

# Particle spectrograph

## Wave operator and propagator

Quadratic (free) Lagrangian density

$$\begin{aligned} &\frac{2}{3}t_3\omega_{\kappa\alpha}^{\alpha\iota}\omega_{\kappa\alpha}^{\kappa}+\frac{2}{3}t_2\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\kappa\lambda}^{\iota}+\frac{1}{3}t_2\omega_{\kappa\lambda}^{\kappa\lambda}\omega_{\kappa\lambda}^{\iota}+\omega_{\kappa\lambda}^{\kappa\lambda}+f^{\alpha\beta}\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi}+ \\ &2r_1\partial_{\iota}\omega_{\kappa}^{\kappa\lambda}\partial_{\iota}\omega_{\lambda}^{\alpha}-\frac{2}{3}r_1\partial^{\beta}\omega_{\kappa}^{\beta\alpha}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}+\frac{2}{3}r_2\partial^{\beta}\omega_{\kappa}^{\beta\alpha}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}- \\ &\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta}-\frac{1}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta}+\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta}- \\ &\frac{2}{3}r_2\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta}+2r_1\partial_a\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}-2r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\theta\kappa\lambda}+ \\ &2r_1\partial_a\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\theta}^{\kappa\lambda\theta}-4r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega^{\kappa\lambda\theta}+\frac{1}{6}t_2\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{6}t_2\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}^{\theta}+ \\ &\frac{1}{6}t_2\partial^{\alpha}f_{\kappa}^{\lambda}\partial^{\kappa}f_{\alpha\lambda}^{\lambda}-\frac{2}{3}t_3\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\iota}^{\iota}-\frac{2}{3}t_3\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\iota}^{\iota}-\frac{4}{3}t_3\partial^{\alpha}f_{\kappa\alpha}\partial^{\kappa}f_{\iota}^{\iota}+ \\ &\frac{2}{3}t_3\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\iota}^{\iota}+\frac{1}{3}t_2\omega_{\iota\theta\kappa}\partial^{\kappa}f^{\iota\theta}-\frac{2}{3}t_2\omega_{\iota\kappa\theta}\partial^{\kappa}f^{\iota\theta}-\frac{1}{3}t_2\omega_{\theta\iota\kappa}\partial^{\kappa}f^{\iota\theta}+ \\ &\frac{2}{3}t_2\omega_{\theta\kappa\iota}\partial^{\kappa}f^{\iota\theta}+\frac{2}{3}t_3\omega_{\iota\alpha}^{\alpha}\partial^{\kappa}f_{\kappa}^{\iota}+\frac{2}{3}t_3\omega_{\iota\lambda}^{\lambda}\partial^{\kappa}f_{\kappa}^{\iota}-\frac{1}{6}t_2\partial^{\alpha}f_{\kappa}^{\lambda}\partial^{\kappa}f_{\lambda\alpha}^{\lambda}- \\ &\frac{1}{6}t_2\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{1}{6}t_2\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{2}{3}t_3\partial^{\alpha}f_{\alpha}^{\lambda}\partial^{\kappa}f_{\lambda\kappa}^{\lambda}+\frac{2}{3}r_1\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}+ \\ &\frac{1}{3}r_2\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}-\frac{2}{3}r_1\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}+\frac{2}{3}r_2\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}+ \\ &\frac{2}{3}r_1\partial^{\beta}\omega_{\iota}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\iota}-\frac{2}{3}r_2\partial^{\beta}\omega_{\iota}^{\alpha\lambda}\partial_{\lambda}\omega_{\alpha\beta}^{\iota}-\frac{8}{3}r_1\partial^{\beta}\omega_{\iota}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\iota}+ \\ &\frac{2}{3}r_2\partial^{\beta}\omega_{\iota}^{\lambda\alpha}\partial_{\lambda}\omega_{\alpha\beta}^{\iota}-2r_1\partial_a\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega^{\theta\kappa}_{\kappa}+2r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega^{\theta\kappa}_{\kappa} \end{aligned}$$

	$\sigma_{1+}^{\#1}+\alpha\beta$	$\sigma_{1+}^{\#2}+\alpha\beta$	$\tau_{1+}^{\#1}+\alpha\beta$	$\sigma_{1-}^{\#1}-\alpha$	$\sigma_{1-}^{\#2}-\alpha$	$\tau_{1-}^{\#1}-\alpha$	$\tau_{1-}^{\#2}-\alpha$
$\sigma_{1+}^{\#1}+\alpha\beta$	$\frac{6}{(3+k^2)^2}t_2$	$\frac{3\sqrt{2}}{(3+k^2)^2}t_2$	$\frac{3i\sqrt{2}k}{(3+k^2)^2}t_2$	0	0	0	0
$\sigma_{1+}^{\#2}+\alpha\beta$	$\frac{3\sqrt{2}}{(3+k^2)^2}t_2$	$\frac{3}{(3+k^2)^2}t_2$	$\frac{3ik}{(3+k^2)^2}t_2$	0	0	0	0
$\tau_{1+}^{\#1}+\alpha\beta$	$-\frac{3i\sqrt{2}k}{(3+k^2)^2}t_2$	$-\frac{3ik}{(3+k^2)^2}t_2$	$\frac{3k^2}{(3+k^2)^2}t_2$	0	0	0	0
$\sigma_{1-}^{\#1}-\alpha$	0	0	0	$-\frac{1}{k^2}r_1$	$-\frac{\sqrt{2}}{k^2r_1+2k^4r_1}$	0	$-\frac{2i}{kr_1+2k^3r_1}$
$\sigma_{1-}^{\#2}-\alpha$	0	0	0	$-\frac{\sqrt{2}}{k^2r_1+2k^4r_1}$	$\frac{3k^2r_1-2t_3}{(k+2k^3)^2r_1t_3}$	0	$\frac{i\sqrt{2}(3k^2r_1-2t_3)}{k(1+2k^2)^2r_1t_3}$
$\tau_{1-}^{\#1}-\alpha$	0	0	0	0	0	0	0
$\tau_{1-}^{\#2}-\alpha$	0	0	0	$\frac{2i}{kr_1+2k^3r_1}$	$-\frac{i\sqrt{2}(3k^2r_1-2t_3)}{k(1+2k^2)^2r_1t_3}$	0	$\frac{6k^2r_1-4t_3}{(1+2k^2)^2r_1t_3}$

	$\omega_{1+}^{\#1}+\alpha\beta$	$\omega_{1+}^{\#2}+\alpha\beta$	$f_{1+}^{\#1}+\alpha\beta$	$\omega_{1-}^{\#1}-\alpha$	$\omega_{1-}^{\#2}-\alpha$	$f_{1-}^{\#1}-\alpha$	$f_{1-}^{\#2}-\alpha$
$\omega_{1+}^{\#1}+\alpha\beta$	$\frac{2t_2}{3}$	$\frac{\sqrt{2}t_2}{3}$	$\frac{1}{3}i\sqrt{2}kt_2$	0	0	0	0
$\omega_{1+}^{\#2}+\alpha\beta$	$\frac{\sqrt{2}t_2}{3}$	$\frac{t_2}{3}$	$\frac{ikt_2}{3}$	0	0	0	0
$f_{1+}^{\#1}+\alpha\beta$	$-\frac{1}{3}i\sqrt{2}kt_2$	$-\frac{1}{3}ikt_2$	$\frac{k^2t_2}{3}$	0	0	0	0
$\omega_{1-}^{\#1}-\alpha$	0	0	0	$-k^2r_1+\frac{2t_3}{3}$	$-\frac{\sqrt{2}t_3}{3}$	0	$-\frac{2}{3}ikt_3$
$\omega_{1-}^{\#2}-\alpha$	0	0	0	$-\frac{\sqrt{2}t_3}{3}$	$\frac{t_3}{3}$	0	$\frac{1}{3}i\sqrt{2}kt_3$
$f_{1-}^{\#1}-\alpha$	0	0	0	0	0	0	0
$f_{1-}^{\#2}-\alpha$	0	0	0	$\frac{2ikt_3}{3}$	$-\frac{1}{3}i\sqrt{2}kt_3$	0	$\frac{2k^2t_3}{3}$

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2}==0$	1
$\tau_{0+}^{\#1}-2ik\sigma_{0+}^{\#1}==0$	1
$\tau_{1-}^{\#2\alpha}+2ik\sigma_{1-}^{\#2\alpha}==0$	3
$\tau_{1-}^{\#1\alpha}==0$	3
$\tau_{1+}^{\#1\alpha\beta}+ik\sigma_{1+}^{\#1\alpha\beta}==0$	3
$\sigma_{1+}^{\#1\alpha\beta}==\sigma_{1+}^{\#2\alpha\beta}$	3
$\tau_{2+}^{\#1\alpha\beta}==0$	5
$\sigma_{2+}^{\#1\alpha\beta}==0$	5
Total constraints:	24

$$\begin{matrix} \omega_{0+}^{\#1}+ \\ t_3 \\ f_{0+}^{\#1}+ \\ i\sqrt{2}kt_3 \\ f_{0+}^{\#2}+ \\ 0 \\ \omega_{0-}^{\#1}+ \\ 0 \end{matrix} \quad \begin{matrix} f_{0+}^{\#1} \\ -i\sqrt{2}kt_3 \\ 2k^2t_3 \\ 0 \\ 0 \\ 0 \end{matrix} \quad \begin{matrix} \omega_{0+}^{\#1} \\ 0 \\ 0 \\ 0 \\ 0 \\ k^2r_2+t_2 \end{matrix}$$

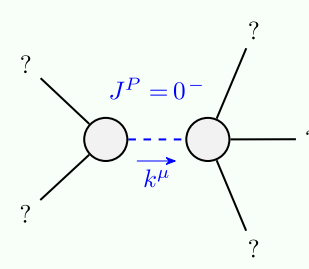
$$\omega_{2+}^{\#1}+\alpha\beta \quad f_{2+}^{\#1}+\alpha\beta \quad \omega_{2-}^{\#1}+\alpha\beta\chi$$

$\omega_{2+}^{\#1}+\alpha\beta$	0	0	0
$f_{2+}^{\#1}+\alpha\beta$	0	0	0
$\omega_{2-}^{\#1}+\alpha\beta\chi$	0	0	$k^2r_1$

$$\sigma_{2+}^{\#1}+\alpha\beta\chi \quad \tau_{2+}^{\#1}+\alpha\beta \quad \sigma_{2-}^{\#1}+\alpha\beta\chi$$

$\sigma_{2+}^{\#1}+\alpha\beta\chi$	0	0	$\frac{1}{k^2r_1}$
$\tau_{2+}^{\#1}+\alpha\beta$	0	0	0
$\sigma_{2-}^{\#1}+\alpha\beta\chi$	0	0	0

## Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(seipitless parled on)

## Unitarity conditions

$r_2 < 0 \&\& t_2 > 0$