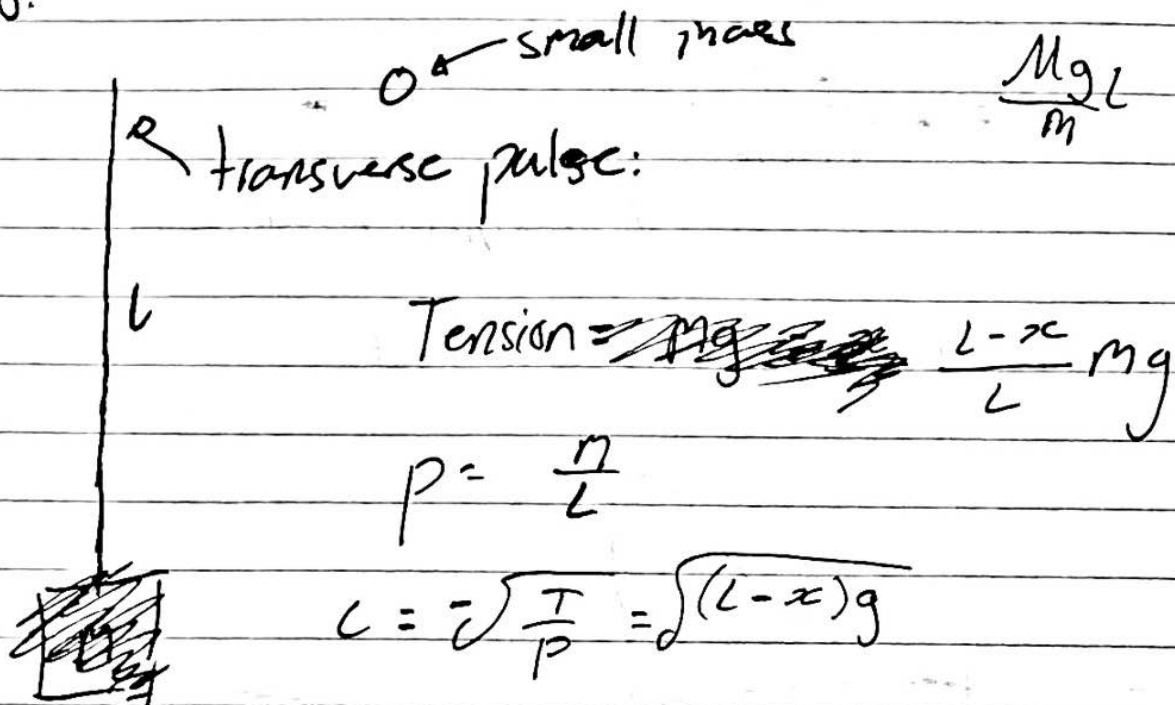


Q8:



$$\dot{x} = \sqrt{(L-x)g} \quad \frac{dx}{dt} = \sqrt{(L-x)g}$$

The ball: $\ddot{x} = -g \quad \frac{dx}{dt} = -tg \quad x = -\frac{t^2}{2}g + L$

$$\frac{dt}{dx} = \int \frac{1}{\sqrt{(L-x)g}} dx \quad t = +2\sqrt{L-x} \sqrt{g} + C$$

$$\left(\frac{t}{2\sqrt{g}}\right)^2 = L-x$$

$$x = L - \left(\frac{t}{2\sqrt{g}}\right)^2$$

$$\dot{x} = -\sqrt{xg} \quad \int dt = \int \frac{1}{\sqrt{xg}}$$

$$t=0 \quad x=L \quad C = +2\sqrt{Lg} \quad t = 2\sqrt{xg} + C$$

$$t = 2(\sqrt{xg} - \sqrt{Lg})$$

$\left(\frac{t}{2\sqrt{g}}\right)^2 = L-x$

$$x = L - \frac{t^2}{2}g \quad \text{for the falling body.}$$

$$\frac{dx}{dt} = -\sqrt{xg} \quad \int_L^x \frac{1}{\sqrt{x}} dx = \int_0^t -\sqrt{g} dt$$

$$2\sqrt{x} - 2\sqrt{L} = -t\sqrt{g}$$

$$x = \left(\sqrt{L} - \frac{t\sqrt{g}}{2} \right)^2$$

$$\left(\sqrt{L} - \frac{t\sqrt{g}}{2} \right)^2 = -\frac{t^2}{2}g + L$$

$$\left(\frac{t\sqrt{g}}{2} \right)^2 = \left(\sqrt{L} - \frac{t\sqrt{g}}{2} \right)^2$$

$$\sqrt{L} = \frac{\sqrt{L} + \frac{t\sqrt{g}}{2}}{2}$$

$$t = \sqrt{\frac{L}{g}} \frac{2}{\sqrt{2}+1}$$

$$x = L - \frac{Lg}{2(\sqrt{2}+1)^2}$$

$$\frac{t^2 g}{4} + \frac{t^2 g}{2} - t\sqrt{Lg} = 0$$

$$\frac{3g}{4}t = \sqrt{Lg}$$

$$t = \frac{\sqrt{L}}{\sqrt{g}} \frac{4}{3}$$

$$x = L - L \left(\frac{16}{9} \right) = \frac{1}{9}L$$

or $\frac{8}{9}L$ down.

now adding weight.

$$x = L - \frac{t^2}{2}g$$

$$\frac{dx}{dt} = -\sqrt{xg + \frac{MgL}{m}}$$

$$\frac{t^2}{2}g = L$$

$$t = \sqrt{\frac{2L}{g}}$$

~~$$\frac{dx}{dt} = -\sqrt{xg + \frac{MgL}{m}}$$~~

$$\int_L^x \frac{1}{\sqrt{x + \frac{M}{m}L}} dx = \int -\sqrt{g} dt$$

$$2\sqrt{x + \frac{M}{m}L} - 2\sqrt{L + \frac{M}{m}L} = -\sqrt{g}t$$

~~$$2\sqrt{\frac{M}{m}L} - 2\sqrt{L + \frac{M}{m}L} = -\sqrt{g}t$$~~

$$M = \frac{1}{g}m$$

Q9.

diameter = 0.5 mm

length = 700 mm

$\rho = 7800 \text{ kg m}^{-3}$

$\gamma = 200 \text{ GPa}$

so $\frac{\sigma}{\epsilon} = \frac{\text{stress}}{\text{strain}}$

$$\frac{T}{p} = v^2$$

frequency = 261 Hz

$\lambda = 1.4 \text{ m}$

$$v = \lambda f$$

$$T = p \lambda^2 f^2$$

$$p = 7800 \times (\pi \times (0.0005)^2)$$

$$p = 1.53 \times 10^{-3} \text{ kg m}^{-1}$$

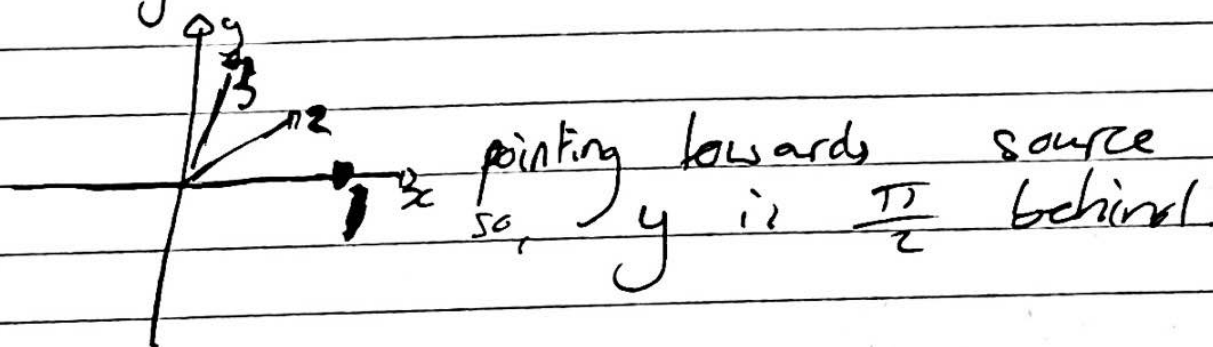
$$T = 204 \text{ N}$$

\rightarrow Which is $\approx 1 \text{ GPa}$.

This is a third of the yield

Q 10. a) $\psi_{y,1} = a \sin \theta \cos(kz - \omega t + \phi_0)$

$\psi_{y,2} = b \cos(kz - \omega t - \frac{\pi}{2}) = b \sin(kz - \omega t)$



$\psi_{y,3} = \frac{1}{2} c \sin(kz - \omega t)$

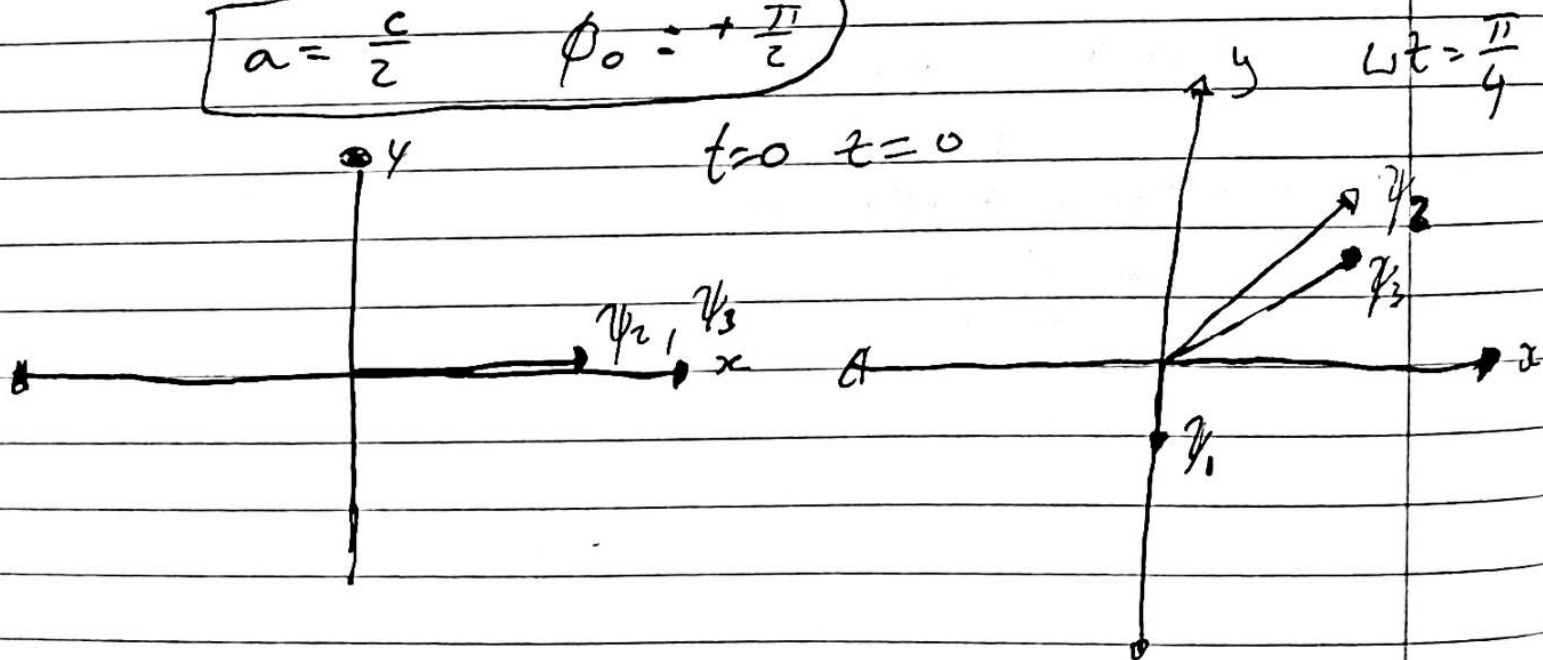
b) $\psi_{y,3} = \psi_{y,2} + \psi_{y,1}$

$a \cos \theta \cos(kz - \omega t + \phi_0) + b \cos(kz - \omega t) = c \cos(kz - \omega t)$

$b = c, \quad \theta = \frac{\pi}{2}$

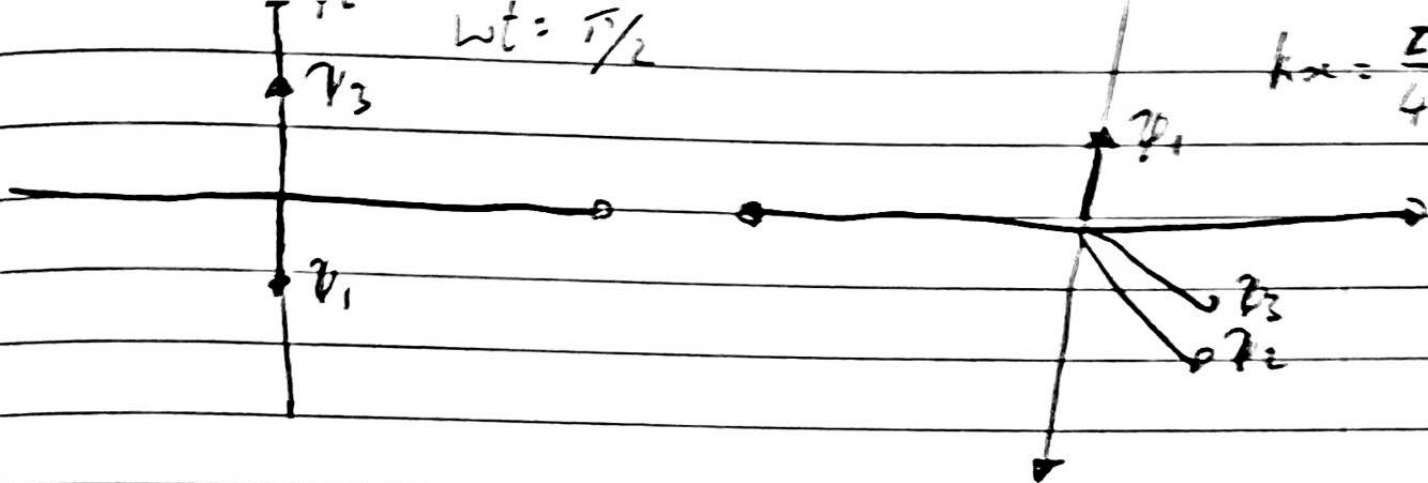
$\frac{1}{2} c \sin(kz - \omega t) = a \cos(kz - \omega t + \phi_0) + b \sin(kz - \omega t)$

$a = \frac{c}{2}, \quad \phi_0 = +\frac{\pi}{2}$



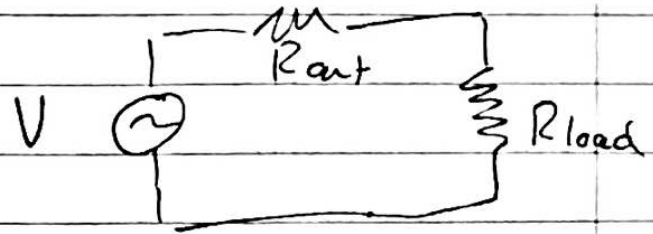
$$\omega t = \pi/2$$

$$kx = \frac{\pi}{4}$$



Example sheet for experimental methods.

1. a)



i)

$$V_{load} = V \frac{R_{load}}{R_{out} + R_{load}} \approx V$$

ii) $I = \frac{V}{R_{load} + R_{out}}$

$$P = IV = \frac{V}{R_{load} + R_{out}} \times \frac{V R_{load}}{R_{out} + R_{load}}$$

$$P = \frac{V^2 R_{load}}{(R + R_{load})^2}$$

$$\frac{dP}{dR_{load}} = \frac{1}{(R + R_{load})^2} - 2 \frac{R_{load}}{(R + R_{load})^3} = 0$$

$$R + R_{load} - 2 R_{load} = 0 \quad \boxed{R_{load} = R_{out}}$$

If the receiving device is an oscilloscope, want accurate reading of power.

b) at switch 1:

$$i_1 = \frac{V_{in}}{R}$$

$$i_2 = \frac{V_{out}}{R}$$

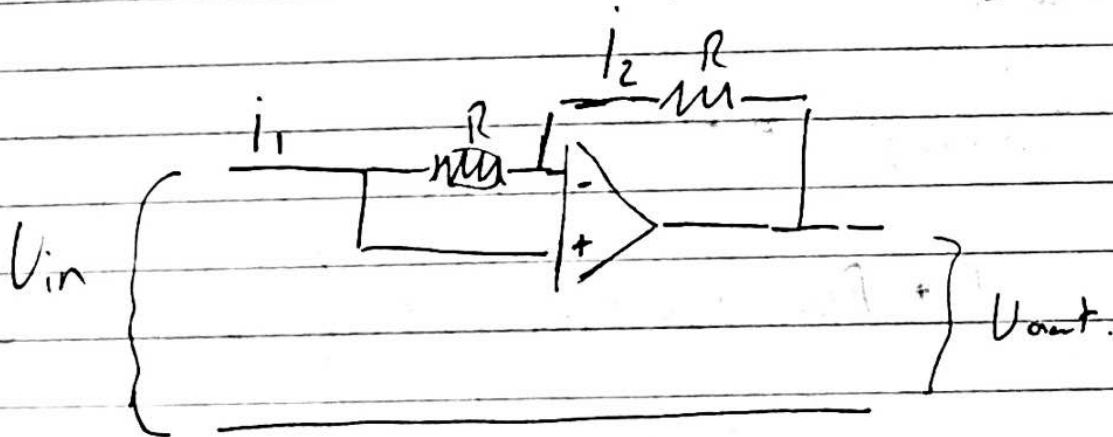
$i_1 = i_2 \therefore V_{in} = V_{out}$ if the two resistors are equal.

Frequency should have no impact, as the impedance is not complex.

At position 2:

$$V_- - V_+ = i_1 R \quad \text{So it is not 0.$$

that means that, the voltage



$$i_1 = \frac{V_{in} - V_-}{R}$$

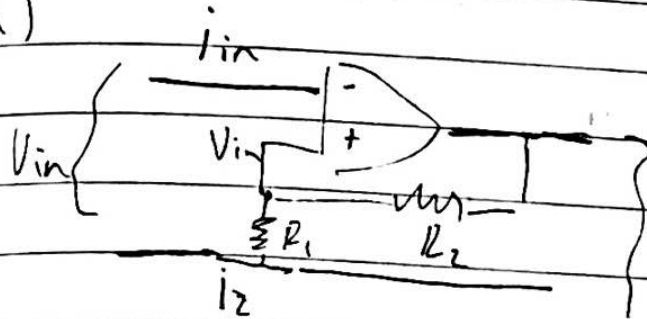
$$i_2 = \frac{V_- - V_{out}}{R}$$

$$\frac{V_{in} - V_-}{R} = \frac{V_- - V_{out}}{R}$$

$$V_+ - V_- = V_{in} - V_-$$

$$V_+ = V_{in}$$

1d)



$$i_{in} = 0$$

$$\frac{V_{in} - V_{out}}{R_2} = i_2$$

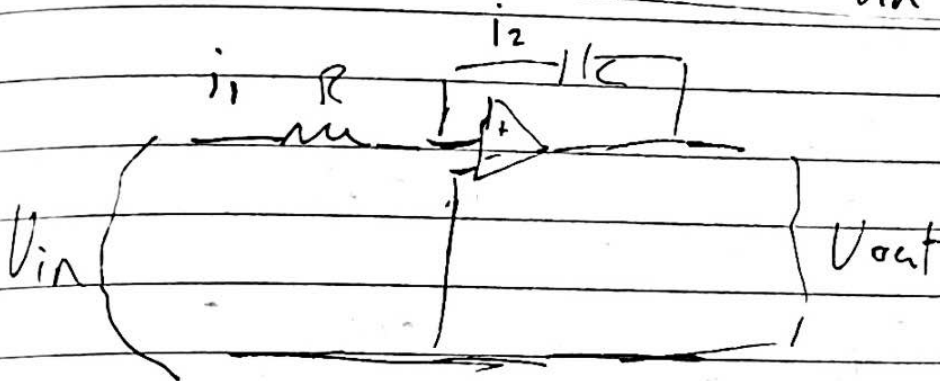
~~V_{out}~~ \rightarrow ~~V_{out}~~ \approx V_{PS}

$$\frac{V_{in}}{R_1} = \frac{V_{out} - V_{in}}{R_2}$$

$$-\frac{V_{in}}{R_1} = i_2$$

$$V_{in} \left(\frac{R_2}{R_1} + 1 \right) = V_{out}$$

$$\frac{V_{out}}{V_{in}} = \left(1 + \frac{R_2}{R_1} \right)$$

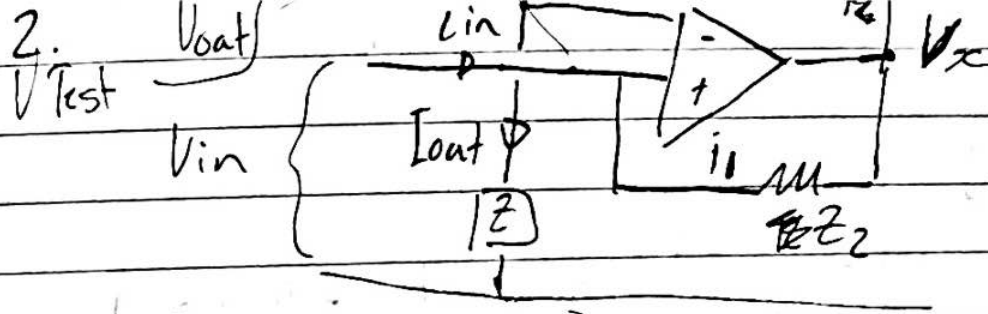


So analysis ~~would~~ gives $\frac{V_{out}}{V_{in}} = \frac{1}{i R \omega C}$

$$i_1 = \frac{V_{in} - V_{-}}{R}$$

$$i_2 = \frac{(V_{-} - V_{out})}{R} (i \omega C)$$

So ~~current is~~
So voltage depends on the frequency.



$$Z_{in} = \frac{V_{in}}{I_{in}}$$

(1) golden rule 2:

$$V_+ = V_- = V_{in} \quad (2)$$

$$I_{out} = \frac{V_{out}}{Z_3} = \frac{V_{in}}{Z_3} \quad (3)$$

Then $I_{in} = \frac{V_{in} - V_x}{Z_2}$

$$V_x = \frac{Z_1 + Z_2}{Z_2} V_{in}$$

$$I_{in} = V_{in} \left(\frac{1 - \frac{Z_1 + Z_2}{Z_2}}{Z_2} \right)$$

$$= V_{in} \left(\frac{-Z_1}{Z_2} \right)$$

$$Z_{in} = - \frac{Z_1 Z_2}{Z_1}$$

now replace:

$$Z_{in} = - \frac{R \left(\frac{1}{j\omega C} \right)}{R}$$

$$Z_{in} = + j \frac{1}{\omega C}$$

so Voltage is ahead of current.