# Particle spectrograph

## Wave operator and propagator

| | $\sigma_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^+}^{\#2}{}_{\alpha\beta}$ | $\tau_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^-}^{\#1}{}_{\alpha}$ | $\sigma_{1^-}^{\#2}{}_{\alpha}$ | $\tau_{1^-}^{\#1}{}_{\alpha}$ | $\tau_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1}{}^{\dagger\alpha\beta}$ | $\frac{2(t_1+t_2)}{3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2)}$ | $\frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | $\frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2}{}^{\dagger\alpha\beta}$ | $\frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | $\frac{6k^2(2r_1+r_5)+t_1+4t_2}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | $\frac{ik(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1}{}^{\dagger\alpha\beta}$ | $-\frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | $-\frac{ik(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | $\frac{k^2(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1}{}^{\dagger\alpha}$ | 0 | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2t_1}$ | 0 | $\frac{2ik}{t_1+2k^2t_1}$ |
| $\sigma_{1^-}^{\#2}{}^{\dagger\alpha}$ | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2t_1}$ | $\frac{-2k^2(r_1+r_5)+t_1}{(t_1+2k^2t_1)^2}$ | 0 | $-\frac{i\sqrt{2}k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2t_1)^2}$ |
| $\tau_{1^-}^{\#1}{}^{\dagger\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau_{1^-}^{\#2}{}^{\dagger\alpha}$ | 0 | 0 | 0 | $-\frac{2ik}{t_1+2k^2t_1}$ | $\frac{i\sqrt{2}k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2t_1)^2}$ | 0 | $\frac{-4k^4(r_1+r_5)+2k^2t_1}{(t_1+2k^2t_1)^2}$ |

| | | |
|---|---|---|
| 0 | 0 | $k^2 r_1 + \frac{t_1}{2}$ |
| $-\frac{ikt_1}{\sqrt{2}}$ | $k^2 t_1$ | 0 |
| $\frac{t_1}{2}$ | $\frac{ikt_1}{\sqrt{2}}$ | 0 |

### Source constraints

| SO(3) irreps | Fundamental fields | Multiplicities |
|---|---|---|
| $\tau_{0^+}^{\#2} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$ | 1 |
| $\tau_{0^+}^{\#1} - 2ik\,\sigma_{0^+}^{\#1} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha + 2\,\partial_\chi\partial^\chi\partial_\beta\sigma^{\alpha\beta}{}_\alpha$ | 1 |
| $\tau_{1^-}^{\#2\alpha} + 2ik\,\sigma_{1^-}^{\#2\alpha} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$ | 3 |
| $\tau_{1^+}^{\#1\alpha} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$ | 3 |
| $\tau_{1^+}^{\#1\alpha\beta} + ik\,\sigma_{1^+}^{\#2\alpha\beta} == 0$ | $\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} + 2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$ | 3 |
| $\tau_{2^+}^{\#1\alpha\beta} - 2ik\,\sigma_{2^+}^{\#1\alpha\beta} == 0$ | $-i(4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} + 4\,i\,k^\chi\,\partial_\epsilon\partial_\chi\partial^\beta\partial^\alpha\sigma^{\delta\epsilon}{}_\delta - 6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\delta\epsilon} - 6\,i\,k^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} + 6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\alpha\delta\beta} + 6\,i\,k^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\beta\delta\alpha} - 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi - 4\,i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\chi\sigma^{\delta\epsilon}{}_\delta) == 0$ | 5 |
| Total constraints/gauge generators: | | 16 |

| | $\omega_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^+}^{\#2}{}_{\alpha\beta}$ | $f_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^-}^{\#1}{}_{\alpha}$ | $\omega_{1^-}^{\#2}{}_{\alpha}$ | $f_{1^-}^{\#1}{}_{\alpha}$ | $f_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1}{}^{\dagger\alpha\beta}$ | $\frac{1}{6}(6k^2(2r_1+r_5)+t_1+4t_2)$ | $-\frac{t_1-2t_2}{3\sqrt{2}}$ | $\frac{ik(t_1-2t_2)}{3\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2}{}^{\dagger\alpha\beta}$ | $-\frac{t_1-2t_2}{3\sqrt{2}}$ | $\frac{t_1+t_2}{3}$ | $-\frac{1}{3}ik(t_1+t_2)$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1}{}^{\dagger\alpha\beta}$ | $\frac{ik(t_1-2t_2)}{3\sqrt{2}}$ | $-\frac{1}{3}ik(t_1+t_2)$ | $\frac{1}{3}k^2(t_1+t_2)$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1}{}^{\dagger\alpha}$ | 0 | 0 | 0 | $k^2(r_1+r_5)-\frac{t_1}{2}$ | $\frac{t_1}{\sqrt{2}}$ | 0 | $-ikt_1$ |
| $\omega_{1^-}^{\#2}{}^{\dagger\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 |
| $f_{1^-}^{\#1}{}^{\dagger\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2}{}^{\dagger\alpha}$ | 0 | 0 | 0 | $ikt_1$ | 0 | 0 | 0 |

| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1}{}^{\dagger}$ | $-t_1$ | $i\sqrt{2}\,kt_1$ | 0 | 0 |
| $f_{0^+}^{\#1}{}^{\dagger}$ | $-i\sqrt{2}\,kt_1$ | $-2k^2t_1$ | 0 | 0 |
| $f_{0^+}^{\#2}{}^{\dagger}$ | 0 | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1}{}^{\dagger}$ | 0 | 0 | 0 | $t_2$ |

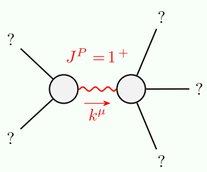
| | $\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$ | $\tau_{2^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{2^+}^{\#1}{}_{\alpha\beta}$ |
|---|---|---|---|
| $\sigma_{2^-}^{\#1}{}^{\dagger\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{2k^2r_1+t_1}$ |
| $\tau_{2^+}^{\#1}{}^{\dagger\alpha\beta}$ | $-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$ | $\frac{4k^2}{(1+2k^2)^2t_1}$ | 0 |
| $\sigma_{2^+}^{\#1}{}^{\dagger\alpha\beta}$ | $\frac{2}{(1+2k^2)^2t_1}$ | $\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$ | 0 |

| | $\sigma_{0^-}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\tau_{0^+}^{\#1}$ | $\sigma_{0^+}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^-}^{\#1}{}^{\dagger}$ | 0 | 0 | 0 | $\frac{1}{t_2}$ |
| $\tau_{0^+}^{\#2}{}^{\dagger}$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#1}{}^{\dagger}$ | $\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$ | $-\frac{2k^2}{(1+2k^2)^2t_1}$ | 0 | 0 |
| $\sigma_{0^+}^{\#1}{}^{\dagger}$ | $-\frac{1}{(1+2k^2)^2t_1}$ | $\frac{ik}{(1+2k^2)^2t_1}$ | 0 | 0 |

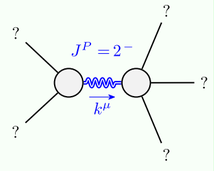
### Quadratic (free) action

$$S == \iiiint(\tfrac{1}{6}(6\,t_1\,\omega^{\alpha\prime}{}_\alpha\,\omega_\prime{}^\theta{}_\theta + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 12\,t_1\,\omega_\alpha{}^\theta{}_\theta\,\partial_\prime f^{\alpha\prime} + 12\,t_1\,\omega_\prime{}^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha - 6\,t_1\,\partial_\prime f^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha - 6\,t_1\,\partial_\prime f^{\alpha\prime}\,\partial_\theta f_\alpha{}^\theta + 12\,t_1\,\partial^\prime f^\alpha{}_\alpha\,\partial_\theta f_\prime{}^\theta + 4\,t_1\,\omega_{\prime\theta\alpha}\,\partial^\theta f^{\alpha\prime} + 4\,t_2\,\omega_{\prime\theta\alpha}\,\partial^\theta f^{\alpha\prime} - 4\,t_1\,\partial_\alpha f_{\prime\theta}\,\partial^\theta f^{\alpha\prime} + 2\,t_2\,\partial_\alpha f_{\prime\theta}\,\partial^\theta f^{\alpha\prime} - 4\,t_1\,\partial_\alpha f_{\theta\prime}\,\partial^\theta f^{\alpha\prime} - t_2\,\partial_\alpha f_{\theta\prime}\,\partial^\theta f^{\alpha\prime} + 2\,t_1\,\partial_\prime f_{\alpha\theta}\,\partial^\theta f^{\alpha\prime} - t_2\,\partial_\prime f_{\alpha\theta}\,\partial^\theta f^{\alpha\prime} + 4\,t_1\,\partial_\theta f_{\alpha\prime}\,\partial^\theta f^{\alpha\prime} + t_2\,\partial_\theta f_{\alpha\prime}\,\partial^\theta f^{\alpha\prime} + 2\,t_1\,\partial_\theta f_{\prime\alpha}\,\partial^\theta f^{\alpha\prime} - t_2\,\partial_\theta f_{\prime\alpha}\,\partial^\theta f^{\alpha\prime} + 2\,(t_1+t_2)\,\omega_{\alpha\prime\theta}\,(\omega^{\alpha\prime\theta} + 2\,\partial^\theta f^{\alpha\prime}) + 2\,\omega_{\alpha\theta\prime}\,((t_1-2t_2)\,\omega^{\alpha\prime\theta} + 2\,(2t_1-t_2)\,\partial^\theta f^{\alpha\prime}) - 8\,r_1\,\partial_\beta\omega_{\alpha\prime\theta}\,\partial^\theta\omega^{\alpha\beta\prime} + 4\,r_1\,\partial_\beta\omega_{\alpha\theta\prime}\,\partial^\theta\omega^{\alpha\beta\prime} - 16\,r_1\,\partial_\beta\omega_{\prime\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\prime} - 4\,r_1\,\partial_\prime\omega_{\alpha\beta\theta}\,\partial^\theta\omega^{\alpha\beta\prime} + 4\,r_1\,\partial_\theta\omega_{\alpha\beta\prime}\,\partial^\theta\omega^{\alpha\beta\prime} + 4\,r_1\,\partial_\theta\omega_{\alpha\prime\beta}\,\partial^\theta\omega^{\alpha\beta\prime} + 6\,r_5\,\partial_\prime\omega_\theta{}^\kappa{}_\kappa\,\partial^\theta\omega^{\alpha\prime}{}_\alpha - 6\,r_5\,\partial_\theta\omega_\prime{}^\kappa{}_\kappa\,\partial^\theta\omega^{\alpha\prime}{}_\alpha - 6\,r_5\,\partial_\alpha\omega^{\alpha\prime\theta}\,\partial_\kappa\omega_\prime{}^\kappa{}_\theta + 12\,r_5\,\partial^\theta\omega^{\alpha\prime}{}_\alpha\,\partial_\kappa\omega_\prime{}^\kappa{}_\theta + 6\,r_5\,\partial_\alpha\omega^{\alpha\prime\theta}\,\partial_\kappa\omega_\theta{}^\kappa{}_\prime - 12\,r_5\,\partial^\theta\omega^{\alpha\prime}{}_\alpha\,\partial_\kappa\omega_\theta{}^\kappa{}_\prime))[t, x, y, z]\,dt\,dx\,dy\,dz$$

## Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $\frac{-3t_1t_2(t_1+t_2)+6r_1(t_1{}^2+2t_2{}^2)+3r_5(t_1{}^2+2t_2{}^2)}{(2r_1+r_5)(t_1+t_2)(-3t_1t_2+4r_1(t_1+t_2)+2r_5(t_1+t_2))} > 0$ |
| Polarisations: | 3 |
| Square mass: | $-\frac{3t_1t_2}{2(2r_1+r_5)(t_1+t_2)} > 0$ |
| Spin: | 1 |
| Parity: | Even |




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_1} > 0$ |
| Polarisations: | 5 |
| Square mass: | $-\frac{t_1}{2r_1} > 0$ |
| Spin: | 2 |
| Parity: | Odd |

(No massless particles)

## Unitarity conditions

$r_1 < 0$ && $r_5 > -2\,r_1$ && $t_1 > 0$ && $-t_1 < t_2 < 0$