## Particle spectrograph

## Wave operator and propagator

$ au_1^{\#2}$	0	0	0	$\frac{2ik}{t_1 + 2k^2t_1}$	$-\frac{i\sqrt{2}k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2t_1)^2}$	0	$\frac{-4 k^4 (r_1 + r_5) + 2 k^2 t_1}{(t_1 + 2 k^2 t_1)^2}$
$\tau_{1^{-}}^{\#1}\alpha$	0	0	0	0	0	0	0
$\sigma_{1}^{\#2}{}_{\alpha}$	0	0	0	$\frac{\sqrt{2}}{t_1 + 2k^2t_1}$	$\frac{-2k^2(r_1+r_5)+t_1}{(t_1+2k^2t_1)^2}$	0	$\frac{i\sqrt{2} k(2k^2(r_1+r_5)-t_1)}{(t_1+2k^2t_1)^2}$
$\sigma_{1}^{\#1}{}_{\alpha}$	0	0	0	0	$\frac{\sqrt{2}}{t_1 + 2 k^2 t_1}$	0	$-\frac{2ik}{t_1+2k^2t_1}$
${\mathfrak l}_1^{\#1}_+ \alpha_{\beta}$	$i \sqrt{2} k (t_1 - 2t_2) $ $(1 + k^2) (3t_1 t_2 + 2k^2 (2t_1 + t_5) (t_1 + t_2))$	$\frac{i k (6 k^2 (2 r_1 + r_5) + t_1 + 4 t_2)}{(1 + k^2)^2 (3 t_1 t_2 + 2 k^2 (2 r_1 + r_5) (t_1 + t_2))}$	$\frac{k^2 \left(6  k^2  (2  r_1 + r_5) + t_1 + 4  t_2\right)}{(1 + k^2)^2  (3  t_1  t_2 + 2  k^2  (2  r_1 + r_5)  (t_1 + t_2))}$	0	0	0	0
$\sigma_{1}^{\#2}{}_{\alpha\beta}$	$\frac{\sqrt{2} (t_1 - 2t_2)}{(1 + k^2) (3t_1 t_2 + 2k^2 (2r_1 + r_5) (t_1 + t_2))}$	$\frac{6 k^2 (2 r_1 + r_5) + t_1 + 4 t_2}{(1 + k^2)^2 (3 t_1 t_2 + 2 k^2 (2 r_1 + r_5) (t_1 + t_2))}$	$-\frac{ik(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))} \frac{k^2(6k^2(2r_1+r_5)+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2(2r_1+r_5)(t_1+t_2))}$	0	0	0	0
$\sigma_{1}^{\#1}{}_{\alpha\beta}$	3 t 1 t 2 + 2	$\frac{\sqrt{2} (t_1 - 2t_2)}{(1 + k^2) (3t_1 t_2 + 2k^2 (2r_1 + r_5) (t_1 + t_2))}$	$i \sqrt{2} k(t_1-2t_2) $ $(1+k^2) (3t_1t_2+2k^2 (2r_1+r_5) (t_1+t_2))$	0	0	0	0
	$\sigma_{1}^{\#1} + \alpha \beta$	$\sigma_1^{\#2} + \alpha \beta$	$\tau_1^{\#1} + \alpha \beta$	$\sigma_{1}^{\#_{1}} +^{lpha}$	$\sigma_1^{\#2} +^{\alpha}$	$\tau_{1}^{\#1} + ^{lpha}$	$\tau_1^{\#2} + ^{\alpha}$

luge generators	Multiplicities	1	1	3	3	3	2	16
Source constraints/gauge generators	SO(3) irreps	$\tau_{0+}^{#2} == 0$	$\tau_{0+}^{\#1} - 2  i  k  \sigma_{0+}^{\#1} == 0$	$t_1^{\#2}\alpha + 2ik \sigma_1^{\#2}\alpha = 0$	$\tau_{1}^{\#1}{}^{\alpha} == 0$	$\tau_{1+}^{\#1}\alpha\beta + ik \ \sigma_{1+}^{\#2}\alpha\beta == 0 \ \ 3$	$\tau_{2+}^{\#1}\alpha\beta$ - 2 j k $\sigma_{2+}^{\#1}\alpha\beta$ == 0	Total constraints:

$\sigma_{2^{-}}^{\#1} lpha eta \chi$	0	0	$\frac{2}{2k^2r_1+t_1}$
$\tau_2^{\#1}_{2^+}\alpha\beta$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\sigma_{2}^{\#1}{}_{\alpha\beta}$		$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
	$\sigma_{2}^{\#1} + \alpha^{\beta}$	$\tau_{2+}^{\#1} + \alpha \beta$	$\sigma_{2}^{\#1} +^{lphaeta\chi}$

$\omega_{2^{-}}^{\#1}\alpha\beta\chi$	0	0	$k^2 r_1 + \frac{t_1}{2}$
$f_{2}^{\#1}\alpha\beta$	$-\frac{ikt_1}{\sqrt{2}}$	$k^2 t_1$	0
$\omega_{2}^{\#1}{}_{\alpha\beta}\;f_{2}^{\#1}{}_{\alpha\beta}$	$\frac{t_1}{2}$	$\frac{ikt_1}{\sqrt{2}}$	0
	$\omega_2^{\#1} +^{\alpha\beta}$	$f_{2}^{#1} + \alpha \beta$	$\omega_{2^{-}}^{\#1} +^{lphaeta\chi}$

$\sigma_0^{\#1}$	$ au_{0}^{\#1}$	$\tau_{0}^{\#2}$	$\sigma_0^{\#1}$
$-\frac{1}{(1+2k^2)^2t_1}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	0	0
$-\frac{i\sqrt{2} k}{(1+2k^2)^2 t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0	0
0	0	0	0
0	0	0	$\frac{1}{t_2}$
	i √2 k	$-\frac{1}{(1+2k^2)^2 t_1} \frac{i\sqrt{2} k}{(1+2k^2)^2 t_1}$ $-\frac{i\sqrt{2} k}{2k^2} \frac{2k^2}{(1+2k^2)^2 t_1}$	$-\frac{1}{(1+2k^2)^2 t_1} \frac{i\sqrt{2}k}{(1+2k^2)^2 t_1} 0$ $-\frac{i\sqrt{2}k}{(1+2k^2)^2 t_1} -\frac{2k^2}{(1+2k^2)^2 t_1} 0$

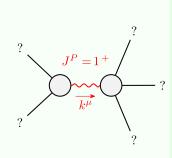
	$\omega_0^{\sharp 1}$	$f_{0}^{\#1}$	$f_{0}^{#2}$	$\omega_0^{\#1}$
$\omega_{0^+}^{\sharp 1}\dagger$	-t <sub>1</sub>	$i \sqrt{2} kt_1$	0	0
$f_{0}^{#1}$ †	$-i \sqrt{2} kt_1$	$-2 k^2 t_1$	0	0
$f_{0}^{#2}$ †	0	0	0	0
$\omega_{0}^{\sharp 1}$ †	0	0	0	$t_2$

Quad	ratic	(fre	ee)	Lag	ırar	ngia	n	der	nsit	y
	0/1	.,	1		<i>u</i> 1			2		.,

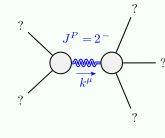
$-t_1 \omega_i^{\alpha i} \omega_{\kappa \alpha}^{\kappa} - \frac{1}{3} t_1 \omega_i^{\kappa \lambda} \omega_{\kappa \lambda}^{i} + \frac{2}{3} t_2 \omega_i^{\kappa \lambda} \omega_{\kappa \lambda}^{i} + \frac{1}{3} t_1 \omega_{\kappa \lambda}^{i} \omega^{\kappa \lambda}_{i} +$
$\frac{1}{3} t_2 \ \omega_{\kappa\lambda}^{\ \prime} \ \omega^{\kappa\lambda}_{\ \ \prime} + f^{\alpha\beta} \ \tau_{\alpha\beta} + \omega^{\alpha\beta\chi} \ \sigma_{\alpha\beta\chi} - r_5 \ \partial_{\prime} \omega^{\kappa\lambda}_{\ \ \kappa} \ \partial^{\prime} \omega_{\lambda}^{\ \alpha} - \frac{2}{3} \ r_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} \partial_{\theta} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\theta\alpha}_{\ \kappa} \partial_{\phi} \omega_{\alpha\beta}^{\ \kappa} \partial_{\phi} \omega_{\alpha\beta}^{\ \kappa} - \frac{2}{3} c_1 \ \partial^{\beta} \omega^{\phi\alpha}_{\ \kappa} \partial_{\phi} \omega_{\alpha\beta}^{\ \kappa$
$\frac{2}{3} r_1 \partial_{\theta} \omega_{\alpha\beta}^{\ \ \kappa} \partial_{\kappa} \omega^{\alpha\beta\theta} + \frac{2}{3} r_1 \partial_{\theta} \omega_{\alpha\beta}^{\ \ \kappa} \partial_{\kappa} \omega^{\theta\alpha\beta} - r_5 \partial_{\alpha} \omega_{\lambda}^{\ \alpha}_{\ \ \theta} \partial_{\kappa} \omega^{\theta\kappa\lambda} +$
$r_5 \partial_{\theta} \omega_{\lambda \alpha}^{\alpha} \partial_{\kappa} \omega^{\theta \kappa \lambda} - r_5 \partial_{\alpha} \omega_{\lambda \theta}^{\alpha} \partial_{\kappa} \omega^{\kappa \lambda \theta} + 2 r_5 \partial_{\theta} \omega_{\lambda \alpha}^{\alpha} \partial_{\kappa} \omega^{\kappa \lambda \theta} - \frac{1}{3} t_1 \partial^{\alpha} f_{\theta \kappa} \partial^{\kappa} f_{\alpha}^{\theta} +$
$\frac{1}{6} t_2 \partial^{\alpha} f_{\theta \kappa} \partial^{\kappa} f_{\alpha}^{\theta} - \frac{2}{3} t_1 \partial^{\alpha} f_{\kappa \theta} \partial^{\kappa} f_{\alpha}^{\theta} - \frac{1}{6} t_2 \partial^{\alpha} f_{\kappa \theta} \partial^{\kappa} f_{\alpha}^{\theta} - \frac{1}{3} t_1 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\alpha \lambda} +$
$\frac{1}{6} t_2 \partial^{\alpha} f^{\lambda}_{\kappa} \partial^{\kappa} f_{\alpha\lambda} + t_1 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f'_{i} + t_1 \omega_{\kappa\lambda}^{\lambda} \partial^{\kappa} f'_{i} + 2 t_1 \partial^{\alpha} f_{\kappa\alpha} \partial^{\kappa} f'_{i} -$
$t_1 \partial_{\kappa} f^{\lambda}_{\lambda} \partial^{\kappa} f'_{,} + \frac{1}{3} t_1 \omega_{i\theta\kappa} \partial^{\kappa} f^{i\theta} + \frac{1}{3} t_2 \omega_{i\theta\kappa} \partial^{\kappa} f^{i\theta} + \frac{4}{3} t_1 \omega_{i\kappa\theta} \partial^{\kappa} f^{i\theta} -$
$\frac{2}{3}t_2 \omega_{IK\theta} \partial^K f^{I\theta} - \frac{1}{3}t_1 \omega_{\theta IK} \partial^K f^{I\theta} - \frac{1}{3}t_2 \omega_{\theta IK} \partial^K f^{I\theta} + \frac{2}{3}t_1 \omega_{\theta KI} \partial^K f^{I\theta} +$
$\frac{2}{3} t_2 \omega_{\theta \kappa \iota} \partial^{\kappa} f^{\iota \theta} - t_1 \omega_{\iota \alpha}^{\alpha} \partial^{\kappa} f^{\prime}_{\kappa} - t_1 \omega_{\iota \lambda}^{\lambda} \partial^{\kappa} f^{\prime}_{\kappa} + \frac{1}{3} t_1 \partial^{\alpha} f^{\lambda}_{\kappa} \partial^{\kappa} f_{\lambda \alpha} -$
$\frac{1}{6} t_2 \partial^{\alpha} f^{\lambda}_{\kappa} \partial^{\kappa} f_{\lambda \alpha} + \frac{1}{3} t_1 \partial_{\kappa} f^{\lambda}_{\theta} \partial^{\kappa} f^{\theta}_{\lambda} - \frac{1}{6} t_2 \partial_{\kappa} f^{\lambda}_{\theta} \partial^{\kappa} f^{\theta}_{\lambda} + \frac{2}{3} t_1 \partial_{\kappa} f^{\lambda}_{\theta} \partial^{\kappa} f^{\theta}_{\lambda} +$
$\frac{1}{6} t_2 \partial_{\kappa} f^{\lambda}_{\theta} \partial^{\kappa} f_{\lambda}^{\theta} - t_1 \partial^{\alpha} f^{\lambda}_{\alpha} \partial^{\kappa} f_{\lambda \kappa} + \frac{2}{3} r_1 \partial_{\kappa} \omega^{\alpha \beta \theta} \partial^{\kappa} \omega_{\alpha \beta \theta} - \frac{2}{3} r_1 \partial_{\kappa} \omega^{\theta \alpha \beta} \partial^{\kappa} \omega_{\alpha \beta \theta} +$
$\frac{2}{3} r_1 \partial^{\beta} \omega_{I}^{\alpha \lambda} \partial_{\lambda} \omega_{\alpha \beta}^{\ \prime} - \frac{8}{3} r_1 \partial^{\beta} \omega_{I}^{\lambda \alpha} \partial_{\lambda} \omega_{\alpha \beta}^{\ \prime} + r_5 \partial_{\alpha} \omega_{\lambda}^{\ \alpha} \partial^{\lambda} \omega^{\theta \kappa}_{\ \kappa} - r_5 \partial_{\theta} \omega_{\lambda}^{\ \alpha} \partial^{\lambda} \omega^{\theta \kappa}_{\ \kappa}$

	$\omega_{1}^{\#1}{}_{\alpha\beta}$	$\omega_{1}^{\#2}{}_{\alpha\beta}$	$f_{1}^{\#1}$	$\omega_{1^-}^{\#1}{}_{\alpha}$	$\omega_{1}^{\#2}{}_{\alpha} f_{1}^{\#1}{}_{\alpha} f_{1}^{\#2}{}_{\alpha}$	$f_{1^{}}^{\#1}{}_{\alpha}$	$f_{1^-}^{\#2}\alpha$
$\omega_1^{\#1} + \alpha^{\beta}$	$\omega_{1}^{\#1} + \alpha \beta \left[ \frac{1}{6} \left( 6 k^2 \left( 2 r_1 + r_5 \right) + t_1 + 4 t_2 \right) \right]$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$-\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	0	0	0	0
$\omega_{1}^{#2} + \alpha^{\beta}$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$\frac{t_1+t_2}{3}$	$\frac{1}{3}\bar{l}k(t_1+t_2)$	0	0	0	0
$f_1^{#1} + ^{\alpha \beta}$	$\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	$-\frac{1}{3}ik(t_1+t_2)\left \frac{1}{3}k^2(t_1+t_2)\right $	$\frac{1}{3} k^2 (t_1 + t_2)$	0	0	0	0
$\omega_{1}^{\#1} +^{\alpha}$	0	0	0	$k^2 (r_1 + r_5) - \frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$ikt_1$
$\omega_{1}^{\#2} +^{\alpha}$	0	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
$f_1^{\#1} +^{\alpha}$	0	0	0	0	0	0	0
$f_1^{#2} + \alpha$	0	0	0	- <i>ī</i> k t <sub>1</sub>	0	0	0

## Massive and massless spectra



Massive partic	Massive particle					
Pole residue:	$\left  \frac{-3t_1t_2(t_1+t_2)+6r_1(t_1^2+2t_2^2)+3r_5(t_1^2+2t_2^2)}{(2r_1+r_5)(t_1+t_2)(-3t_1t_2+4r_1(t_1+t_2)+2r_5(t_1+t_2))} > 0 \right $					
Polarisations:	3					
Square mass:	$-\frac{3t_1t_2}{2(2r_1+r_5)(t_1+t_2)} > 0$					
Spin:	1					
Parity:	Even					



Massive particle						
Pole residue: $\left  -\frac{1}{r_1} > 0 \right $						
Polarisations:	5					
Square mass: $-\frac{t_1}{2r_1} > 0$						
Spin: 2						
Parity:	Odd					

(No massless particles)

## Unitarity conditions