

# Particle spectrograph

## Wave operator and propagator

	$\sigma_{1^+}^{\#1} \dagger \alpha\beta$	$\sigma_{1^+}^{\#2} \dagger \alpha\beta$	$\tau_{1^+}^{\#1} \dagger \alpha\beta$	$\sigma_{1^+}^{\#1} \dagger \alpha$	$\sigma_{1^+}^{\#2} \dagger \alpha$	$\tau_{1^+}^{\#1} \dagger \alpha$	$\tau_{1^+}^{\#2} \dagger \alpha$
$\sigma_{1^+}^{\#1} \dagger \alpha\beta$	0	$-\frac{\sqrt{2}}{t_1+k^2}t_1$	$-\frac{i\sqrt{2}k}{t_1+k^2}t_1$	0	0	0	0
$\sigma_{1^+}^{\#2} \dagger \alpha\beta$	$-\frac{\sqrt{2}}{t_1+k^2}t_1$	$-\frac{2k^2r_5+t_1}{(1+k^2)^2}t_1^2$	$-\frac{i(2k^3r_5-kt_1)}{(1+k^2)^2}t_1^2$	0	0	0	0
$\tau_{1^+}^{\#1} \dagger \alpha\beta$	$\frac{i\sqrt{2}k}{t_1+k^2}t_1$	$\frac{i(2k^3r_5-kt_1)}{(1+k^2)^2}t_1^2$	$\frac{-2k^4r_5+k^2t_1}{(1+k^2)^2}t_1^2$	0	0	0	0
$\sigma_{1^+}^{\#1} \dagger \alpha$	0	0	0	$\frac{2(t_1+t_3)}{3t_1t_3+2k^2r_5(t_1+t_3)}$	$-\frac{\sqrt{2}(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{2ik(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{2ik^2r_5+t_1+4t_3}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$
$\sigma_{1^+}^{\#2} \dagger \alpha$	0	0	0	$-\frac{\sqrt{2}(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{6k^2r_5+t_1+4t_3}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	$\frac{i\sqrt{2}k(6k^2r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	$\frac{2k^2(6k^2r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$
$\tau_{1^+}^{\#1} \dagger \alpha$	0	0	0	0	0	0	0
$\tau_{1^+}^{\#2} \dagger \alpha$	0	0	0	$-\frac{2ik(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2r_5(t_1+t_3))}$	$-\frac{i\sqrt{2}k(6k^2r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2r_5(t_1+t_3))}$	0	0

Quadratic (free) action

$$S_F = \iiint \left( \frac{1}{6} (-2(t_1-2t_3)) \omega_{\lambda'}^{\alpha'} \omega_{\kappa\alpha}^{\kappa} - 6t_1 \omega_{\kappa\lambda'}^{\kappa\lambda} \omega_{\alpha\beta}^{\alpha\beta} \tau_{\alpha\beta} + 6 \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} - \right. \\ \left. 6r_5 \partial_{\lambda} \omega_{\kappa\lambda}^{\kappa\lambda} \partial' \omega_{\lambda}^{\alpha} + 4r_2 \partial^{\beta} \omega_{\alpha}^{\beta\alpha} + 4r_2 \partial_{\theta} \omega_{\alpha\beta}^{\beta\alpha} - 2r_2 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\alpha\beta\theta} - \right. \\ \left. 4r_2 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\theta\alpha\beta} - 6r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\theta\kappa\lambda} + 6r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\theta\kappa\lambda} - \right. \\ \left. 6r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\kappa\lambda\theta} + 12r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega_{\lambda}^{\kappa\lambda\theta} - 3t_1 \partial^{\alpha} f_{\theta\kappa}^{\kappa} \partial_{\alpha}^{\kappa} f_{\alpha}^{\theta} - 3t_1 \partial^{\alpha} f_{\kappa\theta}^{\theta} \partial_{\alpha}^{\kappa} f_{\alpha}^{\theta} - \right. \\ \left. 3t_1 \partial^{\alpha} f_{\lambda}^{\lambda} \partial_{\kappa}^{\kappa} f_{\alpha\lambda}^{\alpha} + 2t_1 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f_{\lambda'}^{\alpha} - 4t_3 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f_{\lambda'}^{\alpha} + 2t_1 \omega_{\kappa\lambda}^{\lambda} \partial^{\kappa} f_{\lambda'}^{\alpha} - \right. \\ \left. 4t_3 \omega_{\kappa\lambda}^{\lambda} \partial^{\kappa} f_{\lambda'}^{\alpha} + 4t_1 \partial^{\alpha} f_{\kappa\alpha}^{\kappa} \partial_{\alpha}^{\kappa} f_{\lambda'}^{\alpha} - 8t_3 \partial^{\alpha} f_{\kappa\alpha}^{\kappa} \partial_{\alpha}^{\kappa} f_{\lambda'}^{\alpha} - 2t_1 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda'}^{\alpha} + \right. \\ \left. 4t_3 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda'}^{\alpha} + 12t_1 \omega_{\kappa\theta}^{\theta} \partial^{\kappa} f_{\lambda}^{\theta} - 2t_1 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f_{\lambda}^{\alpha} + 4t_3 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f_{\lambda}^{\alpha} - \right. \\ \left. 2t_1 \omega_{\lambda\alpha}^{\alpha} \partial^{\kappa} f_{\kappa}^{\alpha} + 4t_3 \omega_{\lambda\alpha}^{\alpha} \partial^{\kappa} f_{\kappa}^{\alpha} + 3t_1 \partial^{\alpha} f_{\lambda}^{\lambda} \partial_{\kappa} f_{\theta}^{\theta} + 3t_1 \partial_{\kappa} f_{\theta}^{\theta} \partial^{\alpha} f_{\lambda}^{\lambda} - 2t_1 \partial^{\alpha} f_{\lambda}^{\lambda} \partial_{\kappa} f_{\theta}^{\theta} + \right. \\ \left. 4r_2 \partial_{\kappa} \omega^{\theta\alpha\beta} \partial^{\kappa} \omega_{\alpha\beta\theta} - 4r_2 \partial^{\beta} \omega_{\lambda'}^{\alpha\lambda} \partial_{\lambda} \omega_{\alpha\beta}^{\alpha} + 4r_2 \partial^{\beta} \omega_{\lambda'}^{\lambda\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\alpha} + \right. \\ \left. 6r_5 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\theta\kappa}^{\theta\kappa} - 6r_5 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\alpha}^{\theta\kappa} \right) [t, x, y, z] dz dy dx dt$$

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\tau_{0^+}^{\#2} == 0$	1
$\tau_{0^+}^{\#1} - 2ik\sigma_{0^+}^{\#1} == 0$	1
$\tau_{1^+}^{\#2\alpha} + 2ik\sigma_{1^+}^{\#2\alpha} == 0$	3
$\tau_{1^+}^{\#1\alpha} == 0$	3
$\tau_{1^+}^{\#1\alpha\beta} + ik\sigma_{1^+}^{\#2\alpha\beta} == 0$	3
$\tau_{2^+}^{\#1\alpha\beta} - 2ik\sigma_{2^+}^{\#1\alpha\beta} == 0$	5
Total constraints:	16

$\sigma_{2^+}^{\#1} \dagger \alpha\beta$	$\tau_{2^+}^{\#1} \dagger \alpha\beta$	$\sigma_{2^+}^{\#1} \dagger \alpha\beta\chi$
$\frac{2}{(1+2k^2)^2}t_1$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$	0
$\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$	$\frac{4k^2}{(1+2k^2)^2}t_1$	0
0	0	$\frac{2}{t_1}$

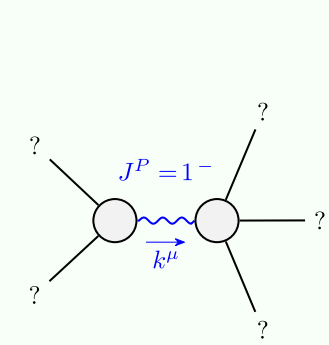
$\sigma_{0^+}^{\#1} \dagger$	$\tau_{0^+}^{\#1} \dagger$	$\tau_{0^+}^{\#2} \dagger$	$\sigma_{0^+}^{\#1} \dagger$
$\frac{1}{(1+2k^2)^2}t_3$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_3$	0	0
$\frac{i\sqrt{2}k}{(1+2k^2)^2}t_3$	$\frac{2k^2}{(1+2k^2)^2}t_3$	0	0
0	0	0	0
0	0	0	$\frac{1}{k^2r_2-t_1}$

$\omega_{2^+}^{\#1} \dagger \alpha\beta$	$f_{2^+}^{\#1} \dagger \alpha\beta$	$\omega_{2^+}^{\#1} \dagger \alpha\beta\chi$
$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$\frac{ikt_1}{\sqrt{2}}$	$k^2t_1$	0
$\omega_{2^+}^{\#1} \dagger \alpha\beta\chi$	0	$\frac{t_1}{2}$

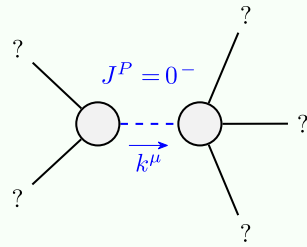
	$\omega_{1^+}^{\#1} \dagger \alpha\beta$	$\omega_{1^+}^{\#2} \dagger \alpha\beta$	$f_{1^+}^{\#1} \dagger \alpha\beta$	$\omega_{1^+}^{\#1} \dagger \alpha$	$\omega_{1^+}^{\#2} \dagger \alpha$	$f_{1^+}^{\#1} \dagger \alpha$	$f_{1^+}^{\#2} \dagger \alpha$
$\omega_{1^+}^{\#1} \dagger \alpha\beta$	$k^2r_5 - \frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0
$\omega_{1^+}^{\#2} \dagger \alpha\beta$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_{1^+}^{\#1} \dagger \alpha\beta$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_{1^+}^{\#1} \dagger \alpha$	0	0	0	$\frac{1}{6}(6k^2r_5+t_1+4t_3)$	$\frac{t_1-2t_3}{3\sqrt{2}}$	0	$\frac{1}{3}ik(t_1-2t_3)$
$\omega_{1^+}^{\#2} \dagger \alpha$	0	0	0	$\frac{t_1-2t_3}{3\sqrt{2}}$	$\frac{t_1+t_3}{3}$	0	$\frac{1}{3}i\sqrt{2}k(t_1+t_3)$
$f_{1^+}^{\#1} \dagger \alpha$	0	0	0	0	0	0	0
$f_{1^+}^{\#2} \dagger \alpha$	0	0	0	$-\frac{1}{3}ik(t_1-2t_3)$	$-\frac{1}{3}i\sqrt{2}k(t_1+t_3)$	0	$\frac{2}{3}k^2(t_1+t_3)$

$\omega_{0^+}^{\#1} \dagger$	$f_{0^+}^{\#1} \dagger$	$f_{0^+}^{\#2} \dagger$	$\omega_{0^+}^{\#1} \dagger$
$t_3$	$-i\sqrt{2}kt_3$	0	0
$i\sqrt{2}kt_3$	$2k^2t_3$	0	0
0	0	0	0
0	0	0	$k^2r_2-t_1$

## Massive and massless spectra



Massive particle	
Pole residue:	$\frac{6t_1t_3(t_1+t_3)-3r_5(t_1^2+2t_3^2)}{2r_5(t_1+t_3)(-3t_1t_3+r_5(t_1+t_3))} > 0$
Polarisations:	3
Square mass:	$-\frac{3t_1t_3}{2r_5t_1+2r_5t_3} > 0$
Spin:	1
Parity:	Odd



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$\frac{t_1}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

## Unitarity conditions

$$r_2 < 0 \ \&\& \ r_5 < 0 \ \&\& \ t_1 < 0 \ \&\& \ 0 < t_3 < -t_1$$