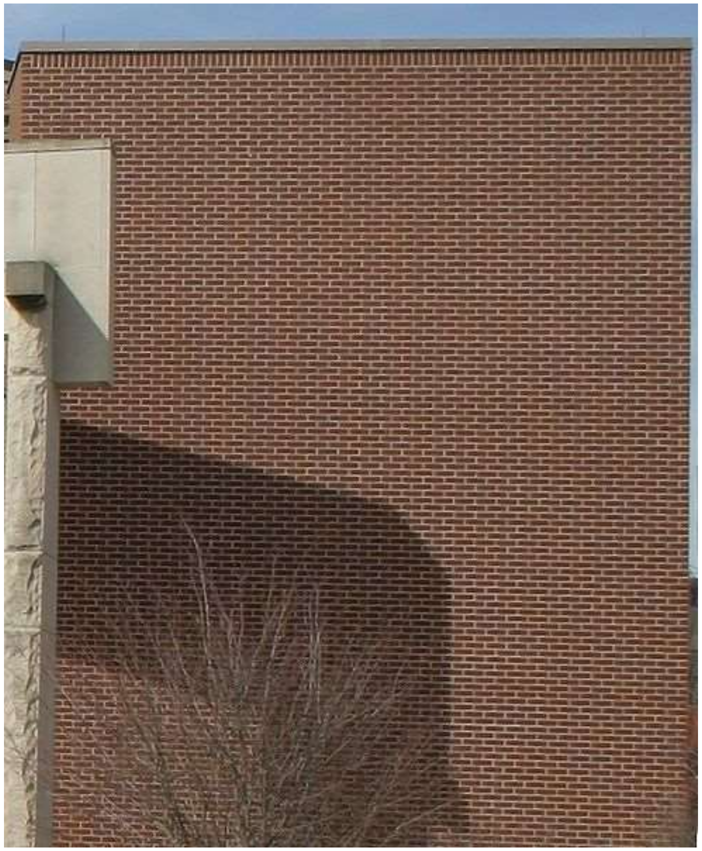
Lecture 5 – coping with unwanted influences

- Filtering:
  - General ideas.
  - Phase sensitive detection.
- Isolation
- Differential measurements.
- Shielding of E + B fields.

Example of aliasing



Tripos Part IB – Physics A

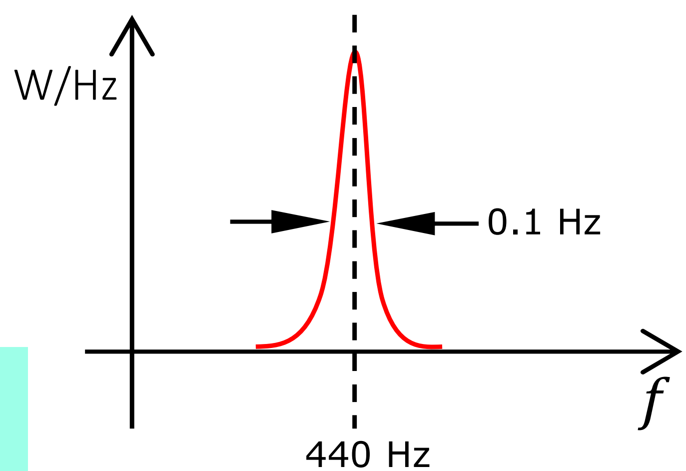
6 A Fourier Transform Spectrometer digitises the sound made by a tuning fork having a fundamental frequency of 440Hz and a bandwidth of 0.1Hz.

What are the properties and restrictions of the data sampling in order that the fundamental frequency and bandwidth are measured accurately? [4]

- To measure a frequency of $f = 440$ Hz faithfully, Nyquist's criterion tells us

$$f_{sample} > 880 \text{ Hz (i.e. } \Delta t < 0.0011 \text{ s).}$$

- The smallest frequency step we must resolve $\Delta f < 0.1$ Hz. *This corresponds to a total measurement time of >10 s.*



So, to sample a signal at $f = 440$ Hz with $\Delta f = 0.1$ Hz we must measure it for perhaps a few times 10s using samples separated by no more than 1.1 ms

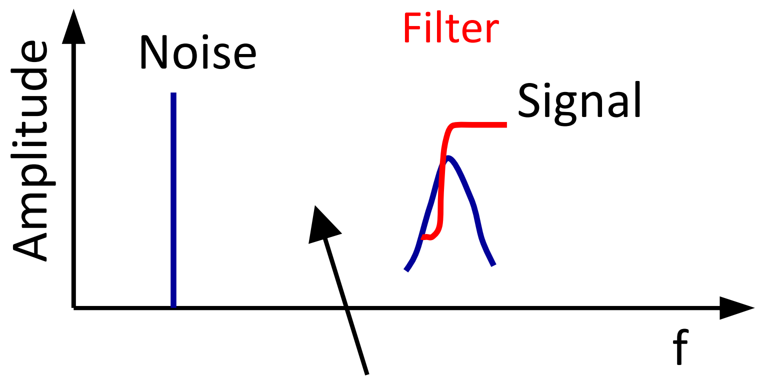
Good experiments (e.g. LIGO) exploit a wide spectrum of methods for coping with unwanted influences

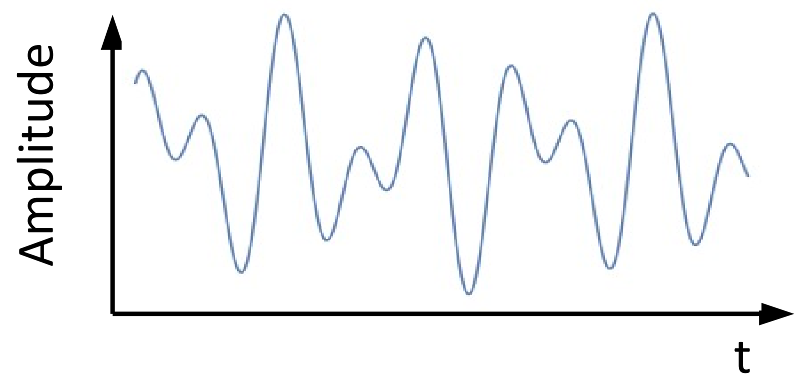
- Filtering:
  - removing noise that otherwise can't be eliminated.
- Differential experiments:
  - Same unwanted influence affects both parts (e.g. $\Delta l = l_1 - l_2$).
- Shielding:
  - For E and B fields and for heat.
- Eliminate noise at source:
  - Remote – away from elec. interference & vibr.
  - High – above much of the atmosphere.
  - Antarctic – cold/dry/high.
  - Space – all the above and "gravity-free".

more expensive/ more hassle/more effective

Filtering relies upon there being a different frequency content between the signal and your noise

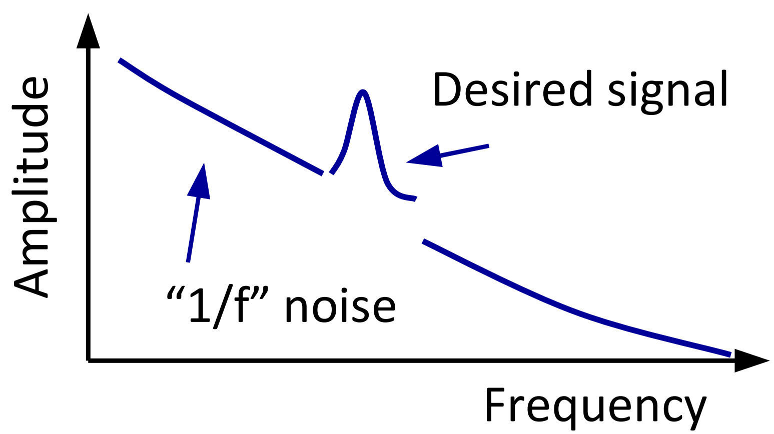
- Most effective if the signal & noise have non-overlapping spectra.





The problem is to make this rise sharply enough

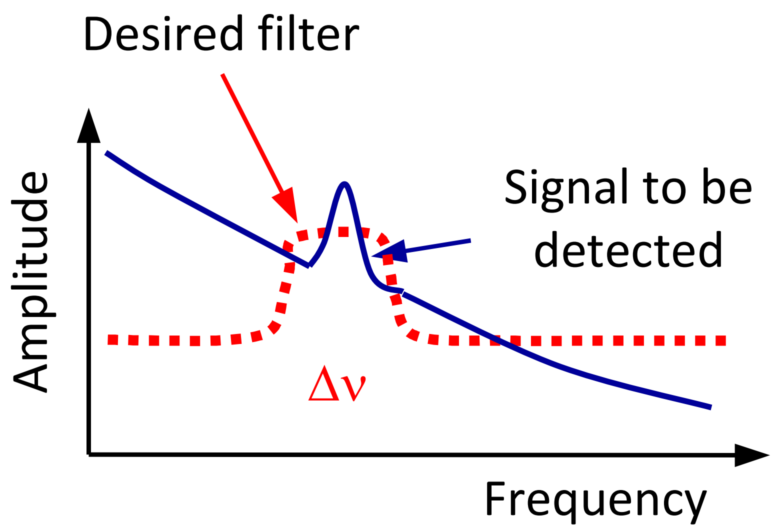
- This is a more problematic case:



- NB: All this applies to non-electrical signals too – e.g. acoustic and optical.
- It also applies to signals in the spatial (not just temporal) domain.

A “matched” filter is desirable

- Ideally we would like $\Delta\nu$ small, and the filter “notch-like”.



- Indeed, an optimal filter would have $\Delta\nu$ equal to the intrinsic width of the signal.

- In sophisticated experiments, one often encodes the quantity of interest with a given frequency and then transmits and measures it using a filter that only allows that frequency through.

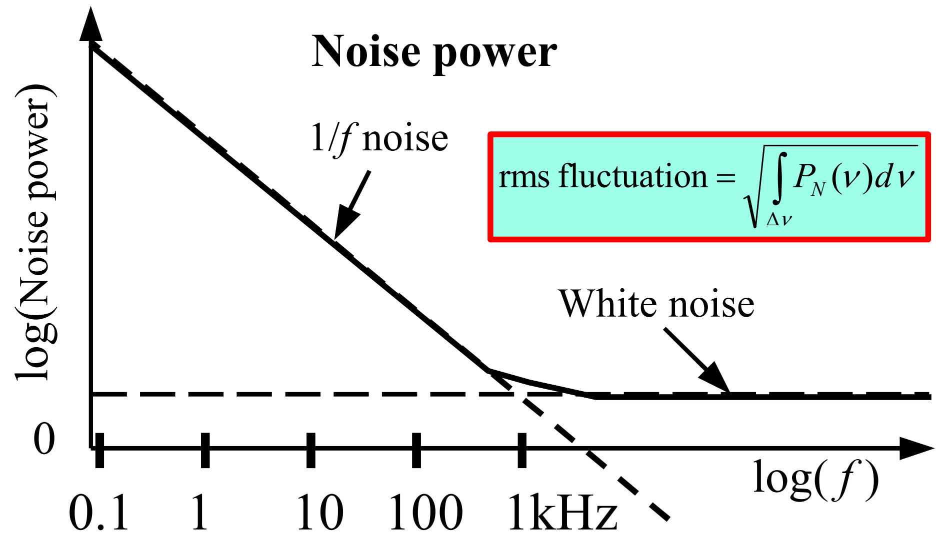
This goes by the name of “lock-in” or “phase-sensitive” detection.

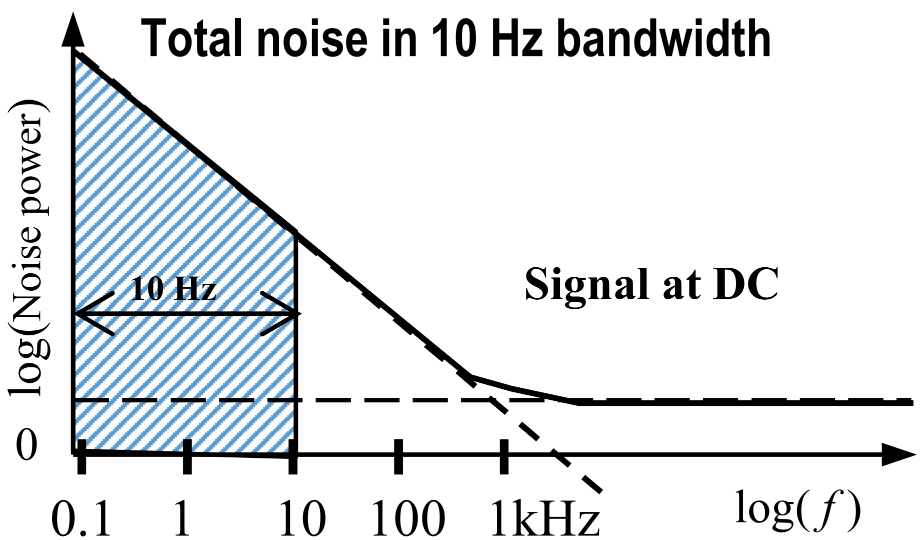
But how do we choose the frequency to encode at?

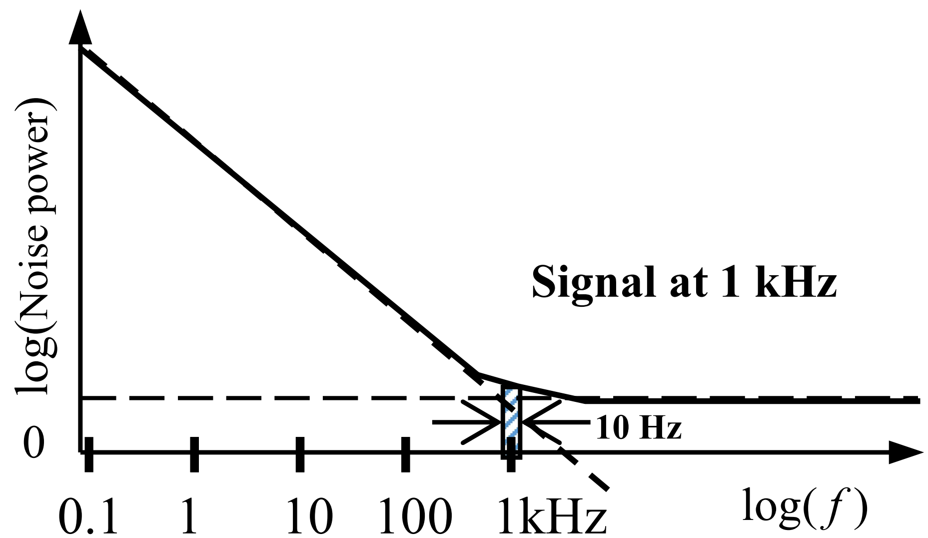
We need to pay attention to the frequency dependence of the noise

- At low frequencies often ~ f^{-1}:
  - Temp (0.1 Hz), pressure (1Hz)
- At high frequencies often ~ f^{0}:
  - Shot noise, Johnson noise

- Effect of noise depends on the signal freq:
  - Often worst at DC, where most signals are.

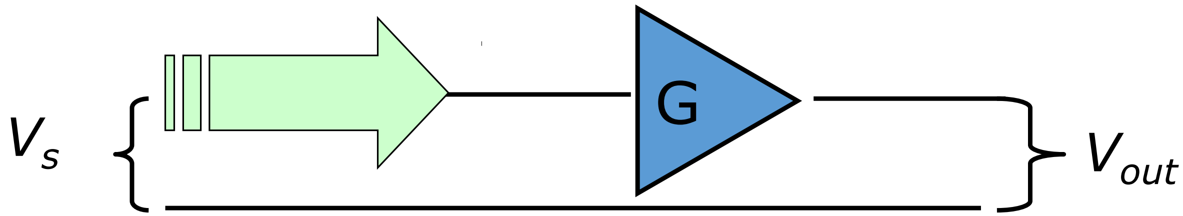




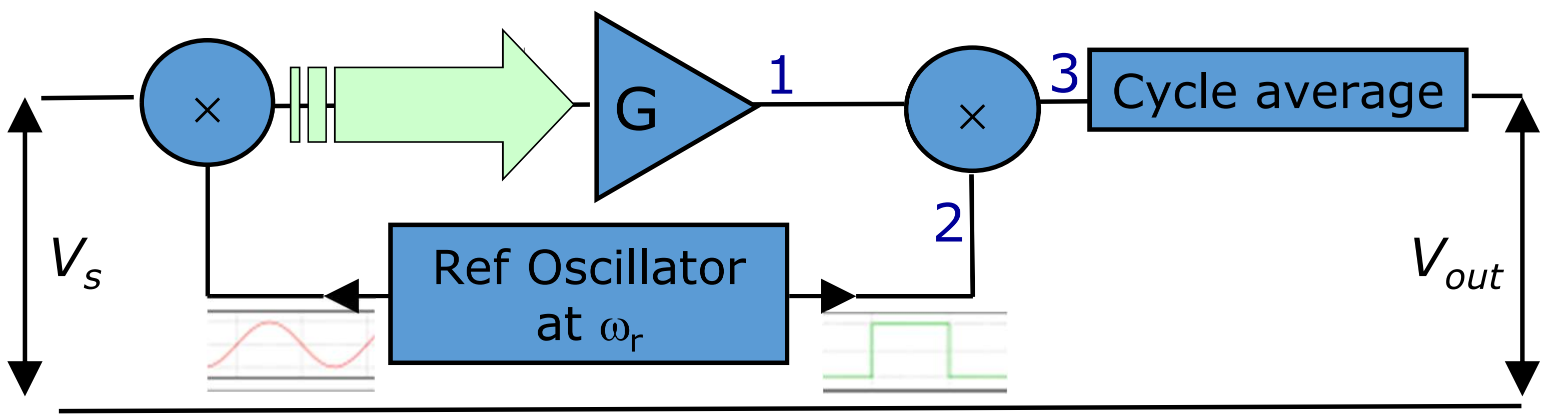


So how does "phase-sensitive detection" work?

- Consider the measurement of a small signal voltage V_s (assume DC at present) from a transducer with an amplifier.



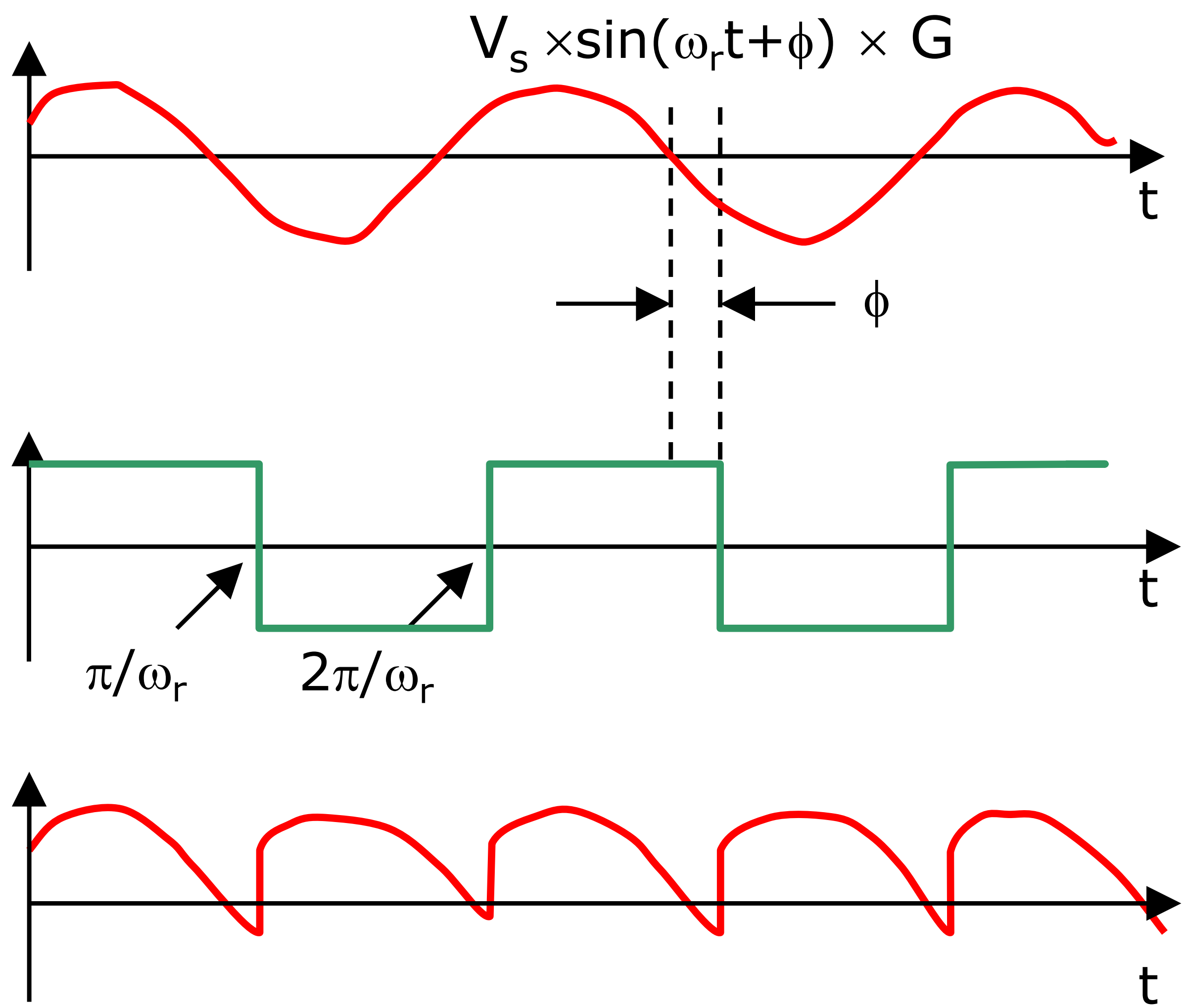
- Suppose we now add a reference oscillator and two modulators (=multipliers) to the system:



- Ref introduces sine wave at 1st modulator
- Ref introduces square wave at 2nd modulator
- This is where detection takes place
- Let phase diff. between signals at 1 & 2 be ϕ.

Let's consider the signals at locations 1, 2, and 3 (where we detect) to see what's going on

- At 1 – the output of the amplifier:
  - The DC signal is now "carried" at ω_r.
- At 2 – the reference to the 2nd modulator:
  - This signal is used to "de-modulate" the amplified signal.
- At 3 – the output of the 2nd modulator:
  - This is what is detected and time averaged.



- Note that the output of 2nd modulator does not have mean=0.

If we average the output over a cycle ($T=2\pi/\omega$) we get our first key result:

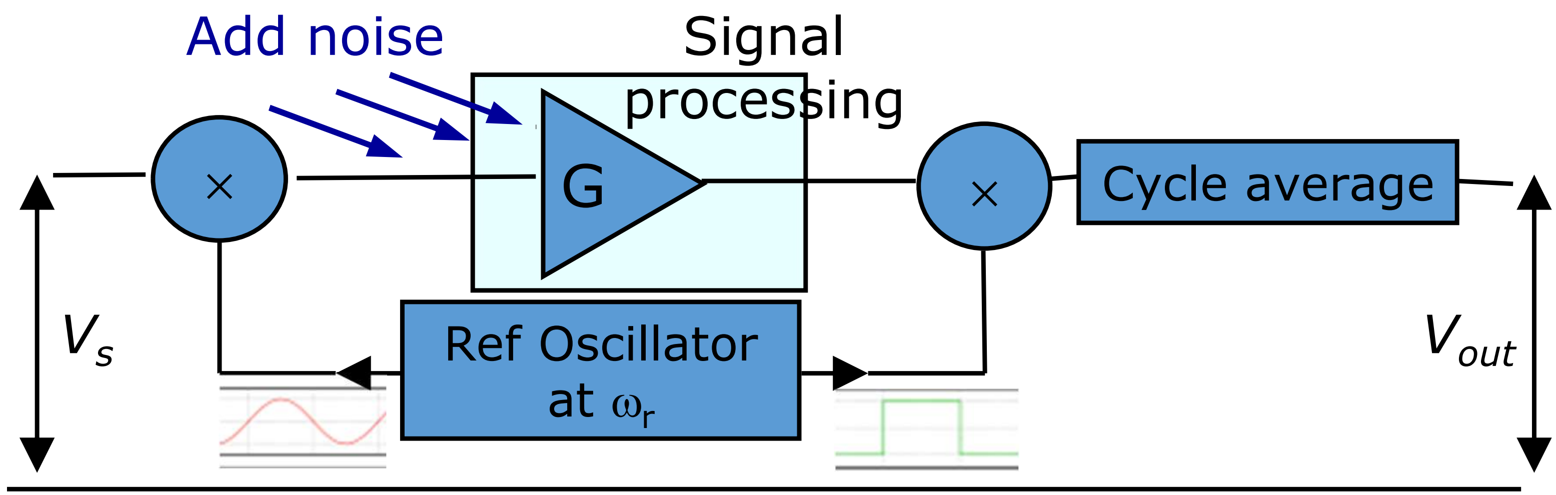
$$
\begin{aligned}
\langle V_{out} \rangle &= \frac{G}{T}\left[\int_0^{\pi/\omega_r} V_s \sin(\omega_r t+\phi)dt + \int_{\pi/\omega_r}^{2\pi/\omega_r} -V_s \sin(\omega_r t+\phi)dt\right] \\
&= \frac{G}{T}\frac{V_s}{\omega_r}\left\{ \left[-\cos(\omega_r t+\phi)\right]_0^{\pi/\omega_r} + \left[+\cos(\omega_r t+\phi)\right]_{\pi/\omega_r}^{2\pi/\omega_r} \right\} \\
&= \frac{2}{\pi} V_s G \cos(\phi).
\end{aligned}
$$

- So $\langle V_{out} \rangle \propto V_s$
  $\langle V_{out} \rangle \propto \cos(\phi)$.

This is the "phase-sensitive" bit

- At detection, replace ϕ with $\phi+\pi/2$, then $\langle V_{out} \rangle = -2/\pi\, V_s G \sin(\phi)$
  Allows to solve for V_s and ϕ.
- This is called measuring the "quadrature-component" (as opposed to the "in-phase" component) of the signal.
- This ϕ-sensitivity can form the basis for very precise measurement.

How does this help? Let's consider the impact of noise added after the first modulator



- At the output, noise coming in after the 1st modulator will be randomized by the reference signal at the 2nd modulator and will average towards zero.
- This works even for offsets at zero frequency.
- The modulated signal will still give an output $= \frac{2}{\pi} V_s G \cos(\phi).$
- So, unwanted influences entering after the 1st modulator are eliminated.

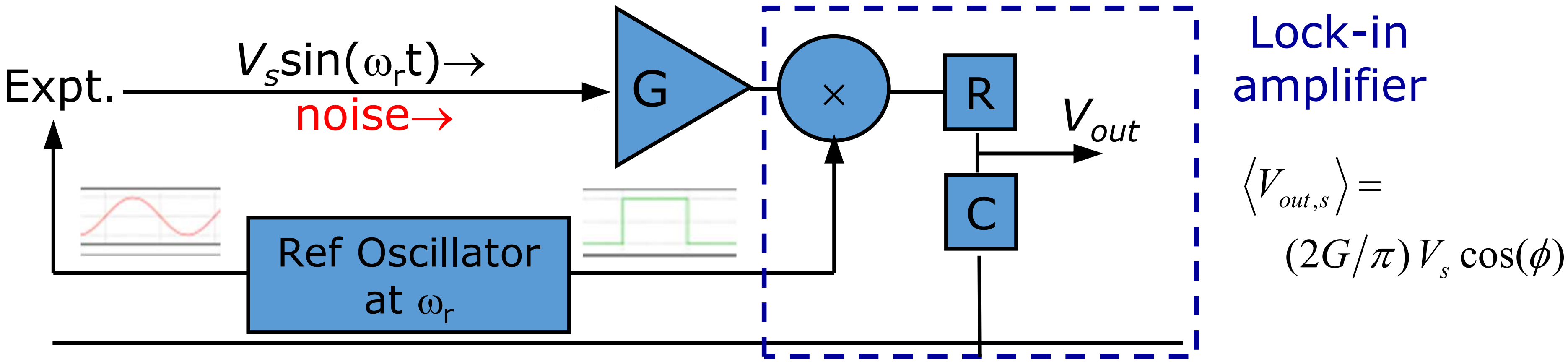
A key requirement is for the correct delivery of the two modulation waveforms:

- The modulation setup can be different. We used sine modulation 1st and then square wave modulation 2nd. But both could have been sinusoidal. (c.f. Fourier analysis)

- However, both modulator waveforms must have identical frequencies and a fixed phase relationship. In practice, they often come from the same oscillator.

- From an "experimental methods" perspective we want to select the most suitable frequency, ω_r, with which to encode our D.C. signal (V_s).
  - Since in many experiments, 1/f noise is the limiting factor, it often helps to encode the signals at a high frequency.

A second key feature is that the output is averaged



- Consider unwanted noise injected before the lock-in, with voltage $V_n\sin\big((\omega_r+\Delta\omega)t+\theta\big)$ at a frequency close to ω_r.
- This gives a noise output: $\langle V_{out,n}\rangle \propto (2G/\pi)V_n\langle cos(\Delta\omega t+\theta)\rangle$
  - i.e. downshifted in frequency from $\omega_r+\Delta\omega$

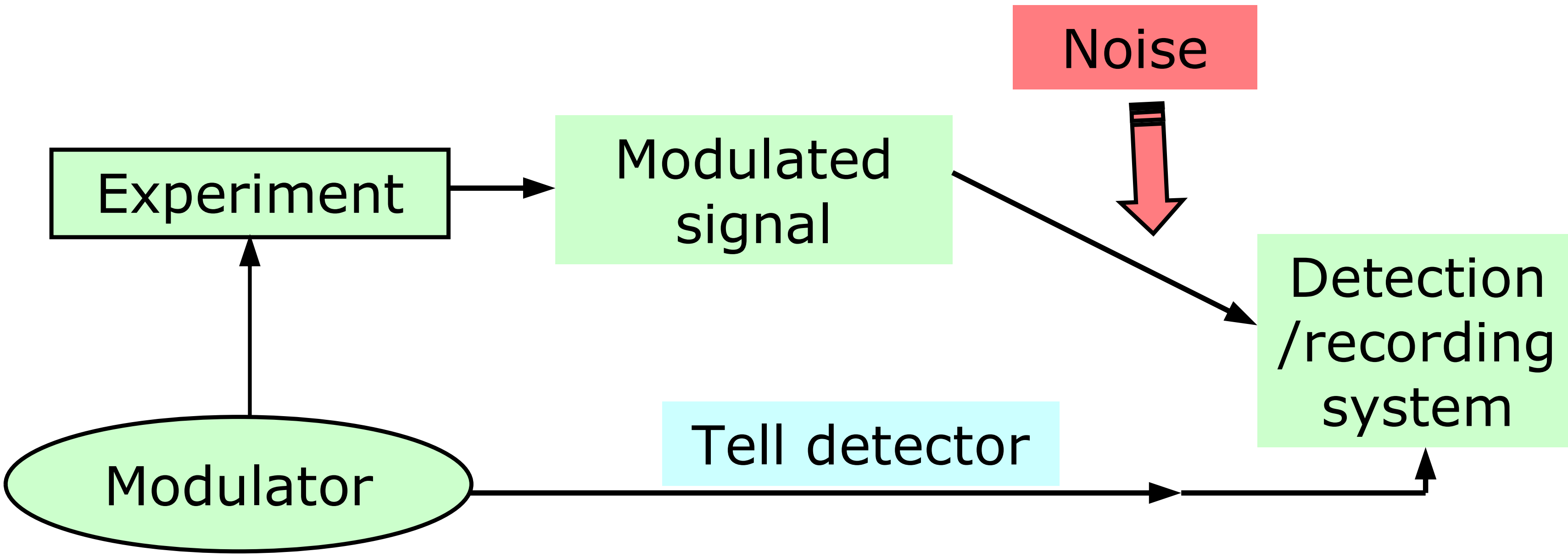
- For any off-carrier noise (i.e. $\Delta\omega\neq 0$), $<\cos(\Delta\omega t)>\rightarrow 0$ provided $\Delta\omega$ is large compared with $1/\tau \equiv 1/RC$. So, if τ is big enough, even noise close in frequency to ω_r will be removed.

A lock-in has an effective "bandwidth" related to the averaging time

- Even noise at ω_r will average out if it consists of components with different phases (i.e. it's incoherent), since $< \Sigma \cos(\theta) > = 0.$

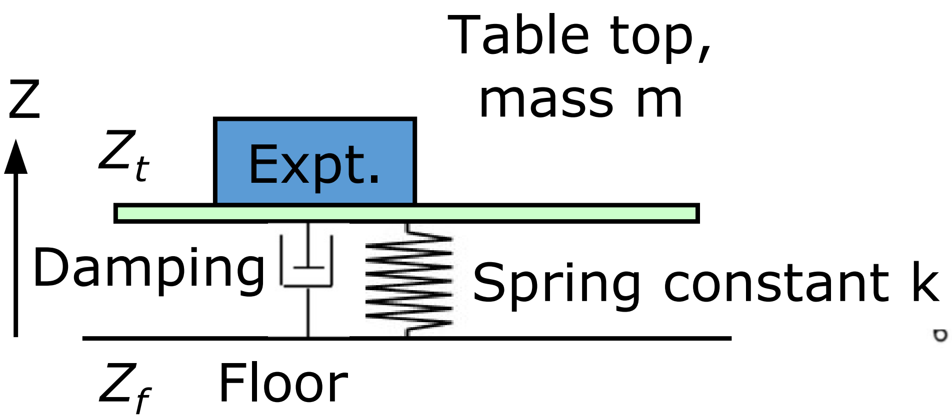
- τ can easily be, say, 1s, so a lock-in is an extremely narrow-band filter.
  - NB the reference oscillator must be phase stable for this length of time.

Summary of phase-sensitive detection



- Dramatically reduces the effect of noise added after modulation because noise isn't modulated.
- At heart, based on the idea of orthogonal function decomposition.
- Also, capitalizes on encoding the signal at a freq where noise disturbances are small.

Eliminating mechanical/vibration noise

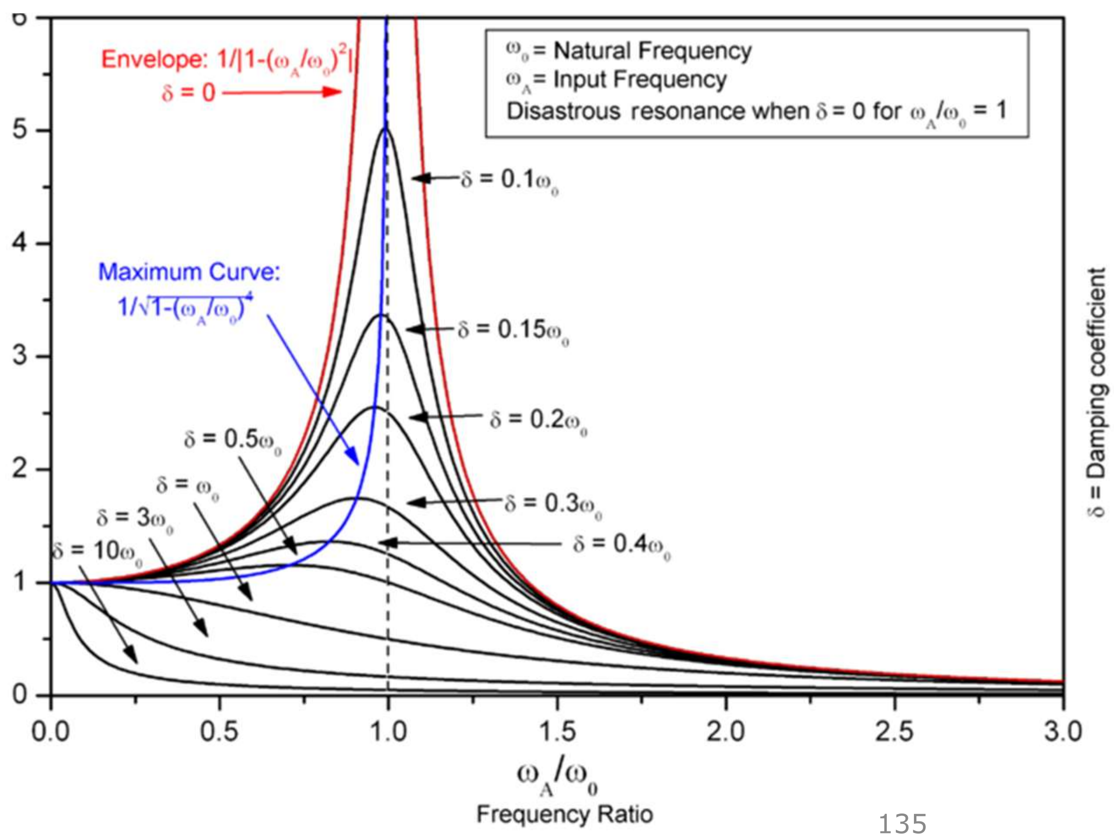


Simple model of an experiment mounted on a vibrating floor

- As the floor vibrates a distance dz_f, the table top (at height z_t) will be in forced vibration.

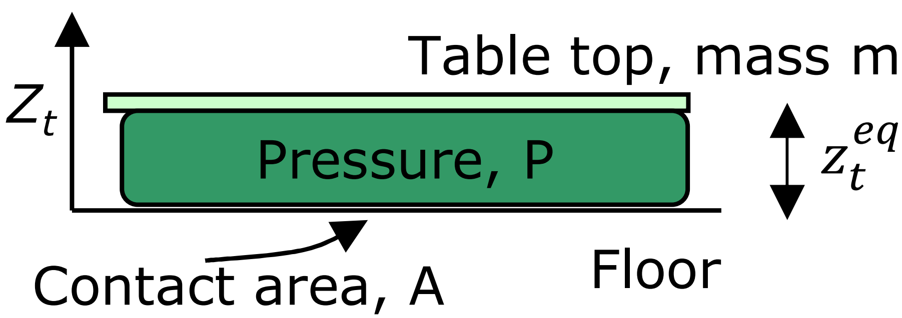
Examine the "forced oscillator" response: at low frequencies the table will follow the floor.

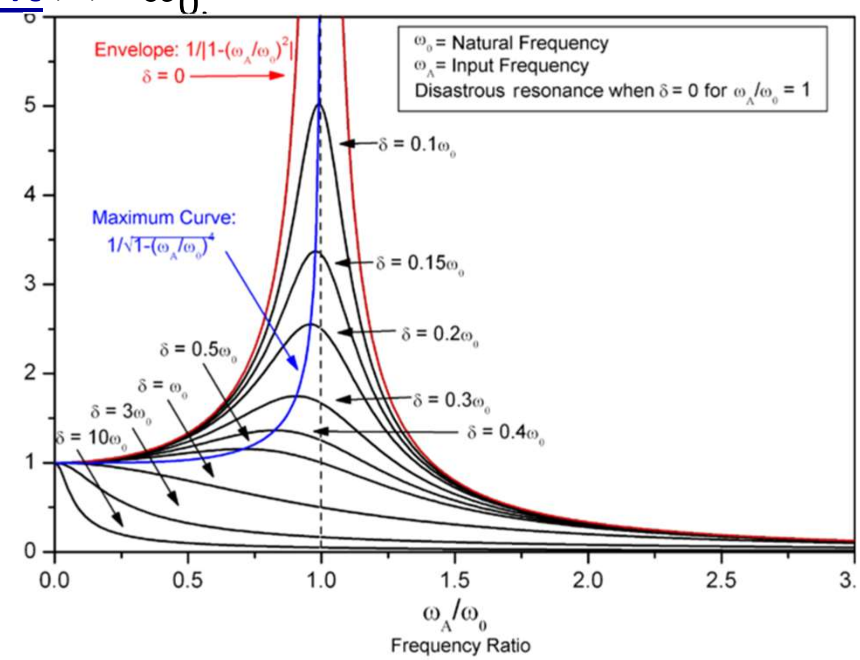
There will be a resonant frequency at $\omega_0^2 = k/m$, when the coupling between the ground and the floor will be maximized.



Adjusting the resonant responses of different parts of the system is the key

- Make ω_0 low by decreasing k: an air cushion is ideal
- Use damping to reduce the peak response of the table at ω_0.
- Make resonant frequencies of experiment >> ω_0.





- Now, F = mg = PA
- And, for compression at vibration speed, the air in the cushion experiences an adiabatic change:

$\Rightarrow$ P (Volume in air cushion)$^\gamma$ = constant. ← *Adiabatic condition*

$$\gamma_{air} = 1.4$$

An air cushion has just the right behaviour to limit the transfer of vibrations to an optical table

$$\boxed{P \propto V^{-\gamma}}$$

So, during one cycle of vibration *(F = mg = PA)*:

$$\frac{dF}{F} = \frac{AdP}{AP} = -\gamma\frac{dV}{V} \approx -\gamma\frac{dz_t}{z_t^{\text{eq}}}$$

$$\Rightarrow dF = -\gamma\frac{dz_t}{z_t^{\text{eq}}}mg.$$

This provides a restoring force = mass × acceleration $= m\ddot{z}_t$

$$\Rightarrow SHM, \text{ with } \omega_0^2 = \left(\frac{\gamma\, g}{z_t^{\text{eq}}}\right).$$

For $z_t^{\text{eq}} = 0.2\text{m}$, say,

$$\omega_0 = \left(\frac{1.4\times10}{0.2}\right)^{1/2} = 70^{1/2} \approx 8\text{ rad s}^{-1}$$

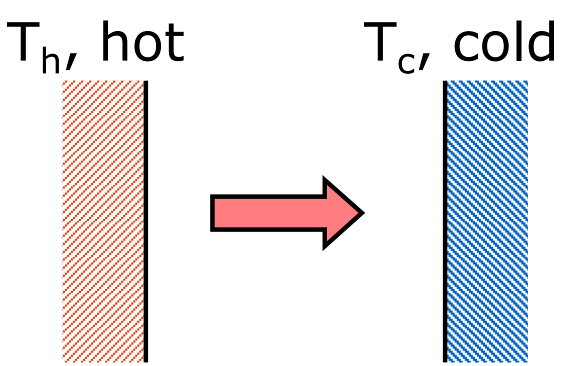
≈ 1 Hz, nicely low.

Eliminating thermal noise

- Very many applications, especially those that involve study of quantum systems. Key is to limit thermal transport.

1. First step to maintaining a temperature: reduce evaporation (lid), conduction (insulate/vacuum), and convection (vacuum).
2. Then reduce radiation.

   - Now power/unit surface area radiated by a body $= \sigma_{Stephan}\ \varepsilon\ T^4,$ where ε = emissivity (=1-reflectivity). ε =1 for a black-body.
   - So, for two bodies of emissivity ε:



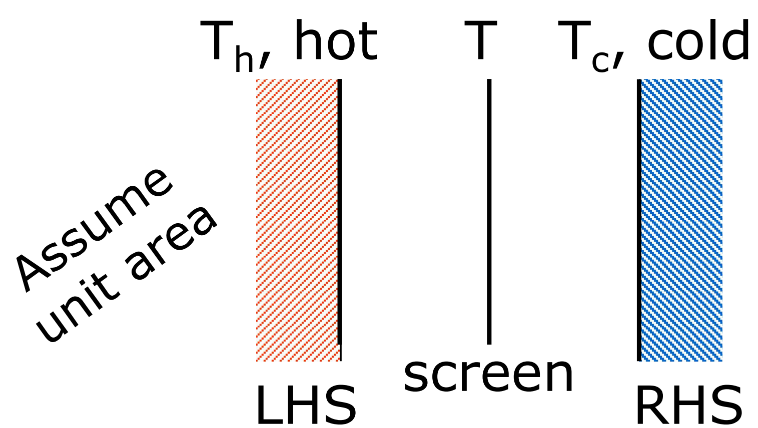
Net radiation flow $\Rightarrow$ per unit area

$= \sigma\varepsilon(T_h^4 - T_c^4).$

Often, limiting radiation transport is critical

- A good strategy is to reduce the emissivity by making the surfaces shiny.
- Matt black has ε~0.95, whereas polished anodized Al has ε~0.32 and polished Al or Au foil has ε~0.03.
  - But beware: you need performance at the peak of the BB curve.

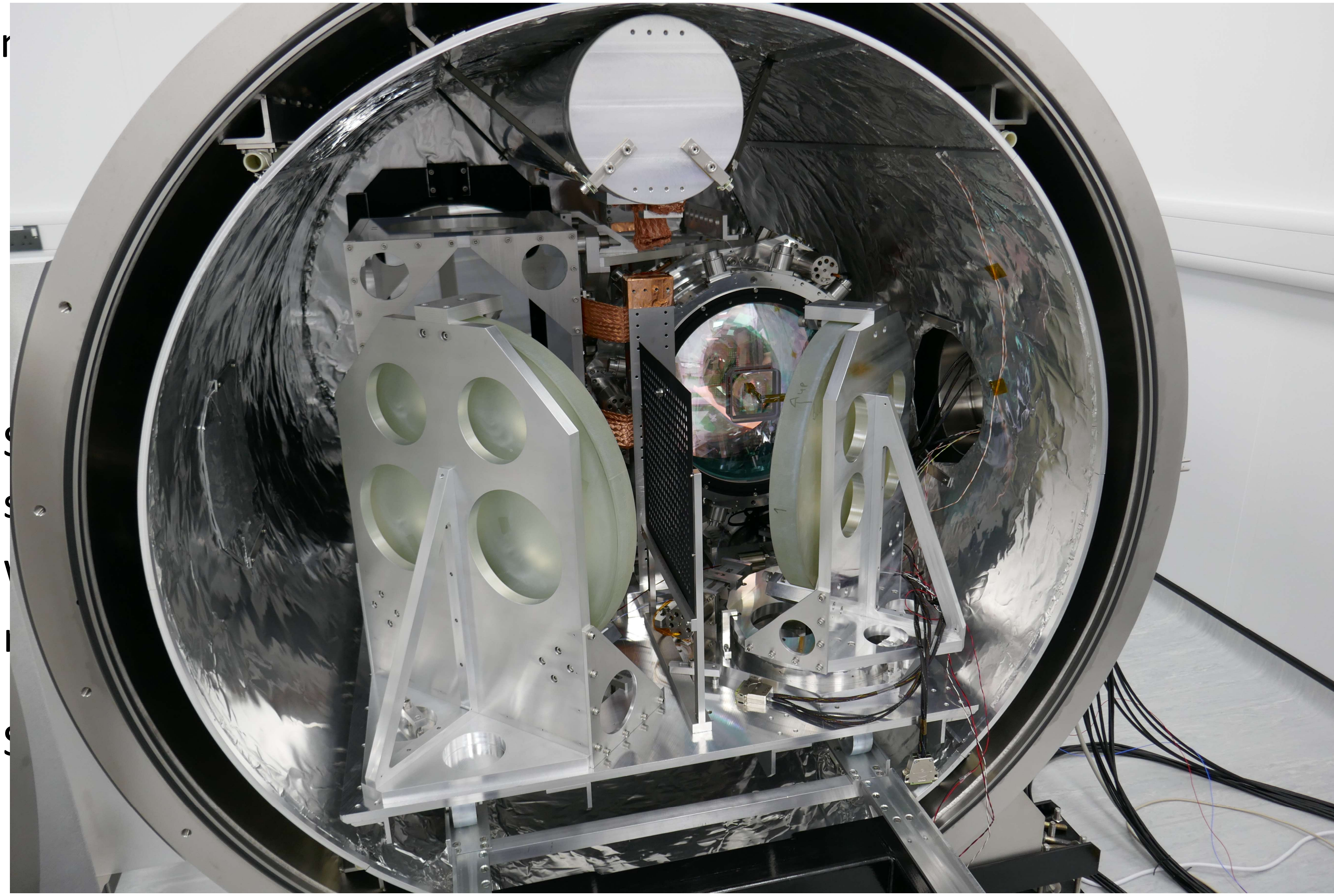
- An additional gain can then be had by inserting a “floating” shield between the two surfaces:



net heat flow $\rightarrow$ on LHS	$=$	$\sigma\varepsilon T_h^4 - \sigma\varepsilon T^4$	(1)
net heat flow $\rightarrow$ on RHS	$=$	$\sigma\varepsilon T^4 - \sigma\varepsilon T_c^4$	(2)

An example of such a radiation shield

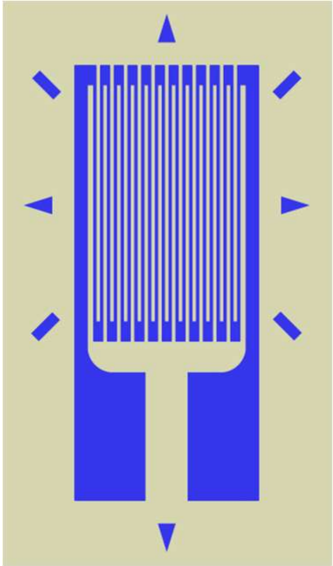
How does this shiny “barrier” help?



In electrical circuits one often uses multiple strategies

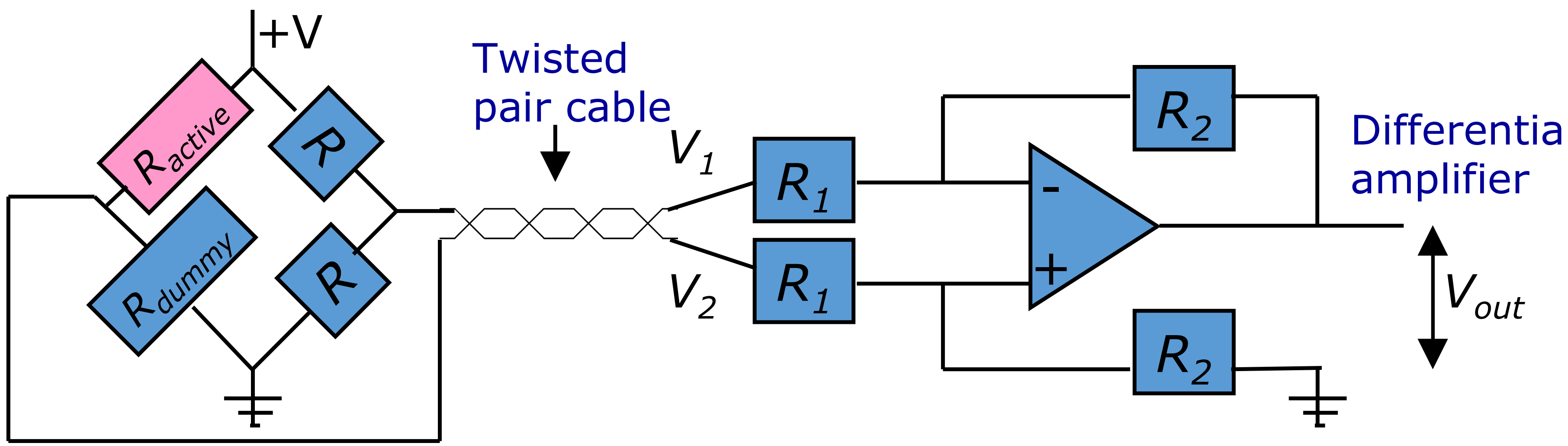
- We can use differential techniques and capitalize on noise that is picked up being correlated.
- We can be careful about picking up electrical noise in the first place.
- We can attempt to shield our system completely from electro-magnetic fields.

- The following slide shows how you might wire up a strain gauge to limit certain types of noise.



A simple foil strain gauge. The resistance of the long foil conductor (stuck on an adhesive sheet) changes if extended in the vertical direction.

This circuit uses four different “tricks”

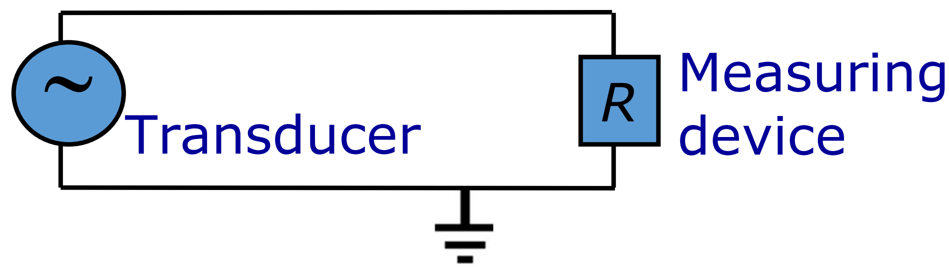


- Use of a bridge to compare the resistances.
- Use of two strain gauges, once active and the other just used to calibrate for the environment.
- Use of a twisted pair: E & B will induce ≈ the same currents in each lead because they follow almost the same path through space.
- Use of a differential amplifier: ignores all “common-mode” induced signals because:

$$V_{out} = R_2 / R_1 (V_2 - V_1).$$

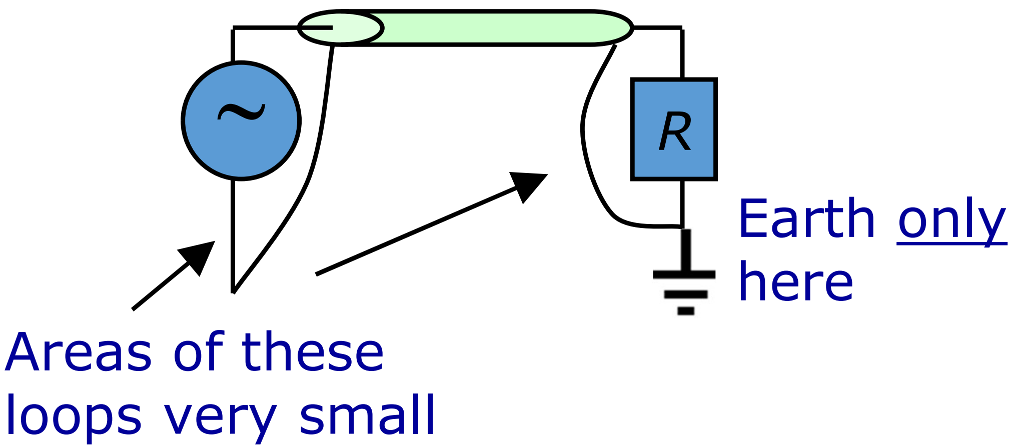
Eliminating electrical pickup

- Consider the effect of a changing magnetic field on
  a typical transducer or instrument set-up:

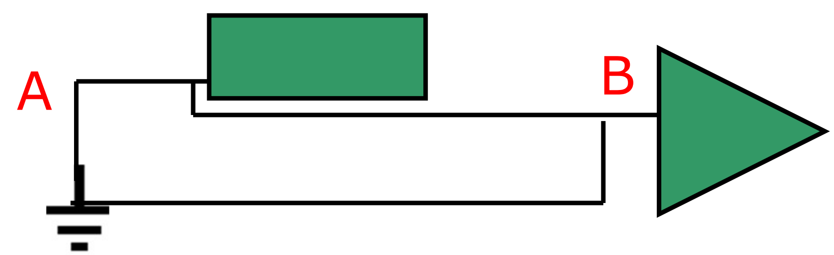


- This gives an unwanted EMF = -d/dt (B.loop area) induced, and hence induced noise across R.

  “Easily” mitigated:

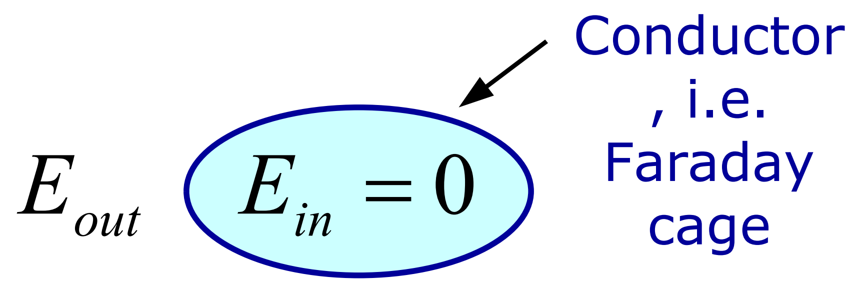


- Don’t create “earth loops”:
  - Induced EMF in the unnecessary loop ⇒ neither A nor B is at ground – so varying V_B ⇒ noise.

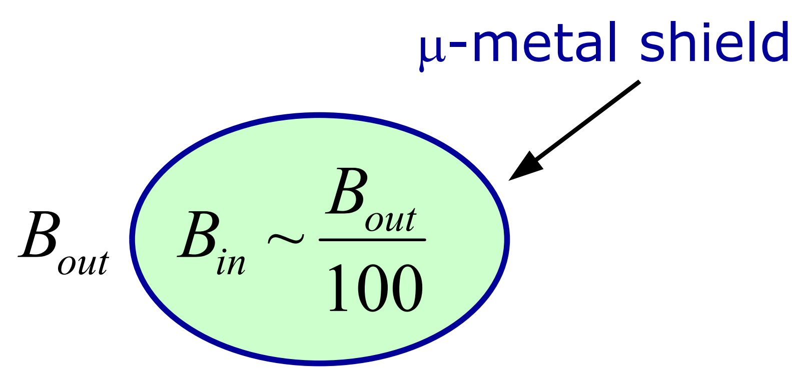


Sometimes only complete shielding solves the problem

- For E fields, use a Faraday cage.
- Re-arrangement of charges within conductor lead to no enclosed overall field.



- For B fields, the situation is less straightforward.
- Use shield made of high permeability metal, e.g. "μ-metal", a Ni/Fe alloy with $\mu_r > 10^4$.
- Provides a low reluctance path for the B field lines.



- As a result, it's fashionable now to transport signals via optical fibres.

Summary so far

- Coping with unwanted influences:
  - Filtering – relies upon knowledge of spectral content:
    - Phase-sensitive detection (and the lock-in amplifier)
  - Vibration filtering – resonant response is key.
  - Thermal shielding – surprisingly easy.
  - Differential measurement – rejection of common mode interference.
  - Electric and magnetic shielding – and avoiding earth loops.

Next lecture(s) we will consider data analysis, notably useful probability distributions for physicists and the concept of inference.