

Wave operator and propagator

Quadratic (free) action

$$S_{==}$$

$$\iiint \int (\beta (h_{\alpha\beta} h^{\alpha\beta} - h^{\alpha}_{\alpha} h^{\beta}_{\beta}) + h^{\alpha\beta} \mathcal{T}_{\alpha\beta} + \frac{1}{2} \alpha (\partial_{\beta} h^{\chi}_{\chi} \partial^{\beta} h^{\alpha}_{\alpha} + 2 \partial_{\alpha} h^{\alpha\beta} \partial_{\chi} h^{\chi}_{\beta} - 2 \partial^{\beta} h^{\alpha}_{\alpha} \partial_{\chi} h^{\chi}_{\beta} - \partial_{\chi} h^{\alpha\beta} \partial^{\chi} h^{\alpha\beta})) [t, x, y, z] dz dy dx dt$$

Diagram illustrating the construction of the source-free constraint matrix for the 2D case. The matrix is assembled from various terms, including source constraints (highlighted in red) and non-source terms (highlighted in blue).

Source Constraints (Red Boxes):

- $\tau_{0+}^{\#1} + \tau_{0+}^{\#2}$ (top-left)
- $\tau_{1+}^{\#1} + \tau_{1+}^{\#2}$ (bottom-right)
- $\tau_{2+}^{\#1} + \tau_{2+}^{\#2}$ (top-right)
- $\tau_{1-}^{\#1} + \tau_{1-}^{\#2}$ (bottom-left)

Non-Source Terms (Blue Boxes):

- $\frac{1}{\beta - \frac{\alpha k^2}{2}}$ (top-middle)
- $\beta - \frac{\alpha k^2}{2}$ (middle-right)
- $\frac{1}{\beta}$ (bottom-middle)
- β (middle-left)

Final Matrix Structure (2x2 blocks):

- Top-Left Block:**

$$\begin{bmatrix} -2\beta + \alpha k^2 & -\sqrt{3}\beta \\ -\sqrt{3}\beta & 0 \end{bmatrix}$$
- Top-Right Block:**

$$\begin{bmatrix} -\frac{1}{\sqrt{3}\beta} & 0 \\ \frac{2\beta - \alpha k^2}{3\beta^2} & -\frac{1}{\sqrt{3}\beta} \end{bmatrix}$$
- Bottom-Left Block:**

$$\begin{bmatrix} \frac{1}{\beta} & \beta \end{bmatrix}$$
- Bottom-Right Block:**

$$\begin{bmatrix} \frac{1}{\beta - \frac{\alpha k^2}{2}} & \beta - \frac{\alpha k^2}{2} \end{bmatrix}$$

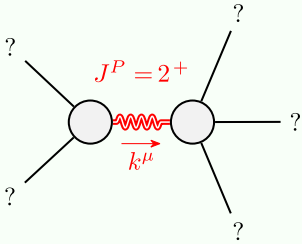
Labels and Constraints:

- Top Labels:** $h_{0+}^{\#1}$, $h_{0+}^{\#2}$, $\tau_{2+}^{\#1} + \alpha\beta$, $h_{2+}^{\#1} + \alpha\beta$, $h_{2+}^{\#1} + \alpha\beta$, $\tau_{0+}^{\#2} + \tau_{0+}^{\#1}$
- Bottom Labels:** $h_{0+}^{\#1} +$, $h_{0+}^{\#2} +$, $\tau_{2+}^{\#1} + \alpha\beta$, $h_{2+}^{\#1} + \alpha\beta$, $h_{2+}^{\#1} + \alpha\beta$, $\tau_{0+}^{\#2} + \tau_{0+}^{\#1}$
- Left Labels:** $h_{0+}^{\#1} +$, $h_{0+}^{\#2} +$, $\tau_{2+}^{\#1} + \alpha\beta$, $h_{2+}^{\#1} + \alpha\beta$, $h_{2+}^{\#1} + \alpha\beta$, $\tau_{0+}^{\#2} + \tau_{0+}^{\#1}$
- Right Labels:** $h_{0+}^{\#1} +$, $h_{0+}^{\#2} +$, $\tau_{2+}^{\#1} + \alpha\beta$, $h_{2+}^{\#1} + \alpha\beta$, $h_{2+}^{\#1} + \alpha\beta$, $\tau_{0+}^{\#2} + \tau_{0+}^{\#1}$

Legend:

- (No source constraints)

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{2}{\alpha} > 0$
Polarisations:	5
Square mass:	$\frac{2\beta}{\alpha} > 0$
Spin:	2
Parity:	Even

(No massless particles)

Unitarity conditions

$$\alpha < 0 \ \&\& \ \beta < 0$$