11)

$$1+z = \frac{1}{\left(1-\frac{2\mu}{r}\right)^{1/2}} = \frac{1}{\left(1-\frac{1}{3}\right)^{1/2}} = \frac{1}{\left(\frac{2}{3}\right)^{1/2}}$$

$$\omega_\infty = \frac{\omega_0}{1+z} = \sqrt{2/3}\,\omega \quad // \qquad \text{Gravitational redshift.}$$

Doppler shift: $r^2\dot\phi = h$

orbit eq^n $\frac{d^2u}{d\phi^2} + u = \frac{GM}{h^2} + \frac{3GM}{c^2}u^2$

with r constant $(u = 1/r)$ yields

$$h^2 = \frac{GMr^2}{r - 3GM/c^2}.$$

$\Rightarrow$ $h = \frac{cr}{\sqrt{3}}$ and $r\dot\phi = \gamma v = c/\sqrt{3}$ Tangential velocity.

$\Rightarrow v = c/2$, $\gamma = 2/\sqrt{3}$

Therefore ~~z_{to}~~ $z_{total} = \frac{\sqrt{3}}{2} \times \sqrt{\frac{2}{3}} = 1/\sqrt{2}$.

In the plane, doppler shift is

$$\left(\frac{1+v/c}{1-v/c}\right)^{1/2} = \begin{cases} \sqrt{3} & \text{towards} \\ 1/\sqrt{3} & \text{away} \end{cases} \qquad \times \text{ by } \sqrt{2/3}$$

So totals of $\sqrt{2}$ and $\frac{\sqrt{2}}{3}$