

Wave operator and propagator

Quadratic (free) Lagrangian density

$$\begin{aligned}
& -t_1 \omega'_{\lambda} \omega^{\alpha\lambda}_{\kappa} \omega^{\kappa-\frac{1}{3}}_{\kappa} t_1 \omega^{\kappa\lambda}_{\lambda} \omega^{\lambda'+\frac{2}{3}}_{\kappa} t_2 \omega^{\kappa\lambda}_{\lambda} \omega^{\lambda'+\frac{2}{3}}_{\kappa} \tau_{\alpha\beta} + \omega^{\alpha\beta\chi}_{\alpha} \sigma_{\alpha\beta\chi} + \\
& \frac{1}{3} t_1 \omega^{\lambda}_{\lambda} \omega^{\lambda\kappa}_{\lambda} + \frac{1}{3} t_2 \omega^{\lambda}_{\lambda} \omega^{\lambda\kappa}_{\lambda} + f^{\alpha\beta} \tau_{\alpha\beta} + \omega^{\alpha\beta\chi}_{\alpha} \sigma_{\alpha\beta\chi} + \\
& 2 r_1 \partial_{\lambda} \omega^{\kappa\lambda}_{\kappa} \partial^{\lambda}_{\lambda} \omega^{\alpha}_{\alpha} - \frac{2}{3} r_1 \partial^{\beta} \omega^{\theta\alpha}_{\kappa} \partial_{\theta} \omega^{\kappa}_{\kappa} + \frac{2}{3} r_2 \partial^{\theta} \omega^{\theta\alpha}_{\kappa} \partial_{\theta} \omega^{\kappa}_{\kappa} - \\
& \frac{2}{3} r_1 \partial_{\theta} \omega^{\kappa}_{\alpha\beta} \partial^{\kappa}_{\kappa} \omega^{\alpha\beta\theta}_{\alpha} - \frac{1}{3} r_2 \partial_{\theta} \omega^{\kappa}_{\alpha\beta} \partial^{\kappa}_{\kappa} \omega^{\alpha\beta\theta}_{\alpha} + \frac{2}{3} r_1 \partial_{\theta} \omega^{\kappa}_{\alpha\beta} \partial^{\kappa}_{\kappa} \omega^{\theta\alpha\beta}_{\alpha} - \\
& \frac{2}{3} r_2 \partial_{\theta} \omega^{\kappa}_{\alpha\beta} \partial^{\kappa}_{\kappa} \omega^{\theta\alpha\beta}_{\alpha} + 2 r_1 \partial_{\alpha} \omega^{\alpha}_{\lambda} \partial_{\lambda} \omega^{\theta\kappa\lambda}_{\theta} - 2 r_1 \partial_{\theta} \omega^{\alpha}_{\lambda} \partial_{\lambda} \omega^{\theta\kappa\lambda}_{\theta} + \\
& 2 r_1 \partial_{\alpha} \omega^{\alpha}_{\lambda} \partial_{\lambda} \omega^{\kappa\lambda\theta}_{\theta} - 4 r_1 \partial_{\theta} \omega^{\alpha}_{\lambda} \partial_{\lambda} \omega^{\kappa\lambda\theta}_{\theta} - \frac{1}{3} t_1 \partial^{\alpha} f_{\theta\kappa} \partial^{\kappa} f^{\theta}_{\alpha} + \\
& \frac{1}{6} t_2 \partial^{\alpha} f_{\theta\kappa} \partial^{\kappa} f^{\theta}_{\alpha} - \frac{2}{3} t_1 \partial^{\alpha} f_{\kappa\theta} \partial^{\kappa} f^{\theta}_{\alpha} - \frac{1}{6} t_2 \partial^{\alpha} f_{\kappa\theta} \partial^{\kappa} f^{\theta}_{\alpha} - \frac{1}{3} t_1 \partial^{\alpha} f_{\lambda} \partial^{\kappa} f^{\lambda}_{\kappa} + \\
& \frac{1}{6} t_2 \partial^{\alpha} f_{\lambda} \partial^{\kappa} f^{\lambda}_{\kappa} + t_1 \omega^{\alpha}_{\kappa} \partial^{\kappa} f^{\lambda}_{\lambda} + t_1 \omega^{\lambda}_{\kappa} \partial^{\kappa} f^{\lambda}_{\lambda} + 2 t_1 \partial^{\alpha} f_{\kappa\alpha} \partial^{\kappa} f^{\lambda}_{\lambda} - \\
& \frac{2}{3} t_2 \omega^{\lambda}_{\kappa\theta} \partial^{\kappa} f^{\lambda\theta}_{\theta} - \frac{1}{3} t_1 \omega^{\theta\kappa}_{\lambda} \partial^{\kappa} f^{\lambda\theta}_{\theta} - \frac{1}{3} t_2 \omega^{\theta\kappa}_{\lambda} \partial^{\kappa} f^{\lambda\theta}_{\theta} + \frac{4}{3} t_1 \omega^{\lambda}_{\kappa\theta} \partial^{\kappa} f^{\lambda\theta}_{\theta} - \\
& \frac{2}{3} t_2 \omega^{\theta\kappa}_{\lambda} \partial^{\kappa} f^{\lambda\theta}_{\theta} - t_1 \omega^{\alpha}_{\lambda} \partial^{\kappa} f^{\lambda}_{\lambda} - t_1 \omega^{\lambda}_{\lambda} \partial^{\kappa} f^{\lambda}_{\lambda} + \frac{1}{3} t_1 \partial^{\alpha} f_{\lambda\alpha} \partial^{\kappa} f^{\lambda}_{\lambda} - \\
& \frac{1}{6} t_2 \partial^{\alpha} f_{\lambda\alpha} \partial^{\kappa} f^{\lambda}_{\lambda} + \frac{1}{3} t_1 \partial_{\lambda} f^{\lambda}_{\theta} \partial^{\kappa} f^{\theta}_{\theta} - \frac{1}{6} t_2 \partial_{\lambda} f^{\lambda}_{\theta} \partial^{\kappa} f^{\theta}_{\theta} + \frac{2}{3} t_1 \partial_{\kappa} f^{\lambda}_{\theta} \partial^{\kappa} f^{\theta}_{\theta} + \\
& \frac{1}{6} t_2 \partial_{\kappa} f^{\lambda}_{\theta} \partial^{\kappa} f^{\theta}_{\theta} - t_1 \partial^{\alpha} f^{\lambda}_{\alpha} \partial^{\kappa} f^{\lambda}_{\kappa} + \frac{2}{3} r_1 \partial_{\kappa} \omega^{\alpha\beta\theta}_{\alpha} \partial^{\kappa} \omega_{\alpha\beta\theta} + \\
& \frac{1}{3} r_2 \partial_{\kappa} \omega^{\alpha\beta\theta}_{\alpha} \partial^{\kappa} \omega_{\alpha\beta\theta} - \frac{2}{3} r_1 \partial_{\kappa} \omega^{\theta\alpha\beta}_{\alpha} \partial^{\kappa} \omega_{\alpha\beta\theta} + \frac{2}{3} r_2 \partial_{\kappa} \omega^{\theta\alpha\beta}_{\alpha} \partial^{\kappa} \omega_{\alpha\beta\theta} + \\
& \frac{2}{3} r_1 \partial^{\beta} \omega^{\alpha\lambda}_{\lambda} \partial_{\lambda} \omega^{\lambda}_{\alpha\beta} - \frac{2}{3} r_2 \partial^{\beta} \omega^{\alpha\lambda}_{\lambda} \partial_{\lambda} \omega^{\lambda}_{\alpha\beta} - \frac{8}{3} r_1 \partial^{\beta} \omega^{\lambda\alpha}_{\lambda} \partial_{\lambda} \omega^{\lambda}_{\alpha\beta} + \\
& \frac{2}{3} r_2 \partial^{\beta} \omega^{\lambda\alpha}_{\lambda} \partial_{\lambda} \omega^{\lambda}_{\alpha\beta} - 2 r_1 \partial_{\alpha} \omega^{\alpha}_{\lambda} \partial^{\lambda} \omega^{\theta\kappa}_{\theta} + 2 r_1 \partial_{\theta} \omega^{\alpha}_{\lambda} \partial^{\lambda} \omega^{\theta\kappa}_{\theta} - \\
& \frac{2}{3} r_2 \partial_{\theta} \omega^{\alpha}_{\lambda} \partial^{\lambda} \omega^{\theta\kappa}_{\theta} + 2 r_1 \partial_{\theta} \omega^{\alpha}_{\lambda} \partial^{\lambda} \omega^{\theta\kappa}_{\theta} - \frac{2}{3} r_2 \partial_{\theta} \omega^{\alpha}_{\lambda} \partial^{\lambda} \omega^{\theta\kappa}_{\theta}
\end{aligned}$$

$\sigma_1^{\#1} + \alpha\beta$	$\frac{2(t_1+t_2)}{3t_1t_2}$	$\frac{\sqrt{2}(t_1-2t_2)}{3(1+\kappa^2)t_1t_2}$	$\frac{i\sqrt{2}\kappa(t_1-2t_2)}{3(1+\kappa^2)t_1t_2}$	0	0	0	0
$\sigma_1^{\#2} + \alpha\beta$	$\frac{\sqrt{2}(t_1-2t_2)}{3(1+\kappa^2)t_1t_2}$	$\frac{t_1+4t_2}{3(1+\kappa^2)^2t_1t_2}$	$\frac{ik(t_1+4t_2)}{3(1+\kappa^2)^2t_1t_2}$	0	0	0	0
$\tau_1^{\#1} + \alpha\beta$	$-\frac{i\sqrt{2}\kappa(t_1-2t_2)}{3(1+\kappa^2)^2t_1t_2}$	$-\frac{ik(t_1+4t_2)}{3(1+\kappa^2)^2t_1t_2}$	$\frac{\kappa^2(t_1+4t_2)}{3(1+\kappa^2)^2t_1t_2}$	0	0	0	0
$\sigma_1^{\#1} + \alpha$	0	0	0	0	$\frac{\sqrt{2}}{t_1+2\kappa^2t_1}$	0	$\frac{2ik}{t_1+2\kappa^2t_1}$
$\sigma_1^{\#2} + \alpha$	0	0	0	$\frac{\sqrt{2}}{t_1+2\kappa^2t_1}$	$\frac{2\kappa^2t_1+t_1}{(t_1+2\kappa^2t_1)^2}$	0	$\frac{i\sqrt{2}\kappa(2\kappa^2t_1+t_1)}{(t_1+2\kappa^2t_1)^2}$
$\tau_1^{\#1} + \alpha$	0	0	0	0	0	0	0
$\tau_1^{\#2} + \alpha$	0	0	0	$-\frac{2ik}{t_1+2\kappa^2t_1}$	$-\frac{i\sqrt{2}\kappa(2\kappa^2t_1+t_1)}{(t_1+2\kappa^2t_1)^2}$	0	$\frac{2\kappa^2(2\kappa^2t_1+t_1)}{(t_1+2\kappa^2t_1)^2}$

$\omega_1^{\#1} + \alpha\beta$	$\frac{1}{6}(t_1 + 4t_2)$	$-\frac{t_1-2t_2}{3\sqrt{2}}$	$-\frac{\bar{t}_1-2\bar{t}_2}{3\sqrt{2}}$	$-\frac{\bar{t}_1-2\bar{t}_2}{3\sqrt{2}}$	$\omega_1^{\#2} + \alpha\beta$	$\frac{1}{3}\bar{t}_1 k(t_1 + t_2)$	$\omega_1^{\#1} - \alpha$	$\omega_1^{\#2} - \alpha$	$f_1^{\#1} - \alpha$	$f_1^{\#2} - \alpha$
$\omega_1^{\#1} + \alpha$	0	0	0	0	$\omega_1^{\#2} + \alpha$	$-\bar{t}_1 k t_1 - \frac{\bar{t}_1}{2}$	$\omega_1^{\#1} - \alpha$	$\omega_1^{\#2} - \alpha$	$f_1^{\#1} - \alpha$	$f_1^{\#2} - \alpha$
$\omega_1^{\#2} + \alpha$	0	0	0	0	$\omega_1^{\#2} + \alpha$	$\frac{t_1}{\sqrt{2}}$	$\omega_1^{\#1} - \alpha$	$\omega_1^{\#2} - \alpha$	$f_1^{\#1} - \alpha$	$f_1^{\#2} - \alpha$
$f_1^{\#1} + \alpha$	0	0	0	0	$f_1^{\#1} + \alpha$	0	$\omega_1^{\#1} - \alpha$	$\omega_1^{\#2} - \alpha$	$f_1^{\#1} - \alpha$	$f_1^{\#2} - \alpha$
$f_1^{\#2} + \alpha$	0	0	0	0	$f_1^{\#2} + \alpha$	$-\bar{t}_1 k t_1$	$\omega_1^{\#1} - \alpha$	$\omega_1^{\#2} - \alpha$	$f_1^{\#1} - \alpha$	$f_1^{\#2} - \alpha$

$\sigma_2^{\#1} + \alpha\beta$	$\frac{2}{(1+2k^2)^2t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\sigma_2^{\#1}$
$\tau_2^{\#1} + \alpha\beta$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$	$\frac{4k^2}{(1+2k^2)^2t_1}$	$\sigma_2^{\#1}$
$\sigma_2^{\#1} + \alpha\beta\chi$	0	0	$\frac{2}{2k^2t_1+t_1}$

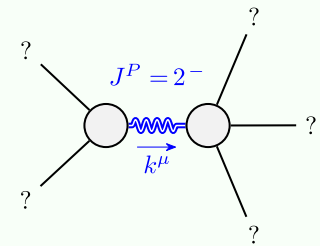
$\omega_0^{*1} +$	$-t_1$	$i\sqrt{2} \, k t_1$	f_0^{*1}	f_0^{*2}	ω_0^{*1}
$f_0^{*1} +$	$-i\sqrt{2} \, k t_1$	$-2k^2 t_1$	0	0	0
$f_0^{*2} +$	0	0	0	0	0
$\omega_0^{*1} +$	0	0	0	0	$k^2 r_2 +$

$\omega_2^{\#1} + \alpha\beta$	$\frac{t_1}{2}$	$-\frac{i k t_1}{\sqrt{2}}$	0	$\omega_2^{\#1} \alpha\beta$
$f_2^{\#1} + \alpha\beta$	$\frac{i k t_1}{\sqrt{2}}$	$k^2 t_1$	0	
$\omega_2^{\#1} + \alpha\beta\chi$	0	0	$k^2 r_1 + \frac{t_1}{2}$	

SO(3) irreps	Multiplicities
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} - 2 \, i \, k \, \sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2 \, i \, k \, \sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + i \, k \, \sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2 \, i \, k \, \sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total constraints:	16

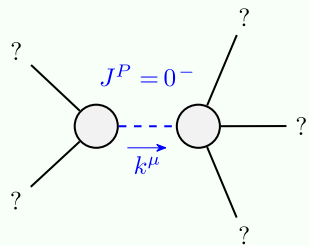
$\sigma_0^{#1}+$	$-\frac{1}{(1+2k^2)^2t_1}$	$\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	0	$\sigma_0^{#1}$
$\tau_0^{#1}+$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$	$-\frac{2k^2}{(1+2k^2)^2t_1}$	0	0
$\tau_0^{#2}+$	0	0	0	0
$\sigma_0^{#1}+$	0	0	0	$\frac{1}{k^2t_2+t_2}$

Massive and massless spectra



Massive particle	
Pole residue:	$-\frac{1}{r_1} > 0$
Polarisations:	5
Square mass:	$-\frac{t_1}{2r_1} > 0$
Spin:	2
Parity:	Odd

(No massless particles)



Massive particle	
Pole residue:	$-\frac{1}{r_2} > 0$
Polarisations:	1
Square mass:	$-\frac{t_2}{r_2} > 0$
Spin:	0
Parity:	Odd

Unitarity conditions

$$r_1 < 0 \ \&\& \ r_2 < 0 \ \&\& \ t_1 > 0 \ \&\& \ t_2 > 0$$