6.) lowest level: $\frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega = \frac{3}{2}\hbar\omega$ degen: 1

2nd lowest: $\frac{3}{2}\hbar\omega + \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega = \frac{5}{2}\hbar\omega$ degen: 3

3th: $\frac{5}{2}\hbar\omega + \frac{1}{2}\hbar\omega + \frac{1}{2}\hbar\omega$ (+) $\frac{3}{2}\hbar\omega + \frac{3}{2}\hbar\omega + \frac{1}{2}\hbar\omega = \frac{7}{2}\hbar\omega$ degen: 6

nth: $\sum_{i=1}^{n+1} i = \frac{1}{2}(n+1)(n+2)$

7.) $|E_{n,1}|^2 + |E_{n,2}|^2 = 2|E_n|^2$

↳ energy of 1 particle HO

$\Rightarrow E_n = \sqrt{2}\,(n+\frac{1}{2})\hbar\omega$

8.) $[\hat{R}, \hat{r}] = \left[\frac{m_a \hat{r}_a + m_b \hat{r}_b}{m_a + m_b}, \hat{r}_a - \hat{r}_b\right] = \left[\frac{m_a \hat{r}_a}{m_a + m_b}, \hat{r}_a\right] - \left[\frac{m_b \hat{r}_b}{m_a + m_b}, \hat{r}_b\right] = 0$

~~what is~~ $\hat{p}$?

Sheet 4:

1.) $L_x = \frac{L_+ + L_-}{2}$

$L_x|\phi_{-1}\rangle = \frac{L_+|\phi_{-1}\rangle}{2} + \frac{L_-|\phi_{-1}\rangle}{2} = \frac{\sqrt{2}}{2}|\phi_0\rangle$

$L_x|\phi_1\rangle = \frac{\sqrt{2}}{2}|\phi_0\rangle$

$L_x|\phi_0\rangle = \frac{\sqrt{2}}{2}|\phi_1\rangle + \frac{\sqrt{2}}{2}|\phi_{-1}\rangle$

$\Rightarrow \frac{1}{\sqrt{2}}|\phi_{-1} - \phi_1\rangle$ is eigenvector with value 0

$\Rightarrow L_x(|\phi_1\rangle + |\phi_{-1}\rangle \pm \sqrt{2}|\phi_0\rangle)$

$= \pm\sqrt{2}\phi_0 + |\phi_1\rangle + |\phi_{-1}\rangle \Rightarrow$ eigenvalues ± 1

$\Rightarrow (|\phi_1\rangle + |\phi_{-1}\rangle + \sqrt{2}|\phi_0\rangle) \Rightarrow$ 3 beams with intensities 1:2:1

same for $m=-1$ beam, for $m_\ell = 0$ beam: 1:0:1

$\ell = 1,\ s = \frac{1}{2} \rightarrow\ j = \frac{3}{2}, \frac{1}{2}$

$m_j = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$

$|\frac{1}{2}, \frac{1}{2}\rangle = \sqrt{\frac{1}{3}}\, Y_{1,0}|\uparrow\rangle - \sqrt{\frac{2}{3}}\, Y_{1,1}|$

$|\frac{1}{2}, -\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}\, Y_{1,-1}|\uparrow\rangle - \sqrt{\frac{1}{3}}\, Y_{1,0}$

$|\frac{3}{2}, \frac{3}{2}\rangle = Y_{1,1}|\uparrow\rangle$

$J_-|\frac{3}{2}, \frac{3}{2}\rangle = \sqrt{3}\,\hbar|\frac{3}{2}, \frac{1}{2}\rangle$

$\Rightarrow |\frac{3}{2}, \frac{1}{2}\rangle = \frac{\sqrt{2}}{\sqrt{3}} Y_{1,0}|\uparrow\rangle + \frac{1}{\sqrt{3}} Y_{1,1}|\downarrow\rangle$

$|\frac{3}{2}, -\frac{3}{2}\rangle = Y_{1,-1}|\downarrow\rangle$

$|\frac{3}{2}, -\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}\, Y_{1,-1}|\uparrow\rangle + \sqrt{\frac{2}{3}}\, Y_{1,0}|\downarrow\rangle$

1.) $L_+ = L_x + i L_y$

$L_- = L_x - i L_y$

$\Rightarrow L_y = \dfrac{L_+ - L_-}{2i}$

$L_x = \dfrac{L_+ + L_-}{2}$

$L_x |\phi_1\rangle = \frac{1}{2} |\phi_0\rangle$

$L_x |\phi_{-1}\rangle = \frac{1}{2} |\phi_0\rangle$ $\Rightarrow L_x \left|\dfrac{\phi_1 + \phi_{-1}}{\sqrt{2}}\right\rangle = 0 \Rightarrow$ eigenfunction

$L_x |\phi_0\rangle = \frac{\sqrt{2}}{2} |\phi_1\rangle + \frac{\sqrt{2}}{2} |\phi_{-1}\rangle$ $\cancel{L_x \left|\dfrac{\phi_1 + \phi_{-1} \pm \sqrt{2}\, \phi_0}{2}\right\rangle = \frac{1}{4}|\phi_0\rangle + \frac{1}{4}|\phi_0\rangle \pm \frac{\sqrt{2}}{2} \left|\frac{\sqrt{2}}{2}\phi_1\right\rangle + \frac{\sqrt{2}}{2}|\phi_{-1}\rangle}$

$L_x |\phi_1 + \phi_{-1} \pm \sqrt{2}\, \phi_0\rangle = \frac{1}{2} |\phi_0\rangle + \frac{1}{2} \phi_0.$