

# Particle spectrograph

## Wave operator and propagator

Quadratic (free) action

$$\begin{aligned} S_F = & \int \int \int \left[ \left( \frac{1}{6} (-6 t_1 \omega_{\kappa\alpha}^{\alpha i} \omega_{\kappa\lambda}^{\kappa} - 2 (t_1 - 2 t_2) \omega_{\kappa\lambda}^{\kappa\lambda} \omega_{\kappa\lambda}^{\lambda} + 2 t_1 \omega_{\kappa\lambda}^{\lambda} \omega_{\kappa\lambda}^{\kappa\lambda} + 2 t_2 \omega_{\kappa\lambda}^{\lambda} \omega_{\kappa\lambda}^{\kappa\lambda} \right. \right. \\ & \omega_{\kappa\lambda}^{\kappa\lambda} + 6 f^{\alpha\beta} \tau_{\alpha\beta} + 6 \omega^{\alpha\beta\chi} \sigma_{\alpha\beta\chi} + 12 r_1 \partial_1 \omega_{\kappa\lambda}^{\kappa\lambda} \partial_1 \omega_{\lambda\alpha}^{\alpha} - 4 r_1 \partial_1^{\beta} \omega_{\kappa}^{\beta\theta\alpha} \partial_{\theta} \omega_{\alpha\beta}^{\kappa} - \\ & 4 r_1 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\alpha\beta\theta} + 4 r_1 \partial_{\theta} \omega_{\alpha\beta}^{\kappa} \partial_{\kappa} \omega^{\theta\alpha\beta} + 12 r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\theta\kappa\lambda} - \\ & 12 r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\theta\kappa\lambda} + 12 r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\kappa\lambda\theta} - 24 r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial_{\kappa} \omega^{\kappa\lambda\theta} - \\ & 2 t_1 \partial^{\alpha} f_{\theta\kappa} \partial^{\kappa} f_{\alpha}^{\theta} + t_2 \partial^{\alpha} f_{\theta\kappa} \partial^{\kappa} f_{\alpha}^{\theta} - 4 t_1 \partial^{\alpha} f_{\kappa\theta} \partial^{\kappa} f_{\alpha}^{\theta} - t_2 \partial^{\alpha} f_{\kappa\theta} \partial^{\kappa} f_{\alpha}^{\theta} - \\ & 2 t_1 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\alpha\lambda} + t_2 \partial^{\alpha} f_{\kappa}^{\lambda} \partial^{\kappa} f_{\alpha\lambda} + 6 t_1 \omega_{\kappa\alpha}^{\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} + 6 t_1 \omega_{\kappa\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + \\ & 12 t_1 \partial^{\alpha} f_{\kappa\alpha} \partial^{\kappa} f_{\lambda}^{\lambda} - 6 t_1 \partial_{\kappa} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + 2 t_1 \omega_{\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} + 2 t_2 \omega_{\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} + \\ & 8 t_1 \omega_{\kappa\theta} \partial^{\kappa} f_{\lambda}^{\lambda} - 4 t_2 \omega_{\kappa\theta} \partial^{\kappa} f_{\lambda}^{\lambda} - 2 t_1 \omega_{\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} - 2 t_2 \omega_{\theta\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} + \\ & 4 t_1 \omega_{\theta\kappa\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} + 4 t_2 \omega_{\theta\kappa\lambda} \partial^{\kappa} f_{\lambda}^{\lambda} - 6 t_1 \omega_{\lambda\alpha}^{\alpha} \partial^{\kappa} f_{\kappa}^{\lambda} - 6 t_1 \omega_{\lambda\kappa}^{\kappa} \partial^{\kappa} f_{\lambda}^{\lambda} + \\ & 2 t_1 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda\alpha} - t_2 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda\alpha} + 2 t_1 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} - t_2 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} + \\ & 4 t_1 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} + t_2 \partial_{\kappa} f_{\theta}^{\lambda} \partial^{\kappa} f_{\lambda}^{\theta} - 6 t_1 \partial^{\alpha} f_{\lambda}^{\lambda} \partial^{\kappa} f_{\lambda\kappa} + 4 r_1 \partial_{\kappa} \omega^{\alpha\beta\theta} \partial^{\kappa} \omega_{\alpha\beta\theta} - \\ & 4 r_1 \partial_{\kappa} \omega^{\theta\alpha\beta} \partial^{\kappa} \omega_{\alpha\beta\theta} + 4 r_1 \partial_1^{\beta} \omega_{\alpha\lambda}^{\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\lambda} - 16 r_1 \partial_1^{\beta} \omega_{\alpha\lambda}^{\alpha} \partial_{\lambda} \omega_{\alpha\beta}^{\lambda} - \\ & 12 r_1 \partial_{\alpha} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\kappa}^{\theta\kappa} + 12 r_1 \partial_{\theta} \omega_{\lambda}^{\alpha} \partial^{\lambda} \omega_{\alpha}^{\theta\kappa} \Big) \Big] [t, x, y, z] dz dy dx dt \end{aligned}$$

$\sigma_{1+}^{\#1} \dagger \alpha\beta$	$\sigma_{1+}^{\#2} \dagger \alpha\beta$	$\tau_{1+}^{\#1} \dagger \alpha\beta$	$\sigma_{1-}^{\#1} \dagger \alpha$	$\sigma_{1-}^{\#2} \dagger \alpha$	$\tau_{1-}^{\#1} \dagger \alpha$	$\tau_{1-}^{\#2} \dagger \alpha$
$\sigma_{1+}^{\#1} \dagger \alpha\beta$	$\frac{2(t_1+t_2)}{3t_1t_2}$	$\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	0	0	0
$\sigma_{1+}^{\#2} \dagger \alpha\beta$	$\frac{\sqrt{2}(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$\frac{t_1+4t_2}{3(1+k^2)^2t_1t_2}$	$\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0
$\tau_{1+}^{\#1} \dagger \alpha\beta$	$-\frac{i\sqrt{2}k(t_1-2t_2)}{3(1+k^2)t_1t_2}$	$-\frac{ik(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	$\frac{k^2(t_1+4t_2)}{3(1+k^2)^2t_1t_2}$	0	0	0
$\sigma_{1-}^{\#1} \dagger \alpha$	0	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	0	$\frac{2ik}{t_1+2k^2t_1}$
$\sigma_{1-}^{\#2} \dagger \alpha$	0	0	$\frac{\sqrt{2}}{t_1+2k^2t_1}$	$\frac{2k^2r_1+t_1}{(t_1+2k^2t_1)^2}$	0	$\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$
$\tau_{1-}^{\#1} \dagger \alpha$	0	0	0	0	0	0
$\tau_{1-}^{\#2} \dagger \alpha$	0	0	$-\frac{2ik}{t_1+2k^2t_1}$	$-\frac{i\sqrt{2}k(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$	0	$\frac{2k^2(2k^2r_1+t_1)}{(t_1+2k^2t_1)^2}$

$\omega_{1+}^{\#1} \dagger \alpha\beta$	$\omega_{1+}^{\#2} \dagger \alpha\beta$	$f_{1+}^{\#1} \dagger \alpha\beta$	$\omega_{1-}^{\#1} \dagger \alpha$	$\omega_{1-}^{\#2} \dagger \alpha$	$f_{1-}^{\#1} \dagger \alpha$	$f_{1-}^{\#2} \dagger \alpha$
$\omega_{1+}^{\#1} \dagger \alpha\beta$	$\frac{1}{6}(t_1+4t_2)$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$-\frac{ik(t_1-2t_2)}{3\sqrt{2}}$	0	0	0
$\omega_{1+}^{\#2} \dagger \alpha\beta$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$\frac{t_1+t_2}{3}$	$\frac{1}{3}ik(t_1+t_2)$	0	0	0
$f_{1+}^{\#1} \dagger \alpha\beta$	$-\frac{1}{3}ik(t_1-2t_2)$	$\frac{1}{3}k^2(t_1+t_2)$	$\frac{1}{3}k^2(t_1+t_2)$	0	0	0
$\omega_{1-}^{\#1} \dagger \alpha$	0	0	$-k^2r_1-\frac{t_1}{2}$	$\frac{t_1}{\sqrt{2}}$	0	$ik t_1$
$\omega_{1-}^{\#2} \dagger \alpha$	0	0	$\frac{t_1}{\sqrt{2}}$	0	0	0
$f_{1-}^{\#1} \dagger \alpha$	0	0	0	0	0	0
$f_{1-}^{\#2} \dagger \alpha$	0	0	$-ik t_1$	0	0	0

Source constraints/gauge generators

SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} - 2ik\sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2ik\sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + ik\sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2ik\sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	16

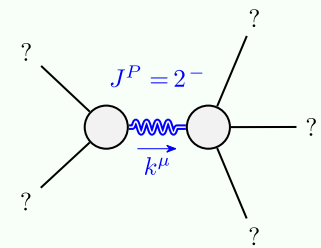
$\omega_{2+}^{\#1} \dagger \alpha\beta$	$f_{2+}^{\#1} \dagger \alpha\beta$	$\omega_{2-}^{\#1} \dagger \alpha\beta\chi$	
$\omega_{2+}^{\#1} \dagger \alpha\beta$	$\frac{t_1}{2}$	$-\frac{ik t_1}{\sqrt{2}}$	0
$f_{2+}^{\#1} \dagger \alpha\beta$	$\frac{ik t_1}{\sqrt{2}}$	$k^2 t_1$	0
$\omega_{2-}^{\#1} \dagger \alpha\beta\chi$	0	0	$k^2 r_1 + \frac{t_1}{2}$

$\omega_{0+}^{\#1} \dagger$	$\omega_{0+}^{\#1}$	$f_{0+}^{\#1} \dagger$	$f_{0+}^{\#1}$	$f_{0+}^{\#2} \dagger$	$\omega_{0+}^{\#1}$
$\omega_{0+}^{\#1} \dagger$	$-t_1$	$i\sqrt{2}k t_1$	0	0	0
$f_{0+}^{\#1} \dagger$	$-i\sqrt{2}k t_1$	$-2k^2 t_1$	0	0	0
$f_{0+}^{\#2} \dagger$	0	0	0	0	0
$\omega_{0-}^{\#1} \dagger$	0	0	0	$t_2$	0

$\sigma_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#1} \dagger$	$\tau_{0+}^{\#2} \dagger$	$\sigma_{0-}^{\#1} \dagger$
$\sigma_{0+}^{\#1} \dagger$	$-\frac{1}{(1+2k^2)^2t_1}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
$\tau_{0+}^{\#1} \dagger$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0
$\tau_{0+}^{\#2} \dagger$	0	0	0
$\sigma_{0-}^{\#1} \dagger$	0	0	$\frac{1}{t_2}$

$\sigma_2^{\#1} \dagger \alpha\beta$	$\sigma_2^{\#1}$	$\tau_2^{\#1} \dagger \alpha\beta$	$\sigma_2^{\#1} \alpha\beta\chi$
$\sigma_2^{\#1} \dagger \alpha\beta$	$\frac{2}{(1+2k^2)^2 t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	0
$\tau_2^{\#1} \dagger \alpha\beta$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	$\frac{4k^2}{(1+2k^2)^2 t_1}$	0
$\sigma_2^{\#1} \dagger \alpha\beta\chi$	0	0	$\frac{2}{2k^2 r_1+t_1}$

## Massive and massless spectra



Massive particle

Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)

## Unitarity conditions

$$r_1 < 0 \&\& t_1 > 0$$