# Particle spectrograph

## Wave operator and propagator

$$\mathcal{S} = \iiint \left( \phi\, \mathcal{J} + h^{\alpha\beta}\, \mathcal{T}_{\alpha\beta} + \tfrac{1}{2}\beta\, \partial_\alpha\phi\, \partial^\alpha\phi + \tfrac{1}{8}\alpha\,(24(1+\phi)\,\partial_\alpha\partial^\alpha\phi - 8\,\partial_\alpha h^\beta{}_\beta\,\partial^\alpha\phi + 8\,\partial^\alpha\phi\,\partial_\beta h_\alpha{}^\beta - 4\,\partial_\beta\partial_\alpha h^{\alpha\beta} + \right.$$
$$4\,\partial_\beta\partial^\beta h^\alpha{}_\alpha - \partial_\beta h^\chi{}_\chi\,\partial^\beta h^\alpha{}_\alpha + 2\,\partial^\beta h^\alpha{}_\alpha\,\partial_\chi h_\beta{}^\chi - 2\,\partial_\beta h_{\alpha\chi}\,\partial^\chi h^{\alpha\beta} + \partial_\chi h_{\alpha\beta}\,\partial^\chi h^{\alpha\beta}) +$$
$$\epsilon\,(-8\,\partial_\beta\partial_\alpha\phi\,\partial^\beta\partial^\alpha\phi - 4\,\partial_\beta\partial_\alpha h^\chi{}_\chi\,\partial^\beta\partial^\alpha\phi + 4\,\partial^\beta\partial^\alpha\phi\,\partial_\chi\partial_\alpha h_\beta{}^\chi + 4\,\partial^\beta\partial^\alpha\phi\,\partial_\chi\partial_\beta h_\alpha{}^\chi -$$
$$4\,\partial^\beta\partial^\alpha\phi\,\partial_\chi\partial^\chi h_{\alpha\beta} + 4\,\partial_\alpha\partial^\alpha\phi\,(2\,\partial_\beta\partial^\beta\phi - \partial_\chi\partial_\beta h^{\beta\chi} + \partial_\chi\partial^\chi h^\beta{}_\beta) - \partial_\chi\partial_\beta h^\delta{}_\delta\,\partial^\chi\partial^\beta h^\alpha{}_\alpha -$$
$$2\,\partial^\chi\partial_\alpha h^{\alpha\beta}\,\partial_\delta\partial_\beta h_\chi{}^\delta - 2\,\partial^\chi\partial_\alpha h^{\alpha\beta}\,\partial_\delta\partial_\chi h_\beta{}^\delta + 4\,\partial^\chi\partial^\beta h^\alpha{}_\alpha\,\partial_\delta\partial_\chi h_\beta{}^\delta +$$
$$\partial_\beta\partial_\alpha h^{\alpha\beta}\,\partial_\delta\partial_\chi h^{\chi\delta} - 2\,\partial_\beta\partial^\beta h^\alpha{}_\alpha\,\partial_\delta\partial_\chi h^{\chi\delta} - \partial_\chi\partial^\chi h^{\alpha\beta}\,\partial_\delta\partial^\delta h_{\alpha\beta} + 4\,\partial^\chi\partial_\alpha h^{\alpha\beta}\,\partial_\delta\partial^\delta h_{\beta\chi} -$$
$$2\,\partial^\chi\partial^\beta h^\alpha{}_\alpha\,\partial_\delta\partial^\delta h_{\beta\chi} + \partial_\beta\partial^\beta h^\alpha{}_\alpha\,\partial_\delta\partial^\delta h^\chi{}_\chi + \partial_\beta\partial_\alpha h_{\chi\delta}\,\partial^\delta\partial^\chi h^{\alpha\beta} - \partial_\chi\partial_\beta h_{\alpha\delta}\,\partial^\delta\partial^\chi h^{\alpha\beta} -$$
$$\left. \partial_\delta\partial_\beta h_{\alpha\chi}\,\partial^\delta\partial^\chi h^{\alpha\beta} + \partial_\delta\partial_\chi h_{\alpha\beta}\,\partial^\delta\partial^\chi h^{\alpha\beta}))[t, x, y, z]\,dz\,dy\,dx\,dt \right.$$

| Spin-parity    form | Covariant    form | Multiplicities |
|---|---|---|
| $\overset{\#2}{0^+\mathcal{T}} == 0$ | $\partial_\beta\partial_\alpha\mathcal{T}^{\alpha\beta} == 0$ | 1 |
| $\overset{\#1}{1^-\mathcal{T}}{}^\alpha == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\mathcal{T}^{-\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\mathcal{T}^{-\alpha\beta}$ | 3 |
| Total expected gauge generators: | | 4 |

|  | $\overset{\#1}{0^+\phi\dagger}$ | $\overset{\#2}{0^+h\dagger}$ | $\overset{\#1}{0^+h\dagger}$ |
|---|---|---|---|
| $\overset{\#1}{0^+\phi}$ | $-\tfrac{1}{2}\tfrac{1}{i}\sqrt{3}\,\alpha\,\bar{k}^2$ | 0 | $-\tfrac{\alpha\,\bar{k}^2}{4}$ |
| $\overset{\#2}{0^+h}$ | 0 | 0 | 0 |
| $\overset{\#1}{0^+h}$ | $\tfrac{1}{2}\tfrac{1}{i}(-6\,\alpha+\beta)\,k^2$ | 0 | $-\tfrac{1}{2}\tfrac{1}{i}\sqrt{3}\,\alpha\,\bar{k}^2$ |

|  | $\overset{\#1}{0^+\mathcal{T}}$ | $\overset{\#2}{0^+\mathcal{T}}$ | $\overset{\#1}{0^+\mathcal{T}}$ |
|---|---|---|---|
| $\overset{\#1}{0^+\mathcal{T}\dagger}$ | $\tfrac{24\,\alpha-4\,\beta}{\alpha\,\beta\,\bar{k}^2}$ | 0 | $-\tfrac{4\sqrt{3}}{\beta\,\bar{k}^2}$ |
| $\overset{\#2}{0^+\mathcal{T}\dagger}$ | 0 | 0 | 0 |
| $\overset{\#1}{0^+\mathcal{T}\dagger}$ | $-\tfrac{4\sqrt{3}}{\beta\,\bar{k}^2}$ | 0 | $\tfrac{2}{\beta\,\bar{k}^2}$ |

|  | $\overset{\#1}{2^+h\dagger}{}^{\alpha\beta}$ |
|---|---|
| $\overset{\#1}{2^+h_{\alpha\beta}}$ | $\tfrac{\alpha\,\bar{k}^2}{8}$ |

|  | $\overset{\#1}{1^-h\dagger}{}^\alpha$ |
|---|---|
| $\overset{\#1}{1^-h_\alpha}$ | 0 |

|  | $\overset{\#1}{2^+\mathcal{T}_{\alpha\beta}}$ |
|---|---|
| $\overset{\#1}{2^+\mathcal{T}\dagger}{}^{\alpha\beta}$ | $\tfrac{8}{\alpha\,\bar{k}^2}$ |

|  | $\overset{\#1}{1^-\mathcal{T}}{}^\alpha$ |
|---|---|
| $\overset{\#1}{1^-\mathcal{T}_\alpha}$ | 0 |

## Massive and massless spectra

| Polarisations: | Poleresidue: | Massless particle |
|---|---|---|
| 1 | $\tfrac{1}{\beta} > 0$ | |



$k^\mu = (p, 0, 0, p)$

(No particles)

| Polarisations: | Poleresidue: | Massless particle |
|---|---|---|
| 2 | $\tfrac{1}{\alpha} > 0$ | |



$k^\mu = (p, 0, 0, p)$

## Unitarity conditions