Supplemental materials

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Introduction — In this supplement we provide the full particle spectrum analyses referred to in Appendix B, and explore the possibility of other EP-invariant operators. We will refer directly to equations, references etc. using the numbering in the main body of the text. The particle spectrum analysis is performed using the *PSALTer* software (see previous uses [235, 236]). Within this software, we load a kinematic module associated with the metric-affine d.o.f. This module was prepared using the conventions of [35]. Therefore, before presenting the results, we need to provide a full tranlation between those conventions and ours.

Matching conventions — The independent connection is $A_{\mu}{}^{\nu}{}_{\rho}$ and the torsion and non-metricity tensors are

$$\begin{aligned}\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu} &\equiv 2A_{[\mu|}{}^{\alpha}{}_{|\nu]},\\ \mathcal{Q}_{\lambda\mu\nu} &\equiv -\partial_{\lambda}g_{\mu\nu} + 2A_{\lambda}{}^{\alpha}{}_{(\mu|}g_{\alpha|\nu)}.\end{aligned} \tag{SM1}$$

The non-Riemannian curvature is

$$\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma} \equiv 2\left(\partial_{[\mu}A_{\nu]}{}^{\rho}{}_{\sigma} + A_{[\mu|}{}^{\rho}{}_{\alpha}A_{|\nu]}{}^{\alpha}{}_{\sigma}\right). \tag{SM2}$$

The various contractions formed from Eqs. (SM1) and (SM2) are

$$\begin{gathered}\mathcal{T}_{\mu} \equiv \mathcal{T}_{\alpha}{}^{\alpha}{}_{\mu}, \quad \mathcal{Q}_{\mu} \equiv \mathcal{Q}_{\mu\alpha}{}^{\alpha}, \quad \tilde{\mathcal{Q}}_{\mu} \equiv \mathcal{Q}_{\alpha}{}^{\alpha}{}_{\mu},\\ \mathcal{F} \equiv \mathcal{F}_{\mu\nu}{}^{\mu\nu}, \quad \mathcal{F}_{\mu\nu} \equiv \mathcal{F}_{\mu\nu\alpha}{}^{\alpha}\\ \mathcal{F}^{(14)}{}_{\mu\nu} \equiv \mathcal{F}_{\alpha\mu\nu}{}^{\alpha}, \quad \mathcal{F}^{(13)}{}_{\mu\nu} \equiv \mathcal{F}_{\alpha\mu}{}^{\alpha}{}_{\nu}.\end{gathered} \tag{SM3}$$

By comparing Eq. (SM1) with Eq. (1), and Eq. (SM2) with Eq. (2), and finally Eq. (SM3) with Eq. (3), with various other inline definitions throughout the body of the text, we can obtain the following equivalences

$$\begin{gathered}A_{\mu}{}^{\rho}{}_{\nu} \equiv \Gamma^{\rho}{}_{\mu\nu}, \quad \mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma} \equiv R^{\rho}{}_{\sigma\mu\nu}, \quad \mathcal{F}_{\mu\nu} \equiv \hat{R}_{\mu\nu},\\ \mathcal{F}^{(14)}{}_{\mu\nu} \equiv \check{R}_{\nu\mu}, \quad \mathcal{F}^{(13)}{}_{\mu\nu} \equiv R_{\nu\mu}, \quad \mathcal{F} \equiv R,\\ \mathcal{Q}_{\lambda\mu\nu} \equiv -Q_{\lambda\mu\nu}, \quad \mathcal{Q}_{\mu} \equiv -Q_{\mu}, \quad \tilde{\mathcal{Q}}_{\mu} \equiv -\hat{Q}_{\mu},\\ \mathcal{T}_{\mu}{}^{\lambda}{}_{\nu} \equiv T^{\lambda}{}_{\mu\nu}, \quad \mathcal{T}_{\mu} \equiv -T_{\mu}.\end{gathered} \tag{SM4}$$

In Eq. (SM4), we mark in red the quantities which have a change in sign or index structure between conventions, beyond just a notational change.

Spectral analyses — Having introduced the conventions of [35], we turn to the linearised particle spectrum analysis. The metric perturbation is $h_{\mu\nu}$ and $A_{\mu}{}^{\rho}{}_{\nu}$ is assumed to be inherently perturbative. To reach the second-order formulation, we only have to edit the quadratic action before substituting into the `ParticleSpectrum` function (which is the main function provided by the *PSALTer* package). The reparameterisation used to transform the quadratic action is

$$A_{\mu}{}^{\rho}{}_{\nu} \mapsto A_{\mu}{}^{\rho}{}_{\nu} + \frac{1}{2}\left(2\partial_{(\mu|}h_{\lambda|\nu)} - \partial_{\lambda}h_{\mu\nu}\right). \tag{SM5}$$

To lowest order in perturbative fields, Eq. (SM5) captures the transition from $\Gamma_{\mu}{}^{\rho}{}_{\nu}$ to $T_{\mu}{}^{\rho}{}_{\nu}$ and $Q_{\mu\rho\nu}$ set out in Eq. (SM1).

Conjugate to the metric perturbation $h_{\mu\nu}$ and the affine connection $A_{\mu}{}^{\rho}{}_{\nu}$ are the (symmetric) stress-energy tensor $T^{\mu\nu}$ and the current $W^{\mu}{}_{\rho}{}^{\nu}$ which in MAG has become known as the *hypermomentum*. A more in-depth treatment of the metric-affine implementation in *PSALTer* can be found in the appendices to [235]. In Fig. SM1 we display the metric-affine gravity kinematic module. All the analyses are represented in Figs. SM2 to SM6. Further details can be found in the accompanying file `ParticleSpectra.nb`.

Invariant terms — In the accompanying file `Invariants.nb` we examine the possibility of other EP-, IW-, or concurrent-invariant terms in the quadratic curvature sector of metric-affine gravity.

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PSALTer kinematic panel

Momentum	Norm	Frame
k^{μ}	$k^2 == k_{\mu}\ k^{\mu}$	$n^{\mu} == \frac{k^{\mu}}{k}$

Fundamental fields

Fields	Symmetries	SO(3)	Sources
$h_{\alpha\beta}$	StrongGenSet[{1,2},GenSet[(1,2)]]	$\frac{1}{3}\eta_{\alpha\beta}\ {}^{0^+}h^{\parallel} + {}^{2^+}h^{\parallel}{}_{\alpha\beta} + {}^{1^-}h^{\perp}{}_{\beta}\ n_{\alpha} + {}^{1^-}h^{\perp}{}_{\alpha}\ n_{\beta} - \frac{1}{3}\ {}^{0^+}h^{\parallel}\ n_{\alpha}\ n_{\beta} + {}^{0^+}h^{\perp}\ n_{\alpha}\ n_{\beta}$	$\mathcal{T}_{\alpha\beta}$
$\mathcal{A}_{\alpha\beta\chi}$	StrongGenSet[{},GenSet[]]	$\frac{4}{3}\ {}^{2^-}\mathcal{A}_{a}{}^{\parallel}{}_{\beta\chi\alpha} + \frac{1}{2}\ {}^{2^-}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta\chi} + \frac{1}{2}\ {}^{2^-}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\chi\beta} + {}^{3^-}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta\chi} + \frac{1}{2}\ {}^{1^-}\mathcal{A}_{a}{}^{\parallel}{}_{\chi}\ \eta_{\alpha\beta} - \frac{1}{6}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\chi}\ \eta_{\alpha\beta} + \frac{1}{15}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\chi}\ \eta_{\alpha\beta} - \frac{1}{2}\ {}^{1^-}\mathcal{A}_{a}{}^{\parallel}{}_{\beta}\ \eta_{\alpha\chi} - \frac{1}{6}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\beta}\ \eta_{\alpha\chi} + \frac{1}{15}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\beta}\ \eta_{\alpha\chi} + \frac{1}{3}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\alpha}\ \eta_{\beta\chi} + \frac{1}{15}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\alpha}\ \eta_{\beta\chi} + {}^{1^+}\mathcal{A}_{a}{}^{\perp}{}_{\beta\chi}\ n_{\alpha} + \frac{1}{3}\ {}^{2^+}\mathcal{A}_{s}{}^{\parallel}{}_{\beta\chi}\ n_{\alpha} + \frac{2}{3}\ {}^{2^+}\mathcal{A}_{s}{}^{\perp}{}_{\beta\chi}\ n_{\alpha} + \frac{1}{9}\ {}^{0^+}\mathcal{A}_{s}{}^{\parallel}\ \eta_{\beta\chi}\ n_{\alpha} + \frac{2}{9}\ {}^{0^+}\mathcal{A}_{s}{}^{\perp h}\ \eta_{\beta\chi}\ n_{\alpha} - {}^{1^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\chi}\ n_{\beta} + {}^{2^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\chi}\ n_{\beta} - \frac{1}{2}\ {}^{1^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\chi}\ n_{\beta} + \frac{1}{3}\ {}^{2^+}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\chi}\ n_{\beta} - \frac{1}{3}\ {}^{2^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\chi}\ n_{\beta} + \frac{1}{3}\ {}^{0^+}\mathcal{A}_{a}{}^{\parallel}\ \eta_{\alpha\chi}\ n_{\beta} + \frac{1}{9}\ {}^{0^+}\mathcal{A}_{s}{}^{\parallel}\ \eta_{\alpha\chi}\ n_{\beta} - \frac{1}{9}\ {}^{0^+}\mathcal{A}_{s}{}^{\perp h}\ \eta_{\alpha\chi}\ n_{\beta} - \frac{1}{2}\ {}^{1^-}\mathcal{A}_{a}{}^{\parallel}{}_{\chi}\ n_{\alpha}\ n_{\beta} - {}^{1^-}\mathcal{A}_{a}{}^{\perp}{}_{\chi}\ n_{\alpha}\ n_{\beta} + \frac{1}{6}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\chi}\ n_{\alpha}\ n_{\beta} - \frac{1}{15}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\chi}\ n_{\alpha}\ n_{\beta} - \frac{1}{3}\ {}^{1^-}\mathcal{A}_{s}{}^{\perp h}{}_{\chi}\ n_{\alpha}\ n_{\beta} + \frac{1}{3}\ {}^{1^-}\mathcal{A}_{s}{}^{\perp t}{}_{\chi}\ n_{\alpha}\ n_{\beta} + {}^{1^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\beta}\ n_{\chi} - {}^{2^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\beta}\ n_{\chi} - \frac{1}{2}\ {}^{1^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\beta}\ n_{\chi} + \frac{1}{3}\ {}^{2^+}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta}\ n_{\chi} - \frac{1}{3}\ {}^{2^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\beta}\ n_{\chi} - \frac{1}{3}\ {}^{0^+}\mathcal{A}_{a}{}^{\parallel}\ \eta_{\alpha\beta}\ n_{\chi} + \frac{1}{9}\ {}^{0^+}\mathcal{A}_{s}{}^{\parallel}\ \eta_{\alpha\beta}\ n_{\chi} - \frac{1}{9}\ {}^{0^+}\mathcal{A}_{s}{}^{\perp h}\ \eta_{\alpha\beta}\ n_{\chi} + \frac{1}{2}\ {}^{1^-}\mathcal{A}_{a}{}^{\parallel}{}_{\beta}\ n_{\alpha}\ n_{\chi} + {}^{1^-}\mathcal{A}_{a}{}^{\perp}{}_{\beta}\ n_{\alpha}\ n_{\chi} + \frac{1}{6}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\beta}\ n_{\alpha}\ n_{\chi} - \frac{1}{15}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\beta}\ n_{\alpha}\ n_{\chi} - \frac{1}{3}\ {}^{1^-}\mathcal{A}_{s}{}^{\perp h}{}_{\beta}\ n_{\alpha}\ n_{\chi} + \frac{1}{3}\ {}^{1^-}\mathcal{A}_{s}{}^{\perp t}{}_{\beta}\ n_{\alpha}\ n_{\chi} - \frac{1}{3}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\alpha}\ n_{\beta}\ n_{\chi} - \frac{1}{15}\ {}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\alpha}\ n_{\beta}\ n_{\chi} + \frac{2}{3}\ {}^{1^-}\mathcal{A}_{s}{}^{\perp h}{}_{\alpha}\ n_{\beta}\ n_{\chi} + \frac{1}{3}\ {}^{1^-}\mathcal{A}_{s}{}^{\perp t}{}_{\alpha}\ n_{\beta}\ n_{\chi} - \frac{1}{3}\ {}^{0^+}\mathcal{A}_{s}{}^{\parallel}\ n_{\alpha}\ n_{\beta}\ n_{\chi} + {}^{0^+}\mathcal{A}_{s}{}^{\perp t}\ n_{\alpha}\ n_{\beta}\ n_{\chi} - \frac{1}{6}\ {}^{0^-}\mathcal{A}_{a}{}^{\parallel}\ \epsilon\eta_{\alpha\beta\chi\delta}\ n^{\delta}$	$\mathcal{W}_{\alpha\beta\chi}$

SO(3) irreps

SO(3)	Symmetries	Expansion	Sources
${}^{0^+}h^{\perp}$	StrongGenSet[{},GenSet[]]	$h_{\alpha\beta}\ n^{\alpha}\ n^{\beta}$	${}^{0^+}\mathcal{T}^{\perp}$
${}^{0^+}h^{\parallel}$	StrongGenSet[{},GenSet[]]	$h^{\alpha}{}_{\alpha} - h_{\alpha\beta}\ n^{\alpha}\ n^{\beta}$	${}^{0^+}\mathcal{T}^{\parallel}$
${}^{1^-}h^{\perp}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$h_{\alpha\beta}\ n^{\beta} - h_{\beta\chi}\ n_{\alpha}\ n^{\beta}\ n^{\chi}$	${}^{1^-}\mathcal{T}^{\perp}{}_{\alpha}$
${}^{2^+}h^{\parallel}{}_{\alpha\beta}$	StrongGenSet[{1,2},GenSet[(1,2)]]	$h_{\alpha\beta} - \frac{1}{3}\ \eta_{\alpha\beta}\ h^{\chi}{}_{\chi} + \frac{1}{3}\ h^{\chi}{}_{\chi}\ n_{\alpha}\ n_{\beta} - h_{\beta\chi}\ n_{\alpha}\ n^{\chi} - h_{\alpha\chi}\ n_{\beta}\ n^{\chi} + \frac{1}{3}\ \eta_{\alpha\beta}\ h_{\chi\delta}\ n^{\chi}\ n^{\delta} + \frac{2}{3}\ h_{\chi\delta}\ n_{\alpha}\ n_{\beta}\ n^{\chi}\ n^{\delta}$	${}^{2^+}\mathcal{T}^{\parallel}{}_{\alpha\beta}$
${}^{0^+}\mathcal{A}_{a}{}^{\parallel}$	StrongGenSet[{}, GenSet[]] -	$\frac{1}{2}\ \mathcal{A}^{\alpha}{}_{\alpha}{}^{\beta}\ n_{\beta} + \frac{1}{2}\ \mathcal{A}^{\alpha\beta}{}_{\alpha}\ n_{\beta}$	${}^{0^+}\mathcal{W}_{a}{}^{\parallel}$
${}^{0^+}\mathcal{A}_{s}{}^{\perp t}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}^{\alpha\beta\chi}\ n_{\alpha}\ n_{\beta}\ n_{\chi}$	${}^{0^+}\mathcal{W}_{s}{}^{\perp t}$
${}^{0^+}\mathcal{A}_{s}{}^{\parallel}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}^{\alpha\beta}{}_{\beta}\ n_{\alpha} + \mathcal{A}^{\alpha}{}_{\alpha}{}^{\beta}\ n_{\beta} + \mathcal{A}^{\alpha\beta}{}_{\alpha}\ n_{\beta} - 3\ \mathcal{A}^{\alpha\beta\chi}\ n_{\alpha}\ n_{\beta}\ n_{\chi}$	${}^{0^+}\mathcal{W}_{s}{}^{\parallel}$
${}^{0^+}\mathcal{A}_{s}{}^{\perp h}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}^{\alpha\beta}{}_{\beta}\ n_{\alpha} - \frac{1}{2}\ \mathcal{A}^{\alpha}{}_{\alpha}{}^{\beta}\ n_{\beta} - \frac{1}{2}\ \mathcal{A}^{\alpha\beta}{}_{\alpha}\ n_{\beta}$	${}^{0^+}\mathcal{W}_{s}{}^{\perp h}$
${}^{0^-}\mathcal{A}_{a}{}^{\parallel}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}^{\alpha\beta\chi}\ \epsilon\eta_{\alpha\beta\chi\delta}\ n^{\delta}$	${}^{0^-}\mathcal{W}_{a}{}^{\parallel}$
${}^{1^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\beta}$	StrongGenSet[{1,2},GenSet[-(1,2)]]	$\frac{1}{4}\ \mathcal{A}_{\alpha\beta}{}^{\chi}\ n_{\chi} - \frac{1}{4}\ \mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\ n_{\chi} - \frac{1}{4}\ \mathcal{A}_{\beta\alpha}{}^{\chi}\ n_{\chi} + \frac{1}{4}\ \mathcal{A}_{\beta}{}^{\chi}{}_{\alpha}\ n_{\chi} - \frac{1}{4}\ \mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} + \frac{1}{4}\ \mathcal{A}^{\chi\delta}{}_{\beta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} + \frac{1}{4}\ \mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\ n_{\beta}\ n_{\chi}\ n_{\delta} - \frac{1}{4}\ \mathcal{A}^{\chi\delta}{}_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}$	${}^{1^+}\mathcal{W}_{a}{}^{\parallel}{}_{\alpha\beta}$
${}^{1^+}\mathcal{A}_{a}{}^{\perp}{}_{\alpha\beta}$	StrongGenSet[{1,2},GenSet[-(1,2)]]	$\frac{1}{2}\ \mathcal{A}^{\chi}{}_{\alpha\beta}\ n_{\chi} - \frac{1}{2}\ \mathcal{A}^{\chi}{}_{\beta\alpha}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} - \frac{1}{2}\ \mathcal{A}^{\chi\delta}{}_{\beta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} - \frac{1}{2}\ \mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\ n_{\beta}\ n_{\chi}\ n_{\delta} + \frac{1}{2}\ \mathcal{A}^{\chi\delta}{}_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}$	${}^{1^+}\mathcal{W}_{a}{}^{\perp}{}_{\alpha\beta}$
${}^{1^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\beta}$	StrongGenSet[{1,2 }, GenSet[-(1,2)]] -	$\frac{1}{2}\ \mathcal{A}_{\alpha\beta}{}^{\chi}\ n_{\chi} - \frac{1}{2}\ \mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}_{\beta\alpha}{}^{\chi}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}_{\beta}{}^{\chi}{}_{\alpha}\ n_{\chi} - \mathcal{A}_{\beta}{}^{\chi\delta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} + \frac{1}{2}\ \mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} + \frac{1}{2}\ \mathcal{A}^{\chi\delta}{}_{\beta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} + \mathcal{A}_{\alpha}{}^{\chi\delta}\ n_{\beta}\ n_{\chi}\ n_{\delta} - \frac{1}{2}\ \mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\ n_{\beta}\ n_{\chi}\ n_{\delta} - \frac{1}{2}\ \mathcal{A}^{\chi\delta}{}_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}$	${}^{1^+}\mathcal{W}_{s}{}^{\perp}{}_{\alpha\beta}$
${}^{1^-}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha}$	StrongGenSet[{}, GenSet[]] -	$\frac{1}{2}\ \mathcal{A}^{\beta}{}_{\alpha\beta} + \frac{1}{2}\ \mathcal{A}^{\beta}{}_{\beta\alpha} - \frac{1}{2}\ \mathcal{A}^{\beta}{}_{\beta}{}^{\chi}\ n_{\alpha}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}^{\beta\chi}{}_{\beta}\ n_{\alpha}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\ n_{\beta}\ n_{\chi} - \frac{1}{2}\ \mathcal{A}^{\beta\chi}{}_{\alpha}\ n_{\beta}\ n_{\chi}$	${}^{1^-}\mathcal{W}_{a}{}^{\parallel}{}_{\alpha}$
${}^{1^-}\mathcal{A}_{a}{}^{\perp}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$\frac{1}{2}\ \mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\ n_{\beta}\ n_{\chi} - \frac{1}{2}\ \mathcal{A}^{\beta\chi}{}_{\alpha}\ n_{\beta}\ n_{\chi}$	${}^{1^-}\mathcal{W}_{a}{}^{\perp}{}_{\alpha}$
${}^{1^-}\mathcal{A}_{s}{}^{\perp t}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}_{\alpha}{}^{\beta\chi}\ n_{\beta}\ n_{\chi} + \mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\ n_{\beta}\ n_{\chi} + \mathcal{A}^{\beta\chi}{}_{\alpha}\ n_{\beta}\ n_{\chi} - 3\ \mathcal{A}^{\beta\chi\delta}\ n_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}$	${}^{1^-}\mathcal{W}_{s}{}^{\perp t}{}_{\alpha}$
${}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}_{\alpha}{}^{\beta}{}_{\beta} + \mathcal{A}^{\beta}{}_{\alpha\beta} + \mathcal{A}^{\beta}{}_{\beta\alpha} - \mathcal{A}^{\beta\chi}{}_{\chi}\ n_{\alpha}\ n_{\beta} - \mathcal{A}^{\beta}{}_{\beta}{}^{\chi}\ n_{\alpha}\ n_{\chi} - \mathcal{A}^{\beta\chi}{}_{\beta}\ n_{\alpha}\ n_{\chi} - \mathcal{A}_{\alpha}{}^{\beta\chi}\ n_{\beta}\ n_{\chi} - \mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\ n_{\beta}\ n_{\chi} - \mathcal{A}^{\beta\chi}{}_{\alpha}\ n_{\beta}\ n_{\chi} + 3\ \mathcal{A}^{\beta\chi\delta}\ n_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}$	${}^{1^-}\mathcal{W}_{s}{}^{\parallel t}{}_{\alpha}$
${}^{1^-}\mathcal{A}_{s}{}^{\perp h}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}_{\alpha}{}^{\beta\chi}\ n_{\beta}\ n_{\chi} - \frac{1}{2}\ \mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\ n_{\beta}\ n_{\chi} - \frac{1}{2}\ \mathcal{A}^{\beta\chi}{}_{\alpha}\ n_{\beta}\ n_{\chi}$	${}^{1^-}\mathcal{W}_{s}{}^{\perp h}{}_{\alpha}$
${}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}_{\alpha}{}^{\beta}{}_{\beta} - \frac{1}{2}\ \mathcal{A}^{\beta}{}_{\alpha\beta} - \frac{1}{2}\ \mathcal{A}^{\beta}{}_{\beta\alpha} - \mathcal{A}^{\beta\chi}{}_{\chi}\ n_{\alpha}\ n_{\beta} + \frac{1}{2}\ \mathcal{A}^{\beta}{}_{\beta}{}^{\chi}\ n_{\alpha}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}^{\beta\chi}{}_{\beta}\ n_{\alpha}\ n_{\chi} - \mathcal{A}_{\alpha}{}^{\beta\chi}\ n_{\beta}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\ n_{\beta}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}^{\beta\chi}{}_{\alpha}\ n_{\beta}\ n_{\chi}$	${}^{1^-}\mathcal{W}_{s}{}^{\parallel h}{}_{\alpha}$
${}^{2^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\beta}$	StrongGenSet[{1, 2}, GenSet[(1,2)]] -	$\frac{1}{4}\ \mathcal{A}_{\alpha\beta}{}^{\chi}\ n_{\chi} + \frac{1}{4}\ \mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\ n_{\chi} - \frac{1}{4}\ \mathcal{A}_{\beta\alpha}{}^{\chi}\ n_{\chi} + \frac{1}{4}\ \mathcal{A}_{\beta}{}^{\chi}{}_{\alpha}\ n_{\chi} + \frac{1}{6}\ \mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\ \eta_{\alpha\beta}\ n_{\delta} - \frac{1}{6}\ \mathcal{A}^{\chi\delta}{}_{\chi}\ \eta_{\alpha\beta}\ n_{\delta} - \frac{1}{6}\ \mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\ n_{\alpha}\ n_{\beta}\ n_{\delta} + \frac{1}{6}\ \mathcal{A}^{\chi\delta}{}_{\chi}\ n_{\alpha}\ n_{\beta}\ n_{\delta} + \frac{1}{4}\ \mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} - \frac{1}{4}\ \mathcal{A}^{\chi\delta}{}_{\beta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} + \frac{1}{4}\ \mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\ n_{\beta}\ n_{\chi}\ n_{\delta} - \frac{1}{4}\ \mathcal{A}^{\chi\delta}{}_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}$	${}^{2^+}\mathcal{W}_{a}{}^{\parallel}{}_{\alpha\beta}$
${}^{2^+}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta}$	StrongGenSet[{1,2},GenSet[(1,2)]]	$\frac{1}{2}\ \mathcal{A}_{\alpha\beta}{}^{\chi}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}_{\beta\alpha}{}^{\chi}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}_{\beta}{}^{\chi}{}_{\alpha}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}^{\chi}{}_{\alpha\beta}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}^{\chi}{}_{\beta\alpha}\ n_{\chi} - \frac{1}{3}\ \mathcal{A}^{\chi\delta}{}_{\delta}\ \eta_{\alpha\beta}\ n_{\chi} + \frac{1}{3}\ \mathcal{A}^{\chi\delta}{}_{\delta}\ n_{\alpha}\ n_{\beta}\ n_{\chi} - \frac{1}{3}\ \mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\ \eta_{\alpha\beta}\ n_{\delta} - \frac{1}{3}\ \mathcal{A}^{\chi\delta}{}_{\chi}\ \eta_{\alpha\beta}\ n_{\delta} + \frac{1}{3}\ \mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\ n_{\alpha}\ n_{\beta}\ n_{\delta} + \frac{1}{3}\ \mathcal{A}^{\chi\delta}{}_{\chi}\ n_{\alpha}\ n_{\beta}\ n_{\delta} - \mathcal{A}_{\beta}{}^{\chi\delta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} - \mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} - \mathcal{A}^{\chi\delta}{}_{\beta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} - \mathcal{A}_{\alpha}{}^{\chi\delta}\ n_{\beta}\ n_{\chi}\ n_{\delta} - \mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\ n_{\beta}\ n_{\chi}\ n_{\delta} - \mathcal{A}^{\chi\delta}{}_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta} + \mathcal{A}^{\chi\delta\epsilon}\ \eta_{\alpha\beta}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + 2\ \mathcal{A}^{\chi\delta\epsilon}\ n_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}\ n_{\epsilon}$	${}^{2^+}\mathcal{W}_{s}{}^{\parallel}{}_{\alpha\beta}$
${}^{2^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\beta}$	StrongGenSet[{1, 2}, GenSet[(1,2)]] -	$\frac{1}{4}\ \mathcal{A}_{\alpha\beta}{}^{\chi}\ n_{\chi} - \frac{1}{4}\ \mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\ n_{\chi} - \frac{1}{4}\ \mathcal{A}_{\beta\alpha}{}^{\chi}\ n_{\chi} - \frac{1}{4}\ \mathcal{A}_{\beta}{}^{\chi}{}_{\alpha}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}^{\chi}{}_{\alpha\beta}\ n_{\chi} + \frac{1}{2}\ \mathcal{A}^{\chi}{}_{\beta\alpha}\ n_{\chi} - \frac{1}{3}\ \mathcal{A}^{\chi\delta}{}_{\delta}\ \eta_{\alpha\beta}\ n_{\chi} + \frac{1}{3}\ \mathcal{A}^{\chi\delta}{}_{\delta}\ n_{\alpha}\ n_{\beta}\ n_{\chi} + \frac{1}{6}\ \mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\ \eta_{\alpha\beta}\ n_{\delta} + \frac{1}{6}\ \mathcal{A}^{\chi\delta}{}_{\chi}\ \eta_{\alpha\beta}\ n_{\delta} - \frac{1}{6}\ \mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\ n_{\alpha}\ n_{\beta}\ n_{\delta} - \frac{1}{6}\ \mathcal{A}^{\chi\delta}{}_{\chi}\ n_{\alpha}\ n_{\beta}\ n_{\delta} + \frac{1}{2}\ \mathcal{A}_{\beta}{}^{\chi\delta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} - \frac{1}{4}\ \mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} - \frac{1}{4}\ \mathcal{A}^{\chi\delta}{}_{\beta}\ n_{\alpha}\ n_{\chi}\ n_{\delta} + \frac{1}{2}\ \mathcal{A}_{\alpha}{}^{\chi\delta}\ n_{\beta}\ n_{\chi}\ n_{\delta} - \frac{1}{4}\ \mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\ n_{\beta}\ n_{\chi}\ n_{\delta} - \frac{1}{4}\ \mathcal{A}^{\chi\delta}{}_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}$	${}^{2^+}\mathcal{W}_{s}{}^{\perp}{}_{\alpha\beta}$
${}^{2^-}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\beta\chi}$	StrongGenSet[{1, 2}, GenSet[-(1,2)]] -	$\frac{1}{8}\ \mathcal{A}_{\alpha\beta\chi} + \frac{1}{8}\ \mathcal{A}_{\alpha\chi\beta} + \frac{1}{8}\ \mathcal{A}_{\beta\alpha\chi} - \frac{1}{8}\ \mathcal{A}_{\beta\chi\alpha} + \frac{1}{4}\ \mathcal{A}_{\chi\alpha\beta} - \frac{1}{4}\ \mathcal{A}_{\chi\beta\alpha} + \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\beta\delta}\ \eta_{\alpha\chi} - \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\delta\beta}\ \eta_{\alpha\chi} - \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\alpha\delta}\ \eta_{\beta\chi} + \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\delta\alpha}\ \eta_{\beta\chi} - \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\beta\delta}\ n_{\alpha}\ n_{\chi} + \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\delta\beta}\ n_{\alpha}\ n_{\chi} + \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\alpha\delta}\ n_{\beta}\ n_{\chi} - \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\delta\alpha}\ n_{\beta}\ n_{\chi} + \frac{1}{8}\ \mathcal{A}_{\beta\chi}{}^{\delta}\ n_{\alpha}\ n_{\delta} - \frac{1}{8}\ \mathcal{A}_{\beta}{}^{\delta}{}_{\chi}\ n_{\alpha}\ n_{\delta} + \frac{1}{4}\ \mathcal{A}_{\chi\beta}{}^{\delta}\ n_{\alpha}\ n_{\delta} - \frac{1}{4}\ \mathcal{A}_{\chi}{}^{\delta}{}_{\beta}\ n_{\alpha}\ n_{\delta} + \frac{1}{8}\ \mathcal{A}^{\delta}{}_{\beta\chi}\ n_{\alpha}\ n_{\delta} - \frac{1}{8}\ \mathcal{A}^{\delta}{}_{\chi\beta}\ n_{\alpha}\ n_{\delta} - \frac{1}{8}\ \mathcal{A}_{\alpha\chi}{}^{\delta}\ n_{\beta}\ n_{\delta} + \frac{1}{8}\ \mathcal{A}_{\alpha}{}^{\delta}{}_{\chi}\ n_{\beta}\ n_{\delta} - \frac{1}{4}\ \mathcal{A}_{\chi\alpha}{}^{\delta}\ n_{\beta}\ n_{\delta} + \frac{1}{4}\ \mathcal{A}_{\chi}{}^{\delta}{}_{\alpha}\ n_{\beta}\ n_{\delta} - \frac{1}{8}\ \mathcal{A}^{\delta}{}_{\alpha\chi}\ n_{\beta}\ n_{\delta} + \frac{1}{8}\ \mathcal{A}^{\delta}{}_{\chi\alpha}\ n_{\beta}\ n_{\delta} + \frac{1}{8}\ \mathcal{A}_{\alpha\beta}{}^{\delta}\ n_{\chi}\ n_{\delta} - \frac{1}{8}\ \mathcal{A}_{\alpha}{}^{\delta}{}_{\beta}\ n_{\chi}\ n_{\delta} - \frac{1}{8}\ \mathcal{A}_{\beta\alpha}{}^{\delta}\ n_{\chi}\ n_{\delta} + \frac{1}{8}\ \mathcal{A}_{\beta}{}^{\delta}{}_{\alpha}\ n_{\chi}\ n_{\delta} - \frac{1}{4}\ \mathcal{A}^{\delta}{}_{\alpha\beta}\ n_{\chi}\ n_{\delta} + \frac{1}{4}\ \mathcal{A}^{\delta}{}_{\beta\alpha}\ n_{\chi}\ n_{\delta} - \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\delta}{}^{\epsilon}\ \eta_{\beta\chi}\ n_{\alpha}\ n_{\epsilon} + \frac{3}{16}\ \mathcal{A}^{\delta\epsilon}{}_{\delta}\ \eta_{\beta\chi}\ n_{\alpha}\ n_{\epsilon} + \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\delta}{}^{\epsilon}\ \eta_{\alpha\chi}\ n_{\beta}\ n_{\epsilon} - \frac{3}{16}\ \mathcal{A}^{\delta\epsilon}{}_{\delta}\ \eta_{\alpha\chi}\ n_{\beta}\ n_{\epsilon} - \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\beta}{}^{\epsilon}\ \eta_{\alpha\chi}\ n_{\delta}\ n_{\epsilon} + \frac{3}{16}\ \mathcal{A}^{\delta\epsilon}{}_{\beta}\ \eta_{\alpha\chi}\ n_{\delta}\ n_{\epsilon} + \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\alpha}{}^{\epsilon}\ \eta_{\beta\chi}\ n_{\delta}\ n_{\epsilon} - \frac{3}{16}\ \mathcal{A}^{\delta\epsilon}{}_{\alpha}\ \eta_{\beta\chi}\ n_{\delta}\ n_{\epsilon} - \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\beta}{}^{\epsilon}\ n_{\alpha}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + \frac{3}{16}\ \mathcal{A}^{\delta\epsilon}{}_{\beta}\ n_{\alpha}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + \frac{3}{16}\ \mathcal{A}^{\delta}{}_{\alpha}{}^{\epsilon}\ n_{\beta}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} - \frac{3}{16}\ \mathcal{A}^{\delta\epsilon}{}_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}\ n_{\epsilon}$	${}^{2^-}\mathcal{W}_{a}{}^{\parallel}{}_{\alpha\beta\chi}$
${}^{2^-}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta\chi}$	StrongGenSet[{1,2},GenSet[-(1,2)]]	$\frac{1}{3}\ \mathcal{A}_{\alpha\beta\chi} + \frac{1}{3}\ \mathcal{A}_{\alpha\chi\beta} - \frac{1}{3}\ \mathcal{A}_{\beta\alpha\chi} - \frac{1}{3}\ \mathcal{A}_{\beta\chi\alpha} + \frac{1}{3}\ \mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\ \eta_{\alpha\chi} - \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\beta\delta}\ \eta_{\alpha\chi} - \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\delta\beta}\ \eta_{\alpha\chi} - \frac{1}{3}\ \mathcal{A}_{\alpha}{}^{\delta}{}_{\delta}\ \eta_{\beta\chi} + \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\alpha\delta}\ \eta_{\beta\chi} + \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\delta\alpha}\ \eta_{\beta\chi} - \frac{1}{3}\ \mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\ n_{\alpha}\ n_{\chi} + \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\beta\delta}\ n_{\alpha}\ n_{\chi} + \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\delta\beta}\ n_{\alpha}\ n_{\chi} + \frac{1}{3}\ \mathcal{A}_{\alpha}{}^{\delta}{}_{\delta}\ n_{\beta}\ n_{\chi} - \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\alpha\delta}\ n_{\beta}\ n_{\chi} - \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\delta\alpha}\ n_{\beta}\ n_{\chi} + \frac{1}{3}\ \mathcal{A}_{\beta\chi}{}^{\delta}\ n_{\alpha}\ n_{\delta} + \frac{1}{3}\ \mathcal{A}_{\beta}{}^{\delta}{}_{\chi}\ n_{\alpha}\ n_{\delta} - \frac{1}{3}\ \mathcal{A}^{\delta}{}_{\beta\chi}\ n_{\alpha}\ n_{\delta} - \frac{1}{3}\ \mathcal{A}^{\delta}{}_{\chi\beta}\ n_{\alpha}\ n_{\delta} + \frac{1}{3}\ \mathcal{A}^{\delta\epsilon}{}_{\epsilon}\ \eta_{\beta\chi}\ n_{\alpha}\ n_{\delta} - \frac{1}{3}\ \mathcal{A}_{\alpha\chi}{}^{\delta}\ n_{\beta}\ n_{\delta} - \frac{1}{3}\ \mathcal{A}_{\alpha}{}^{\delta}{}_{\chi}\ n_{\beta}\ n_{\delta} + \frac{1}{3}\ \mathcal{A}^{\delta}{}_{\alpha\chi}\ n_{\beta}\ n_{\delta} + \frac{1}{3}\ \mathcal{A}^{\delta}{}_{\chi\alpha}\ n_{\beta}\ n_{\delta} - \frac{1}{3}\ \mathcal{A}^{\delta\epsilon}{}_{\epsilon}\ \eta_{\alpha\chi}\ n_{\beta}\ n_{\delta} - \frac{1}{3}\ \mathcal{A}_{\alpha\beta}{}^{\delta}\ n_{\chi}\ n_{\delta} - \frac{1}{3}\ \mathcal{A}_{\alpha}{}^{\delta}{}_{\beta}\ n_{\chi}\ n_{\delta} + \frac{1}{3}\ \mathcal{A}_{\beta\alpha}{}^{\delta}\ n_{\chi}\ n_{\delta} + \frac{1}{3}\ \mathcal{A}_{\beta}{}^{\delta}{}_{\alpha}\ n_{\chi}\ n_{\delta} - \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\delta}{}^{\epsilon}\ \eta_{\beta\chi}\ n_{\alpha}\ n_{\epsilon} - \frac{1}{6}\ \mathcal{A}^{\delta\epsilon}{}_{\delta}\ \eta_{\beta\chi}\ n_{\alpha}\ n_{\epsilon} + \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\delta}{}^{\epsilon}\ \eta_{\alpha\chi}\ n_{\beta}\ n_{\epsilon} + \frac{1}{6}\ \mathcal{A}^{\delta\epsilon}{}_{\delta}\ \eta_{\alpha\chi}\ n_{\beta}\ n_{\epsilon} - \frac{1}{3}\ \mathcal{A}_{\beta}{}^{\delta\epsilon}\ \eta_{\alpha\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\beta}{}^{\epsilon}\ \eta_{\alpha\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{6}\ \mathcal{A}^{\delta\epsilon}{}_{\beta}\ \eta_{\alpha\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{3}\ \mathcal{A}_{\alpha}{}^{\delta\epsilon}\ \eta_{\beta\chi}\ n_{\delta}\ n_{\epsilon} - \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\alpha}{}^{\epsilon}\ \eta_{\beta\chi}\ n_{\delta}\ n_{\epsilon} - \frac{1}{6}\ \mathcal{A}^{\delta\epsilon}{}_{\alpha}\ \eta_{\beta\chi}\ n_{\delta}\ n_{\epsilon} - \frac{1}{3}\ \mathcal{A}_{\beta}{}^{\delta\epsilon}\ n_{\alpha}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\beta}{}^{\epsilon}\ n_{\alpha}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{6}\ \mathcal{A}^{\delta\epsilon}{}_{\beta}\ n_{\alpha}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{3}\ \mathcal{A}_{\alpha}{}^{\delta\epsilon}\ n_{\beta}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} - \frac{1}{6}\ \mathcal{A}^{\delta}{}_{\alpha}{}^{\epsilon}\ n_{\beta}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} - \frac{1}{6}\ \mathcal{A}^{\delta\epsilon}{}_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}\ n_{\epsilon}$	${}^{2^-}\mathcal{W}_{s}{}^{\parallel}{}_{\alpha\beta\chi}$
${}^{3^-}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta\chi}$	StrongGenSet[{1, 2, 3}, GenSet[(1,2), (2,3)]]	$\frac{1}{6}\ \mathcal{A}_{\alpha\beta\chi} + \frac{1}{6}\ \mathcal{A}_{\alpha\chi\beta} + \frac{1}{6}\ \mathcal{A}_{\beta\alpha\chi} + \frac{1}{6}\ \mathcal{A}_{\beta\chi\alpha} + \frac{1}{6}\ \mathcal{A}_{\chi\alpha\beta} + \frac{1}{6}\ \mathcal{A}_{\chi\beta\alpha} - \frac{1}{15}\ \mathcal{A}_{\chi}{}^{\delta}{}_{\delta}\ \eta_{\alpha\beta} - \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\chi\delta}\ \eta_{\alpha\beta} - \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\delta\chi}\ \eta_{\alpha\beta} - \frac{1}{15}\ \mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\ \eta_{\alpha\chi} - \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\beta\delta}\ \eta_{\alpha\chi} - \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\delta\beta}\ \eta_{\alpha\chi} - \frac{1}{15}\ \mathcal{A}_{\alpha}{}^{\delta}{}_{\delta}\ \eta_{\beta\chi} - \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\alpha\delta}\ \eta_{\beta\chi} - \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\delta\alpha}\ \eta_{\beta\chi} + \frac{1}{15}\ \mathcal{A}_{\chi}{}^{\delta}{}_{\delta}\ n_{\alpha}\ n_{\beta} + \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\chi\delta}\ n_{\alpha}\ n_{\beta} + \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\delta\chi}\ n_{\alpha}\ n_{\beta} + \frac{1}{15}\ \mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\ n_{\alpha}\ n_{\chi} + \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\beta\delta}\ n_{\alpha}\ n_{\chi} + \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\delta\beta}\ n_{\alpha}\ n_{\chi} + \frac{1}{15}\ \mathcal{A}_{\alpha}{}^{\delta}{}_{\delta}\ n_{\beta}\ n_{\chi} + \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\alpha\delta}\ n_{\beta}\ n_{\chi} + \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\delta\alpha}\ n_{\beta}\ n_{\chi} - 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\frac{1}{5}\ \mathcal{A}^{\delta}{}_{\delta}{}^{\epsilon}\ n_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\epsilon} - \frac{1}{5}\ \mathcal{A}^{\delta\epsilon}{}_{\delta}\ n_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\epsilon} + \frac{1}{15}\ \mathcal{A}_{\chi}{}^{\delta\epsilon}\ \eta_{\alpha\beta}\ n_{\delta}\ n_{\epsilon} + \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\chi}{}^{\epsilon}\ \eta_{\alpha\beta}\ n_{\delta}\ n_{\epsilon} + \frac{1}{15}\ \mathcal{A}^{\delta\epsilon}{}_{\chi}\ \eta_{\alpha\beta}\ n_{\delta}\ n_{\epsilon} + \frac{1}{15}\ \mathcal{A}_{\beta}{}^{\delta\epsilon}\ \eta_{\alpha\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\beta}{}^{\epsilon}\ \eta_{\alpha\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{15}\ \mathcal{A}^{\delta\epsilon}{}_{\beta}\ \eta_{\alpha\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{15}\ \mathcal{A}_{\alpha}{}^{\delta\epsilon}\ \eta_{\beta\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{15}\ \mathcal{A}^{\delta}{}_{\alpha}{}^{\epsilon}\ \eta_{\beta\chi}\ n_{\delta}\ n_{\epsilon} + \frac{1}{15}\ \mathcal{A}^{\delta\epsilon}{}_{\alpha}\ \eta_{\beta\chi}\ n_{\delta}\ n_{\epsilon} + \frac{4}{15}\ \mathcal{A}_{\chi}{}^{\delta\epsilon}\ n_{\alpha}\ n_{\beta}\ n_{\delta}\ n_{\epsilon} + \frac{4}{15}\ \mathcal{A}^{\delta}{}_{\chi}{}^{\epsilon}\ n_{\alpha}\ n_{\beta}\ n_{\delta}\ n_{\epsilon} + \frac{4}{15}\ \mathcal{A}^{\delta\epsilon}{}_{\chi}\ n_{\alpha}\ n_{\beta}\ n_{\delta}\ n_{\epsilon} + \frac{4}{15}\ \mathcal{A}_{\beta}{}^{\delta\epsilon}\ n_{\alpha}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + \frac{4}{15}\ \mathcal{A}^{\delta}{}_{\beta}{}^{\epsilon}\ n_{\alpha}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + \frac{4}{15}\ \mathcal{A}^{\delta\epsilon}{}_{\beta}\ n_{\alpha}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + \frac{4}{15}\ \mathcal{A}_{\alpha}{}^{\delta\epsilon}\ n_{\beta}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + \frac{4}{15}\ \mathcal{A}^{\delta}{}_{\alpha}{}^{\epsilon}\ n_{\beta}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} + \frac{4}{15}\ \mathcal{A}^{\delta\epsilon}{}_{\alpha}\ n_{\beta}\ n_{\chi}\ n_{\delta}\ n_{\epsilon} - 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FIG. SM1. Kinematic structure of metric-affine gravity. The SO(3) irreps have a different labelling of duplicate J^P states to those in [35]. Note that the 2^- and 3^- states have a hidden multi-term cyclic symmetry on all indices, which is not accommodated by the C language implementation of the Butler-Portugal algorithm. These definitions are used in Figs. SM2 to SM6.

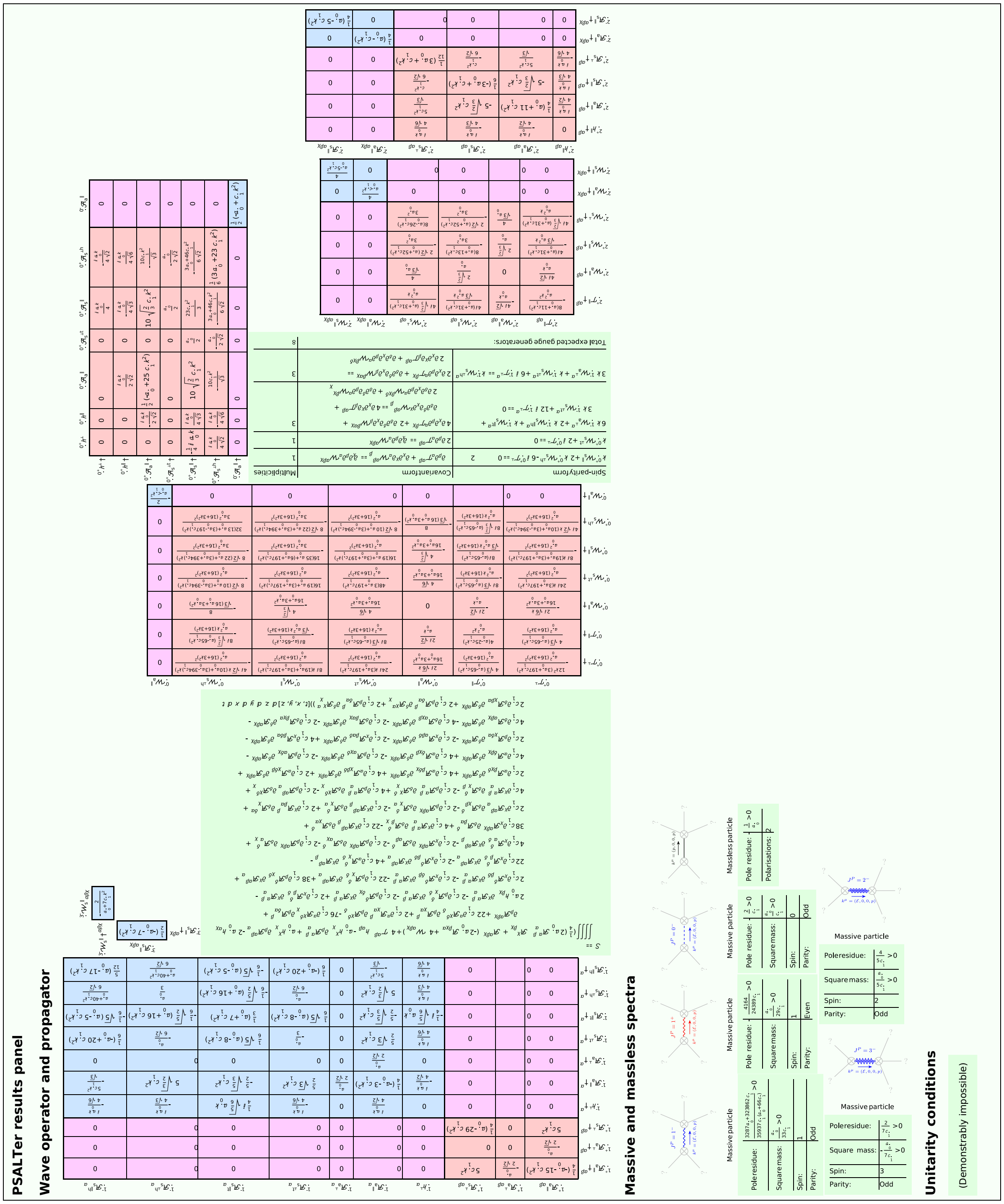

FIG. SM2. Particle spectrum of the (arbitrary) projective-invariant theory defined in Eq. (7), in the first-order formulation, where a_0 corresponds to $-M_{\mathrm{P}}{}^2/2$ and c_1 corresponds to α. Note that there are eight gauge generators in total: these correspond to the d.o.f in the diffeomorphism and projective vector generators. The massive spectrum is shown in Table I, and we note in addition the presence of the massless graviton, whose no-ghost condition requires $a_0 < 0$. All the quantities in this output are defined in Fig. SM1.

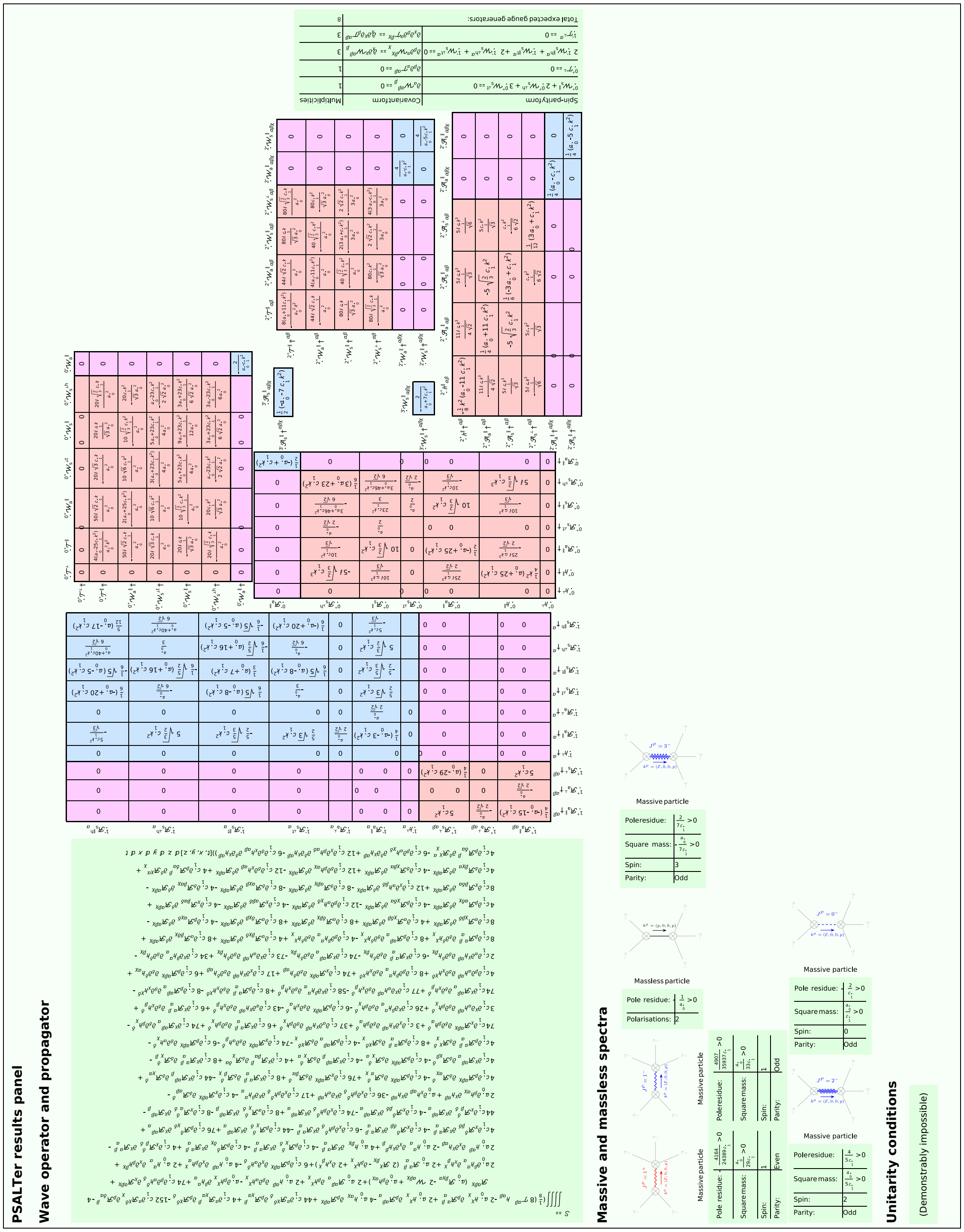

FIG. SM3. The results in Fig. SM2 repeated in the second-order formulation. Note that the quadratic action in this case contains very many more operators than does Fig. SM2. The matrix elements and the forms of the source constraints and pole residues are expected to change, but the mass spectrum and overall (non-)unitarity is the same. All the quantities in this output are defined in Fig. SM1.

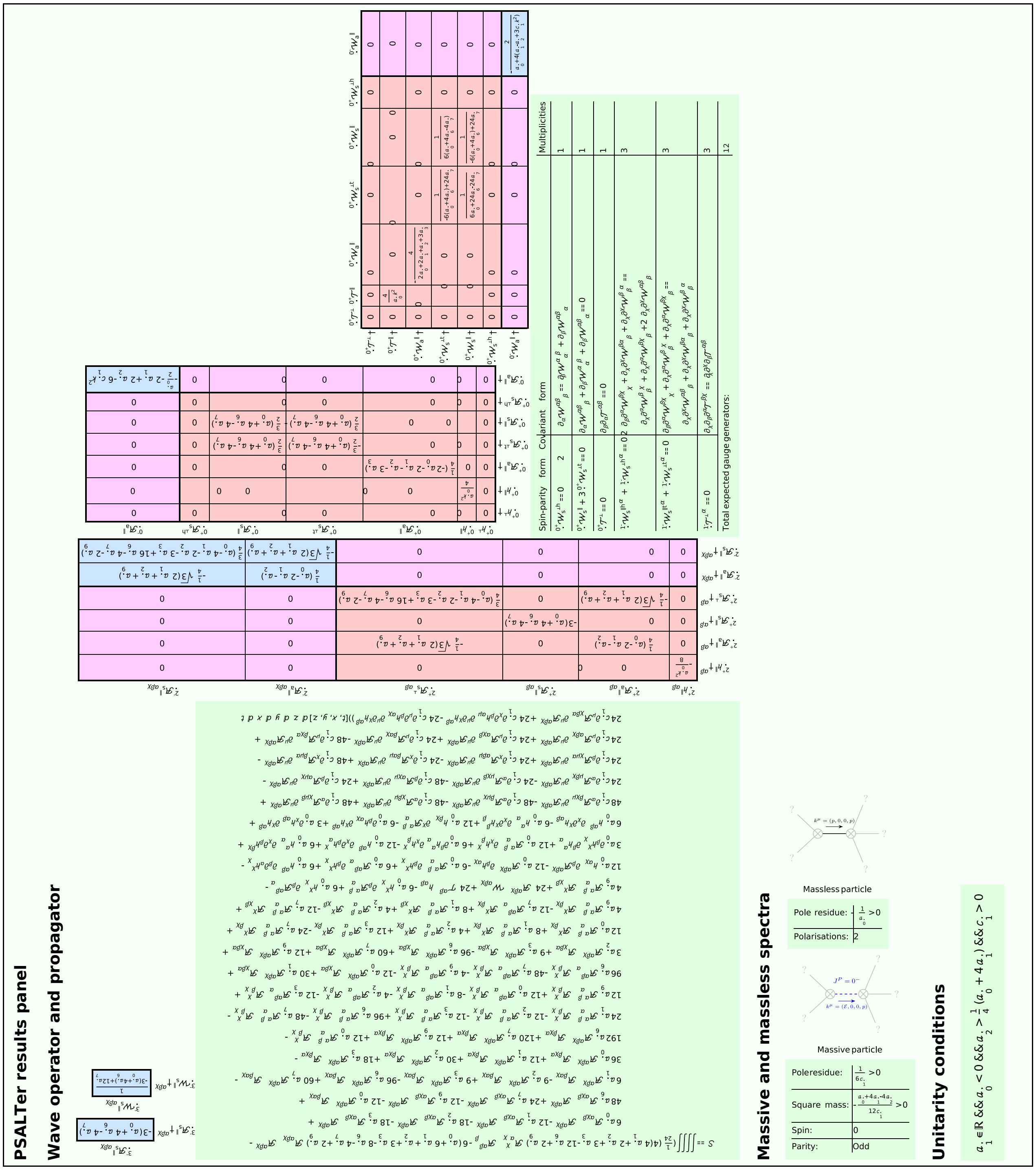

FIG. SM4. Particle spectrum of the (general) EP-invariant theory defined in Eq. (11), in the second-order formulation, where a_0 corresponds to $-M_{\text{P}}{}^2/2$ and c_1 corresponds to α. The parameters b_1, b_2 and b_3 are represented among the many remaining a_i, but in our linearisation we are not careful to eliminate the pure-tensor $t^{\mu}{}_{\nu\sigma}$ and $q^{\mu}{}_{\nu\sigma}$ sector. Note that we now have 12 gauge generators: diffeomorphisms and the two vector generators of the EP symmetry. In the particle spectrum, we see the graviton and the pseudoscalar in Eq. (14a). By linearising Eq. (14b) and carefully matching the parameters we could in principle recover the square of the pseudoscalar mass — in any case we can see that α and c_1 appear in the denominator as expected. The entire theory may be made unitary. All the quantities in this output are defined in Fig. SM1.

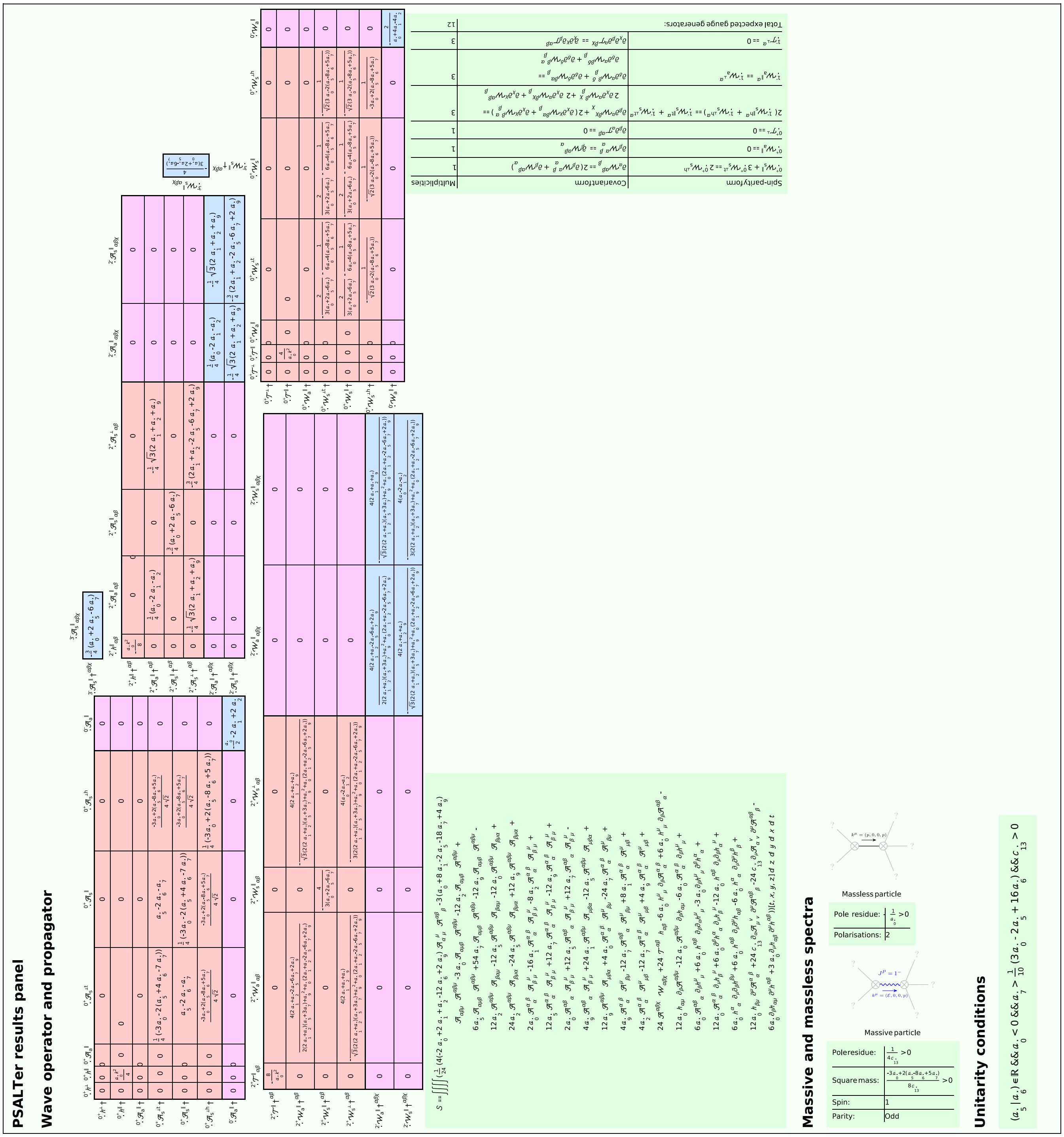

FIG. SM5. Particle spectrum of the (general) IW-invariant theory defined in Eq. (C1), in the second-order formulation, where a_0 corresponds to $-M_{\rm P}{}^2/2$ and c_{13} corresponds to α. The parameters b_1, b_2 and b_3 are again over-represented as in Fig. SM4. Note that we again have 12 gauge generators: diffeomorphisms and the two vector generators of the IW symmetry. In the particle spectrum, we see the graviton and the massive (Weyl) Proca field Q_μ. Again, we expect to recover the mass in Eq. (C2). The entire theory may be made unitary. All the quantities in this output are defined in Fig. SM1.

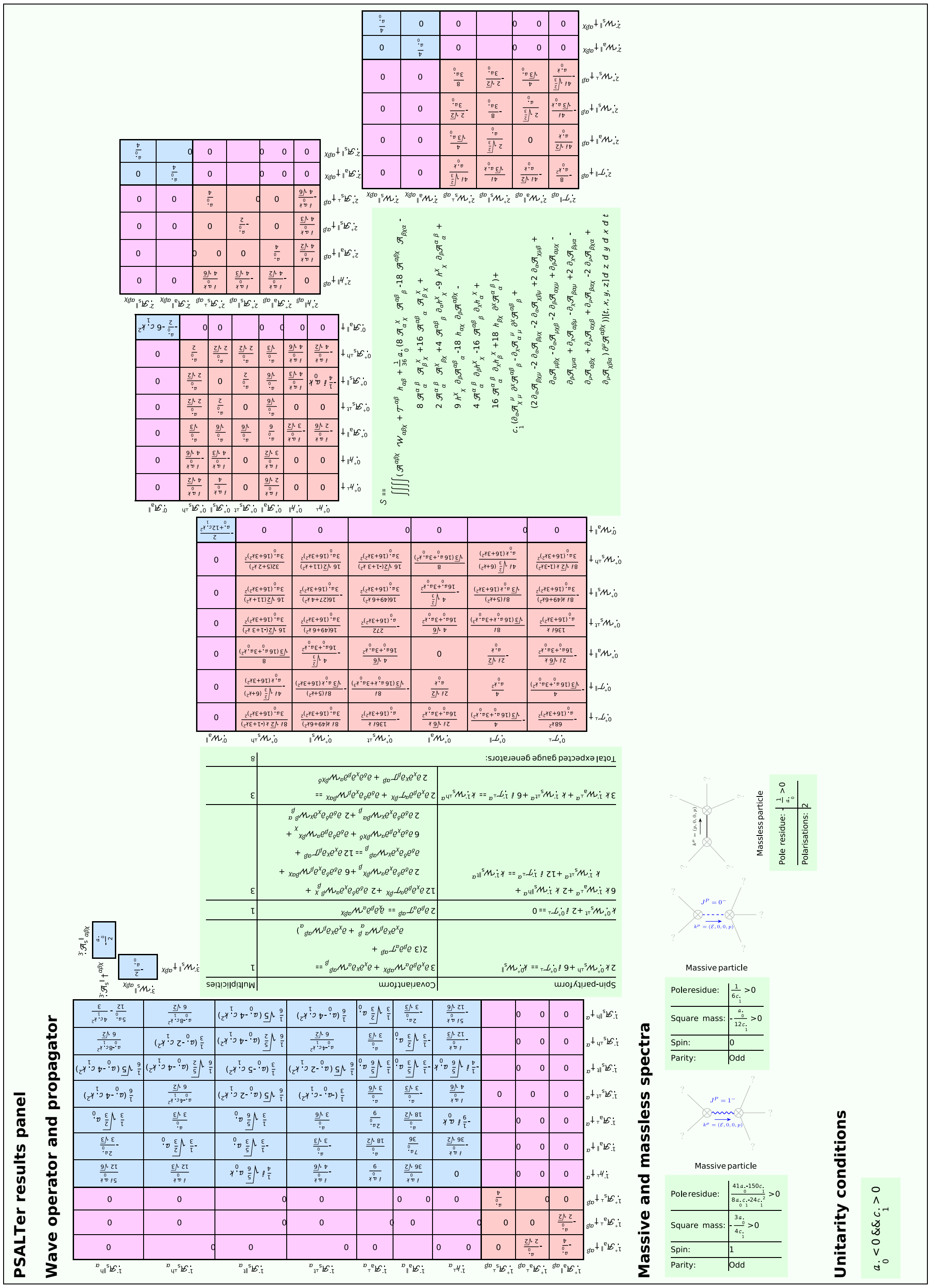

FIG. SM6. Particle spectrum of an arbitrarily-tuned concurrent-invariant theory, derived from the general theory defined in Eq. (C4), in the first-order formulation, where a_0 corresponds to $-M_{\mathrm{P}}{}^2/2$ and c_1 corresponds to α and $\tilde{\alpha}$, and the remaining dimensionful couplings are set arbitrarily to a_0. Note that we have only eight gauge generators: diffeomorphisms and the vector generating the concurrent transformations. In the particle spectrum, we see the graviton is joined by both the massive (Weyl) Proca field in Fig. SM5 and the pseudoscalar in Fig. SM4. Again, we expect to recover the mass in Eq. (C5). The entire theory may be made unitary. All the quantities in this output are defined in Fig. SM1.

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Connecting to external linux executable...
Connection established.
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Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}
CopyRight (C) 2002-2021, Jose M. Martin-Garcia, under the General Public License.
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Package xAct`xPlain` version 0.0.0-developer, {2024, 3, 23}
CopyRight © 2023, Will E. V. Barker and Sebastian Zell, under the General Public License.
------------------------------------------------------------
These packages come with ABSOLUTELY NO WARRANTY; for details type Disclaimer[]. This is free software, and
  you are welcome to redistribute it under certain conditions. See the General Public License for details.
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```

The generality of the EP, IW and concurrent theories

Concrete relation to manuscript: In this script we verify our claims that the actions in Eqs. (11), (C2) an (C4) in our manuscript contain the most general terms which we can write down when we consider operators quadratic in the metric-affine curvature and its contractions.

We will load packages xTensor and xTras.

```
------------------------------------------------------------
Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}
CopyRight (C) 2002-2021, Jose M. Martin-Garcia, under the General Public License.
------------------------------------------------------------
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------------------------------------------------------------
------------------------------------------------------------
Package xAct`xPert`  version 1.0.6, {2018, 2, 28}
CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia and Guillermo A. Mena Marugan, under the General Public License.
** Variable $PrePrint assigned value ScreenDollarIndices
** Variable $CovDFormat changed from Prefix to Postfix
** Option AllowUpperDerivatives of ContractMetric changed from False to True
** Option MetricOn of MakeRule changed from None to All
** Option ContractMetrics of MakeRule changed from False to True
------------------------------------------------------------
Package xAct`Invar`  version 2.0.5, {2013, 7, 1}
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** DefConstantSymbol: Defining constant symbol sigma.
** DefConstantSymbol: Defining constant symbol dim.
** Option CurvatureRelations of DefCovD changed from True to False
** Variable $CommuteCovDsOnScalars changed from True to False
------------------------------------------------------------
Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}
CopyRight (C) 2005-2021, David Yllanes and Jose M. Martin-Garcia, under the General Public License.
------------------------------------------------------------
Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}
CopyRight (C) 2011-2021, Thomas Bäckdahl, under the General Public License.
------------------------------------------------------------
Package xAct`xTras`  version 1.4.2, {2014, 10, 30}
CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.
** Variable $CovDFormat changed from Postfix to Prefix
** Option CurvatureRelations of DefCovD changed from False to True
------------------------------------------------------------
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```

Setting up metric-affine gauge theory

Defining the field strength tensors

First we define the asymmetric connection field.

$$\mathcal{A}_{\mu}{}^{\rho}{}_{\sigma} \tag{1}$$

We want to define the curvature in Equation (2.1) on page 4 of arXiv:1912.01023.

$$\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma} \tag{2}$$

$$\mathcal{A}_{\mu}{}^{\rho}{}_{\alpha}\,\mathcal{A}_{\nu}{}^{\alpha}{}_{\sigma} - \mathcal{A}_{\mu}{}^{\alpha}{}_{\sigma}\,\mathcal{A}_{\nu}{}^{\rho}{}_{\alpha} + \partial_{\mu}\mathcal{A}_{\nu}{}^{\rho}{}_{\sigma} - \partial_{\nu}\mathcal{A}_{\mu}{}^{\rho}{}_{\sigma} \tag{3}$$

Here we can make contact with the curvature as defined by xAct.

$$R[\nabla]_{\nu\mu\sigma}{}^{\rho} \tag{4}$$

$$\Gamma[\nabla]^{\alpha}{}_{\nu\sigma}\,\Gamma[\nabla]^{\rho}{}_{\mu\alpha} - \Gamma[\nabla]^{\alpha}{}_{\mu\sigma}\,\Gamma[\nabla]^{\rho}{}_{\nu\alpha} + \partial_{\mu}\Gamma[\nabla]^{\rho}{}_{\nu\sigma} - \partial_{\nu}\Gamma[\nabla]^{\rho}{}_{\mu\sigma} \tag{5}$$

So, by comparing Eqs. (5), and (3) we can guess that the index configurations in Eqs. (4), and (2) are matching. We will later confirm this more carefully, by checking how to match index configurations between the connections.

Next want to define the torsion in Equation (2.2) on page 5 of arXiv:1912.01023.

$$\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu} \tag{6}$$

$$\mathcal{A}_{\mu\ \nu}^{\ \alpha} - \mathcal{A}_{\nu\ \mu}^{\ \alpha} \tag{7}$$

And finally the non-metricity in Equation (2.3) on page 5 of arXiv:1912.01023. Watch out for the trivial misprint in the trace valence. Also, since the non-metricity only appears via quadratic invariants we don't need to bother about perturbing the metric here.

$$Q_{\lambda\mu\nu} \tag{8}$$

$$\mathcal{A}_{\lambda\ \nu}^{\ \alpha}\ g_{\alpha\mu} + \mathcal{A}_{\lambda\ \mu}^{\ \alpha}\ g_{\alpha\nu} - \partial_\lambda g_{\mu\nu} \tag{9}$$

Check conventions for covariant derivative of vector.

$$\nabla_\mu \mathcal{V}^\nu \tag{10}$$

$$\Gamma[\nabla]^\nu{}_{\mu\alpha}\ \mathcal{V}^\alpha + \partial_\mu \mathcal{V}^\nu \tag{11}$$

Here we can make contact with the covariant derivative.

$$-\nabla_\lambda g_{\mu\nu} \tag{12}$$

$$\Gamma[\nabla]^\alpha{}_{\lambda\mu}\ g_{\alpha\nu} + \Gamma[\nabla]^\alpha{}_{\lambda\nu}\ g_{\mu\alpha} - \partial_\lambda g_{\mu\nu} \tag{13}$$

Comparing Eq. (13) with Eq. (9) we see that the `absolute' connection associated with the covariant derivative (i.e. the connection relative to the partial derivative) is the same as our artificially defined connection tensor Eq. (1) up to exchanging the first two indices.

Now we want to build a new rule.

$$\Gamma[\nabla]^\chi{}_{\chi\mu} \tag{14}$$

$$-\Gamma[\nabla]^\alpha{}_{\mu\alpha} + Q^\alpha{}_{\alpha\mu} \tag{15}$$

$$\mathcal{A}_{\mu\ \nu}^{\ \alpha} \tag{16}$$

$$\Gamma[\nabla]^\alpha{}_{\mu\nu} \tag{17}$$

Therefore by using Eq. (17) and comparing with Eq. (7) we can write following rule.

$$\Gamma[\nabla]^\alpha{}_{\mu\nu} \tag{18}$$

$$\frac{1}{2}\ \Gamma[\nabla]^\alpha{}_{\mu\nu} + \frac{1}{2}\ \Gamma[\nabla]^\alpha{}_{\nu\mu} + \frac{1}{2}\ \mathcal{T}_{\mu\ \nu}^{\ \alpha} \tag{19}$$

The equivalence of Eqs. (16), and (17) also confirms the assumed equivalence between Eqs. (4), and (2). In summary this equivalence reads as follows.

$$\mathcal{F}_{\mu\nu\ \sigma}^{\ \ \rho} \tag{20}$$

$$-R[\nabla]_{\mu\nu\sigma}{}^\rho \tag{21}$$

We also note the definition of the non-metricity through the covariant derivative.

$$-\nabla_\lambda g_{\mu\nu} \tag{22}$$

$$Q_{\lambda\mu\nu} \tag{23}$$

To understand the torsion conventions, we use the following.

$$\frac{1}{2}\left(\nabla_\mu \nabla_\nu \phi - \nabla_\nu \nabla_\mu \phi\right) \tag{24}$$

$$-\frac{1}{2}\ \mathcal{T}_{\mu\alpha\nu}\ \partial^\alpha \phi \tag{25}$$

Now we try the same computation, but we revert to the metric-compatible connection.

$$\frac{1}{2}\left(\nabla_\mu \nabla_\nu \phi - \nabla_\nu \nabla_\mu \phi\right) \tag{26}$$

$$-\frac{1}{2}\ T[\nabla]_{\alpha\mu\nu}\left(\mathring{\nabla}^\alpha \phi\right) \tag{27}$$

So comparing Eq. (25) with Eq. (27) we are justified in constructing the following rule.

$$T[\nabla]_{\alpha\mu\nu} \qquad (28)$$

$$\mathcal{T}_{\mu\alpha\nu} \qquad (29)$$

Later on in the calculations we discover the following connection, which we are now able to completely replace using Eq. (23) and Eq. (29).

$$\Gamma[\mathring{\nabla},\nabla]^{\alpha}{}_{\mu\nu} \qquad (30)$$

$$-\frac{1}{2} g^{\alpha\beta}\, T[\nabla]_{\beta\mu\nu} + \frac{1}{2} g^{\alpha\beta}\, T[\nabla]_{\mu\nu\beta} + \frac{1}{2} g^{\alpha\beta}\, T[\nabla]_{\nu\mu\beta} - \frac{1}{2} g^{\alpha\beta}\, \nabla_{\beta} g_{\mu\nu} + \frac{1}{2} g^{\alpha\beta}\, \nabla_{\mu} g_{\nu\beta} + \frac{1}{2} g^{\alpha\beta}\, \nabla_{\nu} g_{\mu\beta} \qquad (31)$$

$$\frac{1}{2} g^{\alpha\beta}\, Q_{\beta\mu\nu} - \frac{1}{2} g^{\alpha\beta}\, Q_{\mu\nu\beta} - \frac{1}{2} g^{\alpha\beta}\, Q_{\nu\mu\beta} - \frac{1}{2} g^{\alpha\beta}\, \mathcal{T}_{\mu\beta\nu} + \frac{1}{2} g^{\alpha\beta}\, \mathcal{T}_{\mu\nu\beta} + \frac{1}{2} g^{\alpha\beta}\, \mathcal{T}_{\nu\mu\beta} \qquad (32)$$

Traces of the field strength tensors

Now we move on to computing the seven contractions defined in Equation (2.5) on page 5 of arXiv:1912.01023. Most of these contractions only appear in quadratic invariants, so we only need these formulae to be accurate to first order in small quantities.

First comes the torsion contraction.

$$\mathcal{T}_{\mu} \qquad (33)$$

$$\mathcal{T}_{\alpha}{}^{\alpha}{}_{\mu} \qquad (34)$$

Next the (standard) non-metricity contraction.

$$Q_{\mu} \qquad (35)$$

$$Q_{\mu\alpha}{}^{\alpha} \qquad (36)$$

Next the (tilde) non-metricity contraction.

$$\tilde{Q}_{\mu} \qquad (37)$$

$$Q_{\alpha}{}^{\alpha}{}_{\mu} \qquad (38)$$

Next the homothetic curvature.

$$\mathcal{F}_{\mu\nu} \qquad (39)$$

$$\mathcal{F}_{\mu\nu\alpha}{}^{\alpha} \qquad (40)$$

Next the first of the pseudo-Ricci tensors.

$$\mathcal{F}^{(14)}{}_{\mu\nu} \qquad (41)$$

$$\mathcal{F}_{\alpha\mu\nu}{}^{\alpha} \qquad (42)$$

Next the second of the pseudo-Ricci tensors.

$$\mathcal{F}^{(13)}{}_{\mu\nu} \qquad (43)$$

$$\mathcal{F}_{\alpha\mu}{}^{\alpha}{}_{\nu} \qquad (44)$$

Now we move on to computing the (conventional) Ricci scalar.

$$\mathcal{F} \qquad (45)$$

$$\left(\mathcal{F}_{\alpha\beta}{}^{\alpha\beta}\right) \qquad (46)$$

Looking for invariant actions

Operator ansatz

We now construct an ansatz for a Lagrangian density containing both parity-even and parity-odd invariants quadratic in the metric-affine curvature. The ansatz is constructed simply by enumerating all permutations of indices, so it provides an over-complete basis for the parity-odd sector. This over-completeness does not affect the results which follow, it just makes expressions more cumbersome.

$$\sqrt{-\bar{\bar{g}}}\,\Big(U_1\, \mathcal{F}_{\alpha\beta}{}^{\delta}{}_{\delta}\, \mathcal{F}^{\alpha\beta\chi}{}_{\chi} + V_1\, \epsilon g_{\beta\delta\epsilon\phi}\, \mathcal{F}_{\alpha}{}^{\delta\epsilon\phi}\, \mathcal{F}^{\alpha\beta\chi}{}_{\chi} + U_2\, \mathcal{F}_{\alpha\beta\chi\delta}\, \mathcal{F}^{\alpha\beta\chi\delta} + U_3\, \mathcal{F}_{\alpha\beta\delta\chi}\, \mathcal{F}^{\alpha\beta\chi\delta} + V_2\, \epsilon g_{\chi\delta\epsilon\phi}\, \mathcal{F}_{\alpha\beta}{}^{\epsilon\phi}\, \mathcal{F}^{\alpha\beta\chi\delta} + U_4\, \mathcal{F}_{\alpha\chi\beta\delta}\, \mathcal{F}^{\alpha\beta\chi\delta} +$$

$$U_5\,\mathcal{F}_{\alpha\chi\delta\beta}\,\mathcal{F}^{\alpha\beta\chi\delta}+V_3\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}+U_6\,\mathcal{F}_{\alpha\delta\chi\beta}\,\mathcal{F}^{\alpha\beta\chi\delta}+V_4\,\epsilon g_{\beta\chi\epsilon\phi}\,\mathcal{F}_{\alpha\delta}{}^{\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}+V_5\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}+V_6\,\epsilon g_{\beta\chi\epsilon\phi}\,\mathcal{F}_{\alpha}{}^{\epsilon}{}_{\delta}{}^{\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}+$$

$$V_7\,\epsilon g_{\beta\chi\epsilon\phi}\,\mathcal{F}_{\alpha}{}^{\epsilon\phi}{}_{\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}+U_7\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}_{\beta\chi}{}^{\delta}{}_{\delta}+U_8\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}_{\beta\chi}{}^{\delta}{}_{\delta}+U_9\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}_{\beta}{}^{\delta}{}_{\chi\delta}+U_{10}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}_{\beta}{}^{\delta}{}_{\chi\delta}+U_{11}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}_{\beta}{}^{\delta}{}_{\delta\chi}+$$

$$V_8\,\epsilon g_{\chi\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}_{\beta}{}^{\delta\epsilon\phi}+V_9\,\epsilon g_{\chi\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}_{\beta}{}^{\delta\epsilon\phi}+U_{12}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}_{\chi\delta\alpha\beta}+V_{10}\,\epsilon g_{\alpha\beta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}_{\chi\delta}{}^{\epsilon\phi}+U_{13}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}_{\chi}{}^{\delta}{}_{\beta\delta}+U_{14}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}_{\chi}{}^{\delta}{}_{\beta\delta}+$$

$$U_{15}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}_{\chi}{}^{\delta}{}_{\delta\beta}+V_{11}\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}_{\chi}{}^{\delta\epsilon\phi}+V_{12}\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}_{\chi}{}^{\delta\epsilon\phi}+V_{13}\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}_{\chi}{}^{\epsilon}{}_{\alpha}{}^{\phi}+V_{14}\,\epsilon g_{\alpha\beta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}_{\chi}{}^{\epsilon}{}_{\delta}{}^{\phi}+$$

$$V_{15}\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}_{\chi}{}^{\epsilon\phi}{}_{\alpha}+V_{16}\,\epsilon g_{\alpha\beta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}_{\chi}{}^{\epsilon\phi}{}_{\delta}+U_{16}\,\mathcal{F}^{\alpha\beta}{}_{\alpha\beta}\,\mathcal{F}^{\chi\delta}{}_{\chi\delta}+V_{17}\,\epsilon g_{\chi\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta}{}_{\alpha\beta}\,\mathcal{F}^{\chi\delta\epsilon\phi}+V_{18}\,\epsilon g_{\alpha\beta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}_{\delta}{}^{\epsilon}{}_{\chi}{}^{\phi}+$$

$$V_{19}\,\epsilon g_{\beta\chi\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}_{\delta}{}^{\epsilon\phi}{}_{\alpha}+V_{20}\,\epsilon g_{\alpha\beta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}_{\delta}{}^{\epsilon\phi}{}_{\chi}+V_{21}\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{F}^{\delta\epsilon}{}_{\alpha}{}^{\phi}+V_{22}\,\epsilon g_{\chi\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}^{\delta\epsilon}{}_{\beta}{}^{\phi}+V_{23}\,\epsilon g_{\chi\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}^{\delta\epsilon}{}_{\beta}{}^{\phi}+$$

$$V_{24}\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}^{\delta\epsilon}{}_{\chi}{}^{\phi}+V_{25}\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}^{\delta\epsilon}{}_{\chi}{}^{\phi}+V_{26}\,\epsilon g_{\beta\chi\epsilon\phi}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}^{\delta\epsilon}{}_{\delta}{}^{\phi}+V_{27}\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{F}^{\delta\epsilon\phi}{}_{\alpha}+V_{28}\,\epsilon g_{\chi\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}^{\delta\epsilon\phi}{}_{\beta}+$$

$$V_{29}\,\epsilon g_{\chi\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}^{\delta\epsilon\phi}{}_{\beta}+V_{30}\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}^{\delta\epsilon\phi}{}_{\chi}+V_{31}\,\epsilon g_{\beta\delta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}^{\delta\epsilon\phi}{}_{\chi}+V_{32}\,\epsilon g_{\beta\chi\epsilon\phi}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}^{\delta\epsilon\phi}{}_{\delta}+V_{33}\,\epsilon g_{\beta\chi\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}^{\delta\epsilon\phi}{}_{\delta}+$$

$$V_{34}\,\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{F}^{\delta\epsilon\phi}{}_{\phi}+V_{35}\,\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{F}^{\delta\epsilon\phi}{}_{\phi}+V_{36}\,\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{F}^{\delta\epsilon\phi}{}_{\phi}+V_{37}\,\epsilon g_{\alpha\beta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}^{\epsilon\phi}{}_{\chi\delta}+V_{38}\,\epsilon g_{\alpha\beta\epsilon\phi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{F}^{\epsilon\phi}{}_{\delta\chi}\Big)$$

We also define the generator 1-form of a completely generic vector symmetry.

$$\lambda_\mu \tag{48}$$

The projective symmetry

Concrete relation to manuscript: We will now investigate the conventional projective transformations, as set out in Eq. (2.16) on page 7 of arXiv:1912.01023, or Eq. (6) on page 2 of our manuscript.

Infinitesimal transformations of fundamental fields.

$g_{\alpha\beta}\to g_{\alpha\beta}$	$\mathcal{A}_{\mu}{}^{\rho}{}_{\nu}\to\mathcal{A}_{\mu}{}^{\rho}{}_{\nu}+\epsilon\,\delta_{\nu}{}^{\rho}\,\lambda_{\mu}$	(49)

Infinitesimal transformations of field strengths induced by Eq. (49).

$\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma}\to\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma}+\epsilon\,\delta^{\rho}{}_{\sigma}\,\lambda_{\alpha}\,\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}+$ $\epsilon\,\delta^{\rho}{}_{\sigma}\,\nabla_{\mu}\lambda_{\nu}-\epsilon\,\delta^{\rho}{}_{\sigma}\,\nabla_{\nu}\lambda_{\mu}$	$\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}\to\epsilon\,\delta^{\alpha}{}_{\nu}\,\lambda_{\mu}-\epsilon\,\delta^{\alpha}{}_{\mu}\,\lambda_{\nu}+\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}$	$Q_{\lambda\mu\nu}\to 2\,\epsilon\,g_{\mu\nu}\,\lambda_{\lambda}+Q_{\lambda\mu\nu}$
$\mathcal{T}_{\mu}\to -3\,\epsilon\,\lambda_{\mu}+\mathcal{T}_{\mu}$	$Q_{\mu}\to 8\,\epsilon\,\lambda_{\mu}+Q_{\mu}$	$\tilde{Q}_{\mu}\to 2\,\epsilon\,\lambda_{\mu}+\tilde{Q}_{\mu}$
$\mathcal{F}_{\mu\nu}\to\mathcal{F}_{\mu\nu}+4\,\epsilon\,\lambda_{\alpha}\,\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}+4\,\epsilon\nabla_{\mu}\lambda_{\nu}-4\,\epsilon\nabla_{\nu}\lambda_{\mu}$	$\mathcal{F}^{(14)}{}_{\mu\nu}\to\mathcal{F}^{(14)}{}_{\mu\nu}-\epsilon\,\lambda_{\alpha}\,\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}-\epsilon\nabla_{\mu}\lambda_{\nu}+\epsilon\nabla_{\nu}\lambda_{\mu}$	$\mathcal{F}^{(13)}{}_{\mu\nu}\to\mathcal{F}^{(13)}{}_{\mu\nu}-\epsilon\,\lambda_{\alpha}\,\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}-\epsilon\nabla_{\mu}\lambda_{\nu}+\epsilon\nabla_{\nu}\lambda_{\mu}$
$\mathcal{F}\to\mathcal{F}$		

(50)

Invariance of Eq. (47) requires the following.

$$\left(2U_{10}-U_{13}-2U_{14}-2U_4-U_5+4U_8+U_9\right)\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,Q_{\alpha\beta\chi}+$$

$$\left(-8U_1-2U_2-2U_3+U_7+U_8\right)\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\beta}\,Q_{\alpha}{}^{\chi}{}_{\chi}+\frac{1}{2}\left(4V_1+V_{11}+V_{12}+V_3+V_4-2V_5+2V_7-V_8-V_9\right)\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\alpha}{}^{\epsilon}{}_{\epsilon}+$$

$$2\left(-8U_1-2U_2-2U_3+U_7+U_8\right)\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,Q_{\alpha\mu\beta}+\left(4V_1+V_{11}+V_{12}+V_3+V_4-2V_5+2V_7-V_8-V_9\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\alpha\mu}{}^{\epsilon}+$$

$$\left(-2U_{10}+U_{13}+2U_{14}+2U_4+U_5-4U_8-U_9\right)\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,Q_{\beta\alpha\chi}+\frac{1}{2}\left(2U_{11}-U_{13}-2U_{15}-U_5-2U_6+4U_7+U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\,Q_{\beta}{}^{\chi}{}_{\chi}+$$

$$\frac{1}{2}\left(2U_{10}-U_{13}-2U_{14}-2U_4-U_5+4U_8+U_9\right)\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\,Q_{\beta}{}^{\chi}{}_{\chi}+\frac{1}{2}\left(2U_{11}-U_{13}-2U_{15}-U_5-2U_6+4U_7+U_9\right)\mathcal{F}_{\mu}{}^{\alpha}{}_{\alpha}{}^{\beta}\,Q_{\beta}{}^{\chi}{}_{\chi}+$$

$$\frac{1}{2}\left(2U_{10}-U_{13}-2U_{14}-2U_4-U_5+4U_8+U_9\right)\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\alpha}\,Q_{\beta}{}^{\chi}{}_{\chi}+\left(-2U_{11}+U_{13}+2U_{15}+U_5+2U_6-4U_7-U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,Q_{\beta\mu\chi}+$$

$$\left(-2U_{10}+U_{13}+2U_{14}+2U_4+U_5-4U_8-U_9\right)\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,Q_{\beta\mu\chi}+\frac{1}{2}\left(2V_{13}+V_{14}+V_{15}+V_{16}+4V_{21}-V_{22}-V_{23}+V_{24}+V_{25}\right)\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\chi}{}^{\epsilon}{}_{\epsilon}+$$

$$\left(2U_{11}-U_{13}-2U_{15}-U_5-2U_6+4U_7+U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,Q_{\chi\mu\beta}+\left(2U_{10}-U_{13}-2U_{14}-2U_4-U_5+4U_8+U_9\right)\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,Q_{\chi\mu\beta}+$$

$$\left(2V_{13}+V_{14}+V_{15}+V_{16}+4V_{21}-V_{22}-V_{23}+V_{24}+V_{25}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\chi\mu}{}^{\epsilon}+\left(16U_1+4U_2+4U_3-2U_7-2U_8\right)\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\beta}\,Q^{\chi}{}_{\alpha\chi}+$$

$$\left(-2U_{11}+U_{13}+2U_{15}+U_5+2U_6-4U_7-U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\,Q^{\chi}{}_{\beta\chi}+\left(-2U_{10}+U_{13}+2U_{14}+2U_4+U_5-4U_8-U_9\right)\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\,Q^{\chi}{}_{\beta\chi}+$$

$$\left(-2U_{11}+U_{13}+2U_{15}+U_5+2U_6-4U_7-U_9\right)\mathcal{F}_{\mu}{}^{\alpha}{}_{\alpha}{}^{\beta}\,Q^{\chi}{}_{\beta\chi}+\frac{1}{2}\left(V_{15}+V_{18}+2V_{19}+V_{20}+4V_{27}-V_{28}-V_{29}+V_{30}+V_{31}\right)\epsilon g_{\mu\alpha\beta\chi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\delta}{}^{\epsilon}{}_{\epsilon}+$$

$$\left(V_{15}+V_{18}+2V_{19}+V_{20}+4V_{27}-V_{28}-V_{29}+V_{30}+V_{31}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\delta\mu}{}^{\epsilon}-2\left(2V_{13}+V_{14}+V_{15}+V_{16}+4V_{21}-V_{22}-V_{23}+V_{24}+V_{25}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,Q^{\delta}{}_{\alpha}{}^{\epsilon}-$$

$$2\left(V_{14}+V_{18}-V_{22}+V_{24}-2V_{26}-V_{28}+V_{30}-V_{32}+4V_{34}\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,Q^{\delta}{}_{\beta}{}^{\epsilon}-2\left(V_{16}+V_{20}-V_{23}+V_{25}-V_{29}+V_{31}-V_{32}-2V_{33}+4V_{35}\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,Q^{\delta}{}_{\beta}{}^{\epsilon}+$$

$$2\left(V_{14}+V_{18}-V_{22}+V_{24}-2V_{26}-V_{28}+V_{30}-V_{32}+4V_{34}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,Q^{\delta}{}_{\chi}{}^{\epsilon}+2\left(V_{16}+V_{20}-V_{23}+V_{25}-V_{29}+V_{31}-V_{32}-2V_{33}+4V_{35}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,Q^{\delta}{}_{\chi}{}^{\epsilon}+$$

$$\left(-2V_{13}-V_{14}-V_{15}-V_{16}-4V_{21}+V_{22}+V_{23}-V_{24}-V_{25}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,Q^{\delta}{}_{\chi}{}^{\epsilon}-2\left(V_{21}+V_{27}+V_{34}+V_{35}-8V_{36}-2V_{37}-2V_{38}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,Q^{\delta}{}_{\delta}{}^{\epsilon}+$$

$$\left(-V_{15}-V_{18}-2V_{19}-V_{20}-4V_{27}+V_{28}+V_{29}-V_{30}-V_{31}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,Q^{\delta}{}_{\delta}{}^{\epsilon}+\left(-4V_1-V_{11}-V_{12}-V_3-V_4+2V_5-2V_7+V_8+V_9\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,Q^{\delta}{}_{\delta}{}^{\epsilon}+$$

$$\left(V_{21}+V_{27}+V_{34}+V_{35}-8V_{36}-2V_{37}-2V_{38}\right)\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,Q^{\delta\epsilon}{}_{\epsilon}+\frac{1}{2}\left(V_{15}+V_{18}+2V_{19}+V_{20}+4V_{27}-V_{28}-V_{29}+V_{30}+V_{31}\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,Q^{\delta\epsilon}{}_{\epsilon}+$$

$$\frac{1}{2}\left(4V_1+V_{11}+V_{12}+V_3+V_4-2V_5+2V_7-V_8-V_9\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,Q^{\delta\epsilon}{}_{\epsilon}+2\left(V_{21}+V_{27}+V_{34}+V_{35}-8V_{36}-2V_{37}-2V_{38}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,Q^{\delta}{}_{\mu}{}^{\epsilon}+$$

$$2\left(V_{16}+V_{20}-V_{23}+V_{25}-V_{29}+V_{31}-V_{32}-2V_{33}+4V_{35}\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q^{\epsilon}{}_{\alpha\delta}+\left(-4V_1-V_{11}-V_{12}-V_3-V_4+2V_5-2V_7+V_8+V_9\right)\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q^{\epsilon}{}_{\alpha\epsilon}+$$

$$
\begin{aligned}
&\left(-V_{15}-V_{18}-2V_{19}-V_{20}-4V_{27}+V_{28}+V_{29}-V_{30}-V_{31}\right)\epsilon g_{\mu\alpha\beta\chi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\delta\epsilon}+\left(-4V_1-V_{11}-V_{12}-V_3-V_4+2V_5-2V_7+V_8+V_9\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\alpha}+\\
&\left(-2V_{13}-V_{14}-V_{15}-V_{16}-4V_{21}+V_{22}+V_{23}-V_{24}-V_{25}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\chi}+\left(-V_{15}-V_{18}-2V_{19}-V_{20}-4V_{27}+V_{28}+V_{29}-V_{30}-V_{31}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\delta}+\\
&\left(16U_1+4U_2+4U_3-2U_7-2U_8\right)\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\beta}\,\mathcal{T}_{\alpha}{}^{\chi}{}_{\chi}+\left(-4V_1-V_{11}-V_{12}-V_3-V_4+2V_5-2V_7+V_8+V_9\right)\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha}{}^{\epsilon}{}_{\epsilon}+\\
&\left(8U_1+2U_2+2U_3-U_7-U_8\right)\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}_{\alpha\mu\beta}+\left(-4V_1-V_{11}-V_{12}-V_3-V_4+2V_5-2V_7+V_8+V_9\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha\mu}{}^{\epsilon}+\\
&\left(-2U_{11}+U_{13}+2U_{15}+U_5+2U_6-4U_7-U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+\left(-2U_{10}+U_{13}+2U_{14}+2U_4+U_5-4U_8-U_9\right)\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+\\
&\left(-2U_{11}+U_{13}+2U_{15}+U_5+2U_6-4U_7-U_9\right)\mathcal{F}_{\mu}{}^{\alpha}{}_{\alpha}{}^{\beta}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+\left(-2U_{10}+U_{13}+2U_{14}+2U_4+U_5-4U_8-U_9\right)\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\alpha}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+\\
&\left(2U_{11}-U_{13}-2U_{15}-U_5-2U_6+4U_7+U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}_{\beta\mu\chi}+\left(2U_{10}-U_{13}-2U_{14}-2U_4-U_5+4U_8+U_9\right)\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}_{\beta\mu\chi}+\\
&\left(-2V_{13}-V_{14}-V_{15}-V_{16}-4V_{21}+V_{22}+V_{23}-V_{24}-V_{25}\right)\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\chi}{}^{\epsilon}{}_{\epsilon}+\left(-2V_{13}-V_{14}-V_{15}-V_{16}-4V_{21}+V_{22}+V_{23}-V_{24}-V_{25}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\chi\mu}{}^{\epsilon}+\\
&\left(-V_{15}-V_{18}-2V_{19}-V_{20}-4V_{27}+V_{28}+V_{29}-V_{30}-V_{31}\right)\epsilon g_{\mu\alpha\beta\chi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\delta}{}^{\epsilon}{}_{\epsilon}+\left(-V_{15}-V_{18}-2V_{19}-V_{20}-4V_{27}+V_{28}+V_{29}-V_{30}-V_{31}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\delta\mu}{}^{\epsilon}+\\
&\left(2V_{13}+V_{14}+V_{15}+V_{16}+4V_{21}-V_{22}-V_{23}+V_{24}+V_{25}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\alpha}{}^{\epsilon}+\left(V_{14}+V_{18}-V_{22}+V_{24}-2V_{26}-V_{28}+V_{30}-V_{32}+4V_{34}\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\beta}{}^{\epsilon}+\\
&\left(V_{16}+V_{20}-V_{23}+V_{25}-V_{29}+V_{31}-V_{32}-2V_{33}+4V_{35}\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}^{\delta}{}_{\beta}{}^{\epsilon}+\left(-V_{14}-V_{18}+V_{22}-V_{24}+2V_{26}+V_{28}-V_{30}+V_{32}-4V_{34}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+\\
&\left(-V_{16}-V_{20}+V_{23}-V_{25}+V_{29}-V_{31}+V_{32}+2V_{33}-4V_{35}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+\frac{1}{2}\left(2V_{13}+V_{14}+V_{15}+V_{16}+4V_{21}-V_{22}-V_{23}+V_{24}+V_{25}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+\\
&2\left(V_{21}+V_{27}+V_{34}+V_{35}-8V_{36}-2V_{37}-2V_{38}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+\left(V_{15}+V_{18}+2V_{19}+V_{20}+4V_{27}-V_{28}-V_{29}+V_{30}+V_{31}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+\\
&\left(4V_1+V_{11}+V_{12}+V_3+V_4-2V_5+2V_7-V_8-V_9\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+\left(-V_{21}-V_{27}-V_{34}-V_{35}+8V_{36}+2V_{37}+2V_{38}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}^{\delta}{}_{\mu}{}^{\epsilon}+\\
&\left(V_{15}+V_{18}+2V_{19}+V_{20}+4V_{27}-V_{28}-V_{29}+V_{30}+V_{31}\right)g^{\alpha\beta}\,\mathcal{F}^{\chi\delta\epsilon}{}_{\mu}\,\nabla_{\beta}\epsilon g_{\alpha\chi\delta\epsilon}+\left(4V_1+V_{11}+V_{12}+V_3+V_4-2V_5+2V_7-V_8-V_9\right)g^{\alpha\beta}\,\mathcal{F}_{\mu}{}^{\chi\delta\epsilon}\,\nabla_{\beta}\epsilon g_{\alpha\chi\delta\epsilon}+\\
&\left(4V_1+V_{11}+V_{12}+V_3+V_4-2V_5+2V_7-V_8-V_9\right)\mathcal{F}^{\alpha\beta\chi\delta}\,\nabla_{\beta}\epsilon g_{\mu\alpha\chi\delta}-2\left(V_{21}+V_{27}+V_{34}+V_{35}-8V_{36}-2V_{37}-2V_{38}\right)g^{\alpha\beta}\,\mathcal{F}^{\chi\delta\epsilon}{}_{\epsilon}\,\nabla_{\beta}\epsilon g_{\mu\alpha\chi\delta}+\\
&\left(2U_{11}-U_{13}-2U_{15}-U_5-2U_6+4U_7+U_9\right)g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\alpha}{}^{\chi}{}_{\chi\mu}+\left(-4V_1-V_{11}-V_{12}-V_3-V_4+2V_5-2V_7+V_8+V_9\right)\epsilon g_{\mu\chi\delta\epsilon}\,g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\alpha}{}^{\chi\delta\epsilon}+\\
&\left(-V_{15}-V_{18}-2V_{19}-V_{20}-4V_{27}+V_{28}+V_{29}-V_{30}-V_{31}\right)\epsilon g_{\mu\chi\delta\epsilon}\,g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}^{\chi\delta\epsilon}{}_{\alpha}+\left(16U_1+4U_2+4U_3-2U_7-2U_8\right)g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\mu\alpha}{}^{\chi}{}_{\chi}+\\
&\left(-2U_{11}+U_{13}+2U_{15}+U_5+2U_6-4U_7-U_9\right)g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\mu}{}^{\chi}{}_{\chi\alpha}+\left(-2V_{13}-V_{14}-V_{15}-V_{16}-4V_{21}+V_{22}+V_{23}-V_{24}-V_{25}\right)\mathcal{F}^{\alpha\beta\chi\delta}\,\nabla_{\chi}\epsilon g_{\mu\alpha\beta\delta}+\\
&\left(-2V_{13}-V_{14}-V_{15}-V_{16}-4V_{21}+V_{22}+V_{23}-V_{24}-V_{25}\right)\epsilon g_{\mu\alpha\beta\delta}\,\nabla_{\chi}\mathcal{F}^{\alpha\beta\chi\delta}+\left(-2U_{10}+U_{13}+2U_{14}+2U_4+U_5-4U_8-U_9\right)g^{\alpha\beta}\,\nabla_{\chi}\mathcal{F}_{\mu\alpha}{}^{\chi}{}_{\beta}+\\
&\left(-V_{15}-V_{18}-2V_{19}-V_{20}-4V_{27}+V_{28}+V_{29}-V_{30}-V_{31}\right)\mathcal{F}^{\alpha\beta\chi\delta}\,\nabla_{\delta}\epsilon g_{\mu\alpha\beta\chi}+\left(-2U_{10}+U_{13}+2U_{14}+2U_4+U_5-4U_8-U_9\right)g^{\alpha\beta}\,g^{\chi\delta}\,\nabla_{\delta}\mathcal{F}_{\alpha\chi\mu\beta}+\\
&2\left(V_{14}+V_{18}-V_{22}+V_{24}-2V_{26}-V_{28}+V_{30}-V_{32}+4V_{34}\right)\epsilon g_{\mu\beta\chi\delta}\,\nabla^{\delta}\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}-2\left(V_{21}+V_{27}+V_{34}+V_{35}-8V_{36}-2V_{37}-2V_{38}\right)\epsilon g_{\mu\alpha\beta\delta}\,\nabla^{\delta}\mathcal{F}^{\alpha\beta\chi}{}_{\chi}+\\
&\left(-V_{15}-V_{18}-2V_{19}-V_{20}-4V_{27}+V_{28}+V_{29}-V_{30}-V_{31}\right)\epsilon g_{\alpha\beta\chi\delta}\,\nabla^{\delta}\mathcal{F}^{\alpha\beta\chi}{}_{\mu}+\left(-2V_{13}-V_{14}-V_{15}-V_{16}-4V_{21}+V_{22}+V_{23}-V_{24}-V_{25}\right)\epsilon g_{\alpha\beta\chi\delta}\,\nabla^{\delta}\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}+\\
&\left(-4V_1-V_{11}-V_{12}-V_3-V_4+2V_5-2V_7+V_8+V_9\right)\epsilon g_{\alpha\beta\chi\delta}\,\nabla^{\delta}\mathcal{F}_{\mu}{}^{\alpha\beta\chi}+2\left(V_{16}+V_{20}-V_{23}+V_{25}-V_{29}+V_{31}-V_{32}-2V_{33}+4V_{35}\right)\epsilon g_{\mu\chi\delta\epsilon}\,g^{\alpha\beta}\,\nabla^{\epsilon}\mathcal{F}_{\alpha}{}^{\chi\delta}{}_{\beta}==0
\end{aligned}
$$

For general values of the fields, the tensor equation Eq. (51) requires the following conditions among the Lagrangian parameters in the ansatz.

$U_1==-\frac{U_2}{4}-\frac{U_3}{4}+\frac{U_7}{8}+\frac{U_8}{8}$	$U_{10}==\frac{U_{13}}{2}+U_{14}+U_4+\frac{U_5}{2}-2U_8-\frac{U_9}{2}$	$U_{11}==\frac{U_{13}}{2}+U_{15}+\frac{U_5}{2}+U_6-2U_7-\frac{U_9}{2}$
$V_1==-\frac{V_{11}}{4}-\frac{V_{12}}{4}-\frac{V_3}{4}-\frac{V_4}{4}+\frac{V_5}{2}-\frac{V_7}{2}+\frac{V_8}{4}+\frac{V_9}{4}$	$V_{13}==V_{18}+V_{19}+V_{20}-V_{26}+4V_{27}-V_{28}-V_{29}+V_{30}+V_{31}-V_{32}-V_{33}+4V_{34}+4V_{35}-16V_{36}-4V_{37}-4V_{38}$	$V_{14}==-V_{18}+V_{22}-V_{24}+2V_{26}+V_{28}-V_{30}+V_{32}-4V_{34}$
$V_{15}==-V_{18}-2V_{19}-V_{20}-4V_{27}+V_{28}+V_{29}-V_{30}-V_{31}$	$V_{16}==-V_{20}+V_{23}-V_{25}+V_{29}-V_{31}+V_{32}+2V_{33}-4V_{35}$	$V_{21}==-V_{27}-V_{34}-V_{35}+8V_{36}+2V_{37}+2V_{38}$

(52)

As is well-known, projective symmetry allows for numerous curvature-squared terms, reflected in the relatively few (only nine) conditions in Eq. (52). Roughly speaking, more terms means more active particles: this is why projective symmetry is insufficient for generating consistent particle physics with only a few new species.

The concurrent symmetry

Concrete relation to manuscript: We will now investigate the concurrent symmetry, as set out in Eq. (C3) on page 6 of our manuscript.

Infinitesimal transformations of fundamental fields.

$g_{\alpha\beta}\to g_{\alpha\beta}$	$\mathcal{A}_{\mu}{}^{\rho}{}_{\nu}\to\mathcal{A}_{\mu}{}^{\rho}{}_{\nu}-\epsilon\,\delta_{\nu}{}^{\rho}\,\lambda_{\mu}+2\epsilon\,\delta_{\mu}{}^{\rho}\,\lambda_{\nu}+2\epsilon\,g_{\mu\nu}\,\lambda^{\rho}$

(53)

Infinitesimal transformations of field strengths induced by Eq. (49).

$\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma}\to\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma}-2\epsilon\,g^{\rho\alpha}\,\lambda_{\alpha}\,\mathcal{Q}_{\mu\nu\sigma}+2\epsilon\,g^{\rho\alpha}\,\lambda_{\alpha}\,\mathcal{Q}_{\nu\mu\sigma}-\epsilon\,\delta^{\rho}{}_{\sigma}\,\lambda_{\alpha}\,\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}+2\epsilon\,g_{\beta\sigma}\,g^{\rho\alpha}\,\lambda_{\alpha}\,\mathcal{T}_{\mu}{}^{\beta}{}_{\nu}+2\epsilon\,\lambda_{\sigma}\,\mathcal{T}_{\mu}{}^{\rho}{}_{\nu}+2\epsilon\,g_{\nu\sigma}\left(g^{\alpha\chi}\,g^{\rho\beta}\,\lambda_{\alpha}\,\mathcal{Q}_{\mu\beta\chi}+g^{\rho\alpha}\,\nabla_{\mu}\lambda_{\alpha}\right)-\epsilon\,\delta^{\rho}{}_{\sigma}\,\nabla_{\mu}\lambda_{\nu}+2\epsilon\,\delta_{\nu}{}^{\rho}\,\nabla_{\mu}\lambda_{\sigma}-2\epsilon\,g_{\mu\sigma}\left(g^{\alpha\chi}\,g^{\rho\beta}\,\lambda_{\alpha}\,\mathcal{Q}_{\nu\beta\chi}+g^{\rho\alpha}\,\nabla_{\nu}\lambda_{\alpha}\right)+\epsilon\,\delta^{\rho}{}_{\sigma}\,\nabla_{\nu}\lambda_{\mu}-2\epsilon\,\delta_{\mu}{}^{\rho}\,\nabla_{\nu}\lambda_{\sigma}$	$\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}\to-3\epsilon\,\delta^{\alpha}{}_{\nu}\,\lambda_{\mu}+3\epsilon\,\delta^{\alpha}{}_{\mu}\,\lambda_{\nu}+\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}$	$\mathcal{Q}_{\lambda\mu\nu}\to-2\epsilon\,g_{\mu\nu}\,\lambda_{\lambda}+4\epsilon\,g_{\lambda\nu}\,\lambda_{\mu}+4\epsilon\,g_{\lambda\mu}\,\lambda_{\nu}+\mathcal{Q}_{\lambda\mu\nu}$

$\mathcal{T}_\mu \to 9\,\epsilon\,\lambda_\mu + \mathcal{T}_\mu$	$Q_\mu \to Q_\mu$	$\tilde{Q}_\mu \to 18\,\epsilon\,\lambda_\mu + \tilde{Q}_\mu$
$\mathcal{F}_{\mu\nu} \to -2\,\epsilon\, g^{\alpha\beta}\,\lambda_\beta\, Q_{\mu\nu\alpha} + 2\,\epsilon\, g^{\alpha\beta}\,\lambda_\beta\, Q_{\nu\mu\alpha} + \mathcal{F}_{\mu\nu} + 2\,\epsilon\, g_{\nu\alpha}\left(g^{\alpha\chi}\, g^{\beta\delta}\,\lambda_\beta\, Q_{\mu\chi\delta} + g^{\alpha\beta}\,\nabla_\mu\lambda_\beta\right) - 2\,\epsilon\nabla_\mu\lambda_\nu - 2\,\epsilon\, g_{\mu\alpha}\left(g^{\alpha\chi}\, g^{\beta\delta}\,\lambda_\beta\, Q_{\nu\chi\delta} + g^{\alpha\beta}\,\nabla_\nu\lambda_\beta\right) + 2\,\epsilon\nabla_\nu\lambda_\mu$	$\mathcal{F}^{(14)}{}_{\mu\nu} \to -\epsilon\, g^{\alpha\beta}\,\lambda_\nu\, Q_{\alpha\mu\beta} + \epsilon\, g^{\alpha\chi}\, g^{\beta\delta}\, g_{\mu\nu}\,\lambda_\chi\, Q_{\beta\alpha\delta} - \epsilon\, g^{\alpha\beta}\,\lambda_\nu\, Q_{\beta\mu\alpha} - 3\,\epsilon\, g^{\alpha\chi}\, g^{\beta\delta}\, g_{\mu\nu}\,\lambda_\chi\, Q_{\delta\alpha\beta} + 2\,\epsilon\, g^{\alpha\beta}\,\lambda_\nu\, Q_{\mu\beta\alpha} + \mathcal{F}^{(14)}{}_{\mu\nu} - 2\,\epsilon\,\lambda_\nu\,\mathcal{T}_\mu{}^\alpha{}_\alpha + \epsilon\,\lambda_\alpha\,\mathcal{T}_\mu{}^\alpha{}_\nu - 2\,\epsilon\, g^{\alpha\beta}\, g_{\chi\nu}\,\lambda_\beta\,\mathcal{T}_\mu{}^\chi{}_\alpha + 2\,\epsilon\, g_{\alpha\beta}\, g_{\mu\nu}\left(g^{\alpha\delta}\, g^{\beta\phi}\, g^{\chi\epsilon}\,\lambda_\chi\, Q_{\phi\delta\epsilon} + g^{\alpha\chi}\, g^{\beta\gamma}\,\nabla_\gamma\lambda_\chi\right) - 6\,\epsilon\, g_{\nu\alpha}\left(g^{\alpha\chi}\, g^{\beta\delta}\,\lambda_\beta\, Q_{\mu\chi\delta} + g^{\alpha\beta}\,\nabla_\mu\lambda_\beta\right) - \epsilon\nabla_\mu\lambda_\nu - \epsilon\nabla_\nu\lambda_\mu$	$\mathcal{F}^{(13)}{}_{\mu\nu} \to -2\,\epsilon\, g^{\alpha\beta}\,\lambda_\beta\, Q_{\alpha\mu\nu} + 2\,\epsilon\, g^{\alpha\beta}\,\lambda_\beta\, Q_{\mu\nu\alpha} + \mathcal{F}^{(13)}{}_{\mu\nu} - 2\,\epsilon\,\lambda_\nu\,\mathcal{T}_\mu{}^\alpha{}_\alpha + \epsilon\,\lambda_\alpha\,\mathcal{T}_\mu{}^\alpha{}_\nu - 2\,\epsilon\, g^{\alpha\beta}\, g_{\chi\nu}\,\lambda_\beta\,\mathcal{T}_\mu{}^\chi{}_\alpha + 2\,\epsilon\, g_{\mu\nu}\left(g^{\alpha\chi}\, g^{\beta\delta}\,\lambda_\beta\, Q_{\alpha\chi\delta} + g^{\alpha\beta}\,\nabla_\alpha\lambda_\beta\right) - 2\,\epsilon\, g_{\nu\alpha}\left(g^{\alpha\chi}\, g^{\beta\delta}\,\lambda_\beta\, Q_{\mu\chi\delta} + g^{\alpha\beta}\,\nabla_\mu\lambda_\beta\right) - 5\,\epsilon\nabla_\mu\lambda_\nu - \epsilon\nabla_\nu\lambda_\mu$
$\mathcal{F} \to -2\,\epsilon\, g^{\alpha\chi}\, g^{\beta\delta}\,\lambda_\chi\, Q_{\alpha\delta\beta} - 2\,\epsilon\, g^{\alpha\chi}\, g^{\beta\delta}\,\lambda_\chi\, Q_{\beta\alpha\delta} + 10\,\epsilon\, g^{\alpha\chi}\, g^{\beta\delta}\,\lambda_\chi\, Q_{\delta\alpha\beta} + \mathcal{F} + 6\,\epsilon\left(g^{\alpha\chi}\, g^{\beta\delta}\,\lambda_\beta\, Q_{\alpha\chi\delta} + g^{\alpha\beta}\,\nabla_\alpha\lambda_\beta\right) - 6\,\epsilon\, g_{\alpha\beta}\left(g^{\alpha\delta}\, g^{\beta\phi}\, g^{\chi\epsilon}\,\lambda_\chi\, Q_{\phi\delta\epsilon} + g^{\alpha\chi}\, g^{\beta\gamma}\,\nabla_\gamma\lambda_\chi\right)$		

Invariance of Eq. (47) requires the following.

$$
\begin{aligned}
&-2\left(4U_3+U_5\right)\mathcal{F}^{\alpha\beta\chi}{}_\mu\, Q_{\alpha\beta\chi} + \left(14U_{10}+U_{13}+2U_{14}-16U_2-8U_3+2U_4-U_5-8U_6+7U_9\right)\mathcal{F}^{\alpha\beta}{}_\mu{}^\chi\, Q_{\alpha\beta\chi} + 2\left(2U_4+U_5\right)\mathcal{F}_\mu{}^{\alpha\beta\chi}\, Q_{\alpha\beta\chi} + \\
&\left(-8V_2+2V_4\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_{\alpha\beta}{}^\epsilon + \left(-2V_3+2V_6\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_{\alpha\chi}{}^\epsilon + \left(2U_2+2U_3-2U_4-2U_5-2U_6+3U_7+U_8\right)\mathcal{F}_\mu{}^{\alpha\beta}{}_\beta\, Q_\alpha{}^\chi{}_\chi + \\
&\left(-2V_4+4V_7\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_{\alpha\delta}{}^\epsilon + \frac{1}{2}\left(-V_{11}-V_{12}-3V_3-3V_4+6V_5-6V_7-7V_8-3V_9\right)\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_\alpha{}^\epsilon{}_\epsilon + \left(4U_2+4U_3-2U_5-4U_6+6U_7\right)\mathcal{F}^{\alpha\beta\chi}{}_\chi\, Q_{\alpha\mu\beta} + \\
&\left(-V_{11}-V_{12}-8V_2-V_3-3V_4+6V_5-2V_6-2V_7-7V_8-V_9\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_{\alpha\mu}{}^\epsilon + \left(2U_{10}-4U_{12}+7U_{13}+14U_{14}-10U_4-3U_5+U_9\right)\mathcal{F}_\mu{}^{\alpha\beta\chi}\, Q_{\beta\alpha\chi} + \\
&\left(7U_{11}+\frac{U_{13}}{2}+U_{15}-4U_2-4U_3-2U_4-\frac{U_5}{2}+U_6+\frac{3U_9}{2}\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\, Q_\beta{}^\chi{}_\chi + \frac{1}{2}\left(6U_{10}+U_{13}+2U_{14}-8U_2-8U_3+2U_4-U_5-4U_6+7U_9\right)\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\, Q_\beta{}^\chi{}_\chi + \\
&\frac{1}{2}\left(-2U_{11}-3U_{13}-14U_{15}+3U_5+6U_6-U_9\right)\mathcal{F}_\mu{}^\alpha{}_\alpha{}^\beta\, Q_\beta{}^\chi{}_\chi + \frac{1}{2}\left(-2U_{10}-7U_{13}-6U_{14}+6U_4+3U_5-U_9\right)\mathcal{F}_\mu{}^{\alpha\beta}{}_\alpha\, Q_\beta{}^\chi{}_\chi + \\
&\left(-14U_{11}-U_{13}-2U_{15}+8U_2+4U_4-U_5-2U_6-U_9\right)\mathcal{F}^{\alpha\beta}{}_\alpha{}^\chi\, Q_{\beta\mu\chi} + \left(-2U_{10}-U_{13}-2U_{14}+8U_3-2U_4+U_5-7U_9\right)\mathcal{F}^{\alpha\beta\chi}{}_\alpha\, Q_{\beta\mu\chi} + \left(4U_{12}-2U_5\right)\mathcal{F}_\mu{}^{\alpha\beta\chi}\, Q_{\chi\alpha\beta} + \\
&2\left(V_{15}+V_3\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_{\chi\alpha}{}^\epsilon + \left(-2V_{10}+2V_{16}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_{\chi\delta}{}^\epsilon + \frac{1}{2}\left(-6V_{13}-3V_{14}-3V_{15}-3V_{16}-7V_{22}-3V_{23}-V_{24}-V_{25}\right)\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_\chi{}^\epsilon{}_\epsilon + \\
&\left(-2U_{11}+4U_{12}-U_{13}-14U_{15}+U_5+6U_6-U_9\right)\mathcal{F}^{\alpha\beta}{}_\alpha{}^\chi\, Q_{\chi\mu\beta} + \left(-2U_{10}-4U_{12}-7U_{13}-2U_{14}+2U_4+3U_5-U_9\right)\mathcal{F}^{\alpha\beta\chi}{}_\alpha\, Q_{\chi\mu\beta} + \\
&\left(-2V_{10}-6V_{13}-3V_{14}-V_{15}-V_{16}-7V_{22}-V_{23}-V_{24}-V_{25}+2V_3\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_{\chi\mu}{}^\epsilon + \left(-4U_2-4U_3+4U_4+4U_5+4U_6-6U_7-4U_8\right)\mathcal{F}_\mu{}^{\alpha\beta}{}_\beta\, Q^\chi{}_{\alpha\chi} + \\
&\left(-14U_{11}-U_{13}-2U_{15}+8U_2+8U_3+4U_4+U_5-2U_6-5U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\, Q^\chi{}_{\beta\chi} + \left(-10U_{10}-U_{13}-2U_{14}+8U_2+8U_3-2U_4+U_5+4U_6-7U_9\right)\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\, Q^\chi{}_{\beta\chi} + \\
&\left(2U_{11}+5U_{13}+14U_{15}-3U_5-6U_6+U_9\right)\mathcal{F}_\mu{}^\alpha{}_\alpha{}^\beta\, Q^\chi{}_{\beta\chi} - 4U_{14}\,\mathcal{F}_\mu{}^{\alpha\beta}{}_\alpha\, Q^\chi{}_{\beta\chi} + 2\left(-2U_{10}+U_{13}-2U_{14}+8U_{16}+U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\beta}\, Q^\chi{}_{\mu\chi} + \\
&2\left(2V_{19}+V_4\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_{\delta\alpha}{}^\epsilon + 2\left(V_{10}+V_{20}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_{\delta\chi}{}^\epsilon + \frac{1}{2}\left(-3V_{15}-3V_{18}-6V_{19}-3V_{20}-7V_{28}-3V_{29}-V_{30}-V_{31}\right)\epsilon g_{\mu\alpha\beta\chi}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_\delta{}^\epsilon{}_\epsilon + \\
&\left(2V_{10}-3V_{15}-3V_{18}-2V_{19}-V_{20}-7V_{28}-V_{29}-V_{30}-V_{31}+2V_4\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q_{\delta\mu}{}^\epsilon + 2\left(V_1+2V_{27}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_\chi\, Q^\delta{}_\alpha{}^\epsilon + \\
&\left(-4V_{20}+2V_6\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_\mu\, Q^\delta{}_\alpha{}^\epsilon + 2\left(2V_{13}+5V_{14}+V_{15}+V_{16}-2V_{20}-V_{22}-V_{23}-7V_{24}-7V_{25}+8V_{37}+8V_{38}-2V_5-2V_6\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_\mu{}^\chi\, Q^\delta{}_\alpha{}^\epsilon - \\
&2\left(2V_{10}+V_3\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}_\mu{}^{\alpha\beta\chi}\, Q^\delta{}_\alpha{}^\epsilon + 2\left(2V_{13}+V_{14}+V_{15}+V_{18}-3V_{22}+3V_{24}+6V_{26}-V_{28}+3V_{30}+3V_{32}+2V_5+V_6+V_8\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_\alpha{}^\chi\, Q^\delta{}_\beta{}^\epsilon + \\
&2\left(V_{15}+V_{16}+2V_{19}+V_{20}-3V_{23}+3V_{25}-V_{29}+3V_{31}+3V_{32}+6V_{33}+V_6+2V_7+V_9\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_\alpha\, Q^\delta{}_\beta{}^\epsilon + 4\left(V_{13}-V_{14}\right)\epsilon g_{\alpha\chi\delta\epsilon}\,\mathcal{F}_\mu{}^{\alpha\beta\chi}\, Q^\delta{}_\beta{}^\epsilon + \\
&2\left(V_{11}-2V_{13}-V_{14}-V_{15}-V_{18}+3V_{22}-3V_{24}-6V_{26}+3V_{28}-V_{30}-3V_{32}-2V_5-V_6\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_\alpha{}^\chi\, Q^\delta{}_\chi{}^\epsilon + \\
&2\left(V_{12}-V_{15}-V_{16}-2V_{19}-V_{20}+3V_{23}-3V_{25}+3V_{29}-V_{31}-3V_{32}-6V_{33}-V_6-2V_7\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_\alpha\, Q^\delta{}_\chi{}^\epsilon + 2\left(V_{14}-4V_{38}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_\mu\, Q^\delta{}_\chi{}^\epsilon + \\
&\left(2V_{13}+5V_{14}+V_{15}+V_{16}+4V_{18}-2V_{20}-V_{22}-V_{23}-7V_{24}-7V_{25}+8V_{38}-4V_5-2V_6\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_\mu{}^\chi\, Q^\delta{}_\chi{}^\epsilon + 2\left(V_{15}-2V_{16}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}_\mu{}^{\alpha\beta\chi}\, Q^\delta{}_\chi{}^\epsilon + \\
&2V_{32}\,\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta}{}_\alpha{}^\chi\, Q^\delta{}_\delta{}^\epsilon + 4V_{33}\,\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_\alpha\, Q^\delta{}_\delta{}^\epsilon - 2\left(V_{14}+V_{16}+V_{18}+V_{20}-3V_{21}-3V_{27}+3V_{34}+2V_{35}+2V_{37}+2V_{38}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_\chi\, Q^\delta{}_\delta{}^\epsilon + \\
&\left(-2V_{14}+V_{15}+2V_{16}+V_{18}+2V_{19}+5V_{20}-V_{28}-V_{29}-7V_{30}-5V_{31}+8V_{37}+8V_{38}-2V_6-4V_7\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_\mu\, Q^\delta{}_\delta{}^\epsilon + 2V_{25}\,\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta}{}_\mu{}^\chi\, Q^\delta{}_\delta{}^\epsilon + \\
&\left(-7V_{11}-5V_{12}+4V_{13}-4V_{14}+4V_{16}-4V_{18}-4V_{19}+4V_{20}+3V_3+3V_4-2V_5+2V_7-V_8-V_9\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}_\mu{}^{\alpha\beta\chi}\, Q^\delta{}_\delta{}^\epsilon - \\
&2V_{32}\,\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta}{}_\alpha{}^\chi\, Q^{\delta\epsilon}{}_\epsilon - 4V_{33}\,\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi}{}_\alpha\, Q^{\delta\epsilon}{}_\epsilon + \left(V_{14}+V_{16}+V_{18}+V_{20}-3V_{21}-3V_{27}+3V_{34}+V_{35}+2V_{37}+2V_{38}\right)\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi}{}_\chi\, Q^{\delta\epsilon}{}_\epsilon + \\
&\frac{1}{2}\left(2V_{14}-V_{15}-2V_{16}-V_{18}-2V_{19}-5V_{20}+V_{28}+V_{29}+7V_{30}+3V_{31}-8V_{37}-8V_{38}+2V_6+4V_7\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi}{}_\mu\, Q^{\delta\epsilon}{}_\epsilon - 2V_{25}\,\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta}{}_\mu{}^\chi\, Q^{\delta\epsilon}{}_\epsilon + \\
&\frac{1}{2}\left(7V_{11}+3V_{12}-4V_{13}+4V_{14}-4V_{16}+4V_{18}+4V_{19}-4V_{20}-3V_3-3V_4+2V_5-2V_7+V_8+V_9\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}_\mu{}^{\alpha\beta\chi}\, Q^{\delta\epsilon}{}_\epsilon - \\
&2\left(V_{11}+V_{15}-2V_{28}+2V_{30}+3V_{32}+V_6-V_8\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_\alpha{}^\chi\, Q^\delta{}_\mu{}^\epsilon - 2\left(V_{12}+2V_{19}-2V_{29}+2V_{31}+6V_{33}+2V_7-V_9\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_\alpha\, Q^\delta{}_\mu{}^\epsilon + \\
&2\left(V_1+V_{14}+V_{18}-3V_{21}-V_{27}+3V_{34}+2V_{37}+2V_{38}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_\chi\, Q^\delta{}_\mu{}^\epsilon - 2\left(V_{15}+V_6\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\, Q^\epsilon{}_{\alpha\chi} -
\end{aligned}
$$

$$2\left(V_{15}+V_{16}+4V_{19}+V_{20}-3V_{23}+3V_{25}-3V_{29}+3V_{31}+3V_{32}+6V_{33}+V_{6}+4V_{7}\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\alpha\delta}+$$

$$\left(V_{11}+V_{12}+3V_{3}+3V_{4}-6V_{5}+6V_{7}+7V_{8}+5V_{9}\right)\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\alpha\epsilon}-2\left(V_{16}+V_{20}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\chi\delta}-$$

$$2V_{23}\,\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\chi\epsilon}+\left(3V_{15}+3V_{18}+6V_{19}+3V_{20}+7V_{28}+5V_{29}+V_{30}+V_{31}\right)\epsilon g_{\mu\alpha\beta\chi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\delta\epsilon}+$$

$$\left(-4V_{10}-7V_{11}-V_{12}+2V_{15}-4V_{18}-4V_{19}+4V_{20}+V_{3}+3V_{4}-2V_{5}+2V_{7}-V_{8}-V_{9}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\alpha}+$$

$$\left(2V_{13}+5V_{14}+V_{15}+V_{16}-2V_{20}-V_{22}-V_{23}-7V_{24}-V_{25}+8V_{38}-2V_{6}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\chi}+$$

$$\left(V_{15}+2V_{16}+V_{18}+2V_{19}+V_{20}-V_{28}-V_{29}-7V_{30}-V_{31}+8V_{37}-4V_{7}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\delta}-2\left(V_{12}-4V_{17}+V_{23}+V_{25}-V_{29}-V_{31}+V_{9}\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\epsilon}+$$

$$2\left(V_{11}+V_{8}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}_{\mu\alpha}{}^{\epsilon}-4\left(U_{10}-2U_{11}+U_{14}-2U_{15}\right)\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{Q}_{\mu\beta\chi}+2\left(U_{13}+U_{9}\right)\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{Q}_{\mu\beta\chi}+$$

$$2\left(V_{22}+V_{24}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}_{\mu\chi}{}^{\epsilon}+2\left(U_{10}-U_{11}+U_{14}-U_{15}-2U_{16}\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\beta}\,\mathcal{Q}_{\mu}{}^{\chi}{}_{\chi}+2\left(V_{28}+V_{30}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}_{\mu\delta}{}^{\epsilon}+$$

$$\left(V_{11}+V_{12}-2V_{17}+V_{22}+V_{23}+V_{24}+V_{25}-V_{28}-V_{29}-V_{30}-V_{31}+V_{8}+V_{9}\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}_{\mu}{}^{\epsilon}{}_{\epsilon}+2\left(2U_{4}+U_{5}\right)\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}_{\alpha\beta\chi}+$$

$$2\left(U_{5}+2U_{6}\right)\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}_{\alpha\beta\chi}-2\left(U_{5}+2U_{6}\right)\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\alpha\beta\chi}-2\left(V_{3}+V_{4}\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha\beta}{}^{\epsilon}+4\left(U_{2}+U_{3}\right)\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}_{\alpha\chi\beta}+$$

$$4\left(U_{2}+U_{3}\right)\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}_{\alpha\chi\beta}-2\left(2U_{4}+U_{5}\right)\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\alpha\chi\beta}-2\left(2V_{5}+V_{6}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha\chi}{}^{\epsilon}-4\left(U_{2}+U_{3}-U_{4}-U_{5}-U_{6}+U_{7}+U_{8}\right)\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\beta}\,\mathcal{T}_{\alpha}{}^{\chi}{}_{\chi}-$$

$$2\left(V_{6}+2V_{7}\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha\delta}{}^{\epsilon}-2\left(V_{6}+2V_{7}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}_{\alpha}{}^{\delta\epsilon}-2\left(2V_{5}+V_{6}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}_{\alpha}{}^{\delta\epsilon}+2\left(V_{3}+V_{4}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\alpha}{}^{\delta\epsilon}+$$

$$\left(V_{11}+V_{12}+3V_{3}+3V_{4}-6V_{5}+6V_{7}+5V_{8}+5V_{9}\right)\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha}{}^{\epsilon}{}_{\epsilon}+\left(-2U_{2}-2U_{3}+U_{7}+U_{8}\right)\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}_{\alpha\mu\beta}+$$

$$\left(V_{11}+V_{12}+V_{3}+V_{4}-2V_{5}+2V_{7}-V_{8}-V_{9}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha\mu}{}^{\epsilon}+\left(-10U_{11}-U_{13}-2U_{15}+8U_{2}+8U_{3}+4U_{4}+U_{5}-2U_{6}-5U_{9}\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+$$

$$\left(-10U_{10}-U_{13}-2U_{14}+8U_{2}+8U_{3}-2U_{4}+U_{5}+4U_{6}-5U_{9}\right)\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+\left(2U_{11}+5U_{13}+10U_{15}-3U_{5}-6U_{6}+U_{9}\right)\mathcal{F}_{\mu}{}^{\alpha}{}_{\alpha}{}^{\beta}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+$$

$$\left(2U_{10}+5U_{13}+10U_{14}-6U_{4}-3U_{5}+U_{9}\right)\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\alpha}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}-2\left(2V_{13}+V_{15}\right)\epsilon g_{\alpha\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\beta}{}^{\delta\epsilon}+\left(-2U_{11}+U_{13}+2U_{15}+U_{5}+2U_{6}-U_{9}\right)\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}_{\beta\mu\chi}+$$

$$\left(-2U_{10}+U_{13}+2U_{14}+2U_{4}+U_{5}-U_{9}\right)\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}_{\beta\mu\chi}-2\left(2V_{13}+V_{15}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\chi\alpha}{}^{\epsilon}-2\left(V_{14}+V_{16}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\chi\delta}{}^{\epsilon}-2\left(V_{14}+V_{16}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}_{\chi}{}^{\delta\epsilon}-$$

$$2\left(V_{18}+V_{20}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}_{\chi}{}^{\delta\epsilon}-2\left(V_{15}+2V_{19}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\chi}{}^{\delta\epsilon}+\left(6V_{13}+3V_{14}+3V_{15}+3V_{16}+5V_{22}+5V_{23}+V_{24}+V_{25}\right)\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\chi}{}^{\epsilon}{}_{\epsilon}+$$

$$\left(2V_{13}+V_{14}+V_{15}+V_{16}-V_{22}-V_{23}+V_{24}+V_{25}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\chi\mu}{}^{\epsilon}-2\left(V_{24}+V_{30}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\,\mathcal{T}^{\chi\delta\epsilon}-2\left(V_{25}+V_{31}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\,\mathcal{T}^{\chi\delta\epsilon}+$$

$$2\left(V_{22}+V_{28}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha}{}_{\alpha}{}^{\beta}\,\mathcal{T}^{\chi\delta\epsilon}+2\left(V_{23}+V_{29}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\alpha}\,\mathcal{T}^{\chi\delta\epsilon}-2\left(V_{21}+V_{27}\right)\epsilon g_{\alpha\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\beta}\,\mathcal{T}^{\chi\delta\epsilon}-2\left(V_{15}+2V_{19}\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\delta\alpha}{}^{\epsilon}-$$

$$2\left(V_{18}+V_{20}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\delta\chi}{}^{\epsilon}+\left(3V_{15}+3V_{18}+6V_{19}+3V_{20}+5V_{28}+5V_{29}+V_{30}+V_{31}\right)\epsilon g_{\mu\alpha\beta\chi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\delta}{}^{\epsilon}{}_{\epsilon}+$$

$$\left(V_{15}+V_{18}+2V_{19}+V_{20}-V_{28}-V_{29}+V_{30}+V_{31}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\delta\mu}{}^{\epsilon}-2\left(V_{21}+V_{27}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}^{\delta}{}_{\alpha}{}^{\epsilon}+2\left(V_{16}+V_{20}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}^{\delta}{}_{\alpha}{}^{\epsilon}+$$

$$\left(-2V_{13}-3V_{14}-V_{15}-V_{16}+2V_{20}+V_{22}+V_{23}+7V_{24}+7V_{25}-8V_{37}-8V_{38}+4V_{5}+2V_{6}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\alpha}{}^{\epsilon}+$$

$$\left(-2V_{13}-V_{14}-V_{15}-V_{18}+V_{22}-3V_{24}-6V_{26}+V_{28}-3V_{30}-3V_{32}-2V_{5}-V_{6}\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\beta}{}^{\epsilon}+$$

$$\left(-V_{15}-V_{16}-2V_{19}-V_{20}+V_{23}-3V_{25}+V_{29}-3V_{31}-3V_{32}-6V_{33}-V_{6}-2V_{7}\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}^{\delta}{}_{\beta}{}^{\epsilon}+$$

$$2\left(V_{14}+V_{18}\right)\epsilon g_{\alpha\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}^{\delta}{}_{\beta}{}^{\epsilon}+\left(2V_{13}+V_{14}+V_{15}+V_{18}-3V_{22}+V_{24}+6V_{26}-3V_{28}+V_{30}+3V_{32}+2V_{5}+V_{6}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+$$

$$\left(V_{15}+V_{16}+2V_{19}+V_{20}-3V_{23}+V_{25}-3V_{29}+V_{31}+3V_{32}+6V_{33}+V_{6}+2V_{7}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+4\left(V_{37}+V_{38}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+$$

$$\frac{1}{2}\left(-2V_{13}-5V_{14}-V_{15}-V_{16}-2V_{18}+2V_{20}+V_{22}+V_{23}+7V_{24}+7V_{25}+4V_{5}+2V_{6}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+2\left(V_{16}+V_{20}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}-$$

$$2\left(2V_{26}+V_{32}\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}-2\left(V_{32}+2V_{33}\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+2\left(V_{14}+V_{16}+V_{18}+V_{20}-3V_{21}-3V_{27}+2V_{34}+2V_{35}+2V_{37}+2V_{38}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+$$

$$\left(2V_{14}-V_{15}-2V_{16}-V_{18}-2V_{19}-5V_{20}+V_{28}+V_{29}+5V_{30}+5V_{31}-8V_{37}-8V_{38}+2V_{6}+4V_{7}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}-2\left(V_{24}+V_{25}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+$$

$$\left(5V_{11}+5V_{12}-4V_{13}+4V_{14}-4V_{16}+4V_{18}+4V_{19}-4V_{20}-3V_{3}-3V_{4}+2V_{5}-2V_{7}+V_{8}+V_{9}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+$$

$$\left(2V_{13}+V_{15}-2V_{22}+2V_{24}+8V_{26}-2V_{28}+2V_{30}+4V_{32}+2V_{5}+V_{6}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\mu}{}^{\epsilon}+$$

$$\left(V_{15}+2V_{19}-2V_{23}+2V_{25}-2V_{29}+2V_{31}+4V_{32}+8V_{33}+V_{6}+2V_{7}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}^{\delta}{}_{\mu}{}^{\epsilon}+\left(V_{21}+V_{27}+V_{34}+V_{35}-2V_{37}-2V_{38}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}^{\delta}{}_{\mu}{}^{\epsilon}-$$

$$2\left(U_{7}+U_{8}\right)\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}_{\mu\alpha\beta}-2\left(V_{11}+V_{12}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu\alpha}{}^{\epsilon}-2\left(U_{13}+2U_{15}\right)\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}_{\mu\beta\chi}-2\left(U_{13}+2U_{14}\right)\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}_{\mu\beta\chi}-2\left(2U_{11}+U_{9}\right)\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}_{\mu\chi\beta}-$$

$$2\left(2U_{10}+U_{9}\right)\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}_{\mu\chi\beta}-2\left(V_{24}+V_{25}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu\chi}{}^{\epsilon}-4\left(U_{10}-U_{11}+U_{14}-U_{15}\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\beta}\,\mathcal{T}_{\mu}{}^{\chi}{}_{\chi}-2\left(V_{30}+V_{31}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu\delta}{}^{\epsilon}-$$

$$2\left(2V_{26}+V_{32}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}_{\mu}{}^{\delta\epsilon}-2\left(V_{32}+2V_{33}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}_{\mu}{}^{\delta\epsilon}-2\left(V_{34}+V_{35}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}_{\mu}{}^{\delta\epsilon}-2\left(V_{8}+V_{9}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu}{}^{\epsilon}{}_{\alpha}-$$

$$2\left(V_{22}+V_{23}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu}{}^{\epsilon}{}_{\chi}-2\left(V_{28}+V_{29}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu}{}^{\epsilon}{}_{\delta}-2\left(V_{11}+V_{12}+V_{22}+V_{23}+V_{24}+V_{25}-V_{28}-V_{29}-V_{30}-V_{31}+V_{8}+V_{9}\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu}{}^{\epsilon}{}_{\epsilon}+$$

$$\left(2V_{14}-V_{15}-2V_{16}-V_{18}-2V_{19}-5V_{20}+V_{28}+V_{29}+7V_{30}+7V_{31}-8V_{37}-8V_{38}+2V_{6}+4V_{7}\right)g^{\alpha\beta}\,\mathcal{F}^{\chi\delta\epsilon}{}_{\mu}\,\nabla_{\beta}\epsilon g_{\alpha\chi\delta\epsilon}+$$

$$\left(7V_{11}+7V_{12}-4V_{13}+4V_{14}-4V_{16}+4V_{18}+4V_{19}-4V_{20}-3V_{3}-3V_{4}+2V_{5}-2V_{7}+V_{8}+V_{9}\right)g^{\alpha\beta}\,\mathcal{F}_{\mu}{}^{\chi\delta\epsilon}\,\nabla_{\beta}\epsilon g_{\alpha\chi\delta\epsilon}+$$

$$\left(-V_{11}-V_{12}-3V_{3}-3V_{4}+6V_{5}-6V_{7}-7V_{8}-7V_{9}\right)\mathcal{F}^{\alpha\beta\chi\delta}\,\nabla_{\beta}\epsilon g_{\mu\alpha\chi\delta}-2\left(V_{14}+V_{16}+V_{18}+V_{20}-3V_{21}-3V_{27}+3V_{34}+3V_{35}+2V_{37}+2V_{38}\right)g^{\alpha\beta}\,\mathcal{F}^{\chi\delta\epsilon}{}_{\epsilon}\,\nabla_{\beta}\epsilon g_{\mu\alpha\chi\delta}+$$

$$\left(14U_{11}+U_{13}+2U_{15}-8U_{2}-8U_{3}-4U_{4}-U_{5}+2U_{6}+7U_{9}\right)g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\alpha}{}^{\chi}{}_{\chi\mu}+\left(V_{11}+V_{12}+3V_{3}+3V_{4}-6V_{5}+6V_{7}+7V_{8}+7V_{9}\right)\epsilon g_{\mu\chi\delta\epsilon}\,g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\alpha}{}^{\chi\delta\epsilon}+$$

$$\left(3V_{15}+3V_{18}+6V_{19}+3V_{20}+7V_{28}+7V_{29}+V_{30}+V_{31}\right)\epsilon g_{\mu\chi\delta\epsilon}\,g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}^{\chi\delta\epsilon}{}_{\alpha}+\left(-4U_{2}-4U_{3}+4U_{4}+4U_{5}+4U_{6}-6U_{7}-6U_{8}\right)g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\mu\alpha}{}^{\chi}{}_{\chi}+$$

$$\left(2U_{11}+7U_{13}+14U_{15}-3U_{5}-6U_{6}+U_{9}\right)g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\mu}{}^{\chi}{}_{\chi\alpha}+\left(6V_{13}+3V_{14}+3V_{15}+3V_{16}+7V_{22}+7V_{23}+V_{24}+V_{25}\right)\mathcal{F}^{\alpha\beta\chi\delta}\,\nabla_{\chi}\epsilon g_{\mu\alpha\beta\delta}+$$

$$\left(6V_{13}+3V_{14}+3V_{15}+3V_{16}+7V_{22}+7V_{23}+V_{24}+V_{25}\right)\epsilon g_{\mu\alpha\beta\delta}\,\nabla_{\chi}\mathcal{F}^{\alpha\beta\chi\delta}+\left(2U_{10}+7U_{13}+14U_{14}-6U_{4}-3U_{5}+U_{9}\right)g^{\alpha\beta}\,\nabla_{\chi}\mathcal{F}_{\mu\alpha}{}^{\chi}{}_{\beta}+$$

$$\left(3V_{15}+3V_{18}+6V_{19}+3V_{20}+7V_{28}+7V_{29}+V_{30}+V_{31}\right)\mathcal{F}^{\alpha\beta\chi\delta}\,\nabla_{\delta}\epsilon g_{\mu\alpha\beta\chi}+\left(-14U_{10}-U_{13}-2U_{14}+8U_{2}+8U_{3}-2U_{4}+U_{5}+4U_{6}-7U_{9}\right)g^{\alpha\beta}\,g^{\chi\delta}\,\nabla_{\delta}\mathcal{F}_{\alpha\chi\mu\beta}-$$

$$2\left(2V_{13}+V_{14}+V_{15}+V_{18}-3V_{22}+3V_{24}+6V_{26}-3V_{28}+3V_{30}+3V_{32}+2V_{5}+V_{6}\right)\epsilon g_{\mu\beta\chi\delta}\,\nabla^{\delta}\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}-$$

$$2\left(V_{14}+V_{16}+V_{18}+V_{20}-3V_{21}-3V_{27}+3V_{34}+3V_{35}+2V_{37}+2V_{38}\right)\epsilon g_{\mu\alpha\beta\delta}\,\nabla^{\delta}\mathcal{F}^{\alpha\beta\chi}{}_{\chi}+$$

$$\left(-2 V_{14} + V_{15} + 2 V_{16} + V_{18} + 2 V_{19} + 5 V_{20} - V_{28} - V_{29} - 7 V_{30} - 7 V_{31} + 8 V_{37} + 8 V_{38} - 2 V_6 - 4 V_7\right) \epsilon g_{\alpha\beta\chi\delta}\, \nabla^{\delta}\mathcal{F}^{\alpha\beta\chi}{}_{\mu} +$$
$$\left(2 V_{13} + 5 V_{14} + V_{15} + V_{16} + 2 V_{18} - 2 V_{20} - V_{22} - V_{23} - 7 V_{24} - 7 V_{25} + 8 V_{37} + 8 V_{38} - 4 V_5 - 2 V_6\right) \epsilon g_{\alpha\beta\chi\delta}\, \nabla^{\delta}\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi} +$$
$$\left(-7 V_{11} - 7 V_{12} + 4 V_{13} - 4 V_{14} + 4 V_{16} - 4 V_{18} - 4 V_{19} + 4 V_{20} + 3 V_3 + 3 V_4 - 2 V_5 + 2 V_7 - V_8 - V_9\right) \epsilon g_{\alpha\beta\chi\delta}\, \nabla^{\delta}\mathcal{F}_{\mu}{}^{\alpha\beta\chi} -$$
$$2\left(V_{15} + V_{16} + 2 V_{19} + V_{20} - 3 V_{23} + 3 V_{25} - 3 V_{29} + 3 V_{31} + 3 V_{32} + 6 V_{33} + V_6 + 2 V_7\right) \epsilon g_{\mu\chi\delta\epsilon}\, g^{\alpha\beta}\, \nabla^{\epsilon}\mathcal{F}_{\alpha}{}^{\chi\delta}{}_{\beta} -$$
$$2\left(V_{11} + V_{12} + V_{22} + V_{23} + V_{24} + V_{25} - V_{28} - V_{29} - V_{30} - V_{31} + V_8 + V_9\right) \mathcal{F}^{\alpha\beta\chi\delta}\, \nabla_{\mu}\epsilon g_{\alpha\beta\chi\delta} + 4\left(U_{10} - U_{11} + U_{14} - U_{15}\right) g^{\alpha\beta}\, \nabla_{\mu}\mathcal{F}_{\alpha}{}^{\chi}{}_{\chi\beta} -$$
$$2\left(V_{11} + V_{12} + V_{22} + V_{23} + V_{24} + V_{25} - V_{28} - V_{29} - V_{30} - V_{31} + V_8 + V_9\right) \epsilon g_{\alpha\beta\chi\delta}\, \nabla_{\mu}\mathcal{F}^{\alpha\beta\chi\delta} == 0$$

For general values of the fields, the tensor equation Eq. (55) requires the following conditions among the Lagrangian parameters in the ansatz.

$U_{10} == 0$	$U_{11} == 0$	$U_{12} == -U_6$
$U_{13} == 0$	$U_{14} == 0$	$U_{15} == 0$
$U_{16} == 0$	$U_2 == -\frac{U_6}{2}$	$U_3 == \frac{U_6}{2}$
$U_4 == U_6$	$U_5 == -2\, U_6$	$U_7 == 0$
$U_8 == 0$	$U_9 == 0$	$V_1 == -2\, V_{27}$
$V_{10} == V_7$	$V_{11} == 0$	$V_{12} == 0$
$V_{13} == -V_7$	$V_{14} == -V_7$	$V_{15} == 2\, V_7$
$V_{16} == V_7$	$V_{17} == 0$	$V_{18} == V_7$
$V_{19} == -V_7$	$V_2 == \frac{V_7}{2}$	$V_{20} == -V_7$
$V_{21} == -V_{27}$	$V_{22} == 0$	$V_{23} == 0$
$V_{24} == 0$	$V_{25} == 0$	$V_{26} == 0$
$V_{28} == 0$	$V_{29} == 0$	$V_3 == -2\, V_7$
$V_{30} == 0$	$V_{31} == 0$	$V_{32} == 0$
$V_{33} == 0$	$V_{34} == 0$	$V_{35} == 0$
$V_{37} == \frac{V_7}{4}$	$V_{38} == -\frac{V_7}{4}$	$V_4 == 2\, V_7$
$V_5 == V_7$	$V_6 == -2\, V_7$	$V_8 == 0$
$V_9 == 0$		

(56)

Thus our apparent finding is that Eq. (56) are the most general conditions for concurrent symmetry. Later on we'll refine this statement to arrive at the theory in Eq. (C4) our manuscript.

The extended projective symmetry

Concrete relation to manuscript: Now we take Eq. (56) and we combine it with Eq. (52) to produce the following. This is the extended projective symmetry as set out in Eq. (9) on page 3 of our manuscript.

$U_1 == 0$	$U_{10} == 0$	$U_{11} == 0$
$U_{12} == -U_6$	$U_{13} == 0$	$U_{14} == 0$
$U_{15} == 0$	$U_{16} == 0$	$U_2 == -\frac{U_6}{2}$
$U_3 == \frac{U_6}{2}$	$U_4 == U_6$	$U_5 == -2\, U_6$
$U_7 == 0$	$U_8 == 0$	$U_9 == 0$
$V_1 == 0$	$V_{10} == V_7$	$V_{11} == 0$
$V_{12} == 0$	$V_{13} == -V_7$	$V_{14} == -V_7$
$V_{15} == 2\, V_7$	$V_{16} == V_7$	$V_{17} == 0$
$V_{18} == V_7$	$V_{19} == -V_7$	$V_2 == \frac{V_7}{2}$
$V_{20} == -V_7$	$V_{21} == 0$	$V_{22} == 0$
$V_{23} == 0$	$V_{24} == 0$	$V_{25} == 0$
$V_{26} == 0$	$V_{27} == 0$	$V_{28} == 0$
$V_{29} == 0$	$V_3 == -2\, V_7$	$V_{30} == 0$
$V_{31} == 0$	$V_{32} == 0$	$V_{33} == 0$
$V_{34} == 0$	$V_{35} == 0$	$V_{36} == 0$
$V_{37} == \frac{V_7}{4}$	$V_{38} == -\frac{V_7}{4}$	$V_4 == 2\, V_7$
$V_5 == V_7$	$V_6 == -2\, V_7$	$V_8 == 0$
$V_9 == 0$		

(57)

Thus our apparent finding is that Eq. (57) are the most general conditions for extended projective symmetry. Later on we'll refine this statement to arrive at the theory in Eq. (11) our manuscript.

The pure-torsion symmetry

We want to find the iso-Weyl plane of symmetries, just as we did the extended projective plane. For this we need an extra theory in this plane (so far we only have the concurrent theory in Eq. (56)). We will pick (arbitrarily) the theory with a symmetry entirely along the torsion direction.

Infinitesimal transformations of fundamental fields.

$g_{\alpha\beta}\to g_{\alpha\beta}$	$\mathcal{A}_{\mu}{}^{\rho}{}_{\nu}\to\mathcal{A}_{\mu}{}^{\rho}{}_{\nu}+\epsilon\,\delta_{\mu}^{\rho}\,\lambda_{\nu}-\epsilon\,g_{\mu\nu}\,\lambda^{\rho}$

(58)

Infinitesimal transformations of field strengths induced by Eq. (49).

$\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma}\to\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma}+\epsilon\,g^{\rho\alpha}\,\lambda_{\alpha}\,Q_{\mu\nu\sigma}-\epsilon\,g^{\rho\alpha}\,\lambda_{\alpha}\,Q_{\nu\mu\sigma}-\epsilon\,g_{\beta\sigma}\,g^{\rho\alpha}\,\lambda_{\alpha}\,\mathcal{T}_{\mu}{}^{\beta}{}_{\nu}+\epsilon\,\lambda_{\sigma}\,\mathcal{T}_{\mu}{}^{\rho}{}_{\nu}-\epsilon\,g_{\nu\sigma}\,(g^{\alpha\chi}\,g^{\rho\beta}\,\lambda_{\alpha}\,Q_{\mu\beta\chi}+g^{\rho\alpha}\,\nabla_{\mu}\lambda_{\alpha})+\epsilon\,\delta_{\nu}^{\rho}\,\nabla_{\mu}\lambda_{\sigma}+\epsilon\,g_{\mu\sigma}\,(g^{\alpha\chi}\,g^{\rho\beta}\,\lambda_{\alpha}\,Q_{\nu\beta\chi}+g^{\rho\alpha}\,\nabla_{\nu}\lambda_{\alpha})-\epsilon\,\delta_{\mu}^{\rho}\,\nabla_{\nu}\lambda_{\sigma}$	$\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}\to-\epsilon\,\delta^{\alpha}{}_{\nu}\,\lambda_{\mu}+\epsilon\,\delta^{\alpha}{}_{\mu}\,\lambda_{\nu}+\mathcal{T}_{\mu}{}^{\alpha}{}_{\nu}$	$Q_{\lambda\mu\nu}\to Q_{\lambda\mu\nu}$
$\mathcal{T}_{\mu}\to 3\,\epsilon\,\lambda_{\mu}+\mathcal{T}_{\mu}$	$Q_{\mu}\to Q_{\mu}$	$\tilde{Q}_{\mu}\to\tilde{Q}_{\mu}$
$\mathcal{F}_{\mu\nu}\to\epsilon\,g^{\alpha\beta}\,\lambda_{\beta}\,Q_{\mu\nu\alpha}-\epsilon\,g^{\alpha\beta}\,\lambda_{\beta}\,Q_{\nu\mu\alpha}+\mathcal{F}_{\mu\nu}-\epsilon\,g_{\nu\alpha}\,(g^{\alpha\chi}\,g^{\beta\delta}\,\lambda_{\beta}\,Q_{\mu\chi\delta}+g^{\alpha\beta}\,\nabla_{\mu}\lambda_{\beta})+\epsilon\nabla_{\mu}\lambda_{\nu}+\epsilon\,g_{\mu\alpha}\,(g^{\alpha\chi}\,g^{\beta\delta}\,\lambda_{\beta}\,Q_{\nu\chi\delta}+g^{\alpha\beta}\,\nabla_{\nu}\lambda_{\beta})-\epsilon\nabla_{\nu}\lambda_{\mu}$	$\mathcal{F}^{(14)}{}_{\mu\nu}\to\frac{1}{2}\,\epsilon\,g^{\alpha\beta}\,\lambda_{\nu}\,Q_{\alpha\mu\beta}+\frac{1}{2}\,\epsilon\,g^{\alpha\chi}\,g^{\beta\delta}\,g_{\mu\nu}\,\lambda_{\chi}\,Q_{\beta\alpha\delta}+\frac{1}{2}\,\epsilon\,g^{\alpha\beta}\,\lambda_{\nu}\,Q_{\beta\mu\alpha}-\frac{3}{2}\,\epsilon\,g^{\alpha\chi}\,g^{\beta\delta}\,g_{\mu\nu}\,\lambda_{\chi}\,Q_{\delta\alpha\beta}-\epsilon\,g^{\alpha\beta}\,\lambda_{\nu}\,Q_{\mu\beta\alpha}+\mathcal{F}^{(14)}{}_{\mu\nu}+\epsilon\,\lambda_{\nu}\,\mathcal{T}_{\mu}{}^{\alpha}{}_{\alpha}-\epsilon\,g^{\alpha\beta}\,g_{\chi\nu}\,\lambda_{\beta}\,\mathcal{T}_{\mu}{}^{\chi}{}_{\alpha}+\epsilon\,g_{\alpha\beta}\,g_{\mu\nu}\,(g^{\alpha\delta}\,g^{\beta\phi}\,g^{\chi\epsilon}\,\lambda_{\chi}\,Q_{\phi\delta\epsilon}+g^{\alpha\chi}\,g^{\beta\gamma}\,\nabla_{\gamma}\lambda_{\chi})+3\,\epsilon\,g_{\nu\alpha}\,(g^{\alpha\chi}\,g^{\beta\delta}\,\lambda_{\beta}\,Q_{\mu\chi\delta}+g^{\alpha\beta}\,\nabla_{\mu}\lambda_{\beta})-\epsilon\nabla_{\mu}\lambda_{\nu}$	$\mathcal{F}^{(13)}{}_{\mu\nu}\to\epsilon\,g^{\alpha\beta}\,\lambda_{\beta}\,Q_{\alpha\mu\nu}-\epsilon\,g^{\alpha\beta}\,\lambda_{\beta}\,Q_{\mu\nu\alpha}+\mathcal{F}^{(13)}{}_{\mu\nu}-\epsilon\,\lambda_{\nu}\,\mathcal{T}_{\mu}{}^{\alpha}{}_{\alpha}+\epsilon\,g^{\alpha\beta}\,g_{\chi\nu}\,\lambda_{\beta}\,\mathcal{T}_{\mu}{}^{\chi}{}_{\alpha}-\epsilon\,g_{\mu\nu}\,(g^{\alpha\chi}\,g^{\beta\delta}\,\lambda_{\beta}\,Q_{\alpha\chi\delta}+g^{\alpha\beta}\,\nabla_{\alpha}\lambda_{\beta})+\epsilon\,g_{\nu\alpha}\,(g^{\alpha\chi}\,g^{\beta\delta}\,\lambda_{\beta}\,Q_{\mu\chi\delta}+g^{\alpha\beta}\,\nabla_{\mu}\lambda_{\beta})-3\,\epsilon\nabla_{\mu}\lambda_{\nu}$
$\mathcal{F}\to\epsilon\,g^{\alpha\chi}\,g^{\beta\delta}\,\lambda_{\chi}\,Q_{\alpha\delta\beta}-2\,\epsilon\,g^{\alpha\chi}\,g^{\beta\delta}\,\lambda_{\chi}\,Q_{\beta\alpha\delta}+4\,\epsilon\,g^{\alpha\chi}\,g^{\beta\delta}\,\lambda_{\chi}\,Q_{\delta\alpha\beta}+\mathcal{F}-2\,\epsilon\,g^{\alpha\chi}\,\lambda_{\chi}\,\mathcal{T}_{\alpha}{}^{\beta}{}_{\beta}-3\,\epsilon(g^{\alpha\chi}\,g^{\beta\delta}\,\lambda_{\beta}\,Q_{\alpha\chi\delta}+g^{\alpha\beta}\,\nabla_{\alpha}\lambda_{\beta})-3\,\epsilon\,g_{\alpha\beta}\,(g^{\alpha\delta}\,g^{\beta\phi}\,g^{\chi\epsilon}\,\lambda_{\chi}\,Q_{\phi\delta\epsilon}+g^{\alpha\chi}\,g^{\beta\gamma}\,\nabla_{\gamma}\lambda_{\chi})$		

(59)

Invariance of Eq. (47) requires the following.

$$
\begin{aligned}
&(4U_3+U_5)\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,Q_{\alpha\beta\chi}+(-4U_{10}+8U_2-4U_3-U_5+4U_6+2U_9)\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,Q_{\alpha\beta\chi}+(-2U_4-U_5)\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,Q_{\alpha\beta\chi}+(4V_2-V_4)\,\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\alpha\beta}{}^{\epsilon}+\\
&(V_3-V_6)\,\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\alpha\chi}{}^{\epsilon}+(U_4-U_6+U_7)\,\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\beta}\,Q_{\alpha}{}^{\chi}{}_{\chi}+(V_4-2V_7)\,\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\alpha\delta}{}^{\epsilon}+\left(-4V_2+\frac{V_3}{2}-\frac{V_4}{2}+V_5-V_6+V_7-V_8\right)\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\alpha}{}^{\epsilon}{}_{\epsilon}+\\
&(-U_5-2U_6+2U_7+U_8)\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,Q_{\alpha\mu\beta}+(-4V_2-V_4+2V_5-V_6-2V_8-V_9)\,\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\alpha\mu}{}^{\epsilon}+(6U_{12}+2U_{13}-4U_{14}+4U_4-U_5)\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,Q_{\beta\alpha\chi}+\\
&\frac{1}{2}(4U_{11}-4U_2+4U_3-2U_4+U_5)\,\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\,Q_{\beta}{}^{\chi}{}_{\chi}+\left(2U_2-2U_3-\frac{U_5}{2}+U_6+U_9\right)\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\,Q_{\beta}{}^{\chi}{}_{\chi}+\left(2U_{12}-2U_{15}-\frac{U_5}{2}+U_6\right)\mathcal{F}_{\mu}{}^{\alpha}{}_{\alpha}{}^{\beta}\,Q_{\beta}{}^{\chi}{}_{\chi}+\\
&\frac{1}{2}(-4U_{12}-2U_{13}-2U_4+U_5)\,\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\alpha}\,Q_{\beta}{}^{\chi}{}_{\chi}+(-4U_{11}+4U_2+2U_4-U_9)\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,Q_{\beta\mu\chi}+(-2U_{10}+4U_3+U_5-2U_9)\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,Q_{\beta\mu\chi}+(-2U_{12}+U_5)\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,Q_{\chi\alpha\beta}+\\
&(-V_{15}-V_3)\,\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\chi\alpha}{}^{\epsilon}+(V_{10}-V_{16})\,\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\chi\delta}{}^{\epsilon}+\frac{1}{2}(-2V_{10}-2V_{13}-V_{14}+V_{15}+V_{16}-2V_{22}+2V_3)\,\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\chi}{}^{\epsilon}{}_{\epsilon}+\\
&(2U_{12}-U_{13}-4U_{15}+2U_6)\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,Q_{\chi\mu\beta}+(-2U_{12}-2U_{13}-2U_{14}+U_5)\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,Q_{\chi\mu\beta}+(-V_{10}-2V_{13}-V_{14}-2V_{22}-V_{23}+V_3)\,\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\chi\mu}{}^{\epsilon}+\\
&(-2U_4+2U_6-2U_7+U_8)\,\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\beta}\,Q^{\chi}{}_{\alpha\chi}+(-4U_{11}+4U_2-4U_3+2U_4-U_5+U_9)\,\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\,Q^{\chi}{}_{\beta\chi}+(2U_{10}-4U_2+4U_3+U_5-2U_6-2U_9)\,\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\,Q^{\chi}{}_{\beta\chi}+\\
&(-4U_{12}-U_{13}+4U_{15}+U_5-2U_6)\,\mathcal{F}_{\mu}{}^{\alpha}{}_{\alpha}{}^{\beta}\,Q^{\chi}{}_{\beta\chi}+2U_{14}\,\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\alpha}\,Q^{\chi}{}_{\beta\chi}+(-2U_{10}+U_{13}-2U_{14}+4U_{16}+U_9)\,\mathcal{F}^{\alpha\beta}{}_{\alpha\beta}\,Q^{\chi}{}_{\mu\chi}+(-2V_{19}-V_4)\,\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\delta\alpha}{}^{\epsilon}+\\
&(-V_{10}-V_{20})\,\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\delta\chi}{}^{\epsilon}+\left(V_{10}-\frac{V_{15}}{2}-\frac{V_{18}}{2}+V_{19}+\frac{V_{20}}{2}-V_{28}+V_4\right)\epsilon g_{\mu\alpha\beta\chi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\delta}{}^{\epsilon}{}_{\epsilon}+(V_{10}-V_{15}-V_{18}-2V_{28}-V_{29}+V_4)\,\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,Q_{\delta\mu}{}^{\epsilon}+\\
&(-V_1-2V_{27})\,\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,Q^{\delta}{}_{\alpha}{}^{\epsilon}+(2V_{20}-V_6)\,\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,Q^{\delta}{}_{\alpha}{}^{\epsilon}+2(2V_{14}-V_{20}-2V_{24}+2V_{25}-4V_{37}+4V_{38}+V_5-V_6)\,\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,Q^{\delta}{}_{\alpha}{}^{\epsilon}+\\
&(2V_{10}+V_3)\,\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,Q^{\delta}{}_{\alpha}{}^{\epsilon}+(-2V_{11}+2V_{13}-V_{15}-2V_{22}+2V_{24}+4V_{26}-2V_{30}-2V_{32}+2V_5-V_6+V_8)\,\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,Q^{\delta}{}_{\beta}{}^{\epsilon}+\\
&(-2V_{12}+V_{15}-2V_{19}-2V_{23}+2V_{25}-2V_{31}+2V_{32}-4V_{33}+V_6-2V_7+V_9)\,\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,Q^{\delta}{}_{\beta}{}^{\epsilon}+\\
&(-2V_{13}+2V_{14})\,\epsilon g_{\alpha\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,Q^{\delta}{}_{\beta}{}^{\epsilon}+(V_{11}-2V_{13}+V_{15}+2V_{22}-2V_{24}-4V_{26}-2V_{28}+2V_{32}-2V_5+V_6-2V_8)\,\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,Q^{\delta}{}_{\chi}{}^{\epsilon}+\\
&(V_{12}-V_{15}+2V_{19}+2V_{23}-2V_{25}-2V_{29}-2V_{32}+4V_{33}-V_6+2V_7-2V_9)\,\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,Q^{\delta}{}_{\chi}{}^{\epsilon}+(-V_{14}+4V_{38})\,\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,Q^{\delta}{}_{\chi}{}^{\epsilon}+\\
&(2V_{14}-2V_{18}-V_{20}-2V_{24}+2V_{25}+4V_{38}+2V_5-V_6)\,\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,Q^{\delta}{}_{\chi}{}^{\epsilon}+(-V_{15}+2V_{16})\,\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,Q^{\delta}{}_{\chi}{}^{\epsilon}-V_{32}\,\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,Q^{\delta}{}_{\delta}{}^{\epsilon}-2V_{33}\,\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,Q^{\delta}{}_{\delta}{}^{\epsilon}+\\
&(-2V_1-V_{14}+V_{16}-V_{18}+V_{20}+2V_{21}-2V_{27}-2V_{34}+V_{35})\,\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,Q^{\delta}{}_{\delta}{}^{\epsilon}+(V_{14}+V_{16}-2V_{20}-2V_{30}+V_{31}+4V_{37}-4V_{38}+V_6-2V_7)\,\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,Q^{\delta}{}_{\delta}{}^{\epsilon}-\\
&V_{25}\,\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,Q^{\delta}{}_{\delta}{}^{\epsilon}+(-4V_{10}-2V_{11}+V_{12}-2V_{13}+2V_{14}+2V_{15}-2V_{16}-2V_{18}-2V_{19}+2V_{20}-V_3+V_4)\,\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,Q^{\delta}{}_{\delta}{}^{\epsilon}+\\
&V_{32}\,\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,Q^{\delta\epsilon}{}_{\epsilon}+2V_{33}\,\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,Q^{\delta\epsilon}{}_{\epsilon}+\frac{1}{2}(2V_1+V_{14}-V_{16}+V_{18}-V_{20}-2V_{21}+2V_{27}+2V_{34})\,\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,Q^{\delta\epsilon}{}_{\epsilon}+
\end{aligned}
$$

$$\left(-\frac{V_{14}}{2}-\frac{V_{16}}{2}+V_{20}+V_{30}-2V_{37}+2V_{38}-\frac{V_6}{2}+V_7\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{Q}^{\delta\epsilon}{}_{\epsilon}+V_{25}\,\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{Q}^{\delta\epsilon}{}_{\epsilon}+$$

$$\left(2V_{10}+V_{11}+V_{13}-V_{14}-V_{15}+V_{16}+V_{18}+V_{19}-V_{20}+\frac{V_3}{2}-\frac{V_4}{2}\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{Q}^{\delta\epsilon}{}_{\epsilon}+\left(V_{11}+V_{15}-2V_{28}+2V_{30}+3V_{32}+V_6-V_8\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{Q}^{\delta}{}_{\mu}{}^{\epsilon}+$$

$$\left(V_{12}+2V_{19}-2V_{29}+2V_{31}+6V_{33}+2V_7-V_9\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{Q}^{\delta}{}_{\mu}{}^{\epsilon}+\left(V_1+V_{14}+V_{18}-2V_{21}+2V_{34}+V_{35}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{Q}^{\delta}{}_{\mu}{}^{\epsilon}+$$

$$\left(V_{15}+V_6\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\alpha\chi}+\left(2V_{12}-V_{15}+4V_{19}+2V_{23}-2V_{25}-2V_{29}+2V_{31}-2V_{32}+4V_{33}-V_6+4V_7-2V_9\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\alpha\delta}+$$

$$\left(8V_2-V_3+V_4-2V_5+2V_6-2V_7+2V_8-V_9\right)\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\alpha\epsilon}+\left(V_{16}+V_{20}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\chi\delta}+V_{23}\,\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\chi\epsilon}+$$

$$\left(-2V_{10}+V_{15}+V_{18}-2V_{19}-V_{20}+2V_{28}-V_{29}-2V_4\right)\epsilon g_{\mu\alpha\beta\chi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\delta\epsilon}+\left(-2V_{10}-2V_{11}-V_{12}+V_{15}-2V_{18}-2V_{19}+2V_{20}+V_4\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\alpha}+$$

$$\left(2V_{14}-V_{20}-2V_{24}-V_{25}+4V_{38}-V_6\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\chi}+\left(V_{16}-2V_{30}-V_{31}+4V_{37}-2V_7\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\delta}+$$

$$\left(-V_{12}+2V_{17}-V_{23}-V_{25}+V_{29}+V_{31}-V_9\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}^{\epsilon}{}_{\mu\epsilon}+\left(-V_{11}-V_8\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}_{\mu\alpha}{}^{\epsilon}-2\left(U_{10}+2U_{11}-U_{13}+U_{14}+2U_{15}-6U_{16}-U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{Q}_{\mu\beta\chi}+$$

$$\left(-U_{13}-U_9\right)\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{Q}_{\mu\beta\chi}+\left(-V_{22}-V_{24}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}_{\mu\chi}{}^{\epsilon}+\left(U_{10}+U_{11}-U_{13}+U_{14}+U_{15}-4U_{16}-U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\beta}\,\mathcal{Q}_{\mu}{}^{\chi}{}_{\chi}+\left(-V_{28}-V_{30}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}_{\mu\delta}{}^{\epsilon}+$$

$$\frac{1}{2}\left(-V_{11}+V_{12}-4V_{17}-V_{22}+V_{23}-V_{24}+V_{25}+V_{28}-V_{29}+V_{30}-V_{31}-V_8+V_9\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{Q}_{\mu}{}^{\epsilon}{}_{\epsilon}+\left(2U_4-U_5\right)\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}_{\alpha\beta\chi}+\left(U_5-2U_6\right)\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}_{\alpha\beta\chi}+$$

$$\left(U_5-2U_6\right)\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\alpha\beta\chi}+\left(-V_3+V_4\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha\beta}{}^{\epsilon}+2\left(U_2-U_3\right)\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}_{\alpha\chi\beta}+\left(-2U_2+2U_3\right)\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}_{\alpha\chi\beta}+\left(2U_4-U_5\right)\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\alpha\chi\beta}+$$

$$\left(-2V_5+V_6\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha\chi}{}^{\epsilon}+\left(-2U_4+2U_6-U_7+U_8\right)\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\beta}\,\mathcal{T}_{\alpha}{}^{\chi}{}_{\chi}+\left(-V_6+2V_7\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha\delta}{}^{\epsilon}+2V_1\,\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}_{\alpha}{}^{\delta\epsilon}+$$

$$\left(V_6-2V_7\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}_{\alpha}{}^{\delta\epsilon}+\left(2V_5-V_6\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}_{\alpha}{}^{\delta\epsilon}+\left(-V_3+V_4\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\alpha}{}^{\delta\epsilon}-4V_2\,\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha}{}^{\epsilon}{}_{\beta}-$$

$$2V_3\,\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha}{}^{\epsilon}{}_{\chi}-2V_4\,\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha}{}^{\epsilon}{}_{\delta}+\left(8V_2-V_3+V_4-2V_5+2V_6-2V_7+V_8-V_9\right)\epsilon g_{\mu\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\alpha}{}^{\epsilon}{}_{\epsilon}-4U_{12}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\beta\alpha\chi}+$$

$$\left(-2U_{11}+4U_2-4U_3+2U_4-U_5+U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+\left(2U_{10}-4U_2+4U_3+U_5-2U_6-U_9\right)\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+\left(-4U_{12}-U_{13}+2U_{15}+U_5-2U_6\right)\mathcal{F}_{\mu}{}^{\alpha}{}_{\alpha}{}^{\beta}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+$$

$$\left(4U_{12}+U_{13}-2U_{14}+2U_4-U_5\right)\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\alpha}\,\mathcal{T}_{\beta}{}^{\chi}{}_{\chi}+2V_8\,\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}_{\beta}{}^{\delta\epsilon}+2V_9\,\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}_{\beta}{}^{\delta\epsilon}+\left(2V_{13}-V_{15}\right)\epsilon g_{\alpha\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\beta}{}^{\delta\epsilon}+$$

$$\left(-2V_{13}+V_{15}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\chi\alpha}{}^{\epsilon}+\left(-V_{14}+V_{16}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\chi\delta}{}^{\epsilon}+2V_{11}\,\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}_{\chi}{}^{\delta\epsilon}+2V_{12}\,\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}_{\chi}{}^{\delta\epsilon}+$$

$$\left(V_{14}-V_{16}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}_{\chi}{}^{\delta\epsilon}+\left(V_{18}-V_{20}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}_{\chi}{}^{\delta\epsilon}+\left(V_{15}-2V_{19}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}_{\chi}{}^{\delta\epsilon}-2V_{10}\,\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\chi}{}^{\epsilon}{}_{\delta}+$$

$$\left(2V_{10}+2V_{13}+V_{14}-V_{15}-V_{16}+V_{22}-V_{23}-2V_3\right)\epsilon g_{\mu\alpha\beta\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\chi}{}^{\epsilon}{}_{\epsilon}+2V_{17}\,\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha\beta}\,\mathcal{T}^{\chi\delta\epsilon}+\left(V_{24}-V_{30}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha\mu}\,\mathcal{T}^{\chi\delta\epsilon}+$$

$$\left(V_{25}-V_{31}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu\alpha}\,\mathcal{T}^{\chi\delta\epsilon}+\left(-V_{22}+V_{28}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha}{}_{\alpha}{}^{\beta}\,\mathcal{T}^{\chi\delta\epsilon}+\left(-V_{23}+V_{29}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\alpha}\,\mathcal{T}^{\chi\delta\epsilon}+\left(V_{21}-V_{27}\right)\epsilon g_{\alpha\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta}{}_{\beta}\,\mathcal{T}^{\chi\delta\epsilon}+$$

$$\left(-V_{15}+2V_{19}\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\delta\alpha}{}^{\epsilon}+\left(-V_{18}+V_{20}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\delta\chi}{}^{\epsilon}+\left(-2V_{10}+V_{15}+V_{18}-2V_{19}-V_{20}+V_{28}-V_{29}-2V_4\right)\epsilon g_{\mu\alpha\beta\chi}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\delta}{}^{\epsilon}{}_{\epsilon}+$$

$$\left(-V_{21}+V_{27}\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}^{\delta}{}_{\alpha}{}^{\epsilon}+\left(V_{16}-V_{20}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}^{\delta}{}_{\alpha}{}^{\epsilon}+\left(-V_{14}+V_{20}+2V_{24}-2V_{25}+4V_{37}-4V_{38}-2V_5+V_6\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\alpha}{}^{\epsilon}-$$

$$2V_{10}\,\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}^{\delta}{}_{\alpha}{}^{\epsilon}+\left(V_{11}-V_{13}+\frac{V_{15}}{2}-V_{24}-2V_{26}+V_{30}+V_{32}-V_5+\frac{V_6}{2}-V_8\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\beta}{}^{\epsilon}+$$

$$\left(V_{12}-\frac{V_{15}}{2}+V_{19}-V_{25}+V_{31}-V_{32}+2V_{33}-\frac{V_6}{2}+V_7-V_9\right)\epsilon g_{\mu\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}^{\delta}{}_{\beta}{}^{\epsilon}+\left(-V_{14}+V_{18}\right)\epsilon g_{\alpha\chi\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}^{\delta}{}_{\beta}{}^{\epsilon}+$$

$$\left(-V_{11}+V_{13}-\frac{V_{15}}{2}-V_{22}+2V_{26}+V_{28}-V_{32}+V_5-\frac{V_6}{2}+V_8\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+\left(-V_{12}+\frac{V_{15}}{2}-V_{19}-V_{23}+V_{29}+V_{32}-2V_{33}+\frac{V_6}{2}-V_7+V_9\right)\epsilon g_{\mu\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+$$

$$2\left(V_{37}-V_{38}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+\frac{1}{2}\left(-2V_{14}+V_{18}+V_{20}+2V_{24}-2V_{25}-2V_5+V_6\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+\left(-V_{16}+V_{20}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}^{\delta}{}_{\chi}{}^{\epsilon}+$$

$$\left(-2V_{26}+V_{32}\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+\left(-V_{32}+2V_{33}\right)\epsilon g_{\mu\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+\left(2V_1+V_{14}-V_{16}+V_{18}-V_{20}-2V_{21}+2V_{27}+V_{34}-V_{35}\right)\epsilon g_{\mu\alpha\beta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+$$

$$\left(-V_{14}-V_{16}+2V_{20}+V_{30}-V_{31}-4V_{37}+4V_{38}-V_6+2V_7\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\mu}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+\left(-V_{24}+V_{25}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+$$

$$\left(4V_{10}+V_{11}-V_{12}+2V_{13}-2V_{14}-2V_{15}+2V_{16}+2V_{18}+2V_{19}-2V_{20}+V_3-V_4\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}_{\mu}{}^{\alpha\beta\chi}\,\mathcal{T}^{\delta}{}_{\delta}{}^{\epsilon}+$$

$$\left(-V_{11}+V_{13}-\frac{V_{15}}{2}-V_{22}+V_{24}+2V_{26}+V_{28}-V_{30}-V_{32}+V_5-\frac{V_6}{2}+V_8\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}^{\delta}{}_{\mu}{}^{\epsilon}+$$

$$\left(-V_{12}+\frac{V_{15}}{2}-V_{19}-V_{23}+V_{25}+V_{29}-V_{31}+V_{32}-2V_{33}+\frac{V_6}{2}-V_7+V_9\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}^{\delta}{}_{\mu}{}^{\epsilon}+\left(U_7-U_8\right)\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}_{\mu\alpha\beta}+\left(V_{11}-V_{12}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu\alpha}{}^{\epsilon}+$$

$$\left(-U_{13}+2U_{15}\right)\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}_{\mu\beta\chi}+\left(U_{13}-2U_{14}\right)\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}_{\mu\beta\chi}+\left(2U_{11}-U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}_{\mu\chi\beta}+\left(-2U_{10}+U_9\right)\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}_{\mu\chi\beta}+\left(V_{24}-V_{25}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu\chi}{}^{\epsilon}-$$

$$2\left(U_{10}+U_{11}-U_{13}+U_{14}+U_{15}-4U_{16}-U_9\right)\mathcal{F}^{\alpha\beta}{}_{\alpha\beta}\,\mathcal{T}_{\mu}{}^{\chi}{}_{\chi}+\left(V_{30}-V_{31}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu\delta}{}^{\epsilon}+\left(2V_{26}-V_{32}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}\,\mathcal{T}_{\mu}{}^{\delta\epsilon}+$$

$$\left(V_{32}-2V_{33}\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\alpha}\,\mathcal{T}_{\mu}{}^{\delta\epsilon}+\left(V_{34}-V_{35}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\,\mathcal{T}_{\mu}{}^{\delta\epsilon}+\left(V_8-V_9\right)\epsilon g_{\beta\chi\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu}{}^{\epsilon}{}_{\alpha}+\left(V_{22}-V_{23}\right)\epsilon g_{\alpha\beta\delta\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu}{}^{\epsilon}{}_{\chi}+$$

$$\left(V_{28}-V_{29}\right)\epsilon g_{\alpha\beta\chi\epsilon}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu}{}^{\epsilon}{}_{\delta}+\left(V_{11}-V_{12}+4V_{17}+V_{22}-V_{23}+V_{24}-V_{25}-V_{28}+V_{29}-V_{30}+V_{31}+V_8-V_9\right)\epsilon g_{\alpha\beta\chi\delta}\,\mathcal{F}^{\alpha\beta\chi\delta}\,\mathcal{T}_{\mu}{}^{\epsilon}{}_{\epsilon}+$$

$$\left(-V_{14}-V_{16}+2V_{20}+2V_{30}-2V_{31}-4V_{37}+4V_{38}-V_6+2V_7\right)g^{\alpha\beta}\,\mathcal{F}^{\chi\delta\epsilon}{}_{\mu}\,\nabla_{\beta}\epsilon g_{\alpha\chi\delta\epsilon}+$$

$$\left(4V_{10}+2V_{11}-2V_{12}+2V_{13}-2V_{14}-2V_{15}+2V_{16}+2V_{18}+2V_{19}-2V_{20}+V_3-V_4\right)g^{\alpha\beta}\,\mathcal{F}_{\mu}{}^{\chi\delta\epsilon}\,\nabla_{\beta}\epsilon g_{\alpha\chi\delta\epsilon}+$$

$$\left(-8V_2+V_3-V_4+2V_5-2V_6+2V_7-2V_8+2V_9\right)\mathcal{F}^{\alpha\beta\chi\delta}\,\nabla_{\beta}\epsilon g_{\mu\alpha\chi\delta}+\left(-2V_1-V_{14}+V_{16}-V_{18}+V_{20}+2V_{21}-2V_{27}-2V_{34}+2V_{35}\right)g^{\alpha\beta}\,\mathcal{F}^{\chi\delta\epsilon}{}_{\epsilon}\,\nabla_{\beta}\epsilon g_{\mu\alpha\chi\delta}+$$

$$\left(4U_{11}-4U_2+4U_3-2U_4+U_5-2U_9\right)g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\alpha}{}^{\chi}{}_{\chi\mu}+\left(8V_2-V_3+V_4-2V_5+2V_6-2V_7+2V_8-2V_9\right)\epsilon g_{\mu\chi\delta\epsilon}\,g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\alpha}{}^{\chi\delta\epsilon}+$$

$$\left(-2V_{10}+V_{15}+V_{18}-2V_{19}-V_{20}+2V_{28}-2V_{29}-2V_4\right)\epsilon g_{\mu\chi\delta\epsilon}\,g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}^{\chi\delta\epsilon}{}_{\alpha}-2\left(U_4-U_6+U_7-U_8\right)g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\mu\alpha}{}^{\chi}{}_{\chi}+\left(-4U_{12}-2U_{13}+4U_{15}+U_5-2U_6\right)g^{\alpha\beta}\,\nabla_{\beta}\mathcal{F}_{\mu}{}^{\chi}{}_{\chi\alpha}+$$

$$\left(2V_{10}+2V_{13}+V_{14}-V_{15}-V_{16}+2V_{22}-2V_{23}-2V_3\right)\mathcal{F}^{\alpha\beta\chi\delta}\,\nabla_{\chi}\epsilon g_{\mu\alpha\beta\delta}+\left(2V_{10}+2V_{13}+V_{14}-V_{15}-V_{16}+2V_{22}-2V_{23}-2V_3\right)\epsilon g_{\mu\alpha\beta\delta}\,\nabla_{\chi}\mathcal{F}^{\alpha\beta\chi\delta}+$$

$$\left(4U_{12}+2U_{13}-4U_{14}+2U_4-U_5\right)g^{\alpha\beta}\,\nabla_{\chi}\mathcal{F}_{\mu\alpha}{}^{\chi}{}_{\beta}+\left(-2V_{10}+V_{15}+V_{18}-2V_{19}-V_{20}+2V_{28}-2V_{29}-2V_4\right)\mathcal{F}^{\alpha\beta\chi\delta}\,\nabla_{\delta}\epsilon g_{\mu\alpha\beta\chi}+$$

$$\left(4U_{10}-4U_2+4U_3+U_5-2U_6-2U_9\right)g^{\alpha\beta}\,g^{\chi\delta}\,\nabla_{\delta}\mathcal{F}_{\alpha\chi\mu\beta}+\left(2V_{11}-2V_{13}+V_{15}+2V_{22}-2V_{24}-4V_{26}-2V_{28}+2V_{30}+2V_{32}-2V_5+V_6-2V_8\right)\epsilon g_{\mu\beta\chi\delta}\,\nabla^{\delta}\mathcal{F}^{\alpha\beta}{}_{\alpha}{}^{\chi}+$$

$$\left(-2V_1-V_{14}+V_{16}-V_{18}+V_{20}+2V_{21}-2V_{27}-2V_{34}+2V_{35}\right)\epsilon g_{\mu\alpha\beta\delta}\,\nabla^{\delta}\mathcal{F}^{\alpha\beta\chi}{}_{\chi}+$$

$$\begin{aligned}&(V_{14}+V_{16}-2V_{20}-2V_{30}+2V_{31}+4V_{37}-4V_{38}+V_{6}-2V_{7})\,\epsilon g_{\alpha\beta\chi\delta}\,\nabla^{\delta}\mathcal{F}^{\alpha\beta\chi}{}_{\mu}+(2V_{14}-V_{18}-V_{20}-2V_{24}+2V_{25}-4V_{37}+4V_{38}+2V_{5}-V_{6})\,\epsilon g_{\alpha\beta\chi\delta}\,\nabla^{\delta}\mathcal{F}^{\alpha\beta}{}_{\mu}{}^{\chi}+\\&(-4V_{10}-2V_{11}+2V_{12}-2V_{13}+2V_{14}+2V_{15}-2V_{16}-2V_{18}-2V_{19}+2V_{20}-V_{3}+V_{4})\,\epsilon g_{\alpha\beta\chi\delta}\,\nabla^{\delta}\mathcal{F}_{\mu}{}^{\alpha\beta\chi}+\\&(2V_{12}-V_{15}+2V_{19}+2V_{23}-2V_{25}-2V_{29}+2V_{31}-2V_{32}+4V_{33}-V_{6}+2V_{7}-2V_{9})\,\epsilon g_{\mu\chi\delta\epsilon}\,g^{\alpha\beta}\,\nabla^{\epsilon}\mathcal{F}_{\alpha}{}^{\chi\delta}{}_{\beta}+\\&(V_{11}-V_{12}+6V_{17}+V_{22}-V_{23}+V_{24}-V_{25}-V_{28}+V_{29}-V_{30}+V_{31}+V_{8}-V_{9})\,\mathcal{F}^{\alpha\beta\chi\delta}\,\nabla_{\mu}\epsilon g_{\alpha\beta\chi\delta}+2(U_{10}+U_{11}-U_{13}+U_{14}+U_{15}-6U_{16}-U_{9})\,g^{\alpha\beta}\,\nabla_{\mu}\mathcal{F}_{\alpha}{}^{\chi}{}_{\chi\beta}+\\&(V_{11}-V_{12}+6V_{17}+V_{22}-V_{23}+V_{24}-V_{25}-V_{28}+V_{29}-V_{30}+V_{31}+V_{8}-V_{9})\,\epsilon g_{\alpha\beta\chi\delta}\,\nabla_{\mu}\mathcal{F}^{\alpha\beta\chi\delta}==0\end{aligned}$$

For general values of the fields, the tensor equation Eq. (60) requires the following conditions among the Lagrangian parameters in the ansatz.

$U_{10}==0$	$U_{11}==0$	$U_{12}==0$
$U_{13}==0$	$U_{14}==0$	$U_{15}==0$
$U_{16}==0$	$U_{2}==0$	$U_{3}==0$
$U_{4}==0$	$U_{5}==0$	$U_{6}==0$
$U_{7}==0$	$U_{8}==0$	$U_{9}==0$
$V_{1}==0$	$V_{10}==0$	$V_{11}==0$
$V_{12}==0$	$V_{13}==0$	$V_{14}==0$
$V_{15}==0$	$V_{16}==0$	$V_{17}==0$
$V_{18}==0$	$V_{19}==0$	$V_{2}==0$
$V_{20}==0$	$V_{21}==0$	$V_{22}==0$
$V_{23}==0$	$V_{24}==0$	$V_{25}==0$
$V_{26}==0$	$V_{27}==0$	$V_{28}==0$
$V_{29}==0$	$V_{3}==0$	$V_{30}==0$
$V_{31}==0$	$V_{32}==0$	$V_{33}==0$
$V_{34}==0$	$V_{35}==0$	$V_{37}==0$
$V_{38}==0$	$V_{4}==0$	$V_{5}==0$
$V_{6}==0$	$V_{7}==0$	$V_{8}==0$
$V_{9}==0$		

(61)

Thus our apparent finding is that Eq. (61) are the most general conditions for pure-torsion symmetry.

The iso-Weyl symmetry

Key observation: Now we take Eq. (56) and we combine it with Eq. (61) to produce the following. This is the iso-Weyl symmetry as defined in Eq. (16) on page 4 of our manuscript.

$U_{10}==0$	$U_{11}==0$	$U_{12}==0$
$U_{13}==0$	$U_{14}==0$	$U_{15}==0$
$U_{16}==0$	$U_{2}==0$	$U_{3}==0$
$U_{4}==0$	$U_{5}==0$	$U_{6}==0$
$U_{7}==0$	$U_{8}==0$	$U_{9}==0$
$V_{1}==0$	$V_{10}==0$	$V_{11}==0$
$V_{12}==0$	$V_{13}==0$	$V_{14}==0$
$V_{15}==0$	$V_{16}==0$	$V_{17}==0$
$V_{18}==0$	$V_{19}==0$	$V_{2}==0$
$V_{20}==0$	$V_{21}==0$	$V_{22}==0$
$V_{23}==0$	$V_{24}==0$	$V_{25}==0$
$V_{26}==0$	$V_{27}==0$	$V_{28}==0$
$V_{29}==0$	$V_{3}==0$	$V_{30}==0$
$V_{31}==0$	$V_{32}==0$	$V_{33}==0$
$V_{34}==0$	$V_{35}==0$	$V_{37}==0$
$V_{38}==0$	$V_{4}==0$	$V_{5}==0$
$V_{6}==0$	$V_{7}==0$	$V_{8}==0$
$V_{9}==0$		

(62)

Thus our apparent finding is that Eq. (62) are the most general conditions for iso-Weyl symmetry. Later on we'll refine this statement to arrive at the theory in Eq. (C1) our manuscript.

Looking more closely at the extended projective symmetry

Now if we impose Eq. (57) on the ansatz in Eq. (47) we obtain the following theory.

$$\begin{aligned}&\frac{1}{4}\mathcal{F}^{\alpha\beta\chi\delta}\Big(2U_{12}\,\mathcal{F}_{\alpha\beta\chi\delta}-2U_{12}\big(\mathcal{F}_{\alpha\beta\delta\chi}+2(\mathcal{F}_{\alpha\chi\beta\delta}-2\,\mathcal{F}_{\alpha\chi\delta\beta}+\mathcal{F}_{\alpha\delta\chi\beta}-\mathcal{F}_{\chi\delta\alpha\beta})\big)+V_{10}\big(2\,\epsilon g_{\chi\delta\epsilon\phi}\,\mathcal{F}_{\alpha\beta}{}^{\epsilon\phi}+4\,\epsilon g_{\beta\delta\epsilon\phi}(-2\,\mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}+\mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}-\mathcal{F}_{\chi}{}^{\epsilon}{}_{\alpha}{}^{\phi}+2\,\mathcal{F}_{\chi}{}^{\epsilon\phi}{}_{\alpha})+\\&\qquad 4\,\epsilon g_{\beta\chi\epsilon\phi}(2\,\mathcal{F}_{\alpha\delta}{}^{\epsilon\phi}-2\,\mathcal{F}_{\alpha}{}^{\epsilon}{}_{\delta}{}^{\phi}+\mathcal{F}_{\alpha}{}^{\epsilon\phi}{}_{\delta}-\mathcal{F}_{\delta}{}^{\epsilon\phi}{}_{\alpha})+\epsilon g_{\alpha\beta\epsilon\phi}(4\,\mathcal{F}_{\chi\delta}{}^{\epsilon\phi}-4\,\mathcal{F}_{\chi}{}^{\epsilon}{}_{\delta}{}^{\phi}+4\,\mathcal{F}_{\chi}{}^{\epsilon\phi}{}_{\delta}+4\,\mathcal{F}_{\delta}{}^{\epsilon}{}_{\chi}{}^{\phi}-4\,\mathcal{F}_{\delta}{}^{\epsilon\phi}{}_{\chi}+\mathcal{F}^{\epsilon\phi}{}_{\chi\delta}-\mathcal{F}^{\epsilon\phi}{}_{\delta\chi})\big)\Big)\end{aligned} \tag{63}$$

By counting the number of undetermined ansatz parameters, it looks like Eq. (63) contains two non-trivial and novel invariants: one is obviously parity-even, the other is obviously parity-odd. In fact, it will turn out that the parity-even invariant is the square of the Holst term, and we can show that the parity-odd invariant is vanishing.

To show this, we make an ansatz for the parity-odd invariant in which one factor is the Holst scalar (obviously EP-invariant) and the other factor contains two antisymmetric tensors and one curvature tensor, with various contractions all parameterised by new constants. This will be the operator which we want to vanish, so to complete the ansatz we add the square of the Holst (which we already know to be non-vanishing).

$$\mathcal{X}_5\ \epsilon g^{\alpha\beta\chi\delta}\ \epsilon g^{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha\beta\chi\delta}\ \mathcal{F}_{\epsilon\phi\gamma\eta}+\epsilon g^{\alpha\beta\chi\delta}\ \epsilon g_{\omega\pi}{}^{\gamma\eta}\ \epsilon g^{\omega\pi\upsilon\psi}\ \mathcal{F}_{\alpha\beta\chi\delta}\left(\mathcal{X}_1\ \mathcal{F}_{\gamma\eta\upsilon\psi}+\mathcal{X}_3\ \mathcal{F}_{\gamma\upsilon\eta\psi}+\mathcal{X}_2\ \mathcal{F}_{\upsilon\psi\gamma\eta}+\mathcal{X}_4\ \mathcal{F}_{\psi\eta\upsilon\gamma}\right) \tag{64}$$

Our guess is going to be that Eq. (64) actually contains Eq. (63) once the right product of antisymmetric tensors has been expanded out into Kronecker symbols and metrics. Performing this expansion gives the following.

$$\begin{aligned}
&-2\mathcal{X}_5\ \mathcal{F}_{\alpha\chi\epsilon\gamma}\ \mathcal{F}^{\alpha\chi\epsilon\gamma}+2\mathcal{X}_5\ \mathcal{F}_{\alpha\chi\epsilon\gamma}\ \mathcal{F}^{\alpha\chi\gamma\epsilon}-4\mathcal{X}_1\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}\ \mathcal{F}^{\alpha\chi\gamma\eta}+4\mathcal{X}_5\ \mathcal{F}_{\alpha\chi\epsilon\gamma}\ \mathcal{F}^{\alpha\epsilon\chi\gamma}-8\mathcal{X}_5\ \mathcal{F}_{\alpha\chi\epsilon\gamma}\ \mathcal{F}^{\alpha\epsilon\gamma\chi}+8\mathcal{X}_1\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}\ \mathcal{F}^{\alpha\gamma\chi\eta}-\\
&\quad 4\mathcal{X}_3\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}\ \mathcal{F}^{\alpha\gamma\chi\eta}+4\mathcal{X}_3\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\alpha\gamma\chi\eta}+4\mathcal{X}_5\ \mathcal{F}_{\alpha\chi\epsilon\gamma}\ \mathcal{F}^{\alpha\gamma\epsilon\chi}-8\mathcal{X}_1\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}\ \mathcal{F}^{\alpha\gamma\eta\chi}+4\mathcal{X}_4\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}\ \mathcal{F}^{\alpha\gamma\eta\chi}-\\
&\quad 4\mathcal{X}_3\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\alpha\gamma\eta\chi}-4\mathcal{X}_4\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\alpha\gamma\eta\chi}+4\mathcal{X}_4\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon\phi}{}_{\chi}\ \mathcal{F}^{\alpha\gamma\eta\chi}-4\mathcal{X}_3\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\chi\gamma\alpha\eta}+4\mathcal{X}_3\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\chi\gamma\eta\alpha}+\\
&\quad 4\mathcal{X}_4\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\chi\gamma\eta\alpha}-4\mathcal{X}_4\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon\phi}{}_{\chi}\ \mathcal{F}^{\chi\gamma\eta\alpha}-4\mathcal{X}_5\ \mathcal{F}_{\alpha\chi\epsilon\gamma}\ \mathcal{F}^{\epsilon\gamma\alpha\chi}-4\mathcal{X}_1\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}\ \mathcal{F}^{\gamma\eta\alpha\chi}-4\mathcal{X}_2\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}\ \mathcal{F}^{\gamma\eta\alpha\chi}+\\
&\quad 4\mathcal{X}_2\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\gamma\eta\alpha\chi}-2\mathcal{X}_3\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\gamma\eta\alpha\chi}-4\mathcal{X}_2\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon\phi}{}_{\chi}\ \mathcal{F}^{\gamma\eta\alpha\chi}+2\mathcal{X}_4\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon\phi}{}_{\chi}\ \mathcal{F}^{\gamma\eta\alpha\chi}-2\mathcal{X}_2\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}^{\epsilon\phi}{}_{\alpha\chi}\ \mathcal{F}^{\gamma\eta\alpha\chi}-\\
&\quad 4\mathcal{X}_2\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\gamma\eta\chi\alpha}+2\mathcal{X}_3\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\gamma\eta\chi\alpha}+4\mathcal{X}_2\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon\phi}{}_{\chi}\ \mathcal{F}^{\gamma\eta\chi\alpha}-2\mathcal{X}_4\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}_{\alpha}{}^{\epsilon\phi}{}_{\chi}\ \mathcal{F}^{\gamma\eta\chi\alpha}+2\mathcal{X}_2\ \epsilon g_{\epsilon\phi\gamma\eta}\ \mathcal{F}^{\epsilon\phi}{}_{\alpha\chi}\ \mathcal{F}^{\gamma\eta\chi\alpha}
\end{aligned} \tag{65}$$

To show that Eq. (64) contains Eq. (63), we subtract Eq. (63) from Eq. (65) and collect tensors.

$$\begin{aligned}
&\left(-\frac{U_{12}}{2}-2\mathcal{X}_5\right)\mathcal{F}_{\alpha\beta\chi\delta}\ \mathcal{F}^{\alpha\beta\chi\delta}+\frac{1}{2}\left(U_{12}+4\mathcal{X}_5\right)\mathcal{F}_{\alpha\beta\delta\chi}\ \mathcal{F}^{\alpha\beta\chi\delta}+\left(-\frac{V_{10}}{2}-4\mathcal{X}_1\right)\epsilon g_{\chi\delta\epsilon\phi}\ \mathcal{F}_{\alpha\beta}{}^{\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}+\left(U_{12}+4\mathcal{X}_5\right)\mathcal{F}_{\alpha\chi\beta\delta}\ \mathcal{F}^{\alpha\beta\chi\delta}-\\
&\quad 2\left(U_{12}+4\mathcal{X}_5\right)\mathcal{F}_{\alpha\chi\delta\beta}\ \mathcal{F}^{\alpha\beta\chi\delta}+2\left(V_{10}+4\mathcal{X}_1-2\mathcal{X}_3\right)\epsilon g_{\beta\delta\epsilon\phi}\ \mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}+\left(U_{12}+4\mathcal{X}_5\right)\mathcal{F}_{\alpha\delta\chi\beta}\ \mathcal{F}^{\alpha\beta\chi\delta}-2\left(V_{10}+4\mathcal{X}_1-2\mathcal{X}_4\right)\epsilon g_{\beta\chi\epsilon\phi}\ \mathcal{F}_{\alpha\delta}{}^{\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}+\\
&\quad \left(-V_{10}+4\mathcal{X}_3\right)\epsilon g_{\beta\delta\epsilon\phi}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}+2\left(V_{10}-2\left(\mathcal{X}_3+\mathcal{X}_4\right)\right)\epsilon g_{\beta\chi\epsilon\phi}\ \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\delta}{}^{\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}+\left(-V_{10}+4\mathcal{X}_4\right)\epsilon g_{\beta\chi\epsilon\phi}\ \mathcal{F}_{\alpha}{}^{\epsilon\phi}{}_{\delta}\ \mathcal{F}^{\alpha\beta\chi\delta}+\left(-U_{12}-4\mathcal{X}_5\right)\mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}_{\chi\delta\alpha\beta}+\\
&\quad \left(-V_{10}-4\left(\mathcal{X}_1+\mathcal{X}_2\right)\right)\epsilon g_{\alpha\beta\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}_{\chi\delta}{}^{\epsilon\phi}+\left(V_{10}-4\mathcal{X}_3\right)\epsilon g_{\beta\delta\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}_{\chi}{}^{\epsilon}{}_{\alpha}{}^{\phi}+\left(V_{10}+4\mathcal{X}_2-2\mathcal{X}_3\right)\epsilon g_{\alpha\beta\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}_{\chi}{}^{\epsilon}{}_{\delta}{}^{\phi}+\\
&\quad \left(-2V_{10}+4\left(\mathcal{X}_3+\mathcal{X}_4\right)\right)\epsilon g_{\beta\delta\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}_{\chi}{}^{\epsilon\phi}{}_{\alpha}+\left(-V_{10}-4\mathcal{X}_2+2\mathcal{X}_4\right)\epsilon g_{\alpha\beta\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}_{\chi}{}^{\epsilon\phi}{}_{\delta}+\left(-V_{10}-4\mathcal{X}_2+2\mathcal{X}_3\right)\epsilon g_{\alpha\beta\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}_{\delta}{}^{\epsilon}{}_{\chi}{}^{\phi}+\\
&\quad \left(V_{10}-4\mathcal{X}_4\right)\epsilon g_{\beta\chi\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}_{\delta}{}^{\epsilon\phi}{}_{\alpha}+\left(V_{10}+4\mathcal{X}_2-2\mathcal{X}_4\right)\epsilon g_{\alpha\beta\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}_{\delta}{}^{\epsilon\phi}{}_{\chi}+\left(-\frac{V_{10}}{4}-2\mathcal{X}_2\right)\epsilon g_{\alpha\beta\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}^{\epsilon\phi}{}_{\chi\delta}+\frac{1}{4}\left(V_{10}+8\mathcal{X}_2\right)\epsilon g_{\alpha\beta\epsilon\phi}\ \mathcal{F}^{\alpha\beta\chi\delta}\ \mathcal{F}^{\epsilon\phi}{}_{\delta\chi}==0
\end{aligned} \tag{66}$$

Now we solve Eq. (66) for the constants.

$$\left\{\mathcal{X}_1\to-\frac{V_{10}}{8},\mathcal{X}_2\to-\frac{V_{10}}{8},\mathcal{X}_3\to\frac{V_{10}}{4},\mathcal{X}_4\to\frac{V_{10}}{4},\mathcal{X}_5\to-\frac{U_{12}}{4}\right\} \tag{67}$$

We have to check that the rules provided in Eq. (67) do actually solve the system when substituted into Eq. (66).

True (68)

Now we examine in detail the (apparently new) parity-even curvature scalar which multiplies the Holst scalar in Eq. (64) by applying the rules in Eq. (67).

$$\frac{1}{4}V_{10}\left(2\ \epsilon g_{\gamma\pi}{}^{\upsilon\psi}\ \epsilon g_{\eta\omega\upsilon\psi}-\epsilon g_{\gamma\eta}{}^{\upsilon\psi}\ \epsilon g_{\pi\omega\upsilon\psi}\right)\mathcal{F}^{\gamma\eta\pi\omega} \tag{69}$$

However when we expand the antisymmetric tensor in Eq. (69) we see that this new scalar factor is identically zero (this makes sense, since there is only one parity-even scalar linear in the curvature, and that is the Ricci scalar).

0 (70)

Concrete relation to manuscript: The conclusion is that there are no parity-odd EP-invariant terms, only the parity-even Holst-squared term.

Looking more closely at the iso-Weyl symmetry

Now if we impose Eq. (62) on the ansatz in Eq. (47) we obtain the following theory.

$$\mathcal{F}^{\alpha\beta\chi}{}_{\chi}\left(U_1\ \mathcal{F}_{\alpha\beta}{}^{\delta}{}_{\delta}+V_{36}\ \epsilon g_{\alpha\beta\delta\epsilon}\ \mathcal{F}^{\delta\epsilon\phi}{}_{\phi}\right) \tag{71}$$

Clearly the parity-even term is the squared Homothetic curvature. Now we notice that the parity-odd term in Eq. (71) is just the dual of the homothetic curvature, contracted with the homothetic curvature. Since the homothetic curvature has the same structure as the Maxwell tensor, we throw this invariant out as a topological scalar (the same operator is excluded from electrodynamics).

Concrete relation to manuscript: The conclusion is that there are no parity-odd IW-invariant terms, only the parity-even square of the homothetic curvature.

Looking more closely at the concurrent symmetry

Now if we impose Eq. (56) on the ansatz in Eq. (47) we obtain the following theory.

$$U_1\ \mathcal{F}_{\alpha\beta}{}^{\delta}{}_{\delta}\ \mathcal{F}^{\alpha\beta\chi}{}_{\chi}+\frac{1}{2}\ \epsilon g_{\beta\delta\epsilon\phi}\left(V_3\ \mathcal{F}^{\alpha\beta\chi\delta}\left(2\ \mathcal{F}_{\alpha\chi}{}^{\epsilon\phi}-\mathcal{F}_{\alpha}{}^{\epsilon}{}_{\chi}{}^{\phi}+\mathcal{F}_{\chi}{}^{\epsilon}{}_{\alpha}{}^{\phi}-2\ \mathcal{F}_{\chi}{}^{\epsilon\phi}{}_{\alpha}\right)+V_1\ \mathcal{F}^{\alpha\beta\chi}{}_{\chi}\left(2\ \mathcal{F}_{\alpha}{}^{\delta\epsilon\phi}+\mathcal{F}^{\delta\epsilon}{}_{\alpha}{}^{\phi}-\mathcal{F}^{\delta\epsilon\phi}{}_{\alpha}\right)\right)+$$

$$V_{36}\, \epsilon g_{\alpha\beta\delta\epsilon}\, \mathcal{F}^{\alpha\beta\chi}{}_{\chi}\, \mathcal{F}^{\delta\epsilon\phi}{}_{\phi} + \frac{1}{8}\, \mathcal{F}^{\alpha\beta\chi\delta} \left(8\, U_2\, \mathcal{F}_{\alpha\beta\chi\delta} - 8\, U_2 \left(\mathcal{F}_{\alpha\beta\delta\chi} + 2 \left(\mathcal{F}_{\alpha\chi\beta\delta} - 2\, \mathcal{F}_{\alpha\chi\delta\beta} + \mathcal{F}_{\alpha\delta\chi\beta} - \mathcal{F}_{\chi\delta\alpha\beta}\right)\right) - V_3 \left(2\, \epsilon g_{\chi\delta\epsilon\phi}\, \mathcal{F}_{\alpha\beta}{}^{\epsilon\phi} + 4\, \epsilon g_{\beta\chi\epsilon\phi} \left(2\, \mathcal{F}_{\alpha\delta}{}^{\epsilon\phi} - 2\, \mathcal{F}_{\alpha}{}^{\epsilon}{}_{\delta}{}^{\phi} + \mathcal{F}_{\alpha}{}^{\epsilon\phi}{}_{\delta} - \mathcal{F}_{\delta}{}^{\epsilon\phi}{}_{\alpha}\right) + \epsilon g_{\alpha\beta\epsilon\phi} \left(4\, \mathcal{F}_{\chi\delta}{}^{\epsilon\phi} - 4\, \mathcal{F}_{\chi}{}^{\epsilon}{}_{\delta}{}^{\phi} + 4\, \mathcal{F}_{\chi}{}^{\epsilon\phi}{}_{\delta} + 4\, \mathcal{F}_{\delta}{}^{\epsilon}{}_{\chi}{}^{\phi} - 4\, \mathcal{F}_{\delta}{}^{\epsilon\phi}{}_{\chi} + \mathcal{F}^{\epsilon\phi}{}_{\chi\delta} - \mathcal{F}^{\epsilon\phi}{}_{\delta\chi}\right)\right)\right)$$

By comparing carefully with Eqs. (72), and (72) we can see in Eq. (72) the squares of the Holst and homothetic operators, and additionally the two invariants that we threw out from the EP and IW analyses above. There is however a new third invariant, which we see is proportional to the homothetic curvature contracted with the following antisymmetric rank-two tensor.

$$\epsilon g_{\beta\delta\epsilon\phi} \left(2\, \mathcal{F}_{\alpha}{}^{\delta\epsilon\phi} + \mathcal{F}^{\delta\epsilon}{}_{\alpha}{}^{\phi} - \mathcal{F}^{\delta\epsilon\phi}{}_{\alpha}\right) \tag{73}$$

To see that Eq. (73) is actually vanishing, note that the same invariant could be written as the dual of the homothetic curvature, contracted with the dual of Eq. (73), where the latter quantity is as follows.

$$\left(-2\, \epsilon g_{\beta\delta\epsilon\phi}\, \epsilon g_{\xi\varphi\alpha}{}^{\phi} - \epsilon g_{\alpha\beta\epsilon\phi}\, \epsilon g_{\xi\varphi\delta}{}^{\phi} + \epsilon g_{\alpha\beta\delta\phi}\, \epsilon g_{\xi\varphi\epsilon}{}^{\phi}\right) \mathcal{F}^{\alpha\beta\delta\epsilon} \tag{74}$$

Then, by expanding the product of antisymmetric tensors, we can see that Eq. (74) vanishes identically.

$$0 \tag{75}$$

Concrete relation to manuscript: The conclusion is that there are no parity-odd concurrent-invariant terms, only the parity-even Holst- and homothetic-square terms.