Large Scale Structure and Galaxy Formation

Lecture 4

Koen Kuijken

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MSc Astronomy, Leiden Observatory

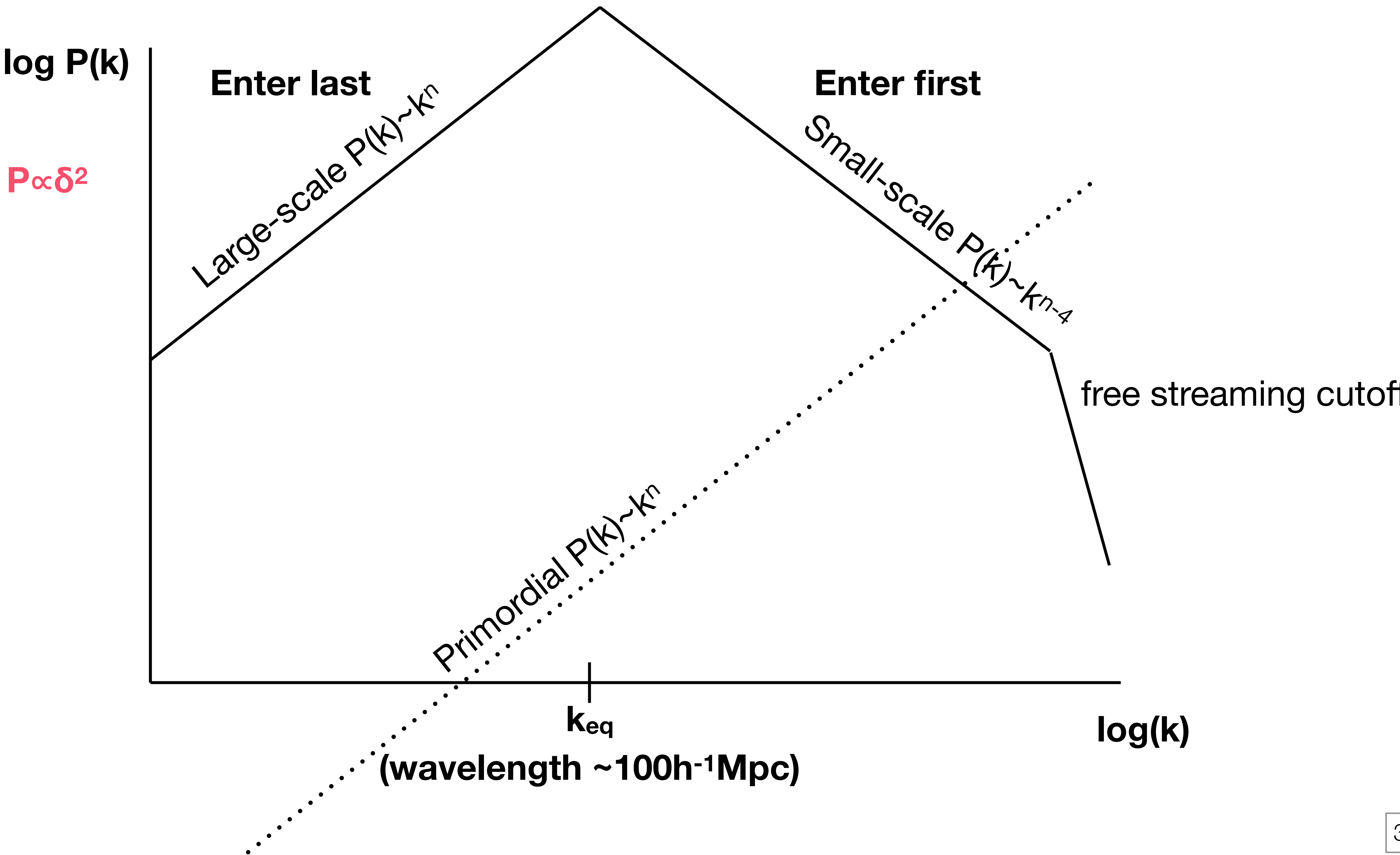
Last lecture:

- Power spectrum $\leftrightarrow$ correlation function describe fluctuation statistics
- Filtered density field $\leftrightarrow$ mass fluctuations, σ_8 parameter
- Harrison-Zel'dovich spectrum $P(k) \propto k^1$ modified by the scale-dependent growth history $\delta_k(t)$ of dark matter fluctuations
  - special role for the scale $k_{\rm eq}$ corresponding to horizon size at $t_{\rm eq}$

- Non-linear evolution from spherical collapse models:
  - density perturbations turn around and collapse+virialize
  - density at collapse = $18\pi^2\rho_{\rm bg} \simeq 178\rho_{\rm bg}$ for Einstein-deSitter b/g
  - corresponding linear-theory overdensity $\delta \equiv \delta\rho/\rho_{\rm bg} - 1 \simeq 1.69$ at collapse, $\delta \simeq 1.06$ at turnaround.

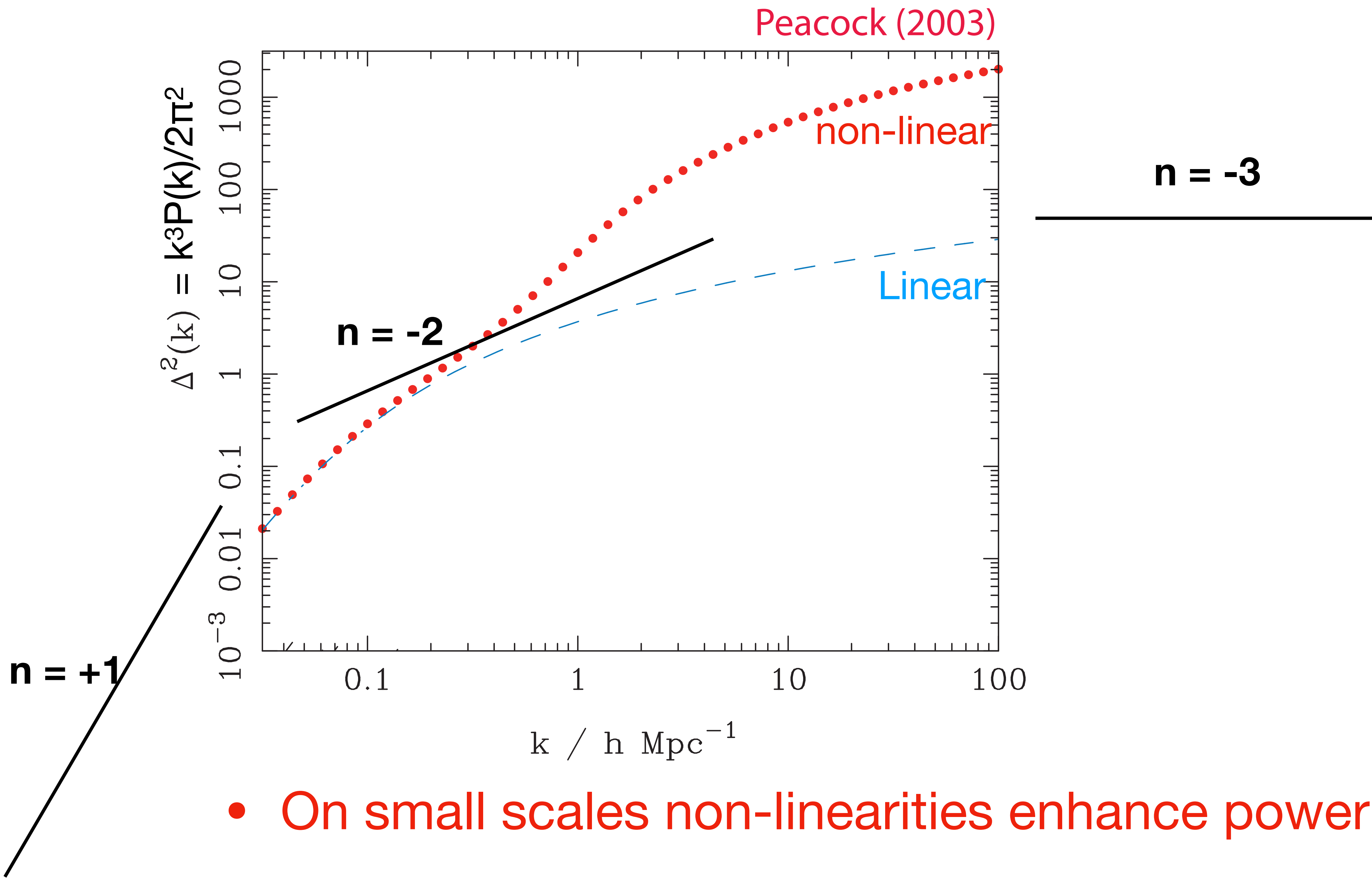
Linear evolution of fluctuations

- The shape of the initial power spectrum is modified because the smaller scales grow more slowly between t_{ent} and t_{eq}:



More accurate calculation

- Linear theory: properly solve perturbation equations
- Non-linear theory: massive N-body simulations



- On small scales non-linearities enhance power

Scaling laws for collapsed objects

- In linear theory we had (after $t = t_{\rm eq}$) $\qquad \delta \propto a \propto t^{2/3}$
- Hence the amplitude of power spectrum and correlation functions grow with time as $\quad P(k), \xi(r) \propto \delta^2 \propto t^{4/3}$
    - and linear r.m.s. fluctuations of the density field filtered on mass scale M grow as as $\quad \sigma_M \propto \delta \propto t^{2/3}$
- ? At what time $t_{\rm NL}$ does a mass scale M "go non-linear"?
    - At any time the fluctuations of mass M for which $\sigma_M \simeq 1$ are the ones that collapse (go non-linear). We saw that the (linear-theory) mass dependence of σ_M is defined by the power spectrum filtered on scale $k_M \propto M^{-1/3}$ as

$$\langle |\delta_{k_M}|^2 \rangle = \sigma_M^2 \equiv \left\langle \left(\frac{\delta M}{\overline{M}} \right)^2 \right\rangle \propto k_M^3 P(k_M) \propto M^{-(n+3)/3} \quad \text{if} \quad P(k_M) \propto k_M^n$$

    - Hence (putting the time dependence back in) $\quad \sigma_M \propto M^{-(n+3)/6} t^{2/3}$
    - Set $\sigma_M \simeq 1$ at $t = t_{\rm NL}$: $\qquad M \propto t_{\rm NL}^{4/(n+3)} \propto a_{\rm NL}^{6/(n+3)} \propto (1 + z_{\rm NL})^{-6/(n+3)}$

Scaling laws for collapsed objects

$$M \propto t_{\rm NL}^{4/(n+3)} \propto a_{\rm NL}^{6/(n+3)} \propto (1+z_{\rm NL})^{-6/(n+3)}$$

$(n=-2):$ $(1+z_{\rm NL})^{-6}$

- NB: n here refers to the power spectrum after t_{eq}, (i.e. n a little above -3 for CDM, not primordial $n \simeq 1$, see slide 3).
- $\Rightarrow$ smaller M halos collapse earlier

- Size and density of collapsed 'virialised' objects?

$$\rho_{\rm vir} = 8\rho_{\rm max} \simeq 8 \times 5.5\rho_{\rm bg}(t_{\rm NL}) \propto t_{\rm NL}^{-2} \quad \propto M^{-(n+3)/2}$$ $M^{-1/2}$

$$R_{\rm vir} \propto (M/\rho_{\rm vir})^{1/3} \quad \propto M^{(n+5)/6}$$ $M^{1/2}$

- Temperature, velocity dispersion, gravitational potential?

$$V_{\rm vir}^2 \propto T \propto \frac{GM}{R_{\rm vir}} \quad \propto M^{(1-n)/6}$$ $M^{1/2}$

- On galaxy scales, $n \simeq -2$ and $V_{\rm Vir} \propto M^{1/4}$

Faber-Jackson (ellipticals)
Tully-Fisher (spirals)
(if luminosity ∝ mass)

The mass function of halos, $f(M)$

- We have seen how
  - initial small density fluctuations grow through gravitational instabilities, which modifies the initial power spectrum
  - fluctuations reach non-linear amplitudes and collapse

- Different initial power spectra and cosmological models therefore predict different distributions of collapsed objects (“halos”) at given mass

- Let $N(>M)$ be the number of halos per unit volume above mass M

- ***Mass function*** $f(M) = -\dfrac{dN(>M)}{dM}$ = distribution over mass:
  - the number density of halos with mass in $[M, M+dM]$ is $f(M)dM$
  - mass density is $\int Mf(M)dM$

Press-Schechter theory

- ***Use linear-theory density field to predict halo mass function, with recipes from non-linear collapse calculations***

- Define a density threshold δ_c and pick a filter scale M
- Over what fraction of space is the filtered density field $\delta_M(\mathbf{x}) > \delta_c$?

- The distribution of δ_M values in the universe is Gaussian, with dispersion σ_M (we saw how σ_M^2 is evaluated as integral over $P(k)$)

$$F(M, \delta_c) \equiv F(\delta_M > \delta_c) = \int_{\delta_c}^{\infty} dx \frac{e^{-x^2/2\sigma_M^2}}{\sqrt{2\pi}\sigma_M} = \frac{1}{2}\mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_M}\right)$$

1.69

- Press-Schechter: "a region with linear density contrast $\delta_M > \delta_c$ collapses into a halo with mass greater than M if δ_c is the linear-theory density threshold for collapse"

(error function)

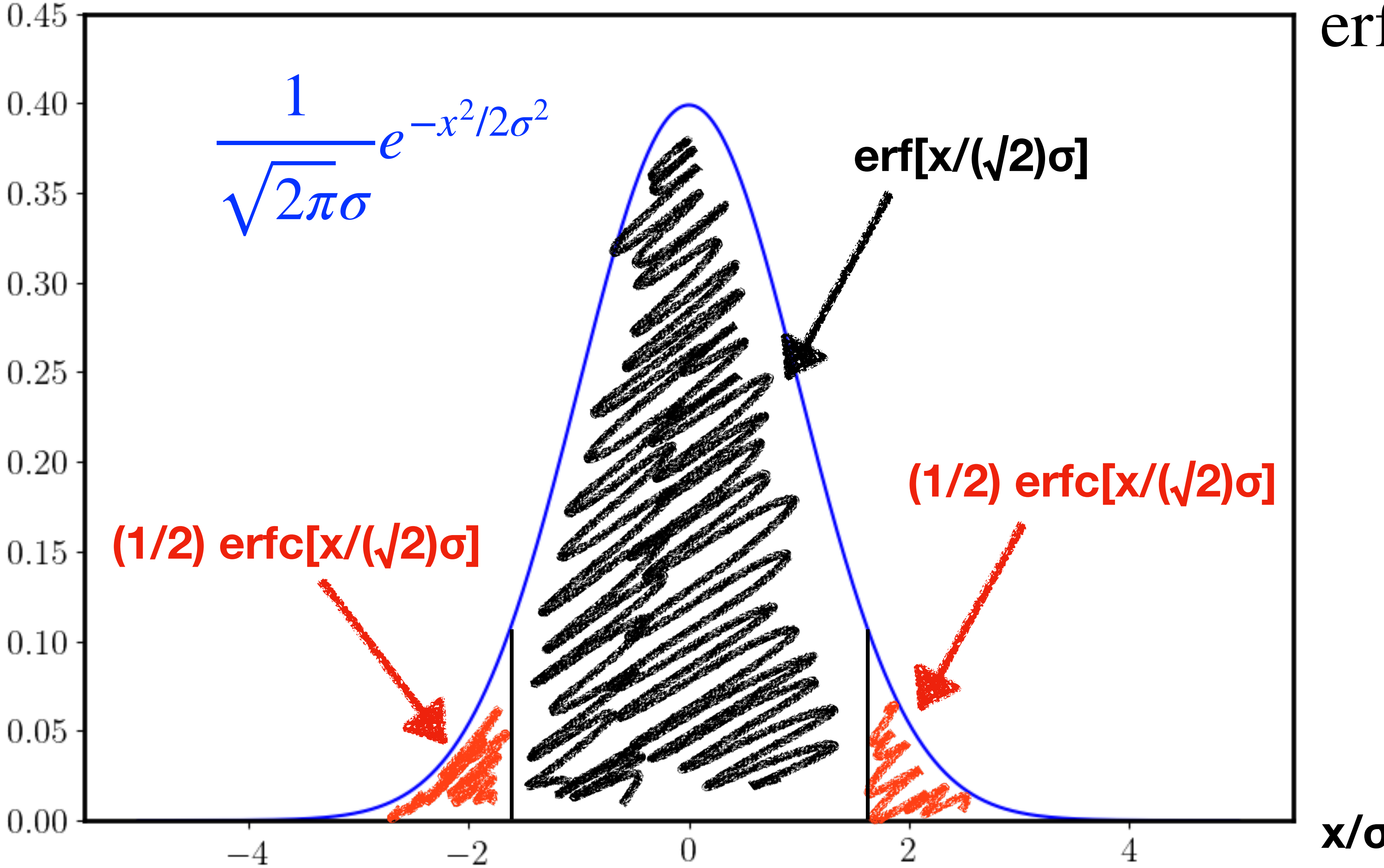
- Integral of Gaussian

$$\mathrm{erf}(x) = \frac{1}{\sqrt{\pi}}\int_{-x}^{x} e^{-t^2}dt = \frac{2}{\sqrt{\pi}}\int_{0}^{x} e^{-t^2}dt \qquad \mathrm{erf}(+\infty) = 1$$

- Complementary error function $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$

$$\mathrm{erfc}(+\infty) = 0$$

$$\mathrm{erfc}(0) = 1$$



Press-Schechter theory

- Mass per unit volume that is in regions with $\delta_M > \delta_c$ is $\rho_{\rm bg} F(M, \delta_c)$
  Hence the mass function satisfies

$$\int_M^\infty M' f(M') dM' = \rho_{\rm bg} F(M, \delta_c) \quad = \frac{\rho_{\rm bg}}{2} \mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_M}\right)$$

- Differentiate with respect to M:

$$\frac{d}{dx}\mathrm{erfc}(x) = -\frac{2}{\sqrt{\pi}} e^{-x^2}$$

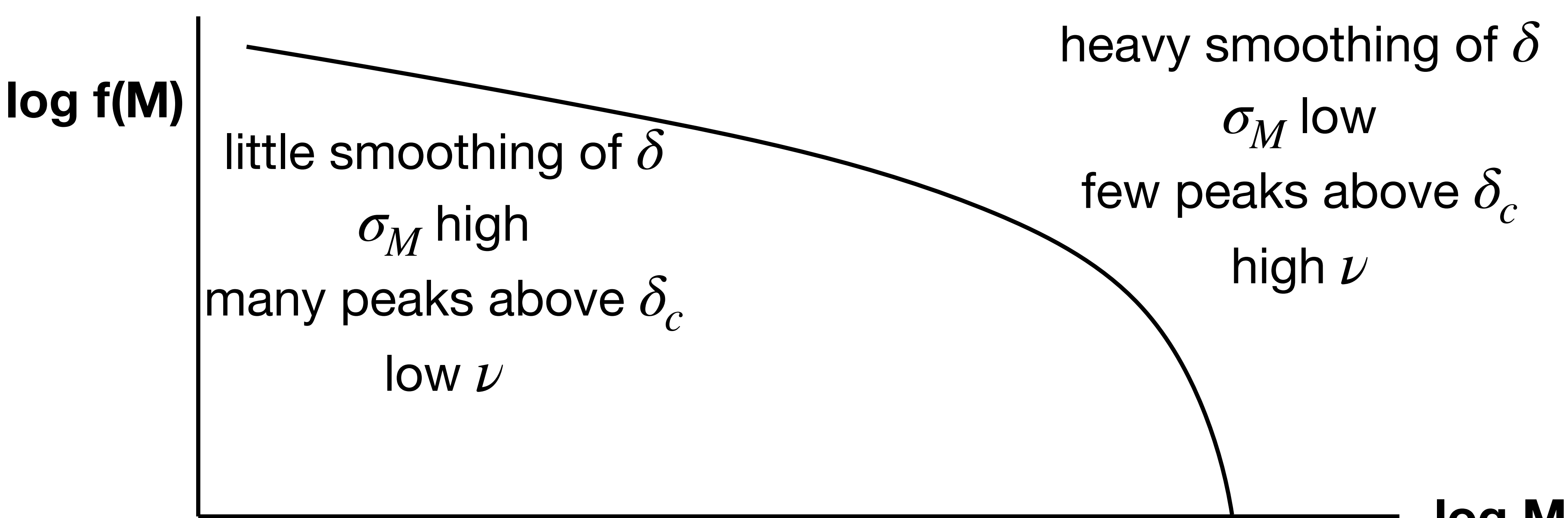
$$M f(M) = -\rho_{\rm bg} \frac{\partial F}{\partial M}$$

$$f(M) = -\frac{\rho_{\rm bg}}{M} \frac{d\sigma_M}{dM} \frac{\delta_c}{\sqrt{2\pi\sigma_M^2}} e^{-\delta_c^2/2\sigma_M^2}$$

- ???: take $M \to 0$ in the integral, then $\sigma_M \to \infty$ and you only get $\rho_{\rm bg}/2$!?
- We are only integrating over the regions with positive overdensity, which is only half of space
    - Underdense for one filter scale may be overdense for another
- *Ad-hoc* solution: insert a factor of 2

Press-Schechter theory

- so $$f(M) = \sqrt{\frac{2}{\pi}}\frac{\rho_{\rm bg}}{M^2}\left|\frac{d\ln\sigma_M}{d\ln M}\right|\frac{\delta_c}{\sigma_M}e^{-\delta_c^2/2\sigma_M^2} = \sqrt{\frac{2}{\pi}}\frac{\rho_{\rm bg}}{M^2}\left|\frac{d\ln\sigma_M}{d\ln M}\right|\nu e^{-\nu^2/2}$$
- where I have inserted the factor 2 and defined the peak height ν that tells you how many standard deviations a collapsing fluctuation represents:
$$\nu = \frac{\delta_c}{\sigma_M}$$
- Note that this mass function is a power law at low mass, but cuts off sharply at higher masses (remember σ_M is a decreasing fn. of M)



P-S evolution of the mass function

- In P-S theory we take the density contrast threshold $\delta_c \simeq 1.69$
- At any given redshift (or time), all regions with $\delta_M > \delta_c$ collapse into haloes.
- Linear perturbation theory predicts how δ evolves in time: write this as

$$\delta(t) = D(t)\delta(t_0) \equiv D(t)\delta_0 \qquad \text{(with } D < 1 \text{ at } t < t_0\text{)}$$

- So if we know the mass function at the present time, at earlier time it follows from replacing $\sigma_M \rightarrow D(t)\sigma_M$, or equivalently $\delta_c \rightarrow \delta_c / D(t)$:

$$f(M,t) = \sqrt{\frac{2}{\pi}} \frac{\rho_{\rm bg}}{M^2} \left| \frac{d \ln \sigma_M}{d \ln M} \right| \frac{\delta_c}{D(t)\sigma_M} \exp\left[-\frac{\delta_c^2}{2D(t)^2\sigma_M^2} \right]$$

- Note that here σ_M is the linear theory prediction for the present time, $z = 0$. All evolution has been expressed with $D(t)$.
  - We simply scale the peak height ν of the present-day density field with $1/D$: a ν-sigma fluctuation that collapses today would have to be a (ν/D)-sigma fluctuation to have collapsed in the past

P-S with power-law $\sigma_M(M)$

- Define characteristic mass $M_\star(z)$ of fluctuations at redshift z such that

$$\sigma(M_\star)D(z) = \delta_c$$

- The P-S mass function then becomes

$$f(M,z) = \sqrt{\frac{2}{\pi}}\frac{\rho_{\rm bg}}{M^2}\left|\frac{d\ln\sigma_M}{d\ln M}\right|\frac{\sigma_{M_\star}}{\sigma_M}e^{-\sigma^2_{M_\star}/2\sigma^2_M}$$

- Now use earlier relation for $P(k) \propto k^n$: $\sigma_M \propto M^{-(n+3)/6}$

$$f(M,z) = \sqrt{\frac{2}{\pi}}\frac{\rho_{\rm bg}}{M_\star^2}\frac{n+3}{6}\left(\frac{M}{M_\star}\right)^{(n-9)/6}\exp\left[-\frac{1}{2}\left(\frac{M}{M_\star}\right)^{(n+3)/3}\right]$$

- Remember that $M_\star$ evolves with time (grows)

P-S with power-law $\sigma_M(M)$

$$f(M,z) = \sqrt{\frac{2}{\pi}} \frac{\rho_{\rm bg}}{M_\star^2} \frac{n+3}{6} \left(\frac{M}{M_\star}\right)^{(n-9)/6} \exp\left[-\frac{1}{2}\left(\frac{M}{M_\star}\right)^{(n+3)/3}\right]$$

- For CDM we have $n \simeq -2$ on galaxy scales, so $f(M) \propto M^{-11/6}$ for small M

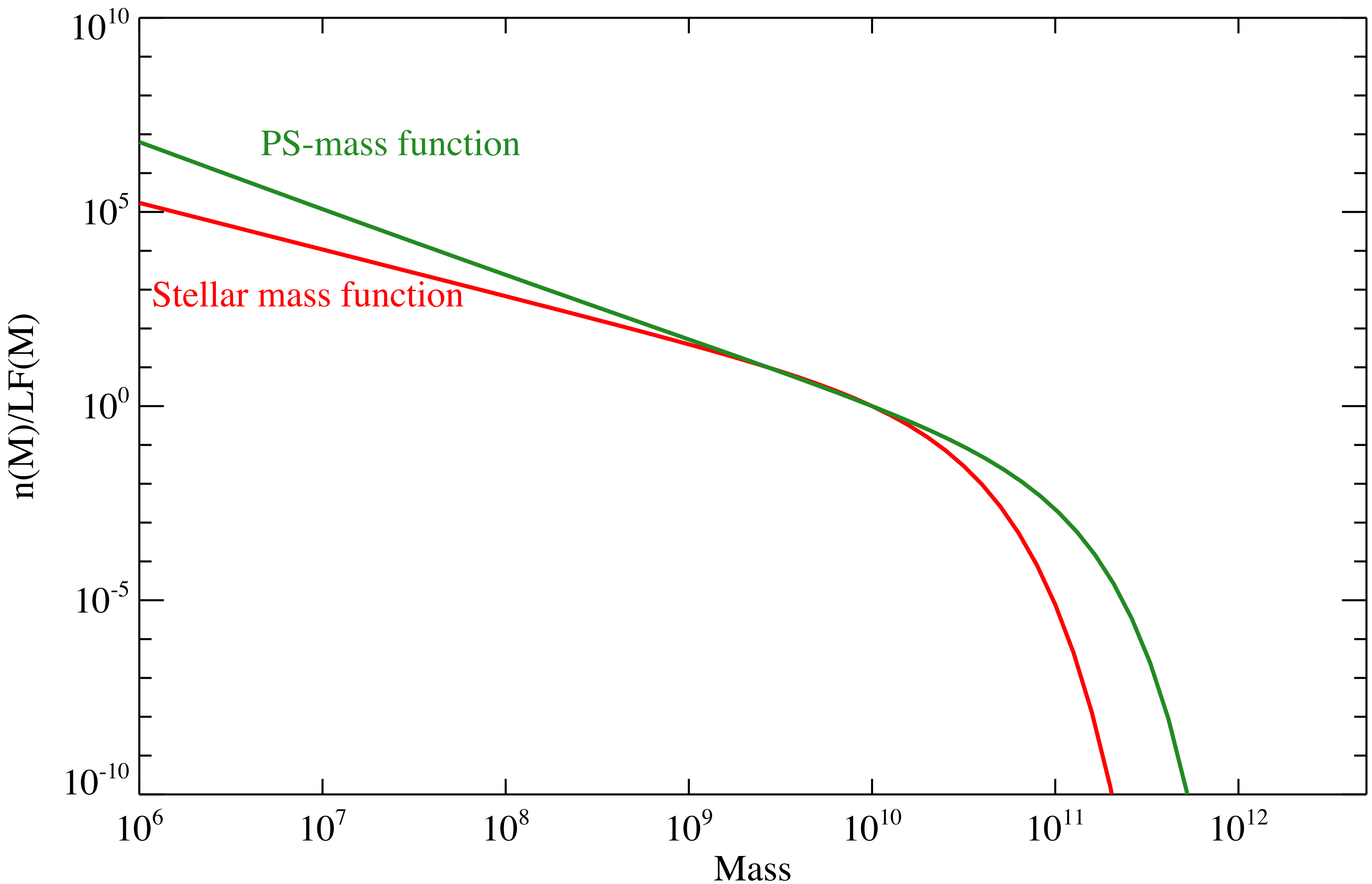
- Can we relate the halo mass function to observed galaxies?
    - The galaxy luminosity function follows the Schechter function,

$$f(L) \propto \left(\frac{L}{L_\star}\right)^{\alpha} e^{-L/L_\star}$$

    - with faint-end power law $\alpha \simeq -1.3$

Comparison to galaxy stellar mass function

- Measurements of the stellar mass function of galaxies vs. P-S halo mass function



plot: J. Brinchmann

- *<u>Many more low-mass halos than faint galaxies.</u>* Why?
- *<u>Many more high-mass halos than bright galaxies.</u>* Why?

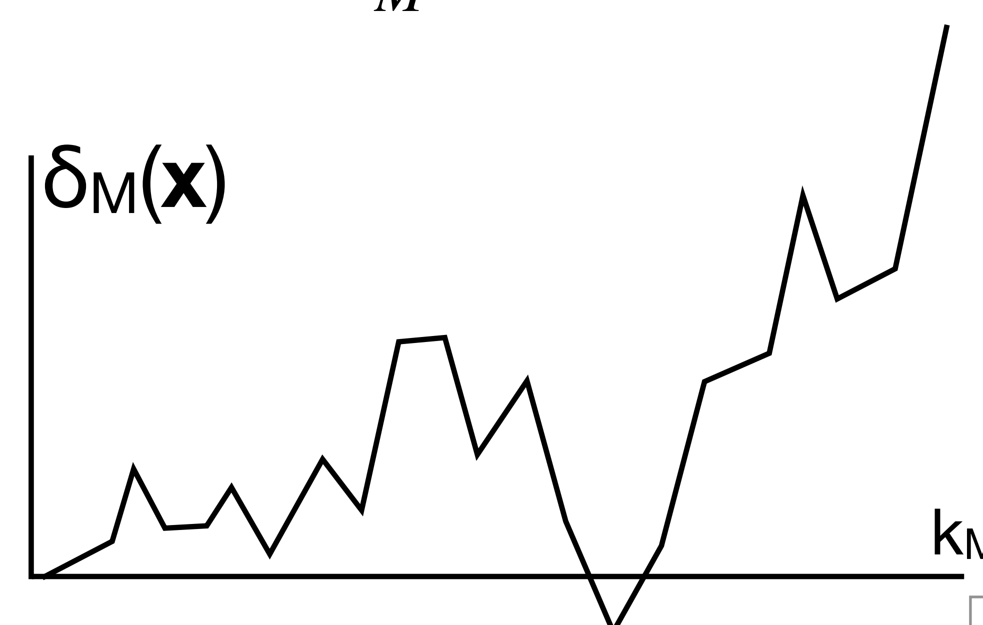
Extended Press-Schechter theory

Excursion sets

- 'Extended Press-Schechter' theory fixes the factor 2 fudge by consistently considering the linear density field smoothed on different scales

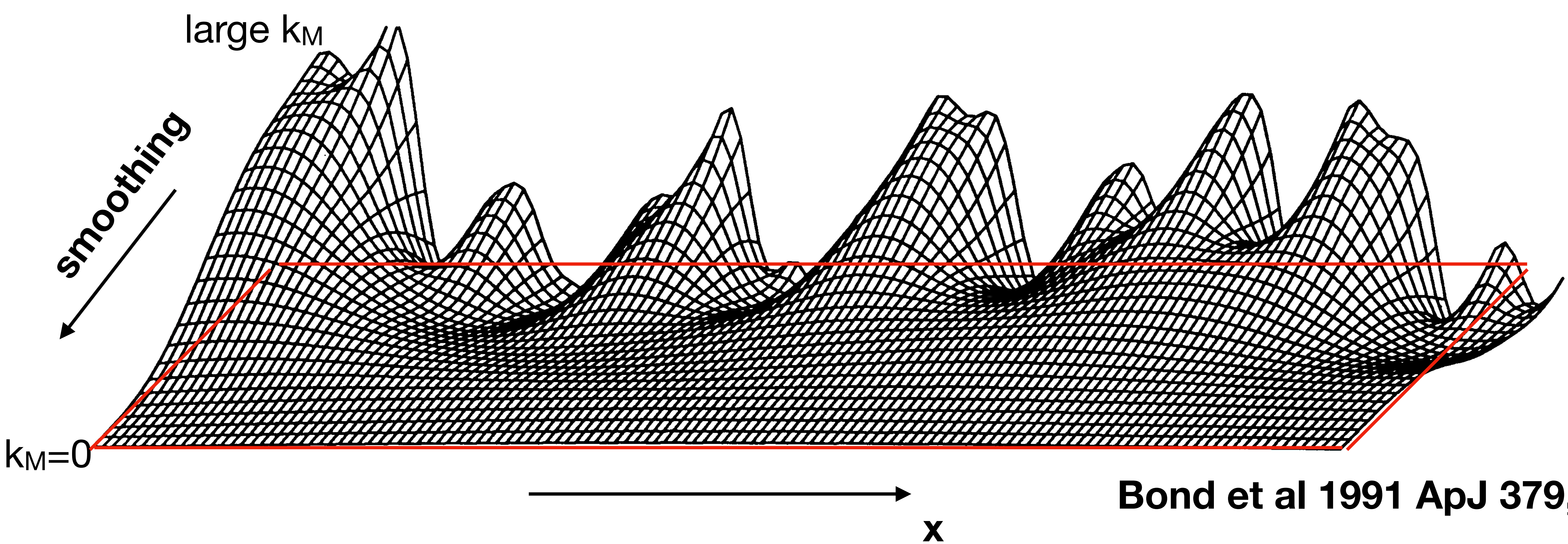
- Imagine a linearly evolved density field $\delta(\mathbf{x})$ *at the present time*
- Smooth it with top-hat filters in k space for all possible mass scales
  - i.e. set $\hat{\delta}(k) = 0$ for all $k > k_M \sim (M/\rho_{\rm bg})^{-1/3}$ and Fourier transform
- Pick a position $\mathbf{x}$
- What happens to the value of $\delta_M(\mathbf{x})$ as function of k_M?

- Variance of $\delta_M(\mathbf{x})$ increases with k_M.
  - (each interval of k adds a random Gaussian number to $\delta_M(\mathbf{x})$)
  - —> random walk starting at 0



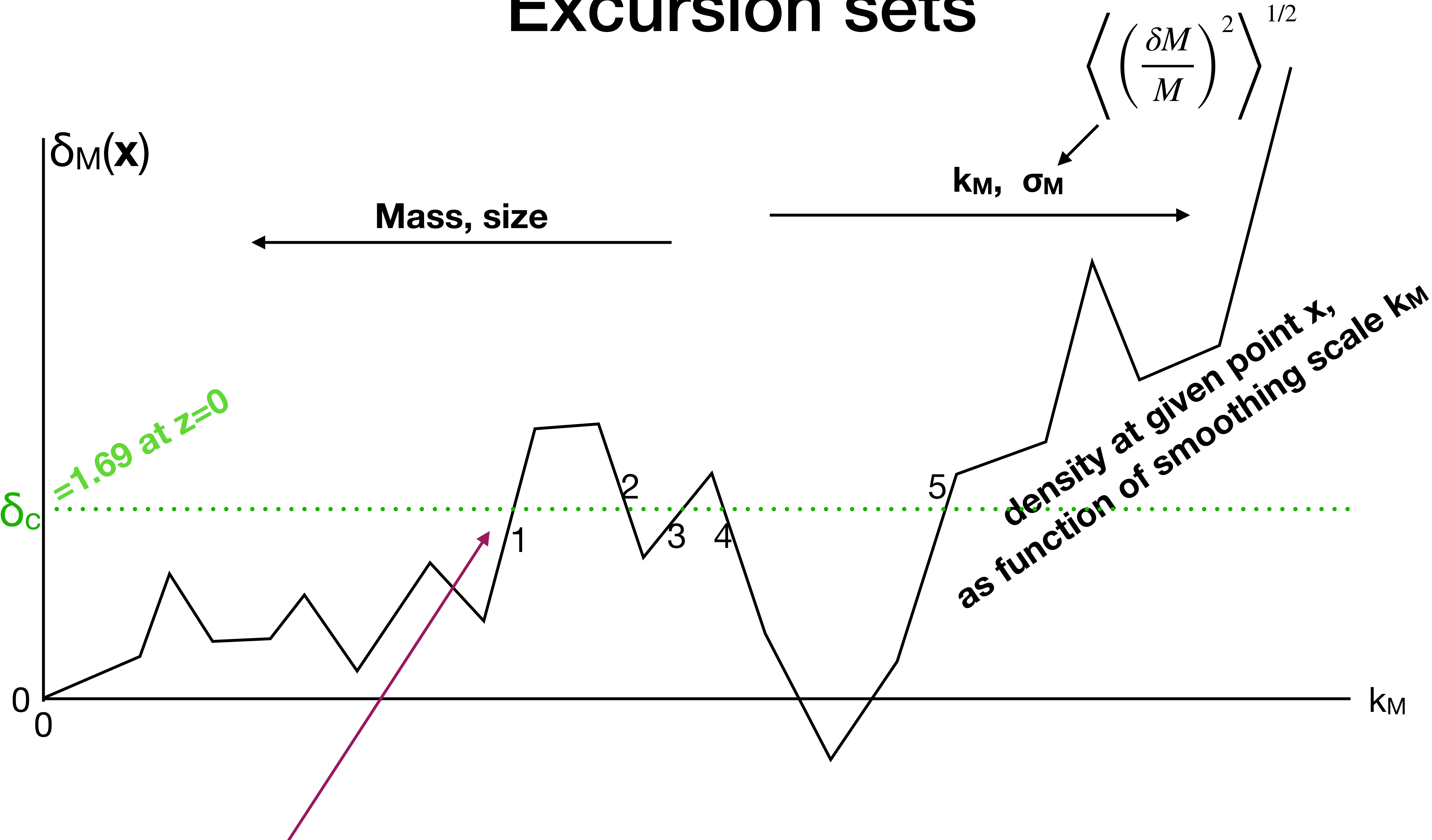
Excursion sets

- Illustration: 1-D density field, smoothed on larger and larger scales
  - wider and wider filter
  - lower and lower k_M.
  - higher and higher length, mass scale
- As you smooth more, peaks merge



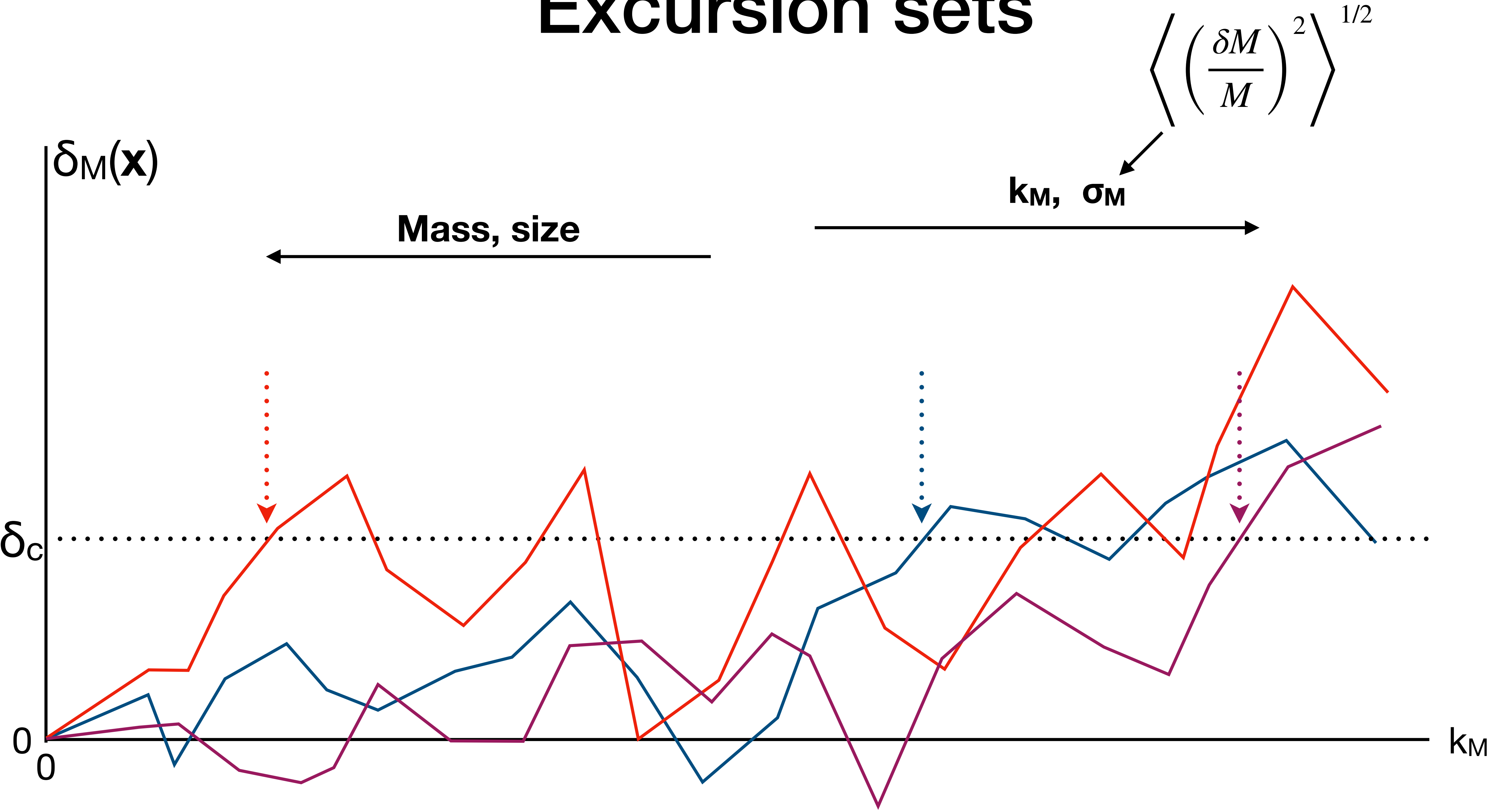
Bond et al 1991 ApJ 379, 441

Excursion sets

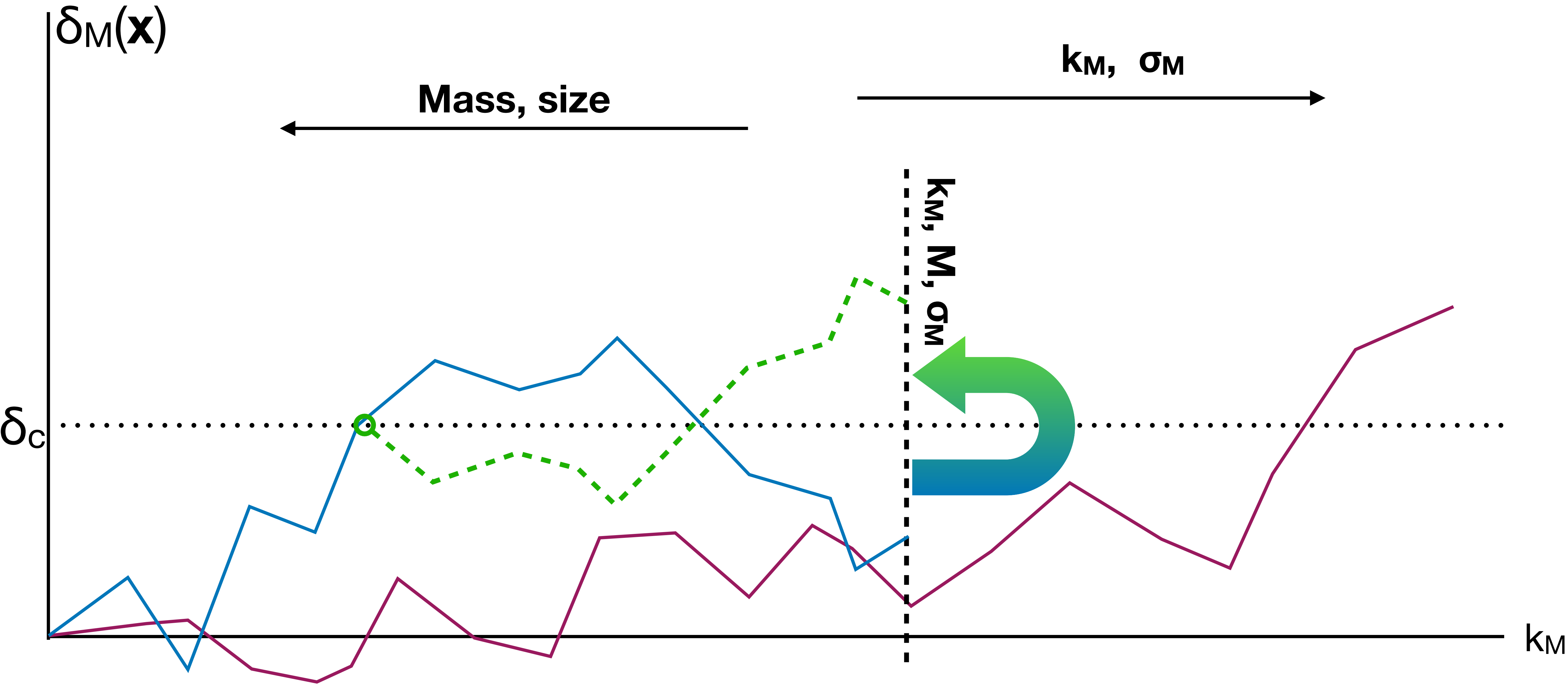


- Region on scale π/k_1, of M_1 has collapsed if $\delta_M(\mathbf{x})>\delta_c$.
- First up-crossing = largest collapsed mass which point **x** can be part of
- P-S doesn't count this point as being in regions of mass $M_{2\text{-}3,4\text{-}5}$, $<M_1$

Excursion sets



- First up-crossing = largest collapsed mass which point **x** can be part of
- extended P-S calculates the probability distribution of this $M_{FirstUp}$
- at higher redshift, threshold δ_c increases as $\propto 1/D(z)$

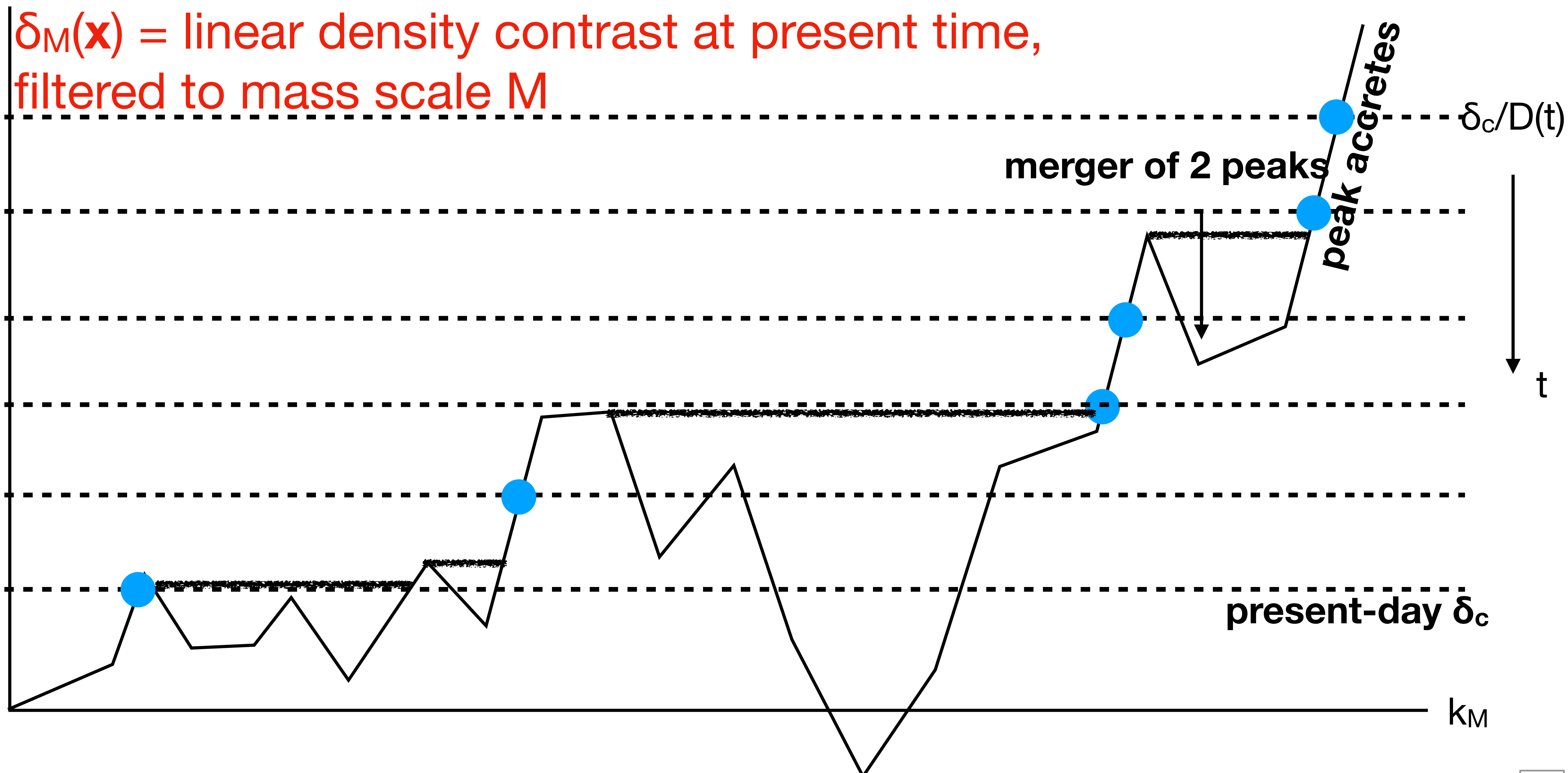


- Prob(M_{1st}<M)=Prob($\delta_M<\delta_c$) - Prob($\delta_m>\delta_c$ for some $k_m<k_M$ & $\delta_M<\delta_c$)
- Each blue path can be mirrored about a crossing point and will then end up with $\delta_M>\delta_c$. Blue and green paths are equally likely as these are random walks.
- Hence Prob($\delta_m>\delta_c$ for some $k_m<k_M$ & $\delta_M<\delta_c$) = Prob($\delta_M>\delta_c$).

$$F(<M) = \int_{-\infty}^{\delta_c} \frac{e^{-x^2/2\sigma_M^2}}{\sqrt{2\pi}\sigma_M} dx - \underline{\int_{\delta_c}^{\infty} \frac{e^{-x^2/2\sigma_M^2}}{\sqrt{2\pi}\sigma_M}} dx = \mathrm{erf}\left[\delta_c/\sqrt{2}\sigma_M\right] \qquad F(>M) = \mathrm{erfc}\left[\delta_c/\sqrt{2}\sigma_M\right]$$

Extended Press-Schechter

- e-PS is useful because it fixes the factor of 2
- Also provides a nice framework for studying evolution of the halo population, and merging of halos



Extended Press-Schechter

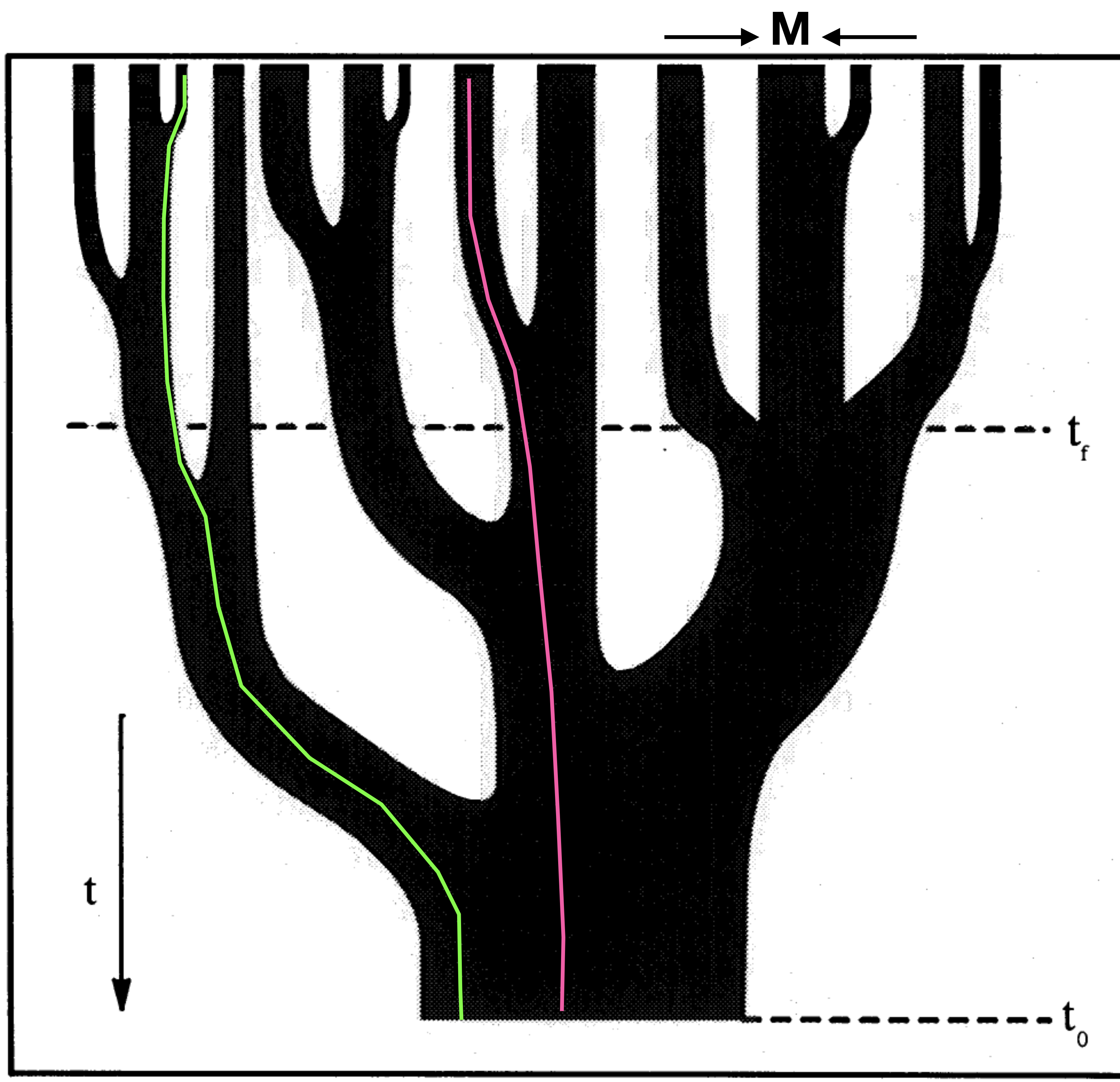
- From the statistics of the random walks, and the dependence of σ_M on the smoothing scale k_M (i.e., the power spectrum), we can answer questions like

    - What is the probability that a halo that has mass M at redshift z ends up in a halo of mass $> 2M$ today?
    - What fraction of halos of mass $10^{10} M_\odot$ live in regions that are underdense on scales of $10^{14} M_\odot$?
    - What is the mass function of halos as a function of z?

- Linear perturbation theory is pretty useful even in the highly non-linear regime!

Halo merger trees

- Follow particles through collapse and merger
- Basis of ‘semi-analytic’ galaxy formation models:
    1. build halos, which are the skeleton
    2. specify recipes for star formation as function of halo mass, formation time, merger history, etc.



Lacey & Cole 1993