Overview of Advanced RL in Finance:

Week 1: The Black-Scholes model, physics and RL

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Economics ended up with the theory of **rational expectations**, which maintains that there is a single optimum view of the future, that which corresponds to it, and eventually all the market participants will converge around that view. This postulate is **absurd**, but it is needed in order to allow economic theory to model itself on **Newtonian Physics**.

George Soros

Writing is easy. All you have to do is cross out the wrong words.

Mark Twain

One should avoid solving more difficult intermediate problems when solving a target problem.

Vladimir Vapnik, *Statistical Learning Theory*, 1998

Ptolemy's Epicycles on Wall Street

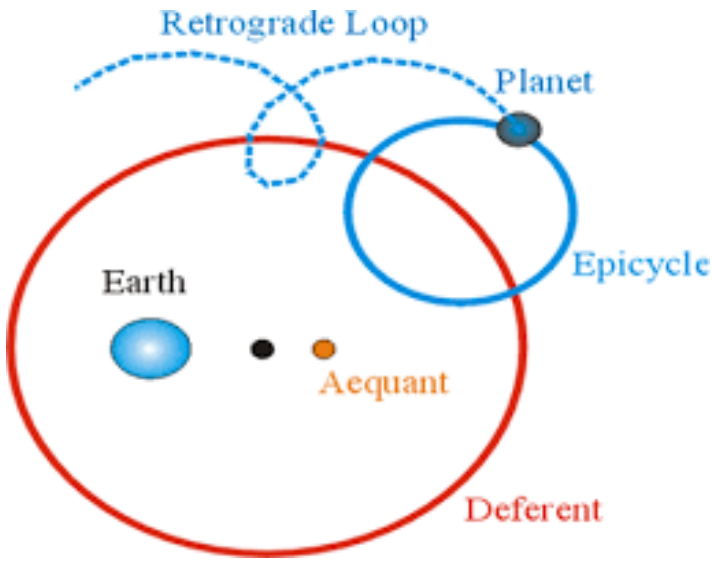

Figure: Ptolemy's geo-centric planetary model: the observed planetary motion is a motion on an epicycle whose center moves on a larger orbit. For more, watch a TEDx talk by Ben Vigoda, CEO of Gamalon,

Control question

Select all correct answers:
1. In Ptolemey's geo-centric system, Earth is located near the center of the Universe.
2. The reason Ptolemey's planetary model stayed unchallenged for about 14 centuries is that it was consistent with all available data.
3. Ptolemey's model used hidden variables, the Kalman filter and the EM algorithm.

Correct answers: 1, 2.

In this presentation:

- Where do we stand in both industry and academia after the Black-Scholes-Merton groundbreaking work of 1973?
- Are there any *viable* (meaningful *and* tractable) alternatives?
- Insights from Physics and Reinforcement Learning
- Model-free option pricing and hedging by Reinforcement Learning (Q-learning and Fitted Q Iteration)
- Summary

PHYSICISTS IN FINANCE

Though the challenges of "quantitative finance" are diverse and often exhilarating, success for the erstwhile physicist is not at all assured. What factors are involved in making the transition to finance?

Joseph M. Pimbley

To a physicist facing significant difficulty in the job market, the allure of a career in finance is obvious: The industry has numerous opportunities that demand the physicist's quantitative skills, and pay handsomely for them. Those contemplating such a move, however, need to look beyond these immediate considerations, for the culture of finance differs markedly from that of physics, having different goals and philosophies, work styles, even dress codes. To be successful on Wall Street, the physicist must willingly adapt to Wall Street's ways.

To add precision to the phrase "physicists in finance," I am using "physicists" to denote PhD recipients and "finance" to refer to the disciplines that require the great-

directly into finance following graduate school or from a postdoctoral position. Less common are the émigrés from full-time "legitimate" physicist positions. Certainly this observation implies that one cause of the physics-to-finance transition is the shortage of jobs in physics, especially for those just starting their careers. But it is regrettable that younger physicists, who have not had the opportunity to explore their chosen disciplines and their abilities on their own, are more likely to shift career goals. Older colleagues, by contrast, have orchestrated successful research projects with lasting contributions, and are therefore much better equipped to contemplate leaving the physics profession. They know what they are forsaking

Figure: "To be successful on Wall Street, the physicist must willingly adapt to Wall Street's ways."
http://www.maxwell-consulting.com/Physicists_Finance_low_mem.pdf

- "In finance, you would be mostly solving a diffusion equation with various boundary conditions."
- Econophysics! (J-P. Bouchaud, E.Stanley and

In this presentation: Option pricing without PDEs or a model?

Sounds like a scientific blasphemy or maybe as Luddites?

Figure: Luddites (1811-1816) protested the use of machinery in a "fraudulent and deceitful manner" to get around standard labor practices.

Control question

Select all correct answers:
1. Econophysics applies methods from physics to analysis of financial markets and asset pricing.
2. Luddites protested Ptolemey's planetary system as incompatible with standard labor practices.
3. Luddites protested the use of machinery in a "fraudulent and deceitful manner".

Correct answers: 1, 3.

Simplicity and beauty in scientific theories

"Everything should be made as simple as possible, but not simpler." (Albert Einstein)

"A physical law must possess mathematical beauty." (Paul Dirac).

"Let's trivialize the problem..." (Lev Landau).

Partial Differential Equations == simplicity and beauty?

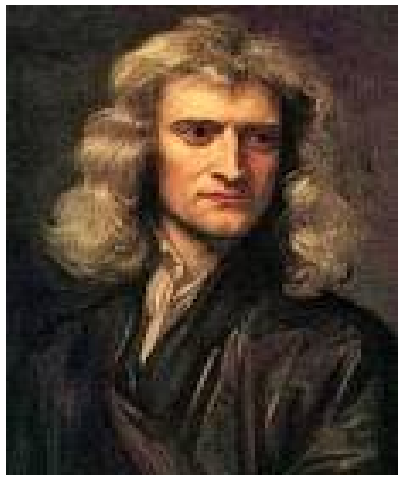

Figure: Isaac Newton

- Newtonian mechanics...
- The diffusion equation...
- The Black-Scholes equation...
- The Schrödinger equation...

Competitive market equilibrium = physics of Newton and Boltzmann

- D. Duffie, "Black, Scholes and Merton - Their Central Contributions to Economics" (1997)
- P. Bernstein, "Capital Ideas: the improbable origins of modern Wall Street", Wiley 2005
- Duffie: "While there are important alternatives, a current basic paradigm for valuation, in both academia and in practice, is that of **competitive market equilibrium**". The price is the price that equates total demand to total supply.

Competitive market equilibrium in Finance

- Three Nobel Prizes in Economics for work based on the paradigm of market equilibrium:
  - Modigliniani-Miller (1958): irrelevance of capital structure for the market value of a corporation.
  - The Capital Asset Pricing Model (CAPM) of William Sharpe (1964).
  - The Black-Scholes option pricing theory (1973) (no-arbitrage as a weaker form of market equilibrium)
- Market equilibrium theories "model themselves on Newtonian physics" (G. Soros).
- More precisely, they describe a thermodynamics equilibrium of statistical mechanics of Ludwig Boltzmann (1844-1906).

Does the Black-Scholes model pass Einstein's test?

Two key elements :

- Option pricing by replication (dynamic hedging)
- Taken to the continuous-time limit $\Delta t \to 0$

Together, these two steps produce the celebrated Black-Scholes equation

$$\frac{\partial C_t}{\partial t} + rS_t \frac{\partial C_t}{\partial S_t} + \frac{1}{2}\sigma^2 S_t^2 \frac{\partial^2 C_t}{\partial S_t^2} - rC_t = 0 \qquad (1)$$

Isn't it simple and beautiful?

"I applied the Capital Asset Pricing Model to *every moment* in a warrant's like, for every possible stock price and warrant value... I stared at the differential equation for many many months..." (Fisher Black).

Black-Scholes model: the main take-aways:

- Data requirements: two numbers: the current stock price S_t and stock volatility σ (plus parameters for an option)
- The option price is *unique* and given by a solution of the BS equation.
- The optimal option hedge (the amount of stock in a replicating portfolio) is obtained *after* the option price is computed.
- Options are **redundant** ($=$ perfectly replicable in terms of stocks and cash - why bother?) and have **instantaneously zero risk**!!
- (What does it even mean? Time in Finance is fundamentally discrete...)
- "When people are seeking profits, equilibrium will prevail" (Fisher Black).

Control question

Select all correct answers:
1. Financial models based on the competitive market equilibrium include, in particular, the CAPM model and Black-Scholes model.
2. Competitive market equilibrium models are models that produce competitive prices.
3. Competitive market equilibrium paradigm builds the financial theory on classical mechanics of Newton and equilibrium thermodynamics of Boltzmann.
. 4. The data requirement in the Black-Scholes model amounts to two numbers: the current stock price and stock volatility.

Correct answers: 1,3, 4.

Black-Scholes model as a model of fake markets?

- Arbitrage pricing gives you an equilibrium price, so that you should not trade below it, and you should not trade above it.
- It only forgot to explain why you should trade at the equilibrium price itself!
- What is the rationale of having entirely redundant financial instruments?
- Options are **not** redundant because they carry **risk**!

"I certainly hope you are wrong, Herr Professor!"

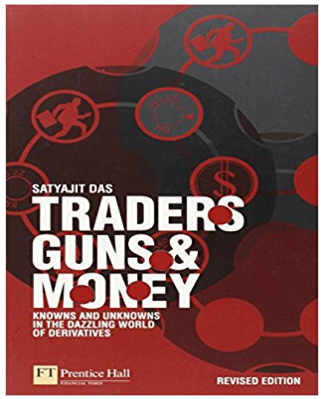

Figure: "A German bank hired a professor from a leading university to help quantify its risk. After some months of extensive analysis, the professor has concluded that the bank had "absolutely no risk". The bank's head of trading responded: "I certainly hope you are wrong, Herr Professor. If you are correct then we can't be making any money!"

What is the **main** problem with the Black-Scholes model?

- Does not match **option** price data?
- Does not match **stock** price data (stock prices are not lognormal)?
- Transaction costs are neglected?
- Discrete hedging?
- Real markets are incomplete?
- **Risk** has disappeared?
- Question: what is a **minimal** change to the BS model so that a new model
  - Will be more useful/meaningful
  - Will have the same or similar level of tractability as the BS model

"Match the market" mantras

The main problem of 'risk-neutral' Quantitative Finance is that it mixes together two problems with the Black-Scholes model:

- It does not incorporate **risk** of hedging in option
- The real-world stock price dynamics are not log-normal
- Risk-neutral models ignore the first problem and pursue the second one (in the "risk-neutral" measure!)
- The end result are "match the market" mantras:
  - *Parametric mantras*: Stochastic volatility models, jump-diffusion, Levy models, etc.
  - *Non-parametric mantras:* Local volatility models, MaxEnt, non-parametric Bayes

Ptolemy's epicycles of "risk-neutral" Mathematical Finance

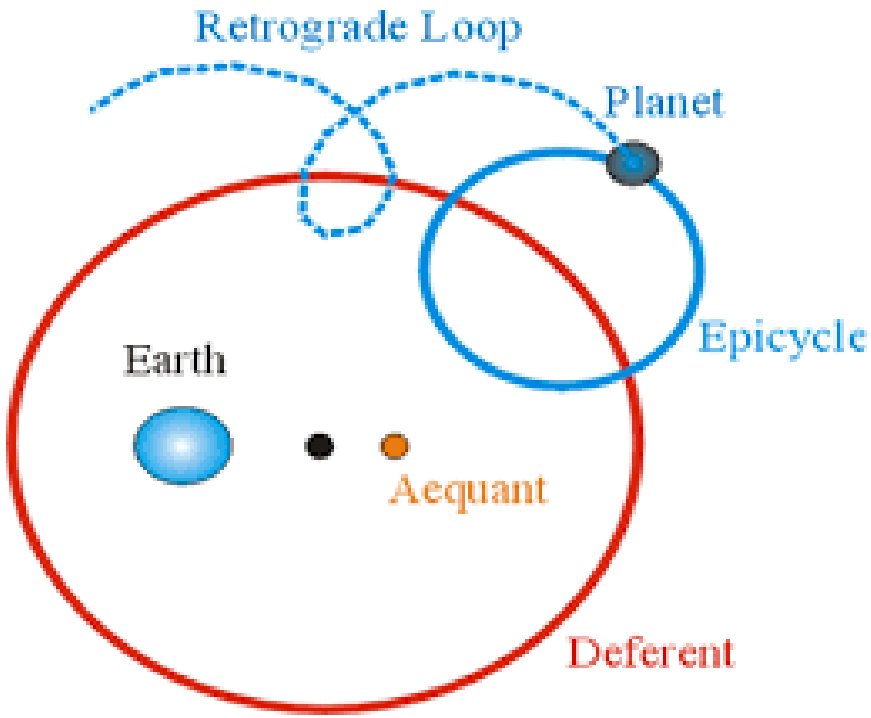

Figure: Ptolemy's model explained 'imperfections' of motion of planets by postulating that the apparently irregular movements were a combination of several regular circular motions seen in perspective from a stationary Earth. Ptolemy had separately fitted model parameters for more than 40 heavenly bodies.

Cargo cult of "risk-neutral" Mathematical Finance

This theory is not even wrong...

Figure: Cult members worshiped certain unspecified Americans having the name "John Frum" who they claimed had brought cargo to their island during World War II and who they identified as being the spiritual entity who would provide cargo to them in the future. (https://en.wikipedia.org/wiki/Cargo_cult)

Control question

Select all correct answers:
1. The main problem with the Black-Scholes model is that the real-world stock prices are not lognormal.
2. In the story of the German math professor, the reply of the head of trading was based on the fact that the only way to make money without risk is by arbitrage.
3. Options are not redundant because stock prices are not lognormal.
4. Options are not redundant because they carry risk of mis-hedging, therefore their prices cannot be summarized in just one number.

Correct answers: 2, 4.

Control question: model building by subtraction

Mark Twain's approach to Quantitative Finance:
What do you think are the wrong words that should be crossed out when trying to improve on the Black-Scholes model? Select all correct answers:

1. No arbitrage pricing
2. Risk-less hedges
3. "Risk-neutral" option valuation
4. The continuous-time limit
5. PDE's
6. All of the above

Correct answers: 6.

Risk is a science of fluctuations

- In the Markowitz portfolio theory: risk is $\mathcal{R}_t = \lambda \mathrm{Var}\left[\Pi_t\right]$
- In statistical mechanics, there are models for both equilibrium and non-equilibrium fluctuations
- The BS model **neglects fluctuations**. This is equivalent to a **thermodynamic limit** in equilibrium statistical mechanics, where all fluctuations die off.
- Competitive market equilibrium models are models where entropy is maximized and does not fluctuate: they are models of a 'heat death' of the Universe as an equilibrium system in a thermodynamic limit.
- A \$50Bn question: Is it a right limit to use as a reference point (or a 'first approximation') to describe a risky business?

"Premature continuous time limit" in the BS model?

- The continuous-time limit is taken from the start
- As pricing by replication becomes exact in this limit, all risk is instantaneously eliminated
- To re-install *option risk* as a first-class citizen of a model, we need to revert back to a discrete-time setting!
- This view will show that the BS equation is just a PDE for a **mean** of the option value in the **mathematical** limit $\Delta t \to 0$
- This limit makes a perfect sense *mathematically* but not *financially*, as it looses risk: the option magically becomes risk-less!
- But shouldn't risk in the option be the *original* purpose, a part of option valuation?

Control question

Select all correct answers:
1. Competitive market equilibrium models are models where entropy is maximized and does not fluctuate.
2. The Black-Scholes model uses a quadratic risk measure, similar to the Markowitz portfolio theory.
3. Risk in the BS model is instantaneously eliminated due to taking the continuous-time limit right from the start.

Correct answers: 1, 3.