Supplemental material

The minisuperspace PGT$^{q,+}$. – As emphasised in the main text, the equations of motion from (12) and (4) should be compared under the spatially-flat SCP to verify the analogue. Here we provide an alternative, heuristic verification based on the reduced symmetry of the Lagrangia. For our previous treatments of the minisuperspace formulation of PGT$^{q,+}$, see [21, 50]. We use an ADM-like interval $\mathrm{d}s^2 = u^2(\mathrm{d}t^2 - v^2\mathrm{d}\mathbf{x}^2)$, where the flat FLRW interval in (6) is recovered by taking $u \mapsto 1$ and $v \mapsto a$. The analogue defined in (9) corresponds to the following choices of gauge, in a further abuse of notation which assumes the holonomic and anholonomic bases to be aligned

$$b^a{}_\mu \overset{\text{an}}{=} u\big(v(\delta^a_\mu - \delta^a_0\eta_{0\mu}) + \delta^a_0\eta_{0\mu}\big), \quad A^{ab}{}_\mu \overset{\text{an}}{=} uv\delta^d_0\big(\phi\delta^{[b}_\mu\delta^{a]}_d - \tfrac{1}{2}\psi\varepsilon_{\mu d}{}^{ab}\big). \tag{S1}$$

The gauge fields in (S1) are then substituted into (3a) and (3b), and then into (4). The Maxwell-like couplings defined in (10), along with a minimal addition of surface terms (including the Gauss-Bonnet derivative) then reduce this to

$$\begin{aligned} L_{\mathrm{T}} \overset{\text{an}}{=}& \big(\tfrac{1}{2}m_{\mathrm{p}}{}^2\upsilon_2 + \sigma_3\phi^2 + \tfrac{1}{2}(\sigma_3-\sigma_2)\psi^2\big)\big[6v^3(\partial_t u)^2 + 12uv^2\partial_t u\partial_t v + 6u^2v(\partial_t v)^2\big] \\ &+ 12\sigma_3\big[uv^3\phi\partial_t u + \tfrac{1}{2}u^2v^3\partial_t\phi + u^2v^2\phi\partial_t v\big]\partial_t\phi + 6(\sigma_3-\sigma_2)\big[uv^3\psi\partial_t u + \tfrac{1}{2}u^2v^3\partial_t\psi + u^2v^2\psi\partial_t v\big]\partial_t\psi \\ &+ 4\sigma_1\big(\psi^2-\phi^2\big)\big[\tfrac{3}{2}u^2v^3\phi\partial_t u + \tfrac{3}{2}u^3v^2\phi\partial_t v + \tfrac{3}{2}u^3v^3\partial_t\phi\big] + 3m_{\mathrm{p}}{}^2(\alpha_0+\upsilon_2)\big[u^2v^3\phi\partial_t u + u^3v^2\phi\partial_t v\big] \\ &+ \tfrac{3}{4}u^4v^3\big[2\sigma_3\phi^4 - 4\sigma_2\phi^2\psi^2 + 2\sigma_3\psi^4 + m_{\mathrm{p}}{}^2(\alpha_0+\upsilon_2)\phi^2 - m_{\mathrm{p}}{}^2(\alpha_0-4\upsilon_1)\psi^2\big] + L_{\mathrm{m}}(\Phi,\Psi;u,v,\phi,\psi). \end{aligned} \tag{S2}$$

The problematic first-order terms associated with the non-canonical sector of (12) are now explicit in the penultimate line of (S2). Further surface terms distinguish (S2) from the minisuperspace Lagrangian of (12).

The autonomous system. – Here we provide an overview of the derivation of the autonomous system illustrated in Fig. 2. The Hamiltonian coordinates of ξ are y and x, while ζ is described by the single variable $z^2 = m_{\mathrm{p}}{}^2W^4\zeta^2/4H^2$. We also define $\lambda = -m_{\mathrm{p}}\partial_\xi V_{\mathrm{T}}/V_{\mathrm{T}}$ and $\mu = W$. While μ is conveniently defined for the specific system in (15), x, y and λ are conventional parameters in the literature, and we will see presently that z is analogous to the conventional matter parameter [54, 67]. From (15), the ξ equation (or alternatively the pressure–$g_{\mu\nu}$ equation) combined with the derivative of the ζ equation (which we give explicitly in (16)) can be expressed as a coupled first-order system in terms of these variables

$$\begin{aligned} \partial_\tau x =& -\big[x\big(2\sqrt{3}\lambda\mu^3xy^2 + 4z^2\big((\mu^4-8)z^2 - 2(\mu^4-4)y^2 - 8\big) + \sqrt{2}\mu\big(\sqrt{3}\lambda\mu^3xy^2z^2 + \mu^4(y-z)^2(y+z)^2 \\ &- 16(y-z)(y+z)\big(y^2-z^2-1\big) + 2\mu^2\big(y^2-1\big)\big(3y^2-z^2\big)\big)\big)\big]/\big[\mu^3\big(\sqrt{2}\big((2+\mu^2)y^2 - \mu^2z^2 - 2\big) - 4\mu z^2\big)\big], \end{aligned} \tag{S3a}$$

$$\begin{aligned} \partial_\tau y =& \, y\big[\mu^2\big(\sqrt{3}\big(2-2y^2+\mu^2z^2\big) - 4\mu z^2\big(3-3y^2+z^2\big)\big) - \sqrt{2}\big(2\mu^2\big(y^2-1\big)\big(3y^2-z^2-3\big) + \sqrt{3}\lambda\mu^3x\big(y^2-2\big)z^2 \\ &- 16\big(1-y^2+z^2\big)^2 + \mu^4\big(y^4+z^2\big(3+z^2\big) - y^2\big(1+2z^2\big)\big)\big)\big)\big]/\big[\mu^2\big(\sqrt{2}\big((2+\mu^2)y^2-\mu^2z^2-2\big) - 4\mu z^2\big)\big]. \end{aligned} \tag{S3b}$$

The dimensionless (Hubble-normalised) time in (S3) is $\mathrm{d}\tau = H\mathrm{d}t$. To obtain the autonomous system in x and y we must eliminate λ, μ and z from (S3a) and (S3b). We first use (15b) and (15c) to solve for λ in terms of μ

$$\lambda = -\frac{\big[4\big(2\Lambda_{\mathrm{b}} + 5\frac{\upsilon_1}{\sigma_1}m_{\mathrm{p}}{}^2\big) + \big(\Lambda_{\mathrm{b}} + 4\frac{\upsilon_1}{\sigma_1}m_{\mathrm{p}}{}^2\big)\mu^2\big]\sqrt{2\big(1+\frac{1}{2}\mu^2\big)}}{\big[8\big(\Lambda_{\mathrm{b}} + \frac{\upsilon_1}{\sigma_1}m_{\mathrm{p}}{}^2\big) + \big(\Lambda_{\mathrm{b}} + 4\frac{\upsilon_1}{\sigma_1}m_{\mathrm{p}}{}^2\big)\mu^2\big]\sqrt{3\big(1+\frac{1}{8}\mu^2\big)}}. \tag{S4}$$

Note that (S4) explicitly incorporates both the bare cosmological constant Λ_{b} and our central combination $\upsilon_1 m_{\mathrm{p}}{}^2/\sigma_1$. As emphasised in the main text, these quantities are on an equal footing. The ζ equation reduces to a quartic in μ

$$\big(x^2-1\big)\mu^4 + 2\sqrt{2}z\mu^3 + 2\big(5x^2-z^2\big)\mu^2 + 16x^2 = 0. \tag{S5}$$

Finally, z is solved for x and y by the density–$g_{\mu\nu}$ equation

$$x^2 + y^2 - z^2 = 0. \tag{S6}$$

Note that (S6) *expels* the physical portions of the phase space from the unit disc; if z were a 'conventional' matter parameter (i.e. obedient to the weak energy condition), it would *confine* them there. The former, more holistic picture (containing all critical points $\partial_\tau x = \partial_\tau y = 0$) can be obtained by taking a simple Möbius transform of the phase space, but this is not done in Fig. 2. The quartic roots of (S5) cause the fully autonomous system to be prohibitively unwieldy. This is a natural consequence of explicitly encoding the *Cuscuton* constraint in the Class ^{2}A* and Class ^{3}C* theories, rather than a generic limitation of the MA in (12).