

Particle spectrograph

Wave operator and propagator

	$\sigma_{1^{+}\alpha\beta}^{\#1}$	$\sigma_{1^{+}\alpha\beta}^{\#2}$	$\tau_{1^{+}\alpha\beta}^{\#1}$	$\sigma_{1^{-}\alpha}^{\#1}$	$\sigma_{1^{-}\alpha}^{\#2}$	$\tau_{1^{-}\alpha}^{\#1}$	$\tau_{1^{-}\alpha}^{\#2}$
$\sigma_{1^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\frac{2\left(t_1+t_2\right)}{3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)}$	$\frac{\sqrt{2}\left(t_1-2t_2\right)}{\left(1+k^2\right)\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	$\frac{i\sqrt{2}k\left(t_1-2t_2\right)}{\left(1+k^2\right)\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	0	0	0	0
$\sigma_{1^{+}}^{\#2}\uparrow^{\alpha\beta}$	$\frac{\sqrt{2}\left(t_1-2t_2\right)}{\left(1+k^2\right)\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	$\frac{6k^2\left(2r_1+r_5\right)+t_1+4t_2}{\left(1+k^2\right)^2\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	$\frac{ik\left(6k^2\left(2r_1+r_5\right)+t_1+4t_2\right)}{\left(1+k^2\right)^2\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	0	0	0	0
$\tau_{1^{+}}^{\#1}\uparrow^{\alpha\beta}$	$-\frac{i\sqrt{2}k\left(t_1-2t_2\right)}{\left(1+k^2\right)\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	$-\frac{ik\left(6k^2\left(2r_1+r_5\right)+t_1+4t_2\right)}{\left(1+k^2\right)^2\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	$\frac{k^2\left(6k^2\left(2r_1+r_5\right)+t_1+4t_2\right)}{\left(1+k^2\right)^2\left(3t_1t_2+2k^2\left(2r_1+r_5\right)\left(t_1+t_2\right)\right)}$	0	0	0	0
$\sigma_{1^{-}}^{\#1}\uparrow^{\alpha}$	0	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	0	$\frac{2ik}{t_1+2k^2t_1}$
$\sigma_{1^{-}}^{\#2}\uparrow^{\alpha}$	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	$\frac{-2k^2\left(r_1+r_5\right)+t_1}{\left(t_1+2k^2t_1\right)^2}$	0	$-\frac{i\sqrt{2}k\left(2k^2\left(r_1+r_5\right)+t_1\right)}{\left(t_1+2k^2t_1\right)^2}$
$\tau_{1^{-}}^{\#1}\uparrow^{\alpha}$	0	0	0	0	0	0	0
$\tau_{1^{-}}^{\#2}\uparrow^{\alpha}$	0	0	0	$-\frac{2ik}{t_1+2k^2t_1}$	$\frac{i\sqrt{2}k\left(2k^2\left(r_1+r_5\right)+t_1\right)}{\left(t_1+2k^2t_1\right)^2}$	0	$\frac{-4k^4\left(r_1+r_5\right)+2k^2t_1}{\left(t_1+2k^2t_1\right)^2}$

$\omega_{2^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\omega_{2^{+}}^{\#1}f_{2^{+}\alpha\beta}$	$\omega_{2^{-}}^{\#1}\uparrow^{\alpha\beta\chi}$
$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{2^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0
$\omega_{2^{-}}^{\#1}\uparrow^{\alpha\beta\chi}$	0	$k^2r_1+\frac{t_1}{2}$

Source constraints		
SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0^{+}}^{\#2}==0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta}==0$	1
$\tau_{0^{+}}^{\#1}-2\,i\,k\,\sigma_{0^{+}}^{\#1}==0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta}==\partial_\beta\partial^\beta\tau^\alpha_\alpha+2\,\partial_\chi\partial^\chi\partial_\beta\sigma^{\alpha\beta}_\alpha$	1
$\tau_{1^{-}}^{\#2\alpha}+2\,i\,k\,\sigma_{1^{-}}^{\#2\alpha}==0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi}==\partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta}+2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$	3
$\tau_{1^{-}}^{\#1\alpha}==0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi}==\partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$	3
$\tau_{1^{+}}^{\#1\alpha\beta}+i\,k\,\sigma_{1^{+}}^{\#2\alpha\beta}==0$	$\partial_\chi\partial^\alpha\tau^{\beta\chi}+\partial_\chi\partial^\beta\tau^{\chi\alpha}+\partial_\chi\partial^\chi\tau^{\alpha\beta}+$ $2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta}+2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi}==$ $\partial_\chi\partial^\alpha\tau^{\chi\beta}+\partial_\chi\partial^\beta\tau^{\alpha\chi}+$ $\partial_\chi\partial^\chi\tau^{\beta\alpha}+2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$	3
$\tau_{2^{+}}^{\#1\alpha\beta}-2\,i\,k\,\sigma_{2^{+}}^{\#1\alpha\beta}==0$	$-i\left(4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta}+2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^{\chi\chi}_\chi\right.$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi}-3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta}-$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi}-3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha}+$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta}+3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha}+$ $4\,i\,k^\chi\partial_\epsilon\partial_\chi\partial^\beta\partial^\alpha\sigma^{\delta\epsilon}_\delta-$ $6\,i\,k^\chi\partial_\epsilon\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\delta\epsilon}-$ $6\,i\,k^\chi\partial_\epsilon\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\delta\epsilon}+$ $2\,\eta^{\alpha\beta}\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta}+$ $6\,i\,k^\chi\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\alpha\delta\beta}+$ $6\,i\,k^\chi\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\beta\delta\alpha}-$ $2\,\eta^{\alpha\beta}\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^{\chi\chi}_\chi-$ $\left.4\,i\,\eta^{\alpha\beta}k^\chi\partial_\phi\partial^\phi\partial_\epsilon\partial_\chi\sigma^{\delta\epsilon}_\delta\right)==0$	5
Total constraints/gauge generators:		16

$\omega_{1^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\omega_{1^{+}}^{\#2}\uparrow^{\alpha\beta}$	$f_{1^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\omega_{1^{-}\alpha}^{\#1}$	$\omega_{1^{-}\alpha}^{\#2}$	$f_{1^{-}\alpha}^{\#1}$	$f_{1^{-}\alpha}^{\#2}$
$\frac{1}{6}\frac{1}{(1+2k^2)^2}t_1$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$-\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	0	0	0	0
$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_1$	$\frac{t_1+t_2}{3}$	$\frac{1}{3}ik(t_1+t_2)$	0	0	0	0
$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_1$	$-\frac{1}{3}ik(t_1+t_2)$	$\frac{1}{3}k^2(t_1+t_2)$	$k^2(r_1+r_5)-\frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$iik t_1$
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

$\omega_{0^{+}}^{\#1}\uparrow$	$f_{0^{+}}^{\#1}\uparrow$	$\omega_{0^{+}}^{\#1}$	$f_{0^{+}}^{\#1}$
-t ₁	-i√2 k t ₁	0	0
-i√2 k t ₁	-2 k ² t ₁	0	0
0	0	0	0
0	0	0	t ₂

$\sigma_{2^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\tau_{2^{+}}^{\#1}\uparrow^{\alpha\beta}$	$\sigma_{2^{-}}^{\#1}\uparrow^{\alpha\beta\chi}$
$\frac{2}{(1+2k^2)^2}t_1$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$	0
$\frac{2i\sqrt{2}k}{(1+2k^2)^2}t_1$	$\frac{4k^2}{(1+2k^2)^2}t_1$	$\frac{2}{2k^2r_1+t_1}$
0	0	0

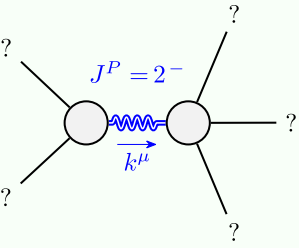
$\sigma_0^{\#1}\uparrow$	$\tau_0^{\#1}\uparrow$	$\sigma_0^{\#1}$	$\tau_0^{\#1}$
$-\frac{1}{(1+2k^2)^2}t_1$	$\frac{i\sqrt{2}k}{(1+2k^2)^2}t_1$	0	0
$-\frac{i\sqrt{2}k}{(1+2k^2)^2}t_1$	$-\frac{2k^2}{(1+2k^2)^2}t_1$	0	0
0	0	0	$\frac{1}{t_2}$

$\sigma_0^{\#1}\uparrow$	$\tau_0^{\#2}\uparrow$	$\sigma_0^{\#2}$	$\tau_0^{\#2}$
0	0	0	0
0	0	0	0
0	0	0	0

Quadratic (free) action
$S==\iiint\!\!\!\int(\frac{1}{6}(6t_1\,\omega^{\alpha i}_\alpha\,\omega_{,~\theta}^\theta+6\,f^{\alpha\beta}\,\tau_{\alpha\beta}+6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}-12t_1\,\omega_\alpha^\theta\,\partial_{,f}f^{\alpha i}+12t_1\,\omega_{,~\theta}^\theta\,\partial'f^\alpha_\alpha-6t_1\partial_{,f}f^\theta_\theta\partial'f^\alpha_\alpha-6t_1\partial_{,f}f^{\alpha i}\partial_\theta f^\theta_\alpha+12t_1\partial'f^\alpha_\alpha\partial_\theta f_{,~\theta}^\theta+4t_1\,\omega_{,\theta\alpha}\,\partial^\theta f^{\alpha i}+4t_2\,\omega_{,\theta\alpha}\,\partial^\theta f^{\alpha i}-4t_1\partial_\alpha f_{,~\theta}\partial^\theta f^{\alpha i}+2t_2\partial_\alpha f_{,~\theta}\partial^\theta f^{\alpha i}-4t_1\partial_\alpha f_{\theta,~i}\partial^\theta f^{\alpha i}+2t_1\partial_{,f}f_{\alpha\theta}\partial^\theta f^{\alpha i}-t_2\partial_{,f}f_{\alpha\theta}\partial^\theta f^{\alpha i}+4t_1\partial_\theta f_{\alpha,~i}\partial^\theta f^{\alpha i}+t_2\partial_\theta f_{\alpha,~i}\partial^\theta f^{\alpha i}+2t_1\partial_\theta f_{i\alpha}\partial^\theta f^{\alpha i}-t_2\partial_\theta f_{i\alpha}\partial^\theta f^{\alpha i}+2(t_1+t_2)\,\omega_{\alpha i\theta}\,(\omega^{\alpha i\theta}+2\,\partial^\theta f^{\alpha i})+2\,\omega_{\alpha\theta,~i}((t_1-2t_2)\,\omega^{\alpha i\theta}+2(2t_1-t_2)\,\partial^\theta f^{\alpha i})-8r_1\partial_\beta\omega_{\alpha i\theta}\partial^\theta\omega^{\alpha\beta i}+4r_1\partial_\beta\omega_{\alpha\theta,~i}\partial^\theta\omega^{\alpha\beta i}-16r_1\partial_\beta\omega_{i\theta\alpha}\partial^\theta\omega^{\alpha\beta i}-4r_1\partial_{,i}\omega_{\alpha\beta\theta}\partial^\theta\omega^{\alpha\beta i}+4r_1\partial_\theta\omega_{\alpha\beta,~i}\partial^\theta\omega^{\alpha\beta i}+4r_1\partial_\theta\omega_{\alpha i\beta}\partial^\theta\omega^{\alpha\beta i}+6r_5\partial_{,i}\omega_\theta^\kappa\partial^\theta\omega^{\alpha i}_\alpha-6r_5\partial_\theta\omega_{,~\kappa}^\kappa\partial^\theta\omega^{\alpha i}_\alpha-6r_5\partial_\alpha\omega^{\alpha i\theta}\partial_\kappa\omega_{,~\theta}^\kappa+12r_5\partial^\theta\omega^{\alpha i}_\alpha\partial_\kappa\omega_{,~\theta}^\kappa+6r_5\partial_\alpha\omega^{\alpha i\theta}\partial_\kappa\omega_\theta^\kappa_{,~i}-12r_5\partial^\theta\omega^{\alpha i}_\alpha\partial_\kappa\omega_\theta^\kappa_{,~i})) [t,\,x,\,y,\,z]dz\,dy\,dx\,dt$

Massive and massless spectra

Massive particle	
Pole residue:	$\frac{-3t_1t_2(t_1+t_2)+6r_1(t_1^2+2t_2^2)+3r_5(t_1^2+2t_2^2)}{(2r_1+r_5)(t_1+t_2)(-3t_1t_2+4r_1(t_1+t_2)+2r_5(t_1+t_2))}>0$
Polarisations:	3
Square mass:	$-\frac{3t_1t_2}{2(2r_1+r_5)(t_1+t_2)}>0$
Spin:	1
Parity:	Even



Massive particle	
Pole residue:	$-\frac{1}{r_1}>0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1}>0$
Spin:	2
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0 \ \&\& \ r_5 > -2\,r_1 \ \&\& \ t_1 > 0 \ \&\& \ -t_1 < t_2 < 0$