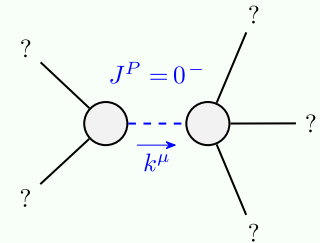# Particle spectrograph

Wave operator and propagator

Quadratic (free) action

$S_F ==$

$$\iiint \Big(\tfrac{1}{6}\big(4t_2\,\omega_i{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}'^i + 2t_2\,\omega_{\kappa\lambda}{}'^i\,\omega^{\kappa\lambda}{}_i + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 6\,r_3\,\partial_i\omega^{\kappa\lambda}{}_\kappa$$
$$\partial^l\omega_{\lambda}{}^\alpha{}_\alpha + 4\,r_2\,\partial^\beta\omega^{\theta\alpha}{}_\kappa\,\partial_\kappa\omega_{\alpha\beta}{}^\kappa - 2\,r_2\,\partial_\theta\omega_{\alpha\beta}{}^\kappa\,\partial_\kappa\omega^{\kappa\lambda\theta} +$$
$$18\,r_3\,\partial_\alpha\omega_\lambda{}^\theta{}_\theta\,\partial_\kappa\omega^{\theta\kappa\lambda} - 18\,r_3\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial_\kappa\omega^{\kappa\lambda\theta} +$$
$$12\,r_3\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial_\kappa\omega^{\kappa\lambda\theta} + t_2\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f_\alpha{}^\theta - t_2\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f^\lambda{}_\alpha + t_2\,\partial^\alpha f^\lambda{}_\kappa\,\partial^\kappa f'^\theta -$$
$$2\,t_2\,\omega_{\theta\kappa}\,\partial^\kappa f'^\theta - 4\,t_2\,\omega_{\iota\kappa\theta}\,\partial^\kappa f'^{l\theta} - 2\,t_2\,\omega_{\theta\iota\kappa}\,\partial^\kappa f'^{l\theta} + 4\,t_2\,\omega_{\theta\kappa\iota}\,\partial^\kappa\omega_{\alpha\beta\theta} +$$
$$t_2\,\partial^\alpha f_{\kappa}{}^\lambda\,\partial^\kappa f_{\lambda\alpha} - t_2\,\partial_\theta f_{\theta}{}^\lambda\,\partial^\kappa f_\lambda{}^\theta + t_2\,\partial_\kappa f^\lambda{}_\theta\,\partial^\kappa f_\lambda{}^\theta + 2\,r_2\,\partial_\kappa\omega^{\alpha\beta\theta}\,\partial^\kappa\omega_{\alpha\beta\theta} +$$
$$4\,r_2\,\partial_\kappa\omega^{\theta\alpha\beta}\,\omega_{\alpha\beta\theta} - 4\,r_2\,\partial^\beta\omega_{\lambda}{}^{\alpha\lambda}\,\partial^\lambda\omega^\beta{}_\iota{}^\lambda + 4\,r_2\,\partial^\beta\omega_{\iota}{}^{\lambda\alpha}\,\partial^\lambda\omega_{\lambda}{}^\alpha{}_{\alpha\beta} - 24\,r_3\,\partial^\beta\omega_{\iota}{}^{\lambda\alpha}$$
$$\partial_\lambda\omega_{\alpha\beta}{}' - 18\,r_3\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial^\lambda\omega_{\theta}{}^{\theta\kappa}{}_\kappa + 18\,r_3\,\partial_\alpha\omega_\lambda{}^\alpha{}_\kappa\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa)\big)[t,x,y,z]\,dz\,dy\,dx\,dt$$

Matrix (0^+, 0^- sector)

	$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^-}^{\#1}$
$\sigma_{0^+}^{\#1}\dagger$	$\frac{1}{6\,k^2 r_3}$	0	0	0
$\tau_{0^+}^{\#1}\dagger$	0	0	0	0
$\tau_{0^+}^{\#2}\dagger$	0	0	0	0
$\sigma_{0^-}^{\#1}\dagger$	0	0	0	$\frac{1}{k^2 r_2 + t_2}$

Matrix (ω, f sector)

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\omega_{0^-}^{\#1}$
$\omega_{0^+}^{\#1}\dagger$	$6\,k^2 r_3$	0	0	0
$f_{0^+}^{\#1}\dagger$	0	0	0	0
$f_{0^+}^{\#2}\dagger$	0	0	0	0
$\omega_{0^-}^{\#1}\dagger$	0	0	0	$k^2 r_2 + t_2$

Spin-1 matrix (σ, τ)

	$\sigma_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\sigma_{1^+}^{\#2}\dagger^{\alpha\beta}$	$\tau_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\sigma_{1^-}^{\#1}\dagger^\alpha$	$\sigma_{1^-}^{\#2}\dagger^\alpha$	$\tau_{1^-}^{\#1}\dagger^\alpha$	$\tau_{1^-}^{\#2}\dagger^\alpha$
$\tau_{1^-}^{\#2}{}_\alpha$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#1}{}_\alpha$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#2}{}_\alpha$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#1}{}_\alpha$	0	0	0	$\frac{1}{k^2 r_3}$	0	0	0
$\tau_{1^+}^{\#1}{}_{\alpha\beta}$	$-\frac{i}{k\,r_3 + k^3 r_3}$	$\frac{i(3k^2 r_3 + 2t_2)}{k(1+k^2)^2 r_3 t_2}$	$\frac{3k^2 r_3 + 2t_2}{(1+k^2)^2 r_3 t_2}$	0	0	0	0
$\sigma_{1^+}^{\#2}{}_{\alpha\beta}$	$-\frac{\sqrt{2}}{k^2 r_3 + k^4 r_3}$	$\frac{3k^2 r_3 + 2t_2}{(k+k^3)^2 r_3 t_2}$	$-\frac{i(3k^2 r_3 + 2t_2)}{k(1+k^2)^2 r_3 t_2}$	0	0	0	0
$\sigma_{1^+}^{\#1}{}_{\alpha\beta}$	$\frac{1}{k^2 r_3}$	$-\frac{\sqrt{2}}{k^2 r_3 + k^4 r_3}$	$\frac{i\sqrt{2}}{k\,r_3 + k^3 r_3}$	0	0	0	0

Spin-1 matrix (ω, f)

	$\omega_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\omega_{1^+}^{\#2}\dagger^{\alpha\beta}$	$f_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\omega_{1^-}^{\#1}\dagger^\alpha$	$\omega_{1^-}^{\#2}\dagger^\alpha$	$f_{1^-}^{\#1}\dagger^\alpha$	$f_{1^-}^{\#2}\dagger^\alpha$
$f_{1^-}^{\#2}{}_\alpha$	0	0	0	0	0	0	0
$f_{1^-}^{\#1}{}_\alpha$	0	0	0	0	0	0	0
$\omega_{1^-}^{\#2}{}_\alpha$	0	0	0	0	0	0	0
$\omega_{1^-}^{\#1}{}_\alpha$	0	0	0	$k^2 r_3$	0	0	0
$f_{1^+}^{\#1}{}_{\alpha\beta}$	$\frac{1}{3} i \sqrt{2}\,k\,t_2$	$\frac{i\,k\,t_2}{3}$	$\frac{k^2 t_2}{3}$	0	0	0	0
$\omega_{1^+}^{\#2}{}_{\alpha\beta}$	$\frac{\sqrt{2}\,t_2}{3}$	$\frac{t_2}{3}$	$-\frac{1}{3} i\,k\,t_2$	0	0	0	0
$\omega_{1^+}^{\#1}{}_{\alpha\beta}$	$k^2 r_3 + \frac{2 t_2}{3}$	$\frac{\sqrt{2}\,t_2}{3}$	$-\frac{1}{3} i \sqrt{2}\,k\,t_2$	0	0	0	0

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau_{0^+}^{\#2} == 0$	1
$\tau_{0^+}^{\#1} == 0$	1
$\tau_{1^-}^{\#2\,\alpha} == 0$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	3
$\sigma_{1^-}^{\#2\,\alpha} == 0$	3
$\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$	3
$\sigma_{2^-}^{\#1\,\alpha\beta\chi} == 0$	5
$\tau_{2^+}^{\#1\,\alpha\beta} == 0$	5
$\sigma_{2^+}^{\#1\,\alpha\beta} == 0$	5
Total constraints:	29

Spin-2 matrix (σ, τ)

	$\sigma_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\tau_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\sigma_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$
$\sigma_{2^+}^{\#1}{}_{\alpha\beta}$	0	0	0
$\tau_{2^+}^{\#1}{}_{\alpha\beta}$	0	0	0
$\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$	0	0	0

Spin-2 matrix (ω, f)

	$\omega_{2^+}^{\#1}\dagger^{\alpha\beta}$	$f_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\omega_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$
$\omega_{2^+}^{\#1}{}_{\alpha\beta}$	0	0	0
$f_{2^+}^{\#1}{}_{\alpha\beta}$	0	0	0
$\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$	0	0	0

Massive and massless spectra

Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_2 < 0$ && $t_2 > 0$