

a) There are $w \cdot L$ options for choosing with the options for each value being 0 or 1. Thus $2^{w \cdot L}$ options.

- Matrix b is of size $L \times 1$ with two options 0 or 1 for each value. Thus there are 2^L .

- Together we have $2^{w \cdot L}$ and 2^L matrices. Thus:

$$2^{w \cdot L} \cdot 2^L = \underline{2^{(w \cdot L) + L}}$$

b) The equation $\Pr_{h \in \mathcal{H}} (h(x_1) = v_1 \wedge h(x_2) = v_2) = 1/m^2$ can be split and re-written as:

$$\Pr_{h \in \mathcal{H}} (h(x_1) = v_1) \cdot \Pr_{h \in \mathcal{H}} (h(x_2) = v_2) = 1/m^2$$

- We know $h(x)$ is a $1 \times L$ matrix of possible values $0 \leq 1$.

- Composing the values of $h(x_1)$ with each value of a pre-determined v_1 (of the same size $1 \times L$ and values of $0 \leq 1$) has the probability $1/2^L$ of being equal.

- Because of the same conditions, the same can be said of $h(x_2)$ equalling v_2 : $1/2^L$

- Thus $\Pr (h(x_1) = v_1) \cdot \Pr (h(x_2) = v_2) = 1/m \cdot 1/m = 1/m^2$, where $2^L = m$

c) - With b set to a matrix of only 0's, we have left over just $A \cdot x$. A is of size $L \cdot w$ and after multiplied by x we are left with matrix of $L \times 1$, meaning there are L numbers with values in the range $[0, w]$ (if all elements are 0, to all elements are 1).

- Thus there are L numbers which all have to be the same odd or even at each space i in order for $h(x_1) = h(x_2)$. With L pairs each with a $1/2$ probability of being the same odd or even (mod 2 gives the same number) there is a $1/2^L$ chance they are equal, or $1/m$.
- Therefore it holds that $\Pr_{h \in \mathcal{H}} (h(x_1) = h(x_2)) \leq 1/2^L = 1/m$