Supplemental material

Introduction. – We present below some cumbersome formulations, to be read in conjunction with the text.

The minisuperspace Lagrangian. – Consider the ADM-like extension $\mathrm{d}s^2 = u^2\left(\mathrm{d}t^2 - v^2\mathrm{d}\mathbf{x}^2\right)$, where the limit $u \mapsto 1$ and $v \mapsto a$ recovers the flat FRW metric. The analogue cleary corresponds to the following gauge

$$b^a{}_\mu = u\big(v(\delta^a_\mu - \delta^a_0\eta_{0\mu}) + \delta^a_0\eta_{0\mu}\big), \quad A^{ab}{}_\mu = uv\delta^d_0\big(2\phi\delta^{[b}_\mu\delta^{a]}_d - \tfrac{1}{2}\psi\varepsilon_{\mu d}{}^{ab}\big). \tag{S1}$$

and substitute these into (**??**) to obtain the minisuperspace Lagrangian of $\mathrm{PGT}^{\mathrm{q},+}$. Once the Gauss-Bonnet term is discarded as a total time derivative, the coupling constants in (**??**) may be used to express this as

$$\begin{aligned}\bar{L}_{\mathrm{T}} = {} & (-3\sigma_2 u^2 v\psi^2 + 24\sigma_3 u^2 v\phi^2 + 3\sigma_3 u^2 v\psi^2 + 3m_{\mathrm{P}}{}^2\upsilon_2 u^2 v)(\partial_t v)^2 + (-6\sigma_2 uv^2\psi^2 + 48\sigma_3 uv^2\phi^2 + 6\sigma_3 uv^2\psi^2 + \\ & 6m_{\mathrm{P}}{}^2\upsilon_2 uv^2)\partial_t v\partial_t u + 48v^2\sigma_3 u^2\phi\partial_t v\partial_t\phi + (-6\sigma_2 u^2 v^2\psi + 6\sigma_3 u^2 v^2\psi)\partial_t v\partial_t\psi + (-3\sigma_2 v^3\psi^2 + 24\sigma_3 v^3\phi^2 + \\ & 3\sigma_3 v^3\psi^2 + 3m_{\mathrm{P}}{}^2\upsilon_2 v^3)(\partial_t u)^2 + 48v^3\sigma_3 u\phi\partial_t u\partial_t\phi + (-6\sigma_2 uv^3\psi + 6\sigma_3 uv^3\psi)\partial_t u\partial_t\psi + 24v^3\sigma_3 u^2(\partial_t\phi)^2 + \\ & (-3\sigma_2 u^2 v^3 + 3\sigma_3 u^2 v^3)(\partial_t\psi)^2 + (-48\sigma_1 u^3 v^3\phi^2 + 12\sigma_1 u^3 v^3\psi^2 - 3m_{\mathrm{P}}{}^2\alpha_0 u^3 v^3)\partial_t\phi + (-48\sigma_1 u^3 v^2\phi^3 + \\ & 12\sigma_1 u^3 v^2\phi\psi^2 - 3m_{\mathrm{P}}{}^2\alpha_0 u^3 v^2\phi + 6m_{\mathrm{P}}{}^2\upsilon_2 u^3 v^2\phi)\partial_t v + (-48\sigma_1 u^2 v^3\phi^3 + 12\sigma_1 u^2 v^3\phi\psi^2 - \\ & 3m_{\mathrm{P}}{}^2\alpha_0 u^2 v^3\phi + 6m_{\mathrm{P}}{}^2\upsilon_2 u^2 v^3\phi)\partial_t u + 3m_{\mathrm{P}}{}^2\upsilon_2 v^3 u^4\phi^2 + 3v^3 m_{\mathrm{P}}{}^2\alpha_0 u^4\phi^2 + 24v^3\sigma_3 u^4\phi^4 + \\ & 3v^3 u^4\psi^2 m_{\mathrm{P}}{}^2\upsilon_1 - 3/4v^3 m_{\mathrm{P}}{}^2\alpha_0 u^4\psi^2 + 3/2v^3\sigma_3 u^4\psi^4 - 12\sigma_2 v^3 u^4\phi^2\psi^2 + \bar{L}_{\mathrm{m}}(\Phi,\Psi;u,v,\phi,\psi).\end{aligned} \tag{S2}$$

The autonomous system. – The variables x and y encode the momentum and position of the canonical inflaton ξ. The *Cuscuton* ζ constrains the system, and is described by $z^2 = m_{\mathrm{p}}{}^2W^4\zeta^2/4H^2$. Note that z adopts the rôle of the matter parameter. We further define $\lambda = -m_{\mathrm{p}}\partial_\xi V/V$ and $\mu = W$. As with x and y, note that λ is a conventional parameter. The $g_{\mu\nu}$ and ξ equations are readily expressed as a coupled first order system in terms of these variables

$$\begin{aligned}H^{-1}\partial_t x = {} & \big(-16\sqrt{2}\mu xy^2 + 6\sqrt{2}\mu^3 xy^2 - 2\sqrt{3}\lambda\mu^3 x^2y^2 + 16\sqrt{2}\mu xy^4 - 6\sqrt{2}\mu^3 xy^4 - \sqrt{2}\mu^5 xy^4 + 32xz^2 + \\ & 16\sqrt{2}\mu xz^2 - 2\sqrt{2}\mu^3 xz^2 - 32xy^2z^2 - 32\sqrt{2}\mu xy^2z^2 + 2\sqrt{2}\mu^3 xy^2z^2 + 8\mu^4 xy^2z^2 + 2\sqrt{2}\mu^5 xy^2z^2 - \\ & \sqrt{6}\lambda\mu^4 x^2y^2z^2 + 32xz^4 + 16\sqrt{2}\mu xz^4 - 4\mu^4 xz^4 - \sqrt{2}\mu^5 xz^4\big)/\big(-2\sqrt{2}\mu^3 + 2\sqrt{2}\mu^3y^2 + \sqrt{2}\mu^5y^2 - \\ & 4\mu^4z^2 - \sqrt{2}\mu^5z^2\big),\end{aligned} \tag{S3a}$$

$$\begin{aligned}H^{-1}\partial_t y = {} & \big(16\sqrt{2}y - 6\sqrt{2}\mu^2 y + 2\sqrt{3}\lambda\mu^2 xy - 32\sqrt{2}y^3 + 12\sqrt{2}\mu^2y^3 + \sqrt{2}\mu^4y^3 - 2\sqrt{3}\lambda\mu^2 xy^3 + 16\sqrt{2}y^5 - \\ & 6\sqrt{2}\mu^2y^5 - \sqrt{2}\mu^4y^5 + 32\sqrt{2}yz^2 - 2\sqrt{2}\mu^2yz^2 - 12\mu^3yz^2 - 3\sqrt{2}\mu^4yz^2 + 2\sqrt{6}\lambda\mu^3xyz^2 + \\ & \sqrt{3}\lambda\mu^4xyz^2 - 32\sqrt{2}y^3z^2 + 2\sqrt{2}\mu^2y^3z^2 + 12\mu^3y^3z^2 + 2\sqrt{2}\mu^4y^3z^2 - \sqrt{6}\lambda\mu^3xy^3z^2 + 16\sqrt{2}yz^4 - \\ & 4\mu^3yz^4 - \sqrt{2}\mu^4yz^4\big)/\big(-2\sqrt{2}\mu^2 + 2\sqrt{2}\mu^2y^2 + \sqrt{2}\mu^4y^2 - 4\mu^3z^2 - \sqrt{2}\mu^4z^2\big).\end{aligned} \tag{S3b}$$

To obtain the autonomous system, we must eliminate λ, μ and z. Since W and V depend explicitly on ξ, we easily solve for λ in terms of μ

$$\lambda = -\frac{2\sqrt{2/3}\big(8\Lambda - 60\upsilon_1 + (\Lambda - 12\upsilon_1)\mu^2\big)\sqrt{16 + 10\mu^2 + \mu^4}}{(8+\mu^2)\big(8\Lambda - 24\upsilon_1 + (\Lambda - 12\upsilon_1)\mu^2\big)}, \tag{S4}$$

while the ζ equation reduces to a quartic constraint in μ

$$(x^1 - 1)\mu^4 + 2\sqrt{2}z\mu^3 + 2(5x^2 - z^2)\mu^2 + 16x^2 = 0. \tag{S5}$$

Finally z is eliminated for x and y by the $g_{\mu\nu}$ equation, which contains the constraint $x^2 + y^2 - z^2$ and expells the physical portions of the phase space from the unit disc. The overall construction – in particular the root system of (**??**) – causes the autonomous system to be quite cumbersome: this is an unavoidable feature of Class ^{2}A* and Class ^{3}C*.