

# Particle spectrograph

## Wave operator and propagator

$\Delta_{1^+}^{\#1} \quad \Delta_{1^+}^{\#2} \quad \Delta_{1^+}^{\#3} \quad \Delta_{1^+}^{\#1} \alpha \quad \Delta_{1^+}^{\#2} \alpha \quad \Delta_{1^+}^{\#3} \alpha \quad \Delta_{1^+}^{\#4} \alpha \quad \Delta_{1^+}^{\#5} \alpha \quad \Delta_{1^+}^{\#6} \alpha \quad \mathcal{T}_{1^+}^{\#1} \alpha$	$\Delta_{1^+}^{\#1} \alpha \beta$	$\Delta_{1^+}^{\#2} \alpha \beta$	$\Delta_{1^+}^{\#3} \alpha \beta$	$\Delta_{1^+}^{\#1} \alpha$	$\Delta_{1^+}^{\#2} \alpha$	$\Delta_{1^+}^{\#3} \alpha$	$\Delta_{1^+}^{\#4} \alpha$	$\Delta_{1^+}^{\#5} \alpha$	$\Delta_{1^+}^{\#6} \alpha$	$\mathcal{T}_{1^+}^{\#1} \alpha$
$\Delta_{1^+}^{\#1} \dagger \alpha \beta$	0	$-\frac{2\sqrt{2}}{a_0}$	0	0	0	0	0	0	0	0
$\Delta_{1^+}^{\#2} \dagger \alpha \beta$	$-\frac{2\sqrt{2}}{a_0}$	$\frac{2}{a_0}$	0	0	0	0	0	0	0	0
$\Delta_{1^+}^{\#3} \dagger \alpha \beta$	0	0	$\frac{4}{a_0}$	0	0	0	0	0	0	0
$\Delta_{1^+}^{\#1} \dagger \alpha$	0	0	0	0	$\frac{2\sqrt{2}}{a_0}$	0	0	0	0	0
$\Delta_{1^+}^{\#2} \dagger \alpha$	0	0	0	$\frac{2\sqrt{2}}{a_0}$	$\frac{2}{a_0}$	0	0	0	0	0
$\Delta_{1^+}^{\#3} \dagger \alpha$	0	0	0	0	0	$-\frac{19}{12a_0}$	$-\frac{5\sqrt{5}}{12a_0}$	$-\frac{1}{6\sqrt{2}a_0}$	$-\frac{1}{6a_0}$	0
$\Delta_{1^+}^{\#4} \dagger \alpha$	0	0	0	0	0	$\frac{5\sqrt{5}}{12a_0}$	$\frac{1}{12a_0}$	$-\frac{\sqrt{5}}{6a_0}$	$-\frac{\sqrt{5}}{6a_0}$	0
$\Delta_{1^+}^{\#5} \dagger \alpha$	0	0	0	0	0	$-\frac{1}{6\sqrt{2}a_0}$	$-\frac{\sqrt{5}}{6a_0}$	$-\frac{7}{3\sqrt{2}a_0}$	$-\frac{7}{3\sqrt{2}a_0}$	0
$\Delta_{1^+}^{\#6} \dagger \alpha$	0	0	0	0	0	$-\frac{1}{6a_0}$	$-\frac{\sqrt{5}}{6a_0}$	$-\frac{7}{3\sqrt{2}a_0}$	$\frac{5}{3a_0}$	0
$\mathcal{T}_{1^+}^{\#1} \dagger \alpha$	0	0	0	0	0	0	0	0	0	0

$\Gamma_{1^+}^{\#1} \quad \Gamma_{1^+}^{\#2} \quad \Gamma_{1^+}^{\#3} \quad \Gamma_{1^+}^{\#1} \alpha \beta \quad \Gamma_{1^+}^{\#2} \alpha \beta \quad \Gamma_{1^+}^{\#3} \alpha \beta \quad \Gamma_{1^+}^{\#1} \alpha \quad \Gamma_{1^+}^{\#2} \alpha \quad \Gamma_{1^+}^{\#3} \alpha \quad \Gamma_{1^+}^{\#4} \alpha \quad \Gamma_{1^+}^{\#5} \alpha \quad \Gamma_{1^+}^{\#6} \alpha \quad h_{1^+}^{\#1} \alpha$	$\Gamma_{1^+}^{\#1} \alpha \beta$	$\Gamma_{1^+}^{\#2} \alpha \beta$	$\Gamma_{1^+}^{\#3} \alpha \beta$	$\Gamma_{1^+}^{\#1} \alpha$	$\Gamma_{1^+}^{\#2} \alpha$	$\Gamma_{1^+}^{\#3} \alpha$	$\Gamma_{1^+}^{\#4} \alpha$	$\Gamma_{1^+}^{\#5} \alpha$	$\Gamma_{1^+}^{\#6} \alpha$	$h_{1^+}^{\#1} \alpha$
$\Gamma_{1^+}^{\#1} \dagger \alpha \beta$	$-\frac{a_0}{4}$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0	0	0
$\Gamma_{1^+}^{\#2} \dagger \alpha \beta$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0	0	0	0
$\Gamma_{1^+}^{\#3} \dagger \alpha \beta$	0	0	$\frac{a_0}{4}$	0	0	0	0	0	0	0
$\Gamma_{1^+}^{\#1} \dagger \alpha$	0	0	0	$-\frac{a_0}{4}$	$\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0
$\Gamma_{1^+}^{\#2} \dagger \alpha$	0	0	0	$\frac{a_0}{2\sqrt{2}}$	0	0	0	0	0	0
$\Gamma_{1^+}^{\#3} \dagger \alpha$	0	0	0	0	0	$-\frac{a_0}{3}$	$\frac{\sqrt{5}a_0}{6}$	$-\frac{a_0}{6\sqrt{2}}$	$-\frac{a_0}{6}$	0
$\Gamma_{1^+}^{\#4} \dagger \alpha$	0	0	0	0	0	$\frac{\sqrt{5}a_0}{6}$	$\frac{a_0}{3}$	$-\frac{1}{6}\sqrt{\frac{5}{2}}a_0$	$-\frac{\sqrt{5}a_0}{6}$	0
$\Gamma_{1^+}^{\#5} \dagger \alpha$	0	0	0	0	0	$-\frac{a_0}{6\sqrt{2}}$	$-\frac{1}{6}\sqrt{\frac{5}{2}}a_0$	$\frac{a_0}{3}$	$\frac{a_0}{6\sqrt{2}}$	0
$\Gamma_{1^+}^{\#6} \dagger \alpha$	0	0	0	0	0	$-\frac{a_0}{6}$	$-\frac{\sqrt{5}a_0}{6}$	$\frac{a_0}{12}$	$\frac{5a_0}{12}$	0
$h_{1^+}^{\#1} \dagger \alpha$	0	0	0	0	0	0	0	0	0	0

Quadratic (free) Lagrangian density

$$\begin{aligned} &-\frac{1}{2}a_0\Gamma^{\alpha\beta\chi}\Gamma_{\beta\chi\alpha}+\frac{1}{2}a_0\Gamma_{\alpha}^{\alpha\beta}\Gamma_{\beta\chi}^{\chi}+h^{\alpha\beta}\mathcal{T}_{\alpha\beta}+\Gamma^{\alpha\beta\chi}\Delta_{\alpha\beta\chi}^- \\ &\frac{1}{2}a_0\Gamma^{\alpha\beta\chi}\partial_{\beta}h_{\alpha\chi}-\frac{1}{4}a_0\Gamma_{\alpha}^{\alpha\beta}\partial_{\beta}h^{\chi}+\frac{1}{4}a_0\Gamma_{\alpha}^{\alpha\beta}\partial_{\beta}h^{\chi}-\frac{1}{4}a_0h^{\chi}\partial_{\beta}\Gamma_{\chi}^{\alpha\beta}+\frac{1}{4}a_0h^{\chi}\partial_{\beta}\Gamma_{\chi}^{\alpha\beta}-\frac{1}{2}a_0h_{\alpha\chi}\partial_{\beta}\Gamma^{\alpha\beta\chi}+\frac{1}{2}a_0h^{\alpha\beta}\partial_{\beta}h_{\alpha}^{\chi}-\frac{1}{8}a_0\partial_{\beta}h^{\chi}\partial_{\chi}h^{\alpha}+\frac{1}{2}a_0\Gamma_{\alpha}^{\alpha\beta}\partial_{\chi}h_{\beta}^{\chi}-\frac{1}{2}a_0\partial_{\alpha}h^{\alpha\beta}\partial_{\chi}h_{\beta}^{\chi}+\frac{1}{2}a_0\partial_{\beta}h_{\alpha}^{\chi}\partial_{\chi}h^{\alpha\beta}-\frac{1}{4}a_0h^{\alpha}\partial_{\chi}h_{\alpha}^{\beta}\partial_{\beta}h^{\chi}+\frac{1}{4}a_0\partial_{\beta}h_{\alpha\chi}\partial^{\chi}h^{\alpha\beta}+\frac{3}{8}a_0\partial_{\chi}h_{\alpha\beta}\partial^{\chi}h^{\alpha\beta}+\frac{1}{2}a_0h_{\beta\chi}\partial^{\chi}\Gamma_{\alpha}^{\alpha\beta} \end{aligned}$$

$\Delta_3^{\#1} \dagger \alpha \beta \chi$ 

$-\frac{2}{a_0}$

$\Gamma_3^{\#1} \dagger \alpha \beta \chi$ 

$-\frac{a_0}{2}$

$\Delta_{0^+}^{\#1} \quad \Delta_{0^+}^{\#2} \quad \Delta_{0^+}^{\#3} \quad \Delta_{0^+}^{\#4} \quad \mathcal{T}_{0^+}^{\#1} \quad \mathcal{T}_{0^+}^{\#2} \quad \Delta_{0^+}^{\#1} \alpha$	$\Delta_{0^+}^{\#1} \alpha \beta$	$\Delta_{0^+}^{\#2} \alpha \beta$	$\Delta_{0^+}^{\#3} \alpha \beta$	$\Delta_{0^+}^{\#4} \alpha \beta$	$\mathcal{T}_{0^+}^{\#1} \alpha$	$\mathcal{T}_{0^+}^{\#2} \alpha$
$\Delta_{0^+}^{\#1} \dagger$	$-\frac{2}{a_0}$	0	0	0	0	0
$\Delta_{0^+}^{\#2} \dagger$	0	$-\frac{3}{4a_0}$	$\frac{5}{4a_0}$	$-\frac{1}{2\sqrt{2}a_0}$	0	0
$\Delta_{0^+}^{\#3} \dagger$	0	$\frac{5}{4a_0}$	$-\frac{3}{4a_0}$	$-\frac{1}{2\sqrt{2}a_0}$	0	0
$\Delta_{0^+}^{\#4} \dagger$	0	$-\frac{1}{2\sqrt{2}a_0}$	$-\frac{1}{2\sqrt{2}a_0}$	$\frac{1}{2a_0}$	0	0
$\mathcal{T}_{0^+}^{\#1} \dagger$	0	0	0	0	$\frac{4}{a_0k^2}$	0
$\mathcal{T}_{0^+}^{\#2} \dagger$	0	0	0	0	0	0
$\Delta_{0^+}^{\#1} \dagger \alpha$	0	0	0	0	0	$-\frac{2}{a_0}$

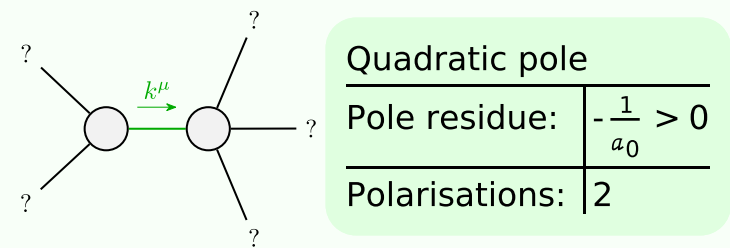
$\Delta_{2^+}^{\#1} \alpha \beta \quad \Delta_{2^+}^{\#2} \alpha \beta \quad \Delta_{2^+}^{\#3} \alpha \beta \quad \mathcal{T}_{2^+}^{\#1} \alpha \beta \quad \Delta_{2^+}^{\#1} \alpha \beta \chi \quad \Delta_{2^+}^{\#2} \alpha \beta \chi$	$\Delta_{2^+}^{\#1} \alpha \beta$	$\Delta_{2^+}^{\#2} \alpha \beta$	$\Delta_{2^+}^{\#3} \alpha \beta$	$\mathcal{T}_{2^+}^{\#1} \alpha \beta$	$\Delta_{2^+}^{\#1} \alpha \beta \chi$	$\Delta_{2^+}^{\#2} \alpha \beta \chi$
$\Delta_{2^+}^{\#1} \dagger \alpha \beta$	$\frac{4}{a_0}$	0	0	0	0	0
$\Delta_{2^+}^{\#2} \dagger \alpha \beta$	0	$-\frac{2}{a_0}$	0	0	0	0
$\Delta_{2^+}^{\#3} \dagger \alpha \beta$	0	0	$\frac{4}{a_0}$	0	0	0
$\mathcal{T}_{2^+}^{\#1} \dagger \alpha \beta$	0	0	0	$-\frac{8}{a_0k^2}$	0	0
$\Delta_{2^+}^{\#1} \dagger \alpha \beta \chi$	0	0	0	0	$\frac{4}{a_0}$	0
$\Delta_{2^+}^{\#2} \dagger \alpha \beta \chi$	0	0	0	0	0	$\frac{4}{a_0}$

$\Gamma_{2^+}^{\#1} \quad \Gamma_{2^+}^{\#2} \quad \Gamma_{2^+}^{\#3} \quad h_{2^+}^{\#1} \alpha \beta \quad \Gamma_{2^+}^{\#1} \alpha \beta \chi \quad \Gamma_{2^+}^{\#2} \alpha \beta \chi$	$\Gamma_{2^+}^{\#1} \alpha \beta$	$\Gamma_{2^+}^{\#2} \alpha \beta$	$\Gamma_{2^+}^{\#3} \alpha \beta$	$h_{2^+}^{\#1} \alpha \beta$	$\Gamma_{2^+}^{\#1} \alpha \beta \chi$	$\Gamma_{2^+}^{\#2} \alpha \beta \chi$
$\Gamma_{2^+}^{\#1} \dagger \alpha \beta$	$\frac{a_0}{4}$	0	0	0	0	0
$\Gamma_{2^+}^{\#2} \dagger \alpha \beta$	0	$-\frac{a_0}{2}$	0	0	0	0
$\Gamma_{2^+}^{\#3} \dagger \alpha \beta$	0	0	$\frac{a_0}{4}$	0	0	0
$h_{2^+}^{\#1} \dagger \alpha \beta$	0	0	0	$-\frac{a_0k^2}{8}$	0	0
$\Gamma_{2^+}^{\#1} \dagger \alpha \beta \chi$	0	0	0	0	$\frac{a_0}{4}$	0
$\Gamma_{2^+}^{\#2} \dagger \alpha \beta \chi$	0	0	0	0	0	$\frac{a_0}{4}$

	$\Gamma_{0^+}^{\#1} \dagger$	$\Gamma_{0^+}^{\#2} \dagger$	$\Gamma_{0^+}^{\#3} \dagger$	$\Gamma_{0^+}^{\#4} \dagger$	$h_{0^+}^{\#1} \dagger$	$h_{0^+}^{\#2} \dagger$	$\Gamma_{0^+}^{\#1} \dagger$
$\Gamma_{0^+}^{\#1} \dagger$	$-\frac{a_0}{2}$	0	0	0	0	0	0
$\Gamma_{0^+}^{\#2} \dagger$	0	0	$\frac{a_0}{2}$	$-\frac{a_0}{2\sqrt{2}}$	0	0	0
$\Gamma_{0^+}^{\#3} \dagger$	0	$\frac{a_0}{2}$	0	$-\frac{a_0}{2\sqrt{2}}$	0	0	0
$\Gamma_{0^+}^{\#4} \dagger$	0	$-\frac{a_0}{2\sqrt{2}}$	$-\frac{a_0}{2\sqrt{2}}$	$\frac{a_0}{2}$	0	0	0
$h_{0^+}^{\#1} \dagger$	0	0	0	0	$\frac{a_0k^2}{4}$	0	0
$h_{0^+}^{\#2} \dagger$	0	0	0	0	0	0	0
$\Gamma_{0^+}^{\#1} \dagger$	0	0	0	0	0	0	$-\frac{a_0}{2}$

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$\mathcal{T}_{0^+}^{\#2} == 0$	1
$\Delta_{0^+}^{\#3} + 2\Delta_{0^+}^{\#4} + 3\Delta_{0^+}^{\#2} == 0$	1
$\mathcal{T}_{1^+}^{\#1\alpha} == 0$	3
$2\Delta_{1^+}^{\#6\alpha} + \Delta_{1^+}^{\#4\alpha} + 2\Delta_{1^+}^{\#5\alpha} + \Delta_{1^+}^{\#3\alpha} == 0$	3
Total constraints:	8

## Massive and massless spectra



(No massive particles)

## Unitarity conditions

$a_0 < 0$