$$G = a\left[\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right]^{1/2} = 1$$

and $$\frac{d}{ds}\left(\frac{\partial G}{\partial \dot{x}^\mu}\right) - \frac{\partial G}{\partial x^\mu} = 0$$

$$\frac{d}{ds}\left(\frac{\partial G}{\partial \dot{\theta}}\right) - \frac{\partial G}{\partial \theta} = 0$$

$$\frac{d}{ds}\left(G^{-1}\dot{\theta}\right) - G^{-1}\sin\theta\cos\theta\,\dot{\phi}^2 = 0$$

and $G = 1 \Rightarrow \ddot{\theta} - \sin\theta\cos\theta\,\dot{\phi}^2 = 0$

Similarly, get $\sin^2\theta\,\dot{\phi} = L$, where $L$ = constant.

This has general solution $\cot\theta = \tan\Psi \sin\phi$, for constant $\Psi$.

Geodesics on a cylinder are helices.