Particle spectrograph

Wave operator and propagator

Massive and massless spectra

Source constraints

SO(3) irreps	Fundamental fields	Multiplicities
$\sigma_{0^-}^{\#1} == 0$	$\epsilon\,\eta_{\alpha\beta\chi\delta}\,\partial^\delta\sigma^{\alpha\beta\chi} == 0$	1
$\tau_{0^+}^{\#2} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$	1
$\tau_{0^+}^{\#1} - 2\,i\,k\,\sigma_{0^-}^{\#1} == 0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha + 2\,\partial_\chi\partial^\chi\partial_\beta\sigma^{\alpha\beta}{}_\alpha$	1
$\tau_{1^-}^{\#2\,\alpha} + 2\,i\,k\,\sigma_{1^-}^{\#2\,\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$	3
$\tau_{1^-}^{\#1\,\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$	3
$\tau_{1^+}^{\#1\,\alpha\beta} == 0$	$\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha}$	3
$\sigma_{1^+}^{\#2\,\alpha\beta} == 0$	$\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} == \partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$	3
$\sigma_{2^-}^{\#1\,\alpha\beta\chi} == 0$	$3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\alpha\sigma^{\beta\delta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\alpha\sigma^{\beta\delta}{}_\delta + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\delta\chi} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\delta\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\delta\alpha}{}_\delta + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\beta\delta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\delta\beta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\beta\chi\alpha} + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\alpha\sigma^{\delta\epsilon}{}_\delta + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\beta\delta\epsilon} + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\alpha\delta}{}_\delta == 3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\beta\sigma^{\alpha\delta}{}_\delta + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\chi\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\delta\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\beta\delta\alpha} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\delta\alpha\beta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\beta\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\chi\beta} + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\beta\sigma^{\delta\epsilon}{}_\delta + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\alpha\delta\epsilon} + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\beta\delta}{}_\delta$	5
$\tau_{2^+}^{\#1\,\alpha\beta} == 0$	$4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} == 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi$	5
Total constraints/gauge generators:		25

	$\sigma_{1^+}^{\#1\,\alpha\beta}$	$\sigma_{1^+}^{\#2\,\alpha\beta}$	$\tau_{1^+}^{\#1\,\alpha\beta}$	$\sigma_{1^-}^{\#1\,\alpha}$	$\sigma_{1^-}^{\#2\,\alpha}$	$\tau_{1^-}^{\#1\,\alpha}$	$\tau_{1^-}^{\#2\,\alpha}$
$\sigma_{1^+}^{\#1\,\alpha\beta}\,\dagger$	$\frac{1}{k^2\,(2\,r_3+r_5)}$	0	0	0	0	0	0
$\sigma_{1^+}^{\#2\,\alpha\beta}\,\dagger$	0	0	0	0	0	0	0
$\tau_{1^+}^{\#1\,\alpha\beta}\,\dagger$	0	0	0	0	0	0	0
$\sigma_{1^-}^{\#1\,\alpha}\,\dagger$	0	0	0	$\frac{2}{k^2\,(r_3+2\,r_5)}$	$\frac{2\sqrt{2}}{k^2\,(1+2\,k^2)\,(r_3+2\,r_5)}$	0	$\frac{4\,i}{k\,(1+2\,k^2)\,(r_3+2\,r_5)}$
$\sigma_{1^-}^{\#2\,\alpha}\,\dagger$	0	0	0	$\frac{2\sqrt{2}}{k^2\,(1+2\,k^2)\,(r_3+2\,r_5)}$	$\frac{3\,k^2\,(r_3+2\,r_5)+4\,t_3}{(k+2\,k^3)^2\,(r_3+2\,r_5)\,t_3}$	0	$\frac{i\,\sqrt{2}\,(3\,k^2\,(r_3+2\,r_5)+4\,t_3)}{k\,(1+2\,k^2)^2\,(r_3+2\,r_5)\,t_3}$
$\tau_{1^-}^{\#1\,\alpha}\,\dagger$	0	0	0	0	0	0	0
$\tau_{1^-}^{\#2\,\alpha}\,\dagger$	0	0	0	$-\frac{4\,i}{k\,(1+2\,k^2)\,(r_3+2\,r_5)}$	$-\frac{i\,\sqrt{2}\,(3\,k^2\,(r_3+2\,r_5)+4\,t_3)}{k\,(1+2\,k^2)^2\,(r_3+2\,r_5)\,t_3}$	0	$\frac{6\,k^2\,(r_3+2\,r_5)+8\,t_3}{(1+2\,k^2)^2\,(r_3+2\,r_5)\,t_3}$

Quadratic (free) action

$$\mathcal{S} == \iiiint (f^{\alpha\beta}\,\tau_{\alpha\beta} + \mathcal{A}^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - \frac{2}{3}\,t_3\,(\mathcal{A}^{\alpha\prime}{}_\alpha\,\mathcal{A}_\prime{}^\theta{}_\theta - 2\,\mathcal{A}_\alpha{}^\theta{}_\theta\,\partial_\prime f^{\alpha\prime} + 2\,\mathcal{A}_\prime{}^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha - \partial_\prime f^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha - \partial_\prime f^{\alpha\prime}\,\partial_\theta f_\alpha{}^\theta + 2\,\partial^\prime f^\alpha{}_\alpha\,\partial_\theta f_\prime{}^\theta) - \frac{1}{2}\,r_3\,(\partial_\beta\mathcal{A}_\prime{}^\theta{}_\theta\,\partial^\prime\mathcal{A}^{\alpha\beta}{}_\alpha + \partial_\prime\mathcal{A}_\beta{}^\theta{}_\theta\,\partial^\prime\mathcal{A}^{\alpha\beta}{}_\alpha + \partial_\alpha\mathcal{A}^{\alpha\beta\prime}\,\partial_\theta\mathcal{A}_\beta{}^\theta{}_\prime - 2\,\partial^\prime\mathcal{A}^{\alpha\beta}{}_\alpha\,\partial_\theta\mathcal{A}_\beta{}^\theta{}_\prime + \partial_\alpha\mathcal{A}^{\alpha\beta\prime}\,\partial_\theta\mathcal{A}_\prime{}^\theta{}_\beta - 2\,\partial^\prime\mathcal{A}^{\alpha\beta}{}_\alpha\,\partial_\theta\mathcal{A}_\prime{}^\theta{}_\beta + 8\,\partial_\beta\mathcal{A}_{\prime\theta\alpha}\,\partial^\theta\mathcal{A}^{\alpha\beta\prime}) + r_5\,(\partial_\prime\mathcal{A}_\theta{}^\kappa{}_\kappa\,\partial^\theta\mathcal{A}^{\alpha\prime}{}_\alpha - \partial_\theta\mathcal{A}_\prime{}^\kappa{}_\kappa\,\partial^\theta\mathcal{A}^{\alpha\prime}{}_\alpha - (\partial_\alpha\mathcal{A}^{\alpha\prime\theta} - 2\,\partial^\theta\mathcal{A}^{\alpha\prime}{}_\alpha)(\partial_\kappa\mathcal{A}_\prime{}^\kappa{}_\theta - \partial_\kappa\mathcal{A}_\theta{}^\kappa{}_\prime)))[t, x, y, z]\,dt\,dz\,dy\,dx\,dt$$

	$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^-}^{\#1}$
$\sigma_{0^+}^{\#1}\,\dagger$	$\frac{1}{(1+2\,k^2)^2\,t_3}$	$-\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_3}$	0	0
$\tau_{0^+}^{\#1}\,\dagger$	$\frac{i\,\sqrt{2}\,k}{(1+2\,k^2)^2\,t_3}$	$\frac{2\,k^2}{(1+2\,k^2)^2\,t_3}$	0	0
$\tau_{0^+}^{\#2}\,\dagger$	0	0	0	0
$\sigma_{0^-}^{\#1}\,\dagger$	0	0	0	0

	$\mathcal{A}_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\mathcal{A}_{0^-}^{\#1}$
$\mathcal{A}_{0^+}^{\#1}\,\dagger$	t_3	$-i\,\sqrt{2}\,k\,t_3$	0	0
$f_{0^+}^{\#1}\,\dagger$	$i\,\sqrt{2}\,k\,t_3$	$2\,k^2\,t_3$	0	0
$f_{0^+}^{\#2}\,\dagger$	0	0	0	0
$\mathcal{A}_{0^-}^{\#1}\,\dagger$	0	0	0	0

	$\mathcal{A}_{2^+}^{\#1\,\alpha\beta}\,\dagger$	$f_{2^+}^{\#1\,\alpha\beta}\,\dagger$	$\mathcal{A}_{2^+}^{\#1\,\alpha\beta}\,\dagger$
$\mathcal{A}_{2^+}^{\#1\,\alpha\beta}$	0	0	$-\frac{3\,k^2\,r_3}{2}$
$f_{2^+}^{\#1\,\alpha\beta}$	0	0	0
$\mathcal{A}_{2^+}^{\#1\,\alpha\beta}$	0	0	0

	$\sigma_{2^+}^{\#1\,\alpha\beta}\,\dagger$	$\tau_{2^+}^{\#1\,\alpha\beta}\,\dagger$	$\sigma_{2^-}^{\#1\,\alpha\beta\chi}\,\dagger$
$\sigma_{2^+}^{\#1\,\alpha\beta}$	$-\frac{2}{3\,k^2\,r_3}$	0	0
$\tau_{2^+}^{\#1\,\alpha\beta}$	0	0	0
$\sigma_{2^-}^{\#1\,\alpha\beta\chi}$	0	0	0

	$\mathcal{A}_{1^+}^{\#1\,\alpha\beta}\,\dagger$	$\mathcal{A}_{1^+}^{\#2\,\alpha\beta}\,\dagger$	$f_{1^+}^{\#1\,\alpha\beta}\,\dagger$	$\mathcal{A}_{1^-}^{\#1\,\alpha}\,\dagger$	$\mathcal{A}_{1^-}^{\#2\,\alpha}\,\dagger$	$f_{1^-}^{\#1\,\alpha}\,\dagger$	$f_{1^-}^{\#2\,\alpha}\,\dagger$
$\mathcal{A}_{1^+}^{\#1\,\alpha\beta}$	$k^2\,(2\,r_3+r_5)$	0	0	0	0	0	0
$\mathcal{A}_{1^+}^{\#2\,\alpha\beta}$	0	0	0	0	0	0	0
$f_{1^+}^{\#1\,\alpha\beta}$	0	0	0	0	0	0	0
$\mathcal{A}_{1^-}^{\#1\,\alpha}$	0	0	0	$k^2\,(\frac{\sqrt{2}}{3}+r_5)+\frac{2\,t_3}{3}$	$-\frac{\sqrt{2}\,t_3}{3}$	0	$\frac{2\,i\,k\,t_3}{3}$
$\mathcal{A}_{1^-}^{\#2\,\alpha}$	0	0	0	$-\frac{\sqrt{2}\,t_3}{3}$	$\frac{t_3}{3}$	0	$-\frac{1}{3}\,i\,\sqrt{2}\,k\,t_3$
$f_{1^-}^{\#1\,\alpha}$	0	0	0	0	0	0	0
$f_{1^-}^{\#2\,\alpha}$	0	0	0	$-\frac{2}{3}\,i\,k\,t_3$	$\frac{1}{3}\,i\,\sqrt{2}\,k\,t_3$	0	$\frac{2\,k^2\,t_3}{3}$

Unitarity conditions

$r_3 < 0 \;\&\&\; (r_5 < -\frac{r_3}{2} \;||\; r_5 > -2\,r_3) \;||\; r_3 > 0 \;\&\&\; -2\,r_3 < r_5 < -\frac{r_3}{2}$

Quadratic pole	
Pole residue:	$-\frac{1}{r_3\,(2\,r_3+r_5)\,(r_3+2\,r_5)\,p^2} > 0$
Polarisations:	2

(No massive particles)