In[∘]:= `Get@FileNameJoin@{NotebookDirectory[], "SupplementalMaterials.m"};`

```
------------------------------------------------------------

Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}

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Connecting to external linux executable...

Connection established.

------------------------------------------------------------

Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}

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------------------------------------------------------------

Package xAct`xPlain` version 1.0.0-developer, {2023, 5, 24}

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  it under certain conditions. See the General Public License for details.

------------------------------------------------------------
```

Supplemental materials: Hamiltonian analysis

Introduction

How to use this document

These calculations are designed to accompany our manuscript in the form of supplemental materials, for the sake of reproducibility. Throughout, commentary by the authors takes the form of green text. Citations, where needed, will be managed by direct reference to arXiv numbers, and all such references are already provided in full within the body of our manuscript. One exception is the source referred to throughout as `Blagojević'; this pertains to the book `Gravitation and Gauge Symmetries', which is also referenced within the manuscript.

Concrete relation to manuscript: In boxes like this, we will make specific connections between a result which is obtained in the supplemental material and a claim which is made in the manuscript. These points of contact are not always numbered equations, they could be textual.

Note that a programmatical session in the Wolfram language does not really correspond to the clean flow of thoughts in a LaTeX document: there are differences that can't (and shouldn't) be ignored. Thus, whilst this document should be at least readable in standalone format, the reader is encouraged to follow it in tandem with the Wolfram language source codes, so as to avoid ambiguities. In this way, the specific operations and manipulations of quantities will become absolutely clear.

Loading HiGGS

For these calculations, we will use the HiGGS package. The version of HiGGS used for the computations here are both developer-only, and so we include copies of the sources with these supplemental materials.

```
------------------------------------------------------------

Package xAct`xPert`  version 1.0.6, {2018, 2, 28}

CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia
  and Guillermo A. Mena Marugan, under the General Public License.

** Variable $PrePrint assigned value ScreenDollarIndices

** Variable $CovDFormat changed from Prefix to Postfix

** Option AllowUpperDerivatives of ContractMetric changed from False to True

** Option MetricOn of MakeRule changed from None to All

** Option ContractMetrics of MakeRule changed from False to True

------------------------------------------------------------

Package xAct`Invar`  version 2.0.5, {2013, 7, 1}

CopyRight (C) 2006-2020, J. M. Martin-Garcia,
  D. Yllanes and R. Portugal, under the General Public License.

** DefConstantSymbol: Defining constant symbol sigma.

** DefConstantSymbol: Defining constant symbol dim.

** Option CurvatureRelations of DefCovD changed from True to False

** Variable $CommuteCovDsOnScalars changed from True to False

------------------------------------------------------------

Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}

CopyRight (C) 2005-2021, David Yllanes and
  Jose M. Martin-Garcia, under the General Public License.

------------------------------------------------------------

Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}

CopyRight (C) 2011-2021, Thomas Bäckdahl, under the General Public License.

------------------------------------------------------------
```

```
Package xAct`xTras`  version 1.4.2, {2014, 10, 30}

CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.

** Variable $CovDFormat changed from Postfix to Prefix

** Option CurvatureRelations of DefCovD changed from False to True

------------------------------------------------------------

Package xAct`HiGGS`  version 2.0.0-developer, {2023, 6, 13}

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------------------------------------------------------------

HiGGS incorporates code by Cyril Pitrou.

------------------------------------------------------------

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------------------------------------------------------------
```

The HiGGS package is now loaded. We are interested only in the computations which lead to our Fig. 3 and Fig. 5. (in the appendix) in the manuscript. Two theories are considered. To perform the Hamiltonian analysis we use the StudyTheory[] command in HiGGS, specifying the parameters of the theory. For details of this command, and for all notation used in this supplement, see arXiv:2206.00658. The results were precomputed on a desktop computer. This is because the full Hamiltonian analysis of a theory, even run on a modern desktop computer with multiple processor cores, still takes up to half an hour. Below we will display the results using the ViewTheory[] command, which takes the pre-computed binary files (.thr.mx) and formats them. If you want to re-compute the analysis locally, the few lines of code you will need are stored at the top of this (.m) file, so just un-comment them. If there are problems with the execution, please refer to the HiGGS docs, or email wb263@cam.ac.uk.

Singular spectrum without multipliers

Introductory comments

In the first instance we perform the Hamiltonian analysis of the theory without multipliers. As we know, this theory suffers from strong coupling. Accordingly, in Eq. (4) in arXiv:2205.13534 we set $\hat{\alpha}_{1}$, $\hat{\alpha}_{2}$, $\hat{\alpha}_{3}$, $\hat{\alpha}_{4}$, $\hat{\alpha}_{6}$, $\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{1}$, $\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{2}$, $\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{3}$, to zero, suppressing most of the higher-curvature invariants, two of the three torsion invariants, and all of the Lagrange multipliers, but allow for nonvanishing $\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\alpha}_{0}$, (Einstein-Hilbert term) $\hat{\alpha}_{5}$ (Riemann-Cartan-Maxwell invariant of curvature) and $\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{3}$.

The results of the analysis are presented below; the HiGGS output is designed to be reasonably self-explanatory, but we will nonetheless add some guiding commentary.

The first set of data are the primary constraints ϕ of the theory. These are implied by the impossibility of inverting the conjugate field-momenta for the velocities of the fields. As is now common in the PGT literature, we refer to these as `if-constraints', i.e. their existence depends on the linear relations assumed among the Lagrangian parameters.

Next, the Legendre-transformed Lagrangian is presented. This is broken into terms; the super-Hamiltonian (i.e. the coefficient $\overset{0^+}{\cdot}\mathcal{H}_b$ of the lapse function $\mathcal{N}$), the linear super-momentum (i.e. the coefficient $\overset{1^-}{\cdot}\mathcal{H}_{b\,l}$ of the shift vector) and the angular super-momentum (i.e. the antisymmetric coefficient $\overset{1^+}{\cdot}\mathcal{H}_{\mathcal{A}\,nm}$ of the timelike part of the rotational gauge field). These concepts are introduced in Blagojević.

Once the primary constraints are listed, the matrix $\{\phi, \ \phi\}$ of Poisson brackets between the primaries is computed. Of course, these brackets only need to be computed for the upper triangular part of the matrix. The resulting entries lie behind the red, white and yellow squares in the relevant panel of Fig. 3.

Finally, the velocities of these primaries ϕ are computed, i.e. their Poisson brackets with the super-Hamiltonian $\overset{0^+}{\cdot}\mathcal{H}_b$. These velocities serve two purposes; (i) they can be used to determine the `missing velocities' or `Hamiltonian multipliers' u (not to be confused with the geometric multipliers λ used in the manuscript), or (ii) they can define the secondary constraints of the theory. Whichever of these applies depends of course on $\{\phi, \ \phi\}$, according to Eq. (9) in our manuscript.

Lists of constraints and Poisson brackets from HiGGS

Having briefly recounted the notation of arXiv:2206.00658, we make a call to the HiGGS command ViewTheory[] on the appropriate model. This command presents all the results which were pre-computed by the earlier call to StudyTheory[].

```
** DefTheory: Incorporating the binary at ConstraintAlgebraAlp0Alp5Bet3.thr.mx
```

```
** DefTheory: Found the following primary if-constraints:
```

$$\overset{0^+}{\cdot}\phi_b \equiv \frac{\overset{0^+}{\cdot}\hat{\pi}_b}{\mathcal{J}} \approx 0$$

$$\overset{1^-}{\cdot}\phi_{b\,i} \equiv \frac{\overset{1^-}{\cdot}\hat{\pi}_{b\,i}}{\mathcal{J}} \approx 0$$

$$\overset{2^+}{\cdot}\phi_{b\,i\,j} \equiv \frac{\overset{2^+}{\cdot}\hat{\pi}_{b\,i\,j}}{\mathcal{J}} \approx 0$$

$$\overset{0^+}{\cdot}\phi_{\mathcal{A}} \equiv 3\,M_{Pl}{}^2\,\hat{\alpha}_{\underset{0}{\cdot}} + \frac{\overset{0^+}{\cdot}\hat{\pi}_{\mathcal{A}}}{\mathcal{J}} \approx 0$$

$$\overset{0^-}{\cdot}\phi_{\mathcal{A}} \equiv \frac{\overset{0^-}{\cdot}\hat{\pi}_{\mathcal{A}}}{\mathcal{J}} \approx 0$$

$$\overset{2^+}{\cdot}\phi_{\mathcal{A}\,i\,j} \equiv \frac{\overset{2^+}{\cdot}\hat{\pi}_{\mathcal{A}\,i\,j}}{\mathcal{J}} \approx 0$$

$${}^{2^-}\phi_{\mathcal{A}ijh} \equiv \frac{2\,{}^{2^-}\hat{\pi}_{\mathcal{A}ijh}}{3\mathcal{J}} + \frac{{}^{2^-}\hat{\pi}_{\mathcal{A}ihj}}{3\mathcal{J}} - \frac{{}^{2^-}\hat{\pi}_{\mathcal{A}jhi}}{3\mathcal{J}} \approx 0$$

** DefTheory: Found the following secondary perpendicular if-constraints:

** DefTheory: Found the following secondary parallel if-constraints:

** DefTheory: Found the following secondary singular if-constraints:

** DefTheory: Defining association key $IfConstraints
for the theory association ConstraintAlgebraAlp0Alp5Bet3

** ViewTheory: The super-Hamiltonian is:

$$\begin{aligned}
{}^{0^+}\mathcal{H}_b \equiv\; & \frac{{}^{1^-}\hat{\pi}_{\mathcal{A}a}\,{}^{1^-}\hat{\pi}_{\mathcal{A}}{}^{a}}{16\,\hat{\alpha}_{5}\,\mathcal{J}} + \frac{{}^{1^+}\hat{\pi}_{\mathcal{A}ab}\,{}^{1^+}\hat{\pi}_{\mathcal{A}}{}^{ab}}{8\,\hat{\alpha}_{5}\,\mathcal{J}} + \frac{3\,{}^{1^+}\hat{\pi}_{bab}\,{}^{1^+}\hat{\pi}_{b}{}^{ab}}{16\,M_{Pl}^{2}\,\hat{\beta}_{3}\,\mathcal{J}} + \frac{{}^{1^-}\hat{\pi}_{b}{}^{a}\,P_{\eta^\parallel}\left[{}^{3}\mathcal{D}^\parallel_{a}\mathcal{J}\right]}{\mathcal{J}} + \\
& \frac{M_{Pl}^{2}\,\hat{\alpha}_{0}\,\mathcal{J}\,{}^{0^+}\mathcal{R}^\parallel}{2} - \frac{1}{2}\,{}^{1^-}\hat{\pi}_{\mathcal{A}}{}^{a}\,{}^{1^-}\mathcal{R}^\parallel_{a} + {}^{1^+}\hat{\pi}_{\mathcal{A}}{}^{ab}\,{}^{1^+}\mathcal{R}^\parallel_{ab} + \frac{1}{6}\,M_{Pl}^{2}\,\hat{\beta}_{3}\,\mathcal{J}\,{}^{0^-}\mathcal{T}^{\parallel 2} - {}^{1^-}\hat{\pi}_{b}{}^{a}\,{}^{1^-}\mathcal{T}^\parallel_{a} + \\
& \frac{1}{2}\,{}^{1^+}\hat{\pi}_{b}{}^{ab}\,{}^{1^+}\mathcal{T}^\parallel_{ab} - \frac{16}{27}\,M_{Pl}^{2}\,\hat{\beta}_{3}\,\mathcal{J}\,{}^{2^-}\mathcal{T}^\parallel_{abb'}\,{}^{2^-}\mathcal{T}^{\parallel abb'} + \frac{32}{27}\,M_{Pl}^{2}\,\hat{\beta}_{3}\,\mathcal{J}\,{}^{2^-}\mathcal{T}^\parallel_{ab'b}\,{}^{2^-}\mathcal{T}^{\parallel abb'} - \\
& n^{a}\left({}^{3}\mathcal{D}^\parallel_{b}\,{}^{1^+}\hat{\pi}_{ba}{}^{b}\right) - n^{a}\left({}^{3}\mathcal{D}^\parallel_{b}\,{}^{2^+}\hat{\pi}_{ba}{}^{b}\right) - \eta^\parallel_{ab}\left({}^{3}\mathcal{D}^{\parallel b}\,{}^{1^-}\hat{\pi}_{b}{}^{a}\right) + \frac{1}{3}\,\eta^\parallel_{ab}\,{}^{0^+}\hat{\pi}_{b}\left({}^{3}\mathcal{D}^{\parallel b}n^{a}\right) \approx 0
\end{aligned}$$

** ViewTheory: The linear super-momentum is:

$$\begin{aligned}
{}^{1^-}\mathcal{H}_{bl} \equiv\; & -\frac{1}{2}\,{}^{1^-}\hat{\pi}_{b}{}^{a}\,P_{\eta^\parallel}\left[{}^{3}\mathcal{D}^\parallel_{a}n_{l}\right] + \frac{{}^{1^+}\hat{\pi}_{bla}\,P_{\eta^\parallel}\left[{}^{3}\mathcal{D}^{\parallel a}\mathcal{J}\right]}{\mathcal{J}} + \frac{{}^{2^+}\hat{\pi}_{bla}\,P_{\eta^\parallel}\left[{}^{3}\mathcal{D}^{\parallel a}\mathcal{J}\right]}{\mathcal{J}} - P_{\eta^\parallel}\left[\eta^{\parallel aa'}\left({}^{3}\mathcal{D}^\parallel_{a'}\,{}^{1^+}\hat{\pi}_{bla}\right)\right] - \\
& P_{\eta^\parallel}\left[\eta^{\parallel aa'}\left({}^{3}\mathcal{D}^\parallel_{a'}\,{}^{2^+}\hat{\pi}_{bla}\right)\right] + \frac{{}^{0^+}\hat{\pi}_{b}\,P_{\eta^\parallel}\left[{}^{3}\mathcal{D}^\parallel_{l}\mathcal{J}\right]}{3\mathcal{J}} - \frac{1}{3}\,P_{\eta^\parallel}\left[{}^{3}\mathcal{D}^\parallel_{l}\,{}^{0^+}\hat{\pi}_{b}\right] - \frac{1}{2}\,{}^{1^-}\hat{\pi}_{b}{}^{a}\,P_{\eta^\parallel}\left[{}^{3}\mathcal{D}^\parallel_{l}n_{a}\right] + \frac{1}{6}\,\epsilon^\parallel_{laa'}\,{}^{1^+}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,{}^{0^-}\mathcal{R}^\parallel - \\
& \frac{{}^{1^-}\hat{\pi}_{\mathcal{A}l}\,{}^{0^+}\mathcal{R}^\parallel}{6} - \frac{1}{2}\,{}^{1^+}\hat{\pi}_{\mathcal{A}la}\,{}^{1^-}\mathcal{R}^{\parallel a} - \frac{1}{2}\,{}^{2^+}\hat{\pi}_{\mathcal{A}la}\,{}^{1^-}\mathcal{R}^{\parallel a} + \frac{{}^{0^+}\hat{\pi}_{\mathcal{A}}\,{}^{1^-}\mathcal{R}^\parallel_{l}}{3} - \frac{4}{3}\,{}^{2^-}\hat{\pi}_{\mathcal{A}l}{}^{aa'}\,{}^{1^+}\mathcal{R}^\parallel_{aa'} + \\
& \frac{1}{6}\,\epsilon^\parallel_{laa'}\,{}^{0^-}\hat{\pi}_{\mathcal{A}}\,{}^{1^+}\mathcal{R}^{\parallel aa'} - \frac{1}{2}\,{}^{1^-}\hat{\pi}_{\mathcal{A}}{}^{a}\,{}^{1^+}\mathcal{R}^\parallel_{la} - \frac{4}{3}\,{}^{1^+}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,{}^{2^-}\mathcal{R}^\parallel_{laa'} + \frac{4}{3}\,{}^{2^+}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,{}^{2^-}\mathcal{R}^\parallel_{laa'} + \\
& \frac{4}{3}\,{}^{2^-}\hat{\pi}_{\mathcal{A}l}{}^{aa'}\,{}^{2^+}\mathcal{R}^\parallel_{aa'} - \frac{1}{2}\,{}^{1^-}\hat{\pi}_{\mathcal{A}}{}^{a}\,{}^{2^+}\mathcal{R}^\parallel_{la} + \frac{1}{6}\,\epsilon^\parallel_{laa'}\,{}^{1^+}\hat{\pi}_{b}{}^{aa'}\,{}^{0^-}\mathcal{T}^\parallel - \frac{3}{2}\,{}^{1^+}\hat{\pi}_{bla}\,{}^{1^-}\mathcal{T}^{\parallel a} - \\
& \frac{3}{2}\,{}^{2^+}\hat{\pi}_{bla}\,{}^{1^-}\mathcal{T}^{\parallel a} + \frac{1}{2}\,{}^{1^-}\hat{\pi}_{b}{}^{a}\,{}^{1^+}\mathcal{T}^\parallel_{la} - \frac{4}{3}\,{}^{1^+}\hat{\pi}_{b}{}^{aa'}\,{}^{2^-}\mathcal{T}^\parallel_{laa'} + \frac{4}{3}\,{}^{2^+}\hat{\pi}_{b}{}^{aa'}\,{}^{2^-}\mathcal{T}^\parallel_{laa'} \approx 0
\end{aligned}$$

** ViewTheory: The 1+ part of the angular super-momentum is:

$$\overset{1^+}{\cdot}\mathcal{H}_{\mathcal{A}nm} \equiv -2\,\overset{1^+}{\cdot}\hat{\pi}_{bmn} - \frac{\epsilon^{\parallel}{}_{mna}\;\overset{0^-}{\cdot}\hat{\pi}_{\mathcal{A}}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\,\mathcal{T}\right]}{6\,\mathcal{T}} + \frac{4\;\overset{2^-}{\cdot}\hat{\pi}_{\mathcal{A}mna}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\,\mathcal{T}\right]}{3\,\mathcal{T}} + \frac{1}{6}\,\epsilon^{\parallel}{}_{mna}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\;\overset{0^-}{\cdot}\hat{\pi}_{\mathcal{A}}\right] -$$

$$\frac{1}{2}\;\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}na}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\,n_m\right] - \frac{1}{2}\;\overset{2^+}{\cdot}\hat{\pi}_{\mathcal{A}na}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\,n_m\right] + \frac{1}{2}\;\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}ma}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\,n_n\right] + \frac{1}{2}\;\overset{2^+}{\cdot}\hat{\pi}_{\mathcal{A}ma}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\,n_n\right] -$$

$$\frac{4}{3}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel aa'}\;\left(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\;\overset{2^-}{\cdot}\hat{\pi}_{\mathcal{A}mna}\right)\right] + \frac{\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}n}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{m}\,\mathcal{T}\right]}{2\,\mathcal{T}} - \frac{1}{2}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{m}\;\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}n}\right] - \frac{1}{2}\;\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}na}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{m}\,n^a\right] -$$

$$\frac{1}{2}\;\overset{2^+}{\cdot}\hat{\pi}_{\mathcal{A}na}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{m}\,n^a\right] - \frac{\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}m}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{n}\,\mathcal{T}\right]}{2\,\mathcal{T}} + \frac{1}{2}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{n}\;\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}m}\right] + \frac{1}{2}\;\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}ma}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{n}\,n^a\right] +$$

$$\frac{1}{2}\;\overset{2^+}{\cdot}\hat{\pi}_{\mathcal{A}ma}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{n}\,n^a\right] + \frac{1}{6}\,\epsilon^{\parallel}{}_{mna}\;\overset{0^-}{\cdot}\hat{\pi}_{\mathcal{A}}\;\overset{1^-}{\cdot}\mathcal{T}^{\parallel a} - \frac{4}{3}\;\overset{2^-}{\cdot}\hat{\pi}_{\mathcal{A}mna}\;\overset{1^-}{\cdot}\mathcal{T}^{\parallel a} - \frac{1}{2}\;\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}n}\;\overset{1^-}{\cdot}\mathcal{T}^{\parallel}{}_{m} + \frac{1}{2}\;\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}m}\;\overset{1^-}{\cdot}\mathcal{T}^{\parallel}{}_{n} -$$

$$\frac{1}{2}\;\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}n}{}^{a}\;\overset{1^+}{\cdot}\mathcal{T}^{\parallel}{}_{ma} - \frac{1}{2}\;\overset{2^+}{\cdot}\hat{\pi}_{\mathcal{A}n}{}^{a}\;\overset{1^+}{\cdot}\mathcal{T}^{\parallel}{}_{ma} - \frac{1}{3}\;\overset{0^+}{\cdot}\hat{\pi}_{\mathcal{A}}\;\overset{1^+}{\cdot}\mathcal{T}^{\parallel}{}_{mn} + \frac{1}{2}\;\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}m}{}^{a}\;\overset{1^+}{\cdot}\mathcal{T}^{\parallel}{}_{na} + \frac{1}{2}\;\overset{2^+}{\cdot}\hat{\pi}_{\mathcal{A}m}{}^{a}\;\overset{1^+}{\cdot}\mathcal{T}^{\parallel}{}_{na} \approx 0$$

** ViewTheory: The on-shell brackets between the various if-constraints are:

$$\left\{\iiint\left(\overset{0^+}{\cdot}\phi_b\,.\,\mathcal{S}_{(1)}\right)[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1,\;\iiint\left(\overset{0^+}{\cdot}\phi_b\,.\,\mathcal{S}_{(2)}\right)[x^0,\,y^1,\,y^2,\,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left(\overset{0^+}{\cdot}\phi_b\,.\,\mathcal{S}_{(1)}\right)[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1,\;\iiint\left(\overset{1^-}{\cdot}\phi_{b\,l}\,.\,\mathcal{S}_{(2)}{}^{l}\right)[x^0,\,y^1,\,y^2,\,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left(\overset{0^+}{\cdot}\phi_b\,.\,\mathcal{S}_{(1)}\right)[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1,\;\iiint\left(\overset{2^+}{\cdot}\phi_{b\,lm}\,.\,\mathcal{S}_{(2)}{}^{lm}\right)[x^0,\,y^1,\,y^2,\,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left(\overset{0^+}{\cdot}\phi_b\,.\,\mathcal{S}_{(1)}\right)[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1,\;\iiint\left(\overset{0^+}{\cdot}\phi_{\mathcal{A}}\,.\,\mathcal{S}_{(2)}\right)[x^0,\,y^1,\,y^2,\,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\left(-\frac{6\,\mathcal{M}_{Pl}{}^{2}\,\underset{0}{\hat{\alpha}}\,.\,\mathcal{S}_{(1)}\,\mathcal{S}_{(2)}}{\mathcal{T}}\right)[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left(\overset{0^+}{\cdot}\phi_b\,.\,\mathcal{S}_{(1)}\right)[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1,\;\iiint\left(\overset{0^-}{\cdot}\phi_{\mathcal{A}}\,.\,\mathcal{S}_{(2)}\right)[x^0,\,y^1,\,y^2,\,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left(\overset{0^+}{\cdot}\phi_b\,.\,\mathcal{S}_{(1)}\right)[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1,\;\iiint\left(\overset{2^+}{\cdot}\phi_{\mathcal{A}\,lm}\,.\,\mathcal{S}_{(2)}{}^{lm}\right)[x^0,\,y^1,\,y^2,\,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left(\overset{0^+}{\cdot}\phi_b\,.\,\mathcal{S}_{(1)}\right)[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1,\;\iiint\left(\overset{2^-}{\cdot}\phi_{\mathcal{A}\,lmn}\,.\,\mathcal{S}_{(2)}{}^{lmn}\right)[x^0,\,y^1,\,y^2,\,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left(\overset{1^-}{\cdot}\phi_{b\,i}\,.\,\mathcal{S}_{(1)}{}^{i}\right)[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1,\;\iiint\left(\overset{1^-}{\cdot}\phi_{b\,l}\,.\,\mathcal{S}_{(2)}{}^{l}\right)[x^0,\,y^1,\,y^2,\,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\frac{2\;\overset{1^+}{\cdot}\hat{\pi}_{b\,aa'}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{a}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{a'}\right]}{\mathcal{T}^2}[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left(\overset{1^-}{\cdot}\phi_{b\,i}\,.\,\mathcal{S}_{(1)}{}^{i}\right)[x^0,\,x^1,\,x^2,\,x^3]\,dx^3\,dx^2\,dx^1,\;\iiint\left(\overset{2^+}{\cdot}\phi_{b\,lm}\,.\,\mathcal{S}_{(2)}{}^{lm}\right)[x^0,\,y^1,\,y^2,\,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left(\overset{1^{-}}{\cdot}\phi_{b\,i}\cdot\mathcal{S}_{(1)}{}^{i}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{0^{+}}{\cdot}\phi_{\mathcal{A}}\cdot\mathcal{S}_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\left(-\frac{\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\ \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)a}\right]\mathcal{S}_{(2)}}{\mathcal{J}^2}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left(\overset{1^{-}}{\cdot}\phi_{b\,i}\cdot\mathcal{S}_{(1)}{}^{i}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{0^{-}}{\cdot}\phi_{\mathcal{A}}\cdot\mathcal{S}_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\frac{2\ \epsilon\gamma_{aa'iZa1}\ \overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{iZa1}\ \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{a'}\right]\mathcal{S}_{(2)}\ n^{a}}{\mathcal{J}^2}[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left(\overset{1^{-}}{\cdot}\phi_{b\,i}\cdot\mathcal{S}_{(1)}{}^{i}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{2^{+}}{\cdot}\phi_{\mathcal{A}\,lm}\cdot\mathcal{S}_{(2)}{}^{lm}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\frac{\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\left(3\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{a'}\right]\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)aa'}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a'a}\right]\right)-2\ \eta^{\parallel}{}_{a'b}\ \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)a}\right]\ \mathcal{S}_{(2)}{}^{a'b}\right)}{12\,\mathcal{J}^2}[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left(\overset{1^{-}}{\cdot}\phi_{b\,i}\cdot\mathcal{S}_{(1)}{}^{i}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{2^{-}}{\cdot}\phi_{\mathcal{A}\,lmn}\cdot\mathcal{S}_{(2)}{}^{lmn}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\frac{1}{8\,\mathcal{J}^2}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{a}\right]\left(\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a'b}\left(-2\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a}{}^{a'b}\right]+2\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{a'}{}_{a}{}^{b}\right]+4\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{a'b}{}_{a}\right]\right)+\right.$$
$$\left.3\ \overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a}{}^{a'}\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{bb'}\ \mathcal{S}_{(2)a'}{}^{bb'}\right]-\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{bb'}\ \mathcal{S}_{(2)}{}^{b}{}_{a'}{}^{b'}\right]\right)\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left(\overset{2^{+}}{\cdot}\phi_{b\,ij}\cdot\mathcal{S}_{(1)}{}^{ij}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{2^{+}}{\cdot}\phi_{b\,lm}\cdot\mathcal{S}_{(2)}{}^{lm}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\frac{\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'}\right]\left(\overset{1^{+}}{\cdot}\hat{\pi}_{ba'b}\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a}{}^{b}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b}{}_{a}\right]\right)+\overset{1^{+}}{\cdot}\hat{\pi}_{bab}\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a'}{}^{b}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b}{}_{a'}\right]\right)\right)}{2\,\mathcal{J}^2}[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left(\overset{2^{+}}{\cdot}\phi_{b\,ij}\cdot\mathcal{S}_{(1)}{}^{ij}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{0^{+}}{\cdot}\phi_{\mathcal{A}}\cdot\mathcal{S}_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left(\overset{2^{+}}{\cdot}\phi_{b\,ij}\cdot\mathcal{S}_{(1)}{}^{ij}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{0^{-}}{\cdot}\phi_{\mathcal{A}}\cdot\mathcal{S}_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left(\overset{2^{+}}{\cdot}\phi_{b\,ij}\cdot\mathcal{S}_{(1)}{}^{ij}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{2^{+}}{\cdot}\phi_{\mathcal{A}\,lm}\cdot\mathcal{S}_{(2)}{}^{lm}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\frac{1}{12\,\mathcal{J}^2}\left(3\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'}\right]\left(2\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\underset{0}{\hat{\alpha}}\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)aa'}\right]+2\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\underset{0}{\hat{\alpha}}\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a'a}\right]+\right.\right.$$
$$\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}ab}\ \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a'}{}^{b}\right]+\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a'b}\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a}{}^{b}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b}{}_{a}\right]\right)+\left.\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}ab}\ \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b}{}_{a'}\right]\right)-$$
$$\left.4\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\underset{0}{\hat{\alpha}}\ \eta^{\parallel}{}_{aa'}\ \eta^{\parallel}{}_{bb'}\ \mathcal{J}\ \mathcal{S}_{(1)}{}^{aa'}\ \mathcal{S}_{(2)}{}^{bb'}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left({}^{2^+}\phi_{b\,ij}\cdot S_{(1)}{}^{ij}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{2^-}\phi_{\mathcal{A}\,lmn}\cdot S_{(2)}{}^{lmn}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\frac{1}{32\,\mathcal{J}^2}3\,{}^{1^-}\hat{\pi}_{\mathcal{A}}{}^{a}\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[S_{(1)}{}^{a'}{}_{a}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{bb'}\ S_{(2)}{}_{a'}{}^{bb'}\right]+2\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[S_{(1)}{}^{a'b}\right]\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[S_{(2)}{}_{aa'b}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[S_{(2)}{}_{aba'}\right]-\underset{\eta^{\parallel}}{\mathrm{P}}\left[S_{(2)}{}_{a'ab}\right]-\underset{\eta^{\parallel}}{\mathrm{P}}\left[S_{(2)}{}_{baa'}\right]\right)+\underset{\eta^{\parallel}}{\mathrm{P}}\left[S_{(1)}{}_{a}{}^{a'}\right]\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{bb'}\ S_{(2)}{}_{a'}{}^{bb'}\right]-\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{bb'}\ S_{(2)}{}^{b}{}_{a'}{}^{b'}\right]\right)-\underset{\eta^{\parallel}}{\mathrm{P}}\left[S_{(1)}{}^{a'}{}_{a}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{bb'}\ S_{(2)}{}^{b}{}_{a'}{}^{b'}\right]-2\,\eta^{\parallel}{}_{a'b}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\ S_{(2)}{}_{a}{}^{b'c}\right]S_{(1)}{}^{a'b}+2\,\eta^{\parallel}{}_{a'b}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\ S_{(2)}{}^{b'}{}_{a}{}^{c}\right]S_{(1)}{}^{a'b}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left({}^{0^+}\phi_{\mathcal{A}}\cdot S_{(1)}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{0^+}\phi_{\mathcal{A}}\cdot S_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left({}^{0^+}\phi_{\mathcal{A}}\cdot S_{(1)}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{0^-}\phi_{\mathcal{A}}\cdot S_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left({}^{0^+}\phi_{\mathcal{A}}\cdot S_{(1)}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{2^+}\phi_{\mathcal{A}\,lm}\cdot S_{(2)}{}^{lm}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left({}^{0^+}\phi_{\mathcal{A}}\cdot S_{(1)}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{2^-}\phi_{\mathcal{A}\,lmn}\cdot S_{(2)}{}^{lmn}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left({}^{0^-}\phi_{\mathcal{A}}\cdot S_{(1)}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{0^-}\phi_{\mathcal{A}}\cdot S_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left({}^{0^-}\phi_{\mathcal{A}}\cdot S_{(1)}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{2^+}\phi_{\mathcal{A}\,lm}\cdot S_{(2)}{}^{lm}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left({}^{0^-}\phi_{\mathcal{A}}\cdot S_{(1)}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{2^-}\phi_{\mathcal{A}\,lmn}\cdot S_{(2)}{}^{lmn}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left({}^{2^+}\phi_{\mathcal{A}\,ij}\cdot S_{(1)}{}^{ij}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{2^+}\phi_{\mathcal{A}\,lm}\cdot S_{(2)}{}^{lm}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left({}^{2^+}\phi_{\mathcal{A}\,ij}\cdot S_{(1)}{}^{ij}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{2^-}\phi_{\mathcal{A}\,lmn}\cdot S_{(2)}{}^{lmn}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

$$\left\{\iiint\left({}^{2^-}\phi_{\mathcal{A}\,ijk}\cdot S_{(1)}{}^{ijk}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{2^-}\phi_{\mathcal{A}\,lmn}\cdot S_{(2)}{}^{lmn}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx 0$$

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** ViewTheory: The commutators between
   the various if-constraints and the super-Hamiltonian are:
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$$\left\{\iiint\left({}^{0^+}\phi_{b}\cdot S_{(1)}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{0^+}\mathcal{H}_{b}\cdot S_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\frac{1}{144\,\hat{\alpha}_{5}\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{3}\,\mathcal{J}^2}S_{(1)}\,S_{(2)}\Big(9\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{3}\,{}^{1^-}\hat{\pi}_{\mathcal{A}a}\,{}^{1^-}\hat{\pi}_{\mathcal{A}}{}^{a}+18\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{3}\,{}^{1^+}\hat{\pi}_{\mathcal{A}aa'}\,{}^{1^+}\hat{\pi}_{\mathcal{A}}{}^{aa'}+81\,\hat{\alpha}_{5}\,{}^{1^+}\hat{\pi}_{baa'}\,{}^{1^+}\hat{\pi}_{b}{}^{aa'}-8\,\hat{\alpha}_{5}\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{3}\,\mathcal{J}\Big(9\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\alpha}_{0}\,\mathcal{J}\,{}^{0^+}\mathcal{R}^{\parallel}+9\,{}^{1^-}\hat{\pi}_{\mathcal{A}}{}^{a}\,{}^{1^-}\mathcal{R}^{\parallel}{}_{a}+54\,{}^{1^+}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,{}^{1^+}\mathcal{R}^{\parallel}{}_{aa'}+3\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{3}\,\mathcal{J}\,{}^{0^-}\mathcal{T}^{\parallel 2}-9\,{}^{1^+}\hat{\pi}_{b}{}^{aa'}\,{}^{1^+}\mathcal{T}^{\parallel}{}_{aa'}-32\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{3}\,\mathcal{J}\,{}^{2^-}\mathcal{T}^{\parallel}{}_{aa'Za1}\,{}^{2^-}\mathcal{T}^{\parallel aa'Za1}+64\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{3}\,\mathcal{J}\,{}^{2^-}\mathcal{T}^{\parallel}{}_{aZa1a'}\,{}^{2^-}\mathcal{T}^{\parallel aa'Za1}+18\,\eta^{\parallel a'Za1}\,n^{a}\left({}^{3}\mathcal{D}^{\parallel}{}_{Za1}\,{}^{1^+}\hat{\pi}_{baa'}\right)\Big)\Big)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left({}^{1^-}_{\bullet}\phi_{b\,i}\cdot \mathcal{S}_{(1)}{}^{i}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{0^+}_{\bullet}\mathcal{H}_b\cdot\mathcal{S}_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint \frac{1}{12\,\hat\alpha_5\, \mathcal{J}^2}\mathcal{S}_{(2)}\Big(-\mathcal{J}\ {}^{1^-}_{\bullet}\hat\pi_{\mathcal{A}}{}^{a}\ \Big(\underset{\eta^\parallel}{P}[\mathcal{S}_{(1)a}]\Big(3\, M_{Pl}^2\,\hat\alpha_0 + 2\,\hat\alpha_5\ {}^{0^+}_{\bullet}\mathcal{R}^\parallel\Big) + 6\,\hat\alpha_5\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a'}]\ {}^{2^+}_{\bullet}\mathcal{R}^\parallel{}_{aa'}\Big) +$$
$$2\,\hat\alpha_5\Big(12\ {}^{1^+}_{\bullet}\hat\pi_{baa'}\ \Big(\underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a}]\ \underset{\eta^\parallel}{P}[{}^{3}_{\bullet}\mathcal{D}^{\parallel a'}\,\mathcal{J}] + \mathcal{J}\,\underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a'}]\ {}^{1^-}_{\bullet}\mathcal{T}^{\parallel a}\Big) -$$
$$4\,\mathcal{J}\,\underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a}]\Big(3\,\underset{\eta^\parallel}{P}\Big[\eta^{\parallel a'b}\ \Big({}^{3}_{\bullet}\mathcal{D}^\parallel{}_b\ {}^{1^+}_{\bullet}\hat\pi_{baa'}\Big)\Big] - 2\ {}^{1^+}_{\bullet}\hat\pi_{\mathcal{A}}{}^{a'b}\ \ {}^{2^-}_{\bullet}\mathcal{R}^\parallel{}_{aa'b} + M_{Pl}^2\,\hat\beta_3\ \epsilon^\parallel{}_{aa'b}\ \mathcal{J}\ {}^{0^-}_{\bullet}\mathcal{T}^\parallel\ \ {}^{1^+}_{\bullet}\mathcal{T}^{\parallel a'b}\Big) +$$
$$\mathcal{J}\,\underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a'}]\Big(6\ {}^{1^+}_{\bullet}\hat\pi_{\mathcal{A}aa'}\ \ {}^{1^-}_{\bullet}\mathcal{R}^{\parallel a} + \epsilon\gamma_{aa'bc}\ \ {}^{1^+}_{\bullet}\hat\pi_{\mathcal{A}}{}^{bc}\ \ {}^{0^-}_{\bullet}\mathcal{R}^\parallel\ n^a\Big) -$$
$$6\,\mathcal{J}\ {}^{1^+}_{\bullet}\hat\pi_{ba}{}^{a'}\ \underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a}]\Big(\underset{\eta^\parallel}{P}\Big[n^b\ \Big({}^{3}_{\bullet}\mathcal{D}^\parallel{}_{a'}\,n_b\Big)\Big] - 2\Big({}^{3}_{\bullet}\mathcal{D}^\parallel{}_b\,\eta^\parallel{}_{a'}{}^{b}\Big)\Big)\Big)\Big)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left({}^{2^+}_{\bullet}\phi_{b\,i\,j}\cdot \mathcal{S}_{(1)}{}^{i\,j}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{0^+}_{\bullet}\mathcal{H}_b\cdot\mathcal{S}_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint \frac{1}{144\,\hat\alpha_5\,\mathcal{J}^2}\mathcal{S}_{(2)}$$
$$\Big(3\ {}^{1^-}_{\bullet}\hat\pi_{\mathcal{A}}{}^{a}\ \Big(3\ {}^{1^-}_{\bullet}\hat\pi_{\mathcal{A}}{}^{a'}\ \underset{\eta^\parallel}{P}[\mathcal{S}_{(1)aa'}] - 12\,\hat\alpha_5\,\mathcal{J}\,\underset{\eta^\parallel}{P}[\mathcal{S}_{(1)aa'}]\ {}^{1^-}_{\bullet}\mathcal{R}^{\parallel a'} - 12\,\hat\alpha_5\,\mathcal{J}\,\underset{\eta^\parallel}{P}[\mathcal{S}_{(1)a'a}]\ {}^{1^-}_{\bullet}\mathcal{R}^{\parallel a'} - 16\,\hat\alpha_5\,\mathcal{J}\,\underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a'b}]$$
$${}^{2^-}_{\bullet}\mathcal{R}^\parallel{}_{aa'b} - 16\,\hat\alpha_5\,\mathcal{J}\,\underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a'b}]\ {}^{2^-}_{\bullet}\mathcal{R}^\parallel{}_{aba'} - \eta^\parallel{}_{a'b}\ \ {}^{1^-}_{\bullet}\hat\pi_{\mathcal{A}a}\ \ \mathcal{S}_{(1)}{}^{a'b} + 8\,\hat\alpha_5\ \eta^\parallel{}_{a'b}\ \mathcal{J}\ {}^{1^-}_{\bullet}\mathcal{R}^\parallel{}_a\ \ \mathcal{S}_{(1)}{}^{a'b}\Big) +$$
$$4\Big(-9\ {}^{1^+}_{\bullet}\hat\pi_{\mathcal{A}a}{}^{b}\ \underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{aa'}]\Big({}^{1^+}_{\bullet}\hat\pi_{\mathcal{A}a'b} + 2\,\hat\alpha_5\,\mathcal{J}\ {}^{2^+}_{\bullet}\mathcal{R}^\parallel{}_{a'b}\Big) + \hat\alpha_5\,\mathcal{J}\,\underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{aa'}]\Big(18\,\underset{\eta^\parallel}{P}\Big[n^b\ \Big({}^{3}_{\bullet}\mathcal{D}^\parallel{}_a\ {}^{1^+}_{\bullet}\hat\pi_{ba'b}\Big)\Big] +$$
$$18\,\underset{\eta^\parallel}{P}\Big[n^b\ \Big({}^{3}_{\bullet}\mathcal{D}^\parallel{}_{a'}\ {}^{1^+}_{\bullet}\hat\pi_{bab}\Big)\Big] + 9\ {}^{1^+}_{\bullet}\hat\pi_{bab}\ \underset{\eta^\parallel}{P}[{}^{3}_{\bullet}\mathcal{D}^\parallel{}_{a'}\,n^b] + 9\ {}^{1^+}_{\bullet}\hat\pi_{ba'b}\ \Big(\underset{\eta^\parallel}{P}[{}^{3}_{\bullet}\mathcal{D}^\parallel{}_a\,n^b] + \underset{\eta^\parallel}{P}[{}^{3}_{\bullet}\mathcal{D}^{\parallel b}\,n_a]\Big) + 9$$
$${}^{1^+}_{\bullet}\hat\pi_{bab}\ \underset{\eta^\parallel}{P}[{}^{3}_{\bullet}\mathcal{D}^{\parallel b}\,n_{a'}] + 36\,M_{Pl}^2\,\hat\alpha_0\,\mathcal{J}\ {}^{2^+}_{\bullet}\mathcal{R}^\parallel{}_{aa'} - 18\ {}^{1^+}_{\bullet}\hat\pi_{\mathcal{A}a'}{}^{b}\ \ {}^{2^+}_{\bullet}\mathcal{R}^\parallel{}_{ab} + 9\ {}^{1^+}_{\bullet}\hat\pi_{ba'}{}^{b}\ \ {}^{1^+}_{\bullet}\mathcal{T}^\parallel{}_{ab} + 9$$
$${}^{1^+}_{\bullet}\hat\pi_{ba}{}^{b}\ \ {}^{1^+}_{\bullet}\mathcal{T}^\parallel{}_{a'b} + 16\,M_{Pl}^2\,\hat\beta_3\ \epsilon^\parallel{}_{a'bb'}\ \mathcal{J}\ {}^{0^-}_{\bullet}\mathcal{T}^\parallel\ \ {}^{2^-}_{\bullet}\mathcal{T}^\parallel{}_a{}^{bb'} + 16\,M_{Pl}^2\,\hat\beta_3\ \epsilon^\parallel{}_{abb'}\ \mathcal{J}\ {}^{0^-}_{\bullet}\mathcal{T}^\parallel$$
$${}^{2^-}_{\bullet}\mathcal{T}^\parallel{}_{a'}{}^{bb'} - 8\,M_{Pl}^2\,\hat\beta_3\ \epsilon^\parallel{}_{a'bb'}\ \mathcal{J}\ {}^{0^-}_{\bullet}\mathcal{T}^\parallel\ \ {}^{2^-}_{\bullet}\mathcal{T}^{\parallel bb'}{}_a - 8\,M_{Pl}^2\,\hat\beta_3\ \epsilon^\parallel{}_{abb'}\ \mathcal{J}\ {}^{0^-}_{\bullet}\mathcal{T}^\parallel\ \ {}^{2^-}_{\bullet}\mathcal{T}^{\parallel bb'}{}_{a'}\Big) +$$
$$3\ \eta^\parallel{}_{aa'}\ \ \mathcal{S}_{(1)}{}^{aa'}\ \Big({}^{1^+}_{\bullet}\hat\pi_{\mathcal{A}bb'}\ \ {}^{1^+}_{\bullet}\hat\pi_{\mathcal{A}}{}^{bb'} - 2\,\hat\alpha_5\,\mathcal{J}\ {}^{1^+}_{\bullet}\hat\pi_b{}^{bb'}\ \ {}^{1^+}_{\bullet}\mathcal{T}^\parallel{}_{bb'} + 4\,\hat\alpha_5\ \eta^{\parallel b'c}$$
$$\mathcal{J}\ n^b\ \Big({}^{3}_{\bullet}\mathcal{D}^\parallel{}_c\ {}^{1^+}_{\bullet}\hat\pi_{bbb'}\Big)\Big)\Big)\Big)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left({}^{0^+}_{\bullet}\phi_{\mathcal{A}}\cdot \mathcal{S}_{(1)}\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left({}^{0^+}_{\bullet}\mathcal{H}_b\cdot\mathcal{S}_{(2)}\right)[x^0,y^1,y^2,y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint \frac{1}{8\,M_{Pl}^2\,\hat\beta_3\,\mathcal{J}^2}\mathcal{S}_{(1)}\,\mathcal{S}_{(2)}\Big({}^{1^+}_{\bullet}\hat\pi_{\mathcal{A}}{}^{ab}\ \Big(3\ {}^{1^+}_{\bullet}\hat\pi_{bab} + 4\,M_{Pl}^2\,\hat\beta_3\,\mathcal{J}\ {}^{1^+}_{\bullet}\mathcal{T}^\parallel{}_{ab}\Big) + 4\,M_{Pl}^2\,\hat\beta_3\ {}^{1^-}_{\bullet}\hat\pi_{\mathcal{A}}{}^{a}\ \Big(2\,\underset{\eta^\parallel}{P}[{}^{3}_{\bullet}\mathcal{D}^\parallel{}_a\,\mathcal{J}] - \mathcal{J}\ {}^{1^-}_{\bullet}\mathcal{T}^\parallel{}_a +$$
$$2\,\mathcal{J}\Big({}^{3}_{\bullet}\mathcal{D}^\parallel{}_b\,\eta^\parallel{}_a{}^{b}\Big)\Big) - 8\,M_{Pl}^2\,\hat\beta_3\ \eta^\parallel{}_{ab}\ \mathcal{J}\Big({}^{3}_{\bullet}\mathcal{D}^{\parallel b}\ {}^{1^-}_{\bullet}\hat\pi_{\mathcal{A}}{}^{a}\Big)\Big)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left(\overset{0^-}{\cdot}\phi_{\mathcal{A}}\cdot\mathcal{S}_{(1)}\right)[x^0,\ x^1,\ x^2,\ x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{0^+}{\cdot}\mathcal{H}_{b}\cdot\mathcal{S}_{(2)}\right)[x^0,\ y^1,\ y^2,\ y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\frac{1}{24\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}^2}\mathcal{S}_{(1)}\Bigg(24\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\epsilon^{\parallel}{}_{aa'b}\Bigg(\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a'b}\Bigg(4\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\,\mathcal{S}_{(2)}\Big]-4\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\,\mathcal{J}\Big]\mathcal{S}_{(2)}+3\,\mathcal{J}\,\mathcal{S}_{(2)}\,\overset{1^-}{\cdot}\mathcal{T}^{\parallel a}\Bigg)+$$

$$\mathcal{J}\,\mathcal{S}_{(2)}\Bigg(2\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\Big]+\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\overset{1^+}{\cdot}\mathcal{T}^{\parallel a'b}\Bigg)\Bigg)+\mathcal{S}_{(2)}\Bigg(64\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\epsilon^{\parallel}{}_{a'bb'}\,\mathcal{J}\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,\overset{2^-}{\cdot}\mathcal{T}^{\parallel}{}_{a}{}^{bb'}-$$

$$3\Bigg(\epsilon\gamma_{aa'bb'}\Bigg(-\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\Big(3\,\overset{1^+}{\cdot}\hat{\pi}_{b}{}^{bb'}+4\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}\,\overset{1^+}{\cdot}\mathcal{T}^{\parallel bb'}\Big)\,n^{a'}+16\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{bb'}$$

$$\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel a'}\,\mathcal{J}\Big]\,n^{a}+\mathcal{J}\,\overset{1^-}{\cdot}\mathcal{T}^{\parallel a}\,n^{a'}\Big)\Bigg)+8\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}\Bigg(\Big(\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\alpha}_{\underset{0}{\cdot}}+8\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\Big)\mathcal{J}\,\overset{0^-}{\cdot}\mathcal{T}^{\parallel}-$$

$$2\,\epsilon^{\parallel}{}_{aa'b'}\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\Big(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\,\eta^{\parallel bb'}\Big)\Bigg)\Bigg)\Bigg)\Bigg)[x^0,\ x^1,\ x^2,\ x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left(\overset{2^+}{\cdot}\phi_{\mathcal{A}ij}\cdot\mathcal{S}_{(1)}{}^{ij}\right)[x^0,\ x^1,\ x^2,\ x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{0^+}{\cdot}\mathcal{H}_{b}\cdot\mathcal{S}_{(2)}\right)[x^0,\ y^1,\ y^2,\ y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\frac{1}{48\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}^2}\mathcal{S}_{(2)}\Bigg(12\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{aa'}\Big]\Bigg(\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a}\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}a'}\Big]+\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}a}\Big]+\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}ab}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\,n^{b}\Big]+$$

$$\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}a'b}\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a}\,n^{b}\Big]+\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,n_{a}\Big]\Big)+\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}ab}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,n_{a'}\Big]\Bigg)+3\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}a'}{}^{b}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{aa'}\Big]$$

$$\Big(3\,\overset{1^+}{\cdot}\hat{\pi}_{bab}+16\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}\,\overset{1^+}{\cdot}\mathcal{T}^{\parallel}{}_{ab}\Big)+3\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}a}{}^{b}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{aa'}\Big]\Big(3\,\overset{1^+}{\cdot}\hat{\pi}_{ba'b}+16\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}\,\overset{1^+}{\cdot}\mathcal{T}^{\parallel}{}_{a'b}\Big)+$$

$$2\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\Big(4\,\eta^{\parallel}{}_{a'b}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a}\mathcal{J}\Big]\,\mathcal{S}_{(1)}{}^{a'b}-2\,\eta^{\parallel}{}_{a'b}\,\mathcal{J}\,\mathcal{S}_{(1)}{}^{a'b}\,\overset{1^-}{\cdot}\mathcal{T}^{\parallel}{}_{a}+3\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)aa'}\Big]\,\overset{1^-}{\cdot}\mathcal{T}^{\parallel a'}+$$

$$3\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)a'a}\Big]\,\overset{1^-}{\cdot}\mathcal{T}^{\parallel a'}+8\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{a'b}\Big]\,\overset{2^-}{\cdot}\mathcal{T}^{\parallel}{}_{aa'b}+8\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{a'b}\Big]\,\overset{2^-}{\cdot}\mathcal{T}^{\parallel}{}_{aba'}-$$

$$6\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)a}{}^{a'}\Big]\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\,\mathcal{J}\Big]+\mathcal{J}\Big(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\,\eta^{\parallel}{}_{a'}{}^{b}\Big)\Big)-6\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{a'}{}_{a}\Big]\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\,\mathcal{J}\Big]+\mathcal{J}\Big(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\,\eta^{\parallel}{}_{a'}{}^{b}\Big)\Big)+$$

$$4\,\eta^{\parallel}{}_{a'b}\,\mathcal{J}\,\mathcal{S}_{(1)}{}^{a'b}\Big(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b'}\,\eta^{\parallel}{}_{a}{}^{b'}\Big)\Big)-2\,\eta^{\parallel}{}_{aa'}\,\mathcal{S}_{(1)}{}^{aa'}\Big(\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{bb'}\Big(3\,\overset{1^+}{\cdot}\hat{\pi}_{bbb'}+16\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}\,\overset{1^+}{\cdot}\mathcal{T}^{\parallel}{}_{bb'}\Big)+$$

$$4\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\eta^{\parallel}{}_{bb'}\,\mathcal{J}\Big(\overset{3}{\cdot}\mathcal{D}^{\parallel b'}\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{b}\Big)\Big)\Big)\Big)[x^0,\ x^1,\ x^2,\ x^3]\,dx^3\,dx^2\,dx^1$$

$$\left\{\iiint\left(\overset{2^-}{\cdot}\phi_{\mathcal{A}ijk}\cdot\mathcal{S}_{(1)}{}^{ijk}\right)[x^0,\ x^1,\ x^2,\ x^3]\,dx^3\,dx^2\,dx^1,\ \iiint\left(\overset{0^+}{\cdot}\mathcal{H}_{b}\cdot\mathcal{S}_{(2)}\right)[x^0,\ y^1,\ y^2,\ y^3]\,dy^3\,dy^2\,dy^1\right\}\approx$$

$$\iiint\Bigg(-\frac{1}{384\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}^2}\Bigg(192\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}ab}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{aa'b}\Big]\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\,\mathcal{S}_{(2)}\Big]+384\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}aa'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{aa'b}\Big]$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\,\mathcal{S}_{(2)}\Big]-18\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\overset{1^+}{\cdot}\hat{\pi}_{ba'b}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)a}{}^{a'b}\Big]\mathcal{S}_{(2)}+27\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\overset{1^+}{\cdot}\hat{\pi}_{ba}{}^{a'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{bb'}\,\mathcal{S}_{(1)a'}{}^{bb'}\Big]\mathcal{S}_{(2)}+$$

$$18\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\overset{1^+}{\cdot}\hat{\pi}_{ba'b}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{a'}{}_{a}{}^{b}\Big]\mathcal{S}_{(2)}+36\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\overset{1^+}{\cdot}\hat{\pi}_{ba'b}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{a'b}{}_{a}\Big]\mathcal{S}_{(2)}-$$

$$27\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\overset{1^+}{\cdot}\hat{\pi}_{ba}{}^{a'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{bb'}\,\mathcal{S}_{(1)}{}^{b}{}_{a'}{}^{b'}\Big]\mathcal{S}_{(2)}-96\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{aa'b}\Big]\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a}\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}a'b}\Big]\mathcal{S}_{(2)}+$$

$$36\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\delta_{a'}{}^{b}\,\mathcal{J}\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{b'c}\,\mathcal{S}_{(1)b}{}^{b'c}\Big]\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a}\,n^{a'}\Big]\mathcal{S}_{(2)}-36\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\delta_{a'}{}^{b}\,\mathcal{J}\,\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{b'c}\,\mathcal{S}_{(1)}{}^{b'}{}_{b}{}^{c}\Big]\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a}\,n^{a'}\Big]\mathcal{S}_{(2)}-96\,\mathcal{M}_{\mathrm{Pl}}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}ab}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\mathcal{S}_{(1)}{}^{aa'b}\Big]\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\,\mathcal{J}\Big]\mathcal{S}_{(2)}+$$

$$96\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}ab}\Big]\,\mathcal{S}_{(2)}+72\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}_{a}{}^{a'b}\Big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\,n_{b}\Big]\,\mathcal{S}_{(2)}-$$

$$72\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{a'}{}_{a}{}^{b}\Big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\,n_{b}\Big]\,\mathcal{S}_{(2)}+36\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{bb'}\,{\mathcal{S}_{(1)}}_{a'}{}^{bb'}\Big]$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel a'}\,n_{a}\Big]\,\mathcal{S}_{(2)}-36\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{bb'}\,{\mathcal{S}_{(1)}}^{b}{}_{a'}{}^{b'}\Big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel a'}\,n_{a}\Big]\,\mathcal{S}_{(2)}-$$

$$192\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}aa'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\,\mathcal{J}\Big]\,\mathcal{S}_{(2)}+192\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}aa'}\Big]\,\mathcal{S}_{(2)}+$$

$$72\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}_{a}{}^{a'b}\Big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\,n_{a'}\Big]\,\mathcal{S}_{(2)}-72\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{a'}{}_{a}{}^{b}\Big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\,n_{a'}\Big]\,\mathcal{S}_{(2)}-$$

$$144\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{a'b}\,{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel b'c}\,\big(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{c}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}ab'}\big)\Big]\,\mathcal{S}_{(2)}+144\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{ab}\,{\mathcal{S}_{(1)}}^{aa'b}\Big]$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel b'c}\,\big(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{c}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a'b'}\big)\Big]\,\mathcal{S}_{(2)}-16\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\epsilon^{\parallel}{}_{a'bb'}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a}{}^{b'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{0^{-}}{\cdot}\mathcal{T}^{\parallel}+$$

$$16\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\epsilon^{\parallel}{}_{abb'}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a'}{}^{b'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{0^{-}}{\cdot}\mathcal{T}^{\parallel}+32\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\epsilon^{\parallel}{}_{aa'b'}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}b}{}^{b'}$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{0^{-}}{\cdot}\mathcal{T}^{\parallel}-24\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\epsilon^{\parallel}{}_{aa'}{}^{b}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{b'c}\,{\mathcal{S}_{(1)}}_{b}{}^{b'c}\Big]\,\mathcal{S}_{(2)}\,\overset{0^{-}}{\cdot}\mathcal{T}^{\parallel}+$$

$$24\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\epsilon^{\parallel}{}_{aa'}{}^{b}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{b'c}\,{\mathcal{S}_{(1)}}^{b'}{}_{b}{}^{c}\Big]\,\mathcal{S}_{(2)}\,\overset{0^{-}}{\cdot}\mathcal{T}^{\parallel}+72\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a}{}^{a'}$$

$$\left(\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{bb'}\,{\mathcal{S}_{(1)}}_{a'}{}^{bb'}\Big]-\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{bb'}\,{\mathcal{S}_{(1)}}^{b}{}_{a'}{}^{b'}\Big]\right)\left(4\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\,\mathcal{S}_{(2)}\Big]-2\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\,\mathcal{J}\Big]\,\mathcal{S}_{(2)}+\mathcal{J}\,\mathcal{S}_{(2)}\,\overset{1^{-}}{\cdot}\mathcal{T}^{\parallel a}\right)+$$

$$256\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\alpha}_{\dot{0}}\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}^{2}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{aa'b}+128\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\alpha}_{\dot{0}}\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}^{2}$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{aba'}-192\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{b'c}\,{\mathcal{S}_{(1)}}_{b}{}^{b'c}\Big]\,\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{a}{}^{b}{}_{a'}+$$

$$192\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{b'c}\,{\mathcal{S}_{(1)}}^{b'}{}_{b}{}^{c}\Big]\,\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{a}{}^{b}{}_{a'}+128\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}b}{}^{b'}$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{ab'a'}+256\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a'}{}^{b'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{ab'b}-$$

$$128\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\alpha}_{\dot{0}}\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}^{2}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{a'ba}-128\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}b}{}^{b'}$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{a'b'a}-256\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a}{}^{b'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{a'b'b}+$$

$$128\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a'}{}^{b'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\,\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{bb'a}-128\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a}{}^{b'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]$$

$$\mathcal{S}_{(2)}\,\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{bb'a'}+144\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{b'c}\,{\mathcal{S}_{(1)}}_{a'}{}^{b'c}\Big]\,\mathcal{S}_{(2)}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\,\eta^{\parallel}{}_{a}{}^{b}\right)-$$

$$144\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{b'c}\,{\mathcal{S}_{(1)}}^{b'}{}_{a'}{}^{c}\Big]\,\mathcal{S}_{(2)}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\,\eta^{\parallel}{}_{a}{}^{b}\right)-72\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\eta^{\parallel}{}_{a'b}\,\mathcal{J}\,\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{b'c}\,{\mathcal{S}_{(1)}}_{a}{}^{b'c}\Big]\,\mathcal{S}_{(2)}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,n^{a'}\right)+72\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\eta^{\parallel}{}_{a'b}\,\mathcal{J}\,\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\eta^{\parallel}{}_{b'c}\,{\mathcal{S}_{(1)}}^{b'}{}_{a}{}^{c}\Big]\,\mathcal{S}_{(2)}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,n^{a'}\right)-$$

$$48\,\mathcal{M}_{\mathrm{Pl}}^{2}\,\hat{\beta}_{\dot{3}}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}a'b}\,\Bigg(\mathcal{J}\left(\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}_{a}{}^{a'b}\Big]-\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{a'}{}_{a}{}^{b}\Big]-2\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{a'b}{}_{a}\Big]\right)\mathcal{S}_{(2)}\,\overset{1^{-}}{\cdot}\mathcal{T}^{\parallel a}+$$

$$2\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[{\mathcal{S}_{(1)}}^{aa'b}\Big]\left(2\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a}\,\mathcal{S}_{(2)}\Big]-\underset{\eta^{\parallel}}{\mathrm{P}}\Big[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a}\,\mathcal{J}\Big]\,\mathcal{S}_{(2)}+\mathcal{J}\,\mathcal{S}_{(2)}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b'}\,\eta^{\parallel}{}_{a}{}^{b'}\right)\right)\Bigg)+$$

$$96\,\mathcal{M}_{\text{Pl}}{}^{2}\,\underset{3}{\hat{\beta}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}ab}\,\underset{\eta^{\parallel}}{\text{P}}\left[\mathcal{S}_{(1)}{}^{aa'b}\right]\mathcal{S}_{(2)}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b'}\,\eta^{\parallel}{}_{a'}{}^{b'}\right)+192\,\mathcal{M}_{\text{Pl}}{}^{2}\,\underset{3}{\hat{\beta}}\,\mathcal{J}\,\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}aa'}$$

$$\left.\underset{\eta^{\parallel}}{\text{P}}\left[\mathcal{S}_{(1)}{}^{aa'b}\right]\mathcal{S}_{(2)}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b'}\,\eta^{\parallel}{}_{b}{}^{b'}\right)\right)\right)[x^{0},\,x^{1},\,x^{2},\,x^{3}]\,dx^{3}\,dx^{2}\,dx^{1}$$

This concludes the HiGGS output.

Concrete relation to manuscript: The above constraints and Poisson brackets constitute the data represented by coloured squares in the left-hand panel of Fig. 2 in our Manuscript. The primary constraints ϕ are listed along the top row. The primary Poisson brackets $\{\phi,\ \phi\}$ are used to infer the white, red or yellow squares in the matrix depending on whether they are vanishing, or appear only in proportion to phase space variables, or are order unity (e.g. proportional to the metric or unit-timelike vector). The $\{\phi,\ \overset{0^{+}}{\cdot}\mathcal{H}_{b}\}$ Poisson brackets define the secondary constraints "χ", which appear on the sides of the matrix. Most of the Poisson brackets $\{\chi,\ \phi\}$ need not be computed (grey squares), but it can be noticed without detailed calculation that the remaining required brackets (yellow squares) do indeed arise, and that these determine all the missing velocities and so terminate the Dirac algorithm.

Healthy spectrum with multipliers

Introductory comments

We now turn to the case in which multipliers have been applied. This time we set $\underset{1}{\hat{\alpha}}$, $\underset{2}{\hat{\alpha}}$, $\underset{3}{\hat{\alpha}}$, $\underset{4}{\hat{\alpha}}$, $\underset{6}{\hat{\alpha}}$, $\mathcal{M}_{\text{Pl}}{}^{2}\,\underset{1}{\hat{\beta}}$, $\mathcal{M}_{\text{Pl}}{}^{2}\,\underset{2}{\hat{\beta}}$, $\mathcal{M}_{\text{Pl}}{}^{2}\,\underset{3}{\bar{\beta}}$, to zero, and allow for nonvanishing $\mathcal{M}_{\text{Pl}}{}^{2}\,\underset{0}{\hat{\alpha}}$, $\underset{5}{\hat{\alpha}}$ and $\mathcal{M}_{\text{Pl}}{}^{2}\,\underset{3}{\hat{\beta}}$, but we also admit $\mathcal{M}_{\text{Pl}}{}^{2}\,\underset{1}{\bar{\beta}}$ and $\mathcal{M}_{\text{Pl}}{}^{2}\,\underset{2}{\bar{\beta}}$. These final two couplings switch on the multipliers in the Lagrangian.

Note this time that because some of the Lagrange multiplier fields are present, certain secondary constraints can be anticipated by HiGGS a priori: this is in accordance with the theory set out in arXiv:2205.13534.

Lists of constraints and Poisson brackets from HiGGS

We will again present the results using the ViewTheory[] command.

```
** DefTheory: Incorporating the binary at ConstraintAlgebraAlp0Alp5Bet3cBet1cBet2.thr.mx
```

```
** DefTheory: Found the following primary if-constraints:
```

$$\overset{0^{+}}{\cdot}\phi_{b}\ \equiv\ \frac{\overset{0^{+}}{\cdot}\hat{\pi}_{b}}{\mathcal{J}}+2\,\mathcal{M}_{\text{Pl}}{}^{2}\,\underset{2}{\bar{\beta}}\,\overset{0^{+}}{\cdot}\lambda^{\perp}_{\mathcal{T}}\ \approx\ 0$$

$$\overset{1^-}{\cdot}\phi_{b\,i} \equiv \frac{\overset{1^-}{\cdot}\hat{\pi}_{b\,i}}{\mathcal{J}} - \frac{2}{3}\left(\mathcal{M}_{\text{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}} - \mathcal{M}_{\text{Pl}}{}^2\,\bar{\beta}_{\underset{2}{\cdot}}\right)\overset{1^-}{\cdot}\lambda^{\parallel}_{\mathcal{T}\,i} + \frac{2}{3}\left(2\,\mathcal{M}_{\text{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}} + \mathcal{M}_{\text{Pl}}{}^2\,\bar{\beta}_{\underset{2}{\cdot}}\right)\overset{1^-}{\cdot}\lambda^{\perp}_{\mathcal{T}\,i} \approx 0$$

$$\overset{2^+}{\cdot}\phi_{b\,i\,j} \equiv \frac{\overset{2^+}{\cdot}\hat{\pi}_{b\,i\,j}}{\mathcal{J}} + 2\,\mathcal{M}_{\text{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}\,\overset{2^+}{\cdot}\lambda^{\perp}_{\mathcal{T}\,i\,j} \approx 0$$

$$\overset{0^+}{\cdot}\phi_{\mathcal{A}} \equiv 3\,\mathcal{M}_{\text{Pl}}{}^2\,\hat{\alpha}_{\underset{0}{\cdot}} + \frac{\overset{0^+}{\cdot}\hat{\pi}_{\mathcal{A}}}{\mathcal{J}} \approx 0$$

$$\overset{0^-}{\cdot}\phi_{\mathcal{A}} \equiv \frac{\overset{0^-}{\cdot}\hat{\pi}_{\mathcal{A}}}{\mathcal{J}} \approx 0$$

$$\overset{2^+}{\cdot}\phi_{\mathcal{A}\,i\,j} \equiv \frac{\overset{2^+}{\cdot}\hat{\pi}_{\mathcal{A}\,i\,j}}{\mathcal{J}} \approx 0$$

$$\overset{2^-}{\cdot}\phi_{\mathcal{A}\,i\,j\,h} \equiv \frac{2\,\overset{2^-}{\cdot}\hat{\pi}_{\mathcal{A}\,i\,j\,h}}{3\,\mathcal{J}} + \frac{\overset{2^-}{\cdot}\hat{\pi}_{\mathcal{A}\,i\,h\,j}}{3\,\mathcal{J}} - \frac{\overset{2^-}{\cdot}\hat{\pi}_{\mathcal{A}\,j\,h\,i}}{3\,\mathcal{J}} \approx 0$$

```
** DefTheory: Found the following secondary perpendicular if-constraints:
```

```
** DefTheory: Found the following secondary parallel if-constraints:
```

$$\overset{1^-}{\cdot}\chi^{\parallel}_{b\,i} \equiv \overset{1^-}{\cdot}\mathcal{T}^{\parallel}{}_{i} \approx 0$$

$$\overset{2^-}{\cdot}\chi^{\parallel}_{b\,i\,j\,h} \equiv \overset{2^-}{\cdot}\mathcal{T}^{\parallel}{}_{i\,j\,h} \approx 0$$

```
** DefTheory: Found the following secondary singular if-constraints:
```

$$\overset{1^+}{\cdot}\chi^{\vDash}_{b\,i\,j} \equiv \frac{\overset{1^+}{\cdot}\hat{\pi}_{b\,i\,j}}{\mathcal{J}} - \frac{2}{3}\,\mathcal{M}_{\text{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}\,\overset{1^+}{\cdot}\lambda^{\parallel}_{\mathcal{T}\,i\,j} + \frac{2}{3}\,\mathcal{M}_{\text{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}\,\overset{1^+}{\cdot}\lambda^{\perp}_{\mathcal{T}\,i\,j} + 4\,\mathcal{M}_{\text{Pl}}{}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\overset{1^+}{\cdot}\mathcal{T}^{\parallel}{}_{i\,j} \approx 0$$

```
** DefTheory: Defining association key $IfConstraints
   for the theory association ConstraintAlgebraAlp0Alp5Bet3cBet1cBet2
```

```
** ViewTheory: The super-Hamiltonian is:
```

$$
\begin{aligned}
\overset{0^{+}}{\cdot}\mathcal{H}_{b} \equiv{}& \frac{\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}a}\;\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}}{16\,\hat{\alpha}_{\underset{5}{\cdot}}\,\mathcal{J}} + \frac{\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}ab}\;\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{ab}}{8\,\hat{\alpha}_{\underset{5}{\cdot}}\,\mathcal{J}} + \frac{3\;\overset{1^{+}}{\cdot}\hat{\pi}_{bab}\;\overset{1^{+}}{\cdot}\hat{\pi}_{b}{}^{ab}}{16\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}} + \frac{\overset{1^{-}}{\cdot}\hat{\pi}_{b}{}^{a}\;\underset{\eta^{\parallel}}{\text{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a}\mathcal{J}\right]}{\mathcal{J}} + \frac{\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\alpha}_{\underset{0}{\cdot}}\,\mathcal{J}\;\overset{0^{+}}{\cdot}\mathcal{R}^{\parallel}}{2} - {}\\
& \frac{1}{2}\;\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\;\overset{1^{-}}{\cdot}\mathcal{R}^{\parallel}{}_{a} + \overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{ab}\;\overset{1^{+}}{\cdot}\mathcal{R}^{\parallel}{}_{ab} - \frac{\mathcal{M}_{\text{Pl}}{}^{2}\,\bar{\beta}_{\underset{1}{\cdot}}\;\overset{1^{+}}{\cdot}\hat{\pi}_{b}{}^{ab}\;\overset{1^{+}}{\cdot}\lambda^{\parallel}_{\mathcal{T}ab}}{4\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\beta}_{\underset{3}{\cdot}}} + \frac{\mathcal{M}_{\text{Pl}}{}^{2}\,\bar{\beta}_{\underset{1}{\cdot}}{}^{2}\,\mathcal{J}\;\overset{1^{+}}{\cdot}\lambda^{\parallel}_{\mathcal{T}ab}\;\overset{1^{+}}{\cdot}\lambda^{\parallel}_{\mathcal{T}}{}^{ab}}{12\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\beta}_{\underset{3}{\cdot}}} + {}\\
& \frac{\mathcal{M}_{\text{Pl}}{}^{2}\,\bar{\beta}_{\underset{1}{\cdot}}\;\overset{1^{+}}{\cdot}\hat{\pi}_{b}{}^{ab}\;\overset{1^{+}}{\cdot}\lambda^{\perp}_{\mathcal{T}ab}}{4\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\beta}_{\underset{3}{\cdot}}} - \frac{\mathcal{M}_{\text{Pl}}{}^{2}\,\bar{\beta}_{\underset{1}{\cdot}}{}^{2}\,\mathcal{J}\;\overset{1^{+}}{\cdot}\lambda^{\parallel}_{\mathcal{T}}{}^{ab}\;\overset{1^{+}}{\cdot}\lambda^{\perp}_{\mathcal{T}ab}}{6\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\beta}_{\underset{3}{\cdot}}} + \frac{\mathcal{M}_{\text{Pl}}{}^{2}\,\bar{\beta}_{\underset{1}{\cdot}}{}^{2}\,\mathcal{J}\;\overset{1^{+}}{\cdot}\lambda^{\perp}_{\mathcal{T}ab}\;\overset{1^{+}}{\cdot}\lambda^{\perp}_{\mathcal{T}}{}^{ab}}{12\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\beta}_{\underset{3}{\cdot}}} + \frac{1}{6}\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\beta}_{\underset{3}{\cdot}}\,\mathcal{J}\;\overset{0^{-}}{\cdot}\mathcal{T}^{\parallel 2} - {}\\
& \overset{1^{-}}{\cdot}\hat{\pi}_{b}{}^{a}\;\overset{1^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{a} + \frac{1}{2}\;\overset{1^{+}}{\cdot}\hat{\pi}_{b}{}^{ab}\;\overset{1^{+}}{\cdot}\mathcal{T}^{\parallel}{}_{ab} - \mathcal{M}_{\text{Pl}}{}^{2}\,\bar{\beta}_{\underset{1}{\cdot}}\,\mathcal{J}\;\overset{1^{+}}{\cdot}\lambda^{\parallel}_{\mathcal{T}}{}^{ab}\;\overset{1^{+}}{\cdot}\mathcal{T}^{\parallel}{}_{ab} + \mathcal{M}_{\text{Pl}}{}^{2}\,\bar{\beta}_{\underset{1}{\cdot}}\,\mathcal{J}\;\overset{1^{+}}{\cdot}\lambda^{\perp}_{\mathcal{T}}{}^{ab}\;\overset{1^{+}}{\cdot}\mathcal{T}^{\parallel}{}_{ab} - {}\\
& n^{a}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\;\overset{1^{+}}{\cdot}\hat{\pi}_{ba}{}^{b}\right) - n^{a}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{b}\;\overset{2^{+}}{\cdot}\hat{\pi}_{ba}{}^{b}\right) - \eta^{\parallel}{}_{ab}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel b}\;\overset{1^{-}}{\cdot}\hat{\pi}_{b}{}^{a}\right) + \frac{1}{3}\,\eta^{\parallel}{}_{ab}\;\overset{0^{+}}{\cdot}\hat{\pi}_{b}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,n^{a}\right) \approx 0
\end{aligned}
$$

** ViewTheory: The linear super-momentum is:

$$
\begin{aligned}
\overset{1^{-}}{\cdot}\mathcal{H}_{bl} \equiv{}& -\frac{1}{2}\;\overset{1^{-}}{\cdot}\hat{\pi}_{b}{}^{a}\;\underset{\eta^{\parallel}}{\text{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a}\,n_{l}\right] + \frac{\overset{1^{+}}{\cdot}\hat{\pi}_{bla}\;\underset{\eta^{\parallel}}{\text{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\mathcal{J}\right]}{\mathcal{J}} + \frac{\overset{2^{+}}{\cdot}\hat{\pi}_{bla}\;\underset{\eta^{\parallel}}{\text{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel a}\mathcal{J}\right]}{\mathcal{J}} - \underset{\eta^{\parallel}}{\text{P}}\left[\eta^{\parallel aa'}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\;\overset{1^{+}}{\cdot}\hat{\pi}_{bla}\right)\right] - {}\\
& \underset{\eta^{\parallel}}{\text{P}}\left[\eta^{\parallel aa'}\left(\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{a'}\;\overset{2^{+}}{\cdot}\hat{\pi}_{bla}\right)\right] + \frac{\overset{0^{+}}{\cdot}\hat{\pi}_{b}\;\underset{\eta^{\parallel}}{\text{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{l}\mathcal{J}\right]}{3\,\mathcal{J}} - \frac{1}{3}\,\underset{\eta^{\parallel}}{\text{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{l}\;\overset{0^{+}}{\cdot}\hat{\pi}_{b}\right] - \frac{1}{2}\;\overset{1^{-}}{\cdot}\hat{\pi}_{b}{}^{a}\;\underset{\eta^{\parallel}}{\text{P}}\left[\overset{3}{\cdot}\mathcal{D}^{\parallel}{}_{l}\,n_{a}\right] + \frac{1}{6}\,\epsilon^{\parallel}{}_{laa'}\;\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\;\overset{0^{-}}{\cdot}\mathcal{R}^{\parallel} - {}\\
& \frac{\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}l}\;\overset{0^{+}}{\cdot}\mathcal{R}^{\parallel}}{6} - \frac{1}{2}\;\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}la}\;\overset{1^{-}}{\cdot}\mathcal{R}^{\parallel a} - \frac{1}{2}\;\overset{2^{+}}{\cdot}\hat{\pi}_{\mathcal{A}la}\;\overset{1^{-}}{\cdot}\mathcal{R}^{\parallel a} + \frac{\overset{0^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}\;\overset{1^{-}}{\cdot}\mathcal{R}^{\parallel}{}_{l}}{3} - \frac{4}{3}\;\overset{2^{-}}{\cdot}\hat{\pi}_{\mathcal{A}l}{}^{aa'}\;\overset{1^{+}}{\cdot}\mathcal{R}^{\parallel}{}_{aa'} + {}\\
& \frac{1}{6}\,\epsilon^{\parallel}{}_{laa'}\;\overset{0^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}\;\overset{1^{+}}{\cdot}\mathcal{R}^{\parallel aa'} - \frac{1}{2}\;\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\;\overset{1^{+}}{\cdot}\mathcal{R}^{\parallel}{}_{la} - \frac{4}{3}\;\overset{1^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\;\overset{2^{-}}{\cdot}\mathcal{R}^{\parallel}{}_{laa'} + \frac{4}{3}\;\overset{2^{+}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{aa'}\;\overset{2^{-}}{\cdot}\mathcal{R}^{\parallel}{}_{laa'} + {}\\
& \frac{4}{3}\;\overset{2^{-}}{\cdot}\hat{\pi}_{\mathcal{A}l}{}^{aa'}\;\overset{2^{+}}{\cdot}\mathcal{R}^{\parallel}{}_{aa'} - \frac{1}{2}\;\overset{1^{-}}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\;\overset{2^{+}}{\cdot}\mathcal{R}^{\parallel}{}_{la} + \frac{1}{6}\,\epsilon^{\parallel}{}_{laa'}\;\overset{1^{+}}{\cdot}\hat{\pi}_{b}{}^{aa'}\;\overset{0^{-}}{\cdot}\mathcal{T}^{\parallel} - \frac{3}{2}\;\overset{1^{+}}{\cdot}\hat{\pi}_{bla}\;\overset{1^{-}}{\cdot}\mathcal{T}^{\parallel a} - {}\\
& \frac{3}{2}\;\overset{2^{+}}{\cdot}\hat{\pi}_{bla}\;\overset{1^{-}}{\cdot}\mathcal{T}^{\parallel a} + \frac{1}{2}\;\overset{1^{-}}{\cdot}\hat{\pi}_{b}{}^{a}\;\overset{1^{+}}{\cdot}\mathcal{T}^{\parallel}{}_{la} - \frac{4}{3}\;\overset{1^{+}}{\cdot}\hat{\pi}_{b}{}^{aa'}\;\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{laa'} + \frac{4}{3}\;\overset{2^{+}}{\cdot}\hat{\pi}_{b}{}^{aa'}\;\overset{2^{-}}{\cdot}\mathcal{T}^{\parallel}{}_{laa'} \approx 0
\end{aligned}
$$

** ViewTheory: The 1+ part of the angular super-momentum is:

$$
{}^{1^+}\mathcal{H}_{\mathcal{A}nm} \equiv -2\,{}^{1^+}\hat{\pi}_{bmn} - \frac{\epsilon^{\parallel}{}_{mna}\;{}^{0^-}\hat{\pi}_{\mathcal{A}}\,\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel a}\,\mathcal{J}\right]}{6\,\mathcal{J}} + \frac{4\,{}^{2^-}\hat{\pi}_{\mathcal{A}mna}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel a}\,\mathcal{J}\right]}{3\,\mathcal{J}} + \frac{1}{6}\,\epsilon^{\parallel}{}_{mna}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel a}\;{}^{0^-}\hat{\pi}_{\mathcal{A}}\right] -
$$

$$
\frac{1}{2}\,{}^{1^+}\hat{\pi}_{\mathcal{A}na}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel a}\,n_{m}\right] - \frac{1}{2}\,{}^{2^+}\hat{\pi}_{\mathcal{A}na}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel a}\,n_{m}\right] + \frac{1}{2}\,{}^{1^+}\hat{\pi}_{\mathcal{A}ma}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel a}\,n_{n}\right] + \frac{1}{2}\,{}^{2^+}\hat{\pi}_{\mathcal{A}ma}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel a}\,n_{n}\right] -
$$

$$
\frac{4}{3}\,\underset{\eta^\parallel}{\mathrm{P}}\left[\eta^{\parallel aa'}\left({}^{3}\mathcal{D}^{\parallel}{}_{a'}\;{}^{2^-}\hat{\pi}_{\mathcal{A}mna}\right)\right] + \frac{{}^{1^-}\hat{\pi}_{\mathcal{A}n}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{m}\,\mathcal{J}\right]}{2\,\mathcal{J}} - \frac{1}{2}\,\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{m}\;{}^{1^-}\hat{\pi}_{\mathcal{A}n}\right] - \frac{1}{2}\,{}^{1^+}\hat{\pi}_{\mathcal{A}na}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{m}\,n^{a}\right] -
$$

$$
\frac{1}{2}\,{}^{2^+}\hat{\pi}_{\mathcal{A}na}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{m}\,n^{a}\right] - \frac{{}^{1^-}\hat{\pi}_{\mathcal{A}m}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{n}\,\mathcal{J}\right]}{2\,\mathcal{J}} + \frac{1}{2}\,\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{n}\;{}^{1^-}\hat{\pi}_{\mathcal{A}m}\right] + \frac{1}{2}\,{}^{1^+}\hat{\pi}_{\mathcal{A}ma}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{n}\,n^{a}\right] +
$$

$$
\frac{1}{2}\,{}^{2^+}\hat{\pi}_{\mathcal{A}ma}\;\underset{\eta^\parallel}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{n}\,n^{a}\right] + \frac{1}{6}\,\epsilon^{\parallel}{}_{mna}\;{}^{0^-}\hat{\pi}_{\mathcal{A}}\;{}^{1^-}\mathcal{T}^{\parallel a} - \frac{4}{3}\,{}^{2^-}\hat{\pi}_{\mathcal{A}mna}\;{}^{1^-}\mathcal{T}^{\parallel a} - \frac{1}{2}\,{}^{1^-}\hat{\pi}_{\mathcal{A}n}\;{}^{1^-}\mathcal{T}^{\parallel}{}_{m} + \frac{1}{2}\,{}^{1^-}\hat{\pi}_{\mathcal{A}m}\;{}^{1^-}\mathcal{T}^{\parallel}{}_{n} -
$$

$$
\frac{1}{2}\,{}^{1^+}\hat{\pi}_{\mathcal{A}n}{}^{a}\;{}^{1^+}\mathcal{T}^{\parallel}{}_{ma} - \frac{1}{2}\,{}^{2^+}\hat{\pi}_{\mathcal{A}n}{}^{a}\;{}^{1^+}\mathcal{T}^{\parallel}{}_{ma} - \frac{1}{3}\,{}^{0^+}\hat{\pi}_{\mathcal{A}}\;{}^{1^+}\mathcal{T}^{\parallel}{}_{mn} + \frac{1}{2}\,{}^{1^+}\hat{\pi}_{\mathcal{A}m}{}^{a}\;{}^{1^+}\mathcal{T}^{\parallel}{}_{na} + \frac{1}{2}\,{}^{2^+}\hat{\pi}_{\mathcal{A}m}{}^{a}\;{}^{1^+}\mathcal{T}^{\parallel}{}_{na} \approx 0
$$

IndexForm: Attempting to apply IndexForm on <<2>>.

IndexForm: Attempting to apply IndexForm on <<2>>.

IndexForm: Attempting to apply IndexForm on <<2>>.

General: Further output of IndexForm::nouse will be suppressed during this calculation.

MapThread: Incompatible dimensions of objects at positions {2, 1} and {2, 2} of

MapThread[{#1[[1]], #1[[2]], #1[[3]], #1[[4]], #1[[5]], #2} &, {{≪1≫}, {0, $\iiint \frac{2\,M_{\mathrm{Pl}}{}^{2}\,\bar{\beta}_{1}\,≪1≫\,≪76≫\left({}^{1^-}\lambda_{\mathcal{T}}^{\parallel\,a} - 2\,{}^{1^-}\lambda_{\mathcal{T}}^{\perp\,<<1>>}\right)}{\mathcal{J}}[x^0, x^1, x^2, x^3]\,dx^3\,dx^2\,dx^1$, $\iiint\left(-\frac{6\,≪199≫\,\underset{\eta^\parallel}{\mathrm{P}}\left[≪76≫^{aa'}\right]≪}{\mathcal{J}}\right)[x^0, x^1, x^2, x^3]\,dx^3\,dx^2\,dx^1$, $\iiint$≪1≫, 0, 0, 0, 0, 0, 0, ≪126≫}}];

dimensions are 55 and 136.

```
** ViewTheory: The on-shell brackets between the various if-constraints are:
```

Part: Part 2 of #1 does not exist.

Part: Part 3 of #1 does not exist.

Part: Part 4 of #1 does not exist.

General: Further output of Part::partw will be suppressed during this calculation.

Function: Slot number 2 in xAct`HiGGS`Private`AllocatedPoissonBracket[#2, #3, #6] & cannot be filled from (xAct`HiGGS`Private`AllocatedPoissonBracket[#2, #3, #6] &)[{1, #1[[2]], #1[[3]], #1[[4]], #1[[5]], #2}].

Function: Slot number 3 in xAct`HiGGS`Private`AllocatedPoissonBracket[#2, #3, #6] & cannot be filled from (xAct`HiGGS`Private`AllocatedPoissonBracket[#2, #3, #6] &)[{1, #1[[2]], #1[[3]], #1[[4]], #1[[5]], #2}].

Function: Slot number 6 in xAct`HiGGS`Private`AllocatedPoissonBracket[#2, #3, #6] & cannot be filled from (xAct`HiGGS`Private`AllocatedPoissonBracket[#2, #3, #6] &)[{1, #1[[2]], #1[[3]], #1[[4]], #1[[5]], #2}].

General: Further output of Function::slotn will be suppressed during this calculation.

$$\left\{\iiint(\#2 . \mathcal{S}_{(1)})[x^0, x^1, x^2, x^3]\, dx^3\, dx^2\, dx^1,\ \iiint(\#3 . \mathcal{S}_{(2)})[x^0, y^1, y^2, y^3]\, dy^3\, dy^2\, dy^1\right\} \approx \#6$$

$$\Bigg\{\iiint\Bigg(\Bigg\{0,\ \iiint\frac{2\, M_{Pl}^2\, \bar{\beta}_{1}\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(2)a}]\, \mathcal{S}_{(1)}\left({}^{1^-}\lambda_{\mathcal{T}}^{\parallel\, a} - 2\, {}^{1^-}\lambda_{\mathcal{T}}^{\perp\, a}\right)}{\mathcal{J}}[x^0, x^1, x^2, x^3]\, dx^3\, dx^2\, dx^1,$$

$$\iiint\left(-\frac{6\, M_{Pl}^2\, \bar{\beta}_{1}\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(2)}{}^{aa'}]\, \mathcal{S}_{(1)}\, {}^{2^+}\lambda^{\perp}_{\mathcal{T}\, aa'}}{\mathcal{J}}\right)[x^0, x^1, x^2, x^3]\, dx^3\, dx^2\, dx^1,$$

$$\iiint\left(-\frac{6\, M_{Pl}^2\, \hat{\alpha}_{0}\, \mathcal{S}_{(1)}\, \mathcal{S}_{(2)}}{\mathcal{J}}\right)[x^0, x^1, x^2, x^3]\, dx^3\, dx^2\, dx^1,\ 0,\ 0,\ 0,\ 0,$$

$$0,\ 0,\ \iiint\left(12\, M_{Pl}^2\, \bar{\beta}_{2}^{\,2}\, \mathcal{S}_{(1)}\, \mathcal{S}_{(2)}\right)[x^0, x^1, x^2, x^3]\, dx^3\, dx^2\, dx^1,\ 0,\ 0,$$

$$\iiint\frac{2\, \mathcal{S}_{(1)}\left(-\underset{\eta^\parallel}{P}[\mathcal{S}_{(2)}{}^{a}]\, \underset{\eta^\parallel}{P}[{}^{3}\mathcal{D}^{\parallel}{}_{a}\mathcal{J}] + \eta^{\parallel}{}_{aa'}\, \mathcal{J}\left({}^{3}\mathcal{D}^{\parallel a'}\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(2)}{}^{a}]\right)\right)}{\mathcal{J}^2}[x^0, x^1, x^2, x^3]\, dx^3\, dx^2\, dx^1,\ 0,$$

$$\iiint\frac{2\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(2)}{}^{aa'}]\, \mathcal{S}_{(1)}\left(M_{Pl}^2\, \bar{\beta}_{1}\, {}^{1^+}\lambda^{\parallel}_{\mathcal{T}\, aa'} - M_{Pl}^2\, \bar{\beta}_{1}\, {}^{1^+}\lambda^{\perp}_{\mathcal{T}\, aa'} - 4\, M_{Pl}^2\, \hat{\beta}_{3}\, {}^{1^+}\mathcal{T}^{\parallel}{}_{aa'}\right)}{\mathcal{J}}[x^0, x^1, x^2, x^3]\, dx^3$$

$$dx^2\, dx^1,\ \iiint\left(-\frac{8\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a}]\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(2)}{}^{a'}]\left(M_{Pl}^2\, \bar{\beta}_{1}\, {}^{1^+}\lambda^{\parallel}_{\mathcal{T}\, aa'} - M_{Pl}^2\, \bar{\beta}_{1}\, {}^{1^+}\lambda^{\perp}_{\mathcal{T}\, aa'} + 3\, M_{Pl}^2\, \hat{\beta}_{3}\, {}^{1^+}\mathcal{T}^{\parallel}{}_{aa'}\right)}{3\, \mathcal{J}}\right)[$$

$$x^0, x^1, x^2, x^3]\, dx^3\, dx^2\, dx^1,$$

$$\iiint\frac{1}{6\, \mathcal{J}}\, M_{Pl}^2\, \bar{\beta}_{1}\Big(-8\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a}]\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(2)}{}^{a'b}]\left({}^{2^-}\lambda^{\parallel}_{\mathcal{T}\, aa'b} + {}^{2^-}\lambda^{\parallel}_{\mathcal{T}\, aba'}\right) + 3\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a'}]\left(\underset{\eta^\parallel}{P}[\mathcal{S}_{(2)aa'}] + \underset{\eta^\parallel}{P}[\mathcal{S}_{(2)a'a}]\right)$$

$$\left({}^{1^-}\lambda_{\mathcal{T}}^{\parallel\, a} - 2\, {}^{1^-}\lambda_{\mathcal{T}}^{\perp\, a}\right) - 2\, \eta^{\parallel}{}_{aa'}\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(1)b}]\, \mathcal{S}_{(2)}{}^{aa'}\left({}^{1^-}\lambda_{\mathcal{T}}^{\parallel\, b} - 2\, {}^{1^-}\lambda_{\mathcal{T}}^{\perp\, b}\right)\Big)[x^0, x^1, x^2, x^3]$$

$$dx^3\, dx^2\, dx^1,\ \iiint\left(-\frac{{}^{1^-}\hat{\pi}_{\mathcal{A}}{}^{a}\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(1)a}]\, \mathcal{S}_{(2)}}{\mathcal{J}^2}\right)[x^0, x^1, x^2, x^3]\, dx^3\, dx^2\, dx^1,$$

$$\iiint\frac{2\, \epsilon\gamma_{aa'iZa1}\, {}^{1^+}\hat{\pi}_{\mathcal{A}}{}^{iZa1}\, \underset{\eta^\parallel}{P}[\mathcal{S}_{(1)}{}^{a'}]\, \mathcal{S}_{(2)}\, n^{a}}{\mathcal{J}^2}[x^0, x^1, x^2, x^3]\, dx^3\, dx^2\, dx^1,$$

$$\iiint \frac{\overset{1^-}{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\left(3\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)}{}^{a'}]\left(\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)aa'}]+\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a'a}]\right)-2\,\eta^\parallel{}_{a'b}\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)a}]\,\mathcal{S}_{(2)}{}^{a'b}\right)}{12\,\mathcal{J}^2}[x^0,x^1,x^2,x^3]\,dx^3$$

$$dx^2\,dx^1,\ \iiint \frac{1}{8\,\mathcal{J}^2}\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)}{}^{a}]\left(\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}a'b}\left(-2\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a}{}^{a'b}]+2\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{a'}{}_{a}{}^{b}]+4\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{a'b}{}_{a}]\right)+\right.$$

$$\left.3\,\overset{1^+}{\cdot}\hat{\pi}_{\mathcal{A}a}{}^{a'}\left(\underset{\eta^\parallel}{\mathrm{P}}[\eta^\parallel{}_{bb'}\,\mathcal{S}_{(2)a'}{}^{bb'}]-\underset{\eta^\parallel}{\mathrm{P}}[\eta^\parallel{}_{bb'}\,\mathcal{S}_{(2)}{}^{b}{}_{a'}{}^{b'}]\right)\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,$$

$$0,\ \iiint\left(\frac{4}{3}\left(\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}{}^2-\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{\underset{2}{\cdot}}{}^2\right)\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)}{}^{a}]\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a}]\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ 0,$$

$$0,\ \iiint\left(\frac{4}{3}\left(2\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}{}^2+\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{\underset{2}{\cdot}}{}^2\right)\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)}{}^{a}]\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a}]\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,\ 0,$$

$$\iiint \frac{\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)}{}^{a}]\left(-\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{a'}]\left(\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^\parallel{}_{a}\,n_{a'}]+\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^\parallel{}_{a'}\,n_{a}]-\overset{1^+}{\cdot}\mathcal{T}^\parallel{}_{aa'}\right)+2\,\eta^\parallel{}_{a'b}\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a}]\left(\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,n^{a'}\right)\right)}{2\,\mathcal{J}}[$$

$$x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,$$

$$\iiint \frac{1}{16\,\mathcal{J}}\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)}{}^{a}]\left(3\,\delta_{a'}{}^{b}\left(\underset{\eta^\parallel}{\mathrm{P}}[\eta^\parallel{}_{b'm}\,\mathcal{S}_{(2)b}{}^{b'm}]-\underset{\eta^\parallel}{\mathrm{P}}[\eta^\parallel{}_{b'm}\,\mathcal{S}_{(2)}{}^{b'}{}_{b}{}^{m}]\right)\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^\parallel{}_{a}\,n^{a'}]-\right.$$

$$6\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{a'}{}_{a}{}^{b}]\,\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^\parallel{}_{a'}\,n_{b}]+3\,\underset{\eta^\parallel}{\mathrm{P}}[\eta^\parallel{}_{bb'}\,\mathcal{S}_{(2)a'}{}^{bb'}]\,\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^{\parallel a'}\,n_{a}]-3\,\underset{\eta^\parallel}{\mathrm{P}}[\eta^\parallel{}_{bb'}\,\mathcal{S}_{(2)}{}^{b}{}_{a'}{}^{b'}]$$

$$\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^{\parallel a'}\,n_{a}]-6\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{a'}{}_{a}{}^{b}]\,\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^\parallel{}_{b}\,n_{a'}]-3\,\underset{\eta^\parallel}{\mathrm{P}}[\eta^\parallel{}_{bb'}\,\mathcal{S}_{(2)a'}{}^{bb'}]\,\overset{1^+}{\cdot}\mathcal{T}^\parallel{}_{a}{}^{a'}+$$

$$3\,\underset{\eta^\parallel}{\mathrm{P}}[\eta^\parallel{}_{bb'}\,\mathcal{S}_{(2)}{}^{b}{}_{a'}{}^{b'}]\,\overset{1^+}{\cdot}\mathcal{T}^\parallel{}_{a}{}^{a'}-2\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{a'}{}_{a}{}^{b}]\,\overset{1^+}{\cdot}\mathcal{T}^\parallel{}_{a'b}-4\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{a'b}{}_{a}]\,\overset{1^+}{\cdot}\mathcal{T}^\parallel{}_{a'b}+$$

$$2\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a}{}^{a'b}]\left(3\,\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^\parallel{}_{a'}\,n_{b}]+3\,\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^\parallel{}_{b}\,n_{a'}]+\overset{1^+}{\cdot}\mathcal{T}^\parallel{}_{a'b}\right)-6\,\eta^\parallel{}_{a'b}\,\underset{\eta^\parallel}{\mathrm{P}}[\eta^\parallel{}_{b'm}\,\mathcal{S}_{(2)a}{}^{b'm}]$$

$$\left.\left(\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,n^{a'}\right)+6\,\eta^\parallel{}_{a'b}\,\underset{\eta^\parallel}{\mathrm{P}}[\eta^\parallel{}_{b'm}\,\mathcal{S}_{(2)}{}^{b'}{}_{a}{}^{m}]\left(\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,n^{a'}\right)\right)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,$$

$$\iiint \frac{1}{18\,\mathcal{J}^2}\left(-36\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\delta_{a'}{}^{b}\,\mathcal{J}\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)}{}^{a}]\left(\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)b}{}^{b'}\,n_{b'}]-\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{b'}{}_{b}\,n_{b'}]\right)\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^\parallel{}_{a}\,n^{a'}]-\right.$$

$$9\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}\,\mathcal{J}\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)}{}^{a'}]\left(\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)aa'}]-\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a'a}]\right)\left(\overset{1^-}{\cdot}\lambda^\parallel_{\mathcal{T}}{}^{a}-2\,\overset{1^-}{\cdot}\lambda^\perp_{\mathcal{T}}{}^{a}\right)+4\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)}{}^{a}]\left(18\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{\underset{3}{\cdot}}\right.$$

$$\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a}{}^{a'}]\,\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^\parallel{}_{a'}\,\mathcal{J}]-18\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{a'}{}_{a}]\,\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^\parallel{}_{a'}\,\mathcal{J}]+\mathcal{J}\left(2\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{a'b}]\right.$$

$$\left(\overset{2^-}{\cdot}\lambda^\parallel_{\mathcal{T}aa'b}-\overset{2^-}{\cdot}\lambda^\parallel_{\mathcal{T}aba'}-2\,\overset{2^-}{\cdot}\lambda^\parallel_{\mathcal{T}a'ba}\right)+9\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a'}{}^{b}\,n_{b}]\left(\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^{\parallel a'}\,n_{a}]-\right.$$

$$\left.\overset{1^+}{\cdot}\mathcal{T}^\parallel{}_{a}{}^{a'}\right)+9\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{b}{}_{a'}\,n_{b}]\left(-\underset{\eta^\parallel}{\mathrm{P}}[\overset{3}{\cdot}\mathcal{D}^{\parallel a'}\,n_{a}]+\overset{1^+}{\cdot}\mathcal{T}^\parallel{}_{a}{}^{a'}\right)+18\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{\underset{3}{\cdot}}$$

$$\left.\eta^\parallel{}_{a'b}\left(-\left(\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a}{}^{a'}]\right)+\overset{3}{\cdot}\mathcal{D}^{\parallel b}\,\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{a'}{}_{a}]\right)\right)\Bigg)\Bigg)[x^0,x^1,x^2,x^3]\,dx^3\,dx^2\,dx^1,$$

$$\iiint \frac{1}{3\,\mathcal{J}}\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(1)}{}^{aa'}]\left(\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a'}{}^{b}]\left(\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}\,\overset{1^+}{\cdot}\lambda^\parallel_{\mathcal{T}ab}-\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}\,\overset{1^+}{\cdot}\lambda^\perp_{\mathcal{T}ab}-6\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\overset{1^+}{\cdot}\mathcal{T}^\parallel{}_{ab}\right)+\right.$$

$$\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{b}{}_{a'}]\left(\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}\,\overset{1^+}{\cdot}\lambda^\parallel_{\mathcal{T}ab}-\mathcal{M}_{\mathrm{Pl}}{}^2\,\bar{\beta}_{\underset{1}{\cdot}}\,\overset{1^+}{\cdot}\lambda^\perp_{\mathcal{T}ab}-6\,\mathcal{M}_{\mathrm{Pl}}{}^2\,\hat{\beta}_{\underset{3}{\cdot}}\,\overset{1^+}{\cdot}\mathcal{T}^\parallel{}_{ab}\right)+\left(\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)a}{}^{b}]+\underset{\eta^\parallel}{\mathrm{P}}[\mathcal{S}_{(2)}{}^{b}{}_{a}]\right)$$

$$\left(\mathcal{M}_{\text{Pl}}{}^{2}\,\bar{\beta}_{1}\;{}^{1^{+}}\lambda^{\parallel}_{\mathcal{T}\,a'b}-\mathcal{M}_{\text{Pl}}{}^{2}\,\bar{\beta}_{1}\;{}^{1^{+}}\lambda^{\perp}_{\mathcal{T}\,a'b}-6\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\beta}_{3}\;{}^{1^{+}}\mathcal{T}^{\parallel}{}_{a'b}\right)\Big)\Big)[x^{0},x^{1},x^{2},x^{3}]\,dx^{3}\,dx^{2}$$

$$dx^{1},\;0,\;0,\;\iiint\frac{1}{12\,\mathcal{J}^{2}}\Bigg(3\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'}\right]\Bigg(2\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\alpha}_{0}\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{aa'}\right]+2\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\alpha}_{0}\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'a}\right]+$$

$${}^{1^{+}}\hat{\pi}_{\mathcal{A}\,ab}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'}{}^{b}\right]+{}^{1^{+}}\hat{\pi}_{\mathcal{A}\,a'b}\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a}{}^{b}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b}{}_{a}\right]\right)+{}^{1^{+}}\hat{\pi}_{\mathcal{A}\,ab}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b}{}_{a'}\right]\Bigg)-$$

$$4\,\mathcal{M}_{\text{Pl}}{}^{2}\,\hat{\alpha}_{0}\;\eta^{\parallel}{}_{aa'}\;\;\eta^{\parallel}{}_{bb'}\;\mathcal{J}\;\mathcal{S}_{(1)}{}^{aa'}\;\;\mathcal{S}_{(2)}{}^{bb'}\Bigg)[x^{0},x^{1},x^{2},x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},$$

$$\iiint\frac{1}{32\,\mathcal{J}^{2}}\,3\;{}^{1^{-}}\hat{\pi}_{\mathcal{A}}{}^{a}\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{a'}{}_{a}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{bb'}\;\;\mathcal{S}_{(2)}{}_{a'}{}^{bb'}\right]+2\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{a'b}\right]\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{aa'b}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{aba'}\right]-\right.\right.$$

$$\left.\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'ab}\right]-\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{baa'}\right]\right)+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}_{a}{}^{a'}\right]\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{bb'}\;\;\mathcal{S}_{(2)}{}_{a'}{}^{bb'}\right]-\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{bb'}\;\;\mathcal{S}_{(2)}{}^{b}{}_{a'}{}^{b'}\right]\right)-$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{a'}{}_{a}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{bb'}\;\;\mathcal{S}_{(2)}{}^{b}{}_{a'}{}^{b'}\right]-2\;\eta^{\parallel}{}_{a'b}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\;\;\mathcal{S}_{(2)}{}_{a}{}^{b'c}\right]\;\mathcal{S}_{(1)}{}^{a'b}+$$

$$\left.2\;\eta^{\parallel}{}_{a'b}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\;\;\mathcal{S}_{(2)}{}^{b'}{}_{a}{}^{c}\right]\;\mathcal{S}_{(1)}{}^{a'b}\right)[x^{0},x^{1},x^{2},x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},\;0,\;0,\;0,\;0,\;0,$$

$$\iiint\Bigg(\frac{2}{3}\,\mathcal{M}_{\text{Pl}}{}^{2}\,\bar{\beta}_{1}{}^{2}\left(3\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'}\right]\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{aa'}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'a}\right]\right)-2\;\eta^{\parallel}{}_{aa'}\;\;\eta^{\parallel}{}_{bb'}\;\;\mathcal{S}_{(1)}{}^{aa'}\;\;\mathcal{S}_{(2)}{}^{bb'}\right)\Bigg)\Big[$$

$$x^{0},x^{1},x^{2},x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},$$

$$\iiint\frac{1}{6\,\mathcal{J}^{2}}\Bigg(3\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{ab}\right]\Bigg(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{a}\mathcal{J}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b}\mathcal{J}\right]-\mathcal{J}\left({}^{3}\mathcal{D}^{\parallel}{}_{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b}\right]+{}^{3}\mathcal{D}^{\parallel}{}_{b}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a}\right]\right)\Bigg)+$$

$$2\;\eta^{\parallel}{}_{ab}\;\;\mathcal{S}_{(1)}{}^{ab}\left(-\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{a'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{a'}\mathcal{J}\right]+\eta^{\parallel}{}_{a'b'}\;\mathcal{J}\left({}^{3}\mathcal{D}^{\parallel b'}\;\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{a'}\right]\right)\right)\Bigg)[x^{0},x^{1},x^{2},x^{3}]\,dx^{3}\,dx^{2}$$

$$dx^{1},\;\iiint\frac{1}{96\,\mathcal{J}^{2}}\Bigg(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{ab}\right]\Bigg(-18\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\;\;\mathcal{S}_{(2)}{}_{b}{}^{b'c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{a}\mathcal{J}\right]+18\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\;\;\mathcal{S}_{(2)}{}^{b'}{}_{b}{}^{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{a}\mathcal{J}\right]+$$

$$27\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{bc}{}^{c'}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{a}\,n^{b'}\right]+9\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b}{}^{c'}{}_{c}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{a}\,n^{b'}\right]-$$

$$27\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{cb}{}^{c'}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{a}\,n^{b'}\right]-18\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{c}{}^{c'}{}_{b}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{a}\,n^{b'}\right]-$$

$$9\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c'}{}_{bc}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{a}\,n^{b'}\right]+18\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c'}{}_{cb}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{a}\,n^{b'}\right]-$$

$$18\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\;\;\mathcal{S}_{(2)}{}_{a}{}^{b'c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b}\mathcal{J}\right]+18\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\;\;\mathcal{S}_{(2)}{}^{b'}{}_{a}{}^{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b}\mathcal{J}\right]+$$

$$27\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{ac}{}^{c'}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b}\,n^{b'}\right]+9\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a}{}^{c'}{}_{c}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b}\,n^{b'}\right]-$$

$$27\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{ca}{}^{c'}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b}\,n^{b'}\right]-18\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{c}{}^{c'}{}_{a}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b}\,n^{b'}\right]-$$

$$9\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c'}{}_{ac}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b}\,n^{b'}\right]+18\;\delta_{b'}{}^{c}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c'}{}_{ca}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b}\,n^{b'}\right]+$$

$$36\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a}{}^{b'}{}_{b}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b'}\mathcal{J}\right]+36\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b}{}^{b'}{}_{a}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b'}\mathcal{J}\right]-36\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'}{}_{ab}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b'}\mathcal{J}\right]-$$

$$36\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'}{}_{ba}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel}{}_{b'}\mathcal{J}\right]-27\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{bb'}{}^{c}\;\;n_{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel b'}\;n_{a}\right]-9\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b}{}^{c}{}_{b'}\;\;n_{c}\right]$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel b'}\;n_{a}\right]+27\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b'b}{}^{c}\;\;n_{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel b'}\;n_{a}\right]+18\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b'}{}^{c}{}_{b}\;\;n_{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}\mathcal{D}^{\parallel b'}\;n_{a}\right]+$$

$$9\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{bb'}\;n_{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel b'}\,n_{a}\right]-18\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{b'b}\;n_{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel b'}\,n_{a}\right]-$$

$$27\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)ab'}{}^{c}\;n_{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel b'}\,n_{b}\right]-9\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a}{}^{c}{}_{b'}\;n_{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel b'}\,n_{b}\right]+27\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b'a}{}^{c}\;n_{c}\right]$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel b'}\,n_{b}\right]+18\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b'}{}^{c}{}_{a}\;n_{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel b'}\,n_{b}\right]+9\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{ab'}\;n_{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel b'}\,n_{b}\right]-18\mathcal{J}$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{b'a}\;n_{c}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel b'}\,n_{b}\right]-4\,\epsilon^{\parallel}{}_{bb'c}\;\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a}{}^{b'c}\right]{}^{0^-}_{\bullet}\mathcal{T}^{\parallel}-4\,\epsilon^{\parallel}{}_{ab'c}\;\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b}{}^{b'c}\right]{}^{0^-}_{\bullet}\mathcal{T}^{\parallel}+$$

$$4\,\epsilon^{\parallel}{}_{bb'c}\;\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'}{}_{a}{}^{c}\right]{}^{0^-}_{\bullet}\mathcal{T}^{\parallel}+4\,\epsilon^{\parallel}{}_{ab'c}\;\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'}{}_{b}{}^{c}\right]{}^{0^-}_{\bullet}\mathcal{T}^{\parallel}+8\,\epsilon^{\parallel}{}_{bb'c}\;\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'c}{}_{a}\right]{}^{0^-}_{\bullet}\mathcal{T}^{\parallel}+$$

$$8\,\epsilon^{\parallel}{}_{ab'c}\;\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'c}{}_{b}\right]{}^{0^-}_{\bullet}\mathcal{T}^{\parallel}+27\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)bb'}{}^{c}\;n_{c}\right]{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{a}{}^{b'}+9\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b}{}^{c}{}_{b'}\;n_{c}\right]$$

$${}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{a}{}^{b'}-27\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b'b}{}^{c}\;n_{c}\right]{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{a}{}^{b'}-18\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b'}{}^{c}{}_{b}\;n_{c}\right]{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{a}{}^{b'}-9\mathcal{J}$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{bb'}\;n_{c}\right]{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{a}{}^{b'}+18\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{b'b}\;n_{c}\right]{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{a}{}^{b'}+27\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)ab'}{}^{c}\;n_{c}\right]{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{b}{}^{b'}+$$

$$9\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a}{}^{c}{}_{b'}\;n_{c}\right]{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{b}{}^{b'}-27\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b'a}{}^{c}\;n_{c}\right]{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{b}{}^{b'}-18\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b'}{}^{c}{}_{a}\;n_{c}\right]$$

$${}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{b}{}^{b'}-9\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{ab'}\;n_{c}\right]{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{b}{}^{b'}+18\mathcal{J}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{b'a}\;n_{c}\right]{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{b}{}^{b'}+18\;\eta^{\parallel}{}_{b'c}\;\mathcal{J}$$

$$\left({}^{3}_{\bullet}\mathcal{D}^{\parallel}{}_{a}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b}{}^{b'c}\right]\right)-18\;\eta^{\parallel}{}_{b'c}\;\mathcal{J}\left({}^{3}_{\bullet}\mathcal{D}^{\parallel}{}_{a}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'}{}_{b}{}^{c}\right]\right)+18\;\eta^{\parallel}{}_{b'c}\;\mathcal{J}\left({}^{3}_{\bullet}\mathcal{D}^{\parallel}{}_{b}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a}{}^{b'c}\right]\right)-$$

$$18\;\eta^{\parallel}{}_{b'c}\;\mathcal{J}\left({}^{3}_{\bullet}\mathcal{D}^{\parallel}{}_{b}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'}{}_{a}{}^{c}\right]\right)-36\;\eta^{\parallel}{}_{b'c}\;\mathcal{J}\left({}^{3}_{\bullet}\mathcal{D}^{\parallel c}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a}{}^{b'}{}_{b}\right]\right)-36\;\eta^{\parallel}{}_{b'c}\;\mathcal{J}$$

$$\left({}^{3}_{\bullet}\mathcal{D}^{\parallel c}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b}{}^{b'}{}_{a}\right]\right)+36\;\eta^{\parallel}{}_{b'c}\;\mathcal{J}\left({}^{3}_{\bullet}\mathcal{D}^{\parallel c}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'}{}_{ab}\right]\right)+36\;\eta^{\parallel}{}_{b'c}\;\mathcal{J}\left({}^{3}_{\bullet}\mathcal{D}^{\parallel c}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'}{}_{ba}\right]\right)\Bigg)+$$

$$18\;\eta^{\parallel}{}_{ab}\;\;\mathcal{S}_{(1)}{}^{ab}\;\Bigg(2\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(2)b'}{}^{cc'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel b'}\,\mathcal{J}\right]-2\underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(2)}{}^{c}{}_{b'}{}^{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel b'}\,\mathcal{J}\right]+$$

$$\mathcal{J}\Bigg(\delta_{b'}{}^{c'}\;\left(-\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{a'}{}_{cc'}\;\;n_{a'}\right]-\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{a'}{}_{c'c}\;\;n_{a'}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)c}{}^{a'}{}_{c'}\;\;n_{a'}\right]+\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)c'}{}^{a'}{}_{c}\;\;n_{a'}\right]\right)$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel c}\,n^{b'}\right]-2\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)cb'}{}^{c'}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel c}\,n^{b'}\right]-$$

$$2\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)c}{}^{c'}{}_{b'}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel c}\,n^{b'}\right]+2\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c'}{}_{cb'}\;\;n_{c'}\right]\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel c}\,n^{b'}\right]-$$

$$2\left(\eta^{\parallel}{}_{a'b}\;\;\mathcal{S}_{(2)}{}^{aa'b}\;\;n_{a}\right)\left(\eta^{\parallel}{}_{aa'}\;\;\left({}^{3}_{\bullet}\mathcal{D}^{\parallel a'}\,n^{a}\right)\right)+2\left(\eta^{\parallel}{}_{ab}\;\;\mathcal{S}_{(2)}{}^{aa'b}\;\;n_{a'}\right)\left(\eta^{\parallel}{}_{aa'}\;\;\left({}^{3}_{\bullet}\mathcal{D}^{\parallel a'}\,n^{a}\right)\right)+$$

$$2\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b'c}{}^{c'}\;\;n_{c'}\right]\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel c}\,n^{b'}\right]-\;{}^{1^+}_{\bullet}\mathcal{T}^{\parallel b'c}\right)-\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)b'}{}^{c'}{}_{c}\;\;n_{c'}\right]\;{}^{1^+}_{\bullet}\mathcal{T}^{\parallel b'c}+$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c'}{}_{b'c}\;\;n_{c'}\right]\;{}^{1^+}_{\bullet}\mathcal{T}^{\parallel b'c}+2\;\eta^{\parallel}{}_{b'c'}\;\;\eta^{\parallel}{}_{ca'}\;\left({}^{3}_{\bullet}\mathcal{D}^{\parallel a'}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'cc'}\right]\right)-$$

$$2\;\eta^{\parallel}{}_{b'a'}\;\;\eta^{\parallel}{}_{cc'}\;\left({}^{3}_{\bullet}\mathcal{D}^{\parallel a'}\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'cc'}\right]\right)+2\;\eta^{\parallel}{}_{a'd}\;\;\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(2)}{}^{b'cc'}\;\;n_{b'}\;\left({}^{3}_{\bullet}\mathcal{D}^{\parallel d}\,n^{a'}\right)-$$

$$2\;\eta^{\parallel}{}_{a'd}\;\;\eta^{\parallel}{}_{b'c'}\;\;\mathcal{S}_{(2)}{}^{b'cc'}\;\;n_{c}\;\left({}^{3}_{\bullet}\mathcal{D}^{\parallel d}\,n^{a'}\right)\Bigg)\Bigg)\Bigg)\Bigg)[x^{0},\,x^{1},\,x^{2},\,x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},$$

$$\iiint\Bigg(-\frac{1}{3\mathcal{J}}\Bigg(3\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'}\right]\left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)a'}{}^{b}\right]\left(\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{\bullet 3}\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel}{}_{a}\,n_{b}\right]+\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{\bullet 3}\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel}{}_{b}\,n_{a}\right]+\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\bar{\beta}_{\bullet 1}\;{}^{2^+}_{\bullet}\lambda^{\perp}_{\mathcal{T}ab}-$$

$$\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{\bullet 3}\;{}^{1^+}_{\bullet}\mathcal{T}^{\parallel}{}_{ab}\right)-\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b}{}_{a'}\right]\left(\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{\bullet 3}\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel}{}_{a}\,n_{b}\right]+\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\hat{\beta}_{\bullet 3}\underset{\eta^{\parallel}}{\mathrm{P}}\left[{}^{3}_{\bullet}\mathcal{D}^{\parallel}{}_{b}\,n_{a}\right]+$$

$$\mathcal{M}_{Pl}{}^{2}\,\bar{\beta}_{\overset{\cdot}{1}}\;{}^{2^{+}}_{\cdot}\lambda^{\perp}_{\mathcal{T}\,ab}-\mathcal{M}_{Pl}{}^{2}\,\hat{\beta}_{\overset{\cdot}{3}}\;{}^{1^{+}}_{\cdot}\mathcal{T}^{\parallel}{}_{ab}\Big)+\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a}{}^{b}\big]-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{b}{}_{a}\big]\Big)\Big(\mathcal{M}_{Pl}{}^{2}\,\hat{\beta}_{\overset{\cdot}{3}}$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\big[{}^{3}_{\cdot}\mathcal{D}^{\parallel}{}_{a'}\,n_{b}\big]+\mathcal{M}_{Pl}{}^{2}\,\hat{\beta}_{\overset{\cdot}{3}}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[{}^{3}_{\cdot}\mathcal{D}^{\parallel}{}_{b}\,n_{a'}\big]+\mathcal{M}_{Pl}{}^{2}\,\bar{\beta}_{\overset{\cdot}{1}}\;{}^{2^{+}}_{\cdot}\lambda^{\perp}_{\mathcal{T}\,a'b}-\mathcal{M}_{Pl}{}^{2}\,\hat{\beta}_{\overset{\cdot}{3}}\;{}^{1^{+}}_{\cdot}\mathcal{T}^{\parallel}{}_{a'b}\Big)\Big)+$$

$$4\,\mathcal{M}_{Pl}{}^{2}\,\hat{\beta}_{\overset{\cdot}{3}}\;\eta^{\parallel}{}_{aa'}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{bb'}\big]\;\mathcal{S}_{(1)}{}^{aa'}\;\;{}^{1^{+}}_{\cdot}\mathcal{T}^{\parallel}{}_{bb'}\Big)\Big)[x^{0},\,x^{1},\,x^{2},\,x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},\;0,$$

$$0,\,0,\,0,\,0,\,0,\,0,\,0,\,0,\,0,\,0,\,0,\;\iiint\frac{{}^{1^{+}}_{\cdot}\hat{\pi}_{\mathcal{A}\,aa'}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{aa'}\big]\,\mathcal{S}_{(1)}}{\mathcal{J}^{2}}[x^{0},\,x^{1},\,x^{2},\,x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},$$

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0,
0,

$$\iiint\frac{\epsilon\gamma_{aa'lm}\;\;{}^{1^{-}}_{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{lm}\big]\,\mathcal{S}_{(1)}\;n^{a'}}{\mathcal{J}^{2}}[x^{0},\,x^{1},\,x^{2},\,x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},$$

0, 0, 0,
0, 0, 0,
0, 0, 0, 0,

$$\iiint\frac{1}{12\,\mathcal{J}^{2}}\Big(3\;{}^{1^{+}}_{\cdot}\hat{\pi}_{\mathcal{A}\,a'b}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'}\big]\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a}{}^{b}\big]-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{b}{}_{a}\big]\Big)+3\;{}^{1^{+}}_{\cdot}\hat{\pi}_{\mathcal{A}\,ab}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'}\big]\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a'}{}^{b}\big]-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{b}{}_{a'}\big]\Big)-$$

$$4\;\eta^{\parallel}{}_{aa'}\;\;{}^{1^{+}}_{\cdot}\hat{\pi}_{\mathcal{A}\,bb'}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{bb'}\big]\;\mathcal{S}_{(1)}{}^{aa'}\Big)[x^{0},\,x^{1},\,x^{2},\,x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},\;0,\,0,\,0,\,0,\,0,\,0,$$

$$0,\,0,\;\iiint\frac{1}{32\,\mathcal{J}}\Big(-6\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b}\big]\Big(2\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{aa'b}\big]+\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{aba'}\big]-2\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a'ab}\big]-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a'ba}\big]-$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{baa'}\big]+\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{ba'a}\big]\Big)+9\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{a'b}\;\;\mathcal{S}_{(1)}{}^{aa'b}\big]\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{b'c}\;\;\mathcal{S}_{(2)}{}_{a}{}^{b'c}\big]-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{b'c}\;\;\mathcal{S}_{(2)}{}^{b'}{}_{a}{}^{c}\big]\Big)+$$

$$9\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{ab}\;\;\mathcal{S}_{(1)}{}^{aa'b}\big]\Big(-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{b'c}\;\;\mathcal{S}_{(2)}{}_{a'}{}^{b'c}\big]+\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{b'c}\;\;\mathcal{S}_{(2)}{}^{b'}{}_{a'}{}^{c}\big]\Big)\Big)[x^{0},\,x^{1},\,x^{2},\,x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},$$

$$\iiint\frac{1}{32\,\mathcal{J}^{2}}\;{}^{1^{-}}_{\cdot}\hat{\pi}_{\mathcal{A}}{}^{a}\;\Big(-3\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{bb'}\;\;\mathcal{S}_{(1)}{}_{a'}{}^{bb'}\big]\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a}{}^{a'}\big]-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{a'}{}_{a}\big]\Big)+3\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{bb'}\;\;\mathcal{S}_{(1)}{}^{b}{}_{a'}{}^{b'}\big]$$

$$\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a}{}^{a'}\big]-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{a'}{}_{a}\big]\Big)+2\,\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}_{a}{}^{a'b}\big]-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{a'}{}_{a}{}^{b}\big]-2\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{a'b}{}_{a}\big]\Big)$$

$$\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a'b}\big]-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{ba'}\big]\Big)\Big)[x^{0},\,x^{1},\,x^{2},\,x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},\;0,\,0,\,0,\,0,\,0,\,0,\,0,$$

$$0,\;\iiint\Big(\frac{2}{3}\,\mathcal{M}_{Pl}{}^{2}\,\bar{\beta}_{\overset{\cdot}{1}}{}^{2}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'}\big]\Big(-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{aa'}\big]+\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a'a}\big]\Big)\Big)[x^{0},\,x^{1},\,x^{2},\,x^{3}]\,dx^{3}\,dx^{2}\,dx^{1},$$

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0, 0, 0, 0, 0,
0, 0, 0, 0, 0,
0, 0, 0, 0, 0,
0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0,
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$$\iiint \frac{1}{4\mathcal{J}^2} \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{a}\right] \left(\delta_{a'}{}^{b}\, \mathcal{J} \left(\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b}{}^{b'}\, n_{b'}\right] - \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b'}{}_{b}\, n_{b'}\right] \right) \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a}\, n^{a'}\right] - 2\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a}{}^{a'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a'}\, \mathcal{J}\right] +\right.$$

$$2\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{a'}{}_{a}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a'}\, \mathcal{J}\right] - \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'}{}^{b}\, n_{b}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel a'}\, n_{a}\right] + \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b}{}_{a'}\, n_{b}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel a'}\, n_{a}\right] +$$

$$\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'}{}^{b}\, n_{b}\right] \overset{1^{+}}{:}\mathcal{T}^{\parallel}{}_{a}{}^{a'} - \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{b}{}_{a'}\, n_{b}\right] \overset{1^{+}}{:}\mathcal{T}^{\parallel}{}_{a}{}^{a'} + 2\, \eta^{\parallel}{}_{a'b}\, \mathcal{J} \left(\overset{3}{:}\mathcal{D}^{\parallel b}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a}{}^{a'}\right] \right) -$$

$$\left. 2\, \eta^{\parallel}{}_{a'b}\, \mathcal{J} \left(\overset{3}{:}\mathcal{D}^{\parallel b}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{a'}{}_{a}\right] \right) \right) [x^0, x^1, x^2, x^3]\, dx^3\, dx^2\, dx^1,\ 0,$$

$$\iiint \frac{1}{32\mathcal{J}^2} \left(-6\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\, \mathcal{S}_{(1)}{}_{a'}{}^{b'c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{aa'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a}\mathcal{J}\right] + 6\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\, \mathcal{S}_{(1)}{}^{b'}{}_{a'}{}^{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{aa'}\right]\right.$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a}\mathcal{J}\right] + 4\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a}\mathcal{J}\right] - 4\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b'a'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a}\mathcal{J}\right] +$$

$$4\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b'}{}^{c}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a}\, n_{a'}\right] - 4\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{b'}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a}\, n_{a'}\right] +$$

$$2\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'}{}^{c}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a}\, n_{b'}\right] - 2\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{a'}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a}\, n_{b'}\right] -$$

$$4\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{ab'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a'}\, \mathcal{J}\right] + 6\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\, \mathcal{S}_{(1)}{}_{a}{}^{b'c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{aa'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a'}\, \mathcal{J}\right] -$$

$$6\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{b'c}\, \mathcal{S}_{(1)}{}^{b'}{}_{a}{}^{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{aa'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a'}\, \mathcal{J}\right] + 4\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b'a}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a'}\, \mathcal{J}\right] -$$

$$4\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b'}{}^{c}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a'}\, n_{a}\right] + 4\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{b'}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a'}\, n_{a}\right] -$$

$$2\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a}{}^{c}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a'}\, n_{b'}\right] + 2\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{a}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{a'}\, n_{b'}\right] +$$

$$3\, \delta_{a}{}^{b'}\, \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{c'i}\, \mathcal{S}_{(1)}{}_{b'}{}^{c'i}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'}{}^{c}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel a'}\, n^{a}\right] -$$

$$3\, \delta_{a}{}^{b'}\, \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{c'i}\, \mathcal{S}_{(1)}{}^{c'}{}_{b'}{}^{i}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'}{}^{c}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel a'}\, n^{a}\right] - 3\, \delta_{a}{}^{b'}\, \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{c'i}\, \mathcal{S}_{(1)}{}_{a'}{}^{c'i}\right]$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b'}{}^{c}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel a'}\, n^{a}\right] + 3\, \delta_{a}{}^{b'}\, \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{c'i}\, \mathcal{S}_{(1)}{}^{c'}{}_{a'}{}^{i}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b'}{}^{c}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel a'}\, n^{a}\right] -$$

$$3\, \delta_{a}{}^{b'}\, \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{c'i}\, \mathcal{S}_{(1)}{}_{b'}{}^{c'i}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{a'}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel a'}\, n^{a}\right] + 3\, \delta_{a}{}^{b'}\, \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{c'i}\, \mathcal{S}_{(1)}{}^{c'}{}_{b'}{}^{i}\right]$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{a'}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel a'}\, n^{a}\right] + 3\, \delta_{a}{}^{b'}\, \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{c'i}\, \mathcal{S}_{(1)}{}_{a'}{}^{c'i}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{b'}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel a'}\, n^{a}\right] -$$

$$3\, \delta_{a}{}^{b'}\, \mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\eta^{\parallel}{}_{c'i}\, \mathcal{S}_{(1)}{}^{c'}{}_{a'}{}^{i}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{b'}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel a'}\, n^{a}\right] - 8\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{aa'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{b'}\, \mathcal{J}\right] +$$

$$8\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'a}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{b'}\, \mathcal{J}\right] - 2\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a'}{}^{c}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{b'}\, n_{a}\right] +$$

$$2\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{a'}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{b'}\, n_{a}\right] + 2\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{a}{}^{c}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{b'}\, n_{a'}\right] -$$

$$2\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}^{c}{}_{a}\, n_{c}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\overset{3}{:}\mathcal{D}^{\parallel}{}_{b'}\, n_{a'}\right] + 4\mathcal{J}\, \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(1)}{}^{aa'b'}\right] \underset{\eta^{\parallel}}{\mathrm{P}}\left[\mathcal{S}_{(2)}{}_{b'}{}^{c}\, n_{c}\right] \overset{1^{+}}{:}\mathcal{T}^{\parallel}{}_{aa'} -$$

$$4\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{c}{}_{b'}\;\; n_{c}\big]\;{}^{1^+}\!\!\!\cdot\mathcal{T}^{\parallel}{}_{aa'}+2\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a'}{}^{c}\;\; n_{c}\big]\;{}^{1^+}\!\!\!\cdot\mathcal{T}^{\parallel}{}_{ab'}-$$

$$2\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{c}{}_{a'}\;\; n_{c}\big]\;{}^{1^+}\!\!\!\cdot\mathcal{T}^{\parallel}{}_{ab'}+3\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(1)}{}_{a'}{}^{cc'}\big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a}{}^{b'}\;\; n_{b'}\big]\;{}^{1^+}\!\!\!\cdot\mathcal{T}^{\parallel\,aa'}-3\,\mathcal{J}$$

$$\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(1)}{}^{c}{}_{a'}{}^{c'}\big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a}{}^{b'}\;\; n_{b'}\big]\;{}^{1^+}\!\!\!\cdot\mathcal{T}^{\parallel\,aa'}-3\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(1)}{}_{a'}{}^{cc'}\big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{b'}{}_{a}\;\; n_{b'}\big]\;{}^{1^+}\!\!\!\cdot\mathcal{T}^{\parallel\,aa'}+$$

$$3\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(1)}{}^{c}{}_{a'}{}^{c'}\big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{b'}{}_{a}\;\; n_{b'}\big]\;{}^{1^+}\!\!\!\cdot\mathcal{T}^{\parallel\,aa'}-2\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a}{}^{c}\;\; n_{c}\big]\;{}^{1^+}\!\!\!\cdot\mathcal{T}^{\parallel}{}_{a'b'}+$$

$$2\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{c}{}_{a}\;\; n_{c}\big]\;{}^{1^+}\!\!\!\cdot\mathcal{T}^{\parallel}{}_{a'b'}-4\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\Big({}^{3}\!\!\cdot\mathcal{D}^{\parallel}{}_{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a'b'}\big]\Big)+$$

$$4\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\Big({}^{3}\!\!\cdot\mathcal{D}^{\parallel}{}_{a}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{b'a'}\big]\Big)+4\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\Big({}^{3}\!\!\cdot\mathcal{D}^{\parallel}{}_{a'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{ab'}\big]\Big)-$$

$$4\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\Big({}^{3}\!\!\cdot\mathcal{D}^{\parallel}{}_{a'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{b'a}\big]\Big)+8\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\Big({}^{3}\!\!\cdot\mathcal{D}^{\parallel}{}_{b'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{aa'}\big]\Big)-$$

$$8\,\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'b'}\big]\Big({}^{3}\!\!\cdot\mathcal{D}^{\parallel}{}_{b'}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a'a}\big]\Big)-6\;\eta^{\parallel}{}_{a'b'}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(1)}{}_{a}{}^{cc'}\big]\Big({}^{3}\!\!\cdot\mathcal{D}^{\parallel\,b'}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{aa'}\big]\Big)+6\;\eta^{\parallel}{}_{ab'}$$

$$\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(1)}{}_{a'}{}^{cc'}\big]\Big({}^{3}\!\!\cdot\mathcal{D}^{\parallel\,b'}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{aa'}\big]\Big)+6\;\eta^{\parallel}{}_{a'b'}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(1)}{}^{c}{}_{a}{}^{c'}\big]\Big({}^{3}\!\!\cdot\mathcal{D}^{\parallel\,b'}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{aa'}\big]\Big)-$$

$$6\;\eta^{\parallel}{}_{ab'}\;\mathcal{J}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\eta^{\parallel}{}_{cc'}\;\;\mathcal{S}_{(1)}{}^{c}{}_{a'}{}^{c'}\big]\Big({}^{3}\!\!\cdot\mathcal{D}^{\parallel\,b'}\;\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{aa'}\big]\Big)\Big)\Big)[x^0,\, x^1,\, x^2,\, x^3]\, dx^3\, dx^2\, dx^1,$$

$$\iiint\frac{1}{3\,\mathcal{J}}\,\mathcal{M}_{\mathrm{Pl}}{}^{2}\,\bar{\beta}_{\underset{1}{\cdot}}\,\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(1)}{}^{aa'}\big]\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a'}{}^{b}\big]\big({}^{1^+}\!\!\!\cdot\lambda^{\parallel}_{\mathcal{T}}{}_{ab}-{}^{1^+}\!\!\!\cdot\lambda^{\perp}_{\mathcal{T}}{}_{ab}\big)+\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{b}{}_{a'}\big]\big(-{}^{1^+}\!\!\!\cdot\lambda^{\parallel}_{\mathcal{T}}{}_{ab}+{}^{1^+}\!\!\!\cdot\lambda^{\perp}_{\mathcal{T}}{}_{ab}\big)-$$

$$\Big(\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}_{a}{}^{b}\big]-\underset{\eta^{\parallel}}{\mathrm{P}}\big[\mathcal{S}_{(2)}{}^{b}{}_{a}\big]\Big)\big({}^{1^+}\!\!\!\cdot\lambda^{\parallel}_{\mathcal{T}}{}_{a'b}-{}^{1^+}\!\!\!\cdot\lambda^{\perp}_{\mathcal{T}}{}_{a'b}\big)\Big)[x^0,\, x^1,\, x^2,\, x^3]\, dx^3\, dx^2\, dx^1\Big\}.\mathcal{S}_{(1)}\Big[$$

$$x^0,\, x^1,\, x^2,\, x^3]\, dx^3\, dx^2\, dx^1,\;\iiint(\#3.\mathcal{S}_{(2)})[x^0,\, y^1,\, y^2,\, y^3]\, dy^3$$

$$dy^2$$

$$dy^1\Big\}\approx\#6$$

MapThread: List expected at position 2 in MapThread[Null, Null].

This concludes the HiGGS output.

Concrete relation to manuscript: The above constraints and Poisson brackets constitute the data represented by coloured squares in Fig. 5 in our Manuscript. The primary constraints ϕ are listed along the top row. The primary Poisson brackets $\{\phi,\ \phi\}$ are used to infer the white, red or yellow squares in the matrix depending on whether they are vanishing, or appear only in proportion to phase space variables, or are order unity (e.g. proportional to the metric or unit-timelike vector). The $\{\phi,\ {}^{0^+}\!\!\cdot\mathcal{H}_b\}$ Poisson brackets define the secondary constraints "χ", which appear on the sides of the matrix. Most of the Poisson brackets $\{\chi,\ \phi\}$ need not be computed (grey squares), but it can be noticed without detailed calculation that many of the remaining required brackets (yellow squares) do indeed arise. Unlike in the strongly coupled case, we find that tertiary and quaternary constraints arise in the theory: but these are also not computed in detail above.

Instead, their existence can readily be inferred by considering whether the consistency chain in question can be absorbed by determining a multiplier, or whether it must continue. In case of continuation, the relevant terms in the induced constraints which arise can be understood by considering Eq. (4) in arXiv:2205.13534.