Air Hockey. A small puck is set on an air hockey table, as shown in the figure on the next page. The friction between the puck and the table is negligible, and the collisions between the puck and the rim of the table are elastic.

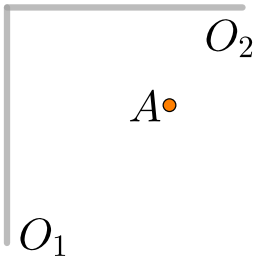
1. Mark all parts of the rim of the table towards which the puck can be directed, so that a goal is scored after exactly three collisions with the rim (not counting own goals).

Auxiliary tasks, for guidance:

A1. What do images represent in optics?

A2. In the figure below, A is a light source. Construct the ray from A which passes through the point B after it is reflected by the mirror O.

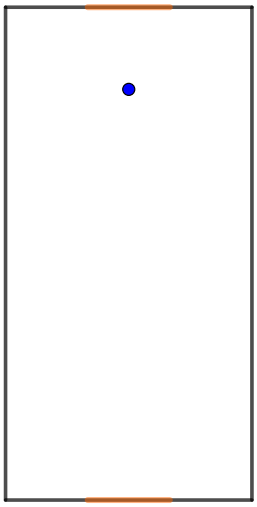
A3. Consider all the rays reflected from the mirrors O_1 and O_2 in the figure below. Construct all the images of the source A due to the two mirrors.



A4. Solve a simplified version of the problem – mark the zones on the rim of the table towards which the puck can be directed, so that a goal is scored after a single collision.

A5. Repeat the same for the case of two reflections.

Worksheet



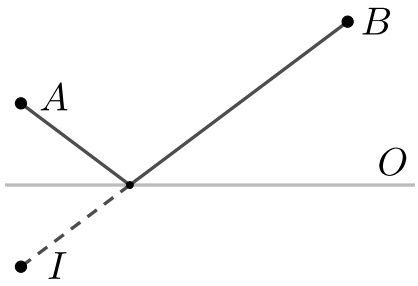
Outline solutions

A1. What do images represent in optics?

In the case of plane mirrors, they are points from which all reflected rays seem to emanate. In other words, if you extend all the reflected rays, they will converge at a single point, called an image.

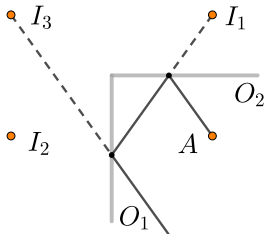
A2. In the figure below, A is a light source. Construct the ray from A which passes through point B after it is reflected by the mirror O.

Any reflected ray must emanate from the image. So, construct the image by simple reflection, and connect it to B. This is what the reflected ray will look like. Now just connect A to the point of contact with the mirror.



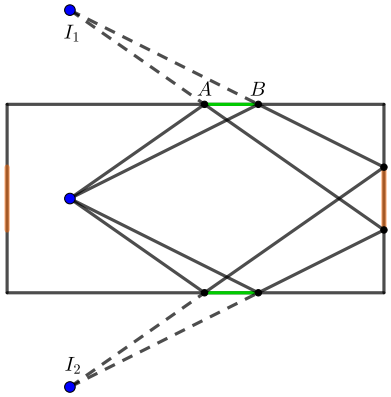
A3. Consider all of the rays reflected from the mirrors O_1 and O_2 in the figure below. Construct all of the images of the source A due to the two mirrors.

After a single reflection, rays will seem to emanate either from I_1 or I_2. However, notice that some rays get reflected twice before leaving the system. After the second reflection, they will seem to originate from the image of their source (that is, I_1 or I_2). For those rays we need to construct another image, I_3. This image is the reflection of I_1 in O_1 *and* the reflection of I_2 in O_2, so there's no need to look for further images.



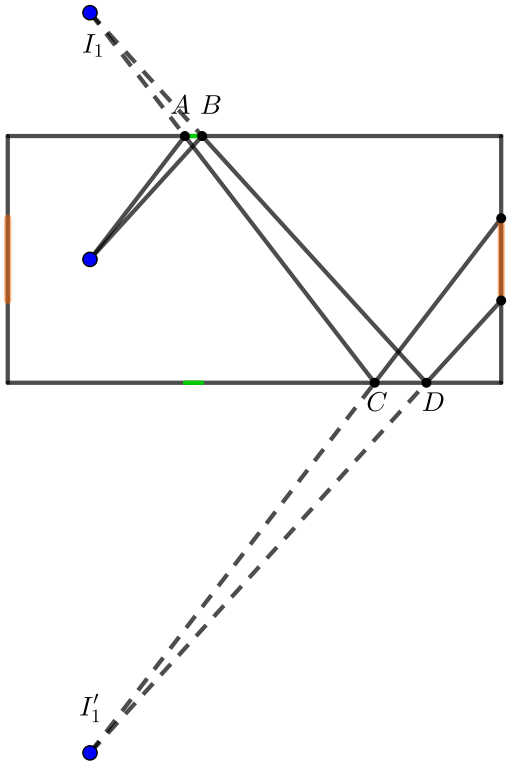
A4. Solve a simplified version of the problem – mark the zones on the rim of the table towards which the puck can be directed, so that a goal is scored after a single collision.

The collisions are elastic, meaning that the angle of reflection equals the angle of incidence. This then turns into an optics problem, the puck being a light source, and collisons being analogous to reflections from plane mirrors.
All paths that terminate in the opponent's goal after a collision with a face must emanate from the image of the source in that face. One can then get the limits for the puck's path by connecting that image with the ends of the opponent's goal:



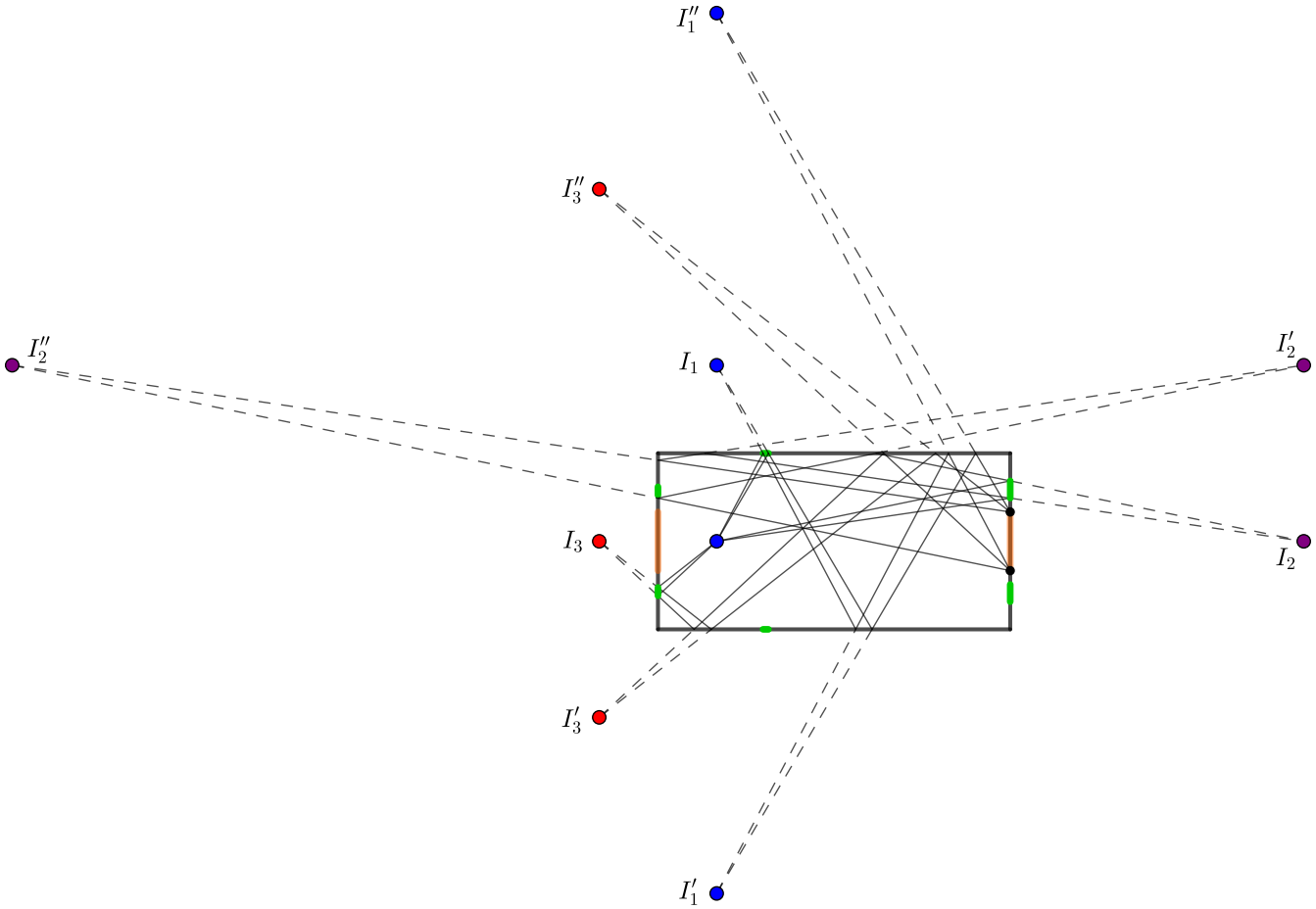
A5. Repeat the same for the case of two reflections.

After the second reflection, rays will seem to originate from an image of an image of the puck. The only way this can happen without scoring an own goal is through reflections from the longer edges of the table. Then, there are again two permitted zones towards which you can launch the puck. The constructions for the case source-top-bottom-goal are shown below.



1. Mark all parts of the rim of the table towards which the puck can be directed, so that a goal is scored after exactly three collisions with the rim (not counting own goals).

Now we need images of images of images. But this time we are able to also score a goal by sending the puck towards the left edge or the right edge. In total, there are three families of images, producing three pairs of permitted zones. The figure below shows the constructions for the cases source-top-bottom-top-goal, source-right-top-left-goal, and source-left-bottom-top-goal, as well as all the permitted zones.



One can check that other paths, such as source-right-left-top-goal, do not work.