

```
In[*]:= Get@FileNameJoin@{NotebookDirectory[], "SupplementalMaterials.m"};
```

```
-----  
Package xAct`xPerm` version 1.2.3, {2015, 8, 23}
```

```
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```

```
Connecting to external linux executable...
```

```
Connection established.  
-----
```

```
Package xAct`xTensor` version 1.2.0, {2021, 10, 17}
```

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```

```
Package xAct`xPlain` version 1.0.0-developer, {2023, 6, 27}
```

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Supplemental materials: Hamiltonian analysis

Introduction

How to use this document

These calculations are designed to accompany our manuscript in the form of supplemental materials, for the sake of reproducibility. Throughout, commentary by the authors takes the form of green text. Citations, where needed, will be managed by direct reference to arXiv numbers, and all such references are already provided in full within the body of our manuscript. One exception is the source referred to throughout as ‘Blagojević’; this pertains to the book ‘Gravitation and Gauge Symmetries’, which is also referenced within the manuscript.

Concrete relation to manuscript: In boxes like this, we will make specific connections between a result which is obtained in the supplemental material and a claim which is made in the manuscript. These points of contact are not always numbered equations, they could be textual.

Note that a programmatical session in the Wolfram language does not really correspond to the clean flow of thoughts in a LaTeX document: there are differences that can't (and shouldn't) be ignored. Thus, whilst this document should be at least readable in standalone format, the reader is encouraged to follow it in tandem with the Wolfram language source codes, so as to avoid ambiguities. In this way, the specific operations and manipulations of quantities will become absolutely clear.

Loading HiGGS

For these calculations, we will use the HiGGS package. The version of HiGGS used for the computations here are both developer-only, and so we include copies of the sources with these supplemental materials.

```
-----
Package xAct`xPert` version 1.0.6, {2018, 2, 28}

Copyright (C) 2005–2020, David Brizuela, Jose M. Martin-Garcia
and Guillermo A. Mena Marugan, under the General Public License.

** Variable $PrePrint assigned value ScreenDollarIndices
** Variable $CovDFormat changed from Prefix to Postfix
** Option AllowUpperDerivatives of ContractMetric changed from False to True
** Option MetricOn of MakeRule changed from None to All
** Option ContractMetrics of MakeRule changed from False to True
-----
```

```
Package xAct`Invar` version 2.0.5, {2013, 7, 1}

Copyright (C) 2006–2020, J. M. Martin-Garcia,
D. Yllanes and R. Portugal, under the General Public License.

** DefConstantSymbol: Defining constant symbol sigma.
** DefConstantSymbol: Defining constant symbol dim.
** Option CurvatureRelations of DefCovD changed from True to False
** Variable $CommuteCovDsOnScalars changed from True to False
-----
```

```
Package xAct`xCoba` version 0.8.6, {2021, 2, 28}

Copyright (C) 2005–2021, David Yllanes and
Jose M. Martin-Garcia, under the General Public License.
-----
```

```
Package xAct`SymManipulator` version 0.9.5, {2021, 9, 14}

Copyright (C) 2011–2021, Thomas Bäckdahl, under the General Public License.
-----
```

```
Package xAct`xTras` version 1.4.2, {2014, 10, 30}
```

```
CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.
```

```
** Variable $CovDFormat changed from Postfix to Prefix
```

```
** Option CurvatureRelations of DefCovD changed from False to True
```

```
-----
Package xAct`HiGGS` version 2.0.0-developer, {2023, 6, 27}
```

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```
-----
HiGGS incorporates code by Cyril Pitrou.
```

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it under certain conditions. See the General Public License for details.
```

The HiGGS package is now loaded. We are interested only in the computations which lead to our Fig. 5. (in the appendix) in the manuscript. Two theories are considered. To perform the Hamiltonian analysis we use the `StudyTheory[]` command in HiGGS, specifying the parameters of the theory. For details of this command, and for all notation used in this supplement, see arXiv:2206.00658. The results were precomputed on a desktop computer. This is because the full Hamiltonian analysis of a theory, even run on a modern desktop computer with multiple processor cores, still takes up to half an hour. Below we will display the results using the `ViewTheory[]` command, which takes the pre-computed binary files (.thr.mx) and formats them. If you want to re-compute the analysis locally, the few lines of code you will need are stored at the top of this (.m) file, so just un-comment them. If there are problems with the execution, please refer to the HiGGS docs, or email wb263@cam.ac.uk.

Singular spectrum without multipliers

Introductory comments

In the first instance we perform the Hamiltonian analysis of the theory without multipliers. As we know, this theory suffers from strong coupling. Accordingly, in Eq. (4) in arXiv:2205.13534 we set $\hat{\alpha}_1^\cdot, \hat{\alpha}_2^\cdot, \hat{\alpha}_3^\cdot,$

$\hat{\alpha}_4^\cdot, \hat{\alpha}_6^\cdot, \mathcal{M}_{Pl}^2 \hat{\beta}_1^\cdot, \mathcal{M}_{Pl}^2 \hat{\beta}_2^\cdot, \mathcal{M}_{Pl}^2 \bar{\beta}_3^\cdot,$ to zero, suppressing most of the higher-curvature invariants, two of the three torsion invariants, and all of the Lagrange multipliers, but allow for nonvanishing $\mathcal{M}_{Pl}^2 \hat{\alpha}_0^\cdot$, (Einstein-Hilbert term) $\hat{\alpha}_5^\cdot$ (Riemann-Cartan-Maxwell invariant of curvature) and $\mathcal{M}_{Pl}^2 \hat{\beta}_3^\cdot$.

The results of the analysis are presented below; the HiGGS output is designed to be reasonably self-explanatory, but we will nonetheless add some guiding commentary.

The first set of data are the primary constraints ϕ of the theory. These are implied by the impossibility of inverting the conjugate field-momenta for the velocities of the fields. As is now common in the PGT literature, we refer to these as 'if-constraints', i.e. their existence depends on the linear relations assumed among the Lagrangian parameters.

Next, the Legendre-transformed Lagrangian is presented. This is broken into terms; the super-Hamiltonian (i.e. the coefficient ${}^{0+}\mathcal{H}_b$ of the lapse function \mathcal{N}), the linear super-momentum (i.e. the coefficient ${}^{1-}\mathcal{H}_{b\ i}$ of the shift vector) and the angular super-momentum (i.e. the antisymmetric coefficient ${}^{2+}\mathcal{H}_{\mathcal{A}\ nm}$ of the timelike part of the rotational gauge field). These concepts are introduced in Blagojević.

Once the primary constraints are listed, the matrix $\{\phi, \phi\}$ of Poisson brackets between the primaries is computed. Of course, these brackets only need to be computed for the upper triangular part of the matrix. The resulting entries lie behind the red, white and yellow squares in the left-hand panel of Fig. 5.

Finally, the velocities of these primaries ϕ are computed, i.e. their Poisson brackets with the super-Hamiltonian ${}^{0+}\mathcal{H}_b$. These velocities serve two purposes; (i) they can be used to determine the 'missing velocities' or 'Hamiltonian multipliers' μ (not to be confused with the geometric multipliers λ used in the manuscript), or (ii) they can define the secondary constraints of the theory. Whichever of these applies depends of course on $\{\phi, \phi\}$, according to the consistency condition.

Lists of constraints and Poisson brackets from HiGGS

Having briefly recounted the notation of arXiv:2206.00658, we make a call to the HiGGS command `ViewTheory[]` on the appropriate model. This command presents all the results which were pre-computed by the earlier call to `StudyTheory[]`.

```
** DefTheory: Incorporating the binary at ConstraintAlgebraAlp0Alp5Bet3.thr.mx
```

```
** DefTheory: Found the following primary if-constraints:
```

$${}^{0+}\phi_b \equiv \frac{{}^{0+}\pi_b}{\mathcal{J}} \approx 0$$

$${}^{1-}\phi_{b\ i} \equiv \frac{{}^{1-}\pi_{b\ i}}{\mathcal{J}} \approx 0$$

$${}^{2+}\phi_{b\ i\ j} \equiv \frac{{}^{2+}\pi_{b\ i\ j}}{\mathcal{J}} \approx 0$$

$${}^{0+}\phi_{\mathcal{A}} \equiv 3\mathcal{M}_{\text{Pl}}^2 \alpha_0 + \frac{{}^{0+}\pi_{\mathcal{A}}}{\mathcal{J}} \approx 0$$

$${}^{0-}\phi_{\mathcal{A}} \equiv \frac{{}^{0-}\pi_{\mathcal{A}}}{\mathcal{J}} \approx 0$$

$${}^{2+}\phi_{\mathcal{A}\ i\ j} \equiv \frac{{}^{2+}\pi_{\mathcal{A}\ i\ j}}{\mathcal{J}} \approx 0$$

$$\overset{2}{\cdot}\phi_{\mathcal{A}ijh} \equiv \frac{2 \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ijh}}{3\mathcal{I}} + \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ihj}}{3\mathcal{I}} - \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}jhi}}{3\mathcal{I}} \approx 0$$

** DefTheory: Found the following primary parallel

if-constraints associated with geometric multiplier momenta (if the sector is singular, the parallel constraint is taken to be the singular constraint and the identical perpendicular constraint below is dropped):

** DefTheory: Found the following primary perpendicular

if-constraints associated with geometric multiplier momenta:

** DefTheory: Found the following secondary perpendicular if-constraints:

** DefTheory: Found the following secondary parallel if-constraints:

** DefTheory: Found the following secondary singular if-constraints:

** DefTheory: Defining association key \$IfConstraints

for the theory association ConstraintAlgebraAlp0Alp5Bet3

** ViewTheory: The super-Hamiltonian is:

$$\begin{aligned} \overset{0}{\cdot}\mathcal{H}_b \equiv & \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}a} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^a}{16 \overset{\wedge}{\alpha}_5 \mathcal{I}} + \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ab} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{ab}}{8 \overset{\wedge}{\alpha}_5 \mathcal{I}} + \frac{3 \overset{1}{\cdot}\overset{\wedge}{\pi}_{b\mathcal{A}b} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^{ab}}{16 \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{I}} + \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \mathcal{I}]}{\mathcal{I}} + \\ & \frac{\mathcal{M}_{Pl}^2 \overset{\wedge}{\alpha}_0 \mathcal{I} \overset{0}{\cdot}\mathcal{R}^{\parallel}}{2} - \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^a \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}}^{\parallel a} + \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{ab} \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}b}^{\parallel} + \frac{1}{6} \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{I} \overset{0}{\cdot}\mathcal{I}^{\parallel 2} - \overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \overset{1}{\cdot}\mathcal{I}_{\mathcal{A}}^{\parallel a} + \\ & \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^{ab} \overset{1}{\cdot}\mathcal{I}_{\mathcal{A}b}^{\parallel} - \frac{16}{27} \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{I} \overset{2}{\cdot}\mathcal{I}_{\mathcal{A}bb'}^{\parallel} \overset{2}{\cdot}\mathcal{I}^{abb'} + \frac{32}{27} \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{I} \overset{2}{\cdot}\mathcal{I}_{\mathcal{A}b'b}^{\parallel} \overset{2}{\cdot}\mathcal{I}^{abb'} - \\ & n^a \left(\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel b} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}a}^b \right) - n^a \left(\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel b} \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}a}^b \right) - \eta^{\parallel \mathcal{A}b} \left(\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel b} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \right) + \frac{1}{3} \eta^{\parallel \mathcal{A}b} \overset{0}{\cdot}\overset{\wedge}{\pi}_b^a \left(\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel b} n^a \right) \approx 0 \end{aligned}$$

** ViewTheory: The linear super-momentum is:

$$\begin{aligned} \overset{1}{\cdot}\mathcal{H}_{b\parallel} \equiv & -\frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel a} \eta_{\parallel}] + \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \mathcal{I}]}{\mathcal{I}} + \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \mathcal{I}]}{\mathcal{I}} - \mathcal{P}_{\eta}[\eta^{aa'} \left(\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel a'} \overset{1}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \right)] - \\ & \mathcal{P}_{\eta}[\eta^{aa'} \left(\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel a'} \overset{2}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \right)] + \frac{\overset{0}{\cdot}\overset{\wedge}{\pi}_b^a \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \mathcal{I}]}{3\mathcal{I}} - \frac{1}{3} \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \overset{0}{\cdot}\overset{\wedge}{\pi}_b^a] - \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \mathcal{P}_{\eta}[\overset{3}{\mathcal{D}}_{\mathcal{A}}^{\parallel} \eta_a] + \frac{1}{6} \epsilon^{\parallel \mathcal{A}aa'} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{aa'} \overset{0}{\cdot}\mathcal{R}^{\parallel} - \\ & \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}\parallel} \overset{0}{\cdot}\mathcal{R}^{\parallel}}{6} - \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}\parallel a} \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}}^{\parallel a} - \frac{1}{2} \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}\parallel a} \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}}^{\parallel a} + \frac{\overset{0}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}} \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}}^{\parallel}}{3} - \frac{4}{3} \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}\parallel}^{aa'} \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}aa'}^{\parallel} + \\ & \frac{1}{6} \epsilon^{\parallel \mathcal{A}aa'} \overset{0}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{aa'} \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}}^{\parallel aa'} - \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^a \overset{1}{\cdot}\mathcal{R}_{\mathcal{A}\parallel a}^{\parallel} - \frac{4}{3} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{aa'} \overset{2}{\cdot}\mathcal{R}_{\mathcal{A}\parallel aa'}^{\parallel} + \frac{4}{3} \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^{aa'} \overset{2}{\cdot}\mathcal{R}_{\mathcal{A}\parallel aa'}^{\parallel} + \\ & \frac{4}{3} \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}\parallel}^{aa'} \overset{2}{\cdot}\mathcal{R}_{\mathcal{A}aa'}^{\parallel} - \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}^a \overset{2}{\cdot}\mathcal{R}_{\mathcal{A}\parallel a}^{\parallel} + \frac{1}{6} \epsilon^{\parallel \mathcal{A}aa'} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^{aa'} \overset{0}{\cdot}\mathcal{I}^{\parallel} - \frac{3}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \overset{1}{\cdot}\mathcal{I}_{\mathcal{A}}^{\parallel a} - \\ & \frac{3}{2} \overset{2}{\cdot}\overset{\wedge}{\pi}_{b\parallel a} \overset{1}{\cdot}\mathcal{I}_{\mathcal{A}}^{\parallel a} + \frac{1}{2} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^a \overset{1}{\cdot}\mathcal{I}_{\mathcal{A}\parallel a}^{\parallel} - \frac{4}{3} \overset{1}{\cdot}\overset{\wedge}{\pi}_b^{aa'} \overset{2}{\cdot}\mathcal{I}_{\mathcal{A}\parallel aa'}^{\parallel} + \frac{4}{3} \overset{2}{\cdot}\overset{\wedge}{\pi}_b^{aa'} \overset{2}{\cdot}\mathcal{I}_{\mathcal{A}\parallel aa'}^{\parallel} \approx 0 \end{aligned}$$

** ViewTheory: The 1+ part of the angular super-momentum is:

$$\begin{aligned}
 {}^1\mathcal{H}_{\mathcal{A}nm} \equiv & -2 \frac{{}^1\hat{\pi}_{bmn}}{\mathcal{J}} - \frac{\epsilon^{\parallel mn a} \frac{0}{\cdot} \hat{\pi}_{\mathcal{A}} P[\frac{3}{\cdot} \mathcal{D}^{\parallel a} \mathcal{J}]}{6 \mathcal{J}} + \frac{4 \frac{2}{\cdot} \hat{\pi}_{\mathcal{A}mn a} P[\frac{3}{\cdot} \mathcal{D}^{\parallel a} \mathcal{J}]}{3 \mathcal{J}} + \frac{1}{6} \epsilon^{\parallel mn a} P[\frac{3}{\cdot} \mathcal{D}^{\parallel a} \frac{0}{\cdot} \hat{\pi}_{\mathcal{A}}] - \\
 & \frac{1}{2} \frac{{}^1\hat{\pi}_{\mathcal{A}n a} P[\frac{3}{\cdot} \mathcal{D}^{\parallel a} n_m]}{\mathcal{J}} - \frac{1}{2} \frac{{}^2\hat{\pi}_{\mathcal{A}n a} P[\frac{3}{\cdot} \mathcal{D}^{\parallel a} n_m]}{\mathcal{J}} + \frac{1}{2} \frac{{}^1\hat{\pi}_{\mathcal{A}m a} P[\frac{3}{\cdot} \mathcal{D}^{\parallel a} n_n]}{\mathcal{J}} + \frac{1}{2} \frac{{}^2\hat{\pi}_{\mathcal{A}m a} P[\frac{3}{\cdot} \mathcal{D}^{\parallel a} n_n]}{\mathcal{J}} - \\
 & \frac{4}{3} P[\eta^{\parallel a a'} (\frac{3}{\cdot} \mathcal{D}^{\parallel a} \frac{2}{\cdot} \hat{\pi}_{\mathcal{A}mn a})] + \frac{{}^1\hat{\pi}_{\mathcal{A}n} P[\frac{3}{\cdot} \mathcal{D}^{\parallel m} \mathcal{J}]}{2 \mathcal{J}} - \frac{1}{2} P[\frac{3}{\cdot} \mathcal{D}^{\parallel m} \frac{1}{\cdot} \hat{\pi}_{\mathcal{A}n}] - \frac{1}{2} \frac{{}^1\hat{\pi}_{\mathcal{A}n a} P[\frac{3}{\cdot} \mathcal{D}^{\parallel m} n^a]}{\mathcal{J}} - \\
 & \frac{1}{2} \frac{{}^2\hat{\pi}_{\mathcal{A}n a} P[\frac{3}{\cdot} \mathcal{D}^{\parallel m} n^a]}{\mathcal{J}} - \frac{{}^1\hat{\pi}_{\mathcal{A}m} P[\frac{3}{\cdot} \mathcal{D}^{\parallel n} \mathcal{J}]}{2 \mathcal{J}} + \frac{1}{2} P[\frac{3}{\cdot} \mathcal{D}^{\parallel n} \frac{1}{\cdot} \hat{\pi}_{\mathcal{A}m}] + \frac{1}{2} \frac{{}^1\hat{\pi}_{\mathcal{A}m a} P[\frac{3}{\cdot} \mathcal{D}^{\parallel n} n^a]}{\mathcal{J}} + \\
 & \frac{1}{2} \frac{{}^2\hat{\pi}_{\mathcal{A}m a} P[\frac{3}{\cdot} \mathcal{D}^{\parallel n} n^a]}{\mathcal{J}} + \frac{1}{6} \epsilon^{\parallel mn a} \frac{0}{\cdot} \hat{\pi}_{\mathcal{A}} \frac{1}{\cdot} \mathcal{T}^{\parallel a} - \frac{4}{3} \frac{{}^2\hat{\pi}_{\mathcal{A}mn a} \frac{1}{\cdot} \mathcal{T}^{\parallel a}}{\mathcal{J}} - \frac{1}{2} \frac{{}^1\hat{\pi}_{\mathcal{A}n} \frac{1}{\cdot} \mathcal{T}^{\parallel m}}{\mathcal{J}} + \frac{1}{2} \frac{{}^1\hat{\pi}_{\mathcal{A}m} \frac{1}{\cdot} \mathcal{T}^{\parallel n}}{\mathcal{J}} - \\
 & \frac{1}{2} \frac{{}^1\hat{\pi}_{\mathcal{A}n}^a \frac{1}{\cdot} \mathcal{T}^{\parallel m a}}{\mathcal{J}} - \frac{1}{2} \frac{{}^2\hat{\pi}_{\mathcal{A}n}^a \frac{1}{\cdot} \mathcal{T}^{\parallel m a}}{\mathcal{J}} - \frac{1}{3} \frac{{}^0\hat{\pi}_{\mathcal{A}} \frac{1}{\cdot} \mathcal{T}^{\parallel mn}}{\mathcal{J}} + \frac{1}{2} \frac{{}^1\hat{\pi}_{\mathcal{A}m}^a \frac{1}{\cdot} \mathcal{T}^{\parallel n a}}{\mathcal{J}} + \frac{1}{2} \frac{{}^2\hat{\pi}_{\mathcal{A}m}^a \frac{1}{\cdot} \mathcal{T}^{\parallel n a}}{\mathcal{J}} \approx 0
 \end{aligned}$$

** ViewTheory: The on-shell brackets between the various if-constraints are:

$$\begin{aligned}
 & \left\{ \iiint \left(\frac{0}{\cdot} \phi_b \cdot \frac{(1)}{\cdot} S \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\frac{0}{\cdot} \phi_b \cdot \frac{(2)}{\cdot} S \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint \left(\frac{0}{\cdot} \phi_b \cdot \frac{(1)}{\cdot} S \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\frac{1}{\cdot} \phi_{b i} \cdot \frac{(2)}{\cdot} S^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint \left(\frac{0}{\cdot} \phi_b \cdot \frac{(1)}{\cdot} S \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\frac{2}{\cdot} \phi_{b i m} \cdot \frac{(2)}{\cdot} S^{i m} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint \left(\frac{0}{\cdot} \phi_b \cdot \frac{(1)}{\cdot} S \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\frac{0}{\cdot} \phi_{\mathcal{A}} \cdot \frac{(2)}{\cdot} S \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
 & \quad \iiint \left(-\frac{6 \mathcal{M}_{Pl}^2 \hat{\alpha}_{\theta} \frac{(1)}{\cdot} S \frac{(2)}{\cdot} S}{\mathcal{J}} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
 & \left\{ \iiint \left(\frac{0}{\cdot} \phi_b \cdot \frac{(1)}{\cdot} S \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\frac{0}{\cdot} \phi_{\mathcal{A}} \cdot \frac{(2)}{\cdot} S \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint \left(\frac{0}{\cdot} \phi_b \cdot \frac{(1)}{\cdot} S \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\frac{2}{\cdot} \phi_{b i m} \cdot \frac{(2)}{\cdot} S^{i m} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint \left(\frac{0}{\cdot} \phi_b \cdot \frac{(1)}{\cdot} S \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\frac{2}{\cdot} \phi_{b i m n} \cdot \frac{(2)}{\cdot} S^{i m n} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
 & \left\{ \iiint \left(\frac{1}{\cdot} \phi_{b i} \cdot \frac{(1)}{\cdot} S^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\frac{1}{\cdot} \phi_{b i} \cdot \frac{(2)}{\cdot} S^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
 & \quad \iiint \frac{2 \frac{1}{\cdot} \hat{\pi}_{b a a'} P[\frac{(1)}{\cdot} S^a] P[\frac{(2)}{\cdot} S^{a'}]}{\mathcal{J}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
 & \left\{ \iiint \left(\frac{1}{\cdot} \phi_{b i} \cdot \frac{(1)}{\cdot} S^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\frac{2}{\cdot} \phi_{b i m} \cdot \frac{(2)}{\cdot} S^{i m} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0
 \end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left(\overset{1}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \left(- \frac{\overset{1}{\cdot} \pi_{\mathcal{A}}^a P_{\eta}^{[(1)S_a]} \overset{(2)}{S}}{\mathcal{J}^2} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{2 \epsilon_{\alpha\alpha' i \alpha 1} \overset{1}{\cdot} \pi_{\mathcal{A}}^i \overset{2\alpha 1}{P_{\eta}^{[(1)S^{a'}]} \overset{(2)}{S} n^a}}{\mathcal{J}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A}lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{\overset{1}{\cdot} \pi_{\mathcal{A}}^a \left(3 P_{\eta}^{[(1)S^{a'}]} \left(P_{\eta}^{[(2)S_{aa'}]} + P_{\eta}^{[(2)S_{a'a}]} \right) - 2 \eta_{a'b} P_{\eta}^{[(1)S_a]} \overset{(2)}{S}^{a'b} \right)}{12 \mathcal{J}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A}lmn} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{8 \mathcal{J}^2} P_{\eta}^{[(1)S^a]} \left(\overset{1}{\cdot} \pi_{\mathcal{A}a'b} \left(-2 P_{\eta}^{[(2)S_a^{a'b}]} + 2 P_{\eta}^{[(2)S^{a'b}]} + 4 P_{\eta}^{[(2)S^{a'b}]} \right) + \right. \\
& \quad \left. 3 \overset{1}{\cdot} \pi_{\mathcal{A}a}^{a'} \left(P_{\eta}^{[\eta_{bb'}^{(2)S_a^{bb'}}]} - P_{\eta}^{[\eta_{bb'}^{(2)S_a^{b'b'}}]} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\cdot} \phi_{ilm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{P_{\eta}^{[(1)S^{aa'}]} \left(\overset{1}{\cdot} \pi_{ba'b} \left(P_{\eta}^{[(2)S_a^b]} + P_{\eta}^{[(2)S^b_a]} \right) + \overset{1}{\cdot} \pi_{ba'b} \left(P_{\eta}^{[(2)S_a^b]} + P_{\eta}^{[(2)S^b_a]} \right) \right)}{2 \mathcal{J}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{2}{\cdot} \phi_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\cdot} \phi_{ilm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{12 \mathcal{J}^2} \left(3 P_{\eta}^{[(1)S^{aa'}]} \left(2 \mathcal{M}_{P1}^2 \overset{\hat{a}}{\alpha}_{\bullet} \mathcal{T} P_{\eta}^{[(2)S_{aa'}]} + 2 \mathcal{M}_{P1}^2 \overset{\hat{a}}{\alpha}_{\bullet} \mathcal{T} P_{\eta}^{[(2)S_{a'a}]} + \right. \right. \\
& \quad \left. \overset{1}{\cdot} \pi_{\mathcal{A}ab} P_{\eta}^{[(2)S_a^b]} + \overset{1}{\cdot} \pi_{\mathcal{A}a'b} \left(P_{\eta}^{[(2)S_a^b]} + P_{\eta}^{[(2)S^b_a]} \right) + \overset{1}{\cdot} \pi_{\mathcal{A}ab} P_{\eta}^{[(2)S^b_a]} \right) - \\
& \quad \left. 4 \mathcal{M}_{P1}^2 \overset{\hat{a}}{\alpha}_{\bullet} \eta_{aa'} \eta_{bb'} \mathcal{T} \overset{(1)}{S}^{aa'} \overset{(2)}{S}^{bb'} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\left\{ \iiint \left(\overset{2}{\cdot} \phi_{b \mid j} \cdot \overset{(1)}{S}^{ij} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid mn} \cdot \overset{(2)}{S}^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx$$

$$\iiint \frac{1}{32 \mathcal{T}^2} 3 \overset{1}{\cdot} \overset{\wedge}{\pi}_{\mathcal{A}}^a \left(P_{\eta}^{(1) S^a} \right) P_{\eta}^{(1) S^a} P_{\eta}^{(2) S_a, bb'} + 2 P_{\eta}^{(1) S^a b} \left(P_{\eta}^{(2) S_{aa'b}} + P_{\eta}^{(2) S_{abaa'}} - P_{\eta}^{(2) S_{a'ab}} - P_{\eta}^{(2) S_{baaa'}} \right) +$$

$$P_{\eta}^{(1) S_a^a} \left(P_{\eta}^{(2) S_a, bb'} - P_{\eta}^{(2) S_a, b'b'} \right) - P_{\eta}^{(1) S^a} P_{\eta}^{(2) S_a, b'b'} -$$

$$2 \eta_{a'b} P_{\eta}^{(2) S_a, b'b'} \overset{(1)}{S}^{a'b} + 2 \eta_{a'b} P_{\eta}^{(2) S_a, b'b'} \overset{(1)}{S}^{a'b} \Big) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1$$

$$\left\{ \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid m} \cdot \overset{(2)}{S}^{lm} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid mn} \cdot \overset{(2)}{S}^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid m} \cdot \overset{(2)}{S}^{lm} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid mn} \cdot \overset{(2)}{S}^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid j} \cdot \overset{(1)}{S}^{ij} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid m} \cdot \overset{(2)}{S}^{lm} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid j} \cdot \overset{(1)}{S}^{ij} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid mn} \cdot \overset{(2)}{S}^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid jh} \cdot \overset{(1)}{S}^{ijh} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A} \mid mn} \cdot \overset{(2)}{S}^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

** ViewTheory: The commutators between

the various if-constraints and the super-Hamiltonian are:

$$\left\{ \iiint \left(\overset{0}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{0}{\cdot} \mathcal{H}_b \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx$$

$$\iiint \frac{1}{144 \overset{\wedge}{\alpha}_5 \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{T}^2} \overset{(1)}{S} \overset{(2)}{S} \left(9 \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \overset{1}{\cdot} \overset{\wedge}{\pi}_{\mathcal{A}a} \overset{1}{\cdot} \overset{\wedge}{\pi}_{\mathcal{A}}^a + 18 \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \overset{1}{\cdot} \overset{\wedge}{\pi}_{\mathcal{A}aa'} \overset{1}{\cdot} \overset{\wedge}{\pi}_{\mathcal{A}}^{aa'} + \right.$$

$$81 \overset{\wedge}{\alpha}_5 \overset{1}{\cdot} \overset{\wedge}{\pi}_{baaa'} \overset{1}{\cdot} \overset{\wedge}{\pi}_{ba}^{aa'} - 8 \overset{\wedge}{\alpha}_5 \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{T} \left(9 \mathcal{M}_{Pl}^2 \overset{\wedge}{\alpha}_6 \mathcal{T}^0 \mathcal{R}^l + 9 \overset{1}{\cdot} \overset{\wedge}{\pi}_{\mathcal{A}}^a \overset{1}{\cdot} \mathcal{R}^l_a + 54 \overset{1}{\cdot} \overset{\wedge}{\pi}_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \mathcal{R}^l_{aa'} + \right.$$

$$3 \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{T}^0 \mathcal{T}^{l2} - 9 \overset{1}{\cdot} \overset{\wedge}{\pi}_{ba}^{aa'} \overset{1}{\cdot} \mathcal{T}^l_{aa'} - 32 \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3 \mathcal{T}^2 \mathcal{T}^l_{aa' Za1} \overset{2}{\cdot} \mathcal{T}^{aa' Za1} + 64 \mathcal{M}_{Pl}^2 \overset{\wedge}{\beta}_3$$

$$\left. \left. \mathcal{T}^2 \mathcal{T}^l_{aZa1a}, \overset{2}{\cdot} \mathcal{T}^{aa' Za1} + 18 \eta^{aa' Za1} n^a \left(\overset{3}{\cdot} \mathcal{D}^l_{Za1} \overset{1}{\cdot} \overset{\wedge}{\pi}_{baaa'} \right) \right) \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1$$

$$\begin{aligned}
& \left\{ \iiint \left(\overset{1}{\mathcal{P}}_{\phi_i} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0}{\mathcal{H}}_b \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{12 \overset{\hat{a}}{\alpha}_5 \mathcal{T}^2} \overset{(2)}{S} \left(-\mathcal{T} \overset{1}{\pi}_{\mathcal{A}}^a \left(P_{\eta}^{(1)} S_a \right) \left(3 \mathcal{M}_{\text{PL}}^2 \overset{\hat{a}}{\alpha}_0 + 2 \overset{\hat{a}}{\alpha}_5 \overset{0}{\mathcal{R}}^{\parallel} \right) + 6 \overset{\hat{a}}{\alpha}_5 P_{\eta}^{(1)} S^{a'} \right) \overset{2}{\mathcal{R}}^{\parallel}_{aa'} \Big) + \\
& 2 \overset{\hat{a}}{\alpha}_5 \left(12 \overset{1}{\pi}_{baa'} \left(P_{\eta}^{(1)} S^a \right) P_{\eta}^{(3)} \mathcal{D}^{a'} \mathcal{T} + \mathcal{T} P_{\eta}^{(1)} S^{a'} \right) \overset{1}{\mathcal{T}}^{\parallel a} \Big) - \\
& 4 \mathcal{T} P_{\eta}^{(1)} S^a \left(3 P_{\eta}^{(1)} \eta^{a'b} \left(\overset{3}{\mathcal{D}}^{\parallel}_b \overset{1}{\pi}_{baa'} \right) \right) - 2 \overset{1}{\pi}_{\mathcal{A}}^{a'b} \overset{2}{\mathcal{R}}^{\parallel}_{aa'b} + \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\beta}_3 \epsilon^{\parallel}_{aa'b} \mathcal{T} \overset{0}{\mathcal{T}}^{\parallel} \overset{1}{\mathcal{T}}^{\parallel a'b} \Big) + \\
& \mathcal{T} P_{\eta}^{(1)} S^{a'} \left(6 \overset{1}{\pi}_{\mathcal{A}aa'} \overset{1}{\mathcal{R}}^{\parallel a} + \epsilon_{aa'bc} \overset{1}{\pi}_{\mathcal{A}}^{bc} \overset{0}{\mathcal{R}}^{\parallel} n^a \right) - \\
& 6 \mathcal{T} \overset{1}{\pi}_{ba}^{a'} P_{\eta}^{(1)} S^a \left(P_{\eta}^{(1)} n^b \left(\overset{3}{\mathcal{D}}^{\parallel}_a \eta_b \right) \right) - 2 \left(\overset{3}{\mathcal{D}}^{\parallel}_b \eta_a^{(1)} \right) \Big) \Big) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{2}{\mathcal{P}}_{\phi_{ij}} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0}{\mathcal{H}}_b \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{144 \overset{\hat{a}}{\alpha}_5 \mathcal{T}^2} \overset{(2)}{S} \\
& \left(3 \overset{1}{\pi}_{\mathcal{A}}^a \left(3 \overset{1}{\pi}_{\mathcal{A}}^{a'} P_{\eta}^{(1)} S_{aa'} \right) - 12 \overset{\hat{a}}{\alpha}_5 \mathcal{T} P_{\eta}^{(1)} S_{aa'} \right) \overset{1}{\mathcal{R}}^{\parallel a'} - 12 \overset{\hat{a}}{\alpha}_5 \mathcal{T} P_{\eta}^{(1)} S_{a'a} \overset{1}{\mathcal{R}}^{\parallel a'} - 16 \overset{\hat{a}}{\alpha}_5 \mathcal{T} P_{\eta}^{(1)} S^{a'b} \Big) \\
& \overset{2}{\mathcal{R}}^{\parallel}_{aa'b} - 16 \overset{\hat{a}}{\alpha}_5 \mathcal{T} P_{\eta}^{(1)} S^{a'b} \overset{2}{\mathcal{R}}^{\parallel}_{abaa'} - \eta_a^{(1)b} \overset{1}{\pi}_{\mathcal{A}a}^{(1)} S^{a'b} + 8 \overset{\hat{a}}{\alpha}_5 \eta_a^{(1)b} \mathcal{T} \overset{1}{\mathcal{R}}^{\parallel}_a \overset{(1)}{S}^{a'b} \Big) + \\
& 4 \left(-9 \overset{1}{\pi}_{\mathcal{A}a}^{(1)b} P_{\eta}^{(1)} S^{aa'} \right) \left(\overset{1}{\pi}_{\mathcal{A}a'}^{(1)b} + 2 \overset{\hat{a}}{\alpha}_5 \mathcal{T} \overset{2}{\mathcal{R}}^{\parallel}_{a'b} \right) + \overset{\hat{a}}{\alpha}_5 \mathcal{T} P_{\eta}^{(1)} S^{aa'} \left(18 P_{\eta}^{(1)} n^b \left(\overset{3}{\mathcal{D}}^{\parallel}_a \overset{1}{\pi}_{baa'} \right) \right) + \\
& 18 P_{\eta}^{(1)} n^b \left(\overset{3}{\mathcal{D}}^{\parallel}_a \overset{1}{\pi}_{baa'} \right) + 9 \overset{1}{\pi}_{baa'}^{(1)b} P_{\eta}^{(1)} \mathcal{D}^{(1)} n^b + 9 \overset{1}{\pi}_{baa'}^{(1)b} \left(P_{\eta}^{(1)} \mathcal{D}^{(1)} n^b + P_{\eta}^{(1)} \mathcal{D}^{(1)b} n_a \right) + 9 \\
& \overset{1}{\pi}_{baa'}^{(1)b} P_{\eta}^{(1)} \mathcal{D}^{(1)b} n_a + 36 \mathcal{M}_{\text{PL}}^2 \overset{\hat{a}}{\alpha}_0 \mathcal{T} \overset{2}{\mathcal{R}}^{\parallel}_{aa'} - 18 \overset{1}{\pi}_{\mathcal{A}a}^{(1)b} \overset{2}{\mathcal{R}}^{\parallel}_{ab} + 9 \overset{1}{\pi}_{baa'}^{(1)b} \overset{1}{\mathcal{T}}^{\parallel}_{ab} + 9 \\
& \overset{1}{\pi}_{baa'}^{(1)b} \overset{1}{\mathcal{T}}^{\parallel}_{a'b} + 16 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\beta}_3 \epsilon^{\parallel}_{a'bb'} \mathcal{T} \overset{0}{\mathcal{T}}^{\parallel} \overset{2}{\mathcal{T}}^{\parallel}_a{}^{bb'} + 16 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\beta}_3 \epsilon^{\parallel}_{abb'} \mathcal{T} \overset{0}{\mathcal{T}}^{\parallel} \\
& \overset{2}{\mathcal{T}}^{\parallel}_a{}^{bb'} - 8 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\beta}_3 \epsilon^{\parallel}_{a'bb'} \mathcal{T} \overset{0}{\mathcal{T}}^{\parallel} \overset{2}{\mathcal{T}}^{\parallel}{}^{bb'}_a - 8 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\beta}_3 \epsilon^{\parallel}_{abb'} \mathcal{T} \overset{0}{\mathcal{T}}^{\parallel} \overset{2}{\mathcal{T}}^{\parallel}{}^{bb'}_a \Big) + \\
& 3 \eta_{aa'}^{(1)} \overset{(1)}{S}^{aa'} \left(\overset{1}{\pi}_{\mathcal{A}bb'}^{(1)b} \overset{1}{\pi}_{\mathcal{A}}^{bb'} - 2 \overset{\hat{a}}{\alpha}_5 \mathcal{T} \overset{1}{\pi}_b^{bb'} \overset{1}{\mathcal{T}}^{\parallel}_{bb'} + 4 \overset{\hat{a}}{\alpha}_5 \eta^{(1)b'c} \right. \\
& \left. \mathcal{T} n^b \left(\overset{3}{\mathcal{D}}^{\parallel}_c \overset{1}{\pi}_{bb'} \right) \right) \Big) \Big) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{0}{\mathcal{P}}_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0}{\mathcal{H}}_b \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{8 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\beta}_3 \mathcal{T}^2} \overset{(1)}{S} \overset{(2)}{S} \left(\overset{1}{\pi}_{\mathcal{A}}^{ab} \left(3 \overset{1}{\pi}_{baa'} + 4 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\beta}_3 \mathcal{T} \overset{1}{\mathcal{T}}^{\parallel}_{ab} \right) + 4 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\beta}_3 \overset{1}{\pi}_{\mathcal{A}}^a \left(2 P_{\eta}^{(1)} \mathcal{D}^{(1)} \mathcal{T} - \mathcal{T} \overset{1}{\mathcal{T}}^{\parallel}_a + \right. \right. \\
& \left. \left. 2 \mathcal{T} \left(\overset{3}{\mathcal{D}}^{\parallel}_b \eta_a^{(1)} \right) \right) - 8 \mathcal{M}_{\text{PL}}^2 \overset{\hat{\beta}}{\beta}_3 \eta_{ab} \mathcal{T} \left(\overset{3}{\mathcal{D}}^{\parallel}_b \overset{1}{\pi}_{\mathcal{A}}^a \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(1)}{S} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{0}{\cdot} \mathcal{H}_b \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{24 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T}^2} \overset{(1)}{S} \left(24 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \epsilon^{\parallel}_{aa'b} \left(\overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'b} \left(4 \mathcal{T} P[\overset{3}{\mathcal{D}}^{\parallel a} \overset{(2)}{S}] - 4 P[\overset{3}{\mathcal{D}}^{\parallel a} \mathcal{T}] \overset{(2)}{S} + 3 \mathcal{T} \overset{(2)}{S} \overset{1}{\cdot} \mathcal{T}^{\parallel a} \right) + \right. \right. \\
& \quad \mathcal{T} \overset{(2)}{S} \left(2 P[\overset{3}{\mathcal{D}}^{\parallel b} \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'}] + \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \mathcal{T}^{\parallel a'b} \right) \left. \right) + \overset{(2)}{S} \left(64 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \epsilon^{\parallel}_{a'bb'} \mathcal{T} \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{2}{\cdot} \mathcal{T}^{\parallel a}{}^{bb'} - \right. \\
& \quad 3 \left(\epsilon \gamma_{aa'bb'} \left(- \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \left(3 \overset{1}{\cdot} \pi_b^{bb'} + 4 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \mathcal{T}^{\parallel bb'} \right) n^{a'} + 16 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \overset{1}{\cdot} \pi_{\mathcal{A}}^{bb'} \right. \\
& \quad \left. \left(P[\overset{3}{\mathcal{D}}^{\parallel a'} \mathcal{T}] n^a + \mathcal{T} \overset{1}{\cdot} \mathcal{T}^{\parallel a} n^{a'} \right) \right) + 8 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \left(\left(\mathcal{M}_{\text{Pl}}^2 \hat{\alpha}_0 + 8 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \right) \mathcal{T} \overset{0}{\cdot} \mathcal{T}^{\parallel} - \right. \\
& \quad \left. \left. 2 \epsilon^{\parallel}_{aa'b'} \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \left(\overset{3}{\mathcal{D}}^{\parallel b} \eta^{\parallel bb'} \right) \right) \right) \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1 \\
& \left\{ \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A}ij} \cdot \overset{(1)}{S}^{ij} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{0}{\cdot} \mathcal{H}_b \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{48 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T}^2} \overset{(2)}{S} \left(12 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} P[\overset{(1)}{S}^{aa'}] \left(P[\overset{3}{\mathcal{D}}^{\parallel a} \overset{1}{\cdot} \pi_{\mathcal{A}a'}] + P[\overset{3}{\mathcal{D}}^{\parallel a'} \overset{1}{\cdot} \pi_{\mathcal{A}a}] + \right. \right. \\
& \quad \left. \overset{1}{\cdot} \pi_{\mathcal{A}ab} P[\overset{3}{\mathcal{D}}^{\parallel a'} n^b] + \overset{1}{\cdot} \pi_{\mathcal{A}a'b} \left(P[\overset{3}{\mathcal{D}}^{\parallel a} n^b] + P[\overset{3}{\mathcal{D}}^{\parallel b} n_a] \right) + \overset{1}{\cdot} \pi_{\mathcal{A}ab} P[\overset{3}{\mathcal{D}}^{\parallel b} n_{a'}] \right) + \\
& \quad 3 \overset{1}{\cdot} \pi_{\mathcal{A}a'}^b P[\overset{(1)}{S}^{aa'}] \left(3 \overset{1}{\cdot} \pi_{ba'b} + 16 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \mathcal{T}^{\parallel a'b} \right) + 3 \overset{1}{\cdot} \pi_{\mathcal{A}a}^b P[\overset{(1)}{S}^{aa'}] \\
& \quad \left(3 \overset{1}{\cdot} \pi_{ba'b} + 16 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \mathcal{T}^{\parallel a'b} \right) + 2 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \\
& \quad \left(4 \eta_{a'b}^{\parallel} P[\overset{3}{\mathcal{D}}^{\parallel a} \mathcal{T}] \overset{(1)}{S}^{a'b} - 2 \eta_{a'b}^{\parallel} \mathcal{T} \overset{(1)}{S}^{a'b} \overset{1}{\cdot} \mathcal{T}^{\parallel a} + 3 \mathcal{T} P[\overset{(1)}{S}_{aa'}] \overset{1}{\cdot} \mathcal{T}^{\parallel a'} + 3 \mathcal{T} P[\overset{(1)}{S}_{a'a}] \overset{1}{\cdot} \mathcal{T}^{\parallel a'} + \right. \\
& \quad \left. 8 \mathcal{T} P[\overset{(1)}{S}^{a'b}] \overset{2}{\cdot} \mathcal{T}^{\parallel aa'b} + 8 \mathcal{T} P[\overset{(1)}{S}^{a'b}] \overset{2}{\cdot} \mathcal{T}^{\parallel a'bb} - 6 P[\overset{(1)}{S}_{aa'}] \left(P[\overset{3}{\mathcal{D}}^{\parallel a'} \mathcal{T}] + \mathcal{T} \left(\overset{3}{\mathcal{D}}^{\parallel b} \eta_{a'}^{\parallel b} \right) \right) - \right. \\
& \quad \left. 6 P[\overset{(1)}{S}^{a'a}] \left(P[\overset{3}{\mathcal{D}}^{\parallel a'} \mathcal{T}] + \mathcal{T} \left(\overset{3}{\mathcal{D}}^{\parallel b} \eta_{a'}^{\parallel b} \right) \right) + 4 \eta_{a'b}^{\parallel} \mathcal{T} \overset{(1)}{S}^{a'b} \left(\overset{3}{\mathcal{D}}^{\parallel b'} \eta_a^{\parallel b'} \right) \right) - \\
& \quad \left. 2 \eta_{aa'}^{\parallel} \overset{(1)}{S}^{aa'} \left(\overset{1}{\cdot} \pi_{\mathcal{A}}^{bb'} \left(3 \overset{1}{\cdot} \pi_{bb'} + 16 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \mathcal{T}^{\parallel bb'} \right) + 4 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \eta_{bb'}^{\parallel} \mathcal{T} \left(\overset{3}{\mathcal{D}}^{\parallel b'} \overset{1}{\cdot} \pi_{\mathcal{A}}^{bb'} \right) \right) \right) [\\
& \quad \chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1 \\
& \left\{ \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{A}ijk} \cdot \overset{(1)}{S}^{ijk} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\overset{0}{\cdot} \mathcal{H}_b \cdot \overset{(2)}{S} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \left\{ - \frac{1}{384 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T}^2} \left(192 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \pi_{\mathcal{A}ab} P[\overset{(1)}{S}^{aa'b}] P[\overset{3}{\mathcal{D}}^{\parallel a'} \overset{(2)}{S}] + 384 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} \overset{1}{\cdot} \pi_{\mathcal{A}aa'} P[\overset{(1)}{S}^{aa'b}] \right. \right. \\
& \quad P[\overset{3}{\mathcal{D}}^{\parallel b} \overset{(2)}{S}] - 18 \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \pi_{ba'b} P[\overset{(1)}{S}_a^{a'b}] \overset{(2)}{S} + 27 \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \pi_{ba'}^{aa'} P[\eta_{bb'}^{\parallel} \overset{(1)}{S}_{a'}^{bb'}] \overset{(2)}{S} + \\
& \quad 18 \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \pi_{ba'b} P[\overset{(1)}{S}_a^{a'b}] \overset{(2)}{S} + 36 \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \pi_{ba'b} P[\overset{(1)}{S}_a^{a'b}] \overset{(2)}{S} - \\
& \quad 27 \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} \overset{1}{\cdot} \pi_{ba'}^{aa'} P[\eta_{bb'}^{\parallel} \overset{(1)}{S}_a^{bb'}] \overset{(2)}{S} - 96 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \mathcal{T} P[\overset{(1)}{S}^{aa'b}] P[\overset{3}{\mathcal{D}}^{\parallel a} \overset{1}{\cdot} \pi_{\mathcal{A}a'b}] \overset{(2)}{S} + \\
& \quad \left. 36 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \delta_{a'}^b \mathcal{T} \overset{1}{\cdot} \pi_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{\parallel} \overset{(1)}{S}_b^{b'c}] P[\overset{3}{\mathcal{D}}^{\parallel a} n^{a'}] \overset{(2)}{S} - 36 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \cdot \delta_{a'}^b \mathcal{T} \right\}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[\eta_{b'c}^{(1)} S_{b'c}^{(1)}] P[\mathcal{D}_a^b n_a^{(2)}]^{(2)} S - 96 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \frac{1}{\eta} \hat{\pi}_{\mathcal{A}ab} P[(^{(1)}S^{aa'b})] P[\mathcal{D}_a^b \mathcal{T}]^{(2)} S + \\
& 96 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} P[(^{(1)}S^{aa'b})] P[\mathcal{D}_a^b \frac{1}{\eta} \hat{\pi}_{\mathcal{A}ab}]^{(2)} S + 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[(^{(1)}S_a^{a'b})] P[\mathcal{D}_a^b n_b]^{(2)} S - \\
& 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[(^{(1)}S_a^{a'b})] P[\mathcal{D}_a^b n_b]^{(2)} S + 36 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[\eta_{bb'}^{(1)} S_a^{bb'}] \\
& P[\mathcal{D}_a^b n_a]^{(2)} S - 36 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[\eta_{bb'}^{(1)} S_a^{bb'}] P[\mathcal{D}_a^b n_a]^{(2)} S - \\
& 192 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \frac{1}{\eta} \hat{\pi}_{\mathcal{A}aa'} P[(^{(1)}S^{aa'b})] P[\mathcal{D}_a^b \mathcal{T}]^{(2)} S + 192 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} P[(^{(1)}S^{aa'b})] P[\mathcal{D}_a^b \frac{1}{\eta} \hat{\pi}_{\mathcal{A}aa'}]^{(2)} S + \\
& 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[(^{(1)}S_a^{a'b})] P[\mathcal{D}_a^b n_a]^{(2)} S - 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[(^{(1)}S_a^{a'b})] P[\mathcal{D}_a^b n_a]^{(2)} S - \\
& 144 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} P[\eta_{a'b}^{(1)} S^{aa'b}] P[\eta^{bb'c} (\mathcal{D}_c^b \frac{1}{\eta} \hat{\pi}_{\mathcal{A}ab})]^{(2)} S + 144 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} P[\eta_{ab}^{(1)} S^{aa'b}] \\
& P[\eta^{bb'c} (\mathcal{D}_c^b \frac{1}{\eta} \hat{\pi}_{\mathcal{A}ab})]^{(2)} S - 16 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \epsilon_{a'bb'}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{b'} P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel} + \\
& 16 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \epsilon_{abb'}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{b'} P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel} + 32 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \epsilon_{aa'b'}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}b}^{b'} \\
& P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel} - 24 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \epsilon_{aa'b}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_b^{b'c}]^{(2)} S \mathcal{T}^{\parallel} + \\
& 24 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \epsilon_{aa'b}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_b^{b'c}]^{(2)} S \mathcal{T}^{\parallel} + 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{a'} \\
& \left(P[\eta_{bb'}^{(1)} S_a^{bb'}] - P[\eta_{bb'}^{(1)} S_a^{bb'}] \right) \left(4 \mathcal{T} P[\mathcal{D}_a^b \mathcal{T}]^{(2)} S - 2 P[\mathcal{D}_a^b \mathcal{T}]^{(2)} S + \mathcal{T}^{(2)} S \frac{1}{\eta} \mathcal{T}^{\parallel} \right) + \\
& 256 \mathcal{M}_{\text{PL}}^2 \hat{\alpha}_0 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T}^2 P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{aa'b} + 128 \mathcal{M}_{\text{PL}}^2 \hat{\alpha}_0 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T}^2 \\
& P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{abb'a} - 192 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_b^{b'c}]^{(2)} S \mathcal{T}^{\parallel}{}_{a'a} + \\
& 192 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_b^{b'c}]^{(2)} S \mathcal{T}^{\parallel}{}_{a'a} + 128 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}b}^{b'} \\
& P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{abb'a} + 256 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{b'} P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{abb'b} - \\
& 128 \mathcal{M}_{\text{PL}}^2 \hat{\alpha}_0 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T}^2 P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{a'b'a} - 128 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}b}^{b'} \\
& P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{a'b'a} - 256 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{b'} P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{a'b'b} + \\
& 128 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{b'} P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{bb'a} - 128 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a}^{b'} \\
& P[(^{(1)}S^{aa'b})]^{(2)} S \mathcal{T}^{\parallel}{}_{bb'a} + 144 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_a^{b'c}]^{(2)} S (\mathcal{D}_b^b \eta_a^b) - \\
& 144 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^{aa'} P[\eta_{b'c}^{(1)} S_a^{b'c}]^{(2)} S (\mathcal{D}_b^b \eta_a^b) - 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \eta_{a'b}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a \\
& P[\eta_{b'c}^{(1)} S_a^{b'c}]^{(2)} S (\mathcal{D}_b^b n_a^{(2)}) + 72 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \eta_{a'b}^{(1)} \mathcal{T} \frac{1}{\eta} \hat{\pi}_{\mathcal{A}}^a P[\eta_{b'c}^{(1)} S_a^{b'c}]^{(2)} S (\mathcal{D}_b^b n_a^{(2)}) - \\
& 48 \mathcal{M}_{\text{PL}}^2 \hat{\beta}_3 \frac{1}{\eta} \hat{\pi}_{\mathcal{A}a'b} \left(\mathcal{T} \left(P[(^{(1)}S_a^{a'b})] - P[(^{(1)}S_a^{a'b})] - 2 P[(^{(1)}S_a^{a'b})] \right) \right)^{(2)} S \frac{1}{\eta} \mathcal{T}^{\parallel}{}^a +
\end{aligned}$$

$$\begin{aligned}
& 2 P_{\eta^l}^{[(1)S^{aa'b}]} \left(2 \mathcal{T} P_{\eta^l}^{[3\mathcal{D}_a^{(2)S}]} - P_{\eta^l}^{[3\mathcal{D}_a^{(2)S}]} \mathcal{T}^{(2)S} \left(3\mathcal{D}_b^{(2)S}, \eta_a^{(2)S}, b' \right) \right) + \\
& 96 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \mathcal{T}^{(1)S^{aa'b}} P_{\eta^l}^{[(1)S^{aa'b}]} \mathcal{T}^{(2)S} \left(3\mathcal{D}_b^{(2)S}, \eta_a^{(2)S}, b' \right) + 192 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \mathcal{T}^{(1)S^{aa'b}} \\
& P_{\eta^l}^{[(1)S^{aa'b}]} \mathcal{T}^{(2)S} \left(3\mathcal{D}_b^{(2)S}, \eta_a^{(2)S}, b' \right) \left[x^0, x^1, x^2, x^3 \right] dx^3 dx^2 dx^1
\end{aligned}$$

This concludes the HiGGS output.

Concrete relation to manuscript: The above constraints and Poisson brackets constitute the data represented by coloured squares in the left-hand panel of Fig. 5 in our Manuscript. The primary constraints ϕ are listed along the top row. The primary Poisson brackets $\{\phi, \phi\}$ are used to infer the white, red or yellow squares in the matrix depending on whether they are vanishing, or appear only in proportion to phase space variables, or are order unity (e.g. proportional to the metric or unit-timelike vector). The $\{\phi, \mathcal{H}_b\}$ Poisson brackets define the secondary constraints " χ ", which appear on the sides of the matrix. Most of the Poisson brackets $\{\chi, \phi\}$ need not be computed (grey squares), but it can be noticed without detailed calculation that the remaining required brackets (yellow squares) do indeed arise, and that these determine all the missing velocities and so terminate the Dirac algorithm.

Healthy spectrum with multipliers

Introductory comments

We now turn to the case in which multipliers have been applied. This time we set $\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4, \hat{\alpha}_6, \mathcal{M}_{Pl}^2 \hat{\beta}_1, \mathcal{M}_{Pl}^2 \hat{\beta}_2, \mathcal{M}_{Pl}^2 \bar{\beta}_3$, to zero, and allow for nonvanishing $\mathcal{M}_{Pl}^2 \hat{\alpha}_0, \hat{\alpha}_5$ and $\mathcal{M}_{Pl}^2 \hat{\beta}_3$, but we also admit $\mathcal{M}_{Pl}^2 \bar{\beta}_1$ and $\mathcal{M}_{Pl}^2 \bar{\beta}_2$. These final two couplings switch on the multipliers in the Lagrangian.

Note this time that because some of the Lagrange multiplier fields are present, certain secondary constraints can be anticipated by HiGGS a priori: this is in accordance with the theory set out in arXiv:2205.13534.

Lists of constraints and Poisson brackets from HiGGS

We will again present the results using the ViewTheory[] command.

```

** DefTheory: Incorporating the binary at ConstraintAlgebraAlp0Alp5Bet3cBet1cBet2.thr.mx
** DefTheory: Found the following primary if-constraints:

```

$$\mathcal{H}_b \equiv \frac{\mathcal{H}_b}{\mathcal{T}} + 2 \mathcal{M}_{Pl}^2 \bar{\beta}_2 \mathcal{H}_b \approx 0$$

$$\overset{1}{\cdot}\phi_{bi} \equiv \frac{\overset{1}{\cdot}\overset{\wedge}{\pi}_{bi}}{\mathcal{I}} - \frac{2}{3} \left(\mathcal{M}_{Pl}^2 \bar{\beta}_1 - \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\lambda_{\mathcal{T}i}^{\parallel} + \frac{2}{3} \left(2 \mathcal{M}_{Pl}^2 \bar{\beta}_1 + \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\lambda_{\mathcal{T}i}^{\perp} \approx 0$$

$$\overset{2}{\cdot}\phi_{bij} \equiv \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{bij}}{\mathcal{I}} + 2 \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{2}{\cdot}\lambda_{\mathcal{T}ij}^{\perp} \approx 0$$

$$\overset{0}{\cdot}\phi_{\mathcal{A}} \equiv 3 \mathcal{M}_{Pl}^2 \overset{\wedge}{\alpha}_0 + \frac{\overset{0}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}}{\mathcal{I}} \approx 0$$

$$\overset{0}{\cdot}\phi_{\mathcal{A}} \equiv \frac{\overset{0}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}}}{\mathcal{I}} \approx 0$$

$$\overset{2}{\cdot}\phi_{\mathcal{A}ij} \equiv \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ij}}{\mathcal{I}} \approx 0$$

$$\overset{2}{\cdot}\phi_{\mathcal{A}ijh} \equiv \frac{2 \overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ijh}}{3\mathcal{I}} + \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}ihj}}{3\mathcal{I}} - \frac{\overset{2}{\cdot}\overset{\wedge}{\pi}_{\mathcal{A}jhi}}{3\mathcal{I}} \approx 0$$

** DefTheory: Found the following primary parallel

if-constraints associated with geometric multiplier momenta (if the sector is singular, the parallel constraint is taken to be the singular constraint and the identical perpendicular constraint below is dropped):

$$\overset{1}{\cdot}\phi_{bij}^{\parallel} \equiv -\frac{2}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{1}{\cdot}\omega_{\mathcal{T}ij}^{\parallel} + \frac{2}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{1}{\cdot}\omega_{\mathcal{T}ij}^{\perp} \approx 0$$

$$\overset{1}{\cdot}\phi_{bi}^{\parallel} \equiv \frac{1}{3} \left(-\mathcal{M}_{Pl}^2 \bar{\beta}_1 - 2 \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\omega_{\mathcal{T}i}^{\parallel} + \frac{2}{3} \left(\mathcal{M}_{Pl}^2 \bar{\beta}_1 - \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\omega_{\mathcal{T}i}^{\perp} \approx 0$$

$$\overset{2}{\cdot}\phi_{bijh}^{\parallel} \equiv -\frac{2}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{2}{\cdot}\omega_{\mathcal{T}ijh}^{\parallel} - \frac{1}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{2}{\cdot}\omega_{\mathcal{T}ihj}^{\parallel} + \frac{1}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{2}{\cdot}\omega_{\mathcal{T}jhi}^{\parallel} \approx 0$$

** DefTheory: Found the following primary perpendicular

if-constraints associated with geometric multiplier momenta:

$$\overset{0}{\cdot}\varphi_b^{\perp} \equiv 2 \mathcal{M}_{Pl}^2 \bar{\beta}_2 \overset{0}{\cdot}\omega_{\mathcal{T}}^{\perp} \approx 0$$

$$\overset{1}{\cdot}\varphi_{bi}^{\perp} \equiv -\frac{2}{3} \left(\mathcal{M}_{Pl}^2 \bar{\beta}_1 - \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\omega_{\mathcal{T}i}^{\parallel} + \frac{2}{3} \left(2 \mathcal{M}_{Pl}^2 \bar{\beta}_1 + \mathcal{M}_{Pl}^2 \bar{\beta}_2 \right) \overset{1}{\cdot}\omega_{\mathcal{T}i}^{\perp} \approx 0$$

$$\overset{2}{\cdot}\varphi_{bij}^{\perp} \equiv 2 \mathcal{M}_{Pl}^2 \bar{\beta}_1 \overset{2}{\cdot}\omega_{\mathcal{T}ij}^{\perp} \approx 0$$

** DefTheory: Found the following secondary perpendicular if-constraints:

** DefTheory: Found the following secondary parallel if-constraints:

$$\overset{1}{\cdot}\chi_{bi}^{\parallel} \equiv \overset{1}{\cdot}\mathcal{T}_{bi}^{\parallel} \approx 0$$

$$\overset{2}{\cdot}\chi_{bijh}^{\parallel} \equiv \overset{2}{\cdot}\mathcal{T}_{bijh}^{\parallel} \approx 0$$

** DefTheory: Found the following secondary singular if-constraints:

$$\mathbf{1}^{\cdot} \chi_{b \mid j}^{\cdot} \equiv \frac{\mathbf{1}^{\cdot} \hat{\pi}_{b \mid j}}{\mathcal{T}} - \frac{2}{3} \mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathbf{1}^{\cdot} \lambda_{\mathcal{T} \mid j}^{\parallel} + \frac{2}{3} \mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathbf{1}^{\cdot} \lambda_{\mathcal{T} \mid j}^{\perp} + 4 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_{\mid j} \approx 0$$

** DefTheory: Defining association key \$IfConstraints

for the theory association ConstraintAlgebraAlp0Alp5Bet3cBet1cBet2

** ViewTheory: The super-Hamiltonian is:

$$\begin{aligned} \mathbf{0}^{\cdot} \mathcal{H}_b \equiv & \frac{\mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}a} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^a}{16 \hat{\alpha}_5 \mathcal{T}} + \frac{\mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}ab} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^{ab}}{8 \hat{\alpha}_5 \mathcal{T}} + \frac{3 \mathbf{1}^{\cdot} \hat{\pi}_{b \mid a} \mathbf{1}^{\cdot} \hat{\pi}_b^{ab}}{16 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \mathcal{T}} + \frac{\mathbf{1}^{\cdot} \hat{\pi}_b^a \mathcal{P}_{\eta}^{\left[\mathbf{3} \mathcal{D}^{\parallel}{}_a \mathcal{T} \right]}}{\mathcal{T}} + \frac{\mathcal{M}_{\text{Pl}}^2 \hat{\alpha}_0 \mathcal{T} \mathbf{0}^{\cdot} \mathcal{R}^{\parallel}}{2} - \\ & \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^a \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_a + \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^{ab} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_{ab} - \frac{\mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathbf{1}^{\cdot} \hat{\pi}_b^{ab} \mathbf{1}^{\cdot} \lambda_{\mathcal{T} \mid ab}^{\parallel}}{4 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3} + \frac{\mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1^2 \mathcal{T} \mathbf{1}^{\cdot} \lambda_{\mathcal{T} \mid ab}^{\parallel} \mathbf{1}^{\cdot} \lambda_{\mathcal{T}}^{ab}}{12 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3} + \\ & \frac{\mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathbf{1}^{\cdot} \hat{\pi}_b^{ab} \mathbf{1}^{\cdot} \lambda_{\mathcal{T} \mid ab}^{\perp}}{4 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3} - \frac{\mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1^2 \mathcal{T} \mathbf{1}^{\cdot} \lambda_{\mathcal{T}}^{ab} \mathbf{1}^{\cdot} \lambda_{\mathcal{T} \mid ab}^{\perp}}{6 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3} + \frac{\mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1^2 \mathcal{T} \mathbf{1}^{\cdot} \lambda_{\mathcal{T} \mid ab}^{\perp} \mathbf{1}^{\cdot} \lambda_{\mathcal{T}}^{ab}}{12 \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3} + \frac{1}{6} \mathcal{M}_{\text{Pl}}^2 \hat{\beta}_3 \mathcal{T} \mathbf{0}^{\cdot} \mathcal{T}^{\parallel 2} - \\ & \mathbf{1}^{\cdot} \hat{\pi}_b^a \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_a + \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_b^{ab} \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_{ab} - \mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathcal{T} \mathbf{1}^{\cdot} \lambda_{\mathcal{T}}^{ab} \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_{ab} + \mathcal{M}_{\text{Pl}}^2 \bar{\beta}_1 \mathcal{T} \mathbf{1}^{\cdot} \lambda_{\mathcal{T}}^{ab} \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_{ab} - \\ & n^a \left(\mathbf{3} \mathcal{D}^{\parallel}{}_b \mathbf{1}^{\cdot} \hat{\pi}_{b \mid a}^{\perp} \right) - n^a \left(\mathbf{3} \mathcal{D}^{\parallel}{}_b \mathbf{2}^{\cdot} \hat{\pi}_{b \mid a}^{\perp} \right) - \eta^{\parallel}{}_{ab} \left(\mathbf{3} \mathcal{D}^{\parallel}{}_b \mathbf{1}^{\cdot} \hat{\pi}_b^a \right) + \frac{1}{3} \eta^{\parallel}{}_{ab} \mathbf{0}^{\cdot} \hat{\pi}_b \left(\mathbf{3} \mathcal{D}^{\parallel}{}_b n^a \right) \approx 0 \end{aligned}$$

** ViewTheory: The linear super-momentum is:

$$\begin{aligned} \mathbf{1}^{\cdot} \mathcal{H}_{b \mid} \equiv & -\frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_b^a \mathcal{P}_{\eta}^{\left[\mathbf{3} \mathcal{D}^{\parallel}{}_a n_{\mid} \right]} + \frac{\mathbf{1}^{\cdot} \hat{\pi}_{b \mid a} \mathcal{P}_{\eta}^{\left[\mathbf{3} \mathcal{D}^{\parallel}{}_a \mathcal{T} \right]}}{\mathcal{T}} + \frac{\mathbf{2}^{\cdot} \hat{\pi}_{b \mid a} \mathcal{P}_{\eta}^{\left[\mathbf{3} \mathcal{D}^{\parallel}{}_a \mathcal{T} \right]}}{\mathcal{T}} - \mathcal{P}_{\eta}^{\left[\eta^{\parallel}{}^{aa'} \left(\mathbf{3} \mathcal{D}^{\parallel}{}_a \mathbf{1}^{\cdot} \hat{\pi}_{b \mid a} \right) \right]} - \\ & \mathcal{P}_{\eta}^{\left[\eta^{\parallel}{}^{aa'} \left(\mathbf{3} \mathcal{D}^{\parallel}{}_a \mathbf{2}^{\cdot} \hat{\pi}_{b \mid a} \right) \right]} + \frac{\mathbf{0}^{\cdot} \hat{\pi}_b \mathcal{P}_{\eta}^{\left[\mathbf{3} \mathcal{D}^{\parallel}{}_b \mathcal{T} \right]}}{3 \mathcal{T}} - \frac{1}{3} \mathcal{P}_{\eta}^{\left[\mathbf{3} \mathcal{D}^{\parallel}{}_b \mathbf{0}^{\cdot} \hat{\pi}_b \right]} - \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_b^a \mathcal{P}_{\eta}^{\left[\mathbf{3} \mathcal{D}^{\parallel}{}_a n_{\mid} \right]} + \frac{1}{6} \epsilon^{\parallel}{}_{\mid aa'} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^{aa'} \mathbf{0}^{\cdot} \mathcal{R}^{\parallel} - \\ & \frac{\mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A} \mid} \mathbf{0}^{\cdot} \mathcal{R}^{\parallel}}{6} - \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A} \mid a} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_a - \frac{1}{2} \mathbf{2}^{\cdot} \hat{\pi}_{\mathcal{A} \mid a} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_a + \frac{\mathbf{0}^{\cdot} \hat{\pi}_{\mathcal{A}} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_{\mid}}{3} - \frac{4}{3} \mathbf{2}^{\cdot} \hat{\pi}_{\mathcal{A} \mid}^{aa'} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_{aa'} + \\ & \frac{1}{6} \epsilon^{\parallel}{}_{\mid aa'} \mathbf{0}^{\cdot} \hat{\pi}_{\mathcal{A}} \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}^{aa'} - \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^a \mathbf{1}^{\cdot} \mathcal{R}^{\parallel}{}_{\mid a} - \frac{4}{3} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^{aa'} \mathbf{2}^{\cdot} \mathcal{R}^{\parallel}{}_{\mid aa'} + \frac{4}{3} \mathbf{2}^{\cdot} \hat{\pi}_{\mathcal{A}}^{aa'} \mathbf{2}^{\cdot} \mathcal{R}^{\parallel}{}_{\mid aa'} + \\ & \frac{4}{3} \mathbf{2}^{\cdot} \hat{\pi}_{\mathcal{A} \mid}^{aa'} \mathbf{2}^{\cdot} \mathcal{R}^{\parallel}{}_{aa'} - \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_{\mathcal{A}}^a \mathbf{2}^{\cdot} \mathcal{R}^{\parallel}{}_{\mid a} + \frac{1}{6} \epsilon^{\parallel}{}_{\mid aa'} \mathbf{1}^{\cdot} \hat{\pi}_b^{aa'} \mathbf{0}^{\cdot} \mathcal{T}^{\parallel} - \frac{3}{2} \mathbf{1}^{\cdot} \hat{\pi}_{b \mid a} \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_a - \\ & \frac{3}{2} \mathbf{2}^{\cdot} \hat{\pi}_{b \mid a} \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_a + \frac{1}{2} \mathbf{1}^{\cdot} \hat{\pi}_b^a \mathbf{1}^{\cdot} \mathcal{T}^{\parallel}{}_{\mid a} - \frac{4}{3} \mathbf{1}^{\cdot} \hat{\pi}_b^{aa'} \mathbf{2}^{\cdot} \mathcal{T}^{\parallel}{}_{\mid aa'} + \frac{4}{3} \mathbf{2}^{\cdot} \hat{\pi}_b^{aa'} \mathbf{2}^{\cdot} \mathcal{T}^{\parallel}{}_{\mid aa'} \approx 0 \end{aligned}$$

** ViewTheory: The 1+ part of the angular super-momentum is:

$$\begin{aligned}
\mathcal{H}_{\mathcal{A}nm} &\equiv -2 \frac{1}{\mathcal{J}} \hat{\pi}_{bmn} - \frac{\epsilon^{lmn} \hat{\pi}_{\mathcal{A}} P[\mathcal{D}^{l0} \mathcal{J}]}{6 \mathcal{J}} + \frac{4}{3 \mathcal{J}} \hat{\pi}_{\mathcal{A}mn} P[\mathcal{D}^{l0} \mathcal{J}] + \frac{1}{6} \epsilon^{lmn} P[\mathcal{D}^{l0} \hat{\pi}_{\mathcal{A}}] - \\
&\frac{1}{2} \frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}n} P[\mathcal{D}^{l0} n_m] - \frac{1}{2} \frac{2}{\mathcal{J}} \hat{\pi}_{\mathcal{A}n} P[\mathcal{D}^{l0} n_m] + \frac{1}{2} \frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}m} P[\mathcal{D}^{l0} n_n] + \frac{1}{2} \frac{2}{\mathcal{J}} \hat{\pi}_{\mathcal{A}m} P[\mathcal{D}^{l0} n_n] - \\
&\frac{4}{3} P[\eta^{aa'} (\mathcal{D}^{l0}, \hat{\pi}_{\mathcal{A}mn})] + \frac{\frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}n} P[\mathcal{D}^{lm} \mathcal{J}]}{2 \mathcal{J}} - \frac{1}{2} P[\mathcal{D}^{lm} \frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}n}] - \frac{1}{2} \frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}n} P[\mathcal{D}^{lm} n^a] - \\
&\frac{1}{2} \frac{2}{\mathcal{J}} \hat{\pi}_{\mathcal{A}n} P[\mathcal{D}^{lm} n^a] - \frac{\frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}m} P[\mathcal{D}^{ln} \mathcal{J}]}{2 \mathcal{J}} + \frac{1}{2} P[\mathcal{D}^{ln} \frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}m}] + \frac{1}{2} \frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}m} P[\mathcal{D}^{ln} n^a] + \\
&\frac{1}{2} \frac{2}{\mathcal{J}} \hat{\pi}_{\mathcal{A}m} P[\mathcal{D}^{ln} n^a] + \frac{1}{6} \epsilon^{lmn} \hat{\pi}_{\mathcal{A}} \mathcal{T}^{lm} - \frac{4}{3} \frac{2}{\mathcal{J}} \hat{\pi}_{\mathcal{A}mn} \mathcal{T}^{lm} - \frac{1}{2} \frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}n} \mathcal{T}^{lm} + \frac{1}{2} \frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}m} \mathcal{T}^{ln} - \\
&\frac{1}{2} \frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}n} \mathcal{T}^{lm} - \frac{1}{2} \frac{2}{\mathcal{J}} \hat{\pi}_{\mathcal{A}n} \mathcal{T}^{lm} - \frac{1}{3} \frac{0}{\mathcal{J}} \hat{\pi}_{\mathcal{A}} \mathcal{T}^{lm} + \frac{1}{2} \frac{1}{\mathcal{J}} \hat{\pi}_{\mathcal{A}m} \mathcal{T}^{ln} + \frac{1}{2} \frac{2}{\mathcal{J}} \hat{\pi}_{\mathcal{A}m} \mathcal{T}^{ln} \approx 0
\end{aligned}$$

** ViewTheory: The on-shell brackets between the various if-constraints are:

$$\begin{aligned}
&\left\{ \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
&\left\{ \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
&\quad \iiint \frac{2 \mathcal{M}_{Pl}^2 \hat{\beta}_1 P[\hat{\mathcal{S}}_a] \hat{\mathcal{S}} (\hat{\mathcal{T}}^{lm} - 2 \hat{\mathcal{T}}^{lm})}{\mathcal{J}} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
&\left\{ \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
&\quad \iiint \left(- \frac{6 \mathcal{M}_{Pl}^2 \hat{\beta}_1 P[\hat{\mathcal{S}}^{aa'}] \hat{\mathcal{S}} \hat{\mathcal{T}}^{aa'}}{\mathcal{J}} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
&\left\{ \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
&\quad \iiint \left(- \frac{6 \mathcal{M}_{Pl}^2 \hat{\alpha}_0 \hat{\mathcal{S}} \hat{\mathcal{S}}}{\mathcal{J}} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
&\left\{ \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
&\left\{ \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
&\left\{ \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
&\left\{ \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
&\left\{ \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint [\hat{\phi}_b \cdot \hat{\mathcal{S}}] [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0
\end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left(\overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2-}{\cdot} \phi_b^{\parallel}{}_{lmn} \cdot \overset{(2)}{S}{}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0+}{\cdot} \phi_b^{\perp} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left(12 \mathcal{M}_{Pl}{}^2 \bar{\beta}_2^{\perp} \overset{(1)}{S} \overset{(2)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{1-}{\cdot} \phi_b^{\perp}{}_{\parallel} \cdot \overset{(2)}{S}{}^{\parallel} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2-}{\cdot} \phi_b^{\perp}{}_{lm} \cdot \overset{(2)}{S}{}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{1-}{\cdot} \chi_b^{\parallel}{}_{\parallel} \cdot \overset{(2)}{S}{}^{\parallel} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \frac{2 \overset{(1)}{S} \left(-P_{\eta^{\parallel}}^{(2)} S^0 \right) P_{\eta^{\parallel}}^{(3)} \mathcal{D}^{\parallel}{}_{\parallel} \mathcal{T} + \eta^{\parallel}{}_{aa'} \mathcal{T} \left(\overset{3}{\mathcal{D}}{}^{\parallel}{}_{\parallel} P_{\eta^{\parallel}}^{(2)} S^0 \right) }{\mathcal{T}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2-}{\cdot} \chi_b^{\parallel}{}_{lmn} \cdot \overset{(2)}{S}{}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{0+}{\cdot} \phi_b \cdot \overset{(1)}{S} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{1-}{\cdot} \chi_b^{\perp}{}_{lm} \cdot \overset{(2)}{S}{}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \frac{2 P_{\eta^{\parallel}}^{(2)} S^{aa'} \overset{(1)}{S} \left(\mathcal{M}_{Pl}{}^2 \bar{\beta}_1^{\perp} \overset{1-}{\lambda}_{\tau aa'}^{\parallel}, -\mathcal{M}_{Pl}{}^2 \bar{\beta}_1^{\perp} \overset{1-}{\lambda}_{\tau aa'}^{\perp}, -4 \mathcal{M}_{Pl}{}^2 \hat{\beta}_3^{\perp} \overset{1-}{\mathcal{T}}{}^{\parallel}{}_{aa'} \right)}{\mathcal{T}} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1-}{\cdot} \phi_b{}_{\parallel} \cdot \overset{(1)}{S}{}^{\parallel} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{1-}{\cdot} \phi_b{}_{\perp} \cdot \overset{(2)}{S}{}^{\perp} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left(-\frac{8 P_{\eta^{\parallel}}^{(1)} S^0 P_{\eta^{\parallel}}^{(2)} S^{a'b}}{3 \mathcal{T}} \left(\mathcal{M}_{Pl}{}^2 \bar{\beta}_1^{\perp} \overset{1-}{\lambda}_{\tau aa'}^{\parallel}, -\mathcal{M}_{Pl}{}^2 \bar{\beta}_1^{\perp} \overset{1-}{\lambda}_{\tau aa'}^{\perp}, +3 \mathcal{M}_{Pl}{}^2 \hat{\beta}_3^{\perp} \overset{1-}{\mathcal{T}}{}^{\parallel}{}_{aa'} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1-}{\cdot} \phi_b{}_{\parallel} \cdot \overset{(1)}{S}{}^{\parallel} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2-}{\cdot} \phi_b{}_{lm} \cdot \overset{(2)}{S}{}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left(\frac{1}{6 \mathcal{T}} \mathcal{M}_{Pl}{}^2 \bar{\beta}_1^{\perp} \left(-8 P_{\eta^{\parallel}}^{(1)} S^0 P_{\eta^{\parallel}}^{(2)} S^{a'b} \right) \left(\overset{2-}{\lambda}_{\tau aa'b}^{\parallel} + \overset{2-}{\lambda}_{\tau aab'a}^{\parallel} \right) + 3 P_{\eta^{\parallel}}^{(1)} S^{a'} \left(P_{\eta^{\parallel}}^{(2)} S_{aa'} \right) + P_{\eta^{\parallel}}^{(2)} S_{a'a} \right) \\
& \quad \left(\overset{1-}{\lambda}_{\tau}{}^{\parallel}{}_{aa'} - 2 \overset{1-}{\lambda}_{\tau}{}^{\perp}{}_{aa'} \right) - 2 \eta^{\parallel}{}_{aa'} P_{\eta^{\parallel}}^{(1)} S_b^{\parallel} \overset{(2)}{S}{}^{aa'} \left(\overset{1-}{\lambda}_{\tau}{}^{\parallel}{}_{aa'} - 2 \overset{1-}{\lambda}_{\tau}{}^{\perp}{}_{aa'} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1-}{\cdot} \phi_b{}_{\parallel} \cdot \overset{(1)}{S}{}^{\parallel} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0+}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left(-\frac{\overset{1-}{\pi}_{\mathcal{A}}{}^0 P_{\eta^{\parallel}}^{(1)} S_a^{\parallel} \overset{(2)}{S}}{\mathcal{T}^2} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1-}{\cdot} \phi_b{}_{\parallel} \cdot \overset{(1)}{S}{}^{\parallel} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0+}{\cdot} \phi_{\mathcal{A}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \frac{2 \epsilon_{\gamma aa' i} \gamma_{aa1} \overset{1-}{\pi}_{\mathcal{A}}{}^i \gamma_{aa1} P_{\eta^{\parallel}}^{(1)} S^{a'} \overset{(2)}{S} n^a}{\mathcal{T}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\phi}_{\mathcal{A}lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{\overset{1}{\pi}_{\mathcal{A}}^a \left(3 P_{\eta}^{(1)} S^{a'} \right) \left(P_{\eta}^{(2)} S_{aa'} + P_{\eta}^{(2)} S_{a'a} \right) - 2 \eta_{a'b} P_{\eta}^{(1)} S_a \overset{(2)}{S}^{a'b} }{12 \mathcal{J}^2} [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\phi}_{lmn} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{8 \mathcal{J}^2 \eta} P_{\eta}^{(1)} S^a \left(\overset{1}{\pi}_{\mathcal{A}a'b} \left(-2 P_{\eta}^{(2)} S_a^{a'b} + 2 P_{\eta}^{(2)} S_a^{a'b} + 4 P_{\eta}^{(2)} S_a^{a'b} \right) + \right. \\
& \quad \left. 3 \overset{1}{\pi}_{\mathcal{A}}^a \left(\eta_{bb'}^{(2)} S_a^{bb'} - P_{\eta}^{(2)} S_a^{b'b'} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\phi}_{lm}^{\parallel} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(2)}{S}^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \left(\frac{4}{3} \left(\mathcal{M}_{\mathbf{P}^1}^2 \bar{\beta}_1^2 - \mathcal{M}_{\mathbf{P}^1}^2 \bar{\beta}_2^2 \right) P_{\eta}^{(1)} S^a P_{\eta}^{(2)} S_a \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\phi}_{lmn}^{\parallel} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0}{\phi}_b^{\perp} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{1}{\phi}_{bi}^{\perp} \cdot \overset{(2)}{S}^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \left(\frac{4}{3} \left(2 \mathcal{M}_{\mathbf{P}^1}^2 \bar{\beta}_1^2 + \mathcal{M}_{\mathbf{P}^1}^2 \bar{\beta}_2^2 \right) P_{\eta}^{(1)} S^a P_{\eta}^{(2)} S_a \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\phi}_{lm}^{\perp} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{1}{\chi}_{bi}^{\parallel} \cdot \overset{(2)}{S}^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{P_{\eta}^{(1)} S^a \left(-P_{\eta}^{(2)} S^{a'} \right) \left(P_{\eta}^{(3)} \mathcal{D}_a n_{a'} + P_{\eta}^{(3)} \mathcal{D}_a n_{a'} - \overset{1}{\mathcal{T}}_{aa'} \right) + 2 \eta_{a'b} P_{\eta}^{(2)} S_a \left(\overset{3}{\mathcal{D}}^b n^{a'} \right)}{2 \mathcal{J}} [x^0, x^1, x^2, x^3] \\
& \quad dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\overset{1}{\phi}_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\chi}_{b|mn}^{\parallel} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{16 \mathcal{J}} P_{\eta}^{(1)} S^a \left(3 \delta_a^b \left(P_{\eta}^{(2)} S_b^{b'm} - P_{\eta}^{(2)} S_b^{b'm} \right) P_{\eta}^{(3)} \mathcal{D}_a n^{a'} - 6 P_{\eta}^{(2)} S_a^{a'b} P_{\eta}^{(3)} \mathcal{D}_a n_b + \right. \\
& \quad 3 P_{\eta}^{(2)} S_a^{bb'} P_{\eta}^{(3)} \mathcal{D}_a n_{a'} - 3 P_{\eta}^{(2)} S_a^{bb'} P_{\eta}^{(3)} \mathcal{D}_a n_{a'} - 6 P_{\eta}^{(2)} S_a^{a'b} P_{\eta}^{(3)} \mathcal{D}_a n_{a'} - \\
& \quad 3 P_{\eta}^{(2)} S_a^{bb'} \overset{1}{\mathcal{T}}_a^{a'} + 3 P_{\eta}^{(2)} S_a^{bb'} \overset{1}{\mathcal{T}}_a^{a'} - 2 P_{\eta}^{(2)} S_a^{a'b} \overset{1}{\mathcal{T}}_{a'b} - \\
& \quad \left. 4 P_{\eta}^{(2)} S_a^{a'b} \overset{1}{\mathcal{T}}_{a'b} + 2 P_{\eta}^{(2)} S_a^{a'b} \right) \left(3 P_{\eta}^{(3)} \mathcal{D}_a n_b + 3 P_{\eta}^{(3)} \mathcal{D}_b n_a + \overset{1}{\mathcal{T}}_{a'b} \right) - \\
& \quad \left. 6 \eta_{a'b} P_{\eta}^{(2)} S_a^{b'm} \left(\overset{3}{\mathcal{D}}^b n^{a'} \right) + 6 \eta_{a'b} P_{\eta}^{(2)} S_a^{b'm} \left(\overset{3}{\mathcal{D}}^b n^{a'} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left(\overset{1}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^i \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{1}{\cdot} \chi_{b|lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{18 \mathcal{T}^2} \left(-36 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \delta_{a,b} \mathcal{T} P_{\eta}^{(1)S^a} \left(P_{\eta}^{(2)S_b^{b'}} n_{b'} - P_{\eta}^{(2)S_b^{b'}} n_{b'} \right) P_{\eta}^{(3)D_a^{a'}} n_{a'} - \right. \\
& 9 \mathcal{M}_{Pl}^2 \hat{\beta}_1 \mathcal{T} P_{\eta}^{(1)S^{a'}} \left(P_{\eta}^{(2)S_{aa'}} - P_{\eta}^{(2)S_{a'a}} \right) \left(\overset{1}{\cdot} \lambda_{\mathcal{T}}^a - 2 \overset{1}{\cdot} \lambda_{\mathcal{T}}^{a'} \right) + \\
& 4 P_{\eta}^{(1)S^a} \left(18 \mathcal{M}_{Pl}^2 \hat{\beta}_3 P_{\eta}^{(2)S_a^{a'}} P_{\eta}^{(3)D_a^{a'}} \mathcal{T} - 18 \mathcal{M}_{Pl}^2 \hat{\beta}_3 P_{\eta}^{(2)S_a^{a'}} P_{\eta}^{(3)D_a^{a'}} \mathcal{T} + \right. \\
& \mathcal{T} \left(2 \mathcal{M}_{Pl}^2 \hat{\beta}_1 P_{\eta}^{(2)S^{a'b}} \left(\overset{2}{\cdot} \lambda_{\mathcal{T}aa'b}^b - \overset{2}{\cdot} \lambda_{\mathcal{T}aba'a}^b - 2 \overset{2}{\cdot} \lambda_{\mathcal{T}a'b_a}^b \right) + 9 \mathcal{M}_{Pl}^2 \hat{\beta}_3 P_{\eta}^{(2)S_a^{b'}} n_{b'} \right) \\
& \left. \left(P_{\eta}^{(3)D^{a'}} n_{a'} - \overset{1}{\cdot} \mathcal{T}^{a'} \right) + 9 \mathcal{M}_{Pl}^2 \hat{\beta}_3 P_{\eta}^{(2)S_b^{b'}} n_{b'} \left(-P_{\eta}^{(3)D^{a'}} n_{a'} + \overset{1}{\cdot} \mathcal{T}^{a'} \right) + \right. \\
& \left. 18 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \eta_{a'b}^b \left(-\left(\overset{3}{D}^b P_{\eta}^{(2)S_a^{a'}} \right) + \overset{3}{D}^b P_{\eta}^{(2)S_a^{a'}} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left(\overset{2}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\cdot} \phi_{b|lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{3 \mathcal{T} \eta} P_{\eta}^{(1)S^{aa'}} \left(P_{\eta}^{(2)S_a^{b'}} \left(\mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}ab}^b - \mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}ab}^{b'} - 6 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \overset{1}{\cdot} \mathcal{T}^{ab} \right) + \right. \\
& P_{\eta}^{(2)S_b^{b'}} \left(\mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}ab}^b - \mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}ab}^{b'} - 6 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \overset{1}{\cdot} \mathcal{T}^{ab} \right) + \left(P_{\eta}^{(2)S_a^{b'}} + P_{\eta}^{(2)S_b^{a'}} \right) \\
& \left. \left(\mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}a'b}^b - \mathcal{M}_{Pl}^2 \hat{\beta}_1 \overset{1}{\cdot} \lambda_{\mathcal{T}a'b}^{b'} - 6 \mathcal{M}_{Pl}^2 \hat{\beta}_3 \overset{1}{\cdot} \mathcal{T}^{a'b} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\left\{ \iiint \left(\overset{2}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{T}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{2}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{0}{\cdot} \phi_{\mathcal{T}} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\begin{aligned}
& \left\{ \iiint \left(\overset{2}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{T}lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{12 \mathcal{T}^2} \left(3 P_{\eta}^{(1)S^{aa'}} \left(2 \mathcal{M}_{Pl}^2 \hat{\alpha}_0 \mathcal{T} P_{\eta}^{(2)S_{aa'}} + 2 \mathcal{M}_{Pl}^2 \hat{\alpha}_0 \mathcal{T} P_{\eta}^{(2)S_a^{a'}} + \right. \right. \\
& \left. \overset{1}{\cdot} \pi_{\mathcal{T}ab}^a P_{\eta}^{(2)S_a^{b'}} + \overset{1}{\cdot} \pi_{\mathcal{T}a'b}^a \left(P_{\eta}^{(2)S_a^{b'}} + P_{\eta}^{(2)S_b^{a'}} \right) + \overset{1}{\cdot} \pi_{\mathcal{T}ab}^a P_{\eta}^{(2)S_b^{a'}} \right) - \\
& \left. 4 \mathcal{M}_{Pl}^2 \hat{\alpha}_0 \eta_{aa'}^a \eta_{bb'}^b \mathcal{T} \overset{(1)}{S}^{aa'} \overset{(2)}{S}^{bb'} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\begin{aligned}
& \left\{ \iiint \left(\overset{2}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{2}{\cdot} \phi_{\mathcal{T}lmn} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iiint \frac{1}{32 \mathcal{T}^2} 3 \overset{1}{\cdot} \pi_{\mathcal{T}}^a \left(P_{\eta}^{(1)S^{a'}} P_{\eta}^{(2)S_{bb'}} \overset{(2)}{S}_{a^{bb'}} + 2 P_{\eta}^{(1)S^{a'b}} \left(P_{\eta}^{(2)S_{aa'b}} + P_{\eta}^{(2)S_{aba'a}} - P_{\eta}^{(2)S_{a'ab}} - P_{\eta}^{(2)S_{baa'a}} \right) + \right. \\
& P_{\eta}^{(1)S^{a'}} \left(P_{\eta}^{(2)S_{bb'}} \overset{(2)}{S}_{a^{bb'}} - P_{\eta}^{(2)S_{bb'}} \overset{(2)}{S}_{a^{b'b'}} \right) - P_{\eta}^{(1)S^{a'}} P_{\eta}^{(2)S_{bb'}} \overset{(2)}{S}_{a^{b'b'}} - \\
& \left. 2 \eta_{a'b}^a P_{\eta}^{(2)S_{b'c}} \overset{(2)}{S}_{a^{b'c}} \overset{(1)}{S}^{a'b} + 2 \eta_{a'b}^a P_{\eta}^{(2)S_{b'c}} \overset{(2)}{S}_{a^{b'c}} \overset{(1)}{S}^{a'b} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

$$\left\{ \iiint \left(\overset{2}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{1}{\cdot} \phi_{b|lm} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\left\{ \iiint \left(\overset{2}{\cdot} \phi_{bi} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left(\overset{1}{\cdot} \phi_{b|l} \cdot \overset{(2)}{S}^l \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0$$

$$\begin{aligned}
& \left\{ \iiint \left(\overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left(\overset{2}{\phi}_{lmn}^{\parallel} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left(\overset{0}{\phi}_b^{\perp} \cdot \overset{(2)}{S} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left(\overset{1}{\phi}_{bi}^{\perp} \cdot \overset{(2)}{S}^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\
& \left\{ \iiint \left(\overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left(\overset{2}{\phi}_{bi}^{\perp} \cdot \overset{(2)}{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left(\frac{2}{3} \mathcal{M}_{Pl}^2 \bar{\beta}_1^2 \left(3 P_{\eta}^{(1)} S^{aa'} \right) \left(P_{\eta}^{(2)} S_{aa'} + P_{\eta}^{(2)} S_{a'a} \right) - 2 \eta_{aa'}^{\parallel} \eta_{bb'}^{\parallel} \overset{(1)}{S}^{aa'} \overset{(2)}{S}^{bb'} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1 \\
& \left\{ \iiint \left(\overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left(\overset{1}{\chi}_{bi}^{\parallel} \cdot \overset{(2)}{S}^i \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left(\frac{1}{6 \mathcal{T}^2} \left(3 P_{\eta}^{(1)} S^{ab} \right) \left(P_{\eta}^{(2)} S_b \right) P_{\eta}^{(3)} \mathcal{D}_a \mathcal{T} + P_{\eta}^{(2)} S_a \right) P_{\eta}^{(3)} \mathcal{D}_b \mathcal{T} - \mathcal{T} \left(\mathcal{D}_a P_{\eta}^{(2)} S_b + \mathcal{D}_b P_{\eta}^{(2)} S_a \right) \right) + \\
& \quad 2 \eta_{ab}^{\parallel} \overset{(1)}{S}^{ab} \left(-P_{\eta}^{(2)} S^{a'} \right) P_{\eta}^{(3)} \mathcal{D}_a \mathcal{T} + \eta_{a'b}^{\parallel} \mathcal{T} \left(\mathcal{D}_b P_{\eta}^{(2)} S^{a'} \right) \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1 \\
& \left\{ \iiint \left(\overset{2}{\phi}_{bij} \cdot \overset{(1)}{S}^{ij} \right) [x^0, x^1, x^2, x^3] d^3x^3 d^2x^2 dx^1, \iiint \left(\overset{2}{\chi}_{lmn}^{\parallel} \cdot \overset{(2)}{S}^{lmn} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \quad \iiint \left(\frac{1}{96 \mathcal{T}^2} \left(P_{\eta}^{(1)} S^{ab} \right) \left(-18 P_{\eta}^{\parallel} \eta_{b,c} \overset{(2)}{S}_b^{b'c'} P_{\eta}^{(3)} \mathcal{D}_a \mathcal{T} + 18 P_{\eta}^{\parallel} \eta_{b,c} \overset{(2)}{S}_b^{b'c'} P_{\eta}^{(3)} \mathcal{D}_a \mathcal{T} + 27 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{bc}^{c'c'} n_{c'} \right) \right. \\
& \quad P_{\eta}^{(3)} \mathcal{D}_a n^{b'} + 9 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{bc}^{c'c'} n_{c'} P_{\eta}^{(3)} \mathcal{D}_a n^{b'} - 27 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{cb}^{c'c'} n_{c'} P_{\eta}^{(3)} \mathcal{D}_a n^{b'} - \\
& \quad 18 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{c'b}^{c'c'} n_{c'} P_{\eta}^{(3)} \mathcal{D}_a n^{b'} - 9 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{bc}^{c'c'} n_{c'} P_{\eta}^{(3)} \mathcal{D}_a n^{b'} + \\
& \quad 18 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{cb}^{c'c'} n_{c'} P_{\eta}^{(3)} \mathcal{D}_a n^{b'} - 18 P_{\eta}^{\parallel} \eta_{b,c} \overset{(2)}{S}_a^{b'c'} P_{\eta}^{(3)} \mathcal{D}_b \mathcal{T} + 18 P_{\eta}^{\parallel} \eta_{b,c} \overset{(2)}{S}_a^{b'c'} P_{\eta}^{(3)} \mathcal{D}_b \mathcal{T} + 27 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{ac}^{c'c'} n_{c'} P_{\eta}^{(3)} \mathcal{D}_b n^{b'} - \\
& \quad 27 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{ca}^{c'c'} n_{c'} P_{\eta}^{(3)} \mathcal{D}_b n^{b'} - 18 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{c'a}^{c'c'} n_{c'} P_{\eta}^{(3)} \mathcal{D}_b n^{b'} - \\
& \quad 9 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{ac}^{c'c'} n_{c'} P_{\eta}^{(3)} \mathcal{D}_b n^{b'} + 18 \delta_{b,c} \mathcal{T} P_{\eta}^{(2)} S_{ca}^{c'c'} n_{c'} P_{\eta}^{(3)} \mathcal{D}_b n^{b'} + \\
& \quad 36 P_{\eta}^{(2)} S_a^{b'b} P_{\eta}^{(3)} \mathcal{D}_b \mathcal{T} + 36 P_{\eta}^{(2)} S_b^{b'a} P_{\eta}^{(3)} \mathcal{D}_b \mathcal{T} - 36 P_{\eta}^{(2)} S^{b'ab} P_{\eta}^{(3)} \mathcal{D}_b \mathcal{T} - \\
& \quad 36 P_{\eta}^{(2)} S^{b'ba} P_{\eta}^{(3)} \mathcal{D}_b \mathcal{T} - 27 \mathcal{T} P_{\eta}^{(2)} S_{bb,c} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_a - 9 \mathcal{T} P_{\eta}^{(2)} S_{b,b}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_a + \\
& \quad 27 \mathcal{T} P_{\eta}^{(2)} S_{b,b}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_a + 18 \mathcal{T} P_{\eta}^{(2)} S_{b,b}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_a + 9 \mathcal{T} P_{\eta}^{(2)} S_{bb,b}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_a - \\
& \quad 18 \mathcal{T} P_{\eta}^{(2)} S_{b,b}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_a - 27 \mathcal{T} P_{\eta}^{(2)} S_{ab,b}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_b - 9 \mathcal{T} P_{\eta}^{(2)} S_{a,b}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_b + \\
& \quad 27 \mathcal{T} P_{\eta}^{(2)} S_{b,a}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_b + 18 \mathcal{T} P_{\eta}^{(2)} S_{b,a}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_b + 9 \mathcal{T} P_{\eta}^{(2)} S_{ab,b}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_b - \\
& \quad 18 \mathcal{T} P_{\eta}^{(2)} S_{b,a}^{c'c'} n_c P_{\eta}^{(3)} \mathcal{D}^{b'} n_b - 4 \epsilon_{bb',c}^{\parallel} \mathcal{T} P_{\eta}^{(2)} S_a^{b'c'} \overset{0}{\mathcal{T}}^{\parallel} - 4 \epsilon_{ab',c}^{\parallel} \mathcal{T} P_{\eta}^{(2)} S_b^{b'c'} \overset{0}{\mathcal{T}}^{\parallel} + \\
& \quad 4 \epsilon_{bb',c}^{\parallel} \mathcal{T} P_{\eta}^{(2)} S^{b'a} \overset{0}{\mathcal{T}}^{\parallel} + 4 \epsilon_{ab',c}^{\parallel} \mathcal{T} P_{\eta}^{(2)} S^{b'b} \overset{0}{\mathcal{T}}^{\parallel} + 8 \epsilon_{bb',c}^{\parallel} \mathcal{T} P_{\eta}^{(2)} S^{b'a} \overset{0}{\mathcal{T}}^{\parallel} + \\
& \quad 8 \epsilon_{ab',c}^{\parallel} \mathcal{T} P_{\eta}^{(2)} S^{b'b} \overset{0}{\mathcal{T}}^{\parallel} + 27 \mathcal{T} P_{\eta}^{(2)} S_{bb,b}^{c'c'} n_c \overset{1}{\mathcal{T}}^{\parallel} + 9 \mathcal{T} P_{\eta}^{(2)} S_{b,b}^{c'c'} n_c \overset{1}{\mathcal{T}}^{\parallel} -
\end{aligned}$$

$$\begin{aligned}
& 27 \mathcal{T} P_{\eta}^{(2)S_{b',b}{}^c} n_c \left[\mathcal{T}_{\eta}^{||a}{}^{b'} - 18 \mathcal{T} P_{\eta}^{(2)S_{b',c}{}^b} n_c \left[\mathcal{T}_{\eta}^{||a}{}^{b'} - 9 \mathcal{T} P_{\eta}^{(2)S_{bb',c}{}^c} n_c \right] \mathcal{T}_{\eta}^{||a}{}^{b'} + \right. \\
& 18 \mathcal{T} P_{\eta}^{(2)S_{b',b}{}^c} n_c \left[\mathcal{T}_{\eta}^{||a}{}^{b'} + 27 \mathcal{T} P_{\eta}^{(2)S_{ab,b}{}^c} n_c \left[\mathcal{T}_{\eta}^{||b}{}^{b'} + 9 \mathcal{T} P_{\eta}^{(2)S_{a,b}{}^c} n_c \right] \mathcal{T}_{\eta}^{||b}{}^{b'} - \right. \\
& 27 \mathcal{T} P_{\eta}^{(2)S_{b',a}{}^c} n_c \left[\mathcal{T}_{\eta}^{||b}{}^{b'} - 18 \mathcal{T} P_{\eta}^{(2)S_{b',c}{}^a} n_c \left[\mathcal{T}_{\eta}^{||b}{}^{b'} - 9 \mathcal{T} P_{\eta}^{(2)S_{ab,b}{}^c} n_c \right] \mathcal{T}_{\eta}^{||b}{}^{b'} + \right. \\
& 18 \mathcal{T} P_{\eta}^{(2)S_{b',a}{}^c} n_c \left[\mathcal{T}_{\eta}^{||b}{}^{b'} + 18 \eta_{b',c}^{||} \mathcal{T} \left({}^3\mathcal{D}_{\eta}^{||a} P_{\eta}^{(2)S_{b'}{}^{b'c}} \right) - 18 \eta_{b',c}^{||} \mathcal{T} \left({}^3\mathcal{D}_{\eta}^{||b} P_{\eta}^{(2)S_{b'}{}^{b'c}} \right) + \right. \\
& 18 \eta_{b',c}^{||} \mathcal{T} \left({}^3\mathcal{D}_{\eta}^{||b} P_{\eta}^{(2)S_{a'}{}^{b'c}} \right) - 18 \eta_{b',c}^{||} \mathcal{T} \left({}^3\mathcal{D}_{\eta}^{||b} P_{\eta}^{(2)S_{a'}{}^{b'c}} \right) - 36 \eta_{b',c}^{||} \mathcal{T} \left({}^3\mathcal{D}_{\eta}^{||c} P_{\eta}^{(2)S_{a'}{}^{b'b}} \right) - \\
& 36 \eta_{b',c}^{||} \mathcal{T} \left({}^3\mathcal{D}_{\eta}^{||c} P_{\eta}^{(2)S_{b'}{}^{b'a}} \right) + 36 \eta_{b',c}^{||} \mathcal{T} \left({}^3\mathcal{D}_{\eta}^{||c} P_{\eta}^{(2)S_{b'}{}^{b'a}} \right) + 36 \eta_{b',c}^{||} \mathcal{T} \left({}^3\mathcal{D}_{\eta}^{||c} P_{\eta}^{(2)S_{b'a}{}^{b'b}} \right) \Big] + \\
& 18 \eta_{ab}^{||} ({}^1S^{ab} \left(2 P_{\eta}^{||\eta_{cc'}} ({}^2S_{b',c}{}^{cc'}) P_{\eta}^{[3\mathcal{D}^{b'}]} \mathcal{T} - 2 P_{\eta}^{||\eta_{cc'}} ({}^2S_{b',c}{}^{cc'}) P_{\eta}^{[3\mathcal{D}^{b'}]} \mathcal{T} + \right. \\
& \mathcal{T} \left(\delta_{b',c'} \left(-P_{\eta}^{(2)S_{cc'}} n_{a'} - P_{\eta}^{(2)S_{c',c}} n_{a'} + P_{\eta}^{(2)S_{c',c}} n_{a'} + P_{\eta}^{(2)S_{c',c}} n_{a'} \right) P_{\eta}^{[3\mathcal{D}^{b'}]} n^{b'} - \right. \\
& 2 P_{\eta}^{(2)S_{cb',c'}} n_{c'} \left[P_{\eta}^{[3\mathcal{D}^{b'}]} n^{b'} - 2 P_{\eta}^{(2)S_{c',b}} n_{c'} \left[P_{\eta}^{[3\mathcal{D}^{b'}]} n^{b'} + 2 P_{\eta}^{(2)S_{c',b}} n_{c'} \left[P_{\eta}^{[3\mathcal{D}^{b'}]} n^{b'} - \right. \right. \\
& 2 \left(\eta_{a',b}^{||} ({}^2S^{aa'b} n_a) \left(\eta_{aa'}^{||} ({}^3\mathcal{D}^{aa'} n^a) \right) + 2 \left(\eta_{ab}^{||} ({}^2S^{aa'b} n_a) \left(\eta_{aa'}^{||} ({}^3\mathcal{D}^{aa'} n^a) \right) + \right. \\
& 2 P_{\eta}^{(2)S_{b',c}} n_{c'} \left(P_{\eta}^{[3\mathcal{D}^{b'}]} n^{b'} - \mathcal{T}_{\eta}^{||b'}{}^{b'c} \right) - P_{\eta}^{(2)S_{b',c}} n_{c'} \left[\mathcal{T}_{\eta}^{||b'}{}^{b'c} + P_{\eta}^{(2)S_{b',c}} n_{c'} \left[\mathcal{T}_{\eta}^{||b'}{}^{b'c} + 2 \right. \right. \\
& \eta_{b',c'}^{||} \eta_{ca'}^{||} \left({}^3\mathcal{D}^{aa'} P_{\eta}^{(2)S_{b'}{}^{b'cc'}} \right) - 2 \eta_{b',a'}^{||} \eta_{cc'}^{||} \left({}^3\mathcal{D}^{aa'} P_{\eta}^{(2)S_{b'}{}^{b'cc'}} \right) + 2 \eta_{a',d}^{||} \eta_{cc'}^{||} ({}^2S^{b'cc'}) \\
& n_b, ({}^3\mathcal{D}^{dd} n^a) - 2 \eta_{a',d}^{||} \eta_{b',c'}^{||} ({}^2S^{b'cc'}) n_c ({}^3\mathcal{D}^{dd} n^a) \Big] \Big] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left({}^2\phi_{bij} \cdot ({}^1S^{ij}) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left({}^1\chi_{b|lm}^{||} \cdot ({}^2S^{lm}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right) \approx \right. \\
& \iiint \left(-\frac{1}{3\mathcal{T}} \left(3 P_{\eta}^{(1)S^{aa'}} \left(P_{\eta}^{(2)S_a}{}^b \right) \left(\mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{[3\mathcal{D}^{||a}]} n_b + \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{[3\mathcal{D}^{||b}]} n_a + \mathcal{M}_{Pl}{}^2 \bar{\beta}_1 {}^2\lambda_{\mathcal{T}ab}^{||} - \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 \mathcal{T}_{\eta}^{||ab} \right) - \right. \\
& P_{\eta}^{(2)S_a}{}^b \left(\mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{[3\mathcal{D}^{||a}]} n_b + \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{[3\mathcal{D}^{||b}]} n_a + \mathcal{M}_{Pl}{}^2 \bar{\beta}_1 {}^2\lambda_{\mathcal{T}ab}^{||} - \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 \mathcal{T}_{\eta}^{||ab} \right) + \\
& \left(P_{\eta}^{(2)S_a}{}^b - P_{\eta}^{(2)S_b}{}^a \right) \left(\mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{[3\mathcal{D}^{||a}]} n_b + \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 P_{\eta}^{[3\mathcal{D}^{||b}]} n_a + \mathcal{M}_{Pl}{}^2 \bar{\beta}_1 {}^2\lambda_{\mathcal{T}ab}^{||} - \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 \mathcal{T}_{\eta}^{||ab} \right. \\
& \left. \left. \mathcal{T}_{\eta}^{||a'b} \right) \right) + 4 \mathcal{M}_{Pl}{}^2 \hat{\beta}_3 \eta_{aa'}^{||} P_{\eta}^{(2)S^{bb'}} ({}^1S^{aa'}) \mathcal{T}_{\eta}^{||bb'} \Big] \Big] [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left({}^0\phi_{\mathcal{A}} \cdot ({}^1S) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left({}^0\phi_{\mathcal{A}} \cdot ({}^2S) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right) \approx 0 \right. \\
& \left\{ \iiint \left({}^0\phi_{\mathcal{A}} \cdot ({}^1S) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left({}^0\phi_{\mathcal{A}} \cdot ({}^2S) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right) \approx 0 \right. \\
& \left\{ \iiint \left({}^0\phi_{\mathcal{A}} \cdot ({}^1S) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left({}^2\phi_{\mathcal{A}|lm} \cdot ({}^2S^{lm}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right) \approx 0 \right. \\
& \left\{ \iiint \left({}^0\phi_{\mathcal{A}} \cdot ({}^1S) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left({}^2\phi_{\mathcal{A}|lmn} \cdot ({}^2S^{lmn}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right) \approx 0 \right. \\
& \left\{ \iiint \left({}^0\phi_{\mathcal{A}} \cdot ({}^1S) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left({}^1\phi_{b|lm}^{||} \cdot ({}^2S^{lm}) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right) \approx 0 \right. \\
& \left\{ \iiint \left({}^0\phi_{\mathcal{A}} \cdot ({}^1S) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iiint \left({}^1\phi_{b|l}^{||} \cdot ({}^2S^l) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right) \approx 0 \right.
\end{aligned}$$

[illegible]

[illegible]

[illegible]

$$\begin{aligned} & \left\{ \iiint \left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \chi_{b,i}^{\dagger} \cdot \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S^i \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \chi_{b,lm}^{\dagger} \cdot \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} S^{lm} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\ & \iint \frac{1}{4\mathcal{J}^2} P^{(1)S^a} \left(\delta_{a,b} \mathcal{J} \left(P_{\eta}^{(2)S_b b'} n_{b,i} \right) - P_{\eta}^{(2)S^b b'} n_{b,i} \right) P_{\eta}^{(3)\mathcal{D}_a n^a} - 2 P_{\eta}^{(2)S_a^a} P_{\eta}^{(3)\mathcal{D}_a \mathcal{J}} + 2 P_{\eta}^{(2)S^a a} P_{\eta}^{(3)\mathcal{D}_a \mathcal{J}} - \\ & \mathcal{J} P_{\eta}^{(2)S_a^b b'} n_{b,i} P_{\eta}^{(3)\mathcal{D}^a n_a} + \mathcal{J} P_{\eta}^{(2)S^b a, b'} n_{b,i} P_{\eta}^{(3)\mathcal{D}^a n_a} + \mathcal{J} P_{\eta}^{(2)S_a^b b'} n_{b,i} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \mathcal{T}_a^{\dagger a'} - \mathcal{J} P_{\eta}^{(2)S^b a, b'} n_{b,i} \\ & \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \mathcal{T}_a^{\dagger a'} + 2 \eta_{a,b}^{\dagger} \mathcal{J} \left(\begin{smallmatrix} 3 \\ 3 \end{smallmatrix} \mathcal{D}^b P_{\eta}^{(2)S_a^a} \right) - 2 \eta_{a,b}^{\dagger} \mathcal{J} \left(\begin{smallmatrix} 3 \\ 3 \end{smallmatrix} \mathcal{D}^b P_{\eta}^{(2)S^a a} \right) \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1 \\ & \left\{ \iiint \left(\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \chi_{b,ijh}^{\dagger} \cdot \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S^{ijh} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \chi_{b,lmn}^{\dagger} \cdot \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} S^{lmn} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx 0 \\ & \left\{ \iiint \left(\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \chi_{b,ijh}^{\dagger} \cdot \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S^{ijh} \right) [\chi^0, \chi^1, \chi^2, \chi^3] d\chi^3 d\chi^2 d\chi^1, \iiint \left(\begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \chi_{b,lm}^{\dagger} \cdot \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} S^{lm} \right) [\chi^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\ & \iint \frac{1}{32\mathcal{J}^2} \left(-6 P_{\eta}^{(2)S_a^b b'} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{a,b}^{\dagger c} P_{\eta}^{(2)S^{aa'}} P_{\eta}^{(3)\mathcal{D}_a \mathcal{J}} + 6 P_{\eta}^{(2)S_a^b b'} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{a,b}^{\dagger c} P_{\eta}^{(2)S^{aa'}} P_{\eta}^{(3)\mathcal{D}_a \mathcal{J}} + 4 P_{\eta}^{(1)S^{aa'b'}} \right. \\ & P_{\eta}^{(2)S_a^b b'} P_{\eta}^{(3)\mathcal{D}_a \mathcal{J}} - 4 P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{b,a}^{\dagger}} P_{\eta}^{(3)\mathcal{D}_a \mathcal{J}} + 4 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{b,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_a n_a} - \\ & 4 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{b,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_a n_a} + 2 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_a n_b} - \\ & 2 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_a n_b} - 4 P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{ab,b}^{\dagger}} P_{\eta}^{(3)\mathcal{D}_a \mathcal{J}} + \\ & 6 P_{\eta}^{(2)S_a^b b'} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{a,b}^{\dagger c} P_{\eta}^{(2)S^{aa'}} P_{\eta}^{(3)\mathcal{D}_a \mathcal{J}} - 6 P_{\eta}^{(2)S_a^b b'} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{a,b}^{\dagger c} P_{\eta}^{(2)S^{aa'}} P_{\eta}^{(3)\mathcal{D}_a \mathcal{J}} + \\ & 4 P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{b,a}^{\dagger}} P_{\eta}^{(3)\mathcal{D}_a \mathcal{J}} - 4 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{b,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_a n_a} + \\ & 4 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{b,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_a n_a} - 2 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_a n_b} + \\ & 2 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_a n_b} + 3 \delta_{a,b} \mathcal{J} P_{\eta}^{(2)S_{c,i}^{\dagger} i} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{b,c}^{\dagger i} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}^a n^a} - \\ & 3 \delta_{a,b} \mathcal{J} P_{\eta}^{(2)S_{c,i}^{\dagger} i} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{b,c}^{\dagger i} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}^a n^a} - 3 \delta_{a,b} \mathcal{J} P_{\eta}^{(2)S_{c,i}^{\dagger} i} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{a,c}^{\dagger i} P_{\eta}^{(2)S_{b,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}^a n^a} + \\ & 3 \delta_{a,b} \mathcal{J} P_{\eta}^{(2)S_{c,i}^{\dagger} i} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{b,c}^{\dagger i} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}^a n^a} + 3 \delta_{a,b} \mathcal{J} P_{\eta}^{(2)S_{c,i}^{\dagger} i} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{a,c}^{\dagger i} P_{\eta}^{(2)S_{b,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}^a n^a} - \\ & 3 \delta_{a,b} \mathcal{J} P_{\eta}^{(2)S_{c,i}^{\dagger} i} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{b,c}^{\dagger i} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}^a n^a} + 3 \delta_{a,b} \mathcal{J} P_{\eta}^{(2)S_{c,i}^{\dagger} i} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{a,c}^{\dagger i} P_{\eta}^{(2)S_{b,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}^a n^a} - \\ & 3 \delta_{a,b} \mathcal{J} P_{\eta}^{(2)S_{c,i}^{\dagger} i} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} S_{b,c}^{\dagger i} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}^a n^a} - 8 P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{aa,b}^{\dagger}} P_{\eta}^{(3)\mathcal{D}_b \mathcal{J}} + \\ & 8 P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{a,a}^{\dagger}} P_{\eta}^{(3)\mathcal{D}_b \mathcal{J}} - 2 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_b n_a} + \\ & 2 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_b n_a} + 2 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_b n_a} - \\ & 2 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} P_{\eta}^{(3)\mathcal{D}_b n_a} + 4 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{b,c}^{\dagger} n_c} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \mathcal{T}_{aa}^{\dagger} - \\ & 4 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{b,c}^{\dagger} n_c} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \mathcal{T}_{aa}^{\dagger} + 2 \mathcal{J} P_{\eta}^{(1)S^{aa'b'}} P_{\eta}^{(2)S_{a,c}^{\dagger} n_c} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \mathcal{T}_{ab}^{\dagger} - \end{aligned}$$

$$\begin{aligned}
& 2 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} P_{\eta}^{[2] S^c_a, n_c} \left[\mathcal{T}^{ab} \right] + 3 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} P_{\eta}^{[2] S^c_a, n_c} \left[\mathcal{T}^{cc'} \right] P_{\eta}^{[2] S^b_a, n_b} \left[\mathcal{T}^{aa'} \right] - \\
& 3 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} P_{\eta}^{[2] S^c_a, n_c} \left[\mathcal{T}^{ab} \right] - 3 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} P_{\eta}^{[2] S^c_a, n_c} \left[\mathcal{T}^{cc'} \right] P_{\eta}^{[2] S^b_a, n_b} \left[\mathcal{T}^{aa'} \right] + \\
& 3 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} P_{\eta}^{[2] S^c_a, n_c} \left[\mathcal{T}^{ab} \right] - 2 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} P_{\eta}^{[2] S^c_a, n_c} \left[\mathcal{T}^{ab} \right] + \\
& 2 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} P_{\eta}^{[2] S^c_a, n_c} \left[\mathcal{T}^{ab} \right] - 4 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} \left(\mathcal{D}^b_a P_{\eta}^{[2] S^c_a, n_c} \right) + \\
& 4 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} \left(\mathcal{D}^b_a P_{\eta}^{[2] S^c_a, n_c} \right) + 4 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} \left(\mathcal{D}^b_a P_{\eta}^{[2] S^c_a, n_c} \right) - \\
& 4 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} \left(\mathcal{D}^b_a P_{\eta}^{[2] S^c_a, n_c} \right) + 8 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} \left(\mathcal{D}^b_a P_{\eta}^{[2] S^c_a, n_c} \right) - \\
& 8 \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} \left(\mathcal{D}^b_a P_{\eta}^{[2] S^c_a, n_c} \right) - 6 \eta^{ab} \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} \left(\mathcal{D}^b_a P_{\eta}^{[2] S^c_a, n_c} \right) + \\
& 6 \eta^{ab} \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} \left(\mathcal{D}^b_a P_{\eta}^{[2] S^c_a, n_c} \right) + 6 \eta^{ab} \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} \left(\mathcal{D}^b_a P_{\eta}^{[2] S^c_a, n_c} \right) - \\
& 6 \eta^{ab} \mathcal{T} P_{\eta}^{[1] S^{aa'b'}} \left(\mathcal{D}^b_a P_{\eta}^{[2] S^c_a, n_c} \right) \left[x^0, x^1, x^2, x^3 \right] dx^3 dx^2 dx^1 \\
& \left\{ \iiint \left(\mathcal{X}^{ij}_{ab} \cdot \mathcal{S}^{ij} \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1, \iint \left(\mathcal{X}^{lm}_{ab} \cdot \mathcal{S}^{lm} \right) [x^0, y^1, y^2, y^3] dy^3 dy^2 dy^1 \right\} \approx \\
& \iint \frac{1}{3 \mathcal{T}} \mathcal{M}_{\text{Pl}}^2 \tilde{\beta}_1 P_{\eta}^{[1] S^{aa'b'}} \left(P_{\eta}^{[2] S^b_a, n_b} \left(\mathcal{T}^{ab} - \mathcal{T}^{ab} \right) + P_{\eta}^{[2] S^b_a, n_b} \left(-\mathcal{T}^{ab} + \mathcal{T}^{ab} \right) - \right. \\
& \left. \left(P_{\eta}^{[2] S^b_a, n_b} - P_{\eta}^{[2] S^b_a, n_b} \right) \left(\mathcal{T}^{ab} - \mathcal{T}^{ab} \right) \right) [x^0, x^1, x^2, x^3] dx^3 dx^2 dx^1
\end{aligned}$$

This concludes the HiGGS output.

Concrete relation to manuscript: The above constraints and Poisson brackets constitute the data represented by coloured squares in the right-hand panel of Fig. 5 in our Manuscript. The primary constraints ϕ are listed along the top row. The primary Poisson brackets $\{\phi, \phi\}$ are used to infer the white, red or yellow squares in the matrix depending on whether they are vanishing, or appear only in proportion to phase space variables, or are order unity (e.g. proportional to the metric or unit-timelike vector). The $\{\phi, \mathcal{H}_b\}$ Poisson brackets define the secondary constraints " χ ", which appear on the sides of the matrix. Most of the Poisson brackets $\{\chi, \phi\}$ need not be computed (grey squares), but it can be noticed without detailed calculation that many of the remaining required brackets (yellow squares) do indeed arise. Unlike in the strongly coupled case, we find that tertiary and quaternary constraints arise in the theory: but these are also not computed in detail above. Instead, their existence can readily be inferred by considering whether the consistency chain in question can be absorbed by determining a multiplier, or whether it must continue. In case of continuation, the relevant terms in the induced constraints which arise can be understood by considering Eq. (4) in arXiv:2205.13534.