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1. $f(x) = 3x(x^3-1)$

$= 3x^4 - 3x$

$f'(x) = ?$

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{3(x+h)^4 - 3(x+h) - (3x^4 - 3x)}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{3(x+h)^4 - 3x^4 + (-3(x+h) + 3x)}{h} \right)$$

$$= \left(\lim_{h \to 0} \frac{3(x+h)^4 - 3x^4}{h} + \lim_{h \to 0} \frac{-3(x+h) + 3x}{h} \right)$$

$$= \left(\lim_{h \to 0} \frac{3(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4) - 3x^4}{h} + \lim_{h \to 0} \frac{-3x - 3h - 3x}{h} \right)$$

$$= \left(\lim_{h \to 0} \frac{3x^4 + 12x^3h + 18x^2h^2 + 12xh^3 + 3h^4 - 3x^4}{h} + \lim_{h \to 0} \frac{-3h}{h} \right)$$

$$= \left(\lim_{h \to 0} \frac{h(12x^3 + 18x^2h + 12xh^2 + 3h^3)}{h} + \lim_{h \to 0} (-3) \right)$$

$$= \left(\lim_{h \to 0} (12x^3 + 18x^2h + 12xh^2 + 3^3) + \lim_{h \to 0} (-3) \right)$$

$$= 12x^3 - 3$$

2. $f(x) = \frac{2}{x} - \frac{1}{x^2}$

$f(x) = 2x^{-1} - x^{-2}$

$f'(x) = ?$

$$f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{2(x+h)^{-1} - (x+h)^{-2} - (2x^{-1} - x^{-2})}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{2(x+h)^{-1} - 2x^{-1} + x^{-2} - (x+h)^{-2}}{h} \right)$$

$$= \left(\lim_{x\to 0} \frac{2(x+h)^{-1} - 2x^{-1}}{h}\right) + \lim_{h\to 0}\left(\frac{x^{-2} - (x+h)^{-2}}{h}\right)$$

$$= \lim_{h\to 0} \frac{2\frac{1}{(x+h)} - \frac{2}{x}}{x} + \lim_{h\to 0} \frac{\frac{1}{x^2} - \frac{1}{x^2+2hx+h^2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-2h}{x^2+hx}}{h} + \lim_{h\to 6} \frac{\frac{h(2x+h)}{x^4+2hx^3+h^2x^2}}{h}$$

$$= \lim_{h\to 0} \frac{-2}{x^2+hx} + \lim_{h\to 0} \frac{2x+h}{x^4+2hx^3+h^2x^2}$$

$$= -2x^{-2} + 2x \cdot x^{-4}$$

$$= -\frac{2}{x^2} + \frac{2}{x^3}$$

3. $f(x) = \frac{1}{2x} + 2x$

$f(x) = \frac{1}{2}x + 2x$

$f'(x) = ?$

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\frac{1}{2}(x+h)^{-1} + 2(x+h) - (\frac{1}{2}x^{-1} + 2x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\frac{1}{2}(x+h)^{-1} - \frac{1}{2}x^{-1} + 2(x+h) - 2x}{h}\right)$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}(x+h)^{-1} - \frac{1}{2}x^{-1}}{h} + \lim_{h\to 0} \frac{2(x+h) - 2x}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{2}\cdot\frac{1}{(x+h)} - \frac{1}{2}x}{h} + \lim_{h\to 0} \frac{2x + 2h - 2x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(2x+2h)} - \frac{1}{2x}}{h} + \lim_{h\to 0} (2)$$

$$= \lim_{h\to 0} \frac{\frac{2x - (2x+2h)}{4x^2+4xh}}{h} + \lim_{h\to 0} (2)$$

$$=$$

$$= \lim_{h\to 0} - \frac{\frac{2h}{4x(x+h)}}{h} + \lim_{h\to 0} (2)$$

$$= \lim_{h\to 0} - \frac{1}{2x^2+2xh} + \lim_{h\to 0} (2)$$

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$$f'(x) = -\frac{1}{2x^2} + 2$$
$$= -\frac{1}{2}x^{-2} + 2$$

4. $f(x) = (2x^2 - x)^2$

$f(x) = 4x^4 - 4x^3 + x^2$

$$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$$

$$f'(x) = \lim_{h \to 0}\left(\frac{4(x+h)^4 - 4(x+h)^3 + (x+h)^2 - (4x^4 - 4x^3 + x^2)}{h}\right)$$

$$= \lim_{h \to 0}\left(\frac{4(x+h)^4 - 4x^4}{h} + \frac{4x^3 - 4(x+h)^3}{h} + \frac{(x+h)^2 - x^2}{h}\right)$$

$$= \lim_{h \to 0}\frac{4(x^4 + 4hx^3 + 6h^2x^2 + 4h^3x + h^4)}{h} + \lim_{h \to 0}\frac{4x^3 - 4(x^3 + 3hx^2 + 3h^2x + h^3)}{h} + \lim_{h \to 0}\frac{(x^2 + 2hx + h^2) - x^2}{h}$$

$$= \lim_{h \to 0}\frac{16hx^3 + 24h^2x^2 + 16h^3x + 4h^4}{h} + \lim_{h \to 0}\frac{-12hx^2 - 12h^2x - 4h^3}{h} + \lim_{h \to 0}\frac{2hx + h^2}{h}$$

$$= \lim_{h \to 0}\frac{h(16x^3 + 24hx^2 + 16h^2x + 4h^3)}{h} + \lim_{h \to 0}\frac{h(-12x^2 - 12hx - 4h^2)}{h} + \lim_{h \to 0}\frac{h(2x+h)}{h}$$

$$= 16x^3 - 12x^2 + 2x$$

5. $f(x) = \dfrac{x^2 - 4x + 4}{x^2 + 2x}$ $\begin{matrix} \to U(x) \\ \to V(x) \end{matrix}$

$U'(x) = 2x - 4$

$V'(x) = 2x + 2$

$$f'(x) = \frac{U'(x) \cdot V(x) - V'(x) \cdot U(x)}{(V(x))^2}$$

$$= \frac{(2x-4) \cdot (x^2 + 2x) - (2x+2)(x^2 - 4x + 4)}{(x^2 + 2x)^2}$$

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$$= \frac{(2x^3+4x^2-4x^2-8x-(2x^3-8x^2+8x+2x-8x+8))}{(x^2+2x)^2}$$

$$= \frac{2x^3-8x-2x^3+6x^2-8}{(x^2+2x)^2}$$

$$= \frac{6x^2-8x-8}{(x^2+2x)^2}$$

6. $F(x) = (x^4+2x)(x^3+2x^2+1)$

$\qquad\quad U(x) \qquad\quad V(x)$

$U'(x) = 4x^3+2$

$V'(x) = 3x^2+4x$

$F'(x) = U'(x)\cdot V(x) + V'(x)\cdot U(x)$

$= (4x^3+2)\cdot(x^3+2x^2+1) + (3x^2+4x)\cdot(x^4+2x)$

$= (4x^6+8x^5+4x^3+2x^3+4x^2+2) + (3x^6+6x^3+4x^5+8x^2)$

$= (4x^6+8x^5+6x^3+4x^2+2) + (3x^6+4x^5+6x^3+8x^2)$

$= 7x^6+12x^5+12x^3+12x^2+2$