

Supplemental material

The following two derivations are provided to supplement the main text of the *Letter*, and share the same notational conventions, equation and reference numbering.

The minisuperspace $PGT^{q,+}$. – For previous treatments of the minisuperspace formulation of $PGT^{q,+}$, see [21, 47]. We use an ADM-like interval $ds^2 = u^2(dt^2 - v^2 d\mathbf{x}^2)$, where the flat FRW interval in (6) is recovered by taking $u \mapsto 1$ and $v \mapsto a$. The analogue defined in (9) corresponds to the following choices of gauge

$$b^a_{\mu} = u(v(\delta^a_{\mu} - \delta^a_0 \eta_{0\mu}) + \delta^a_0 \eta_{0\mu}), \quad A^{ab}_{\mu} = uv\delta^d_0(2\phi\delta^{[b}_{\mu}\delta^{a]}_d - \tfrac{1}{2}\psi\varepsilon_{\mu d}{}^{ab}). \quad (S1)$$

The gauge fields in (S1) are then substituted into (3a) and (3b), and then into (4). Once the Gauss-Bonnet term is discarded as a total time derivative, the coupling constants defined in (10) may be used to express the reduced Lagrangian as

$$\begin{aligned} \bar{L}_T = & -3vu^2(\sigma_2\psi^2 - 8\sigma_3\phi^2 - \sigma_3\psi^2 - m_p^2 v_2)(\partial_t v)^2 - 6v^2 u(\sigma_2\psi^2 - 8\sigma_3\phi^2 - \sigma_3\psi^2 - m_p^2 v_2)\partial_t v \partial_t u \\ & - 3v^3(\sigma_2\psi^2 - 8\sigma_3\phi^2 - \sigma_3\psi^2 - m_p^2 v_2)(\partial_t u)^2 + 48v^2 \sigma_3 u^2 \phi \partial_t v \partial_t \phi + 48v^3 \sigma_3 u \phi \partial_t u \partial_t \phi + 24v^3 \sigma_3 u^2 (\partial_t \phi)^2 \\ & - 6v^2 u^2 \psi(\sigma_2 - \sigma_3)\partial_t v \partial_t \psi - 6v^3 u \psi(\sigma_2 - \sigma_3)\partial_t u \partial_t \psi - 3v^3 u^2 (\sigma_2 - \sigma_3)(\partial_t \psi)^2 \\ & - 3v^2 u^3 \phi(16\sigma_1\phi^2 - 4\sigma_1\psi^2 + m_p^2 \alpha_0 - 2m_p^2 v_2)\partial_t v - 3v^3 u^2 \phi(16\sigma_1\phi^2 - 4\sigma_1\psi^2 + m_p^2 \alpha_0 - 2m_p^2 v_2)\partial_t u \\ & - 3v^3 u^3(16\sigma_1\phi^2 - 4\sigma_1\psi^2 + m_p^2 \alpha_0)\partial_t \phi \\ & + 3/4 v^3 u^4 (-16\sigma_2\phi^2\psi^2 + 32\sigma_3\phi^4 + 2\sigma_3\psi^4 + 4m_p^2 \alpha_0\phi^2 - m_p^2 \alpha_0\psi^2 + 4m_p^2 v_1\psi^2 + 4m_p^2 v_2\phi^2) \\ & + \bar{L}_m(\Phi, \Psi; u, v, \phi, \psi). \end{aligned} \quad (S2)$$

The problematic first-order terms associated with the non-canonical sector of the MA in (12) are now explicit in (S2).

The autonomous system. – The variables x and y encode the momentum and position of the canonical inflaton ξ . The *Cuscuton* ζ constrains the system, and is described by the single variable $z^2 = m_p^2 W^4 \zeta^2 / 4H^2$. We also define $\lambda = -m_p \partial_{\xi} V_T / V_T$ and $\mu = W$. While μ is convenient for the specific system in (15), variables x , y and λ are conventional parameters, and we will see presently that z is analogous to the conventional matter parameter [52, 63]. The pressure- $g_{\mu\nu}$ and ξ equations of (15) are expressed as a coupled first-order system in terms of these variables

$$\begin{aligned} \partial_{\tau} x = & (-16\sqrt{2}\mu xy^2 + 6\sqrt{2}\mu^3 xy^2 - 2\sqrt{3}\lambda\mu^3 x^2 y^2 + 16\sqrt{2}\mu xy^4 - 6\sqrt{2}\mu^3 xy^4 - \sqrt{2}\mu^5 xy^4 + 32xz^2 + 16\sqrt{2}\mu xz^2 - \\ & 2\sqrt{2}\mu^3 xz^2 - 32xy^2 z^2 - 32\sqrt{2}\mu xy^2 z^2 + 2\sqrt{2}\mu^3 xy^2 z^2 + 8\mu^4 xy^2 z^2 + 2\sqrt{2}\mu^5 xy^2 z^2 - \sqrt{6}\lambda\mu^4 x^2 y^2 z^2 + \\ & 32xz^4 + 16\sqrt{2}\mu xz^4 - 4\mu^4 xz^4 - \sqrt{2}\mu^5 xz^4) / (-2\sqrt{2}\mu^3 + 2\sqrt{2}\mu^3 y^2 + \sqrt{2}\mu^5 y^2 - 4\mu^4 z^2 - \sqrt{2}\mu^5 z^2), \end{aligned} \quad (S3a)$$

$$\begin{aligned} \partial_{\tau} y = & (16\sqrt{2}y - 6\sqrt{2}\mu^2 y + 2\sqrt{3}\lambda\mu^2 xy - 32\sqrt{2}y^3 + 12\sqrt{2}\mu^2 y^3 + \sqrt{2}\mu^4 y^3 - 2\sqrt{3}\lambda\mu^2 xy^3 + 16\sqrt{2}y^5 - \\ & 6\sqrt{2}\mu^2 y^5 - \sqrt{2}\mu^4 y^5 + 32\sqrt{2}yz^2 - 2\sqrt{2}\mu^2 yz^2 - 12\mu^3 yz^2 - 3\sqrt{2}\mu^4 yz^2 + 2\sqrt{6}\lambda\mu^3 xy z^2 + \sqrt{3}\lambda\mu^4 xy z^2 - \\ & 32\sqrt{2}y^3 z^2 + 2\sqrt{2}\mu^2 y^3 z^2 + 12\mu^3 y^3 z^2 + 2\sqrt{2}\mu^4 y^3 z^2 - \sqrt{6}\lambda\mu^3 xy^3 z^2 + 16\sqrt{2}yz^4 - 4\mu^3 yz^4 - \\ & \sqrt{2}\mu^4 yz^4) / (-2\sqrt{2}\mu^2 + 2\sqrt{2}\mu^2 y^2 + \sqrt{2}\mu^4 y^2 - 4\mu^3 z^2 - \sqrt{2}\mu^4 z^2). \end{aligned} \quad (S3b)$$

The dimensionless Hubble time is $d\tau = H dt$. To obtain the autonomous system in x and y we must eliminate λ , μ and z . We use (15b) and (15c) to solve for λ in terms of μ

$$\lambda = -\frac{2\sqrt{2/3}(8\Lambda_b + 20m_p^2 v_1/\sigma_1 + (\Lambda_b + 4m_p^2 v_1/\sigma_1)\mu^2)\sqrt{16 + 10\mu^2 + \mu^4}}{(8 + \mu^2)(8\Lambda_b + 8m_p^2 v_1/\sigma_1 + (\Lambda_b + 4m_p^2 v_1/\sigma_1)\mu^2)}, \quad (S4)$$

Note that this explicitly incorporates the bare cosmological constant Λ_b and the central combination $m_p^2 v_1/\sigma_1$, which are on an equal footing. The ζ equation, which we gave explicitly in (16), reduces to a quartic in μ

$$(x^2 - 1)\mu^4 + 2\sqrt{2}z\mu^3 + 2(5x^2 - z^2)\mu^2 + 16x^2 = 0. \quad (S5)$$

Finally z is eliminated for x and y by the density- $g_{\mu\nu}$ equation

$$x^2 + y^2 - z^2 = 0. \quad (S6)$$

Note that (S6) *expells* the physical portions of the phase space from the unit disc, while a conventional matter parameter would *confine* them there. The quartic roots of (S5) cause the fully autonomous system to be unwieldy. Note that this is a generic feature of Class ${}^2A^*$ and Class ${}^3C^*$, rather than of the MA formalism.