



Unitarity conditions

$r_1 < 0 \ \&\& \ t_1 > 0$

$\sigma_1^{\#1} + \alpha\beta$	$\sigma_1^{\#2} + \alpha\beta$	$\tau_1^{\#1} + \alpha\beta$	$\sigma_1^{\#1} - \alpha$	$\sigma_1^{\#2} - \alpha$	$\tau_1^{\#1} - \alpha$	$\tau_1^{\#2} - \alpha$
$\sigma_1^{\#1} + \alpha\beta$	0	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$-\frac{i \sqrt{2} k}{t_1+k^2 t_1}$	0	0	0
$\sigma_1^{\#2} + \alpha\beta$	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$-\frac{2 k^2 r_1+t_1}{(1+k^2)^2 t_1^2}$	$-\frac{i(2 k^3 r_1-k t_1)}{(1+k^2)^2 t_1^2}$	0	0	0
$\tau_1^{\#1} + \alpha\beta$	$-\frac{\sqrt{2}}{t_1+k^2 t_1}$	$-\frac{2 k^4 r_1+k^2 t_1}{(1+k^2)^2 t_1^2}$	0	0	0	0
$\sigma_1^{\#1} - \alpha$	0	0	$\frac{2(t_1+t_3)}{3 t_1 t_3}$	$-\frac{\sqrt{2}\left(t_1-2 t_3\right)}{3\left(1+2 k^2\right) t_1 t_3}$	0	$-\frac{2 i k t_1-4 i k t_3}{3 t_1 t_3+6 k^2 t_1 t_3}$
$\sigma_1^{\#2} - \alpha$	0	0	$-\frac{\sqrt{2}\left(t_1-2 t_3\right)}{3\left(1+2 k^2\right) t_1 t_3}$	$\frac{t_1+4 t_3}{3\left(1+2 k^2\right)^2 t_1 t_3}$	0	$\frac{i \sqrt{2} k\left(t_1+4 t_3\right)}{3\left(1+2 k^2\right)^2 t_1 t_3}$
$\tau_1^{\#1} - \alpha$	0	0	0	0	0	0
$\tau_1^{\#2} - \alpha$	0	0	$\frac{2 i k\left(t_1-2 t_3\right)}{3 t_1 t_3+6 k^2 t_1 t_3}$	$-\frac{i \sqrt{2} k\left(t_1+4 t_3\right)}{3\left(1+2 k^2\right)^2 t_1 t_3}$	0	$\frac{2 k^2\left(t_1+4 t_3\right)}{3\left(1+2 k^2\right)^2 t_1 t_3}$

Lagrangian density

$$\begin{aligned}
& -\frac{1}{3}t_1\omega_{\kappa}^{\alpha\prime}\omega_{\kappa}^{\kappa}+\frac{2}{3}t_3\omega_{\kappa}^{\alpha\prime}\omega_{\kappa}^{\kappa}-t_1\omega_{\kappa}^{\kappa\lambda}\omega_{\kappa}^{\prime}+f^{\alpha\beta}\tau_{\alpha\beta}+\omega^{\alpha\beta X}\sigma_{\alpha\beta X}+r_1\partial_{\lambda}\omega_{\kappa}^{\kappa\lambda}\partial^{\prime}\omega_{\lambda}^{\alpha}-\frac{2}{3}r_1\partial^{\beta}\omega_{\kappa}^{\theta\alpha}\partial_{\theta}\omega_{\alpha\beta}^{\kappa}-\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\alpha\beta\theta}+\frac{2}{3}r_1\partial_{\theta}\omega_{\alpha\beta}^{\kappa}\partial_{\kappa}\omega^{\theta\alpha\beta}+r_1\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\lambda}^{\theta\kappa\lambda}-r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\lambda}^{\theta\kappa\lambda}+r_1\partial_{\alpha}\omega_{\lambda}^{\alpha}\partial_{\theta}\omega_{\lambda}^{\kappa\lambda\theta}-2r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial_{\kappa}\omega_{\alpha}^{\kappa\lambda\theta}-\frac{1}{2}t_1\partial^{\alpha}f_{\theta\kappa}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{2}t_1\partial^{\alpha}f_{\kappa\theta}\partial^{\kappa}f_{\alpha}^{\theta}-\frac{1}{2}t_1\partial^{\alpha}f_{\lambda}^{\kappa}\partial^{\kappa}f_{\alpha}^{\lambda}+\frac{1}{3}t_1\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\prime}^{\prime}-\frac{2}{3}t_3\omega_{\kappa\alpha}^{\alpha}\partial^{\kappa}f_{\prime}^{\prime}+\frac{1}{3}t_1\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\prime}^{\prime}-\frac{2}{3}t_3\omega_{\kappa\lambda}^{\lambda}\partial^{\kappa}f_{\prime}^{\prime}+\frac{2}{3}t_1\partial^{\alpha}f_{\kappa\alpha}\partial^{\kappa}f_{\prime}^{\prime}-\frac{4}{3}t_3\partial^{\alpha}f_{\kappa\alpha}\partial^{\kappa}f_{\prime}^{\prime}-\frac{1}{3}t_1\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\prime}^{\prime}+\frac{2}{3}t_3\partial_{\kappa}f_{\lambda}^{\lambda}\partial^{\kappa}f_{\prime}^{\prime}+\frac{2}{3}t_1\omega_{\prime\kappa\theta}\partial^{\kappa}f^{\prime\theta}-\frac{1}{3}t_1\omega_{\prime\alpha}^{\alpha}\partial^{\kappa}f_{\prime}^{\prime}+\frac{2}{3}t_3\omega_{\prime\alpha}^{\alpha}\partial^{\kappa}f_{\prime}^{\prime}-\frac{1}{3}t_1\omega_{\prime\lambda}^{\lambda}\partial^{\kappa}f_{\prime}^{\prime}+\frac{2}{3}t_3\omega_{\prime\lambda}^{\lambda}\partial^{\kappa}f_{\prime}^{\prime}+\frac{1}{2}t_1\partial^{\alpha}f_{\kappa}^{\lambda}\partial^{\kappa}f_{\lambda\alpha}^{\prime}+\frac{1}{2}t_1\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}+\frac{1}{2}t_1\partial_{\kappa}f_{\theta}^{\lambda}\partial^{\kappa}f_{\lambda}^{\theta}-\frac{1}{3}t_1\partial^{\alpha}f_{\lambda\kappa}^{\lambda}\partial^{\kappa}f_{\alpha}^{\lambda}+\frac{2}{3}t_3\partial^{\alpha}f_{\lambda\kappa}^{\lambda}\partial^{\kappa}f_{\alpha}^{\lambda}+\frac{2}{3}r_1\partial_{\kappa}\omega^{\alpha\beta\theta}\partial^{\kappa}\omega_{\alpha\beta\theta}^{\prime}-\frac{2}{3}r_1\partial_{\kappa}\omega^{\theta\alpha\beta}\partial^{\kappa}\omega_{\alpha\beta\theta}^{\prime}+\frac{2}{3}r_1\partial^{\beta}\omega_{\alpha\lambda}^{\prime}\partial_{\lambda}\omega_{\alpha\beta}^{\prime}-\frac{8}{3}r_1\partial^{\beta}\omega_{\lambda}^{\prime\prime}\partial_{\lambda}\omega_{\alpha\beta}^{\prime}-r_1\partial_{\lambda}\omega_{\alpha\beta}^{\prime\prime}\partial^{\lambda}\omega_{\lambda}^{\theta\kappa}+r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\alpha}^{\theta\kappa}+r_1\partial_{\theta}\omega_{\lambda}^{\alpha}\partial^{\lambda}\omega_{\alpha}^{\theta\kappa}
\end{aligned}$$

	$\omega_1^{\#1} + \alpha\beta$	$\omega_1^{\#2} + \alpha\beta$	$f_1^{\#1} + \alpha\beta$	$\omega_1^{\#1} - \alpha$	$\omega_1^{\#2} - \alpha$	$f_1^{\#1} - \alpha$	$f_1^{\#2} - \alpha$
$\omega_1^{\#1} + \alpha\beta$	$k^2 r_1 - \frac{t_1}{2}$	$-\frac{t_1}{\sqrt{2}}$	$-\frac{ikt_1}{\sqrt{2}}$	0	0	0	0
$\omega_1^{\#2} + \alpha\beta$	$-\frac{t_1}{\sqrt{2}}$	0	0	0	0	0	0
$f_1^{\#1} + \alpha\beta$	$\frac{ikt_1}{\sqrt{2}}$	0	0	0	0	0	0
$\omega_1^{\#1} - \alpha$	0	0	0	$\frac{1}{6}(t_1 + 4t_3)$	$\frac{t_1 - 2t_3}{3\sqrt{2}}$	0	$\frac{1}{3}ik(t_1 - 2t_3)$
$\omega_1^{\#2} - \alpha$	0	0	0	$\frac{t_1 - 2t_3}{3\sqrt{2}}$	$\frac{t_1 + t_3}{3}$	0	$\frac{1}{3}i\sqrt{2}k(t_1 + t_3)$
$f_1^{\#1} - \alpha$	0	0	0	0	0	0	0
$f_1^{\#2} - \alpha$	0	0	0	$-\frac{1}{3}ik(t_1 - 2t_3)$	$-\frac{1}{3}i\sqrt{2}k(t_1 + t_3)$	0	$\frac{2}{3}k^2(t_1 + t_3)$

	$\sigma_{0+}^{\#1}$	$\tau_{0+}^{\#1}$	$\tau_{0+}^{\#2}$	$\sigma_{0-}^{\#1}$
$\sigma_{0+}^{\#1} \dagger$	$\frac{1}{(1+2k^2)^2 t_3}$	$-\frac{i\sqrt{2}k}{(1+2k^2)^2 t_3}$	0	0
$\tau_{0+}^{\#1} \dagger$	$\frac{i\sqrt{2}k}{(1+2k^2)^2 t_3}$	$\frac{2k^2}{(1+2k^2)^2 t_3}$	0	0
$\tau_{0+}^{\#2} \dagger$	0	0	0	0
$\sigma_{0-}^{\#1} \dagger$	0	0	0	$-\frac{1}{t_1}$

$\sigma_{2+}^{\#1} + \alpha\beta$	$\frac{2}{(1+2k^2)^2 t_1}$	$-\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	0	$\sigma_{2-}^{\#1} \alpha\beta\chi$
$\tau_{2+}^{\#1} + \alpha\beta$	$\frac{2i\sqrt{2}k}{(1+2k^2)^2 t_1}$	$\frac{4k^2}{(1+2k^2)^2 t_1}$	0	
$\sigma_{2-}^{\#1} + \alpha\beta\chi$	0	0	$\frac{2}{2k^2 r_1 + t_1}$	

	$\omega_{0+}^{\#1}$	$f_{0+}^{\#1}$	$f_{0+}^{\#2}$	$\omega_{0-}^{\#1}$
$\omega_{0+}^{\#1} \dagger$	t_3	$-i \sqrt{2} k t_3$	0	0
$f_{0+}^{\#1} \dagger$	$i \sqrt{2} k t_3$	$2 k^2 t_3$	0	0
$f_{0+}^{\#2} \dagger$	0	0	0	0
$\omega_{0-}^{\#1} \dagger$	0	0	0	$-t_1$

$$\begin{array}{ccc}
 \omega_{2^+}^{\#1} \alpha \beta & f_{2^+}^{\#1} \alpha \beta & \omega_{2^+}^{\#1} \alpha \beta \chi \\
 \omega_{2^+}^{\#1} \dagger \alpha \beta & \frac{t_1}{2} & -\frac{i k t_1}{\sqrt{2}} & 0 \\
 f_{2^+}^{\#1} \dagger \alpha \beta & \frac{i k t_1}{\sqrt{2}} & k^2 t_1 & 0 \\
 \omega_{2^+}^{\#1} \dagger \alpha \beta \chi & 0 & 0 & k^2 r_1 + \frac{t_1}{2}
 \end{array}$$

Source constraints	
SO(3) irreps	#
$\tau_{0+}^{\#2} == 0$	1
$\tau_{0+}^{\#1} - 2 i k \sigma_{0+}^{\#1} == 0$	1
$\tau_{1-}^{\#2\alpha} + 2 i k \sigma_{1-}^{\#2\alpha} == 0$	3
$\tau_{1-}^{\#1\alpha} == 0$	3
$\tau_{1+}^{\#1\alpha\beta} + i k \sigma_{1+}^{\#2\alpha\beta} == 0$	3
$\tau_{2+}^{\#1\alpha\beta} - 2 i k \sigma_{2+}^{\#1\alpha\beta} == 0$	5
Total #:	16