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1. $F(x) = \begin{cases} ax+1 & x \leq 1 \\ ax^2+bx+5 & 1<x<3 \\ 2 & x \geq 3 \end{cases}$

Jawab:

- $F(x)$ kontinu di $x=1$, $\lim_{x \to 1^-} F(x) = \lim_{x \to 1^+} F(x)$

$\lim_{x \to 1^-} F(x) = \lim_{x \to 1^+} F(x)$

$\lim_{x \to 1^-} (ax+1) = \lim_{x \to 1^+} (ax^2+bx+5)$

$(a(1)+1) = a(1)^2 + b(1) + 5$

$a+1 = a+b+5$

$-b = 5-1$

$-b = 4$

$b = -4 \ldots (1)$

- $F(x)$ kontinu di $x=3$, $\lim_{x \to 3^-} F(x) = \lim_{x \to 3^+} F(x)$

$\lim_{x \to 3^-} F(x) = \lim_{x \to 3^+} F(x)$

$\lim_{x \to 3^-} (ax^2+bx+5) = \lim_{x \to 3^+} (2)$

$a(3)^2 + b(3) + 5 = 2$

$9a + 3b + 5 = 2$

$9a + 3b = 2-5$

$9a + 3b = -3$

$3a + b = -1 \ldots (2)$

$(1) \to (2)$ $3a + (-4) = -1$

$3a = -1+4$

$3a = 3$

$a = 1$

Fungsinya menjadi

$$f(x)=\begin{cases} x+1 & , x\le 1 \\ x^2-4x+5 & 1<x<3 \\ 2 & x\geq 3 \end{cases}$$

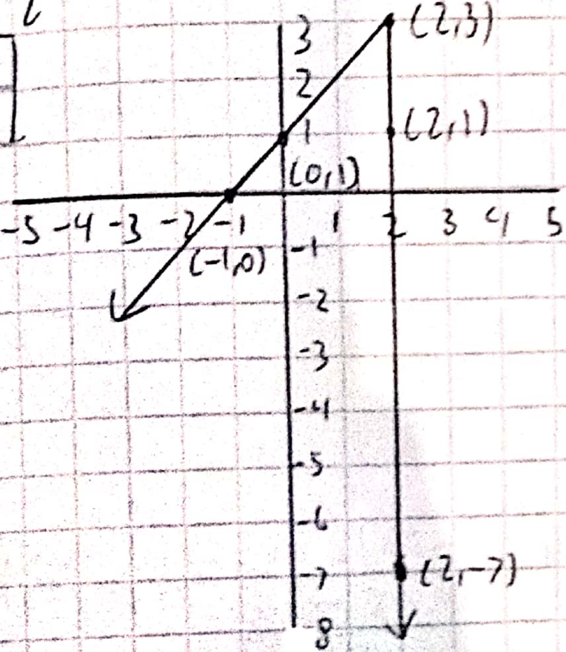
$\rightarrow x^2-4x+5\ , 1<x<3$

$x_p=\frac{4}{2}=2,\ y_p=-7, (2,-7)$

$\rightarrow x+1, x\le 1$

x	0	-1
y	1	0

x	2
y	1



3. $f(x)\begin{cases} ax+2 & x\le -2 \\ -x^2-b & -2<x<2 \\ ax-2 & x\geq 2 \end{cases}$

$\rightarrow f(x)$ kontinu di $x=-2$, $\lim\limits_{x\to -2^-} f(x)=\lim\limits_{x\to -2^+} f(x)$

$\lim\limits_{x\to -2^-} f(x)=\lim\limits_{x\to -2^+} f(x)$

$\lim\limits_{x\to -2^-}(ax+2)=\lim\limits_{x\to -2^+}(-x^2-b)$

$a(-2)+2=-(-2)^2-b$

$-2a+2=-4-b$

$-2a+b=-4-2$

$-2a+b=-6$ ... (1)

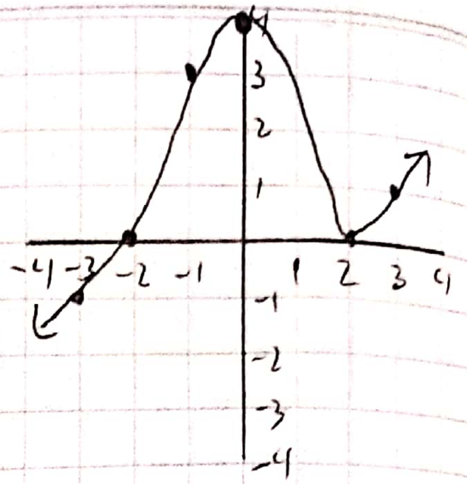
$\rightarrow f(x)$ kontinu di $x=2$, $\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^+} f(x)$

$\lim\limits_{x\to 2^-} f(x)=\lim\limits_{x\to 2^+} f(x)$

$$\lim_{x\to 2^-}(-x^2-b) = \lim_{x\to 2^+}(ax-2)$$
$$-(2)^2-b = a(2)-2$$
$$-4-b = 2a-2$$
$$-4+2 = 2a+b$$
$$-2 = 2a+b \quad \ldots (2)$$

(1) dan (2)
$$-2a+b=-6$$
$$\underline{2a+b=-2} \; -$$
$$-4a=-4$$
$$a=1$$

$* \; 2a+b=-2$
$$2+b=-2$$
$$b=-4$$



Jadi fungsinya

$$f(x) = \begin{cases} x+2, & x \le -2 \\ -x^2+4, & -2<x<2 \\ x-2, & x \geq 2 \end{cases}$$

$\rightarrow -x^2+4, \; -2<x<2 \quad (0,4)$

X	-1	0
Y	3	4

$X_p=0$
$X_T=4$

$\rightarrow x+2, \; x\le -2$

X	-2	-3
Y	0	-1

$\rightarrow x-2, \; x\geq 2$

X	3	2
Y	1	0

4. $$f(x) = \begin{cases} ax+2, & x \le -1 \\ ax^2+bx, & -1<x<1 \\ -ax+8, & x\geq 1 \end{cases}$$

- $f(x)$ kontinu di $x=-1$, $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$$
$$\lim(ax+2) = \lim(ax^2+bx)$$
$$a(-1)+2 = a(-1)^2+b(-1)$$
$$-a+2 = a-b$$
$$-2a+b=-2 \quad \ldots (1)$$

- $f(x)$ kontinu di $x=1$, $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$$
$$\lim(ax^2+bx) = \lim(-ax+8)$$

$a+b = -a+8$

$2a+b = 8 \ldots (2)$

(1) dan (2)

$$\begin{array}{r} -2a+b = -2 \\ 2a+b = 8 \\ \hline -4a = -10 \end{array} \; -$$

$a = -10/-4$

$= 2{,}5$

* $2(2{,}5)+b = 8$

$5+b = 8$

$b = 3$

Jadi fungsinya

$$f(x) = \begin{cases} 2{,}5x+2 & , x \le -1 \\ 2{,}5x^2+3x & , -1 < x < 1 \\ -2{,}5x+8 & , x \ge 1 \end{cases}$$

* $2{,}5x^2+3x \; , -1 < x < 1$

$x_p = -\frac{3}{5} \; , \; y_p = -\frac{9}{10}$

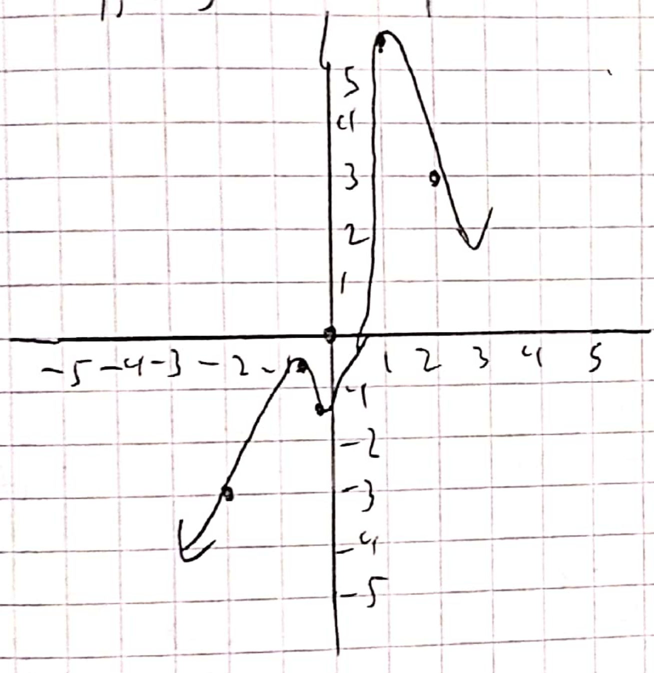
x	0
y	0

* $2{,}5x+2 \; , x \le -1$

x	-1	-2
y	$-\frac{1}{2}$	-3

* $-2{,}5x+8 \; , x \ge 1$

x	1	2
y	5,5	3



5. $$f(x) = \begin{cases} ax+4 & , x \le -2 \\ ax^2+bx & , -2 < x < 1 \\ -ax+8 & , x \ge 1 \end{cases}$$

→ f(x) kontinu di $x = -2$

$\lim\limits_{x \to -2^-} (ax+4) = \lim\limits_{x \to -2^+} (ax^2+bx)$

$-2a+4 = 4a-2b$

$-6a+2b = -4 \ldots (1)$

→ f(x) kontinu di $x = 1$

$\lim\limits_{x \to 1^-} (ax^2+bx) = \lim\limits_{x \to 1^+} (-ax+8)$

$a+b = -a+8$

$2a+b = 8 \ldots (2)$

(1 dan 2)

$-6a+2b = -4 \quad | \times 1$

$2a+b = 8 \quad | \times 2$

$$\begin{array}{r} -6a+2b = -4 \\ 4a+2b = 16 \\ \hline -10a = -20 \end{array} \; -$$

$a = 2$

* $2a+b = 8$

$4+b = 8$

$b = 4$

Jadi fungsinya

$$f(x) = \begin{cases} 2x+4 &, x \le -2 \\ 2x^2+4x &, -2 < x < 1 \\ -2x+8 &, x \ge 1 \end{cases}$$

$\rightarrow 2x^2+4x,\ -2<x<1$

$x_p = -1,\ y_p = -2$

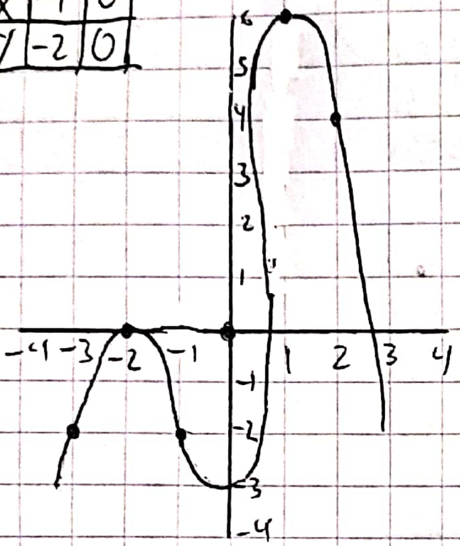
X	-1	0
Y	-2	0

$\rightarrow 2x+4,\ x \le -2$

X	-2	-3
Y	0	-2

$\rightarrow -2x+8,\ x \ge 1$

X	1	2
Y	6	4



6.

$$f(x) = \begin{cases} ax+6 &, x \le -2 \\ -ax^2+bx &, -2 < x < 1 \\ ax-12 &, x \ge 1 \end{cases}$$

$\rightarrow f(x)$ kontinu di $x=-2$

$\lim_{x\to -2^-} (ax+6) = \lim_{x \to -2^+} (-ax^2+bx)$

$-2a+6 = -4a-2b$

$2a+2b = -6$

$a+b = -3$ ... (1)

$\rightarrow f(x)$ kontinu di $x=1$

$\lim_{x\to 1} (-ax^2+bx) = \lim_{x\to 1^+} (ax-12)$

$-a+b = a-12$

$12 = 2a-b$ ... (2)

(1) dan (2)

$a+b=-3$

$2a-b=12\ +$

$3a = 9$

$a = 3$

$a+b=-3$

$3+b=-3$

$b=-6$

Jadi fungsinya

$$f(x) = \begin{cases} 3x+6 & x \le -2 \\ -3x^2-6x & -2 < x < 1 \\ 3x-12 & x \ge 1 \end{cases}$$

$\rightarrow -3x^2-6x$

$x_p = -1\quad y_p = 3$

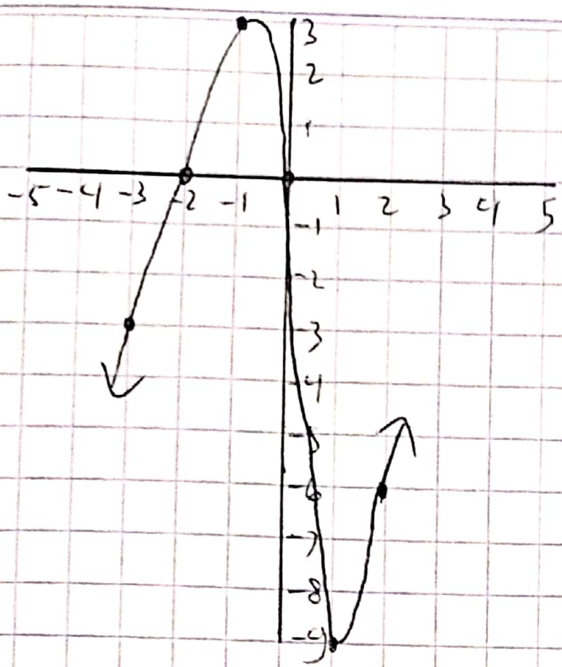
X	-2	-1	0	1
Y	0	3	0	-9

$\rightarrow 3x+6$

X	-2	-3
Y	0	-3

$\rightarrow 3x-12$

X	1	2
Y	-9	-6



2.

$$F(x) = \begin{cases} x+b & ; \quad x \leq 0 \\ x^2-3x+a & ; \quad 0 < x < 2 \\ 2-x & ; \quad x \geq 2 \end{cases}$$

- $F(x)$ kontinu di $x=0$

$$\lim_{x \to 0^-}(x+b) = \lim_{x \to 0^+}(x^2-3x+a)$$

$$b = a \quad \text{---}(1)$$

- $F(x)$ kontinu di $x=2$

$$\lim_{x \to 2^-}(x^2-3x+a) = \lim_{x \to 2^+}(2-x)$$

$$4-6+a = 0$$

$$a = 2$$

- (1) dan (2)

$a=2$

$a=b$

$b=2$

Jadi fungsinya

$$F(x) \begin{cases} x+2 & x \leq 0 \\ x^2-3x+2 & 0 < x < 2 \\ 2-x & x \geq 2 \end{cases}$$

→ x^2-3x+2, $0<x<2$

$X_p = \frac{-b}{2a} = \frac{3}{4} \rightarrow \left(\frac{3}{4}, \frac{-1}{4}\right)$

$Y_p = -\frac{1}{4}$

X	1
Y	0

→ $x+2$, $x \leq 0$

X	0	-1
Y	2	1

→ $2-x$, $x \geq 2$

X	2	3
Y	0	-1