Lecture 2 – measurement in physics

- Why electronic measurement is so useful.
- Input and output impedances. (Measuring with Oscilloscopes)
- Ideal operational amplifiers (op-amp). (Amplifying)
- Non-ideal behavior of op-amps.

The oscilloscope as a "perfect" measurement tool

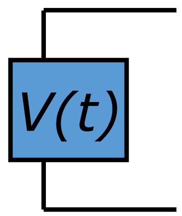
- Probably the most important measuring device in laboratory physics as many transducers produce a voltage output. It's essential to know how to use one and what its limitations are.

- Oscilloscopes measure *V(t)* rather well. At up to few 100 MHz, and at least 500× higher if you are willing to pay money.

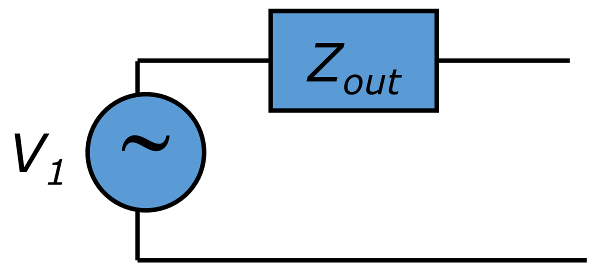
- And critically, they meet point 1 – they don't affect what they're measuring, or do they?

“Black-boxes”: input and output impedances

- Consider a real voltage source, such as a transducer for temperature, e.g. a thermocouple:

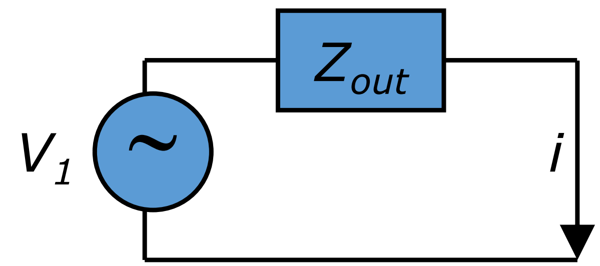


- This can (Thevenin’s theorem) always (in the linear regime) be represented by an equivalent circuit:



Here V_1 is a “perfect” voltage source – it can deliver an infinite current with V_1 constant – in series with an impedance.

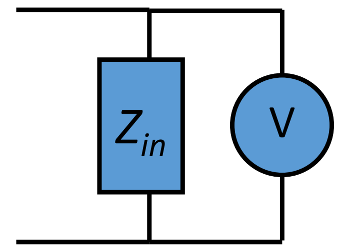
- If the output is shorted, a current i flows (which we can measure with an ammeter).



- The output impedance of the transducer is defined as: $Z_{out} = V_1/i$.

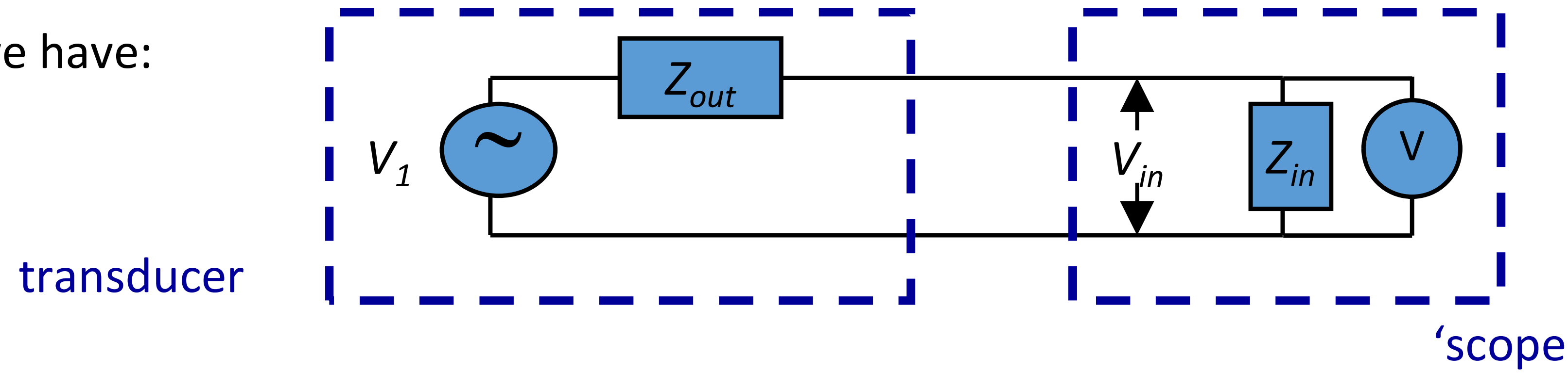
Now consider connecting the transducer to a ‘scope

- The oscilloscope’s equivalent circuit is:



It just consists of an impedance, the “input impedance”+ ideal Voltmeter (which draws no current)

- So we have:



- Current conservation and Ohm’s law $\Rightarrow$

$$\text{current in } Z_{out} = \text{current in } Z_{in} \Rightarrow \frac{V_1 - V_{in}}{Z_{out}} = \frac{V_{in} - 0}{Z_{in}}.$$

$$So \quad V_{in} = V_1 \frac{Z_{in}}{Z_{in} + Z_{out}}.$$

So for measurements with a real transducer and a real oscilloscope we find

- The voltage measured by the ’scope ≠ the voltage produced by the transducer.

$$V_{in} = V_1 \frac{Z_{in}}{Z_{in} + Z_{out}}$$

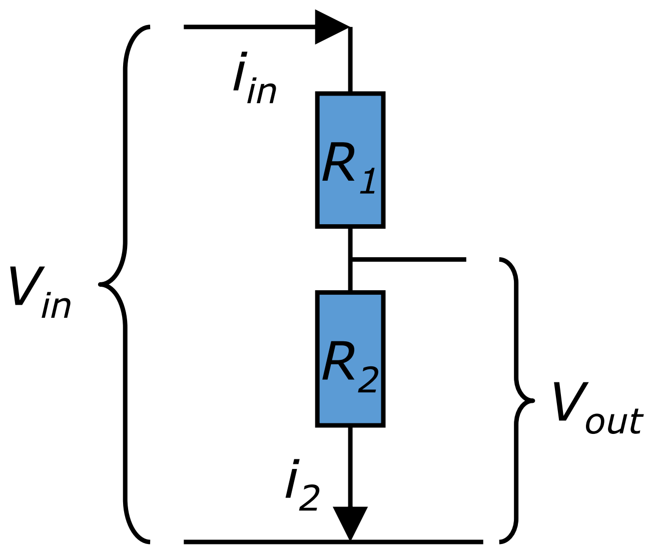
- For measurements to be approximately correct we need
  - Z_{in} very high – so the ’scope draws very little current.
  - Z_{out} very low – so the transducer provides as much current as possible.

Key insights

- It’s easy to measure the wrong thing.
- This impacts both the transducer design and the measurement apparatus.
- For voltage measurements, we’d like:
  - A transducer with a low output impedance.
  - A measurement device with a high input impedance.
  - Typical ´scopes have Z_{in} at DC of 1-10 MΩ.

A second look at our measurement example

- Consider a perfect resistor network with R_1 and R_2, and a perfect measuring instrument.



This means the device measuring V_{out} has $Z_{in} = \infty$ so it takes no current.

$$\text{So, } i_{in} = \frac{V_{in}}{R_1+R_2} = i_2 = \frac{V_{out}}{R_2}.$$

This is called a "potential divider"

$$V_{out} = V_{in} \cdot \frac{R_2}{R_1+R_2} = V_{in} \times \frac{\text{resistance across } V_{out}}{\text{resistance across } V_{in}}.$$

What if the 'scope impedance is complex?

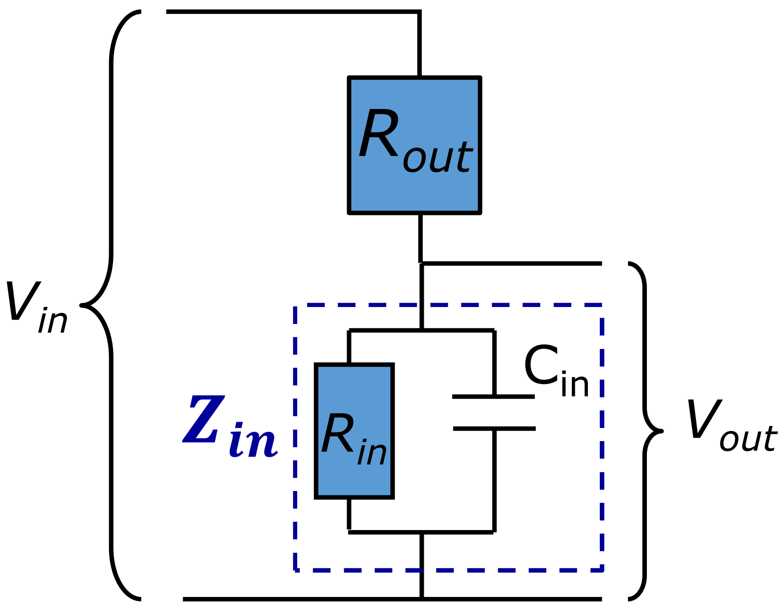
- In real 'scopes Z_{in} is complex.
  - $R_{in} \sim 1M\Omega, C_{in} \sim 20pF$
  - Assume our transducer has $R_{out} \sim 100\Omega$

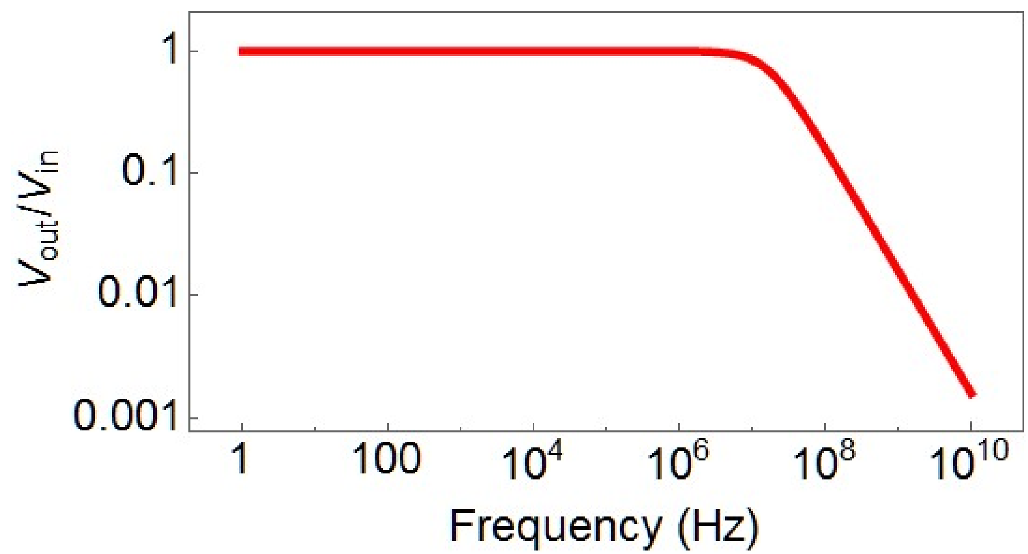
$$Z_{in} = \left(\frac{1}{R_{in}} + i\omega C_{in}\right)^{-1} = \frac{R_{in}}{1 + i\omega C_{in} R_{in}}$$

$$Z_{total} = R_{out} + Z_{in}$$

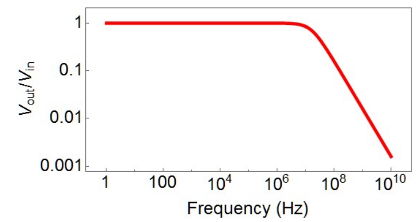
$$V_{out} = V_{in} \frac{Z_{in}}{Z_{total}}$$

$$V_{out} = V_{in} \frac{\frac{R_{in}}{1+i\omega C_{in} R_{in}}}{R_{out} + \frac{R_{in}}{1+i\omega C_{in} R_{in}}} = V_{in} \frac{R_{in}}{R_{in} + R_{out}(1 + i\omega C_{in} R_{in})}$$

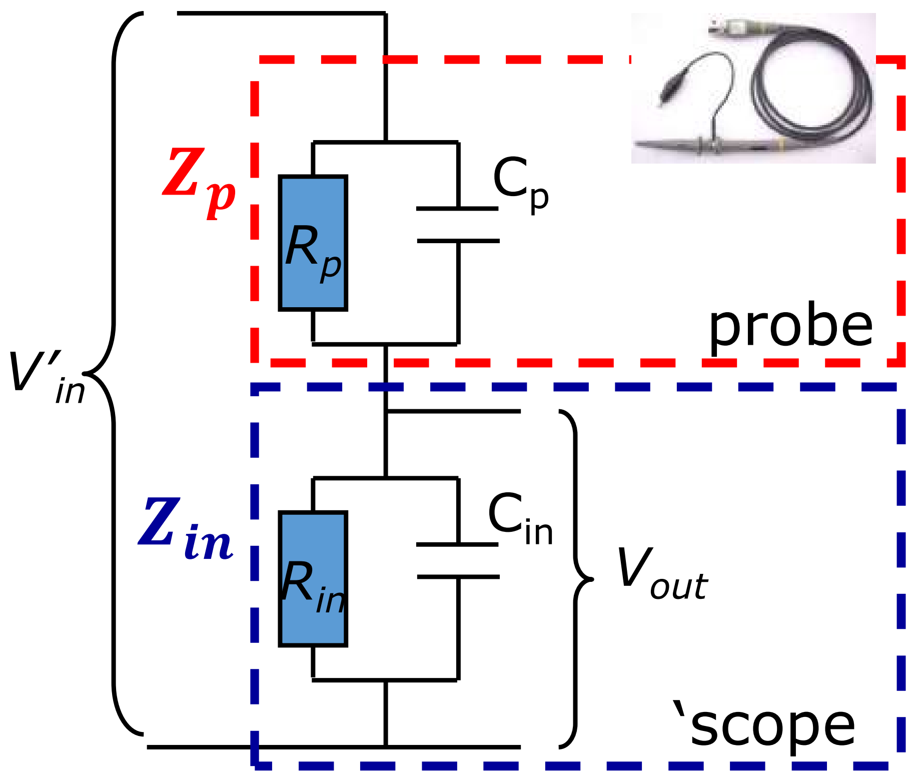




Can we fix this?



This is a problem at high frequencies, but we can compensate to some extent using 'scope probes:



$$Z_{total} = Z_p + Z_{in}$$

$$V_{out} = V'_{in}\frac{Z_{in}}{Z_p + Z_{in}}$$

$$V_{out} = V'_{in}\frac{\frac{R_{in}}{1 + i\omega C_{in}R_{in}}}{\frac{R_p}{1 + i\omega C_p R_p} + \frac{R_{in}}{1 + i\omega C_{in}R_{in}}}$$

If $C_{in}R_{in} = C_pR_p$, $V_{out} = \frac{V'_{in}\ R_{in}}{R_{in}+R_p}$

C_{in} is 'compensated', however $V_{out} \neq V'_{in}$

Some warnings regarding measurement

1. For current measurement, you least disturb the system if the measurement device takes all of the current – low Z_{in}.

2. To transfer maximum power from one system to another – (this is not equivalent to a non-invasive measurement) – then Z_{out} of one system must equal Z_{in} of the next:
   - EM radiation flux into a solar cell.
   - The gel used when having an ultrasound scan.

   (Impedance: ratio of a “potential” to a “flow”)

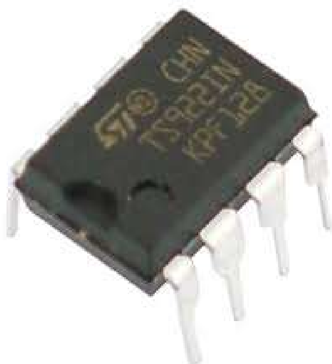
The high Z_{in} of a ’scope is great for measuring voltage signals, but little power gets into the oscilloscope.

In general, we frequently amplify and/or modify them:

Operational amplifier…

Operational Amplifiers – background

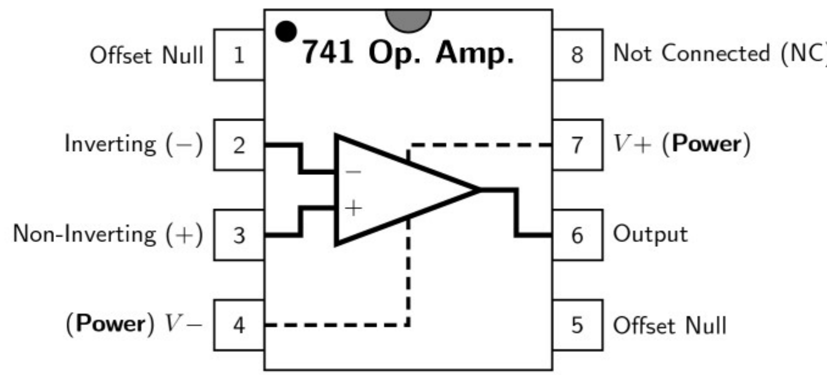
- Packaged, high gain, high input impedance voltage amplifiers:
  - Contain 10-100 transistors, Rs, Cs etc.
  - Contain very clever circuitry. [we are skipping the electronics of discrete transistors!]
  - Draw energy from a power supply.
  - You – as physicists – need to know NONE of the details

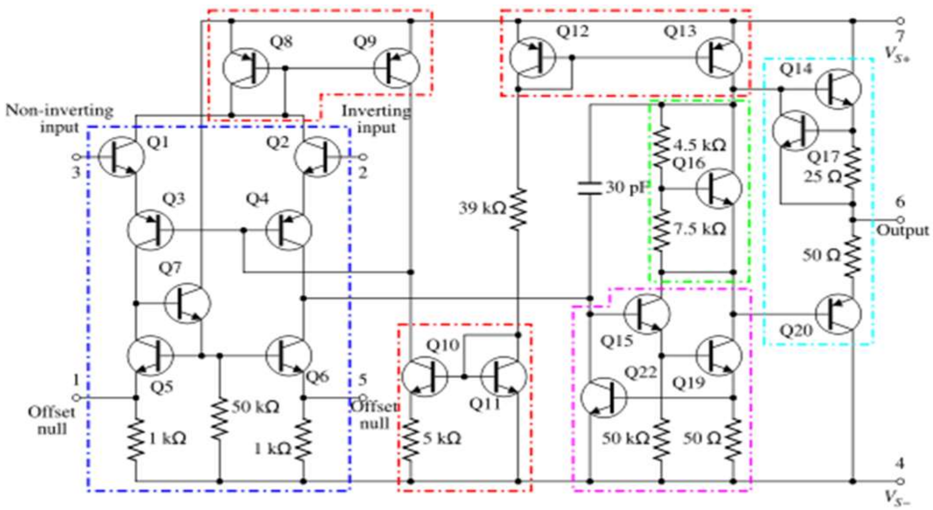


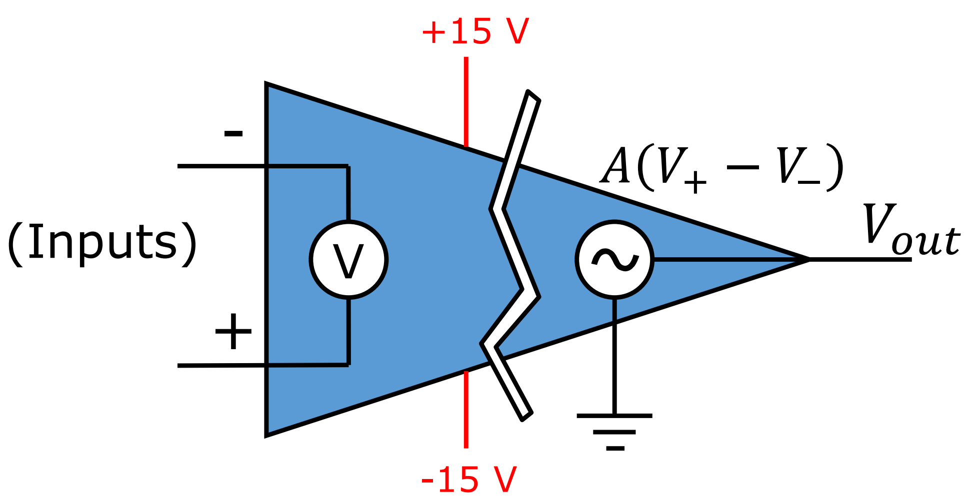
- Why is this?
  - We wish to use op-amps (not design them).
  - In this case we can follow a "systems" approach and assume their behavior is characterized by a small number of properties.
    - This is what physicists routinely do when modelling the real world
    - It's also to some extent necessary (and common) to use equipment where we "trust" previous design/implementation.

  - A real op-amp is a good approximation to an ideal amplifier (we will explore how good later).

How to ‘view’ an op-amp







- Amplifier has an “open loop gain” of A

$$V_{out} = A(V_+ - V_-)$$

- So, for example, if we connect “+” input to a potential V_{in} and connect “-” input to ground:

$$V_{out} = AV_{in}$$

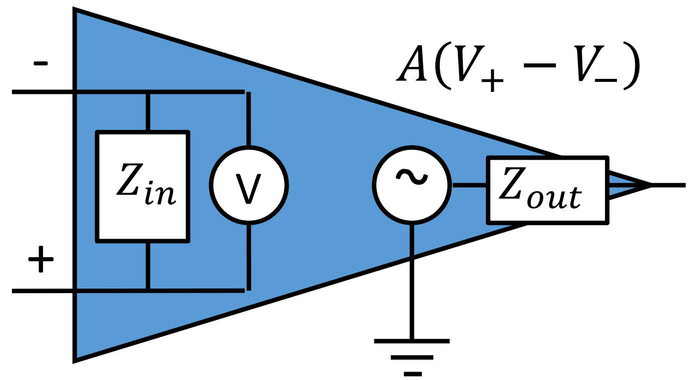
Properties of a model "ideal" voltage amplifier [assume these from now … until we do otherwise!]

$$A = \infty$$

(in reality $A \gg 10^5$)

$$Z_{in} \equiv \frac{\partial V_{in}}{\partial i_{in}} = \infty$$

If the amplifier is fed with a small current that doesn't matter.



$$Z_{out} \equiv \frac{\partial V_{out}}{\partial i_{out}} = 0$$

So it can provide lots of current to what it is connected to.

Let's ask ourselves: would such an ideal amplifier be useful?

If A=∞, even a tiny V_{in} would make V_{out} =∞

(or rather equal to the ±15V supply limit…)

We can make such an infinite gain amplifier useful by using "negative feedback"

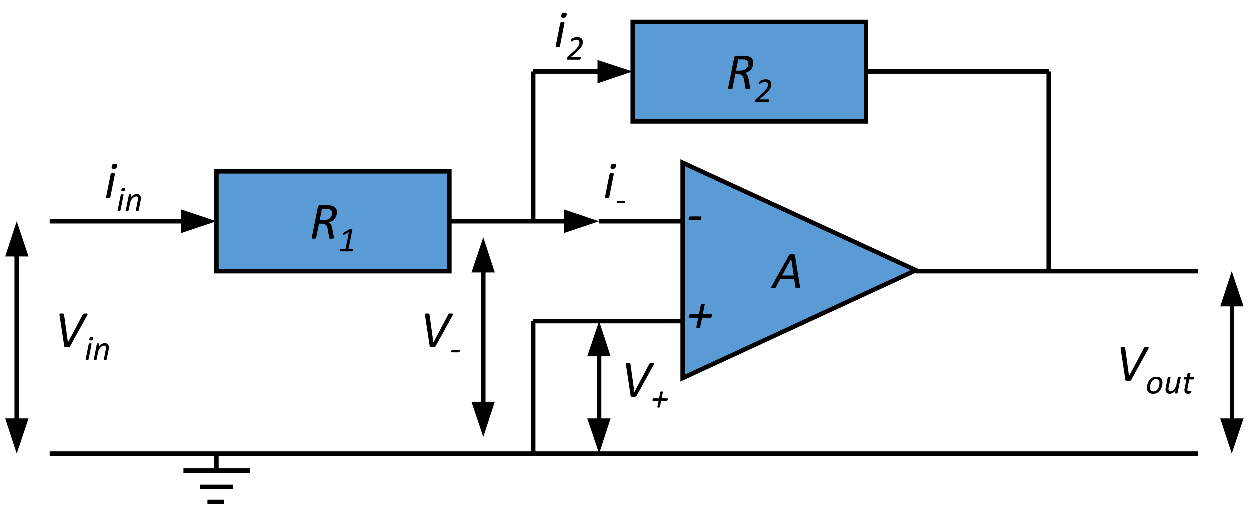
$$V_{out} = A(V_+ - V_-)$$

- We sense a fraction of the output and connect to the –ve input.
- As the output rises, the amplifier starts lowering it.
- An equilibrium is reached that stops the output saturating
  - Under these circumstances, the analysis of circuits involving such amplifiers is relatively straightforward.

GOLDEN RULES FOR ANALYSING IDEAL OP-AMPS

- Golden Rule #1: the inputs draw no current:
  - Because Z_{in} is infinite.
- Golden Rule #2: The voltages on the "+" and "-" inputs are the same, unless the output is saturated:
  - Because A is so high, if the output is not to saturate, the voltages at the "+" and "-" inputs must be *very close* to equal.
  - Note: one must provide *negative feedback* to achieve this.

How to wire up an inverting voltage amplifier

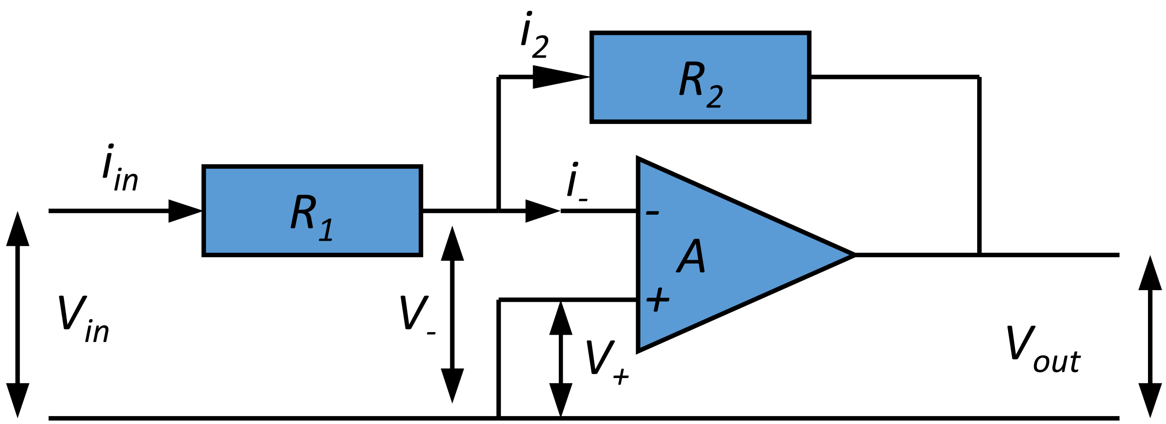


- Want to find the self-consistent solution for V_{out}/V_{in} .
- This is called the "closed-loop" gain, i.e. the gain of the circuit with feedback.

- Step 1:
  - $V_+ = 0$. So GR2 $\Rightarrow V_-=0$.
  - In this situation, the "-" pin is called a "virtual earth".

Step 2: conserve current and use Ohm's law

- GR1 $\Rightarrow$ $i_-=0$. So, $i_{in}=i_2$.



$$i_{in} = \frac{V_{in} - \overset{(=0)}{\cancel{V_-}}}{R_1}$$

$$i_2 = \frac{\cancel{V_-} - V_{out}}{R_2}$$

$$\boxed{\text{gain} \equiv \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$$

i.e, independent of A

- Remember to apply a p.d. sign convention consistently. Here we have taken V_{in} as highest, V_- less, and V_{out} least (hence the current flow directions). Can, e.g., reverse this but the solution will be the same.

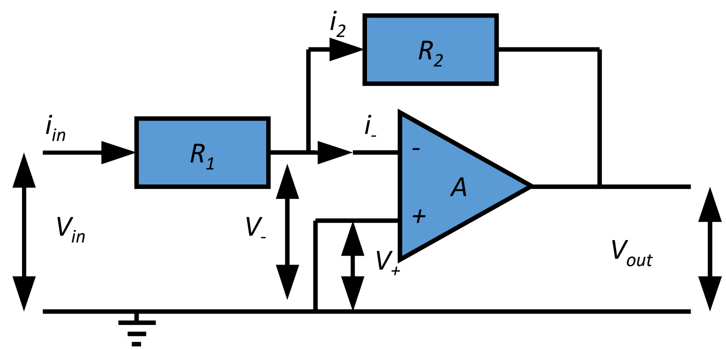
So, what does this mean in practice?

- Provided the op-amp is ideal (or nearly so):
  - We can use the systems approach.
  - The gain of the circuit is set by just the two resistors.
  - With negative feedback present at all frequencies, circuit will amplify provided V_{in} is not so high that V_{out} reaches ±15 V.

Skipping in lecture today

Can we better justify golden rule 2 ($V_+ = V_-$)?

- What if we don't assume it's true? Remember A is very large ($> 10^5$)



Since V_+ is grounded, the op-amp output:

$$V_{out} = A(V_+ - V_-) = -AV_-$$

Ohm's Law across R_1 and R_2:

$$i_{in} = \frac{V_{in} - V_-}{R_1}, \qquad i_2 = \frac{V_- - V_{out}}{R_2}$$

GR1 ($i_- = 0$), so conserving current:

$$i_{in} = i_2 = \frac{V_{in} - V_-}{R_1} = \frac{V_- - V_{out}}{R_2}$$

$$V_{out} = -\frac{R_2}{R_1} V_{in} + \left(1 + \frac{R_2}{R_1}\right) V_-$$

$$V_{out} = -\frac{R_2}{R_1} V_{in} - \left(1 + \frac{R_2}{R_1}\right) \frac{V_{out}}{A}$$

$$V_{out} = -\frac{R_2}{R_1} V_{in} \left(\frac{A}{A + \left(1 + \frac{R_2}{R_1}\right)} \right) \approx -\frac{R_2}{R_1} V_{in}$$

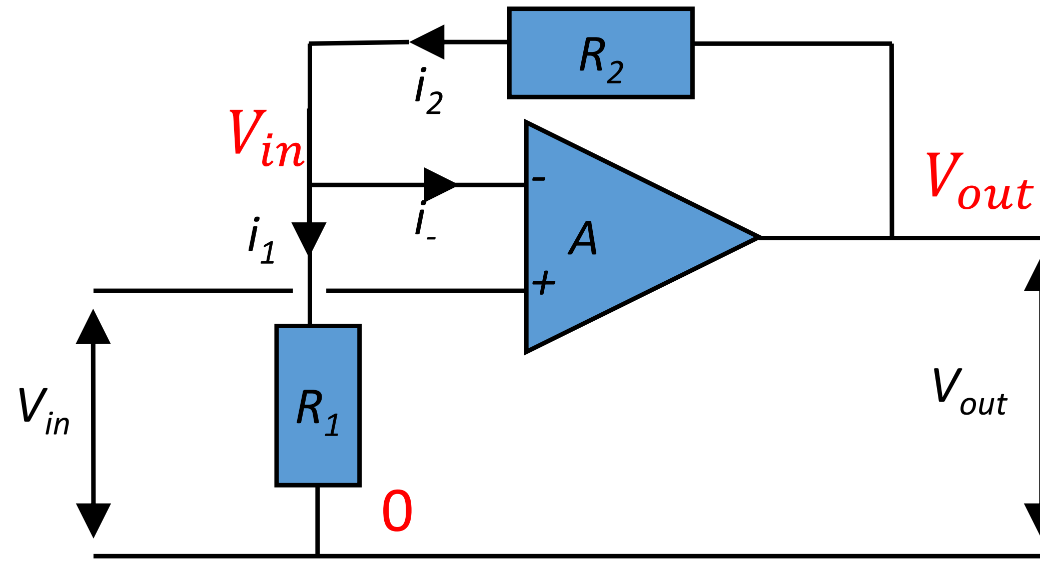
$$\boxed{V_+ - V_- \approx -\frac{1}{A}\frac{R_2}{R_1} V_{in} \approx 0}$$

(a very small number!)

Golden Rule 2!

So feedback + large A → GR2

We can also wire non-inverting amplifier circuits



- Step 1:
    - GR2 $\Rightarrow V_- = V_+ = V_{in}$

- Step 2: conserve currents and use Ohm's law and GR1

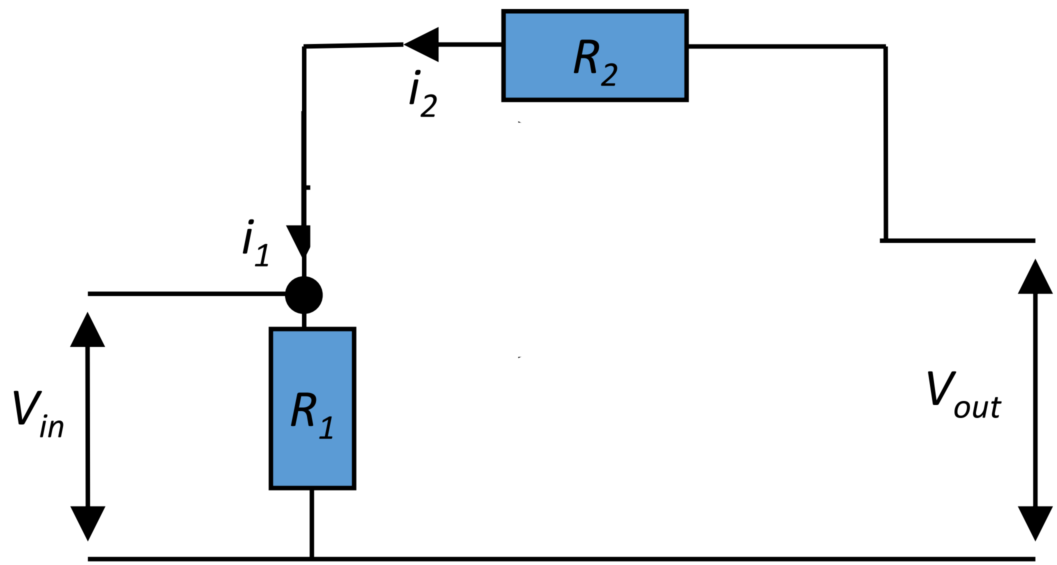
$= V_{in}$

$$i_2 = i_1 + i_- \Rightarrow \frac{V_{out} - V_-}{R_2} = \frac{V_- - 0}{R_1} + 0$$

$$\Rightarrow \text{gain} \equiv \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}$$

We can understand the non-inverting circuit in a different (and helpful) way

- GR1 $\Rightarrow$ $i_- = 0$ & GR2 $\Rightarrow$ $V_- = V_+ = V_{in}$ (as before)



- Can recognize that:
  - R_1 and R_2 are acting as a potential divider across V_{out} and ground (i.e. 0V)

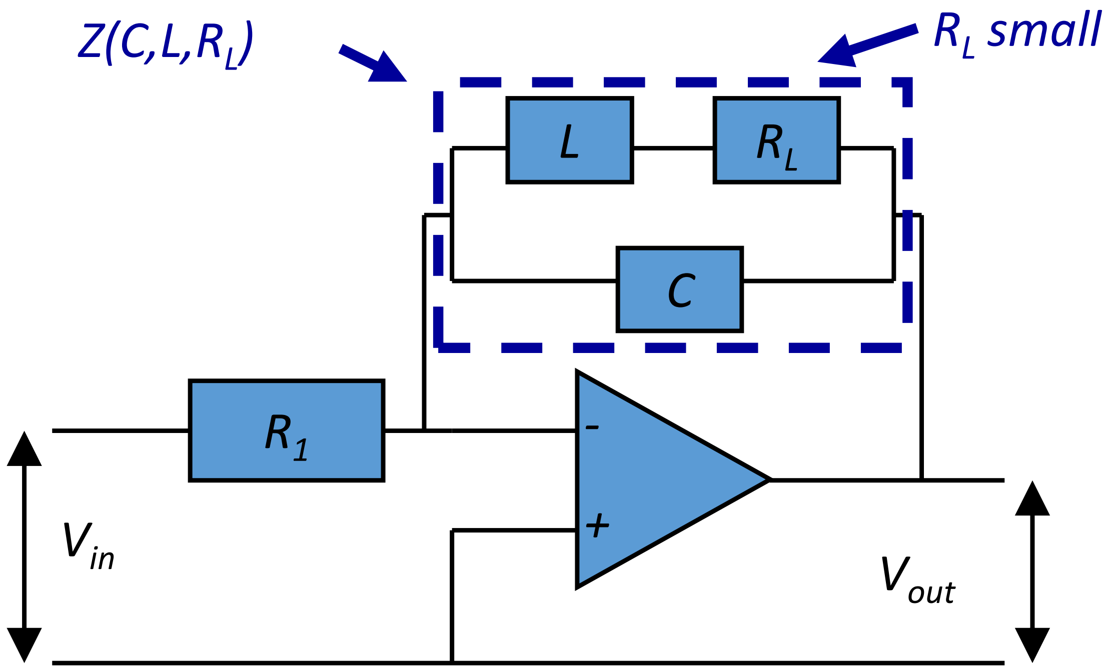
$$V_- = V_{in} = \frac{R_1}{R_1 + R_2} \cdot V_{out} \Rightarrow \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}.$$

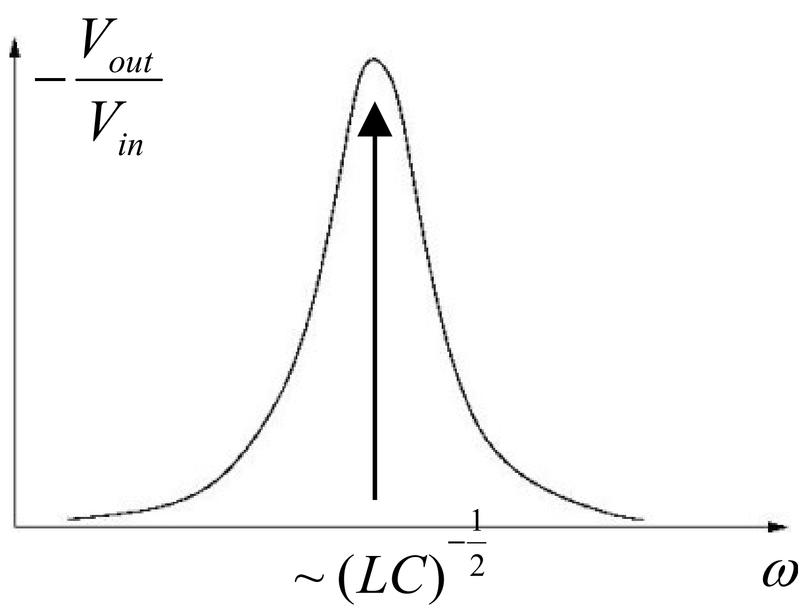
  - Recovering our previous result.

Op-amps can be used to do much more than make amplifiers…

- The external components don't have to be just resistors – can be Rs, Cs, Ls and more

  $\Rightarrow$ e.g. integrators, differentiators, filters…





- This is useful for amplifying signals close to a wanted frequency and filtering out the rest.

Summary so far

- Making a measurement very likely may $\Rightarrow$ measuring a small electrical signal.
- This must be measured faithfully, and possibly amplified and filtered.
- If we have an ideal high gain voltage amplifier we now know how to use it in a circuit.
- The circuit gain (=V_{out}/V_{in}) can be set by a few external components as long as they are arranged to give negative feedback.

Considering non-ideal performance

- How does non-ideal op-amp behaviour affect our previous results and their use in the lab:
  - Modelling non-ideal behavior.
  - Gain, Z_{in}, Z_{out}?

- How do we manage other “failings” when measuring with a real device:
  - For example, frequency response.

What do we mean by a non-ideal op-amp?

A not ∞ but 10^4–10^6.

Z_{in} not ∞ but high, and Z_{out} not 0 but low.

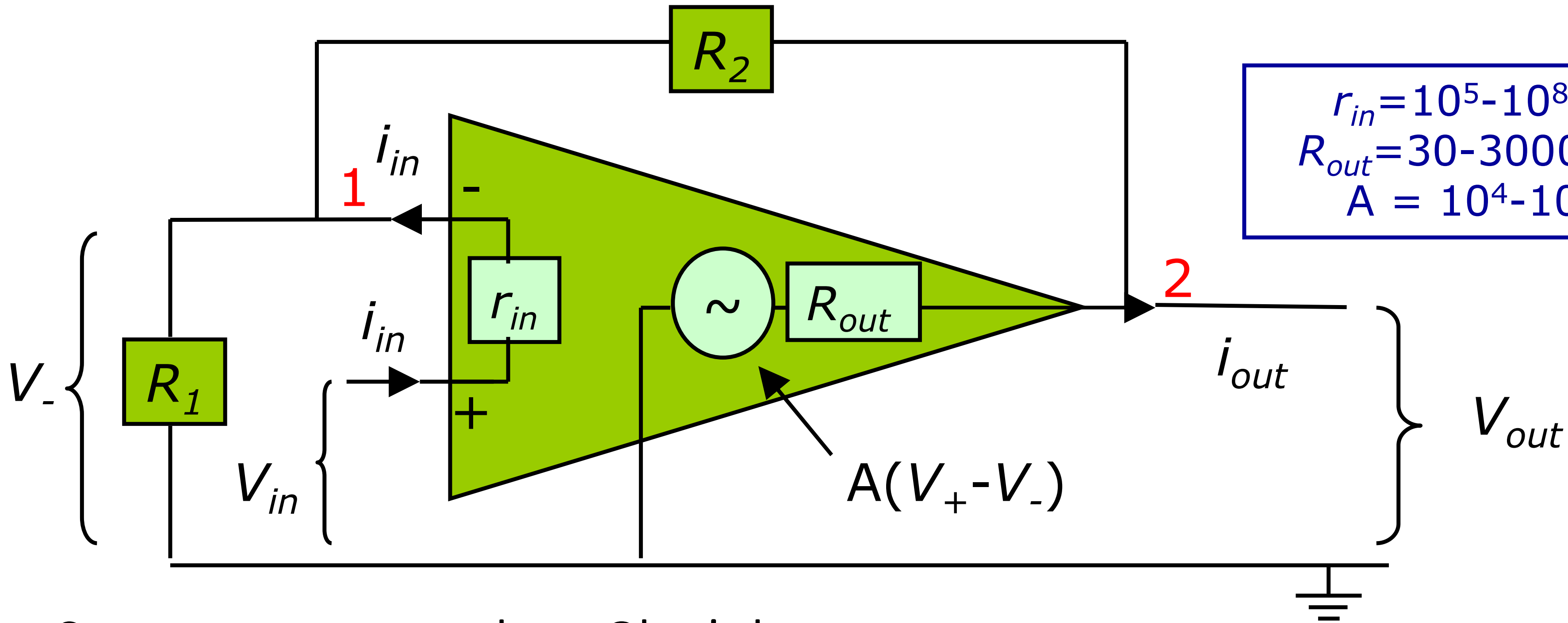
A is complex and a function of frequency.
The amplifying circuitry has a finite slew rate, dV_{out}/dt.

There is an input "bias current" independent of V_{in} (10^{-12}-10^{-7}A) that sets an upper limit to external resistor values.

There is an output voltage independent of $(V_+ - V_-)$ – equivalent to a differential offset of 10^{-3} to 10^{-2} V – that must be balanced with an external potentiometer.

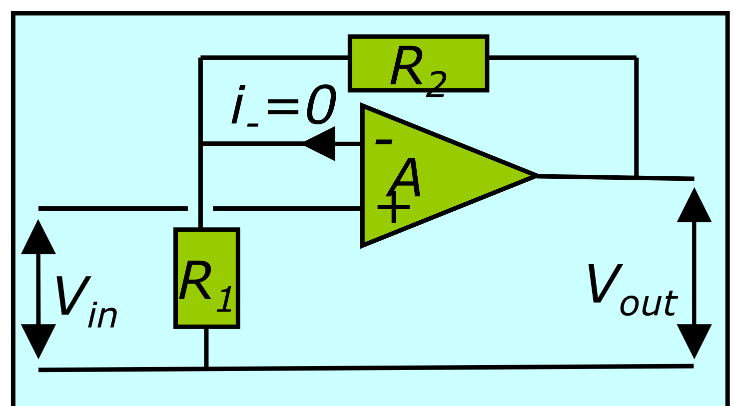
Annoying but not show-stoppers

A reasonable more realistic model for the non-inverting case



$r_{in} = 10^5\text{-}10^8\ \Omega$,
$R_{out} = 30\text{-}3000\ \Omega$,
$A = 10^4\text{-}10^6$

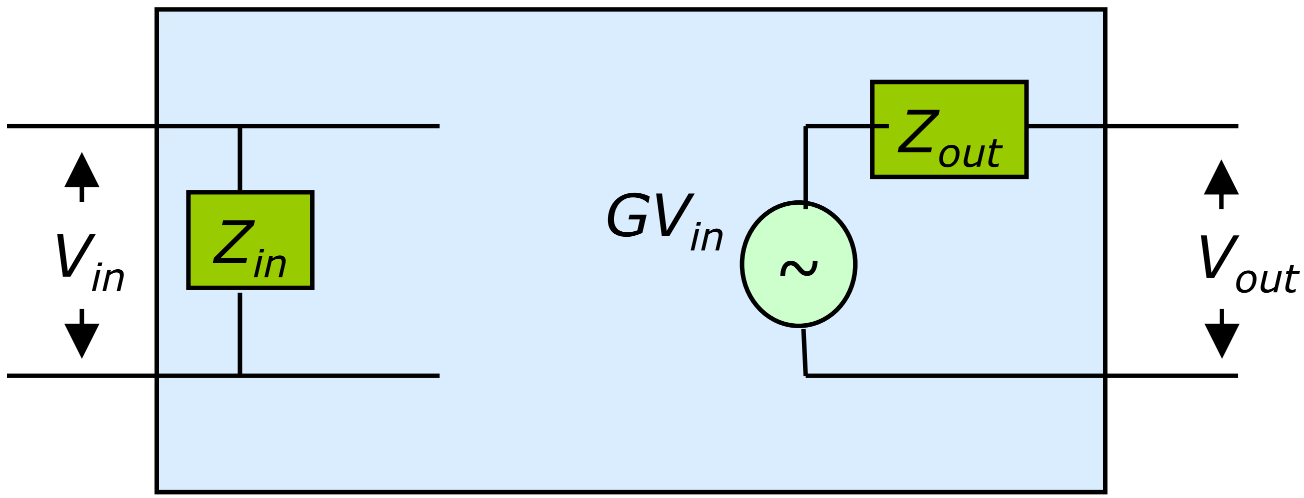
- Conserve current and use Ohm's law:
  - At 1: $$i_{in} = \frac{V_{in} - V_-}{r_{in}} = \frac{V_- - 0}{R_1} + \frac{V_- - V_{out}}{R_2}$$
  - At 2: $$i_{out} = \frac{A(V_{in} - V_-) - V_{out}}{R_{out}} + \frac{V_- - V_{out}}{R_2}$$



What we do now is eliminate the unknowns to get the characteristics of the circuit

- We treat the whole circuit as a "system" with an effective input impedance, an open-loop gain, and an output impedance such that we can write

$$V_{out} = V_{in} \cdot \underbrace{(\text{open loop gain})}_{(G)} - i_{out} \cdot \underbrace{(\text{output impedance})}_{(Z_{out})}$$



- We hope Z_{in} of the "system" is high and Z_{out} for the "system" is low.

Let's do the algebra, how does this "model" circuit compare to our "perfect" op-amp?

- For large A and large r_{in}:

$$V_{out}/V_{in} = 1 + \frac{R_2}{R_1}$$

  - As in the ideal case, like the Golden rules.

$A = 10^4 - 10^6$, $r_{in} = 10^5 - 10^8\ \Omega$, $R_{out} = 30 - 3000\ \Omega$

- For large A and large r_{in}:

$$Z_{out} \rightarrow \frac{R_{out}}{A}\left(1 + \frac{R_2}{R_1}\right)$$

  - So the circuit has a lower Z_{out} than R_{out} of the op-amp itself.

- For large A and low R_{out}:

$$Z_{in} \rightarrow \frac{r_{in}A}{1 + R_2/R_1}$$

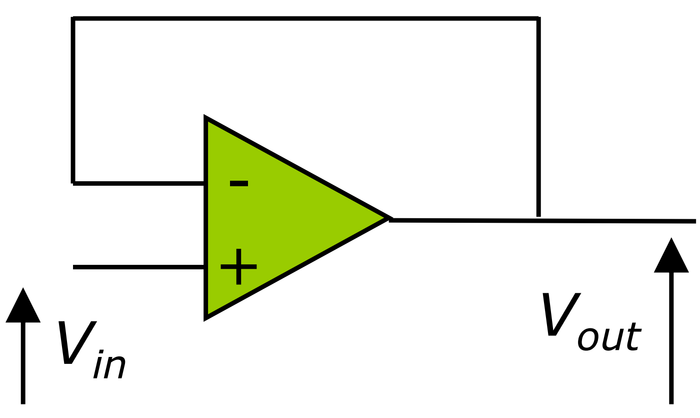
  - So the circuit has a higher Z_{in} than r_{in} of the op-amp itself. This amplifying circuit can have a Z_{in} ~$10^{12}\Omega$.

So basically if A is big, in this configuration the performance of the circuit as a voltage amplifier is almost "perfect"

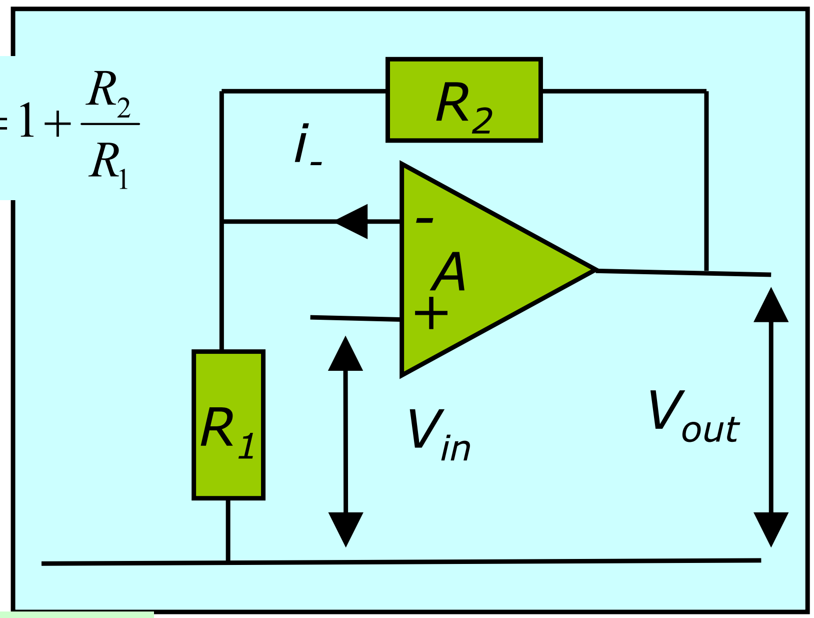
This is because we have negative feedback – it's the <u>crucial</u> aspect of the circuit.

- A useful, though apparently extreme, case of a non inverting amplifier has $R_2/R_1=0$:

<u>The "buffer" or "follower"</u>



$$V_{out}/V_{in} = 1 + \frac{R_2}{R_1}$$

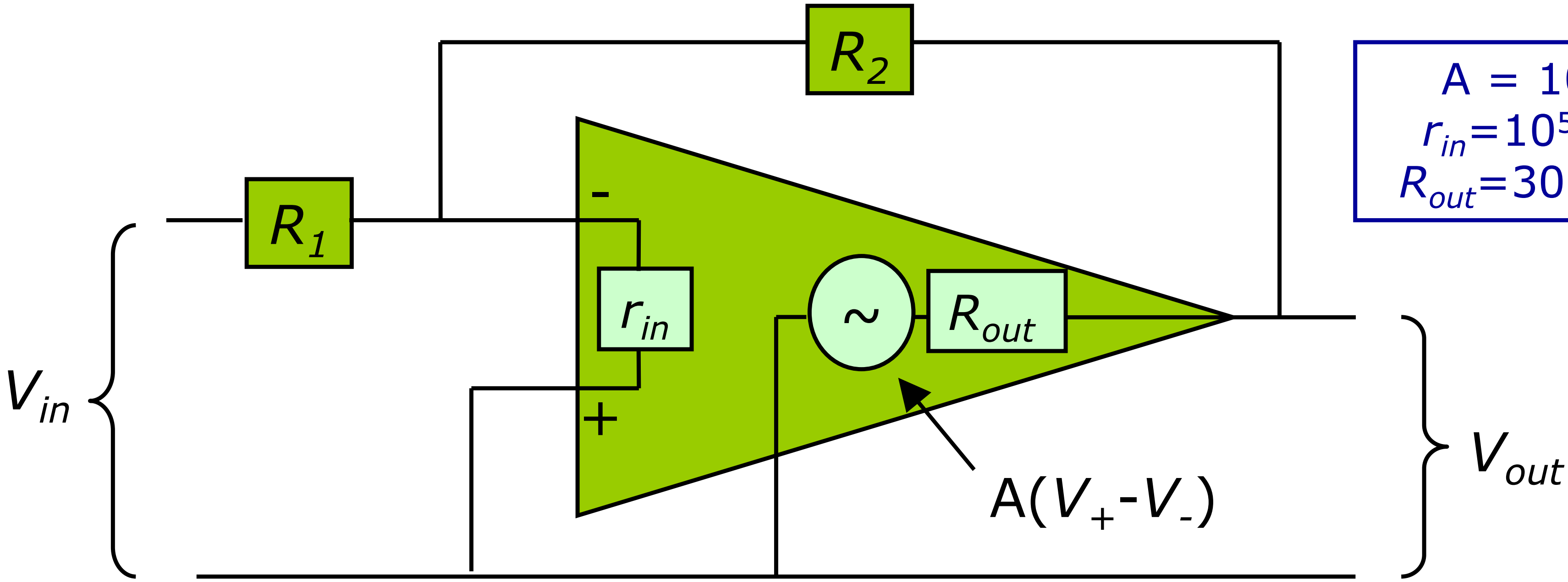


$$V_{in} = V_{out}$$

$$\text{BUT}: \; Z_{in} = r_{in} \cdot A \quad \text{and} \quad Z_{out} = \frac{R_{out}}{A}$$

Use this to "connect" two circuits together

We can similarly model the inverting non-ideal amplifier configuration



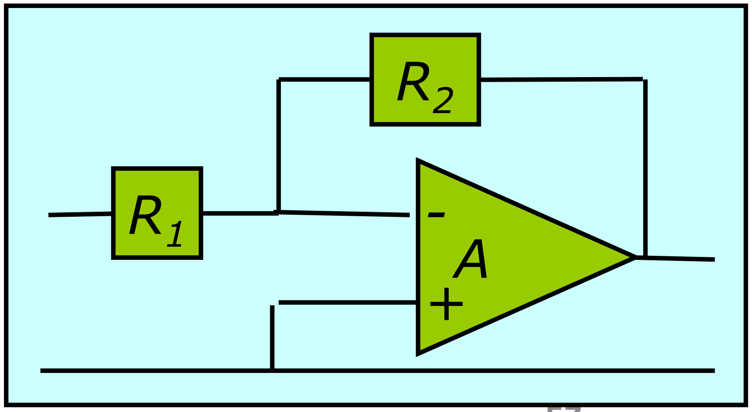
$A = 10^4 - 10^6$,
$r_{in} = 10^5 - 10^8\ \Omega$,
$R_{out} = 30 - 3000\ \Omega$

- Again, don't analyze with the Golden Rules, but just conserve current and use Ohm's law.

  We find, provided *A* is high:

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$

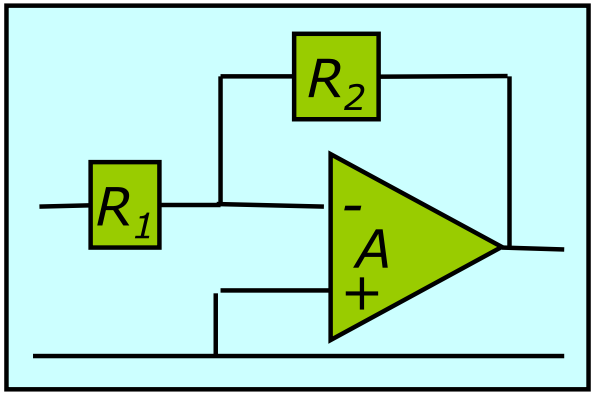
As in the ideal case again.



Similarly, we can solve for the input and output impedances of this circuit. Are Z_{in} high & Z_{out} low?

- $$Z_{out} = \frac{R_{out}}{A} f\left({R_2}/{R_1} \right)$$

  Good – with a typical op amp and R_1=1k and R_2=10k, Z_{out} = $10^{-3}\ \Omega$



- But…

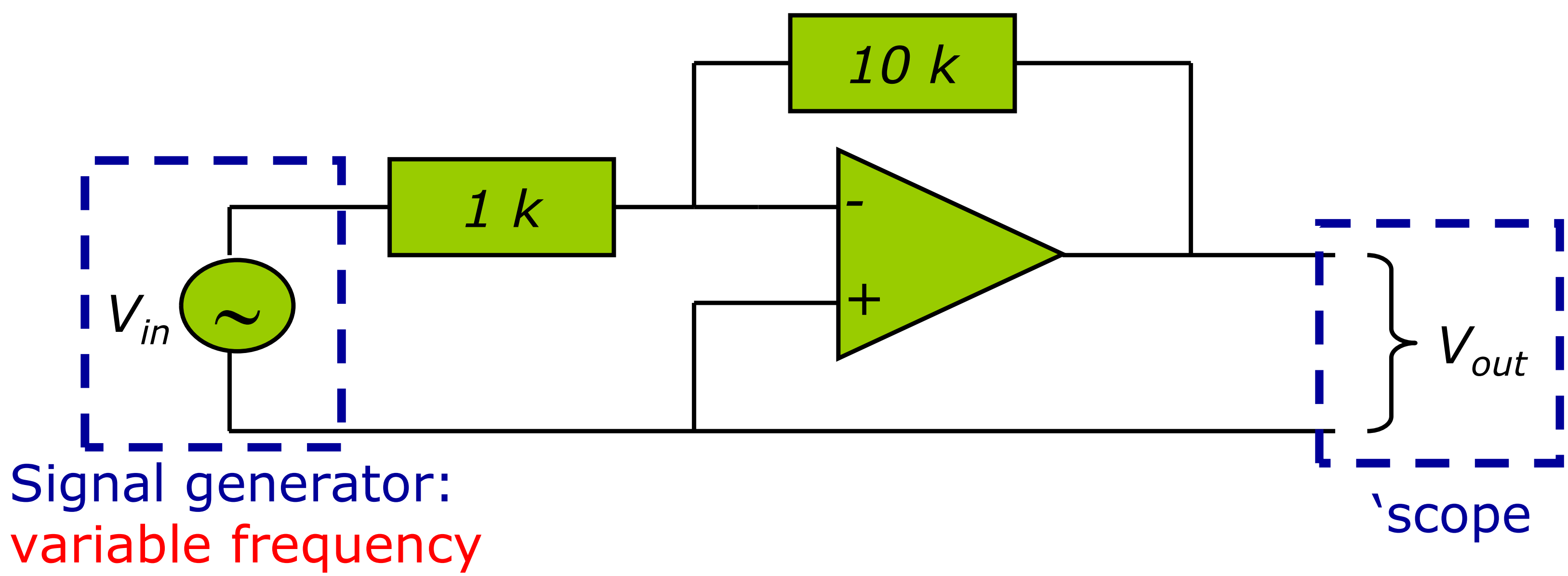
  $$Z_{in} = R_1 + \text{low-valued } f\left(\frac{R_1, R_2, r_{in}}{A} \right)$$

  "virtual-earth" arrangement

  - If R_1 is, say, 1k – then this is small compared with r_{in} (10^5-$10^8\Omega$), though large compared with Z_{out} (~$10^{-3}\ \Omega$) of a previous op-amp stage.
  - If you really need Z_{in} high, then use a non-inverting system.

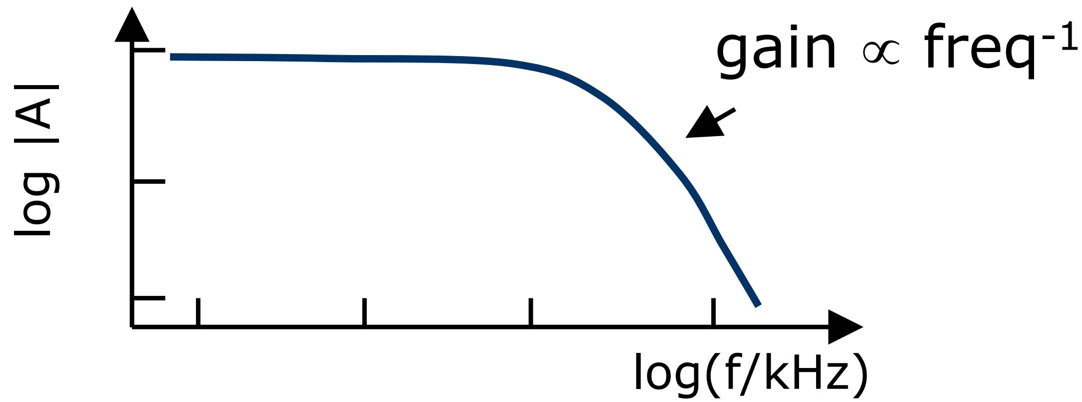
Other shortcomings: frequency dependence

- Most measurements involve a range of frequencies:
  - Seismic waves 0.1 Hz – 10 Hz
  - LIGO 50 Hz – 1 kHz
  - Human hearing 20 Hz – 20 kHz
  - Ultrasound 50 kHz – 200 kHz
- In the practical lab you might explore the frequency response of a typical op-amp circuit by measuring it with a set up with a nominal gain of 10.

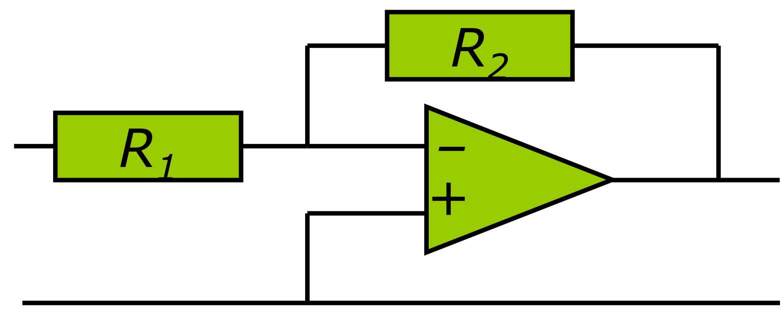


The measured gain of the circuit implies that op-amps have a built in reduction of A – why?

- In general we find:



- Consider the feedback resistor, R_2, in this circuit:

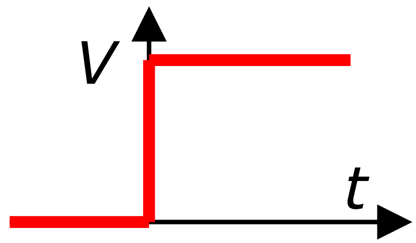


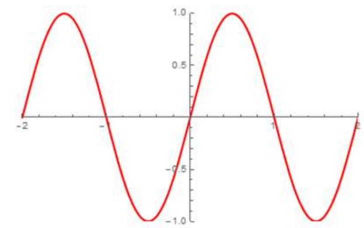
  - A real R_2 will actually be Z(L,C,R,ω), i.e. have a complex impedance

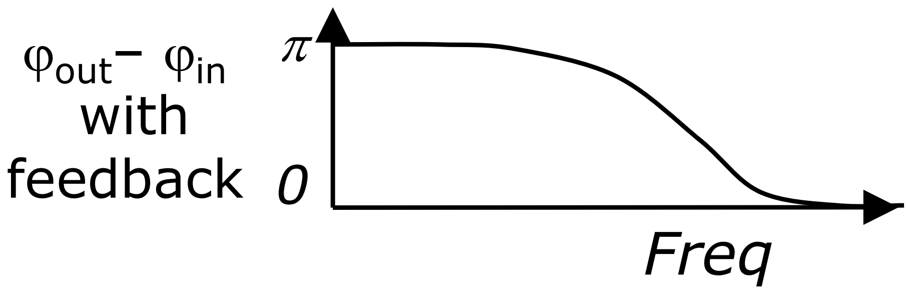
- So, over a big enough frequency range, this can lead to a phase shift of π. This will change –ve feedback into +ve feedback $\Rightarrow$ saturation/oscillation rather than amplification.

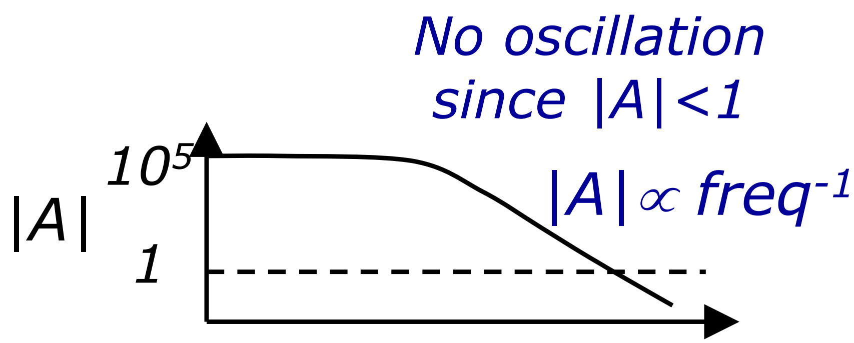
Is this really important for low freq. circuits?

- Consider switching a circuit on:
  - This step has Fourier components at all frequencies, so positive feedback is bound to happen.
  - So, op-amps are designed to have “internal frequency compensation” so that A itself falls at high frequencies.









- In itself, this is not always sufficient. Also need to stop oscillation-inducing signals getting back to the inputs, e.g.:
  - Pick-up from stray EM radiation: screen.
  - Along supply lines – use decoupling capacitors.

Summary

This one op-amp lecture is the bare minimum one needs to know about op-amps

It already allows you to build a vast number of useful circuits.