Seminar 5

General relativity

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University of Cambridge

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What is general relativity?

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- Which was invented first, **special relativity** or **general relativity**?
- **Special relativity** tells us about the **fundamental structure** of spacetime (**Minkowskian signature**, **four dimensions** etc)
- **General relativity** tells us how the **geometry** of the spacetime is affected by **matter**

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High-speed tour of general relativity

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- To start off, I'll try to give an overview of the physics
- **FEEL FREE TO HAVE A NAP IF YOU AREN'T INTERESTED: IT IS FRIDAY. . .**

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- **Square it**!

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High-speed tour of general relativity

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- We also learned that the LHS means **distance** in special relativity, right?
- What do they tell you about how **light** travels in school?
- And what is the **shortest distance** between two points in geometry?
- OK, so the equation we have is just a mathematical statement of the **shortest distance**!

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High-speed tour of general relativity

- If spacetime is **curved**, the **shortest distance** isn't a **straight line**

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High-speed tour of general relativity

- If spacetime is **curved**, the **shortest distance** isn't a **straight line**
- In this case, the nice simple formula gets very horrible:

$$
\begin{gathered}
c^2 g_{tt}\Delta x^2 + g_{xx}\Delta x^2 + g_{yy}\Delta y^2 + g_{zz}\Delta z^2 \\
+2cg_{tx}\Delta t\Delta x + 2cg_{ty}\Delta t\Delta y + 2cg_{tz}\Delta t\Delta z \\
+2g_{xy}\Delta x\Delta y + 2g_{xz}\Delta x\Delta z + 2g_{zy}\Delta z\Delta y = 0
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- We have introduced a new object:

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g_{\mu\nu} = g_{\mu\nu}(ct, x, y, z)
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- We have introduced a new object:

$$g_{\mu\nu} = g_{\mu\nu}(ct, x, y, z)$$

- The **labels** μ and ν could be t, x, y or z

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High-speed tour of general relativity

- What kind of **mathematical object** do you think $g_{\mu\nu}$ is?

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- We say $g_{\mu\nu}$ is the **metric tensor**, it varies with **space** and **time**, and encodes the fact that the **shortest distance** might not be the **straightest line**

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- Therefore it contains **all the information about the curvature, which is equal to the gravitational field**

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- Therefore it contains **all the information about the curvature, which is equal to the gravitational field**
- Mason/Emre or anyone else not napping: can you find $g_{\mu\nu}$ for the original **flat case** – there should only be **four** $g_{\mu\nu}$ components which aren't zero...

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- Now $g_{\mu\nu}$ might contain the information about gravity, but it isn't equal to the curvature immediately...

- Now $g_{\mu\nu}$ might contain the information about gravity, but it isn't equal to the curvature immediately...
- The **curvature** is given by the **curvature tensor** $R_{\mu\nu}$ which depends in a **complicated way** on $g_{\mu\nu}$...

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$$
\begin{aligned}
&R_{\mu\nu} = \\
&\frac{1}{2}\,\partial_\rho g^{\rho\sigma}\,\partial_\nu g_{\mu\sigma} + \frac{1}{2}\,\partial_\rho g^{\rho\sigma}\,\partial_\mu g_{\nu\sigma} - \frac{1}{2}\,\partial_\rho g^{\rho\sigma}\,\partial_\sigma g_{\mu\nu} + \frac{1}{2}\,g^{\rho\sigma}\partial_{\nu\rho} g_{\mu\sigma} \\
&+ \frac{1}{2}\,g^{\rho\sigma}\partial_{\mu\rho} g_{\nu\sigma} - \frac{1}{2}\,g^{\rho\sigma}\partial_{\rho\sigma} g_{\mu\nu} - \frac{1}{2}\,\partial_\nu g^{\rho\sigma}\,\partial_\mu g_{\rho\sigma} - \frac{1}{2}\,g^{\rho\sigma}\partial_{\mu\nu} g_{\rho\sigma} \\
&+ \frac{1}{4}\,g^{\kappa\lambda}\partial_\nu g_{\mu\kappa}\,g^{\rho\sigma}\partial_\lambda g_{\rho\sigma} + \frac{1}{4}\,g^{\kappa\lambda}\partial_\mu g_{\nu\kappa}\,g^{\rho\sigma}\partial_\lambda g_{\rho\sigma} - \frac{1}{4}\,g^{\kappa\lambda}\partial_\kappa g_{\mu\nu}\,g^{\rho\sigma}\partial_\lambda g_{\rho\sigma} \\
&- \frac{1}{4}\,g^{\kappa\lambda}\partial_\mu g_{\kappa\rho}\,g^{\rho\sigma}\partial_\nu g_{\lambda\sigma} - \frac{1}{2}\,g^{\kappa\lambda}\partial_\kappa g_{\mu\rho}\,g^{\rho\sigma}\partial_\sigma g_{\nu\lambda} + \frac{1}{2}\,g^{\kappa\lambda}\partial_\kappa g_{\mu\rho}\,g^{\rho\sigma}\partial_\lambda g_{\nu\sigma}
\end{aligned}
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- But where does the **curvature** come from?

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- But where does the **curvature** come from?
- So somehow we need an **equation** to relate $R_{\mu\nu}$ to **mass-energy**, **momentum** and **stress** (or as Claudia said, **pressure**). . .

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- That formula should tell you something about how smart Einstein was. . .
- But where does the **curvature** come from?
- So somehow we need an **equation** to relate $R_{\mu\nu}$ to **mass-energy**, **momentum** and **stress** (or as Claudia said, **pressure**). . .
- So we need one of these **tensor things** that encodes the **mass-energy**, **momentum** and **stress**. . .

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- All these things which generate **gravity** are described by the **stress-energy tensor** $T_{\mu\nu}$

High-speed tour of general relativity

- All these things which generate **gravity** are described by the **stress-energy tensor** $T_{\mu\nu}$
- Let's write it in **matrix form**:

$$\mathbf{T} = \begin{bmatrix} \mathcal{E} & p_x & p_y & p_z \\ p_x & s_{xx} & s_{xy} & s_{xz} \\ p_y & s_{yx} & s_{yy} & s_{yz} \\ p_z & s_{zx} & s_{zy} & s_{zz} \end{bmatrix}$$

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- Any idea what the components represent?

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High-speed tour of general relativity

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- We just need to define something that depends on the **curvature** tensor, called the **Einstein tensor**:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

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- We are now very close to the fundamental statement behind general relativity!
- We just need to define something that depends on the **curvature** tensor, called the **Einstein tensor**:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

- And finally we get this:

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

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High-speed tour of general relativity

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- These are the **Einstein field equations**:

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- First written down in 1915/1916
- You've seen how complicated they are when expressed with $g_{\mu\nu}$: **it is incredably hard to find solutions to these equations**

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- So matter usually moves in straight lines like light, but if the spacetime is curved, it will move in a curved line, example anyone?
- Something in orbit!
- The formula we began with is a statement of the **geodesic equation**, which encodes all these ideas about how things **move**

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High-speed tour of general relativity

- So we're done! We have **two ideas** out of **general relativity**:
  - **Geodesic equation**: curved spacetime tells matter how to move
  - **Einstein field equations**: matter tells spacetime how to curve

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- So we're done! We have **two ideas** out of **general relativity**:
  - **Geodesic equation**: curved spacetime tells matter how to move
  - **Einstein field equations**: matter tells spacetime how to curve
- If you take these ideas away today, Seminar 5 will have been a success :)

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The Schwarzschild black hole

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- I mentioned it before, anyone rememeber what it is?
- The **curved** spacetime in the **vacuum** around a **spherically symmetric** mass, M
- **Y'all are living inside of the Schwarzschild solution!**

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The Schwarzschild black hole

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The Schwarzschild black hole

- Okay, but if we live in the Schwarzschild solution (e.g. the curved spacetime in the vacuum around the sun through which the Earth orbits) then why is this section titled *Schwarzschild black hole*?

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- When r becomes a certain value, we are looking at a **black hole**!

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- What did I say about a solar-mass black hole?

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- When r becomes a certain value, we are looking at a **black hole**!
- What did I say about a solar-mass black hole?
- So it is the **same** gravity/curvature out here in the Earth's orbit!

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The Schwarzschild black hole

- Note to self: go to whiteboard and talk about **light-cones**, **future**, **past** and **causality**. . .

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The Schwarzschild black hole

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The Schwarzschild black hole

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- Should be t (or ct if we're being careful!) and the **radial** coordinate, r

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The Schwarzschild black hole

- Right, so let's just consider a photon which moves **directly toward** or **directly away from** the black hole
- What two coordinates are we going to need?
- Should be t (or ct if we're being careful!) and the **radial** coordinate, r
- Switch off all the other coordinates, keep it **simple**!

The Schwarzschild black hole

- Just with those **two** coordinates we expect a **complete mess** for the **shortest distance** followed by a photon:

$$c^2 g_{tt} \Delta t^2 + g_{rr} \Delta r^2 + 2c g_{tr} \Delta t \Delta r = 0$$

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- Actually it isn't nearly so bad as we might expect:

$$\left(1 - \frac{r_s}{r}\right) c^2 \Delta t^2 - \frac{1}{\left(1 - \frac{r_s}{r}\right)} \Delta r^2 = 0$$

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- You have **probably heard** of the quantity r_s in science fiction, anyone know what it is?

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The Schwarzschild black hole

- Photon's path towards/away from a black hole:

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$$r_s=\frac{2MG}{c^2}$$

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- So r_s is the **Schwarzschild radius**:

$$r_s=\frac{2MG}{c^2}$$

- What happens if $M\to 0$?
- We just have the photon motion of **flat space**, but written $x=r$:

$$c^2\Delta t^2-\Delta r^2=0$$

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- **Over to you now!**

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- Find this:

$$\frac{c\Delta t}{\Delta r}$$

- Emre: $r=0.9r_s$ and $r=r_s$, Mason: $r=0.1r_s$ and $r=0$, Ali Goktug: $r=100000r_s$, Claudia: $r=2r_s$, Federico: $r=3r_s$, Beltran: $r=4r_s$

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- So what did we find:

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  - When $r \to \infty$ we get the **same** light-cone as with $M \to 0$, i.e. **flat space**, why?

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The Schwarzschild black hole

- So what did we find:
  - When $r \to \infty$ we get the **same** light-cone as with $M \to 0$, i.e. **flat space**, why?
  - When $r \to r_s$ but $r > r_s$ the light-cone becomes **squashed**, we can think of this as the photon being **pulled** by the increasingly curved spacetime, but crucially, **the photon, and any massive particle, can still move towards or away from the black hole**

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The Schwarzschild black hole

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  - When $r \to \infty$ we get the **same** light-cone as with $M \to 0$, i.e. **flat space**, why?
  - When $r \to r_s$ but $r > r_s$ the light-cone becomes **squashed**, we can think of this as the photon being **pulled** by the increasingly curved spacetime, but crucially, **the photon, and any massive particle, can still move towards or away from the black hole**
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  - When $r = 0$ something **terrible** happens, **physics breaks**

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The Schwarzschild black hole

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  - (Ali Goktug asked about Hawking radiation – rememeber to mention this)

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The Schwarzschild black hole

- Another note to self:

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The Schwarzschild black hole

- Another note to self:
  - Find a clip from Interstellar or something?

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Gravity waves

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Gravity waves

- More notes to self:
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  - Talk about the wave equation...

Gravity waves

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- With **weak** gravity the Einstein field equations (with coordinates x and t only) become:

$$g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} \to \frac{1}{c^2}\frac{\partial^2 h_{\mu\nu}(x,t)}{\partial t^2} - \frac{\partial^2 h_{\mu\nu}(x,t)}{\partial x^2} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

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Gravity waves

- So in a **vacuum**, the Einstein field equations predict that the **shape of spacetime** (i.e. shortest distance between points) moves along like a **wave** at speed c

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Final remarks

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- Thanks for your time, enjoy week 2! :)
- (for Uni application references etc, or questions, I'm at wb263@cam.ac.uk)

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