# Particle spectrograph

## Wave operator and propagator

| | $\omega_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^+}^{\#2}{}_{\alpha\beta}$ | $f_{1^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{1^-}^{\#1}{}_{\alpha}$ | $\omega_{1^-}^{\#2}{}_{\alpha}$ | $f_{1^-}^{\#1}{}_{\alpha}$ | $f_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1}{}^{\alpha\beta}$ | $\frac{1}{6}(6k^2 r_5 + t_1 + 4t_2)$ | $-\frac{t_1 - 2t_2}{3\sqrt{2}}$ | $-\frac{ik(t_1-2t_2)}{3\sqrt{2}}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2}{}^{\alpha\beta}$ | $-\frac{t_1-2t_2}{3\sqrt{2}}$ | $\frac{t_1+t_2}{3}$ | $\frac{1}{3}ik(t_1+t_2)$ | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1}{}^{\alpha\beta}$ | $\frac{ik(t_1-2t_2)}{3\sqrt{2}}$ | $-\frac{1}{3}ik(t_1+t_2)$ | $\frac{1}{3}k^2(t_1+t_2)$ | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1}{}^{\alpha}$ | 0 | 0 | 0 | $k^2 r_5 - \frac{t_1}{2}$ | $\frac{t_1}{\sqrt{2}}$ | 0 | $ikt_1$ |
| $\omega_{1^-}^{\#2}{}^{\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{\sqrt{2}}$ | 0 | 0 | 0 |
| $f_{1^-}^{\#1}{}^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2}{}^{\alpha}$ | 0 | 0 | 0 | $-ikt_1$ | 0 | 0 | 0 |

| | $\omega_{2^+}^{\#1}{}_{\alpha\beta}$ | $f_{2^+}^{\#1}{}_{\alpha\beta}$ | $\omega_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1}{}^{\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{ikt_1}{\sqrt{2}}$ | 0 |
| $f_{2^+}^{\#1}{}^{\alpha\beta}$ | $\frac{ikt_1}{\sqrt{2}}$ | $k^2 t_1$ | 0 |
| $\omega_{2^-}^{\#1}{}^{\alpha\beta\chi}$ | 0 | 0 | $\frac{t_1}{2}$ |

| | $\sigma_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^+}^{\#2}{}_{\alpha\beta}$ | $\tau_{1^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{1^-}^{\#1}{}_{\alpha}$ | $\sigma_{1^-}^{\#2}{}_{\alpha}$ | $\tau_{1^-}^{\#1}{}_{\alpha}$ | $\tau_{1^-}^{\#2}{}_{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\sigma_{1^+}^{\#1}{}^{\alpha\beta}$ | $\frac{2(t_1+t_2)}{3t_1t_2+2k^2r_5(t_1+t_2)}$ | $\frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2r_5(t_1+t_2))}$ | $\frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2r_5(t_1+t_2))}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+}^{\#2}{}^{\alpha\beta}$ | $\frac{\sqrt{2}(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2r_5(t_1+t_2))}$ | $\frac{6k^2r_5+t_1+4t_2}{(1+k^2)^2(3t_1t_2+2k^2r_5(t_1+t_2))}$ | $\frac{ik(6k^2r_5+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2r_5(t_1+t_2))}$ | 0 | 0 | 0 | 0 |
| $\tau_{1^+}^{\#1}{}^{\alpha\beta}$ | $\frac{i\sqrt{2}k(t_1-2t_2)}{(1+k^2)(3t_1t_2+2k^2r_5(t_1+t_2))}$ | $\frac{ik(6k^2r_5+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2r_5(t_1+t_2))}$ | $\frac{k^2(6k^2r_5+t_1+4t_2)}{(1+k^2)^2(3t_1t_2+2k^2r_5(t_1+t_2))}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^-}^{\#1}{}^{\alpha}$ | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2t_1}$ | $\frac{-2k^2r_5+t_1}{(t_1+2k^2t_1)^2}$ | 0 | $\frac{ik(2k^2r_5-t_1)}{(t_1+2k^2t_1)^2}$ |
| $\sigma_{1^-}^{\#2}{}^{\alpha}$ | 0 | 0 | 0 | $\frac{\sqrt{2}}{t_1+2k^2t_1}$ | $\frac{-2k^2r_5+t_1}{(t_1+2k^2t_1)^2}$ | 0 | $\frac{ik(2k^2r_5-t_1)}{(t_1+2k^2t_1)^2}$ |
| $\tau_{1^-}^{\#1}{}^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | $-\frac{2ik}{t_1+2k^2t_1}$ |
| $\tau_{1^-}^{\#2}{}^{\alpha}$ | 0 | 0 | 0 | $\frac{2ik}{t_1+2k^2t_1}$ | $-\frac{ik\sqrt{2}(2k^2r_5-t_1)}{(t_1+2k^2t_1)^2}$ | 0 | $\frac{-4k^4r_5+2k^2t_1}{(t_1+2k^2t_1)^2}$ |

| | $\sigma_{2^+}^{\#1}{}_{\alpha\beta}$ | $\tau_{2^+}^{\#1}{}_{\alpha\beta}$ | $\sigma_{2^-}^{\#1}{}_{\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^+}^{\#1}{}^{\alpha\beta}$ | $\frac{2}{(1+2k^2)^2t_1}$ | $-\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$ | 0 |
| $\tau_{2^+}^{\#1}{}^{\alpha\beta}$ | $\frac{2i\sqrt{2}k}{(1+2k^2)^2t_1}$ | $\frac{4k^2}{(1+2k^2)^2t_1}$ | 0 |
| $\sigma_{2^-}^{\#1}{}^{\alpha\beta\chi}$ | 0 | 0 | $\frac{2}{t_1}$ |

| | $\omega_{0^+}^{\#1}$ | $f_{0^+}^{\#1}$ | $f_{0^+}^{\#2}$ | $\omega_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\omega_{0^+}^{\#1}{}^{\dagger}$ | $-t_1$ | $i\sqrt{2}kt_1$ | 0 | 0 |
| $f_{0^+}^{\#1}{}^{\dagger}$ | $-i\sqrt{2}kt_1$ | $-2k^2t_1$ | 0 | 0 |
| $f_{0^+}^{\#2}{}^{\dagger}$ | 0 | 0 | 0 | 0 |
| $\omega_{0^-}^{\#1}{}^{\dagger}$ | 0 | 0 | 0 | $t_2$ |

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}{}^{\dagger}$ | $-\frac{1}{(1+2k^2)^2t_1}$ | $-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$ | 0 | 0 |
| $\tau_{0^+}^{\#1}{}^{\dagger}$ | $-\frac{i\sqrt{2}k}{(1+2k^2)^2t_1}$ | $-\frac{2k^2}{(1+2k^2)^2t_1}$ | 0 | 0 |
| $\tau_{0^+}^{\#2}{}^{\dagger}$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1}{}^{\dagger}$ | 0 | 0 | 0 | $\frac{1}{t_2}$ |

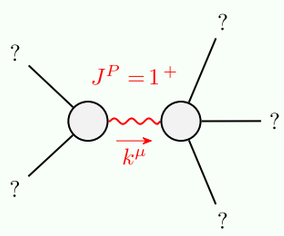
Quadratic (free) action

$$S == \iiint\!\!\iint \left(\tfrac{1}{6}(6t_1\,\omega^{\alpha\prime}{}_{\alpha}\,\omega_{\prime}{}^{\theta}{}_{\theta} + 6\,f^{\alpha\beta}{}_{\tau_{\alpha\beta}} + 6\,\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi} - 12t_1\,\omega_{\alpha}{}^{\theta}{}_{\theta}\,\partial_{\prime}f^{\alpha\prime\prime} + 12t_1\right.$$
$$\omega_{\prime}{}^{\theta}{}_{\theta}\,\partial^{\prime}f^{\alpha}{}_{\alpha} - 6t_1\,\partial_{\prime}f^{\theta}{}_{\theta}\partial^{\prime}f^{\alpha}{}_{\alpha} - 6t_1\,\partial_{\prime}f^{\alpha\prime}\partial_{\theta}f_{\alpha}{}^{\theta} + 12t_1\,\partial^{\prime}f^{\alpha}{}_{\alpha}\partial_{\theta}f_{\prime}{}^{\theta} +$$
$$4t_1\,\omega_{\prime\theta\alpha}\,\partial^{\theta}f^{\alpha\prime} + 4t_2\,\omega_{\prime\theta\alpha}\,\partial^{\theta}f^{\alpha\prime} - 4t_1\,\partial_{\alpha}f_{\prime\theta}\partial^{\theta}f^{\alpha\prime} + 2t_2\,\partial_{\alpha}f_{\prime\theta}\partial^{\theta}f^{\alpha\prime} -$$
$$4t_1\,\partial_{\theta}f_{\alpha\prime}\partial^{\theta}f^{\alpha\prime} - t_2\,\partial_{\alpha}f_{\theta\prime}\partial^{\theta}f^{\alpha\prime} + 2t_1\,\partial_{\prime}f_{\alpha\theta}\partial^{\theta}f^{\alpha\prime} - t_2\,\partial_{\prime}f_{\alpha\theta}\partial^{\theta}f^{\alpha\prime} +$$
$$4t_1\,\partial_{\theta}f_{\alpha\prime}\partial^{\theta}f^{\alpha\prime} + t_2\,\partial_{\theta}f_{\alpha\prime}\partial^{\theta}f^{\alpha\prime} + 2t_1\,\partial_{\theta}f_{\prime\alpha}\partial^{\theta}f^{\alpha\prime} - t_2\,\partial_{\theta}f_{\prime\alpha}\partial^{\theta}f^{\alpha\prime} +$$
$$2(t_1+t_2)\,\omega_{\alpha\prime\theta}\,(\omega^{\alpha\prime\theta} + 2\,\partial^{\theta}f^{\alpha\prime}) + 2\,\omega_{\alpha\theta\prime}\,((t_1-2t_2)\,\omega^{\alpha\prime\theta} + 2(2t_1-t_2)\,\partial^{\theta}f^{\alpha\prime}) +$$
$$6r_5\,\partial_{\prime}\omega_{\theta}{}^{\kappa}{}_{\kappa}\,\partial^{\theta}\omega^{\alpha\prime}{}_{\alpha} - 6r_5\,\partial_{\theta}\omega_{\prime}{}^{\kappa}{}_{\kappa}\,\partial^{\theta}\omega^{\alpha\prime}{}_{\alpha} - 6r_5\,\partial_{\alpha}\omega^{\alpha\prime\theta}\partial_{\kappa}\omega_{\prime}{}^{\kappa}{}_{\theta} + 12r_5\,\partial^{\theta}\omega^{\alpha\prime}{}_{\alpha}$$
$$\left.+ 6r_5\,\partial_{\alpha}\omega^{\alpha\prime\theta}\partial_{\kappa}\omega_{\theta}{}^{\kappa}{}_{\prime} - 12r_5\,\partial^{\theta}\omega^{\alpha\prime}{}_{\alpha}\partial_{\kappa}\omega_{\theta}{}^{\kappa}{}_{\prime}\right)[t,\,x,\,y,\,z]\,dz\,dy\,dx\,dt$$

Source constraints/gauge generators

| SO(3) irreps | Multiplicities |
|---|---|
| $\tau_{0^+}^{\#2} == 0$ | 1 |
| $\tau_{0^+}^{\#1} - 2ik\,\sigma_{0^+}^{\#1} == 0$ | 1 |
| $\tau_{1^-}^{\#2\,\alpha} + 2ik\,\sigma_{1^-}^{\#2\,\alpha} == 0$ | 3 |
| $\tau_{1^-}^{\#1\,\alpha} == 0$ | 3 |
| $\tau_{1^+}^{\#1\,\alpha\beta} + ik\,\sigma_{1^+}^{\#2\,\alpha\beta} == 0$ | 3 |
| $\tau_{2^+}^{\#1\,\alpha\beta} - 2ik\,\sigma_{2^+}^{\#1\,\alpha\beta} == 0$ | 5 |
| Total constraints: | 16 |

## Massive and massless spectra




Massive particle

| | |
|---|---|
| Pole residue: | $\frac{-3t_1t_2(t_1+t_2)+3r_5(t_1{}^2+2t_2{}^2)}{r_5(t_1+t_2)(-3t_1t_2+2r_5(t_1+t_2))} > 0$ |
| Polarisations: | 3 |
| Square mass: | $-\frac{3t_1t_2}{2r_5t_1+2r_5t_2} > 0$ |
| Spin: | 1 |
| Parity: | Even |

(No massless particles)

## Unitarity conditions

$$r_5 > 0 \;\&\&\; (t_1 < 0 \;\&\&\; (t_2 < 0 \,||\, t_2 > -t_1)) \,||\, (t_1 > 0 \;\&\&\; -t_1 < t_2 < 0)$$