Seminar 1

Overview of modern physics

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• Seminar 1: Overview of modern physics





- Seminar 1: Overview of modern physics
- Seminar 2: Overview of Astrophysics







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- Seminar 3: Calculus and classical mechanics







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- Seminar 5: General relativity





• Space?







- Space?
- Time?







- Space?
- Time?
- Energy?







- Space?
- Time?
- Energy?
- Forces?







- Space?
- Time?
- Energy?
- Forces?
- Mathematical description of all these things







• a priori (thermodynamics, statistical physics)





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- a posteriori (almost everything else!)







- a priori (thermodynamics, statistical physics)
- a posteriori (almost everything else!)
- This breakdown of physical laws is often not a clean one...







• We've been concocting physical laws for thousands of years







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- We've been concocting physical laws for thousands of years
- Ancient: strongly representative of everyday world
- Modern: weakly representative of everyday world







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 - 500-700 BCE: Pre-Socratic philosophers (Thales, Anaximenes, Anaximander) express physical laws concerning magnets, electrostatically charged amber, and posit cosmologies







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- 100-200 BCE: Hipparchus makes contributions to astronomy and predicts eclipses





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 - 400-500 BCE: Kuo works on genuine magnetism and optics, developing the pinhole camera





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- Medieval Europe:
 - So far as I can tell, discovered very little. . .





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 - 1600-1800 CE: Descartes gives us Cartesian coordinates, Leibniz and Newton develop calculus, Newton finally develops his three laws of motion along with the corpuscular theory of light, Newton's law of cooling, speed of sound and Newtonian gravity – this was reconciled with Kepler's laws; meanwhile Boyle, Hooke Gay-Lussac, Papin, Watt and others develop certain thermodynamic principles (though they aren't integrated)





• 18th century Europe:







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 - Euler, Legendre, d'Alembert, Laplace and Lagrange begin general application of mathematical methods to mechanics







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 - Röntgen, Becquerel, Curie and Rutherford discover spontaneous radiation





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 - Einstein, building on mathematics developed by Riemann, Lorentz, Cartan and others, proposes general relativity
 - Dirac, Feynman, Schwinger, Gell-Mann, Weinberg, Yang, Mills, Higgs and others merge special relativity and quantum mechanics to produce quantum field theory, gauge field theory and the standard model





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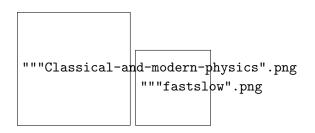


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Regimes in graphics









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 - Mathematical physics
 - Fluids, statistical physics

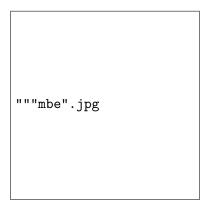












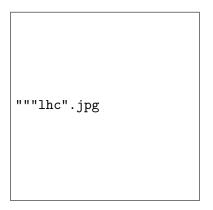




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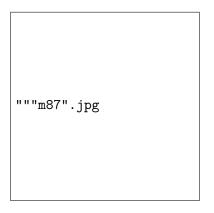




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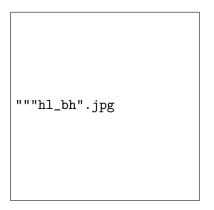




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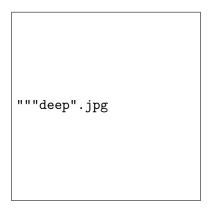




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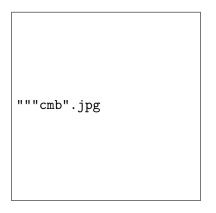
















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 - We don't know where the dimensionless physical constants come from





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 - We don't know why the universe is so flat





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 - We don't know what dark matter is
 - We don't know what dark energy is
 - We don't know if the universe is finite, infinite or superinfinite
 - We don't know why the universe is so flat
 - We don't know for sure if the big bang even happened





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- Idea is to give a basic exposure to these ideas





[A quip] that the physicist Richard Feynman made to the novelist Herman Wouk when they were discussing the Manhattan Project. Wouk was doing research for a big novel he hoped to write about World War II, and he went to Caltech to interview physicists who had worked on the bomb, one of whom was Feynman. After the interview, as they were parting, Feynman asked Wouk if he knew calculus. No, Wouk admitted, he didn't. "You had better learn it," said Feynman. "It's the language God talks."





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- Define the **derivative** of y with respect to x i3- ξ

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- Interpet it as the **rate of change** of y with respect to x





• Properties of differentiation:





- Properties of differentiation:
 - Linearity







- Properties of differentiation:
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 - Leibniz rule







- Properties of differentiation:
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 - Leibniz rule
 - Chain rule







• Simplest example, polynomial function:

$$f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$





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• Take any term and apply the formula for differentiation:

$$\frac{\mathrm{d}c_n x^n}{\mathrm{d}x} = \lim_{\delta x \to 0} \frac{c_n (x + \delta x)^n - c_n x^n}{\delta x}$$





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 Easy to then find derivatives of any polynomial by using the linearity property





• The inverse operation is called **integration** and write it this way

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- We have a formula to go from a function to its derivative, but not the other way around
- The only way to proceed is to **notice** that if f(x) differentiates to g(x), then it must be the correct answer!





• Simplest example is to stick to polynomials and find f(x) such that:

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• The +c part encodes the information lost in the process of differentiation, and we can see this from a discussion of **area under a curve**





• Time to introduce a pair of very important functions

$$\sin(x), \quad \cos(x)$$





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- These are the **trigonometric** functions
- Someone probably told you these have something to do with triangles, which is true but not so important in physics
- Equivalently, someone might have told you that these functions have something to do with waves, which is also not exactly wrong...





• Let's start with sin(x), we can see precisely what it is doing to x by introducing the notion of a **power series**:

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$





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 Don't be too concerned with this sigma and factorial notation, it translates to something very like a polynomial:

$$\sin(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \cdots$$





• Should be able to find the derivative of sin(x) quite easily, just apply the rules:

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• By differentiating $\sin(x)$ we've stumbled on the definition of $\cos(x)$, what happens if we keep differentiating?







 The trigonometric functions are how we rotate points in three-dimensional space (hence their application in triangles), we will touch on how to do this later, but x just becomes the angle of rotation





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 These are known as the hyperbolic functions, they are used in the rotation of four-dimensional space-time





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And this process continues in an obvious manner...





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• The number $e \approx 2.71828$ is known as **Euler's constant** – this particular value enables us to write the following power series:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$





 Now once again we want to differentiate this function to see what it gives:

$$\frac{\mathrm{d}e^x}{\mathrm{d}x} = e^x$$





 Now once again we want to differentiate this function to see what it gives:

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• As with the trigonometric and hyperbolic functions, the exponential function emerges in all areas of physics, you have probably heard already that it describes **radioactive decay**, but also e.g. the behaviour of the size of the universe near its beginning (**inflation**) and end (**de-Sitter expansion**)





 The relation between the exponential and hyperbolic functions is fairly clear from the power series, we just need to get to grips with even and odd functions:

$$\sinh(x) = \frac{1}{2} (e^x - e^{-x}), \quad \cosh(x) = \frac{1}{2} (e^x + e^{-x})$$
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And another less obvious corollary:

$$\cosh^2(x) - \sinh^2(x) = 1$$







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- The only place we really unavoidably need imaginary (or complex) numbers is in quantum mechanics and its special-relativistic generalisation, quantum field theory
- However, keep in mind that physics only works because some fundamental quantities (i.e. wavefunctions) that describe real phenomena, can't be expressed using real numbers





• We find:

$$\sin(x) = \frac{1}{2i} (e^{ix} - e^{-ix}), \quad \cos(x) = \frac{1}{2} (e^{ix} + e^{-ix})$$







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- Almost all **physical laws** need to be cast in the language of calculus to hold any meaning
- Almost all physical laws are expressed in terms of differential equations
- However they are often expressed simply as equations instead:

$$F = \frac{Gm_1m_2}{r^2}$$





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- A more interesting differential equation:

$$\frac{\mathrm{d}^2 f(x)}{\mathrm{d}x^2} = -\lambda^2 f(x)$$





 It is likely you will mostly meet linear differential equations in school and furthermore that these re ordinary differential equations (ODEs), these take the general form:

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- Linear ODEs can be further divided into **homogeneous** and **inhomogeneous** depending on whether g(x) is present. . .
- Luckily, linear ODEs are incredably important in basic physics, and we already have most of the tools we need to solve them!





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• So applying what we learned earlier, we have exponential decay:

$$Q = Q_0 e^{-\frac{t}{RC}}$$





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- Let's do this from the point of view of circuits again, replace the resistor with an **inductor**, $V_L=L\dot{I}$
- Should end up with the following:

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• Note that in general, the coefficient at the front occurs because of the linearity of the ODE, since this ODE is of **2nd order**, we get two constants, i.e. information about Q was lost twice when setting up the system





• When we put the resistor back in what do we find?







• What we've been looking at so far regarding circuits is actually a coupled ODE system, there were two unknown functions: Q(t), I(t)





- What we've been looking at so far regarding circuits is actually a coupled ODE system, there were two unknown functions: Q(t), I(t)
- \bullet Since we were able to eliminate I(t) from the system in a trivial way, we only had to solve one ODE







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 - Foxes don't reproduce either, but they do eat rabbits
 - The rate at which foxes eat rabbits is proportional to the number of foxes and the number of rabbits





• Now add the following feature:





- Now add the following feature:
 - Foxes get to reproduce, but only at a rate proportional to their rabbit-eating





• Lastly:







- Lastly:
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- Lastly:
 - Foxes die naturally and at a rate proportional to their population
 - Rabbits reproduce like foxes do, i.e. by eating in this case they eat at
 a rate proportional to their own population size (i.e. their food source
 is not constrained)







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- The full system of relations results in the Lotka-Volterra equations, these can't generally be solved cleanly but we can get an idea through numerical ODE techniques of how things play out...
- They also apparently have some applications in physics...





Energy transport and confinement in tokamak fusion plasmas is usually determined by the coupled nonlinear interactions of small-scale drift turbulence and larger scale coherent nonlinear structures, such as zonal flows, together with free energy sources such as temperature gradients. Zero-dimensional models, designed to embody plausible physical narratives for these interactions, can help to identify the origin of enhanced energy confinement and of transitions between confinement regimes. A prime zero-dimensional paradigm is predator-prey or Lotka-Volterra. Here, we extend a successful three-variable (temperature gradient; microturbulence level; one class of coherent structure) model in this genre [M. A. Malkov and P. H. Diamond, Phys. Plasmas 16, 012504 (2009)], by adding a fourth variable representing a second class of . . .





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- Unfortunately most of the really interesting differential equations in physics are partial differential equations (PDEs)
- These sound a lot worse than they are, but before we look at them we need to understand partial differentiation
- We can easily think of a function of two or more variables, e.g. f(x, y):

$$f(x,y) = \cosh(x) + 3xy - \sin(y)$$





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So for this function we have

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So for this function we have

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• The only subtlety is the use of ∂ instead of $d\dots$ they mean exactly the same thing





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 - $y \frac{\partial(xy)}{\partial x} x \frac{\partial(xy)}{\partial y}$







Before we get to check out some more physics, try the following examples:

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$$y \frac{\partial(xy)}{\partial x} - x \frac{\partial(xy)}{\partial y}$$

• $\frac{\partial^2 \cosh(x^2 + y^2)}{\partial x \partial y}$

$$\frac{\partial^2 \cosh(x^2 + y^2)}{\partial x \partial y}$$







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- For example, we might have temperature T of air depending simultaneously on time, t, and the position in a room, which we can describe by x, y, and z this means our equations will involve T(t,x,y,z)





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- For example, we might have temperature T of air depending simultaneously on time, t, and the position in a room, which we can describe by x, y, and z this means our equations will involve T(t,x,y,z)
- Generally this sort of thing comes up in **field theories**, such theories have dominated fundamental physics for the last 100 years!





 Most field theories end up producing some kind of wave equation, temperature does not form waves (for reasons you may come across in Week 2)





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- For example pressure, P(t,x,y,z), and the electromagnetic fields $\mathbf{E}(t,x,y,z)$ and $\mathbf{B}(t,x,y,z)$, although the last two are **vector** fields





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- A string has tension τ and mass per unit length μ , it is extended between two points which are spatially separated down x, and its deviation from a straight line at any point x and time t is given by f(t,x)





- To see why the wave equation predicts waves, consider an example in one space dimension
- A string has tension τ and mass per unit length μ , it is extended between two points which are spatially separated down x, and its deviation from a straight line at any point x and time t is given by f(t,x)
- To see how the string behaves, we will need a careful application of Newton's three laws and some trigonometry (perhaps we will get there in Seminar 3)





• The result, however, is the following wave equation:

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 This looks a bit formidable, but notice that it is solved by the following:

$$f(t,x) = \sin\left(x - \sqrt{\frac{\tau}{\mu}}t\right), \quad f(t,x) = \sin\left(x + \sqrt{\frac{\tau}{\mu}}t\right)$$





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• Think about what these functions actually mean, we have a \sin -wave moving with speed $v=\sqrt{\tau/\mu}$ along the string





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- The wave equation for waves on a string is an example of a field theory in time and one spatial dimension, it isn't an expecially fundamental field theory because it relies on non-relativistic notions of mass and acceleration





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- The wave equation for waves on a string is an example of a field theory in time and one spatial dimension, it isn't an expecially fundamental field theory because it relies on non-relativistic notions of mass and acceleration
- A very similar theory applies to gases, and results in the acoustic wave equation,

$$v^2\frac{\partial^2 P(t,x,y,z)}{\partial t^2} = \frac{\partial^2 P(t,x,y,z)}{\partial x^2} + \frac{\partial^2 P(t,x,y,z)}{\partial y^2} + \frac{\partial^2 P(t,x,y,z)}{\partial z^2}$$





• The sound speed is given by the dependence of the pressure P on the density ρ , but the details of **why** will have to wait for Week 2

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• plane waves of the simple form P(t,x,y,z)=f(x-vt)), i.e. moving at speed v down the x direction



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- In Seminar 4 we may be able to talk about the **Maxwell equations**, these are the four equations which govern the elextromagnetic fields, (obviously a **field theory**), and two of them constitute a **coupled** wave equation





- So we have seen that the wave equation takes a predictable form in these theories, only the **speed** of the waves changes due to the details of the **theory**
- In Seminar 4 we may be able to talk about the Maxwell equations, these are the four equations which govern the elextromagnetic fields, (obviously a field theory), and two of them constitute a coupled wave equation
- The result is that these fields can move on their own through space (i.e. without any charges or currents) at a speed known as c, the speed of light





• It doesn't stop there: just as we have a fundamental theory for electromagnetism, the wave equation with v=c also emerges in the most famous theory of **gravity**, i.e. **general relativity** – this gives rise to **gravity waves**





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- The wave equation has a cousin, known as the diffusion equation,
 e.g. for temperature in one spatial dimension:

$$\frac{\partial T(t,x)}{\partial t} = D \frac{\partial^2 T(t,x)}{\partial x^2}$$





 We mentioned before that heat (or temperature) tends not to form waves, but the diffusion equation can give wave-like behaviour if we make time imaginary:

$$i\frac{\partial \Psi(t,x)}{\partial t} = -\frac{\hbar}{2m}\frac{\partial^2 \Psi(t,x)}{\partial x^2}$$





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• This is the **Schrödinger equation**, and it describes the behaviour of the **wavefunction** $\Psi(t,x)$ in one dimension, the fundamental description of a particle in quantum mechanics



