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Optimal Dynamic Hedging of Equity Options: Residual-Risks, Transaction-Costs, & Conditioning

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Abstract. *Attempted dynamic replication* based valuation of equity options is analyzed using the Optimal Hedge Monte-Carlo (OHMC) method. Detailed here are **(1)** the option hedging strategy and its costs; **(2)** *irreducible* hedging errors associated with realistically fat-tailed & asymmetric return distributions; **(3)** impact of transaction costs on hedging costs and hedge-performance; **(4)** impact of *conditioning* hedging strategy on realized volatility. The asset returns are addressed by the **G**eneral **A**uto-**R**egressive **A**sset **M**odel (GARAM, Wang et al [2009]) that employs *two* stochastic processes to model the return magnitude and sign and results in a realistic term-structure of the fat-tails, dynamic-asymmetry, and clustering of volatility. The relationship between the option price and ensuing return versus risk characteristics of the option seller-hedger & buyer-hedger are described for different conditioning regimes in GARAM. A *hurdle return* is employed to assess bounding *values* of options that reflect hedging costs, the inevitable hedge slippage, & transaction costs. The *hurdle return* can also be used to make relative-value inferences (e.g., by comparing to the return-risk profile of a *delta-1* position in the underlying) or even *fit* option values to *market* while still informing the trader about residual risk and its *asymmetry* between option buyer-hedger and seller-hedger. *Tail-risk* measures are shown to diminish by conditioning the hedging strategy and valuation on realized volatility. The role of fat-tails and uncertainty of realized volatility and its temporal persistence in controlling the optimal hedge ratios, irreducible hedging errors, and option-trading risk premiums are delineated.

Keywords: *options, hedging, kurtosis, skewness, residual-risk, transaction-costs, hurdle-return, risk-capital, volatility trading*

JEL Classification: G13: Contingent pricing; Futures Pricing ; G11: Portfolio Choice; Investment Decisions; D81: Criteria for Decision-Making under Risk & Uncertainty

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1. Introduction

1.1 Motivation

Market demand-supply dynamics establish prices of options, as different market agents decide to participate at prices that suit their distinct risk-reward criteria. Mathematical models typically do not directly address the detailed market dynamics that surround the price of a traded option, as that would require modeling a variety of human imperatives – namely their differing and time-varying and possibly market condition dependent objectives, capacities and capabilities, and their varying greed and fear and how that controls the demand and supply of derivative contracts and their underlying assets. The question then arises:

> ***Do models have any insightful role to play in option trading?***

A positive answer to this question is afforded by the *modern* approach to modeling derivatives, i.e., the Optimal Hedge Monte-Carlo (OHMC) approach (Potters & Bouchaud [2001]; Potters et al [2001]; Bouchaud & Potters [2003]). OHMC helps specify hedging strategies and computes their costs, and the distribution of P&L around the expected value in the *real-world*. By directly establishing the *range* of P&L outcomes while trading an option, the results of OHMC enable interpreting prices in terms of an option trader's greed and fear and market demand and supply. In this paper, we apply OHMC to basic equity options and employ the General Auto-Regressive Asset Model (GARAM) (Wang et al [2009]) to describe the underlying asset values. We focus on **(1)** the option hedging strategy and its costs; **(2)** *irreducible* hedging errors associated with realistically fat-tailed & asymmetric return distributions; **(3)** impact of transaction costs on hedging strategy and performance; **(4)** impact of *conditioning* of hedging strategy and option value on realized volatility. Therefore, our approach to answering the question raised in the previous paragraph in affirmative is as follows:

> ***Developing trading strategies & connecting option prices to market dynamics requires directly addressing real-world hedging costs & residual risks.***

The reluctance of quantitative modelers to explicitly acknowledge that human reaction controlled risk premiums have a role to play in option pricing is understandable. After all, these risk premiums are the domain of psychology of greed and fear that the quantitative modeler typically did not ever try to address. However, that limitation of models is no excuse to *cling* to the patent *falsehood* that the value of an option is the unique price of replicating it via dynamic hedging! Seldom is there a way to perfectly replicate an option payoff by dynamically trading the underlying! (see Bouchaud & Potters [2001]; Derman & Taleb [2005]; Haug & Taleb [2009]). Is it not then ridiculous to insist on interpreting option prices as the outcome of a perfect dynamic replication strategy – and hence equal to an *expectation* of the payoff under a *risk-neutral* measure? We reject this naïve approach followed by "mathematical-finance"[1], in favor of an approach that considers *attempted* dynamic replication, and exposes the *residual hedging*

[1] Derivative valuation focused *mathematical finance* takes simple averages of option payoffs and *fits* the *probabilistic measure* of the underlying to observed prices. Large sell-side institutions have sponsored this line of 'analysis' to facilitate booking day-1 P&L on exotic options using the *probabilistic measure* fitted to vanilla options – assuming their value to be 'risk-neutral' expectations of option payoffs. Remarkably, no reference to residual risk and associated risk-capital is made in this large institution sell-side perpetuated "theory" of valuation!

errors. The purpose of this paper is to illustrate how the alternative approach to *risk-neutral* valuation is practically implemented and how it can simultaneously address trading strategy, option-valuation, and risk management imperatives.

The *risk-neutral* derivative valuation approach is tied to the *belief* of perfect dynamic replication that trivializes the option value to a mere expectation of payoff under the *risk-neutral* measure[2]. The risk-neutral practice of option valuation is therefore a *tautology* based on the patent falsehood of a perfect hedge. We find it disturbing that the *practical finance* field of option valuation is tied to *ontological* arguments – that has spurred a pseudo-sophisticated “mathematical-theory” rather than a practical approach that is tied to the directly observable nature of the underlying and that of hedge performance.

We remind the readers that the two tenets enshrined in the constitution of the *risk-neutral nation*, **(1)** underlying driven by *Brownian Motion*, and **(2)** *perfect dynamic replication*, are demonstrably false in commonplace experience. The ensuing *risk-less* view of derivative trading is vacuous and negligent in its failure to highlight the minimum risk-capital associated with any attempted replication. *Financial Engineering*, based on finding expectations of option payoffs under *risk-neutral measures*, is a farce, as it does not address the real-world experience with attempted replication.

While trading derivatives, barring very few trivial situations (e.g., buy & sell an *identical* contract or sell a put & buy a call and short the underlying – put-call parity[3]) accounting for hedge slippage in the *real-world* and developing a hedging strategy in search of profitability requires a probabilistic description of *real-world* returns. Indeed, such *real-world* probabilistic descriptions of the underlying do not pose any special problem for OHMC, as it is independent of the dynamics of the underlying assets and informs the user about residual risks inherent in attempted replication. The flexibility of such a modern derivative analysis approach encourages the development of *realistic* models of the underlying asset evolution that can combine empirically observed features as well as beliefs about the asset[4].

OHMC is not only applicable to vanilla equity options, rather, it has been applied to a wide range of derivative contracts including structured products with equity & credit underlying. The works of Kapoor and co-workers on the OHMC approach encompasses a range of products and underlying: **(1)** equities based structures, including Cliquets & Multi-Asset Options (Kapoor et

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[2]While Black-Scholes [1973] is novel in demonstrating perfect dynamic replication under idealized conditions - that are not remotely realistic - the subsequent *risk-neutral* “*analysis*” has resulted in a blind-faith based valuation framework – invoking theoretical *measures* that cannot be directly observed even in principle. Most *desk quants* spend their time evaluating averages of option payoffs under fitted *measures* with no consideration of hedge performance – and students coming out of business programs often parrot this as the gospel truth! Such a risk-neutral valuation focus has brought disrepute on the *valuation quant* profession as being removed from *reality* and often unwittingly aiding and abetting the taking of unmeasured *risks* while trading derivatives.

[3] Demand-supply variability precludes put-call parity as the sole determinant of price in option markets. What if the demand for puts is larger than the availability of calls to facilitate that approach to hedging? Indeed, the risk-premium associated with options indicates such demand-supply imbalances, and is an avenue for designing investment strategies with interesting risk-return tradeoffs.

[4] In contrast, the *risk-neutral* approach is based on the contrivance of a *perfect hedge* that is based on a narrow and unrealistic description of the underlying. Such *risk-blind* modeling is associated with the lack of risk management and the concomitant widespread *heist* of risk-capital by derivative trading.

al [2003], Petrelli et al [2008] & [2009]); **(2)** derivatives with credit underlying, such as CDS swaptions, & CDOs (Petrelli et al [2006] & Zhang et al [2007]). In all these aforementioned works, hedge slippage en-route to *attempted replication* of derivative contracts was assessed, in addition to the *expected* hedging costs, and *real-world* descriptions of the underlying with fat-tails and jumps to default were considered.

Finding “risk-neutral expectations” of option contract payoffs under arbitrary descriptions of the underlying process to simply fit observed prices does not yield new insights into option markets and the human-market interactions that, ostensibly, have a role to play in establishing the traded prices of options. These formal risk-neutral expectations simply do not describe hedge performance – or even admit to the hedge slippage that would occur in the real world if the underlying followed some of the chosen mathematical processes (e.g., jump-diffusion or variance-gamma processes). A more fruitful line of analysis is to attempt to assess option risk premiums by assessing the attempted replicating strategies *real-world* P&L distribution. Then the option value can be seen to set a return on risk capital or a Sharpe-Ratio (or any other return per unit risk metric). To pursue this line of analysis of options requires a *realistic* model of the underlying. Our recently communicated model, GARAM (Wang et al [2009]), serves this purpose. Other models that represent the probabilistic features of equity assets pertinent to understanding *real-world* option trading are based on multi-fractal cascades (see Borland et al [2009]; Pochart and Bouchaud [2002]; Bouchaud et al, [2001]; Muzy et al [2000]).

Our analysis explicitly accounts for transaction costs and addresses the irreducible *hedge slippage*. To assess risk-premiums associated with hedging realistically jumpy assets, we quantify the expected change in wealth per unit risk measures (e.g., risk-capital; volatility; downside volatility). Now this is where it all comes together- what is a *fair* risk-premium? We do not think there can be, or will be, or even should be, a unique answer to this question! There are a *variety* of arguments for a risk premium at which a trade becomes attractive to *different* market agents. Certainly we would personally not sell and dynamically hedge an option if the expected return per unit risk for doing so is not demonstrably higher than that of a delta-1 trade.

The risk-return of a trader holding a long position in the underlying asset can impose a bounding *hurdle return* for the derivatives trader attempting to replicate. Absolute return hurdles can also be set by the party that supplies the risk-capital[5] (without which one *should not* be able to sell an option). These hurdle returns yield bounds on the option price that reflects the residual risks in addition to the average hedging costs associated with *attempted replication*. In this work, both the option buyer’s and seller’s bounds on valuations are calculated to assess the *range* of option values associated with irreducible hedging errors and transaction costs.

[5] A useful thought experiment to see the connection between valuation, profitability, and residual risk is to try to design a *business-plan* on selling and dynamically hedging vanilla derivatives by trading the underlying. The margin requirements for that activity on an exchange will be driven by the residual risks inherent in such a strategy. Imagine pitching this plan to Warren Buffet to raise capital. Warren’s well known penchant for a *good-deal* can only be met if the option is sold sufficiently *above the expected costs of hedging*, so that he can *expect* return on his capital that is used to post margin for the trading strategy. This simple business reality eludes the *risk-neutral* “mathematical-finance” framework of option valuation with all its *mathematical sophistication!*

As volatility exhibits persistence in time, we examine the impact of conditioning the hedge ratios on spot asset value and realized volatility. We present results on buy and sell positions on Calls and Puts of different maturities and strikes. By quantifying the risk-return of option trading while explicitly accounting for the impact of realized volatility on option pricing and hedging, and employing an empirically realistic description of realized volatility, this work provides a computational platform for *volatility trading.*

1.2 Hedging Financial Derivatives

Hedging Forward Contracts With Underlying

The forward contract is perhaps the only example of a derivative contract where perfect replication is feasible – in principle (!)– and via static hedging. Consider the change in wealth of the forward contract seller, that agrees to deliver n stocks to the forward contract buyer, for an amount F per stock, adjusted for transaction costs. To attempt to *replicate* the forward contract payoff, a trader buys n units of the reference asset and holds it over the time interval $(t_0, T]$. The trader pays transactions costs, financing costs, and receives the dividends from the asset – of an amount denoted by π_i at time instances $t_i \in (t_0, T]$. The change in wealth of the static "Delta-One" trade attempting replication is given by

$$\Delta W_{t_0}^{F_{seller}}(t_0,T) = \left[n\left(F(t_0,T) - \delta_F\right) - \chi_F - \frac{n\left(s(t_0)+\delta_s\right)+\chi_s}{DF(t_0,T)} \right] df(t_0,T) + n\pi_i df(t_0,t_i)$$

In the wealth change representation above, the absolute difference between a "mid" price and a bid/ask price is denoted by δ_F and δ_s for the forward contract and the underlying asset respectively. These *spreads* result in transaction costs that increase with the number of contracts traded. Fixed transaction costs (i.e., independent of the number of shares traded) associated with trading the forward contract and the underlying equity are denoted by χ_F and χ_s. The concept of time value of money is effected via the discount factors $df(t_a, t_b)$ which may be based on basic benchmarks (Treasuries or, say LIBOR). The cost of financing positions may involve a spread over a benchmark, and that is represented in the *funding discount factor* $DF(t_a, t_b)$.

The perfect replication mandate is to have no change in wealth and meet ones obligations of the derivative contract – while accounting for the time-value of money - regardless of the outcome of the underlying:

$$\Delta W_{t_0}^{F_{seller}}(t_0,T) = 0 \Rightarrow \qquad F(t_0,T) = \delta_F + \frac{\chi_F}{n} + \left[\frac{\left(s(t_0)+\delta_s\right)+\chi_s/n}{DF(t_0,T)} \right] - \frac{\pi_i df(t_0,t_i)}{df(t_0,T)}$$

Although this treatment of the forward contract is trivial, it is the starting point of the *idea* that a derivative contract seller can limit the effect of the impact of uncertainty of the payoff he has to make, by holding a position in the asset underlying the derivative. Also, the change in wealth formulation made here is a useful precursor to the more ambitious goal of OHMC undertaken in the subsequent sections of this paper.

We would be remiss in not pointing out that despite its formulaic simplicity, even a forward contract attempted replication is *imperfect* due to uncertainty in dividend payments. Also, the cost of financing for different market agents is different and so can the tax treatment of dividend payments be different. These fundamental and practical impediments to a *unique* price of a derivative are associated with the role of demand and supply in mediating its price and an inherent *variety* in possible prices of derivative contracts. Attempted replication and demand & supply are not mutually exclusive dynamics in determining derivative prices. Quite to the contrary, we think that it is the interaction of these dynamics what makes derivative trading interesting, and potentially profitable, and of course, always requiring weighing potential returns with inevitable risks.

Hedging Options with Options

A broker buying and selling options with no capacity for taking any risk has to buy and sell *identical* contracts and pocket the bid-offer spread, or buy and sell positions that replicate a forward contract (i.e., recognizing put-call parity) that can at least theoretically be replicated by a static position in the underlying. This relatively controlled risk approach to option trading is only possible if the supply-demand are in equilibrium, and the bid-offer spreads will determine the income/loss potential.

There are other basic options that are simple combinations of two options. For example, a put spread or a call spread involves selling and buying puts or calls at two different strikes. Then there are more complex options that *appear* as combinations of multiple options. There are options embedded in structured products that involve look-backs – some of them are called Cliquets (or ratchets). For examples an *85 strike knock-out Cliquet put* pays the investor the amount by which the underlying has dropped below 85% of its value at a prior look-back period and knocks-out after the first occurrence of the payout condition. The periodicity of checking the drop is the roll period. This could possibly be hedged by other more basic forward starting options. However, there is no visible two-way market for *forward starting options*!

This look-back type of option is embedded in many structured products - some with large maturities. There are different variants of look-backs – some looking back to peak-values, and some involving multiple assets simultaneously. Some derivative contracts involve such exotic look-backs and are also coupled with life-insurance (e.g., *variable-annuity*).

In the completely naïve application of *the risk-neutral routine*, an implied volatility *surface* is fitted to vanilla options and that is used to value exotics. For multi-asset options additional correlation inputs are also made – some fitted to basic multi-asset options. The main problem with this approach is that the instruments being used to build the implied volatility surface do not have risk-sensitivities that are similar to the ones being mis-modeled! Specifically, a Cliquet has sensitivity to forward starting volatility that cannot conceivably be captured by a series of fixed strike puts or calls of different maturities and strikes. To address some of these concerns the risk-neutral quants have used *stochastic volatility models.* However these models are of no avail without understanding the risk return of a forward starting option – and that option does not exist as a visible tradable.

Often, the main treachery in exotic options is the dominance of risks that can't be hedged. Stuck to their Brownian motion and other-worldly "Q measures," the risk neutral quants fail to admit that these exotics involve irreducible risks! Forward starting volatility risk in Cliquets and their attempted modeling in the *risk-neutral* framework invoking *complete markets* and *perfect replication* is a compelling exhibit of the lunacy of risk-neutral modeling. This *risk-neutral* farce is not without consequence – as this results in P&L recognition without sizing of risk-capital, or even a modest understanding of risk-return. This practice is not only scientifically bankrupt (despite it being the *prevailing regime* in valuation focused *mathematical finance*) – it can also bring its naïve adherents closer to financial bankruptcy! Even non-adherents to this false dogma are influenced when they are negatively impacted through the systemic consequences of a complete lack of sizing of risks by large market participants – as their counterparty rating or credit spread is not always accounting for such obfuscation about risk exposures.

The list of options that can be perfectly hedged by other options is limited. Moreover, that still does not address the question of how those basic options should be hedged and valued?!

Leaving aside the limited realm of perfect static hedging, one can still consider hedging options with other options. The idea is that the delta hedge can be supplemented by a gamma hedge and a vega hedge. A gamma hedge recognizes the non-linearity of the option payoff with respect to the underlying. A vega hedge is somewhat more convoluted because it potentially wraps together the two different important effects of changing risks and changing risk-premiums:

(1) Realized volatility - a surrogate for risk - is not a constant. Therefore it is natural for market agents to ask more for manufacturing an option in a high realized volatility environment. A rise in implied volatility in of itself does not always indicate an increase in the market price of risk.

(2) The option seller can chose to ask more for manufacturing an option, even if risks and realized volatility are asymptomatic. This can reflect his sudden desire for higher compensation for risk, as well as his responding to a greater demand for options than his available supply.

Clearly **(1)** and **(2)** are different dynamics – but they get inseparably lumped together in the risk-neutral framework that simply fits an implied volatility to a price. This is central to the lack of understanding of risk-return of option trading engendered by the risk-neutral approach. In the fitting mathematics that the risk-neutral 'methodology' embodies, the implied parameters are solely motivated to fit a price – while they clearly embody two distinct dynamics – namely realized risk dynamics and the market greed and fear dynamics. Our work furthers an approach that clearly separates the dynamics **(1)** and **(2)** and hence results in methodologies for *quantification* of option trading risk-premiums that can be used to develop trading strategies.

The OHMC formulation made here encourages exploring hedging options with options, by forwarding a framework that can be applied to a portfolio of options of different strikes and tenors. However we make no pretense of perfect replication, and, we do not ascribe clairvoyance to any mythical volatility surface in being able to forecast the value of options in the future!! In the absence of *blind-faith* about perfect replication and clairvoyant implied volatility surfaces, pursuing attempted replication and discerning residual risks is challenging but worthwhile, as it can result in a meaningful documentation of option risk premiums.

Hedging Options With Position in Underlying

A broker or dealer faced with a demand for, say, a put-option, and with no supplier for it (or for a call-option to exploit put-call parity), cannot pocket the bid-offer spread.   Such a supply-demand imbalance spurred other ways to attempt to "manufacture" the derivative payoff practically and theoretically. The *science fiction* solution was provided by Black-Scholes – a purported recipe for manufacturing the payoff with *certain* costs. We call it a *science* insofar as it is reasonable to *attempt* to mimic payoffs and assess their costs and to *hedge* an option position with a position in the underlying, as suggested by the forward contract argument. We call it *fiction* because there is no asset for which this model works! Where risk-return are even roughly understood, option sellers need capital that is many-many multiples of the average cost of attempted replication. A materially significant chance of significant hedge slippage is the rule rather than the exception! Where risk-return are completely unknown – risk-capital remains un-sized, and severe outcomes occur due to the unchecked accumulation of risks (e.g., the credit derivative bubble burst).

Superficial *valuation modeling* that sidesteps questions of hedging errors seems to serve the purpose of those who get paid on day-1 differences between model price and execution price. Some market players associated with derivatives may simply thrive on the non-estimation of real risks associated with *exotic* structured products and hence sponsor a modeling paradigm that is explicitly based on assuming perfect dynamic replication and does not quantify risk-return tradeoffs.

1.3 Ecology of Contemporary Derivative Analysis

The dysfunctional state of the mainstream option analysis can be better understood by recognizing the players involved and their motivations, predilections, and their reward systems. We present a cartoon of that in **Figure 1** and describe it further in this section.

"Risk-Neutral" Regime

An army of theoreticians has embraced the formal "risk-neutral" approach to valuing derivative contracts - the underlying is *de-trended* and an average of the option payoff is taken and called the "arbitrage-free value." In awe of *mathematical finance*, business-school programs are teaching this theory without sufficient examination of its assumptions and implications. The presumption is that risk-neutral expectations provide the unique cost of replicating the option payoff *perfectly*. This approach has been shown only to work theoretically only when the underlying asset does not exhibit any excess kurtosis. But it is mindlessly used by "risk-neutral quants"[6] for models of situations that obviously defy perfect replication, such as underlying following *jump-diffusion* processes, and even credit derivatives where payoffs only occur following a jump-to-default!

Even the most innocuous assets exhibits excess kurtosis (e.g., S&P500 daily return) - so perfect replication is unattainable. Not enough has been done in mainstream *valuation-modeling* by way of documentation of the limits of replication. Instead, there are plenty of distortions of Black-Scholes [1973] old results to fit prices. Multiple incarnations of volatility have been created to fit prices and "Q" measures (i.e., presumed *risk-neutral* measures governing *precisely* and *perfectly* replicating derivative trading!) are invoked without articulating the mechanics of replication. While perfect replication is thwarted at the whiff of jumpiness in asset returns (excess kurtosis), even when the traditional *valuation models* employ a fat tailed underlying process, they are used to fit prices rather than to document irreducible hedging errors. *Risk-neutral* "jump-diffusion" models are fit to prices without quantifying the residual risk arising due to jump, or by assuming jumps to be a diversifiable risk factor! Mainstream derivative valuation modeling has been clinging to the orthodoxy of perfect replication – in the face of incontrovertible evidence to the contrary. The practical need to quantify the risk-return of attempted replication strategies is not served by the *risk-neutral* model.

[6] *Risk neutral quants* generally report to the "business" and are tasked to provide *easy* and *fast* models that *fit* vanilla options. These models are used to declare day-1 profits that reimburse the personnel responsible for bringing in the business. These models do not admit to replication errors endemic to attempted option replication! By *valuing* derivative contracts *without assessing risk capital*, the risk neutral modeling framework contributes to the lack of effective risk management revealed in many financial institutions, and the regular "surprising" blowups associated with complex derivatives.

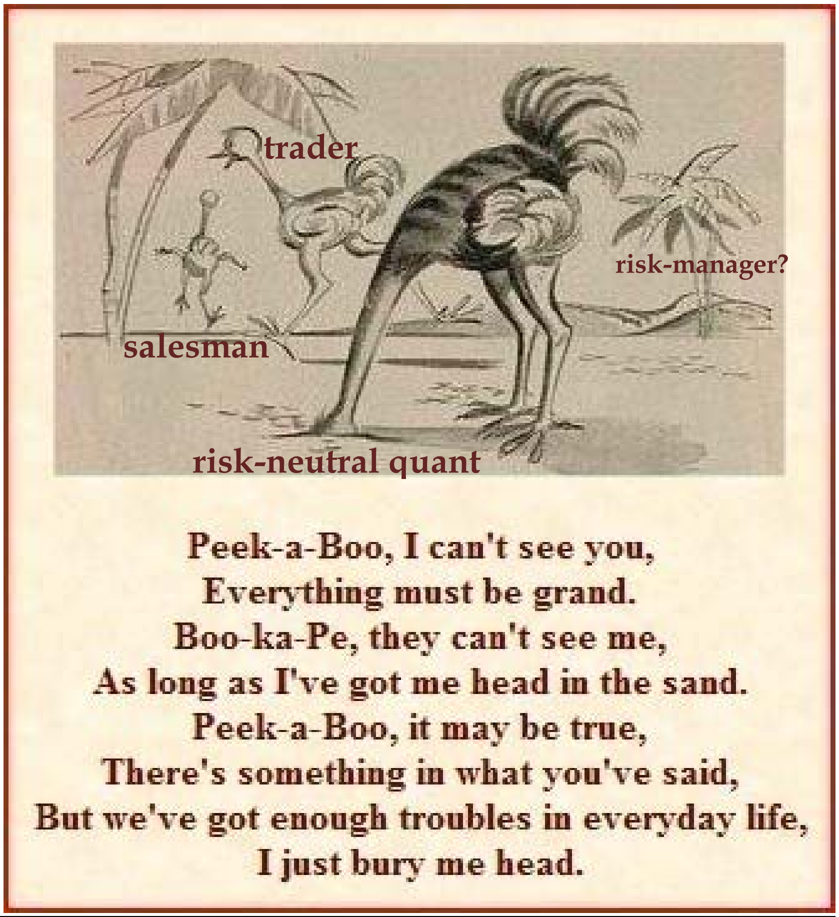

Figure 1. The dysfunctional risk-neutral regime of derivative "analysis."

Derivative traders & risk managers need tools to describe errors endemic to attempted replication in addition to day 1 P&L and carry in a trade!? *Risk-neutral formalism*-based valuation models presume perfect replication & fail to advertise irreducible hedging errors endemic to attempted derivative replication. The practice of fitting parameters to vanilla derivatives and using those parameters to *value* exotics without addressing irreducible hedging errors does not help develop an understanding of risks and associated premiums of *exotics* or vanilla options. The ensuing model-based *upfront* P&L-driven motives for entering into derivative trades are rendered particularly dangerous due to the lack of concurrent analysis of residual risks. By failing to directly address residual risks and therefore the risk-capital of a derivative trade, the risk neutral regime has resulted in a remarkably poor risk-management practice associated with derivative trading. The risk-neutral quant is depicted as searching for some loony other-worldly *measures* to mindlessly *fit* his model, while the *real-world* risks are being taken by sales and trading. The risk-manager is wondering what is going on - from a distance.

credits: original image of ostriches is from http://users.cybertime.net/~ajgood/ostritch.html

An example of the high-handedness of the risk neutral modeling regime is the insistence that a particular *implied volatility surface* reflects the unique costs of replicating the associated vanilla options and therefore must be necessary and sufficient to "pricing" and "valuing" any exotic derivative. This complete denial of irreducible hedging errors of vanilla options in the interpretation of the implied volatility surface supports the caricature of the risk neutral regime sponsoring *mathematical finance* being like a despotic regime in the third world – offering little of value to its citizens, and known mainly for "*its dogmatism and resoluteness*" (Triana [2009]). This dogmatism in valuing exotic derivatives has not resulted in understanding the special risk premiums pertinent to exotic derivatives (e.g., Cliquets, barrier options etc) and is sustained by the way P&L is booked - without assessing hedge slippage & the associated risk capital.

Traders, Accountants, & their "Valuation Quants"

Traders have apparently not explained their real risks or trades to their *valuation quant* – while trading assistants are commanded to book trades into *official* valuation models that are built on the materially false assumption of perfect hedging! The modeled fiction of perfect replication and the associated *unique* price has taken hold of accounting communities, aided and abetted by an academic focus on a *clean* problem that is dangerously and qualitatively different from the real problem. Valuation focused academics still revel in describing "self-financing" models associated with perfect replication, and have done little to address the risk-capital requirements of a derivatives trader, and the relationship with counterparty risk incurred by the option purchaser.

Traders with insights into limits of replication have had little incentive to communicate to the financial community at large that the post-Black Scholes ornate model fitting game has gone on at the cost of quantifying residual risk and developing informed views on risk-return dynamics. At issue is not a higher order correction to pricing models. A lack of quantification of risks associated with option trades and a lack of estimation of risk capital has resulted from the formal risk-neutral models unexamined application. The self-serving sponsorship of valuation models that are based on assuming perfect hedges is a corrupting influence that has stunted the development of realistic and credible derivative analysis tools.

Risk Management?[7]

The builders of *risk-neutral valuation models* take no responsibility for the risk of derivative trading books – they are seldom aware of the rationale for the trades! Contemporary & formal risk-neutral modeling seems to be solely driven by the need to book trades and ascribe certain costs to hedging (*perfect replication*) and to recognize into day 1 P&L any payment received in excess of that amount (i.e., *arbitrage!*).

[7]In large financial institutions the roles of *risk-management* include model *validation*, market-risk, credit-risk, counterparty-risk, risk-capital assessment, regulatory reporting etc.

Vanilla options that trade on exchanges require no valuation model to perform fair and competent accounting. Even these vanilla options have significant residual risk in any attempted dynamic replication strategy. After fitting the parameters of risk-neutral models to vanilla option prices, exotic options become the main purpose for creating the easy to fit "risk-neutral" model. However, the presumption of perfect replication for exotic options is even more outlandish than it is for vanilla options! Notwithstanding the convenience and single mindedness of risk neutral models, they do not shed any light on irreducible hedging errors, while outputting a purported unique option value!

The periodic debacles with derivative trading are in part due to the poor fabric of risk management that results from the use of contemporary and mainstream quantitative valuation models. The standard risk-sensitivities, i.e., the vegas, deltas, and gammas can be rendered quite ineffective in a complex book due to sensitivity hedging and the attendant basis risks (tenor barbells, numerous cross-gammas,……) and no direct quantification of the unavoidable residual risk *en-route* to valuation. When the poor state of risk management is periodically and sometimes catastrophically revealed it is often castigated by valuation quants as a "breakdown of the risk management model" (often there is no "risk-management model"!) or confused with operational-systems issues ("improperly booked trade").

The risk-neutral derivative valuation model has not received enough scrutiny from disinterested yet critically informed parties with respect to its role in building materially false expectations of hedge performance and the impact that has on the quality of risk management of financial institutions. The potential for recognition into P&L of any difference between the traded price and modeled price is based on the *assumption* of the valuation representing the unique hedging costs – that is what makes the use of these naïve models dangerous. While attempting to hedge is not the source of the mischief, *assuming* a unique cost associated with a perfect hedge is a poor starting point for accounting and risk-management.

The lack of highlighting of the leading order risks inherent to any attempted derivative replication is an incompleteness of the Black-Scholes model that has become increasingly dangerous as the volume and complexity of derivatives have increased. The lack of risk management has resulted in credence to the vilification of derivative trading in the media. To thwart that negative perception requires developing a widespread understanding of residual risks associated with trading & hedging derivatives and the associated linkages with risk capital and important counterparty risk inherent to any derivative trade.

1.4 Optimal Hedge Monte-Carlo Approach

The OHMC approach puts itself in the seat of a derivatives trader and attempts to keep a trading book as flat as possible between two hedging intervals. Viewing the mark of the derivative and hedge-ratio as *functions* of spot, and possibly other conditioning variables, OHMC seeks to find the *functions* to accomplish its objectives. If it turns out that the derivative can be perfectly replicated then the residual risk will be zero, and the solution will correspond to the *risk-neutral* solution. This perfect replication can only be achieved if the assets are driven from a Brownian motion process that has no excess kurtosis, and one hedges continuously without any transaction costs.

The premise of continuous hedging is entirely misleading for assets that exhibit jumpiness (they all do!), as the residual risk achieves an irreducible value and after a point hedging more often doesn't decrease risks (see Petrelli et al [2008] for an explicit demonstration of this). Incessant hedging would only explode the transaction costs on top of the irreducible hedging error! OHMC helps address hedging while accounting for transaction costs and the irreducible hedging error simultaneously (see **Figure 2**).

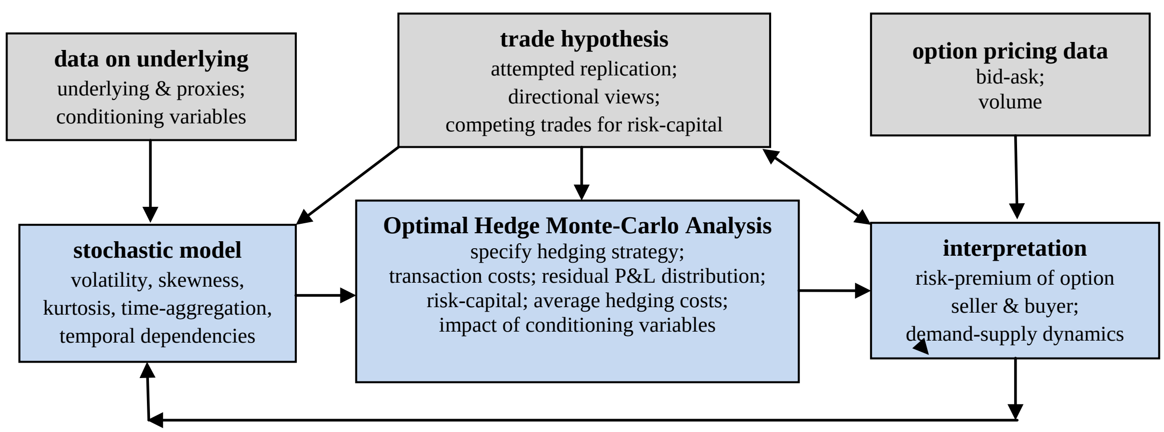

Figure 2. Information flows in Optimal Hedge Monte-Carlo based option trading analysis

We believe that the naiveté of perfect replication has to be discarded by *modern derivative trading* - which has to embrace estimating *attempted replication costs* as much as estimating *irreducible hedging errors*. The forces dictating an increase in the risk awareness of option valuation models are: **(1)** broadly disseminated derivative trading risk-management mishaps; **(2)** advent of financial instruments that enable directly monetizing risk-premiums associated with derivative trading (e.g., variance swaps, covariance swaps); **(3)** proliferation of risk-return savvy hedge funds that are explicitly aware of the relationship between an option price, risk-capital, and profit margin and are on the hunt to exploit slovenly participants who "do not get derivatives;" **(4)** increase in computational power and access to market data; **(5)** documentation of the residual-risks of dynamic hedging schemes.

A comparison of the current *risk-neutral regime* and the modern OHMC approach is given in **Table 1**.

item	Risk-Neutral Model	Optimal Hedge Monte-Carlo
Assume perfect replication	√	X
Assess role of fat-tails (i.e, jumpiness) on hedge performance	X	√
Assess role of uncertainty of realized volatility on hedging	X	√
Ensure option price fits market	√*	√ (so what?)**
Assess residual risk measures employing realistic description of underlying	X	√
Assess option price implied option risk premium	X	√
Develop hedging strategy & assess price at which expected return per risk metric meets hurdle	X	√
Help relate bid-offer to derivative details & underlying	X	√
Incorporate transaction costs and realistic hedge frequency	X	√

Table 1. Comparison of Optimal Hedge Monte Carlo approach to derivative analysis with the risk-neutral approach

*By assuming away residual risks, fitting prices seems to be the only pursuit of "risk-neutral" models. Consequently, upfront P&L can be recognized without assessing risk-capital that hedge slippage must attract!

**OHMC computes hedging costs and hedge slippage, and helps recognize the range of P&L outcomes. "Market-implied" return on risk capital or other risk measures can be inferred from vanilla option bid-offer via OHMC. Exotic OTC options have little market price visibility and are often marked using risk-neutral models with parameters fitted to vanilla options.

The work of Bouchaud and co-workers (Potters & Bouchaud [2001]; Potters et al [2001]; Bouchaud & Potters [2003]) provides a notable exception to works that evoked the critical commentary we make on mainstream derivative valuation modeling. They have pioneered the OHMC method and provided a documentation of hedging errors while employing a variety of models of the underlying, including directly using historical return data. Their work provides a rich commentary and analysis that explains the power and limitations of Black-Scholes results in a real world hedging application.

The series of works by Kapoor and co-workers on optimal hedging - applying OHMC to different derivative problems - are motivated by the following:

(1) understanding the risk-return of derivative trading strategies
(2) developing buy-sell signals in option trading
(3) developing risk metrics that are explicitly cognizant of attempted replication & its limitations.

These works have also expanded the set of financial derivatives for which the irreducible errors in attempting replication are documented. From the perspective of product classes the work of Kapoor and co-workers can be divided as follows

(1) structured products with equity underlying modeled with GARCH models (Kapoor et al [2003]);
(2) static hedging default-risk in CDOs (Petrelli et al [2006]);
(3) dynamic hedging of CDS options (Zhang et al [2007];
(4) Crash-Cliquets (Petrelli et al, [2008]);
(5) Multi-Asset Options (Petrelli et al [2009]).

These works have explored relative value arguments to try to interpret the pricing of derivatives, given that they can never be replicated without error. This paper extends prior analysis by simultaneously addressing transaction costs, risk-premiums, and conditioning the hedging strategy on recently realized volatility for European style Calls and Puts.

1.5 Organization

In **Section 2** the formulation for OHMC is presented. That formulation addresses **(1)** residual risk dependent risk premium; **(2)** transactions costs; and **(3)** dynamically conditioning of hedging strategy and valuation on realized volatility. **Section 3** presents risk-return metrics of a dynamic hedger attempting to replicate an option payoff. In **Section 4** examples and sample calculations are presented. The examples employ our new equity return stochastic model GARAM (Wang et al, [2009]). Key findings of this work are also summarized in **Section 4**. A discussion of the broad implications of this work is provided in **Section 5**.

The appendixes present details and simple cases in support of the analysis in the main sections.

- In **Appendix-A**, we present details of static hedging.
- In **Appendix-B** we present the Basis Functions employed to render the hedging variational problem amenable to a computational solution.
- In **Appendix-C** details of the OHMC numerical solution are presented.
- In **Appendix-D** we document our concept of a hurdle on expected return on risk capital and other risk measures. This is important for derivative trading – and has remained an underdeveloped topic as much 'valuation' modeling over the years has conveniently assumed the existence of a perfect hedge and has not furthered the science and art of expressing relative value arguments that are cognizant of the limits of attempted replication.

2. Optimal Hedge Monte-Carlo Formulation

OHMC analyses the P&L of a derivatives trader in the real-world. In OHMC, all statistical measures invoked in describing the uncertainty of the P&L are real-world measures. We make no loony references to *measures* in *other worlds* where market participants are indifferent to risks[8], because that has no bearing on trading, risk-management, and developing investment strategies in the *real-world*. OHMC is focused on determining the hedge ratio and option value functions of spot and other conditioning variables:

Hedge Ratio $\phi\left(s(t_k),\varpi_1(t_k),\varpi_2(t_k),\ldots\ldots,\varpi_J(t_k),t_k\right)$

spot-value conditioning variables hedging-time

Option Value $C\left(s(t_k),\varpi_1(t_k),\varpi_2(t_k),\ldots\ldots,\varpi_J(t_k),t_k\right)$

The functions $\phi\left(s(t_k),\varpi_1(t_k),\varpi_2(t_k),\ldots\ldots,\varpi_J(t_k),t_k\right)$ and $C\left(s(t_k),\varpi_1(t_k),\varpi_2(t_k),\ldots\ldots,\varpi_J(t_k),t_k\right)$ are found to accomplish certain trading P&L objectives in *attempting* to replicate an option payoff. The underlying asset value represented by $s(t_k)$ is allowed to follow any stochastic evolution – the OHMC formulation does not restrict it or rely on specific unrealistic forms (e.g., Brownian Motion). For brevity, going forward in this paper, we describe the OHMC formulation with one additional conditioning variable, other than the spot value itself, i.e., we focus on determining the functions $\phi\left(s(t_k),\varpi(t_k),t_k\right)$ and $C\left(s(t_k),\varpi(t_k),t_k\right)$.

2.1 Option Hedger's Wealth Change

Consider the P&L of a derivatives trader over the time interval $(t_k, t_{k+1}]$. The option trader's P&L on account of the option sell position is

$$\begin{gathered}\Delta W_{t_k}^{option}\left(t_k,t_{k+1}\right)=C\left(s(t_k),\varpi(t_k),t_k\right)-G(t_k)\\ G(t_k)=C\left(s(t_{k+1}),\varpi(t_{k+1}),t_{k+1}\right)df\left(t_k,t_{k+1}\right)+P(t_{k,i})df\left(t_k,t_{k,i}\right)\end{gathered}\qquad(1)$$

[8] The run of the mill *valuation quant* mantra that the real world probabilistic descriptions of the assets are not needed to *value* derivatives is consistent with their denial of replication errors. This misleading *idea* is repeated by those who stand to lose if the replication errors of their derivative trades were better understood – and often masked with a pseudo sophisticated reference to *risk-neutrality* and associated paraphernalia ("Q" measure; martingale…#@&*) while ignoring the *commonplace* experience of hedging errors experienced by anyone who has ever attempted to hedge/replicate by trading the asset underlying a derivative. This has turned mainstream valuation modeling into a tool of obfuscation & mindless fitting that has not resulted in any improved understanding of risk-return of derivative contracts or realistic stochastic models of assets.

The value of the option is denoted by C, which is explicitly dependent on the asset value $s(t_k)$ and another conditioning variable $\varpi(t_k)$, in addition to time t_k. Risk-free discounting is effected through the discount factors denoted by df. The payoffs promised in the option contract are denoted by $P(t_{k,i})$. To account for the possibility of a series of payoffs associated with either a basket of options or a single option with a more complex payoff, we identify the time instances of possible payoffs by $t_{k,i} \in (t_k, t_{k+1}]$, and a sum over the repeated index i in (1) incorporates all the payoffs relevant to the hedging interval $(t_k, t_{k+1}]$.

The wealth change of the option position expressed in the form (1) is amenable to any option (e.g., Cliquets, Barrier options etc) by incorporating cash-accruals for running premiums, and path-dependent indicator functions to effect knock-in or knock-out conditions. OHMC is readily amenable to complex options.

To *attempt to* hedge the P&L driven from the option position over the hedge interval $(t_k, t_{k+1}]$, the trader holds $\phi(s(t_k), \varpi(t_k), t_k)$ amount of the asset. Over the hedge-interval, the trader pays transactions costs, financing costs, and receives the dividends of the asset (π_i) at time instances $t_i \in (t_k, t_{k+1}]$. Neglecting the transactions costs for the moment, the P&L generated by holding one unit of the underlying asset as an hedge is $H(t_k)$:

$$H(t_k) = \left[s(t_{k+1}) - \frac{s(t_k)}{DF(t_k, t_{k+1})} \right] df(t_k, t_{k+1}) + \pi_i df(t_k, t_i) \tag{2}$$

The costs of funding the hedge position are being accounted for by using funding discount factors, $DF(t_k, t_{k+1})$ which are possibly distinct from the discount factor $df(t_k, t_{k+1})$. The P&L from the hedge (neglecting transaction costs) is therefore

$$\Delta W_{t_k}^{hedge}(t_k, t_{k+1}) = \phi(s(t_k), \varpi(t_k), t_k) H(t_k) \tag{3}$$

The transaction costs - incurred at t_{k+1} - for the hedge rebalancing over the hedge interval $(t_k, t_{k+1}]$ also contributes to the change in wealth:

$$\Delta W_{t_k}^{tc}(t_k, t_{k+1}) = -\left[\delta \left| \phi(s(t_{k+1}), \varpi(t_{k+1}), t_{k+1}) - \phi(s(t_k), \varpi(t_k), t_k) \right| + \chi \right] df(t_k, t_{k+1}) \tag{4}$$

The transaction costs are incurred along with the hedging P&L denoted in (3). We track them separately for computational and pedagogical purposes. The total P&L of the option-hedge position follows

$$\Delta W_{t_k}(t_k, t_{k+1}) = \Delta W_{t_k}^{option}(t_k, t_{k+1}) + \Delta W_{t_k}^{hedge}(t_k, t_{k+1}) + \Delta W_{t_k}^{tc}(t_k, t_{k+1}) \tag{5}$$

2.2 Optimization Problem

In the OHMC approach, a MC simulation of asset evolution is performed and the valuation and hedge ratio *functions* of spot and other conditioning variables are assessed to minimize risks. For compactness of notation we use only one additional conditioning variable in this paper, although the algorithms can be generalized to handle additional conditioning variables. Based on the MC simulation of the underlying (and the conditioning variables, which in our case are derived from the underlying) all terms of the wealth balance (5) can be directly computed, other than the yet unknown functions of value and hedge ratio functions $C(s(t_k),\varpi(t_k),t_k)$ and $\phi(s(t_k),\varpi(t_k),t_k)$. These two functions are found by imposing a constraint on the average change in wealth and to minimize the wealth change variance:

Find $C(s(t_k),\varpi(t_k),t_k)$ and $\phi(s(t_k),\varpi(t_k),t_k)$ so that

$$E[\mathit{\Delta} W_{t_k}(t_k,t_{k+1})] = \overline{\mathit{\Delta} W_{t_k}(t_k,t_{k+1})} \tag{6}$$

$$\text{minimize} \quad \sigma^2_{\mathit{\Delta} W_{t_k}(t_k,t_{k+1})} \equiv E[(\mathit{\Delta} W_{t_k}(t_k,t_{k+1}) - \overline{\mathit{\Delta} W_{t_k}(t_k,t_{k+1})})^2] \tag{7}$$

The closest counter-part to the *risk-neutral* approach is to set the expected wealth change to zero ($\overline{\mathit{\Delta} W_{t_k}}(t_k,t_{k+1}) = 0$) and seek the minimum variance solution. Indeed, if risk can be eliminated then any deviation from zero mean change in wealth is an *arbitrage opportunity*. However that theoretical view is inadequate for real trading because there is no derivative that can be perfectly replicated by dynamically trading the underlying, on account of excess kurtosis of the underlying. So, the real world problem could impose a positive *expected* change in wealth if the market is dominated by a derivative seller-hedger, and especially if the residual risk entails large possible losses and smaller possible gains – i.e., an *insurance seller profile*. The real world problem could also sensibly tolerate a negative expected change in wealth if the residual risk entails surprising gains that are larger than the resulting expected loss – i.e., *lottery buyer profile*. A dynamic option trading strategy can have elements of both an *insurance-seller* and a *lottery buyer* – of course seeking to sell expensive insurance (relative to potential claims!) and buy cheap lotteries simultaneously, and occasionally get paid to own a lottery (good-deal?).

Anecdotally, selling and delta hedging equity index options tends to be a positive *average* wealth change business. This is witnessed also in variance swap based investment products that advertised double digit *expected* returns, attributed to the "discrepancy" between implied volatility and historical volatility – prior to the 2008 fall market correction!

Risk-aware market agents track listed option prices and *hedge slippage* in any replication attempt. If the market pricing becomes rich but the historical hedging costs and hedge slippage are unremarkable then that provides a signal of either a changing risk-premium regime, or the reference assets are being re-appraised by market. Recognizing the irreducible hedging risks and their asymmetry, it is entirely appropriate to explore OHMC with non-zero expected wealth change constraints.

2.3 Numerical Solution

We render the variational-calculus[9] problem (defined by (6) & (7)) finite dimensional by representing the unknown pricing functions and hedge ratio functions in terms of sums of products of unknown time dependent coefficients and state-space dependent Basis Functions:

$$C(s(t_k),\varpi(t_k),t_k)=\hat{\mathbf{C}}(t_k)\bullet\mathbf{\Omega}(s(t_k),\varpi(t_k))\ ;\ \phi(s(t_k),\varpi(t_k),t_k)=\hat{\boldsymbol{\varphi}}(t_k)\bullet\mathbf{\Omega}(s(t_k),\varpi(t_k)) \qquad \text{(8a)}$$

Further details about the Basis Functions are provided in **Appendix-B**. The number of functions can be increased to obtain the desired resolution of the $s(t_k)-\varpi(t_k)$ *state-space*. The notation of (8a) can be altered by representing dot products as sum over repeated indexes

$$C(s(t_k),\varpi(t_k),t_k)=\hat{C}_i(t_k)\Omega_i(s(t_k),\varpi(t_k))\ ;\ \phi(s(t_k),\varpi(t_k),t_k)=\hat{\phi}_i(t_k)\Omega_i(s(t_k),\varpi(t_k)) \qquad \text{(8b)}$$

A further decrease in clutter can be achieved by dropping explicit reference to functional dependence altogether:

$$C(t_k)=\hat{C}_j(t_k)\Omega_j\ ;\ \phi(t_k)=\hat{\phi}_j(t_k)\Omega_j \qquad \text{(8c)}$$

Using this notation in (5), the wealth change perturbation $\Delta\tilde{W}_{t_k}(t_k,t_{k+1})$ around its ensemble mean $\overline{\Delta W_{t_k}}(t_k,t_{k+1})$ follows

$$\begin{aligned}\Delta\tilde{W}_{t_k}(t_k,t_{k+1})&=\hat{C}_j(t_k)\Omega_j-G(t_k)+\hat{\phi}_j(t_k)\Omega_jH(t_k)+N_{t_k}(t_k,t_{k+1})\\ N_{t_k}(t_k,t_{k+1})&=\Delta W^{tc}_{t_k}(t_k,t_{k+1})-\overline{\Delta W_{t_k}}(t_k,t_{k+1})\end{aligned} \qquad \text{(9)}$$

We employ a Lagrange-multiplier technique to solve the constrained optimization problem.

$$F(t_k)=\overline{\left(\Delta\tilde{W}_{t_k}(t_k,t_{k+1})\right)^2}+2\gamma(t_k)\overline{\Delta\tilde{W}_{t_k}(t_k,t_{k+1})} \qquad \text{(10)}$$

With $\gamma(t_k)$ denoting the Lagrange-multiplier, the variational calculus problem defined in (6) & (7) is solved as follows:

$$\frac{\partial F(t_k)}{\partial\hat{C}_i(t_k)}=0\ ;\ \frac{\partial F(t_k)}{\partial\phi_i(t_k)}=0\ ;\ \frac{\partial F(t_k)}{\partial\gamma(t_k)}=0 \qquad \text{(11)}$$

---

[9] The *Brachistochrone Problem* is one of the earliest problems posed in the calculus of variations. This problem is defined as follows: Find the shape of the curve down which a bead sliding from rest and accelerated by gravity will slip (without friction) from one point to another in the least time. Newton was challenged to solve the problem in 1696, and it is believed that he did so the very next day!

Addressing the conditions in (11) we have

$$
\begin{aligned}
&\frac{\partial F(t_k)}{\partial \hat{C}_i(t_k)} = 0 \Rightarrow E\left[\Delta\widetilde{W}_t(t_k,t_{k+1}) \frac{\partial \Delta\widetilde{W}_{t_k}(t_k,t_{k+1})}{\partial \hat{C}_i(t_k)} \right] + \gamma(t_k) E\left[\frac{\partial \Delta\widetilde{W}_{t_k}(t_k,t_{k+1})}{\partial \hat{C}_i(t_k)} \right] = 0 \\
&\frac{\partial F(t_k)}{\partial \hat{\phi}_i(t_k)} = 0 \Rightarrow E\left[\Delta\widetilde{W}_{t_k}(t_k,t_{k+1}) \frac{\partial \Delta\widetilde{W}_{t_k}(t_k,t_{k+1})}{\partial \hat{\phi}_i(t_k)} \right] + \gamma(t_k) E\left[\frac{\partial \Delta\widetilde{W}_{t_k}(t_k,t_{k+1})}{\partial \hat{\phi}_i(t_k)} \right] = 0 \\
&\frac{\partial F(t_k)}{\partial \gamma(t_k)} = 0 \Rightarrow E\left[\Delta\widetilde{W}_{t_k}(t_k,t_{k+1}) \right] = 0
\end{aligned} \tag{12}
$$

To assess the terms in (12) further we have

$$
\begin{aligned}
\frac{\partial \Delta\widetilde{W}_{t_k}(t_k,t_{k+1})}{\partial \hat{C}_i(t_k)} &= \Omega_i + \frac{\partial N_{t_k}(t_k,t_{k+1})}{\partial \hat{C}_i(t_k)} \\
\frac{\partial \Delta\widetilde{W}_{t_k}(t_k,t_{k+1})}{\partial \hat{\phi}_i(t_k)} &= \Omega_i H(t_k) + \frac{\partial N_{t_k}(t_k,t_{k+1})}{\partial \hat{\phi}_i(t_k)}
\end{aligned} \tag{13}
$$

Substituting (12) and (13) in (11) yields the set of equations effecting the constrained optimization of (6) and (7):

$$
\begin{gathered}
\hat{C}_j \overline{\Omega_j \Omega_i} + \hat{\phi}_j(t_k) \overline{\Omega_j \Omega_i H} + \gamma \overline{\Omega_i} = \\
\overline{\left(G - N_{t_k}\right)\left(\Omega_i + \frac{\partial N_{t_k}}{\partial \hat{C}_i}\right)} - \left\{ \hat{C}_j \overline{\Omega_j \frac{\partial N_{t_k}}{\partial \hat{C}_i}} + \hat{\phi}_j \overline{\Omega_j H \frac{\partial N_{t_k}}{\partial \hat{C}_i}} + \gamma \overline{\frac{\partial N_{t_k}}{\partial \hat{C}_i}} \right\}
\end{gathered} \tag{14a}
$$

$$
\begin{gathered}
\hat{C}_j \overline{\Omega_j \Omega_i H} + \hat{\phi}_j \overline{\Omega_j \Omega_i H^2} + \gamma \overline{\Omega_i H} = \\
\overline{\left(G - N_{t_k}\right)\left(\Omega_i H + \frac{\partial N_{t_k}}{\partial \hat{\phi}_i}\right)} - \left\{ \hat{C}_j \overline{\Omega_j \frac{\partial N_{t_k}}{\partial \hat{\phi}_i}} + \hat{\phi}_j \overline{\Omega_j H \frac{\partial N_{t_k}}{\partial \hat{\phi}_i}} + \gamma \overline{\frac{\partial N_{t_k}}{\partial \hat{\phi}_i}} \right\}
\end{gathered} \tag{14b}
$$

$$
\hat{C}_j \overline{\Omega_j} + \hat{\phi}_j \overline{\Omega_j H} = \overline{G - N_{t_k}} \tag{14c}
$$

The term $N_{t_k}(t_k,t_{k+1}) = \Delta W^{tc}_{t_k}(t_k,t_{k+1}) - \overline{\Delta W_{t_k}}(t_k,t_{k+1})$ is the source of *nonlinearity* in this optimization problem. This nonlinearity arises due to the dependence of transaction costs on the change in hedge ratios over a hedging interval and the potential dependence of the mean wealth change constraint on hedge performance over the hedge interval. The linear version of (14) is reported in **Appendix-C** and provides the backbone for the numerical solution, as the nonlinear terms are simply taken on the *right-hand-side* and updated *iteratively*.

The general approach of using basis functions to simplify a variational calculus problem has been employed widely in science and engineering applications (e.g., in finding bounds on quantities in Turbulent flows). Bouchaud and co-workers introduced that technique in their pioneering work on OHMC that yields (14) without the nonlinear transaction-cost and risk-premium terms considered here. Kapoor and co-workers have used the same technique for equity derivatives with GARCH underlying and for CDS swaptions – also considering knockout conditions (also in Crash Cliquets). Kapoor and co-workers have also extended this approach for multi-asset derivatives.

Transaction Cost

In this formulation the transaction costs and risk-premium constraint are embedded in the term denoted by $N_{t_k}(t_k, t_{k+1})$. This is a non-linear term. With $N_{t_k}(t_k, t_{k+1}) = 0$ the set of equations in (14) can be directly solved (**Appendix-C**). So, without the nonlinear term the main approximation in the OHMC solutions is in the basis function representation. With transactions costs, and therefore the nonlinear term the set (14) must be solved iteratively, successively refining the solution and the estimate of the nonlinear term.

The transaction costs arise from changes in the hedge ratio. In the OHMC algorithm, as we march backwards in time from the option contract expiry to the initial time-step, we incorporate the effect of transaction costs into the solution for the unknown functions of hedge ratio and value. Transaction costs contribute to $N_{t_k}(t_k, t_{k+1})$ and to $\partial\left(N_{t_k}(t_k, t_{k+1})\right)/\partial\hat{\phi}_i$. The hedge ratio being zero after the option expires and zero prior to entering into the contract are the temporal boundary conditions on the hedge ratio. In a static hedging problem transaction costs do not introduce nonlinearity (see **Appendix-A**), whereas in a dynamic hedging problem transaction costs result in the nonlinear system given by (14).

Risk-Premium Constraint

The risk-premium constraint contribution to $N_{t_k}(t_k, t_{k+1})$ adds nonlinearity owing to the dependence of risk premium on the residual risks – which can only be assessed after estimating the hedge and value functions. While the transaction cost is path-dependent, the risk premiums we explore later are dependent on the residual risk probability *distribution*. Such risk premiums contribute to the term to $N_{t_k}(t_k, t_{k+1})$ and to $\partial\left(N_{t_k}(t_k, t_{k+1})\right)/\partial\hat{\phi}_i(t_k)$ and $\partial\left(N_{t_k}(t_k, t_{k+1})\right)/\partial\hat{C}_i(t_k)$. This requires that (14) be solved iteratively, where estimates of $\hat{\phi}_i(t_k)$ and $\hat{C}_i(t_k)$ give rise to estimates of $N_{t_k}(t_k, t_{k+1})$and $\partial\left(N_{t_k}(t_k, t_{k+1})\right)/\partial\hat{\phi}_i(t_k)$ and $\partial\left(N_{t_k}(t_k, t_{k+1})\right)/\partial\hat{C}_i(t_k)$, and refinements to $\hat{\phi}_i(t_k)$ and $\hat{C}_i(t_k)$.

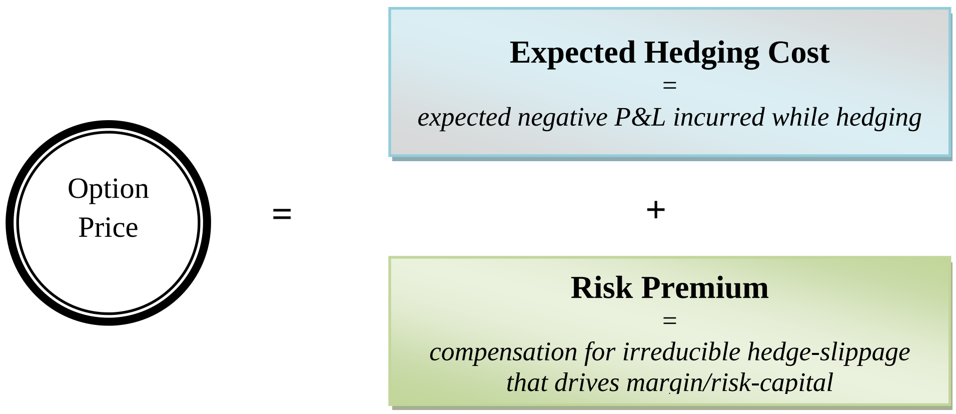

Figure 3. Optimal Hedge Monte-Carlo View of Option Prices.

OHMC utilizes a real-world *conditional* description of the underlying and provides estimates of the expected hedging costs and the irreducible hedge slippage, that can be used to develop a view on the risk-return of option trading. The residual risk is driven by the non-normality of the underlying (e.g., GARAM return model embodies a terms structure of skewness and kurtosis) and interaction of transaction costs and hedging frequency. The OHMC approach helps understand the risk-premium dynamics.

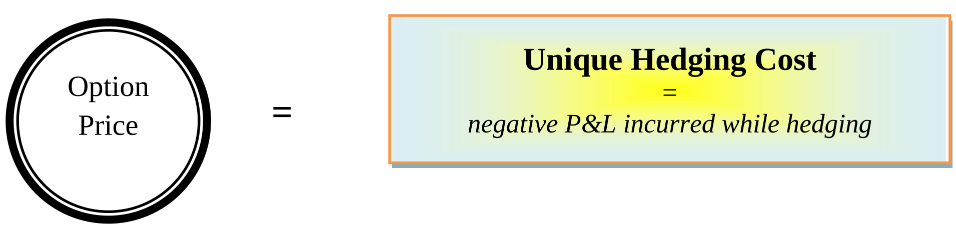

Figure 4. Risk-Neutral Conception of Option Prices.

Risk-Neutral modeling invokes loony measures to fit option prices to a model of hedging that does not admit hedge slippage (i.e., Black-Scholes model and its *mathematical finance* driven risk-neutral theoretical offspring). Often this fitting is achieved by distorting the volatility parameter of the loony measure that cannot be inferred by any direct real-world observations. Risk-neutral pricing is incapable of decomposing an option price into an expected hedging costs and a risk-premium. This approach is primarily used to recognize upfront P&L on bespoke derivative products by comparing execution price to *risk-neutral model* value.

The specific risk-premiums pertinent to option trading-hedging are discussed in **Section 3** and summarized in **Appendix-D**. It needs to be recognized that risk premiums can either be incorporated into the OHMC algorithm at each time step via the corresponding non-linear terms in (14), or, somewhat differently by basing risk-premiums on the global wealth change distribution resulting from an OHMC solution without any risk-premium. For the latter case an add-on to the OHMC *value* is made, and the hedging strategy is independent of the risk-premium. A combination of a local risk-premium and a global risk premium can also be entertained. In this combined approach the hedging strategy and value function seek to attain a specified risk premium each time-step, and a global wealth distribution based add-on to the initial value occurs to attain the desired global wealth change expected value.

Those busy fitting risk-neutral model volatility parameters to observed vanilla option values must wonder why are we unilaterally adding a risk-premium in the option value? Is it not the *market* that determines the risk-premium? To that we answer, of course! Option prices are not the cost of dynamically replicating them. Real options cannot be dynamically replicated perfectly. So the risk-neutral quant deserves no reprieve for this bit of false humility often offered up as the excuse for mindlessly fitting risk-neutral models to *market*.

Option prices are usefully interpreted as a combination of the *expected* cost of attempting to replicate them plus a risk premium charged due to the existence of a fundamentally irreducible hedging error. We find it entirely useless to fit a volatility of a perfect replication model to an observed price, as that does not inform us of the useful decomposition of option price into expected hedging costs and risk-premiums for irreducible risks (see **Figures 3 & 4**).

Our approach to risk-premiums within OHMC can be inverted to yield a risk-premium implied from an observed option price (**Section 3** and **Appendix-D**). That premium is not expressed in terms of a facile implied volatility associated with perfect-replication (i.e., *replifiction*). Rather, that can be expressed as an implied *hurdle* on expected return of the option trader-hedger relative to a risk measure reflecting real-world hedge-slippage. We think that the most exciting use of OHMC is in systematically employing the residual risk to infer risk-premiums from option prices, and developing trading strategies with proprietary targets of return over risk.

Conditioning Hedging Strategy

It is natural to treat recently realized volatility as an explicit determinant of pricing and hedging in addition to the spot value of the underlying. This is because it displays long-range correlations which are addressed in GARAM (Wang et al [2009]) and based on all prior work on option pricing we know it has a central role in determining the costs of hedging! So we view the pricing and hedge ratio as functions of spot asset value and recently realized volatility ϖ :

$$\varpi(t_k) = \frac{1}{m}\sum_{j=k-m}^{k}\left[ln\left(s(t_j)/ s(t_{j-1})\right)\right]^2 \tag{15}$$

This argument of *conditioning* hedging and pricing on realized volatility can be further generalized. In fact, the option value and hedge ratio can be viewed as functions of all the asset values of the past. Furthermore, any related variables could also be considered (e.g., economic activity indicators, credit-spreads, interest rates, etc). After all, a *seasoned trader* purportedly has the wisdom of dealing with hedging over many economic cycles and *conditions* her actions on all available information.

Multi-variate conditioning is a powerful concept. However its implementation is fraught with challenges. It is not straightforward to specify a realistic Monte-Carlo model of the underlying and economic activity indicators, credit-spreads, interest rates! A risk-neutral model compliant artificial description[10] will not suffice – as we are seeking a real-world decision making tool and confronting residual risks directly. Also, the computational costs of the OHMC algorithm dictate that we introduce a small set of conditioning variables. Hence, we are employing spot asset value and recently realized volatility as explicit conditioning variables in the results shown in the next section.

First Hedging Interval

In the OHMC algorithm the treatment of the first hedging interval is simpler than the subsequent ones. This is because we are seeking the price and hedge ratio with information about spot asset value. Having successfully propagated the OHMC solution from option expiry to the first hedging time interval, we solve the ordinary constrained minimization problem to find the hedge ratio and value. Transaction costs introduce nonlinearity. Mean wealth changes that are dependent on residual risks can also be added in assessing the option value.

Similar to any other hedge instance, the wealth change of an option trader has contributions from the option, the hedge, and transaction costs:

$$\Delta W_{t_0}(t_0,t_1) = \Delta W_{t_0}^{option}(t_0,t_1) + \Delta W_{t_0}^{hedge}(t_0,t_1) + \Delta W_{t_0}^{tc}(t_0,t_1) \tag{16}$$

The contributions to the wealth change from the option position are

$$\Delta W_{t_0}^{option}(t_0,t_1) = C(t_0) - G(t_0); \tag{17}$$

$$\overline{\Delta W_{t_0}^{option}}(t_0,t_1) = C(t_0) - \overline{G}(t_0);\ \Delta \tilde{W}_{t_0}^{option}(t_0,t_1) = -\tilde{G}(t_0)$$

The hedging P&L contributes

$$\Delta W_{t_0}^{hedge}(t_0,t_1) = \phi(t_0) H(t_0); \tag{18}$$
$$\overline{\Delta W_{t_0}^{hedge}}(t_0,t_1) = \phi(t_0)\overline{H}(t_0);\ \Delta \tilde{W}_{t_0}^{hedge}(t_0,t_1) = \phi(t_0)\tilde{H}(t_0)$$

The transaction costs contributions to the traders wealth change are

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[10] e.g., Multi-Variate Geometric Brownian Motion!

$$\begin{aligned}
\Delta W_{t_0}^{tc}(t_0,t_1) &= -\left\{\delta\left|\phi(t_0)\right| + \chi\right\}\frac{df(t_0,t_1)}{DF(t_0,t_1)} - \left\{\delta\left|\phi(t_1)-\phi(t_0)\right| + \chi\right\}df(t_0,t_1) \\
\overline{\Delta W_{t_0}^{tc}}(t_0,t_1) &= -\left\{\delta\left|\phi(t_0)\right| + \chi\right\}\frac{df(t_0,t_1)}{DF(t_0,t_1)} - \left\{\delta\overline{\left|\phi(t_1)-\phi(t_0)\right|} + \chi\right\}df(t_0,t_1) \\
\Delta \widetilde{W}_0^{tc}(0,T) &= -\delta\left(\left|\phi(t_1)-\phi(t_0)\right| - \overline{\left|\phi(t_1)-\phi(t_0)\right|}\right)df(t_0,t_1)
\end{aligned} \tag{19}$$

The wealth change deviation from the ensemble average follows:

$$\begin{aligned}
\Delta \widetilde{W}_{t_0}(t_0,t_1) &\equiv \Delta \widetilde{W}_{t_0}^{option}(t_0,t_1) + \Delta \widetilde{W}_{t_0}^{hedge}(t_0,t_1) + \Delta \widetilde{W}_{t_0}^{tc}(t_0,t_1) \\
\Delta \widetilde{W}_{t_0}(t_0,t_1) &= -\widetilde{G}(t_0) + \phi(t_0)\widetilde{H}(t_0) + \Delta \widetilde{W}_{t_0}^{tc}(t_0,t_1)
\end{aligned} \tag{20}$$

The wealth change variance for the first time interval of the dynamic hedging scheme follows

$$\begin{aligned}
\sigma^2_{\Delta W_{t_0}(t_0,t_1)} &= \sigma^2_{G(t_0)} + \phi^2(t_0)\sigma^2_{H(t_0)} + \sigma^2_{\Delta W_{t_0}^{tc}(t_0,t_1)} \\
&- 2\overline{\widetilde{G}(t_0)\Delta \widetilde{W}_{t_0}^{tc}(t_0,t_1)} - 2\phi(t_0)\left\{\overline{\widetilde{G}(t_0)\widetilde{H}(t_0)} - \overline{\widetilde{H}(t_0)\Delta \widetilde{W}_{t_0}^{tc}(t_0,t_1)}\right\}
\end{aligned} \tag{21}$$

The initial hedge ratio $\phi(t_0)$ that minimizes the wealth change variance over the first hedging interval can be numerically assessed based on (21). The corresponding static hedging problem is simpler on account of $\phi(t_1 = T) = 0$, thereby eliminating the non-linearity in the minimization problem arising due to transaction costs (see **Appendix-A**).

Assessing the value of the derivative requires imposing the average wealth change constraint:

$$\begin{aligned}
\overline{\Delta W_{t_0}}(t_0,t_1) &= \overline{\Delta W_{t_0}^{option}}(t_0,t_1) + \overline{\Delta W_{t_0}^{hedge}}(t_0,t_1) + \overline{\Delta W_{t_0}^{tc}}(t_0,t_1) \\
\overline{\Delta W_{t_0}}(t_0,t_1) &= C(t_0) - \overline{G}(t_0) + \phi(t_0)\overline{H}(t_0) + \overline{\Delta W_{t_0}^{tc}}(t_0,t_1) \\
C(t_0) &= \overline{\Delta W_{t_0}}(t_0,t_1) + \overline{G}(t_0) - \phi(t_0)\overline{H}(t_0) - \overline{\Delta W_{t_0}^{tc}}(t_0,t_1)
\end{aligned} \tag{22}$$

The last (but not the least) important input into the pricing model has to be the wealth change average. The rationale for imposing a non-zero average change in wealth is that hedging does not eliminate risk and therefore can attract a risk-premium – i.e., a risk-taker is justified in having an expectation of positive change in wealth.

3. Risk-Return of Dynamic Hedger

The losses incurred by a derivative trading book can jeopardize the solvency of a financial institution – or certainly the employment of a trader or the existence of a trading desk or that of a hedge fund. While ultimately a firm may be interested in its global risk profile, losses at any sub-unit that are disproportionately larger than its size indicate that either extreme odds have been realized and/or that the institution does not understand and can't control the risks of its parts. Reputational damage resulting from financial losses in a subset of a firm can have a detrimental effect on the firm at a global level that go beyond the immediate financial risks. Also, if a clear methodology of understanding risk-return is not expounded at a trade level or a trading desk level, it is unlikely (and dangerous to assume) that risks are understood at a global portfolio level. In this backdrop the unchallenged invocation of *replication* and/or complete *diversification* of residual risks inside a *valuation* model is dangerous and misleading – for both vanilla and exotic options.

Having solved the OHMC problem, the residual P&L, $\Delta W_{t_k}(t_k, t_{k+1})$, over any hedge interval can be found. Now, with the OHMC hedging time-grid specified as $\{t_0 = 0, t_1, \ldots.., t_k, t_{k+1}, t_{K-1} = T\}$ the total change in wealth discounted to t_0 is given by

$$\Delta W_0(0,T) = \sum_{k=0}^{K-2} \Delta W_{t_k}(t_k, t_{k+1}) df(t_0, t_k)$$

The cumulative P&L from trade initiation to t_k (present valued to time t_k) follows

$$\Delta W_{t_k}(0,t_k) = \sum_{j=0}^{k-1} \Delta W_{t_j}(t_j, t_{j+1}) \left[df(t_j, t_k) \right]^{-1}$$
$$= \frac{\Delta W_{t_{k-1}}(0, t_{k-1})}{df(t_{k-1}, t_k)} + \frac{\Delta W_{t_{k-1}}(t_{k-1}, t_k)}{df(t_{k-1}, t_k)}$$

A market participant (e.g., dealer, proprietary trader or hedge fund) can decide to enter a trade when their analysis of risk-return indicates an attractive trade. This must involve comparing some metric of potential gain with some metric of risk? OHMC helps establish the hedging strategy and the associated risk-capital and other residual risk characteristics. By revealing the P&L associated with attempted replications, the OHMC approach arms the trader with analytical tools that connect the option price to the risk-return equation. Of course, different market agents will have qualitatively different risk-return criteria with numerically distinct thresholds or *hurdle returns*, below which they will pass on a trade. The demand and supply of risk capital and the opportunity set available to the market participant will also be important determinants in their trade decision making psychology, in addition to views on the underlying. By unraveling attempted replication risk-return, OHMC opens an interface of quantitative-analysis of derivatives to behavioral-finance.

3.1 Trader-Hedger's Risk Metrics

The expected P&L from a derivative trade should be compared with tail losses to ensure solvency, and profitability. While great trades may come from market insights that are not modeled routinely, it should at least be possible to weed out poor derivative trades systematically. To do so we define risk-return metrics and also assess the risk capital associated with the derivative trade. To compute risk capital over different time intervals (derivative tenor, hedging interval, etc), we employ a *target hazard rate* to consistently assess the target survival probability over different time horizons.

h interval expected P&L $\quad \overline{\Delta W_t(t,t+h)}$

h interval P&L standard-deviation $\sigma_{\Delta W_t(t,t+h)}$

$$\sigma^2_{\Delta W_t(t,t+h)} = E[\left(\Delta W_t(t,t+h) - \overline{\Delta W_t(t,t+h)}\right)^2]$$

h interval annualized Sharpe-Ratio

$$\Lambda = \frac{\overline{\Delta W_t(t,t+h)} \times (1/h)}{\sigma_{\Delta W_t(t,t+h)} \times \sqrt{1/h}}$$

h interval P&L negative semi-deviation $\sigma^-_{\Delta W_t(t,t+h)}$

$$\left(\sigma^-_{\Delta W_t(t,t+h)}\right)^2 = E[\left(\Delta W_t(t,t+h) - \overline{\Delta W_t(t,t+h)}\right)^2 \mid \Delta W_t(t,t+h) < \overline{\Delta W_t(t,t+h)}]$$

h interval annualized Sortino-Ratio

$$\Lambda^- = \frac{\overline{\Delta W_t(t,t+h)} \times (1/h)}{\sigma^-_{\Delta W_t(t,t+h)} \times \sqrt{1/h}}$$

target survival probability/confidence-level over period τ $\quad p_s(\tau)$

τ interval target hazard rate $\quad \lambda_\tau = -\ln\left(p_s(\tau)\right)/\tau$

τ interval hazard rate based h interval target survival probability $\quad p_s(h) = \exp[-\lambda_\tau h]$

h interval $p_s(h)$ confidence-level wealth change

$$q(t;p_s(h)) \ni Probability\{\Delta W_t(t,t+h) > q(t;p_s(h))\} = p_s(h)$$

h interval $p_s(h)$ confidence-level deviation from average wealth change

$$Q(t;p_s(h)) = \overline{\Delta W_t(t,t+h)} - q(t;p_s(h))$$

h interval $p_s(h)$ confidence-level expected return on risk capital[11]

$$\Theta(t,t+h)=\frac{\overline{\Delta W_t(t,t+h)}}{Q(t;p_s(h))}$$

h interval $p_s(h)$ confidence-level rate of expected return on risk capital

$$\theta(t,t+h)=\ln[\Theta(t,t+h)+1]/h$$

As the wealth change is defined by discounting cash-flows under the risk-free rate, throughout this paper, the expected return on risk capital and its rate are in reference to the risk-free rate.

3.2 Pricing to Return on Risk Target

The OHMC analysis yields a hedging strategy that has associated with it an average wealth change and a wealth change distribution, that is characterized in part by the standard deviation, negative semi-deviation, & tail risk measures. This hedging strategy has an upfront estimated cost C – which in the case of a zero mean-change wealth constraint is the average cost of hedging. Now clearly if the average wealth change is constrained to zero, option sell-hedge trades do not make much sense as they have greater downside surprise than upside potential (shown in **Section 4**). It is then conceivable that an option seller-hedger will try to ask for more than C – the cost of hedging associated with zero mean wealth change - to make sense of the option sell-hedge position. Here we consider what an option seller has to charge in addition to the average cost of hedging to accomplish a desired expected return per unit risk target – or a *hurdle return*.

Rate of Return on Risk-Capital

The hedging strategies result in an expected change of wealth over the T period option denoted by $\overline{\Delta W_0(0,T)}$. The T period risk-capital, at the confidence level p_s, for the option seller, is denoted by Q:

$$\text{Probability}[\Delta W_0(0,T)\geq q]=p_s\,;\;Q=\overline{\Delta W_0(0,T)}-q$$

[11] Surprisingly few 'option-valuation' works address the risk-capital of a derivatives trader. While they revel in the perfect hedge and the associated trivially "self-financing" nature of the idealized model, they fail to provide even a rough estimate of the trader's risk-capital. Consequently, derivative traders and their pussyfooting 'valuation quants' may convey a sense of entitlement for selling options – especially if they are housed in highly rated large entities that do not afford them the direct experience of raising *risk-capital*. The risk mismanagement associated with derivative trading cannot be rendered satisfactory without interjecting the usual *valuation* chatter with that of risk-capital *requirements*.

Denoting the desired *rate* of expected return over risk capital by $\hat{\theta}$, the *risk-premium* that needs to be charged by the option seller, in addition to the zero mean hedging strategy cost *C*, to attain the desired rate of expected return, is given by Ψ :

$$\frac{\overline{\Delta W_0(0,T)}+\Psi}{Q}=e^{\hat{\theta}T}-1; \qquad \Psi=\left(e^{\hat{\theta}T}-1\right)Q-\overline{\Delta W_0(0,T)} \tag{23}$$

The same argument can be made by the option buyer-hedger. The difference being that *Q* and *q* will then be the buyers P&L attributes (the wealth change distribution is not symmetric), and that Ψ quantity needs to be added to the negative hedging strategy cost to meet the option buyer-hedgers hurdle rate. In the next section we assess the price of the option that results in a $\hat{\theta}=25\%/yr$ over US Treasury rates (see **Tables 2-7** in **Section 4**).

We are not suggesting that the real option market involves a bunch of option seller-hedgers trading with a bunch of option buyer-hedgers. For instance, the option buyer may not be a hedger at all, and he may have purchased the option from a seller-hedger. The outright option buyer has a fixed upfront cost and has potential upside. There are many different ways to try to represent the psychology[12] of the outright option purchaser. In this work we limit our deliberations to option seller/buyer-hedgers, recognizing that outright option purchasers can also importantly influence the market pricing dynamics.

Now risk-capital is fundamentally important as nobody wants to buy an option from an insolvent counterparty! However risk-capital is not sufficient to describe the whole story – the risk premium can very well be driven by multiple criteria. An investor in an option trading strategy may be sensitive to the *volatility* of the strategy. The volatility tolerance level and hurdle expected return per unit volatility can also be directly addressed in the OHMC analysis, as done next.

Sharpe-Ratio

What does an option seller need to add in addition to the zero mean wealth change hedging costs *C* to achieve a Sharpe-Ratio target of $\hat{\Lambda}$? That amount, again denoted by Ψ follows:

$$\frac{\left(\overline{\Delta W_0(0,T)}+\Psi\right)(1/T)}{\sigma_{\Delta W_0(0,T)}\sqrt{1/T}}=\hat{\Lambda}; \qquad \Psi=\hat{\Lambda}\sigma_{\Delta W_0(0,T)}\sqrt{T}-\overline{\Delta W_0(0,T)} \tag{24}$$

We have taken $\hat{\Lambda}$ to be an annualized Sharpe-Ratio and annualized the expected wealth change by assuming a linear scaling with term and the scaling of the standard deviation as a square root of term. This is simply to follow the convention so that the Sharpe-Ratios of different strategies

[12] OHMC opens the doors of options analysis to behavioral finance insofar as the objectives of different market agents can be linked to the risk-return of their positions. For a general discussion and illustration of the importance of psychology, sociology, and anthropology in economics, see Akerlof and Shiller [2009] and Akerlof [1984].

can be compared. We report on option prices that render the seller/buyer-hedger's Sharpe-Ratio = 1 in the next section (see **Tables 2-7** in **Section 4**).

Sortino-Ratio

Similar to pricing an option to a target return on risk capital or Sharpe-Ratio, as shown above, we could price to a target Sortino-Ratio. This would recognize that typically an option seller has a larger negative semi-deviation than standard deviation, and investment strategies with asymmetrically larger gains than losses are more attractive than the ones with larger potential losses than gains:

$$\frac{\left(\overline{\Delta W_0(0,T)}+\Psi\right)(1/T)}{\sigma^-_{\Delta W_0(0,T)}\sqrt{1/T}}=\hat{\Lambda}^-;\qquad \Psi=\hat{\Lambda}^-\sigma^-_{\Delta W_0(0,T)}\sqrt{T}-\overline{\Delta W_0(0,T)} \tag{25}$$

The Sortino-Ratio is particularly relevant to option trading because the residual risk is asymmetric. Also, an investor in a trading strategy has typically different tolerance to downside volatility relative to upside volatility – a Sharpe-Ratio simply does not afford this differentiation. See **Tables 2-7** for Sortino-Ratio based pricing of option prices.

Return on Risk Metrics of Delta1 Trade

Consider the P&L of the trader who buys a unit of the reference asset and holds it over the time interval (t_0, T]. The trader pays transactions costs, financing costs, and receives the dividends of the asset at time instances $t_i \in (t_0, T]$. The change in wealth of a "Delta-One" trade is given by

$$\Delta W^{delta1}_{t_0}(t_0,T)=\left[\left(s(T)-\delta-\chi\right)-\frac{\left(s(t_0)+\delta+\chi\right)}{DF(t_0,T)}\right]df(t_0,T)+\pi_i df(t_0,t_i)$$

To reinforce the role of transaction costs separately we can write the wealth change above as

$$\Delta W^{delta1}_{t_0}(t_0,T)=\left[s(T)-\frac{s(t_0)}{DF(t_0,T)}\right]df(t_0,T)+\pi_i df(t_0,t_i)-(\delta+\chi)\left(1+\frac{1}{DF(t_0,T)}\right)df(t_0,T)$$

Just like we did for option trader-hedger, we can define the rate of return on risk capital, Sharpe-Ratio, & Sortino-Ratio (θ_{delta1}, Λ_{delta1}, Λ^-_{delta1}) for a delta-1 trader. Any systematic option seller/buyer hedger must ask herself: Should my *expected* return per unit risk be lower than that of a delta-1 trader on the same underlying?

3.3 Jerome-Kerviel Bound

We argue that a derivative trade appears unattractive if the expected return on risk is less than that of a "Delta-One" trade on the same underlying. We define this bound as follows:

> The Jerome-Kerviel (JK) bound is the price of a derivative such that when a market agent attempts to dynamically hedge that derivative, it results in an expected change in wealth per unit risk to be identical to a delta-1 trade on the underlying.

As risk can be measured in different ways there are as many incarnations of this bound as there are pertinent risk measures. We employ the three choices made above, namely rate of return on risk capital, Sharpe-Ratio, & Sortino-Ratio. This bound can be assessed by simply setting the target return on risk metric to the delta-1 amount, i.e., $\hat{\theta} = \theta_{delta1}$, $\hat{\Lambda} = \Lambda_{delta1}$, $\hat{\Lambda}^{-} = \Lambda^{-}_{delta1}$.

Jerome Kerviel was a cash products trader at a bank. He was trading simple products, and not complex derivatives. So in derivative parlance he was a 'Delta-One" trader. It is reported that one of the reason for the occurrence of the financial incident involving the mere delta-1 trader Jerome Kerviel was that to obtain a large bonus – like that of a *profitable derivatives trader* [13]– Jerome had to put on an extremely large delta-1 position.

We find it useful to compare the expected profitability of derivatives trades and compare them with a 'Delta-One' trades. Specifically, we like to compare the two trades on the metric of expected change in wealth per unit risk. We take a long 'Delta-One' position, and its real-world stochastic model and find the θ_{delta1} Λ_{delta1} Λ^{-}_{delta1} quantities – that connote real world rate of expected change of wealth per unit risk. In OHMC, as we always assess hedging errors and we can handle any stochastic model of the underlying, we can calculate how the derivative pricing sets these quantities for the derivative trade. If a derivative trade is priced below its JK bound, then a delta-1 long position in the underlying has a higher expected change in wealth per unit risk measure.

3.4 Demand & Supply Driven Pricing Regimes

The assertion of a unique *arbitrage-free value* of a derivative contract by the mainstream valuation modeling is underpinned by dismissing of residual risks. That dismissal is unacceptable from the point of view of developing a responsible discipline of modeling the economics of derivatives. That discipline must develop views and methodologies for assessing risk-capital while addressing hedging and attempted replication! Without dismissing residual risks we have furthered bounding arguments from the points of view of a derivative seller-buyer

[13] One has to wonder if the reason a derivatives trades appears profitable is an under-sizing of risk-capital – aided and abetted by risk-neutral mindsets, or if the profitability is linked to an *edge* in dealing with risk-return of *sophisticated* instruments?

hedger. Clearly these different agents have different pricing points at which the trades appear attractive – and if both these agents are using the same model and trade strategy, it is unlikely that they will trade with each other! Indeed, we can have a large difference in bounding values above, depending on the confidence level of risk capital and the trade-tenor, among other things. The *range* of pricing is defined as the difference between the seller's price and buyer's price

range = option-seller-hedger-price – option-buyer-hedger-price (26)

In our analysis the bounds on the hedger that is the payoff recipient is relevant in a market dominated by the option buyer (i.e., the recipient of the payoff) whereas the bound on the hedger-option payoff obligor is relevant in a market dominated by the option seller (i.e., the obligor of the payoff). The range of these bounding values is explicitly dependent on the risk metric used to assess the bounding values – i.e., we conclude that the range of rational prices is dependent on the risk-aversion of the market participants.

Certainly in derivative markets one observes the *fitted risk-neutral model parameters* show distinct regimes and a great variation in numerical values of parameters among those regimes. The model fitters mindlessly focused on the goodness of fit on any particular day may not be aware of the flows of different types of market players (or residual risk for that matter!). More informed market participants are aware of the demand and supply dynamics and can sometimes associate the distinct pricing regimes with those dynamics, in addition to their views on the underlying. A prevailing pricing regime reflects a particular demand supply situation which can be disrupted by factors internal to trades (realized volatility, correlation) or external to the trades whose pricing we debate (e.g., *margin calls* on an *unrelated* trade or a general liquidity crunch). Even if under quiescent conditions the market trades around a certain set of implied parameters, in the event of market stress the demand-supply picture gets altered and the pricing migrates to the new regime.

We do not find this large range between option prices to be troubling at all. To the contrary, we think that acknowledging this range prepares one for market realities of derivative trading. Even if the pricing regime change does not happen over ones watch, the range reflects irreducible risks inherent to attempted derivative replication. Responsible trading activity requires quantifying these risks and the implied *range of "fair" prices*. The lack of focus on hedge slippage of mainstream derivative *valuation modeling* and the inability to entertain bid-ask within a theoretical framework are in stark contrast with the OHMC approach that makes it readily possible to describe a range around the average hedging costs.

Real prices are probably driven by a mix of replication arguments, hedge slippage experiences, demand and supply considerations, variety of views on the underlying, and differing psychological predispositions towards risk-return. The *assumption* of a *perfect* dynamic option replication price results in an unacceptably sterile framework for real-world option trading. **Appendix-D** presents a succinct summary of option trading risk-premiums.

4. Analysis of European Options

The OHMC approach integrates tasks of developing a hedging strategy, option valuation, and risk management. The series of analysis envisaged in developing a derivative buying-selling hedging strategy are as follows:

(1) Develop a stochastic model of the underlying reflecting its *real-world* characteristics

The model should reflect as much real *texture* as possible and should also respond to changing environments. For instance, intervals of high and low volatility tend to show temporal persistence, so it seems essential to acknowledge the starting *volatility regime*. There is scope for using econometric models that help a trader reflect his economic views, while creating a realistic stochastic description. There is also role for incorporating features not observed in historical data by a combination of using surrogates and factor models. There is no getting around having an *objective measure* description of the underlying.[14]

(2) Assess the hedging strategy, the average hedging P&L, and hedge slippage

OHMC analysis provides a hedging strategy in pursuit of minimizing a hedge slippage measure. Here we illustrate P&L volatility as the hedge slippage measure to be minimized, and also focus on the asymmetry & tail behavior of the residual risk. One can also attempt to minimize tail risk measures – however that will not eliminate risk and the need to develop a strategy of dealing with it. For instance, if one seeks to minimize a tail risk measure one can end up with a P&L distribution with a *fat-body*. The decision to trade can be made while being cognizant of the risk-return choices available to the market agent and the relative attractiveness of the derivative trade.

(3) Price by articulating profitability & risk-return criterion while allocating risk-capital

Taking risks requires risk capital. Having articulated a hedging strategy and its residual risk, the option price implies a certain expected return on risk capital and other risk-metrics. The profitability criterion could be set by an investor, or by comparing with another trade. In contrast, the risk-neutral view purports option prices as monoliths equal to the unique cost of *perfect replication*.

The perfect replication *mantra* seems to be embraced by poorly informed control functions that are at a distance from the trade – and get to simplify their roles by conjuring absolutes out of

[14] That a real-world description of the asset underlying a derivative contract is not needed for valuing-trading derivatives is the dumbest implication attributed to the *risk-neutral* model by the run of the mill *valuation quant.* That is also irresponsible nonsense - as significant real-world residual risk is the rule rather than the exception! Decisions regarding trading at specific pricing points require views on the underlying, amid the complexity of reconciling multiple possible descriptions of the underlying, while conditioning on pertinent observables.

something inherently uncertain. This is often encouraged by parties who benefit in the short-term by a continuous focus on "price verification" to a level of false numerical precision ritual, often accompanied by a complete lack of admission or recognition of risk capital associated with their businesses[15]. If a trader is buying and selling the *identical* contracts then risks arise mainly from the credit quality of its counterparties. However when a trader sells an option and hedges by trading the underlying or *different* options, the trader incurs residual risks that should be quantified as a part and parcel of *valuing* the derivative contract, as illustrated here.

4.1 Sample Problems

We illustrate the application of OHMC to option trading by showing some examples. The examples are limited to *vanilla* options – they provide an avenue for developing some intuition about risk-return metrics and option trading strategies. However the OHMC formulation is easily generalized to handle more exotic options – and the approach developed here can be quite effectively deployed for such options. We have applied a simpler version of the OHMC approach to Cliquets and Multi-Asset options in our prior work employing a GARCH model for the underlying (Petrelli et al [2008] & [2009]). Here we include effects of **(1)** transactions-costs; **(2)** conditioning hedging on realized volatility; and **(3)** a residual risk-dependent expected wealth change constraint per hedge-interval.

Tenor Range: 10, 21, 42 trading days

Options & Strikes:	Puts	100%	95%	85%
	Calls	100%	105%	115%

Initial Volatility Regime: Low, High

Position: Option Seller-Hedger, Option Buyer-Hedger

Hedging Mode:

	(a)	**(b)**	**(c)**	**(d)**	**(e)**
Transaction Costs	off	*on*	off	off	*on*
Local Risk Premium[16]	off	off	*on*	off	*on*
Hedge Conditioning[17]	off	off	off	*on*	*on*

[15] For *price-verification* many have argued, for example, whether a "super senior" CDO tranche protection is worth 10 bps/yr or 12 bps/yr – while they had no idea about the risk-capital that the negative convex position should attract! Similarly one can argue whether a 1-week look-back 90% strike knockout Cliquet put is worth 100 bps/yr or 120 bps per year – that argument does not begin to address the severely asymmetric P&L distribution of the option seller-hedger. Risk-neutral quants spend their time "calibrating" models to produce prices that are "intuitive" to business heads. Some of this "intuition" is myopically self-serving and disingenuous– especially when there is no hedging strategy or two-way market in the derivative being discussed!

[16]The mean change in wealth constraint of OHMC is used to incorporate a risk-premium *locally* in time. Specifically, the annualized Sortino ratio over the hedging interval (daily in our examples) is set to 1.

[17] We condition hedging and valuation on the 10-day trailing realized volatility, in addition to the spot value.

Interest Rates, Funding, & Dividend Assumptions

The time value of money and the associated discount factor *df* is effected through United States treasury rates. The funding costs for the hedge position can be expressed as a spread over treasury that is reflected in the funding discount factors *DF*. For our examples we take *DF* = *df*.

The consideration of funding rates are tied to the solvency of the option seller. We assess *risk-capital* of option trader-hedgers, and that provides an estimate of the capital required to render them solvent to the degree associated with the confidence level of the risk-capital.

The treatment of dividend in the model is necessarily simplistic, as we do not further a stochastic dividend model here. We use historical treasury rates and trailing annual dividends for the sample calculations. The differences between the results for the low volatility regime and high volatility regime are not substantially influenced by the differences in treasuries, funding and dividend. Rather, they strongly reflect the asset behavior and its conditioning on return observations.

Stochastic Model of Underlying: GARAM

We employ our previously described model GARAM (Wang et al [2009]) to perform Monte-Carlo simulation of the reference asset. GARAM addresses clustering of volatility and the term structures of return skewness and kurtosis by employing two stochastic processes to model the return magnitude and return sign. The auto-covariance and cross-covariance functions of these stochastic processes are empirically based. The marginal density of the squared return over a base time-scale is also based on an empirical transformation of the log-Normal density. We adopt the stylized S&P index based real-world measure parameters of GARAM that are based on data from January 3, 1950 to June 2, 2009.

Historical observations enter into our computations in 2 ways: **(1)** historical data is used calibrate GARAM parameters; **(2)** historical data are used to create *conditional* realizations of the underlying. In principal *all* the historical data can be used to create conditional realizations. However, as the time-lag increases, the *memory* of the stochastic processes building GARAM weaken, and after a large time lag, the information has little *conditioning effect*. This decay of correlation is slow – that is the motivation for using auto-regressive processes to build GARAM. As the number of conditioning data points increases, the computational cost of producing conditional realizations also increases – hence the motivation to limit conditioning data to a *useful window*. In the sample calculations shown here we use the returns for 63 prior trading days as the conditioning information, and generate 1 million conditional realizations of the underlying. For further details of GARAM parameter inference, and the methodology of generating conditional paths, we refer the reader to our recent work (Wang et al [2009]).

Conditioning Regimes

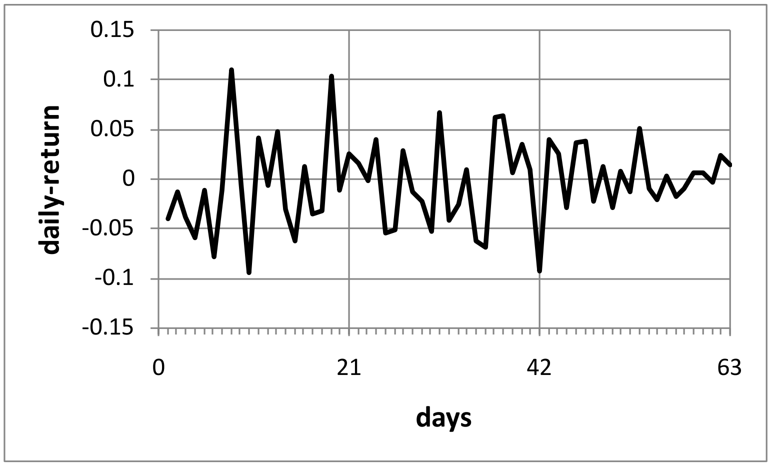


Figure 5. High volatility conditioning return data. The return volatility over the 63 day conditioning window is 67.6% and over the last 10 days 21.6%. The 1 month and 3 month treasury rates corresponding to this regime are both taken as 0.11%/yr. The dividend payments are assumed to be constant and paid uniformly every day over the option horizon (10-42 days) and are taken as 3.28%/yr of spot value at the inception of the option contracts. The fixed transaction costs are set to zero ($\chi = 0$) and the per-share transaction cost (includes commissions and spread from *mid* price) δ = 2.77 bps of spot at inception.

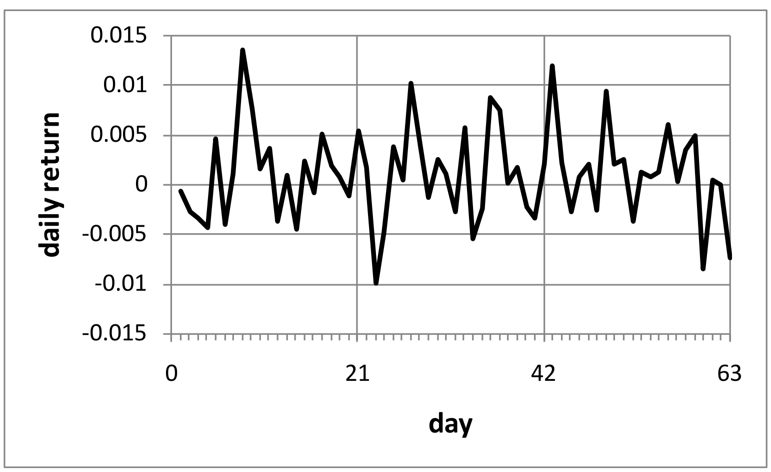


Figure 6. Low volatility conditioning return data. The return volatility over the 63 day conditioning window is 7.56% and over the last 10 days 7.15%. The 1 month and 3 month treasury rates corresponding to this regime are both taken as 5.19%/yr and 5.07 %/yr respectively. The dividend payments are assumed to be constant and paid uniformly every day over the option horizon (10-42 days) and are taken as 1.78%/yr of spot value at the inception of the option contracts. The fixed transaction costs are set to zero ($\chi = 0$) and the per-share transaction cost (includes commissions and spread from *mid* price) δ = 1.83 bps of spot at inception.

The conditioning information for GARAM has a clear role to supply the model with information about the underlying current conditions. The delineation of what is "current" is mediated via the covariance functions that define GARAM. We chose two contrasting 63 day periods of visibly high volatility and discernibly low volatility. By doing so we are able to illustrate the effect of contrasting conditioning information on hedge performance – including *expected* hedging costs and hedge slippage distribution. This conditioning is *central* to developing trading strategies where one seeks to discern risk-premiums in derivative markets. OHMC provides a framework to separate the effects of changing environments for the underlying asset from those arising due to changing risk premiums of market agents.

Summary Results

Now we have assembled all the ingredients for an OHMC application. Key metrics about hedge performance and the implications for option pricing are communicated here. We focus on direct hedge performance metrics and not on how they would be accommodated in a *Black-Scholes* framework – i.e., employing an *implied volatility surface* after presuming perfect replication*!* After building familiarity with the rich hedge performance metrics accessible by OHMC we believe the need for such an *implied volatility surface* is completely obviated. All elements of option trading can be performed without resorting to the falsehood of *perfect replicatio*n and the associated *implied volatility*. We find it possible to work with the following metrics to build intuition about option trading

- realized volatility – long term & short term;
- autocorrelation of realized volatility & the term structure of return kurtosis;
- correlation of return sign-indicator & realized volatility & term structure of return skewness;
- hedge cost and average hedge P&L;
  - when the average hedge P&L = 0 (by OHMC constraint) then hedge cost can also be called average hedge cost
  - when the hedge cost has risk-premium constraint embedded in it then it can also be called option-value associated with the risk-premium (e.g., **hedge mode c** and **e**)
- hedge slippage metrics and their strike and term dependence;
- option-price implied risk premium.[18]

Tables 2-7 present summary hedge performance information for 5 distinct hedging modes. The *hurdle price* of the seller-hedger and buyer-hedger are presented for three different *hurdles*:

- 25%/year rate of return on risk-capital
- Sharpe-Ratio = 1
- Sortino-Ratio = 1

[18] **Appendix-D** provides a useful summary of risk premiums that are introduced in **Section 3**.

maturity	hedge mode	option seller-hedger									option buyer-hedger								
		initial hedge ratio (%)	hedge cost (% of spot)	hedge performance dw (% of spot)				hurdle price (% of spot)			initial hedge ratio (%)	hedge cost (% of spot)	hedge performance dw (% of spot)				hurdle price (% of spot)		
				avg	std dev	neg std dev	risk capital	25% RORC	1 Sharpe	1 Sortino			avg	std dev	neg std dev	risk capital	25% RORC	1 Sharpe	1 Sortino
		low realized volatility regime at inception																	
10 day	a	-47.70	0.51	0.00	0.33	0.42	4.76	0.56	0.57	0.59	47.70	-0.51	0.00	0.33	0.24	0.56	-0.50	-0.44	-0.46
	b	-48.35	0.54	0.00	0.33	0.43	4.80	0.59	0.61	0.62	47.06	-0.48	0.00	0.33	0.24	0.56	-0.47	-0.42	-0.43
	c	-47.71	0.60	0.09	0.33	0.42	4.76	0.56	0.57	0.59	47.70	-0.49	0.02	0.33	0.24	0.56	-0.50	-0.45	-0.46
	d	-47.78	0.51	0.00	0.33	0.42	4.55	0.55	0.57	0.59	47.78	-0.51	0.00	0.33	0.24	0.56	-0.50	-0.44	-0.46
	e	-48.41	0.65	0.10	0.36	0.48	4.88	0.59	0.62	0.64	47.14	-0.45	0.02	0.33	0.24	0.56	-0.47	-0.41	-0.43
21 day	a	-47.13	0.80	0.00	0.47	0.60	5.58	0.92	0.94	0.98	47.13	-0.80	0.00	0.47	0.34	0.90	-0.79	-0.67	-0.71
	b	-47.64	0.84	0.00	0.48	0.62	5.67	0.96	0.98	1.02	46.64	-0.77	0.00	0.46	0.34	0.89	-0.75	-0.64	-0.67
	c	-47.11	0.98	0.17	0.47	0.60	5.59	0.92	0.94	0.98	47.13	-0.77	0.03	0.47	0.34	0.90	-0.79	-0.67	-0.71
	d	-47.68	0.80	0.00	0.46	0.59	4.92	0.90	0.94	0.97	47.68	-0.80	0.00	0.46	0.34	0.89	-0.78	-0.67	-0.70
	e	-48.13	1.06	0.21	0.51	0.66	5.26	0.96	1.00	1.04	47.15	-0.71	0.05	0.46	0.35	0.89	-0.74	-0.63	-0.66
42 day	a	-46.60	1.25	0.00	0.68	0.88	6.72	1.56	1.54	1.62	46.60	-1.25	0.00	0.68	0.50	1.41	-1.19	-0.97	-1.05
	b	-47.00	1.30	0.00	0.69	0.91	6.84	1.61	1.59	1.68	46.22	-1.21	0.00	0.67	0.50	1.41	-1.15	-0.93	-1.00
	c	-46.59	1.57	0.31	0.68	0.88	6.72	1.56	1.54	1.62	46.59	-1.20	0.06	0.68	0.50	1.41	-1.19	-0.97	-1.05
	d	-48.01	1.25	0.00	0.67	0.84	5.83	1.51	1.53	1.60	48.01	-1.25	0.00	0.67	0.51	1.39	-1.19	-0.97	-1.03
	e	-48.37	1.72	0.41	0.69	0.86	5.90	1.57	1.59	1.67	47.62	-1.07	0.13	0.67	0.51	1.39	-1.13	-0.92	-0.98
		high realized volatility regime at inception																	
10 day	a	-47.73	2.24	0.00	1.09	1.42	15.57	2.40	2.46	2.53	47.73	-2.24	0.00	1.09	0.78	2.06	-2.22	-2.02	-2.08
	b	-47.99	2.29	0.00	1.10	1.44	15.78	2.45	2.51	2.58	47.46	-2.19	0.00	1.09	0.78	2.05	-2.17	-1.97	-2.03
	c	-47.83	2.54	0.30	1.09	1.41	15.53	2.40	2.46	2.53	47.75	-2.17	0.08	1.09	0.78	2.06	-2.22	-2.02	-2.08
	d	-47.77	2.24	0.00	1.09	1.42	14.98	2.39	2.46	2.53	47.77	-2.24	0.00	1.09	0.78	2.03	-2.22	-2.02	-2.08
	e	-48.19	2.59	0.30	1.10	1.43	14.81	2.44	2.51	2.58	47.52	-2.10	0.09	1.09	0.78	2.03	-2.17	-1.97	-2.03
21 day	a	-49.00	3.31	0.00	1.40	1.81	16.92	3.69	3.73	3.85	49.00	-3.31	0.00	1.40	1.02	2.87	-3.24	-2.88	-3.00
	b	-49.24	3.36	0.00	1.41	1.84	17.14	3.75	3.79	3.91	48.77	-3.25	0.00	1.39	1.02	2.86	-3.18	-2.83	-2.94
	c	-49.21	3.82	0.51	1.40	1.79	16.84	3.70	3.73	3.85	49.04	-3.19	0.11	1.40	1.02	2.87	-3.24	-2.89	-3.00
	d	-49.23	3.30	0.00	1.40	1.79	15.25	3.65	3.72	3.84	49.23	-3.30	0.00	1.40	1.03	2.75	-3.23	-2.88	-2.99
	e	-49.61	3.93	0.58	1.41	1.81	15.29	3.70	3.78	3.90	49.01	-3.07	0.17	1.39	1.03	2.76	-3.18	-2.82	-2.93
42 day	a	-50.22	4.72	0.00	1.78	2.29	17.66	5.50	5.46	5.67	50.22	-4.72	0.00	1.78	1.31	3.84	-4.55	-3.98	-4.17
	b	-50.45	4.78	0.00	1.80	2.33	17.82	5.57	5.53	5.75	50.00	-4.66	0.00	1.77	1.32	3.83	-4.49	-3.92	-4.11
	c	-50.50	5.50	0.77	1.78	2.26	17.52	5.50	5.47	5.67	50.28	-4.57	0.15	1.78	1.31	3.84	-4.55	-3.98	-4.18
	d	-50.79	4.70	0.00	1.76	2.21	14.82	5.35	5.43	5.62	50.79	-4.70	0.00	1.76	1.35	3.64	-4.54	-3.97	-4.14
	e	-51.19	5.75	0.98	1.78	2.23	14.87	5.43	5.51	5.70	50.57	-4.30	0.34	1.76	1.35	3.65	-4.48	-3.91	-4.08

Table 2. Summary results for 100% strike Put Option

hedge-mode	Trans-costs	risk-premium	conditioning
a	no	no	no
b	**yes**	no	no
c	no	**yes**	no
d	no	no	**yes**
e	**yes**	**yes**	**yes**

maturity	hedge mode	option seller-hedger: initial hedge ratio (%)	option seller-hedger: hedge cost (% of spot)	option seller-hedger: hedge performance dw (% of spot): avg	option seller-hedger: hedge performance dw (% of spot): std dev	option seller-hedger: hedge performance dw (% of spot): neg std dev	option seller-hedger: hedge performance dw (% of spot): risk capital	option seller-hedger: hurdle price (% of spot): 25% RORC	option seller-hedger: hurdle price (% of spot): 1 Sharpe	option seller-hedger: hurdle price (% of spot): 1 Sortino	option buyer-hedger: initial hedge ratio (%)	option buyer-hedger: hedge cost (% of spot)	option buyer-hedger: hedge performance dw (% of spot): avg	option buyer-hedger: hedge performance dw (% of spot): std dev	option buyer-hedger: hedge performance dw (% of spot): neg std dev	option buyer-hedger: hedge performance dw (% of spot): risk capital	option buyer-hedger: hurdle price (% of spot): 25% RORC	option buyer-hedger: hurdle price (% of spot): 1 Sharpe	option buyer-hedger: hurdle price (% of spot): 1 Sortino
		low realized volatility regime at inception																	
10 day	a	-1.41	0.00	0.00	0.08	0.11	3.62	0.04	0.02	0.03	1.41	0.00	0.00	0.08	0.04	0.55	0.00	0.01	0.00
	b	-1.57	0.01	0.00	0.08	0.12	3.66	0.04	0.02	0.03	1.25	0.00	0.00	0.08	0.05	0.56	0.00	0.01	0.01
	c	-1.63	0.02	0.01	0.08	0.10	3.61	0.04	0.02	0.03	1.59	0.00	0.01	0.08	0.05	0.55	0.00	0.01	0.00
	d	-1.33	0.00	0.00	0.07	0.10	3.32	0.04	0.02	0.02	1.33	0.00	0.00	0.07	0.04	0.53	0.00	0.01	0.00
	e	-1.51	0.02	0.02	0.07	0.11	3.27	0.04	0.02	0.03	1.18	0.00	0.01	0.07	0.04	0.53	0.00	0.01	0.01
21 day	a	-5.90	0.05	0.00	0.21	0.29	4.54	0.14	0.11	0.13	5.90	-0.05	0.00	0.21	0.14	1.13	-0.02	0.01	-0.01
	b	-6.30	0.05	0.00	0.21	0.32	4.62	0.15	0.12	0.15	5.50	-0.04	0.00	0.21	0.16	1.15	-0.02	0.02	0.01
	c	-6.03	0.12	0.07	0.21	0.28	4.56	0.14	0.11	0.13	6.02	-0.01	0.03	0.21	0.15	1.13	-0.02	0.01	-0.01
	d	-5.52	0.04	0.00	0.20	0.29	4.32	0.13	0.10	0.13	5.52	-0.04	0.00	0.20	0.12	0.92	-0.03	0.01	-0.01
	e	-5.34	0.14	0.08	0.22	0.32	4.38	0.15	0.12	0.15	5.19	-0.01	0.03	0.20	0.12	0.93	-0.02	0.02	0.00
42 day	a	-14.09	0.23	0.00	0.42	0.58	5.58	0.48	0.40	0.47	14.09	-0.23	0.00	0.42	0.28	1.66	-0.15	-0.05	-0.11
	b	-14.66	0.24	0.00	0.43	0.64	5.68	0.50	0.42	0.51	13.54	-0.21	0.00	0.41	0.31	1.68	-0.13	-0.04	-0.08
	c	-14.30	0.42	0.20	0.42	0.56	5.55	0.48	0.40	0.46	14.14	-0.17	0.06	0.42	0.28	1.66	-0.15	-0.05	-0.11
	d	-13.33	0.22	0.00	0.40	0.59	5.24	0.45	0.39	0.46	13.33	-0.22	0.00	0.40	0.25	1.37	-0.16	-0.05	-0.11
	e	-13.61	0.50	0.25	0.44	0.64	5.48	0.50	0.44	0.52	12.89	-0.11	0.08	0.41	0.27	1.41	-0.13	-0.02	-0.08
		high realized volatility regime at inception																	
10 day	a	-19.99	0.65	0.00	0.80	1.11	15.05	0.80	0.81	0.87	19.99	-0.65	0.00	0.80	0.51	2.02	-0.63	-0.49	-0.55
	b	-20.38	0.67	0.00	0.81	1.18	15.32	0.83	0.83	0.91	19.60	-0.63	0.00	0.79	0.53	2.03	-0.61	-0.47	-0.52
	c	-20.36	0.87	0.21	0.80	1.09	14.92	0.81	0.81	0.87	20.12	-0.59	0.06	0.80	0.51	2.02	-0.63	-0.49	-0.55
	d	-19.94	0.64	0.00	0.80	1.12	13.81	0.79	0.81	0.87	19.94	-0.64	0.00	0.80	0.50	1.99	-0.62	-0.48	-0.54
	e	-20.73	0.89	0.23	0.83	1.19	14.67	0.82	0.83	0.91	19.72	-0.56	0.07	0.80	0.54	2.02	-0.61	-0.47	-0.52
21 day	a	-28.13	1.45	0.00	1.15	1.55	14.69	1.78	1.79	1.91	28.13	-1.45	0.00	1.15	0.78	2.79	-1.38	-1.10	-1.21
	b	-28.52	1.48	0.00	1.16	1.62	14.85	1.82	1.83	1.97	27.73	-1.41	0.00	1.14	0.81	2.82	-1.35	-1.07	-1.17
	c	-28.68	1.88	0.42	1.15	1.50	14.53	1.79	1.80	1.91	28.26	-1.35	0.10	1.15	0.79	2.80	-1.39	-1.10	-1.21
	d	-28.01	1.43	0.00	1.15	1.55	13.32	1.73	1.78	1.90	28.01	-1.43	0.00	1.15	0.79	2.72	-1.37	-1.08	-1.19
	e	-28.98	1.94	0.48	1.18	1.60	13.43	1.77	1.82	1.95	27.72	-1.25	0.15	1.15	0.81	2.76	-1.34	-1.06	-1.16
42 day	a	-34.54	2.65	0.00	1.55	2.05	16.21	3.36	3.29	3.50	34.54	-2.65	0.00	1.55	1.10	3.75	-2.48	-2.00	-2.19
	b	-34.90	2.70	0.00	1.57	2.12	16.37	3.42	3.35	3.58	34.18	-2.60	0.00	1.54	1.13	3.78	-2.44	-1.96	-2.13
	c	-35.10	3.34	0.68	1.55	1.98	16.11	3.38	3.31	3.49	34.69	-2.51	0.15	1.55	1.11	3.76	-2.49	-2.01	-2.19
	d	-34.42	2.61	0.00	1.55	2.01	13.91	3.22	3.25	3.44	34.42	-2.61	0.00	1.55	1.13	3.55	-2.45	-1.96	-2.14
	e	-35.16	3.53	0.87	1.57	2.04	14.02	3.28	3.32	3.51	34.15	-2.27	0.30	1.54	1.16	3.61	-2.41	-1.93	-2.09

Table 3. Summary results for 95% strike Put Option

hedge-mode	Trans-costs	risk-premium	conditioning
a	no	no	no
b	**yes**	no	no
c	no	**yes**	no
d	no	no	**yes**
e	**yes**	**yes**	**yes**

		option seller-hedger									option buyer-hedger								
				hedge performance dw (% of spot)				hurdle price (% of spot)					hedge performance dw (% of spot)				hurdle price (% of spot)		
maturity	hedge mode	initial hedge ratio (%)	hedge cost (% of spot)	avg	std dev	neg std dev	risk capital	25% RORC	1 Sharpe	1 Sortino	initial hedge ratio (%)	hedge cost (% of spot)	avg	std dev	neg std dev	risk capital	25% RORC	1 Sharpe	1 Sortino
		low realized volatility regime at inception																	
21 day	a	-0.075	0.000	0.000	0.034	0.046	0.759	0.016	0.010	0.014	0.075	0.000	0.000	0.034	0.013	0.328	0.007	0.010	0.003
	b	-0.090	0.000	0.000	0.034	0.049	0.762	0.016	0.010	0.015	0.060	0.000	0.000	0.034	0.014	0.331	0.007	0.010	0.004
	c	-0.076	0.004	0.004	0.034	0.046	0.760	0.016	0.010	0.013	0.077	0.002	0.002	0.034	0.013	0.327	0.007	0.010	0.004
	d	-0.043	0.000	0.000	0.029	0.037	0.702	0.015	0.009	0.011	0.043	0.000	0.000	0.029	0.014	0.418	0.008	0.008	0.004
	e	-0.050	0.006	0.005	0.029	0.040	0.699	0.015	0.009	0.012	0.031	0.003	0.003	0.029	0.014	0.420	0.009	0.009	0.004
42 day	a	-0.851	0.007	0.000	0.163	0.221	5.401	0.249	0.075	0.100	0.851	-0.007	0.000	0.163	0.091	1.086	0.041	0.061	0.031
	b	-0.934	0.010	0.000	0.162	0.234	5.408	0.252	0.078	0.108	0.768	-0.005	0.000	0.163	0.098	1.095	0.044	0.063	0.036
	c	-1.147	0.045	0.037	0.164	0.201	5.418	0.251	0.077	0.093	1.042	0.015	0.023	0.164	0.097	1.024	0.038	0.060	0.032
	d	-0.599	0.006	0.000	0.145	0.209	4.790	0.221	0.067	0.094	0.599	-0.006	0.000	0.145	0.078	1.172	0.046	0.054	0.026
	e	-0.645	0.052	0.044	0.146	0.209	4.806	0.224	0.070	0.096	0.561	0.024	0.028	0.145	0.077	1.148	0.047	0.056	0.028
		high realized volatility regime at inception																	
10 day	a	-1.89	0.04	0.00	0.28	0.40	11.06	0.15	0.10	0.12	1.89	-0.04	0.00	0.28	0.16	1.82	-0.02	0.02	-0.01
	b	-1.98	0.04	0.00	0.28	0.42	11.05	0.16	0.10	0.13	1.81	-0.04	0.00	0.28	0.17	1.83	-0.02	0.02	0.00
	c	-1.99	0.10	0.06	0.28	0.39	11.12	0.15	0.10	0.12	1.99	-0.01	0.03	0.28	0.17	1.82	-0.02	0.02	0.00
	d	-1.84	0.04	0.00	0.27	0.39	9.78	0.14	0.09	0.12	1.84	-0.04	0.00	0.27	0.14	1.68	-0.02	0.02	-0.01
	e	-1.93	0.11	0.07	0.27	0.40	9.74	0.14	0.10	0.12	1.78	0.00	0.03	0.27	0.16	1.69	-0.02	0.02	0.00
21 day	a	-6.12	0.21	0.00	0.58	0.81	11.36	0.47	0.39	0.45	6.12	-0.21	0.00	0.58	0.39	3.09	-0.14	-0.03	-0.09
	b	-6.31	0.22	0.00	0.59	0.86	11.42	0.48	0.40	0.48	5.92	-0.20	0.00	0.58	0.42	3.13	-0.13	-0.03	-0.08
	c	-6.56	0.42	0.20	0.58	0.75	11.46	0.48	0.39	0.44	6.47	-0.12	0.10	0.58	0.41	3.10	-0.15	-0.04	-0.09
	d	-5.85	0.20	0.00	0.56	0.82	10.99	0.45	0.37	0.45	5.85	-0.20	0.00	0.56	0.34	2.56	-0.14	-0.04	-0.10
	e	-6.20	0.43	0.22	0.57	0.85	11.17	0.47	0.39	0.47	5.76	-0.09	0.10	0.56	0.36	2.59	-0.13	-0.03	-0.08
42 day	a	-12.55	0.69	0.00	0.95	1.30	11.95	1.22	1.08	1.23	12.55	-0.69	0.00	0.95	0.63	4.02	-0.51	-0.30	-0.43
	b	-12.83	0.71	0.00	0.96	1.39	12.14	1.24	1.11	1.28	12.26	-0.67	0.00	0.94	0.68	4.06	-0.50	-0.29	-0.39
	c	-13.23	1.13	0.42	0.94	1.21	11.87	1.23	1.10	1.21	12.85	-0.54	0.15	0.94	0.66	4.04	-0.52	-0.31	-0.42
	d	-11.85	0.66	0.00	0.93	1.34	11.96	1.18	1.05	1.21	11.83	-0.44	0.22	0.94	0.65	3.46	-0.50	-0.26	-0.38
	e	-12.67	1.21	0.52	0.95	1.37	12.15	1.22	1.08	1.26	11.83	-0.44	0.22	0.94	0.65	3.46	-0.50	-0.26	-0.38

Table 4. Summary results for 85% strike Put Option

hedge-mode	Trans-costs	risk-premium	conditioning
a	no	no	no
b	**yes**	no	no
c	no	**yes**	no
d	no	no	**yes**
e	**yes**	**yes**	**yes**

maturity	hedge mode	*option seller-hedger*									*option buyer-hedger*								
		initial hedge ratio (%)	hedge cost (% of spot)	hedge performance dw (% of spot)				hurdle price (% of spot)			initial hedge ratio (%)	hedge cost (% of spot)	hedge performance dw (% of spot)				hurdle price (% of spot)		
				avg	std dev	neg std dev	risk capital	25% RORC	1 Sharpe	1 Sortino			avg	std dev	neg std dev	risk capital	25% RORC	1 Sharpe	1 Sortino
		low realized volatility regime at inception																	
10 day	a	52.30	0.64	0.00	0.33	0.42	4.76	0.69	0.70	0.72	-52.30	-0.64	0.00	0.33	0.24	0.56	-0.63	-0.58	-0.59
	b	52.43	0.67	0.00	0.33	0.43	4.79	0.72	0.74	0.76	-52.15	-0.61	0.00	0.33	0.24	0.56	-0.60	-0.54	-0.56
	c	52.29	0.73	0.09	0.33	0.42	4.76	0.69	0.70	0.72	-52.30	-0.62	0.02	0.33	0.24	0.56	-0.63	-0.58	-0.59
	d	52.22	0.64	0.00	0.33	0.42	4.55	0.68	0.70	0.72	-52.22	-0.64	0.00	0.33	0.24	0.56	-0.63	-0.58	-0.59
	e	52.44	0.78	0.10	0.37	0.49	4.93	0.73	0.75	0.78	-52.04	-0.58	0.02	0.33	0.24	0.56	-0.60	-0.54	-0.56
21 day	a	52.87	1.09	0.00	0.47	0.60	5.58	1.20	1.22	1.26	-52.87	-1.09	0.00	0.47	0.34	0.90	-1.07	-0.95	-0.99
	b	52.86	1.13	0.00	0.48	0.62	5.66	1.25	1.26	1.31	-52.84	-1.05	0.00	0.46	0.34	0.89	-1.03	-0.91	-0.95
	c	52.88	1.26	0.17	0.47	0.60	5.59	1.20	1.22	1.26	-52.86	-1.05	0.03	0.47	0.34	0.90	-1.07	-0.95	-0.99
	d	52.32	1.09	0.00	0.46	0.59	4.92	1.19	1.22	1.25	-52.32	-1.09	0.00	0.46	0.34	0.89	-1.07	-0.95	-0.99
	e	52.39	1.35	0.21	0.51	0.66	5.26	1.25	1.29	1.33	-52.30	-0.99	0.05	0.46	0.35	0.89	-1.02	-0.91	-0.94
42 day	a	53.40	1.86	0.00	0.68	0.88	6.72	2.16	2.14	2.22	-53.40	-1.86	0.00	0.68	0.50	1.41	-1.79	-1.57	-1.65
	b	53.31	1.91	0.00	0.69	0.91	6.84	2.21	2.20	2.29	-53.45	-1.80	0.00	0.67	0.51	1.40	-1.74	-1.52	-1.59
	c	53.38	2.17	0.31	0.68	0.88	6.72	2.16	2.14	2.22	-53.39	-1.80	0.06	0.68	0.50	1.41	-1.79	-1.57	-1.65
	d	51.99	1.85	0.00	0.67	0.84	5.83	2.11	2.13	2.20	-51.99	-1.85	0.00	0.67	0.51	1.39	-1.79	-1.57	-1.64
	e	52.02	2.33	0.41	0.69	0.87	5.90	2.18	2.20	2.27	-52.02	-1.67	0.13	0.67	0.52	1.39	-1.73	-1.51	-1.58
		high realized volatility regime at inception																	
10 day	a	52.27	2.11	0.00	1.09	1.42	15.57	2.28	2.34	2.40	-52.27	-2.11	0.00	1.09	0.78	2.06	-2.09	-1.89	-1.96
	b	52.34	2.16	0.00	1.10	1.44	15.71	2.33	2.39	2.45	-52.23	-2.06	0.00	1.09	0.79	2.05	-2.04	-1.84	-1.91
	c	52.25	2.41	0.30	1.09	1.41	15.54	2.28	2.34	2.40	-52.25	-2.04	0.07	1.09	0.78	2.06	-2.09	-1.89	-1.96
	d	52.23	2.11	0.00	1.09	1.42	14.98	2.27	2.33	2.40	-52.23	-2.11	0.00	1.09	0.78	2.03	-2.09	-1.89	-1.95
	e	52.27	2.47	0.31	1.10	1.44	14.81	2.31	2.38	2.45	-52.17	-1.98	0.09	1.09	0.78	2.03	-2.04	-1.84	-1.90
21 day	a	51.00	3.04	0.00	1.40	1.81	16.92	3.43	3.46	3.59	-51.00	-3.04	0.00	1.40	1.02	2.87	-2.98	-2.62	-2.74
	b	51.00	3.10	0.00	1.41	1.84	17.09	3.49	3.52	3.65	-50.99	-2.99	0.00	1.39	1.02	2.86	-2.92	-2.57	-2.68
	c	51.03	3.55	0.51	1.40	1.80	16.87	3.43	3.46	3.58	-50.96	-2.93	0.11	1.40	1.02	2.87	-2.98	-2.62	-2.74
	d	50.77	3.03	0.00	1.40	1.79	15.25	3.38	3.45	3.57	-50.77	-3.03	0.00	1.40	1.03	2.75	-2.97	-2.61	-2.72
	e	50.82	3.67	0.58	1.41	1.82	15.23	3.44	3.51	3.63	-50.75	-2.81	0.17	1.39	1.03	2.76	-2.91	-2.56	-2.67
42 day	a	49.78	4.19	0.00	1.78	2.29	17.66	4.97	4.93	5.14	-49.78	-4.19	0.00	1.78	1.31	3.84	-4.02	-3.45	-3.65
	b	49.73	4.26	0.00	1.80	2.33	17.77	5.04	5.00	5.23	-49.82	-4.13	0.00	1.77	1.32	3.83	-3.96	-3.39	-3.58
	c	49.97	4.97	0.79	1.78	2.28	17.59	4.96	4.93	5.13	-49.74	-4.04	0.15	1.78	1.31	3.84	-4.02	-3.45	-3.65
	d	49.21	4.17	0.00	1.76	2.21	14.82	4.83	4.90	5.09	-49.21	-4.17	0.00	1.76	1.35	3.64	-4.01	-3.44	-3.61
	e	49.28	5.22	0.99	1.78	2.24	14.88	4.89	4.97	5.16	-49.26	-3.77	0.33	1.76	1.35	3.65	-3.95	-3.38	-3.55

Table 5. Summary results for 100% strike Call-Option

hedge-mode	Trans-costs	risk-premium	conditioning
a	no	no	no
b	**yes**	no	no
c	no	**yes**	no
d	no	no	**yes**
e	**yes**	**yes**	**yes**

maturity	hedge mode	option seller-hedger									option buyer-hedger								
		initial hedge ratio (%)	hedge cost (% of spot)	hedge performance dw (% of spot)				hurdle price (% of spot)			initial hedge ratio (%)	hedge cost (% of spot)	hedge performance dw (% of spot)				hurdle price (% of spot)		
				avg	std dev	neg std dev	risk capital	25% RORC	1 Sharpe	1 Sortino			avg	std dev	neg std dev	risk capital	25% RORC	1 Sharpe	1 Sortino
		low realized volatility regime at inception																	
10 day	a	1.24	0.00	0.00	0.08	0.11	4.07	0.04	0.02	0.03	-1.24	0.00	0.00	0.08	0.04	0.50	0.00	0.01	0.00
	b	1.39	0.01	0.00	0.08	0.11	4.09	0.04	0.02	0.03	-1.11	0.00	0.00	0.08	0.04	0.49	0.00	0.01	0.01
	c	1.28	0.02	0.02	0.08	0.11	4.08	0.04	0.02	0.03	-1.25	0.01	0.01	0.08	0.04	0.50	0.00	0.01	0.00
	d	1.20	0.00	0.00	0.08	0.11	3.61	0.04	0.02	0.03	-1.20	0.00	0.00	0.08	0.04	0.47	0.00	0.01	0.00
	e	1.37	0.02	0.02	0.08	0.11	3.63	0.04	0.02	0.03	-1.07	0.00	0.01	0.08	0.04	0.47	0.00	0.01	0.00
21 day	a	5.31	0.04	0.00	0.22	0.30	4.72	0.14	0.10	0.13	-5.31	-0.04	0.00	0.22	0.14	1.03	-0.02	0.02	0.00
	b	5.68	0.05	0.00	0.22	0.30	4.76	0.15	0.11	0.13	-4.94	-0.03	0.00	0.22	0.14	1.01	-0.01	0.03	0.01
	c	5.36	0.12	0.08	0.22	0.29	4.70	0.14	0.10	0.12	-5.32	-0.01	0.03	0.22	0.14	1.03	-0.02	0.02	0.00
	d	4.84	0.04	0.00	0.21	0.32	4.46	0.14	0.10	0.13	-4.84	-0.04	0.00	0.21	0.12	0.83	-0.03	0.02	-0.01
	e	5.18	0.13	0.08	0.21	0.31	4.49	0.14	0.11	0.14	-4.53	-0.01	0.03	0.21	0.11	0.81	-0.02	0.02	0.00
42 day	a	14.60	0.25	0.00	0.50	0.66	6.10	0.53	0.46	0.53	-14.60	-0.25	0.00	0.50	0.34	1.40	-0.19	-0.05	-0.11
	b	15.12	0.28	0.00	0.51	0.66	6.15	0.55	0.49	0.55	-14.06	-0.23	0.00	0.49	0.33	1.37	-0.17	-0.03	-0.10
	c	14.58	0.49	0.24	0.49	0.66	6.10	0.53	0.46	0.53	-14.59	-0.20	0.05	0.50	0.34	1.40	-0.19	-0.05	-0.11
	d	12.19	0.26	0.00	0.47	0.66	5.38	0.50	0.46	0.54	-12.19	-0.26	0.00	0.47	0.30	1.18	-0.21	-0.06	-0.13
	e	12.64	0.57	0.28	0.48	0.67	5.38	0.53	0.49	0.57	-11.72	-0.15	0.09	0.47	0.29	1.16	-0.19	-0.04	-0.12
		high realized volatility regime at inception																	
10 day	a	23.09	0.53	0.00	0.97	1.27	16.85	0.71	0.73	0.79	-23.09	-0.53	0.00	0.97	0.68	2.13	-0.51	-0.34	-0.40
	b	23.39	0.56	0.00	0.98	1.27	16.95	0.74	0.76	0.82	-22.80	-0.51	0.00	0.97	0.67	2.10	-0.49	-0.31	-0.37
	c	23.17	0.80	0.26	0.97	1.27	16.83	0.71	0.73	0.79	-23.12	-0.46	0.07	0.97	0.68	2.13	-0.51	-0.34	-0.40
	d	23.07	0.54	0.00	0.97	1.28	16.21	0.71	0.73	0.80	-23.07	-0.54	0.00	0.97	0.66	2.11	-0.52	-0.34	-0.40
	e	23.42	0.83	0.26	0.97	1.28	16.21	0.73	0.76	0.82	-22.79	-0.43	0.08	0.96	0.65	2.09	-0.49	-0.32	-0.38
21 day	a	29.57	1.21	0.00	1.37	1.75	17.45	1.61	1.62	1.74	-29.57	-1.21	0.00	1.37	0.99	2.93	-1.15	-0.80	-0.91
	b	29.79	1.26	0.00	1.38	1.76	17.44	1.65	1.67	1.78	-29.34	-1.17	0.00	1.36	0.98	2.90	-1.10	-0.76	-0.88
	c	29.70	1.71	0.50	1.37	1.74	17.44	1.61	1.62	1.74	-29.59	-1.10	0.11	1.37	0.99	2.93	-1.15	-0.80	-0.91
	d	29.23	1.22	0.00	1.34	1.73	15.44	1.57	1.62	1.74	-29.23	-1.22	0.00	1.34	0.97	2.82	-1.16	-0.82	-0.93
	e	29.55	1.81	0.55	1.35	1.74	15.39	1.61	1.67	1.79	-29.03	-1.01	0.16	1.33	0.96	2.80	-1.11	-0.78	-0.89
42 day	a	34.03	2.21	0.00	1.82	2.31	18.17	3.01	2.97	3.17	-34.03	-2.21	0.00	1.82	1.34	3.91	-2.04	-1.46	-1.65
	b	34.16	2.27	0.00	1.83	2.33	18.28	3.07	3.03	3.23	-33.89	-2.16	0.00	1.80	1.33	3.89	-1.98	-1.41	-1.60
	c	34.29	3.00	0.80	1.82	2.30	18.19	3.01	2.96	3.16	-34.04	-2.05	0.16	1.82	1.34	3.91	-2.04	-1.46	-1.65
	d	32.99	2.22	0.00	1.76	2.22	15.40	2.90	2.95	3.14	-32.99	-2.22	0.00	1.76	1.33	3.66	-2.06	-1.49	-1.67
	e	33.24	3.25	0.97	1.78	2.23	15.42	2.95	3.01	3.20	-32.90	-1.83	0.33	1.76	1.32	3.64	-2.00	-1.43	-1.61

Table 6. Summary results for 105% strike Call Option

hedge-mode	Trans-costs	risk-premium	conditioning
a	no	no	no
b	**yes**	no	no
c	no	**yes**	no
d	no	no	**yes**
e	**yes**	**yes**	**yes**

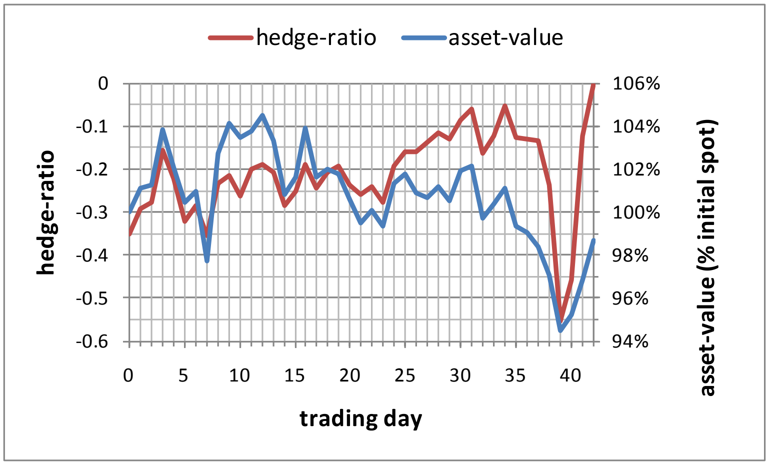
maturity	hedge mode	option seller-hedger: initial hedge ratio (%)	option seller-hedger: hedge cost (% of spot)	option seller-hedger: hedge performance dw (% of spot): avg	option seller-hedger: hedge performance dw (% of spot): std dev	option seller-hedger: hedge performance dw (% of spot): neg std dev	option seller-hedger: hedge performance dw (% of spot): risk capital	option seller-hedger: hurdle price (% of spot): 25% RORC	option seller-hedger: hurdle price (% of spot): 1 Sharpe	option seller-hedger: hurdle price (% of spot): 1 Sortino	option buyer-hedger: initial hedge ratio (%)	option buyer-hedger: hedge cost (% of spot)	option buyer-hedger: hedge performance dw (% of spot): avg	option buyer-hedger: hedge performance dw (% of spot): std dev	option buyer-hedger: hedge performance dw (% of spot): neg std dev	option buyer-hedger: hedge performance dw (% of spot): risk capital	option buyer-hedger: hurdle price (% of spot): 25% RORC	option buyer-hedger: hurdle price (% of spot): 1 Sharpe	option buyer-hedger: hurdle price (% of spot): 1 Sortino
low realized volatility regime at inception																			
21 day	a	0.031	0.000	0.000	0.020	0.028	0.164	0.004	0.006	0.008	-0.031	0.000	0.000	0.020	0.006	0.176	0.004	0.006	0.002
	b	0.038	0.000	0.000	0.020	0.029	0.166	0.004	0.006	0.008	-0.024	0.000	0.000	0.020	0.007	0.178	0.004	0.006	0.002
	c	0.032	0.002	0.002	0.020	0.027	0.165	0.004	0.006	0.008	-0.032	0.001	0.001	0.020	0.007	0.175	0.004	0.006	0.002
	d	0.019	0.000	0.000	0.019	0.027	0.252	0.005	0.006	0.008	-0.019	0.000	0.000	0.019	0.007	0.221	0.004	0.005	0.002
	e	0.022	0.004	0.004	0.019	0.029	0.255	0.006	0.006	0.008	-0.012	0.002	0.002	0.019	0.007	0.223	0.005	0.005	0.002
42 day	a	0.420	0.003	0.000	0.116	0.158	4.110	0.187	0.052	0.069	-0.420	-0.003	0.000	0.116	0.058	0.669	0.027	0.045	0.021
	b	0.468	0.004	0.000	0.116	0.159	4.109	0.189	0.053	0.071	-0.371	-0.002	0.000	0.115	0.058	0.664	0.028	0.046	0.022
	c	0.460	0.029	0.026	0.116	0.146	4.202	0.191	0.052	0.064	-0.418	0.013	0.016	0.116	0.056	0.630	0.025	0.045	0.020
	d	0.257	0.004	0.000	0.110	0.172	3.895	0.178	0.050	0.076	-0.257	-0.004	0.000	0.110	0.050	0.827	0.033	0.042	0.017
	e	0.281	0.036	0.031	0.111	0.173	3.904	0.180	0.051	0.077	-0.240	0.016	0.019	0.110	0.048	0.800	0.033	0.043	0.017
high realized volatility regime at inception																			
10 day	a	2.88	0.03	0.00	0.45	0.59	15.99	0.19	0.12	0.15	-2.88	-0.03	0.00	0.45	0.29	2.40	0.00	0.06	0.03
	b	2.99	0.03	0.00	0.45	0.59	15.90	0.19	0.12	0.15	-2.76	-0.02	0.00	0.45	0.28	2.38	0.00	0.07	0.04
	c	2.99	0.12	0.10	0.45	0.58	16.08	0.19	0.12	0.14	-2.92	0.03	0.06	0.45	0.29	2.39	0.00	0.07	0.03
	d	2.89	0.03	0.00	0.42	0.58	14.55	0.18	0.11	0.14	-2.89	-0.03	0.00	0.42	0.24	2.17	0.00	0.06	0.02
	e	3.09	0.13	0.10	0.42	0.57	14.55	0.18	0.12	0.15	-2.81	0.02	0.05	0.42	0.24	2.15	0.00	0.06	0.03
21 day	a	6.87	0.15	0.00	0.85	1.11	16.00	0.51	0.40	0.48	-6.87	-0.15	0.00	0.85	0.58	3.86	-0.06	0.11	0.03
	b	7.06	0.16	0.00	0.85	1.11	16.07	0.52	0.41	0.49	-6.67	-0.13	0.00	0.84	0.58	3.83	-0.05	0.12	0.04
	c	7.13	0.44	0.30	0.85	1.07	15.95	0.50	0.40	0.46	-6.95	-0.02	0.12	0.85	0.59	3.87	-0.05	0.11	0.04
	d	6.77	0.15	0.00	0.80	1.14	14.84	0.49	0.39	0.50	-6.77	-0.15	0.00	0.80	0.50	3.22	-0.08	0.09	-0.01
	e	7.09	0.47	0.31	0.81	1.12	14.90	0.50	0.41	0.50	-6.65	-0.02	0.13	0.80	0.50	3.21	-0.07	0.10	0.01
42 day	a	12.42	0.50	0.00	1.39	1.81	17.62	1.27	1.07	1.25	-12.42	-0.50	0.00	1.39	0.98	4.31	-0.31	0.08	-0.09
	b	12.66	0.53	0.00	1.40	1.81	17.67	1.30	1.11	1.28	-12.17	-0.47	0.00	1.38	0.96	4.28	-0.28	0.10	-0.07
	c	12.96	1.10	0.62	1.39	1.73	17.40	1.25	1.06	1.20	-12.53	-0.33	0.16	1.39	0.99	4.33	-0.30	0.08	-0.08
	d	11.58	0.53	0.00	1.30	1.81	15.64	1.22	1.07	1.28	-11.58	-0.53	0.00	1.30	0.85	3.69	-0.36	0.01	-0.17
	e	11.99	1.25	0.70	1.31	1.79	15.62	1.24	1.09	1.29	-11.44	-0.23	0.26	1.29	0.84	3.70	-0.33	0.04	-0.15

Table 7. Summary results for 115% strike Call Option

hedge-mode	Trans-costs	risk-premium	conditioning
a	no	no	no
b	**yes**	no	no
c	no	**yes**	no
d	no	no	**yes**
e	**yes**	**yes**	**yes**

4.2 Hedge Performance

Figures 7 & 8 depict hedge performance experienced in two distinct Monte-Carlo paths within OHMC.



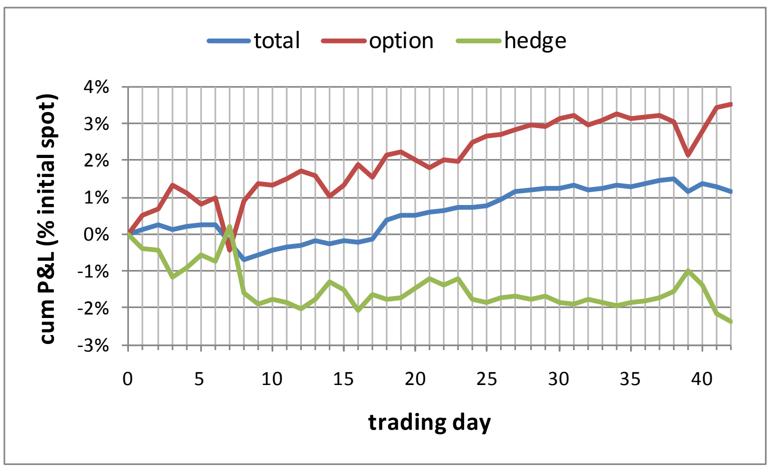
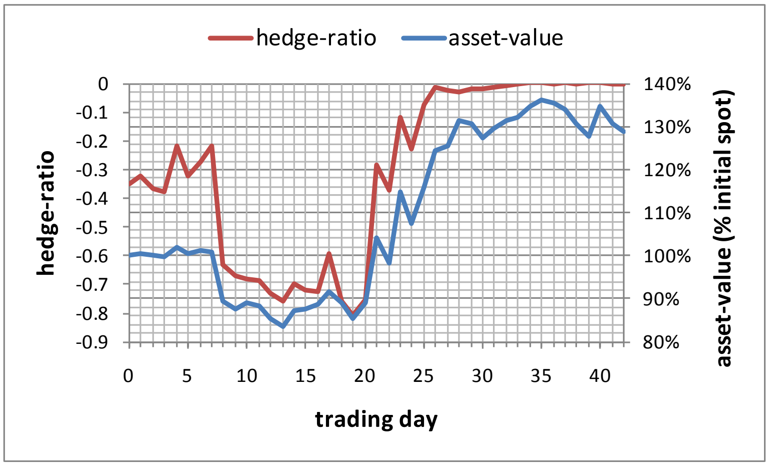


Figure 7. Median total P&L sample path hedge performance from OHMC analysis of sell-hedge 42 day 95% strike put in high volatility regime (**hedge mode e**). The hedging strategy is cognizant of transaction costs and is conditioned on the trailing 10 day realized volatility. The hedging strategy also seeks to maintain an *expected* P&L over each hedging interval to achieve a target Sortino-Ratio of 1. The attempted replication is full of slips even in this relatively benign outcome. The risk-premium charged by the seller hedger towards the goal of maintaining a Sortino-Ratio of 1 every day is fulfilled insofar as the total P&L at the end of 42 days slightly exceeds the initially expected P&L in pricing the put. The P&L outcome is of course uncertain, and can be far less favorable if the underlying moves sharply, as shown in **Figure 8**. The hedging P&L also includes the impact of transaction costs. For summary hedge performance statistics see **Table 2**.



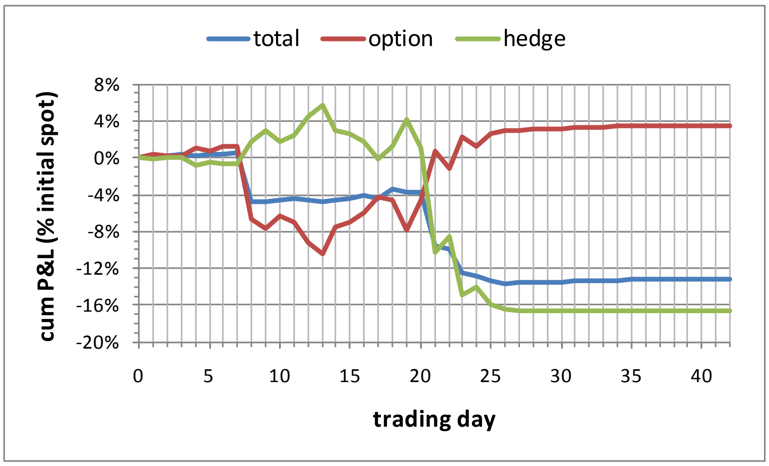


Figure 8. Tail loss scenario (1 yr 99.9 confidence level) total P&L sample path hedge performance from OHMC analysis of sell-hedge 42 day 95% strike put in high volatility regime (**hedge mode e**). While the hedging strategy seeks to maintain an *expected* P&L over a hedging interval such that the Sortino-Ratio is 1 (identical strategy as shown in **Figure 8**), the total P&L outcome is negative. The sudden drop in asset value at around the 7th trading day, and the upswing of asset values around the 20th trading day are associated with significant hedge slippage and losses. Due to the sudden drop in the asset around the 7th day the losses incurred on the sell option position are greater in magnitude than the gains from the hedge position in the underlying. In the sharp upswing in the asset near the 20th trading day, the gains arising from the sell option position are less than the losses incurred due to the hedge position. Such steep losses incurred by the derivative seller-hedger outline the need for risk-capital by the seller-hedger. The OHMC framework provides a unified platform for valuation, risk-capital assessment, and developing trading strategies. For summary hedge performance statistics see **Table 2**.

Asymmetry of Residual Risks

As a rule, the residual risks of the hedging strategy are significant and fat-tailed. For the basic hedging strategy, **hedge mode a**, both the buyer and seller share a common residual wealth change standard deviation. However that is where the similarities between the buyer-hedger and seller-hedger ends! The residual risks are asymmetric (**Figures 9 & 10**).

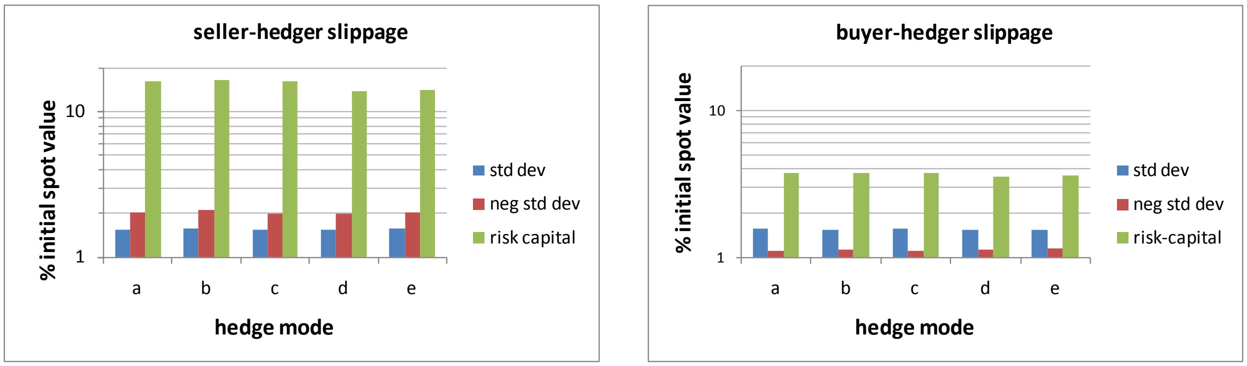


Figure 9. Hedge slippage measures for a 42 day 95% strike put in high volatility regime (see **Table 2**).

The seller-hedger has a negative semi-deviation of residual wealth change that is always larger than her standard-deviation, while the buyer-hedger has a negative semi-deviation that is always smaller than the residual standard deviation (**Figure 9**). This differentiation between the seller-hedger and buyer-hedger remain for all the hedging strategies (**hedge mode a-e**). Transaction-costs and their explicit consideration in the hedging strategy, considered in **hedge mode b**, exacerbate this difference for the seller hedger. Conditioning the hedging strategy on realized volatility (**hedge mode d**) can, in certain circumstances, modestly decrease this asymmetry for the option seller-hedger, but does not eliminate it.

The more extreme risks (i.e., tail-risks) are dramatically different for the option seller-hedger and buyer hedger (**Figures 9 & 10**). Conditioning the hedging strategy on realized volatility can significantly reduce the tail-risks for the option seller-hedger. The buyer-hedger's tail-risks, smaller to begin with, are not significantly reduced by conditioning the hedging strategy on realized volatility.

Insofar as attempted replication is always significantly imperfect, the residual risk and its asymmetries are central to developing a view of the option prices. In this pricing dynamic, a seller-hedger has a different risk profile than a buyer-hedger. Any *implied volatility surface* or set of implied parameters in a risk-neutral framework fails to shed light on the asymmetries of residual risk.

Opportunistically selling and buying options to achieve attractive returns while limiting downside risks is the goal of *volatility* investment strategies. The dissection of risks and its asymmetries afforded by OHMC can be a valuable aid in developing such strategies.

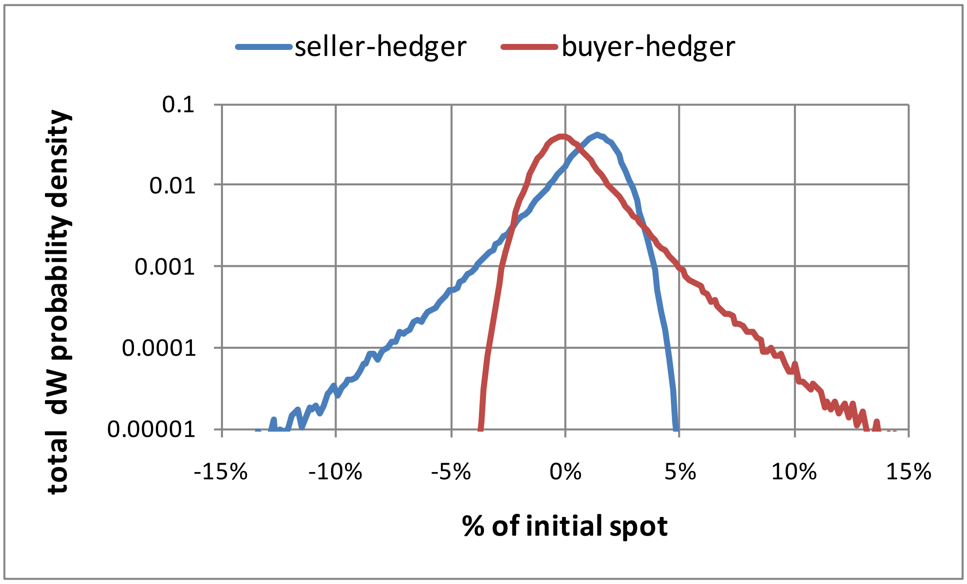


Figure 10. Comparison of seller-hedger and buyer-hedger total wealth change distribution for 42 day 95% strike put (see **Table 2 hedge mode e**). The hedging strategy includes local risk-premium constraints on the expected change in wealth, in addition to considering transaction costs and conditioning on 10-day realized volatility. Note that the positive modal wealth change for the option seller-hedger is also accompanied by a punishing loss-tail, and limited upside. Conversely, the option buyer-hedger's wealth change distribution has a modal wealth change value that is slightly negative, but is also accompanied by a limited loss-tail, and larger potential gains. Armed with such information one can develop investment strategies employing vanilla options. Furthermore, exotic options also have embedded in them a series of buy and sell positions and understanding the risk-return profile of such exotics is aided by understanding the asymmetry between a vanilla option seller-hedger and buyer-hedger. By assuming Normal returns and immaculate replication, the risk neutral approach fails to inform about these residual risk characteristics that are central in modulating the greed and fear of option market participants. Understanding these asymmetries is central to developing successful investment & trading strategies involving options.

Volatility Regime

Information about the volatility regime enters into our analysis through the GARAM simulations of the asset returns. The conditioning information in GARAM is a time-series of returns. Implicitly, this transmits information about the realized volatility in the conditioning window into the simulated asset returns, and OHMC further propagates that information into hedge performance assessment.

The volatility regime determines the magnitude of residual risks (e.g., see **Figure 11**), the average hedging costs, and the hedge ratios. For more out-of-money strikes the volatility regime has a larger impact (see **Tables 2 through 8**).

From a historical perspective, an explicit incorporation of a volatility regime into an option analysis is significant insofar as it generalizes the BS approach to a realistic description of the underlying where volatility exhibits variability & temporal persistence. OHMC enables generalizing the BS approach to attempted replication strategies that are imperfect, with quantifiable residual risks that can be assessed while conditioning on recently realized volatility.

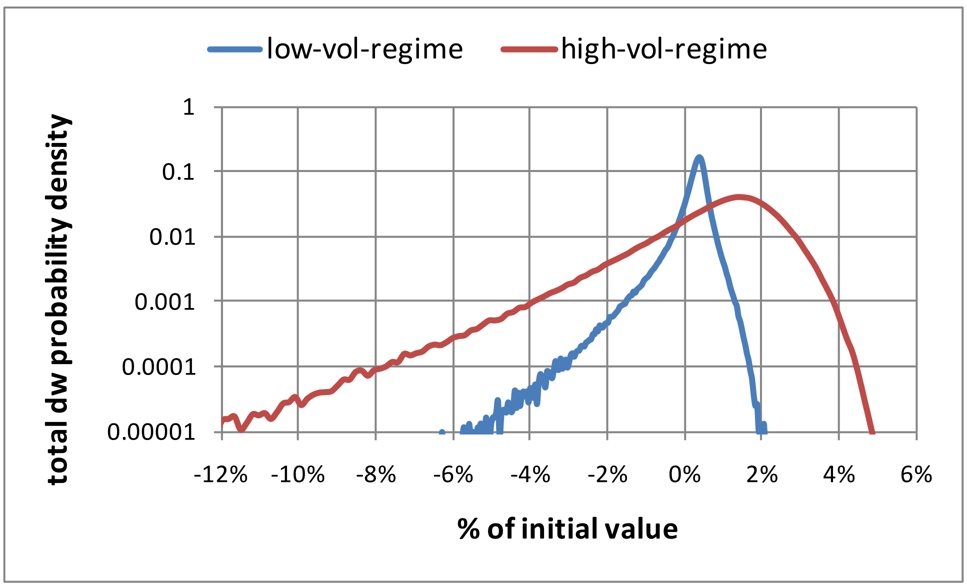


Figure 11. Comparison of option sell-hedge total wealth change distribution in low and high volatility regime, for 42 day 95% strike put (see **Table 2 hedge mode e).** The hedging strategy imposed the same risk-premium constraints at each hedge time step (Sortino-Ratio = 1). The resultant total wealth change distributions are widely different, owing to different regimes of realized volatility. OHMC helps define trading strategies that are explicitly informed of such differences. For example, to limit the volatility of a option trading strategy, the *gearing* of the trading strategy needs to respond to changing regimes.

Transaction Costs

The impact of transaction costs can be appreciated by comparing **hedge mode a** and **hedge mode b**. Here are listed the impacts of transaction costs arising from the *bid-offer* in trading the underlying asset:

(1) The average hedge cost increases due to transactions costs. The option seller expects to spend more money in attempting to replicate and the option buyer expects to receive less money in doing the same.

(2) The option seller's residual risk measures increase due to transaction-costs.

(3) The option seller-hedgers increase in risk-capital due to transaction costs is larger than the increase in his negative semi-deviation.

(4) The option seller-hedgers increase in the negative semi-deviation of wealth change due to transaction costs is larger than the increase in the standard deviation.

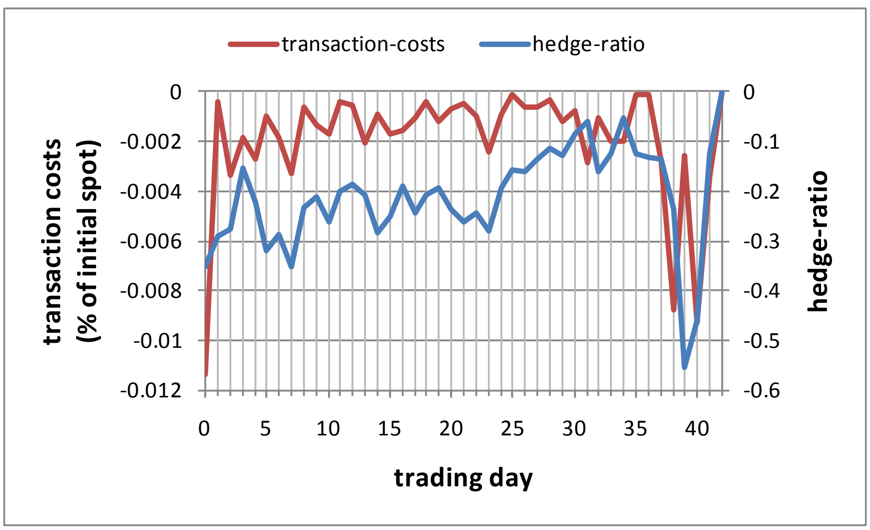


Figure 12. Median wealth change scenario transaction costs for sell-hedge 42 day 95% strike put (see **Table 2 hedge mode e**). There are two large contributions to transactions costs while hedging a sold option: **(1)** initial hedge position which involves going from no position in the underlying to the initial hedge position; **(2)** large hedge adjustment accompanied by sharp moves in the underlying.

Risk-Premium Constraints

The motivation for imposing a risk-premium constraint at each hedging interval is to try to get reimbursed for the irreducible hedging risks over each interval. We have effected a local Sortino Ratio of 1 (annualized) in imposing the local risk premium constraint. We can assess the impact of this by comparing **hedge mode a** and **hedge mode c**.

The mean change in wealth becomes positive when we apply the risk-premium constraint. For an introduction to considerations of an option trader-hedger's expected P&L the reader is referred to **Section 3** and to **Appendix-D** for a summary.

The square root of term dependence assumed here in annualizing the Sortino-Ratio is simply following convention - the actual residual risks can scale differently. We see that in many cases the option seller hedging cost that incorporates a daily annualized Sortino-Ratio of 1 in **hedge mode c** is less than the one found by annualizing the option life annualized Sortino-Ratio (see **Tables 2-7**). So, adding risk premium locally to a target hedge interval annualized Sortino-Ratio does not guarantee the same annualized Sortino-Ratio over the option life.

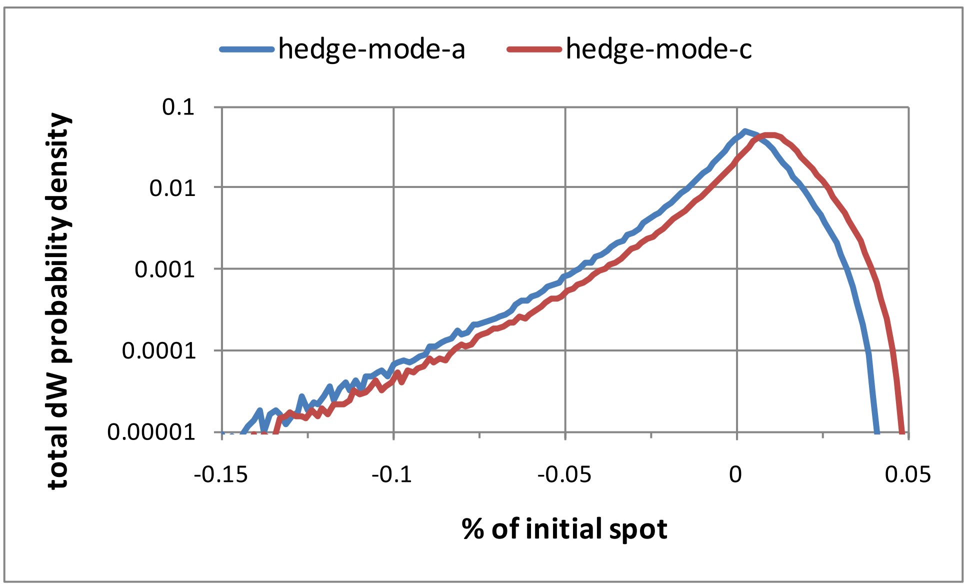


Figure 13. Impact of local risk premium constraint on total wealth change distribution for sell-hedge 42 day 95% strike put (see **Table 2**).

The facility to add a local risk-premium constraint and a global return-risk target in pricing the option helps a trader inject his profitability doctrine into the option trading strategy.

The idea of "adding a risk-premium" to the option seller hedger's "average cost of hedging" can also be applied to the option buyer-hedger too. In that case, we end up subtracting a risk measure from the buyer-hedger's average expected gains due to hedging.

In our OHMC computational solution the buyer-hedger is represented as a seller-hedger that receives the option payoff, and we have maintained all the sign conventions for a natural seller-hedger. As a result, for **hedge mode a**, the hedge cost of the buyer-hedger is always non-positive - which simply says the buyer hedger expects a positive P&L due to his act of attempting to hedge the contract with a possibly positive payoff. So, to incorporate a risk-premium we end up *adding* a non-negative quantity to the options buyers negative cost of hedging. As such there is no guarantee on the sign of the *sum* of the options buyer’s average hedging costs and risk premium, as they have opposite signs. As a result, in some situations the risk-premium adjusted buyer-hedger’s axe is a positive quantity - that indicates that he should not buy the option at any price!

A relative value based reasoning that fails to provide an argument for a buyer-hedger to purchase an option (at any positive cost) does not rule out the viability of an un-hedged buy option position. We have not explored the risk-return of such naked option positions, or statically-hedged option positions, in this work.

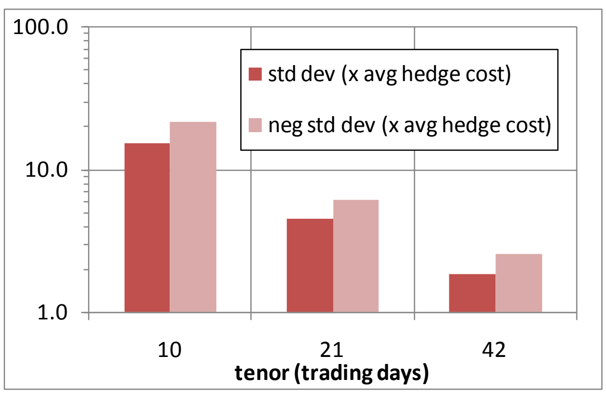
Term Dependence

The shorter the option tenor is, the greater the hedging errors are compared to the average hedge cost (see **Tables 2-7**). **Figure 14** depicts the term dependence of hedge-slippage measures.

As longer options typically incur average higher hedging costs, the hedge-slippage associated with sharp moves in the underlying, that can be particularly troublesome to hedge if the option expiry is imminent, becomes a smaller fraction of the average hedging costs.

Low-Volatility Regime

tenor (days)	std dev (x avg hedge cost)	neg std dev (x avg hedge cost)	skewness	kurtosis
10	15.5	21.6	-23	1061
21	4.5	6.1	-6.1	113
42	1.85	2.56	-3.8	42



High-Volatility Regime

tenor (days)	std dev (x avg hedge cost)	neg std dev (x avg hedge cost)	skewness	kurtosis
10	1.23	1.72	-3.8	38
21	0.79	1.07	-2.9	30
42	0.59	0.77	-2.4	18

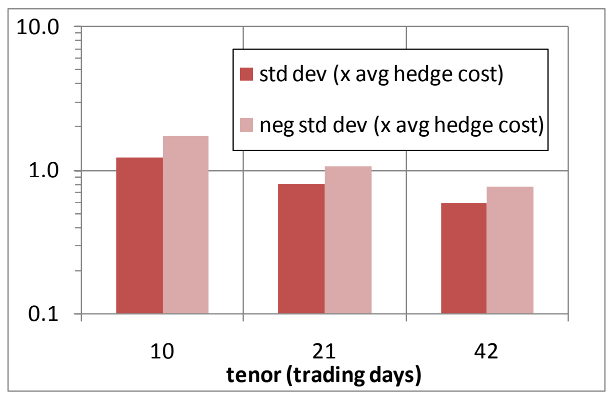


Figure 14. Term-dependence of residual risk for a seller-hedger of a 95% strike put (**hedge mode a**; see **Table 2**).

The kurtosis and skewness of the option seller-hedgers wealth change distributions are large! The kurtosis and skewness are extreme in low volatility regimes. These measures of fat tails and asymmetry decrease with increasing term, as do the body risk measures of standard deviation and negative semi-deviation. The extreme values of kurtosis and skewness in the low volatility regime, compared to the high volatility regime, is explained by the prevalence of a few outcomes of extreme wealth change and a general prevalence of modest or little wealth change.

The term dependence of hedge-slippage measures along with the average hedging costs helps understand option pricing term-dependence. This understanding can be put to uses to develop relative value metrics and buy and sell signals in option trading strategies.

Strike Dependence

Out of money options have a larger hedging error compared to average hedging costs than options with strikes closer to the spot asset value at inception (see **Tables 2-7**).

While the body residual-risk measures are increasing multiples of the average hedging costs as one looks at strikes further out of money, the tail risks increase even more, relative to the average hedging costs. This feature points out the potential dangers in under-sizing risk-capital for more out of the money options. Indeed, the recent financial crisis revealed that poorly managed financial institutions severely underestimated risk-capital associated with out of money puts.

Low-Volatility Regime

strike (% spot)	std dev (x avg hedge cost)	neg std dev (x avg hedge cost)	skewness	kurtosis
85	147	200	-168	41086
95	4.5	6.1	-6.1	113
100	0.58	0.74	-2.3	19

High-Volatility Regime

strike	std dev (x avg hedge cost)	neg std dev (x avg hedge cost)	skewness	kurtosis
85	2.8	3.8	-5.5	91
95	0.79	1.1	-2.9	30
100	0.42	0.55	-2.4	21

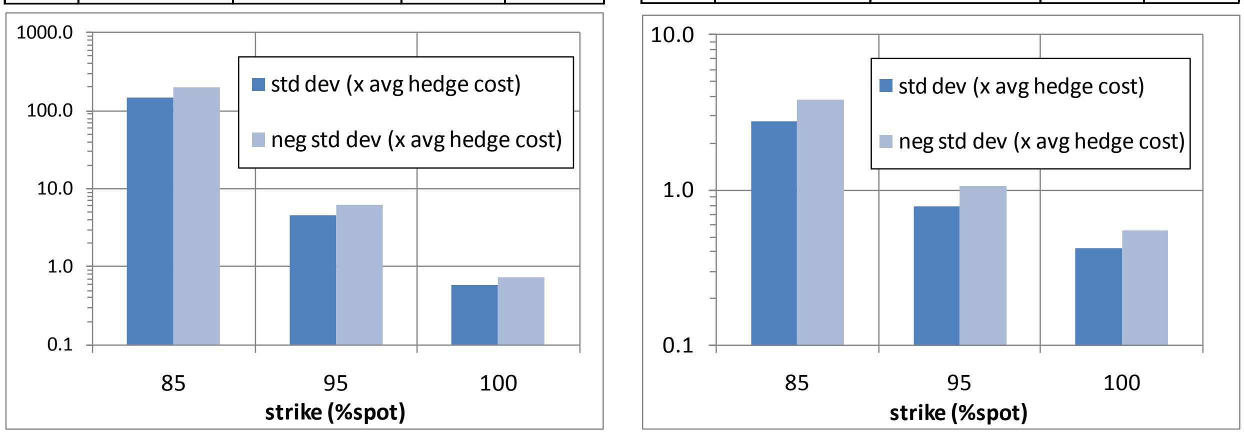


Figure 15. Strike-dependence of residual risk for a seller-hedger of a 21 day put (**hedge mode-a**, see **Tables 2,3,& 4**).

The extreme values of skewness and kurtosis, especially for the out of money sell-hedge positions, reflects the situation where in most possible outcomes the wealth change is modest, but, in a small fraction of outcomes, extreme losses occur. This fractious behavior of the wealth change distribution is exacerbated as the sell option is more out of money and/or its duration is smaller. We believe this feature is central to understanding option risk premiums and traded option prices, in addition to the average cost of hedging.

Conditioning on Realized Volatility

The motivation of conditioning the hedging strategy on realized volatility explicitly is to acknowledge the central role of volatility in hedge performance and to be responsive to its changes. In the sample problems we use a trailing 10-day realized volatility as the conditioning variable in addition to the usual conditioning on spot asset value. The impact of conditioning can be discerned by comparing **hedge mode a** with **hedge mode b**. Risk measures can shrink due to conditioning. Tail risk measure shrinks most notably, for both seller-hedger (**see Figure 16**) and buyer hedger, although more pronounced for the former.

To the extent the 10-day realized volatility has some information about the future return distribution, conditioning on it leads to a dependence of valuation and hedging on realized volatility (**Figure 17)**, and leads to a more anticipatory hedging strategy. That is evidenced in the lower 'gamma-profile' of the hedging strategy. The hedge ratio with volatility conditioning evolves more smoothly with spot than without conditioning. This is associated not only with shrinkage of the residual risk-tails, but also accompanied by limiting the transaction costs associated with hedge rebalancing.

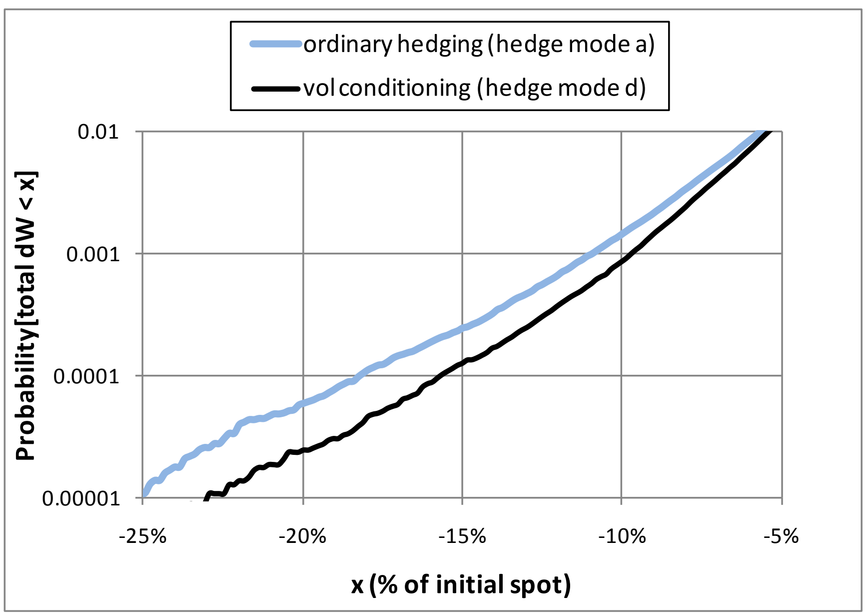


Figure 16. Impact of conditioning hedging and valuation on 10 day trailing realized volatility for a sell 95% strike 42 day put in a high volatility regime (**see Table 2**). A comparison of **hedge mode a** and **hedge mode d** total wealth change cumulative wealth change distribution is shown above. At high confidence levels, conditioning on trailing volatility shrinks the tail losses. This is due to the anticipatory nature of hedging while conditioning on realized volatility – owing its origin in the temporal persistence of the squared returns.

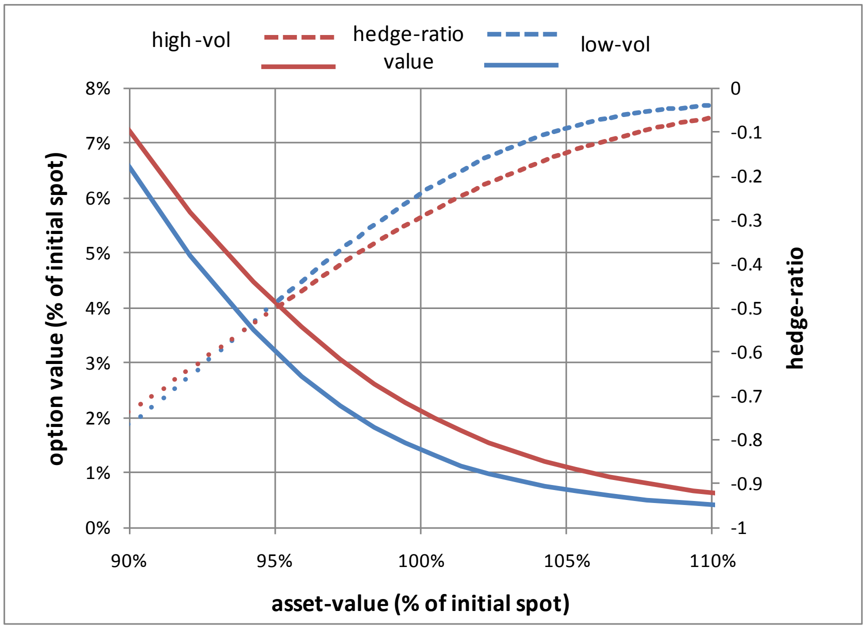


Figure 17. Impact of conditioning hedging and valuation on 10 day trailing realized volatility forward value and hedge ratio of a 42 day 95% strike put seller-hedger in a high volatility regime (see **Table 2**, **hedge mode e**). The results of OHMC on the 21st day are depicted above. The 10-day realized volatility in the low volatility regime is 17% and the 10-day realized volatility in the high volatility regime is 32%. By accounting for such *forward realized volatility sensitivity* of the hedging strategy, OHMC with conditioning on realized volatility results in a trading strategy with thinner tail-losses. While accounting for the forward realized volatility dynamics yields benefits even for simple European style options, they can be even more pertinent for derivative contracts with embedded forward starting options (e.g., Cliquets).

The conditioning of the hedging strategy on realized volatility is above and beyond the feature that our Monte-Carlo simulation of the underlying is *conditioned* on recent realized asset returns. If the options tenor is small enough such that the volatility regime that is used to condition the asset simulation persists throughout the term of the option, then the impact of conditioning the hedging strategy on realized volatility is small. Indeed we see greater impact on longer dated options than short-term options (see **Tables 2-7**).

The conditioning of the hedging strategy on realized volatility does not complicate the overall conceptual approach in applying OHMC, as it does not presume perfect replication and readily advertizes residual risks.[19] The conditioning of the hedging strategy on realized volatility is a natural refinement to OHMC.

[19] In contrast, the risk-neutral treatment of stochastic volatility breaks the original Black-Scholes replication argument and ends up introducing the notion of the *market price of volatility risk* – that is naively set to zero in the mindless fitting exercise that is the staple activity of the risk-neutral "quant."

Key Results

Conceptual Approach to Option Modeling

- Direct hedge optimization and assessment of residual risks enables explicitly decoupling impact of risk of the underlying and risk premiums on option hedging and valuation.

- Dynamics of underlying are incorporated in our analysis in two ways: **(a)** generating MC realizations of underlying that are conditioned on observations; **(b)** hedging optimally recognizing the *forward* realized volatility of the underlying.

- Residual risk assessment enables designing trading strategy – i.e., buy-sell signals, capital, gearing for target volatility & return.

- OHMC enables inferring option price implied risk-premium and imposing an investor's return over risk hurdle into trading decision making.

Risk-Return of Vanilla Options Hedged With Underlying

- Residual risks are significant, fat-tailed, and highly asymmetric
  - The option seller-hedger has a much wider loss-tail than gain-tail and the option buyer-hedger has a wider gain-tail than loss-tail

- Explicit conditioning of hedging strategy on forward realized volatility shrinks tail hedge slippage

- Transaction costs result in a wider residual risk distribution
  - asset evolution paths with high realized volatility have larger changes in hedge ratios, and therefore greater incurred transaction costs
  - conditioning hedging & valuation on realized volatility limits transactions costs

- Hedge slippage as multiple of average hedging costs increases as the option is more out of the money

- Hedge slippage as multiple of average hedging costs decreases as the option maturity increases

- Residual risk-dependent expected P&L constraints can be incorporated in the hedging strategy

6. Discussion

Risk Taking Culture & Risk Neutral Models

The current regime of *risk-neutral* valuation modeling represents a symbiosis between some constituencies that want trivial ways to control (and exaggerate?) the profitability of a business and largely naïve and misguided mathematical modeling whose purveyors are paid for providing valuation models that serve the myopic purpose of such constituencies. The reason we label that modeling effort *naïve* and not *malicious* is because the *risk-neutral valuation quant effort* is often uninformed of how P&L is being booked and risk capital is being understated!

The idea of an *immaculate* hedging cost – courtesy *Black-Scholes* - is being used to recognize upfront P&L on complex derivatives. A model that provides a business head an ability to recognize P&L on day 1 of a derivative contract that will last for 10 years is quite convenient! Even more so if that model is based on assuming perfect hedges – so nobody has a clue about the risk capital of the trade at the time of booking the upfront P&L – aided and abetted by a risk-neutral model!! A business head with short term interests can get the better of the often *hapless* and under-armed risk-manager as the risk neutral valuation quant bamboozles him with obfuscation and nonsense about “Q measures” and “martingales” that have no basis in observable reality! As these complex trades pile-up, the ability of any control function to intelligently articulate risk-capital requirements decreases. All this obfuscation lasts until all the cross-gamma-forward realized volatility and dispersion sensitivity result in a big *blow-up!*

The motivation of hedging a derivative by holding a *delta* position in the underlying is not malicious per se. Our accusation is that the anvil of *perfect replication* - that is being used to hammer out increasingly complex derivative products - is a source of mischief rather than guidance to derivative trading. This is because of the following reasons:

1. Materially significant hedging errors are endemic to even attempting to replicate vanilla options via dynamic delta hedging. Fitting volatility parameters fails to inform about the risk-return profile of even vanilla options. The risk-return profile of a delta hedged vanilla option is important if a demand supply imbalance precludes selling and covering with identical contracts, and earning bid-offer, *or*, selling puts and buying calls of identical maturity and strikes and going short the underlying (i.e., put-call parity).

2. The use of models based on perfect replication with parameters fitted to vanilla options, to book P&L on exotic options, fails to recognize the special risk premiums that the exotic option should attract. In exotic options there are often multiple embedded buy and sell positions. The exotic risk premium can’t be intelligently described by a mythical mid-volatility-surface that may reflect neither the greed or fear of the buyer or seller!

We believe that clinging to the orthodoxy of perfect delta hedges is a mistake of historical proportions. While the Black-Scholes derivation of the perfect delta hedge is theoretically interesting, the real world underlying are never described by a Geometric-Brownian-Motion! The reason that the vacuous risk-neutral regime has not been eliminated is the alignment between the myopic interests[20] of some business heads and the naiveté of accountants and run of the mill *valuation quants* who are largely oblivious to any element of reality.

We see clear alternatives to the perfect hedge *belief* based approach – the OHMC based approach presented in this paper is an example. There exist some business heads that are clearly interested in both the short term and long term health of their businesses, and who believe that assessing risk-return should be part and parcel of pricing-trading-hedging derivatives, i.e., integrated into the valuation approach.

Solvency of the Derivative Trader

Risk management of financial institutions is charged with ensuring solvency with a certain confidence level. To the extent risk management activities help making choices of trades that have a higher P&L expectation relative to risk, *risk management* can have a direct role in *alpha generation*. As the OHMC trading platform outputs integrated measures of residual risks en-route to *pricing*, it provides a means for integrating trade strategy-development, trade valuation, and risk management.

We anticipate that well publicized disasters in derivative risk management, search for profitability, a rich variety of hedge funds pursuing attractive return-risk tradeoffs, and maybe even central regulatory oversight needs for large financial institutions will result in a greater acceptance of valuation methods that simultaneously highlight *residual risks* over methods that misleadingly invoke *perfect replication* without any direct analysis of hedging errors and create a false perception of risk-control. We think that it is a grave mistake to view valuation modeling as a search for *risk-neutral distributions/parameters* to fit market prices with the tacit fueling of the incorrect perception of almost perfect replication-hedging. Models that highlight residual risks and enable viewing derivative value as a means of setting a risk-return trade-off should gain in popularity.

All too often oversight of derivative trading is obsessed with enforcing *fair-valuation*. We think that quantification of residual risks and risk capital needs greater attention. The false notion of a monolithic derivative price and any deviation from it as "arbitrage" (and therefore day 1 P&L) encourages taking outsized trading bets relative to risk capital – that remains un-sized at the time trade motivations are conceived by using a risk-neutral model. As the distance from the intimate

[20] The risk-neutral valuation modeling bureaucracy and the associated regalia (e.g., 'model validation,' 'valuation quant') are often sponsored by the very elements that show utter indifference to the *risks* they expose their firms, shareholders, or investors to. These elements are challenged by models that are more transparent and realistic about hedge performance - hence their convenient sponsorship of misguided academics that cling to the *complete-markets* & *perfect replication* framework of option pricing.

details of the derivative contract increases, personnel responsible for risk-control are at a disadvantage in their jobs of sizing risk capital. Why should every trading strategy in a firm not know their stand-alone risk-capital needs? Why should the risk-capital of every trade not be known? Why should the high resolution depiction of the trade in the valuation model not be leveraged to understand risk-return? Why should valuation be based on *assuming* perfect replication?

Without satisfactory answers to aforementioned questions significant trading counterparties simply do not know the risk-return tradeoffs of the bets they have entered into. This is a recipe for uncontrolled fear when the risks manifest themselves in the P&L in a negative way. That uncontrolled fear manifests itself as illiquidity and further widening of bid-offer and the difficulty of *valuing* derivative books.

After large negative P&L moves sometimes "quants" are asked to analyze P&L using all possible "greeks" in order to understand the failure of the replication strategy. This often reveals the limitations of the "greeks" in the face of large and sharp market moves. One hears concessions that one cannot hedge "jumps," "highly correlated moves," or "six standard deviation" moves! That all maybe true – but why is that not addressed as a part and parcel of understanding the hedging strategy *en-route* to derivative valuation?!

In contrast to the prevailing *risk-neutral regime*, a direct admission of the impossibility of perfect replication with *a-priori* estimates of average hedging costs and hedging error and the associated risk-capital, as enabled by OHMC, can contribute in a more rational derivative trading dynamic, where trade volumes and pricing reflect the risk-capital that addresses the solvency of the trading counterparties.

Volatility Trading

Recognizing irreducible risks of option trading and the associated need for capital and return on capital is central to our conception of a volatility trading system. Coupled with a realistic conditional stochastic model of the reference asset, OHMC provides an in-depth dissection of an *optimal* hedging strategy that minimizes some specified hedging error measure while imposing a specified mean expected change in wealth. This information results in being able to decompose an option price into *expected* hedging costs, and compensation for residual risks (**Figures 3 & 4**). Each option trader-hedger armed with this information can opine on if that compensation is sufficient for them, and whether it is an attractive deployment of their resources – namely time and *risk-capital*. Indeed, many hedge funds make available to their investors the returns associated with volatility risk-premiums (**Figure 18**). As hedge funds can-not rely on their credit rating while selling options, they end up being important repositories of experience and data on hedge performance and associated margining while trading options.

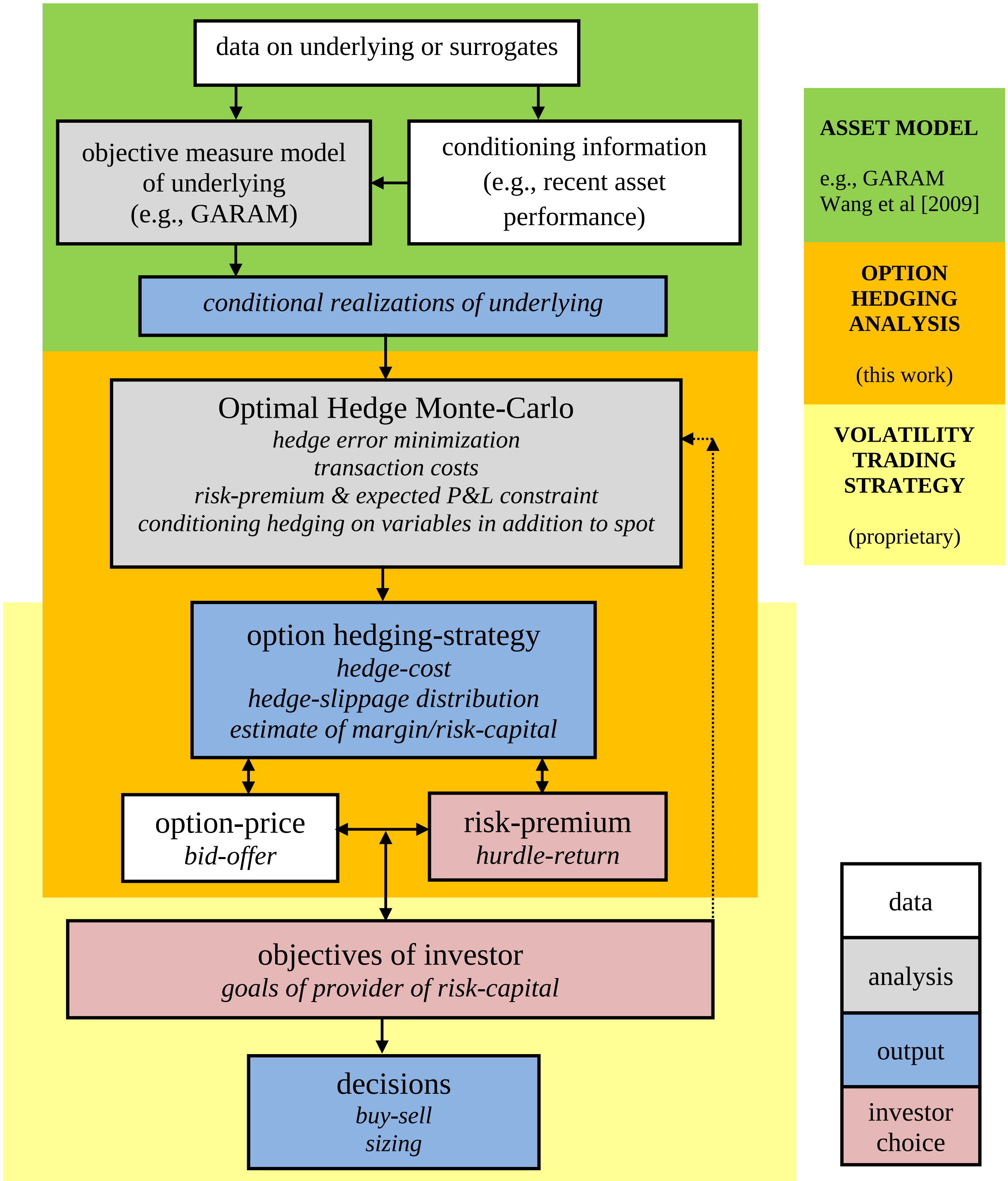


Figure 18. Optimal-Hedge Monte Carlo Based *Volatility-Trading* System.

Having to post carefully calculated margin against a specific trading strategy incentivizes its proponents to understand risk-return tradeoffs. In contrast, if the perpetrators of a trading strategy have no direct experience with their risk-capital/margin requirements, because it is done at a 'global-level' with hundreds of strategies, they tend to become less aware of risk-return and can even view selling an option to be their entitlement! Such slovenly market participants tend to be associated with oversized losses for their parent firms in whose name they sell options. By accepting 'the unique-price' of a derivative contract without directly addressing hedging and the potential for large residual risk, the risk-neutral model adherents are incapable of discerning a derivative trading strategy risk-return.

In the common option trading vernacular an opinion on the option price is loosely described by comments on the *richness* of implied volatility. By decomposing the option price into a component driven from realized asset returns, and a component driven from compensation for residual risk that is in fact conditioned on realized returns, OHMC plus GARAM completely obviate the need for the limited crutch of *implied volatility*. Instead, armed with expected hedging costs and implied risk-premiums expressed in terms of key investment strategy metrics of risk-capital, Sharpe, and Sortino-Ratios, the option trader-hedger can pursue *volatility trading* via simple put and call options, or more complex ones like Cliquets and barrier options.

Future Work

Exotic Options

Cliquets & Barrier Options are examples of equity derivatives that are embedded in a variety structured products. They can be analyzed in the OHMC approach with no more effort than the vanilla options addressed here. That is because OHMC confronts all aspects of risk return, and the GARAM approach is sufficiently rich to explore more complex products. The OHMC framework has already been demonstrated for Cliquets (Petrelli et al [2008]), albeit the treatment of volatility conditioning, transaction costs, and the GARAM asset representation provide the backdrop for a more thorough evaluation of risk-return tradeoffs for both Cliquets and Barrier options. The conditioning of the hedging strategy on realized volatility – that enabled decreasing tail-risks and understanding forward realized volatility sensitivity – should be particularly important to analyzing Cliquets.

Multi-Asset Options

OHMC is readily extendible to multiple dimensions, with modest dimensional problems readily solvable on ordinary computing environments, as outlined in Petrelli et al [2009]. The enhanced analysis demonstrated in this paper – addressing transactions costs, risk-premiums, and conditioning on realized volatility - can be readily applied to multi-asset options.

The framework for multi-asset options can also be used to understand options with payoffs that depend on more than one attribute of the underlying. For example, *target-volatility* strategies are built by combining a risky asset with cash, based on realized volatility experienced and the target volatility. In a similar vein, option payoffs can be specified that depend on the spot value of the underlying, as well as its realized volatility. Hedging of such options requires considering the option value and hedge ratios as functions of the underling and its realized volatility – like done in this work for conditioning on realized volatility.

Static Option Hedge Overlays

Elementary options are often used to hedge exotic options, in addition to hedging with the underlying. The OHMC results of the exotic option provide all the information needed to examine the efficacy of such vanilla hedge overlays. While in this paper we only showed sample results of single vanilla options, the computational framework can readily handle multiple options simultaneously. There is no extra computational cost for this as the OHMC methodology is readily applicable to a portfolio of options.

Extensions to Model of Underlying

The real-world trading calendar controls many important frequencies of information embedded in the empirical auto-covariance function of return magnitude (or squared). Therefore a stochastic model for the asset that is explicitly *aware* of the trading calendar and the import of certain dates, could be of great practical use. Such a melding of financial and econometric information could provide the next level of realism in GARAM, in addition to more sophisticated ways of coupling the discrete return sign to return squared (i.e., return magnitude).

Dynamics of Risk-Premiums

It is our contention that the risk-neutral modeling approach is patently disingenuous about hedge performance and has resulted in derivative trading earning the murky reputation that in the popular press is being portrayed as being uncomfortably similar to a crime scene. After-all, a widespread heist of risk-capital is apparent to the casual newspaper reader with common sense. The disconnect of risk from pricing under the fictitious perfect hedge assumption is the main reason for periodic "surprising" blow-ups that give credence to derivatives being labeled as "financial weapons of mass destruction."[21] Models and derivative trading personnel that develop and use them cannot escape taking responsibility for the resulting breakdown in trust in the capital markets due to the recognition of the potential enormity of the counterparty risks

[21] Warren Buffet in annual letter to his shareholders in 2003. For a juxtaposition of Buffet's philosophy with large institutional participants on many matters – including derivatives – we refer the reader to Tavakoli [2009].

associated with derivative trading.[22] This poor state of affairs results from the "risk neutral" approach that is falsely associated with "pricing risks," whereas it actually completely hides risks because of the purported unique hedging costs associated with perfect replication!

Contrary to the false replication mantra of risk neutral models, the Optimal Hedging Monte-Carlo (OHMC) approach based analysis concludes that attempted replication is accompanied with significant hedging errors, and it presents the average hedging costs and the residual risk. Then any market observed price can be associated with a *risk-premium*. That is, a hedger-seller of an option is viewed as charging for the average hedging costs and a risk-premium arising due to the inevitable residual risks. The documentation of the historical behavior of that risk premium can result in important insights into market behavior, and also help take the analysis to the next step, where the temporal variability of the risk-premium is itself a factor in developing a view of option trading risk-return and *pricing* risks. To deny the existence of and the need to explicitly analyze option risk premiums is to side-step the psychological reality of market participants in favor of sterile & trivial mathematics of the risk neutral regime based on completely false assumptions. There is no need to hide residual risk and invoke the magic of replication to invest in, risk-manage, and trade, strategies involving derivative contracts. The OHMC approach provides a complete alternative to the risk neutral approach, and it enables integrating hedging, risk management, and trading strategy development imperatives.

[22] Exchange traded standardized derivatives with sensible margining requirements alleviate counterparty risk concerns.

Appendix-A

Optimal Static Hedging Analysis

Static hedging analysis is a useful first step to start estimating hedging costs and hedge performance for any derivative contract. We describe static hedging of equity derivatives in this section.

In credit markets, the cash-flow plus time decay (carry) versus default event risk relationship is an important trading motivator. Relative value metrics showing the carry-default-risk relationship can be easily unraveled in a static hedging framework without ignoring residual risks[23]. Therefore static hedging is also particularly relevant to understanding credit derivative trading and provides direct insights into pricing and bounds on pricing (e.g., Petrelli [2006]).

For a basic introduction to static hedging and further discussion of the role of the minimum variance hedge the reader is referred to Luenberger [1998] (Chapter 10). We present static hedging here to recognize its role as a precursor to the multi-period optimal hedging analysis via Optimal Hedge Monte-Carlo (OHMC).

A key feature to note is that unless the asset randomness is extremely simple, a static hedge cannot eliminate risk. The only circumstance in which static hedging results in complete risk elimination is when the asset random outcome is restricted to two values. Even a third possible random outcome precludes the perfect hedge. We describe that in detail below with examples in the sub-sections on *Two-State Asset & the Perfect Static Hedge*, and *Three-State Asset & the Imperfect Static Hedge*.

Now just because you cannot eliminate risks in static hedging should not make you more susceptible to believing the mainstream *continuous time mathematical finance* results that presume perfect *dynamic* hedging! It turns out when the asset exhibits *jumpiness* (i.e., excess kurtosis), even as the hedging frequency is increased unboundedly, residual risk persists. So, due to *jumpiness*, even continuous dynamic hedging behaves like static hedging insofar as residual risks remain. In fact, an even more dramatic argument in support of significant residual risks under dynamic hedging arises - realistic assets exhibits greater kurtosis over smaller time-scale! [24] (see Wang et al [2009] for the term-structure of skewness and kurtosis).

---

[23] The mechanical use of the risk-neutral model in credit derivatives contributed to the credit bubble as regulators failed to look beyond simple delta-risk (e.g., CS01, the sensitivity of P&L to 1 bps spread widening), and valuation "quants" kept *validating* risk-neutral models that failed to advertise the impossibility of replication of derivatives whose payoff is determined by the occurrence of defaults or a combination of default and spread move!

[24] It is a mistake to think that the only idealization in *continuous time mathematical finance analysis of derivatives* is continuous hedging. The *'classic' continuous time mathematical finance* results are limited to underlying driven by a Brownian motion with no excess kurtosis and no transactions costs.

Static Hedging of an Equity Derivative

The wealth change of an option trader over the time interval [t_0, T] has contributions from the option, the hedge, and transaction costs:

$$\Delta W_{t_0}(t_0,T) = \Delta W_{t_0}^{option}(t_0,T) + \Delta W_{t_0}^{hedge}(t_0,T) + \Delta W_{t_0}^{tc}(t_0,T) \tag{A1}$$

Quantities with tilda (i.e., ~) on their top will represent deviations from ensemble averages, that are crowned with an over-bar. Denoting the option payoffs at time $t_{0,i} \in (t_0,T]$ by $P(t_{0,i})$, and the dividend payments at time $t_i \in (t_0,T]$ by π_i, the contributions to the change in wealth from the option, the hedge and the transaction costs are described here. The contributions from the option are

$$\begin{gathered}\Delta W_{t_0}^{option}(t_0,T) = C(t_0) - G(t_0);\ G(t_0) = P(t_{0,i})df(t_0,t_{0,i}) \\ \overline{\Delta W_{t_0}^{option}(t_0,T)} = C(t_0) - \overline{G}(t_0) \\ \Delta\tilde{W}_{t_0}^{option}(t_0,T) = -\tilde{G}(t_0)\end{gathered} \tag{A2}$$

The wealth change contributions coming from the hedge (excluding transaction costs that are addresses separately right after) are:

$$\begin{gathered}\Delta W_{t_0}^{hedge}(t_0,T) = \phi(t_0)H(t_0);\ H(t_0) = (s(T) - s(t_0)/DF(t_0,T))df(t_0,T) + \pi_i df(t_0,t_i) \\ \overline{\Delta W_{t_0}^{hedge}(t_0,T)} = \phi(t_0)\overline{H}(t_0) \\ \Delta\tilde{W}_{t_0}^{hedge}(t_0,T) = \phi(t_0)\tilde{H}(t_0)\end{gathered} \tag{A3}$$

The transaction cost contributions from the hedge are written below. Note that for a single period static hedging problem the transaction costs are not dependent on the random asset return, given spot value.

$$\begin{gathered}\Delta W_{t_0}^{tc}(t_0,T) = -\{\delta|\phi(t_0)| + \chi\}\left(1 + \frac{1}{DF(t_0,T)}\right)df(t_0,t_1) \\ \overline{\Delta W_{t_0}^{tc}(t_0,T)} = -\{\delta|\phi(t_0)| + \chi\}\left(1 + \frac{1}{DF(t_0,T)}\right)df(t_0,t_1) \\ \Delta\tilde{W}_{t_0}^{tc}(t_0,T) = 0\end{gathered} \tag{A4}$$

The total wealth change deviation from the ensemble average follows:

$$\begin{aligned} \Delta\tilde{W}_{t_0}(t_0,T) &\equiv \Delta\tilde{W}_{t_0}^{option}(t_0,T) + \Delta\tilde{W}_{t_0}^{hedge}(t_0,T) + \Delta\tilde{W}_{t_0}^{tc}(t_0,T) \\ \Delta\tilde{W}_{t_0}(t_0,T) &= -\tilde{G}(t_0) + \phi(t_0)\tilde{H}(t_0) \end{aligned} \tag{A5}$$

The variance of the change in wealth is

$$\overline{\Delta\tilde{W}_{t_0}^2(t_0,T)} = \overline{\tilde{G}^2(t_0)} + \phi(t_0)^2\,\overline{\tilde{H}^2(t_0)} - 2\phi(t_0)\overline{\tilde{G}(t_0)\tilde{H}(t_0)} \tag{A6}$$

From (A6) we can find the wealth change variance minimizing hedge ratio:

$$\frac{d\,\overline{\Delta\tilde{W}_{t_0}^2(t_0,T)}}{d\phi(t_0)} = 2\phi(t_0)\overline{\tilde{H}^2(t_0)} - 2\overline{\tilde{G}(t_0)\tilde{H}(t_0)};\ \ \frac{d^2\,\overline{\Delta\tilde{W}_{t_0}^2(t_0,T)}}{d\phi^2(t_0)} = 2\overline{\tilde{H}^2(t_0)} \geq 0$$

$$\frac{d\,\overline{\Delta\tilde{W}_{t_0}^2(t_0,T)}}{d\phi(t_0)} = 0 \Rightarrow \phi(t_0) = \overline{\tilde{G}(t_0)\tilde{H}(t_0)} / \overline{\tilde{H}^2(t_0)} \tag{A7}$$

The residual variance of the static hedger's wealth change follows

$$\sigma^2_{\Delta W_{t_0}(t_0,t_1)} = \sigma^2_{G(t_0)} - \frac{\left(\overline{\tilde{G}(t_0)\tilde{H}(t_0)}\right)^2}{\sigma^2_{H(t_0)}} \tag{A8}$$

Now finding the price of the derivative requires imposing the average wealth change constraint:

$$\begin{aligned} \overline{\Delta W_{t_0}(t_0,T)} &= C(t_0) - \overline{G}(t_0) + \phi(t_0)\overline{H}(t_0) + \overline{\Delta W_{t_0}^{tc}(t_0,T)} \\ C(t_0) &= \overline{\Delta W_{t_0}(t_0,T)} + \overline{G}(t_0) - \phi(t_0)\overline{H}(t_0) - \overline{\Delta W_{t_0}^{tc}(t_0,T)} \end{aligned} \tag{A9}$$

The last (but not the least) important input into the pricing model has to be the wealth change average. When risks can be completely eliminated (a remote hypothetical) the wealth change can be set to zero and the unique cost of hedging is the option price. The rationale for imposing a non-zero average change in wealth is that under realistic conditions hedging does not eliminate risk and therefore can attract a risk-premium – i.e., a risk-taker is justified in having an expectation of positive change in wealth. Would you continually enter into a contract where you could lose money without any *expectation* of a positive outcome and if the surprise losses are typically larger than the surprise gains? Probably not. The circumstance in which a market agent *expects* to lose money can be rationalized if the surprise gains are much larger than the surprise losses and the expected losses. We touch upon risk premiums from an option trader hedger perspective in the main text in **Section-3**, and **Appendix-D**. This area has remained insufficiently explored due to the artificial sterility imposed on the topic of derivative valuation, by the risk-neutral approach that does not admit to any residual risks! We believe that much works remain to be done in understanding the dynamics of risk and risk-premiums in option trading.

Two-State Asset & the Perfect Static Hedge

Consider an asset that at t_0 has the value of s_0 and can take the value s_1 and s_2 with probability p_1 and p_2 respectively ($p_1 + p_2 = 1$). For the sake of brevity and to focus on the key differences between two-state and three-state static hedging we are setting rates to zero (for time-value of money and funding), and the transaction costs are ignored.

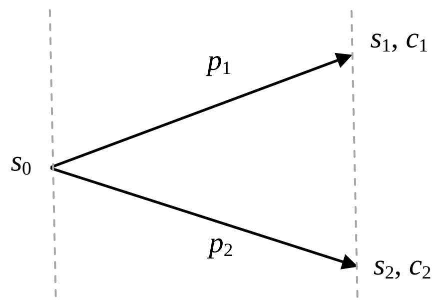


Figure A1. Two-state static hedging problem.

Although this problem can be solved quite directly, we follow the general approach laid out in this Appendix. The variance of the payoff follows

$$\sigma_G^2 = E\left[G^2\right] - E[G]^2 = \left(p_1 c_1^2 + p_2 c_2^2\right) - \left(p_1 c_1 + p_2 c_2\right)^2$$
$$= p_1 c_1^2 (1 - p_1) + p_2 c_2^2 (1 - p_2) - 2 p_1 p_2 c_1 c_2 = p_1 p_2 (c_1 - c_2)^2$$

The variance of a unit of hedge follows

$$\sigma_H^2 = E\left[H^2\right] - E[H]^2 = \left[p_1 (s_1 - s_0)^2 + p_2 (s_2 - s_0)^2\right] - \left[p_1 (s_1 - s_0) + p_2 (s_2 - s_0)\right]^2$$
$$= p_1 (s_1 - s_0)^2 (1 - p_1) + p_2 (s_2 - s_0)^2 (1 - p_2) - 2 p_1 p_2 (s_1 - s_0)(s_2 - s_0)$$
$$= p_1 p_2 \left[(s_1 - s_0) - (s_2 - s_0)\right]^2 = p_1 p_2 (s_1 - s_2)^2$$

The covariance between the payoff and the unit hedge P&L follows

$$
\begin{aligned}
\overline{\tilde{G}\tilde{H}} &= E[GH]-E[G]E[H] = p_1c_1(s_1-s_0)+p_2c_2(s_2-s_0)-\left[(p_1c_1+p_2c_2)(p_1(s_1-s_0)+p_2(s_2-s_0))\right] \\
&= p_1c_1(s_1-s_0)+p_2c_2(s_2-s_0)-\left[p_1^2c_1(s_1-s_0)+p_2^2c_2(s_2-s_0)+p_1c_1p_2(s_2-s_0)+p_2c_2p_1(s_1-s_0)\right] \\
&= p_1c_1(s_1-s_0)(1-p_1)+p_2c_2(s_2-s_0)(1-p_2)-p_1p_2c_1(s_2-s_0)-p_1p_2c_2(s_1-s_0) \\
&= p_1p_2c_1(s_1-s_0)+p_1p_2c_2(s_2-s_0)-p_1p_2c_1(s_2-s_0)-p_1p_2c_2(s_1-s_0) = p_1p_2(c_2-c_1)(s_2-s_1)
\end{aligned}
$$

Note that in the simplifications for the expressions above we have explicitly used the two-state nature of the problem by setting $p_1 = 1 - p_2$ and $p_2 = 1 - p_1$. Per the prior analysis, the residual risk follows

$$
\sigma_{\Delta W}^2 = \sigma_G^2 - \frac{\left(\overline{\tilde{G}\tilde{H}}\right)^2}{\sigma_H^2} = p_1p_2(c_1-c_2)^2 - \frac{\left[p_1p_2(c_2-c_1)(s_2-s_1)\right]^2}{p_1p_2(s_1-s_2)^2} = 0
$$

Three-State Asset & the Imperfect Static Hedge

We consider a slight change to the two-state asset problem above, by the introduction of a third state the asset can evolve to, as depicted in **Figure A2**.

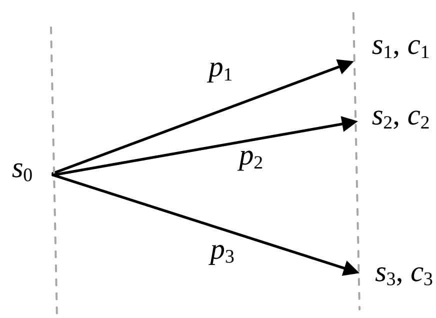


Figure A2. Three-state static hedging problem.

The variance of the payoff follows

$$
\sigma_G^2 = E\left[G^2\right]-E[G]^2 = p_1{c_1}^2 + p_2{c_2}^2 + p_3{c_3}^2 - \left[p_1c_1+p_2c_2+p_3c_3\right]^2
$$

The variance of a unit of hedge follows

$$\sigma_H^2 = E[H^2] - E[H]^2$$
$$= [p_1(s_1 - s_0)^2 + p_2(s_2 - s_0)^2 + p_3(s_3 - s_0)^2] - [p_1(s_1 - s_0) + p_2(s_2 - s_0) + p_3(s_3 - s_0)]^2$$

The covariance between the payoff and the unit hedge P&L is

$$\overline{\tilde{G}\tilde{H}} = E[GH] - E[G]E[H] = [p_1c_1(s_1 - s_0) + p_2c_2(s_2 - s_0) + p_3c_3(s_3 - s_0)] - [p_1c_1 + p_2c_2 + p_3c_3][p_1(s_1 - s_0) + p_2(s_2 - s_0) + p_3(s_3 - s_0)]$$

The residual risk can then be calculated as

$$\sigma_{\Delta W}^2 = \sigma_G^2 - \frac{\left(\overline{\tilde{G}\tilde{H}}\right)^2}{\sigma_H^2}$$

The three-state problem does not simplify and give zero residual risk - by using the basic probability definition ($p_1+p_2+p_3 = 1$) - as occurs in the two-state problem.

Example: Static Hedge Performance for a Three State Asset

$$s_0 = 1;\ s_1 = 2;\ s_2 = 0.5;\ s_3 = 0;$$

$$c_1 = 2;\ c_2 = 1;\ c_3 = 0$$

$$p_1 = p_2 = (1-p_3)/2$$

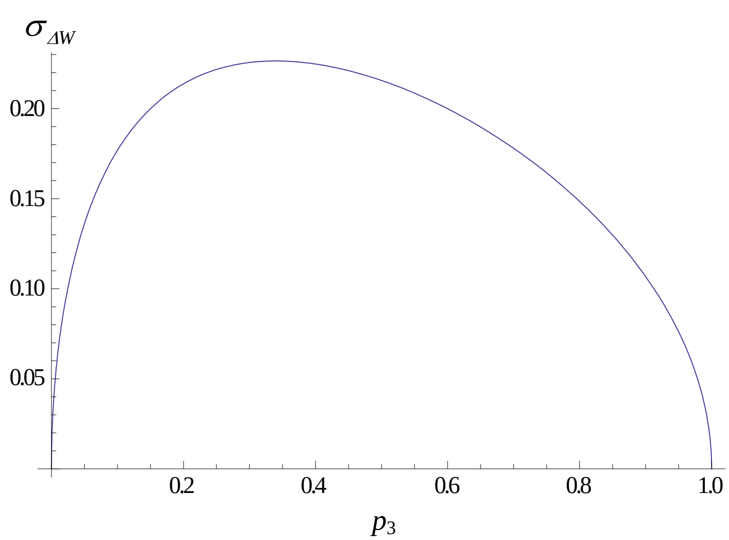


Figure A3. Example computation of residual risk for three-state static hedging problem.

Of course, a two state world is not sufficient to describe real market conditions. Even in the simple-most caricature of the market, on a given day, it may move discernibly up, it may move discernibly down or it may move sideways – i.e., not discernibly up or down. Such a three-state model precludes static hedging error elimination. Knowing this and that the two-state static hedging problem has been used to build the argument for a perfect hedge under continuous hedging for a Brownian motion underlying - by using a binomial lattice - one should be quite wary of the resulting *risk-free* model.

The promise of perfect option replication is modern day financial voodoo that parallels ancient claims of turning common metals into gold. The main difference is that many in the mainstream academic and practitioner establishment have embraced the fiction of perfect option replication. As a result, the casual observer may be deceived by the self-anointed officiousness of the "*Q-measure*" thumping, *complete-market* worshipping, *risk-neutral* belt that has become a part of financial bureaucracies in the *modern* cities of New-York, London, Paris, and Tokyo!

The differences between the two-state and the three state static hedging problem should be a sufficient antidote against that *risk-neutral voodoo* that ails much of derivative pricing *analytics* and reinforce the fundamental presence of *residual-risk* associated with any *real-world* attempted replication. The main text addresses this issue in a dynamic hedging analysis framework.

Appendix-B

Basis Functions

In the optimization problems of the main section we are finding value-hedge-ratio functions of spot values - and possibly other conditioning variables - that minimize P&L volatility subject to an expected wealth change constraint. The optimization problem posed in the main section is infinite dimensional: it is a *variational-calculus* problem. By replacing the unknown functions with sum-products of unknown coefficients and basis functions we turn the problem into a discrete one, where we numerically solve search for the unknown coefficients.

The range of asset values for which a basis function has a non-zero value describes the support of the basis functions. By choosing basis functions of limited support we can render sparse the matrix defining the set of linear equations to be solved to determine the unknown coefficients (Equation 15 of main section). Here we document two types of basis functions that we have employed in our OHMC applications.

B.1 Linear Basis Functions

Linear basis functions are easy to define and use. The unknown coefficients are the values of the function at the nodal locations and specification of boundary conditions is easy.

One-Dimensional Linear Basis Functions

This representation involves dividing the state space into non-overlapping *elements*, which may be defined by the *nodal* locations s_j. The basis functions maybe then associated with each node

$$\Omega_j(s) = \begin{cases} \dfrac{s - s_{j-1}}{s_j - s_{j-1}}, & s_{j-1} \le s \le s_j \\ \dfrac{s_{j+1} - s}{s_{j+1} - s_j}, & s_j \le s \le s_{j+1} \\ 0, & \text{otherwise} \end{cases} \tag{B1}$$

With N nodes in 1-D, the function is represented as

$$f(s) = \sum_{k=0}^{N-1} \hat{f}_k \Omega_k(s) \tag{B2}$$

Recognizing the limited support of the basis functions and identifying the adjacent nodal locations k^* and k^*+1 that s lies in between ($s_{k^*} \le s \le s_{k^*+1}$) we can represent the function as

$$f(s) = \hat{f}_{k^*}\Omega_{k^*}(s) + \hat{f}_{k^*+1}\Omega_{k^*+1}(s) \tag{B3}$$

M-Dimensional Linear Basis Functions

This representation involves dividing the state space into non-overlapping *elements*, which may be defined by the *nodal* locations defined by $\mathbf{s}_k \equiv \left(s_{1,j_1}, s_{2,j_2}, ..., s_{M,j_M}\right)$. The basis functions are then associated with each node and assessed by multiplying the univariate basis functions associated with each dimension

$$\Omega_k(\mathbf{s}) = \prod_{n=1}^{M} \Omega_{j_n}(s_n) \tag{B4}$$

With N_m nodes being used to discretize the state space in the *m*th dimension, the function is represented as

$$f(\mathbf{s}) = \sum_{k=0}^{N_1 N_2 \ldots N_M - 1} \hat{f}_k \Omega_k(\mathbf{s}) \tag{B5}$$

Recognizing the limited support of the basis functions and identifying the adjacent nodal locations in the *m*th dimension that s_m lies in between ($s_{j_m^*} \le s_m \le s_{j_m^*+1}$), the nodes of the *element* of the *state-space* that encompass a point (s_1, s_2, .., s_M) are identified as $K^* = \left\{k_1^*, k_2^*, k_3^*, ...k_{2^M}^*\right\}$ and the value of the function being approximated is

$$f(\mathbf{s}) = \sum_{k \in K^*} \hat{f}_k \Omega_k(\mathbf{s}) \tag{B6}$$

Two-Dimensional Linear Basis Functions

This representation involves dividing the state space into non-overlapping *elements*, which may be defined by the *nodal* locations defined by $\mathbf{s}_k \equiv \left(s_{1,j_1}, s_{2,j_2}\right)$ The basis functions maybe then associated with each node as the product of the 1-D basis functions associated with each dimension

$$\Omega_k(\mathbf{s}) = \Omega_{j_1}(s_1)\Omega_{j_2}(s_2) \tag{B7}$$

With N_m being the number of nodes in the *m*th dimension the function is represented as

$$f(s_1, s_2) = \sum_{k=0}^{N_1 N_2 - 1} \hat{f}_k \Omega_k(s_1, s_2) \tag{B8}$$

The adjacent nodal locations in the *m*th dimension that s_m lies in between ($s_{j_m^*} \le s_m \le s_{j_m^*+1}$) help define the nodes of the *element* of the *state-space* that encompass a point (s_1, s_2): $k_1^* = j_1^* + j_2^* N_1$;

$k_2^* = k_1^* + 1;\ k_3^* = k_2^* + N_1;\ k_4^* = k_1^* + N_1$. The set of the corner nodes of this element is $K^* = \{k_1^*, k_2^*, k_3^*, k_4^*\}$ and the value of the function being approximated is

$$f(s_1, s_2) = \sum_{k \in K^*} \hat{f}_k \Omega_k(s_1, s_2) \tag{B9}$$

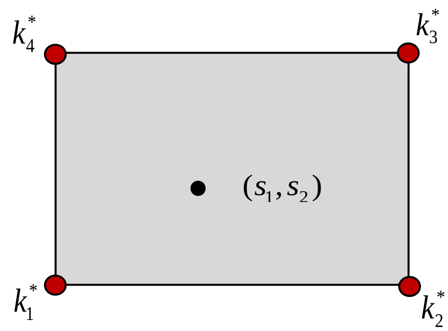


Figure B1. Local definition of nodes of element encompassing $\mathbf{s}_k = (s_1, s_2)$

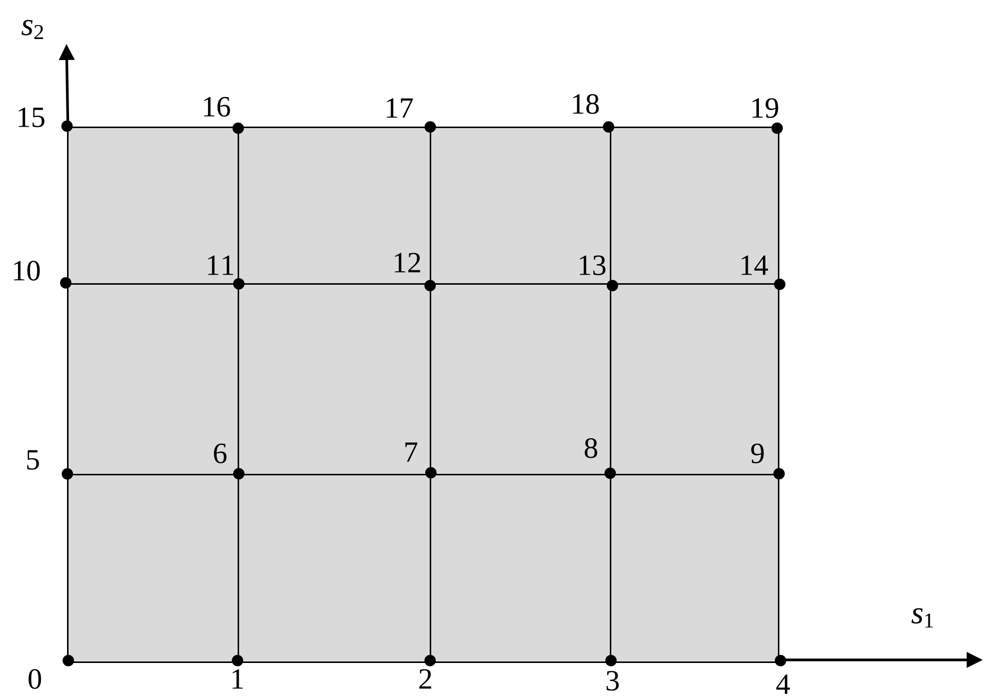


Figure B2. Illustration of Global node numbering for 2-D problem

B.2 Hermite Cubic Basis Functions

More complex than linear basis functions, Hermite cubic polynomials enable specifying boundary conditions in terms of function values and/or its gradient directly. While linear basis functions involve a basis function and unknown coefficient per node, for Hermite cubic basis functions there are more than one unknown coefficient and basis function associated with each node.

One-Dimensional Hermite Cubic Basis Functions

This representation involves dividing the state space into non-overlapping *elements*, which may be defined by the *nodal* locations s_k. The basis functions maybe then associated with each node

$$\omega_j(s) = \begin{cases} \dfrac{(s - s_{j-1})^2}{(s_j - s_{j-1})^3}\left[2(s_j - s) + (s_j - s_{j-1})\right] & s_{j-1} \le s \le s_j \\ \dfrac{(s - s_{j+1})^2}{(s_{j+1} - s_j)^3}\left[2(s - s_j) + (s_{j+1} - s_j)\right] & s_j \le s \le s_{j+1} \\ 0 & \text{otherwise} \end{cases} \tag{B10}$$

$$\tilde{\omega}_j(s) = \begin{cases} \dfrac{(s - s_{j-1})^2 (s - s_j)}{(s_j - s_{j-1})^2} & s_{j-1} \le s \le s_j \\ \dfrac{(s - s_{j+1})^2 (s - s_j)}{(s_{j+1} - s_j)^2} & s_j \le s \le s_{j+1} \\ 0 & \text{otherwise} \end{cases}$$

With N nodes in 1-D, the function is represented as

$$f(s) = \sum_{k=0}^{N-1}\left(\hat{f}_k \omega_k(s) + \frac{d\hat{f}_k}{ds}\tilde{\omega}_k(s) \right) \tag{B11}$$

Recognizing the limited support of the basis functions and identifying the adjacent nodal locations k^* and k^*+1 that s lies in between ($s_{k^*} \le s \le s_{k^*+1}$) we can represent the function as

$$f(s) = \hat{f}_{k^*}\omega_{k^*}(s) + \frac{d\hat{f}_{k^*}}{ds}\tilde{\omega}_{k^*}(s) + \hat{f}_{k^*+1}\omega_{k^*+1}(s) + \frac{d\hat{f}_{k^*+1}}{ds}\tilde{\omega}_{k^*+1}(s) \tag{B12}$$

Two-Dimensional Hermite Cubic Basis Functions

This representation involves dividing the state space into non-overlapping *elements*, which may be defined by the *nodal* locations defined by $\mathbf{s}_k \equiv (s_{1,j1}, s_{2,j2})$. The basis functions maybe then associated with each node as the product of the 1-D basis functions associated with each dimension. With N_m being the number of nodes in the mth dimension the function is represented as

$$f(s_1, s_2) = \sum_{k=0}^{N_1 N_2 - 1} \left(\hat{f}_k \omega_{j_1}(s_1)\omega_{j_2}(s_2) + \frac{\partial \hat{f}_k}{\partial s_1} \tilde{\omega}_{j_1}(s_1)\omega_{j_2}(s_2) + \frac{\partial \hat{f}_k}{\partial s_2} \tilde{\omega}_{j_2}(s_2)\omega_{j_1}(s_1) + \frac{\partial^2 \hat{f}_k}{\partial s_1 \partial s_2} \tilde{\omega}_{j_1}(s_1)\tilde{\omega}_{j_2}(s_2) \right) \quad \text{(B13)}$$

The adjacent nodal locations in the mth dimension that s_m lies in between ($s_{j_m^*} \le s_m \le s_{j_m^*+1}$) help define the nodes of the *element* of the *state-space* that encompass a point (s_1, s_2): $k_1^* = j_1^* + j_2^* N_1$; $k_2^* = k_1^* + 1$; $k_3^* = k_2^* + N_1$; $k_4^* = k_1^* + N_1$. The set of the corner nodes of this element is $K* = \{k_1^*, k_2^*, k_3^*, k_4^*\}$ and the value of the function being approximated is

$$f(s_1, s_2) = \sum_{k \in K*} \left(\hat{f}_k \omega_{j_1^*}(s_1)\omega_{j_2^*}(s_2) + \frac{\partial \hat{f}_k}{\partial s_1} \tilde{\omega}_{j_1^*}(s_1)\omega_{j_2^*}(s_2) + \frac{\partial \hat{f}_k}{\partial s_2} \tilde{\omega}_{j_2^*}(s_2)\omega_{j_1^*}(s_1) + \frac{\partial^2 \hat{f}_k}{\partial s_1 \partial s_2} \tilde{\omega}_{j_1^*}(s_1)\tilde{\omega}_{j_2^*}(s_2) \right) \quad \text{(B14)}$$

Appendix-C

OHMC Numerical Solution

Case 1

No Transaction Costs; No Conditioning on Realized Volatility; Zero Mean Change in Wealth; 1D Linear Basis Functions

The set of linear equations effecting the constrained optimization are:

$$\overline{\Omega_i\Omega_j}\hat{C}_j + \overline{H\Omega_i\Omega_j}\hat{\phi}_j + \gamma\overline{\Omega_i} = \overline{G\Omega_i}\ ;$$
$$\overline{H\Omega_i\Omega_j}\hat{C}_j + \overline{H^2\Omega_i\Omega_j}\hat{\phi}_j + \overline{\Omega_i H}\gamma = \overline{GH\Omega_i}\ ;$$
$$\overline{\Omega_j}\hat{C}_j + \overline{H\Omega_j}\hat{\phi}_j = \overline{G}$$

The quantities $\hat{C}_j$, $\hat{\phi}_j$, G, H, and γ all are associated with time t_k, and in the main section that was denoted by t_k in parenthesis. The system above is represented as $A_{ij}y_j = q_i$ where the *rhs* is stored in the vector q_i and the unknowns ($\hat{C}_i(t_k)$ $\hat{\phi}_i(t_k)$ γ) are represented by the vector y_i.

1D Linear Basis Functions

number of nodes spanning asset value state space: N
total number of unknowns: $2N+1$

For linear basis function there is one basis function per node, so the repeated index (summation over j) also denotes the nodes used to divide the asset state-space. The different terms of the set of linear equations are defined below (with $0 \le i \le N-1$ and $0 \le j \le N-1$):

$$A_{2i,2j} = \overline{\Omega_i\Omega_j} \quad A_{2i,2j+1} = \overline{H\Omega_i\Omega_j} \quad A_{2i,2N} = \overline{\Omega_i} \quad y_{2j} = \hat{C}_j \quad q_{2i} = \overline{G\Omega_i}$$
$$A_{2i+1,2j} = \overline{H\Omega_i\Omega_j} \quad A_{2i+1,2j+1} = \overline{H^2\Omega_i\Omega_j} \quad A_{2i+1,2N} = \overline{H\Omega_i} \quad y_{2j+1} = \hat{\phi}_j \quad q_{2i+1} = \overline{GH\Omega_i}$$
$$A_{2N,2j} = \overline{\Omega_j} \quad A_{2N,2j+1} = \overline{\Omega_j H} \quad A_{2N,2N} = 0 \quad y_{2N} = \gamma \quad q_{2N} = \overline{G}$$

Case 2

No Transaction Costs; Conditioning on Realized Volatility; Zero Mean Change in Wealth; 2D Linear Basis Functions

2D Linear Basis Functions

number of nodes spanning asset value state space:	N_1
number of nodes spanning asset volatility state space:	N_2
total number of nodes:	N_1N_2
total number of unknowns:	$2N_1N_2+1$

Similar to Case 1, for linear basis function there is one basis function per node, so the repeated index (summation over *j*) also denotes the nodes used to divide the asset state-space. The set of linear equation effecting the OHMC solution with conditioning (represented as $A_{ij}y_j = q_i$) are given below, with $0 \le i \le N_1N_2 - 1$ and $0 \le j \le N_1N_2 - 1$:

$$
\begin{array}{lllll}
A_{2i,2j} = \overline{\Omega_j\Omega_i} & A_{2i,2j+1} = \overline{\Omega_j\Omega_i H} & A_{2i,2N_1N_2} = \overline{\Omega_i} & y_{2j} = \hat{C}_j & q_{2i} = \overline{G\Omega_i} \\
A_{2i+1,2j} = \overline{\Omega_j\Omega_i H} & A_{2i+1,2j+1} = \overline{\Omega_j\Omega_i H^2} & A_{2i+1,2N_1N_2} = \overline{\Omega_i H} & y_{2j+1} = \hat{\phi}_j & q_{2i+1} = \overline{GH\Omega_i} \\
A_{2N_1N_2,2j} = \overline{\Omega_j} & A_{2N_1N_2,2j+1} = \overline{\Omega_j H} & A_{2N_1N_2,2N_1N_2} = 0 & y_{2N_1N_2} = \gamma & q_{2N_1N_2} = \overline{G}
\end{array}
$$

Conditioning on an additional variable, other than sport asset value, increases the dimensionality the OHMC problem.

Case 3

Transaction Costs; Conditioning on Realized Volatility; Residual Risk Dependent Mean Change in Wealth; 2D Linear Basis Functions

The set of linear equations then effecting the constrained optimization are:

$$
\begin{aligned}
\hat{C}_j\overline{\Omega_j\Omega_i} + \hat{\phi}_j\overline{\Omega_j\Omega_i H} + \gamma\overline{\Omega_i} &= \overline{G\Omega_i} + h_i \\
\hat{C}_j\overline{\Omega_j\Omega_i H} + \hat{\phi}_j\overline{\Omega_j\Omega_i H^2} + \gamma\overline{\Omega_i H} &= \overline{G\Omega_i H} + l_i \\
\hat{C}_j\overline{\Omega_j} + \hat{\phi}_j\overline{\Omega_j H} &= \overline{G} - \overline{N}
\end{aligned}
$$

The *rhs* terms are as follows

$$
h_i = \overline{(G-N)\frac{\partial N}{\partial \hat{C}_i}} - \overline{N\Omega_i} - \left\{\hat{C}_j\overline{\Omega_j\frac{\partial N}{\partial \hat{C}_i}} + \hat{\phi}_j\overline{\Omega_j H\frac{\partial N}{\partial \hat{C}_i}} + \gamma\overline{\frac{\partial N}{\partial \hat{C}_i}}\right\}
$$

$$
l_i = \overline{(G-N)\frac{\partial N}{\partial \hat{\phi}_i}} - \overline{N\Omega_i H} - \left\{\hat{C}_j\overline{\Omega_j\frac{\partial N}{\partial \hat{\phi}_i}} + \hat{\phi}_j\overline{\Omega_j H\frac{\partial N}{\partial \hat{\phi}_i}} + \gamma\overline{\frac{\partial N}{\partial \hat{\phi}_i}}\right\}
$$

Recall that $N = \Delta W^{tc} - \overline{\Delta W}$ and the time subscripts have been dropped to simplify the algorithm description. The set of linear equation effecting the OHMC solution with conditioning (represented as $A_{ij} y_j = q_i$) are given below, with $0 \le i \le N_1 N_2 - 1$ and $0 \le j \le N_1 N_2 - 1$:

$$
\begin{array}{lllll}
A_{2i,2j} = \overline{\Omega_j \Omega_i} & A_{2i,2j+1} = \overline{\Omega_j \Omega_i H} & A_{2i,2N_1N_2} = \overline{\Omega_i} & y_{2j} = \hat{C}_j & q_{2i} = \overline{G\Omega_i} + h_i \\
A_{2i+1,2j} = \overline{\Omega_j \Omega_i H} & A_{2i+1,2j+1} = \overline{\Omega_j \Omega_i H^2} & A_{2i+1,2N_1N_2} = \overline{\Omega_i H} & y_{2j+1} = \hat{\phi}_j & q_{2i+1} = \overline{G\Omega_i H} + l_i \\
A_{2N_1N_2,2j} = \overline{\Omega_j} & A_{2N_1N_2,2j+1} = \overline{\Omega_j H} & A_{2N_1N_2,2N_1N_2} = 0 & y_{2N_1N_2} = \gamma & q_{2N_1N_2} = \overline{G} - \overline{N}
\end{array}
$$

Transaction costs and risk-premiums introduce nonlinearity into the OHMC algorithm, via the terms h_i, l_i, and N. This is dealt with by first solving the problem ignoring transaction costs and risk-premiums, and then evaluating the nonlinear terms, and updating the solution.

Hermite-Cubic Basis Functions

In using Hermite-Cubic Basis Functions there are the multiple basis functions (and corresponding unknown coefficients) associated with each nodal local in the asset state-space. This is the main difference with linear basis functions, where each nodal location has one basis function and one unknown coefficient associated with it.

For 1D problems there are two basis functions associated with each node, and for 2D problems there are four. The algorithm for setting up the set of linear equations can follow an approach similar to the linear basis functions, recognizing the multiplicity of basis functions per state-space nodal location.

2D Case Hermite-Cubic Basis Functions

number of nodes spanning asset value state space:	N_1
number of nodes spanning asset volatility state space:	N_2
total number of nodes:	N_1N_2
total number of unknowns:	$8N_1N_2+1$

Computational Solution Structure

Temporal Modularity

As the OHMC approach adopted here is local in time, computing the solution only requires simultaneously assessing a pairs of successive simulated asset values at the start and end of the hedge interval. Recognizing this feature results in a temporally modular memory efficient implementation – where the storage requirements are much less than that needed for all the MC realizations at all time-steps.

Local Support Basis Functions

Each Monte-Carlo path contributes to a few elements of the matrix A_{ij}, as determined by the support of the basis functions. Recognizing this feature is key to assembling the set of equations quickly. Also, for repeated calculations in a time-step, it is advantageous to first assess the elements that are pertinent to each path, and store that information in memory and use it for all subsequent evaluations.

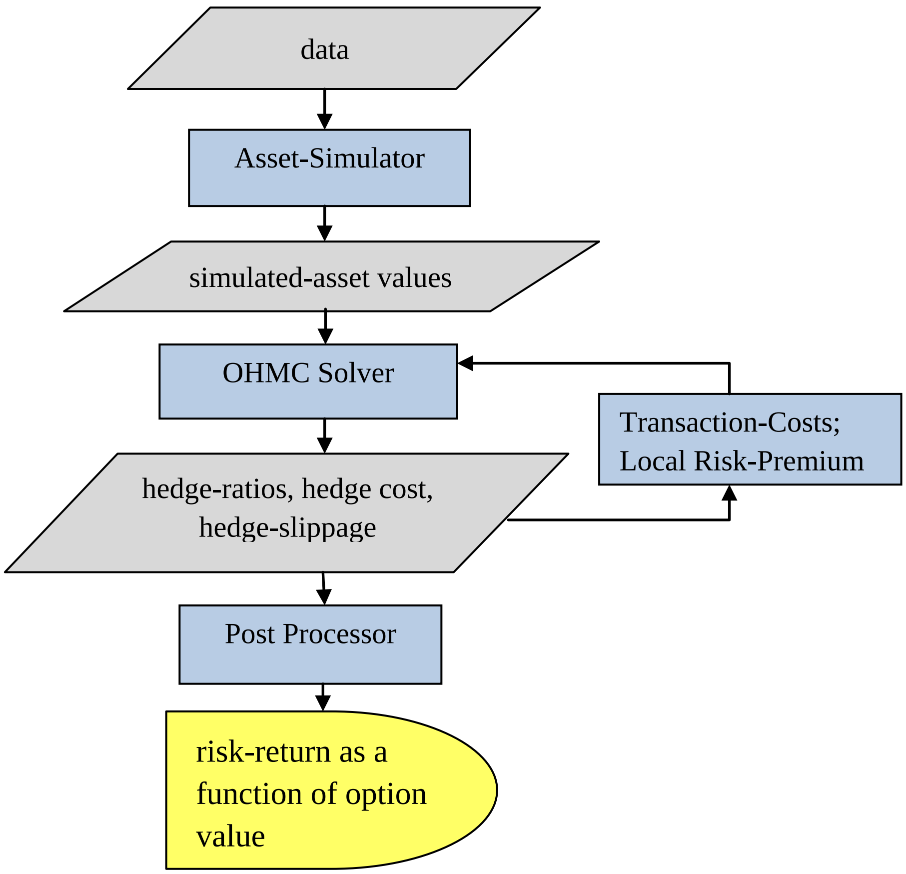


Figure C1. Schematic of OHMC algorithm implementation

Admittedly more involved than merely taking *risk-neutral* expectations of option payoffs, we think that the computational complexity of OHMC is manageable for a practical trading book. Moreover, the task of risk management, hedging strategy development, and *valuation* can all be performed within the integrated framework. The salesman, structurer, trader, product-controller, risk-manager can simultaneously observe an analysis of a derivative trade.

We think that *interested parties* that myopically benefit from lack of estimates of risk-capital needs of a trading book are complicit in limiting the development of financial product analysis, often under false and self-serving guise of *simplicity*. A *realistic* revelation of hedge slippage en-route to *valuation* directly limits the amount of upfront P&L that should be recognized in a complex derivative transaction. These myopic interested parties would rather not have that distraction when declaring instantaneous *risk-free profits* on *complex* derivative transactions – hence their sponsorship of *simplistic analysis*! The 'zero-risk" presumption of the risk-neutral approach makes it an unacceptable simplification of option analysis. OHMC relaxes that assumption and yields estimates of residual risk without limiting the underlying to Brownian motion driven processes. The implementation of OHMC algorithm detailed in this appendix is practically feasible.

Appendix-D

Option Trading Risk-Premium

Communicating about the characteristics of real-world attempted replication strategies requires a common language that *makes sense* without invoking *perfect replication*. We present here terminology in support of that and for developing models to infer risk-premiums from observed option prices. We take the viewpoint of the option *seller* hedger – although that can be easily changed to take the view-point of the option buyer-hedger.

Terminology for Real-World Option Attempted Replication

Characteristics of Option Trade (per contract)	
Trading Calendar	$\{t_0 = 0, t_1, t_2, \ldots.., t_k, t_{k+1}, \ldots., t_{K-1} = T\}$
Time t_a Value of Unit Cash Inflow at t_b	$df(t_a, t_b)$
Asset Value	$s(t_k)$
Asset Evolution Joint Density Function	$f_{s(t_1),s(t_2),\ldots.s(T)}(s(t_1), s(t_2), \ldots., s(T))$
Conditioning Variables	$\varpi_j(t_k);\ 1 \le j \le J$
Hedging Strategy	$\phi(s(t_k), \varpi_1(t_k), \varpi_2(t_k), \ldots.., \varpi_J(t_k), t_k)$
Cost of Hedging Strategy[25]	$C(t_0)$
Change in Wealth in Hedge Interval	$\Delta W_{t_j}(t_j, t_{j+1})$
Total Wealth Change	$\Delta W = \Delta W_{t_0}(t_0, T) = \sum_{j=0}^{K-2} \Delta W_{t_j}(t_j, t_{j+1}) df(t_0, t_j)$
Probability Density of Change in Wealth	$f_{\Delta W}(x)$
Average Change in Wealth[25]	$\overline{\Delta W} = \int_{-\infty}^{\infty} \Delta W f_{\Delta W}(x) dx$
Actual Traded Option Price[25]	$V(t_0)$
Option Seller's Average Change in Wealth	$\Psi = V(t_0) - C(t_0) + \overline{\Delta W}$
Risk-Capital at Confidence Level $p_S(T)$	$Q = \{\overline{\Delta W} - q\}^{+}$ э Probability$\{\Delta W > q\} = p_S(T)$
Implied Return on Risk-Capital	$\Theta = \Psi / Q$
Implied Rate of Return on Risk-Capital	$\theta = (1/T)\ln(\Theta + 1)$
Standard Deviation of Wealth Change	$(\sigma_{\Delta W})^2 = \int_{-\infty}^{\infty} (x - \overline{\Delta W})^2 f_{\Delta W}(x) dx$
Implied Sharpe-Ratio	$\Lambda = (\Psi / \sigma_{\Delta W})\sqrt{1/T}$

[25] The OHMC hedging strategy involves the option-seller receiving the amount $C(t_0)$, and hedging with hedge ratio φ and experiencing a total wealth change of ΔW. The expected change in wealth of the OHMC strategy is denoted by $\overline{\Delta W}$. The actual option is sold at a *market* determined price of $V(t_0)$.

Negative Semi-Deviation of Wealth Change: $\left(\sigma_{\Delta W}^{-}\right)^2 = \dfrac{\int_{-\infty}^{\infty} \min\left(x-\overline{\Delta W},0\right)^2 f_{\Delta W}(x)dx}{\int_{-\infty}^{\overline{\Delta W}} f_{\Delta W}(x)dx}$

Implied Sortino-Ratio: $\Lambda^{-} = \left(\Psi / \sigma_{\Delta W}^{-}\right)\sqrt{1/T}$

Positive Semi-Deviation of Wealth Change: $\left(\sigma_{\Delta W}^{+}\right)^2 = \dfrac{\int_{-\infty}^{\infty} \max\left(0, x-\overline{\Delta W}\right)^2 f_{\Delta W}(x)dx}{\int_{\overline{\Delta W}}^{\infty} f_{\Delta W}(x)dx}$

Implied Artemis-Ratio: $\Lambda^{+} = \left(\Psi / \sigma_{\Delta W}^{+}\right)\sqrt{1/T}$

The return rates formulated above have the sense of return over the discounting rate to effect time-value of money (e.g., US Treasury rates, or LIBOR). The risk-capital, denoted by Q, is defined above as the difference between the expected value and the tail loss value of the probabilistic description of the total wealth change of the optimal hedging strategy. This definition can be altered in any way the user sees fit. For instance, if one is sensitive to re-pricing of market risk premiums in the short term, then the definition above should be augmented by the additional terms introduced in **Section 3.1** of the main text on pricing to return on risk target. Of course, trading an option and a hedge on an exchange will incur margin per the rules of the exchange, which will limit the gearing of the investment strategy. We further define important attributes of an option trading-hedging investment strategy below.

Characteristics of Investment Strategy

Available Risk-Capital Z

Gearing Fraction Ξ

Number of Contracts Sold $\Xi \dfrac{Z}{Q}$

Annualized Mean Return $\dfrac{\Xi}{Q}\Psi \times (1/T)$

Volatility (annualized) $\Xi \dfrac{\sigma_{\Delta W}\sqrt{1/T}}{Q}$

Target Volatility Limit σ_{trg}

Target Volatility Limit Gearing $\Xi_{\sigma_{trg}} = \dfrac{\sigma_{trg}}{(\sigma_{\Delta W}\sqrt{1/T})/Q}$

Downside Volatility(annualized) $\Xi \dfrac{\sigma_{\Delta W}^{-}\sqrt{1/T}}{Q}$

Target Downside Volatility Limit σ_{trg}^{-}

Downside Volatility Limit Gearing $\Xi_{\sigma_{trg}^{-}} = \dfrac{\sigma_{trg}^{-}}{(\sigma_{\Delta W}^{-}\sqrt{1/T})/Q}$

An option-trader hedger can explicitly tailor their trading decisions to the risk-return appetite of the investor insofar as a risk taking trades can be undertaken only when the investor's *hurdles* on risk-return metrics are met, within the confines of available risk-capital.

Model of Underlying Asset

Having accepted the reality of the impossibility of perfect hedges, we seek to deal with the outcome of imperfect attempted replication. All statistical descriptions are in the *real-world* measure for the underlying. In this paper we used GARAM (Wang et al [2009]) to model the asset value probabilistically. We remind the reader that our description is *conditioned* on observations – i.e., it is cognizant of the asset behavior of the past. This is pertinent because of the gross deviations of returns from Normality on account of excess kurtosis and skewness, and subtle temporal dependencies of asset returns that render Brownian motion to be a rather unreal model that is only impressive for its sterility in comparison with reality![26] For option trading models that have taken on the challenge of understanding real-world risk-return, a realistic conditioned description of the underlying is required – as the option price at any instance is conditioned on all available information up to that point.

Specification of Hedging Strategy

Using the stochastic model for the underlying, the Optimal Hedge Monte-Carlo (OHMC) method provides the hedging strategy. That hedging strategy can be molded to account for factors that are deemed important conditioning variables, and the constraints and the minimizing objectives can also be chosen to ones preference. The hedging strategy is not perfect! It has a cost associated with it, and it results in a P&L distribution that is exposed by OHMC.

Role of Residual Risk Information in Trading Strategy

Now comes the tricky part. One needs to recognize how imperfectly these hedging strategies work! Other than some insular "quants" hiding behind the jargon of unearthly *measures*, the whole world knows that – exchanges do require significant margin for a *hedged* option sell position! It is rather immature for "quants" to be involved in derivative trading and pretend that there is a perfect dynamic replication strategy when there simply is not one! Rather, one can put the understanding of these hedging strategies and their imperfections to work in developing a trading strategy, or an investment strategy. This can be done by associating a traded price with a return over risk metric. In this way one can ascribe richness or cheapness to an option price. In the case of customized derivatives one can price to a return over risk target while being cognizant of the residual risk that arise due to special features of the customized contract (e.g., Cliquets, Barrier Options, etc). One can analyze the listed market to discern the prevailing levels of compensation for risk in the options market. More importantly, one can price and trade customized contracts to ones return hurdle mandate. Also, one can assess *risk-capital* that is cognizant of the option contract, the underlying asset, and the hedging strategy.

[26] Taleb [2007] debunks the mythology of the applicability of the Normal distribution in financial markets.

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