Particle spectrograph

Wave operator and propagator

Source constraints			
SO(3) irreps	Fundamental fields		Multiplicities
$\tau_{0^+}^{\#2} == 0$	$\partial_\beta \partial_\alpha t^{\alpha\beta} == 0$		1
$\tau_{0^+}^{\#1} - 2\,i\,k\,\sigma_{0^+}^{\#1} == 0$	$\partial_\alpha \partial_\beta t^{\alpha\beta} == \partial_\beta \partial^\beta t^\alpha{}_\alpha + 2\,\partial_\chi \partial^\chi \partial_\alpha \partial_\beta t^{\alpha\beta}$		1
$\tau_{1^+}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^+}^{\#2\,\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha t^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta t^{\alpha\beta} + 2\,\partial_\alpha \partial^\delta \partial_\chi \partial_\beta t^{\alpha\beta\chi}$		3
$\tau_{1^-}^{\#1\,\alpha} == 0$	$\partial_\chi \partial_\beta \partial^\alpha t^{\beta\chi} == \partial_\chi \partial^\chi \partial_\beta t^{\beta\alpha}$		3
$\tau_{1^+}^{\#1\,\alpha\beta} + i\,k\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$	$\partial_\chi \partial^\alpha t^{\beta\chi} + \partial_\chi \partial^\beta t^{\alpha\chi} + \partial_\chi \partial^\chi t^{\alpha\beta} + 2\,\partial_\alpha \partial_\chi \partial^\chi \sigma^{\beta\chi\delta} + 2\,\partial_\beta \partial^\delta \partial_\chi \sigma^{\alpha\beta\chi} == \partial_\chi \partial^\chi t^{\chi\beta} + \partial_\chi \partial^\beta \sigma^{\alpha\chi} + \partial_\chi \partial^\chi \sigma^{\beta\alpha}$		3
$\tau_{2^+}^{\#1\,\alpha\beta} - 2\,i\,k\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$	$-i\,(4\,\partial_\alpha \partial_\chi \partial^\beta \partial^\delta t^{\chi\delta} + 2\,\partial_\alpha \partial^\delta \partial^\beta \partial^\chi t^\chi{}_\chi - 3\,\partial_\alpha \partial^\delta \partial_\chi \partial^\chi t^{\beta\chi} - 3\,\partial_\alpha \partial^\beta \partial_\chi \partial^\chi t^{\alpha\chi} - 3\,\partial_\alpha \partial^\delta \partial_\chi \partial^\chi t^{\alpha\beta} + 4\,i\,k^\chi\,\partial_\varepsilon \partial_\chi \partial^\beta \partial^\alpha \sigma^{\delta\varepsilon} - 6\,i\,k^\chi\,\partial_\varepsilon \partial_\alpha \partial_\chi \partial^\beta \sigma^{\delta\varepsilon} - 6\,i\,k^\chi\,\partial_\varepsilon \partial_\alpha \partial_\chi \partial^\delta \sigma^{\beta\varepsilon} + 2\,\eta^{\alpha\beta}\,\partial_\varepsilon \partial^\varepsilon \partial_\alpha \partial_\chi t^{\chi\delta} + 6\,i\,k^\chi\,\partial_\varepsilon \partial^\varepsilon \partial_\alpha \partial_\chi \sigma^{\alpha\delta\beta} + 6\,i\,k^\chi\,\partial_\varepsilon \partial^\varepsilon \partial_\alpha \partial_\chi \sigma^{\beta\delta\alpha} - 2\,\eta^{\alpha\beta}\,\partial_\varepsilon \partial^\varepsilon \partial_\delta \partial^\delta t^\chi{}_\chi + 4\,i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi \partial^\phi \partial_\varepsilon \partial_\chi \sigma^{\delta\varepsilon}) == 0$		5
Total constraints/gauge generators:			16

Quadratic (free) action

$$\mathcal{S} == \iiiint \left(\frac{1}{6}\,(6\,t_1\,\omega^{\alpha\prime}{}_\alpha\,\omega_\prime{}^\theta{}_\theta + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 12\,t_1\,\omega_\alpha{}^\theta{}_\theta\,\partial_\prime f^{\alpha\prime} + 12\,t_1\,\omega_\prime{}^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha - 6\,t_1\,\partial_\prime f^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha - 12\,r_1\,\partial_\beta \omega_\prime{}^\theta{}_\theta\,\partial^\prime \omega^{\alpha\beta}{}_\alpha + 12\,r_1\,\partial_\prime \omega_\beta{}^\theta{}_\theta\,\partial^\prime \omega^{\alpha\beta}{}_\alpha - 6\,t_1\,\partial_\prime f^{\alpha\prime}\,\partial_\theta f_\alpha{}^\theta + 12\,t_1\,\partial^\prime f^\alpha{}_\alpha\,\partial_\theta f_\prime{}^\theta + 12\,r_1\,\partial_\alpha \omega^{\alpha\beta\prime}\,\partial_\theta \omega_\beta{}^\theta{}_\prime - 24\,r_1\,\partial^\prime \omega^{\alpha\beta}{}_\alpha\,\partial_\theta \omega_\beta{}^\theta{}_\prime - 12\,r_1\,\partial_\alpha \omega^{\alpha\beta\prime}\,\partial_\theta \omega_\prime{}^\theta{}_\beta + 24\,r_1\,\partial^\prime \omega^{\alpha\beta}{}_\alpha\,\partial_\theta \omega_\prime{}^\theta{}_\beta + 4\,t_1\,\omega_{\prime\theta\alpha}\,\partial^\theta f^{\alpha\prime} + 4\,t_2\,\omega_{\prime\theta\alpha}\,\partial^\theta f^{\alpha\prime} - 4\,t_1\,\partial_\alpha f_{\prime\theta}\,\partial^\theta f^{\alpha\prime} + 2\,t_2\,\partial_\alpha f_{\prime\theta}\,\partial^\theta f^{\alpha\prime} - 4\,t_1\,\partial_\alpha f_{\theta\prime}\,\partial^\theta f^{\alpha\prime} - t_2\,\partial_\alpha f_{\theta\prime}\,\partial^\theta f^{\alpha\prime} + 2\,t_1\,\partial_\prime f_{\alpha\theta}\,\partial^\theta f^{\alpha\prime} - t_2\,\partial_\prime f_{\alpha\theta}\,\partial^\theta f^{\alpha\prime} + 4\,t_1\,\partial_\theta f_{\alpha\prime}\,\partial^\theta f^{\alpha\prime} + t_2\,\partial_\theta f_{\alpha\prime}\,\partial^\theta f^{\alpha\prime} + 2\,t_1\,\partial_\theta f_{\prime\alpha}\,\partial^\theta f^{\alpha\prime} - t_2\,\partial_\theta f_{\prime\alpha}\,\partial^\theta f^{\alpha\prime} + 2\,(t_1+t_2)\,\omega_{\alpha\prime\theta}\,(\omega^{\alpha\prime\theta} + 2\,\partial^\theta f^{\alpha\prime}) + 2\,\omega_{\alpha\theta\prime}\,((t_1-2\,t_2)\,\omega^{\alpha\prime\theta} + 2\,(2\,t_1-t_2)\,\partial^\theta f^{\alpha\prime}) - 8\,r_1\,\partial_\beta \omega_{\alpha\prime\theta}\,\partial^\theta \omega^{\alpha\beta\prime} + 8\,r_2\,\partial_\beta \omega_{\alpha\prime\theta}\,\partial^\theta \omega^{\alpha\beta\prime} + 4\,r_1\,\partial_\beta \omega_{\alpha\theta\prime}\,\partial^\theta \omega^{\alpha\beta\prime} - 4\,r_2\,\partial_\beta \omega_{\alpha\theta\prime}\,\partial^\theta \omega^{\alpha\beta\prime} - 16\,r_1\,\partial_\beta \omega_{\prime\theta\alpha}\,\partial^\theta \omega^{\alpha\beta\prime} + 4\,r_2\,\partial_\beta \omega_{\prime\theta\alpha}\,\partial^\theta \omega^{\alpha\beta\prime} - 4\,r_1\,\partial_\prime \omega_{\alpha\beta\theta}\,\partial^\theta \omega^{\alpha\beta\prime} + 2\,r_2\,\partial_\prime \omega_{\alpha\beta\theta}\,\partial^\theta \omega^{\alpha\beta\prime} + 4\,r_1\,\partial_\theta \omega_{\alpha\beta\prime}\,\partial^\theta \omega^{\alpha\beta\prime} + 2\,r_2\,\partial_\theta \omega_{\alpha\beta\prime}\,\partial^\theta \omega^{\alpha\beta\prime} + 4\,r_1\,\partial_\theta \omega_{\alpha\prime\beta}\,\partial^\theta \omega^{\alpha\beta\prime} - 4\,r_2\,\partial_\theta \omega_{\alpha\prime\beta}\,\partial^\theta \omega^{\alpha\beta\prime}))[t, x, y, z]\,dt\,dz\,dy\,dx\,dt$$

$\sigma_{1^+}^{\#1\,\alpha\beta}$

$\frac{2\,(t_1+t_2)}{3\,t_1\,t_2}$	$\frac{\sqrt{2}\,(t_1-2\,t_2)}{3\,t_1\,t_2}$	$-\frac{i\,\sqrt{2}\,k\,(t_1-2\,t_2)}{3\,(1+k^2)\,t_1\,t_2}$	0	0	0	0	0	0	0
$\frac{\sqrt{2}\,(t_1-2\,t_2)}{3\,t_1\,t_2}$	$\frac{t_1+4\,t_2}{3\,(1+k^2)^2\,t_1\,t_2}$	$-\frac{i\,k\,(t_1+4\,t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$	0	0	0	0	0	0	0
$-\frac{i\,\sqrt{2}\,k\,(t_1-2\,t_2)}{3\,(1+k^2)\,t_1\,t_2}$	$\frac{i\,k\,(t_1+4\,t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$	$\frac{k^2\,(t_1+4\,t_2)}{3\,(1+k^2)^2\,t_1\,t_2}$	0	0	0	0	0	0	0

$\sigma_{1^-}^{\#2\,\alpha}$

0	0	0	$\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$
0	$\frac{\sqrt{2}}{t_1+2\,k^2\,t_1}$	0	$\frac{i\,\sqrt{2}\,k\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$
0	0	0	0
0	$-\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$	0	$\frac{2\,k^2\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$

$\tau_{1^-}^{\#1\,\alpha}$

0	0	0	$\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$
0	$\frac{\sqrt{2}}{t_1+2\,k^2\,t_1}$	0	$\frac{i\,\sqrt{2}\,k\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$
0	0	0	0
0	$-\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$	0	$\frac{2\,k^2\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$

$\tau_{1^-}^{\#2\,\alpha}$

0	0	0	$\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$
0	$\frac{\sqrt{2}}{t_1+2\,k^2\,t_1}$	0	$\frac{i\,\sqrt{2}\,k\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$
0	0	0	0
0	$-\frac{2\,i\,k}{t_1+2\,k^2\,t_1}$	0	$\frac{2\,k^2\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$

$\omega_{1^+}^{\#1\,\alpha\beta}$

0	0	0	0
0	0	0	0
0	0	0	0
$-k^2\,r_1 - \frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$i\,k\,t_1$

$\omega_{1^+}^{\#2\,\alpha\beta}$

0	0	0	0
0	0	0	0
0	0	0	0
$\frac{t_1}{\sqrt{2}}$	0	0	0

$f_{1^+}^{\#1\,\alpha\beta}$

$-\frac{i\,k\,(t_1-2\,t_2)}{3\,\sqrt{2}}$	0	0	0
$\frac{1}{3}\,i\,k\,(t_1+t_2)$	0	0	0
$\frac{1}{3}\,i\,k\,(t_1+t_2)$	0	0	0

$f_{1^+}^{\#2\,\alpha\beta}$

0	0	0	0
0	0	0	0
0	0	0	0
$\frac{t_1}{\sqrt{2}}$	0	0	0

$\omega_{1^+}^{\#1\,\alpha\beta}$

$\frac{1}{6}\,(t_1+4\,t_2)$	$-\frac{t_1-2\,t_2}{3\,\sqrt{2}}$	$\frac{i\,k\,(t_1-2\,t_2)}{3\,\sqrt{2}}$	0
$-\frac{t_1-2\,t_2}{3\,\sqrt{2}}$	$\frac{t_1+2\,t_2}{3}$	$-\frac{1}{3}\,i\,k\,(t_1+t_2)$	0
$\frac{i\,k\,(t_1-2\,t_2)}{3\,\sqrt{2}}$	$-\frac{1}{3}\,i\,k\,(t_1+t_2)$	$\frac{1}{3}\,k^2\,(t_1+t_2)$	0
0	0	0	0

$\omega_{1^-}^{\#2\,\alpha}$

0	0	0	$i\,k\,t_1$
0	0	0	0
0	0	0	0
$\frac{t_1}{\sqrt{2}}$	0	0	0

$f_{1^-}^{\#1\,\alpha}$

0	0	0	0
0	0	0	0
0	0	0	0
0	$\frac{t_1}{\sqrt{2}}$	0	$-i\,k\,t_1$

$f_{1^-}^{\#2\,\alpha}$

0	0	0	0
0	0	0	0
0	0	0	0
$-\frac{i\,\sqrt{2}\,k\,(2\,k^2\,r_1+t_1)}{(t_1+2\,k^2\,t_1)^2}$	0	0	0

$\omega_{0^+}^{\#1}$

$-t_1$	$i\,\sqrt{2}\,k\,t_1$	0	0
$-i\,\sqrt{2}\,k\,t_1$	$-2\,k^2\,t_1$	0	0
0	0	0	0
0	0	0	$k^2\,r_2 + t_2$

$\omega_{0^-}^{\#1}$

$-\frac{1}{(1+2\,k^2)^2\,t_1}$	$\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	0	0
$-\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	$-\frac{2\,k^2}{(1+2\,k^2)^2\,t_1}$	0	0
0	0	0	0
0	0	0	$\frac{1}{k^2\,r_2+t_2}$

$\tau_{2^+}^{\#1\,\alpha\beta}$

0	0	0	$\frac{2}{2\,k^2\,r_1+t_1}$
0	$\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	$\frac{4\,k^2}{(1+2\,k^2)^2\,t_1}$	0
$\frac{2}{(1+2\,k^2)^2\,t_1}$	$-\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	0	0
0	0	0	0

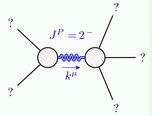
$\omega_{2^+}^{\#1\,\alpha\beta}$

0	0	0	$\frac{t_1}{2}$
0	$-\frac{i\,k\,t_1}{\sqrt{2}}$	$k^2\,t_1$	0
0	$\frac{i\,k\,t_1}{\sqrt{2}}$	0	0
$\frac{t_1}{2}$	0	0	$k^2\,r_1 + \frac{t_1}{2}$

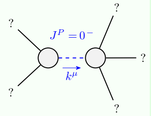
$\sigma_{2^+}^{\#1\,\alpha\beta}$

0	0	0	$\frac{2}{2\,k^2\,r_1+t_1}$
0	$\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	$\frac{4\,k^2}{(1+2\,k^2)^2\,t_1}$	0
$\frac{2}{(1+2\,k^2)^2\,t_1}$	$-\frac{2\,i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_1}$	0	0
0	0	0	0

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2\,r_1} > 0$
Spin:	2
Parity:	Odd



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0 \;\&\&\; r_2 < 0 \;\&\&\; t_1 > 0 \;\&\&\; t_2 > 0$