

```
In[ ]:= Get@FileNameJoin@{NotebookDirectory[], "SupplementalMaterials.m"};
```

```
-----  
Package xAct`xPerm` version 1.2.3, {2015, 8, 23}
```

```
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```

```
Connecting to external linux executable...
```

```
Connection established.  
-----
```

```
Package xAct`xTensor` version 1.2.0, {2021, 10, 17}
```

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```

```
Package xAct`xPlain` version 1.0.0-developer, {2023, 5, 6}
```

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# Supplemental materials: Hamiltonian analysis

## Introduction

### How to use this document

These calculations are designed to accompany our manuscript in the form of supplemental materials, for the sake of reproducibility. Throughout, commentary by the authors takes the form of green text. Citations, where needed, will be managed by direct reference to arXiv numbers, and all such references are already provided in full within the body of our manuscript. One exception is the source referred to throughout as ‘Blagojević’; this pertains to the book ‘Gravitation and Gauge Symmetries’, which is also referenced within the manuscript.

**Concrete relation to manuscript:** In boxes like this, we will make specific connections between a result which is obtained in the supplemental material and a claim which is made in the manuscript. These points of contact are not always numbered equations, they could be textual.

Note that a programmatical session in the Wolfram language does not really correspond to the clean flow of thoughts in a LaTeX document: there are differences that can't (and shouldn't) be ignored. Thus, whilst this document should be at least readable in standalone format, the reader is encouraged to follow it in tandem with the Wolfram language source codes, so as to avoid ambiguities. In this way, the specific operations and manipulations of quantities will become absolutely clear.

## Loading HiGGS

For these calculations, we will use the HiGGS package. The version of HiGGS used for the computations here are both developer-only, and so we include copies of the sources with these supplemental materials.

```
-----
Package xAct`xPert` version 1.0.6, {2018, 2, 28}

Copyright (C) 2005–2020, David Brizuela, Jose M. Martin-Garcia
and Guillermo A. Mena Marugan, under the General Public License.

** Variable $PrePrint assigned value ScreenDollarIndices
** Variable $CovDFormat changed from Prefix to Postfix
** Option AllowUpperDerivatives of ContractMetric changed from False to True
** Option MetricOn of MakeRule changed from None to All
** Option ContractMetrics of MakeRule changed from False to True
-----
```

```
Package xAct`Invar` version 2.0.5, {2013, 7, 1}

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D. Yllanes and R. Portugal, under the General Public License.

** DefConstantSymbol: Defining constant symbol sigma.
** DefConstantSymbol: Defining constant symbol dim.
** Option CurvatureRelations of DefCovD changed from True to False
** Variable $CommuteCovDsOnScalars changed from True to False
-----
```

```
Package xAct`xCoba` version 0.8.6, {2021, 2, 28}

Copyright (C) 2005–2021, David Yllanes and
Jose M. Martin-Garcia, under the General Public License.
-----
```

```
Package xAct`SymManipulator` version 0.9.5, {2021, 9, 14}

Copyright (C) 2011–2021, Thomas Bäckdahl, under the General Public License.
-----
```

```
Package xAct`xTras` version 1.4.2, {2014, 10, 30}
```

```
CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.
```

```
** Variable $CovDFormat changed from Postfix to Prefix
```

```
** Option CurvatureRelations of DefCovD changed from False to True
```

```
Package xAct`HiGGS` version 2.0.0-developer, {2023, 5, 18}
```

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```

```
HiGGS incorporates code by Cyril Pitrou.
```

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```

The HiGGS package is now loaded. We are interested only in the computations which lead to our Fig. 3 and Fig. 5. (in the appendix) in the manuscript. Two theories are considered. To perform the Hamiltonian analysis we use the `StudyTheory[]` command in HiGGS, specifying the parameters of the theory. For details of this command, and for all notation used in this supplement, see arXiv:2206.00658. The results were precomputed on a desktop computer. This is because the full Hamiltonian analysis of a theory, even run on a modern desktop computer with multiple processor cores, still takes up to half an hour. Below we will display the results using the `ViewTheory[]` command, which takes the pre-computed binary files (.thr.mx) and formats them. If you want to re-compute the analysis locally, the few lines of code you will need are stored at the top of this (.m) file, so just un-comment them. If there are problems with the execution, please refer to the HiGGS docs, or email wb263@cam.ac.uk.

## Singular spectrum without multipliers

### Introductory comments

In the first instance we perform the Hamiltonian analysis of the theory without multipliers. As we know, this theory suffers from strong coupling. Accordingly, in Eq. (4) in arXiv:2205.13534 we set  $\hat{\alpha}_1^\cdot, \hat{\alpha}_2^\cdot, \hat{\alpha}_3^\cdot,$

$\hat{\alpha}_4^\cdot, \hat{\alpha}_6^\cdot, \mathcal{M}_{Pl}^2 \hat{\beta}_1^\cdot, \mathcal{M}_{Pl}^2 \hat{\beta}_2^\cdot, \mathcal{M}_{Pl}^2 \bar{\beta}_3^\cdot,$  to zero, suppressing most of the higher-curvature invariants, two of the three torsion invariants, and all of the Lagrange multipliers, but allow for nonvanishing  $\mathcal{M}_{Pl}^2 \hat{\alpha}_0^\cdot$ , (Einstein-Hilbert term)  $\hat{\alpha}_5^\cdot$  (Riemann-Cartan-Maxwell invariant of curvature) and  $\mathcal{M}_{Pl}^2 \hat{\beta}_3^\cdot$ .

The results of the analysis are presented below; the HiGGS output is designed to be reasonably self-explanatory, but we will nonetheless add some guiding commentary.

The first set of data are the primary constraints  $\phi$  of the theory. These are implied by the impossibility of inverting the conjugate field-momenta for the velocities of the fields. As is now common in the PGT literature, we refer to these as 'if-constraints', i.e. their existence depends on the linear relations assumed among the Lagrangian parameters.

Next, the Legendre-transformed Lagrangian is presented. This is broken into terms; the super-Hamiltonian (i.e. the coefficient  ${}^{0+}\mathcal{H}_b$  of the lapse function  $\mathcal{N}$ ), the linear super-momentum (i.e. the coefficient  ${}^{1-}\mathcal{H}_{b\ i}$  of the shift vector) and the angular super-momentum (i.e. the antisymmetric coefficient  ${}^{2+}\mathcal{H}_{\mathcal{A}\ nm}$  of the timelike part of the rotational gauge field). These concepts are introduced in Blagojević.

Once the primary constraints are listed, the matrix  $\{\phi, \phi\}$  of Poisson brackets between the primaries is computed. Of course, these brackets only need to be computed for the upper triangular part of the matrix. The resulting entries lie behind the red, white and yellow squares in the relevant panel of Fig. 3.

Finally, the velocities of these primaries  $\phi$  are computed, i.e. their Poisson brackets with the super-Hamiltonian  ${}^{0+}\mathcal{H}_b$ . These velocities serve two purposes; (i) they can be used to determine the 'missing velocities' or 'Hamiltonian multipliers'  $\mu$  (not to be confused with the geometric multipliers  $\lambda$  used in the manuscript), or (ii) they can define the secondary constraints of the theory. Whichever of these applies depends of course on  $\{\phi, \phi\}$ , according to the consistency condition.

### Lists of constraints and Poisson brackets from HiGGS

Having briefly recounted the notation of arXiv:2206.00658, we make a call to the HiGGS command ViewTheory[] on the appropriate model. This command presents all the results which were pre-computed by the earlier call to StudyTheory[].

```
** DefTheory: Incorporating the binary at ConstraintAlgebraAlp0Alp5Bet3.thr.mx
```

```
** DefTheory: Found the following primary if-constraints:
```

$${}^{0+}\phi_b \equiv \frac{{}^{0+}\pi_b}{\mathcal{J}} \approx 0$$

$${}^{1-}\phi_{b\ i} \equiv \frac{{}^{1-}\pi_{b\ i}}{\mathcal{J}} \approx 0$$

$${}^{2+}\phi_{b\ i\ j} \equiv \frac{{}^{2+}\pi_{b\ i\ j}}{\mathcal{J}} \approx 0$$

$${}^{0+}\phi_{\mathcal{A}} \equiv 3\mathcal{M}_{\text{Pl}}^2\alpha_0 + \frac{{}^{0+}\pi_{\mathcal{A}}}{\mathcal{J}} \approx 0$$

$${}^{0-}\phi_{\mathcal{A}} \equiv \frac{{}^{0-}\pi_{\mathcal{A}}}{\mathcal{J}} \approx 0$$

$${}^{2+}\phi_{\mathcal{A}\ i\ j} \equiv \frac{{}^{2+}\pi_{\mathcal{A}\ i\ j}}{\mathcal{J}} \approx 0$$

$$\overset{2}{\cdot}\phi_{\mathcal{A}ijh} \equiv \frac{2 \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ijh}}{3\mathcal{J}} + \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ihj}}{3\mathcal{J}} - \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}jhi}}{3\mathcal{J}} \approx 0$$

\*\* DefTheory: Found the following primary parallel

if-constraints associated with geometric multiplier momenta (if the sector is singular, the parallel constraint is taken to be the singular constraint and the identical perpendicular constraint below is dropped):

\*\* DefTheory: Found the following primary perpendicular

if-constraints associated with geometric multiplier momenta:

\*\* DefTheory: Found the following secondary perpendicular if-constraints:

\*\* DefTheory: Found the following secondary parallel if-constraints:

\*\* DefTheory: Found the following secondary singular if-constraints:

\*\* DefTheory: Defining association key \$IfConstraints

for the theory association ConstraintAlgebraAlp0Alp5Bet3

\*\* ViewTheory: The super-Hamiltonian is:

$$\begin{aligned} \overset{0}{\cdot}\mathcal{H}_b \equiv & \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}a} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^a}{16 \overset{\wedge}{\alpha}_5 \mathcal{J}} + \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ab} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{ab}}{8 \overset{\wedge}{\alpha}_5 \mathcal{J}} + \frac{3 \overset{1}{\cdot}\overset{\wedge}{\pi}_{b\mathcal{A}b} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^{ab}}{16 \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{J}} + \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \mathcal{J}]}{\mathcal{J}} + \\ & \frac{\mathcal{M}_{Pl}^2 \overset{\wedge}{\alpha}_0 \mathcal{J} \overset{0}{\cdot}\mathcal{R}^{\parallel}}{2} - \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^a \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}}^{\parallel a} + \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{ab} \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}b}^{\parallel} + \frac{1}{6} \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{J} \overset{0}{\cdot}\mathcal{T}^{\parallel 2} - \overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \overset{1}{\cdot}\mathcal{T}_{\mathcal{A}}^{\parallel a} + \\ & \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^{ab} \overset{1}{\cdot}\mathcal{T}_{\mathcal{A}b}^{\parallel} - \frac{16}{27} \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{J} \overset{2}{\cdot}\mathcal{T}_{\mathcal{A}bb'}^{\parallel} \overset{2}{\cdot}\mathcal{T}^{\parallel ab b'} + \frac{32}{27} \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{J} \overset{2}{\cdot}\mathcal{T}_{\mathcal{A}b'b}^{\parallel} \overset{2}{\cdot}\mathcal{T}^{\parallel abb'} - \\ & n^a \left( \overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel b} \overset{1}{\cdot}\overset{\wedge}{\pi}_{b\mathcal{A}}^b \right) - n^a \left( \overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel b} \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^b \right) - \eta^{\parallel \mathcal{A}b} \left( \overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel b} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \right) + \frac{1}{3} \eta^{\parallel \mathcal{A}b} \overset{0}{\cdot}\overset{\wedge}{\pi}_b^a \left( \overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel b} n^a \right) \approx 0 \end{aligned}$$

\*\* ViewTheory: The linear super-momentum is:

$$\begin{aligned} \overset{1}{\cdot}\mathcal{H}_{b\parallel} \equiv & -\frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel a} \eta_{\parallel}] + \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \mathcal{J}]}{\mathcal{J}} + \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \mathcal{J}]}{\mathcal{J}} - \mathcal{P}_{\eta}[\eta^{aa'} \left( \overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel a'} \overset{1}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \right)] - \\ & \mathcal{P}_{\eta}[\eta^{aa'} \left( \overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel a'} \overset{2}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \right)] + \frac{\overset{0}{\cdot}\overset{\wedge}{\pi}_b^a \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \mathcal{J}]}{3\mathcal{J}} - \frac{1}{3} \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \overset{0}{\cdot}\overset{\wedge}{\pi}_b^a] - \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \eta_a] + \frac{1}{6} \epsilon^{\parallel \mathcal{A}aa'} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{aa'} \overset{0}{\cdot}\mathcal{R}^{\parallel} - \\ & \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}\parallel} \overset{0}{\cdot}\mathcal{R}^{\parallel}}{6} - \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}\parallel a} \overset{1}{\cdot}\mathcal{R}^{\parallel a} - \frac{1}{2} \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}\parallel a} \overset{1}{\cdot}\mathcal{R}^{\parallel a} + \frac{\overset{0}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}} \overset{1}{\cdot}\mathcal{R}_{\parallel}^{\parallel}}{3} - \frac{4}{3} \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}\parallel}^{aa'} \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}a'}^{\parallel} + \\ & \frac{1}{6} \epsilon^{\parallel \mathcal{A}aa'} \overset{0}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{aa'} \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}}^{\parallel aa'} - \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^a \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}}^{\parallel a} - \frac{4}{3} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{aa'} \overset{2}{\cdot}\mathcal{R}_{\mathcal{A}\parallel aa'}^{\parallel} + \frac{4}{3} \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{aa'} \overset{2}{\cdot}\mathcal{R}_{\mathcal{A}\parallel aa'}^{\parallel} + \\ & \frac{4}{3} \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}\parallel}^{aa'} \overset{2}{\cdot}\mathcal{R}_{\mathcal{A}a'}^{\parallel} - \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^a \overset{2}{\cdot}\mathcal{R}_{\mathcal{A}}^{\parallel a} + \frac{1}{6} \epsilon^{\parallel \mathcal{A}aa'} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^{aa'} \overset{0}{\cdot}\mathcal{T}^{\parallel} - \frac{3}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \overset{1}{\cdot}\mathcal{T}_{\mathcal{A}}^{\parallel a} - \\ & \frac{3}{2} \overset{2}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \overset{1}{\cdot}\mathcal{T}_{\mathcal{A}}^{\parallel a} + \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \overset{1}{\cdot}\mathcal{T}_{\mathcal{A}}^{\parallel a} - \frac{4}{3} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^{aa'} \overset{2}{\cdot}\mathcal{T}_{\mathcal{A}\parallel aa'}^{\parallel} + \frac{4}{3} \overset{2}{\cdot}\overset{\wedge}{\pi}_b^{aa'} \overset{2}{\cdot}\mathcal{T}_{\mathcal{A}\parallel aa'}^{\parallel} \approx 0 \end{aligned}$$

\*\* ViewTheory: The 1+ part of the angular super-momentum is:

$$\begin{aligned}
 \mathbf{1}^+ \mathcal{H}_{\mathcal{A}nm} \equiv & -2 \mathbf{1}^+ \hat{\pi}_{bmn} - \frac{\epsilon^{\parallel mn a} \mathbf{0}^+ \hat{\pi}_{\mathcal{A}} P[\mathbf{3} \mathcal{D}^{\parallel a} \mathcal{T}]}{6 \mathcal{T}} + \frac{4 \mathbf{2}^+ \hat{\pi}_{\mathcal{A}mn a} P[\mathbf{3} \mathcal{D}^{\parallel a} \mathcal{T}]}{3 \mathcal{T}} + \frac{1}{6} \epsilon^{\parallel mn a} P[\mathbf{3} \mathcal{D}^{\parallel a} \mathbf{0}^+ \hat{\pi}_{\mathcal{A}}] - \\
 & \frac{1}{2} \mathbf{1}^+ \hat{\pi}_{\mathcal{A}n a} P[\mathbf{3} \mathcal{D}^{\parallel a} n_m] - \frac{1}{2} \mathbf{2}^+ \hat{\pi}_{\mathcal{A}n a} P[\mathbf{3} \mathcal{D}^{\parallel a} n_m] + \frac{1}{2} \mathbf{1}^+ \hat{\pi}_{\mathcal{A}m a} P[\mathbf{3} \mathcal{D}^{\parallel a} n_n] + \frac{1}{2} \mathbf{2}^+ \hat{\pi}_{\mathcal{A}m a} P[\mathbf{3} \mathcal{D}^{\parallel a} n_n] - \\
 & \frac{4}{3} P[n^{\parallel a a'} (\mathbf{3} \mathcal{D}^{\parallel a}, \mathbf{2}^+ \hat{\pi}_{\mathcal{A}mn a})] + \frac{\mathbf{1}^+ \hat{\pi}_{\mathcal{A}n} P[\mathbf{3} \mathcal{D}^{\parallel m} \mathcal{T}]}{2 \mathcal{T}} - \frac{1}{2} P[\mathbf{3} \mathcal{D}^{\parallel m} \mathbf{1}^+ \hat{\pi}_{\mathcal{A}n}] - \frac{1}{2} \mathbf{1}^+ \hat{\pi}_{\mathcal{A}n a} P[\mathbf{3} \mathcal{D}^{\parallel m} n^a] - \\
 & \frac{1}{2} \mathbf{2}^+ \hat{\pi}_{\mathcal{A}n a} P[\mathbf{3} \mathcal{D}^{\parallel m} n^a] - \frac{\mathbf{1}^+ \hat{\pi}_{\mathcal{A}m} P[\mathbf{3} \mathcal{D}^{\parallel n} \mathcal{T}]}{2 \mathcal{T}} + \frac{1}{2} P[\mathbf{3} \mathcal{D}^{\parallel n} \mathbf{1}^+ \hat{\pi}_{\mathcal{A}m}] + \frac{1}{2} \mathbf{1}^+ \hat{\pi}_{\mathcal{A}m a} P[\mathbf{3} \mathcal{D}^{\parallel n} n^a] + \\
 & \frac{1}{2} \mathbf{2}^+ \hat{\pi}_{\mathcal{A}m a} P[\mathbf{3} \mathcal{D}^{\parallel n} n^a] + \frac{1}{6} \epsilon^{\parallel mn a} \mathbf{0}^+ \hat{\pi}_{\mathcal{A}} \mathbf{1}^+ \mathcal{T}^{\parallel a} - \frac{4}{3} \mathbf{2}^+ \hat{\pi}_{\mathcal{A}mn a} \mathbf{1}^+ \mathcal{T}^{\parallel a} - \frac{1}{2} \mathbf{1}^+ \hat{\pi}_{\mathcal{A}n} \mathbf{1}^+ \mathcal{T}^{\parallel m} + \frac{1}{2} \mathbf{1}^+ \hat{\pi}_{\mathcal{A}m} \mathbf{1}^+ \mathcal{T}^{\parallel n} - \\
 & \frac{1}{2} \mathbf{1}^+ \hat{\pi}_{\mathcal{A}n}^a \mathbf{1}^+ \mathcal{T}^{\parallel m a} - \frac{1}{2} \mathbf{2}^+ \hat{\pi}_{\mathcal{A}n}^a \mathbf{1}^+ \mathcal{T}^{\parallel m a} - \frac{1}{3} \mathbf{0}^+ \hat{\pi}_{\mathcal{A}} \mathbf{1}^+ \mathcal{T}^{\parallel mn} + \frac{1}{2} \mathbf{1}^+ \hat{\pi}_{\mathcal{A}m}^a \mathbf{1}^+ \mathcal{T}^{\parallel n a} + \frac{1}{2} \mathbf{2}^+ \hat{\pi}_{\mathcal{A}m}^a \mathbf{1}^+ \mathcal{T}^{\parallel n a} \approx 0
 \end{aligned}$$

\*\* ViewTheory: The on-shell brackets between the various if-constraints are:

$$\begin{aligned}
 & \left\{ \iiint (\mathbf{0}^+ \phi_b \cdot \mathbf{(1)}^+ \mathcal{S}) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint (\mathbf{0}^+ \phi_b \cdot \mathbf{(2)}^+ \mathcal{S}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint (\mathbf{0}^+ \phi_b \cdot \mathbf{(1)}^+ \mathcal{S}) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint (\mathbf{1}^+ \phi_{b i} \cdot \mathbf{(2)}^+ \mathcal{S}^i) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint (\mathbf{0}^+ \phi_b \cdot \mathbf{(1)}^+ \mathcal{S}) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint (\mathbf{2}^+ \phi_{b l m} \cdot \mathbf{(2)}^+ \mathcal{S}^{lm}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint (\mathbf{0}^+ \phi_b \cdot \mathbf{(1)}^+ \mathcal{S}) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint (\mathbf{0}^+ \phi_{\mathcal{A}} \cdot \mathbf{(2)}^+ \mathcal{S}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
 & \quad \iiint \left( -\frac{6 \mathcal{M}_{Pl}^2 \hat{\alpha} \cdot \mathbf{(1)}^+ \mathcal{S} \mathbf{(2)}^+ \mathcal{S}}{\mathcal{T}} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
 & \left\{ \iiint (\mathbf{0}^+ \phi_b \cdot \mathbf{(1)}^+ \mathcal{S}) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint (\mathbf{0}^+ \phi_{\mathcal{A}} \cdot \mathbf{(2)}^+ \mathcal{S}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint (\mathbf{0}^+ \phi_b \cdot \mathbf{(1)}^+ \mathcal{S}) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint (\mathbf{2}^+ \phi_{b l m} \cdot \mathbf{(2)}^+ \mathcal{S}^{lm}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint (\mathbf{0}^+ \phi_b \cdot \mathbf{(1)}^+ \mathcal{S}) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint (\mathbf{2}^+ \phi_{b l m n} \cdot \mathbf{(2)}^+ \mathcal{S}^{lmn}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint (\mathbf{1}^+ \phi_{b i} \cdot \mathbf{(1)}^+ \mathcal{S}^i) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint (\mathbf{1}^+ \phi_{b i} \cdot \mathbf{(2)}^+ \mathcal{S}^i) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
 & \quad \iiint \frac{2 \mathbf{1}^+ \hat{\pi}_{b a a'} P[\mathbf{(1)}^+ \mathcal{S}^a] P[\mathbf{(2)}^+ \mathcal{S}^{a'}]}{\mathcal{T}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
 & \left\{ \iiint (\mathbf{1}^+ \phi_{b i} \cdot \mathbf{(1)}^+ \mathcal{S}^i) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint (\mathbf{2}^+ \phi_{b l m} \cdot \mathbf{(2)}^+ \mathcal{S}^{lm}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0
 \end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left( \overset{1}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \left( - \frac{\overset{1}{\cdot} \pi_{\mathcal{A}}^a P_{\eta}^{[(1)S_a]} \overset{(2)}{S}}{\mathcal{J}^2} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{2 \epsilon_{\alpha\alpha' i \alpha 1} \overset{1}{\cdot} \pi_{\mathcal{A}}^i \overset{2\alpha 1}{P_{\eta}^{[(1)S^{a'}]} \overset{(2)}{S} n^a}}{\mathcal{J}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\cdot} \phi_{\mathcal{A}lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{\overset{1}{\cdot} \pi_{\mathcal{A}}^a \left( 3 P_{\eta}^{[(1)S^{a'}]} \left( P_{\eta}^{[(2)S_{aa'}]} + P_{\eta}^{[(2)S_{a'a}]} \right) - 2 \eta_{a'b} P_{\eta}^{[(1)S_a]} \overset{(2)}{S}^{a'b} \right)}{12 \mathcal{J}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\cdot} \phi_{\mathcal{A}lmn} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{8 \mathcal{J}^2} P_{\eta}^{[(1)S^a]} \left( \overset{1}{\cdot} \pi_{\mathcal{A}a'b} \left( -2 P_{\eta}^{[(2)S_a^{a'b}]} + 2 P_{\eta}^{[(2)S^{a'b}]} + 4 P_{\eta}^{[(2)S^{a'b}]} \right) + \right. \\
& \quad \left. 3 \overset{1}{\cdot} \pi_{\mathcal{A}a}^{a'} \left( P_{\eta}^{[\eta_{bb'}^{(2)S_a^{bb'}}]} - P_{\eta}^{[\eta_{bb'}^{(2)S_a^{b'b'}}]} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\cdot} \phi_{ilm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{P_{\eta}^{[(1)S^{aa'}]} \left( \overset{1}{\cdot} \pi_{ba'b} \left( P_{\eta}^{[(2)S_a^b]} + P_{\eta}^{[(2)S^b_a]} \right) + \overset{1}{\cdot} \pi_{ba'b} \left( P_{\eta}^{[(2)S_a^b]} + P_{\eta}^{[(2)S^b_a]} \right) \right)}{2 \mathcal{J}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\cdot} \phi_{ilm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{12 \mathcal{J}^2} \left( 3 P_{\eta}^{[(1)S^{aa'}]} \left( 2 \mathcal{M}_{P1}^2 \overset{\hat{a}}{\alpha}_{\bullet} \mathcal{T} P_{\eta}^{[(2)S_{aa'}]} + 2 \mathcal{M}_{P1}^2 \overset{\hat{a}}{\alpha}_{\bullet} \mathcal{T} P_{\eta}^{[(2)S_{a'a}]} + \right. \right. \\
& \quad \left. \overset{1}{\cdot} \pi_{\mathcal{A}ab} P_{\eta}^{[(2)S_a^b]} + \overset{1}{\cdot} \pi_{\mathcal{A}a'b} \left( P_{\eta}^{[(2)S_a^b]} + P_{\eta}^{[(2)S^b_a]} \right) + \overset{1}{\cdot} \pi_{\mathcal{A}ab} P_{\eta}^{[(2)S^b_a]} \right) - \\
& \quad \left. 4 \mathcal{M}_{P1}^2 \overset{\hat{a}}{\alpha}_{\bullet} \eta_{aa'} \eta_{bb'} \mathcal{T} \overset{(1)}{S}^{aa'} \overset{(2)}{S}^{bb'} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\left\{ \iiint \left( \overset{2}{\phi}_{\mathcal{A}ij} \cdot \overset{(1)}{S}^{ij} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{2}{\phi}_{\mathcal{A}lmn} \cdot \overset{(2)}{S}^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx$$

$$\iiint \frac{1}{32 \mathcal{T}^2} 3 \overset{1}{\pi}_{\mathcal{A}}^a \left( P_{\eta}^{(1)S^a} \right) P_{\eta}^{(1)S^a} P_{\eta}^{(2)S^a} P_{\eta}^{(2)S^a} + 2 P_{\eta}^{(1)S^a} \left( P_{\eta}^{(2)S^a} P_{\eta}^{(2)S^a} + P_{\eta}^{(2)S^a} P_{\eta}^{(2)S^a} \right) - P_{\eta}^{(2)S^a} P_{\eta}^{(2)S^a} - P_{\eta}^{(2)S^a} P_{\eta}^{(2)S^a} \right) +$$

$$P_{\eta}^{(1)S^a} \left( P_{\eta}^{(2)S^a} P_{\eta}^{(2)S^a} - P_{\eta}^{(2)S^a} P_{\eta}^{(2)S^a} \right) - P_{\eta}^{(1)S^a} P_{\eta}^{(2)S^a} P_{\eta}^{(2)S^a} -$$

$$2 \eta_{a'b} P_{\eta}^{(1)S^a} P_{\eta}^{(2)S^a} P_{\eta}^{(2)S^a} \left( \overset{(1)}{S}^{a'b} + 2 \eta_{a'b} P_{\eta}^{(1)S^a} P_{\eta}^{(2)S^a} \right) \left( \overset{(1)}{S}^{a'b} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1$$

$$\left\{ \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{2}{\phi}_{\mathcal{A}lm} \cdot \overset{(2)}{S}^{lm} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{2}{\phi}_{\mathcal{A}lmn} \cdot \overset{(2)}{S}^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{2}{\phi}_{\mathcal{A}lm} \cdot \overset{(2)}{S}^{lm} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{2}{\phi}_{\mathcal{A}lmn} \cdot \overset{(2)}{S}^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left( \overset{2}{\phi}_{\mathcal{A}ij} \cdot \overset{(1)}{S}^{ij} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{2}{\phi}_{\mathcal{A}lm} \cdot \overset{(2)}{S}^{lm} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left( \overset{2}{\phi}_{\mathcal{A}ij} \cdot \overset{(1)}{S}^{ij} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{2}{\phi}_{\mathcal{A}lmn} \cdot \overset{(2)}{S}^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left( \overset{2}{\phi}_{\mathcal{A}ijh} \cdot \overset{(1)}{S}^{ijh} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{2}{\phi}_{\mathcal{A}lmn} \cdot \overset{(2)}{S}^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

\*\* ViewTheory: The commutators between

the various if-constraints and the super-Hamiltonian are:

$$\left\{ \iiint \left( \overset{0}{\phi}_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{0}{\mathcal{H}}_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx$$

$$\iiint \frac{1}{144 \overset{\hat{a}}{\alpha}_5 \overset{\hat{a}}{\mathcal{M}}_{\mathcal{P}L}^2 \overset{\hat{a}}{\beta}_3 \mathcal{T}^2} \left( \overset{(1)}{S} \overset{(2)}{S} \left( 9 \overset{\hat{a}}{\mathcal{M}}_{\mathcal{P}L}^2 \overset{\hat{a}}{\beta}_3 \overset{\hat{a}}{\pi}_{\mathcal{A}}^a + 18 \overset{\hat{a}}{\mathcal{M}}_{\mathcal{P}L}^2 \overset{\hat{a}}{\beta}_3 \overset{\hat{a}}{\pi}_{\mathcal{A}aa} + \overset{\hat{a}}{\pi}_{\mathcal{A}}^{aa} + \right.$$

$$81 \overset{\hat{a}}{\alpha}_5 \overset{\hat{a}}{\pi}_{\mathcal{A}aa} \overset{\hat{a}}{\pi}_{\mathcal{A}}^{aa} - 8 \overset{\hat{a}}{\alpha}_5 \overset{\hat{a}}{\mathcal{M}}_{\mathcal{P}L}^2 \overset{\hat{a}}{\beta}_3 \mathcal{T} \left( 9 \overset{\hat{a}}{\mathcal{M}}_{\mathcal{P}L}^2 \overset{\hat{a}}{\alpha}_5 \overset{\hat{a}}{\mathcal{T}}^0 \overset{\hat{a}}{\mathcal{R}}^a + 9 \overset{\hat{a}}{\pi}_{\mathcal{A}}^a \overset{\hat{a}}{\mathcal{R}}^a + 54 \overset{\hat{a}}{\pi}_{\mathcal{A}}^{aa} \overset{\hat{a}}{\mathcal{R}}^{aa} + \right.$$

$$3 \overset{\hat{a}}{\mathcal{M}}_{\mathcal{P}L}^2 \overset{\hat{a}}{\beta}_3 \overset{\hat{a}}{\mathcal{T}}^0 \overset{\hat{a}}{\mathcal{T}}^2 - 9 \overset{\hat{a}}{\pi}_{\mathcal{A}}^{aa} \overset{\hat{a}}{\mathcal{T}}^0 \overset{\hat{a}}{\mathcal{T}}^2 - 32 \overset{\hat{a}}{\mathcal{M}}_{\mathcal{P}L}^2 \overset{\hat{a}}{\beta}_3 \overset{\hat{a}}{\mathcal{T}}^2 \overset{\hat{a}}{\mathcal{T}}^0 \overset{\hat{a}}{\mathcal{T}}^2 + 64 \overset{\hat{a}}{\mathcal{M}}_{\mathcal{P}L}^2 \overset{\hat{a}}{\beta}_3$$

$$\left. \left. \overset{\hat{a}}{\mathcal{T}}^2 \overset{\hat{a}}{\mathcal{T}}^0 \overset{\hat{a}}{\mathcal{T}}^2 + 18 \overset{\hat{a}}{\pi}_{\mathcal{A}}^{aa} \overset{\hat{a}}{\mathcal{T}}^2 + 18 \overset{\hat{a}}{\pi}_{\mathcal{A}}^{aa} \overset{\hat{a}}{\mathcal{T}}^2 \right) \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1$$



$$\begin{aligned}
& \left\{ \iiint \left( \overset{1}{\mathcal{P}}_{\phi_i} \cdot \overset{(1)}{\mathcal{S}}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0}{\mathcal{H}}_b \cdot \overset{(2)}{\mathcal{S}} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{12 \overset{\hat{a}}{\mathcal{A}}_5 \mathcal{T}^2} \overset{(2)}{\mathcal{S}} \left( -\mathcal{T} \overset{1}{\pi}_{\mathcal{A}}^a \left( P_{\eta}^{(1)} \mathcal{S}_a \right) \left( 3 \mathcal{M}_{\text{PL}}^2 \overset{\hat{a}}{\mathcal{A}}_0 + 2 \overset{\hat{a}}{\mathcal{A}}_5 \overset{0}{\mathcal{R}}^{\parallel} \right) + 6 \overset{\hat{a}}{\mathcal{A}}_5 P_{\eta}^{(1)} \mathcal{S}^{a'} \right) \overset{2}{\mathcal{R}}^{\parallel}_{aa'} \Big) + \\
& 2 \overset{\hat{a}}{\mathcal{A}}_5 \left( 12 \overset{1}{\pi}_{baa'} \left( P_{\eta}^{(1)} \mathcal{S}^a \right) P_{\eta}^{(3)} \mathcal{D}^{a'} \mathcal{T} + \mathcal{T} P_{\eta}^{(1)} \mathcal{S}^{a'} \right) \overset{1}{\mathcal{T}}^{\parallel a} \Big) - \\
& 4 \mathcal{T} P_{\eta}^{(1)} \mathcal{S}^a \left( 3 P_{\eta}^{(1)} \eta^{a'b} \left( \overset{3}{\mathcal{D}}^{\parallel}_b \overset{1}{\pi}_{baa'} \right) \right) - 2 \overset{1}{\pi}_{\mathcal{A}}^{a'b} \overset{2}{\mathcal{R}}^{\parallel}_{aa'b} + \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\mathcal{B}}_3 \epsilon^{\parallel}_{aa'b} \mathcal{T} \overset{0}{\mathcal{T}}^{\parallel} \overset{1}{\mathcal{T}}^{\parallel a'b} \Big) + \\
& \mathcal{T} P_{\eta}^{(1)} \mathcal{S}^{a'} \left( 6 \overset{1}{\pi}_{\mathcal{A}aa'} \overset{1}{\mathcal{R}}^{\parallel a} + \epsilon_{aa'bc} \overset{1}{\pi}_{\mathcal{A}}^{bc} \overset{0}{\mathcal{R}}^{\parallel} n^a \right) - \\
& 6 \mathcal{T} \overset{1}{\pi}_{ba}^{a'} P_{\eta}^{(1)} \mathcal{S}^a \left( P_{\eta}^b \left( \overset{3}{\mathcal{D}}^{\parallel}_a \eta_b \right) \right) - 2 \left( \overset{3}{\mathcal{D}}^{\parallel}_b \eta_a^b \right) \Big) \Big) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{2}{\mathcal{P}}_{\phi_{ij}} \cdot \overset{(1)}{\mathcal{S}}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0}{\mathcal{H}}_b \cdot \overset{(2)}{\mathcal{S}} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{144 \overset{\hat{a}}{\mathcal{A}}_5 \mathcal{T}^2} \overset{(2)}{\mathcal{S}} \\
& \left( 3 \overset{1}{\pi}_{\mathcal{A}}^a \left( 3 \overset{1}{\pi}_{\mathcal{A}}^{a'} P_{\eta}^{(1)} \mathcal{S}_{aa'} \right) - 12 \overset{\hat{a}}{\mathcal{A}}_5 \mathcal{T} P_{\eta}^{(1)} \mathcal{S}_{aa'} \right) \overset{1}{\mathcal{R}}^{\parallel a'} - 12 \overset{\hat{a}}{\mathcal{A}}_5 \mathcal{T} P_{\eta}^{(1)} \mathcal{S}_{a'a} \overset{1}{\mathcal{R}}^{\parallel a'} - 16 \overset{\hat{a}}{\mathcal{A}}_5 \mathcal{T} P_{\eta}^{(1)} \mathcal{S}^{a'b} \Big) \\
& \overset{2}{\mathcal{R}}^{\parallel}_{aa'b} - 16 \overset{\hat{a}}{\mathcal{A}}_5 \mathcal{T} P_{\eta}^{(1)} \mathcal{S}^{a'b} \overset{2}{\mathcal{R}}^{\parallel}_{abaa'} - \eta_{a'b} \overset{1}{\pi}_{\mathcal{A}a}^{(1)} \mathcal{S}^{a'b} + 8 \overset{\hat{a}}{\mathcal{A}}_5 \eta_{a'b} \mathcal{T} \overset{1}{\mathcal{R}}^{\parallel}_a \overset{(1)}{\mathcal{S}}^{a'b} \Big) + \\
& 4 \left( -9 \overset{1}{\pi}_{\mathcal{A}a}^{ab} P_{\eta}^{(1)} \mathcal{S}^{aa'} \right) \left( \overset{1}{\pi}_{\mathcal{A}a'b} + 2 \overset{\hat{a}}{\mathcal{A}}_5 \mathcal{T} \overset{2}{\mathcal{R}}^{\parallel}_{a'b} \right) + \overset{\hat{a}}{\mathcal{A}}_5 \mathcal{T} P_{\eta}^{(1)} \mathcal{S}^{aa'} \left( 18 P_{\eta}^b \left( \overset{3}{\mathcal{D}}^{\parallel}_a \overset{1}{\pi}_{ba'b} \right) \right) + \\
& 18 P_{\eta}^b \left( \overset{3}{\mathcal{D}}^{\parallel}_a \overset{1}{\pi}_{ba'b} \right) + 9 \overset{1}{\pi}_{ba'b} P_{\eta}^{(3)} \mathcal{D}^{\parallel}_a n^b + 9 \overset{1}{\pi}_{ba'b} \left( P_{\eta}^{(3)} \mathcal{D}^{\parallel}_a n^b + P_{\eta}^{(3)} \mathcal{D}^{\parallel}_b n_a \right) + 9 \\
& \overset{1}{\pi}_{ba'b} P_{\eta}^{(3)} \mathcal{D}^{\parallel}_b n_a + 36 \mathcal{M}_{\text{PL}}^2 \overset{\hat{a}}{\mathcal{A}}_0 \mathcal{T} \overset{2}{\mathcal{R}}^{\parallel}_{aa'} - 18 \overset{1}{\pi}_{\mathcal{A}a}^{ab} \overset{2}{\mathcal{R}}^{\parallel}_{ab} + 9 \overset{1}{\pi}_{ba}^{ab} \overset{1}{\mathcal{T}}^{\parallel}_{ab} + 9 \\
& \overset{1}{\pi}_{ba}^{ab} \overset{1}{\mathcal{T}}^{\parallel}_{a'b} + 16 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\mathcal{B}}_3 \epsilon^{\parallel}_{a'bb'} \mathcal{T} \overset{0}{\mathcal{T}}^{\parallel} \overset{2}{\mathcal{T}}^{\parallel}_a{}^{bb'} + 16 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\mathcal{B}}_3 \epsilon^{\parallel}_{abb'} \mathcal{T} \overset{0}{\mathcal{T}}^{\parallel} \\
& \overset{2}{\mathcal{T}}^{\parallel}_{a'}{}^{bb'} - 8 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\mathcal{B}}_3 \epsilon^{\parallel}_{a'bb'} \mathcal{T} \overset{0}{\mathcal{T}}^{\parallel} \overset{2}{\mathcal{T}}^{\parallel}{}^{bb'}_a - 8 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\mathcal{B}}_3 \epsilon^{\parallel}_{abb'} \mathcal{T} \overset{0}{\mathcal{T}}^{\parallel} \overset{2}{\mathcal{T}}^{\parallel}{}^{bb'}_{a'} \Big) + \\
& 3 \eta_{aa'}^{(1)} \mathcal{S}^{aa'} \left( \overset{1}{\pi}_{\mathcal{A}bb'} \overset{1}{\pi}_{\mathcal{A}}^{bb'} - 2 \overset{\hat{a}}{\mathcal{A}}_5 \mathcal{T} \overset{1}{\pi}_b^{bb'} \overset{1}{\mathcal{T}}^{\parallel}_{bb'} + 4 \overset{\hat{a}}{\mathcal{A}}_5 \eta^{b'c} \right. \\
& \left. \mathcal{T} n^b \left( \overset{3}{\mathcal{D}}^{\parallel}_c \overset{1}{\pi}_{bb'} \right) \right) \Big) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{0}{\mathcal{P}}_{\mathcal{A}} \cdot \overset{(1)}{\mathcal{S}} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0}{\mathcal{H}}_b \cdot \overset{(2)}{\mathcal{S}} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{8 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\mathcal{B}}_3 \mathcal{T}^2} \overset{(1)}{\mathcal{S}} \overset{(2)}{\mathcal{S}} \left( \overset{1}{\pi}_{\mathcal{A}}^{ab} \left( 3 \overset{1}{\pi}_{ba'b} + 4 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\mathcal{B}}_3 \mathcal{T} \overset{1}{\mathcal{T}}^{\parallel}_{ab} \right) + 4 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\mathcal{B}}_3 \overset{1}{\pi}_{\mathcal{A}}^a \left( 2 P_{\eta}^{(3)} \mathcal{D}^{\parallel}_a \mathcal{T} - \mathcal{T} \overset{1}{\mathcal{T}}^{\parallel}_a + \right. \right. \\
& \left. \left. 2 \mathcal{T} \left( \overset{3}{\mathcal{D}}^{\parallel}_b \eta_a^b \right) \right) - 8 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\mathcal{B}}_3 \eta_{ab} \mathcal{T} \left( \overset{3}{\mathcal{D}}^{\parallel}_b \overset{1}{\pi}_{\mathcal{A}}^a \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left( \overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{0}{\cdot} \mathcal{H}_b \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{24 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T}^2} \overset{(1)}{S} \left( 24 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \epsilon^{\parallel}_{aa'b} \left( \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'b} \left( 4 \mathcal{T} P[\overset{3}{\mathcal{D}}^{\parallel a} \overset{(2)}{S}] - 4 P[\overset{3}{\mathcal{D}}^{\parallel a} \mathcal{T}] \overset{(2)}{S} + 3 \mathcal{T} \overset{(2)}{S} \overset{1}{\cdot} \mathcal{T}^{\parallel a} \right) + \right. \right. \\
& \quad \mathcal{T} \overset{(2)}{S} \left( 2 P[\overset{3}{\mathcal{D}}^{\parallel b} \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'}] + \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \mathcal{T}^{\parallel a'b} \right) \Big) + \overset{(2)}{S} \left( 64 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \epsilon^{\parallel}_{a'bb'} \mathcal{T} \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{2}{\cdot} \mathcal{T}^{\parallel a}{}^{bb'} - \right. \\
& \quad 3 \left( \epsilon_{aa'bb'} \left( - \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \left( 3 \overset{1}{\cdot} \pi_b^{bb'} + 4 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \mathcal{T}^{\parallel bb'} \right) n^{a'} + 16 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \overset{1}{\cdot} \pi_{\mathcal{A}}^{bb'} \right. \\
& \quad \left. \left. \left( P[\overset{3}{\mathcal{D}}^{\parallel a'} \mathcal{T}] n^a + \mathcal{T} \overset{1}{\cdot} \mathcal{T}^{\parallel a} n^{a'} \right) \right) + 8 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \left( \left( \mathcal{M}_{\text{Pl}}^2 \hat{\alpha}_0 + 8 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \right) \mathcal{T} \overset{0}{\cdot} \mathcal{T}^{\parallel} - \right. \right. \\
& \quad \left. \left. 2 \epsilon^{\parallel}_{aa'b'} \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \left( \overset{3}{\mathcal{D}}^{\parallel b} \eta^{\parallel bb'} \right) \right) \right) \Big) \Big) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{\mathcal{A}ij} \cdot \overset{(1)}{S}^{ij} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{0}{\cdot} \mathcal{H}_b \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{48 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T}^2} \overset{(2)}{S} \left( 12 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} P[\overset{(1)}{S}^{aa'}] \left( P[\overset{3}{\mathcal{D}}^{\parallel a} \overset{1}{\cdot} \pi_{\mathcal{A}a'}] + P[\overset{3}{\mathcal{D}}^{\parallel a'} \overset{1}{\cdot} \pi_{\mathcal{A}a}] + \right. \right. \\
& \quad \overset{1}{\cdot} \pi_{\mathcal{A}ab} P[\overset{3}{\mathcal{D}}^{\parallel a'} n^b] + \overset{1}{\cdot} \pi_{\mathcal{A}a'b} \left( P[\overset{3}{\mathcal{D}}^{\parallel a} n^b] + P[\overset{3}{\mathcal{D}}^{\parallel b} n_a] \right) + \overset{1}{\cdot} \pi_{\mathcal{A}ab} P[\overset{3}{\mathcal{D}}^{\parallel b} n_{a'}] \Big) + \\
& \quad 3 \overset{1}{\cdot} \pi_{\mathcal{A}a'}^b P[\overset{(1)}{S}^{aa'}] \left( 3 \overset{1}{\cdot} \pi_{ba'b} + 16 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \mathcal{T}^{\parallel a'b} \right) + 3 \overset{1}{\cdot} \pi_{\mathcal{A}a}^b P[\overset{(1)}{S}^{aa'}] \\
& \quad \left( 3 \overset{1}{\cdot} \pi_{ba'b} + 16 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \mathcal{T}^{\parallel a'b} \right) + 2 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \\
& \quad \left( 4 \eta_{a'b}^{\parallel} P[\overset{3}{\mathcal{D}}^{\parallel a} \mathcal{T}] \overset{(1)}{S}^{a'b} - 2 \eta_{a'b}^{\parallel} \mathcal{T} \overset{(1)}{S}^{a'b} \overset{1}{\cdot} \mathcal{T}^{\parallel a} + 3 \mathcal{T} P[\overset{(1)}{S}_{aa'}] \overset{1}{\cdot} \mathcal{T}^{\parallel a'} + 3 \mathcal{T} P[\overset{(1)}{S}_{a'a}] \overset{1}{\cdot} \mathcal{T}^{\parallel a'} + \right. \\
& \quad 8 \mathcal{T} P[\overset{(1)}{S}^{a'b}] \overset{2}{\cdot} \mathcal{T}^{\parallel aa'b} + 8 \mathcal{T} P[\overset{(1)}{S}^{a'b}] \overset{2}{\cdot} \mathcal{T}^{\parallel a'bb} - 6 P[\overset{(1)}{S}_{aa'}] \left( P[\overset{3}{\mathcal{D}}^{\parallel a'} \mathcal{T}] + \mathcal{T} \left( \overset{3}{\mathcal{D}}^{\parallel b} \eta_{a'}^{\parallel b} \right) \right) - \\
& \quad 6 P[\overset{(1)}{S}^{a'a}] \left( P[\overset{3}{\mathcal{D}}^{\parallel a'} \mathcal{T}] + \mathcal{T} \left( \overset{3}{\mathcal{D}}^{\parallel b} \eta_{a'}^{\parallel b} \right) \right) + 4 \eta_{a'b}^{\parallel} \mathcal{T} \overset{(1)}{S}^{a'b} \left( \overset{3}{\mathcal{D}}^{\parallel b'} \eta_a^{\parallel b'} \right) \Big) - \\
& \quad \left. 2 \eta_{aa'}^{\parallel} \overset{(1)}{S}^{aa'} \left( \overset{1}{\cdot} \pi_{\mathcal{A}}^{bb'} \left( 3 \overset{1}{\cdot} \pi_{bb'} + 16 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \mathcal{T}^{\parallel bb'} \right) + 4 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \eta_{bb'}^{\parallel} \mathcal{T} \left( \overset{3}{\mathcal{D}}^{\parallel b'} \overset{1}{\cdot} \pi_{\mathcal{A}}^{bb'} \right) \right) \right) \Big) \\
& \quad [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{\mathcal{A}ijk} \cdot \overset{(1)}{S}^{ijk} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left( \overset{0}{\cdot} \mathcal{H}_b \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \left\{ - \frac{1}{384 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T}^2} \left( 192 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \pi_{\mathcal{A}ab} P[\overset{(1)}{S}^{aa'b}] P[\overset{3}{\mathcal{D}}^{\parallel a'} \overset{(2)}{S}] + 384 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \pi_{\mathcal{A}aa'} P[\overset{(1)}{S}^{aa'b}] \right. \right. \\
& \quad P[\overset{3}{\mathcal{D}}^{\parallel b} \overset{(2)}{S}] - 18 \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \pi_{ba'b} P[\overset{(1)}{S}_a^{a'b}] \overset{(2)}{S} + 27 \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \pi_{ba'}^{aa'} P[\eta_{bb'}^{\parallel} \overset{(1)}{S}_{a'}^{bb'}] \overset{(2)}{S} + \\
& \quad 18 \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \pi_{ba'b} P[\overset{(1)}{S}_a^{a'b}] \overset{(2)}{S} + 36 \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \pi_{ba'b} P[\overset{(1)}{S}_a^{a'b}] \overset{(2)}{S} - \\
& \quad 27 \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \pi_{ba'}^{aa'} P[\eta_{bb'}^{\parallel} \overset{(1)}{S}_a^{bb'}] \overset{(2)}{S} - 96 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} P[\overset{(1)}{S}^{aa'b}] P[\overset{3}{\mathcal{D}}^{\parallel a} \overset{1}{\cdot} \pi_{\mathcal{A}a'b}] \overset{(2)}{S} + \\
& \quad \left. 36 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \delta_{a'}^b \mathcal{T} \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{\parallel} \overset{(1)}{S}_b^{b'c}] P[\overset{3}{\mathcal{D}}^{\parallel a} n^{a'}] \overset{(2)}{S} - 36 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \delta_{a'}^b \mathcal{T} \right. \\
& \quad \left. \left. \left. \right. \right. \right\}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[\eta_{b'c}^{(1)} S_{b'c}^{(1)}] P[\mathcal{D}_a^b n^{a'}]^{(2)} S - 96 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \frac{1}{\eta} \hat{\pi}_{\mathcal{A}ab} P[(^{(1)}S^{aa'b})] P[\mathcal{D}_a^b \mathcal{T}]^{(2)} S + \\
& 96 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} P[(^{(1)}S^{aa'b})] P[\mathcal{D}_a^b \frac{1}{\eta} \hat{\pi}_{\mathcal{A}ab}]^{(2)} S + 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[(^{(1)}S_a^{a'b})] P[\mathcal{D}_a^b n_b]^{(2)} S - \\
& 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[(^{(1)}S_a^{a'b})] P[\mathcal{D}_a^b n_b]^{(2)} S + 36 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[\eta_{bb'}^{(1)} S_a^{bb'}] \\
& P[\mathcal{D}_a^b n_a]^{(2)} S - 36 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[\eta_{bb'}^{(1)} S_a^{bb'}] P[\mathcal{D}_a^b n_a]^{(2)} S - \\
& 192 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \frac{1}{\eta} \hat{\pi}_{\mathcal{A}aa'} P[(^{(1)}S^{aa'b})] P[\mathcal{D}_a^b \mathcal{T}]^{(2)} S + 192 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} P[(^{(1)}S^{aa'b})] P[\mathcal{D}_a^b \frac{1}{\eta} \hat{\pi}_{\mathcal{A}aa'}]^{(2)} S + \\
& 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[(^{(1)}S_a^{a'b})] P[\mathcal{D}_a^b n_a]^{(2)} S - 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[(^{(1)}S_a^{a'b})] P[\mathcal{D}_a^b n_a]^{(2)} S - \\
& 144 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} P[\eta_{a'b}^{(1)} S^{aa'b}] P[\eta^{b'c} (\mathcal{D}_c^b \frac{1}{\eta} \hat{\pi}_{\mathcal{A}ab})]^{(2)} S + 144 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} P[\eta_{ab}^{(1)} S^{aa'b}] \\
& P[\eta^{b'c} (\mathcal{D}_c^b \frac{1}{\eta} \hat{\pi}_{\mathcal{A}ab})]^{(2)} S - 16 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \epsilon_{a'bb'}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{b'} P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel} + \\
& 16 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \epsilon_{abb'}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{b'} P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel} + 32 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \epsilon_{aa'b'}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}b}^{b'} \\
& P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel} - 24 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \epsilon_{aa'b}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_b^{b'c}]^{(2)} S \mathcal{T}^{\parallel} + \\
& 24 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \epsilon_{aa'b}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_b^{b'c}]^{(2)} S \mathcal{T}^{\parallel} + 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a \\
& \left( P[\eta_{bb'}^{(1)} S_a^{bb'}] - P[\eta_{bb'}^{(1)} S_a^{b'b}] \right) \left( 4 \mathcal{T} P[\mathcal{D}_a^b \mathcal{T}]^{(2)} S - 2 P[\mathcal{D}_a^b \mathcal{T}]^{(2)} S + \mathcal{T}^{(2)} S \frac{1}{\eta} \mathcal{T}^{\parallel} \right) + \\
& 256 \mathcal{M}_{\text{PL}}^2 \hat{\alpha}_0 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T}^2 P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{aa'b} + 128 \mathcal{M}_{\text{PL}}^2 \hat{\alpha}_0 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T}^2 \\
& P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{abb'a} - 192 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_b^{b'c}]^{(2)} S \mathcal{T}^{\parallel}{}_{a'a} + \\
& 192 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_b^{b'c}]^{(2)} S \mathcal{T}^{\parallel}{}_{a'a} + 128 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}b}^{b'} \\
& P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{abb'a} + 256 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{b'} P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{abb'b} - \\
& 128 \mathcal{M}_{\text{PL}}^2 \hat{\alpha}_0 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T}^2 P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{a'b'a} - 128 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}b}^{b'} \\
& P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{a'b'a} - 256 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{b'} P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{a'b'b} + \\
& 128 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{b'} P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{bb'a} - 128 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{b'} \\
& P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{bb'a} + 144 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_a^{b'c}]^{(2)} S (\mathcal{D}_a^b \eta_a^b) - \\
& 144 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_a^{b'c}]^{(2)} S (\mathcal{D}_a^b \eta_a^b) - 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \eta_{a'b} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a \\
& P[\eta_{b'c}^{(1)} S_a^{b'c}]^{(2)} S (\mathcal{D}_a^b n^{a'}) + 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \eta_{a'b} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[\eta_{b'c}^{(1)} S_a^{b'c}]^{(2)} S (\mathcal{D}_a^b n^{a'}) - \\
& 48 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a'b} \left( \mathcal{T} \left( P[(^{(1)}S_a^{a'b})] - P[(^{(1)}S_a^{a'b})] - 2 P[(^{(1)}S_a^{a'b})] \right) \right)^{(2)} S \frac{1}{\eta} \mathcal{T}^{\parallel} +
\end{aligned}$$

$$\begin{aligned}
& 2 P_{\eta^1}^{[(1)S^{aa'b}]} \left( 2 \mathcal{T} P_{\eta^1}^{[3\mathcal{D}_a^{(2)S}] - P_{\eta^1}^{[3\mathcal{D}_a^{\parallel} \mathcal{T}]}^{(2)S} + \mathcal{T}^{(2)S} \left( 3\mathcal{D}_b^{\parallel}, \eta_a^{\parallel}, b' \right) \right) + \\
& 96 \mathcal{M}_{Pl}^2 \hat{\beta}_3^{\cdot} \mathcal{T}^{1^{\cdot} \hat{\pi}_{aa'b}} P_{\eta^1}^{[(1)S^{aa'b}]}^{(2)S} \left( 3\mathcal{D}_b^{\parallel}, \eta_a^{\parallel}, b' \right) + 192 \mathcal{M}_{Pl}^2 \hat{\beta}_3^{\cdot} \mathcal{T}^{1^{\cdot} \hat{\pi}_{aa'b}}, \\
& P_{\eta^1}^{[(1)S^{aa'b}]}^{(2)S} \left( 3\mathcal{D}_b^{\parallel}, \eta_b^{\parallel}, b' \right) \left[ x^0, x^1, x^2, x^3 \right] dx^3 dx^2 dx^1
\end{aligned}$$

This concludes the HiGGS output.

**Concrete relation to manuscript:** The above constraints and Poisson brackets constitute the data represented by coloured squares in the left-hand panel of Fig. 2 in our Manuscript. The primary constraints  $\phi$  are listed along the top row. The primary Poisson brackets  $\{\phi, \phi\}$  are used to infer the white, red or yellow squares in the matrix depending on whether they are vanishing, or appear only in proportion to phase space variables, or are order unity (e.g. proportional to the metric or unit-timelike vector). The  $\{\phi, {}^{0^*}\mathcal{H}_b\}$  Poisson brackets define the secondary constraints " $\chi$ ", which appear on the sides of the matrix. Most of the Poisson brackets  $\{\chi, \phi\}$  need not be computed (grey squares), but it can be noticed without detailed calculation that the remaining required brackets (yellow squares) do indeed arise, and that these determine all the missing velocities and so terminate the Dirac algorithm.

## Healthy spectrum with multipliers

### Introductory comments

We now turn to the case in which multipliers have been applied. This time we set  $\hat{\alpha}_1^{\cdot}, \hat{\alpha}_2^{\cdot}, \hat{\alpha}_3^{\cdot}, \hat{\alpha}_4^{\cdot}, \hat{\alpha}_6^{\cdot}, \mathcal{M}_{Pl}^2 \hat{\beta}_1^{\cdot}, \mathcal{M}_{Pl}^2 \hat{\beta}_2^{\cdot}, \mathcal{M}_{Pl}^2 \bar{\beta}_3^{\cdot}$ , to zero, and allow for nonvanishing  $\mathcal{M}_{Pl}^2 \hat{\alpha}_0^{\cdot}, \hat{\alpha}_5^{\cdot}$  and  $\mathcal{M}_{Pl}^2 \bar{\beta}_3^{\cdot}$ , but we also admit  $\mathcal{M}_{Pl}^2 \bar{\beta}_1^{\cdot}$  and  $\mathcal{M}_{Pl}^2 \bar{\beta}_2^{\cdot}$ . These final two couplings switch on the multipliers in the Lagrangian.

Note this time that because some of the Lagrange multiplier fields are present, certain secondary constraints can be anticipated by HiGGS a priori: this is in accordance with the theory set out in arXiv:2205.13534.

### Lists of constraints and Poisson brackets from HiGGS

We will again present the results using the ViewTheory[] command.

```

** DefTheory: Incorporating the binary at ConstraintAlgebraAlp0Alp5Bet3cBet1cBet2.thr.mx
** DefTheory: Found the following primary if-constraints:

```

$${}^{0^*}\phi_b \equiv \frac{{}^{0^*}\pi_b}{\mathcal{T}} + 2 \mathcal{M}_{Pl}^2 \bar{\beta}_2^{\cdot} {}^{0^*}\lambda_{\mathcal{T}}^{\perp} \approx 0$$

$$\overset{1}{\cdot}\phi_{bi} \equiv \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_{bi}}{\mathcal{I}} - \frac{2}{3} \left( \mathcal{M}_{Pl}^2 \bar{\beta}_1 - \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\lambda_{\mathcal{T}i}^{\parallel} + \frac{2}{3} \left( 2 \mathcal{M}_{Pl}^2 \bar{\beta}_1 + \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\lambda_{\mathcal{T}i}^{\perp} \approx 0$$

$$\overset{2}{\cdot}\phi_{bij} \equiv \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{bij}}{\mathcal{I}} + 2 \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{2}{\cdot}\lambda_{\mathcal{T}ij}^{\perp} \approx 0$$

$$\overset{0}{\cdot}\phi_{\mathcal{A}} \equiv 3 \mathcal{M}_{Pl}^2 \overset{\wedge}{\alpha}_0 + \frac{\overset{0}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}}{\mathcal{I}} \approx 0$$

$$\overset{0}{\cdot}\phi_{\mathcal{A}} \equiv \frac{\overset{0}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}}{\mathcal{I}} \approx 0$$

$$\overset{2}{\cdot}\phi_{\mathcal{A}ij} \equiv \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ij}}{\mathcal{I}} \approx 0$$

$$\overset{2}{\cdot}\phi_{\mathcal{A}ijh} \equiv \frac{2 \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ijh}}{3\mathcal{I}} + \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ihj}}{3\mathcal{I}} - \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}jhi}}{3\mathcal{I}} \approx 0$$

\*\* DefTheory: Found the following primary parallel

if-constraints associated with geometric multiplier momenta (if the sector is singular, the parallel constraint is taken to be the singular constraint and the identical perpendicular constraint below is dropped):

$$\overset{1}{\cdot}\phi_{bij}^{\parallel} \equiv -\frac{2}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{1}{\cdot}\omega_{\mathcal{T}ij}^{\parallel} + \frac{2}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{1}{\cdot}\omega_{\mathcal{T}ij}^{\perp} \approx 0$$

$$\overset{1}{\cdot}\phi_{bi}^{\parallel} \equiv \frac{1}{3} \left( -\mathcal{M}_{Pl}^2 \bar{\beta}_1 - 2 \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\omega_{\mathcal{T}i}^{\parallel} + \frac{2}{3} \left( \mathcal{M}_{Pl}^2 \bar{\beta}_1 - \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\omega_{\mathcal{T}i}^{\perp} \approx 0$$

$$\overset{2}{\cdot}\phi_{bijh}^{\parallel} \equiv -\frac{2}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{2}{\cdot}\omega_{\mathcal{T}ijh}^{\parallel} - \frac{1}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{2}{\cdot}\omega_{\mathcal{T}ihj}^{\parallel} + \frac{1}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{2}{\cdot}\omega_{\mathcal{T}jhi}^{\parallel} \approx 0$$

\*\* DefTheory: Found the following primary perpendicular

if-constraints associated with geometric multiplier momenta:

$$\overset{0}{\cdot}\varphi_b^{\perp} \equiv 2 \mathcal{M}_{Pl}^2 \bar{\beta}_2 \overset{0}{\cdot}\omega_{\mathcal{T}}^{\perp} \approx 0$$

$$\overset{1}{\cdot}\varphi_{bi}^{\perp} \equiv -\frac{2}{3} \left( \mathcal{M}_{Pl}^2 \bar{\beta}_1 - \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\omega_{\mathcal{T}i}^{\parallel} + \frac{2}{3} \left( 2 \mathcal{M}_{Pl}^2 \bar{\beta}_1 + \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\omega_{\mathcal{T}i}^{\perp} \approx 0$$

$$\overset{2}{\cdot}\varphi_{bij}^{\perp} \equiv 2 \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{2}{\cdot}\omega_{\mathcal{T}ij}^{\perp} \approx 0$$

\*\* DefTheory: Found the following secondary perpendicular if-constraints:

\*\* DefTheory: Found the following secondary parallel if-constraints:

$$\overset{1}{\cdot}\chi_{bi}^{\parallel} \equiv \overset{1}{\cdot}\mathcal{T}_i^{\parallel} \approx 0$$

$$\overset{2}{\cdot}\chi_{bijh}^{\parallel} \equiv \overset{2}{\cdot}\mathcal{T}_{ijh}^{\parallel} \approx 0$$

\*\* DefTheory: Found the following secondary singular if-constraints:

$$\mathbf{1}^{\cdot} \chi_{b \mid j}^{\cdot} \equiv \frac{\mathbf{1}^{\cdot} \hat{\pi}_{b \mid j}}{\mathcal{T}} - \frac{2}{3} \mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathbf{1}^{\cdot} \lambda_{\mathcal{T} \mid j}^{\parallel} + \frac{2}{3} \mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathbf{1}^{\cdot} \lambda_{\mathcal{T} \mid j}^{\perp} + 4 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_{\mid j} \approx 0$$

\*\* DefTheory: Defining association key \$IfConstraints

for the theory association ConstraintAlgebraAlp0Alp5Bet3cBet1cBet2

\*\* ViewTheory: The super-Hamiltonian is:

$$\begin{aligned} \mathbf{0}^{\cdot} \mathcal{H}_b \equiv & \frac{\mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A} a} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^a}{16 \hat{\alpha}_5 \mathcal{T}} + \frac{\mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A} a b} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^{ab}}{8 \hat{\alpha}_5 \mathcal{T}} + \frac{3 \mathbf{1}^{\cdot} \hat{\pi}_{b a b} \mathbf{1}^{\cdot} \hat{\pi}_b^{ab}}{16 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \mathcal{T}} + \frac{\mathbf{1}^{\cdot} \hat{\pi}_b^a \mathcal{P}_{\eta}^{\left[ \mathbf{3} \mathcal{D}^{\parallel}{}_a \mathcal{T} \right]}}{\mathcal{T}} + \frac{\mathcal{M}_{\text{Pl}}^2 \hat{\alpha}_0 \mathcal{T} \mathbf{0}^{\cdot} \mathcal{R}^{\parallel}}{2} - \\ & \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^a \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_a + \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^{ab} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_{ab} - \frac{\mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathbf{1}^{\cdot} \hat{\pi}_b^{ab} \mathbf{1}^{\cdot} \lambda_{\mathcal{T} ab}^{\parallel}}{4 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3} + \frac{\mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1^2 \mathcal{T} \mathbf{1}^{\cdot} \lambda_{\mathcal{T} ab}^{\parallel} \mathbf{1}^{\cdot} \lambda_{\mathcal{T}}^{ab}}{12 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3} + \\ & \frac{\mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathbf{1}^{\cdot} \hat{\pi}_b^{ab} \mathbf{1}^{\cdot} \lambda_{\mathcal{T} ab}^{\perp}}{4 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3} - \frac{\mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1^2 \mathcal{T} \mathbf{1}^{\cdot} \lambda_{\mathcal{T}}^{ab} \mathbf{1}^{\cdot} \lambda_{\mathcal{T} ab}^{\perp}}{6 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3} + \frac{\mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1^2 \mathcal{T} \mathbf{1}^{\cdot} \lambda_{\mathcal{T} ab}^{\perp} \mathbf{1}^{\cdot} \lambda_{\mathcal{T}}^{ab}}{12 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3} + \frac{1}{6} \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \mathcal{T} \mathbf{0}^{\cdot} \mathcal{T}^{\parallel 2} - \\ & \mathbf{1}^{\cdot} \hat{\pi}_b^a \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_a + \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_b^{ab} \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_{ab} - \mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathcal{T} \mathbf{1}^{\cdot} \lambda_{\mathcal{T}}^{ab} \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_{ab} + \mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathcal{T} \mathbf{1}^{\cdot} \lambda_{\mathcal{T}}^{ab} \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_{ab} - \\ & n^a \left( \mathbf{3} \mathcal{D}^{\parallel}{}_b \mathbf{1}^{\cdot} \hat{\pi}_{b a}^{\perp} \right) - n^a \left( \mathbf{3} \mathcal{D}^{\parallel}{}_b \mathbf{2}^{\cdot} \hat{\pi}_{b a}^{\perp} \right) - \eta^{\parallel}{}_{ab} \left( \mathbf{3} \mathcal{D}^{\parallel}{}_b \mathbf{1}^{\cdot} \hat{\pi}_b^a \right) + \frac{1}{3} \eta^{\parallel}{}_{ab} \mathbf{0}^{\cdot} \hat{\pi}_b \left( \mathbf{3} \mathcal{D}^{\parallel}{}_b n^a \right) \approx 0 \end{aligned}$$

\*\* ViewTheory: The linear super-momentum is:

$$\begin{aligned} \mathbf{1}^{\cdot} \mathcal{H}_{b \mid} \equiv & -\frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_b^a \mathcal{P}_{\eta}^{\left[ \mathbf{3} \mathcal{D}^{\parallel}{}_a n_{\mid} \right]} + \frac{\mathbf{1}^{\cdot} \hat{\pi}_{b \mid a} \mathcal{P}_{\eta}^{\left[ \mathbf{3} \mathcal{D}^{\parallel}{}_a \mathcal{T} \right]}}{\mathcal{T}} + \frac{\mathbf{2}^{\cdot} \hat{\pi}_{b \mid a} \mathcal{P}_{\eta}^{\left[ \mathbf{3} \mathcal{D}^{\parallel}{}_a \mathcal{T} \right]}}{\mathcal{T}} - \mathcal{P}_{\eta}^{\left[ \eta^{\parallel}{}^{aa'} \left( \mathbf{3} \mathcal{D}^{\parallel}{}_a \mathbf{1}^{\cdot} \hat{\pi}_{b \mid a} \right) \right]} - \\ & \mathcal{P}_{\eta}^{\left[ \eta^{\parallel}{}^{aa'} \left( \mathbf{3} \mathcal{D}^{\parallel}{}_a \mathbf{2}^{\cdot} \hat{\pi}_{b \mid a} \right) \right]} + \frac{\mathbf{0}^{\cdot} \hat{\pi}_b \mathcal{P}_{\eta}^{\left[ \mathbf{3} \mathcal{D}^{\parallel}{}_b \mathcal{T} \right]}}{3 \mathcal{T}} - \frac{1}{3} \mathcal{P}_{\eta}^{\left[ \mathbf{3} \mathcal{D}^{\parallel}{}_b \mathbf{0}^{\cdot} \hat{\pi}_b \right]} - \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_b^a \mathcal{P}_{\eta}^{\left[ \mathbf{3} \mathcal{D}^{\parallel}{}_a n_{\mid} \right]} + \frac{1}{6} \epsilon^{\parallel}{}_{l a a'} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^{aa'} \mathbf{0}^{\cdot} \mathcal{R}^{\parallel} - \\ & \frac{\mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A} \mid} \mathbf{0}^{\cdot} \mathcal{R}^{\parallel}}{6} - \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A} \mid a} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_a - \frac{1}{2} \mathbf{2}^{\cdot} \hat{\pi}_{\mathcal{A} \mid a} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_a + \frac{\mathbf{0}^{\cdot} \hat{\pi}_{\mathcal{A}} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_{\mid}}{3} - \frac{4}{3} \mathbf{2}^{\cdot} \hat{\pi}_{\mathcal{A} \mid}^{aa'} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_{aa'} + \\ & \frac{1}{6} \epsilon^{\parallel}{}_{l a a'} \mathbf{0}^{\cdot} \hat{\pi}_{\mathcal{A}} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}^{aa'} - \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^a \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_{\mid a} - \frac{4}{3} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^{aa'} \mathbf{2}^{\cdot} \mathcal{R}^{\parallel}{}_{\mid aa'} + \frac{4}{3} \mathbf{2}^{\cdot} \hat{\pi}_{\mathcal{A}}^{aa'} \mathbf{2}^{\cdot} \mathcal{R}^{\parallel}{}_{\mid aa'} + \\ & \frac{4}{3} \mathbf{2}^{\cdot} \hat{\pi}_{\mathcal{A} \mid}^{aa'} \mathbf{2}^{\cdot} \mathcal{R}^{\parallel}{}_{aa'} - \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^a \mathbf{2}^{\cdot} \mathcal{R}^{\parallel}{}_{\mid a} + \frac{1}{6} \epsilon^{\parallel}{}_{l a a'} \mathbf{1}^{\cdot} \hat{\pi}_b^{aa'} \mathbf{0}^{\cdot} \mathcal{T}^{\parallel} - \frac{3}{2} \mathbf{1}^{\cdot} \hat{\pi}_{b \mid a} \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_a - \\ & \frac{3}{2} \mathbf{2}^{\cdot} \hat{\pi}_{b \mid a} \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_a + \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_b^a \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_{\mid a} - \frac{4}{3} \mathbf{1}^{\cdot} \hat{\pi}_b^{aa'} \mathbf{2}^{\cdot} \mathcal{T}^{\parallel}{}_{\mid aa'} + \frac{4}{3} \mathbf{2}^{\cdot} \hat{\pi}_b^{aa'} \mathbf{2}^{\cdot} \mathcal{T}^{\parallel}{}_{\mid aa'} \approx 0 \end{aligned}$$

\*\* ViewTheory: The 1+ part of the angular super-momentum is:

$$\begin{aligned}
{}^{1\cdot}\mathcal{H}_{\mathcal{A}nm} \equiv & -2 \frac{{}^{1\cdot}\hat{\pi}_{bmn}}{\mathcal{T}} - \frac{\epsilon^{lmn} {}^{0\cdot}\hat{\pi}_{\mathcal{A}} P[{}^3\mathcal{D}^{l0}\mathcal{T}]}{6\mathcal{T}} + \frac{4 {}^{2\cdot}\hat{\pi}_{\mathcal{A}mna} P[{}^3\mathcal{D}^{l0}\mathcal{T}]}{3\mathcal{T}} + \frac{1}{6} \epsilon^{lmn} P[{}^3\mathcal{D}^{l0} {}^{0\cdot}\hat{\pi}_{\mathcal{A}}] - \\
& \frac{1}{2} \frac{{}^{1\cdot}\hat{\pi}_{\mathcal{A}na}}{\mathcal{T}} P[{}^3\mathcal{D}^{l0} n_m] - \frac{1}{2} \frac{{}^{2\cdot}\hat{\pi}_{\mathcal{A}na}}{\mathcal{T}} P[{}^3\mathcal{D}^{l0} n_m] + \frac{1}{2} \frac{{}^{1\cdot}\hat{\pi}_{\mathcal{A}ma}}{\mathcal{T}} P[{}^3\mathcal{D}^{l0} n_n] + \frac{1}{2} \frac{{}^{2\cdot}\hat{\pi}_{\mathcal{A}ma}}{\mathcal{T}} P[{}^3\mathcal{D}^{l0} n_n] - \\
& \frac{4}{3} P[\eta^{aa'} ({}^3\mathcal{D}^{l0}, {}^{2\cdot}\hat{\pi}_{\mathcal{A}mna})] + \frac{{}^{1\cdot}\hat{\pi}_{\mathcal{A}n} P[{}^3\mathcal{D}^{lm}\mathcal{T}]}{2\mathcal{T}} - \frac{1}{2} P[{}^3\mathcal{D}^{lm} {}^{1\cdot}\hat{\pi}_{\mathcal{A}n}] - \frac{1}{2} \frac{{}^{1\cdot}\hat{\pi}_{\mathcal{A}na}}{\mathcal{T}} P[{}^3\mathcal{D}^{lm} n^a] - \\
& \frac{1}{2} \frac{{}^{2\cdot}\hat{\pi}_{\mathcal{A}na}}{\mathcal{T}} P[{}^3\mathcal{D}^{lm} n^a] - \frac{{}^{1\cdot}\hat{\pi}_{\mathcal{A}m} P[{}^3\mathcal{D}^{ln}\mathcal{T}]}{2\mathcal{T}} + \frac{1}{2} P[{}^3\mathcal{D}^{ln} {}^{1\cdot}\hat{\pi}_{\mathcal{A}m}] + \frac{1}{2} \frac{{}^{1\cdot}\hat{\pi}_{\mathcal{A}ma}}{\mathcal{T}} P[{}^3\mathcal{D}^{ln} n^a] + \\
& \frac{1}{2} \frac{{}^{2\cdot}\hat{\pi}_{\mathcal{A}ma}}{\mathcal{T}} P[{}^3\mathcal{D}^{ln} n^a] + \frac{1}{6} \epsilon^{lmn} {}^{0\cdot}\hat{\pi}_{\mathcal{A}} {}^{1\cdot}\mathcal{T}^{lm} - \frac{4}{3} \frac{{}^{2\cdot}\hat{\pi}_{\mathcal{A}mna}}{\mathcal{T}} {}^{1\cdot}\mathcal{T}^{lm} - \frac{1}{2} \frac{{}^{1\cdot}\hat{\pi}_{\mathcal{A}n}}{\mathcal{T}} {}^{1\cdot}\mathcal{T}^{lm} + \frac{1}{2} \frac{{}^{1\cdot}\hat{\pi}_{\mathcal{A}m}}{\mathcal{T}} {}^{1\cdot}\mathcal{T}^{ln} - \\
& \frac{1}{2} \frac{{}^{1\cdot}\hat{\pi}_{\mathcal{A}n}^a}{{}^{1\cdot}\mathcal{T}^{ma}} - \frac{1}{2} \frac{{}^{2\cdot}\hat{\pi}_{\mathcal{A}n}^a}{{}^{1\cdot}\mathcal{T}^{ma}} - \frac{1}{3} \frac{{}^{0\cdot}\hat{\pi}_{\mathcal{A}} {}^{1\cdot}\mathcal{T}^{lm}}{\mathcal{T}} + \frac{1}{2} \frac{{}^{1\cdot}\hat{\pi}_{\mathcal{A}m}^a}{{}^{1\cdot}\mathcal{T}^{na}} + \frac{1}{2} \frac{{}^{2\cdot}\hat{\pi}_{\mathcal{A}m}^a}{{}^{1\cdot}\mathcal{T}^{na}} \approx 0
\end{aligned}$$

\*\* ViewTheory: The on-shell brackets between the various if-constraints are:

$$\begin{aligned}
& \left\{ \iiint [{}^{0\cdot}\phi_b \cdot ({}^1\mathcal{S})][x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [{}^{0\cdot}\phi_b \cdot ({}^2\mathcal{S})][x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint [{}^{0\cdot}\phi_b \cdot ({}^1\mathcal{S})][x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [{}^{1\cdot}\phi_{b|} \cdot ({}^2\mathcal{S}^l)][x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left[ \frac{2 \mathcal{M}_{Pl}^2 \bar{\beta}_1 P[{}^{(2)}\mathcal{S}_a]}{\mathcal{T}} ({}^{1\cdot}\lambda_{\mathcal{T}}^{la} - 2 {}^{1\cdot}\lambda_{\mathcal{T}}^{la}) \right] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint [{}^{0\cdot}\phi_b \cdot ({}^1\mathcal{S})][x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [{}^{2\cdot}\phi_{b|} \cdot ({}^2\mathcal{S}^{lm})][x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left[ - \frac{6 \mathcal{M}_{Pl}^2 \bar{\beta}_1 P[{}^{(2)}\mathcal{S}^{aa'}]}{\mathcal{T}} ({}^{1\cdot}\lambda_{\mathcal{T}}^{aa'}) \right] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint [{}^{0\cdot}\phi_b \cdot ({}^1\mathcal{S})][x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [{}^{0\cdot}\phi_{\mathcal{A}} \cdot ({}^2\mathcal{S})][x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left[ - \frac{6 \mathcal{M}_{Pl}^2 \hat{\alpha}_0 ({}^{(1)}\mathcal{S}^{(2)}\mathcal{S})}{\mathcal{T}} \right] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint [{}^{0\cdot}\phi_b \cdot ({}^1\mathcal{S})][x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [{}^{0\cdot}\phi_{\mathcal{A}} \cdot ({}^2\mathcal{S})][x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint [{}^{0\cdot}\phi_b \cdot ({}^1\mathcal{S})][x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [{}^{2\cdot}\phi_{b|} \cdot ({}^2\mathcal{S}^{lm})][x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint [{}^{0\cdot}\phi_b \cdot ({}^1\mathcal{S})][x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [{}^{2\cdot}\phi_{\mathcal{A}|} \cdot ({}^2\mathcal{S}^{lmn})][x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint [{}^{0\cdot}\phi_b \cdot ({}^1\mathcal{S})][x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [{}^{1\cdot}\phi_{b|} \cdot ({}^2\mathcal{S}^{lm})][x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint [{}^{0\cdot}\phi_b \cdot ({}^1\mathcal{S})][x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [{}^{1\cdot}\phi_{b|} \cdot ({}^2\mathcal{S}^l)][x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0
\end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left( \overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2-}{\cdot} \phi_b^{\parallel}{}_{lmn} \cdot \overset{(2)}{S}{}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0+}{\cdot} \phi_b^{\perp} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left( 12 \mathcal{M}_{Pl}{}^2 \bar{\beta}_2^{\perp} \overset{(1)}{S} \overset{(2)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{1-}{\cdot} \phi_b^{\perp}{}_{\parallel} \cdot \overset{(2)}{S}{}^{\parallel} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2-}{\cdot} \phi_b^{\perp}{}_{lm} \cdot \overset{(2)}{S}{}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{1-}{\cdot} \chi_b^{\parallel}{}_{\parallel} \cdot \overset{(2)}{S}{}^{\parallel} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \frac{2 \overset{(1)}{S} \left( -P_{\eta^{\parallel}}^{(2)} S^0 \right) P_{\eta^{\parallel}}^{(3)} \mathcal{D}^{\parallel}{}_{\parallel} \mathcal{T} + \eta^{\parallel}{}_{aa'} \mathcal{T} \left( \overset{3}{\mathcal{D}}{}^{\parallel}{}_{\parallel} P_{\eta^{\parallel}}^{(2)} S^0 \right) }{\mathcal{T}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2-}{\cdot} \chi_b^{\parallel}{}_{lmn} \cdot \overset{(2)}{S}{}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{1-}{\cdot} \chi_b^{\perp}{}_{lm} \cdot \overset{(2)}{S}{}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \frac{2 P_{\eta^{\parallel}}^{(2)} S^{aa'} \overset{(1)}{S} \left( \mathcal{M}_{Pl}{}^2 \bar{\beta}_1^{\perp} \overset{1-}{\lambda}_{\tau aa'}^{\parallel}, -\mathcal{M}_{Pl}{}^2 \bar{\beta}_1^{\perp} \overset{1-}{\lambda}_{\tau aa'}^{\perp}, -4 \mathcal{M}_{Pl}{}^2 \hat{\beta}_3^{\perp} \overset{1-}{\mathcal{T}}{}^{\parallel}{}_{aa'} \right)}{\mathcal{T}} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1-}{\cdot} \phi_b{}_{\parallel} \cdot \overset{(1)}{S}{}^{\parallel} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{1-}{\cdot} \phi_b{}_{\perp} \cdot \overset{(2)}{S}{}^{\perp} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left( -\frac{8 P_{\eta^{\parallel}}^{(1)} S^0 P_{\eta^{\parallel}}^{(2)} S^{a'b}}{3 \mathcal{T}} \left( \mathcal{M}_{Pl}{}^2 \bar{\beta}_1^{\perp} \overset{1-}{\lambda}_{\tau aa'}^{\parallel}, -\mathcal{M}_{Pl}{}^2 \bar{\beta}_1^{\perp} \overset{1-}{\lambda}_{\tau aa'}^{\perp}, +3 \mathcal{M}_{Pl}{}^2 \hat{\beta}_3^{\perp} \overset{1-}{\mathcal{T}}{}^{\parallel}{}_{aa'} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1-}{\cdot} \phi_b{}_{\parallel} \cdot \overset{(1)}{S}{}^{\parallel} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2-}{\cdot} \phi_b{}_{lm} \cdot \overset{(2)}{S}{}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left( \frac{1}{6 \mathcal{T}} \mathcal{M}_{Pl}{}^2 \bar{\beta}_1^{\perp} \left( -8 P_{\eta^{\parallel}}^{(1)} S^0 P_{\eta^{\parallel}}^{(2)} S^{a'b} \right) \left( \overset{2-}{\lambda}_{\tau aa'b}^{\parallel} + \overset{2-}{\lambda}_{\tau aab'a}^{\parallel} \right) + 3 P_{\eta^{\parallel}}^{(1)} S^{a'} \left( P_{\eta^{\parallel}}^{(2)} S_{aa'} \right) + P_{\eta^{\parallel}}^{(2)} S_{a'a} \right) \\
& \quad \left( \overset{1-}{\lambda}_{\tau}{}^a - 2 \overset{1-}{\lambda}_{\tau}{}^{\perp} \right) - 2 \eta^{\parallel}{}_{aa'} P_{\eta^{\parallel}}^{(1)} S_b \overset{(2)}{S}{}^{aa'} \left( \overset{1-}{\lambda}_{\tau}{}^b - 2 \overset{1-}{\lambda}_{\tau}{}^{\perp} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1-}{\cdot} \phi_b{}_{\parallel} \cdot \overset{(1)}{S}{}^{\parallel} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0+}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left( -\frac{\overset{1-}{\pi}_{\mathcal{A}}{}^a P_{\eta^{\parallel}}^{(1)} S_a \overset{(2)}{S}}{\mathcal{T}^2} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1-}{\cdot} \phi_b{}_{\parallel} \cdot \overset{(1)}{S}{}^{\parallel} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0+}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \frac{2 \epsilon_{\gamma aa' i} \gamma_{a1} \overset{1-}{\pi}_{\mathcal{A}}{}^i \gamma_{a1} P_{\eta^{\parallel}}^{(1)} S^{a'} \overset{(2)}{S} n^a}{\mathcal{T}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$



$$\begin{aligned}
& \left\{ \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\phi}_{\mathcal{A}lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{\overset{1}{\pi}_{\mathcal{A}}^a \left( 3 P_{\eta}^{(1)} [S^a] \left( P_{\eta}^{(2)} [S_{aa}] + P_{\eta}^{(2)} [S_a^a] \right) - 2 \eta_{a,b}^{\parallel} P_{\eta}^{(1)} [S_a] \overset{(2)}{S}^{a'b} \right)}{12 \mathcal{J}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\phi}_{lmn} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{8 \mathcal{J}^2 \eta^{\parallel}} P_{\eta}^{(1)} [S^a] \left( \overset{1}{\pi}_{\mathcal{A}a,b} \left( -2 P_{\eta}^{(2)} [S_a^{a'b}] + 2 P_{\eta}^{(2)} [S_a^a] \overset{(2)}{S}^b + 4 P_{\eta}^{(2)} [S_a^{a'b}] \right) + \right. \\
& \quad \left. 3 \overset{1}{\pi}_{\mathcal{A}a}^{a'} \left( P_{\eta}^{\parallel} [\eta_{bb'}^{\parallel} \overset{(2)}{S}_{a,b}^{bb'}] - P_{\eta}^{\parallel} [\eta_{bb'}^{\parallel} \overset{(2)}{S}_a^{b'b'}] \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\phi}_{lm}^{\parallel} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(2)}{S}^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \left( \frac{4}{3} \left( \mathcal{M}_{\mathbf{P}^1}^2 \bar{\beta}_1^2 - \mathcal{M}_{\mathbf{P}^1}^2 \bar{\beta}_2^2 \right) P_{\eta}^{(1)} [S^a] P_{\eta}^{(2)} [S_a] \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\phi}_{lmn}^{\parallel} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0}{\phi}_b^{\perp} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{1}{\phi}_{bi}^{\perp} \cdot \overset{(2)}{S}^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \left( \frac{4}{3} \left( 2 \mathcal{M}_{\mathbf{P}^1}^2 \bar{\beta}_1^2 + \mathcal{M}_{\mathbf{P}^1}^2 \bar{\beta}_2^2 \right) P_{\eta}^{(1)} [S^a] P_{\eta}^{(2)} [S_a] \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\phi}_{lm}^{\perp} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{1}{\chi}_{bi}^{\parallel} \cdot \overset{(2)}{S}^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{P_{\eta}^{(1)} [S^a] \left( -P_{\eta}^{(2)} [S^{a'}] \left( P_{\eta}^{(3)} [\mathcal{D}_a^{\parallel} n_a] + P_{\eta}^{(3)} [\mathcal{D}_a^{\parallel} n_a] - \overset{1}{\mathcal{T}}_{aa}^{\parallel} \right) + 2 \eta_{a,b}^{\parallel} P_{\eta}^{(2)} [S_a] \left( \overset{3}{\mathcal{D}}^b n^{a'} \right) \right)}{2 \mathcal{J}} [x^0, x^1, x^2, x^3] \\
& \quad dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\chi}_{b|mn}^{\parallel} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{16 \mathcal{J}} P_{\eta}^{(1)} [S^a] \left( 3 \delta_a^{\parallel b} \left( P_{\eta}^{\parallel} [\eta_{b'm}^{\parallel} \overset{(2)}{S}_b^{b'm}] - P_{\eta}^{\parallel} [\eta_{b'm}^{\parallel} \overset{(2)}{S}^{b'b'm}] \right) P_{\eta}^{(3)} [\mathcal{D}_a^{\parallel} n^{a'}] - 6 P_{\eta}^{(2)} [S_a^{a'b}] P_{\eta}^{(3)} [\mathcal{D}_a^{\parallel} n_b] + \right. \\
& \quad 3 P_{\eta}^{\parallel} [\eta_{bb'}^{\parallel} \overset{(2)}{S}_a^{bb'}] P_{\eta}^{(3)} [\mathcal{D}^{a'} n_a] - 3 P_{\eta}^{\parallel} [\eta_{bb'}^{\parallel} \overset{(2)}{S}_a^{b'b'}] P_{\eta}^{(3)} [\mathcal{D}^{a'} n_a] - 6 P_{\eta}^{(2)} [S_a^{a'b}] P_{\eta}^{(3)} [\mathcal{D}_b^{\parallel} n_a] - \\
& \quad 3 P_{\eta}^{\parallel} [\eta_{bb'}^{\parallel} \overset{(2)}{S}_a^{bb'}] \overset{1}{\mathcal{T}}_a^{a'} + 3 P_{\eta}^{\parallel} [\eta_{bb'}^{\parallel} \overset{(2)}{S}_a^{b'b'}] \overset{1}{\mathcal{T}}_a^{a'} - 2 P_{\eta}^{(2)} [S_a^{a'b}] \overset{1}{\mathcal{T}}_{a,b}^{\parallel} - \\
& \quad 4 P_{\eta}^{(2)} [S_a^{a'b}] \overset{1}{\mathcal{T}}_{a,b}^{\parallel} + 2 P_{\eta}^{(2)} [S_a^{a'b}] \left( 3 P_{\eta}^{(3)} [\mathcal{D}_a^{\parallel} n_b] + 3 P_{\eta}^{(3)} [\mathcal{D}_b^{\parallel} n_a] + \overset{1}{\mathcal{T}}_{a,b}^{\parallel} \right) - \\
& \quad \left. 6 \eta_{a,b}^{\parallel} P_{\eta}^{\parallel} [\eta_{b'm}^{\parallel} \overset{(2)}{S}_a^{b'm}] \left( \overset{3}{\mathcal{D}}^b n^{a'} \right) + 6 \eta_{a,b}^{\parallel} P_{\eta}^{\parallel} [\eta_{b'm}^{\parallel} \overset{(2)}{S}_a^{b'b'm}] \left( \overset{3}{\mathcal{D}}^b n^{a'} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left( \overset{1}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{1}{\cdot} \chi_{b|lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{18 \mathcal{T}^2} \left( -36 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \delta_{a,b} \mathcal{T} P_{\eta}^{(1)} S^a \left( P_{\eta}^{(2)} S_b^{b'} n_{b'} \right) - P_{\eta}^{(2)} S_b^{b'} n_{b'} \right) P_{\eta}^{(3)} \mathcal{D}_a n^a - \\
& 9 \mathcal{M}_{Pl}^2 \hat{\beta}_1 \mathcal{T} P_{\eta}^{(1)} S^{aa'} \left( P_{\eta}^{(2)} S_{aa'} - P_{\eta}^{(2)} S_{a'a} \right) \left( \overset{1}{\cdot} \lambda_{\mathcal{T}}^a - 2 \overset{1}{\cdot} \lambda_{\mathcal{T}}^{a'} \right) + \\
& 4 P_{\eta}^{(1)} S^a \left( 18 \mathcal{M}_{Pl}^2 \hat{\beta}_3 P_{\eta}^{(2)} S_a^{a'} \right) P_{\eta}^{(3)} \mathcal{D}_a \mathcal{T} - 18 \mathcal{M}_{Pl}^2 \hat{\beta}_3 P_{\eta}^{(2)} S_a^{a'} P_{\eta}^{(3)} \mathcal{D}_a \mathcal{T} + \\
& \mathcal{T} \left( 2 \mathcal{M}_{Pl}^2 \hat{\beta}_1 P_{\eta}^{(2)} S^{a'b} \left( \overset{2}{\cdot} \lambda_{\mathcal{T}aa'b} - \overset{2}{\cdot} \lambda_{\mathcal{T}ab'a} - 2 \overset{2}{\cdot} \lambda_{\mathcal{T}a'b'a} \right) + 9 \mathcal{M}_{Pl}^2 \hat{\beta}_3 P_{\eta}^{(2)} S_a^{b'} n_{b'} \right. \\
& \left. \left( P_{\eta}^{(3)} \mathcal{D}^{aa'} n_a \right) - \overset{1}{\cdot} \mathcal{T}^{aa'} \right) + 9 \mathcal{M}_{Pl}^2 \hat{\beta}_3 P_{\eta}^{(2)} S_a^{b'} n_{b'} \left( -P_{\eta}^{(3)} \mathcal{D}^{aa'} n_a \right) + \overset{1}{\cdot} \mathcal{T}^{aa'} \right) + \\
& 18 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \eta_{a'b} \left( -\left( \overset{3}{\cdot} \mathcal{D}^{ab} P_{\eta}^{(2)} S_a^{a'} \right) + \overset{3}{\cdot} \mathcal{D}^{ab} P_{\eta}^{(2)} S_a^{a'} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\cdot} \phi_{b|lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{3 \mathcal{T} \eta} P_{\eta}^{(1)} S^{aa'} \left( P_{\eta}^{(2)} S_a^{b'} \left( \mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}ab} - \mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}ab} - 6 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \overset{1}{\cdot} \mathcal{T}^{ab} \right) + \right. \\
& \left. P_{\eta}^{(2)} S_a^{b'} \left( \mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}ab} - \mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}ab} - 6 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \overset{1}{\cdot} \mathcal{T}^{ab} \right) + \left( P_{\eta}^{(2)} S_a^{b'} + P_{\eta}^{(2)} S_a^{b'} \right) \right. \\
& \left. \left( \mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}a'b} - \mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}a'b} - 6 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \overset{1}{\cdot} \mathcal{T}^{a'b} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0}{\cdot} \phi_{\mathcal{T}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{0}{\cdot} \phi_{\mathcal{T}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\cdot} \phi_{\mathcal{T}lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{12 \mathcal{T}^2} \left( 3 P_{\eta}^{(1)} S^{aa'} \left( 2 \mathcal{M}_{Pl}^2 \hat{\alpha}_0 \mathcal{T} P_{\eta}^{(2)} S_{aa'} + 2 \mathcal{M}_{Pl}^2 \hat{\alpha}_0 \mathcal{T} P_{\eta}^{(2)} S_{a'a} \right) + \right. \\
& \left. \overset{1}{\cdot} \pi_{\mathcal{T}ab} P_{\eta}^{(2)} S_a^{b'} + \overset{1}{\cdot} \pi_{\mathcal{T}a'b} \left( P_{\eta}^{(2)} S_a^{b'} + P_{\eta}^{(2)} S_a^{b'} \right) + \overset{1}{\cdot} \pi_{\mathcal{T}ab} P_{\eta}^{(2)} S_a^{b'} \right) - \\
& 4 \mathcal{M}_{Pl}^2 \hat{\alpha}_0 \eta_{aa'} \eta_{bb'} \mathcal{T} \overset{(1)}{S}^{aa'} \overset{(2)}{S}^{bb'} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{2}{\cdot} \phi_{lmn} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{32 \mathcal{T}^2} 3 \overset{1}{\cdot} \pi_{\mathcal{T}}^a \left( P_{\eta}^{(1)} S_a^{a'} P_{\eta}^{(2)} S_{bb'} \overset{(2)}{S}_{a'}^{bb'} + 2 P_{\eta}^{(1)} S_a^{a'b} \left( P_{\eta}^{(2)} S_{aa'b} + P_{\eta}^{(2)} S_{abaa'} - P_{\eta}^{(2)} S_{a'ab} - P_{\eta}^{(2)} S_{baaa'} \right) \right) + \\
& P_{\eta}^{(1)} S_a^{a'} \left( P_{\eta}^{(2)} S_{bb'} \overset{(2)}{S}_{a'}^{bb'} - P_{\eta}^{(2)} S_{bb'} \overset{(2)}{S}_{a'}^{b'b'} \right) - P_{\eta}^{(1)} S_a^{a'} P_{\eta}^{(2)} S_{bb'} \overset{(2)}{S}_{a'}^{b'b'} - \\
& 2 \eta_{a'b} P_{\eta}^{(1)} S_{b'c} \overset{(2)}{S}_a^{b'c} \overset{(1)}{S}^{a'b} + 2 \eta_{a'b} P_{\eta}^{(1)} S_{b'c} \overset{(2)}{S}_a^{b'c} \overset{(1)}{S}^{a'b} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{1}{\cdot} \phi_{b|lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( \overset{1}{\cdot} \phi_{b|l} \cdot \overset{(2)}{S}^l \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0
\end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left( \overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left( \overset{2}{\phi}_{lmn}^{\parallel} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left( \overset{0}{\phi}_b^{\perp} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left( \overset{1}{\phi}_{bi}^{\perp} \cdot \overset{(2)}{S}^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( \overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left( \overset{2}{\phi}_{bi}^{\perp} \cdot \overset{(2)}{S}^{im} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left( \frac{2}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1^2 \left( 3 P_{\eta}^{(1)S^{aa'}} \left( P_{\eta}^{(2)S_{aa'}} + P_{\eta}^{(2)S_{a'a}} \right) - 2 \eta_{aa'}^{\parallel} \eta_{bb'}^{\parallel} \overset{(1)}{S}^{aa'} \overset{(2)}{S}^{bb'} \right) \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1 \\
& \left\{ \iiint \left( \overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left( \overset{1}{\chi}_{bi}^{\parallel} \cdot \overset{(2)}{S}^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left( \frac{1}{6 \mathcal{T}^2} \left( 3 P_{\eta}^{(1)S^{ab}} \left( P_{\eta}^{(2)S_b} P_{\eta}^{(3)\mathcal{D}_a \mathcal{T}} + P_{\eta}^{(2)S_a} P_{\eta}^{(3)\mathcal{D}_b \mathcal{T}} \right) - \mathcal{T} \left( \overset{3}{\mathcal{D}_a} P_{\eta}^{(2)S_b} + \overset{3}{\mathcal{D}_b} P_{\eta}^{(2)S_a} \right) \right) \right. \\
& \quad \left. + 2 \eta_{ab}^{\parallel} \overset{(1)}{S}^{ab} \left( -P_{\eta}^{(2)S^{a'}} P_{\eta}^{(3)\mathcal{D}_a \mathcal{T}} + \eta_{a'b'}^{\parallel} \mathcal{T} \left( \overset{3}{\mathcal{D}_b} P_{\eta}^{(2)S^{a'}} \right) \right) \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1 \\
& \left\{ \iiint \left( \overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left( \overset{2}{\chi}_{lmn}^{\parallel} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left( \frac{1}{96 \mathcal{T}^2} \left( P_{\eta}^{(1)S^{ab}} \left( -18 P_{\eta}^{(2)S_{b'c}} \overset{(2)}{S}_b^{b'c} P_{\eta}^{(3)\mathcal{D}_a \mathcal{T}} + 18 P_{\eta}^{(2)S_{b'c}} \overset{(2)}{S}_b^{b'c} P_{\eta}^{(3)\mathcal{D}_a \mathcal{T}} + 27 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{bc}^{c'}} n_{c'} \right] \right. \right. \\
& \quad P_{\eta}^{(3)\mathcal{D}_a n^{b'}} + 9 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{b'c}} n_{c'} P_{\eta}^{(3)\mathcal{D}_a n^{b'}} - 27 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{bc}^{c'}} n_{c'} P_{\eta}^{(3)\mathcal{D}_a n^{b'}} - \\
& \quad 18 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{c'b}} n_{c'} P_{\eta}^{(3)\mathcal{D}_a n^{b'}} - 9 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{b'c}} n_{c'} P_{\eta}^{(3)\mathcal{D}_a n^{b'}} + \\
& \quad 18 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{c'b}} n_{c'} P_{\eta}^{(3)\mathcal{D}_a n^{b'}} - 18 P_{\eta}^{(2)S_{b'c}} \overset{(2)}{S}_a^{b'c} P_{\eta}^{(3)\mathcal{D}_b \mathcal{T}} + 18 P_{\eta}^{(2)S_{b'c}} \overset{(2)}{S}_a^{b'c} P_{\eta}^{(3)\mathcal{D}_b \mathcal{T}} + 27 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{ac}^{c'}} n_{c'} P_{\eta}^{(3)\mathcal{D}_b n^{b'}} - \\
& \quad 27 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{ca}^{c'}} n_{c'} P_{\eta}^{(3)\mathcal{D}_b n^{b'}} - 18 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{c'a}} n_{c'} P_{\eta}^{(3)\mathcal{D}_b n^{b'}} - \\
& \quad 9 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{ac}^{c'}} n_{c'} P_{\eta}^{(3)\mathcal{D}_b n^{b'}} + 18 \delta_{b'c}^{\perp} \mathcal{T} P_{\eta}^{(2)S_{c'a}} n_{c'} P_{\eta}^{(3)\mathcal{D}_b n^{b'}} + \\
& \quad 36 P_{\eta}^{(2)S_{a'b}} \overset{(2)}{S}_b^{b'a} P_{\eta}^{(3)\mathcal{D}_b \mathcal{T}} + 36 P_{\eta}^{(2)S_{b'a}} \overset{(2)}{S}_b^{b'a} P_{\eta}^{(3)\mathcal{D}_b \mathcal{T}} - 36 P_{\eta}^{(2)S^{b'a}} P_{\eta}^{(3)\mathcal{D}_b \mathcal{T}} - \\
& \quad 36 P_{\eta}^{(2)S^{b'a}} P_{\eta}^{(3)\mathcal{D}_b \mathcal{T}} - 27 \mathcal{T} P_{\eta}^{(2)S_{bb'c}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_a} - 9 \mathcal{T} P_{\eta}^{(2)S_{b'b}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_a} + \\
& \quad 27 \mathcal{T} P_{\eta}^{(2)S_{b'b}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_a} + 18 \mathcal{T} P_{\eta}^{(2)S_{b'b}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_a} + 9 \mathcal{T} P_{\eta}^{(2)S_{bb'b}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_a} - \\
& \quad 18 \mathcal{T} P_{\eta}^{(2)S_{b'b}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_a} - 27 \mathcal{T} P_{\eta}^{(2)S_{abb'c}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_b} - 9 \mathcal{T} P_{\eta}^{(2)S_{a'b}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_b} + \\
& \quad 27 \mathcal{T} P_{\eta}^{(2)S_{b'a}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_b} + 18 \mathcal{T} P_{\eta}^{(2)S_{b'a}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_b} + 9 \mathcal{T} P_{\eta}^{(2)S_{abb'c}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_b} - \\
& \quad 18 \mathcal{T} P_{\eta}^{(2)S_{b'a}} n_{c'} P_{\eta}^{(3)\mathcal{D}^{b'} n_b} - 4 \epsilon_{bb'c}^{\parallel} \mathcal{T} P_{\eta}^{(2)S_a^{b'c}} \overset{0}{\mathcal{T}}^{\parallel} - 4 \epsilon_{ab'c}^{\parallel} \mathcal{T} P_{\eta}^{(2)S_b^{b'c}} \overset{0}{\mathcal{T}}^{\parallel} + \\
& \quad 4 \epsilon_{bb'c}^{\parallel} \mathcal{T} P_{\eta}^{(2)S^{b'a}} \overset{0}{\mathcal{T}}^{\parallel} + 4 \epsilon_{ab'c}^{\parallel} \mathcal{T} P_{\eta}^{(2)S^{b'a}} \overset{0}{\mathcal{T}}^{\parallel} + 8 \epsilon_{bb'c}^{\parallel} \mathcal{T} P_{\eta}^{(2)S^{b'a}} \overset{0}{\mathcal{T}}^{\parallel} + \\
& \quad 8 \epsilon_{ab'c}^{\parallel} \mathcal{T} P_{\eta}^{(2)S^{b'a}} \overset{0}{\mathcal{T}}^{\parallel} + 27 \mathcal{T} P_{\eta}^{(2)S_{bb'c}} n_{c'} \overset{1}{\mathcal{T}}_a^{b'} + 9 \mathcal{T} P_{\eta}^{(2)S_{b'b}} n_{c'} \overset{1}{\mathcal{T}}_a^{b'} -
\end{aligned}$$

$$\begin{aligned}
& 27 \mathcal{T} P_{\eta}^{(2)S_{b',b}{}^c} n_c \left[ \mathcal{T}_{\eta}^{||a}{}^{b'} - 18 \mathcal{T} P_{\eta}^{(2)S_{b',c}{}^c} n_c \left[ \mathcal{T}_{\eta}^{||a}{}^{b'} - 9 \mathcal{T} P_{\eta}^{(2)S_{bb',c}{}^c} n_c \right] \mathcal{T}_{\eta}^{||a}{}^{b'} + \right. \\
& 18 \mathcal{T} P_{\eta}^{(2)S_{b',b}{}^c} n_c \left[ \mathcal{T}_{\eta}^{||a}{}^{b'} + 27 \mathcal{T} P_{\eta}^{(2)S_{ab,b}{}^c} n_c \left[ \mathcal{T}_{\eta}^{||b}{}^{b'} + 9 \mathcal{T} P_{\eta}^{(2)S_{a,b,b}{}^c} n_c \right] \mathcal{T}_{\eta}^{||b}{}^{b'} - \right. \\
& 27 \mathcal{T} P_{\eta}^{(2)S_{b',a}{}^c} n_c \left[ \mathcal{T}_{\eta}^{||b}{}^{b'} - 18 \mathcal{T} P_{\eta}^{(2)S_{b',c}{}^c} n_c \left[ \mathcal{T}_{\eta}^{||b}{}^{b'} - 9 \mathcal{T} P_{\eta}^{(2)S_{ab,b}{}^c} n_c \right] \mathcal{T}_{\eta}^{||b}{}^{b'} + \right. \\
& 18 \mathcal{T} P_{\eta}^{(2)S_{b',a}{}^c} n_c \left[ \mathcal{T}_{\eta}^{||b}{}^{b'} + 18 \eta_{b',c}^{||} \mathcal{T} \left( {}^3\mathcal{D}_{\eta}^{||} P_{\eta}^{(2)S_{b'}{}^{b'c}} \right) - 18 \eta_{b',c}^{||} \mathcal{T} \left( {}^3\mathcal{D}_{\eta}^{||} P_{\eta}^{(2)S_{b'}{}^{b'c}} \right) + \right. \\
& 18 \eta_{b',c}^{||} \mathcal{T} \left( {}^3\mathcal{D}_{\eta}^{||} P_{\eta}^{(2)S_{a'}{}^{b'c}} \right) - 18 \eta_{b',c}^{||} \mathcal{T} \left( {}^3\mathcal{D}_{\eta}^{||} P_{\eta}^{(2)S_{a'}{}^{b'c}} \right) - 36 \eta_{b',c}^{||} \mathcal{T} \left( {}^3\mathcal{D}_{\eta}^{||c} P_{\eta}^{(2)S_{a'}{}^{b'b}} \right) - \\
& 36 \eta_{b',c}^{||} \mathcal{T} \left( {}^3\mathcal{D}_{\eta}^{||c} P_{\eta}^{(2)S_{b'}{}^{ba}} \right) + 36 \eta_{b',c}^{||} \mathcal{T} \left( {}^3\mathcal{D}_{\eta}^{||c} P_{\eta}^{(2)S_{b'}{}^{ba}} \right) \left. \right] + \\
& 18 \eta_{ab}^{||} ({}^1S^{ab} \left( 2 P_{\eta}^{||\eta_{cc'}, (2)S_{b',c}{}^{c'}} \right) P_{\eta}^{(3)D^{b'}} \mathcal{T} - 2 P_{\eta}^{||\eta_{cc'}, (2)S_{b',c}{}^{c'}} \left[ P_{\eta}^{(3)D^{b'}} \mathcal{T} + \right. \\
& \mathcal{T} \left( \delta_{b',c'} \left( -P_{\eta}^{(2)S_{cc'}, (2)S_{a',c}{}^{c'}} - P_{\eta}^{(2)S_{a',c}{}^{c'}} n_{a'} \right) + P_{\eta}^{(2)S_{c',a'}{}^{c'}} n_{a'} + P_{\eta}^{(2)S_{c',a'}{}^{c'}} n_{a'} \right) P_{\eta}^{(3)D^{b'}} \mathcal{T} - \\
& 2 P_{\eta}^{(2)S_{cb',c'}{}^{c'}} n_{c'} \left[ P_{\eta}^{(3)D^{b'}} \mathcal{T} - 2 P_{\eta}^{(2)S_{c',b',c}{}^{c'}} n_{c'} \left[ P_{\eta}^{(3)D^{b'}} \mathcal{T} + 2 P_{\eta}^{(2)S_{c',b',c}{}^{c'}} n_{c'} \right] P_{\eta}^{(3)D^{b'}} \mathcal{T} - \right. \\
& 2 \left( \eta_{a',b}^{||} ({}^2S^{aa'b} n_a) \left( \eta_{aa'}^{||} ({}^3D^{aa'} n^a) \right) + 2 \left( \eta_{ab}^{||} ({}^2S^{aa'b} n_a) \left( \eta_{aa'}^{||} ({}^3D^{aa'} n^a) \right) + \right. \\
& 2 P_{\eta}^{(2)S_{b',c}{}^{c'}} n_{c'} \left( P_{\eta}^{(3)D^{b'}} \mathcal{T} - \mathcal{T}_{\eta}^{||b'c} \right) - P_{\eta}^{(2)S_{b',c}{}^{c'}} n_{c'} \left[ \mathcal{T}_{\eta}^{||b'c} + P_{\eta}^{(2)S_{b',c}{}^{c'}} n_{c'} \right] \mathcal{T}_{\eta}^{||b'c} + 2 \\
& \eta_{b',c'}^{||} \eta_{ca'}^{||} \left( {}^3D^{aa'} P_{\eta}^{(2)S_{b'}{}^{cc'}} \right) - 2 \eta_{b',a'}^{||} \eta_{cc'}^{||} \left( {}^3D^{aa'} P_{\eta}^{(2)S_{b'}{}^{cc'}} \right) + 2 \eta_{a',d}^{||} \eta_{cc'}^{||} ({}^2S^{b'cc'}) \\
& n_{b'} ({}^3D^{dd'} n^{a'}) - 2 \eta_{a',d}^{||} \eta_{b',c'}^{||} ({}^2S^{b'cc'}) n_c ({}^3D^{dd'} n^{a'}) \left. \right) \left. \right] \left[ x^0, x^1, x^2, x^3 \right] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( {}^2\phi_{bij} \cdot ({}^1S^{ij}) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( {}^1\chi_{b|lm}^{||} \cdot ({}^2S^{lm}) \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \left( -\frac{1}{3\mathcal{T}} \left( 3 P_{\eta}^{(1)S^{aa'}} \right) \left( P_{\eta}^{(2)S_a{}^b} \right) \left( \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{(3)D^a} n_b + \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{(3)D^b} n_a + \mathcal{M}_{Pl}{}^2 \bar{\beta}_1 {}^2\lambda_{\mathcal{T}_{ab}}^{||} - \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 \mathcal{T}_{\eta}^{||ab} \right) - \right. \\
& P_{\eta}^{(2)S_a{}^b} \left( \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{(3)D^a} n_b + \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{(3)D^b} n_a + \mathcal{M}_{Pl}{}^2 \bar{\beta}_1 {}^2\lambda_{\mathcal{T}_{ab}}^{||} - \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 \mathcal{T}_{\eta}^{||ab} \right) + \\
& \left( P_{\eta}^{(2)S_a{}^b} - P_{\eta}^{(2)S_b{}^a} \right) \left( \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{(3)D^a} n_b + \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{(3)D^b} n_a + \mathcal{M}_{Pl}{}^2 \bar{\beta}_1 {}^2\lambda_{\mathcal{T}_{ab}}^{||} - \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 \mathcal{T}_{\eta}^{||ab} \right. \\
& \left. \left. \mathcal{T}_{\eta}^{||a'b} \right) \right) + 4 \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 \eta_{aa'}^{||} P_{\eta}^{(2)S^{bb'}} ({}^1S^{aa'}) \mathcal{T}_{\eta}^{||bb'} \left. \right) \left[ x^0, x^1, x^2, x^3 \right] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( {}^0\phi_{\mathcal{A}} \cdot ({}^1S) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( {}^0\phi_{\mathcal{A}} \cdot ({}^2S) \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( {}^0\phi_{\mathcal{A}} \cdot ({}^1S) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( {}^0\phi_{\mathcal{A}} \cdot ({}^2S) \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( {}^0\phi_{\mathcal{A}} \cdot ({}^1S) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( {}^2\phi_{\mathcal{A}|lm} \cdot ({}^2S^{lm}) \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( {}^0\phi_{\mathcal{A}} \cdot ({}^1S) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( {}^2\phi_{\mathcal{A}|lmn} \cdot ({}^2S^{lmn}) \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( {}^0\phi_{\mathcal{A}} \cdot ({}^1S) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( {}^1\phi_{b|lm}^{||} \cdot ({}^2S^{lm}) \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left( {}^0\phi_{\mathcal{A}} \cdot ({}^1S) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left( {}^1\phi_{b|l}^{||} \cdot ({}^2S^l) \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0
\end{aligned}$$

[illegible]



[illegible]







$$\begin{aligned}
& 2 \mathcal{T} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S^c_a, n_c} \left[ \mathcal{T}^{I_{ab}} + 3 \mathcal{T} P_{\eta}^{(1)S^c_{cc'}, (1)S^c_a, cc'} P_{\eta}^{(2)S^b_a, n_b} \right] \mathcal{T}^{I_{aa'}} - \\
& 3 \mathcal{T} P_{\eta}^{(1)S^c_{cc'}, (1)S^c_a, cc'} P_{\eta}^{(2)S^b_a, n_b} \mathcal{T}^{I_{aa'}} - 3 \mathcal{T} P_{\eta}^{(1)S^c_{cc'}, (1)S^c_a, cc'} P_{\eta}^{(2)S^b_a, n_b} \mathcal{T}^{I_{aa'}} + \\
& 3 \mathcal{T} P_{\eta}^{(1)S^c_{cc'}, (1)S^c_a, cc'} P_{\eta}^{(2)S^b_a, n_b} \mathcal{T}^{I_{aa'}} - 2 \mathcal{T} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S^c_a, n_c} \mathcal{T}^{I_{a'b}} + \\
& 2 \mathcal{T} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S^c_a, n_c} \mathcal{T}^{I_{a'b}} - 4 \mathcal{T} P_{\eta}^{(1)S^{aa'b'}} \left( \mathcal{D}^I_{a'} P_{\eta}^{(2)S^c_a, n_c} \right) + \\
& 4 \mathcal{T} P_{\eta}^{(1)S^{aa'b'}} \left( \mathcal{D}^I_{a'} P_{\eta}^{(2)S^b_a, n_b} \right) + 4 \mathcal{T} P_{\eta}^{(1)S^{aa'b'}} \left( \mathcal{D}^I_{a'} P_{\eta}^{(2)S^c_a, n_c} \right) - \\
& 4 \mathcal{T} P_{\eta}^{(1)S^{aa'b'}} \left( \mathcal{D}^I_{a'} P_{\eta}^{(2)S^b_a, n_b} \right) + 8 \mathcal{T} P_{\eta}^{(1)S^{aa'b'}} \left( \mathcal{D}^I_{b'} P_{\eta}^{(2)S^c_a, n_c} \right) - \\
& 8 \mathcal{T} P_{\eta}^{(1)S^{aa'b'}} \left( \mathcal{D}^I_{b'} P_{\eta}^{(2)S^c_a, n_c} \right) - 6 \eta^I_{a'b'} \mathcal{T} P_{\eta}^{(1)S^c_{cc'}, (1)S^c_a, cc'} \left( \mathcal{D}^{I_{b'}} P_{\eta}^{(2)S^{aa'}} \right) + \\
& 6 \eta^I_{ab'} \mathcal{T} P_{\eta}^{(1)S^c_{cc'}, (1)S^c_a, cc'} \left( \mathcal{D}^{I_{b'}} P_{\eta}^{(2)S^{aa'}} \right) + 6 \eta^I_{a'b'} \mathcal{T} P_{\eta}^{(1)S^c_{cc'}, (1)S^c_a, cc'} \left( \mathcal{D}^{I_{b'}} P_{\eta}^{(2)S^{aa'}} \right) - \\
& 6 \eta^I_{ab'} \mathcal{T} P_{\eta}^{(1)S^c_{cc'}, (1)S^c_a, cc'} \left( \mathcal{D}^{I_{b'}} P_{\eta}^{(2)S^{aa'}} \right) \Big] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left( \mathcal{X}^{i,j}_{b'} \cdot \mathcal{S}^{i,j} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iint \left( \mathcal{X}^{i,m}_{b'} \cdot \mathcal{S}^{i,m} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{3 \mathcal{T}} \mathcal{M}_{Pl}^2 \tilde{\beta}_1 P_{\eta}^{(1)S^{aa'}} \left( P_{\eta}^{(2)S^b_a, n_b} \left( \mathcal{L}^{I_{ab}}_{\mathcal{T}} - \mathcal{L}^{I_{ab}}_{\mathcal{T}} \right) + P_{\eta}^{(2)S^b_a, n_b} \left( -\mathcal{L}^{I_{ab}}_{\mathcal{T}} + \mathcal{L}^{I_{ab}}_{\mathcal{T}} \right) - \right. \\
& \left. \left( P_{\eta}^{(2)S^b_a, n_b} - P_{\eta}^{(2)S^b_a, n_b} \right) \left( \mathcal{L}^{I_{a'b}}_{\mathcal{T}} - \mathcal{L}^{I_{a'b}}_{\mathcal{T}} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

This concludes the HiGGS output.

**Concrete relation to manuscript:** The above constraints and Poisson brackets constitute the data represented by coloured squares in Fig. 5 in our Manuscript. The primary constraints  $\phi$  are listed along the top row. The primary Poisson brackets  $\{\phi, \phi\}$  are used to infer the white, red or yellow squares in the matrix depending on whether they are vanishing, or appear only in proportion to phase space variables, or are order unity (e.g. proportional to the metric or unit-timelike vector). The  $\{\phi, \mathcal{H}_b\}$  Poisson brackets define the secondary constraints " $\chi$ ", which appear on the sides of the matrix. Most of the Poisson brackets  $\{\chi, \phi\}$  need not be computed (grey squares), but it can be noticed without detailed calculation that many of the remaining required brackets (yellow squares) do indeed arise. Unlike in the strongly coupled case, we find that tertiary and quaternary constraints arise in the theory: but these are also not computed in detail above. Instead, their existence can readily be inferred by considering whether the consistency chain in question can be absorbed by determining a multiplier, or whether it must continue. In case of continuation, the relevant terms in the induced constraints which arise can be understood by considering Eq. (4) in arXiv:2205.13534.