7) Schwarzschild metric:

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right)dt^2 - \frac{1}{c^2}\left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2$$
$$- \frac{r^2}{c^2} d\theta^2 - \frac{r^2 \sin^2\theta \, d\phi^2}{c^2}$$

radially, $d\phi = d\theta = 0$

for photons, $ds^2 = 0$

$$\Rightarrow \left(1 - \frac{2GM}{rc^2}\right)dt^2 = \frac{1}{c^2}\left(1 - 2\frac{GM}{c^2 r}\right)^{-1} dr^2$$

$$\Rightarrow c^2\left(1 - \frac{2GM}{rc^2}\right)^2 = \frac{dr^2}{dt^2}$$

$$\frac{dr}{dt} = \pm c\left(1 - \frac{2GM}{rc^2}\right)$$

for in travelling γ: $\displaystyle\int_{r_0}^{r}\left(1 - \frac{2GM}{r'c^2}\right)^{-1} dr' = c\int_{t_0}^{t} dt'$

This has solution $ct = ct_0 - r + r_0 - R_s \ln\left(\frac{r - R_s}{r_0 - R_s}\right)$

Takes infinite time to reach horizon (asymptote).