1.

Because I am the kind of person who always standing flags (this is an internet slang which means that somebody set a rule for him/herself but always violates it in a short period of time)and I wish that I will not ‘’kick out’’ my flag anymore,I build a flag on which is an infinity mark and expect the flag will never fall down in my movie and my life!!

There are four polygons in my model

M1=([[−6.5, −6.5, −4.5, −4.5, −6.5],
[4.5, −4.5, −4.5, 4.5, 4.5],
[0, 0, 0, 0, 0,]])

M2 =[[−2.5, −2.5, −1.5, −1.5, −2.5],
[1, −1, 1, −1, 1],
[0, 0, 0, 0, 0]])

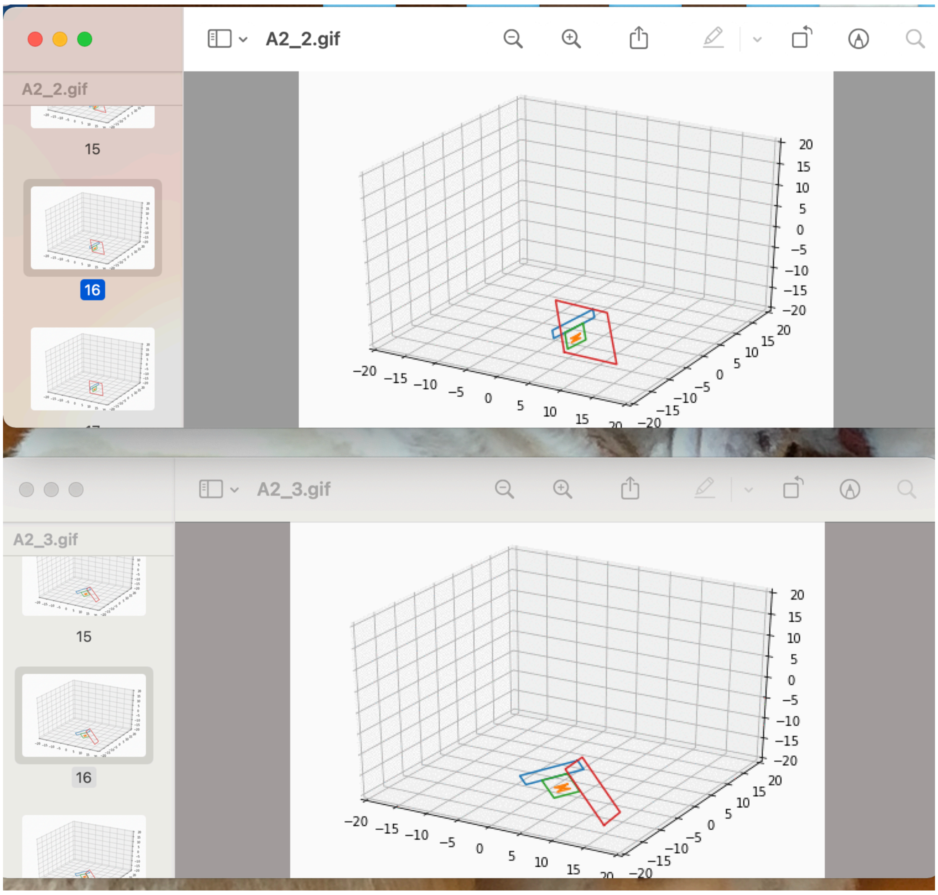
M3 =([[−4.5, −4.5, −0.5, −0.5, −4.5],
[2, −2, −2, 2, 2],
[0, 0, 0, 0, 0]])

M4 =([[6, −6.5, −6.5, 6, 6],
[4.5, 4.5, 4.5, 4.5, 4.5],
[6.5, 6.5, −2.5, −2.5, 6.5]])

where M1 represents the flagpole,M2 represents the mark on the flag,M3 represents the flag and M4 is the plane where flag is standing on it, and I have 200 frames in this movie.

3.

I switch the order to R=RrollRyawRpitch while the original movie is R=RyawRpitchRroll, and we can see the difference by the storyboard



In the frame 16 ,we can see that the objects of A2_2 and A2_3 are near to the x−y plane, but the direction they rotate are totally different, and there will be some angles that object in A2_3 will not rotate to ; that is ,it will not rotate the whole trajectory we designates it to be.

4.(1)

4.

(1) $A^TA = AA^T \Rightarrow A^T = A^{-1}$

Since there are 3 kinds of rotations, we have $R_{roll} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$

$R_{yaw} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $R_{pitch} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$

$$R_{roll} R_{roll}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \cos\theta\sin\theta \\ 0 & \cos\theta\sin\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad - ①$$

$$R_{yaw} R_{yaw}^T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta & 0 \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad - ②$$

$$R_{pitch} R_{pitch}^T = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta + \sin^2\theta & 0 & \cos\theta\sin\theta - \sin\theta\cos\theta \\ 0 & 1 & 0 \\ \sin\theta\sin\theta - \sin\theta\cos\theta & 0 & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad - ③$$

By ① ∧ ② ∧ ③, we know that 3x3 rotation matrices are orthogonal matrices

(2)

(2) we know that by QR decomposition, A matrix A can be expressed as the product of an orthogonal matrix Q and an upper triangular matrix R, then we have to show that let $P = R_{pitch} R_{yaw} R_{roll}$ and P is an orthogonal matrix so that for any 3×3 matrix A can be expressed as

$A = R_{pitch} R_{yaw} R_{roll} U = PU$.

In order to show P is an orthogonal matrix, we have to prove that $P^T P = P P^T = I$

$$P^T = (R_{pitch} R_{yaw} R_{roll})^T = R_{roll}^T R_{yaw}^T R_{pitch}^T$$

$$P^T P = (R_{roll}^T R_{yaw}^T R_{pitch}^T)(R_{pitch} R_{yaw} R_{roll})$$

$$= R_{roll}^T R_{yaw}^T I R_{yaw} R_{roll} \quad \text{(By the result of (1))}$$

$$= R_{roll}^T I R_{roll} = I$$, so we have $P^T = P^{-1}$,

then we can also have $PP^T = P^T P = I$. so we complete this proof

Proof of QR decomposition

Given independent vectors a_1, a_2, a_3.

Gram-Schmidt constructs q_1, q_2, q_3

A_1, A_2, A_3 are orthogonal

a_1, A_1, q_1 span the same subspace

a_1, a_2 and A_1, A_2 and q_1, q_2 span the same subspace

a_1, a_2, a_3 and A_1, A_2, A_3 and q_1, q_2, q_3 span the same subspace

Therefore, we can have

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$$a_1 = (q_1^T a_1) q_1$$

$$a_2 = (q_1^T a_2) q_1 + (q_2^T a_2) q_2$$

$$a_3 = (q_1^T a_3) q_1 + (q_2^T a_3) q_2 + (q_3^T a_3) q_3$$

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}$$

$$Q = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$$

upper triangular

$\Uparrow$

$$\underbrace{\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}}_{A} = \underbrace{\begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} q_1^T a_1 & q_1^T a_2 & q_1^T a_3 \\ 0 & q_2^T a_2 & q_2^T a_3 \\ 0 & 0 & q_3^T a_3 \end{bmatrix}}_{R}$$

5.

As long as we choose +−90(degree) as the angle in Rpitch ,it will cause the first rotation matrix have the same affection to the object as the third rotation matrix , so we will lose one dimension in the whole trajectory !

Just as A2_5, the object just move up and down but doesn’t rotate.