| | $\sigma^{\#1}_{1^+}{}_{\alpha\beta}$ | $\sigma^{\#2}_{1^+}{}_{\alpha\beta}$ | $\tau^{\#1}_{1^+}{}_{\alpha\beta}$ | $\sigma^{\#1}_{1^-}{}_{\alpha}$ | $\sigma^{\#2}_{1^-}{}_{\alpha}$ | $\tau^{\#1}_{1^-}{}_{\alpha}$ | $\tau^{\#2}_{1^-}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma^{\#1}_{1^+}{}^\dagger{}^{\alpha\beta}$ | 0 | $\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+k^2)}$ | $\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+k^2)}$ | 0 | 0 | 0 | 0 |
| $\sigma^{\#2}_{1^+}{}^\dagger{}^{\alpha\beta}$ | $\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+k^2)}$ | $-\frac{2}{(\alpha_0-4\beta_1)(1+k^2)^2}$ | $-\frac{2ik}{(\alpha_0-4\beta_1)(1+k^2)^2}$ | 0 | 0 | 0 | 0 |
| $\tau^{\#1}_{1^+}{}^\dagger{}^{\alpha\beta}$ | $-\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+k^2)}$ | $\frac{2ik}{(\alpha_0-4\beta_1)(1+k^2)^2}$ | $-\frac{2k^2}{(\alpha_0-4\beta_1)(1+k^2)^2}$ | 0 | 0 | 0 | 0 |
| $\sigma^{\#1}_{1^-}{}^\dagger{}^{\alpha}$ | 0 | 0 | 0 | 0 | $-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+2k^2)}$ | 0 | $-\frac{4ik}{(\alpha_0-4\beta_1)(1+2k^2)}$ |
| $\sigma^{\#2}_{1^-}{}^\dagger{}^{\alpha}$ | 0 | 0 | 0 | $-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+2k^2)}$ | $-\frac{2}{(\alpha_0-4\beta_1)(1+2k^2)^2}$ | 0 | $-\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+2k^2)^2}$ |
| $\tau^{\#1}_{1^-}{}^\dagger{}^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\tau^{\#2}_{1^-}{}^\dagger{}^{\alpha}$ | 0 | 0 | 0 | $\frac{4ik}{(\alpha_0-4\beta_1)(1+2k^2)}$ | $\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+2k^2)^2}$ | 0 | $-\frac{4k^2}{(\alpha_0-4\beta_1)(1+2k^2)^2}$ |

| | $\omega^{\#1}_{0^+}$ | $f^{\#1}_{0^+}$ | $f^{\#2}_{0^+}$ | $\omega^{\#1}_{0^-}$ |
|---|---|---|---|---|
| $\omega^{\#1}_{0^+}{}^\dagger$ | $\frac{1}{2}(\alpha_0-4\beta_1)$ | $-\frac{i(\alpha_0-4\beta_1)k}{\sqrt{2}}$ | 0 | 0 |
| $f^{\#1}_{0^+}{}^\dagger$ | $\frac{i(\alpha_0-4\beta_1)k}{\sqrt{2}}$ | $-4\beta_1 k^2$ | 0 | 0 |
| $f^{\#2}_{0^+}{}^\dagger$ | 0 | 0 | 0 | 0 |
| $\omega^{\#1}_{0^-}{}^\dagger$ | 0 | 0 | 0 | $\frac{\alpha_0}{2}-2\beta_1+\alpha_3 k^2$ |

## Lagrangian density

$$-\frac{1}{2}\alpha_0\,\omega_{\alpha\chi\beta}\,\omega^{\alpha\beta\chi}-\frac{1}{2}\alpha_0\,\omega^{\alpha\beta}{}_{\alpha}\,\omega_{\beta}{}^{\chi}{}_{\chi}+2\beta_1\,\omega^{\alpha\beta}{}_{\alpha}\,\omega_{\beta}{}^{\chi}{}_{\chi}-$$
$$2\beta_1\,\omega_{\alpha}{}^{\chi\delta}\,\omega_{\chi\delta}{}^{\alpha}-2\beta_1\,\omega_{\alpha}{}^{\chi}{}_{\chi}\,\partial_\beta f^{\alpha\beta}-2\beta_1\,\omega_{\alpha}{}^{\delta}{}_{\delta}\,\partial_\beta f^{\alpha\beta}-\alpha_0\,f^{\alpha\beta}\,\partial_\beta\omega_{\alpha}{}^{\chi}{}_{\chi}+$$
$$\alpha_0\,\partial_\beta\omega^{\alpha\beta}{}_{\alpha}+\frac{2}{3}\alpha_3\,\partial^\alpha\omega^{\beta\zeta}{}_{\chi}\,\partial_\beta\omega_{\zeta\alpha}{}^{\chi}+2\beta_1\,\omega_{\beta}{}^{\chi}{}_{\chi}\,\partial^\beta f^{\alpha}{}_{\alpha}+$$
$$2\beta_1\,\omega_{\beta}{}^{\delta}{}_{\delta}\,\partial^\beta f^{\alpha}{}_{\alpha}-2\beta_1\,\partial_\beta f^{\chi}{}_{\chi}\,\partial^\beta f^{\alpha}{}_{\alpha}+\alpha_0\,f^{\alpha\beta}\,\partial_\chi\omega_{\alpha}{}^{\chi}{}_{\beta}-$$
$$\alpha_0\,f^{\alpha}{}_{\alpha}\,\partial_\chi\omega^{\beta\chi}{}_{\beta}-\frac{2}{3}\alpha_3\,\partial_\beta\omega_{\zeta\alpha}{}^{\chi}\,\partial_\chi\omega^{\beta\zeta\alpha}-\frac{1}{3}\alpha_3\,\partial_\beta\omega_{\zeta\alpha}{}^{\chi}\,\partial_\chi\omega^{\zeta\alpha\beta}+$$
$$4\beta_1\,\omega_{\alpha\chi\beta}\,\partial^\chi f^{\alpha\beta}+\beta_1\,\partial_\chi f_{\beta}{}^{\delta}\,\partial^\chi f_{\delta}{}^{\beta}+\beta_1\,\partial_\chi f^{\delta}{}_{\beta}\,\partial^\chi f_{\delta}{}^{\beta}+$$
$$\frac{2}{3}\alpha_3\,\partial_\chi\omega^{\beta\zeta\alpha}\,\partial^\chi\omega_{\zeta\alpha\beta}+\frac{1}{3}\alpha_3\,\partial_\chi\omega^{\zeta\alpha\beta}\,\partial^\chi\omega_{\zeta\alpha\beta}+4\beta_1\,\partial^\beta f^{\alpha}{}_{\alpha}\,\partial_\delta f_{\beta}{}^{\delta}-$$
$$2\beta_1\,\partial_\beta f_{\chi}{}^{\beta}\,\partial_\delta f^{\chi\delta}+\frac{2}{3}\alpha_3\,\partial^\beta\omega_{\alpha}{}^{\delta\zeta}\,\partial_\delta\omega_{\zeta\beta}{}^{\alpha}-\frac{2}{3}\alpha_3\,\partial^\beta\omega_{\alpha}{}^{\zeta\delta}\,\partial_\delta\omega_{\zeta\beta}{}^{\alpha}-$$
$$\beta_1\,\partial^\chi f_{\zeta}{}^{\beta}\,\partial^\zeta f_{\beta\chi}-\beta_1\,\partial^\chi f_{\zeta}{}^{\beta}\,\partial^\zeta f_{\chi\beta}+\beta_1\,\partial^\chi f_{\delta\zeta}\,\partial^\zeta f^{\delta}{}_{\chi}-\beta_1\,\partial^\chi f_{\zeta\delta}\,\partial^\zeta f^{\delta}{}_{\chi}$$

Added source term: $f^{\alpha\beta}\,\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}$

| | $\sigma^{\#1}_{2^+}{}_{\alpha\beta}$ | $\tau^{\#1}_{2^+}{}_{\alpha\beta}$ | $\sigma^{\#1}_{2^-}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma^{\#1}_{2^+}{}^\dagger{}^{\alpha\beta}$ | $-\frac{16\beta_1}{\alpha_0{}^2-4\alpha_0\beta_1}$ | $\frac{2i\sqrt{2}}{\alpha_0 k}$ | 0 |
| $\tau^{\#1}_{2^+}{}^\dagger{}^{\alpha\beta}$ | $-\frac{2i\sqrt{2}}{\alpha_0 k}$ | $\frac{2}{\alpha_0 k^2}$ | 0 |
| $\sigma^{\#1}_{2^-}{}^\dagger{}^{\alpha\beta\chi}$ | 0 | 0 | $\frac{1}{-\frac{\alpha_0}{4}+\beta_1}$ |

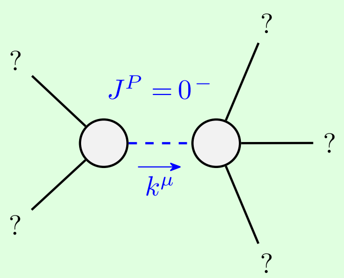
| | $\omega^{\#1}_{2^+}{}_{\alpha\beta}$ | $f^{\#1}_{2^+}{}_{\alpha\beta}$ | $\omega^{\#1}_{2^-}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega^{\#1}_{2^+}{}^\dagger{}^{\alpha\beta}$ | $-\frac{\alpha_0}{4}+\beta_1$ | $\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$ | 0 |
| $f^{\#1}_{2^+}{}^\dagger{}^{\alpha\beta}$ | $-\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$ | $2\beta_1 k^2$ | 0 |
| $\omega^{\#1}_{2^-}{}^\dagger{}^{\alpha\beta\chi}$ | 0 | 0 | $-\frac{\alpha_0}{4}+\beta_1$ |

Source constraints

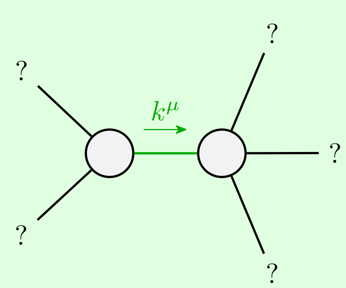
| SO(3) irreps | # |
|---|---|
| $\tau^{\#2}_{0^+}==0$ | 1 |
| $\tau^{\#2}_{1^-}{}^{\alpha}+2ik\,\sigma^{\#2}_{1^-}{}^{\alpha}==0$ | 3 |
| $\tau^{\#1}_{1^-}{}^{\alpha}==0$ | 3 |
| $\tau^{\#1}_{1^+}{}^{\alpha\beta}+ik\,\sigma^{\#2}_{1^+}{}^{\alpha\beta}==0$ | 3 |
| Total #: | 10 |

| | $\sigma^{\#1}_{0^+}$ | $\tau^{\#1}_{0^+}$ | $\tau^{\#2}_{0^+}$ | $\sigma^{\#1}_{0^-}$ |
|---|---|---|---|---|
| $\sigma^{\#1}_{0^+}{}^\dagger$ | $\frac{8\beta_1}{\alpha_0{}^2-4\alpha_0\beta_1}$ | $-\frac{i\sqrt{2}}{\alpha_0 k}$ | 0 | 0 |
| $\tau^{\#1}_{0^+}{}^\dagger$ | $\frac{i\sqrt{2}}{\alpha_0 k}$ | $-\frac{1}{\alpha_0 k^2}$ | 0 | 0 |
| $\tau^{\#2}_{0^+}{}^\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma^{\#1}_{0^-}{}^\dagger$ | 0 | 0 | 0 | $\frac{2}{\alpha_0-4\beta_1+2\alpha_3 k^2}$ |

| | $\omega^{\#1}_{1^+}{}_{\alpha\beta}$ | $\omega^{\#2}_{1^+}{}_{\alpha\beta}$ | $f^{\#1}_{1^+}{}_{\alpha\beta}$ | $\omega^{\#1}_{1^-}{}_{\alpha}$ | $\omega^{\#2}_{1^-}{}_{\alpha}$ | $f^{\#1}_{1^-}{}_{\alpha}$ | $f^{\#2}_{1^-}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega^{\#1}_{1^+}{}^\dagger{}^{\alpha\beta}$ | $\frac{1}{4}(\alpha_0-4\beta_1)$ | $\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$ | $\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $\omega^{\#2}_{1^+}{}^\dagger{}^{\alpha\beta}$ | $\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $f^{\#1}_{1^+}{}^\dagger{}^{\alpha\beta}$ | $-\frac{i(\alpha_0-4\beta_1)k}{2\sqrt{2}}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega^{\#1}_{1^-}{}^\dagger{}^{\alpha}$ | 0 | 0 | 0 | $\frac{1}{4}(\alpha_0-4\beta_1)$ | $-\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$ | 0 | $-\frac{1}{2}i(\alpha_0-4\beta_1)k$ |
| $\omega^{\#2}_{1^-}{}^\dagger{}^{\alpha}$ | 0 | 0 | 0 | $-\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$ | 0 | 0 | 0 |
| $f^{\#1}_{1^-}{}^\dagger{}^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f^{\#2}_{1^-}{}^\dagger{}^{\alpha}$ | 0 | 0 | 0 | $\frac{1}{2}i(\alpha_0-4\beta_1)k$ | 0 | 0 | 0 |




| Massive particle | |
|---|---|
| Pole residue: | $-\frac{1}{\alpha_3}>0$ |
| Polarisations: | 1 |
| Square mass: | $-\frac{\alpha_0-4\beta_1}{2\alpha_3}>0$ |
| Spin: | 0 |
| Parity: | Odd |




| Quadratic pole | |
|---|---|
| Pole residue: | $\frac{1}{\alpha_0}>0$ |
| Polarisations: | 2 |

Unitarity conditions

$\alpha_0>0\ \&\&\ \alpha_3<0\ \&\&\ \beta_1<\frac{\alpha_0}{4}$