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1. $\lim\limits_{h \to 0} \dfrac{f(x+h)-f(x)}{h}$

A. $f(x) = 2x^2 - 3x$

$$f'(x) = \lim_{h\to 0} = \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} = \left(\frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h}\right)$$

$$= \lim_{h\to 0} = \left(\frac{2(x+h)^2 - 2x^2 + 3x - 3(x+h)}{h}\right)$$

$$= \lim_{h\to 0} = \left(\frac{2(x^2+2xh+h^2) - 2x^2 + 3x - 3(x+h)}{h}\right)$$

$$= \left(\lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2}{h} + \lim_{h\to 0} \frac{3x - 3x - 3h}{h}\right)$$

$$= \left(\lim_{h\to 0} \frac{h(4x+2h)}{h} + \lim_{h\to 0} \frac{-3h}{h}\right)$$

$$= \left(\lim_{h\to 0} 4x + 2h + \lim_{h\to 0} (-3)\right)$$

$$= 4x - 3,\quad f'(2) = 4\cdot(2) - 3 = 5$$

B. $f(x) = \dfrac{1}{x} = x^{-1}$

$$f'(x) = \lim_{h\to 0} \left(\frac{(x+h)^{-1} - x^{-1}}{h}\right)$$

$$= \lim_{h\to 0} \left(\frac{\frac{1}{(x+h)} - \frac{1}{x}}{h}\right)$$

$$= \lim_{h\to 0} \left(\frac{\frac{x-(x+h)}{x^2+hx}}{h}\right)$$

$$= \lim_{h\to 0} \left(\frac{\frac{-h}{x(x+h)}}{h}\right)$$

$$= \lim_{h\to 0} \left(\frac{-1}{x(x+h)}\right)$$

$$= \frac{-1}{x^2} = -x^{-2}$$

2. A. $F(x) = \underbrace{(x^4+2x)}_{u(x)}\underbrace{(x^3+2x^2+1)}_{v(x)}$

$u'(x) = 4x^3+2$

$v'(x) = 3x^2+4x$

$F'(x) = u' \cdot v + v' \cdot u$

$= (4x^3+2)(x^3+2x^2+1)+(3x^2+4x)(x^4+2x)$

$= (4x^6+8x^5+4x^3+2x^2+4x^2+2)+(3x^6+6x^3+4x^5+8x^2)$

$= 4x^6+3x^6+8x^5+4x^5+6x^3+6x^3+4x^2+8x^2+2$

$= 7x^6+12x^5+12x^3+12x^2+2$

B. $F(x) = \dfrac{2x^2-3}{2x} \begin{matrix} \to u(x) \\ \to v(x) \end{matrix}$

$u'(x) = 4x$

$v'(x) = 2$

$F'(x) = \dfrac{u' \cdot v - v' \cdot u}{v^2}$

$= \dfrac{4x(2x)-2(2x^2-3)}{(2x)^2}$

$= \dfrac{8x^2-4x^2+6}{4x^2}$

$= \dfrac{4x^2+6}{4x^2}$

3. A. $2x^4y^4+2xy^2 = 3xy^3+y-3$

$2x^4y^4+2xy^2-3xy^3-y+3=0$

$= \left(8x^3y^4+8x^4y^3\dfrac{dy}{dx}\right)+\left(2y^2+4xy\dfrac{dy}{dx}\right)-\left(-3y^3+9xy^2\dfrac{dy}{dx}\right)-1\dfrac{dy}{dx}=0$

$= \dfrac{dy}{dx}\left(8x^4y^3+4xy-9xy^2-1\right) = -8x^3y^4-2y^2+3y^3$

$= \dfrac{dy}{dx} = \dfrac{-8x^3y^4-2y^2+3y^3}{8x^4y^3+4xy-9xy^2-1}$

B. $4x^2y^2+9xy^3 = 2xy+2x$

$4x^2y^2+9xy^3-2xy-2x=0$

$= \left(8xy^2+8x^2y\dfrac{dy}{dx}\right)+\left(9y^3+27xy^2\dfrac{dy}{dx}\right)-\left(2y+2x\dfrac{dy}{dx}\right)-2=0$

$\dfrac{dy}{dx}(8x^2y+27xy^2-2x) = -8xy^2-9y^3+2y+2$

$$\frac{dy}{dx} = \frac{-8xy^2-9y^3+2y+2}{8x^2y+27xy^2-2x}$$

4\. $f(x) = x^3-6x^2+9x-4$

A. Titik kritis

$f'(x) = 3x^2-12x+9$

$f'(x) = 0$

$3x^2-12x+9=0$

$3(x^2-4x+3)$

$3(x-1)(x-3)$

$x=1 \vee x=3 \rightarrow$ Titik kritis

B. $\underset{x<1}{+}\quad \underset{1}{\bullet}\quad \underset{1<x<3}{-}\quad \underset{3}{\bullet}\quad \underset{x>3}{+}$

$f'(x) = 3x^2-12x+9$

$f'(0) = 3\cdot(0)^2-12(0)+9$

$= 9\ (+)$

$f(x)$ naik pada interval $x<1 \vee x>3$

$f(x)$ turun pada interval $1<x<3$

C. Gambar grafik

✱ $x=1$

$\rightarrow f(x) = x^3-6x^2+9x-4$

$f(1) = (1)^3-6(1)^2+9(1)-4$

$= 1-6+9-4$

$= 0 \rightarrow$ koordinat titik maks $(1,0)$

✱ $x=3$

$\rightarrow f(x) = x^3-6x^2+9x-4$

$f(3) = (3)^3-6(3)^2+9(3)-4$

$= 27-54+27-4$

$= -4 \rightarrow$ koordinat min $(3,-4)$