

$\omega_{2+}^{\#1} \dagger \alpha\beta$	$f_{2+}^{\#1}$	$\omega_{2-}^{\#1} \alpha\beta\chi$
$-\frac{3k^2r_3}{2}$	0	0
0	0	0
0	0	0

$\omega_{0+}^{\#1} \dagger$	$f_{0+}^{\#1}$	$f_{0+}^{\#2}$	$\omega_{0-}^{\#1}$
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	$k^2r_2+t_2$

$\omega_{1+}^{\#1} \dagger \alpha\beta$	$\omega_{1+}^{\#2} \alpha\beta$	$f_{1+}^{\#1} \alpha\beta$	$f_{1-}^{\#2} \alpha$
$k^2(2r_3+r_5)+\frac{2t_2}{3}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{1}{3}i\sqrt{2}kt_2$	0
$\frac{\sqrt{2}t_2}{3}$	$\frac{t_2}{3}$	$\frac{ikt_2}{3}$	0
$-\frac{1}{3}i\sqrt{2}kt_2$	$-\frac{1}{3}iikt_2$	$\frac{k^2t_2}{3}$	0
0	0	0	$\frac{1}{2}k^2(r_3+2r_5)$
0	0	0	0
0	0	0	0
0	0	0	0

$\sigma_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
$\sigma_{0-}^{\#1} \dagger$	0	0	$\frac{1}{k^2r_2+t_2}$

Source constraints

$\omega_{1+}^{\#1} \dagger \alpha\beta$	$\omega_{1+}^{\#2} \dagger \alpha\beta$	$f_{1+}^{\#1} \dagger \alpha\beta$	$\omega_{1-}^{\#1} \alpha$	$\omega_{1-}^{\#2} \alpha$	$f_{1-}^{\#1} \alpha$	$f_{1-}^{\#2} \alpha$
$k^2(2r_3+r_5)+\frac{2t_2}{3}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{1}{3}i\sqrt{2}kt_2$	0	0	0	0
$\frac{\sqrt{2}t_2}{3}$	$\frac{t_2}{3}$	$\frac{ikt_2}{3}$	0	0	0	0
$-\frac{1}{3}i\sqrt{2}kt_2$	$-\frac{1}{3}iikt_2$	$\frac{k^2t_2}{3}$	0	0	0	0
$\omega_{1+}^{\#1} \dagger \alpha\beta$	$\omega_{1+}^{\#2} \dagger \alpha\beta$	$f_{1+}^{\#1} \dagger \alpha\beta$	$\omega_{1-}^{\#1} \dagger \alpha$	$\omega_{1-}^{\#2} \dagger \alpha$	$f_{1-}^{\#1} \dagger \alpha$	$f_{1-}^{\#2} \dagger \alpha$
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

SO(3) irreps	#
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} == 0$	1
$\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2} \alpha == 0$	3
$\tau_{1-}^{\#1} \alpha == 0$	3
$\sigma_{1-}^{\#2} \alpha == 0$	3
$\tau_{1+}^{\#1} \alpha\beta + i k \sigma_{1+}^{\#2} \alpha\beta == 0$	3
$\sigma_{2-}^{\#1} \alpha\beta\chi == 0$	5
$\tau_{2+}^{\#1} \alpha\beta == 0$	5
Total #:	25

$\sigma_{2+}^{\#1} \dagger \alpha\beta$	$\tau_{2+}^{\#1} \dagger \alpha\beta$	$\sigma_{2-}^{\#1} \alpha\beta\chi$
$-\frac{2}{3k^2r_3}$	0	0
0	0	0
0	0	0

$\sigma_{1-}^{\#1} \alpha$	$\sigma_{1-}^{\#2} \alpha$	$\tau_{1-}^{\#1} \alpha$	$\tau_{1-}^{\#2} \alpha$
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$\sigma_{1+}^{\#1} \alpha\beta$	$\sigma_{1+}^{\#2} \alpha\beta$	$\tau_{1+}^{\#1} \alpha\beta$	$\sigma_{1-}^{\#1} \alpha$	$\sigma_{1-}^{\#2} \alpha$	$\tau_{1-}^{\#1} \alpha$	$\tau_{1-}^{\#2} \alpha$
$\frac{1}{k^2(2r_3+r_5)}$	$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$	$-\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$	0	0	0	0
$-\frac{\sqrt{2}}{k^2(1+k^2)(2r_3+r_5)}$	$\frac{3k^2(2r_3+r_5)+2t_2}{(k+k^3)^2(2r_3+r_5)t_2}$	$\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$	0	0	0	0
$\frac{i\sqrt{2}}{k(1+k^2)(2r_3+r_5)}$	$-\frac{i(3k^2(2r_3+r_5)+2t_2)}{k(1+k^2)^2(2r_3+r_5)t_2}$	$\frac{3k^2(2r_3+r_5)+2t_2}{(1+k^2)^2(2r_3+r_5)t_2}$	0	0	0	0
0	0	0	$\frac{2}{k^2(r_3+2r_5)}$	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Lagrangian density

$$\begin{aligned} &\frac{2}{3}t_2\omega_{\lambda'}^{\kappa\lambda}\omega_{\kappa\lambda}'+\frac{1}{3}t_2\omega_{\kappa\lambda}'\omega_{\lambda'}^{\kappa\lambda}-\frac{1}{2}r_3\partial_{\lambda}\omega_{\kappa\lambda}'\partial'_{\lambda}\omega_{\lambda'}^{\alpha}- \\ &r_5\partial_{\lambda}\omega_{\kappa\lambda}'\partial'_{\lambda}\omega_{\lambda'}^{\alpha}+\frac{2}{3}r_2\partial^{\beta}\omega_{\kappa}^{\theta\alpha}\partial_{\theta\omega_{\alpha\beta}}^{\kappa}-\frac{1}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta}+\frac{1}{2}r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}-r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}- \\ &\frac{1}{2}r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}+r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}-\frac{1}{2}r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega^{\kappa\lambda\theta}-\frac{1}{2}r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}- \\ &r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}+r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}+2r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}+ \\ &\frac{1}{6}t_2\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{6}t_2\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}^{\theta}+\frac{1}{6}t_2\partial^{\alpha}f_{\alpha\kappa}\partial^{\kappa}f_{\alpha\lambda}+ \\ &\frac{1}{3}t_2\omega_{\theta\kappa}\partial^{\kappa}f^{\theta}-\frac{2}{3}t_2\omega_{\kappa\theta}\partial^{\kappa}f^{\theta}-\frac{1}{3}t_2\omega_{\theta\kappa}\partial^{\kappa}f^{\theta}+\frac{2}{3}t_2\omega_{\theta\kappa\lambda}\partial^{\kappa}f^{\theta}- \\ &\frac{1}{6}t_2\partial^{\alpha}f_{\lambda}^{\kappa}\partial^{\kappa}f_{\lambda\alpha}-\frac{1}{6}t_2\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{1}{6}t_2\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+ \\ &\frac{1}{3}r_2\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+\frac{2}{3}r_2\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}-\frac{2}{3}r_2\partial^{\beta}\omega_{\lambda'}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}' + \\ &\frac{2}{3}r_2\partial^{\beta}\omega_{\lambda'}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}'-4r_3\partial^{\beta}\omega_{\lambda'}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}'-\frac{1}{2}r_3\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega^{\theta\kappa}_{\kappa}+ \\ &r_5\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega^{\theta\kappa}_{\kappa}+\frac{1}{2}r_3\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega^{\theta\kappa}_{\kappa}-r_5\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega^{\theta\kappa}_{\kappa} \end{aligned}$$

$$\text{Added source term: } f^{\alpha\beta}_{\alpha\beta} + \omega^{\alpha\beta\chi}_{\alpha\beta\chi} + \sigma^{\alpha\beta\chi}_{\alpha\beta\chi}$$

Quadratic pole	
Pole residue:	$-\frac{1}{r_3(2r_3+r_5)(r_3+2r_5)p^2}$
Polarisations:	2

Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

Unitarity conditions

$$r_2 < 0 \&\& r_3 < 0 \&\& r_5 < -\frac{r_3}{2} \&\& t_2 > 0 \parallel r_2 < 0 \&\& r_3 < 0 \&\& r_5 > -2r_3 \&\& t_2 > 0 \parallel$$

$$r_2 < 0 \&\& r_3 > 0 \&\& -2r_3 < r_5 < -\frac{r_3}{2} \&\& t_2 > 0$$

