Natural Sciences IB: Physics(A)

Instructions

Time 1.5 hours, Answer **ALL QUESTIONS** Calculators are allowed.

Question 1

Explain how the χ^2 test can be used to compare theoretical models with data, including a discussion of "degrees of freedom"

In an experiment, a radioactive source is observed through a varying number, n of sheets of material, and the following counts are measured over one second intervals

number of sheets	1	2	3	4	5	6	7	8
counts in 1s	103	80	79	71	50	52	46	51

The exact nature of the material is unknown, but there are two possibilities, each of which predict a count rate of

$$N_0 e^{-na}$$

where $N_0 = 104.5$ counts s^{-1}, has been determined from a 900 s long integration observation with no sheets of material (i.e. $n = 0$) and a could be either 0.15 or 0.19 depending on the material.

Explain why the value of N_0 is accurately known compared with the counts in 1 s given above

Carry out a χ^2 test to see which value of a is preferred

Plot a suitable graph of the data, including appropriate error bars and the preferred model predictions. Use this to suggest how the model might be improved.

Question 2

Define the phase velocity and group velocity for a dispersive wave

Show that if the phase velocity, v_p is proportional to ω^n, where ω is the angular frequency of the wave, the the group velocity , v_g is given by

$$v_g = \frac{v_p}{1-n}$$

For ocean waves in deep water, where viscosity and surface tension can be neglected, the phase velocity can be reasonably expected to depend on the frequency, ω, on the density of the water, ρ, and the acceleration due to gravity, g. Use dimensional analysis to establish the form of this relationship.

Given that waves with a period of 8 s are found to have a wavelength of close to 100 m, determine the constant of proportionality in the expression for the phase velocity. What is the group velocity for waves of this period?

A short lived violent storm in the Northern Pacific Ocean causes waves to arrive in Hawaii. on a certain day, the typical period of the waves is measured to be 12 , while three days later it has fallen to 8s. How far was the storm from Hawaii, and approximately when did it occur relative to the day on which the first measurement was made?

Question 3

Explain the conditions under which Fresnel diffraction applies

Show that the complex amplitude of the radiation received at an on-axis point a distance z from an annular aperture of radius $r(\ll z)$ and thickness dr illuminated normally by a parallel beam of wavelength λ is given by

$$d\psi \propto K \exp\left(\frac{i\pi r^2}{\lambda z}\right) 2\pi r dr$$

where K is an obliquity factor

Explain with the aid of a phasor diagram how this expression can be used to derive the amplitude received when the annulus is replaced with a circular aperture of radius R. Include in your discussion a definition of Fresnel half-period zones and show that the radius of the n^{th} such zone is given by $\sqrt{n\lambda z}$

A circular aperture 3.8 mm in diameter is illuminated normally with a parallel beam of white light containing radiation over the wavelength range $\lambda = 400$ nm to 700 nm. What wavelengths are missing from the light received on-axis 1 m away from the aperture?