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4102 / A11.2021.13254

1. $\int \frac{2}{3} dx^3 dx = \frac{\frac{2}{3}}{3+1} x^{3+1} + C$

$$= \frac{\frac{2}{3}}{4} x^4 + C$$

$$= \frac{2}{12} x^4 + C$$

$$= \frac{1}{6} x^4 + C$$

2. $\int x^2 \sqrt{3+5x} \, dx$
$= \int x^2 (3+5x^2)^{\frac{1}{2}} dx$
$= \frac{x^3 (3+5x)^2}{2} + C$

$= 2x^3 (3+5x)^2 + C$

3. $\int \frac{x^2 - 2x + 1}{(x^3 - 3x^2 + 3x)^4} dx$

$\rightarrow U = x^3 - 3x^2 + 3x$

$du = 3x^2 - 6x + 3 \, dx$

$dx = \frac{du}{3x^2 - 6x + 3}$

$= \int \frac{x^2 - 2x + 1}{u^4} \cdot \frac{du}{3x^2 - 6x + 3}$

$= \int \frac{x^2 - 2x + 1}{u^4} \cdot \frac{du}{3(x^2 - 2x + 1)}$

$= \int \frac{1}{u^4} \cdot \frac{du}{3}$

$= \frac{1}{3} \int \frac{1}{u^4} \cdot du$

$= \frac{1}{3} \int u^{-4} \cdot du$

$= \frac{1}{3} \cdot \frac{1}{-3} u^{-3} + C$

$= \frac{1}{-9} \cdot \frac{1}{u^3} + C$

$= \frac{1}{-9} \cdot \frac{1}{(x^3 - 3x^2 + 3x)^3} + C$

4. $\int (x^2 - 1) \cos x \, dx$
$= \int x^2 \cos(x) - \cos(x) \, dx$
$= \int x^2 \cos(x) dx - \int \cos(x) dx$
$= x^2 \sin(x) + 2x \cos(x) - 2\sin(x) - \sin(x)$
$= x^2 \sin(x) + 2x \cos(x) - 3\sin(x)$

5. $\int \frac{x+2}{x^2 - 6x + 8} dx$

$\rightarrow \frac{x+2}{x^2 - 6x + 8} = \frac{A}{(x-4)} + \frac{B}{(x-2)}$

$\frac{x+2}{x^2 - 6x + 8} = \frac{A(x-2) + B(x-4)}{(x-4)(x-2)}$

$* (x-2) = 0$
$\quad x = 2$
$\quad 2 + 2 = A(0) + B(-2)$
$\quad 4 = -2B$
$\quad -2 = B$

$* (x-4) = 0$
$\quad x = 4$
$\quad 4 + 2 = A(4-2) + B(0)$

$$6 = 2A$$
$$3 = A$$
$$= \frac{3}{x-4} + \left(\frac{-2}{x-2}\right)$$
$$= \int \frac{x+2}{x^2-6x+8} dx$$
$$= \int \frac{3}{x-4} dx + \int \frac{-2}{x-2} dx$$
$$= 3\ln(x-4) - 2\ln(x-2) + C$$

6.

$$\begin{array}{r}
x-1 \\
x^2+x-2 \overline{\smash{\big)}\ x^3-2x+1} \\
\underline{x^3+x^2-2x} \\
-x^2+1 \\
\underline{-x^2-x+2} \\
x-1
\end{array}$$

$$= \int (x-1) dx + \int \frac{(x-1)}{(x^2+x-2)} dx$$

$$= \int (x-1) dx + \int \frac{A}{(x+2)} dx + \int \frac{B}{(x-1)} dx$$

$$= \frac{x-1}{x^2+x-2} = \frac{A(x-1) + B(x+2)}{(x+2)(x-1)}$$

$$\rightarrow (x-1) = 0$$
$$x = 0$$
$$1-1 = A(1-1) + B(1+2)$$
$$0 = 0 + 3B$$
$$0 = 3B$$
$$B = 0$$
$$\rightarrow x+2 = 0$$
$$x = -2$$
$$-2-1 = A(-3) + 0$$
$$\frac{-3}{-3} = A$$
$$1 = A$$

$$\int \frac{x^3-2x+1}{x^2+x-2} dx$$
$$= \int (x-1) dx + \int \frac{1}{x+2} dx$$
$$= \int x\, dx - \int 1 dx + \int \frac{1}{x+2} dx$$
$$= \frac{x^2}{2} - \int 1 dx + \int \frac{1}{x+2} dx$$
$$= \frac{x^2}{2} - x + \ln(x+2) + C$$