

Package xAct`xPerm` version 1.2.3, {2015, 8, 23}

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Connecting to external linux executable...

Connection established.

Package xAct`xTensor` version 1.2.0, {2021, 10, 17}

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Package xAct`xPlain` version 0.0.0–developer, {2025, 8, 24}

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General solution

Original basis used for our work

We first consider the basis.

$$\lambda_0 O_0 + \lambda_1 O_1 + \lambda_2 O_2 + \lambda_3 O_3 + \lambda_4 O_4 + \lambda_5 O_5$$

(1)

Here is the general solution.

$$\lambda_2 \rightarrow -\frac{\lambda_0(-6+\mathcal{D}+2s)+\lambda_1(-2+s(-3+\mathcal{D}+s))}{(-1+s)(-4+\mathcal{D}+s)} \quad (2)$$

$$\lambda_3 \rightarrow -\lambda_5 \quad (3)$$

$$\lambda_4 \rightarrow \frac{2\lambda_0+2\lambda_1-\lambda_5(-1+s)(-4+\mathcal{D}+s)}{2(-1+s)(-4+\mathcal{D}+s)} \quad (4)$$

Here is the case of "s=1". Note that there are fewer operators than six in the case of "s=1" and "s=2".

$$\lambda_1 \rightarrow -\lambda_0 \quad (5)$$

The specific case of "s=3".

$$\lambda_2 \rightarrow \frac{\mathcal{D}\lambda_0-2\lambda_1+3\mathcal{D}\lambda_1}{2-2\mathcal{D}} \quad (6)$$

$$\lambda_5 \rightarrow \frac{\lambda_0+\lambda_1-2(-1+\mathcal{D})(\lambda_3+\lambda_4)}{3(-1+\mathcal{D})} \quad (7)$$

Alternative Fronsdal-type basis

Here we define a basis, where the Fronsdal combination and the trace of Fronsdal have their own coefficients. This makes it manifest that these coefficients completely disappear from the system which you provided.

$$\lambda_0 \rightarrow \mathcal{F}_1 \quad (8)$$

$$\lambda_1 \rightarrow c_1 - \mathcal{F}_1 \quad (9)$$

$$\lambda_2 \rightarrow c_2 + \mathcal{F}_1 \quad (10)$$

$$\lambda_3 \rightarrow \mathcal{F}_2 \quad (11)$$

$$\lambda_4 \rightarrow \frac{1}{2}(2c_4 + \mathcal{F}_2) \quad (12)$$

$$\lambda_5 \rightarrow c_5 - \mathcal{F}_2 \quad (13)$$

Here is the transformation of the operator sum.

$$c_1 O_1 + c_2 O_2 + \mathcal{F}_1 (O_0 - O_1 + O_2) + c_4 O_4 + \mathcal{F}_2 \left(O_3 + \frac{O_4}{2} - O_5 \right) + c_5 O_5 \quad (14)$$

Here is the general solution. Notice how the Fronsdal-type coefficients completely disappear.

$$c_2 \rightarrow -\frac{c_1 (-2 + s (-3 + \mathcal{D} + s))}{(-1 + s) (-4 + \mathcal{D} + s)} \quad (15)$$

$$c_4 \rightarrow \frac{c_1}{(-1 + s) (-4 + \mathcal{D} + s)} \quad (16)$$

$$c_5 \rightarrow 0 \quad (17)$$

Here is the case of "s=1". Note that there are fewer operators than six in the case of "s=1" and "s=2".

$$c_1 \rightarrow 0 \quad (18)$$

The specific case of "s=3".

$$c_2 \rightarrow \frac{c_1 (2 - 3 \mathcal{D})}{2 (-1 + \mathcal{D})} \quad (19)$$

$$c_5 \rightarrow \frac{1}{3} \left(-2 c_4 + \frac{c_1}{-1 + \mathcal{D}} \right) \quad (20)$$

System coefficients

Here are the coefficients of the system of equations for general spin and dimension.

$$\left(\begin{array}{l} \frac{2^{-n} (-2n+s)}{\text{Pochhammer}\left[3-\frac{\mathcal{D}}{2}-s, n\right]} \\ \frac{2^{-1-n} (-2n+s) (2+2n (-3+\mathcal{D}+s)-s (-3+\mathcal{D}+s)) \text{Gamma}\left[3-\frac{\mathcal{D}}{2}-s\right]}{\text{Gamma}\left[4-\frac{\mathcal{D}}{2}+n-s\right]} \\ - \frac{2^{-1-n} (2n-s) (1+2n-s) (-4+\mathcal{D}+s) \text{Gamma}\left[3-\frac{\mathcal{D}}{2}-s\right]}{\text{Gamma}\left[4-\frac{\mathcal{D}}{2}+n-s\right]} \\ - \frac{2^{1-n} n (-4+\mathcal{D}+s) \text{Gamma}\left[3-\frac{\mathcal{D}}{2}-s\right]}{\text{Gamma}\left[3-\frac{\mathcal{D}}{2}+n-s\right]} \\ - \frac{2^{-n} n (2n-s) (-5+\mathcal{D}+s) (-4+\mathcal{D}+s) \text{Gamma}\left[3-\frac{\mathcal{D}}{2}-s\right]}{\text{Gamma}\left[4-\frac{\mathcal{D}}{2}+n-s\right]} \\ \frac{2^{-1-n} n (-4+\mathcal{D}+s) (-2n (-3+s)+(-4+s) (3+s)+\mathcal{D} (2-2n+s)) \text{Gamma}\left[3-\frac{\mathcal{D}}{2}-s\right]}{\text{Gamma}\left[4-\frac{\mathcal{D}}{2}+n-s\right]} \end{array} \right)$$

Here is a grid of discrete ranks.

⋮ **Power:** Infinite expression $\frac{1}{0}$ encountered. ⓘ

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⋮ **First:** {} has zero length and no first element.

$$\left(\begin{array}{ccccccc} 0 & 1 & 2 & X & 3 & 3 & 3 \\ 0 & 1 & 2 & 1 & 2 & 2 & 3 \\ 0 & 1 & 2 & 2 & 3 & 3 & 3 \\ 0 & 1 & 2 & 2 & 3 & 3 & 3 \\ 0 & 1 & 2 & 2 & 3 & 3 & 3 \\ 0 & 1 & 2 & 2 & 3 & 3 & 3 \\ 0 & 1 & 2 & 2 & 3 & 3 & 3 \\ 0 & 1 & 2 & 2 & 3 & 3 & 3 \end{array} \right)$$

(22)

Key observation: This is the end of the script.