# Particle spectrograph

## Wave operator and propagator

| | $\sigma^{\#1}_{2^+\,\alpha\beta}$ | $\tau^{\#1}_{2^+\,\alpha\beta}$ | $\sigma^{\#1}_{2^-\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\frac{2}{(1+2k^2)^2 t_1}$ | $-\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | 0 |
| $\tau^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\frac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | $\frac{4k^2}{(1+2k^2)^2 t_1}$ | 0 |
| $\sigma^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{t_1}$ |

| | $\omega^{\#1}_{0^+}$ | $f^{\#1}_{0^+}$ | $f^{\#2}_{0^+}$ | $\omega^{\#1}_{0^-}$ |
|---|---|---|---|---|
| $\omega^{\#1}_{0^+}\dagger$ | $t_3$ | $-i\sqrt{2}\,k t_3$ | 0 | 0 |
| $f^{\#1}_{0^+}\dagger$ | $i\sqrt{2}\,k t_3$ | $2k^2 t_3$ | 0 | 0 |
| $f^{\#2}_{0^+}\dagger$ | 0 | 0 | 0 | 0 |
| $\omega^{\#1}_{0^-}\dagger$ | 0 | 0 | 0 | $k^2 r_2 - t_1$ |

| | $\omega^{\#1}_{2^+\,\alpha\beta}$ | $f^{\#1}_{2^+\,\alpha\beta}$ | $\omega^{\#1}_{2^-\,\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{ikt_1}{\sqrt{2}}$ | 0 |
| $f^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\frac{ikt_1}{\sqrt{2}}$ | $k^2 t_1$ | 0 |
| $\omega^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$ | 0 | 0 | $\frac{t_1}{2}$ |

### Source constraints

| SO(3) irreps | Fundamental fields | Multiplicities |
|---|---|---|
| $\tau^{\#2}_{0^+} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == 0$ | 1 |
| $\tau^{\#1}_{0^+} - 2ik\,\sigma^{\#1}_{0^+} == 0$ | $\partial_\beta\partial_\alpha\tau^{\alpha\beta} == \partial_\beta\partial^\beta\tau^\alpha{}_\alpha + 2\,\partial_\chi\partial^\chi\partial_\beta\sigma^{\alpha\beta}{}_\alpha$ | 1 |
| $\tau^{\#2\,\alpha}_{1^-} + 2ik\,\sigma^{\#2\,\alpha}_{1^-} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$ | 3 |
| $\tau^{\#1\,\alpha}_{1^-} == 0$ | $\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau^{\beta\alpha}$ | 3 |
| $\tau^{\#1\,\alpha\beta}_{1^+} + ik\,\sigma^{\#2\,\alpha\beta}_{1^+} == 0$ | $\partial_\chi\partial^\alpha\tau^{\beta\chi} + \partial_\chi\partial^\beta\tau^{\chi\alpha} + \partial_\chi\partial^\chi\tau^{\alpha\beta} + 2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} == \partial_\chi\partial^\alpha\tau^{\chi\beta} + \partial_\chi\partial^\beta\tau^{\alpha\chi} + \partial_\chi\partial^\chi\tau^{\beta\alpha} + 2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$ | 3 |
| $\tau^{\#1\,\alpha\beta}_{2^+} - 2ik\,\sigma^{\#1\,\alpha\beta}_{2^+} == 0$ | $-i(4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta} + 2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^\chi{}_\chi - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha} - 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\alpha\beta} + 3\,\partial_\delta\partial^\delta\partial_\chi\partial^\chi\tau^{\beta\alpha} + 4ik^\chi\,\partial_\epsilon\partial_\chi\partial^\beta\partial^\alpha\sigma^{\delta\epsilon}{}_\delta - 6ik^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\delta\epsilon} - 6ik^\chi\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\delta\epsilon} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta} + 6ik^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\alpha\delta\beta} + 6ik^\chi\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\beta\delta\alpha} - 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^\chi{}_\chi - 4i\,\eta^{\alpha\beta}\,k^\chi\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\chi\sigma^{\delta\epsilon}{}_\delta) == 0$ | 5 |
| Total constraints/gauge generators: | | 16 |

### Quadratic (free) action

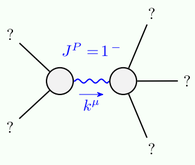
$$\mathcal{S} == \iiiint\left(\frac{1}{6}\,(2\,\omega^{\alpha\prime}{}_\alpha\,(t_1\,\omega_\prime{}^\theta{}_\theta - 2t_3\,\omega_\prime{}^\kappa{}_\kappa) + 6\,f^{\alpha\beta}\,\tau_{\alpha\beta} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 4t_1\,\omega_\alpha{}^\theta{}_\theta\,\partial_\prime f^{\alpha\prime} + 8t_3\,\omega_\alpha{}^\kappa{}_\kappa\,\partial_\prime f^{\alpha\prime} + 4t_1\,\omega_\prime{}^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha - 8t_3\,\omega_\prime{}^\kappa{}_\kappa\,\partial^\prime f^\alpha{}_\alpha - 2t_1\,\partial_\prime f^\theta{}_\theta\,\partial^\prime f^\alpha{}_\alpha + 4t_3\,\partial_\prime f^\kappa{}_\kappa\,\partial^\prime f^\alpha{}_\alpha - 2t_1\,\partial_\prime f^{\alpha\prime}\,\partial_\theta f_\alpha{}^\theta + 4t_1\,\partial^\prime f^\alpha{}_\alpha\,\partial_\theta f_\prime{}^\theta - 6t_1\,\partial_\alpha f_{\prime\theta}\,\partial^\theta f^{\alpha\prime} - 3t_1\,\partial_\alpha f_{\theta\prime}\,\partial^\theta f^{\alpha\prime} + 3t_1\,\partial_\prime f_{\alpha\theta}\,\partial^\theta f^{\alpha\prime} + 3t_1\,\partial_\theta f_{\alpha\prime}\,\partial^\theta f^{\alpha\prime} + 3t_1\,\partial_\theta f_{\prime\alpha}\,\partial^\theta f^{\alpha\prime} + 6t_1\,\omega_{\alpha\theta\prime}\,(\omega^{\alpha\prime\theta} + 2\,\partial^\theta f^{\alpha\prime}) + 8r_2\,\partial_\beta\omega_{\alpha\prime\theta}\,\partial^\theta\omega^{\alpha\beta\prime} - 4r_2\,\partial_\beta\omega_{\alpha\theta\prime}\,\partial^\theta\omega^{\alpha\beta\prime} + 4r_2\,\partial_\beta\omega_{\prime\theta\alpha}\,\partial^\theta\omega^{\alpha\beta\prime} - 2r_2\,\partial_\prime\omega_{\alpha\beta\theta}\,\partial^\theta\omega^{\alpha\beta\prime} + 2r_2\,\partial_\theta\omega_{\alpha\beta\prime}\,\partial^\theta\omega^{\alpha\beta\prime} - 4r_2\,\partial_\theta\omega_{\alpha\prime\beta}\,\partial^\theta\omega^{\alpha\beta\prime} + 6r_5\,\partial_\prime\omega_\theta{}^\kappa{}_\kappa\,\partial^\theta\omega^{\alpha\prime}{}_\alpha - 6r_5\,\partial_\theta\omega_\prime{}^\kappa{}_\kappa\,\partial^\theta\omega^{\alpha\prime}{}_\alpha + 4t_3\,\partial_\prime f^{\alpha\prime}\,\partial_\kappa f_\alpha{}^\kappa - 8t_3\,\partial^\prime f^\alpha{}_\alpha\,\partial_\kappa f_\prime{}^\kappa - 6r_5\,\partial_\alpha\omega^{\alpha\prime\theta}\,\partial_\kappa\omega_\prime{}^\kappa{}_\theta + 12r_5\,\partial^\theta\omega^{\alpha\prime}{}_\alpha\,\partial_\kappa\omega_\prime{}^\kappa{}_\theta + 6r_5\,\partial_\alpha\omega^{\alpha\prime\theta}\,\partial_\kappa\omega_\theta{}^\kappa{}_\prime - 12r_5\,\partial^\theta\omega^{\alpha\prime}{}_\alpha\,\partial_\kappa\omega_\theta{}^\kappa{}_\prime)\right)[t, x, y, z]\,dt\,dx\,dy\,dz$$

| | $\sigma^{\#1}_{0^+}$ | $\tau^{\#2}_{0^+}$ | $\tau^{\#1}_{0^+}$ | $\sigma^{\#1}_{0^-}$ |
|---|---|---|---|---|
| $\sigma^{\#1}_{0^+}\dagger$ | $\frac{1}{(1+2k^2)^2 t_3}$ | $\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3}$ | 0 | 0 |
| $\tau^{\#1}_{0^+}\dagger$ | $-\frac{i\sqrt{2}\,k}{(1+2k^2)^2 t_3}$ | $\frac{2k^2}{(1+2k^2)^2 t_3}$ | 0 | 0 |
| $\tau^{\#2}_{0^+}\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma^{\#1}_{0^-}\dagger$ | 0 | 0 | 0 | $\frac{1}{k^2 r_2 - t_1}$ |

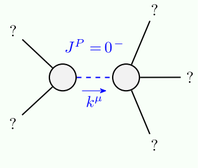
| | $\sigma^{\#1}_{1^+\,\alpha\beta}$ | $\sigma^{\#2}_{1^+\,\alpha\beta}$ | $\tau^{\#1}_{1^+\,\alpha\beta}$ | $\sigma^{\#1}_{1^-\,\alpha}$ | $\sigma^{\#2}_{1^-\,\alpha}$ | $\tau^{\#1}_{1^-\,\alpha}$ | $\tau^{\#2}_{1^-\,\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma^{\#1}_{1^+}\dagger^{\alpha\beta}$ | 0 | $-\frac{\sqrt{2}}{t_1+k^2 t_1}$ | $-\frac{i\sqrt{2}\,k}{t_1+k^2 t_1}$ | 0 | 0 | 0 | 0 |
| $\sigma^{\#2}_{1^+}\dagger^{\alpha\beta}$ | $-\frac{\sqrt{2}}{t_1+k^2 t_1}$ | $\frac{-2k^2 r_5+t_1}{(1+k^2)^2 {t_1}^2}$ | $-\frac{i(2k^3 r_5-kt_1)}{(1+k^2)^2 {t_1}^2}$ | 0 | 0 | 0 | 0 |
| $\tau^{\#1}_{1^+}\dagger^{\alpha\beta}$ | $\frac{i\sqrt{2}\,k}{t_1+k^2 t_1}$ | $\frac{i(2k^3 r_5-kt_1)}{(1+k^2)^2 {t_1}^2}$ | $\frac{-2k^4 r_5+k^2 t_1}{(1+k^2)^2 {t_1}^2}$ | 0 | 0 | 0 | 0 |
| $\sigma^{\#1}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{2(t_1+t_3)}{3t_1t_3+2k^2 r_5(t_1+t_3)}$ | $-\frac{\sqrt{2}\,(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2 r_5(t_1+t_3))}$ | 0 | $-\frac{2ik(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2 r_5(t_1+t_3))}$ |
| $\sigma^{\#2}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | $-\frac{\sqrt{2}\,(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2 r_5(t_1+t_3))}$ | $\frac{6k^2 r_5+t_1+4t_3}{(1+2k^2)^2(3t_1t_3+2k^2 r_5(t_1+t_3))}$ | 0 | $\frac{i\sqrt{2}\,k(6k^2 r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2 r_5(t_1+t_3))}$ |
| $\tau^{\#1}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau^{\#2}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{2ik(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2 r_5(t_1+t_3))}$ | $-\frac{i\sqrt{2}\,k(6k^2 r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2 r_5(t_1+t_3))}$ | 0 | $\frac{2k^2(6k^2 r_5+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2 r_5(t_1+t_3))}$ |

| | $f^{\#2}_{1^-}{}_\alpha$ | $f^{\#1}_{1^-}{}_\alpha$ | $\omega^{\#2}_{1^-}{}_\alpha$ | $\omega^{\#1}_{1^-}{}_\alpha$ | $f^{\#1}_{1^+}{}_{\alpha\beta}$ | $\omega^{\#2}_{1^+}{}_{\alpha\beta}$ | $\omega^{\#1}_{1^+}{}_{\alpha\beta}$ |
|---|---|---|---|---|---|---|---|
| $\omega^{\#1}_{1^+}\dagger^{\alpha\beta}$ | 0 | 0 | 0 | 0 | $\frac{ikt_1}{\sqrt{2}}$ | $\frac{t_1}{\sqrt{2}}$ | $k^2 r_5 - \frac{t_1}{2}$ |
| $\omega^{\#2}_{1^+}\dagger^{\alpha\beta}$ | 0 | 0 | 0 | 0 | $\frac{ikt_1}{\sqrt{2}}$ | $\frac{t_1}{\sqrt{2}}$ | $\frac{t_1}{2}$ |
| $f^{\#1}_{1^+}\dagger^{\alpha\beta}$ | 0 | 0 | 0 | 0 | $-\frac{ikt_1}{\sqrt{2}}$ | 0 | 0 |
| $\omega^{\#1}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{1}{6}(6k^2 r_5 + t_1 + 4t_3)$ | 0 | 0 | 0 |
| $\omega^{\#2}_{1^-}\dagger^{\alpha}$ | 0 | 0 | $\frac{t_1-2t_3}{3\sqrt{2}}$ | $\frac{t_1-2t_3}{3\sqrt{2}}$ | 0 | 0 | 0 |
| $f^{\#1}_{1^-}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f^{\#2}_{1^-}\dagger^{\alpha}$ | 0 | $\frac{2}{3}k^2(t_1+t_3)$ | 0 | $-\frac{1}{3}ik(t_1-2t_3)$ | 0 | 0 | 0 |

## Massive and massless spectra




| Massive particle | |
|---|---|
| Pole residue: | $\frac{6t_1t_3(t_1+t_3)-3r_5({t_1}^2+2{t_3}^2)}{2r_5(t_1+t_3)(-3t_1t_3+r_5(t_1+t_3))} > 0$ |
| Polarisations: | 3 |
| Square mass: | $-\frac{3t_1t_3}{2r_5t_1+2r_5t_3} > 0$ |
| Spin: | 1 |
| Parity: | Odd |




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $\frac{t_1}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

(No massless particles)

## Unitarity conditions

$r_2 < 0 \;\&\&\; r_5 < 0 \;\&\&\; t_1 < 0 \;\&\&\; 0 < t_3 < -t_1$