Linear gravity from lightlike sources

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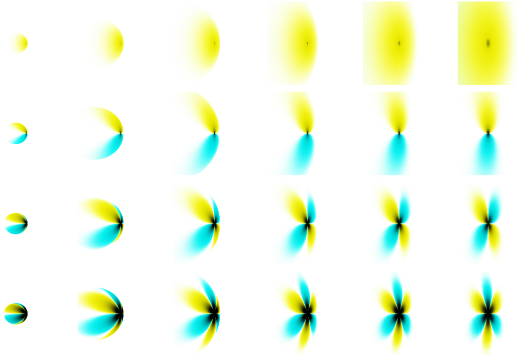
Supervised by Anthony Lasenby, Mike Hobson & Will Handley



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  "*...but slow light pulses do.*"

Lightlike source: the retarded perspective

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$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2) \tag{1}$$

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- However the notion of retarded time is complicated for sources moving at the fundamenetal velocity

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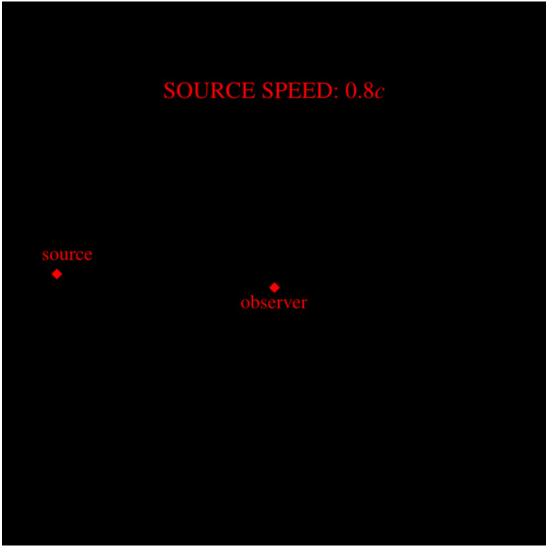
- In traditional calculations at $v = 1$, advanced position implicitly used for half of all coordinate time...
- ...end up with gauge waves but no physical fields:

$$h_{\mu\nu} \propto k_\mu k_\nu f\left(k_\alpha x^\alpha\right) \tag{3}$$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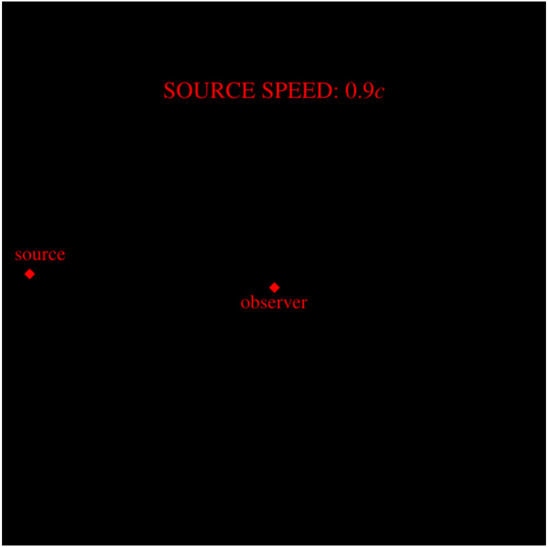
source
observer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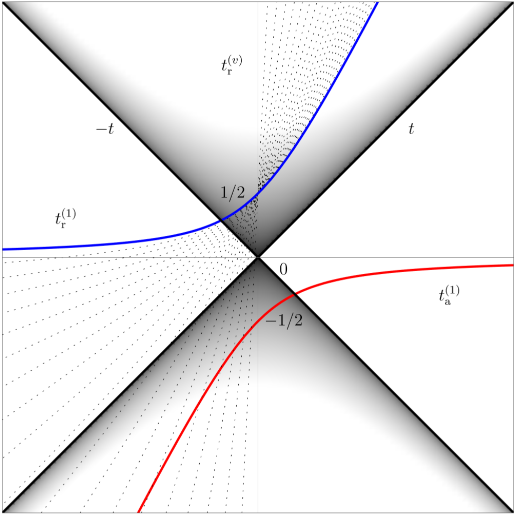

Cylindrical axis, z

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- But we established this portion of spacetime was not associated with any finite retarded time!

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- An ultraboost takes us back to the laboratory frame and the GEM multipoles become flattened abreast of the source trajectory

Flattening gravitoelectromagnetic multipoles

- Final result for the electrical fields

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\begin{aligned}
e^{\mu\nu} = -8\frac{k^{\mu}k^{\nu}}{k^2}\Bigg[& a_0^{(\Phi)}(z-t)\ln\frac{1}{\rho} \\
& + \sum_{l=1}^{\infty}\frac{a_l^{(\Phi)}(z-t)\cos l\varphi + b_l^{(\Phi)}(z-t)\sin l\varphi}{l\rho^l}\Bigg],
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- Magnetic fields are more complicated, but have the same 2D structure

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- According to the retarded potential, the emission signal embedded in this shell is **not radiative**
- At large times and close to the pulse trajectory, the known gravitational fields are resurrected

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- Hence the gravitational fields of lightlike sources are just an afterglow of their emission process
- The fields accompany the sources as if produced by them, but can be thought of as inevitable ‘decoration’
- Observable field strength:

Equivalent gravity of a half-infinite beam of null waves with constant spin/energy profile equal to that of the part of the source undergoing emission at retarded time

Test geodesics

- Magnetic fields perturb the Lagrangian of a test particle, e.g. for a thin circularly polarized wavepulse of intensity profile $f(z)$

$$\mathcal{L} = \frac{1}{2}\left(\dot{w}_+\dot{w}_- - \dot{\rho}^2 - \rho^2\dot{\varphi}^2\right) - f\left(w_-\right)\dot{w}_-\dot{\varphi} \tag{7}$$

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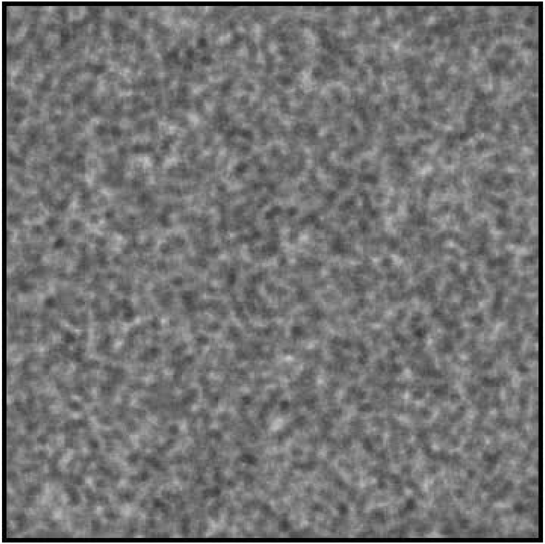
- Hence spinning waves imply velocity memory



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- But the process might play a role in the depolarization and absorbtion of gravitational waves passing through the ISM, in this context the calculation is an exercies in quadratic gravity