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1. $F(x) = 2x+2$, sumbu x, $x=0$ $x=2$

$- \Delta x = \frac{x_n - x_0}{n}$

$= \frac{2-0}{n} = \frac{2}{n}$

$- x_1 = 0 + \frac{2}{n}$

$- x_2 = \frac{2}{n} + \frac{2}{n} = 2\left(\frac{2}{n}\right)$

$- x_3 = 2\left(\frac{2}{n}\right) + \frac{2}{n} = 3\left(\frac{2}{n}\right)$

$x_i = \left(\frac{2}{n}\right) i$

$- F(x) = 2x+2$

$= 2\left(\left(\frac{2}{n}\right) + i\right) + 2$

$= \frac{4}{n} i + 2$

$- A = \sum_{i=1}^{n} \Delta x \cdot F(x_i)$

$= \sum_{i=1}^{n} \frac{2}{n} \cdot \left(\left(\frac{4}{n}\right) i + 2\right)$

$= \sum_{i=1}^{n} \left(\frac{8}{n^2}\right) i + \frac{4}{n}$

$= \left(\frac{8}{n^2}\right) \sum_{i=1}^{n} i + \frac{4}{n}$

$= \left(\frac{8}{n^2}\right)\left(\frac{n(n+1)}{2}\right) + 4$

$= \left(\frac{8}{n^2}\right)\left(\frac{n^2+n}{2}\right) + 4$

$= 4 + \frac{4}{n} + 4$

$\lim_{n\to\infty} A = \lim_{n\to\infty} \left(4 + \frac{4}{n} + 4\right)$

$= 4+4$

$= 8$

2. $F(x) = -x^2 + 2x$, interval $[1,2]$

$- \Delta x = \frac{2-1}{n}$

$= \frac{1}{n}$

$- x_1 = 1 + \frac{1}{n}$

$- x_2 = 1 + \frac{1}{n} + \frac{1}{n} = 1 + 2\left(\frac{1}{n}\right)$

$- x_3 = 1 + 2\left(\frac{1}{n}\right) + \frac{1}{n} = 1 + 3\left(\frac{1}{n}\right)$

$x_i = 1 + \left(\frac{1}{n}\right) i$

$- F(x_i) = -\left(1 + \left(\frac{1}{n}\right) i\right)^2 + 2\left(1 + \left(\frac{1}{n}\right) i\right)$

$= -\frac{i^2}{n^2} - \cancel{\frac{2i}{n}} - 1 + 2 + \cancel{\frac{2i}{n}}$

$= -\frac{i^2}{n^2} + 1$

$- A = \sum_{i=1}^{n} \Delta x \cdot F(x_i)$

$= \sum_{i=1}^{n} \frac{1}{n} \cdot \left(-\frac{i^2}{n^2} + 1\right)$

$= \sum_{i=1}^{n} -\frac{i^2}{n^2} + \frac{1}{n}$

$$= \left(-\frac{1}{n^3}\right)\sum_{i=1}^{n} i^2 + \frac{1}{n}$$

$$= \left(-\frac{1}{n^3}\right)\left(\frac{2n^3+3n^2+n}{6}\right) + 1$$

$$= -\frac{1}{3} - \frac{1}{2n} - \frac{1}{6n^2} + 1$$

$$\lim_{n\to\infty} A = \lim_{n\to\infty}\left(-\frac{1}{3} - \frac{1}{2n} - \frac{1}{6n^2} + 1\right)$$

$$= -\frac{1}{3} + 1$$

$$= \frac{2}{3}$$