

Particle spectrograph

Wave operator and propagator

	$\sigma_{1^+\alpha\beta}^{\#1}$	$\sigma_{1^+\alpha\beta}^{\#2}$	$\tau_{1^+\alpha\beta}^{\#1}$	$\sigma_{1^-\alpha}^{\#1}$	$\sigma_{1^-\alpha}^{\#2}$	$\tau_{1^-\alpha}^{\#1}$	$\tau_{1^-\alpha}^{\#2}$
$\sigma_{1^+}^{\#1}\dagger^{\alpha\beta}$	0	$-\frac{\sqrt{2}}{t_1+k^2t_1}$	$-\frac{i\sqrt{2}k}{t_1+k^2t_1}$	0	0	0	0
$\sigma_{1^+}^{\#2}\dagger^{\alpha\beta}$	$-\frac{\sqrt{2}}{t_1+k^2t_1}$	$\frac{-2k^2(2r_1+r_5)+t_1}{(1+k^2)^2t_1^2}$	$\frac{-2ik^3(2r_1+r_5)+ikt_1}{(1+k^2)^2t_1^2}$	0	0	0	0
$\tau_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{i\sqrt{2}k}{t_1+k^2t_1}$	$\frac{i(2k^3(2r_1+r_5)-kt_1)}{(1+k^2)^2t_1^2}$	$\frac{-2k^4(2r_1+r_5)+k^2t_1}{(1+k^2)^2t_1^2}$	0	0	0	0
$\sigma_{1^+}^{\#1}\dagger^\alpha$	0	0	0	$\frac{2(t_1+t_3)}{3t_1t_3+2k^2(r_1+r_5)(t_1+t_3)}$	$-\frac{\sqrt{2}(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$	0	$-\frac{2ik(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$
$\sigma_{1^+}^{\#2}\dagger^\alpha$	0	0	0	$-\frac{\sqrt{2}(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$	$\frac{6k^2(r_1+r_5)+t_1+4t_3}{(1+2k^2)^2(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$	0	$\frac{i\sqrt{2}k(6k^2(r_1+r_5)+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$
$\tau_{1^+}^{\#1}\dagger^\alpha$	0	0	0	0	0	0	0
$\tau_{1^+}^{\#2}\dagger^\alpha$	0	0	0	$\frac{2ik(t_1-2t_3)}{(1+2k^2)(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$	$-\frac{i\sqrt{2}k(6k^2(r_1+r_5)+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$	0	$\frac{2k^2(6k^2(r_1+r_5)+t_1+4t_3)}{(1+2k^2)^2(3t_1t_3+2k^2(r_1+r_5)(t_1+t_3))}$

Source constraints		
SO(3) irreps	Fundamental fields	Multiplicities
$\tau_{0^+}^{\#2}==0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta}==0$	1
$\tau_{0^+}^{\#1}-2\,i\,k\,\sigma_{0^+}^{\#1}==0$	$\partial_\beta\partial_\alpha\tau^{\alpha\beta}==\partial_\beta\partial^\beta\tau^\alpha_\alpha+2\,\partial_\chi\partial^X\partial_\beta\sigma^{\alpha\beta}_\alpha$	1
$\tau_1^{\#2\alpha}+2\,i\,k\,\sigma_1^{\#2\alpha}==0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi}==\partial_\chi\partial^X\partial_\beta\tau^{\alpha\beta}+2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\alpha\beta\chi}$	3
$\tau_1^{\#1\alpha}==0$	$\partial_\chi\partial_\beta\partial^\alpha\tau^{\beta\chi}==\partial_\chi\partial^X\partial_\beta\tau^{\beta\alpha}$	3
$\tau_{1^+}^{\#1\alpha\beta}+i\,k\,\sigma_{1^+}^{\#2\alpha\beta}==0$	$\partial_\chi\partial^\alpha\tau^{\beta\chi}+\partial_\chi\partial^\beta\tau^{\chi\alpha}+\partial_\chi\partial^\chi\tau^{\alpha\beta}+$ $2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\chi\delta}+2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi}==$ $\partial_\chi\partial^\alpha\tau^{\chi\beta}+\partial_\chi\partial^\beta\tau^{\alpha\chi}+$ $\partial_\chi\partial^\chi\tau^{\beta\alpha}+2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\chi\delta}$	3
$\tau_{2^+}^{\#1\alpha\beta}-2\,i\,k\,\sigma_{2^+}^{\#1\alpha\beta}==0$	$-i\,(4\,\partial_\delta\partial_\chi\partial^\beta\partial^\alpha\tau^{\chi\delta}+2\,\partial_\delta\partial^\delta\partial^\beta\partial^\alpha\tau^{\chi\chi}_\chi-$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\beta\chi}-3\,\partial_\delta\partial^\delta\partial_\chi\partial^\alpha\tau^{\chi\beta}-$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\alpha\chi}-3\,\partial_\delta\partial^\delta\partial_\chi\partial^\beta\tau^{\chi\alpha}+$ $3\,\partial_\delta\partial^\delta\partial_\chi\partial^X\tau^{\alpha\beta}+3\,\partial_\delta\partial^\delta\partial_\chi\partial^X\tau^{\beta\alpha}+$ $4\,i\,k^X\,\partial_\epsilon\partial_\chi\partial^\beta\partial^\alpha\sigma^{\delta\epsilon}_\delta-$ $6\,i\,k^X\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\alpha\sigma^{\beta\delta\epsilon}-$ $6\,i\,k^X\,\partial_\epsilon\partial_\delta\partial_\chi\partial^\beta\sigma^{\alpha\delta\epsilon}+$ $2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\tau^{\chi\delta}+$ $6\,i\,k^X\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\alpha\delta\beta}+$ $6\,i\,k^X\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial_\chi\sigma^{\beta\delta\alpha}-$ $2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\tau^{\chi\chi}_\chi-$ $4\,i\,\eta^{\alpha\beta}\,k^X\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\chi\sigma^{\delta\epsilon}_\delta)==0$	5
Total constraints/gauge generators:		16

	$\omega_{0^+}^{\#1}$	$f_{0^+}^{\#1}$	$f_{0^+}^{\#2}$	$\omega_{0^+}^{\#1}$		$\omega_{2^+}^{\#1}\alpha\beta$	$f_{2^+}^{\#1}\alpha\beta$	$\omega_{2^+}^{\#1}\alpha\beta\chi$
$\omega_{0^+}^{\#1}\dagger$	$t_3$	$-i\sqrt{2}kt_3$	0	0	$\omega_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{t_1}{2}$	$-\frac{ikt_1}{\sqrt{2}}$	0
$f_{0^+}^{\#1}\dagger$	$i\sqrt{2}kt_3$	$2k^2t_3$	0	0	$f_{2^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	$k^2t_1$	0
$f_{0^+}^{\#2}\dagger$	0	0	0	0	$\omega_{2^+}^{\#1}\dagger^{\alpha\beta\chi}$	0	0	$k^2r_1+\frac{t_1}{2}$
$\omega_{0^+}^{\#1}\dagger$	0	0	0	$-t_1$				
	$\omega_{1^+}^{\#1}\alpha\beta$	$\omega_{1^+}^{\#2}\alpha\beta$	$f_{1^+}^{\#1}\alpha\beta$		$\omega_{1^+}^{\#1}\alpha$	$\omega_{1^+}^{\#2}\alpha$	$f_{1^+}^{\#1}\alpha$	$f_{1^+}^{\#2}\alpha$
$\omega_{1^+}^{\#1}\dagger^{\alpha\beta}$	$k^2(2r_1+r_5)-\frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$		0	0	0	0
$\omega_{1^+}^{\#2}\dagger^{\alpha\beta}$	$-\frac{t_1}{\sqrt{2}}$	0	0		0	0	0	0
$f_{1^+}^{\#1}\dagger^{\alpha\beta}$	$\frac{ikt_1}{\sqrt{2}}$	0	0		0	0	0	0
$\omega_{1^+}^{\#1}\dagger^\alpha$	0	0	0		$\frac{1}{6}(6k^2(r_1+r_5)+t_1+4t_3)$	$\frac{t_1-2t_3}{3\sqrt{2}}$	0	$\frac{1}{3}ik(t_1-2t_3)$
$\omega_{1^+}^{\#2}\dagger^\alpha$	0	0	0		$\frac{t_1-2t_3}{3\sqrt{2}}$	$\frac{t_1+t_3}{3}$	0	$\frac{1}{3}i\sqrt{2}k(t_1+t_3)$
$f_{1^+}^{\#1}\dagger^\alpha$	0	0	0		0	0	0	0
$f_{1^+}^{\#2}\dagger^\alpha$	0	0	0		$-\frac{1}{3}ik(t_1-2t_3)$	$-\frac{1}{3}i\sqrt{2}k(t_1+t_3)$	0	$\frac{2}{3}k^2(t_1+t_3)$

Quadratic (free) action
$S==\iiint\!\!\!\int(\frac{1}{6}(2\,\omega^{\alpha i}_\alpha(t_1\,\omega_{\mid\theta}^\theta-2\,t_3\,\omega_{\mid\kappa}^\kappa)+6\,f^{\alpha\beta}\,\tau_{\alpha\beta}+6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}-$ $4\,t_1\,\omega_\alpha^\theta\partial_{\mid}f^{\alpha i}+8\,t_3\,\omega_\alpha^\kappa\partial_{\mid}f^{\alpha i}+4\,t_1\,\omega_{\mid\theta}^\theta\partial'f^\alpha_\alpha-$ $8\,t_3\,\omega_{\mid\kappa}^\kappa\partial'f^\alpha_\alpha-2\,t_1\partial_{\mid}f_{\mid\theta}^\theta\partial'f^\alpha_\alpha+4\,t_3\partial_{\mid}f_{\mid\kappa}^\kappa\partial'f^\alpha_\alpha-$ $2\,t_1\partial_{\mid}f^{\alpha i}\partial_{\theta}f^\alpha_\theta+4\,t_1\partial'f^\alpha_\alpha\partial_{\theta}f_{\mid\theta}^\theta-6\,t_1\partial_{\alpha}f_{\mid\theta}^\theta\partial^\theta f^{\alpha i}-$ $3\,t_1\partial_{\alpha}f_{\theta\mid}\partial^\theta f^{\alpha i}+3\,t_1\partial_{\mid}f_{\alpha\theta}\partial^\theta f^{\alpha i}+3\,t_1\partial_{\theta}f_{\alpha i}\partial^\theta f^{\alpha i}+$ $3\,t_1\partial_{\theta}f_{\mid\alpha}\partial^\theta f^{\alpha i}+6\,t_1\,\omega_{\alpha\theta\mid}(\,\omega^{\alpha i\theta}+2\,\partial^\theta f^{\alpha i})-$ $8\,r_1\,\partial_\beta\omega_{\alpha i\theta}\partial^\theta\omega^{\alpha\beta i}+4\,r_1\,\partial_\beta\omega_{\alpha\theta\mid}\partial^\theta\omega^{\alpha\beta i}-$ $16\,r_1\,\partial_\beta\omega_{\mid\theta\alpha}\partial^\theta\omega^{\alpha\beta i}-4\,r_1\,\partial_{\mid}\omega_{\alpha\beta\theta}\partial^\theta\omega^{\alpha\beta i}+$ $4\,r_1\,\partial_\theta\omega_{\alpha\beta i}\partial^\theta\omega^{\alpha\beta i}+4\,r_1\,\partial_\theta\omega_{\alpha i\beta}\partial^\theta\omega^{\alpha\beta i}+$ $6\,r_5\,\partial_{\mid}\omega_{\theta\mid\kappa}^\kappa\partial^\theta\omega^{\alpha i}_\alpha-6\,r_5\,\partial_\theta\omega_{\mid\kappa}^\kappa\partial^\theta\omega^{\alpha i}_\alpha+$ $4\,t_3\partial_{\mid}f^{\alpha i}\partial_{\kappa}f^\alpha_\kappa-8\,t_3\partial'f^\alpha_\alpha\partial_{\kappa}f_{\mid\kappa}^\kappa-6\,r_5\,\partial_\alpha\omega^{\alpha i\theta}\partial_{\kappa}\omega_{\mid\theta}^\kappa+$ $12\,r_5\,\partial^\theta\omega^{\alpha i}_\alpha\partial_{\kappa}\omega_{\mid\theta}^\kappa+6\,r_5\,\partial_\alpha\omega^{\alpha i\theta}\partial_{\kappa}\omega_{\theta\mid\kappa}^\kappa-$ $12\,r_5\,\partial^\theta\omega^{\alpha i}_\alpha\partial_{\kappa}\omega_{\theta\mid\kappa}^\kappa)) [t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt$

$\sigma_{2^+}^{\#1}\alpha\beta\chi$	$\sigma_{2^+}^{\#1}\alpha\beta$	$\sigma_{2^+}^{\#1}\alpha$
0	0	$\frac{2}{2k^2r_1+t_1}$
$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	0
$\frac{2}{(1+2k^2)^2t_1}$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	0
$\sigma_{2^+}^{\#1}+\alpha\beta$	$\tau_{2^+}^{\#1}+\alpha\beta$	$\sigma_{2^+}^{\#1}+\alpha\beta\chi$

$\sigma_{0^+}^{\#1}$	$\tau_{0^+}^{\#2}$	$\sigma_{0^+}^{\#1}$
0	0	$-\frac{1}{t_1}$
$\frac{1}{(1+2k^2)^2t_3}$	$\frac{1}{(1+2k^2)^2t_3}$	0
$\frac{i\sqrt{2}k}{(1+2k^2)^2t_3}$	$\frac{2k^2}{(1+2k^2)^2t_3}$	0
$\sigma_{0^+}^{\#1}+$	$\tau_{0^+}^{\#2}+$	$\sigma_{0^+}^{\#1}+$

Massive and massless spectra

Massive particle	
Pole residue:	$-\frac{3(-2t_1t_3(t_1+t_3)+r_1(t_1^2+2t_3^2)+r_5(t_1^2+2t_3^2))}{2(r_1+r_5)(t_1+t_3)(-3t_1t_3+r_1(t_1+t_3)+r_5(t_1+t_3))}>0$
Polarisations:	3
Square mass:	$-\frac{3t_1t_3}{2(r_1+r_5)(t_1+t_3)}>0$
Spin:	1
Parity:	Odd

Massive particle		(No massless particles)
Pole residue:	$-\frac{1}{r_1}>0$	
Polarisations:	5	
Square mass:	$-\frac{t_1}{2r_1}>0$	
Spin:	2	
Parity:	Odd	

Unitarity conditions

$r_1 < 0 \ \&\& \ r_5 < -r_1 \ \&\& \ t_1 > 0 \ \&\& \ t_3 < -t_1 \ || \ t_3 > 0$