

Supplemental material

The following two derivations are provided to supplement the main text of the *Letter*, and share the same notational conventions, equation and reference numbering.

The minisuperspace PGT^{a+} . – For previous treatments of the minisuperspace formulation of PGT^{a+} , see [21, 48]. We use an ADM-like interval $ds^2 = u^2(dt^2 - v^2 d\mathbf{x}^2)$, where the flat FRW interval in (6) is recovered by taking $u \mapsto 1$ and $v \mapsto a$. The analogue defined in (9) corresponds to the following choices of gauge

$$b^a_\mu = u(v(\delta^a_\mu - \delta^a_0\eta_{0\mu}) + \delta^a_0\eta_{0\mu}), \quad A^{ab}_\mu = uv\delta^d_0(2\phi\delta^{[b}_\mu\delta^{a]}_d - \frac{1}{2}\psi\varepsilon_{\mu d}{}^{ab}). \quad (S1)$$

The gauge fields in (S1) are then substituted into (3a) and (3b), and then into (4). Once the Gauss-Bonnet term is discarded as a total time derivative, the coupling constants defined in (10) may be used to express the reduced Lagrangian as

$$\begin{aligned} \bar{L}_T = & (-3\sigma_2 u^2 v \psi^2 + 24\sigma_3 u^2 v \phi^2 + 3\sigma_3 u^2 v \psi^2 + 3m_P^2 v_2 u^2 v)(\partial_t v)^2 + (-6\sigma_2 uv^2 \psi^2 + 48\sigma_3 uv^2 \phi^2 + 6\sigma_3 uv^2 \psi^2 + \\ & 6m_P^2 v_2 uv^2) \partial_t v \partial_t u + 48v^3 \sigma_3 u^2 \phi \partial_t v \partial_t \phi + (-6\sigma_2 u^2 v^2 \psi + 6\sigma_3 u^2 v^2 \psi) \partial_t v \partial_t \psi + (-3\sigma_2 v^3 \psi^2 + 24\sigma_3 v^3 \phi^2 + \\ & 3\sigma_3 v^3 \psi^2 + 3m_P^2 v_2 v^3)(\partial_t u)^2 + 48v^3 \sigma_3 u \phi \partial_t u \partial_t \phi + (-6\sigma_2 uv^3 \psi + 6\sigma_3 uv^3 \psi) \partial_t u \partial_t \psi + 24v^3 \sigma_3 u^2 (\partial_t \phi)^2 + \\ & (-3\sigma_2 u^2 v^3 + 3\sigma_3 u^2 v^3)(\partial_t \psi)^2 + (-48\sigma_1 u^3 v^3 \phi^2 + 12\sigma_1 u^3 v^3 \psi^2 - 3m_P^2 \alpha_0 u^3 v^3) \partial_t \phi + (-48\sigma_1 u^3 v^2 \phi^3 + \\ & 12\sigma_1 u^3 v^2 \phi \psi^2 - 3m_P^2 \alpha_0 u^3 v^2 \phi + 6m_P^2 v_2 u^3 v^2 \phi) \partial_t v + (-48\sigma_1 u^2 v^3 \phi^3 + 12\sigma_1 u^2 v^3 \phi \psi^2 - \\ & 3m_P^2 \alpha_0 u^2 v^3 \phi + 6m_P^2 v_2 u^2 v^3 \phi) \partial_t u + 3m_P^2 v_2 v^3 u^4 \phi^2 + 3v^3 m_P^2 \alpha_0 u^4 \phi^2 + 24v^3 \sigma_3 u^4 \phi^4 + \\ & 3v^3 u^4 \psi^2 m_P^2 v_1 - 3/4 v^3 m_P^2 \alpha_0 u^4 \psi^2 + 3/2 v^3 \sigma_3 u^4 \psi^4 - 12\sigma_2 v^3 u^4 \phi^2 \psi^2 + \bar{L}_m(\Phi, \Psi; u, v, \phi, \psi). \end{aligned} \quad (S2)$$

The first-order terms associated with the non-canonical sector of the full MA in (12) are now explicit in (S2).

The autonomous system. – The variables x and y encode the momentum and position of the canonical inflaton ξ . The *Cuscuton* ζ constrains the system, and is described by the single variable $z^2 = m_P^2 W^4 \zeta^2 / 4H^2$. We also define $\lambda = -m_P \partial_\xi V_T / V_T$ and $\mu = W$. While μ is convenient for the specific system in (15), variables x , y and λ are conventional parameters, and we will see presently that z is analogous to the conventional matter parameter [52, 63]. The pressure- $g_{\mu\nu}$ and ξ equations of (15) are expressed as a coupled first order system in terms of these variables

$$\partial_\tau x = (-16\sqrt{2}\mu xy^2 + 6\sqrt{2}\mu^3 xy^2 - 2\sqrt{3}\lambda\mu^3 x^2 y^2 + 16\sqrt{2}\mu xy^4 - 6\sqrt{2}\mu^3 xy^4 - \sqrt{2}\mu^5 xy^4 + 32xz^2 + 16\sqrt{2}\mu xz^2 - \\ 2\sqrt{2}\mu^3 xz^2 - 32xy^2 z^2 - 32\sqrt{2}\mu xy^2 z^2 + 2\sqrt{2}\mu^3 xy^2 z^2 + 8\mu^4 xy^2 z^2 + 2\sqrt{2}\mu^5 xy^2 z^2 - \sqrt{6}\lambda\mu^4 x^2 y^2 z^2 + \\ 32xz^4 + 16\sqrt{2}\mu xz^4 - 4\mu^4 xz^4 - \sqrt{2}\mu^5 xz^4) / (-2\sqrt{2}\mu^3 + 2\sqrt{2}\mu^3 y^2 + \sqrt{2}\mu^5 y^2 - 4\mu^4 z^2 - \sqrt{2}\mu^5 z^2), \quad (S3a)$$

$$\partial_\tau y = (16\sqrt{2}y - 6\sqrt{2}\mu^2 y + 2\sqrt{3}\lambda\mu^2 xy - 32\sqrt{2}y^3 + 12\sqrt{2}\mu^2 y^3 + \sqrt{2}\mu^4 y^3 - 2\sqrt{3}\lambda\mu^2 xy^3 + 16\sqrt{2}y^5 - \\ 6\sqrt{2}\mu^2 y^5 - \sqrt{2}\mu^4 y^5 + 32\sqrt{2}yz^2 - 2\sqrt{2}\mu^2 yz^2 - 12\mu^3 yz^2 - 3\sqrt{2}\mu^4 yz^2 + 2\sqrt{6}\lambda\mu^3 xyz^2 + \sqrt{3}\lambda\mu^4 xyz^2 - \\ 32\sqrt{2}y^3 z^2 + 2\sqrt{2}\mu^2 y^3 z^2 + 12\mu^3 y^3 z^2 + 2\sqrt{2}\mu^4 y^3 z^2 - \sqrt{6}\lambda\mu^3 xy^3 z^2 + 16\sqrt{2}yz^4 - 4\mu^3 yz^4 - \\ \sqrt{2}\mu^4 yz^4) / (-2\sqrt{2}\mu^2 + 2\sqrt{2}\mu^2 y^2 + \sqrt{2}\mu^4 y^2 - 4\mu^3 z^2 - \sqrt{2}\mu^4 z^2). \quad (S3b)$$

The dimensionless Hubble time is $d\tau = H dt$. To obtain the autonomous system in x and y we must eliminate λ , μ and z . We use (15b) and (15c) to solve for λ in terms of μ

$$\lambda = -\frac{2\sqrt{2/3}(8\Lambda_b + 20m_P^2 v_1/\sigma_1 + (\Lambda_b + 4m_P^2 v_1/\sigma_1)\mu^2)\sqrt{16 + 10\mu^2 + \mu^4}}{(8 + \mu^2)(8\Lambda_b + 8m_P^2 v_1/\sigma_1 + (\Lambda_b + 4m_P^2 v_1/\sigma_1)\mu^2)}, \quad (S4)$$

Note that this explicitly incorporates the bare cosmological constant Λ_b and the central combination $m_P^2 v_1/\sigma_1$, which are on an equal footing. The ζ equation, which we gave explicitly in (16), reduces to a quartic in μ

$$(x^1 - 1)\mu^4 + 2\sqrt{2}z\mu^3 + 2(5x^2 - z^2)\mu^2 + 16x^2 = 0. \quad (S5)$$

Finally z is eliminated for x and y by the density- $g_{\mu\nu}$ equation

$$x^2 + y^2 - z^2 = 0. \quad (S6)$$

Note that (S6) *expells* the physical portions of the phase space from the unit disc, while a conventional matter parameter would *confine* them there. The quartic roots of (S5) cause the fully autonomous system to be unwieldy. Note that this is a generic feature of Class $^2A^*$ and Class $^3C^*$, rather than of the MA formalism.