## Lagrangian density

$$-t_1\,\omega_I{}^{\alpha I}\,\omega_{\kappa\alpha}{}^\kappa - t_1\,\omega_I{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}^I + f^{\alpha\beta}\,\tau_{\alpha\beta} + \omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - r_5\,\partial_I\omega^{\kappa\lambda}{}_\kappa\,\partial^I\omega_\lambda{}^\alpha{}_\alpha +$$
$$\tfrac{2}{3}r_2\,\partial^\beta\omega_I{}^{\theta\alpha}\,\partial_\theta\omega_{\alpha\beta}{}^\kappa{}_\kappa - \tfrac{1}{3}r_2\,\partial_\theta\omega_{\alpha\beta}{}^\theta - \tfrac{2}{3}r_2\,\partial_\theta\omega_{\alpha\beta}{}^\kappa\,\partial_\kappa\omega^{\theta\alpha\beta} -$$
$$r_5\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial_\kappa\omega^{\theta\kappa\lambda} + r_5\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial_\kappa\omega^{\kappa\lambda\theta} + 2r_5\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial_\kappa\omega^{\kappa\lambda\theta} -$$
$$\tfrac{1}{2}t_1\,\partial^\alpha f_{\theta\kappa}\,\partial^\kappa f_\alpha{}^\theta - \tfrac{1}{2}t_1\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f^\theta{}_\alpha - \tfrac{1}{2}t_1\,\partial^\alpha f^\lambda{}_\kappa\,\partial^\kappa f^I{}_I + t_1\,\omega_{\kappa\alpha}{}^\alpha\,\partial^\kappa f^{I}{}_I -$$
$$t_1\,\omega_{\kappa\lambda}{}^\lambda\,\partial^\kappa f^I{}_I + 2t_1\,\partial^\alpha f_{\kappa\alpha}\,\partial^\kappa f^I{}_I - t_1\,\partial_\kappa f^\lambda{}_\lambda\,\partial^\kappa f^I{}_I + 2t_1\,\omega_{I\kappa\theta}\,\partial^\kappa f^{I\theta} -$$
$$t_1\,\omega_{I\alpha}{}^\alpha\,\partial^\kappa f^I{}_I - t_1\,\omega_{I\lambda}{}^\alpha\,\partial^\kappa f^I{}_\kappa + \tfrac{1}{2}t_1\,\partial^{\alpha}f^\lambda{}_\kappa\,\partial^\kappa f_{\lambda\alpha} + \tfrac{1}{2}t_1\,\partial_\lambda f_\theta{}^\lambda\,\partial^\kappa f_\lambda{}^\theta +$$
$$\tfrac{1}{2}t_1\,\partial_\kappa f^\lambda{}_\theta\,\partial^\kappa f_\lambda{}^\theta - t_1\,\partial^\alpha f_\alpha{}^\kappa\,\partial^\kappa f_{\lambda\kappa} + \tfrac{1}{3}r_2\,\partial_\kappa\omega^{\theta\alpha\beta}\,\partial_\theta\omega_{\alpha\beta\theta} + \tfrac{2}{3}r_2\,\partial_\kappa\omega^{\theta\alpha\beta}\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa -$$
$$\tfrac{2}{3}r_2\,\partial^\beta\omega_I{}^{\alpha\lambda}\,\partial_\lambda\omega_{\alpha\beta}{}^I + \tfrac{2}{3}r_2\,\partial^\beta\omega^{\lambda\alpha}{}_I\,\partial_\lambda\omega_{\alpha\beta}{}^I + r_5\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa - r_5\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa$$

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| | $\sigma^{\#1}_{0^+}$ | $\tau^{\#1}_{0^+}$ | $\tau^{\#2}_{0^+}$ | $\sigma^{\#1}_{0^-}$ |
|---|---|---|---|---|
| $\sigma^{\#1}_{0^+}\dagger$ | $\dfrac{1}{(1+2k^2)^2 t_1}$ | $\dfrac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | $0$ | $0$ |
| $\tau^{\#1}_{0^+}\dagger$ | $-\dfrac{i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | $-\dfrac{2k^2}{(1+2k^2)^2 t_1}$ | $0$ | $0$ |
| $\tau^{\#2}_{0^+}\dagger$ | $0$ | $0$ | $0$ | $0$ |
| $\sigma^{\#1}_{0^-}\dagger$ | $0$ | $0$ | $0$ | $\dfrac{1}{k^2 r_2-t_1}$ |

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## Source constraints

| SO(3) irreps | # |
|---|---|
| $\tau^{\#2}_{0^+}==0$ | 1 |
| $\tau^{\#1}_{0^+}-2ik\,\sigma^{\#1}_{0^+}==0$ | 1 |
| $\tau^{\#2}_{1^-}{}^\alpha+2ik\,\sigma^{\#2}_{1^-}{}^\alpha==0$ | 3 |
| $\tau^{\#1}_{1^-}{}^\alpha==0$ | 3 |
| $\tau^{\#1}_{1^+}{}^{\alpha\beta}+ik\,\sigma^{\#1}_{1^+}{}^{\alpha\beta}==0$ | 3 |
| $\tau^{\#1}_{2^+}{}^{\alpha\beta}-2ik\,\sigma^{\#1}_{2^+}{}^{\alpha\beta}==0$ | 5 |
| Total #: | 16 |

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| | $\omega^{\#1}_{0^-}$ | $f^{\#2}_{0^+}$ | $f^{\#1}_{0^+}$ | $\omega^{\#1}_{0^+}$ |
|---|---|---|---|---|
| $\omega^{\#1}_{0^+}\dagger$ | $0$ | $0$ | $0$ | $-t_1$ |
| $f^{\#1}_{0^+}\dagger$ | $0$ | $i\sqrt{2}\,k t_1$ | $-2k^2 t_1$ | $\sqrt{2}\,k t_1$ |
| $f^{\#2}_{0^+}\dagger$ | $0$ | $0$ | $0$ | $0$ |
| $\omega^{\#1}_{0^-}\dagger$ | $k^2 r_2-t_1$ | $0$ | $0$ | $0$ |

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| | $\sigma^{\#1}_{1^+}\dagger^{\alpha\beta}$ | $\sigma^{\#2}_{1^+}\dagger^{\alpha\beta}$ | $\tau^{\#1}_{1^+}\dagger^{\alpha\beta}$ | $\sigma^{\#1}_{1^-}\dagger^{\alpha}$ | $\sigma^{\#2}_{1^-}\dagger^{\alpha}$ | $\tau^{\#1}_{1^-}\dagger^{\alpha}$ | $\tau^{\#2}_{1^-}\dagger^{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\tau^{\#2}_{1^-}{}_\alpha$ | $0$ | $0$ | $0$ | $\dfrac{2ik}{t_1+2k^2 t_1}$ | $\dfrac{i\sqrt{2}\,k(2k^2 r_5-t_1)}{(t_1+2k^2 t_1)^2}$ | $0$ | $\dfrac{-4k^4 r_5+2k^2 t_1}{(t_1+2k^2 t_1)^2}$ |
| $\tau^{\#1}_{1^-}{}_\alpha$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $\sigma^{\#2}_{1^-}{}_\alpha$ | $0$ | $0$ | $0$ | $\dfrac{\sqrt{2}}{t_1+2k^2 t_1}$ | $\dfrac{-2k^2 r_5+t_1}{(t_1+2k^2 t_1)^2}$ | $0$ | $\dfrac{i\sqrt{2}\,k(2k^2 r_5-t_1)}{(t_1+2k^2 t_1)^2}$ |
| $\sigma^{\#1}_{1^-}{}_\alpha$ | $0$ | $0$ | $0$ | $0$ | $\dfrac{\sqrt{2}}{t_1+2k^2 t_1}$ | $0$ | $\dfrac{2ik}{t_1+2k^2 t_1}$ |
| $\tau^{\#1}_{1^+}{}_{\alpha\beta}$ | $\dfrac{i\sqrt{2}\,k}{t_1+k^2 t_1}$ | $-\dfrac{i(2k^3 r_5-k t_1)}{(1+k^2)^2 t_1^2}$ | $\dfrac{-2k^4 r_5+k^2 t_1}{(1+k^2)^2 t_1^2}$ | $0$ | $0$ | $0$ | $0$ |
| $\sigma^{\#2}_{1^+}{}_{\alpha\beta}$ | $-\dfrac{\sqrt{2}}{t_1+k^2 t_1}$ | $\dfrac{-2k^2 r_5+t_1}{(1+k^2)^2 t_1^2}$ | $\dfrac{i(2k^3 r_5-k t_1)}{(1+k^2)^2 t_1^2}$ | $0$ | $0$ | $0$ | $0$ |
| $\sigma^{\#1}_{1^+}{}_{\alpha\beta}$ | $0$ | $-\dfrac{\sqrt{2}}{t_1+k^2 t_1}$ | $\dfrac{i\sqrt{2}\,k}{t_1+k^2 t_1}$ | $0$ | $0$ | $0$ | $0$ |

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| | $\omega^{\#1}_{1^+}\dagger^{\alpha\beta}$ | $\omega^{\#2}_{1^+}\dagger^{\alpha\beta}$ | $f^{\#1}_{1^+}\dagger^{\alpha\beta}$ | $\omega^{\#1}_{1^-}\dagger^{\alpha}$ | $\omega^{\#2}_{1^-}\dagger^{\alpha}$ | $f^{\#1}_{1^-}\dagger^{\alpha}$ | $f^{\#2}_{1^-}\dagger^{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $f^{\#2}_{1^-}{}_\alpha$ | $0$ | $0$ | $0$ | $ik t_1$ | $0$ | $0$ | $0$ |
| $f^{\#1}_{1^-}{}_\alpha$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $\omega^{\#2}_{1^-}{}_\alpha$ | $0$ | $0$ | $0$ | $0$ | $\dfrac{t_1}{\sqrt{2}}$ | $0$ | $0$ |
| $\omega^{\#1}_{1^-}{}_\alpha$ | $0$ | $0$ | $0$ | $k^2 r_5-\dfrac{t_1}{2}$ | $\dfrac{t_1}{\sqrt{2}}$ | $-ik t_1$ | $0$ |
| $f^{\#1}_{1^+}{}_{\alpha\beta}$ | $\dfrac{ik t_1}{\sqrt{2}}$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $\omega^{\#2}_{1^+}{}_{\alpha\beta}$ | $-\dfrac{t_1}{\sqrt{2}}$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $\omega^{\#1}_{1^+}{}_{\alpha\beta}$ | $k^2 r_5-\dfrac{t_1}{2}$ | $-\dfrac{t_1}{\sqrt{2}}$ | $\dfrac{ik t_1}{\sqrt{2}}$ | $0$ | $0$ | $0$ | $0$ |

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| | $\omega^{\#1}_{2^+}{}_{\alpha\beta}$ | $f^{\#1}_{2^+}{}_{\alpha\beta}$ | $\omega^{\#1}_{2^-}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\dfrac{t_1}{2}$ | $-\dfrac{ik t_1}{\sqrt{2}}$ | $0$ |
| $f^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\dfrac{ik t_1}{\sqrt{2}}$ | $k^2 t_1$ | $0$ |
| $\omega^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$ | $0$ | $0$ | $\dfrac{t_1}{2}$ |

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| | $\sigma^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\tau^{\#1}_{2^+}\dagger^{\alpha\beta}$ | $\sigma^{\#1}_{2^-}\dagger^{\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma^{\#1}_{2^-}{}_{\alpha\beta\chi}$ | $0$ | $0$ | $\dfrac{2}{t_1}$ |
| $\tau^{\#1}_{2^+}{}_{\alpha\beta}$ | $\dfrac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | $\dfrac{4k^2}{(1+2k^2)^2 t_1}$ | $0$ |
| $\sigma^{\#1}_{2^+}{}_{\alpha\beta}$ | $\dfrac{2}{(1+2k^2)^2 t_1}$ | $-\dfrac{2i\sqrt{2}\,k}{(1+2k^2)^2 t_1}$ | $0$ |

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### Massive particle

$J^P = 0^-$, $k^\mu$

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|---|---|
| Pole residue: | $-\dfrac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $\dfrac{t_1}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

(No massless particles)

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## Unitarity conditions

$r_2 < 0$ && $t_1 < 0$