AST356 / 326

Coherent Detection & Thermal Noise

Oct 16, 2017

Announcements

- Lab #1 optional extension, check your email.
- You're stuck with me for the next 3 weeks, office hours still Fri @ 3-4pm, now in AB126.
- Lab #2 begins today, due Nov 15th.
  - Go download, print, and read the lab manual if you haven't yet!
  - Don't leave things to the last minute, try to pace yourself. Not going to be extended like #1…
- Presentation groups next Monday: **`EGADIJ`**

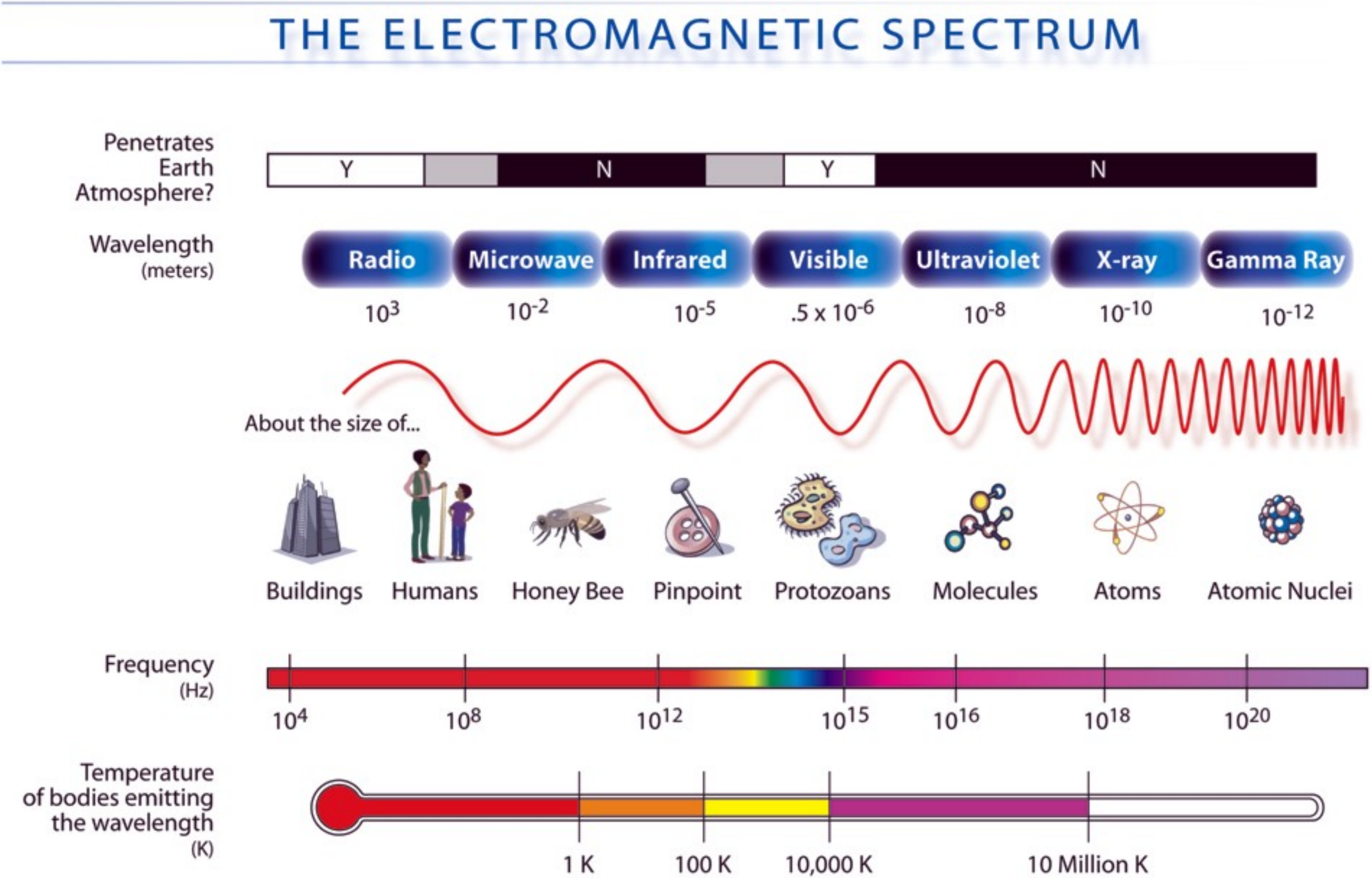
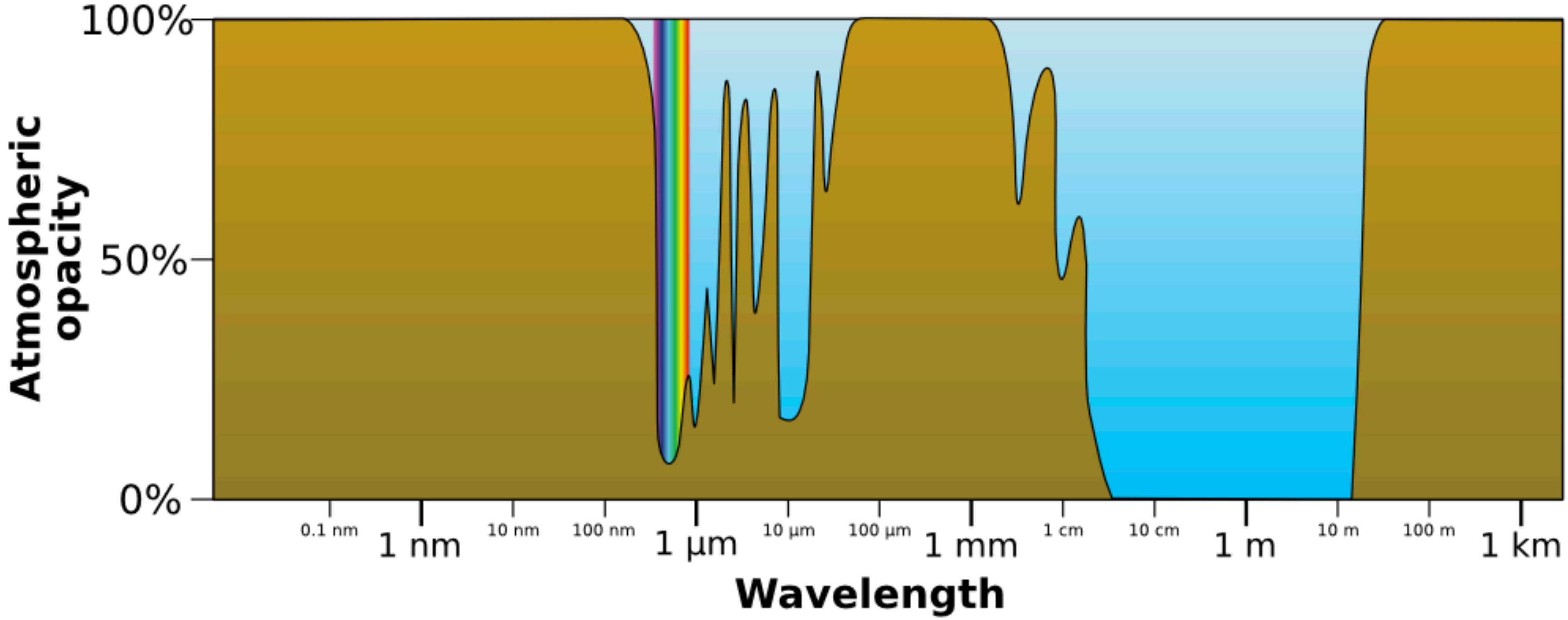


Figure courtesy NASA

Atmopsheric Windows



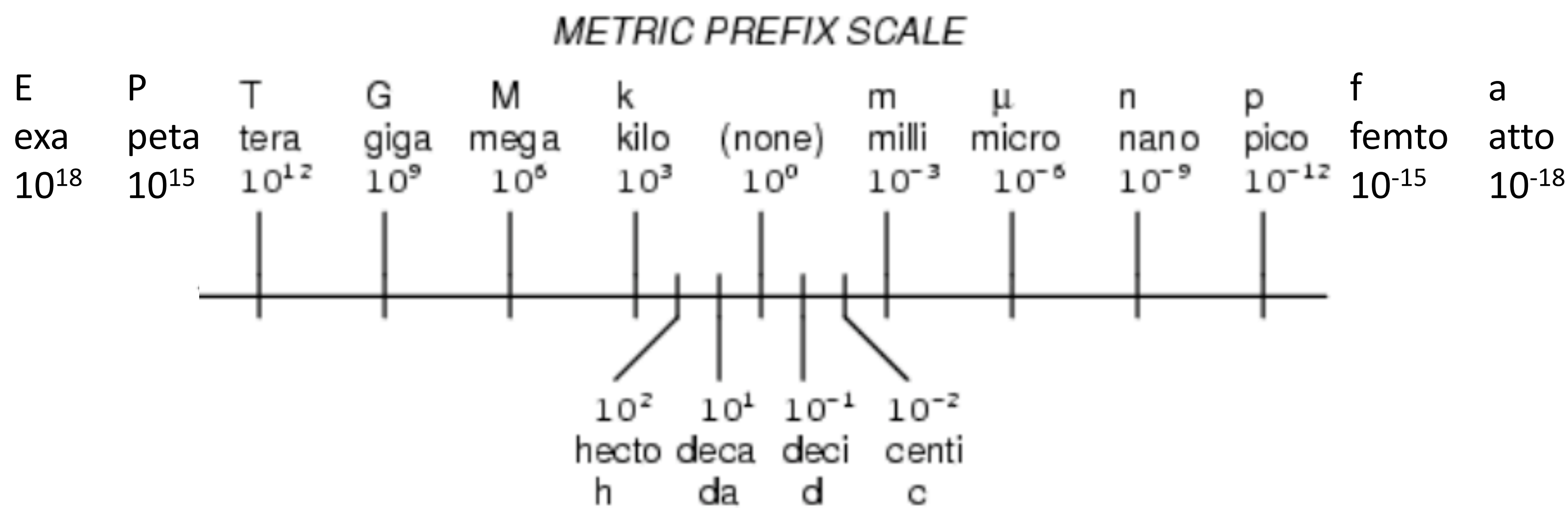
1mm – 10m
(300GHz – 30MHz)

Excellent seeing over a huge band!

Wavelengths vs. Frequencies

Unfortunately, radio people tend to use both.

Remember: $c = \lambda\nu = 3 \times 10^8$ m/s



Handy reference points: ν = 300 MHz = 3×10^8 Hz ⇔ λ = 1 m

ν = 300 GHz = 3×10^{11} Hz ⇔ λ = 1 mm

Low Energy Light

$$E = h\nu$$

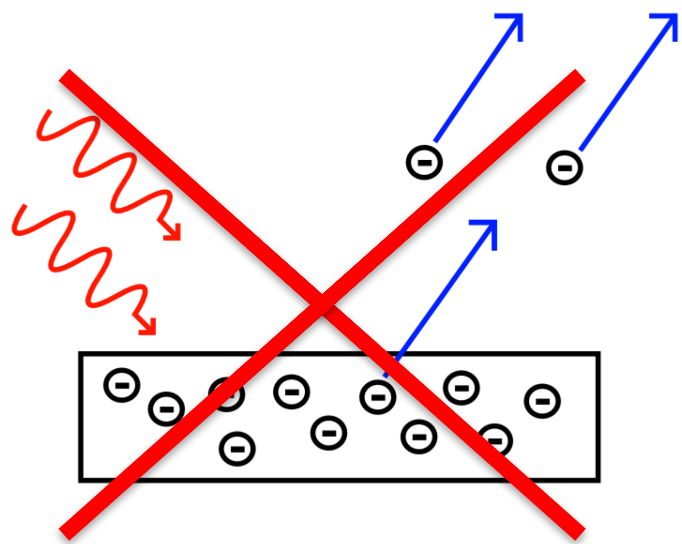
1mm (300GHz) photon:

- 200 yoctojoules
- 2×10^{-22} Joules
- 0.001eV

1m (300MHz) photon:

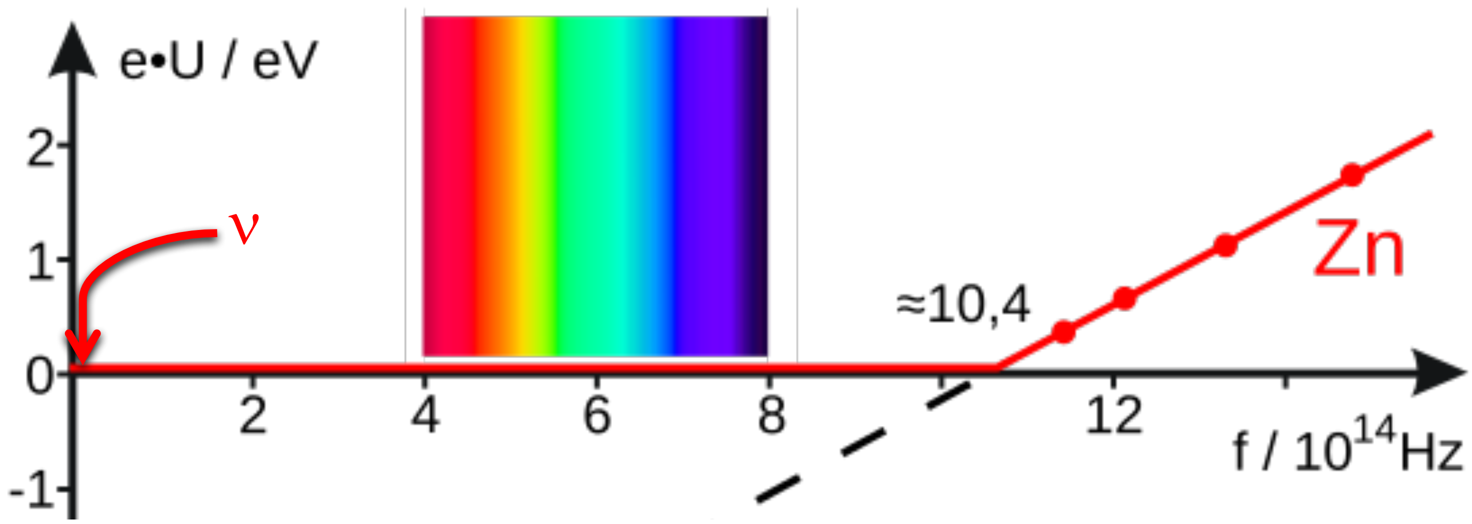
- 0.2 yoctojoules
- 2×10^{-25} Joules
- 0.000 001eV

Insufficient for photoelectric effect!



Three Ways to Detect Light

1. ~~Photon Detectors~~
2. Thermal Detectors
3. Coherent Detectors



Detecting Radio Light: Thermal

Bolometers:
- just fancy thermometers
- heat up in response to power
- Presently the most sensitive detectors for microwave light
- Still not enough power in most radio signals to be viable

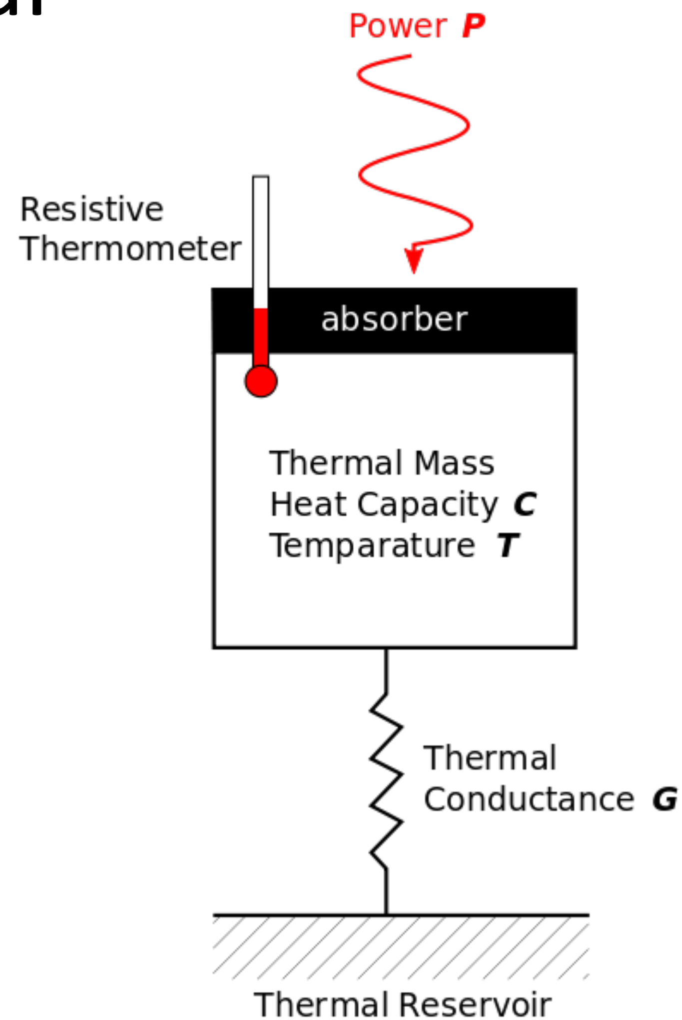
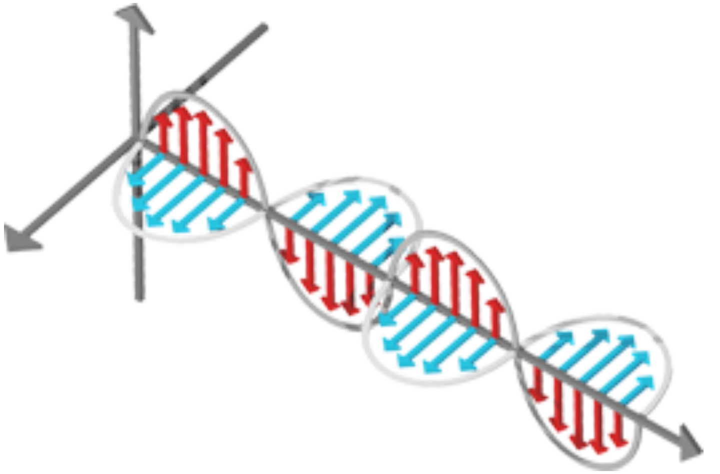


Image from Wikipedia

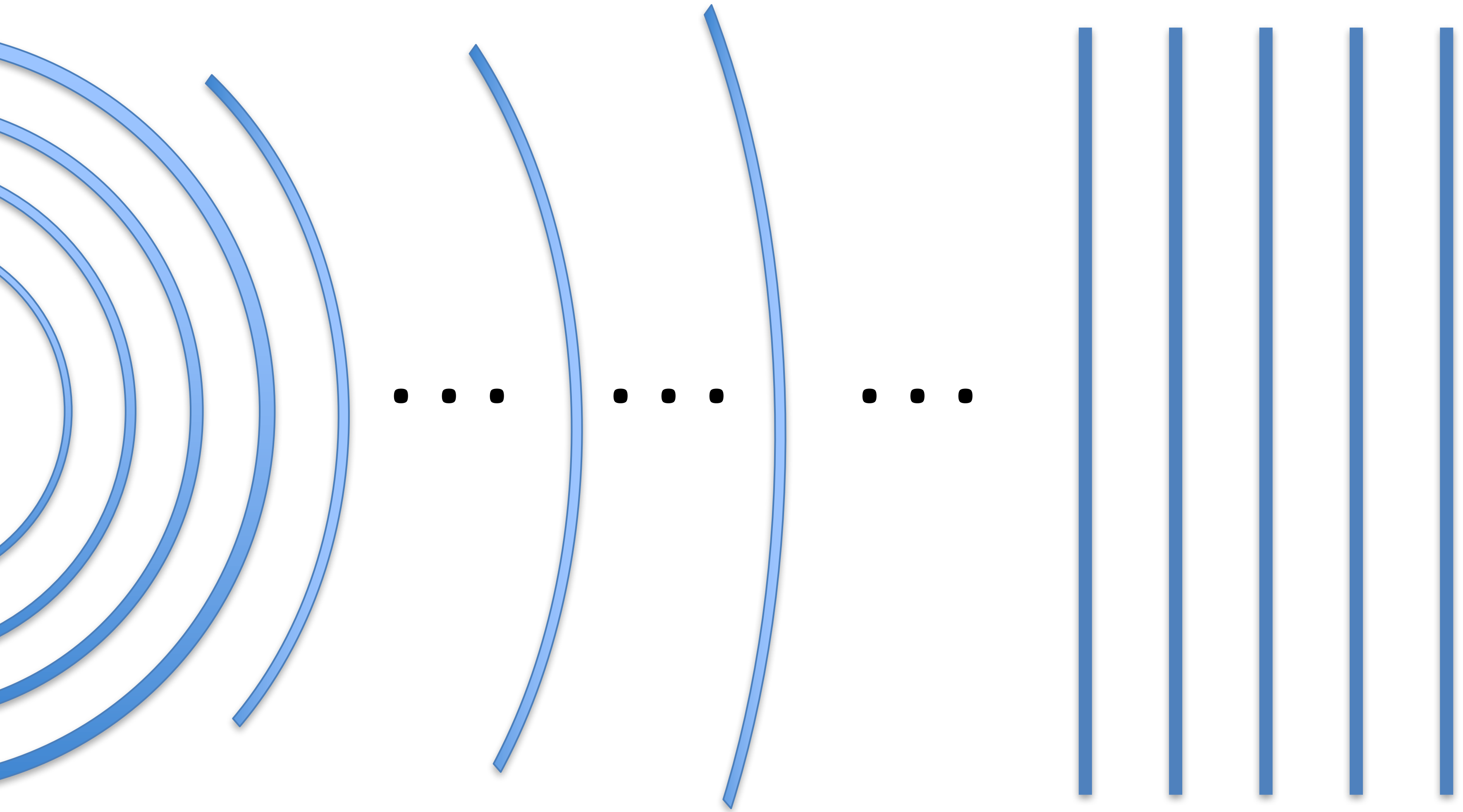
Wave-particle Duality

- Optical: eV – can photo-ionize
- Radio: μeV – negligible energy / photon

Think *waves*,
not particles.



Waves



Detecting Radio Light: Coherent

Forget particles and power – **just record the waves**.

A ***feed*** couples free space waves into ***waveguide***.
That makes it easier control and manipulate the light.

From there, we can:

- amplify
- delay (phase shift)
- filter
- mix
- sample
- correlate

Coherent detection records **Electric fields**, both **amplitude** and **phase**.

Coherent & Incoherent

Coherent is unlike photon counting or thermal detection. Those each measure the ***power*** in the incoming light, and are *incoherent* forms of detection.

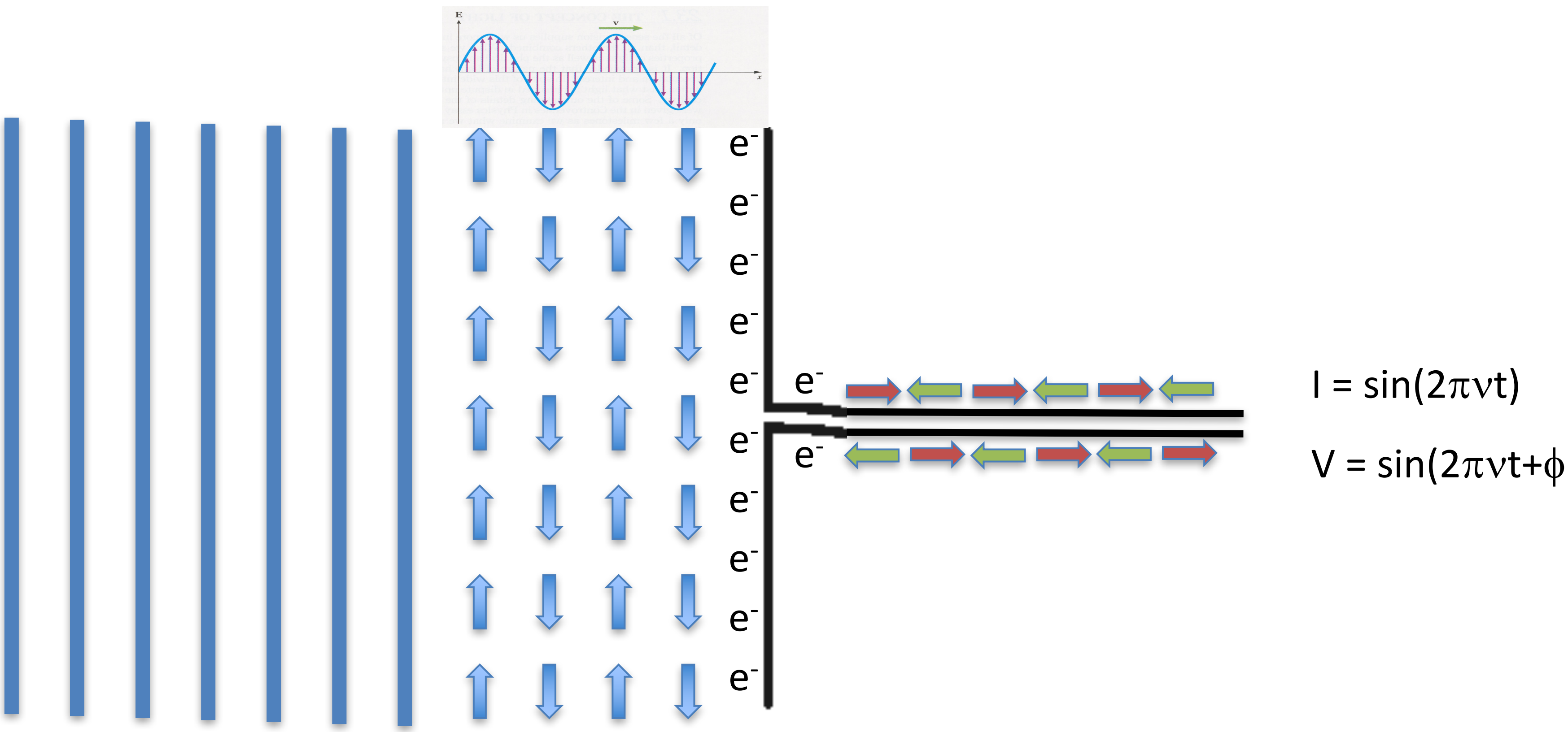
Coherent detectors measure $E(t) = \Sigma_{\omega} A \sin(\omega t + \phi_{\omega,t})$. Variations in phase ϕ contain lots of information, but the overall ϕ is usually arbitrary & meaningless.

The power of the source $\mathbf{P} \propto \mathbf{\langle E^2 \rangle} \propto \mathbf{|A|^2}$ is often more useful than the raw amplitudes, as it relates to source temperature, energy conservation, …

Feeds

In radio, large light, small focal planes – no Megapixel CCDs.

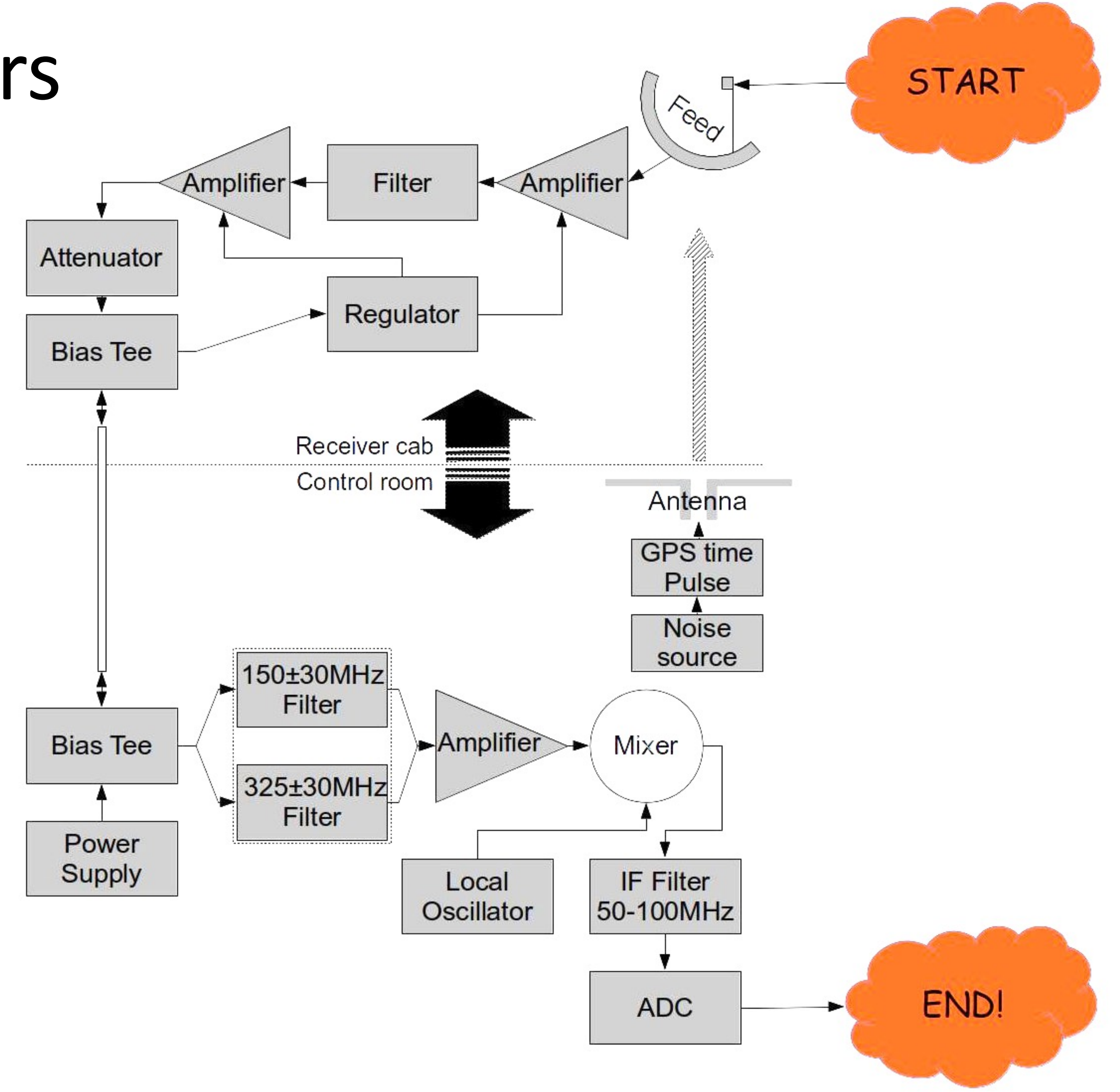
- Radio "cameras" are still in the single- (or few-) pixel regime.
- Feeds collect radiation from free space



For high efficiency (to be resonant), a feed's size must be of order λ.

Receivers

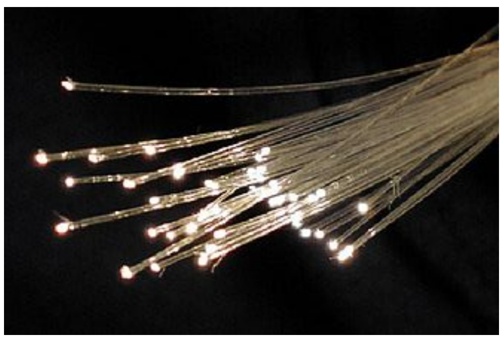
(Everything behind the feed.)



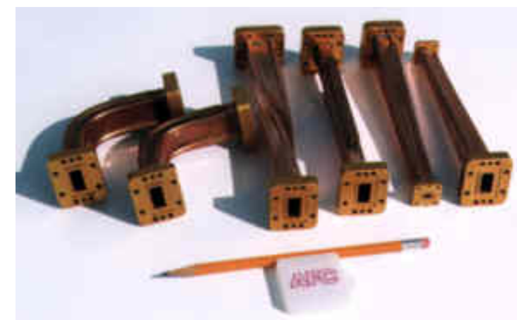
Waveguide

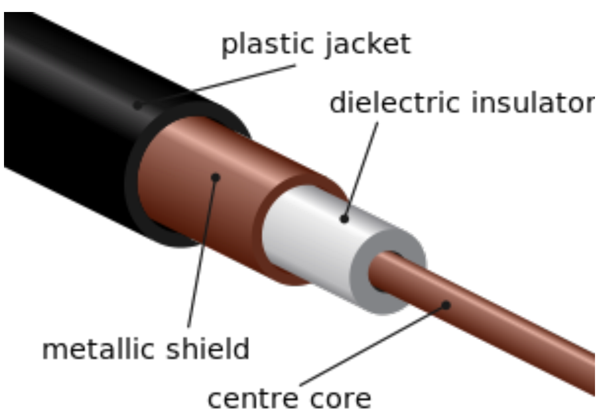
We usually think of light in free space or bulk material.
EM fields will also propagate in **waveguides**.
A waveguide confines the signal to travel in one direction.

Optical:
Fiber

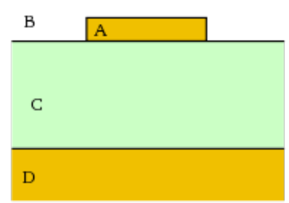


Microwave:
Metal Tubes
/ microstrip





Radio:
Transmission
Lines / Cables /
Microstrip

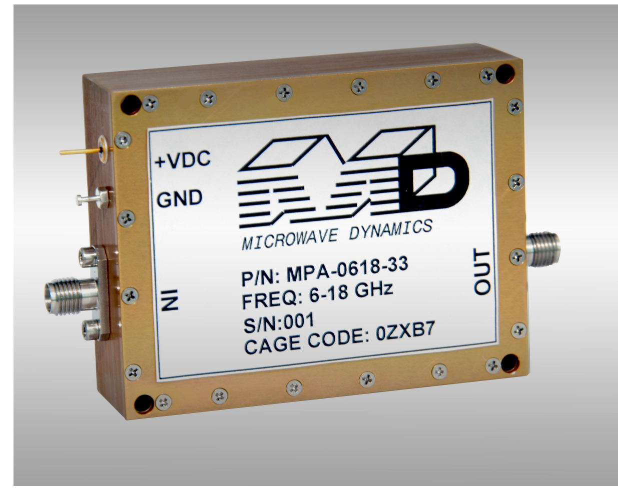


RF Amplifiers

Semiconductors are fast now.

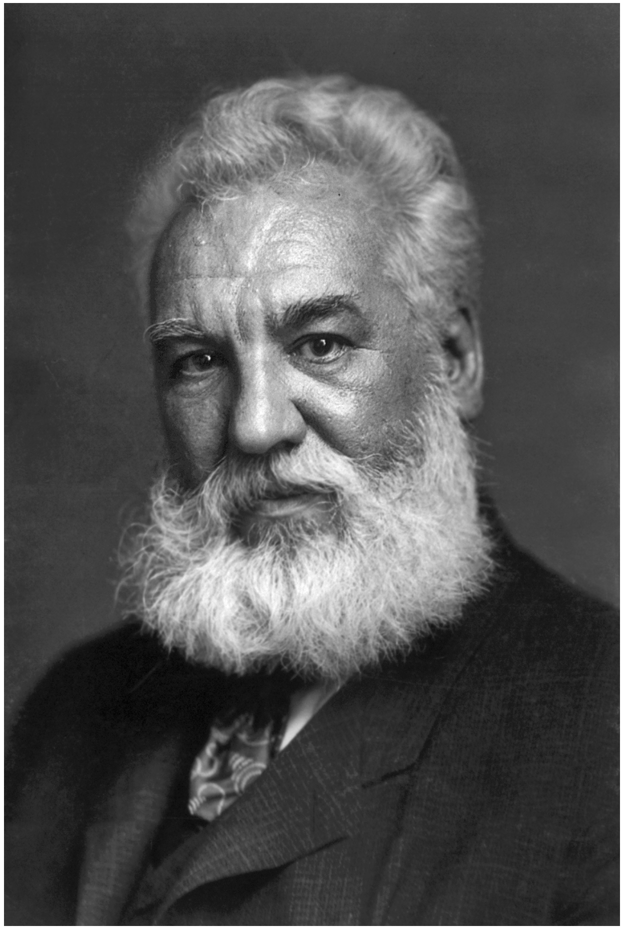
Many commercial amplifiers exist, capable of amplifying signals up to ≈100GHz.

In coherent detection, we *directly amplify* an EM wave in waveguide.



Microwave Dynamics

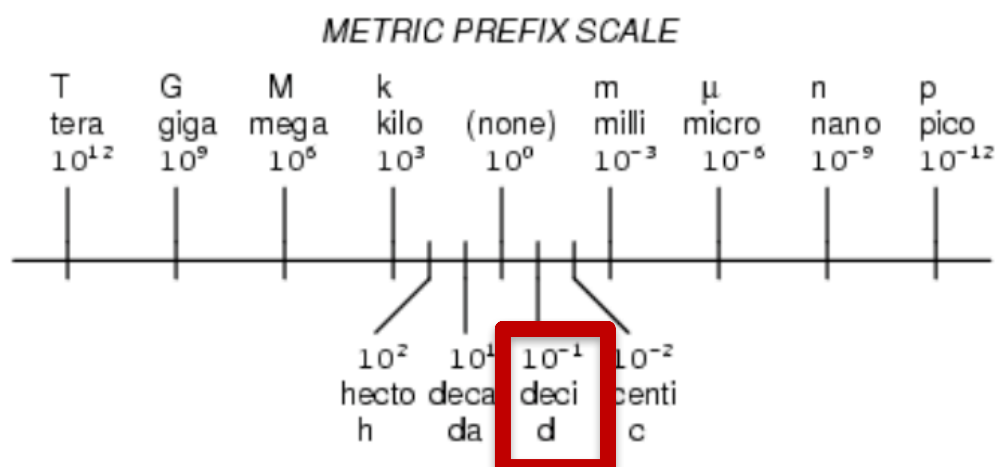
DeciBels



Alexander Graham Bell

Bell

- **Hugely** influential in radio.
- Unit of signal attenuation named after him.
- 1**Bel** = 10-fold loss of signal
- More commonly use **dB**



- Now widely used log-relative

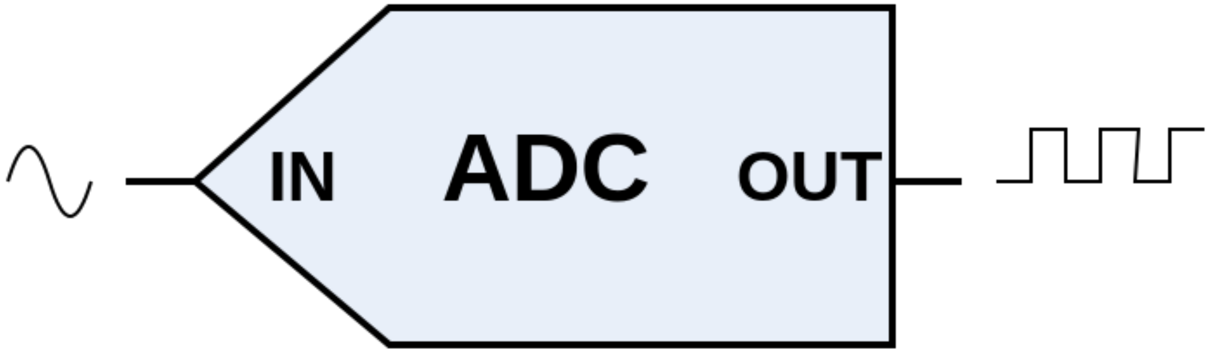
Practical dB

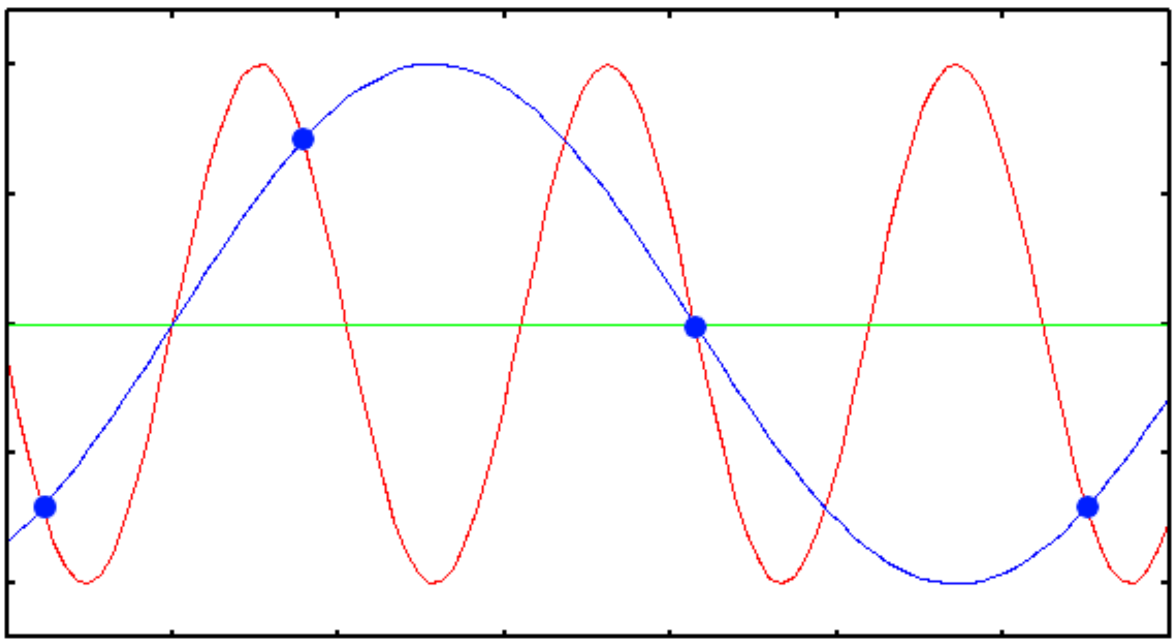
- dB now used everywhere log scales are needed.
- cosmic time often given in dBs, decibels relative to a second
- Radio power given in dBm (dB-milliWatts) or dBW (dB-Watts).
  1dBW = +30dBm

dB	Relative Level
0	1x
+10	10x
+20	100x
+30	1000x
-10	0.1x
-20	0.01x
-30	0.001x
+3	2x
+26	400x

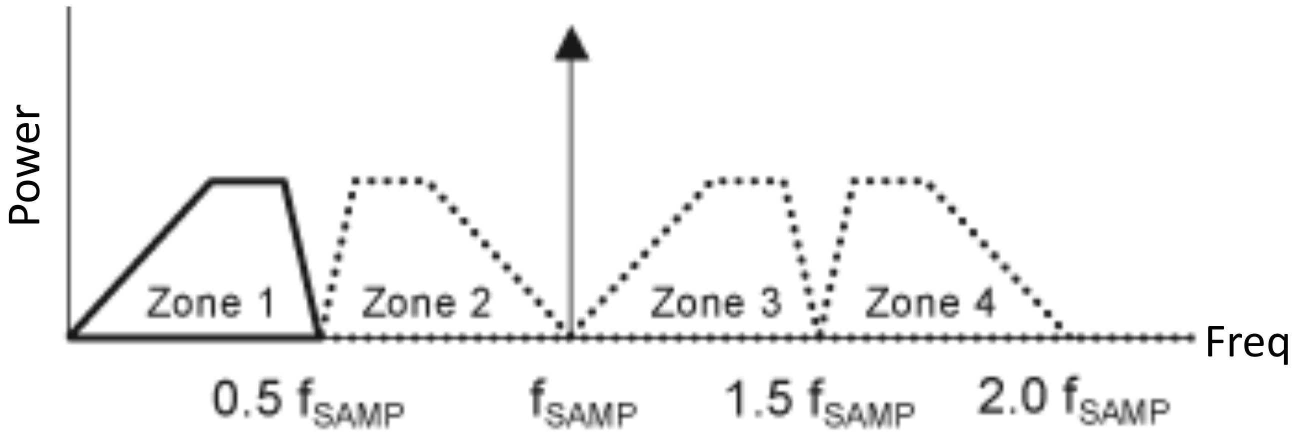
Digitizers

Once it's a nice big signal, we connect a high speed voltmeter (an **Analog to Digital Converter**, ADC) to read it out!





Aliasing & Nyquist

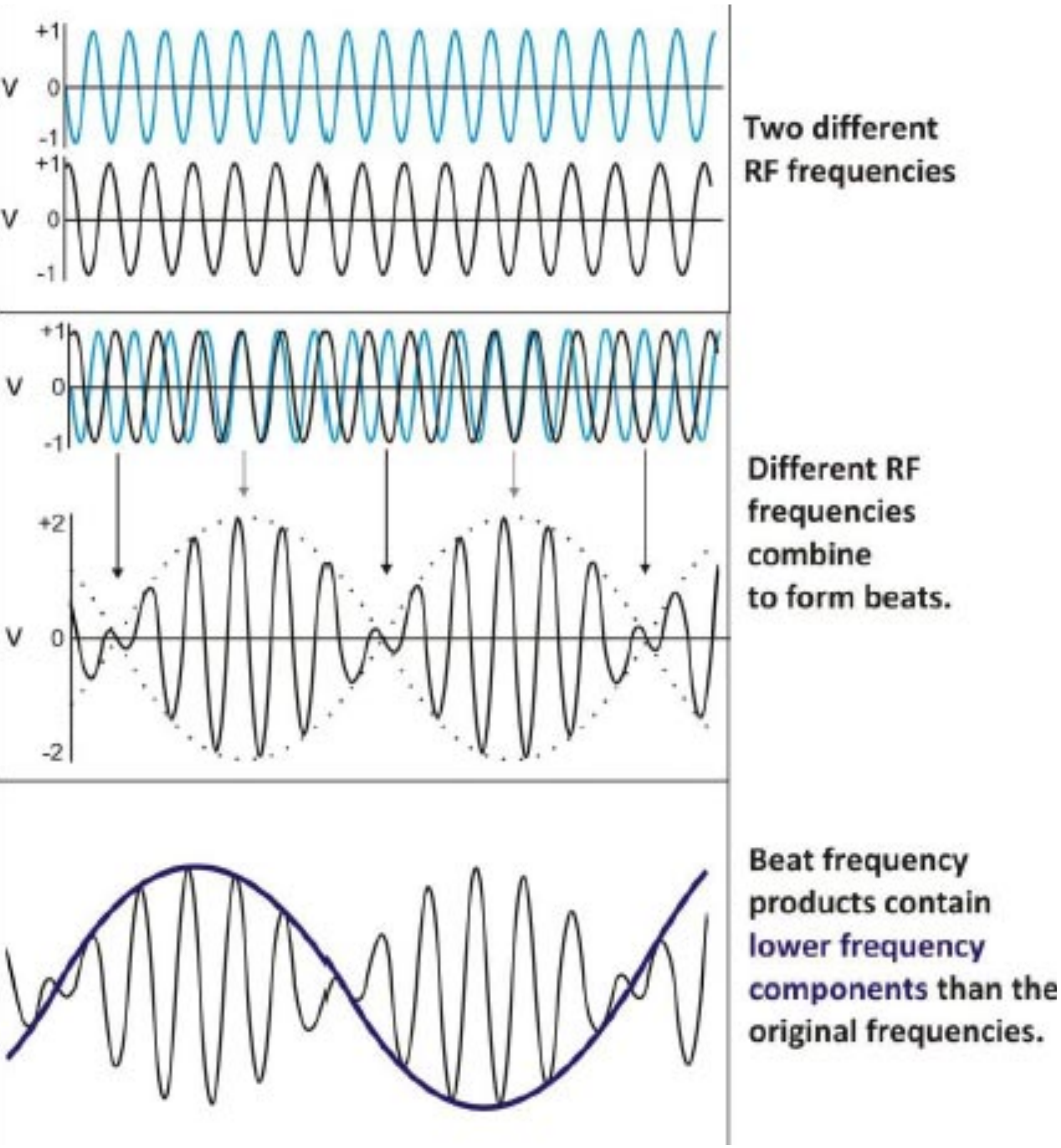


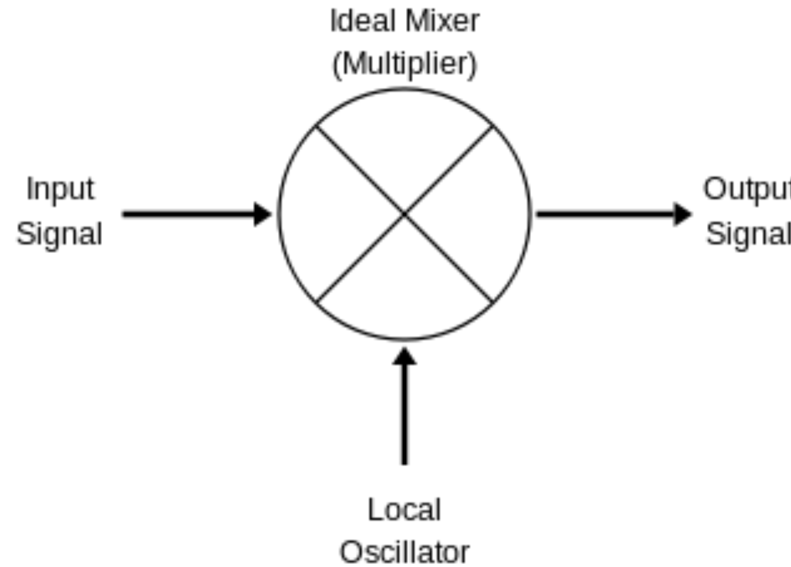
Can only distinguish $f_{samp}/2$ total bandwidth.

Adequately Sampled Signal

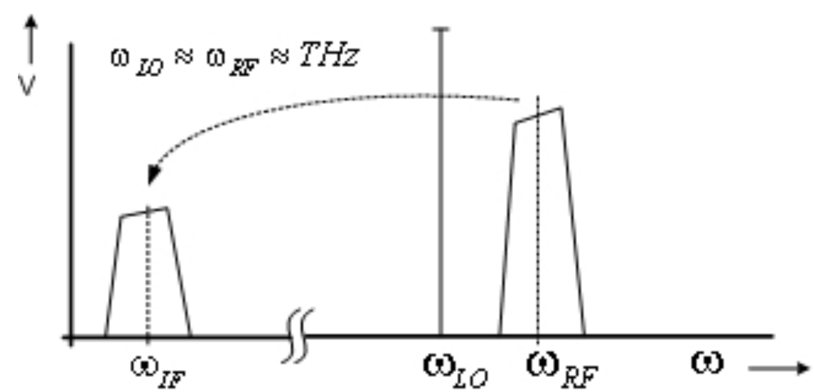
Aliased Signal Due to Undersampling

Mixers

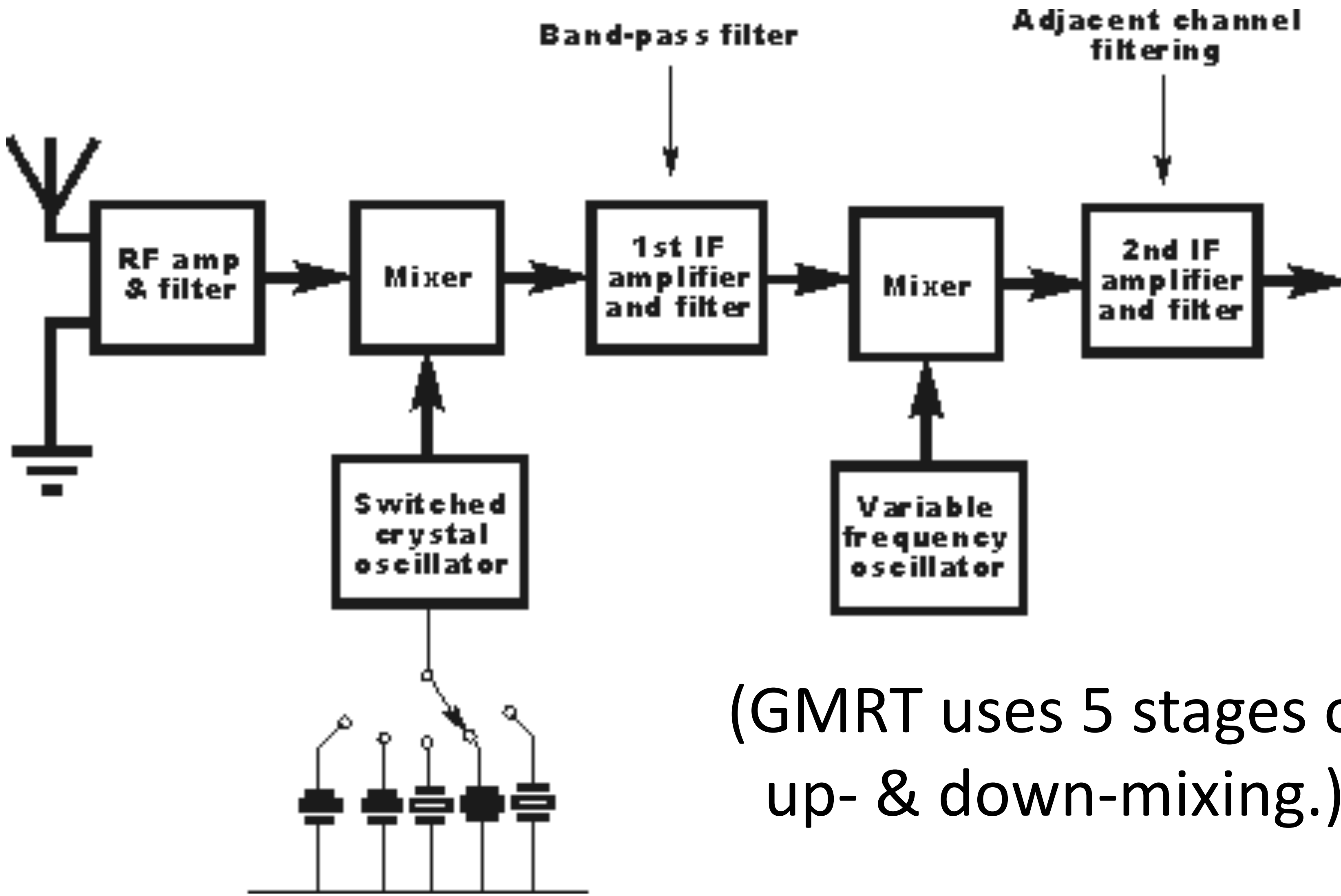




$$\sin(\omega_1 t+\phi_1)\,\sin(\omega_2 t+\phi_2) = \tfrac{1}{2}\left[\cos((\omega_1-\omega_2)t + (\phi_1-\phi_2)) - \cos((\omega_1+\omega_2)t + (\phi_1+\phi_2))\right]$$

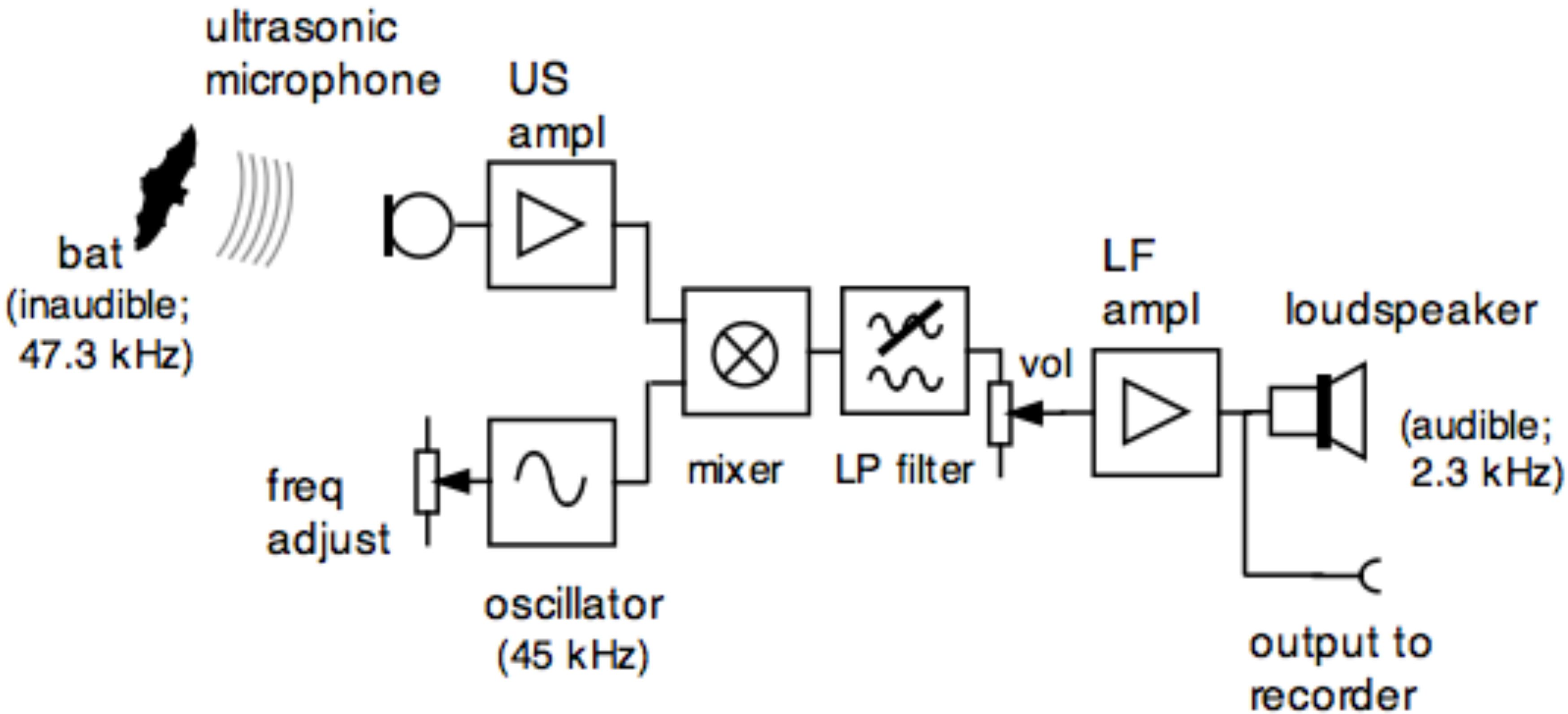


Heterodyne Detectors



(GMRT uses 5 stages of up- & down-mixing.)

Heterodyne Bat Detector



Design Problems!

I want to record 120-150MHz as efficiently as possible.
What mixers and sampling rate do I use?

30 MHz Bandwidth → 60 MSps
120-150MHz band → LO at 120 MHz, 150MHz, or 90MHz.

I want to record 20-220MHz as efficiently as possible.
What mixers and sampling rate do I use?

200 MHz Bandwidth → 400 MSps
20-220MHz band → LO at 220MHz.

Digital Light

After digitization, do whatever you want to the light.

For example:

- **Channelize**: Fourier Transform to split frequencies
  (Arbitrarily fine $\Delta\nu$ resolution, no need for prisms.)

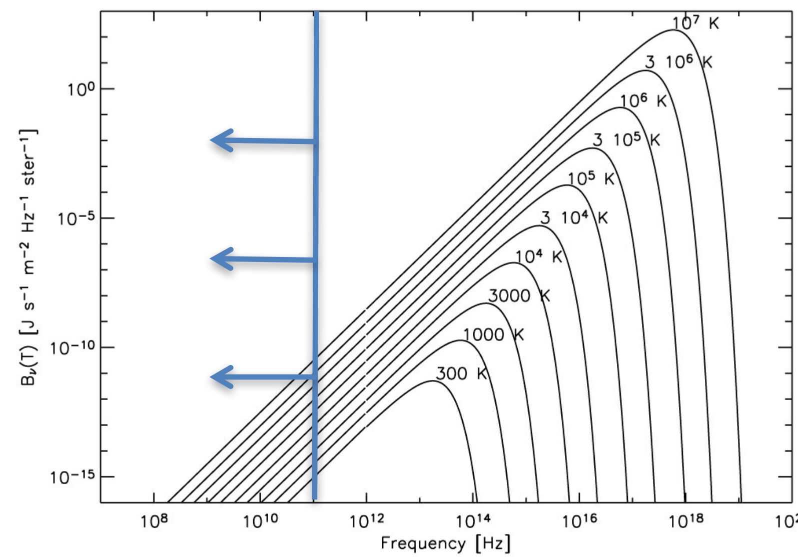
- **Integrate**: sum up incoming power $|E|^2$
  (Equivalent to a thermal detection.)

- **Correlate**: combine the light from faraway antennas
  (More if you ever study interferometry.)

RJ Temperature

For radio & mm, usually kT >> hν, so we're in Rayleigh-Jeans regime.

$$B_\nu(T) = \frac{2\nu^2 kT}{c^2}$$



Within a band, blackbody power is *linearly* related to temperature.

$$\boxed{P = kT\Delta\nu}$$

(Also means thermal noise from our instrument T_{rcvr} is significant.)

"Temperature" is a frequent unit of signal & noise power.

System Temperature

We ***want*** to measure $\boxed{P_{source} = kT_{source}\Delta\nu}$

We ***can*** measure the total system power:

$$T_{\mathrm{sys}} = T_{\mathrm{cmb}} + \Delta T_{\mathrm{source}} + T_{\mathrm{atm}} + T_{\mathrm{spillover}} + T_{\mathrm{rcvr}} + \ \ldots$$

Includes all power before and after the antenna, including amplifiers, cables, pickup, crosstalk, ADCs, etc.
At low frequencies, *everything* is glowing thermally.

How do we get T_{source}?

Radiometer Equation

Low-energy detectors don't have "read noise" in electrons.
They measure light in your waveguide.

$$T_{\rm sys} = T_{\rm cmb} + \Delta T_{\rm source} + T_{\rm atm} + T_{\rm spillover} + T_{\rm rcvr} + \ \ldots$$

Bandwidth ($\Delta\nu$) and time (τ) give independent measurements, and the ***Radiometer Equation*** gives fundamental noise:

$$\boxed{\boldsymbol{N} \approx \frac{T_{\rm sys}}{\sqrt{\Delta\nu_{\rm RF}\tau}}}$$

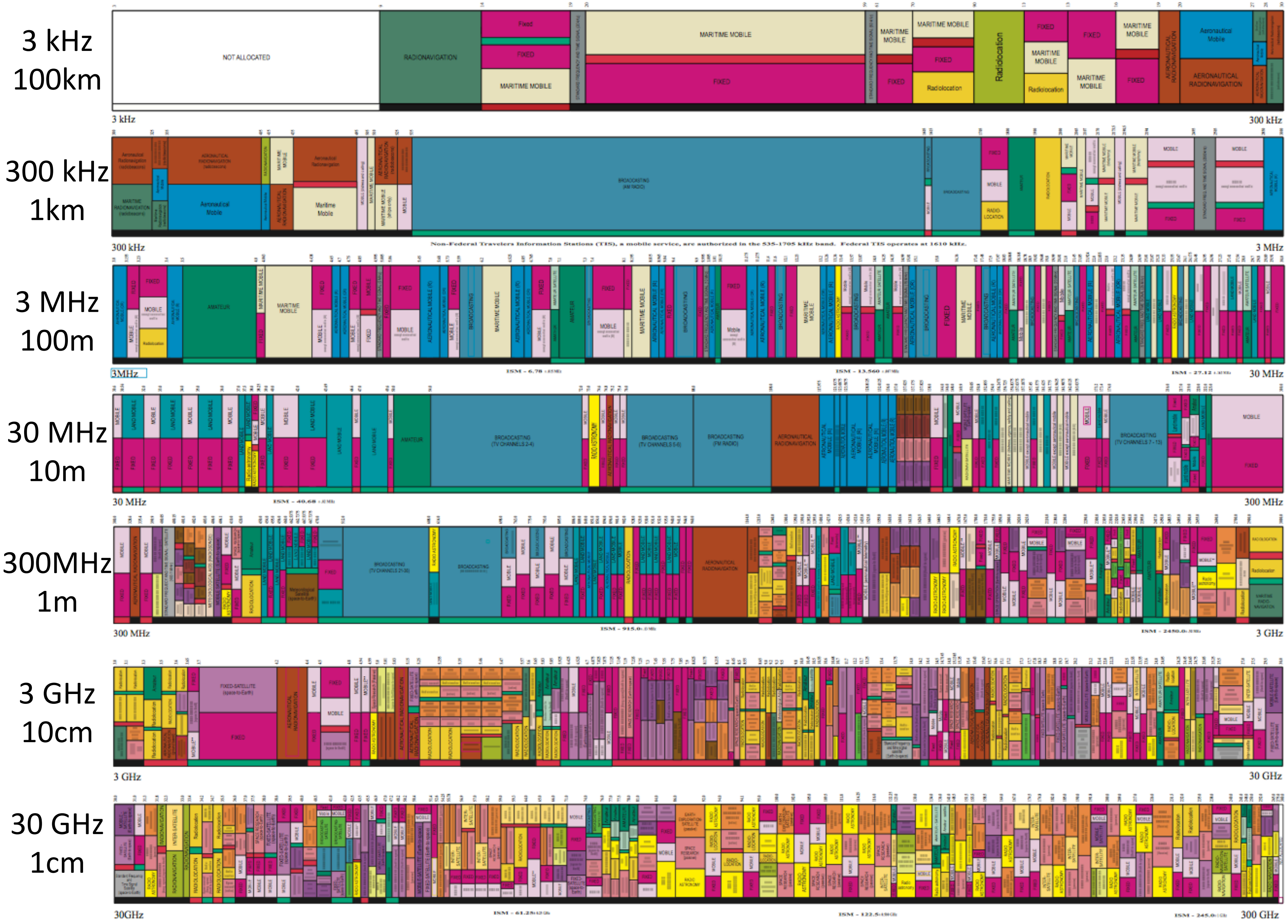
Note that ***ALL*** power in the system adds to the noise.
The goal for a low-noise system is to minimize power.

Radio Frequency Interference (RFI)

Light pollution at long wavelengths.

- AM Radio
- FM Radio
- Broadcast TV
- RADAR
- Cell Phones
- CB Radio
- Electronics
- Etc…

United States RF Spectrum Allocation



How bad is it?

How hot are the FM broadcasts from the CN tower?

Wikipedia

Frequency	kW	Callsign[46]	Branding
91.1 MHz	40	CJRT	JAZZ.FM91
94.1 MHz	38	CBL	CBC Radio 2
96.3 MHz	38	CFMZ	Classical 96
97.3 MHz	28.9	CHBM	boom 97.3
98.1 MHz	44	CHFI	98.1 CHFI
99.9 MHz	40	CKFM	Virgin Radio 99.9FM
100.7 MHz	4	CHIN	CHIN Radio
102.1 MHz	35	CFNY	102.1 the Edge
104.5 MHz	40	CHUM	104.5 CHUM FM
107.1 MHz	40	CILQ	Classic Rock Q 107

Wikipedia

Total: 350 kW

Back-of-the-Envelope!

What is the flux (in Jy) of the 350kW of FM radio coming off the CN tower at UofT? Assume an isotropic transmitter.

Bandwidth of FM: 20 MHz
Distance to the CN: ≈2km
$1\text{Jy} = 10^{-26}$ W/m^2/Hz

$S = 350\text{kW} / (4\pi \times (2\text{km})^2) / 20\text{MHz}$
$= 3 \times 10^{-10}$ W/m^2/Hz
$= 3 \times 10^{16}$ Jy = 30 PJy

Bright celestial sources are of order 1 Jy.
The brightest celestial source is Cassiopeia A, about 10^5 Jy at 100MHz.
The CN tower is ***BRIGHT***.

Wikipedia

INTERMISSION

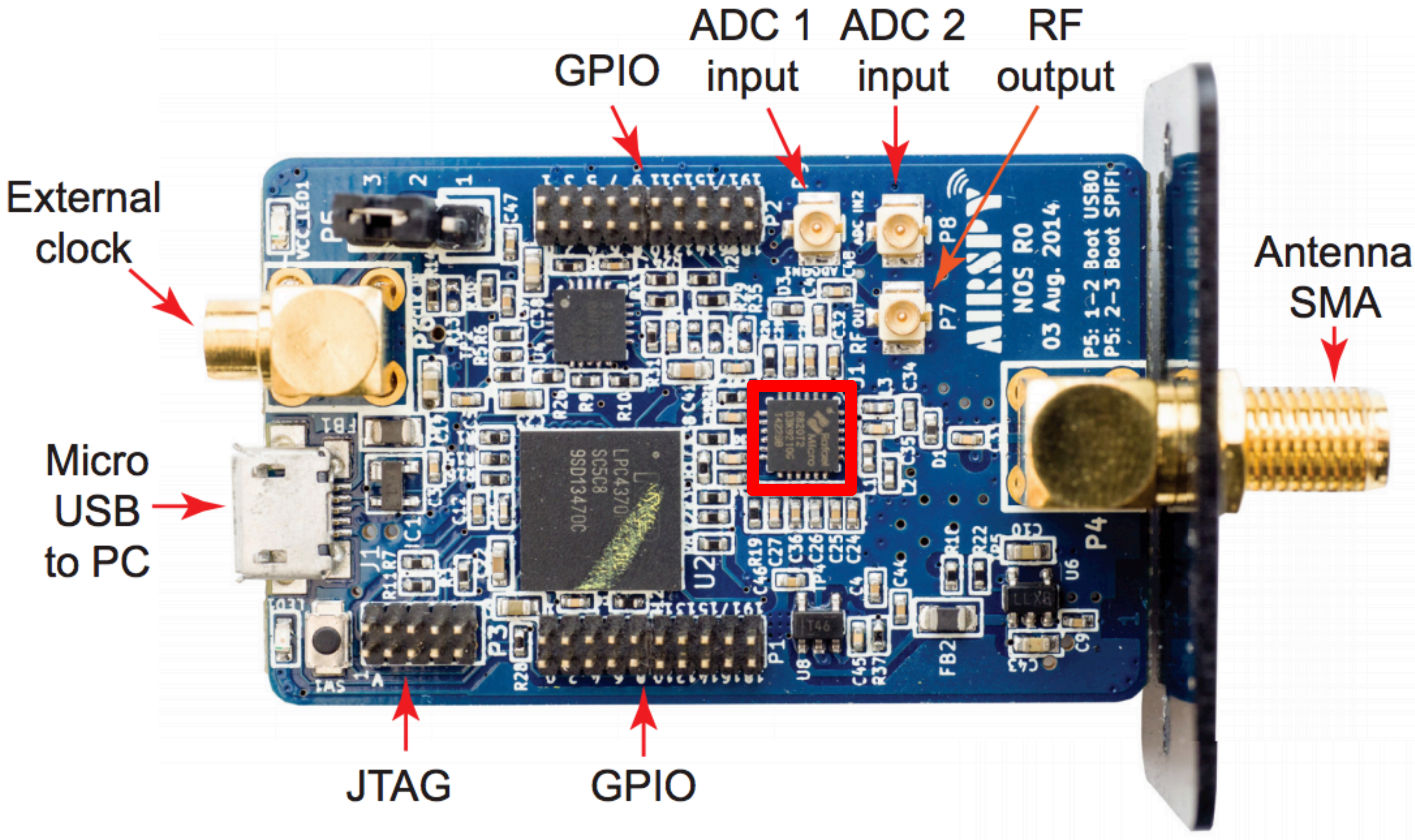
Lab #2

Radiometers & Thermal Noise

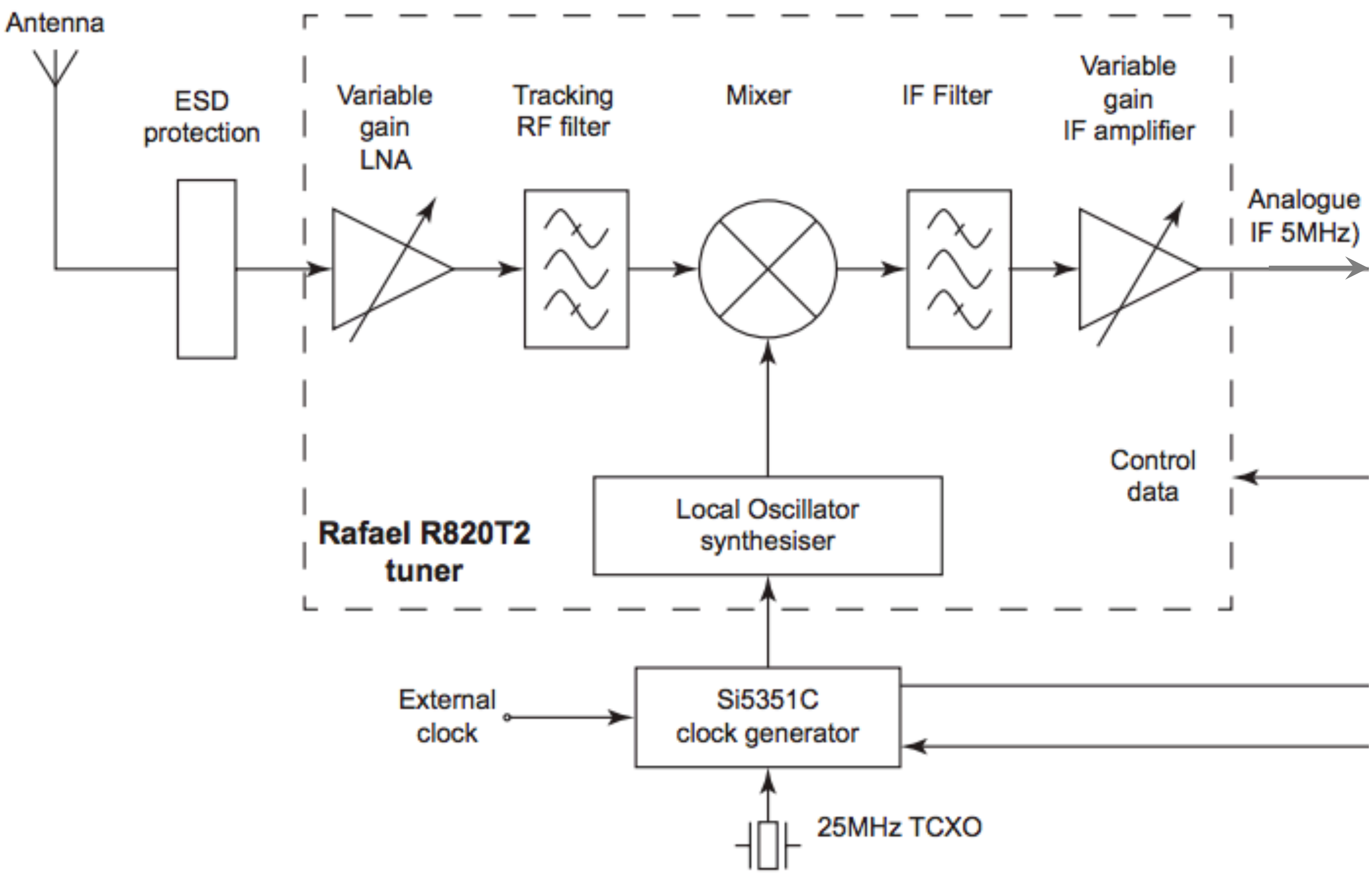
Part I: Receivers, Stats & Spectra

The AirSpy Receiver

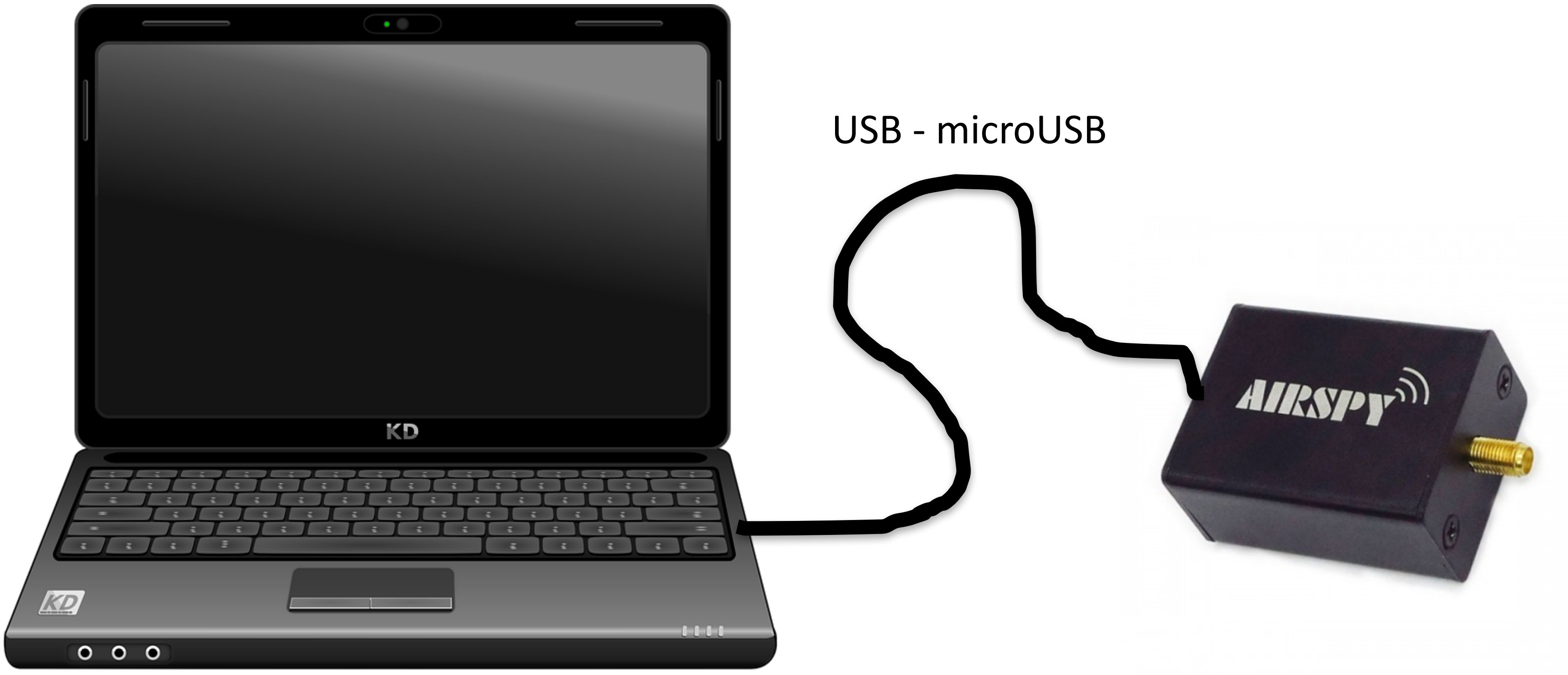
The AirSpy Receiver



The AirSpy Receiver

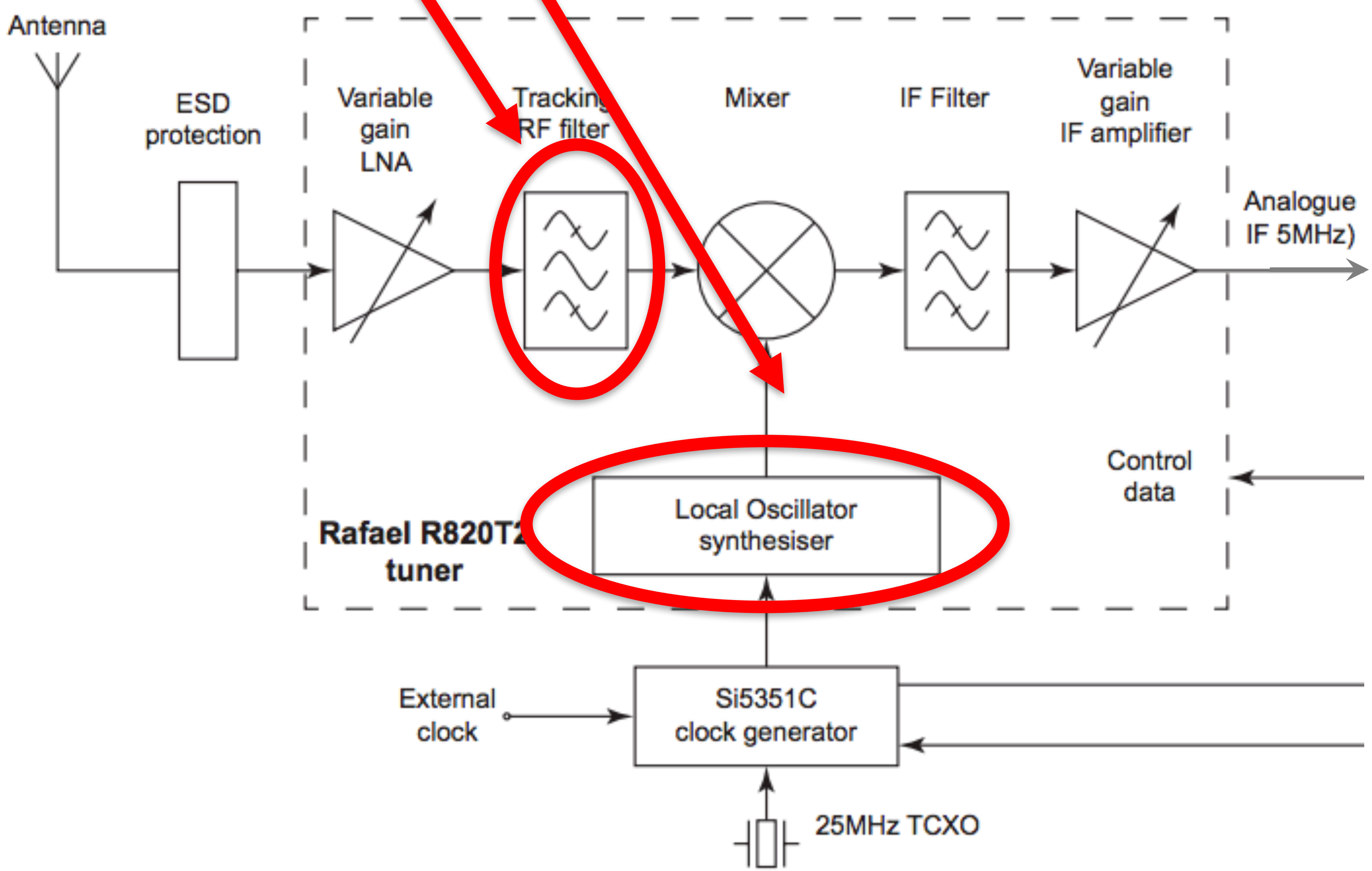


```
airspy_rx –f 1000 –a 1 –t 4 –v 15 –m 15 -l 14
–n 1000000 –d –r my_data.dat
```



IMPORTANT: RECORD DATA INTO /tmp ON THE LAPTOPS! YOUR HOME DIRECTORY WILL NOT WORK. You can copy it over for posterity once you have a good data set.

```
airspy_rx –f 1000 –a 1 –t 4 –v 15 –m 15 -l 14
      –n 1000000 –d –r my_data.dat
```



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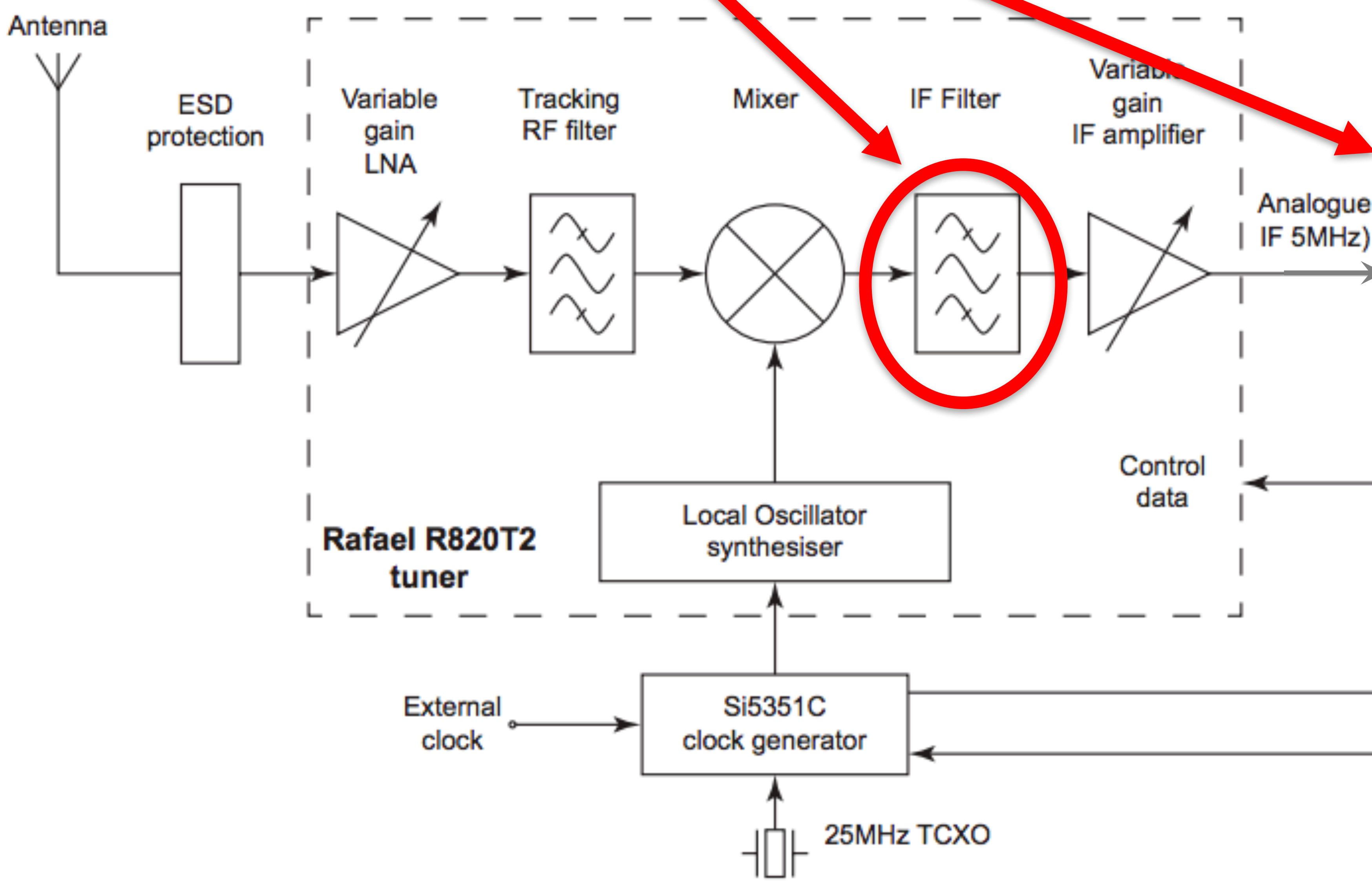

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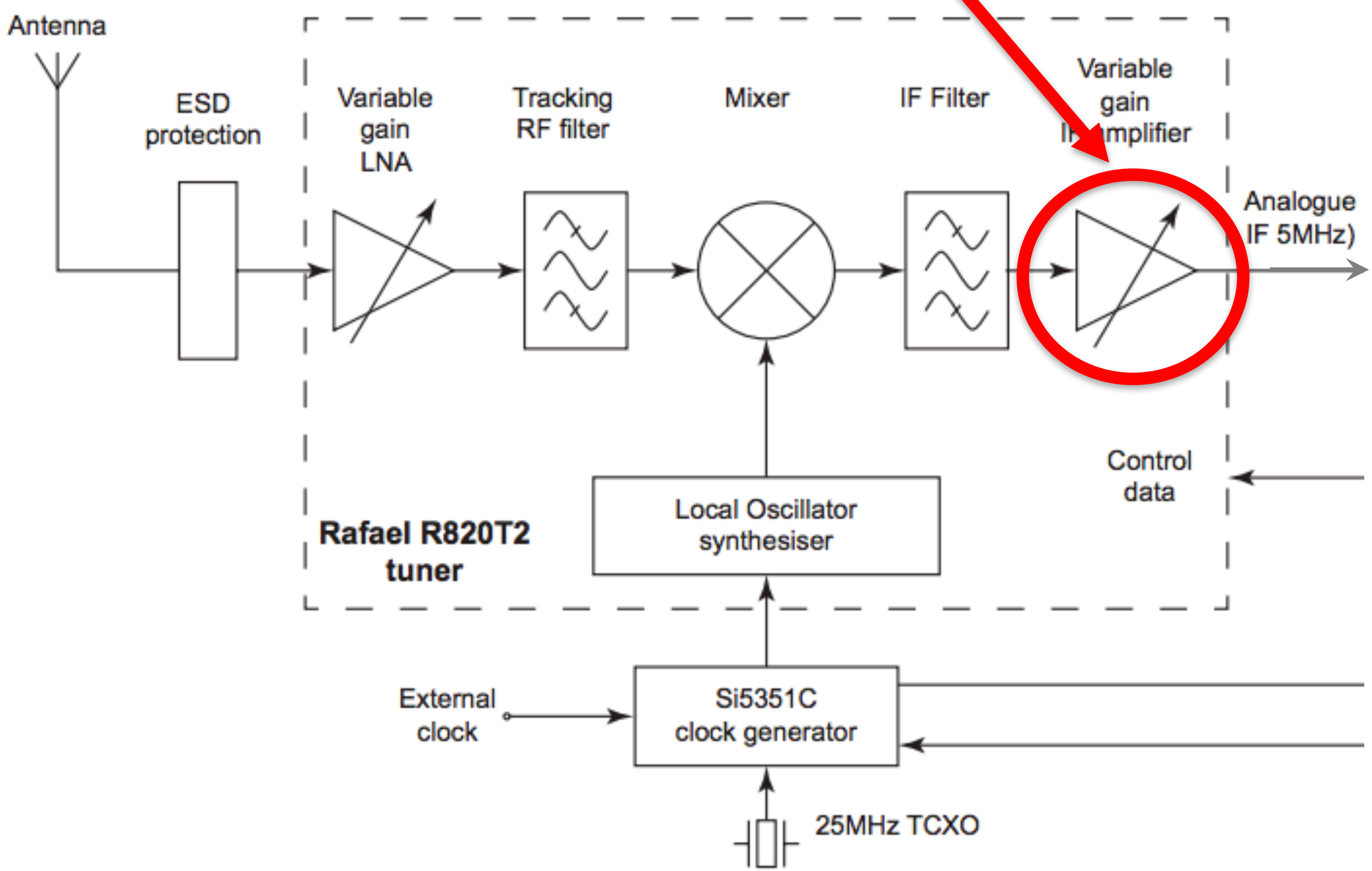

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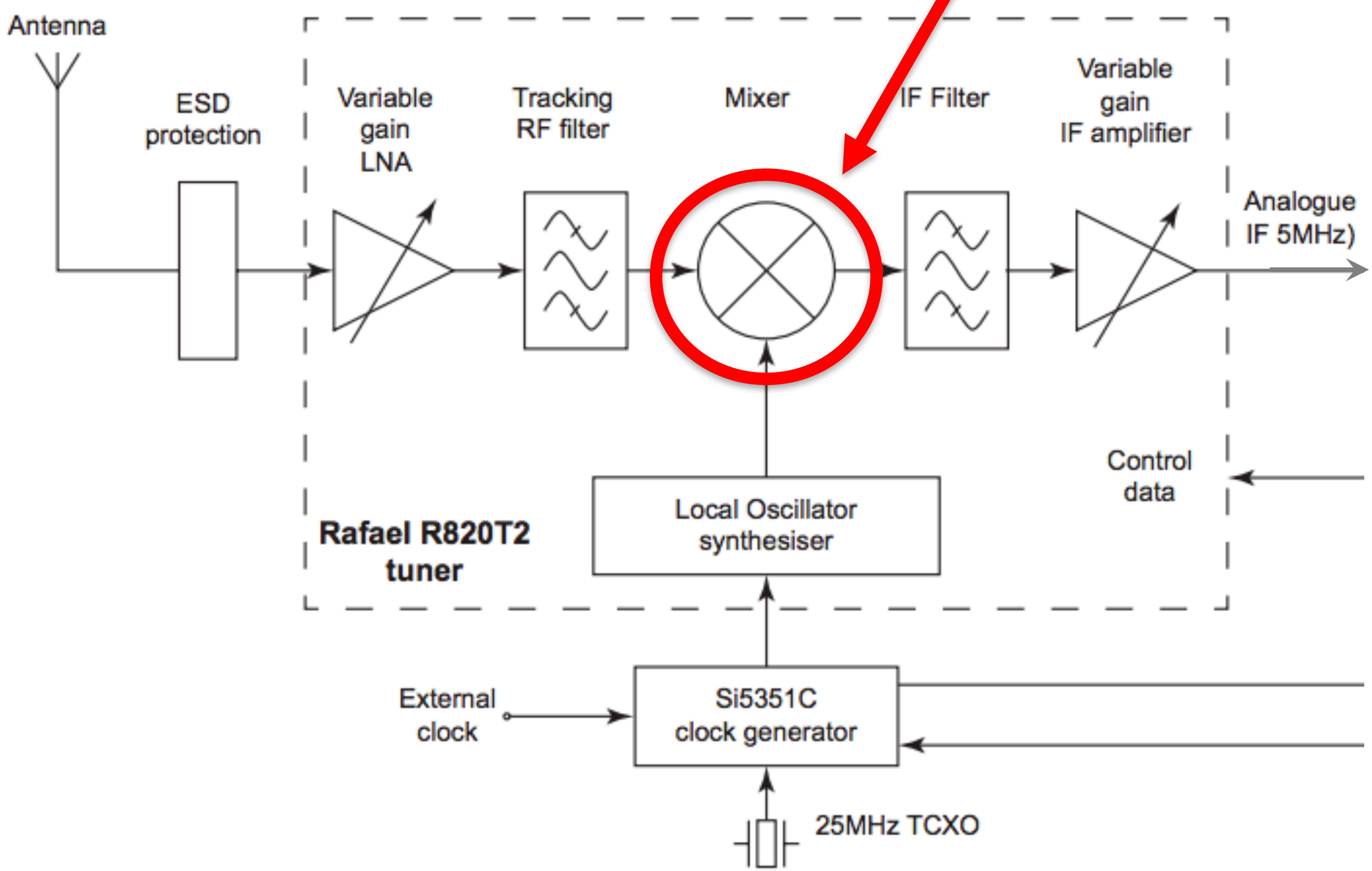

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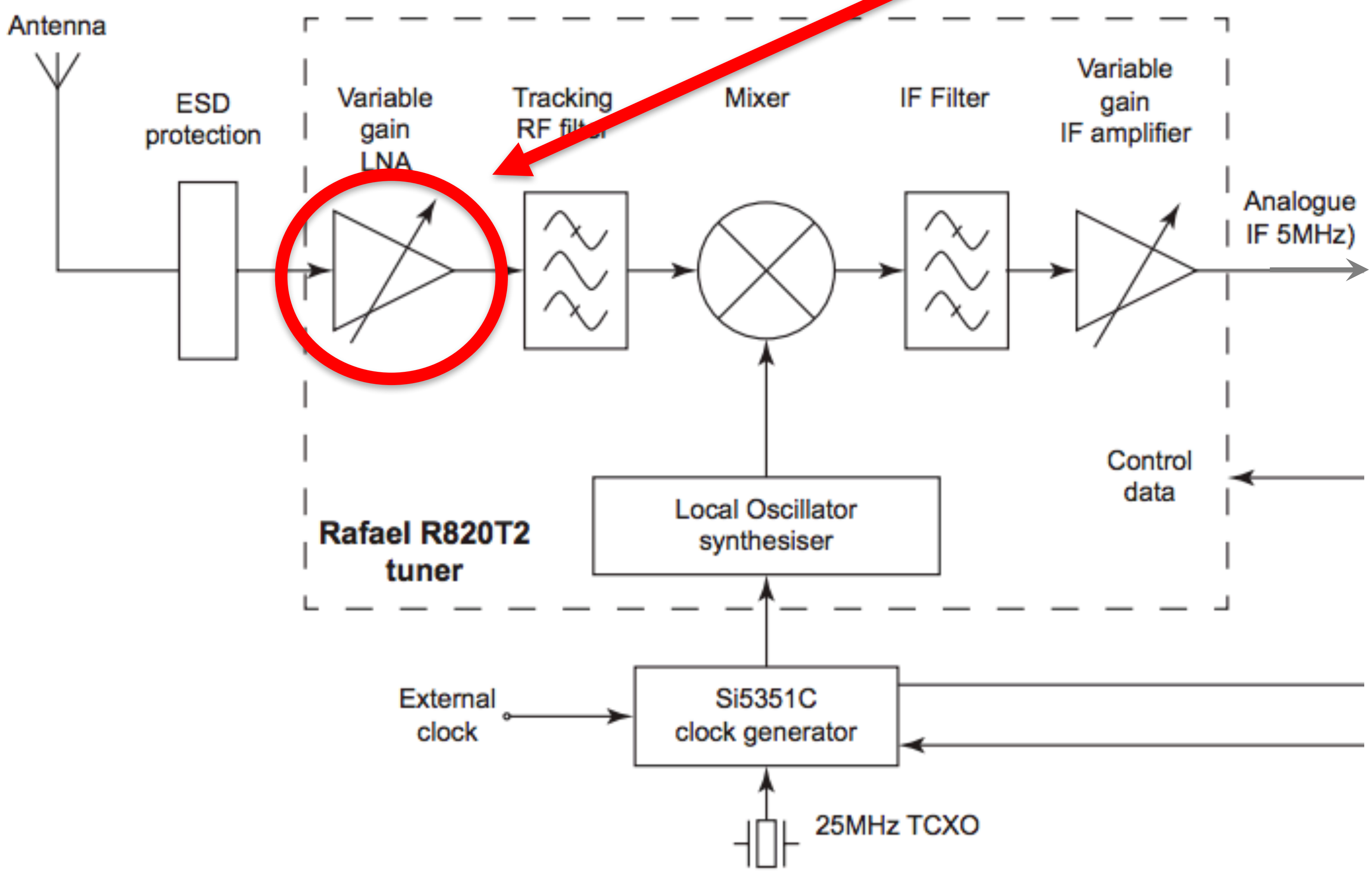

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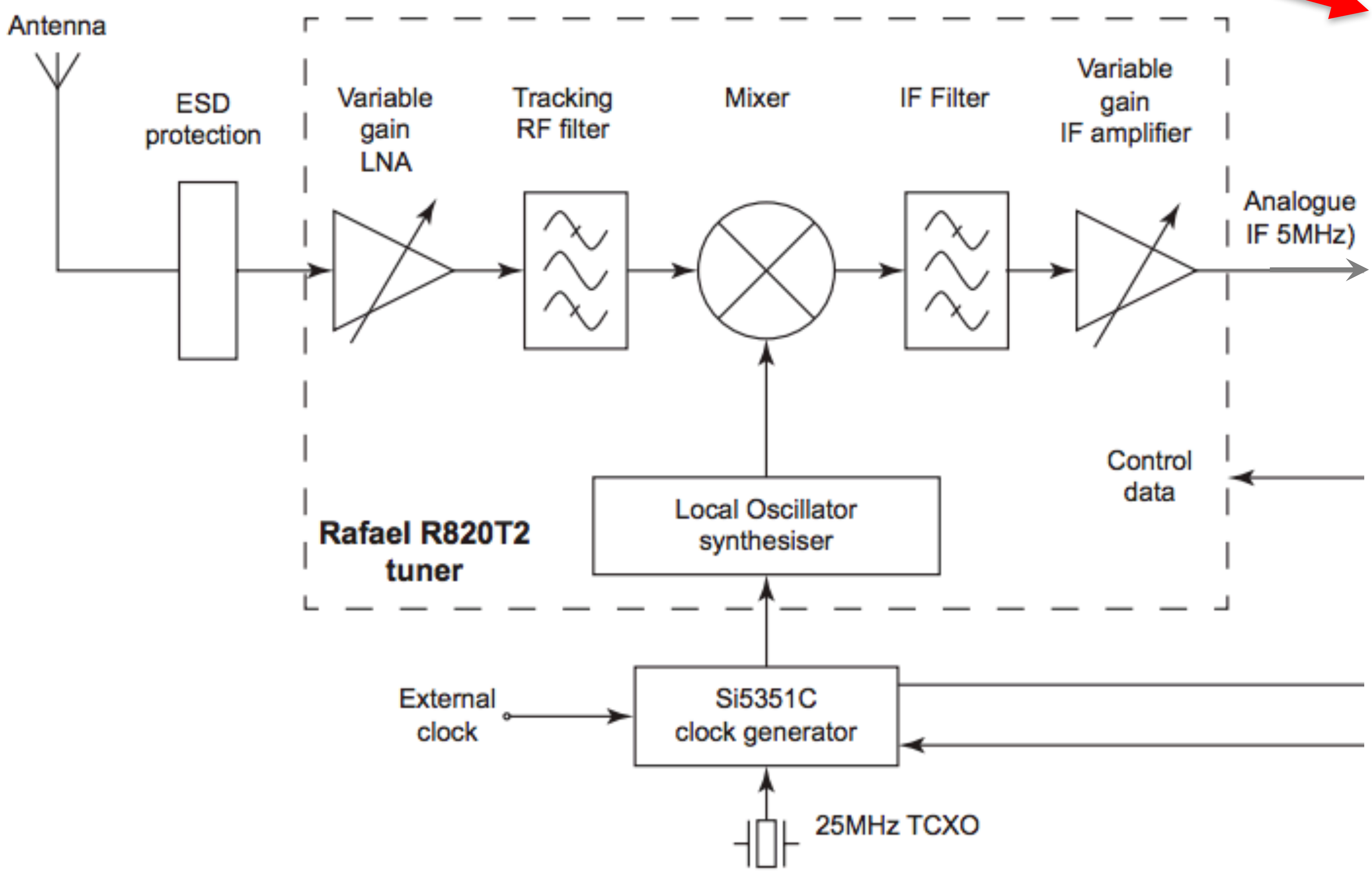

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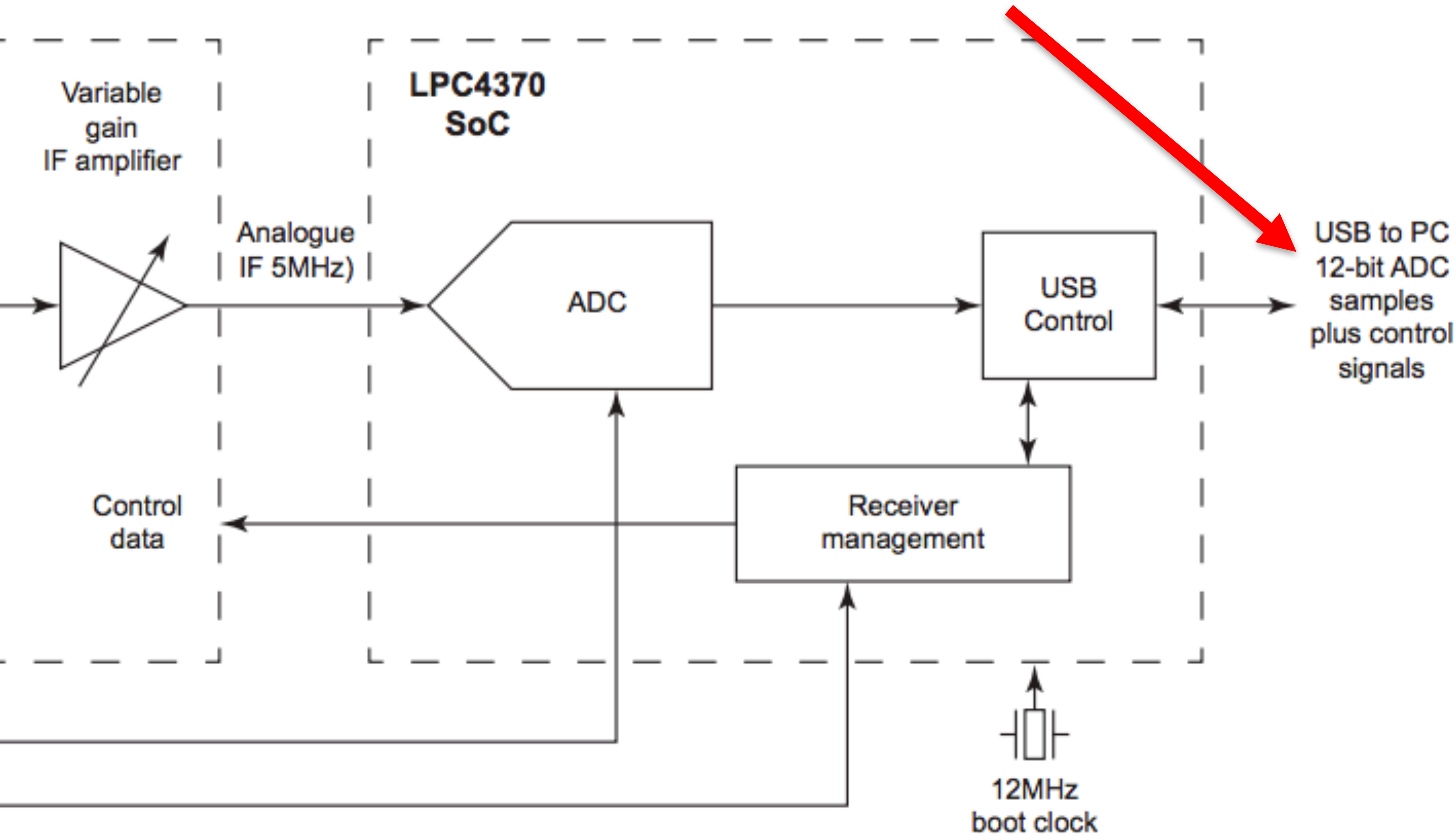

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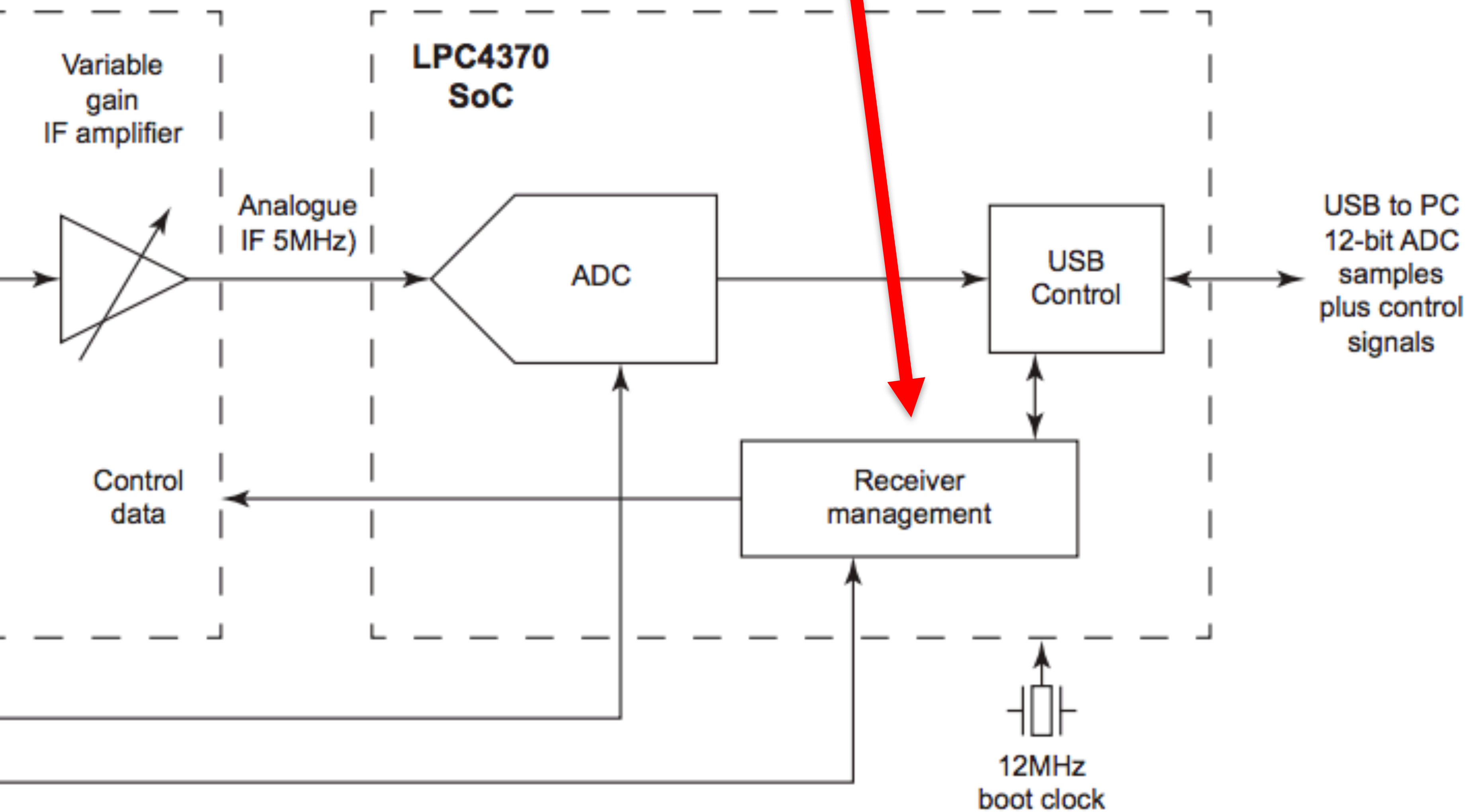
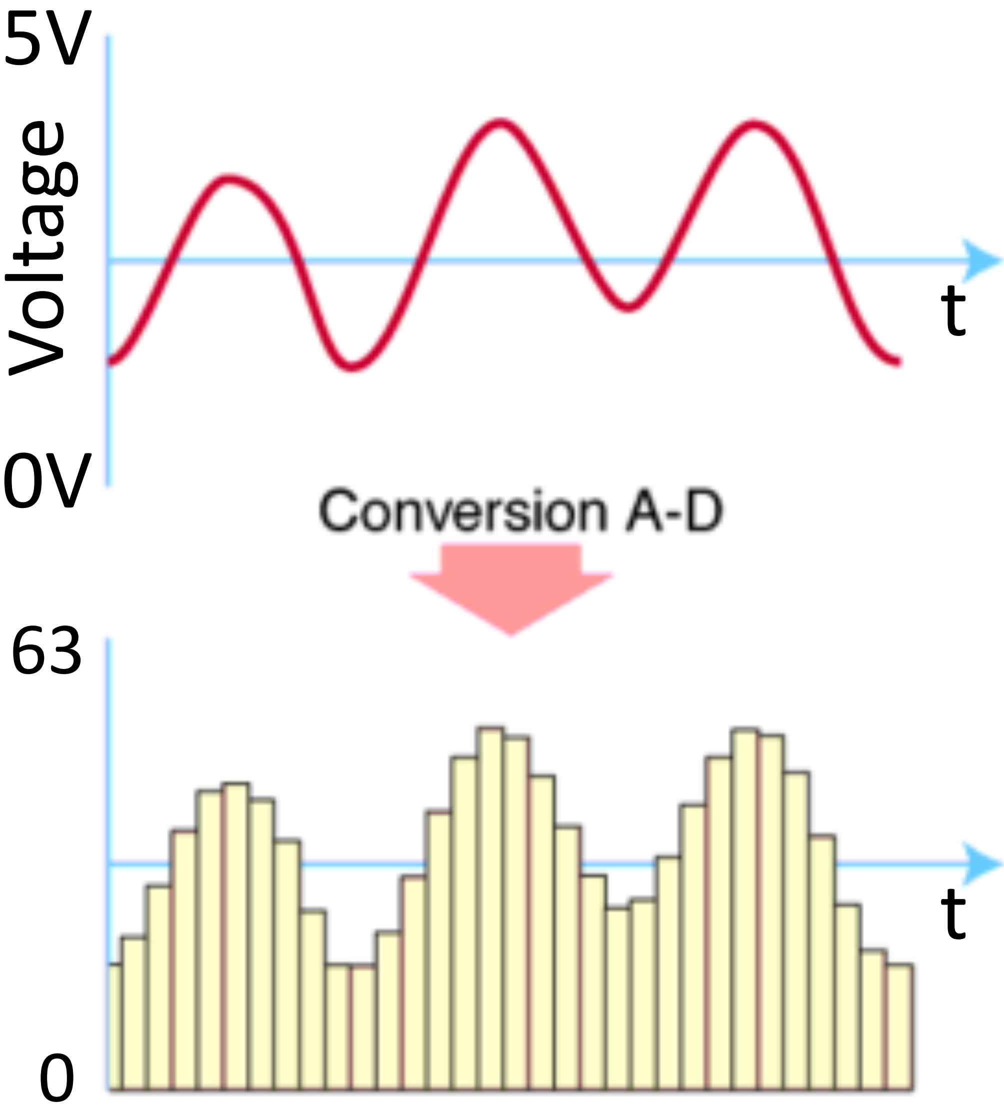
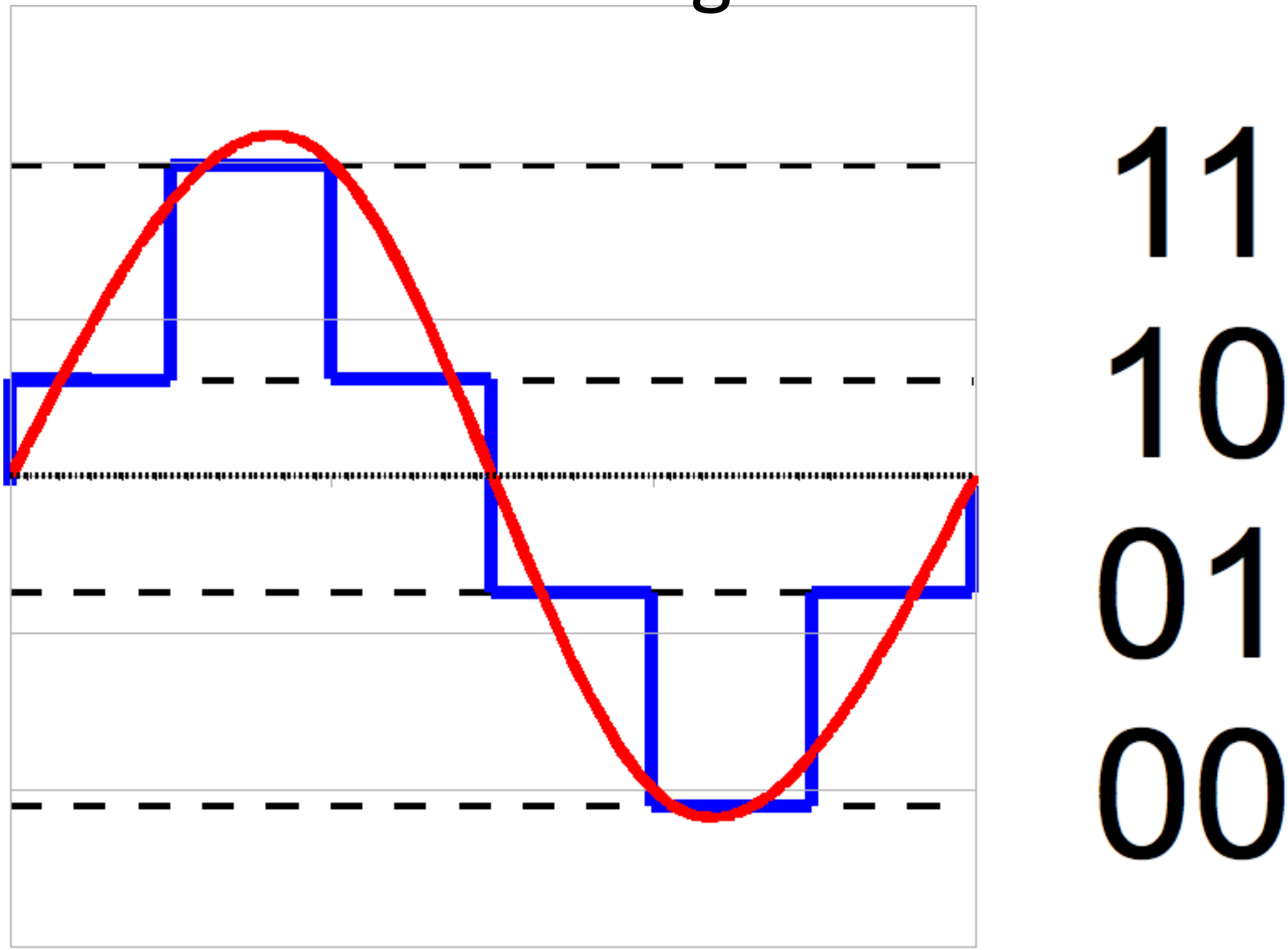


ADCs Revisited

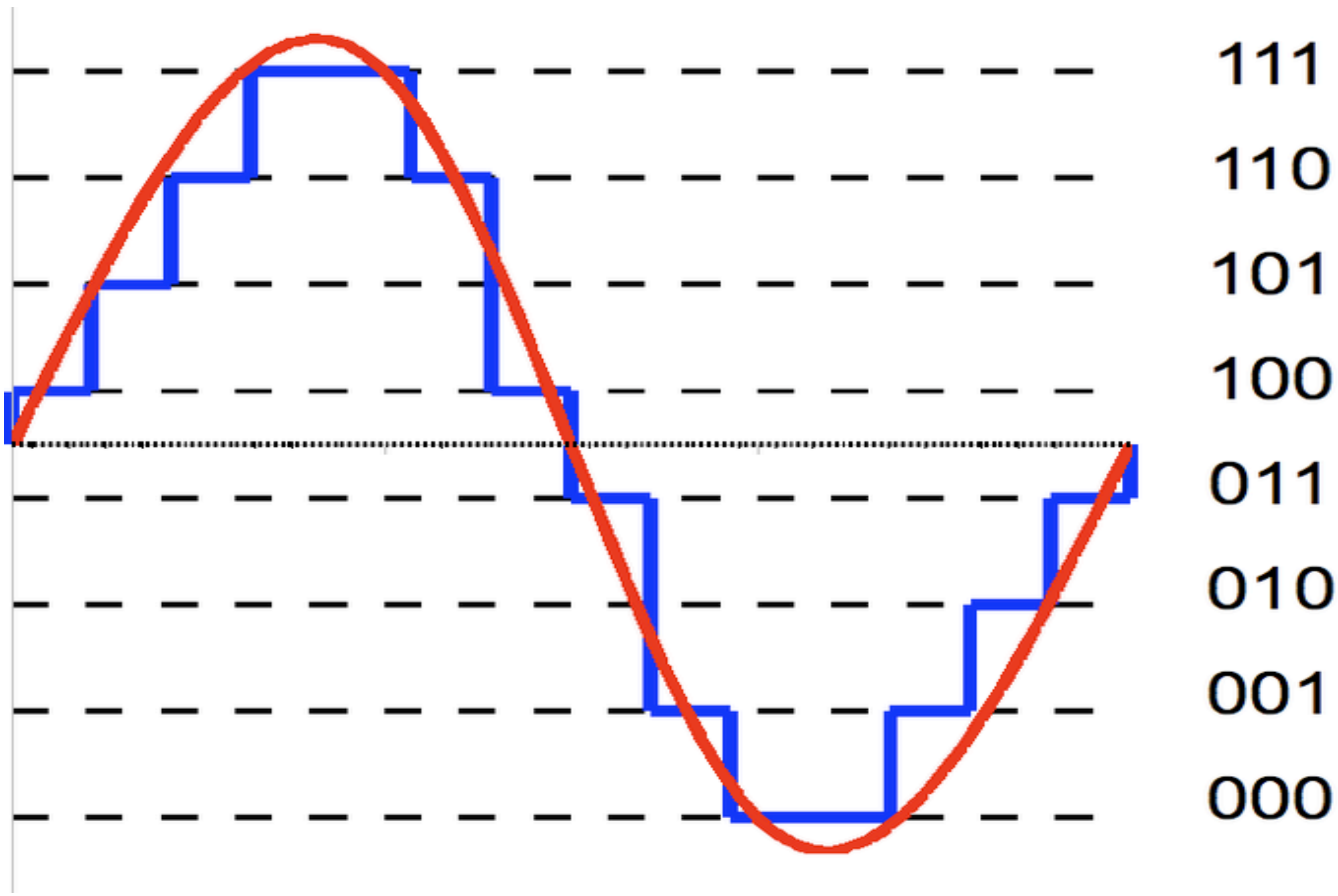


- Measure Voltage, express in bits.
- Repeat after some time, typically ≈ms.
- Used everywhere: thermometers, motors, magnetic fields, etc.
- Radio just does it faster.

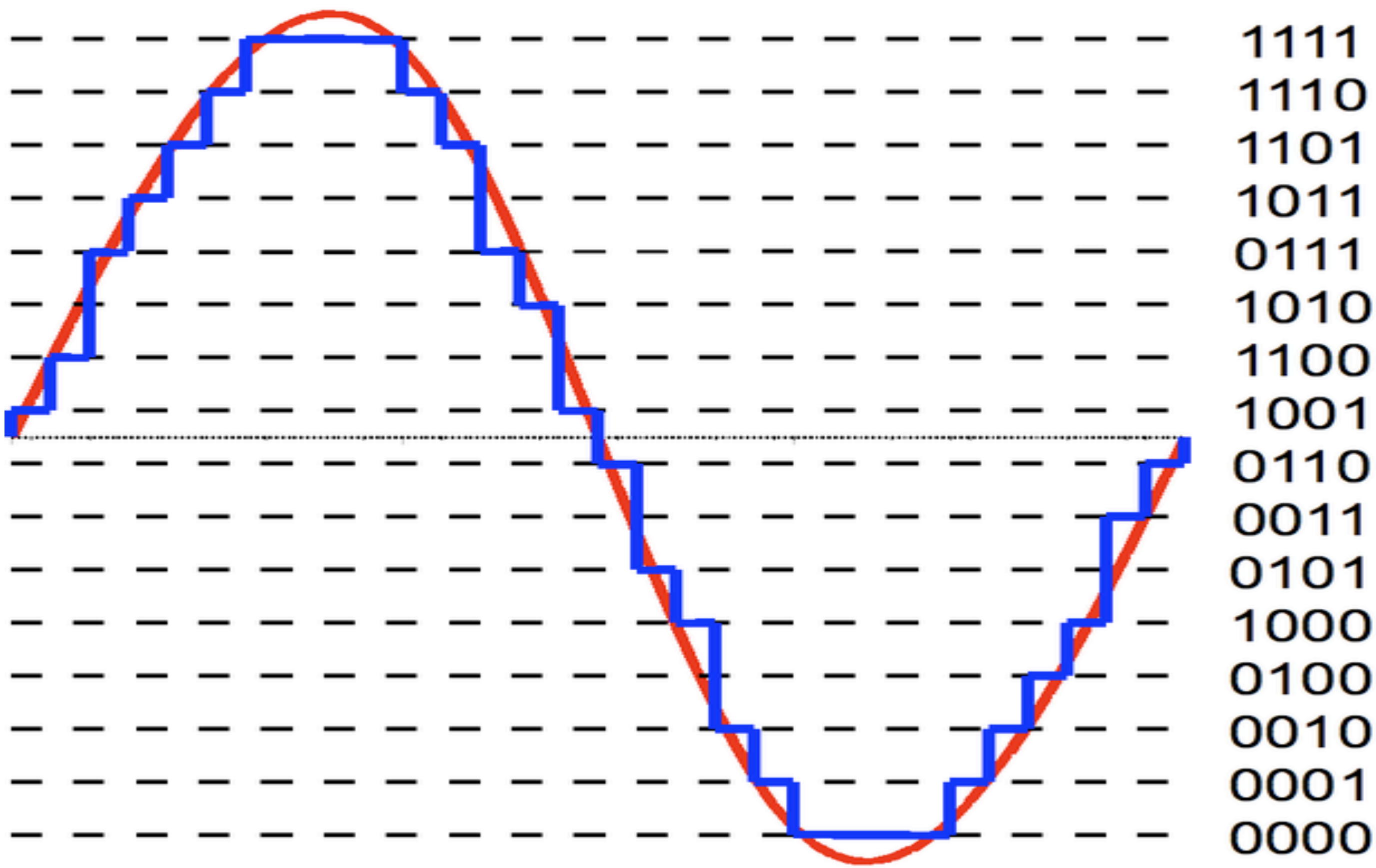
ADCs Revisited
2-bit Digitizer



ADCs Revisited
3-bit Digitizer



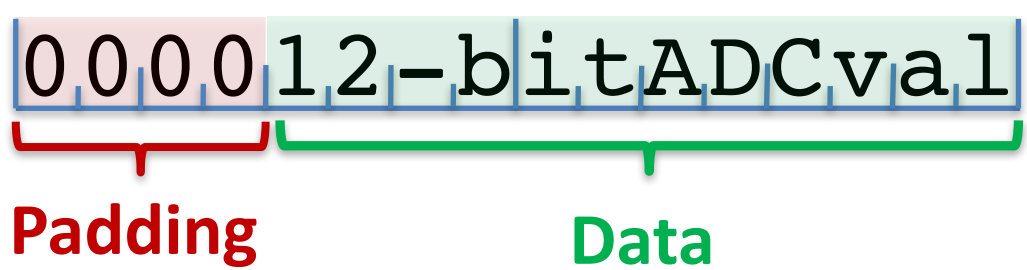
ADCs Revisited
4-bit Digitizer



```
airspy_rx –f 1000 –a 1 –t 4 –v 15 –m 15 -l 14
–n 1000000 –d –r my_data.dat
```

my_data.dat

- AirSpy uses 12-bit ADC.
- PCs usually deal with 8bit = 1Byte chunks.
- AirSpy packs its 12b numbers into 16b = 2B, so PCs can handle more easily.



```
airspy_rx –f 1000 –a 1 –t 4 –v 15 –m 15 -l 14
–n 1000000 –d –r my_data.dat
```

`my_data.dat`

- AirSpy ADC is symmetric: 0V is half-way between min and max values.
- 12-bit range: 0-4095
- 0V $\leftarrow\rightarrow$ $2^{11} = 2048$
- To get voltage in bits, need

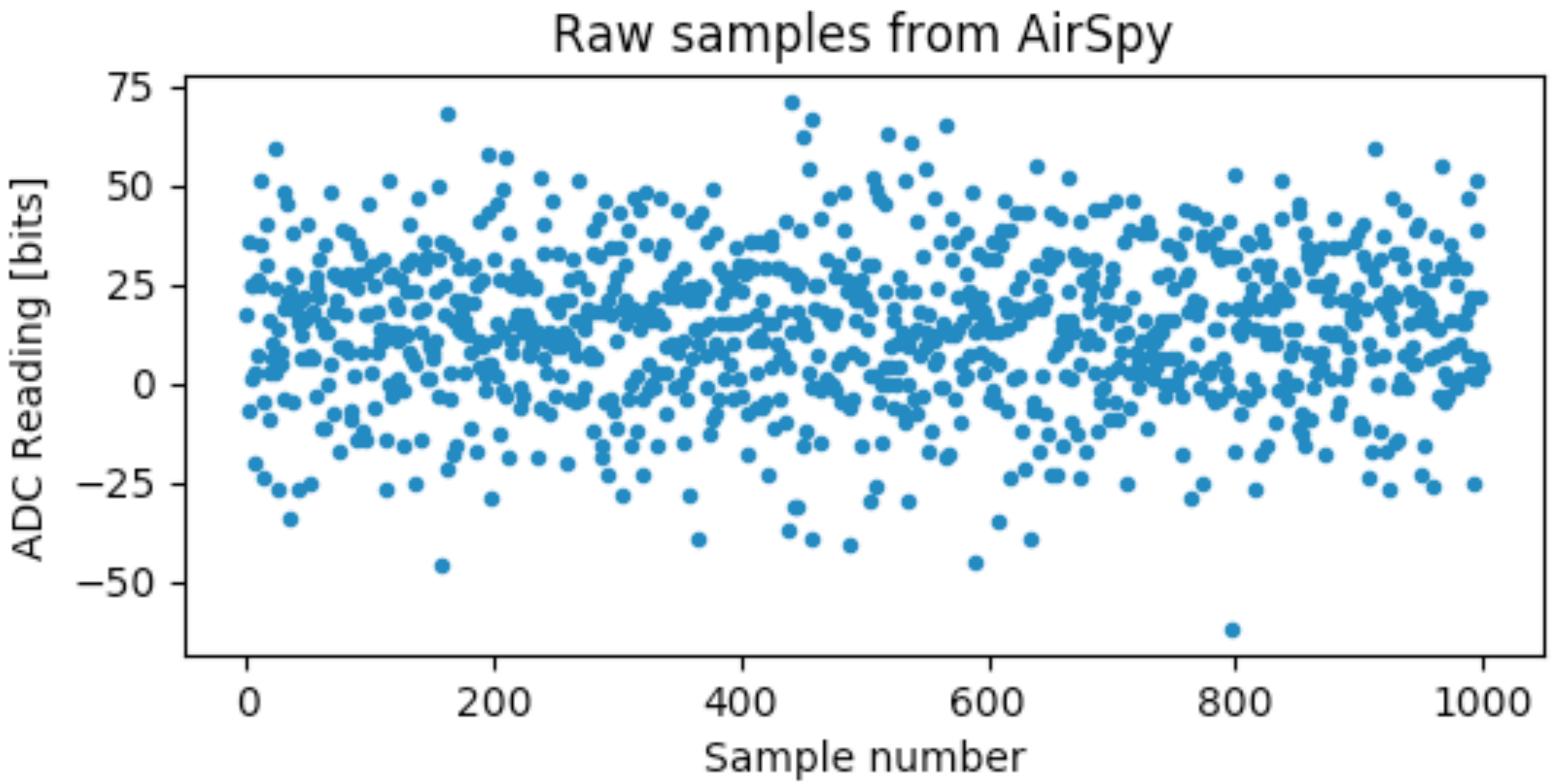
  $$\mathtt{ADCval} - 2^{11}$$

```
airspy_rx –f 1000 –a 1 –t 4 –v 15 –m 15 -l 14
–n 1000000 –d –r my_data.dat
```

my_data.dat

```
In [1]: import numpy as np

In [2]: data =
np.fromfile('my_data.dat',
dtype='int16') - 2**11;

In [3]: data.shape
Out[3]:(1000000,)

In [4]: data.mean()
Out[4]: 13.603
```

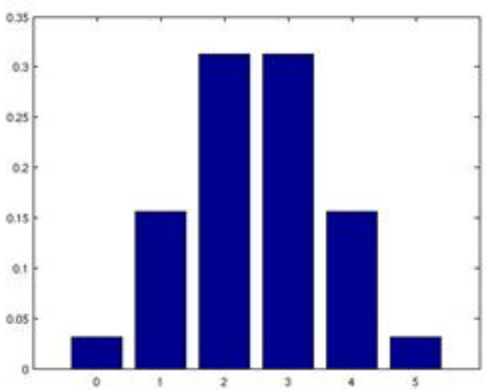
```
In [5]: import matplotlib.pyplot as plt

In [6]: plt.plot(data[10000,11000],'.');
In [7]: plt.xlabel('Sample number');
In [8]: plt.ylabel('ADC Reading [bits]');
In [9]: plt.title('Raw samples from AirSpy');
In [10]: plt.show();
```



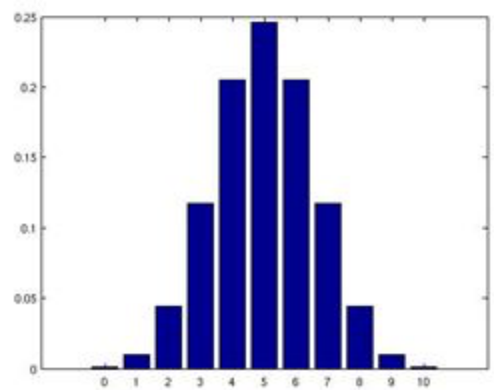
Central Limit Theorem

For **any** underlying probability distribution, the distribution of a sum of realizations tends toward Gaussian as the number becomes large.

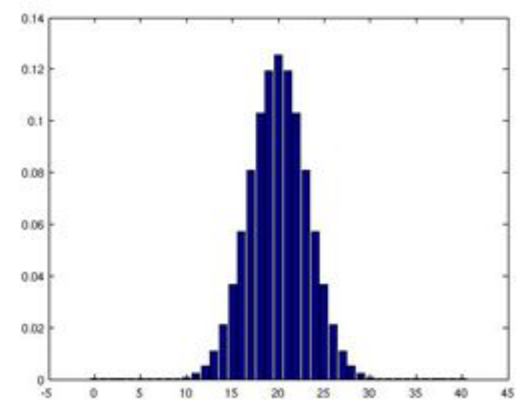
Example: (fair) coin flip. heads=0, tails=1.



N = 5



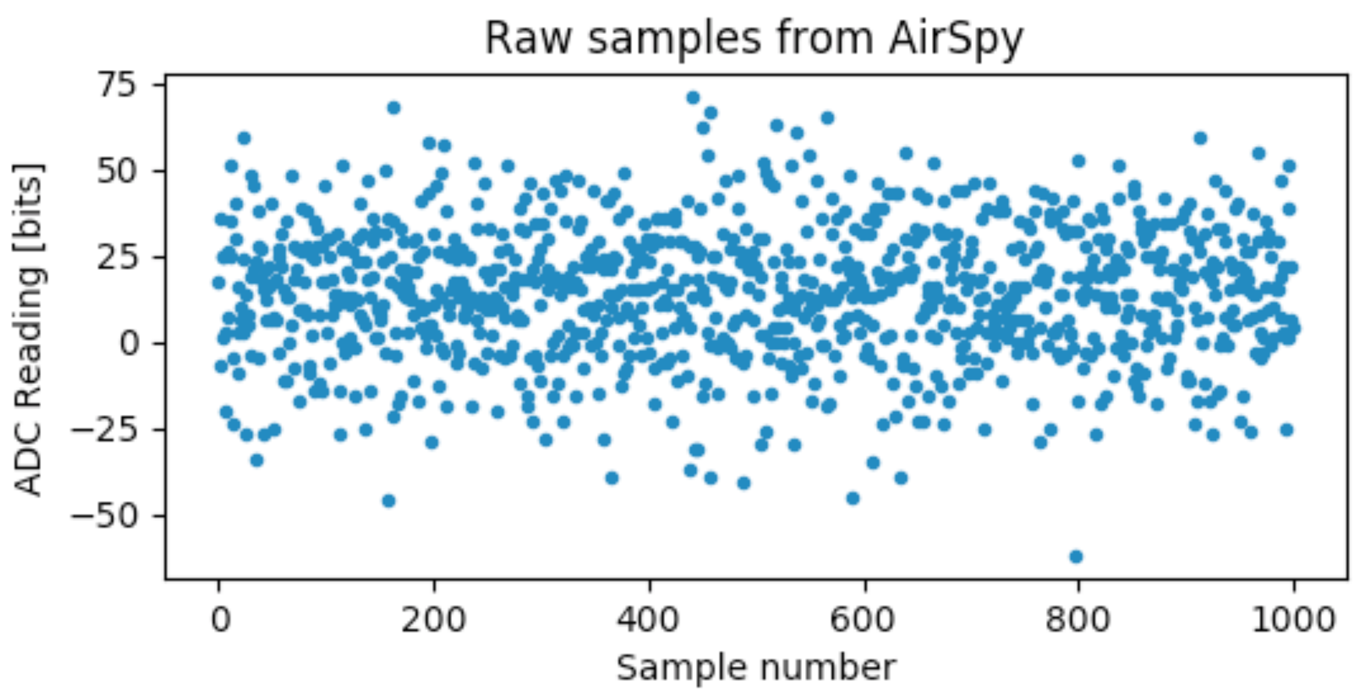
N = 10



N = 40

Normal 'ol EM Waves

- Most (natural) things that generate radio photons are complicated, driven by QM and thermal processes.
- CLT: net effect will be a normally distributed E-field.
- Average and deviation may vary with frequency, time, direction, polarization, ….
- CLT: sum still normal!



Temperature of a Radio Wave

$$\mathbf{T} \propto \mathbf{P} \propto <\mathbf{E}^2>$$

$$\begin{aligned} Var(E) &\equiv \langle (E - \langle E \rangle)^2 \rangle \\ &= \langle E^2 \rangle - \langle E \rangle^2 \\ &= \langle E^2 \rangle \\ &\propto T \end{aligned}$$

RJ temperature is the variance
of our coherent signal!

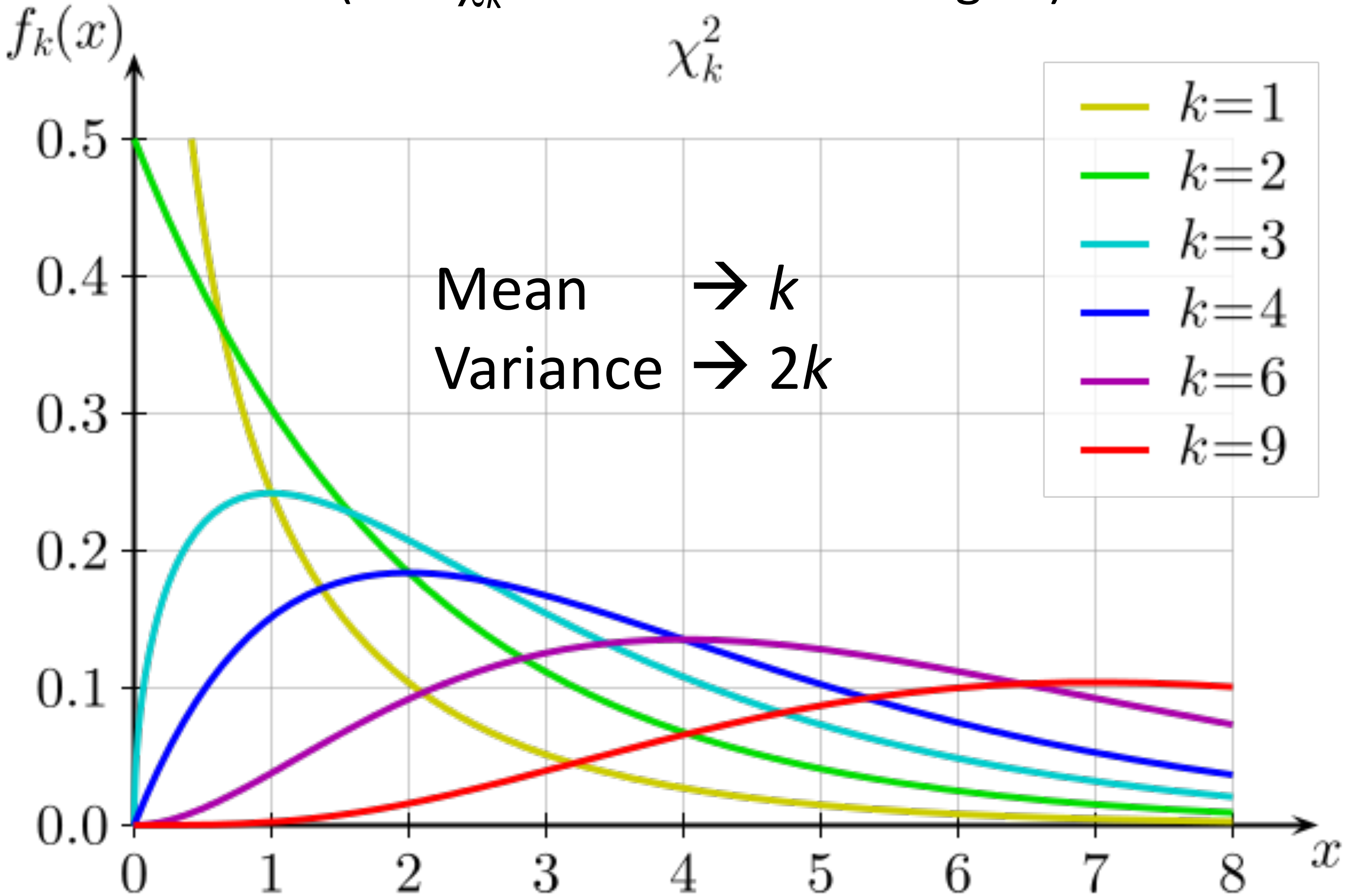
$$T_{\rm sys} = \cancel{T_{\rm cmb}} + \Delta T_{\rm source} + \cancel{T_{\rm atm}} + \cancel{T_{\rm spillover}} + T_{\rm rcvr} + \ \ldots$$

χ^2 Distributions

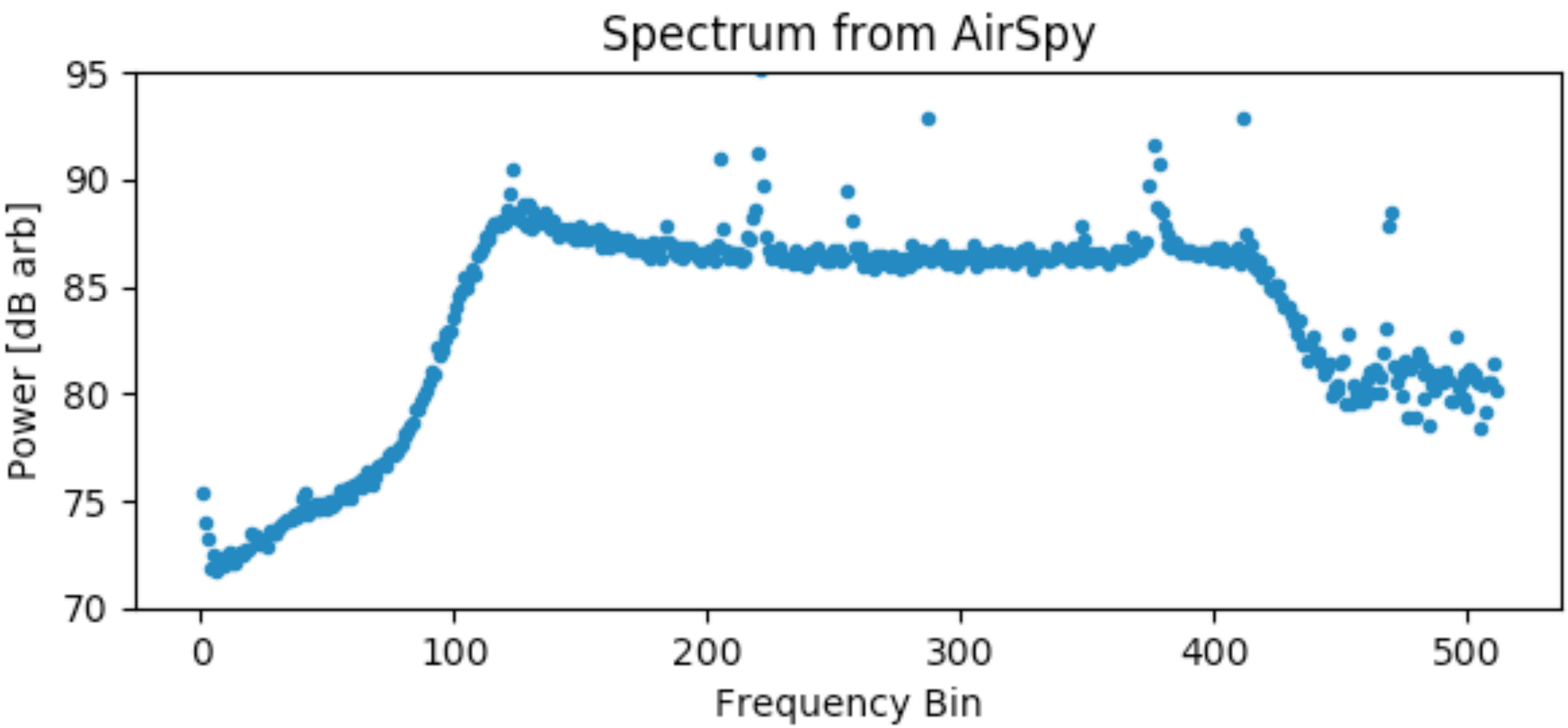
Sum-of-squares of *k* samples drawn from a normal distribution.

Come up frequently in error analysis!

(CLT: $\chi_k^2 \rightarrow$ Gaussian for large *k*)



```
In [11]: f=np.fft.fft(d[0:2**19].reshape(-1,1024),\
                          axis=1 );
In [12]: s=(f.real**2+f.imag**2).sum(axis=0);
In [13]: plt.plot(10*np.log10(s[0:512]), '.');
In [14]: plt.xlabel('Frequency Bin');
In [15]: plt.ylabel('Power [dB arb]');
In [16]: plt.title('Spectrum from AirSpy');
In [17]: plt.show();
```



Testing Normality

Low-energy detectors don't have "read noise" in electrons.
They measure light in your waveguide.

If the signal is Gaussian, the ***Radiometer Equation*** will hold:

$$\boxed{\boldsymbol{N} \approx \frac{T_{\mathrm{sys}}}{\sqrt{\Delta\nu_{\mathrm{RF}}\tau}}}$$

$$T_{\mathrm{sys}} = \cancel{T_{\mathrm{cmb}}} + \Delta T_{\mathrm{source}} + \cancel{T_{\mathrm{atm}}} + \cancel{T_{\mathrm{spillover}}} + T_{\mathrm{rcvr}} + \ \ldots$$

ALL power in the system adds to the noise.
You should check the relation in narrow or broad freq bands.

Good Luck!