

Algebra 1: Quadratic Equation and Linear Equation

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本节课我们学习如何解一元二次方程和一个二元一次方程组, 在实际解题过程中我们需要对题目所描述的实际问题进行建模, 然后去求解.

Example

解下面的方程:

$$\begin{cases} x + 3y = 5 \\ 2x - 7y = 3 \end{cases}$$

我们一般采用消元法消去其中一个未知数, 从而得到另一个两个未知数的具体取值.

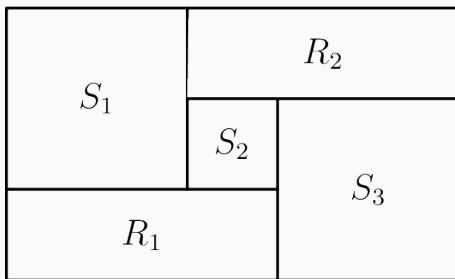
Frog

A group of frogs (called an army) is living in a tree. A frog turns green when in the shade and turns yellow when in the sun. Initially, the ratio of green to yellow frogs was $3 : 1$. Then 3 green frogs moved to the sunny side and 5 yellow frogs moved to the shady side. Now the ratio is $4 : 1$. What is the difference between the number of green frogs and the number of yellow frogs now?

(A) 10 (B) 12 (C) 16 (D) 20 (E) 24

Rectangles(AMC 8, 2020-25)

S_1, S_2, S_3 为三个正方形, 大矩形长 3322, 宽 2020, 求 S_2 边长 (S_1 和 S_3 边长不一定相等).



二次方程求根公式

当 a, b, c 为实数时, 记二次方程 $ax^2 + bx + c = 0, a \neq 0$ 的判别式为 $D = b^2 - 4ac$

- ① $D > 0$ 时, 有两个不同的实数解.
- ② $D = 0$ 时, 只有一个实数解.
- ③ $D < 0$ 时, 没有实数解.

Theorem (Quadratic Formula)

$D > 0$ 时候, 二次方程 $ax^2 + bx + c = 0, a \neq 0$ 的解由如下公式给出:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Vieta's Theorem

Theorem (Vieta's Theorem for quadratic polynomials)

If x_1 and x_2 are two roots of the quadratic polynomial $ax^2 + bx + c$, we have

$$x_1 + x_2 = -\frac{b}{a}, x_1 x_2 = \frac{c}{a}$$

Example

Assume α and β are two roots of the equation $2x^2 + x - 7 = 0$. Find

$$\alpha^2 + \beta^2, \frac{1}{\alpha} + \frac{1}{\beta}, \alpha^3 + \beta^3$$

Vieta's Theorem(韦达定理)

Theorem (Vieta's Theorem for quadratic polynomials)

如果 x_1 和 x_2 是二次方程 $ax^2 + bx + c = 0$ 的两个根, 我们有

$$x_1 + x_2 = -\frac{b}{a}, x_1 x_2 = \frac{c}{a}$$

Example

Assume α and β are two roots of the equation $2x^2 + x - 7 = 0$. Find

$$\alpha^2 + \beta^2, \frac{1}{\alpha} + \frac{1}{\beta}$$

Exercise

Question

Suppose that the equation $x^2 - px + q = 0$ has solutions $x = a$ and $x = b$. What's the equations has solutions $x = a + \frac{1}{b}$ and $x = b + \frac{1}{a}$.

Exercise

Question (AMC 8, 2019-20)

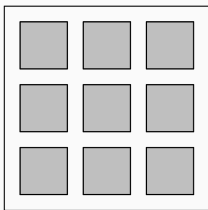
How many different real numbers x satisfy the equation

有多少不同的实数 x 满足这个方程

$$(x^2 - 5)^2 = 16$$

Gray Square Tiles(AMC 8, 2020-24)

A large square region is paved with n^2 gray square tiles, each measuring s inches on a side. A border d inches wide surrounds each tile. The figure below shows the case for $n = 3$. When $n = 24$, the 576 gray tiles cover 64% of the area of the large square region. What is the ratio $\frac{d}{s}$ for this larger value of n ? (补充说明: d 是灰色瓷砖之间间隔的距离, 也是外围瓷砖与黑色边框之间的距离)



- (A) $\frac{6}{25}$ (B) $\frac{1}{4}$ (C) $\frac{9}{25}$ (D) $\frac{7}{16}$ (E) $\frac{9}{16}$

注意到 $n = 24$ 时, 横行共有 25 个长为 d 的间隔, 共 24 个长为 s 的瓷砖边长, 因此我们有

$$\frac{s^2 \times 24^2}{(24s + 25d)^2} = \frac{64}{100}$$

从而

$$\frac{24^2}{(24 + 25d/s)^2} = \frac{64}{100}$$

Homework

Question (AMC 8, 2019-22)

A store increased the original price of a shirt by a certain percent and then lowered the new price by the same amount. Given that the resulting price was 84% of the original price, by what percent was the price increased and decreased? (A) 16 (B) 20 (C) 28 (D) 36 (E) 40

Question

Find the solution of the following equation:

解下面的方程:

- $2x^2 - 9x = 5$



$$\frac{5}{x} + \frac{6}{x+1} = 2$$

Homework

Question

For what values of the real number k does $x^2 + kx + 4k = 0$ have no real solutions.

k 取什么的时候 $x^2 + kx + 4k = 0$ 没有实根.

Question

x 是一个实数, 满足 $|2x^2 - 9x + 6| = 2x^2 - 9x + 6$, 求 x 的所有可能取值.