Supplemental materials

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Introduction — In this supplement we provide the full particle spectrum analyses referred to in Appendix B, and explore the possibility of other EP-invariant operators. We will refer directly to equations, references etc. using the numbering in the main body of the text. The particle spectrum analysis is performed using the *PSALTer* software [230, 231]. Within this software, we load a kinematic module associated with the metric-affine d.o.f. This module was prepared using the conventions of [35]. Therefore, before presenting the results, we need to provide a full tranlation between those conventions and ours.

Matching conventions — The independent connection is $A_{\mu}{}^{\nu}{}_{\rho}$ and the torsion and non-metricity tensors are

$$\begin{aligned} \mathcal{T}_{\mu}{}^{\alpha}{}_{\nu} &\equiv 2A_{[\mu|}{}^{\alpha}{}_{|\nu]}, \\ \mathcal{Q}_{\lambda\mu\nu} &\equiv -\partial_{\lambda} g_{\mu\nu} + 2A_{\lambda}{}^{\alpha}{}_{(\mu|} g_{\alpha|\nu)}. \end{aligned} \tag{SM1}$$

The non-Riemannian curvature is

$$\mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma} \equiv 2\left(\partial_{[\mu} A_{\nu]}{}^{\rho}{}_{\sigma} + A_{[\mu|}{}^{\rho}{}_{\alpha} A_{|\nu]}{}^{\alpha}{}_{\sigma}\right). \tag{SM2}$$

The various contractions formed from Eqs. (SM1) and (SM2) are

$$\begin{gathered} \mathcal{T}_{\mu} \equiv \mathcal{T}_{\alpha}{}^{\alpha}{}_{\mu}, \quad \mathcal{Q}_{\mu} \equiv \mathcal{Q}_{\mu\alpha}{}^{\alpha}, \quad \tilde{\mathcal{Q}}_{\mu} \equiv \mathcal{Q}_{\alpha}{}^{\alpha}{}_{\mu}, \\ \mathcal{F} \equiv \mathcal{F}_{\mu\nu}{}^{\mu\nu}, \quad \mathcal{F}_{\mu\nu} \equiv \mathcal{F}_{\mu\nu\alpha}{}^{\alpha} \\ \mathcal{F}^{(14)}{}_{\mu\nu} \equiv \mathcal{F}_{\alpha\mu\nu}{}^{\alpha}, \quad \mathcal{F}^{(13)}{}_{\mu\nu} \equiv \mathcal{F}_{\alpha\mu}{}^{\alpha}{}_{\nu}. \end{gathered} \tag{SM3}$$

By comparing Eq. (SM1) with Eq. (1), and Eq. (SM2) with Eq. (2), and finally Eq. (SM3) with Eq. (3), with various other inline definitions throughout the body of the text, we can obtain the following equivalences

$$\begin{gathered} A_{\mu}{}^{\rho}{}_{\nu} \equiv \Gamma^{\rho}{}_{\mu\nu}, \quad \mathcal{F}_{\mu\nu}{}^{\rho}{}_{\sigma} \equiv R^{\rho}{}_{\sigma\mu\nu}, \quad \mathcal{F}_{\mu\nu} \equiv \hat{R}_{\mu\nu}, \\ \mathcal{F}^{(14)}{}_{\mu\nu} \equiv \check{R}_{\nu\mu}, \quad \mathcal{F}^{(13)}{}_{\mu\nu} \equiv R_{\nu\mu}, \quad \mathcal{F} \equiv R, \\ \mathcal{Q}_{\lambda\mu\nu} \equiv -Q_{\lambda\mu\nu}, \quad \mathcal{Q}_{\mu} \equiv -Q_{\mu}, \quad \tilde{\mathcal{Q}}_{\mu} \equiv -\hat{Q}_{\mu}, \\ \mathcal{T}_{\mu}{}^{\lambda}{}_{\nu} \equiv T^{\lambda}{}_{\mu\nu}, \quad \mathcal{T}_{\mu} \equiv -T_{\mu}. \end{gathered} \tag{SM4}$$

In Eq. (SM4), we mark in red the quantities which have a change in sign or index structure between conventions, beyond just a notational change.

Spectral analyses — Having introduced the conventions of [35], we turn to the linearised particle spectrum analysis. The metric perturbation is $h_{\mu\nu}$ and $A_{\mu}{}^{\rho}{}_{\nu}$ is assumed to be inherently perturbative. To reach the second-order formulation, we only have to edit the quadratic action before substituting into the `ParticleSpectrum` function (which is the main function provided by the *PSALTer* package). The reparameterisation used to transform the quadratic action is

$$A_{\mu}{}^{\rho}{}_{\nu} \mapsto A_{\mu}{}^{\rho}{}_{\nu} + \frac{1}{2}\left(2\partial_{(\mu|} h_{\lambda|\nu)} - \partial_{\lambda} h_{\mu\nu}\right). \tag{SM5}$$

To lowest order in perturbative fields, Eq. (SM5) captures the transition from $\Gamma_{\mu}{}^{\rho}{}_{\nu}$ to $T_{\mu}{}^{\rho}{}_{\nu}$ and $Q_{\mu\rho\nu}$ set out in Eq. (SM1). Conjugate to the metric perturbation $h_{\mu\nu}$ and the affine connection $A_{\mu}{}^{\rho}{}_{\nu}$ are the (symmetric) stress-energy tensor $T^{\mu\nu}$ and the current $W^{\mu}{}_{\rho}{}^{\nu}$ which in MAG has become known as the *hypermomentum*. A more in-depth treatment of the metric-affine implementation in *PSALTer* can be found in the appendices to [230]. In Fig. SM1 we display the metric-affine gravity kinematic module. All the analyses are represented in Figs. SM2 to SM6. Further details can be found in the accompanying file `ParticleSpectra.nb`.

EP-invariant terms — In the accompanying file `Invariants.nb` we examine the possibility of other EP-invariant terms in the quadratic curvature sector of metric-affine gravity.

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PSALTer kinematic panel

Momentum	Norm	Frame
k^{μ}	$k^2 == k_{\mu}\ k^{\mu}$	$n^{\mu} == \frac{k^{\mu}}{k}$

Fundamental fields

Fields	Symmetries	SO(3)	Sources
$h_{\alpha\beta}$	StrongGenSet[{1,2},GenSet[(1,2)]]	$\frac{1}{3}\eta_{\alpha\beta}\,{}^{0^+}h^{\parallel} + {}^{2^+}h^{\parallel}{}_{\alpha\beta} + {}^{1^-}h^{\perp}{}_{\beta}\,n_{\alpha} + {}^{1^-}h^{\perp}{}_{\alpha}\,n_{\beta} - \frac{1}{3}\,{}^{0^+}h^{\parallel}\,n_{\alpha}\,n_{\beta} + {}^{0^+}h^{\perp}\,n_{\alpha}\,n_{\beta}$	$\mathcal{T}_{\alpha\beta}$
$\mathcal{A}_{\alpha\beta\chi}$	StrongGenSet[{},GenSet[]]	$\frac{4}{3}\,{}^{2^-}\mathcal{A}_{a}{}^{\parallel}{}_{\beta\chi\alpha} + \frac{1}{2}\,{}^{2^-}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta\chi} + \frac{1}{2}\,{}^{2^-}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\chi\beta} + {}^{3^-}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta\chi} + \frac{1}{2}\,{}^{1^-}\mathcal{A}_{a}{}^{\parallel}{}_{\chi}\,\eta_{\alpha\beta} - \frac{1}{6}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\chi}\,\eta_{\alpha\beta} + \frac{1}{15}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\chi}\,\eta_{\alpha\beta} - \frac{1}{2}\,{}^{1^-}\mathcal{A}_{a}{}^{\parallel}{}_{\beta}\,\eta_{\alpha\chi} - \frac{1}{6}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\beta}\,\eta_{\alpha\chi} + \frac{1}{15}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\beta}\,\eta_{\alpha\chi} + \frac{1}{3}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\alpha}\,\eta_{\beta\chi} + \frac{1}{15}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\alpha}\,\eta_{\beta\chi} +$ ${}^{1^+}\mathcal{A}_{a}{}^{\perp}{}_{\beta\chi}\,n_{\alpha} + \frac{1}{3}\,{}^{2^+}\mathcal{A}_{s}{}^{\parallel}{}_{\beta\chi}\,n_{\alpha} + \frac{2}{3}\,{}^{2^+}\mathcal{A}_{s}{}^{\perp}{}_{\beta\chi}\,n_{\alpha} + \frac{1}{9}\,{}^{0^+}\mathcal{A}_{s}{}^{\parallel}\,\eta_{\beta\chi}\,n_{\alpha} + \frac{2}{9}\,{}^{0^+}\mathcal{A}_{s}{}^{\perp h}\,\eta_{\beta\chi}\,n_{\alpha} - {}^{1^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\chi}\,n_{\beta} + {}^{2^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\chi}\,n_{\beta} - \frac{1}{2}\,{}^{1^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\chi}\,n_{\beta} + \frac{1}{3}\,{}^{2^+}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\chi}\,n_{\beta} - \frac{1}{3}\,{}^{2^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\chi}\,n_{\beta} + \frac{1}{3}\,{}^{0^+}\mathcal{A}_{a}{}^{\parallel}\,\eta_{\alpha\chi}\,n_{\beta} +$ $\frac{1}{9}\,{}^{0^+}\mathcal{A}_{s}{}^{\parallel}\,\eta_{\alpha\chi}\,n_{\beta} - \frac{1}{9}\,{}^{0^+}\mathcal{A}_{s}{}^{\perp h}\,\eta_{\alpha\chi}\,n_{\beta} - \frac{1}{2}\,{}^{1^-}\mathcal{A}_{a}{}^{\parallel}{}_{\chi}\,n_{\alpha}\,n_{\beta} - {}^{1^-}\mathcal{A}_{a}{}^{\perp}{}_{\chi}\,n_{\alpha}\,n_{\beta} + \frac{1}{6}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\chi}\,n_{\alpha}\,n_{\beta} - \frac{1}{15}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\chi}\,n_{\alpha}\,n_{\beta} - \frac{1}{3}\,{}^{1^-}\mathcal{A}_{s}{}^{\perp h}{}_{\chi}\,n_{\alpha}\,n_{\beta} + \frac{1}{3}\,{}^{1^-}\mathcal{A}_{s}{}^{\perp t}{}_{\chi}\,n_{\alpha}\,n_{\beta} + {}^{1^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\beta}\,n_{\chi} - {}^{2^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\beta}\,n_{\chi} -$ $\frac{1}{2}\,{}^{1^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\beta}\,n_{\chi} + \frac{1}{3}\,{}^{2^+}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta}\,n_{\chi} - \frac{1}{3}\,{}^{2^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\beta}\,n_{\chi} - \frac{1}{3}\,{}^{0^+}\mathcal{A}_{a}{}^{\parallel}\,\eta_{\alpha\beta}\,n_{\chi} + \frac{1}{9}\,{}^{0^+}\mathcal{A}_{s}{}^{\parallel}\,\eta_{\alpha\beta}\,n_{\chi} - \frac{1}{9}\,{}^{0^+}\mathcal{A}_{s}{}^{\perp h}\,\eta_{\alpha\beta}\,n_{\chi} + \frac{1}{2}\,{}^{1^-}\mathcal{A}_{a}{}^{\parallel}{}_{\beta}\,n_{\alpha}\,n_{\chi} + {}^{1^-}\mathcal{A}_{a}{}^{\perp}{}_{\beta}\,n_{\alpha}\,n_{\chi} + \frac{1}{6}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\beta}\,n_{\alpha}\,n_{\chi} - \frac{1}{15}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\beta}\,n_{\alpha}\,n_{\chi} -$ $\frac{1}{3}\,{}^{1^-}\mathcal{A}_{s}{}^{\perp h}{}_{\beta}\,n_{\alpha}\,n_{\chi} + \frac{1}{3}\,{}^{1^-}\mathcal{A}_{s}{}^{\perp t}{}_{\beta}\,n_{\alpha}\,n_{\chi} - \frac{1}{3}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\alpha}\,n_{\beta}\,n_{\chi} - \frac{1}{15}\,{}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\alpha}\,n_{\beta}\,n_{\chi} + \frac{2}{3}\,{}^{1^-}\mathcal{A}_{s}{}^{\perp h}{}_{\alpha}\,n_{\beta}\,n_{\chi} + \frac{1}{3}\,{}^{1^-}\mathcal{A}_{s}{}^{\perp t}{}_{\alpha}\,n_{\beta}\,n_{\chi} - \frac{1}{3}\,{}^{0^+}\mathcal{A}_{s}{}^{\parallel}\,n_{\alpha}\,n_{\beta}\,n_{\chi} + {}^{0^+}\mathcal{A}_{s}{}^{\perp t}\,n_{\alpha}\,n_{\beta}\,n_{\chi} - \frac{1}{6}\,{}^{0^-}\mathcal{A}_{a}{}^{\parallel}\,\epsilon\eta_{\alpha\beta\chi\delta}\,n^{\delta}$	$\mathcal{W}_{\alpha\beta\chi}$

SO(3) irreps

SO(3)	Symmetries	Expansion	Sources
${}^{0^+}h^{\perp}$	StrongGenSet[{},GenSet[]]	$h_{\alpha\beta}\,n^{\alpha}\,n^{\beta}$	${}^{0^+}\mathcal{T}^{\perp}$
${}^{0^+}h^{\parallel}$	StrongGenSet[{},GenSet[]]	$h^{\alpha}{}_{\alpha} - h_{\alpha\beta}\,n^{\alpha}\,n^{\beta}$	${}^{0^+}\mathcal{T}^{\parallel}$
${}^{1^-}h^{\perp}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$h_{\alpha\beta}\,n^{\beta} - h_{\beta\chi}\,n_{\alpha}\,n^{\beta}\,n^{\chi}$	${}^{1^-}\mathcal{T}^{\perp}{}_{\alpha}$
${}^{2^+}h^{\parallel}{}_{\alpha\beta}$	StrongGenSet[{1,2},GenSet[(1,2)]]	$h_{\alpha\beta} - \frac{1}{3}\eta_{\alpha\beta}\,h^{\chi}{}_{\chi} + \frac{1}{3}h^{\chi}{}_{\chi}\,n_{\alpha}\,n_{\beta} - h_{\beta\chi}\,n_{\alpha}\,n^{\chi} - h_{\alpha\chi}\,n_{\beta}\,n^{\chi} + \frac{1}{3}\eta_{\alpha\beta}\,h_{\chi\delta}\,n^{\chi}\,n^{\delta} + \frac{2}{3}h_{\chi\delta}\,n_{\alpha}\,n_{\beta}\,n^{\chi}\,n^{\delta}$	${}^{2^+}\mathcal{T}^{\parallel}{}_{\alpha\beta}$
${}^{0^+}\mathcal{A}_{a}{}^{\parallel}$	StrongGenSet[{}, GenSet[]] -	$\frac{1}{2}\mathcal{A}^{\alpha}{}_{\alpha}{}^{\beta}\,n_{\beta} + \frac{1}{2}\mathcal{A}^{\alpha\beta}{}_{\alpha}\,n_{\beta}$	${}^{0^+}\mathcal{W}_{a}{}^{\parallel}$
${}^{0^+}\mathcal{A}_{s}{}^{\perp t}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}^{\alpha\beta\chi}\,n_{\alpha}\,n_{\beta}\,n_{\chi}$	${}^{0^+}\mathcal{W}_{s}{}^{\perp t}$
${}^{0^+}\mathcal{A}_{s}{}^{\parallel}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}^{\alpha\beta}{}_{\beta}\,n_{\alpha} + \mathcal{A}^{\alpha}{}_{\alpha}{}^{\beta}\,n_{\beta} + \mathcal{A}^{\alpha\beta}{}_{\alpha}\,n_{\beta} - 3\,\mathcal{A}^{\alpha\beta\chi}\,n_{\alpha}\,n_{\beta}\,n_{\chi}$	${}^{0^+}\mathcal{W}_{s}{}^{\parallel}$
${}^{0^+}\mathcal{A}_{s}{}^{\perp h}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}^{\alpha\beta}{}_{\beta}\,n_{\alpha} - \frac{1}{2}\mathcal{A}^{\alpha}{}_{\alpha}{}^{\beta}\,n_{\beta} - \frac{1}{2}\mathcal{A}^{\alpha\beta}{}_{\alpha}\,n_{\beta}$	${}^{0^+}\mathcal{W}_{s}{}^{\perp h}$
${}^{0^-}\mathcal{A}_{a}{}^{\parallel}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}^{\alpha\beta\chi}\,\epsilon\eta_{\alpha\beta\chi\delta}\,n^{\delta}$	${}^{0^-}\mathcal{W}_{a}{}^{\parallel}$
${}^{1^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\beta}$	StrongGenSet[{1,2},GenSet[-(1,2)]]	$\frac{1}{4}\mathcal{A}_{\alpha\beta}{}^{\chi}\,n_{\chi} - \frac{1}{4}\mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\,n_{\chi} - \frac{1}{4}\mathcal{A}_{\beta\alpha}{}^{\chi}\,n_{\chi} + \frac{1}{4}\mathcal{A}_{\beta}{}^{\chi}{}_{\alpha}\,n_{\chi} - \frac{1}{4}\mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} + \frac{1}{4}\mathcal{A}^{\chi\delta}{}_{\beta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} + \frac{1}{4}\mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\,n_{\beta}\,n_{\chi}\,n_{\delta} - \frac{1}{4}\mathcal{A}^{\chi\delta}{}_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}$	${}^{1^+}\mathcal{W}_{a}{}^{\parallel}{}_{\alpha\beta}$
${}^{1^+}\mathcal{A}_{a}{}^{\perp}{}_{\alpha\beta}$	StrongGenSet[{1,2},GenSet[-(1,2)]]	$\frac{1}{2}\mathcal{A}^{\chi}{}_{\alpha\beta}\,n_{\chi} - \frac{1}{2}\mathcal{A}^{\chi}{}_{\beta\alpha}\,n_{\chi} + \frac{1}{2}\mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} - \frac{1}{2}\mathcal{A}^{\chi\delta}{}_{\beta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} - \frac{1}{2}\mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\,n_{\beta}\,n_{\chi}\,n_{\delta} + \frac{1}{2}\mathcal{A}^{\chi\delta}{}_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}$	${}^{1^+}\mathcal{W}_{a}{}^{\perp}{}_{\alpha\beta}$
${}^{1^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\beta}$	StrongGenSet[{1,2}, GenSet[-(1,2)]] -	$\frac{1}{2}\mathcal{A}_{\alpha\beta}{}^{\chi}\,n_{\chi} - \frac{1}{2}\mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\,n_{\chi} - \frac{1}{2}\mathcal{A}_{\beta\alpha}{}^{\chi}\,n_{\chi} + \frac{1}{2}\mathcal{A}_{\beta}{}^{\chi}{}_{\alpha}\,n_{\chi} - \mathcal{A}_{\beta}{}^{\chi\delta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} + \frac{1}{2}\mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} + \frac{1}{2}\mathcal{A}^{\chi\delta}{}_{\beta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} + \mathcal{A}_{\alpha}{}^{\chi\delta}\,n_{\beta}\,n_{\chi}\,n_{\delta} - \frac{1}{2}\mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\,n_{\beta}\,n_{\chi}\,n_{\delta} - \frac{1}{2}\mathcal{A}^{\chi\delta}{}_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}$	${}^{1^+}\mathcal{W}_{s}{}^{\perp}{}_{\alpha\beta}$
${}^{1^-}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha}$	StrongGenSet[{}, GenSet[]] -	$\frac{1}{2}\mathcal{A}^{\beta}{}_{\alpha\beta} + \frac{1}{2}\mathcal{A}^{\beta}{}_{\beta\alpha} - \frac{1}{2}\mathcal{A}^{\beta}{}_{\beta}{}^{\chi}\,n_{\alpha}\,n_{\chi} + \frac{1}{2}\mathcal{A}^{\beta\chi}{}_{\beta}\,n_{\alpha}\,n_{\chi} + \frac{1}{2}\mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\,n_{\beta}\,n_{\chi} - \frac{1}{2}\mathcal{A}^{\beta\chi}{}_{\alpha}\,n_{\beta}\,n_{\chi}$	${}^{1^-}\mathcal{W}_{a}{}^{\parallel}{}_{\alpha}$
${}^{1^-}\mathcal{A}_{a}{}^{\perp}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$\frac{1}{2}\mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\,n_{\beta}\,n_{\chi} - \frac{1}{2}\mathcal{A}^{\beta\chi}{}_{\alpha}\,n_{\beta}\,n_{\chi}$	${}^{1^-}\mathcal{W}_{a}{}^{\perp}{}_{\alpha}$
${}^{1^-}\mathcal{A}_{s}{}^{\perp t}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}_{\alpha}{}^{\beta\chi}\,n_{\beta}\,n_{\chi} + \mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\,n_{\beta}\,n_{\chi} + \mathcal{A}^{\beta\chi}{}_{\alpha}\,n_{\beta}\,n_{\chi} - 3\,\mathcal{A}^{\beta\chi\delta}\,n_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}$	${}^{1^-}\mathcal{W}_{s}{}^{\perp t}{}_{\alpha}$
${}^{1^-}\mathcal{A}_{s}{}^{\parallel t}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}_{\alpha}{}^{\beta}{}_{\beta} + \mathcal{A}^{\beta}{}_{\alpha\beta} + \mathcal{A}^{\beta}{}_{\beta\alpha} - \mathcal{A}^{\beta\chi}{}_{\chi}\,n_{\alpha}\,n_{\beta} - \mathcal{A}^{\beta}{}_{\beta}{}^{\chi}\,n_{\alpha}\,n_{\chi} - \mathcal{A}^{\beta\chi}{}_{\beta}\,n_{\alpha}\,n_{\chi} - \mathcal{A}_{\alpha}{}^{\beta\chi}\,n_{\beta}\,n_{\chi} - \mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\,n_{\beta}\,n_{\chi} - \mathcal{A}^{\beta\chi}{}_{\alpha}\,n_{\beta}\,n_{\chi} + 3\,\mathcal{A}^{\beta\chi\delta}\,n_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}$	${}^{1^-}\mathcal{W}_{s}{}^{\parallel t}{}_{\alpha}$
${}^{1^-}\mathcal{A}_{s}{}^{\perp h}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}_{\alpha}{}^{\beta\chi}\,n_{\beta}\,n_{\chi} - \frac{1}{2}\mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\,n_{\beta}\,n_{\chi} - \frac{1}{2}\mathcal{A}^{\beta\chi}{}_{\alpha}\,n_{\beta}\,n_{\chi}$	${}^{1^-}\mathcal{W}_{s}{}^{\perp h}{}_{\alpha}$
${}^{1^-}\mathcal{A}_{s}{}^{\parallel h}{}_{\alpha}$	StrongGenSet[{},GenSet[]]	$\mathcal{A}_{\alpha}{}^{\beta}{}_{\beta} - \frac{1}{2}\mathcal{A}^{\beta}{}_{\alpha\beta} - \frac{1}{2}\mathcal{A}^{\beta}{}_{\beta\alpha} - \mathcal{A}^{\beta\chi}{}_{\chi}\,n_{\alpha}\,n_{\beta} + \frac{1}{2}\mathcal{A}^{\beta}{}_{\beta}{}^{\chi}\,n_{\alpha}\,n_{\chi} + \frac{1}{2}\mathcal{A}^{\beta\chi}{}_{\beta}\,n_{\alpha}\,n_{\chi} - \mathcal{A}_{\alpha}{}^{\beta\chi}\,n_{\beta}\,n_{\chi} + \frac{1}{2}\mathcal{A}^{\beta}{}_{\alpha}{}^{\chi}\,n_{\beta}\,n_{\chi} + \frac{1}{2}\mathcal{A}^{\beta\chi}{}_{\alpha}\,n_{\beta}\,n_{\chi}$	${}^{1^-}\mathcal{W}_{s}{}^{\parallel h}{}_{\alpha}$
${}^{2^+}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\beta}$	StrongGenSet[{1, 2}, GenSet[(1,2)]] -	$\frac{1}{4}\mathcal{A}_{\alpha\beta}{}^{\chi}\,n_{\chi} + \frac{1}{4}\mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\,n_{\chi} - \frac{1}{4}\mathcal{A}_{\beta\alpha}{}^{\chi}\,n_{\chi} + \frac{1}{4}\mathcal{A}_{\beta}{}^{\chi}{}_{\alpha}\,n_{\chi} + \frac{1}{6}\mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\,\eta_{\alpha\beta}\,n_{\delta} - \frac{1}{6}\mathcal{A}^{\chi\delta}{}_{\chi}\,\eta_{\alpha\beta}\,n_{\delta} -$ $\frac{1}{6}\mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\,n_{\alpha}\,n_{\beta}\,n_{\delta} + \frac{1}{6}\mathcal{A}^{\chi\delta}{}_{\chi}\,n_{\alpha}\,n_{\beta}\,n_{\delta} + \frac{1}{4}\mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} - \frac{1}{4}\mathcal{A}^{\chi\delta}{}_{\beta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} + \frac{1}{4}\mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\,n_{\beta}\,n_{\chi}\,n_{\delta} - \frac{1}{4}\mathcal{A}^{\chi\delta}{}_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}$	${}^{2^+}\mathcal{W}_{a}{}^{\parallel}{}_{\alpha\beta}$
${}^{2^+}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta}$	StrongGenSet[{1,2},GenSet[(1,2)]]	$\frac{1}{2}\mathcal{A}_{\alpha\beta}{}^{\chi}\,n_{\chi} + \frac{1}{2}\mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\,n_{\chi} + \frac{1}{2}\mathcal{A}_{\beta\alpha}{}^{\chi}\,n_{\chi} + \frac{1}{2}\mathcal{A}_{\beta}{}^{\chi}{}_{\alpha}\,n_{\chi} + \frac{1}{2}\mathcal{A}^{\chi}{}_{\alpha\beta}\,n_{\chi} + \frac{1}{2}\mathcal{A}^{\chi}{}_{\beta\alpha}\,n_{\chi} - \frac{1}{3}\mathcal{A}^{\chi\delta}{}_{\delta}\,\eta_{\alpha\beta}\,n_{\chi} + \frac{1}{3}\mathcal{A}^{\chi\delta}{}_{\delta}\,n_{\alpha}\,n_{\beta}\,n_{\chi} - \frac{1}{3}\mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\,\eta_{\alpha\beta}\,n_{\delta} - \frac{1}{3}\mathcal{A}^{\chi\delta}{}_{\chi}\,\eta_{\alpha\beta}\,n_{\delta} + \frac{1}{3}\mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\,n_{\alpha}\,n_{\beta}\,n_{\delta} +$ $\frac{1}{3}\mathcal{A}^{\chi\delta}{}_{\chi}\,n_{\alpha}\,n_{\beta}\,n_{\delta} - \mathcal{A}_{\beta}{}^{\chi\delta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} - \mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} - \mathcal{A}^{\chi\delta}{}_{\beta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} - \mathcal{A}_{\alpha}{}^{\chi\delta}\,n_{\beta}\,n_{\chi}\,n_{\delta} - \mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\,n_{\beta}\,n_{\chi}\,n_{\delta} - \mathcal{A}^{\chi\delta}{}_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta} + \mathcal{A}^{\chi\delta\epsilon}\,\eta_{\alpha\beta}\,n_{\chi}\,n_{\delta}\,n_{\epsilon} + 2\,\mathcal{A}^{\chi\delta\epsilon}\,n_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}\,n_{\epsilon}$	${}^{2^+}\mathcal{W}_{s}{}^{\parallel}{}_{\alpha\beta}$
${}^{2^+}\mathcal{A}_{s}{}^{\perp}{}_{\alpha\beta}$	StrongGenSet[{1, 2}, GenSet[(1,2)]] -	$\frac{1}{4}\mathcal{A}_{\alpha\beta}{}^{\chi}\,n_{\chi} - \frac{1}{4}\mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\,n_{\chi} - \frac{1}{4}\mathcal{A}_{\beta\alpha}{}^{\chi}\,n_{\chi} - \frac{1}{4}\mathcal{A}_{\beta}{}^{\chi}{}_{\alpha}\,n_{\chi} + \frac{1}{2}\mathcal{A}^{\chi}{}_{\alpha\beta}\,n_{\chi} + \frac{1}{2}\mathcal{A}^{\chi}{}_{\beta\alpha}\,n_{\chi} - \frac{1}{3}\mathcal{A}^{\chi\delta}{}_{\delta}\,\eta_{\alpha\beta}\,n_{\chi} + \frac{1}{3}\mathcal{A}^{\chi\delta}{}_{\delta}\,n_{\alpha}\,n_{\beta}\,n_{\chi} + \frac{1}{6}\mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\,\eta_{\alpha\beta}\,n_{\delta} + \frac{1}{6}\mathcal{A}^{\chi\delta}{}_{\chi}\,\eta_{\alpha\beta}\,n_{\delta} -$ $\frac{1}{6}\mathcal{A}^{\chi}{}_{\chi}{}^{\delta}\,n_{\alpha}\,n_{\beta}\,n_{\delta} - \frac{1}{6}\mathcal{A}^{\chi\delta}{}_{\chi}\,n_{\alpha}\,n_{\beta}\,n_{\delta} + \frac{1}{2}\mathcal{A}_{\beta}{}^{\chi\delta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} - \frac{1}{4}\mathcal{A}^{\chi}{}_{\beta}{}^{\delta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} - \frac{1}{4}\mathcal{A}^{\chi\delta}{}_{\beta}\,n_{\alpha}\,n_{\chi}\,n_{\delta} + \frac{1}{2}\mathcal{A}_{\alpha}{}^{\chi\delta}\,n_{\beta}\,n_{\chi}\,n_{\delta} - \frac{1}{4}\mathcal{A}^{\chi}{}_{\alpha}{}^{\delta}\,n_{\beta}\,n_{\chi}\,n_{\delta} - \frac{1}{4}\mathcal{A}^{\chi\delta}{}_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}$	${}^{2^+}\mathcal{W}_{s}{}^{\perp}{}_{\alpha\beta}$
${}^{2^-}\mathcal{A}_{a}{}^{\parallel}{}_{\alpha\beta\chi}$	StrongGenSet[{1, 2}, GenSet[-(1,2)]] -	$\frac{1}{8}\mathcal{A}_{\alpha\beta\chi} + \frac{1}{8}\mathcal{A}_{\alpha\chi\beta} + \frac{1}{8}\mathcal{A}_{\beta\alpha\chi} - \frac{1}{8}\mathcal{A}_{\beta\chi\alpha} + \frac{1}{4}\mathcal{A}_{\chi\alpha\beta} - \frac{1}{4}\mathcal{A}_{\chi\beta\alpha} + \frac{3}{16}\mathcal{A}^{\delta}{}_{\beta\delta}\,\eta_{\alpha\chi} - \frac{3}{16}\mathcal{A}^{\delta}{}_{\delta\beta}\,\eta_{\alpha\chi} - \frac{3}{16}\mathcal{A}^{\delta}{}_{\alpha\delta}\,\eta_{\beta\chi} + \frac{3}{16}\mathcal{A}^{\delta}{}_{\delta\alpha}\,\eta_{\beta\chi} - \frac{3}{16}\mathcal{A}^{\delta}{}_{\beta\delta}\,n_{\alpha}\,n_{\chi} +$ $\frac{3}{16}\mathcal{A}^{\delta}{}_{\delta\beta}\,n_{\alpha}\,n_{\chi} + \frac{3}{16}\mathcal{A}^{\delta}{}_{\alpha\delta}\,n_{\beta}\,n_{\chi} - \frac{3}{16}\mathcal{A}^{\delta}{}_{\delta\alpha}\,n_{\beta}\,n_{\chi} + \frac{1}{8}\mathcal{A}_{\beta\chi}{}^{\delta}\,n_{\alpha}\,n_{\delta} - \frac{1}{8}\mathcal{A}_{\beta}{}^{\delta}{}_{\chi}\,n_{\alpha}\,n_{\delta} + \frac{1}{4}\mathcal{A}_{\chi\beta}{}^{\delta}\,n_{\alpha}\,n_{\delta} - \frac{1}{4}\mathcal{A}_{\chi}{}^{\delta}{}_{\beta}\,n_{\alpha}\,n_{\delta} + \frac{1}{8}\mathcal{A}^{\delta}{}_{\beta\chi}\,n_{\alpha}\,n_{\delta} - \frac{1}{8}\mathcal{A}^{\delta}{}_{\chi\beta}\,n_{\alpha}\,n_{\delta} -$ $\frac{1}{8}\mathcal{A}_{\alpha\chi}{}^{\delta}\,n_{\beta}\,n_{\delta} + \frac{1}{8}\mathcal{A}_{\alpha}{}^{\delta}{}_{\chi}\,n_{\beta}\,n_{\delta} - \frac{1}{4}\mathcal{A}_{\chi\alpha}{}^{\delta}\,n_{\beta}\,n_{\delta} + \frac{1}{4}\mathcal{A}_{\chi}{}^{\delta}{}_{\alpha}\,n_{\beta}\,n_{\delta} - \frac{1}{8}\mathcal{A}^{\delta}{}_{\alpha\chi}\,n_{\beta}\,n_{\delta} + \frac{1}{8}\mathcal{A}^{\delta}{}_{\chi\alpha}\,n_{\beta}\,n_{\delta} + \frac{1}{8}\mathcal{A}_{\alpha\beta}{}^{\delta}\,n_{\chi}\,n_{\delta} - \frac{1}{8}\mathcal{A}_{\alpha}{}^{\delta}{}_{\beta}\,n_{\chi}\,n_{\delta} - \frac{1}{8}\mathcal{A}_{\beta\alpha}{}^{\delta}\,n_{\chi}\,n_{\delta} +$ $\frac{1}{8}\mathcal{A}_{\beta}{}^{\delta}{}_{\alpha}\,n_{\chi}\,n_{\delta} - \frac{1}{4}\mathcal{A}^{\delta}{}_{\alpha\beta}\,n_{\chi}\,n_{\delta} + \frac{1}{4}\mathcal{A}^{\delta}{}_{\beta\alpha}\,n_{\chi}\,n_{\delta} - \frac{3}{16}\mathcal{A}^{\delta}{}_{\delta}{}^{\epsilon}\,\eta_{\beta\chi}\,n_{\alpha}\,n_{\epsilon} + \frac{3}{16}\mathcal{A}^{\delta\epsilon}{}_{\delta}\,\eta_{\beta\chi}\,n_{\alpha}\,n_{\epsilon} + \frac{3}{16}\mathcal{A}^{\delta}{}_{\delta}{}^{\epsilon}\,\eta_{\alpha\chi}\,n_{\beta}\,n_{\epsilon} - \frac{3}{16}\mathcal{A}^{\delta\epsilon}{}_{\delta}\,\eta_{\alpha\chi}\,n_{\beta}\,n_{\epsilon} - \frac{3}{16}\mathcal{A}^{\delta}{}_{\beta}{}^{\epsilon}\,\eta_{\alpha\chi}\,n_{\delta}\,n_{\epsilon} +$ $\frac{3}{16}\mathcal{A}^{\delta\epsilon}{}_{\beta}\,\eta_{\alpha\chi}\,n_{\delta}\,n_{\epsilon} + \frac{3}{16}\mathcal{A}^{\delta}{}_{\alpha}{}^{\epsilon}\,\eta_{\beta\chi}\,n_{\delta}\,n_{\epsilon} - \frac{3}{16}\mathcal{A}^{\delta\epsilon}{}_{\alpha}\,\eta_{\beta\chi}\,n_{\delta}\,n_{\epsilon} - \frac{3}{16}\mathcal{A}^{\delta}{}_{\beta}{}^{\epsilon}\,n_{\alpha}\,n_{\chi}\,n_{\delta}\,n_{\epsilon} + \frac{3}{16}\mathcal{A}^{\delta\epsilon}{}_{\beta}\,n_{\alpha}\,n_{\chi}\,n_{\delta}\,n_{\epsilon} + \frac{3}{16}\mathcal{A}^{\delta}{}_{\alpha}{}^{\epsilon}\,n_{\beta}\,n_{\chi}\,n_{\delta}\,n_{\epsilon} - \frac{3}{16}\mathcal{A}^{\delta\epsilon}{}_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}\,n_{\epsilon}$	${}^{2^-}\mathcal{W}_{a}{}^{\parallel}{}_{\alpha\beta\chi}$
${}^{2^-}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta\chi}$	StrongGenSet[{1,2},GenSet[-(1,2)]]	$\frac{1}{3}\mathcal{A}_{\alpha\beta\chi} + \frac{1}{3}\mathcal{A}_{\alpha\chi\beta} - \frac{1}{3}\mathcal{A}_{\beta\alpha\chi} - \frac{1}{3}\mathcal{A}_{\beta\chi\alpha} + \frac{1}{3}\mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\,\eta_{\alpha\chi} - \frac{1}{6}\mathcal{A}^{\delta}{}_{\beta\delta}\,\eta_{\alpha\chi} - \frac{1}{6}\mathcal{A}^{\delta}{}_{\delta\beta}\,\eta_{\alpha\chi} - \frac{1}{3}\mathcal{A}_{\alpha}{}^{\delta}{}_{\delta}\,\eta_{\beta\chi} + \frac{1}{6}\mathcal{A}^{\delta}{}_{\alpha\delta}\,\eta_{\beta\chi} + \frac{1}{6}\mathcal{A}^{\delta}{}_{\delta\alpha}\,\eta_{\beta\chi} - \frac{1}{3}\mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\,n_{\alpha}\,n_{\chi} + \frac{1}{6}\mathcal{A}^{\delta}{}_{\beta\delta}\,n_{\alpha}\,n_{\chi} +$ $\frac{1}{6}\mathcal{A}^{\delta}{}_{\delta\beta}\,n_{\alpha}\,n_{\chi} + \frac{1}{3}\mathcal{A}_{\alpha}{}^{\delta}{}_{\delta}\,n_{\beta}\,n_{\chi} - \frac{1}{6}\mathcal{A}^{\delta}{}_{\alpha\delta}\,n_{\beta}\,n_{\chi} - \frac{1}{6}\mathcal{A}^{\delta}{}_{\delta\alpha}\,n_{\beta}\,n_{\chi} + \frac{1}{3}\mathcal{A}_{\beta\chi}{}^{\delta}\,n_{\alpha}\,n_{\delta} + \frac{1}{3}\mathcal{A}_{\beta}{}^{\delta}{}_{\chi}\,n_{\alpha}\,n_{\delta} - \frac{1}{3}\mathcal{A}^{\delta}{}_{\beta\chi}\,n_{\alpha}\,n_{\delta} - \frac{1}{3}\mathcal{A}^{\delta}{}_{\chi\beta}\,n_{\alpha}\,n_{\delta} + \frac{1}{3}\mathcal{A}^{\delta\epsilon}{}_{\epsilon}\,\eta_{\beta\chi}\,n_{\alpha}\,n_{\delta} - \frac{1}{3}\mathcal{A}_{\alpha\chi}{}^{\delta}\,n_{\beta}\,n_{\delta} -$ $\frac{1}{3}\mathcal{A}_{\alpha}{}^{\delta}{}_{\chi}\,n_{\beta}\,n_{\delta} + \frac{1}{3}\mathcal{A}^{\delta}{}_{\alpha\chi}\,n_{\beta}\,n_{\delta} + \frac{1}{3}\mathcal{A}^{\delta}{}_{\chi\alpha}\,n_{\beta}\,n_{\delta} - \frac{1}{3}\mathcal{A}^{\delta\epsilon}{}_{\epsilon}\,\eta_{\alpha\chi}\,n_{\beta}\,n_{\delta} - \frac{1}{3}\mathcal{A}_{\alpha\beta}{}^{\delta}\,n_{\chi}\,n_{\delta} - \frac{1}{3}\mathcal{A}_{\alpha}{}^{\delta}{}_{\beta}\,n_{\chi}\,n_{\delta} + \frac{1}{3}\mathcal{A}_{\beta\alpha}{}^{\delta}\,n_{\chi}\,n_{\delta} + \frac{1}{3}\mathcal{A}_{\beta}{}^{\delta}{}_{\alpha}\,n_{\chi}\,n_{\delta} - \frac{1}{6}\mathcal{A}^{\delta}{}_{\delta}{}^{\epsilon}\,\eta_{\beta\chi}\,n_{\alpha}\,n_{\epsilon} -$ $\frac{1}{6}\mathcal{A}^{\delta\epsilon}{}_{\delta}\,\eta_{\beta\chi}\,n_{\alpha}\,n_{\epsilon} + \frac{1}{6}\mathcal{A}^{\delta}{}_{\delta}{}^{\epsilon}\,\eta_{\alpha\chi}\,n_{\beta}\,n_{\epsilon} + \frac{1}{6}\mathcal{A}^{\delta\epsilon}{}_{\delta}\,\eta_{\alpha\chi}\,n_{\beta}\,n_{\epsilon} - \frac{1}{3}\mathcal{A}_{\beta}{}^{\delta\epsilon}\,\eta_{\alpha\chi}\,n_{\delta}\,n_{\epsilon} + \frac{1}{6}\mathcal{A}^{\delta}{}_{\beta}{}^{\epsilon}\,\eta_{\alpha\chi}\,n_{\delta}\,n_{\epsilon} + \frac{1}{6}\mathcal{A}^{\delta\epsilon}{}_{\beta}\,\eta_{\alpha\chi}\,n_{\delta}\,n_{\epsilon} + \frac{1}{3}\mathcal{A}_{\alpha}{}^{\delta\epsilon}\,\eta_{\beta\chi}\,n_{\delta}\,n_{\epsilon} - \frac{1}{6}\mathcal{A}^{\delta}{}_{\alpha}{}^{\epsilon}\,\eta_{\beta\chi}\,n_{\delta}\,n_{\epsilon} -$ $\frac{1}{6}\mathcal{A}^{\delta\epsilon}{}_{\alpha}\,\eta_{\beta\chi}\,n_{\delta}\,n_{\epsilon} - \frac{1}{3}\mathcal{A}_{\beta}{}^{\delta\epsilon}\,n_{\alpha}\,n_{\chi}\,n_{\delta}\,n_{\epsilon} + \frac{1}{6}\mathcal{A}^{\delta}{}_{\beta}{}^{\epsilon}\,n_{\alpha}\,n_{\chi}\,n_{\delta}\,n_{\epsilon} + \frac{1}{6}\mathcal{A}^{\delta\epsilon}{}_{\beta}\,n_{\alpha}\,n_{\chi}\,n_{\delta}\,n_{\epsilon} + \frac{1}{3}\mathcal{A}_{\alpha}{}^{\delta\epsilon}\,n_{\beta}\,n_{\chi}\,n_{\delta}\,n_{\epsilon} - \frac{1}{6}\mathcal{A}^{\delta}{}_{\alpha}{}^{\epsilon}\,n_{\beta}\,n_{\chi}\,n_{\delta}\,n_{\epsilon} - \frac{1}{6}\mathcal{A}^{\delta\epsilon}{}_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}\,n_{\epsilon}$	${}^{2^-}\mathcal{W}_{s}{}^{\parallel}{}_{\alpha\beta\chi}$
${}^{3^-}\mathcal{A}_{s}{}^{\parallel}{}_{\alpha\beta\chi}$	StrongGenSet[{1, 2, 3}, GenSet[(1,2), (2,3)]]	$\frac{1}{6}\mathcal{A}_{\alpha\beta\chi} + \frac{1}{6}\mathcal{A}_{\alpha\chi\beta} + \frac{1}{6}\mathcal{A}_{\beta\alpha\chi} + \frac{1}{6}\mathcal{A}_{\beta\chi\alpha} + \frac{1}{6}\mathcal{A}_{\chi\alpha\beta} + \frac{1}{6}\mathcal{A}_{\chi\beta\alpha} - \frac{1}{15}\mathcal{A}_{\chi}{}^{\delta}{}_{\delta}\,\eta_{\alpha\beta} - \frac{1}{15}\mathcal{A}^{\delta}{}_{\chi\delta}\,\eta_{\alpha\beta} - \frac{1}{15}\mathcal{A}^{\delta}{}_{\delta\chi}\,\eta_{\alpha\beta} - \frac{1}{15}\mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\,\eta_{\alpha\chi} - \frac{1}{15}\mathcal{A}^{\delta}{}_{\beta\delta}\,\eta_{\alpha\chi} - \frac{1}{15}\mathcal{A}^{\delta}{}_{\delta\beta}\,\eta_{\alpha\chi} - \frac{1}{15}\mathcal{A}_{\alpha}{}^{\delta}{}_{\delta}\,\eta_{\beta\chi} -$ $\frac{1}{15}\mathcal{A}^{\delta}{}_{\alpha\delta}\,\eta_{\beta\chi} - \frac{1}{15}\mathcal{A}^{\delta}{}_{\delta\alpha}\,\eta_{\beta\chi} + \frac{1}{15}\mathcal{A}_{\chi}{}^{\delta}{}_{\delta}\,n_{\alpha}\,n_{\beta} + \frac{1}{15}\mathcal{A}^{\delta}{}_{\chi\delta}\,n_{\alpha}\,n_{\beta} + \frac{1}{15}\mathcal{A}^{\delta}{}_{\delta\chi}\,n_{\alpha}\,n_{\beta} + \frac{1}{15}\mathcal{A}_{\beta}{}^{\delta}{}_{\delta}\,n_{\alpha}\,n_{\chi} + \frac{1}{15}\mathcal{A}^{\delta}{}_{\beta\delta}\,n_{\alpha}\,n_{\chi} + \frac{1}{15}\mathcal{A}^{\delta}{}_{\delta\beta}\,n_{\alpha}\,n_{\chi} + \frac{1}{15}\mathcal{A}_{\alpha}{}^{\delta}{}_{\delta}\,n_{\beta}\,n_{\chi} +$ $\frac{1}{15}\mathcal{A}^{\delta}{}_{\alpha\delta}\,n_{\beta}\,n_{\chi} + \frac{1}{15}\mathcal{A}^{\delta}{}_{\delta\alpha}\,n_{\beta}\,n_{\chi} - 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\frac{1}{5}\mathcal{A}^{\delta\epsilon\phi}\,\eta_{\beta\chi}\,n_{\alpha}\,n_{\delta}\,n_{\epsilon}\,n_{\phi} - \frac{1}{5}\mathcal{A}^{\delta\epsilon\phi}\,\eta_{\alpha\chi}\,n_{\beta}\,n_{\delta}\,n_{\epsilon}\,n_{\phi} - \frac{1}{5}\mathcal{A}^{\delta\epsilon\phi}\,\eta_{\alpha\beta}\,n_{\chi}\,n_{\delta}\,n_{\epsilon}\,n_{\phi} - \frac{2}{5}\mathcal{A}^{\delta\epsilon\phi}\,n_{\alpha}\,n_{\beta}\,n_{\chi}\,n_{\delta}\,n_{\epsilon}\,n_{\phi}$	${}^{3^-}\mathcal{W}_{s}{}^{\parallel}{}_{\alpha\beta\chi}$

FIG. SM1. Kinematic structure of metric-affine gravity. The SO(3) irreps have a different labelling of duplicate J^P states to those in [35]. Note that the 2^- and 3^- states have a hidden multi-term cyclic symmetry on all indices, which is not accommodated by the C language implementation of the Butler-Portugal algorithm. These definitions are used in Figs. SM2 to SM6.

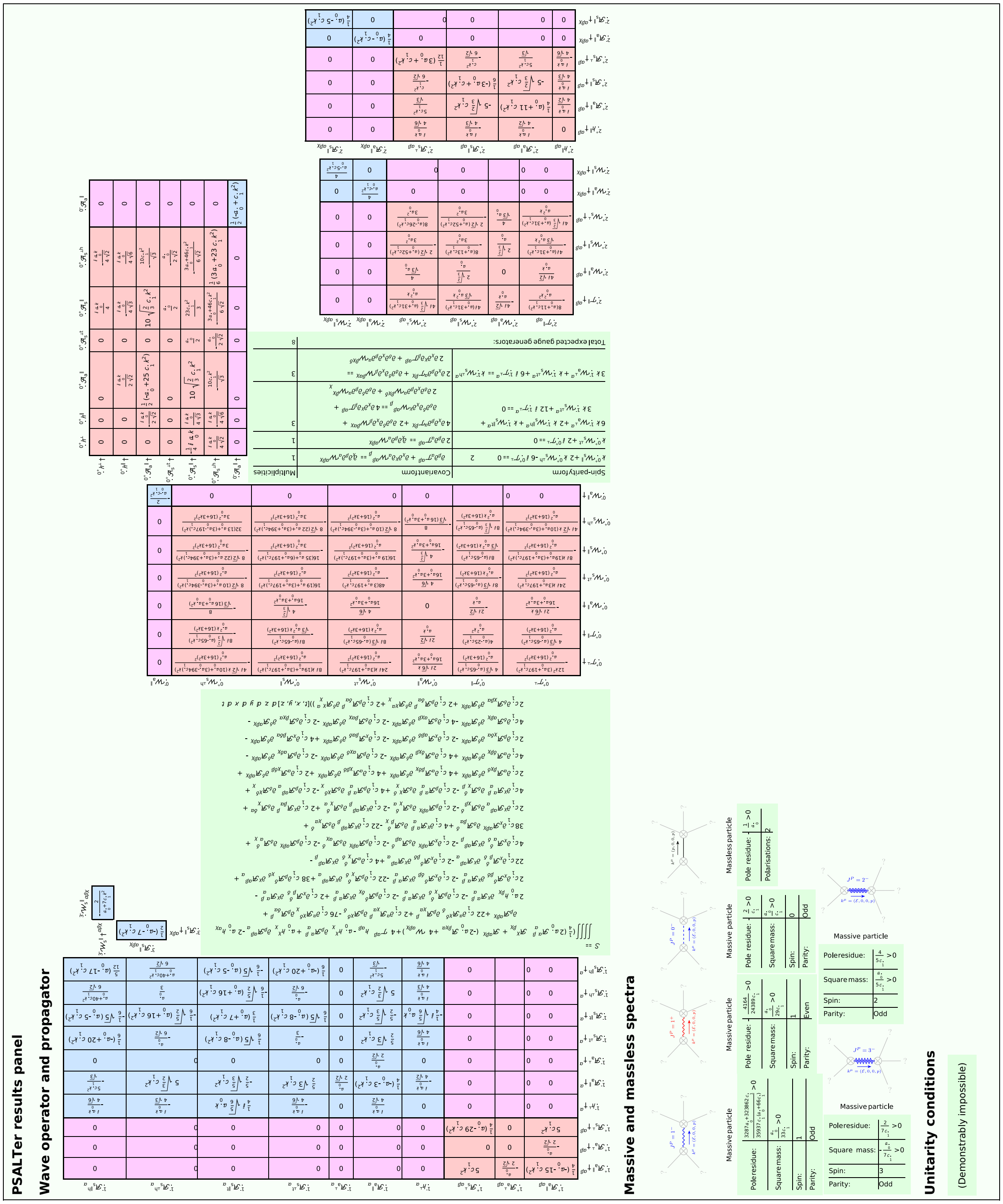

FIG. SM2. Particle spectrum of the (arbitrary) projective-invariant theory defined in Eq. (7), in the first-order formulation, where a_0 corresponds to $-M_{\mathrm{P}}{}^2/2$ and c_1 corresponds to α. Note that there are eight gauge generators in total: these correspond to the d.o.f in the diffeomorphism and projective vector generators. The massive spectrum is shown in Table I, and we note in addition the presence of the massless graviton, whose no-ghost condition requires $a_0 < 0$. All the quantities in this output are defined in Fig. SM1.

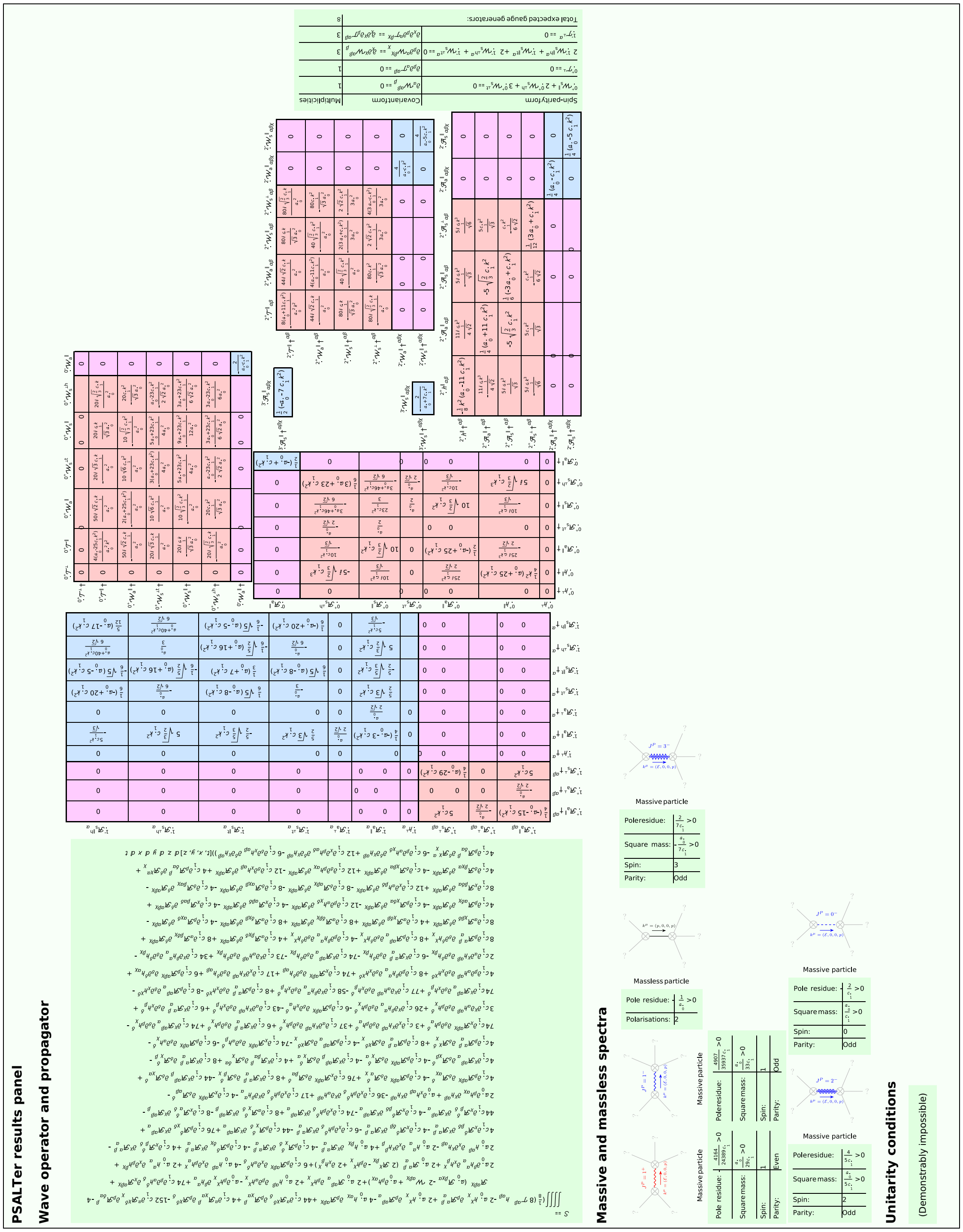

FIG. SM3. The results in Fig. SM2 repeated in the second-order formulation. Note that the quadratic action in this case contains very many more operators than does Fig. SM2. The matrix elements and the forms of the source constraints and pole residues are expected to change, but the mass spectrum and overall (non-)unitarity is the same. All the quantities in this output are defined in Fig. SM1.

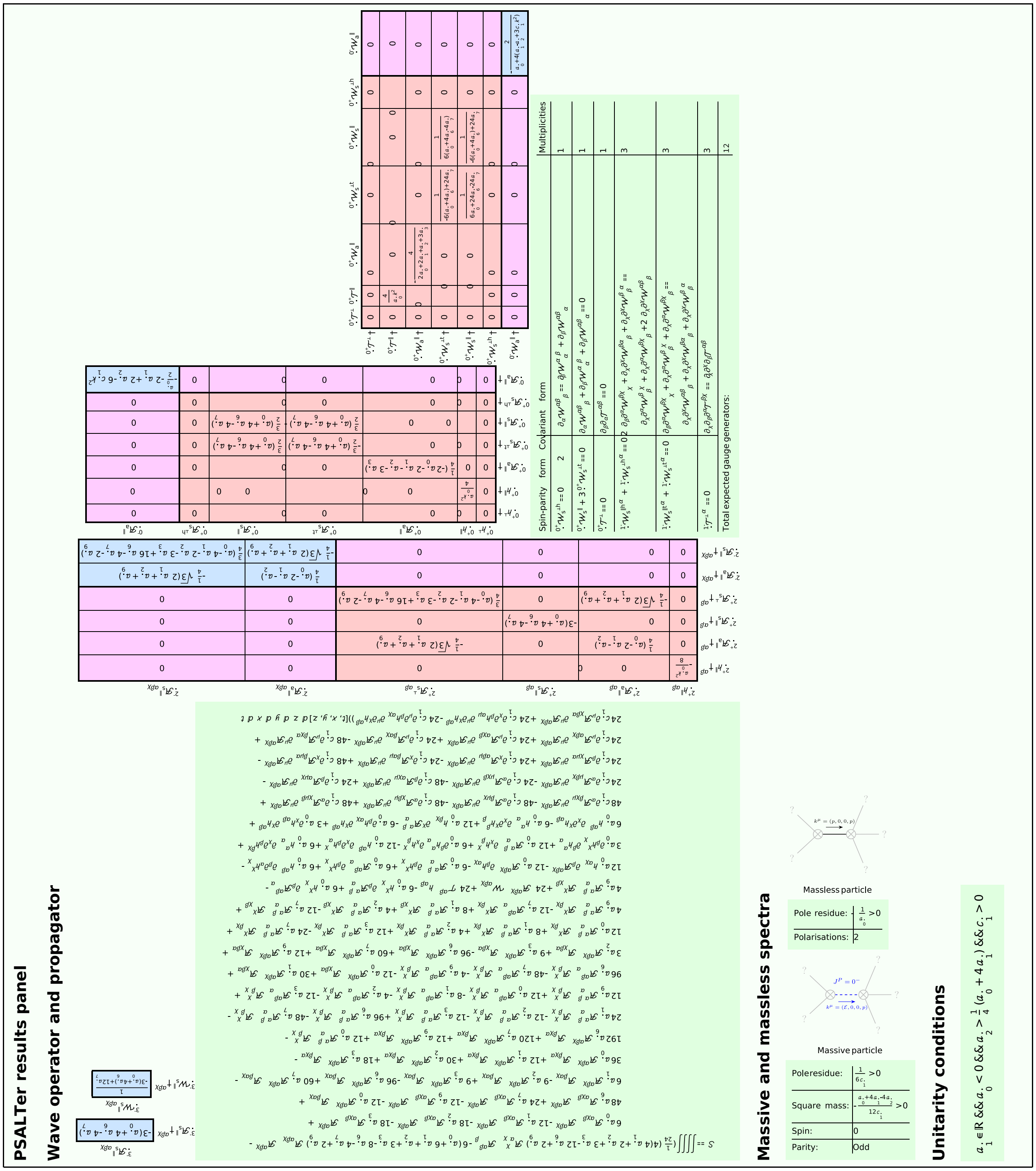

FIG. SM4. Particle spectrum of the (general) EP-invariant theory defined in Eq. (11), in the second-order formulation, where a_0 corresponds to $-M_{\mathrm{P}}{}^2/2$ and c_1 corresponds to α. The parameters b_1, b_2 and b_3 are represented among the many remaining a_i, but in our linearisation we are not careful to eliminate the pure-tensor $t^{\mu}{}_{\nu\sigma}$ and $q^{\mu}{}_{\nu\sigma}$ sector. Note that we now have 12 gauge generators: diffeomorphisms and the two vector generators of the EP symmetry. In the particle spectrum, we see the graviton and the pseudoscalar in Eq. (14a). By linearising Eq. (14b) and carefully matching the parameters we could in principle recover the square of the pseudoscalar mass — in any case we can see that α and c_1 appear in the denominator as expected. The entire theory may be made unitary. All the quantities in this output are defined in Fig. SM1.

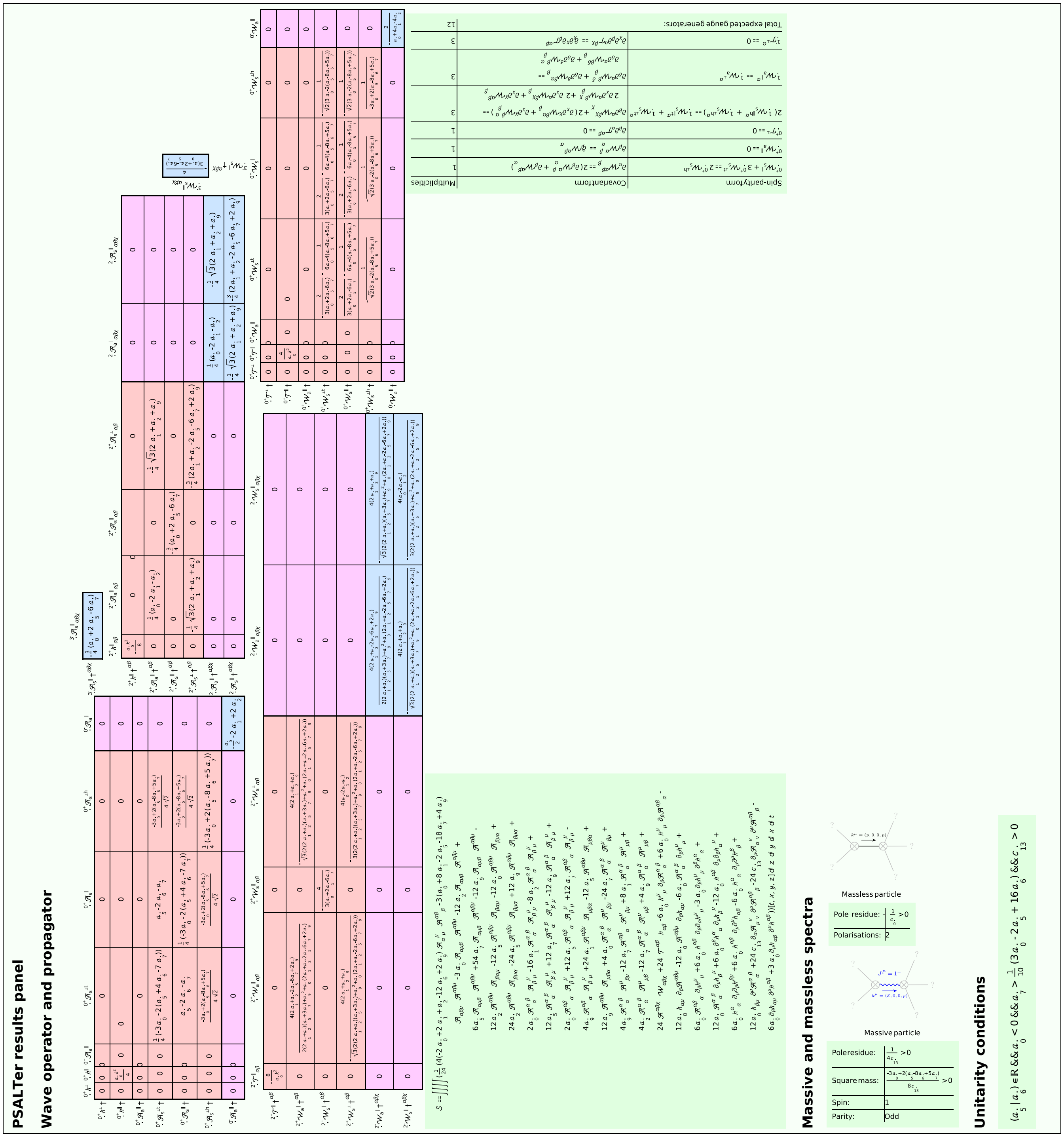

FIG. SM5. Particle spectrum of the (general) IW-invariant theory defined in Eq. (C1), in the second-order formulation, where a_0 corresponds to $-M_{\rm P}{}^2/2$ and c_{13} corresponds to α. The parameters b_1, b_2 and b_3 are again over-represented as in Fig. SM4. Note that we again have 12 gauge generators: diffeomorphisms and the two vector generators of the IW symmetry. In the particle spectrum, we see the graviton and the massive (Weyl) Proca field Q_μ. Again, we expect to recover the mass in Eq. (C2). The entire theory may be made unitary. All the quantities in this output are defined in Fig. SM1.

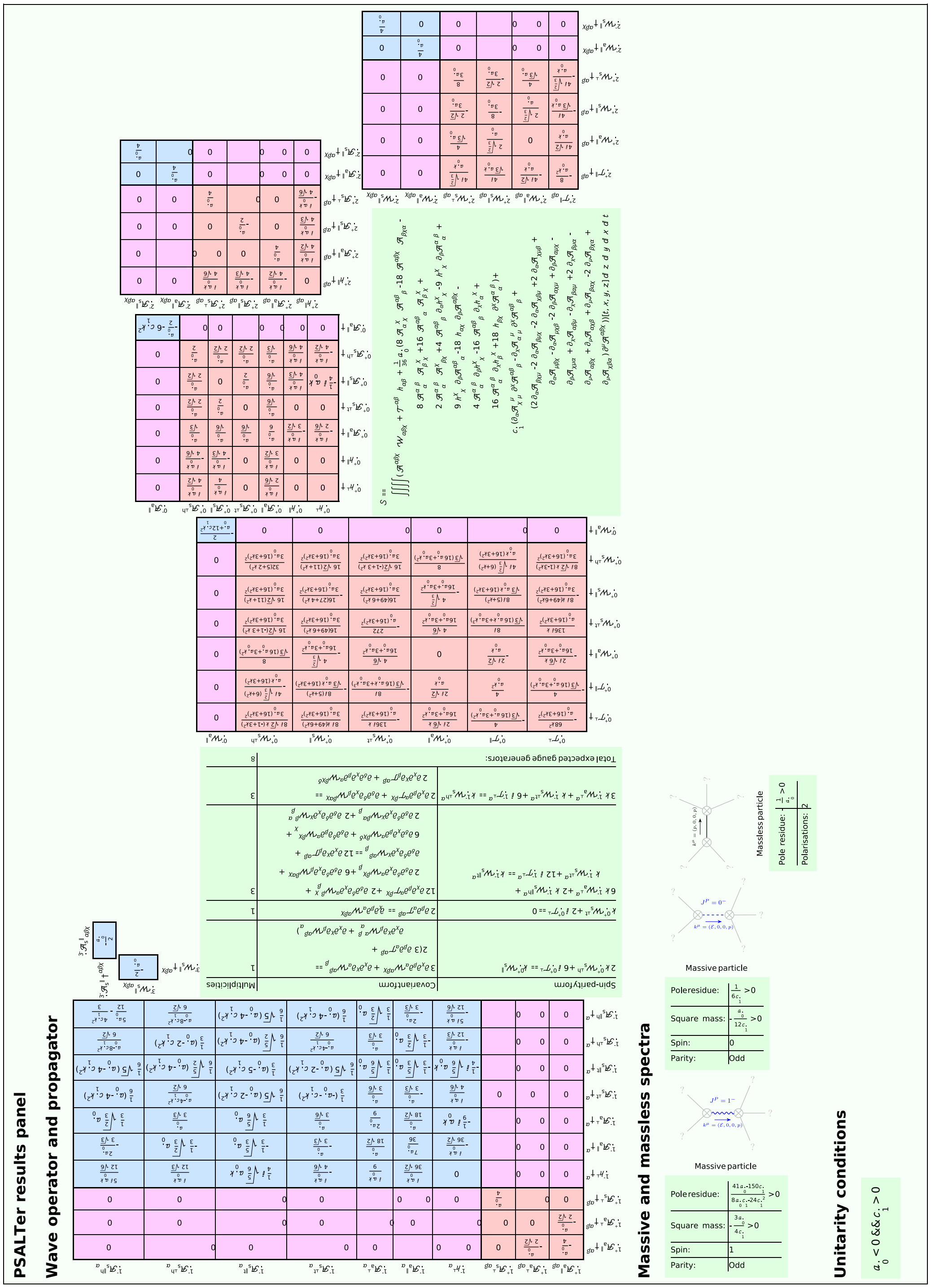

FIG. SM6. Particle spectrum of an arbitrarily-tuned concurrent-invariant theory, derived from the general theory defined in Eq. (C4), in the first-order formulation, where a_0 corresponds to $-M_{\mathrm{P}}^2/2$ and c_1 corresponds to α and $\tilde{\alpha}$, and the remaining dimensionful couplings are set arbitrarily to a_0. Note that we have only eight gauge generators: diffeomorphisms and the vector generating the concurrent transformations. In the particle spectrum, we see the graviton is joined by both the massive (Weyl) Proca field in Fig. SM5 and the pseudoscalar in Fig. SM4. Again, we expect to recover the mass in Eq. (C5). The entire theory may be made unitary. All the quantities in this output are defined in Fig. SM1.