Logistic Regression with Softmax

Reference:
[1] http://ufldl.stanford.edu/tutorial/supervised/SoftmaxRegression/ (http://ufldl.stanford.edu/tutorial/supervised/SoftmaxRegression/)

[2] https://houxianxu.github.io/2015/04/23/logistic-softmax-regression/ (https://houxianxu.github.io/2015/04/23/logistic-softmax-regression/)

[3] https://zhuanlan.zhihu.com/p/98061179?utm_source=wechat_session&utm_medium=social&utm_oi=777418892074061824 (https://zhuanlan.zhihu.com/p/98061179?utm_source=wechat_session&utm_medium=social&utm_oi=777418892074061824)

[4] https://github.com/hankcs/CS224n/tree/master/assignment1 (https://github.com/hankcs/CS224n/tree/master/assignment1)

[5] https://github.com/hartikainen/stanford-cs224n/tree/master/assignment1 (https://github.com/hartikainen/stanford-cs224n/tree/master/assignment1)

In [ ]:
```
from google.colab import drive
drive.mount('/content/drive')
```

```
Mounted at /content/drive
```

In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

In [ ]:
```
data = pd.read_csv('/content/drive/My Drive/Competition/train.csv')
```

In [ ]:
```
y_all_train = data.iloc[:, -1]
```

In [ ]:
```
y_all_train.shape
```

Out[ ]:
```
(47760,)
```

In [ ]:
```
def label_percentages(labels):
  n0 = 0
  n1 = 0
  n2 = 0
  total = labels.shape[0]
  for label in labels:
    if label == 0:
      n0 += 1
    elif label == 1:
      n1 += 1
    elif label == 2:
      n2 += 1

  return (n0, n1, n2), (n0/total, n1/total, n2/total), total
```

In [ ]:
```
label_percentages(y_all_train)
```

Out[ ]:
```
((37535, 2002, 8223),
 (0.7859087102177554, 0.0419179229480737, 0.17217336683417087),
 47760)
```

In [ ]:
```
test = pd.read_csv('/content/drive/My Drive/Competition/test.csv')
```

In [ ]:
```
all_features = pd.concat([data.iloc[:, :-1], test]).reset_index(drop=True)
```

In [ ]:
```
def preprocessing(features):
  X = features.copy()
  X = X.iloc[:, 1:19]
  X = X.drop(columns="PS")
  X = X.drop(columns="PRECT")
  X.insert(0, 'bias', 1)
  X_means = np.mean(X)
  X_std = np.std(X)
  X_scale = (X - X_means) / X_std
  X_scale.iloc[:, 0] = np.ones((X_scale.shape[0], 1))
  return X_scale
```

In [ ]:
```
features_scale = preprocessing(all_features)
features_scale
```

Out[ ]:

	bias	lat	lon	TMQ	U850	V850	UBOT	VBOT	QREFH
0	1.0	-0.952799	-0.659826	-1.456660	-0.544970	1.082394	-0.689690	1.750971	-1.01902
1	1.0	1.167947	0.208099	0.980482	1.330311	2.126323	1.095283	2.212530	1.02801
2	1.0	1.167947	0.185259	-1.798887	-0.270104	-1.403102	-0.220842	-1.095445	-1.71571
3	1.0	0.717160	-0.393358	1.393384	0.109542	-0.322441	0.959048	-0.313340	1.09709
4	1.0	-0.942553	-0.682666	-0.894244	-0.988140	-1.196643	-1.202690	-0.838749	-0.67312
...	...	...	...	...	...	...	...	...	.
55075	1.0	1.178192	0.177646	1.257555	-0.070115	-1.906459	0.752188	-2.054562	1.62259
55076	1.0	1.178192	0.185259	1.331331	0.049520	-2.030020	0.963159	-2.102031	1.73687
55077	1.0	1.178192	0.192872	1.504772	0.199574	-2.240993	1.175167	-2.110102	1.82509
55078	1.0	1.178192	0.200486	1.618416	0.349145	-2.518317	1.395306	-2.125182	1.84868
55079	1.0	1.178192	0.208099	1.677245	0.493976	-2.823162	1.595011	-2.079873	1.94768

55080 rows × 17 columns

In [ ]:
```
len_data = data.shape[0]
```

In [ ]:
```
# separate data and test data
train_data = features_scale.iloc[0:len_data, :]
test_data = features_scale.iloc[len_data:features_scale.shape[0], :]
train_data.shape, test_data.shape
```

Out[ ]:
```
((47760, 17), (7320, 17))
```

In [ ]:
```
train_data
```

Out[ ]:

	bias	lat	lon	TMQ	U850	V850	UBOT	VBOT	QREFH
0	1.0	-0.952799	-0.659826	-1.456660	-0.544970	1.082394	-0.689690	1.750971	-1.01902
1	1.0	1.167947	0.208099	0.980482	1.330311	2.126323	1.095283	2.212530	1.02801
2	1.0	1.167947	0.185259	-1.798887	-0.270104	-1.403102	-0.220842	-1.095445	-1.71571
3	1.0	0.717160	-0.393358	1.393384	0.109542	-0.322441	0.959048	-0.313340	1.09709
4	1.0	-0.942553	-0.682666	-0.894244	-0.988140	-1.196643	-1.202690	-0.838749	-0.67312
...	...	...	...	...	...	...	...	...	.
47755	1.0	-1.208927	2.050537	-1.559338	0.542162	0.960851	0.282271	0.865639	-1.50132
47756	1.0	-0.963044	-0.659826	-0.419844	1.410663	-0.103645	1.506244	-1.000442	-0.18382
47757	1.0	-1.208927	2.050537	-1.075436	1.198620	-1.466387	0.950071	-1.395281	-0.95668
47758	1.0	1.157701	0.177646	0.818513	-0.347110	0.046155	-0.779050	0.281748	1.55431
47759	1.0	1.178192	0.185259	1.114539	-0.284359	-0.212958	-0.678358	-0.274555	0.78549

47760 rows × 17 columns

In [ ]:
```
test_data
```

Out[ ]:

	bias	lat	lon	TMQ	U850	V850	UBOT	VBOT	QREFHT
47760	1.0	-1.229417	2.042923	-0.969527	-0.199640	-0.019019	-0.908190	-0.249263	-1.53332
47761	1.0	-1.229417	2.050537	-1.033650	-0.208858	0.073115	-1.022537	-0.056414	-1.61189
47762	1.0	-1.229417	2.058150	-1.115755	-0.218882	0.146943	-1.088501	0.171496	-1.65935
47763	1.0	-1.229417	2.065764	-1.153955	-0.218557	0.240390	-1.077335	0.400787	-1.70300
47764	1.0	-1.229417	2.073377	-1.182211	-0.208022	0.401727	-1.028380	0.603821	-1.75904
...	...	...	...	...	...	...	...	...	..
55075	1.0	1.178192	0.177646	1.257555	-0.070115	-1.906459	0.752188	-2.054562	1.62259
55076	1.0	1.178192	0.185259	1.331331	0.049520	-2.030020	0.963159	-2.102031	1.73687
55077	1.0	1.178192	0.192872	1.504772	0.199574	-2.240993	1.175167	-2.110102	1.82509
55078	1.0	1.178192	0.200486	1.618416	0.349145	-2.518317	1.395306	-2.125182	1.84868
55079	1.0	1.178192	0.208099	1.677245	0.493976	-2.823162	1.595011	-2.079873	1.94768

7320 rows × 17 columns

Logistic Regression with Softmax

Given a test input x, we want our hypothesis to estimate the probability that $P(y=k|x)$ for each value of $k=1,\ldots,K$. I.e., we want to estimate the probability of the class label taking on each of the K different possible values. Thus, our hypothesis will output a K-dimensional vector (whose elements sum to 1) giving us our K estimated probabilities. Concretely, our hypothesis $h_\theta(x)$ takes the form:

$$h_\theta(x) = \begin{bmatrix} P(y=1|x;\theta) \\ P(y=2|x;\theta) \\ \vdots \\ P(y=K|x;\theta) \end{bmatrix} = \frac{1}{\sum_{j=1}^{K} \exp(\theta^{(j)\top}x)} \begin{bmatrix} \exp(\theta^{(1)\top}x) \\ \exp(\theta^{(2)\top}x) \\ \vdots \\ \exp(\theta^{(K)\top}x) \end{bmatrix}$$

Here $\theta^{(1)},\theta^{(2)},\ldots,\theta^{(K)} \in \Re^n$ are the parameters of our model. Notice that the term $\frac{1}{\sum_{j=1}^{K}\exp(\theta^{(j)\top}x)}$ normalizes the distribution, so that it sums to one.

([http://ufldl.stanford.edu/tutorial/supervised/SoftmaxRegression/ (http://ufldl.stanford.edu/tutorial/supervised/SoftmaxRegression/)](http://ufldl.stanford.edu/tutorial/supervised/SoftmaxRegression/))

(5 points) Prove that softmax is invariant to constant offsets in the input, that is, for any input vector $\boldsymbol{x}$ and any constant c,

$$\text{softmax}(\boldsymbol{x}) = \text{softmax}(\boldsymbol{x}+c)$$

where $\boldsymbol{x}+c$ means adding the constant c to every dimension of $\boldsymbol{x}$. Remember that

$$softmax(\boldsymbol{x})_i = \frac{e^{\boldsymbol{x}_i}}{\sum_j e^{\boldsymbol{x}_j}} \tag{1}$$

Note: In practice, we make use of this property and choose $c = -\max_i x_i$ when computing softmax probabilities for numerical stability (i.e., subtracting its maximum element from all elements of $\boldsymbol{x}$).

(Standford CS 224n 2017W, assignement 1)

In [ ]:

```python
# input product = X * theta
def softmax(product):
    if len(product.shape) > 1:
      max_each_row = np.max(product, axis=1, keepdims=True)
      exps = np.exp(product - max_each_row)
      sum_exps = np.sum(exps, axis=1, keepdims=True)
      res = exps / sum_exps

    else:
        product_max = np.max(product)
        product = product - product_max
        numerator = np.exp(product)
        denominator = 1.0 / np.sum(numerator)
        res = numerator.dot(denominator)
    return res
```

Cost Function

We now describe the cost function that we'll use for softmax regression. In the equation below, $1\{\cdot\}$ is the ""indicator function,"" so that $1\{\text{a true statement}\} = 1$, and $1\{\text{a false statement}\} = 0$. For example, $1\{2+2=4\}$ evaluates to 1; whereas $1\{1+1=5\}$ evaluates to 0. Our cost function will be:

$$J(\theta) = -\left[\sum_{i=1}^{m}\sum_{k=1}^{K} 1\left\{y^{(i)} = k\right\} \log \frac{\exp(\theta^{(k)\top} x^{(i)})}{\sum_{j=1}^{K} \exp(\theta^{(j)\top} x^{(i)})}\right]$$

Notice that this generalizes the logistic regression cost function, which could also have been written:

$$\begin{aligned} J(\theta) &= -\left[\sum_{i=1}^{m} (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)})) + y^{(i)} \log h_\theta(x^{(i)})\right] \\ &= -\left[\sum_{i=1}^{m}\sum_{k=0}^{1} 1\left\{y^{(i)} = k\right\} \log P(y^{(i)} = k | x^{(i)}; \theta)\right] \end{aligned}$$

The softmax cost function is similar, except that we now sum over the K different possible values of the class label. Note also that in softmax regression, we have that

$$P(y^{(i)} = k | x^{(i)}; \theta) = \frac{\exp(\theta^{(k)\top} x^{(i)})}{\sum_{j=1}^{K} \exp(\theta^{(j)\top} x^{(i)})}$$

(http://ufldl.stanford.edu/tutorial/supervised/SoftmaxRegression/ (http://ufldl.stanford.edu/tutorial/supervised/SoftmaxRegression/))

Regularized Cost Function

$$\text{Cost} = -\frac{1}{m}\left\{\left[\sum_{i=1}^{m}\sum_{j=1}^{K} 1\{y_i = j\} \log \underbrace{\frac{e^{\theta_j^T x_i}}{\sum_{l=1}^{K} e^{\theta_l^T x_i}}}_{\text{softmax}}\right] + \lambda \cdot \sum_{i=1}^{K}\sum_{j=1}^{n} \theta_{ij}^2\right\}$$

m: number of samples n: number of features

k: number of classes

We should not regularize the θ_0

In [ ]:
```
def reg_cost_softmax(X, y_onehot, theta, lambda_):
  n_samples = X.shape[0]
  softmax_res = softmax(np.dot(X, theta.T))  # (n_samples, n_classes)
  cost = - (1.0 / n_samples) * np.sum(y_onehot * np.log(softmax_res))

  theta_without_bias = theta[:, 1:theta.shape[1]]
  reg = lambda_ / n_samples  * np.sum(theta_without_bias ** 2)
  return cost + reg
```

Gradient with L2 Regularization

$$\frac{\partial\, cost}{\partial \theta_j} = -\frac{1}{m}\left\{\left[\sum_{i=1}^{m} x_i \left(1\{y_i=j\} - p(y_i=j \mid x_i;\theta)\right)\right] - \lambda \cdot \theta_j\right\}$$

for $j \geq 1$.

$$\frac{\partial\, Cost}{\partial \theta} = -\frac{1}{m}\left[(y-p)^T X + \lambda\theta\right]$$

$\hookrightarrow$ softmax

We should not regularize the θ_0

In [ ]:
```
def reg_gradient_softmax(X, y_onehot, theta, lambda_):
  n_samples = X.shape[0]
  softmax_res = softmax(np.dot(X, theta.T))

  gradient = (-1.0 / n_samples) * np.dot((y_onehot - softmax_res).T, X)
  # (n_classes, n_features)

  theta_without_bias = theta[:, 1:theta.shape[1]]
  # theta: (n_classes, n_features)
  # n_feautres = X features + 1(bias term)
  # theta_without_bias: (n_classes, n_features - 1)
  reg = -lambda_ / n_samples * theta_without_bias

  gradient[:, 1:gradient.shape[1]] = gradient[:, 1:gradient.shape[1]] + reg

  return gradient
```

Gradient Descent

In [ ]:
```
# alpha is learning rate
def gradient_descent(X, y_onehot, theta, lambda_, eps, alpha, max_iter):
  losses = []
  i = 0
  print("Iteration: Cost")

  while(i < max_iter):
    i += 1
    grad = reg_gradient_softmax(X, y_onehot, theta, lambda_)
    theta -= alpha * grad

    loss = reg_cost_softmax(X, y_onehot, theta, lambda_)
    if (i % 1000 == 0):
      print("{}: {:.8f}".format(i, loss))

    len_losses = len(losses)
    if (len_losses == 0):
      print("{}: {:.8f}".format(i, loss))
      diff = np.abs(loss)
    else :
      diff = np.abs(losses[len_losses-1] - loss)

    losses.append(loss)
    if(diff < eps):
      return theta, losses

  return theta, losses
```

Trainining model

In [ ]:
```
y_all_train.shape
```

Out[ ]:
```
(47760,)
```

In [ ]:
```
def split_train_test(X, y, training_size, val_size):
  m = X.shape[0]
  nb_train = (int) (m * training_size)
  X_train = X.iloc[0:nb_train, :]
  y_train = y[0:nb_train]

  nb_val = (int) (m * val_size)

  val_index = nb_train + nb_val
  X_val = X.iloc[nb_train : val_index, :]
  y_val = y[nb_train : val_index]

  X_test = X.iloc[val_index : m, :]
  y_test = y[val_index : m]
  return X_train, y_train, X_val, y_val, X_test, y_test
```

In [ ]:
```
def onehot_y(labels, classes):
    size = labels.shape[0]
    result = np.zeros((size, classes))
    for i in range(size):
        cl = int(labels[i])
        result[i][cl] = 1
    return result
```

In [ ]:
```
X_train, y_train, X_val, y_val, X_test, y_test = split_train_test(train_data,
y_all_train, 0.8, 0.1)
```

In [ ]:
```
X_train.shape, y_train.shape, X_val.shape, y_val.shape, X_test.shape, y_test.s
hape
```

Out[ ]:
```
((38208, 17), (38208,), (4776, 17), (4776,), (4776, 17), (4776,))
```

In [ ]:
```
y_label = pd.Series.to_numpy(y_train.copy())
```

In [ ]:
```
y_onehot = onehot_y(y_label, 3)
y_onehot
```

Out[ ]:
```
array([[1., 0., 0.],
       [0., 1., 0.],
       [1., 0., 0.],
       ...,
       [1., 0., 0.],
       [0., 1., 0.],
       [1., 0., 0.]])
```

In [ ]:
```
y_onehot.shape
```

Out[ ]:
```
(38208, 3)
```

In [ ]:
```
X_train_array = X_train.copy().to_numpy()
X_train_array
```

Out[ ]:
```
array([[ 1.        , -0.95279851, -0.65982622, ...,  1.42921582,
        -0.4285615 , -0.9506135 ],
       [ 1.        ,  1.16794656,  0.20809904, ..., -1.47779616,
         1.27702377,  1.39059963],
       [ 1.        ,  1.16794656,  0.1852589 , ...,  0.56987529,
        -1.43640373, -1.10560504],
       ...,
       [ 1.        ,  1.09623054, -0.61414595, ..., -0.33071481,
        -0.02934039, -0.88646481],
       [ 1.        ,  0.69666987, -0.38574456, ..., -1.14146989,
         0.86990277,  1.06717879],
       [ 1.        ,  0.70691502, -0.40097132, ..., -1.08116037,
         0.6735045 ,  0.990746  ]])
```

In [ ]:
```
X_train_array.shape
```

Out[ ]:
```
(38208, 17)
```

In [ ]:
```
# y_train: onehot of y
# lambda_: hyperparameter for regularization (or penalty)
# alpha: learning rate
# theta0: dim is (n, nb_classes) n is number of features including bias term
# return theta
# X, y, theta, lambda_, eps, alpha, max_iter, batch_size for sgd

def train(X_train, y_train, theta0, lambda_, eps, alpha, max_iter, nb_classes
):
  n_features = X_train.shape[1]  # number of features including bias term
  theta, losses = gradient_descent(X_train, y_train, theta0, lambda_, eps, alp
ha, max_iter)
  return theta, losses
```

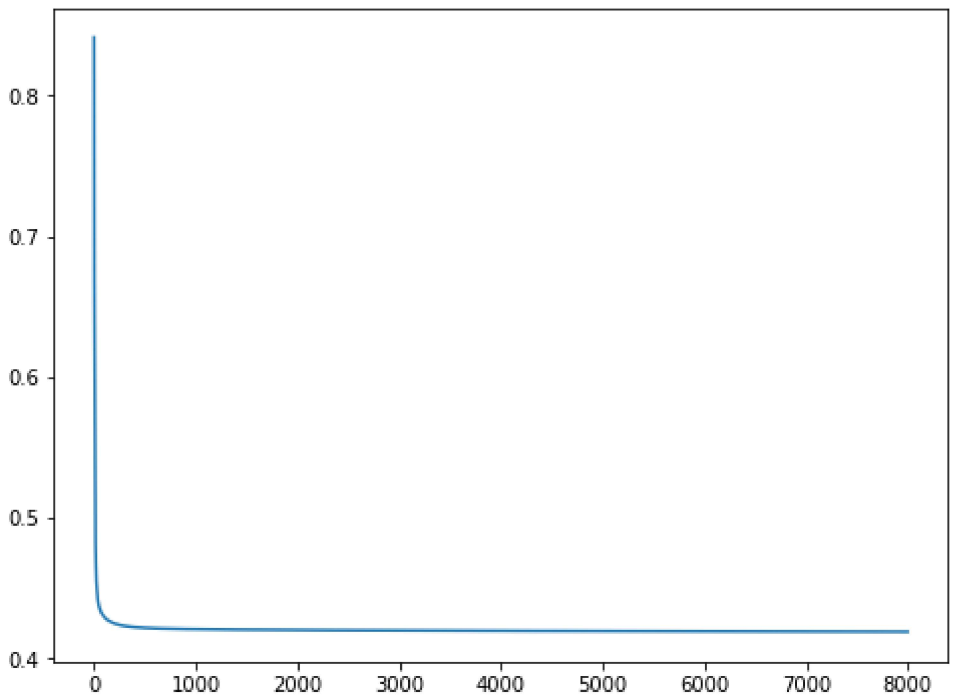
In [ ]:
```
def plot_loss(losses):
  plt.figure(figsize=(8, 6))
  plt.plot([i for i in range(len(losses))], losses)
  plt.show()
```

In [ ]:
```
theta0 = np.zeros((3, 17))
```

In [ ]:
```
# Learning rate = 0.85 and Lambda = 0
final_theta_0_85, losses_0_85 = train(X_train_array, y_onehot, theta0, 0, 10^-6, 0.85, 8000, 3)
```

```
Iteration: Cost
1: 0.84122577
1000: 0.42055552
2000: 0.41996294
3000: 0.41963985
4000: 0.41939887
5000: 0.41919385
6000: 0.41901017
7000: 0.41884251
8000: 0.41868849
```

In [ ]:
```
plot_loss(losses_0_85)
```



In [ ]:
```
def calculate_accuracy(X_test, y_test, theta):
  X_test_array = X_test.to_numpy()
  mat = X_test_array.dot(theta.T)
  y_pred = np.argmax(mat, axis=1)
  y_test_array = y_test.to_numpy()
  accuracy_rate = np.sum(y_test_array == y_pred) / y_test_array.shape[0]
  return accuracy_rate
```

In [ ]:
```
accuracy_on_train = calculate_accuracy(X_train, y_train, final_theta_0_85)
accuracy_on_train
```

Out[ ]: 0.8234401172529313

In [ ]:
```
accuracy_on_val = calculate_accuracy(X_val, y_val, final_theta_0_85)
accuracy_on_val
```

Out[ ]: 0.8140703517587939

In [ ]:
```
accuracy_on_test = calculate_accuracy(X_test, y_test, final_theta_0_85)
accuracy_on_test
```

Out[ ]: 0.8134422110552764

Hyperparameter Tuning

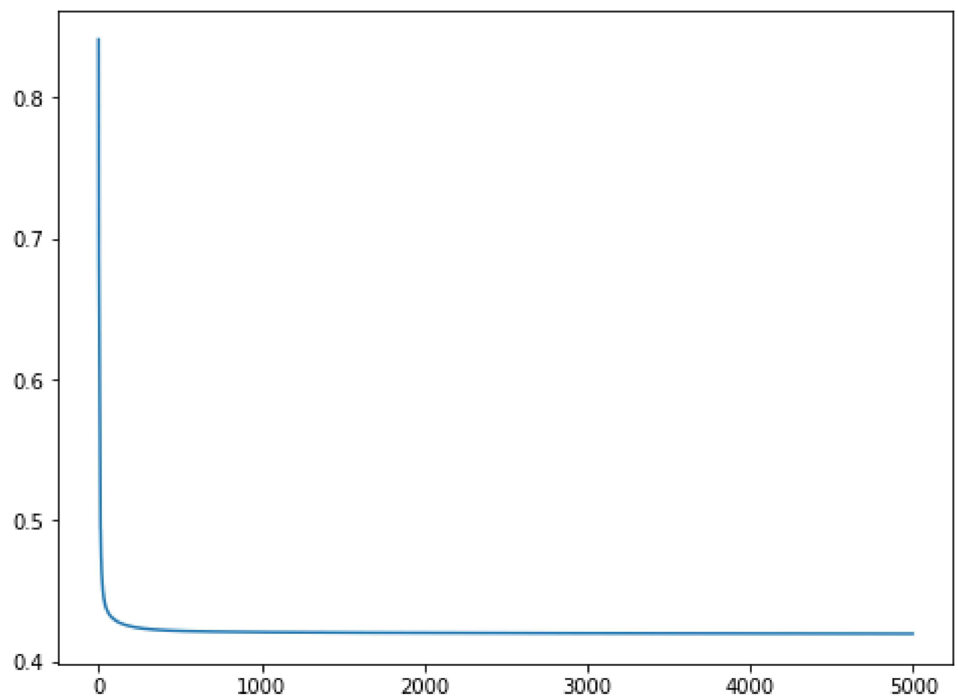
In [ ]:
```
def hyperparameter_tuning(lambda_list, X_train, y_onehot, X_test, y_test, eps, 
alpha, max_iter, nb_classes):
  n = X_train.shape[1]
  all_theta = {}
  all_losses = {}
  print("Hyperparameter tuning: Lambda")
  for each_lambda in lambda_list:
    theta0 = np.zeros((3, 17))
    print(each_lambda)
    theta, loss_dict = train(X_train, y_onehot, theta0, each_lambda, eps, alph
a, max_iter, nb_classes)
    all_theta[each_lambda] = theta
    all_losses[each_lambda] = loss_dict
    accuracy = calculate_accuracy(X_test, y_test, theta)
    print("accuracy for lambda = {}: {:.8f}".format(each_lambda, accuracy))
    print("-------------------------------------------------")

  return all_theta, all_losses
```

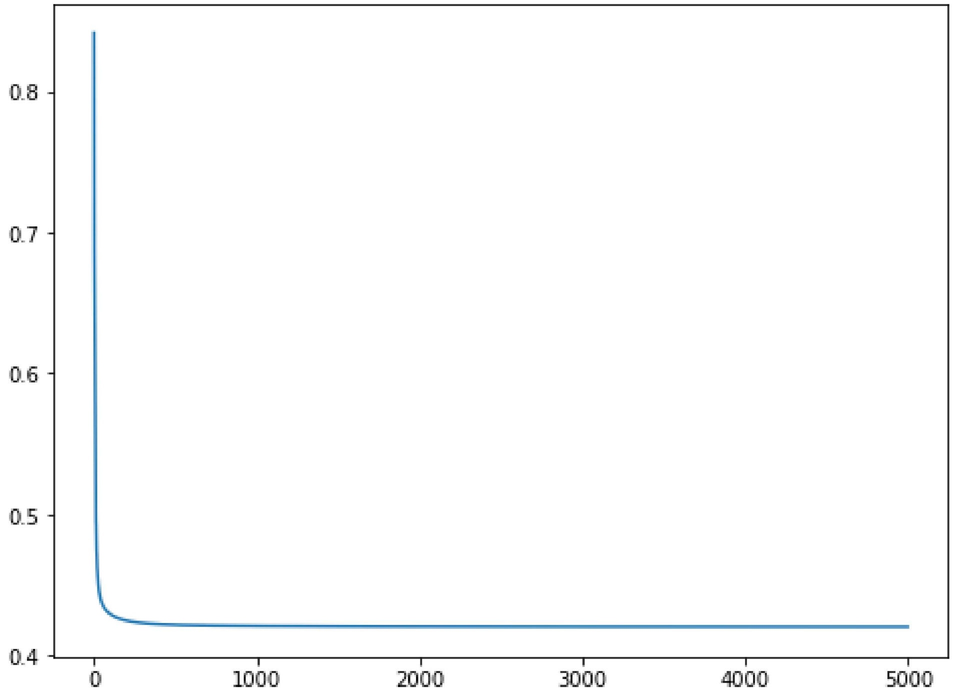
In [ ]:
```
all_theta, all_losses = hyperparameter_tuning([1, 3], X_train, y_onehot, X_val
, y_val, 10^-6, 0.85, 5000, 3)
```

```
Hyperparameter tuning: Lambda
1
Iteration: Cost
1: 0.84122942
1000: 0.42076321
2000: 0.42022391
3000: 0.41994593
4000: 0.41974874
5000: 0.41958812
accuracy for lambda = 1: 0.81344221
-------------------------------------------------
3
Iteration: Cost
1: 0.84123672
1000: 0.42119166
2000: 0.42077951
3000: 0.42062294
4000: 0.42055778
5000: 0.42054738
accuracy for lambda = 3: 0.81386097
-------------------------------------------------
```

In [ ]:
```
plot_loss(all_losses[1])
```



In [ ]:
```
plot_loss(all_losses[3])
```



In [ ]:
```
def check_accuracy(X_train, y_train, X_val, y_val, X_test, y_test, theta):
  train_acc = calculate_accuracy(X_train, y_train, theta)
  print("accuracy on X_train =  {:.8f}".format(train_acc))
  val_acc = calculate_accuracy(X_val, y_val, theta)
  print("accuracy on X_val = {:.8f}".format(val_acc))
  test_acc = calculate_accuracy(X_test, y_test, theta)
  print("accuracy on X_test = {:.8f}".format(test_acc))
```

In [ ]:
```
check_accuracy(X_train, y_train, X_val, y_val, X_test, y_test, all_theta[1])
```

```
accuracy on X_train =  0.82312605
accuracy on X_val = 0.81344221
accuracy on X_test = 0.81344221
```

In [ ]:
```
check_accuracy(X_train, y_train, X_val, y_val, X_test, y_test, all_theta[3])
```

```
accuracy on X_train =  0.82328308
accuracy on X_val = 0.81386097
accuracy on X_test = 0.81323283
```

Prediction

In [ ]:
```
test_data
```

Out[ ]:

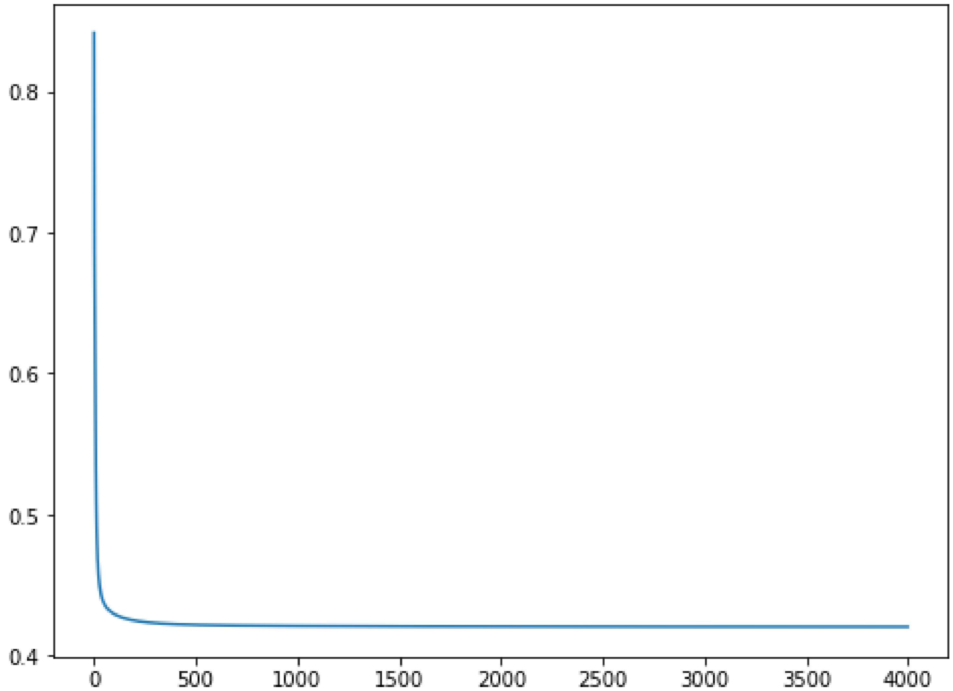
	bias	lat	lon	TMQ	U850	V850	UBOT	VBOT	QREFHT
47760	1.0	-1.229417	2.042923	-0.969527	-0.199640	-0.019019	-0.908190	-0.249263	-1.53332
47761	1.0	-1.229417	2.050537	-1.033650	-0.208858	0.073115	-1.022537	-0.056414	-1.61189
47762	1.0	-1.229417	2.058150	-1.115755	-0.218882	0.146943	-1.088501	0.171496	-1.65935
47763	1.0	-1.229417	2.065764	-1.153955	-0.218557	0.240390	-1.077335	0.400787	-1.70300
47764	1.0	-1.229417	2.073377	-1.182211	-0.208022	0.401727	-1.028380	0.603821	-1.75904
...	...	...	...	...	...	...	...	...	..
55075	1.0	1.178192	0.177646	1.257555	-0.070115	-1.906459	0.752188	-2.054562	1.62259
55076	1.0	1.178192	0.185259	1.331331	0.049520	-2.030020	0.963159	-2.102031	1.73687
55077	1.0	1.178192	0.192872	1.504772	0.199574	-2.240993	1.175167	-2.110102	1.82509
55078	1.0	1.178192	0.200486	1.618416	0.349145	-2.518317	1.395306	-2.125182	1.84868
55079	1.0	1.178192	0.208099	1.677245	0.493976	-2.823162	1.595011	-2.079873	1.94768

7320 rows × 17 columns

In [ ]:
```
n = test_data.shape[1]
nb_classes = 3
theta0 = np.zeros((3, 17))
# X_train, y_onehot, theta0, each_lambda, eps, alpha, max_iter, nb_classes
lambda_ = 3
eps = 10^-6
alpha = 0.85
max_iter = 4000
final_theta, loss_final = train(X_train, y_onehot, theta0, lambda_, eps, alpha
, max_iter, nb_classes)
```

```
Iteration: Cost
1: 0.84123672
1000: 0.42119166
2000: 0.42077951
3000: 0.42062294
4000: 0.42055778
```

In [ ]:
```
plot_loss(loss_final)
```



In [ ]:
```
# accuracy on X_train
calculate_accuracy(X_train, y_train, final_theta)
```

Out[ ]: 0.8234662897822446

In [ ]:
```
# accuracy on split X_val
calculate_accuracy(X_val, y_val, final_theta)
```

Out[ ]: 0.8138609715242882

In [ ]:
```
# accuracy on split X_test
calculate_accuracy(X_test, y_test, final_theta)
```

Out[ ]: 0.8138609715242882

In [ ]:
```
mat_prob_test = test_data.dot(final_theta.T)
mat_prob_test
```

Out[ ]:

	0	1	2
47760	4.556023	-7.667289	3.111266
47761	4.602816	-7.756571	3.153755
47762	4.688239	-7.892798	3.204560
47763	4.742801	-7.966166	3.223365
47764	4.720127	-7.910267	3.190139
...	...	...	...
55075	1.889559	0.603445	-2.493003
55076	1.822941	0.651805	-2.474746
55077	1.709969	0.691091	-2.401060
55078	1.600662	0.744508	-2.345170
55079	1.574041	0.721485	-2.295525

7320 rows × 3 columns

In [ ]:
```
mat_prob_test_array = mat_prob_test.to_numpy()
mat_prob_test_array
```

Out[ ]:
```
array([[ 4.55602338, -7.66728914,  3.11126576],
       [ 4.60281606, -7.75657117,  3.15375511],
       [ 4.68823894, -7.89279848,  3.20455953],
       ...,
       [ 1.70996931,  0.69109068, -2.40105999],
       [ 1.60066155,  0.74450838, -2.34516993],
       [ 1.57404053,  0.72148489, -2.29552542]])
```

In [ ]:
```
pred_test = np.argmax(mat_prob_test_array, axis=1)
pred_test
```

Out[ ]:
```
array([0, 0, 0, ..., 0, 0, 0])
```

In [ ]:
```
label_percentages(pred_test)
```

Out[ ]:
```
((6408, 209, 703),
 (0.8754098360655738, 0.028551912568306012, 0.09603825136612022),
 7320)
```

In [ ]:
```
submission = pd.read_csv('/content/drive/My Drive/Competition/sample_submissio
n.csv')
submission
```

Out[ ]:

	S.No	LABELS
0	0	1
1	1	1
2	2	1
3	3	1
4	4	1
...	...	...
7315	7315	1
7316	7316	1
7317	7317	1
7318	7318	1
7319	7319	1

7320 rows × 2 columns

In [ ]:
```
submission.iloc[:,1] = pred_test
submission
```

Out[ ]:

	S.No	LABELS
0	0	0
1	1	0
2	2	0
3	3	0
4	4	0
...	...	...
7315	7315	0
7316	7316	0
7317	7317	0
7318	7318	0
7319	7319	0

7320 rows × 2 columns

In [ ]:
```
from google.colab import files
submission.to_csv('submission_pred.csv', index=False)
files.download('submission_pred.csv')
```