

# Particle spectrograph

## Wave operator and propagator

$\Delta_1^{\#1} \dagger \alpha\beta$	$\Delta_1^{\#2} \dagger \alpha\beta$	$\Delta_1^{\#3} \dagger \alpha\beta$	$\Delta_1^{\#1-} \alpha$	$\Delta_1^{\#2-} \alpha$	$\Delta_1^{\#3-} \alpha$	$\Delta_1^{\#4-} \alpha$	$\Delta_1^{\#5-} \alpha$	$\Delta_1^{\#6-} \alpha$	$\mathcal{T}_1^{\#1-} \alpha$
$\Delta_1^{\#1} \dagger \alpha\beta$	$0$	$-\frac{2\sqrt{2}}{a_0}$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Delta_1^{\#2} \dagger \alpha\beta$	$-\frac{2\sqrt{2}}{a_0}$	$\frac{2}{a_0}$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Delta_1^{\#3} \dagger \alpha\beta$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Delta_1^{\#1-} \alpha$	$0$	$0$	$0$	$\frac{\sqrt{2}(4+k^2)}{a_0(2+k^2)}$	$-\frac{2k^2}{\sqrt{3}a_0(2+k^2)}$	$0$	$\frac{\sqrt{2}k^2}{a_0(2+k^2)}$	$0$	$-\frac{2i\sqrt{2}k}{a_0(2+k^2)}$
$\Delta_1^{\#2-} \alpha$	$0$	$0$	$\frac{\sqrt{2}(4+k^2)}{a_0(2+k^2)}$	$\frac{(4+k^2)^2}{2a_0(2+k^2)^2}$	$\frac{k^2(-2+k^2)}{2\sqrt{6}a_0(2+k^2)^2}$	$-\frac{\sqrt{5}k^2}{4a_0+2a_0k^2}$	$\frac{k^2(5+2k^2)}{\sqrt{3}a_0(2+k^2)^2}$	$-\frac{k^2}{\sqrt{6}a_0(2+k^2)^2}$	$-\frac{i k(4+k^2)}{a_0(2+k^2)^2}$
$\Delta_1^{\#3-} \alpha$	$0$	$0$	$-\frac{2k^2}{\sqrt{3}(2a_0+a_0k^2)}$	$\frac{k^2(-2+k^2)}{2\sqrt{6}a_0(2+k^2)^2}$	$\frac{76+52k^2+3k^4}{12a_0(2+k^2)^2}$	$\frac{\sqrt{5}(10+3k^2)}{12a_0(2+k^2)}$	$\frac{-2+k^2}{3\sqrt{2}a_0(2+k^2)^2}$	$\frac{1}{-2a_0-\frac{8a_0}{2+3k^2}}$	$\frac{i k(6+5k^2)}{\sqrt{6}a_0(2+k^2)^2}$
$\Delta_1^{\#4-} \alpha$	$0$	$0$	$0$	$-\frac{\sqrt{5}k^2}{4a_0+2a_0k^2}$	$\frac{\sqrt{5}(10+3k^2)}{12a_0(2+k^2)}$	$\frac{1}{12a_0}$	$-\frac{\sqrt{5}}{6a_0+3a_0k^2}$	$-\frac{\sqrt{5}}{6a_0}$	$-\frac{i\sqrt{\frac{5}{6}}k}{a_0(2+k^2)}$
$\Delta_1^{\#5-} \alpha$	$0$	$0$	$\frac{\sqrt{2}k^2}{2a_0+a_0k^2}$	$\frac{k^2(5+2k^2)}{\sqrt{3}a_0(2+k^2)^2}$	$\frac{-2+k^2}{3\sqrt{2}a_0(2+k^2)^2}$	$\frac{\sqrt{5}}{6a_0+3a_0k^2}$	$-\frac{\sqrt{2}(7+14k^2+3k^4)}{3a_0(2+k^2)^2}$	$\frac{5}{3a_0}$	$-\frac{i\sqrt{\frac{2}{3}}k}{a_0(2+k^2)^2}$
$\Delta_1^{\#6-} \alpha$	$0$	$0$	$0$	$-\frac{k^2}{\sqrt{6}(2a_0+a_0k^2)}$	$\frac{i k(4+k^2)}{a_0(2+k^2)^2}$	$-\frac{\sqrt{5}}{\sqrt{6}a_0(2+k^2)^2}$	$-\frac{2ik(3+k^2)}{\sqrt{3}a_0(2+k^2)^2}$	$\frac{5}{3a_0}$	$-\frac{i\sqrt{\frac{2}{3}}k}{a_0(2+k^2)^2}$
$\mathcal{T}_1^{\#1-} \alpha$	$0$	$0$	$\frac{2i\sqrt{2}k}{2a_0+a_0k^2}$	$-\frac{ik(4+k^2)}{a_0(2+k^2)^2}$	$-\frac{ik(6+5k^2)}{\sqrt{6}a_0(2+k^2)^2}$	$\frac{i\sqrt{\frac{5}{6}}k}{2a_0+a_0k^2}$	$-\frac{2ik(3+k^2)}{\sqrt{3}a_0(2+k^2)^2}$	$\frac{i\sqrt{\frac{2}{3}}k}{2a_0+a_0k^2}$	$\frac{2k^2}{a_0(2+k^2)^2}$

$\Delta_0^{\#1} \dagger$	$\Delta_0^{\#2} \dagger$	$\Delta_0^{\#3} \dagger$	$\mathcal{T}_0^{\#1} \dagger$	$\Delta_0^{\#2-} \alpha$	$\Delta_0^{\#3-} \alpha$	$\Delta_0^{\#4-} \alpha$	$\Delta_0^{\#5-} \alpha$	$\Delta_0^{\#6-} \alpha$	$\mathcal{T}_0^{\#1-} \alpha$
$\Delta_0^{\#1} \dagger$	$0$	$-\frac{4\sqrt{6}}{16a_0+3a_0k^2}$	$-\frac{8}{\sqrt{3}(16a_0+3a_0k^2)}$	$-\frac{4\sqrt{\frac{2}{3}}}{16a_0+3a_0k^2}$	$-\frac{8}{\sqrt{3}(16a_0+3a_0k^2)}$	$-\frac{2i\sqrt{2}k}{a_0k}$	$-\frac{2i\sqrt{6}k}{16a_0+3a_0k^2}$	$0$	$-\frac{2i\sqrt{6}k}{16a_0+3a_0k^2}$
$\Delta_0^{\#2} \dagger$	$\frac{4\sqrt{6}}{16a_0+3a_0k^2}$	$-\frac{144}{a_0(16+3k^2)^2}$	$-\frac{8\sqrt{2}(10+3k^2)}{a_0(16+3k^2)^2}$	$-\frac{16(19+3k^2)}{a_0(16+3k^2)^2}$	$-\frac{8\sqrt{2}(10+3k^2)}{a_0(16+3k^2)^2}$	$-\frac{8i\sqrt{3}}{16a_0k+3a_0k^3}$	$\frac{72ik}{a_0(16+3k^2)^2}$	$0$	$\frac{72ik}{a_0(16+3k^2)^2}$
$\Delta_0^{\#3} \dagger$	$-\frac{4\sqrt{\frac{2}{3}}}{16a_0+3a_0k^2}$	$\frac{16(19+3k^2)}{a_0(16+3k^2)^2}$	$-\frac{16(35+6k^2)}{3a_0(16+3k^2)^2}$	$-\frac{16(19+3k^2)}{3a_0(16+3k^2)^2}$	$-\frac{8\sqrt{2}(22+3k^2)}{3a_0(16+3k^2)^2}$	$\frac{8i}{\sqrt{3}(16a_0k+3a_0k^3)}$	$-\frac{8ik(19+3k^2)}{a_0(16+3k^2)^2}$	$0$	$-\frac{8ik(19+3k^2)}{a_0(16+3k^2)^2}$
$\Delta_0^{\#4} \dagger$	$-\frac{8}{\sqrt{3}(16a_0+3a_0k^2)}$	$-\frac{8\sqrt{2}(10+3k^2)}{a_0(16+3k^2)^2}$	$-\frac{8\sqrt{2}(22+3k^2)}{3a_0(16+3k^2)^2}$	$-\frac{32(13+3k^2)}{3a_0(16+3k^2)^2}$	$-\frac{8\sqrt{\frac{2}{3}}}{16a_0k+3a_0k^3}$	$\frac{8i\sqrt{\frac{2}{3}}}{16a_0k+3a_0k^3}$	$\frac{4i\sqrt{2}k(10+3k^2)}{a_0(16+3k^2)^2}$	$0$	$\frac{4i\sqrt{2}k(10+3k^2)}{a_0(16+3k^2)^2}$
$\mathcal{T}_0^{\#1} \dagger$	$\frac{2i\sqrt{2}}{a_0k}$	$\frac{8i\sqrt{3}}{16a_0k+3a_0k^3}$	$-\frac{8i}{\sqrt{3}(16a_0k+3a_0k^3)}$	$-\frac{8i\sqrt{\frac{2}{3}}}{16a_0k+3a_0k^3}$	$-\frac{4}{a_0k^2}$	$-\frac{4\sqrt{3}}{16a_0+3a_0k^2}$	$\frac{4\sqrt{3}}{16a_0+3a_0k^2}$	$0$	$\frac{4\sqrt{3}}{16a_0+3a_0k^2}$
$\mathcal{T}_0^{\#2} \dagger$	$\frac{2i\sqrt{6}k}{16a_0+3a_0k^2}$	$-\frac{72ik}{a_0(16+3k^2)^2}$	$-\frac{8ik(19+3k^2)}{a_0(16+3k^2)^2}$	$-\frac{4i\sqrt{2}k(10+3k^2)}{a_0(16+3k^2)^2}$	$-\frac{4\sqrt{3}}{16a_0+3a_0k^2}$	$-\frac{36k^2}{a_0(16+3k^2)^2}$	$-\frac{36k^2}{a_0(16+3k^2)^2}$	$0$	$-\frac{36k^2}{a_0(16+3k^2)^2}$
$\Delta_0^{\#1-} \dagger$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$-\frac{2}{a_0}$	$-\frac{2}{a_0}$

$\Delta_3^{\#1-} \alpha\beta\chi$

$\Delta_3^{\#1-} \dagger \alpha\beta\chi$

$-\frac{2}{a_0}$

$\Gamma_3^{\#1-} \alpha\beta\chi$

$\Gamma_3^{\#1-} \dagger \alpha\beta\chi$

$-\frac{a_0}{2}$

$\Gamma_2^{\#1} \dagger \alpha\beta$	$\Gamma_2^{\#2} \dagger \alpha\beta$	$\Gamma_2^{\#3} \dagger \alpha\beta$	$h_2^{\#1} \dagger \alpha\beta$	$\Gamma_2^{\#1-} \alpha\beta\chi$	$\Gamma_2^{\#2-} \alpha\beta\chi$
$\Gamma_2^{\#1} \dagger \alpha\beta$	$\frac{a_0}{4}$	$0$	$0$	$\frac{ia_0k}{4\sqrt{2}}$	$0$
$\Gamma_2^{\#2} \dagger \alpha\beta$	$0$	$-\frac{a_0}{2}$	$0$	$\frac{ia_0k}{4\sqrt{3}}$	$0$
$\Gamma_2^{\#3} \dagger \alpha\beta$	$0$	$0$	$\frac{a_0}{4}$	$-\frac{ia_0k}{4\sqrt{6}}$	$0$
$h_2^{\#1} \dagger \alpha\beta$	$-\frac{ia_0k}{4\sqrt{2}}$	$-\frac{ia_0k}{4\sqrt{3}}$	$\frac{ia_0k}{4\sqrt{6}}$	$0$	$0$
$\Gamma_2^{\#1-} \dagger \alpha\beta\chi$	$0$	$0$	$0$	$\frac{16}{4}$	$0$
$\Gamma_2^{\#2-} \dagger \alpha\beta\chi$	$0$	$0$	$0$	$0$	$\frac{16}{4}$

$\Gamma_1^{\#1} \dagger \alpha\beta$	$\Gamma_1^{\#2} \dagger \alpha\beta$	$\Gamma_1^{\#3} \dagger \alpha\beta$	$\Gamma_1^{\#1-} \alpha$	$\Gamma_1^{\#2-} \alpha$	$\Gamma_1^{\#3-} \alpha$	$\Gamma_1^{\#4-} \alpha$	$\Gamma_1^{\#5-} \alpha$	$\Gamma_1^{\#6-} \alpha$	$h_1^{\#1-} \alpha$
$\Gamma_1^{\#1} \dagger \alpha\beta$	$-\frac{a_0}{4}$	$-\frac{a_0}{2\sqrt{2}}$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_1^{\#2} \dagger \alpha\beta$	$-\frac{a_0}{2\sqrt{2}}$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_1^{\#3} \dagger \alpha\beta$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_1^{\#1-} \alpha$	$0$	$0$	$0$	$-\frac{a_0}{4}$	$\frac{a_0}{2\sqrt{2}}$	$0$	$0$	$0$	$-\frac{ia_0k}{4\sqrt{2}}$
$\Gamma_1^{\#2-} \alpha$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_1^{\#3-} \alpha$	$0$	$0$	$0$	$0$	$-\frac{a_0}{3}$	$\frac{\sqrt{5}a_0}{6}$	$-\frac{a_0}{6\sqrt{2}}$	$-\frac{a_0}{6}$	$\frac{ia_0k}{4\sqrt{6}}$
$\Gamma_1^{\#4-} \alpha$	$0$	$0$	$0$	$0$	$\frac{\sqrt{5}a_0}{6}$	$\frac{a_0}{3}$	$-\frac{\sqrt{5}a_0}{6}$	$-\frac{1}{6}\sqrt{\frac{5}{2}}a_0$	$-\frac{1}{4}i\sqrt{\frac{5}{6}}a_0k$
$\Gamma_1^{\#5-} \alpha$	$0$	$0$	$0$	$0$	$-\frac{a_0}{6\sqrt{2}}$	$\frac{a_0}{3}$	$\frac{a_0}{6}$	$\frac{a_0}{6\sqrt{2}}$	$\frac{ia_0k}{4\sqrt{3}}$
$\Gamma_1^{\#6-} \alpha$	$0$	$0$	$0$	$0$	$-\frac{a_0}{6}$	$\frac{a_0}{6\sqrt{2}}$	$-\frac{\sqrt{5}a_0}{6}$	$\frac{5a_0}{12}$	$\frac{ia_0k}{4\sqrt{6}}$
$h_1^{\#1-} \alpha$	$0$	$0$	$0$	$-\frac{ia_0k}{4\sqrt{2}}$	$-\frac{ia_0k}{4\sqrt{6}}$	$-\frac{1}{4}i\sqrt{\frac{5}{6}}a_0k$	$-\frac{ia_0k}{4\sqrt{3}}$	$-\frac{ia_0k}{4\sqrt{6}}$	$0$

$\Gamma_0^{\#1} \dagger$	$\Gamma_0^{\#2} \dagger$	$\Gamma_0^{\#3} \dagger$	$\Gamma_0^{\#4} \dagger$	$\Gamma_0^{\#1-} \alpha$	$\Gamma_0^{\#2-} \alpha$	$\Gamma_0^{\#3-} \alpha$	$\Gamma_0^{\#4-} \alpha$	$\Gamma_0^{\#5-} \alpha$	$\Gamma_0^{\#6-} \alpha$	$h_0^{\#1-} \alpha$
$\Gamma_0^{\#1} \dagger$	$-\frac{a_0}{2}$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#2} \dagger$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#3} \dagger$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#4} \dagger$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$h_0^{\#1} \dagger$	$\frac{ia_0k}{2\sqrt{2}}$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#1-} \alpha$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#2-} \alpha$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#3-} \alpha$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#4-} \alpha$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$
$h_0^{\#1-} \alpha$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$	$0$

Source constraints/gauge generators	
SO(3) irreps	Multiplicities
$2\mathcal{T}_{0+}^{\#2}-i k\Delta_{0+}^{\#2}==0$	1
$\Delta_{0+}^{\#3}+2\Delta_{0+}^{\#4}+3\Delta_{0+}^{\#2}==0$	1
$6\mathcal{T}_1^{\#1\alpha}-i k(3\Delta_1^{\#2\alpha}-\Delta_1^{\#5\alpha}+\Delta_1^{\#3\alpha})==0$	3
$2\Delta_1^{\#6\alpha}+\Delta_1^{\#4\alpha}+2\Delta_1^{\#5\alpha}+\Delta_1^{\#3\alpha}==0$	3
Total constraints:	8

Quadratic (free) action

$$S_F == \iiint\iiint(\frac{1}{4}$$
$$(2a_0\Gamma_a^{\alpha\beta}\Gamma_{\beta\chi}^{\chi}+4h^{\alpha\beta}\mathcal{T}_{\alpha\beta}+\Gamma^{\alpha\beta\chi}(-2a_0\Gamma_{\beta\chi\alpha}+4\Delta_{\alpha\beta\chi})-a_0h_{\chi}^{\chi}\partial_{\beta}\Gamma_a^{\alpha\beta}+$$
$$a_0h_{\chi}^{\chi}\partial_{\beta}\Gamma^{\alpha\beta}_{\alpha}-2a_0h_{\alpha\chi}\partial_{\beta}\Gamma^{\alpha\beta\chi}+2a_0h_{\beta\chi}\partial^{\chi}\Gamma^{\alpha\beta}_{\alpha}))[t,x,y,z]dzdydxdt$$

$\Gamma_0^{\#1} \dagger$	$\Gamma_0^{\#2} \dagger$	$\Gamma_0^{\#3} \dagger$	$\Gamma_0^{\#4} \dagger$	$h_0^{\#1} \dagger$	$\Gamma_0^{\#1-} \alpha$
$\Gamma_0^{\#1} \dagger$	$-\frac{a_0}{2}$	$0$	$0$	$-\frac{ia_0k}{2\sqrt{2}}$	$0$
$\Gamma_0^{\#2} \dagger$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#3} \dagger$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#4} \dagger$	$0$	$0$	$0$	$0$	$0$
$h_0^{\#1} \dagger$	$\frac{ia_0k}{2\sqrt{2}}$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#1-} \alpha$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#2-} \alpha$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#3-} \alpha$	$0$	$0$	$0$	$0$	$0$
$\Gamma_0^{\#4-} \alpha$	$0$	$0$	$0$	$0$	$0$
$h_0^{\#1-} \alpha$	$0$	$0$	$0$	$0$	$0$

$\Delta_2^{\#1} \dagger \alpha\beta$	$\Delta_2^{\#2} \dagger \alpha\beta$	$\Delta_2^{\#3} \dagger \alpha\beta$	$\mathcal{T}_2^{\#1} \dagger \alpha\beta$	$\Delta_2^{\#1-} \alpha\beta\chi$	$\Delta_2^{\#2-} \alpha\beta\chi$
$\Delta_2^{\#1} \dagger \alpha\beta$	$0$	$\frac{2\sqrt{\frac{2}{3}}}{a_0}$	$\frac{4}{\sqrt{3}a_0}$	$\frac{4i\sqrt{2}}{a_0k}$	$0$
$\Delta_2^{\#2} \dagger \alpha\beta$	$\frac{2\sqrt{\frac{2}{3}}}{a_0}$	$-\frac{8}{3a_0}$	$-\frac{2\sqrt{2}}{3a_0}$	$-\frac{4i}{\sqrt{3}a_0k}$	$0$
$\Delta_2^{\#3} \dagger \alpha\beta$	$\frac{4}{\sqrt{3}a_0}$	$-\frac{2\sqrt{2}}{3a_0}$	$\frac{8}{3a_0}$	$-\frac{4i\sqrt{\frac{2}{3}}}{a_0k}$	$0$
$\mathcal{T}_2^{\#1} \dagger \alpha\beta$	$-\frac{4i\sqrt{2}}{a_0k}$	$\frac{4i}{\sqrt{3}a_0k}$	$\frac{4i\sqrt{\frac{2}{3}}}{a_0k}$	$-\frac{8}{a_0k^2}$	$0$
$\Delta_2^{\#1-} \alpha\beta\chi$	$0$	$0$	$0$	$\frac{4}{a_0}$	$0$
$\Delta_2^{\#2-} \alpha\beta\chi$	$0$	$0$	$0$	$0$	$\frac{4}{a_0}$

## Massive and massless spectra

Quadratic pole

Pole residue:  $-\frac{1}{a_0} > 0$

Polarisations: 2

(No massive particles)

## Unitarity conditions

$a_0 < 0$