Introduction to birth-death processes

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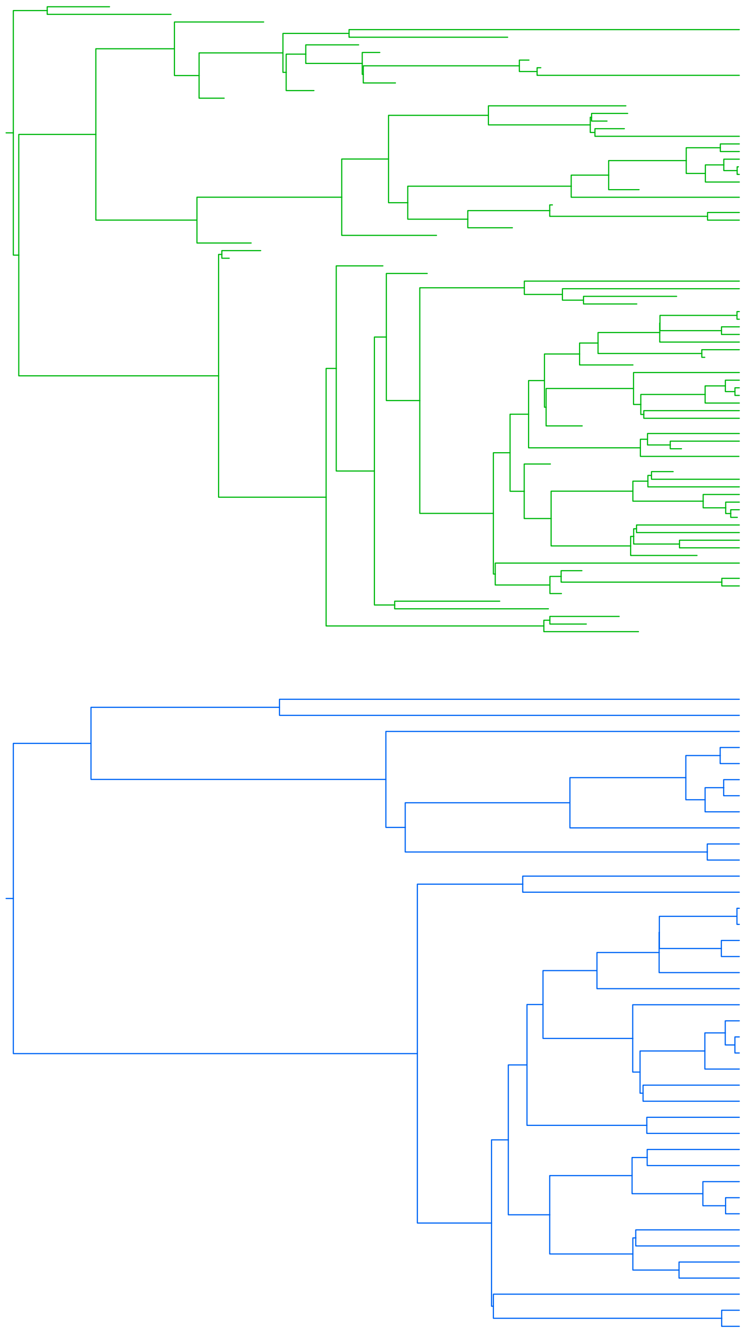
IB200, Spring 2016

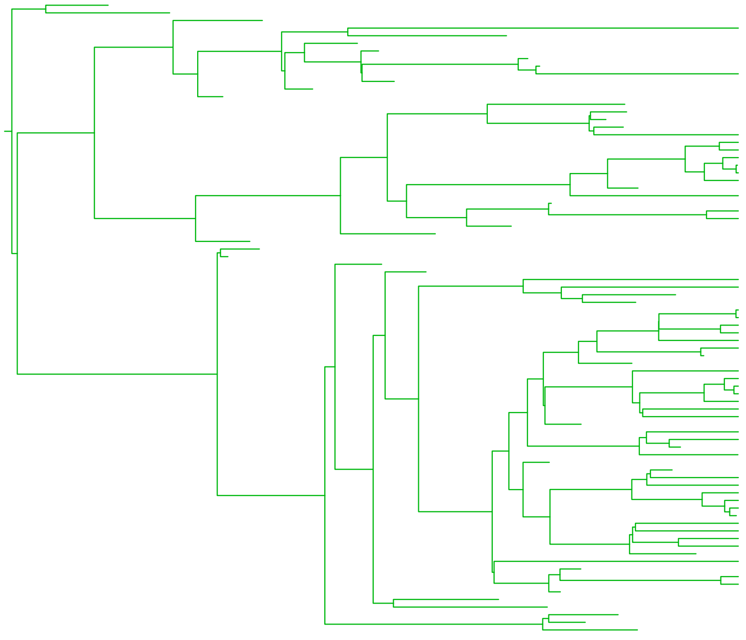


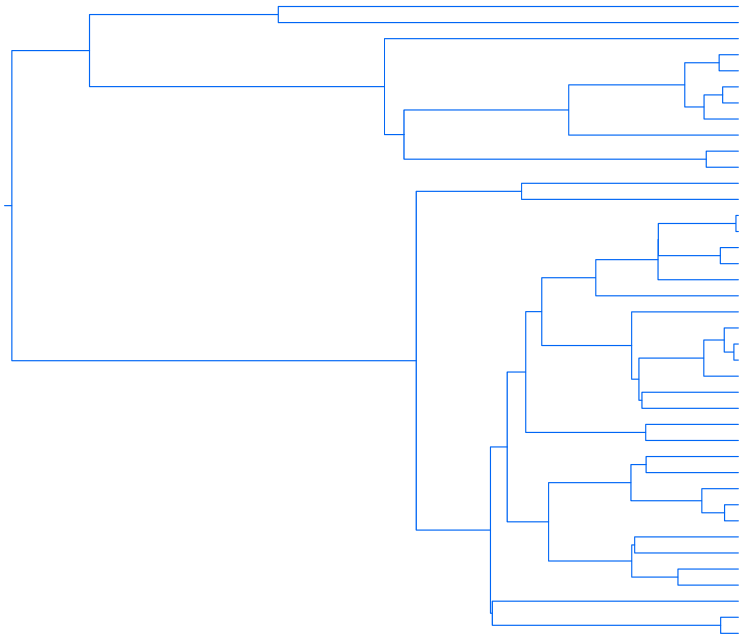
birth-death processes:

1. constant rate

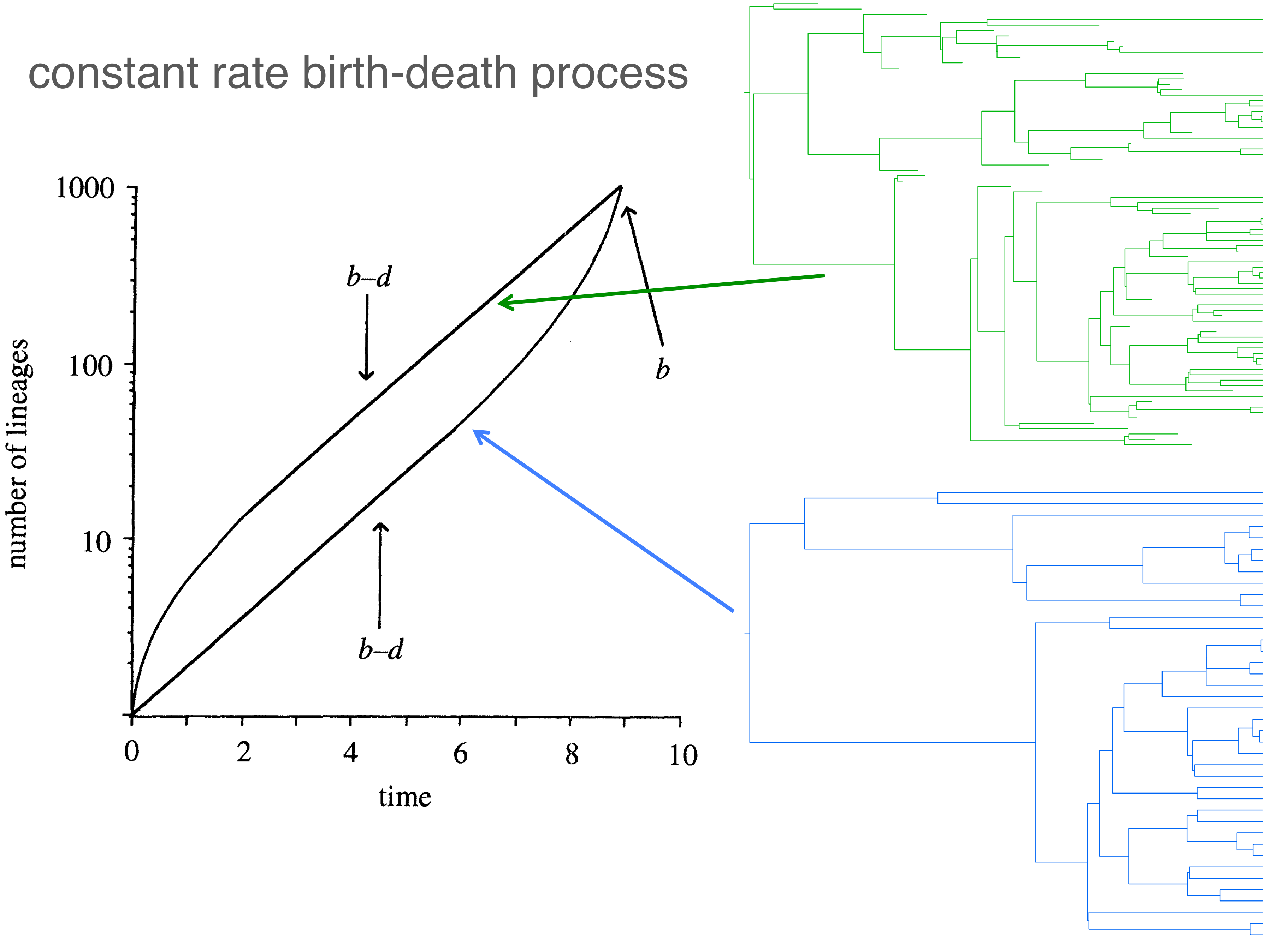
2. character dependent

3. branch heterogenous





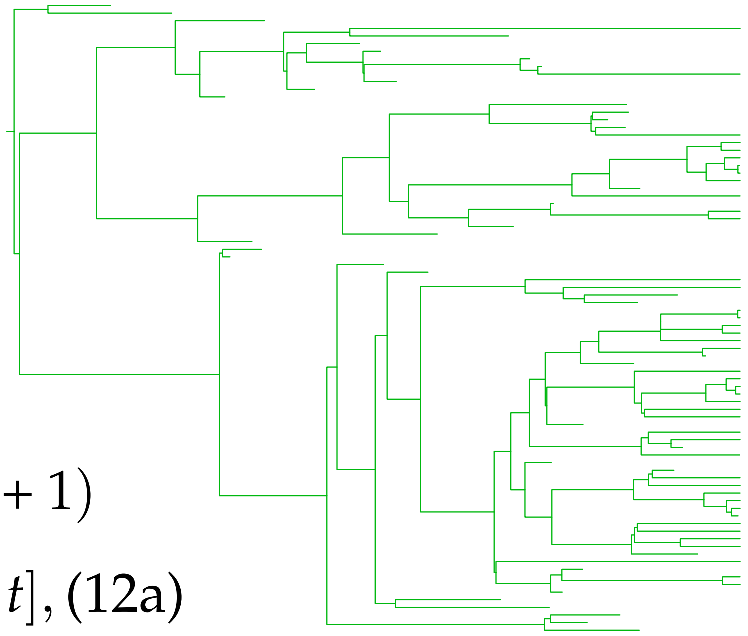
constant rate birth-death process



constant rate birth-death process

Probabilitiy of N lineages at time t:

$$\frac{\mathrm{d}\mathbb{P}[N_{\mathrm{g}}\,;t]}{\mathrm{d}t} = \lambda_1(N_{\mathrm{g}}-1)\mathbb{P}[N_{\mathrm{g}}-1\,;t] + \mu_1(N_{\mathrm{g}}+1)$$
$$\times\mathbb{P}[N_{\mathrm{g}}+1\,;t] - (\lambda_1+\mu_1)N_{\mathrm{g}}\mathbb{P}[N_{\mathrm{g}}\,;t], \quad (12a)$$

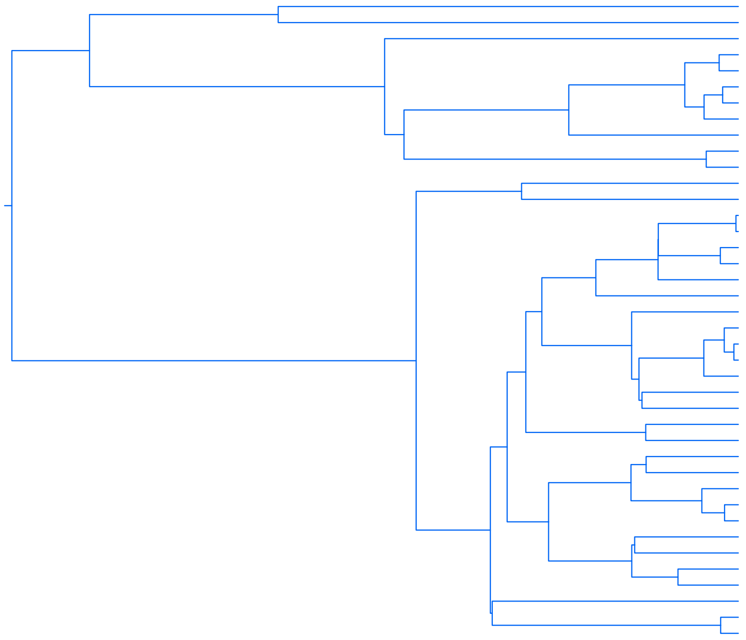


Initial conditions:

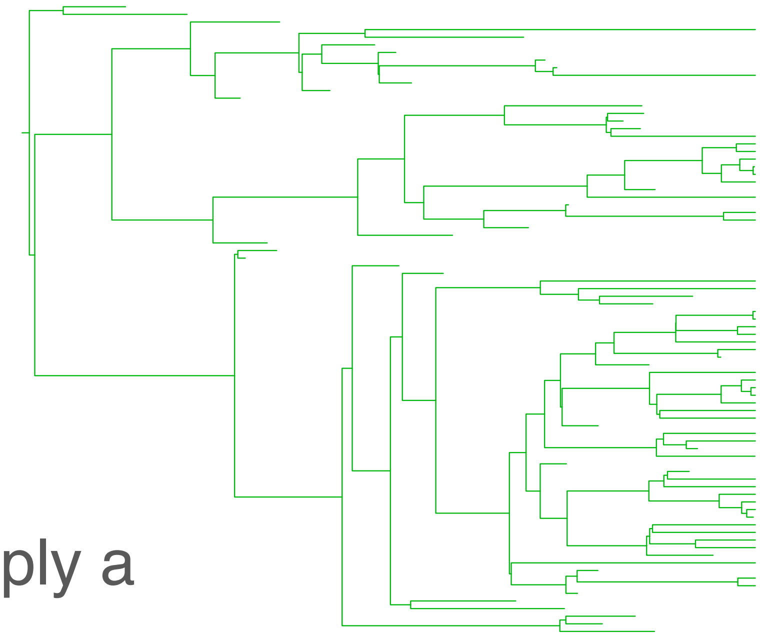
$$\mathbb{P}[N_{\mathrm{g}} = N_{\mathrm{g}}(0)\,;t=0] = 1$$

Expected number of lineages at time t:

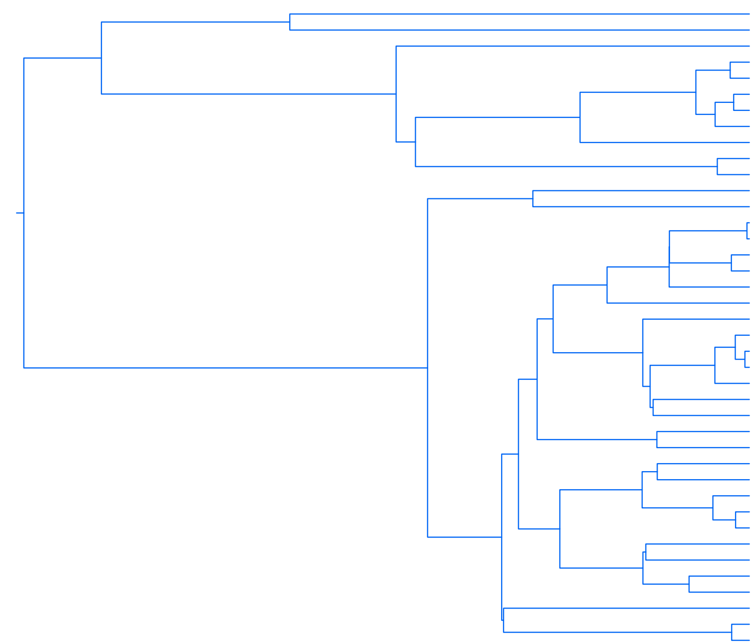
$$\mathbb{E}[N_{\mathrm{g}}, t] = N_{\mathrm{g}}(0)\mathrm{e}^{(\lambda_1-\mu_1)t}$$



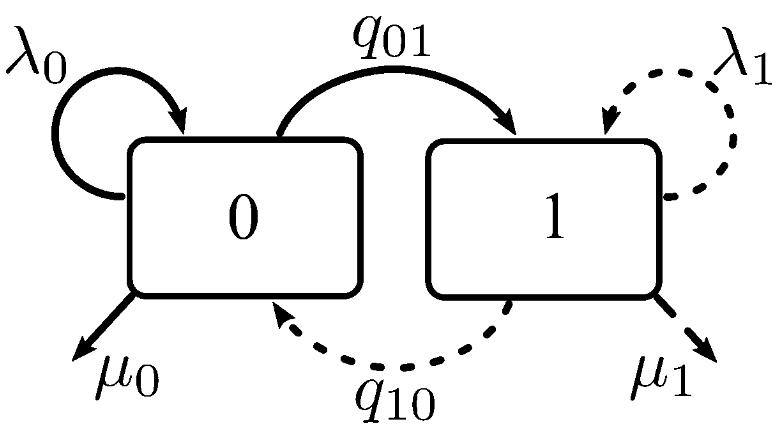
constant rate birth-death process

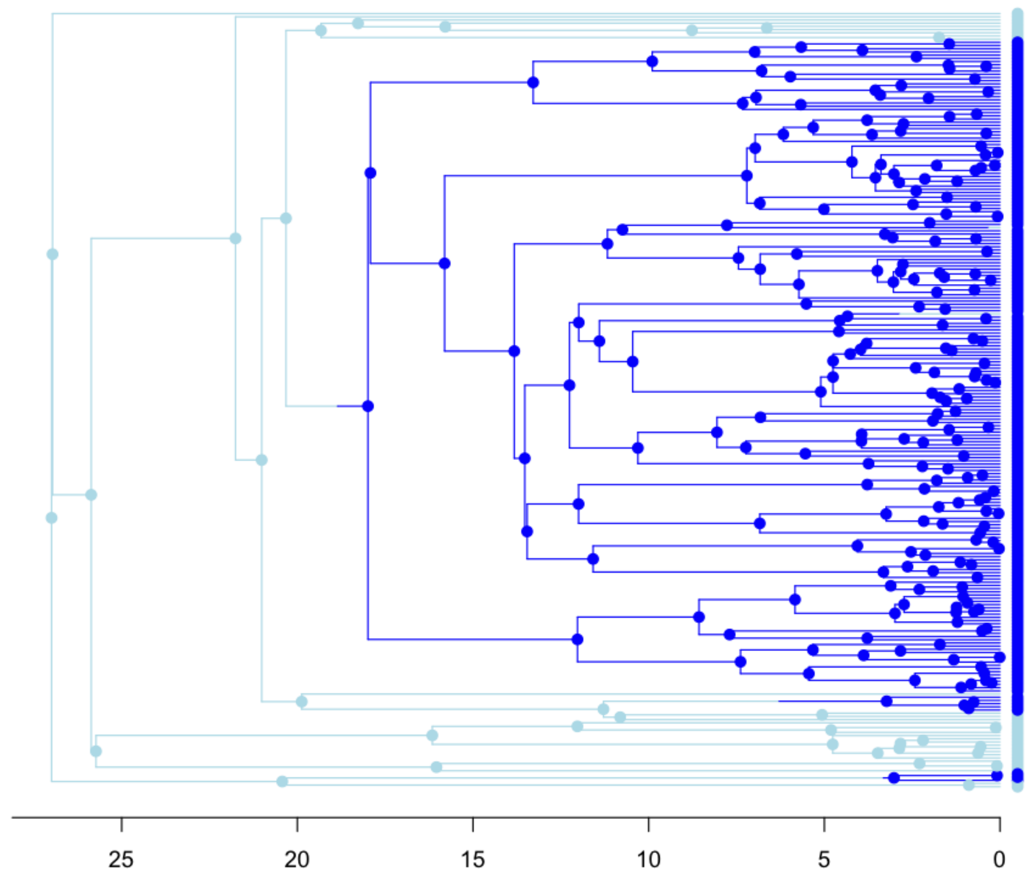


Yule (pure birth) processes are simply a special case where $\mu = 0$



character dependent birth-death processes





character dependent birth-death processes

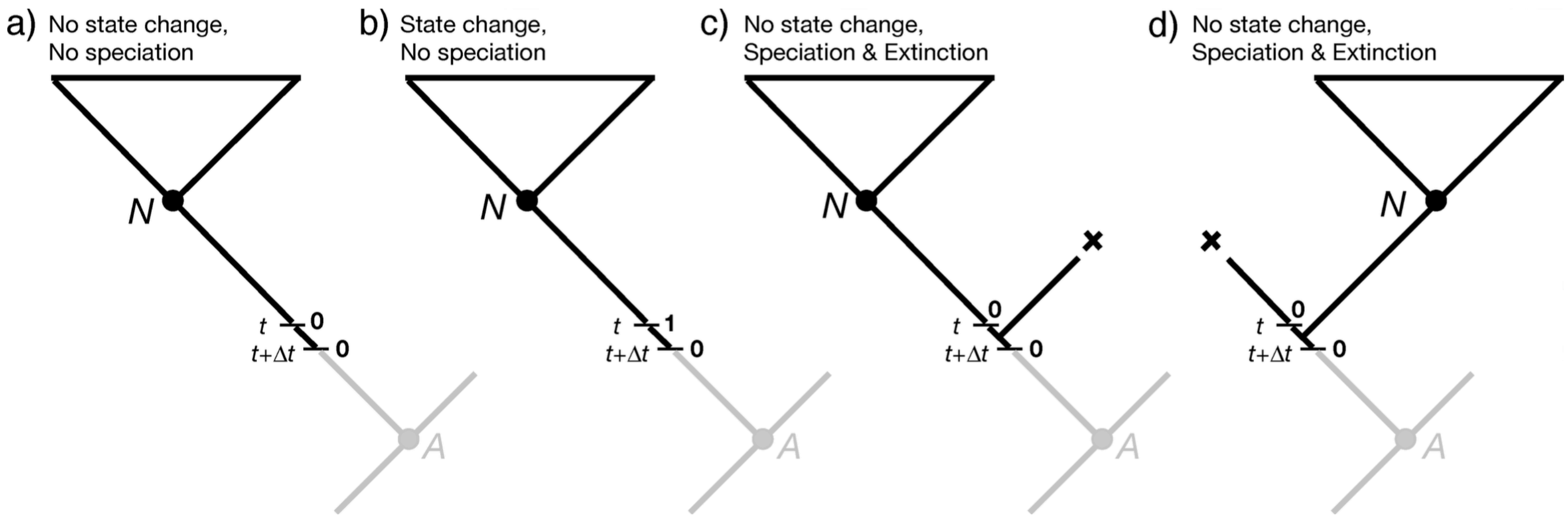


FIGURE 2. Alternative scenarios by which a lineage with state 0 at time $t+\Delta t$ on the branch might yield clade decended from node N but no other living descendants.

$D_{N0}(t+\Delta t) =$

$$
\begin{array}{ll}
(1-\mu_0\Delta t)\times & \text{(in all cases no extinction in } \Delta t\text{)} \\
\quad [(1-q_{01}\Delta t)(1-\lambda_0\Delta t)D_{N0}(t) & \text{(see Fig. 2a: No state change, no speciation)} \\
\quad +(q_{01}\Delta t)(1-\lambda_0\Delta t)D_{N1}(t) & \text{(see Fig. 2b: State change, no speciation)} \\
\quad +(1-q_{01}\Delta t)(\lambda_0\Delta t)E_0(t)D_{N0}(t) & \text{(see Fig. 2c: No state change, speciation, extinction)} \\
\quad +(1-q_{01}\Delta t)(\lambda_0\Delta t)E_0(t)D_{N0}(t)] & \text{(see Fig. 2d: No state change, speciation, extinction)} \\
\quad +(\mu_0\Delta t)\times 0 & \text{(if the lineage went extinct, the clade has zero probability of being observed)}
\end{array}
\quad (1)
$$

character dependent birth-death processes

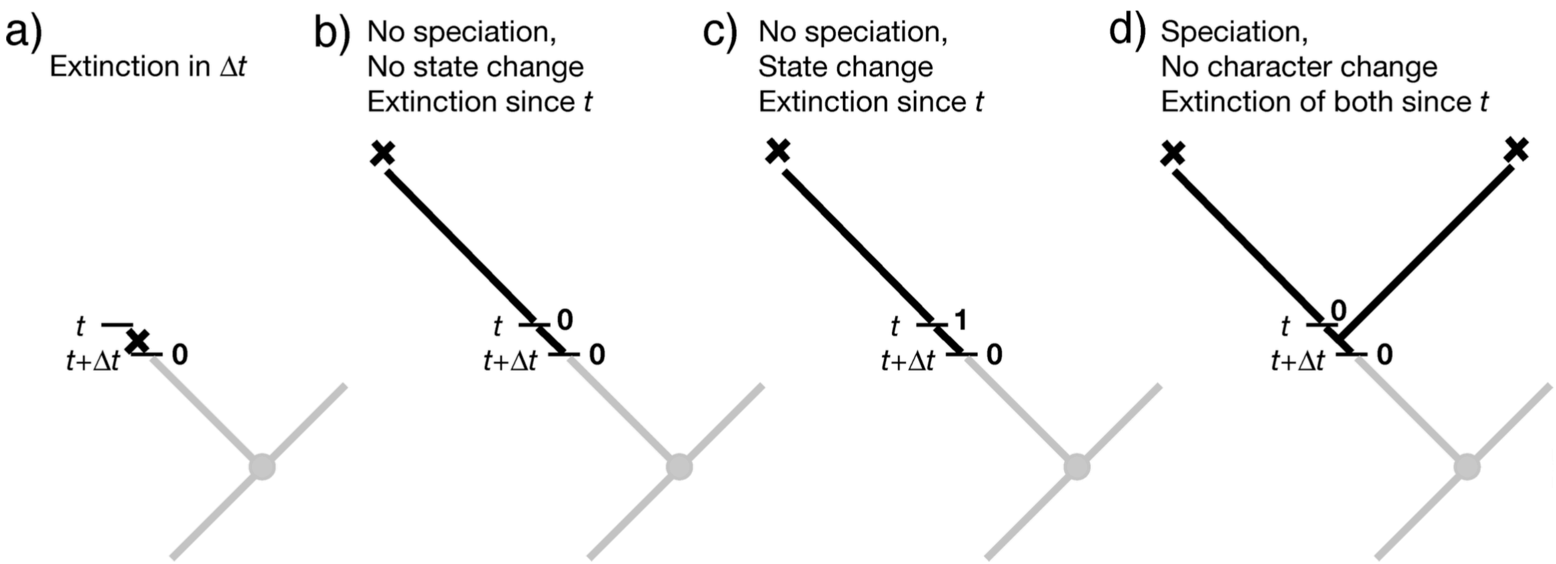
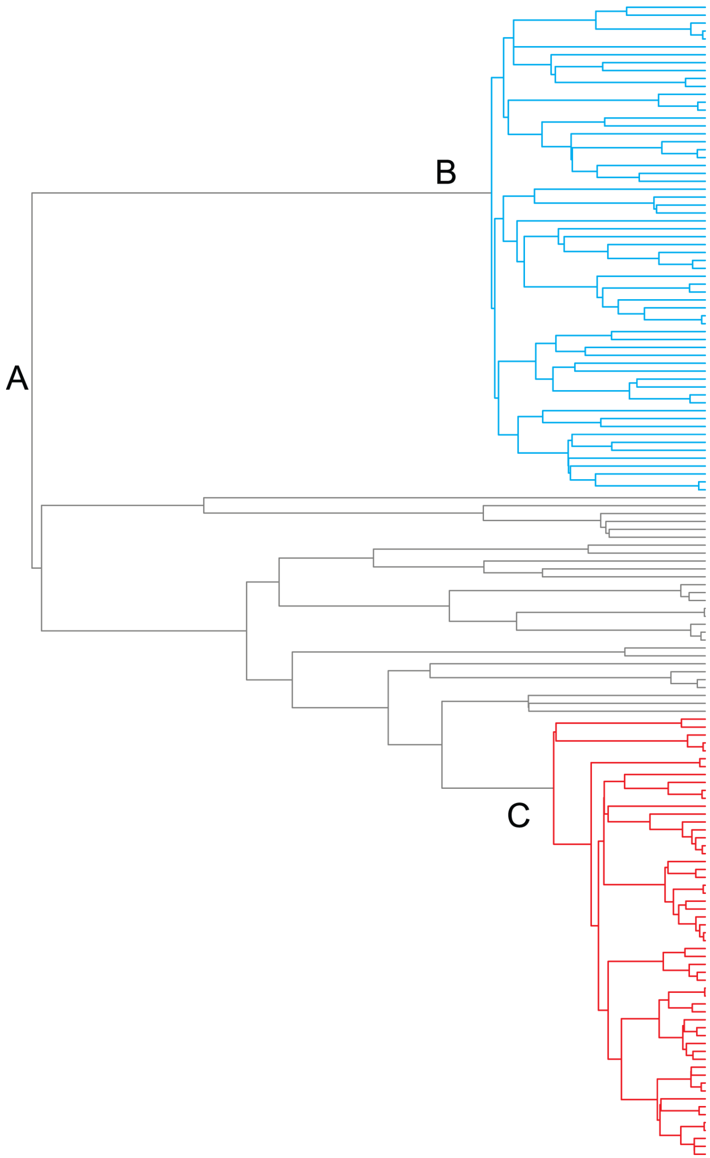


FIGURE 3. Alternative scenarios by which a lineage at time t with state 0 might go extinct.

$$
\begin{aligned}
&E_0(t+\Delta t) = \\
&\mu_0 \Delta t && \text{(see Fig. 3a: Extinction in } \Delta t\text{)} \\
&+(1-\mu_0\Delta t)(1-q_{01}\Delta t)(1-\lambda_0\Delta t)E_0(t) && \text{(see Fig. 3b: No state change, no speciation)} \\
&+(1-\mu_0\Delta t)(q_{01}\Delta t)(1-\lambda_0\Delta t)E_1(t) && \text{(see Fig. 3c: State change, no speciation)} \\
&+(1-\mu_0\Delta t)(1-q_{01}\Delta t)(\lambda_0\Delta t)E_0(t)^2 && \text{(see Fig. 3d: No state change, speciation)}
\end{aligned}
$$

branch heterogenous birth-death processes



A compound Poisson process introduces shifts in the diversification process across lineages and through time.

Rate shift events occur according to a Poisson process

$$P(k \text{ events in interval}) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where the rate parameter λ equals the average number of events in an interval.

This assumes that descendant lineages inherit diversification regimes.