Particle spectrograph

Wave operator and propagator

Quadratic (free) action

$$
\begin{aligned}
S ={}& \iiint\int \Big(\tfrac{1}{3}\big(3 t_1\,\omega^{\alpha\iota}{}_\alpha\,\omega_{\iota\theta}{}^\theta + 3 f^{\alpha\beta}\tau_{\alpha\beta} + 3\,\omega^{\alpha\beta\chi}\sigma_{\alpha\beta\chi} - 6 t_1\,\omega_{\alpha\theta}{}^\theta\,\partial_\iota f^{\alpha\iota} + 6 t_1\,\omega_{\iota\theta}{}^\theta\, \\
& \partial_\iota f^\alpha{}_\alpha - 3 t_1\,\partial_\iota f^\alpha{}_\theta\,\partial^\theta f^\alpha{}_\iota - 6 r_1\,\partial_\beta\omega_{\iota\theta}{}^\theta\,\partial^\iota\omega^{\alpha\beta}{}_\alpha + 6 r_1\,\omega_{\beta\theta}{}^\theta\,\partial_\iota\partial^\iota\omega^{\alpha\beta}{}_\alpha - \\
& 3 t_1\,\partial_\iota f^\alpha{}_\alpha\,\partial_\theta f^{\alpha\theta} + 6 t_1\,\partial_\iota f^\alpha{}_\theta\,\partial^\theta f^\alpha{}_\iota + 6 r_1\,\partial_\alpha\omega^{\alpha\beta}{}_\theta\,\partial_\beta\omega_{\iota}{}^{\alpha\iota} - 12 r_1\,\partial^\iota\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_{\beta\iota}{}^\theta - \\
& 6 r_1\,\partial_\alpha\omega^{\alpha\beta}{}_\theta\,\partial_\theta\omega_{\iota,\beta}{}^\theta + 12 r_1\,\partial_\iota\omega^{\alpha\beta}{}_\alpha\,\partial_\theta\omega_{\iota,\beta}{}^\theta + 2 t_1\,\partial_\iota\partial^\theta f^{\alpha\iota} - \\
& 2 t_1\,\partial_\alpha\partial^\theta f^\alpha{}_\iota - 2 t_1\,\partial_\alpha f_{\iota\theta}\,\partial^\theta f^\alpha{}_\iota + t_1\,\partial_\iota f_{\alpha\theta}\,\partial^\theta f^{\alpha\iota} + 2 t_1\,\partial_\theta f_{\alpha\iota}\,\partial^\theta f^{\alpha\iota} + \\
& t_1\,\partial_\theta f_{\iota\alpha}\,\partial^\theta f^{\alpha\iota} + t_1\,\omega_{\alpha\iota\theta}\big(\omega^{\alpha\iota\theta} + 2\partial^\theta f^{\alpha\iota}\big) + t_1\,\omega_{\alpha\theta\iota}\big(\omega^{\alpha\iota\theta} + 4\partial^\theta f^{\alpha\iota}\big) - \\
& 4 r_1\,\partial_\beta\omega_{\alpha\iota\theta}\,\partial^\theta\omega^{\alpha\beta\iota} + 4 r_2\,\partial_\beta\omega_{\alpha\iota\theta}\,\partial^\theta\omega^{\alpha\beta\iota} + 2 r_1\,\partial_\beta\omega_{\alpha\theta\iota}\,\partial^\theta\omega^{\alpha\beta\iota} - \\
& 2 r_2\,\partial_\beta\omega_{\alpha\theta\iota}\,\partial^\theta\omega^{\alpha\beta\iota} - 8 r_1\,\partial_\beta\omega_{\iota\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\iota} + 2 r_2\,\partial_\beta\omega_{\iota\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\iota} + \\
& 2 r_1\,\partial_\iota\omega_{\alpha\beta\theta}\,\partial^\theta\omega^{\alpha\beta\iota} - r_2\,\partial_\iota\omega_{\alpha\beta\theta}\,\partial^\theta\omega^{\alpha\beta\iota} + r_2\,\partial_\theta\omega_{\alpha\beta\iota}\,\partial^\theta\omega^{\alpha\beta\iota} + \\
& 2 r_1\,\partial_\theta\omega_{\alpha\iota\beta}\,\partial^\theta\omega^{\alpha\beta\iota} - 2 r_2\,\partial_\theta\omega_{\alpha\iota\beta}\,\partial^\theta\omega^{\alpha\beta\iota}\big)\Big)[t,x,y,z]\,dz\,dy\,dx\,dt
\end{aligned}
$$

Matrix (σ_1/τ_1 block)

	$\sigma_1^{\#1}\dagger^{\alpha\beta}$	$\sigma_1^{\#2}\dagger^{\alpha\beta}$	$\tau_1^{\#1}\dagger^{\alpha\beta}$	$\sigma_1^{\#1}\dagger^{\alpha}$	$\sigma_1^{\#2}\dagger^{\alpha}$	$\tau_1^{\#1}\dagger^{\alpha}$	$\tau_1^{\#2}\dagger^{\alpha}$
$\tau_1^{\#2}{}_\alpha$	0	0	0	$\dfrac{2ik}{t_1+2k^2 t_1}$	$\dfrac{i\sqrt2\,k(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2}$	0	$\dfrac{2k^2(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2}$
$\tau_1^{\#1}{}_\alpha$	0	0	0	0	0	0	0
$\sigma_1^{\#2}{}_\alpha$	0	0	0	$\dfrac{\sqrt2}{t_1+2k^2 t_1}$	$\dfrac{2k^2 r_1+t_1}{(t_1+2k^2 t_1)^2}$	0	$-\dfrac{i\sqrt2\,k(2k^2 r_1+t_1)}{(t_1+2k^2 t_1)^2}$
$\sigma_1^{\#1}{}_\alpha$	0	0	0	0	$\dfrac{\sqrt2}{t_1+2k^2 t_1}$	0	$-\dfrac{2ik}{t_1+2k^2 t_1}$
$\tau_1^{\#1}{}_{\alpha\beta}$	$-\dfrac{6i\sqrt2\,k}{(3+2k^2)^2 t_1}$	$\dfrac{12ik}{(3+2k^2)^2 t_1}$	$\dfrac{12k^2}{(3+2k^2)^2 t_1}$	0	0	0	0
$\sigma_1^{\#2}{}_{\alpha\beta}$	$-\dfrac{6\sqrt2}{(3+2k^2)^2 t_1}$	$\dfrac{12}{(3+2k^2)^2 t_1}$	$\dfrac{12ik}{(3+2k^2)^2 t_1}$	0	0	0	0
$\sigma_1^{\#1}{}_{\alpha\beta}$	$\dfrac{6}{(3+2k^2)^2 t_1}$	$\dfrac{6\sqrt2}{(3+2k^2)^2 t_1}$	$\dfrac{6i\sqrt2\,k}{(3+2k^2)^2 t_1}$	0	0	0	0

Matrix (σ_0 block)

	$\sigma_0^{\#1}{}_{0^+}$	$\tau_0^{\#1}{}_{0^+}$	$\tau_0^{\#2}{}_{0^+}$	$\sigma_0^{\#1}{}_{0^-}$
$\sigma_0^{\#1}{}_{0^+}\dagger$	$-\dfrac{1}{(1+2k^2)^2 t_1}$	$\dfrac{i\sqrt2\,k}{(1+2k^2)^2 t_1}$	0	0
$\tau_0^{\#1}{}_{0^+}\dagger$	$-\dfrac{i\sqrt2\,k}{(1+2k^2)^2 t_1}$	$-\dfrac{2k^2}{(1+2k^2)^2 t_1}$	0	0
$\tau_0^{\#2}{}_{0^+}\dagger$	0	0	0	0
$\sigma_0^{\#1}{}_{0^-}\dagger$	0	0	0	$\dfrac{1}{k^2 r_2}$

Matrix (σ_2 block)

	$\sigma_2^{\#1}{}_{2^+\alpha\beta}$	$\tau_2^{\#1}{}_{2^+\alpha\beta}$	$\sigma_2^{\#1}{}_{2^-\alpha\beta\chi}$
$\sigma_2^{\#1}{}_{2^+}\dagger^{\alpha\beta}$	$\dfrac{2}{(1+2k^2)^2 t_1}$	$-\dfrac{2i\sqrt2\,k}{(1+2k^2)^2 t_1}$	0
$\tau_2^{\#1}{}_{2^+}\dagger^{\alpha\beta}$	$\dfrac{2i\sqrt2\,k}{(1+2k^2)^2 t_1}$	$\dfrac{4k^2}{(1+2k^2)^2 t_1}$	0
$\sigma_2^{\#1}{}_{2^-}\dagger^{\alpha\beta\chi}$	0	0	$\dfrac{2}{2k^2 r_1+t_1}$

Matrix (ω_0/f_0 block)

	$\omega_0^{\#1}{}_{0^+}\dagger$	$f_0^{\#1}{}_{0^+}\dagger$	$f_0^{\#2}{}_{0^+}\dagger$	$\omega_0^{\#1}{}_{0^-}\dagger$
$\omega_0^{\#1}{}_{0^-}$	0	0	0	$k^2 r_2$
$f_0^{\#2}{}_{0^+}$	0	0	0	0
$f_0^{\#1}{}_{0^+}$	$i\sqrt2\,k t_1$	$-2k^2 t_1$	0	0
$\omega_0^{\#1}{}_{0^+}$	$-t_1$	$-i\sqrt2\,k t_1$	0	0

Matrix (ω_1/f_1 block)

	$\omega_1^{\#1}{}_{1^+}\dagger^{\alpha\beta}$	$\omega_1^{\#2}{}_{1^+}\dagger^{\alpha\beta}$	$f_1^{\#1}{}_{1^+}\dagger^{\alpha\beta}$	$\omega_1^{\#1}{}_{1^-}\dagger^{\alpha}$	$\omega_1^{\#2}{}_{1^-}\dagger^{\alpha}$	$f_1^{\#1}{}_{1^-}\dagger^{\alpha}$	$f_1^{\#2}{}_{1^-}\dagger^{\alpha}$
$f_1^{\#1}{}_\alpha\ f_1^{\#2}{}_\alpha$	0	0	0	$i k t_1$	0	0	0
$\omega_1^{\#2}{}_\alpha\ f_1^{\#1}{}_\alpha$	0	0	0	$\dfrac{t_1}{\sqrt2}$	0	0	0
$\omega_1^{\#1}{}_\alpha$	0	0	0	$-k^2 r_1 - \dfrac{t_1}{2}$	$\dfrac{t_1}{\sqrt2}$	0	$-i k t_1$
$f_1^{\#1}{}_{\alpha\beta}$	$\dfrac{i k t_1}{3\sqrt2}$	$\dfrac{i k t_1}{3}$	$\dfrac{k^2 t_1}{3}$	0	0	0	0
$\omega_1^{\#2}{}_{\alpha\beta}$	$-\dfrac{t_1}{3\sqrt2}$	$\dfrac{t_1}{3}$	$-\dfrac{1}{3}i k t_1$	0	0	0	0
$\omega_1^{\#1}{}_{\alpha\beta}$	$\dfrac{t_1}{6}$	$-\dfrac{t_1}{3\sqrt2}$	$\dfrac{i k t_1}{3\sqrt2}$	0	0	0	0

Matrix (ω_2/f_2 block)

	$\omega_2^{\#1}{}_{2^+\alpha\beta}$	$f_2^{\#1}{}_{2^+\alpha\beta}$	$\omega_2^{\#1}{}_{2^-\alpha\beta\chi}$
$\omega_2^{\#1}{}_{2^+}\dagger^{\alpha\beta}$	$\dfrac{t_1}{2}$	$\dfrac{i k t_1}{\sqrt2}$	0
$f_2^{\#1}{}_{2^+}\dagger^{\alpha\beta}$	$-\dfrac{i k t_1}{\sqrt2}$	$k^2 t_1$	0
$\omega_2^{\#1}{}_{2^-}\dagger^{\alpha\beta\chi}$	0	0	$k^2 r_1 + \dfrac{t_1}{2}$

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau_0^{\#2}{}_{0^+} == 0$	1
$\tau_0^{\#1}{}_{0^+} - 2 i k\,\sigma_0^{\#1}{}_{0^+} == 0$	1
$\tau_1^{\#2\alpha} + 2 i k\,\sigma_1^{\#2\alpha} == 0$	3
$\tau_1^{\#1\alpha} == 0$	3
$\tau_1^{\#1\alpha\beta} - 2 i k\,\sigma_1^{\#1\alpha\beta} == 0$	3
$2\,\sigma_1^{\#1\alpha\beta} + \sigma_1^{\#2\alpha\beta} == 0$	3
$\tau_2^{\#1\alpha\beta} - 2 i k\,\sigma_2^{\#1\alpha\beta} == 0$	5
Total constraints:	19

Massive and massless spectra

$J^P = 2^-$, $\vec{k^\mu}$
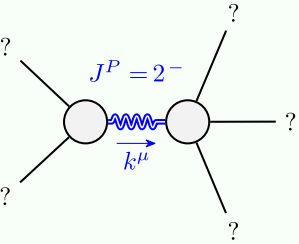
Massive particle	
Pole residue:	$-\dfrac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\dfrac{t_1}{2 r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

Unitarity conditions

$r_1 < 0 \ \&\&\ t_1 > 0$