# Particle spectrograph

## Wave operator and propagator

Quadratic (free) Lagrangian density

$$\frac{1}{3} t_2\ \omega_{\iota}{}^{\kappa\lambda}\ \omega_{\kappa\lambda}{}' + \frac{1}{3} t_2\ \omega_{\kappa\lambda}{}'\ \omega^{\kappa\lambda} + f^{\alpha\beta}\ \tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\ \sigma_{\alpha\beta\chi} + 2\ r_1\ \partial_{\iota}\omega^{\kappa\lambda}\ \partial'\omega_{\lambda}{}^{\alpha}{}_{\kappa} -$$
$$\frac{2}{3} r_1\ \partial^{\beta}\omega^{\theta\alpha}\ \partial_{\theta}\omega_{\kappa}{}^{\kappa}{}_{\alpha\beta} + \frac{2}{3} r_2\ \partial^{\beta}\omega^{\theta\alpha}\ \partial_{\theta}\omega_{\kappa}{}^{\kappa}{}_{\alpha\beta} - \frac{2}{3} r_1\ \partial_{\theta}\omega_{\kappa}{}^{\kappa}{}_{\alpha\beta}\omega^{\alpha\beta\theta} -$$
$$\frac{1}{3} r_2\ \partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\ \partial_{\kappa}\omega^{\alpha\beta\theta} - \frac{2}{3} r_1\ \partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\ \partial_{\kappa}\omega^{\theta\alpha\beta} + \frac{2}{3} r_2\ \partial_{\theta}\omega_{\alpha\beta}{}^{\kappa}\ \partial_{\kappa}\omega^{\theta\alpha\beta} +$$
$$2\ r_1\ \partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\ \partial_{\kappa}\omega^{\theta\kappa\lambda} - 2\ r_1\ \partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\ \partial_{\kappa}\omega^{\theta\kappa\lambda} + 2\ r_1\ \partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\ \partial_{\kappa}\omega^{\kappa\lambda\theta} -$$
$$4\ r_1\ \partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\ \partial_{\kappa}\omega^{\kappa\lambda\theta} + \frac{1}{6} t_2\ \partial^{\alpha}f_{\theta\kappa}\ \partial^{\kappa}f_{\alpha}{}^{\theta} - \frac{1}{6} t_2\ \partial^{\alpha}f_{\kappa\theta}\ \partial^{\kappa}f_{\alpha}{}^{\theta} + \frac{1}{6} t_2\ \partial^{\alpha}f_{\kappa}{}^{\lambda}\ \partial^{\kappa}f_{\alpha\lambda} +$$
$$\frac{1}{3} t_2\ \omega_{\iota\theta\kappa}\ \partial^{\kappa}f'^{\iota\theta} - \frac{2}{3} t_2\ \omega_{\iota\kappa\theta}\ \partial^{\kappa}f'^{\iota\theta} - \frac{1}{3} t_2\ \omega_{\theta\kappa\iota}\ \partial^{\kappa}f'^{\iota\theta} +$$
$$\frac{1}{6} t_2\ \partial_{\kappa}f_{\theta}{}^{\lambda}\ \partial^{\kappa}f_{\lambda}{}^{\theta} + \frac{1}{6} t_2\ \partial_{\kappa}f^{\lambda}{}_{\theta}\ \partial^{\kappa}f_{\lambda}{}^{\theta} + \frac{2}{3} r_1\ \partial_{\kappa}\omega^{\alpha\beta\theta}\ \partial^{\kappa}\omega_{\alpha\beta\theta} +$$
$$\frac{1}{3} r_2\ \partial_{\kappa}\omega^{\alpha\beta\theta}\ \partial^{\kappa}\omega_{\alpha\beta\theta} - \frac{2}{3} r_1\ \partial_{\kappa}\omega^{\theta\alpha\beta}\ \partial^{\kappa}\omega_{\alpha\beta\theta} + \frac{2}{3} r_2\ \partial_{\kappa}\omega^{\theta\alpha\beta}\ \partial^{\kappa}\omega_{\alpha\beta\theta} +$$
$$\frac{2}{3} r_1\ \partial^{\beta}\omega_{\iota}{}^{\alpha\lambda}\ \partial_{\lambda}\omega_{\alpha\beta}{}' - \frac{2}{3} r_2\ \partial^{\beta}\omega_{\iota}{}^{\alpha\lambda}\ \partial_{\lambda}\omega_{\alpha\beta}{}' - \frac{8}{3} r_1\ \partial^{\beta}\omega_{\iota}{}^{\lambda\alpha}\ \partial_{\lambda}\omega_{\alpha\beta}{}' +$$
$$\frac{2}{3} r_2\ \partial^{\beta}\omega_{\iota}{}^{\lambda\alpha}\ \partial_{\lambda}\omega_{\alpha\beta}{}' - 2\ r_1\ \partial_{\alpha}\omega_{\lambda}{}^{\alpha}{}_{\theta}\ \partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa} + 2\ r_1\ \partial_{\theta}\omega_{\lambda}{}^{\alpha}{}_{\alpha}\ \partial^{\lambda}\omega^{\theta\kappa}{}_{\kappa}$$

| | $\omega_{1^+}^{\#1}{}^{\alpha\beta}$ | $\omega_{1^+}^{\#2}{}^{\alpha\beta}$ | $f_{1^+}^{\#1}{}^{\alpha\beta}$ | $\omega_{1^-}^{\#1}{}^{\alpha}$ | $\omega_{1^-}^{\#2}{}^{\alpha}$ | $f_{1^-}^{\#1}{}^{\alpha}$ | $f_{1^-}^{\#2}{}^{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1}{}^{\alpha\beta}\,\dagger$ | $\frac{2 t_2}{3}$ | $\frac{\sqrt{2}\ t_2}{3}$ | $-\frac{1}{3} i\ \sqrt{2}\ k t_2$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2}{}^{\alpha\beta}\,\dagger$ | $\frac{\sqrt{2}\ t_2}{3}$ | $\frac{t_2}{3}$ | $\frac{i k t_2}{3}$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1}{}^{\alpha\beta}\,\dagger$ | $-\frac{1}{3} i\ \sqrt{2}\ k t_2$ | $-\frac{1}{3} i k t_2$ | $\frac{k^2 t_2}{3}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1}{}^{\alpha}\,\dagger$ | 0 | 0 | 0 | $-k^2\ r_1$ | 0 | 0 | 0 |
| $\omega_{1^-}^{\#2}{}^{\alpha}\,\dagger$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#1}{}^{\alpha}\,\dagger$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2}{}^{\alpha}\,\dagger$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | $\sigma_{1^+}^{\#1}{}^{\alpha\beta}$ | $\sigma_{1^+}^{\#2}{}^{\alpha\beta}$ | $\tau_{1^+}^{\#1}{}^{\alpha\beta}$ | $\sigma_{1^-}^{\#1}{}^{\alpha}$ | $\sigma_{1^-}^{\#2}{}^{\alpha}$ | $\tau_{1^-}^{\#1}{}^{\alpha}$ | $\tau_{1^-}^{\#2}{}^{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1}{}^{\alpha\beta}\,\dagger$ | $\frac{6}{(3+k^2)^2 t_2}$ | $\frac{3\ \sqrt{2}}{(3+k^2)^2 t_2}$ | $-\frac{3 i\ \sqrt{2}\ k}{(3+k^2)^2 t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2}{}^{\alpha\beta}\,\dagger$ | $\frac{3\ \sqrt{2}}{(3+k^2)^2 t_2}$ | $\frac{3}{(3+k^2)^2 t_2}$ | $-\frac{3 i k}{(3+k^2)^2 t_2}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1}{}^{\alpha\beta}\,\dagger$ | $-\frac{3 i\ \sqrt{2}\ k}{(3+k^2)^2 t_2}$ | $-\frac{3 i k}{(3+k^2)^2 t_2}$ | $\frac{3 k^2}{(3+k^2)^2 t_2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1}{}^{\alpha}\,\dagger$ | 0 | 0 | 0 | $-\frac{1}{k^2 r_1}$ | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#2}{}^{\alpha}\,\dagger$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#1}{}^{\alpha}\,\dagger$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2}{}^{\alpha}\,\dagger$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1}\,\dagger$ | 0 | 0 | 0 | 0 |
| $f_{0^+}^{\#1}\,\dagger$ | 0 | 0 | 0 | 0 |
| $f_{0^+}^{\#2}\,\dagger$ | 0 | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1}\,\dagger$ | 0 | 0 | 0 | $k^2\ r_2 + t_2$ |

| | $\omega_{2^+}^{\#1}{}^{\alpha\beta}$ | $f_{2^+}^{\#1}{}^{\alpha\beta}$ | $\omega_{2^-}^{\#1}{}^{\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1}\,\dagger^{\alpha\beta}$ | 0 | 0 | 0 |
| $f_{2^+}^{\#1}\,\dagger^{\alpha\beta}$ | 0 | 0 | 0 |
| $\omega_{2^-}^{\#1}\,\dagger^{\alpha\beta\chi}$ | 0 | 0 | $k^2\ r_1$ |

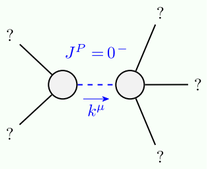
Source constraints/gauge generators

| SO(3) irreps | Multiplicities |
|---|---|
| $\tau_{0^+}^{\#2} == 0$ | 1 |
| $\tau_{0^+}^{\#1} == 0$ | 1 |
| $\sigma_{0^+}^{\#1} == 0$ | 1 |
| $\tau_{1^-}^{\#2}{}^{\alpha} == 0$ | 3 |
| $\tau_{1^-}^{\#1}{}^{\alpha} == 0$ | 3 |
| $\sigma_{1^-}^{\#2}{}^{\alpha} == 0$ | 3 |
| $\tau_{1^+}^{\#1}{}^{\alpha\beta} + i\ k\ \sigma_{1^+}^{\#1}{}^{\alpha\beta} == 0$ | 3 |
| $\sigma_{1^+}^{\#1}{}^{\alpha\beta} == \sigma_{1^+}^{\#2}{}^{\alpha\beta}$ | 3 |
| $\tau_{2^+}^{\#1}{}^{\alpha\beta} == 0$ | 5 |
| $\sigma_{2^+}^{\#1}{}^{\alpha\beta} == 0$ | 5 |
| Total constraints: | 28 |

| | $\sigma_{2^+}^{\#1}{}^{\alpha\beta}$ | $\tau_{2^+}^{\#1}{}^{\alpha\beta}$ | $\sigma_{2^-}^{\#1}{}^{\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1}\,\dagger^{\alpha\beta}$ | 0 | 0 | $\frac{1}{k^2 r_1}$ |
| $\tau_{2^+}^{\#1}\,\dagger^{\alpha\beta}$ | 0 | 0 | 0 |
| $\sigma_{2^-}^{\#1}\,\dagger^{\alpha\beta\chi}$ | 0 | 0 | 0 |

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}\,\dagger$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#1}\,\dagger$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#2}\,\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1}\,\dagger$ | 0 | 0 | 0 | $\frac{1}{k^2 r_2 + t_2}$ |

## Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $-\frac{t_2}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

(No massless particles)

## Unitarity conditions

$r_2 < 0\ \&\&\ t_2 > 0$