F Henry PhysA Waves Probs ①

7) $\lambda^2 = -\frac{g}{l}\sin\lambda = -\sin\lambda$

$X = \dot{\theta}$ $\therefore \dot{X} = \ddot{\theta}$

$\therefore \dot{X} = -\sin\theta$

8a) $s = x$ $\quad x = \frac{1}{2} \times g t^2$ ① for free mass

$u = 0$

$v = x$

$a = g$

$t = t$

$x = vt$ ② for wave speed

$x = \frac{1}{2} g \times \frac{x^2}{v^2}$

$v^2 = \frac{T}{\rho} = \frac{Tl}{m}$

$T = mg$

$\therefore v^2 = \frac{mgl}{m} = gl$ $\quad \therefore x = \frac{\frac{1}{2} g x^2}{gl} = \frac{x^2}{2l}$

$x = 0$ or $\boxed{x = 2l}$ — ? longer than string

b) $T = g(m+M)$ and calculate $x = l$

~~but maths wrong earlier so can't do~~

$x\left(\frac{g(m+M)l}{m}\right) = \frac{1}{2} g x^2$ $\quad x = 0$ or $x = \frac{2l(m+M)}{m}$

$x = l \therefore m = 2m + 2M$ $\quad \boxed{M = -\frac{m}{2}}$

wrong earlier on so can't work

9a) m ~~#~~ of string $= \pi \times \left(\frac{0.5\times10^{-3}}{2}\right)^2 \times 700\times10^{-3} \times 7800$

$$= 1.07\times10^{-3}\ kg$$

$$\frac{m}{L} = 1.53\times10^{-3}\ kg\,m^{-1} = \rho$$

$$f_1 = \frac{v}{2L} = \frac{\sqrt{\frac{T}{\rho}}}{2L}$$

$$\therefore \rho(2Lf_1)^2 = \boxed{T = 205N}$$

stress at this tuning $= \frac{T}{A} = \frac{T}{\pi\left(\frac{0.5\times10^{-3}}{2}\right)^2} = \boxed{1.04\times10^9\ Nm^{-2}}$

piano tuning could be dangerous as could yield or snap if tuned to higher frequencies than material can handle. (high Tension string snaps could be dangerous)

b) ~~261.6Hz~~ ? $200\times10^9 = \frac{1.04\times10^9}{\text{strain}}$

$$\text{strain} = 5.2\times10^{-3}$$

$$\therefore \boxed{\Delta x = 3.63\times10^{-3}\ m}$$

$$T = k\Delta x$$

$$\therefore k = 5.63\times10^4$$

assuming SHM in string acting as spring

$$w^2 = \frac{k}{m} \quad \therefore \quad w = \sqrt{\frac{k}{m}} = \boxed{7250}\ rad\,s^{-1}$$

$$\boxed{f = 1150\ s^{-1}}$$

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9c) $\cancel{T = kx \text{ strain} \times L}$ strain = ε

$$T = k\varepsilon L$$
$$k = \frac{T}{\varepsilon L} \qquad \therefore f_L = \frac{1}{2\pi}\sqrt{\frac{T}{\varepsilon L m}}$$

$$f_T = \frac{1}{2L}\sqrt{\frac{TL}{m}} = \frac{1}{2}\sqrt{\frac{T}{mL}}$$

$$\frac{f_T}{f_L} = \frac{\frac{1}{2}\sqrt{\frac{T}{mL}}}{\frac{1}{2\pi}\sqrt{\frac{T}{\varepsilon mL}}} = \frac{1}{\frac{1}{\pi}\times\frac{1}{\sqrt{\varepsilon}}} = \pi\sqrt{\varepsilon}$$

$\cancel{\frac{f_T}{f_L} > 1}$ If $\frac{f_T}{f_L} > 1$ then transverse waves will be faster than than compression waves

$$\therefore \pi\sqrt{\varepsilon} > 1$$
$$\varepsilon > \frac{1}{\pi^2}$$
$$\varepsilon > 0.10 \qquad \therefore \text{strain} > 10\%$$

as it ~~w~~ breaks usually if more than 1-2% this means transvese almost always slower than compression.

10a) $\Psi_{y,1} = a\sin\theta\cos(kz - wt + \phi_0)$

$\Psi_{y,2} = b\cos(kz - wt + \frac{\pi}{2})$ is this right hand?

$\cancel{\Psi_{y,3} = A_x = 2A_y}$ $\cancel{\Psi_{y,3} = 2A_y\left(\frac{\Psi_x}{2A_y}\cos\phi - \sqrt{\frac{4A_y^2 - \Psi_x^2}{4A_y^2}}\sin\phi\right)}$

$\cancel{\Psi_{y,3} = \frac{\Psi_{x,3}\cos\phi}{2} - \frac{\sqrt{4A_y^2 - \Psi_{x,3}^2}}{2}\sin\phi}$

$\Psi_{y,3} = \frac{c}{2}\cos(kz - wt - \phi)$

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10b) $\Psi_{x,1} + \Psi_{x,2} = a\cos\theta\left(\cos(kz-\omega t)\cos\phi_0 - \sin(kz-\omega t)\sin\phi_0\right) + b\cos(kz-\omega t)$

$= \cos(kz-\omega t)(a\cos\theta\cos\phi_0 + b) - a\cos\theta\sin\phi_0\sin(kz-\omega t)$

$\Rightarrow$ compare coefficients to $\Psi_{x,3}$

$a\cos\theta\cos\phi_0 + b = c$ (1) $\quad a\cos\theta\sin\phi_0 = 0$ (2)

$\Psi_{y,1} + \Psi_{y,2} = a\sin\theta\left(\cos(kz-\omega t)\cos\phi_0 - \sin(kz-\omega t)\sin\phi_0\right) + b\left(\cancel{\cos(kz-\omega t)\cos\frac{\pi}{2}} - \sin(kz-\omega t)\sin\frac{\pi}{2}\right)$

~~$\therefore$ compare coefficients to $\Psi_{y,3}$~~

$= a\sin\theta\cos\phi_0\cos(kz-\omega t) - \sin(kz-\omega t)(a\sin\theta\sin\phi_0 + b)$

compare coefficients to $\Psi_{y,3}$

$\Psi_{y,3} = \frac{c}{2}\left(\cos(kz-\omega t)\cos\phi + \sin(kz-\omega t)\sin\phi\right)$

$\frac{c}{2}\cos\phi = a\sin\theta\cos\phi_0$ (3) $\quad -\frac{c}{2}\sin\phi = a\sin\theta\sin\phi_0 + b$ (4)

this is true if (1), (2), (3), (4) all hold

~~$(3)^2 + (4)^2: \frac{c^2}{4} = a^2\sin^2\theta\cos^2\phi_0 + a^2\sin^2\theta\sin^2\phi_0 + 2ab\sin\theta\sin\phi_0 + b^2$~~

~~$\frac{c^2}{4} = a^2\sin^2\theta + b^2 + 2ab\sin\theta\sin\phi_0$~~

~~From (2) $\cos\theta = 0$ or $\sin\phi_0 = 0$~~ ~~sub (2) into (3)~~

$$\cancel{Z = \frac{\frac{\partial \Psi}{\partial z}}{\frac{\partial \Psi}{\partial t}}} \qquad \cancel{P = -T\frac{\partial \Psi}{\partial z} \times \frac{\partial \Psi}{\partial t}}$$

11) $Z = \frac{F}{u}$ $\qquad P = Fu = Z\left(\frac{\partial \Psi}{\partial t}\right)^2$

$\cancel{P(t) = Z}$ $C = A + Bi$

$$\cancel{\frac{\partial \Psi}{\partial t} = R} \quad \Psi = R\left((A+Bi)(e^{i(wt-kz)})\right)$$
$$= A\cos(wt-kz) - B\sin(wt-kz)$$

$$\frac{\partial \Psi}{\partial t} = -wA\sin(wt-kz) - wB\cos(wt-kz)$$

$$\left(\frac{\partial \Psi}{\partial t}\right)^2 = (-w)^2\left(A^2\sin^2(wt-kz) + B^2\cos^2(wt-kz) + 2AB\sin('' '')\cos('' '')\right)$$

$$\therefore \langle P \rangle = Z \times w^2\left(\tfrac{1}{2}A^2 + \tfrac{1}{2}B^2 + 0\right)$$
$$= Z \times w^2\left(\tfrac{1}{2}\right)(|C|^2)$$
$$\boxed{= \tfrac{1}{2} Z w^2 (|C|)^2}$$

If Z is complex $\quad Z = Z_0 e^{iwt}$

and with same frequency as sinusoidal wave

Real

Means Impedance value changes with wave ~~[illegible]~~ (blocks power more at certain phases of the wave)

$\langle P \rangle = \frac{1}{4} Z_0 w^2 (|C|)^2$?

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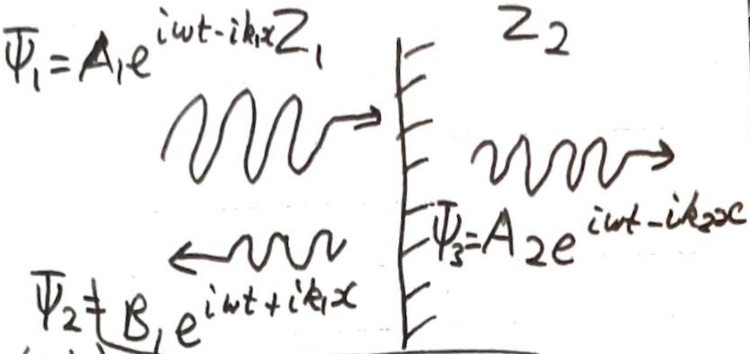
12) $v=\sqrt{\frac{T}{\mu}}$ $\qquad \frac{\partial \Psi}{\partial t} = u_v$

at $x=0$



$A_1 + B_1 = A_2$ 1

$\cancel{T(-ik_1)A_1 + T(ik_1)B_1 = T(-ik_2)A_2 - \alpha}$

$\cancel{TA_2(-ik_2)}$

$\cancel{k_1 = \frac{w}{v_1} \quad k_2 = \frac{w}{v_2}}$

$\cancel{-\frac{T}{v_1}wA_1 + \frac{T}{v_1}wB_1 = -\frac{T}{v_2}wA_2 + \alpha i}$

$$-T\left(\frac{\partial \Psi_1}{\partial t}\right) - T\left(\frac{\partial \Psi_2}{\partial t}\right) = -T\left(\frac{\partial \Psi_3}{\partial t}\right) - \alpha\left(\frac{\partial \Psi_3}{\partial t}\right)$$

$x=0$

$$T(-k_1)A_1 + T(k_1)B_1 = T(-k_2)A_2 - \alpha(-k_2)A_2$$

$$k_1 = \frac{w}{v_1} \qquad k_2 = \frac{w}{v_2}$$

$$\therefore -\frac{T}{v_1}A_1 + \frac{T}{v_1}B_1 = -\frac{T}{v_2}A_2 - \frac{\alpha}{v_2}A_2$$

$\left(\frac{T}{v_1} = Z_1\right)$ $\left(\frac{T}{v_2} = Z_2\right)$

$$\therefore -Z_1A_1 + Z_1B_1 = -Z_2A_2 - \frac{Z_2\alpha}{T}A_2$$

$$Z_1(A_1 - B_1) = Z_2A_2\left(1+\frac{\alpha}{T}\right) \quad 2$$

Sub 1 into 2 $Z_1(A_1-B_1) = Z_2\left(1+\frac{\alpha}{T}\right)(A_1+B_1)$

$$A_1\left(Z_1 - Z_2\left(1+\frac{\alpha}{T}\right)\right) = B_1\left(Z_2\left(1+\frac{\alpha}{T}\right) + Z_1\right)$$

$$\frac{B_1}{A_1} = \frac{Z_1 - Z_2\left(1+\frac{\alpha}{T}\right)}{Z_1 + Z_2\left(1+\frac{\alpha}{T}\right)} = r$$ ~~as $\alpha \to 0$~~

12) done wrong α should be ~~a~~ scale factor of Z_2

~~is ...~~ ∴ If α is large then would be node
If α is small then would be antinode

can't do rest as maths is wrong

7) $\ddot{\theta} = -\sin\theta$

$x = \dot{\theta}$ & $\dot{x} = \ddot{\theta} = -\sin\theta$

$$\begin{pmatrix} \dot{\theta} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} x \\ -\sin\theta \end{pmatrix}$$

$$\dot{\vec{y}} = \vec{F}$$

$$\frac{\vec{y}_{t+1} - \vec{y}_t}{\Delta t} = \vec{F} \Rightarrow \vec{y}_{t+1} = \vec{y}_t + \Delta t \cdot \vec{F}$$

$$\vec{y} = \begin{pmatrix} \theta \\ x \end{pmatrix} = \begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix}$$