In the instantaneous rest$_\xi$ frame, $\underline{d}$ can be aligned with z axis such that $\underline{d} = R\hat{\underline{k}}$

$$\Rightarrow \quad \underline{\xi}\cdot\underline{d} = \xi_z R$$

$$\ddot{\underline{\xi}} = -\frac{GM}{R^3}\left(\underline{\xi} - \frac{3\xi_z \cancel{R}}{R^2}\cancel{R}\hat{\underline{k}}\right)$$

$$\Rightarrow \quad \ddot{\xi}_x = -\frac{GM}{R^3}\xi_x \qquad \ddot{\xi}_y = -\frac{GM}{R^3}\xi_y$$

$$\text{and} \quad \ddot{\xi}_z = \frac{2GM}{R^3}\xi_z$$

This equation has solution $\xi_z(t) = \xi_z(0)\cosh(\alpha t)$ (assuming R constant) where $\alpha = \frac{2GM}{R^3}$

Numbers $\Rightarrow$ $\sim 26\,s$. at $R \doteq 6.4 \times 10^3$ km.

$\xi_x \to \xi_x(1-\Delta)$, where $\Delta = \frac{GM}{2R^3}\delta t^2$

same for ξ_y.

$\xi_z \to \xi_z(1+2\Delta)$ This gives an ellipsoid:

$$\frac{r^2}{(1-\Delta)^2} + \frac{z^2}{(1+2\Delta)^2} = r^2$$

with volume

$$\frac{4}{3}\pi R^3(1-\Delta)^2(1+2\Delta)$$

$$= \frac{4}{3}\pi R^3(1-2\Delta+\Delta^2)(1+2\Delta)$$

$$= \frac{4}{3}\pi R^3\left(1+\cancel{2\Delta}-\cancel{2\Delta}+-4\Delta^2+2\Delta^3\right)$$

$$\simeq \frac{4}{3}\pi R^3$$