha\chi} + \frac{2}{3}\left(\beta_{1} - \beta_{2}\right)\mathcal{T}^{\alpha}{}_{\alpha}{}^{\beta}\,\mathcal{T}^{\chi}{}_{\beta\chi} \tag{21}$$

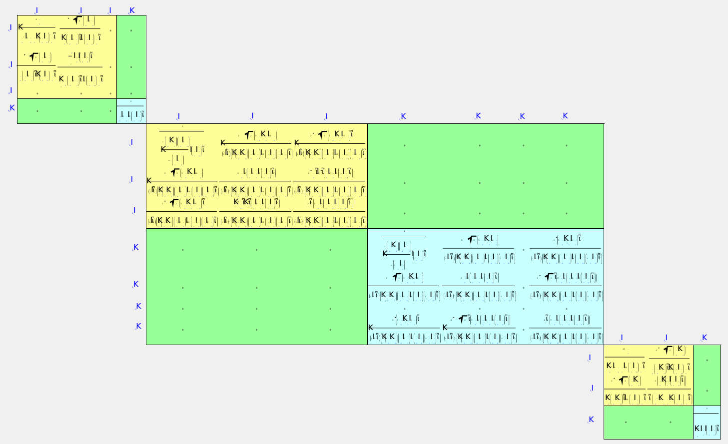
PSALTer results panel

$$
\begin{aligned}
\mathcal{S} == \iiiint \Bigg(\tfrac{1}{6} \Big(& -3\, \alpha_{0}\, \mathcal{A}^{\alpha\beta}{}_{\alpha}\, \mathcal{A}_{\beta}{}^{\chi}{}_{\chi} + 4\, \beta_{1}\, \mathcal{A}^{\alpha\beta}{}_{\alpha}\, \mathcal{A}_{\beta}{}^{\chi}{}_{\chi} - 4\, \beta_{2}\, \mathcal{A}^{\alpha\beta}{}_{\alpha}\, \mathcal{A}_{\beta}{}^{\chi}{}_{\chi} + 6\, \mathcal{A}^{\alpha\beta\chi}\, \sigma_{\alpha\beta\chi} + 6\, f^{\alpha\beta}\, \tau(\Delta+\mathcal{K})_{\alpha\beta} - 6\, \alpha_{0}\, f^{\alpha\beta}\, \partial_{\beta}\mathcal{A}_{\alpha}{}^{\chi}{}_{\chi} + 6\, \alpha_{0}\, \partial_{\beta}\mathcal{A}^{\alpha\beta}{}_{\alpha} - 8\, \beta_{1}\, \mathcal{A}_{\alpha}{}^{\chi}{}_{\chi}\, \partial_{\beta}f^{\alpha\beta} + 8\, \beta_{2}\, \mathcal{A}_{\alpha}{}^{\chi}{}_{\chi}\, \partial_{\beta}f^{\alpha\beta} + 8\, \beta_{1}\, \mathcal{A}_{\beta}{}^{\chi}{}_{\chi}\, \partial^{\beta}f^{\alpha}{}_{\alpha} - 8\, \beta_{2}\, \mathcal{A}_{\beta}{}^{\chi}{}_{\chi}\, \partial^{\beta}f^{\alpha}{}_{\alpha} - 4\, \beta_{1}\, \partial_{\beta}f^{\chi}{}_{\chi}\, \partial^{\beta}f^{\alpha}{}_{\alpha} + \\
& 4\, \beta_{2}\, \partial_{\beta}f^{\chi}{}_{\chi}\, \partial^{\beta}f^{\alpha}{}_{\alpha} + 6\, \alpha_{0}\, f^{\alpha\beta}\, \partial_{\chi}\mathcal{A}_{\alpha}{}^{\chi}{}_{\beta} - 6\, \alpha_{0}\, f^{\alpha}{}_{\alpha}\, \partial_{\chi}\mathcal{A}^{\beta\chi}{}_{\beta} - 4\, \beta_{1}\, \partial_{\beta}f^{\alpha\beta}\, \partial_{\chi}f_{\alpha}{}^{\chi} + 4\, \beta_{2}\, \partial_{\beta}f^{\alpha\beta}\, \partial_{\chi}f_{\alpha}{}^{\chi} + 8\, \beta_{1}\, \partial^{\beta}f^{\alpha}{}_{\alpha}\, \partial_{\chi}f_{\beta}{}^{\chi} - 8\, \beta_{2}\, \partial^{\beta}f^{\alpha}{}_{\alpha}\, \partial_{\chi}f_{\beta}{}^{\chi} + 6\, \alpha_{1}\, \partial_{\beta}\mathcal{A}_{\chi}{}^{\delta}{}_{\delta}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha} - 6\, \alpha_{2}\, \partial_{\beta}\mathcal{A}_{\chi}{}^{\delta}{}_{\delta}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha} - 6\, \alpha_{4}\, \partial_{\beta}\mathcal{A}_{\chi}{}^{\delta}{}_{\delta}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha} + 6\, \alpha_{5}\, \partial_{\beta}\mathcal{A}_{\chi}{}^{\delta}{}_{\delta}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha} + \\
& 6\, \alpha_{1}\, \partial_{\chi}\mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha} + 6\, \alpha_{2}\, \partial_{\chi}\mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha} - 6\, \alpha_{4}\, \partial_{\chi}\mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha} - 6\, \alpha_{5}\, \partial_{\chi}\mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha} + 8\, \beta_{1}\, \mathcal{A}_{\beta\chi\alpha}\, \partial^{\chi}f^{\alpha\beta} + 16\, \beta_{3}\, \mathcal{A}_{\beta\chi\alpha}\, \partial^{\chi}f^{\alpha\beta} - 8\, \beta_{1}\, \partial_{\alpha}f_{\beta\chi}\, \partial^{\chi}f^{\alpha\beta} + 8\, \beta_{3}\, \partial_{\alpha}f_{\beta\chi}\, \partial^{\chi}f^{\alpha\beta} - 8\, \beta_{1}\, \partial_{\alpha}f_{\chi\beta}\, \partial^{\chi}f^{\alpha\beta} - 4\, \beta_{3}\, \partial_{\alpha}f_{\chi\beta}\, \partial^{\chi}f^{\alpha\beta} + \\
& 4\, \beta_{1}\, \partial_{\beta}f_{\alpha\chi}\, \partial^{\chi}f^{\alpha\beta} - 4\, \beta_{3}\, \partial_{\beta}f_{\alpha\chi}\, \partial^{\chi}f^{\alpha\beta} + 8\, \beta_{1}\, \partial_{\chi}f_{\alpha\beta}\, \partial^{\chi}f^{\alpha\beta} + 4\, \beta_{3}\, \partial_{\chi}f_{\alpha\beta}\, \partial^{\chi}f^{\alpha\beta} + 4\, \beta_{1}\, \partial_{\chi}f_{\beta\alpha}\, \partial^{\chi}f^{\alpha\beta} - 4\, \beta_{3}\, \partial_{\chi}f_{\beta\alpha}\, \partial^{\chi}f^{\alpha\beta} + 4\left(\beta_{1} + 2\, \beta_{3}\right) \mathcal{A}_{\alpha\beta\chi} \left(\mathcal{A}^{\alpha\beta\chi} + 2\, \partial^{\chi}f^{\alpha\beta}\right) + \mathcal{A}_{\alpha\chi\beta} \left(\left(-3\, \alpha_{0} + 4\, \beta_{1} - 16\, \beta_{3}\right) \mathcal{A}^{\alpha\beta\chi} + 16\left(\beta_{1} - \beta_{3}\right) \partial^{\chi}f^{\alpha\beta}\right) + \\
& 6\, \alpha_{1}\, \partial_{\alpha}\mathcal{A}^{\alpha\beta\chi}\, \partial_{\delta}\mathcal{A}_{\beta}{}^{\delta}{}_{\chi} + 6\, \alpha_{2}\, \partial_{\alpha}\mathcal{A}^{\alpha\beta\chi}\, \partial_{\delta}\mathcal{A}_{\beta}{}^{\delta}{}_{\chi} - 6\, \alpha_{4}\, \partial_{\alpha}\mathcal{A}^{\alpha\beta\chi}\, \partial_{\delta}\mathcal{A}_{\beta}{}^{\delta}{}_{\chi} - 6\, \alpha_{5}\, \partial_{\alpha}\mathcal{A}^{\alpha\beta\chi}\, \partial_{\delta}\mathcal{A}_{\beta}{}^{\delta}{}_{\chi} - 12\, \alpha_{1}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}_{\beta}{}^{\delta}{}_{\chi} - 12\, \alpha_{2}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}_{\beta}{}^{\delta}{}_{\chi} + 12\, \alpha_{4}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}_{\beta}{}^{\delta}{}_{\chi} + 12\, \alpha_{5}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}_{\beta}{}^{\delta}{}_{\chi} + \\
& 6\, \alpha_{1}\, \partial_{\alpha}\mathcal{A}^{\alpha\beta\chi}\, \partial_{\delta}\mathcal{A}_{\chi}{}^{\delta}{}_{\beta} - 6\, \alpha_{2}\, \partial_{\alpha}\mathcal{A}^{\alpha\beta\chi}\, \partial_{\delta}\mathcal{A}_{\chi}{}^{\delta}{}_{\beta} - 6\, \alpha_{4}\, \partial_{\alpha}\mathcal{A}^{\alpha\beta\chi}\, \partial_{\delta}\mathcal{A}_{\chi}{}^{\delta}{}_{\beta} + 6\, \alpha_{5}\, \partial_{\alpha}\mathcal{A}^{\alpha\beta\chi}\, \partial_{\delta}\mathcal{A}_{\chi}{}^{\delta}{}_{\beta} - 12\, \alpha_{1}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}_{\chi}{}^{\delta}{}_{\beta} + 12\, \alpha_{2}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}_{\chi}{}^{\delta}{}_{\beta} + 12\, \alpha_{4}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}_{\chi}{}^{\delta}{}_{\beta} - 12\, \alpha_{5}\, \partial^{\chi}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}_{\chi}{}^{\delta}{}_{\beta} + 8\, \alpha_{1}\, \partial_{\beta}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}^{\chi\delta}{}_{\chi} - \\
& 12\, \alpha_{4}\, \partial_{\beta}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}^{\chi\delta}{}_{\chi} + 4\, \alpha_{6}\, \partial_{\beta}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}^{\chi\delta}{}_{\chi} - 8\, \alpha_{1}\, \partial_{\beta}\mathcal{A}_{\alpha\chi\delta}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} + 8\, \alpha_{3}\, \partial_{\beta}\mathcal{A}_{\alpha\chi\delta}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} + 4\, \alpha_{1}\, \partial_{\beta}\mathcal{A}_{\alpha\delta\chi}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} - 4\, \alpha_{3}\, \partial_{\beta}\mathcal{A}_{\alpha\delta\chi}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} + 8\, \alpha_{1}\, \partial_{\beta}\mathcal{A}_{\chi\delta\alpha}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} - 12\, \alpha_{2}\, \partial_{\beta}\mathcal{A}_{\chi\delta\alpha}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} + 4\, \alpha_{3}\, \partial_{\beta}\mathcal{A}_{\chi\delta\alpha}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} - \\
& 4\, \alpha_{1}\, \partial_{\chi}\mathcal{A}_{\alpha\beta\delta}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} - 6\, \alpha_{2}\, \partial_{\chi}\mathcal{A}_{\alpha\beta\delta}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} - 2\, \alpha_{3}\, \partial_{\chi}\mathcal{A}_{\alpha\beta\delta}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} + 4\, \alpha_{1}\, \partial_{\delta}\mathcal{A}_{\alpha\beta\chi}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} + 6\, \alpha_{2}\, \partial_{\delta}\mathcal{A}_{\alpha\beta\chi}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} + 2\, \alpha_{3}\, \partial_{\delta}\mathcal{A}_{\alpha\beta\chi}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} + 4\, \alpha_{1}\, \partial_{\delta}\mathcal{A}_{\alpha\chi\beta}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} - 4\, \alpha_{3}\, \partial_{\delta}\mathcal{A}_{\alpha\chi\beta}\, \partial^{\delta}\mathcal{A}^{\alpha\beta\chi} \Big) \Bigg)[t, x, y, z]\, dz\, dy\, dx\, dt
\end{aligned}
$$

Wave operator



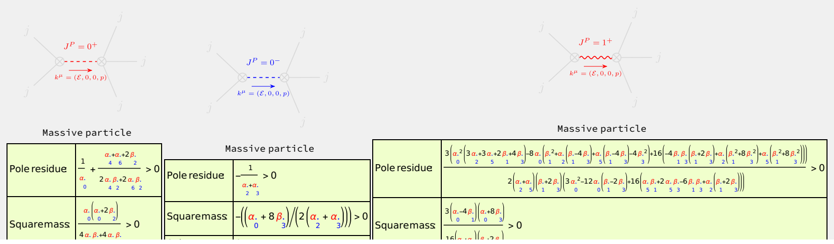
Saturated propagator



Source constraints

$0^{+}\tau^{\perp} == 0$	$2 i k\, 1^{-}\sigma^{\perp\alpha} + 1^{-}\tau^{\perp\alpha} == 0$
$1^{-}\tau^{\parallel\alpha} == 0$	$i k\, 1^{+}\sigma^{\perp\alpha\beta} + 1^{+}\tau^{\parallel\alpha\beta} == 0$

Particle spectrum



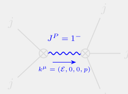
Gauge symmetries

Spin:	0
Parity:	Even

Spin:	0
Parity:	Odd

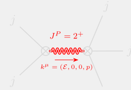
Spin:	1
Parity:	Even

(Not yet implemented)



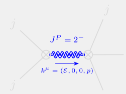
Massive particle

Pole residue:	$-\left(\left(3\left(\alpha_{0}^2\left(3\alpha_{4}+3\alpha_{5}+4\beta_{1}+2\beta_{2}\right)+4\alpha_{0}\left(-2\alpha_{4}\beta_{1}-2\alpha_{5}\beta_{1}-4\beta_{1}^2+2\alpha_{4}\beta_{2}+2\alpha_{5}\beta_{2}+\beta_{2}^2\right)+8\left(-2\beta_{1}\beta_{2}\left(2\beta_{1}+\beta_{2}\right)+\alpha_{4}\left(2\beta_{1}^2+\beta_{2}^2\right)+\alpha_{5}\left(2\beta_{1}^2+\beta_{2}^2\right)\right)\right)\right)\Big/\left(2\left(\alpha_{4}+\alpha_{5}\right)\left(2\beta_{1}+\beta_{2}\right)\left(3\alpha_{0}^2+6\alpha_{0}\left(-2\beta_{1}+\beta_{2}\right)+4\left(2\alpha_{5}\beta_{1}+\alpha_{5}\beta_{2}-6\beta_{1}\beta_{2}+\alpha_{4}\left(2\beta_{1}+\beta_{2}\right)\right)\right)\right)\right)>0$
Square mass:	$\frac{3\left(\alpha_{0}-4\beta_{1}\right)\left(\alpha_{0}+2\beta_{2}\right)}{8\left(\alpha_{4}+\alpha_{5}\right)\left(2\beta_{1}+\beta_{2}\right)}>0$
Spin:	1
Parity:	Odd



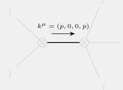
Massive particle

Pole residue:	$-\frac{2}{\alpha_{0}}+\frac{\alpha_{1}+\alpha_{4}+2\beta_{1}}{2\alpha_{1}\beta_{1}+2\alpha_{4}\beta_{1}}>0$
Square mass:	$\frac{\alpha_{0}\left(\alpha_{0}-4\beta_{1}\right)}{16\left(\alpha_{1}+\alpha_{4}\right)\beta_{1}}>0$
Spin:	2
Parity:	Even



Massive particle

Pole residue:	$-\left(1\Big/\left(\alpha_{1}+\alpha_{2}\right)\right)>0$
Square mass:	$\frac{\alpha_{0}-4\beta_{1}}{4\left(\alpha_{1}+\alpha_{2}\right)}>0$
Spin:	2
Parity:	Odd



Massless particle

Pole residue:	$\frac{p^2}{\alpha_{0}}>0$
Polarisations:	2

Unitarity conditions

(Impossible)

Assumptions

(Not yet implemented)

Key observation: These results should be compared with the Hayashi and Shirafuji papers, in particular Eqs. (4.11) in Prog. Theor. Phys. 64 (1980) 2222.

Einstein-Cartan theory (ECT)

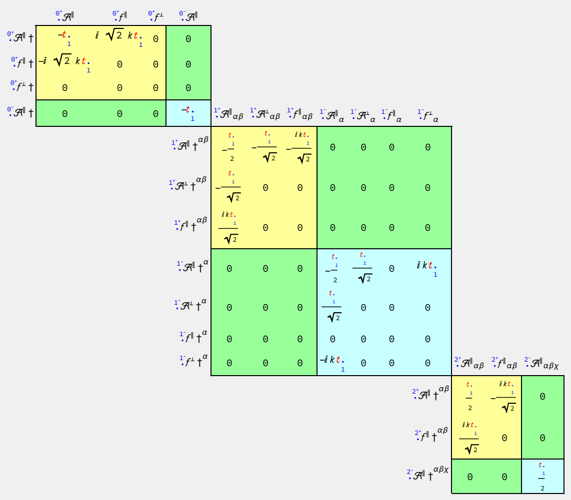
Now we would like to check the basic Einstein-Cartan theory. Here is the full nonlinear Lagrangian:

$$t.\,\mathcal{R}^{\iota\theta}{}_{\iota\theta} \qquad (22)$$

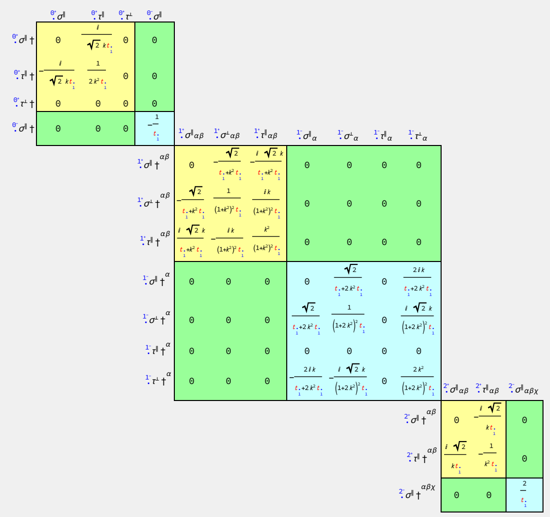
PSALTer results panel

$$S = \iiiint \left(\mathcal{A}^{\alpha\beta\chi}\ \sigma_{\alpha\beta\chi} + f^{\alpha\beta}\ \tau(\Delta+\mathcal{K})_{\alpha\beta} + t_{1} \left(\mathcal{A}_{\iota\zeta\theta}\ \mathcal{A}^{\iota\theta\zeta} + \mathcal{A}^{\iota\theta}{}_{\iota}\ \mathcal{A}_{\theta}{}^{\zeta}{}_{\zeta} + 2 f^{\iota\theta}\ \partial_{\theta}\mathcal{A}_{\iota}{}^{\zeta}{}_{\zeta} - 2\,\partial_{\theta}\mathcal{A}^{\iota\theta}{}_{\iota} - 2 f^{\iota\theta}\ \partial_{\zeta}\mathcal{A}_{\iota}{}^{\zeta}{}_{\theta} + 2 f^{\iota}{}_{\iota}\ \partial_{\zeta}\mathcal{A}^{\theta\zeta}{}_{\theta} \right)\right)[t, x, y, z]\, dz\, dy\, dx\, dt$$

Wave operator



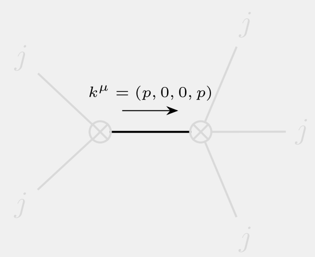
Saturated propagator



Source constraints

$0^{+}\cdot\tau^{\perp} == 0$	$2\,i\,k\ 1^{-}\cdot\sigma^{\perp}{}^{\alpha} + 1^{-}\cdot\tau^{\perp}{}^{\alpha} == 0$
$1^{-}\cdot\tau^{\parallel}{}^{\alpha} == 0$	$i\,k\ 1^{+}\cdot\sigma^{\perp}{}^{\alpha\beta} + 1^{+}\cdot\tau^{\parallel}{}^{\alpha\beta} == 0$

Particle spectrum



Massless particle

Pole residue:	$-\frac{p^2}{t_{1}} > 0$
Polarisations:	2

Gauge symmetries

(Not yet implemented)

Unitarity conditions

$t_{1} < 0$

Assumptions

(Not yet implemented)

Okay, so that is the end of the PSALTer output for Einstein-Cartan gravity. What we find are no propagating massive modes, but instead two degrees of freedom in the massive sector. The no-ghost conditions on these massless d.o.f restrict the sign in front of the Einstein-Hilbert term to be negative (which is what we expect for our conventions).

General relativity (GR)

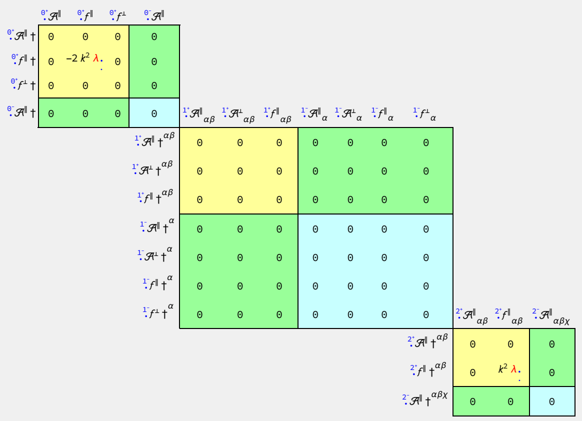
Using Karananas' coefficients, it is particularly easy to also look at GR, instead of Einstein-Cartan theory. The difference here is that the quadratic torsion coefficients are manually removed. Here is the nonlinear Lagrangian:

$$-\lambda.\,\mathcal{R}^{\iota\theta}{}_{\iota\theta}+\frac{1}{4}\lambda.\,\mathcal{T}_{\iota\theta\kappa}\,\mathcal{T}^{\iota\theta\kappa}+\frac{1}{2}\lambda.\,\mathcal{T}^{\iota\theta\kappa}\,\mathcal{T}_{\theta\iota\kappa}+\lambda.\,\mathcal{T}^{\iota}{}_{\iota}{}^{\theta}\,\mathcal{T}^{\kappa}{}_{\theta\kappa} \quad (23)$$

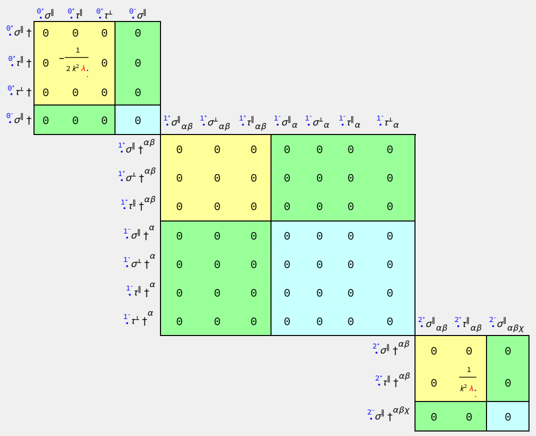
PSALTer results panel

$$\mathcal{S} == \iiiint \Big(\mathcal{A}^{\alpha\beta\chi}\ \sigma_{\alpha\beta\chi} + f^{\alpha\beta}\ \tau(\Delta+\mathcal{K})_{\alpha\beta} + \tfrac{1}{2}\lambda_{.}\Big(4\,\partial_{\iota}\mathcal{A}^{\alpha\iota}{}_{\alpha} - 4\ \mathcal{A}_{\alpha}{}^{\theta}{}_{\theta}\ \partial_{\iota}f^{\alpha\iota} + 4\ \mathcal{A}_{\iota}{}^{\theta}{}_{\theta}\ \partial^{\iota}f^{\alpha}{}_{\alpha} - 2\,\partial_{\iota}f^{\theta}{}_{\theta}\ \partial^{\iota}f^{\alpha}{}_{\alpha} - 4\ f^{\alpha\iota}\left(\partial_{\iota}\mathcal{A}_{\alpha}{}^{\theta}{}_{\theta} - \partial_{\theta}\mathcal{A}_{\alpha}{}^{\theta}{}_{\iota}\right) - 4\ f^{\alpha}{}_{\alpha}\ \partial_{\theta}\mathcal{A}^{\iota\theta}{}_{\iota} - 2\,\partial_{\iota}f^{\alpha\iota}\ \partial_{\theta}f_{\alpha}{}^{\theta} + 4\,\partial^{\iota}f^{\alpha}{}_{\alpha}\ \partial_{\theta}f_{\iota}{}^{\theta} + 4\ \mathcal{A}_{\alpha\theta\iota}\ \partial^{\theta}f^{\alpha\iota} - 2\,\partial_{\alpha}f_{\iota\theta}\ \partial^{\theta}f^{\alpha\iota} - \partial_{\alpha}f_{\theta\iota}\ \partial^{\theta}f^{\alpha\iota} + \partial_{\iota}f_{\alpha\theta}\ \partial^{\theta}f^{\alpha\iota} + \partial_{\theta}f_{\alpha\iota}\ \partial^{\theta}f^{\alpha\iota} + \partial_{\theta}f_{\iota\alpha}\ \partial^{\theta}f^{\alpha\iota}\Big)\Big)[t,\ x,\ y,\ z]\,dz\,dy\,dx\,dt$$

Wave operator



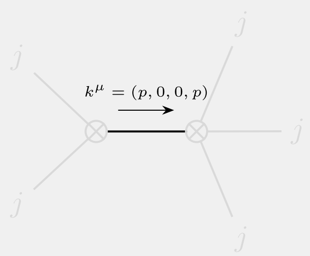
Saturated propagator



Source constraints

${}^{0^-}_{\cdot}\sigma^{\parallel} == 0$	${}^{0^+}_{\cdot}\tau^{\perp} == 0$
${}^{0^+}_{\cdot}\sigma^{\parallel} == 0$	${}^{1^-}_{\cdot}\tau^{\perp}{}^{\alpha} == 0$
${}^{1^-}_{\cdot}\tau^{\parallel}{}^{\alpha} == 0$	${}^{1^-}_{\cdot}\sigma^{\perp}{}^{\alpha} == 0$
${}^{1^-}_{\cdot}\sigma^{\parallel}{}^{\alpha} == 0$	${}^{1^+}_{\cdot}\tau^{\parallel}{}^{\alpha\beta} == 0$
${}^{1^+}_{\cdot}\sigma^{\perp}{}^{\alpha\beta} == 0$	${}^{1^+}_{\cdot}\sigma^{\parallel}{}^{\alpha\beta} == 0$
${}^{2^-}_{\cdot}\sigma^{\parallel}{}^{\alpha\beta\chi} == 0$	${}^{2^+}_{\cdot}\sigma^{\parallel}{}^{\alpha\beta} == 0$

Particle spectrum



Massless particle

Pole residue:	$\frac{p^2}{\lambda_{.}} > 0$
Polarisations:	2

Gauge symmetries

(Not yet implemented)

Unitarity conditions

$\lambda_{.} > 0$

Assumptions

(Not yet implemented)

Thus, the conclusions are the same, as expected.

Key observation: We have now reached the end of the PSALTer calibration script.