Supplemental material

The following two derivations are provided to supplement the main text of the *Letter*, and share the same notational conventions, equation and reference numbering.

The minisuperspace PGT$^{\mathrm{q},+}$. – For previous treatments of the minisuperspace formulation of PGT$^{\mathrm{q},+}$, see [21, 47]. We use an ADM-like interval $\mathrm{d}s^2 = u^2(\mathrm{d}t^2 - v^2\mathrm{d}\mathbf{x}^2)$, where the flat FRW interval in (6) is recovered by taking $u \mapsto 1$ and $v \mapsto a$. The analogue defined in (9) corresponds to the following choices of gauge, with holonomic and anholonomic bases aligned

$$b^a{}_\mu = u\big(v(\delta^a_\mu - \delta^a_0\eta_{0\mu}) + \delta^a_0\eta_{0\mu}\big), \quad A^{ab}{}_\mu = uv\delta^d_0\big(2\phi\delta^{[b}_\mu\delta^{a]}_d - \tfrac{1}{2}\psi\varepsilon_{\mu d}{}^{ab}\big). \tag{S1}$$

The gauge fields in (S1) are then substituted into (3a) and (3b), and then into (4). The Maxwell-like couplings defined in (10), along with a minimal addition of surface terms (including the Gauss-Bonnet derivative) then reduces this to

$$\begin{aligned} L_{\mathrm{T}} = &-3\big((\sigma_2-\sigma_3)\psi^2 - 2\sigma_3\phi^2 - m_{\mathrm{p}}{}^2\upsilon_2\big)\big(v^3(\partial_t u)^2 + 2uv^2\partial_t u\partial_t v + u^2 v(\partial_t v)^2\big) \\ &+ 6\sigma_3\big(2uv^3\phi\partial_t u + u^2v^3\partial_t\phi + 2u^2v^2\phi\partial_t v\big)\partial_t\phi - 3(\sigma_2-\sigma_3)\big(2uv^3\psi\partial_t u + u^2v^3\partial_t\psi + 2u^2v^2\psi\partial_t v\big)\partial_t\psi \\ &- 6\sigma_1\big(\phi^2-\psi^2\big)\big(u^2v^3\phi\partial_t u + u^3v^2\phi\partial_t v + u^3v^3\partial_t\phi\big) + 3m_{\mathrm{p}}{}^2(\alpha_0+\upsilon_2)\big(u^2v^3\phi\partial_t u + u^3v^2\phi\partial_t v\big) \\ &+ \tfrac{3}{4}u^4v^3\big(2\sigma_3\phi^4 - 4\sigma_2\phi^2\psi^2 + 2\sigma_3\psi^4 + m_{\mathrm{p}}{}^2(\alpha_0+\upsilon_2)\phi^2 - m_{\mathrm{p}}{}^2(\alpha_0-4\upsilon_1)\psi^2\big) + L_{\mathrm{m}}(\Phi,\Psi;u,v,\phi,\psi). \end{aligned} \tag{S2}$$

The problematic first-order terms associated with the non-canonical sector of (12) are now explicit in the penultimate line of (S2). Further surface terms distinguish (S2) from the minisuperspace Lagrangian of (12).

The autonomous system. – The variables x and y encode the momentum and position of the canonical inflaton ξ. The *Cuscuton* ζ constrains the system, and is described by the single variable $z^2 = m_{\mathrm{p}}{}^2W^4\zeta^2/4H^2$. We also define $\lambda = -m_{\mathrm{p}}\partial_\xi V_{\mathrm{T}}/V_{\mathrm{T}}$ and $\mu = W$. While μ is convenient for the specific system in (15), variables x, y and λ are conventional parameters, and we will see presently that z is analogous to the conventional matter parameter [52, 63]. The pressure–$g_{\mu\nu}$ and ξ equations of (15) are expressed as a coupled first-order system in terms of these variables

$$\begin{aligned} \partial_\tau x = \big(&-16\sqrt{2}\mu xy^2 + 6\sqrt{2}\mu^3xy^2 - 2\sqrt{3}\lambda\mu^3x^2y^2 + 16\sqrt{2}\mu xy^4 - 6\sqrt{2}\mu^3xy^4 - \sqrt{2}\mu^5xy^4 + 32xz^2 + 16\sqrt{2}\mu xz^2 - \\ &2\sqrt{2}\mu^3xz^2 - 32xy^2z^2 - 32\sqrt{2}\mu xy^2z^2 + 2\sqrt{2}\mu^3xy^2z^2 + 8\mu^4xy^2z^2 + 2\sqrt{2}\mu^5xy^2z^2 - \sqrt{6}\lambda\mu^4x^2y^2z^2 + \\ &32xz^4 + 16\sqrt{2}\mu xz^4 - 4\mu^4xz^4 - \sqrt{2}\mu^5xz^4\big)/\big(-2\sqrt{2}\mu^3 + 2\sqrt{2}\mu^3y^2 + \sqrt{2}\mu^5y^2 - 4\mu^4z^2 - \sqrt{2}\mu^5z^2\big), \end{aligned} \tag{S3a}$$

$$\begin{aligned} \partial_\tau y = \big(&16\sqrt{2}y - 6\sqrt{2}\mu^2y + 2\sqrt{3}\lambda\mu^2xy - 32\sqrt{2}y^3 + 12\sqrt{2}\mu^2y^3 + \sqrt{2}\mu^4y^3 - 2\sqrt{3}\lambda\mu^2xy^3 + 16\sqrt{2}y^5 - \\ &6\sqrt{2}\mu^2y^5 - \sqrt{2}\mu^4y^5 + 32\sqrt{2}yz^2 - 2\sqrt{2}\mu^2yz^2 - 12\mu^3yz^2 - 3\sqrt{2}\mu^4yz^2 + 2\sqrt{6}\lambda\mu^3xyz^2 + \sqrt{3}\lambda\mu^4xyz^2 - \\ &32\sqrt{2}y^3z^2 + 2\sqrt{2}\mu^2y^3z^2 + 12\mu^3y^3z^2 + 2\sqrt{2}\mu^4y^3z^2 - \sqrt{6}\lambda\mu^3xy^3z^2 + 16\sqrt{2}yz^4 - 4\mu^3yz^4 - \\ &\sqrt{2}\mu^4yz^4\big)/\big(-2\sqrt{2}\mu^2 + 2\sqrt{2}\mu^2y^2 + \sqrt{2}\mu^4y^2 - 4\mu^3z^2 - \sqrt{2}\mu^4z^2\big). \end{aligned} \tag{S3b}$$

The dimensionless Hubble time is $\mathrm{d}\tau = H\mathrm{d}t$. To obtain the autonomous system in x and y we must eliminate λ, μ and z. We use (15b) and (15c) to solve for λ in terms of μ

$$\lambda = -\frac{2\sqrt{2/3}\big(8\Lambda_{\mathrm{b}} + 20m_{\mathrm{p}}{}^2\upsilon_1/\sigma_1 + (\Lambda_{\mathrm{b}} + 4m_{\mathrm{p}}{}^2\upsilon_1/\sigma_1)\mu^2\big)\sqrt{16+10\mu^2+\mu^4}}{(8+\mu^2)\big(8\Lambda_{\mathrm{b}} + 8m_{\mathrm{p}}{}^2\upsilon_1/\sigma_1 + (\Lambda_{\mathrm{b}} + 4m_{\mathrm{p}}{}^2\upsilon_1/\sigma_1)\mu^2\big)}, \tag{S4}$$

Note that this explicitly incorporates the bare cosmological constant Λ_{b} and the central combination $m_{\mathrm{p}}{}^2\upsilon_1/\sigma_1$, which are on an equal footing. The ζ equation, which we gave explicitly in (16), reduces to a quartic in μ

$$(x^2-1)\mu^4 + 2\sqrt{2}z\mu^3 + 2(5x^2-z^2)\mu^2 + 16x^2 = 0. \tag{S5}$$

Finally z is eliminated for x and y by the density–$g_{\mu\nu}$ equation

$$x^2 + y^2 - z^2 = 0. \tag{S6}$$

Note that (S6) *expells* the physical portions of the phase space from the unit disc, while a conventional matter parameter would *confine* them there. The quartic roots of (S5) cause the fully autonomous system to be unwieldy. Note that this is a generic feature of Class $^2\mathrm{A}^*$ and Class $^3\mathrm{C}^*$, rather than of the MA formalism.