ECE 611 – final exam project

A very important nonlinear equation in science is the Nonlinear Schrodinger equation (NLS)

$$i\frac{\partial \psi}{\partial t} + \frac{\partial^2 \psi}{\partial x^2} + |\psi|^2\psi = 0 \tag{1}$$

This is a partial differential equation. Devise a numerical scheme to solve this equation. Apply periodic boundary conditions for all time:

$$\psi(0,t) = \psi(L,t) \tag{2}$$

L is the length of the integration domain.

From the literature you can find an analytic solution to Eq. (1) that is called a **soliton**. It will have a form like $exp(..)sech(...)$ with 2 free parameters. These parameters control the soliton speed and amplitude.

Problem

(a) Device a numerical scheme that will successfully follow the evolution of the exact 1-soliton solution of Eq. (1). Follow this soliton as it performs at least 3 transits of the spatial region $[0, L]$. The soliton should travel without any distortion. Plot the time evolution of your numerical solution.

(b) Now consider a 2-soliton solution -having different amplitude and velocities of different sign. That is, the 2 solitons do not overlap (by choice of their initial location and width) and move towards each other for a soliton collision. Numerical follow the 2-soliton dynamics and show that the post-collision solitons retain they shape and velocity. Numerically follow this dynamics for at least 12 soliton-soliton collisions. Plot some of these results. What one should find is that there is a distinctive spatial induced in the post-collision state. Verify that the spatial shift is the same for each soliton-soliton collision

(c) NLS has an infinite number of conservation integrals (it is an integrable system). Numerically devise a scheme to evaluate the integrals (as a function of time)

$$normalization : S_0(t) = \int_0^L |\psi(x,t)|^2 \, dx \tag{3}$$

$$energy : S_2(t) = \int_0^L \left[2\left|\frac{\partial \psi(x,t)}{\partial x}\right|^2 - \frac{1}{2}\left|\psi(x,t)\right|^4 \right] \tag{4}$$

First verify analytically that $S_0(t)$ and $S_2(t)$ are indeed conserved. Now plot the numerical solution to these integrals. Comment on your numerical accuracy.

You will need to determine the appropriate length L, the amplitude and speed parameters of the solitons – appropriate for the convergence of your numerical scheme.