## Lagrangian density

$$-\frac{1}{3}t_1\,\omega_i{}^{\alpha i}\,\omega_{\kappa\alpha}{}^\kappa - t_1\,\omega_i{}^{\kappa\lambda}\,\omega_{\kappa\lambda}{}^i - 2r_3\,\partial_i\omega_\kappa{}^{\kappa\lambda}\,\partial^i\omega_\lambda{}^\alpha{}_\alpha -$$
$$r_5\,\partial_i\omega^{\kappa\lambda}{}_\kappa\,\partial^i\omega_{\lambda}{}^\alpha{}_\alpha + \frac{2}{3}r_2\,\partial_\theta\omega_{\alpha\beta}{}^\alpha\,\partial^\beta\omega^{\theta\alpha}{}_\alpha - \frac{1}{3}r_2\,\partial_\theta\omega_{\alpha\beta}{}^\kappa\,\partial_\kappa\omega^{\alpha\beta\theta} -$$
$$\frac{2}{3}r_2\,\partial_\theta\omega_{\alpha\beta}{}^\kappa\,\partial_\kappa\omega^{\theta\alpha\beta} + 2r_3\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial_\kappa\omega^{\theta\kappa\lambda} - r_5\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial_\kappa\omega^{\kappa\lambda\theta} -$$
$$2r_3\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial_\kappa\omega^{\theta\kappa\lambda} + r_5\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial_\kappa\omega^{\kappa\lambda\theta} +$$
$$r_5\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial_\kappa\omega^{\kappa\lambda\theta} + 4r_3\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial_\kappa\omega^{\kappa\lambda\theta} + 2r_5\,\partial_\theta\omega_\lambda{}^\alpha{}_\alpha\,\partial_{\alpha}\omega^f{}_{\alpha\lambda} +$$
$$\frac{1}{2}t_1\,\partial^\alpha f_{\theta\kappa}\,\partial^\kappa f_\alpha{}^\theta - \frac{1}{3}t_1\,\partial^\alpha f_{\kappa\theta}\,\partial^\kappa f_\alpha{}^\theta - \frac{1}{3}t_1\,\partial_\alpha f^\lambda{}_\lambda\,\partial^\kappa f^i{}_i +$$
$$\frac{1}{3}t_1\,\omega_{\kappa\alpha}{}^\alpha\,\partial^\kappa f^i{}_i + \frac{1}{3}t_1\,\omega_{i\lambda}{}^\lambda\,\partial^\kappa f_\kappa{}^i - \frac{1}{3}t_1\,\partial^\alpha f_\kappa{}^\lambda\,\partial^\kappa f_{\lambda\alpha} +$$
$$2t_1\,\omega_{i\kappa\theta}\,\partial^\kappa f^{i\theta} - \frac{1}{3}t_1\,\omega_{i\alpha}{}^\alpha\,\partial^\kappa f^i{}_\kappa - \frac{1}{3}t_1\,\partial^\alpha f^\lambda{}_\lambda\,\partial^\kappa f_{\kappa\alpha} +$$
$$\frac{1}{2}t_1\,\partial_\kappa f_\lambda{}^\lambda\,\partial^\kappa f_{\lambda}{}^\theta + \frac{1}{2}t_1\,\partial_\kappa f^\lambda{}_\theta\,\partial^\kappa f_{\lambda}{}^\theta - \frac{1}{3}t_1\,\partial^\alpha f^\lambda{}_\lambda\,\partial^\kappa f_{\lambda\kappa} +$$
$$\frac{1}{3}r_2\,\partial_\kappa\omega^{\alpha\beta\theta}\,\partial^\kappa\omega_{\alpha\beta\theta} + \frac{2}{3}r_2\,\partial^\beta\omega_i{}^{\lambda\alpha}\,\partial_\lambda\omega_{\alpha\beta}{}^i - \frac{2}{3}r_2\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa +$$
$$\frac{2}{3}r_2\,\partial^\beta\omega_i{}^{\lambda\alpha}\,\partial_\lambda\omega_{\alpha\beta}{}^i - 4r_3\,\partial^\beta\omega_i{}^{\lambda\alpha}\,\partial_\lambda\omega_{\alpha\beta}{}^i - 2r_3\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa +$$
$$r_5\,\partial_\alpha\omega_\lambda{}^\alpha{}_\theta\,\partial^\lambda\omega^{\theta\kappa}{}_\kappa + 2r_3\,\partial_\theta\omega_{\alpha\beta}{}^\alpha\,\partial^\lambda\omega^{\theta\beta}{}_\lambda$$

**Added source term:** $\quad f^{\alpha\beta}\,\tau_{\alpha\beta}+\omega^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}$

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### Table (top centre)

| | $\omega_{1^+\ \alpha\beta}^{\#1}$ | $\omega_{1^+\ \alpha\beta}^{\#2}$ | $f_{1^+\ \alpha\beta}^{\#1}$ | $\omega_{1^-\ \alpha}^{\#1}$ | $\omega_{1^-\ \alpha}^{\#2}$ | $f_{1^-\ \alpha}^{\#1}$ | $f_{1^-\ \alpha}^{\#2}$ |
|---|---|---|---|---|---|---|---|
| $\omega_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $k^2(2r_3+r_5)-\frac{t_1}{2}$ | $-\frac{t_1}{\sqrt2}$ | $-\frac{ikt_1}{\sqrt2}$ | 0 | 0 | 0 | 0 |
| $\omega_{1^+}^{\#2}\dagger^{\alpha\beta}$ | $-\frac{t_1}{\sqrt2}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{ikt_1}{\sqrt2}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\omega_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | $k^2(2r_3+r_5)+\frac{t_1}{6}$ | $\frac{t_1}{3\sqrt2}$ | 0 | $\frac{ikt_1}{3}$ |
| $\omega_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | $\frac{t_1}{3\sqrt2}$ | $\frac{t_1}{3}$ | 0 | $\frac{1}{3}ii\sqrt2\,kt_1$ |
| $f_{1^-}^{\#1}\dagger^{\alpha}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $f_{1^-}^{\#2}\dagger^{\alpha}$ | 0 | 0 | 0 | $-\frac{1}{3}iikt_1$ | $-\frac{1}{3}ii\sqrt2\,kt_1$ | 0 | $\frac{2k^2t_1}{3}$ |

### Table (top right)

| | $\omega_{2^+\ \alpha\beta}^{\#1}$ | $f_{2^+\ \alpha\beta}^{\#1}$ | $\omega_{2^-\ \alpha\beta\chi}^{\#1}$ |
|---|---|---|---|
| $\omega_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{t_1}{2}$ | $-\frac{ikt_1}{\sqrt2}$ | 0 |
| $f_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\frac{ikt_1}{\sqrt2}$ | $k^2t_1$ | 0 |
| $\omega_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$ | 0 | 0 | $\frac{t_1}{2}$ |

### Table (right)

| | $\sigma_{0^+}^{\#1}$ | $\tau_{0^+}^{\#1}$ | $\tau_{0^+}^{\#2}$ | $\sigma_{0^-}^{\#1}$ |
|---|---|---|---|---|
| $\sigma_{0^+}^{\#1}\dagger$ | $\frac{1}{6k^2r_3}$ | 0 | 0 | 0 |
| $\tau_{0^+}^{\#1}\dagger$ | 0 | 0 | 0 | 0 |
| $\tau_{0^+}^{\#2}\dagger$ | 0 | 0 | 0 | 0 |
| $\sigma_{0^-}^{\#1}\dagger$ | 0 | 0 | 0 | $\frac{1}{k^2r_2-t_1}$ |

### Table (centre-right, spin-2)

| | $\sigma_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\tau_{2^+}^{\#1}\dagger^{\alpha\beta}$ | $\sigma_{2^-}^{\#1}\dagger^{\alpha\beta\chi}$ |
|---|---|---|---|
| $\sigma_{2^-\ \alpha\beta\chi}^{\#1}$ | 0 | 0 | $\frac{2}{t_1}$ |
| $\tau_{2^+\ \alpha\beta}^{\#1}$ | $-\frac{2i\sqrt2\,k}{(1+2k^2)^2t_1}$ | $\frac{4k^2}{(1+2k^2)^2t_1}$ | 0 |
| $\sigma_{2^+\ \alpha\beta}^{\#1}$ | $\frac{2}{(1+2k^2)^2t_1}$ | $\frac{2i\sqrt2\,k}{(1+2k^2)^2t_1}$ | 0 |

### Table (right, spin-0)

| | $\omega_{0^+}^{\#1}\dagger$ | $f_{0^+}^{\#1}\dagger$ | $f_{0^+}^{\#2}\dagger$ | $\omega_{0^-}^{\#1}\dagger$ |
|---|---|---|---|---|
| $\omega_{0^-}^{\#1}$ | 0 | 0 | 0 | $k^2r_2-t_1$ |
| $\omega_{0^+}^{\#1}$ | $6k^2r_3$ | 0 | 0 | 0 |
| $f_{0^+}^{\#1}$ | 0 | 0 | 0 | 0 |
| $f_{0^+}^{\#2}$ | 0 | 0 | 0 | 0 |

### Table (left)

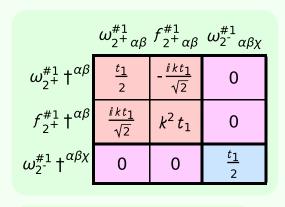| | $\sigma_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\sigma_{1^+}^{\#2}\dagger^{\alpha\beta}$ | $\tau_{1^+}^{\#1}\dagger^{\alpha\beta}$ | $\sigma_{1^-}^{\#1}\dagger^{\alpha}$ | $\sigma_{1^-}^{\#2}\dagger^{\alpha}$ | $\tau_{1^-}^{\#1}\dagger^{\alpha}$ | $\tau_{1^-}^{\#2}\dagger^{\alpha}$ |
|---|---|---|---|---|---|---|---|
| $\tau_{1^-\ \alpha}^{\#2}$ | 0 | 0 | 0 | $-\frac{i}{k(1+2k^2)(2r_3+r_5)}$ | $\frac{i(6k^2(2r_3+r_5)+t_1)}{\sqrt2\,k(1+2k^2)^2(2r_3+r_5)t_1}$ | 0 | $\frac{6k^2(2r_3+r_5)+t_1}{(1+2k^2)^2(2r_3+r_5)t_1}$ |
| $\tau_{1^-\ \alpha}^{\#1}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\sigma_{1^-\ \alpha}^{\#2}$ | 0 | 0 | 0 | $-\frac{1}{\sqrt2(k^2+2k^4)(2r_3+r_5)}$ | $\frac{6k^2(2r_3+r_5)+t_1}{2(k+2k^3)^2(2r_3+r_5)t_1}$ | 0 | $-\frac{i(6k^2(2r_3+r_5)+t_1)}{\sqrt2\,k(1+2k^2)^2(2r_3+r_5)t_1}$ |
| $\sigma_{1^-\ \alpha}^{\#1}$ | 0 | 0 | 0 | $\frac{1}{k^2(2r_3+r_5)}$ | $-\frac{1}{\sqrt2(k^2+2k^4)(2r_3+r_5)}$ | 0 | $\frac{i}{k(1+2k^2)(2r_3+r_5)}$ |
| $\tau_{1^+\ \alpha\beta}^{\#1}$ | $-\frac{i\sqrt2\,k}{t_1+k^2t_1}$ | $\frac{-2ik^3(2r_3+r_5)+ikt_1}{(1+k^2)^2t_1^2}$ | $\frac{-2k^4(2r_3+r_5)+k^2t_1}{(1+k^2)^2t_1^2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+\ \alpha\beta}^{\#2}$ | $-\frac{\sqrt2}{t_1+k^2t_1}$ | $-\frac{2k^2(2r_3+r_5)+t_1}{(1+k^2)^2t_1^2}$ | $\frac{i(2k^3(2r_3+r_5)+kt_1)}{(1+k^2)^2t_1^2}$ | 0 | 0 | 0 | 0 |
| $\sigma_{1^+\ \alpha\beta}^{\#1}$ | 0 | $-\frac{\sqrt2}{t_1+k^2t_1}$ | $\frac{i\sqrt2\,k}{t_1+k^2t_1}$ | 0 | 0 | 0 | 0 |

### Source constraints

| SO(3) irreps | # |
|---|---|
| $\tau_{0^+}^{\#2}==0$ | 1 |
| $\tau_{0^+}^{\#1}==0$ | 1 |
| $\tau_{1^-}^{\#2\ \alpha}+2ik\,\sigma_{1^-}^{\#2\ \alpha}==0$ | 3 |
| $\tau_{1^-}^{\#1\ \alpha}==0$ | 3 |
| $\tau_{1^+}^{\#1\ \alpha\beta}+ik\,\sigma_{1^+}^{\#2\ \alpha\beta}==0$ | 3 |
| $\tau_{2^+}^{\#1\ \alpha\beta}-2ik\,\sigma_{2^+}^{\#1\ \alpha\beta}==0$ | 5 |
| Total #: | 16 |

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### Massive particle

$J^P = 0^-$, $\;k^\mu$

| | |
|---|---|
| Pole residue: | $-\frac{1}{r_2} > 0$ |
| Polarisations: | 1 |
| Square mass: | $\frac{t_1}{r_2} > 0$ |
| Spin: | 0 |
| Parity: | Odd |

### Quadratic pole

$k^\mu$

| | |
|---|---|
| Pole residue: | $-\frac{1}{(2r_3+r_5)t_1^2} > 0$ |
| Polarisations: | 2 |

### Unitarity conditions

$$r_2 < 0\ \&\&\ r_5 < -2r_3\ \&\&\ t_1 < 0$$