Analytically $\int_{-3}^{5} (4x-3)^3 \, dx$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$$\Rightarrow \int_{-3}^{5} (64x^3 - 144x^2 + 108x - 27) \, dx$$

Integrate each term seperately

$$\int (64x^3 - 144x^2 + 108x - 27) \, dx = 64 \cdot \frac{x^4}{4} - 144 \frac{x^3}{3} + 108 \cdot \frac{x^2}{2} - 27x$$

$$= 16x^4 - 48x^3 + 54x^2 - 27x$$

Evaluate the term from -3 to 5:

$$\Rightarrow \left[16 \cdot (5)^4 - 48 \, (5)^3 + 54 \, (5)^2 - 27 \cdot 5\right] - \left[16 \cdot (-3)^4 - 48 \cdot (-3)^3 + 54 \, (-3)^2 - 2\right.$$

$$\Rightarrow \left[10000 - 6000 + 1350 - 135\right] - \left[1296 + 1296 + 486 + 81\right]$$

$$\Rightarrow 5215 - 3159$$

$$\Rightarrow \boxed{2056} \quad \text{Ans.}$$

Trapezoidal $T_n = \int_a^b f(x) \, dx$, here, $b = 5$, $a = -3$

let, $n = 4$

$$\Delta x = \frac{b-a}{n} = \frac{5-(-3)}{4} = \frac{8}{4} = 2$$

$$f(x) = (4x-3)^3$$

$$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)\right]$$

$$= \frac{2}{2}\left[f(-3) + 2f(-1) + 2f(1) + 2f(3) + f(5)\right]$$

$$f(-3) = \left[4\cdot(-3)-3\right]^3 = -3375$$

$$f(5) = (20-3)^3 = 4913$$

$$f(-3) + f(5) = -3375 + 4913 = 1538$$

$$f(-1) = \left[4\cdot(-1)-3\right]^3 = \left[-4-3\right]^3 = -343$$

$$f(1) = 1$$

$$f(3) = 729$$

$$= 1\left[1538 + 2^*(-343) + 2^*(1) + 2^*729\right]$$

$$= \left[1538 + 2 + 1458 - 686\right]$$

$$= \boxed{2312}$$

Simpson's $\frac{1}{3}$ Rule :

$$S_n = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \quad + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

$$= \frac{2}{3}\left[f(-3) + 4 * f(-1) + 2 * f(1) + 4 * f(3) + f(5)\right]$$

$$f(-3) + f(5) = 1538$$

$$= \frac{2}{3}\left[1538 + 4 * (-343) + 2 * (1) + 4 * (729)\right]$$

$$= \frac{2}{3}\left[4456 - 1372\right]$$

$$= \frac{2}{3}(3084)$$

$$= \boxed{2056}$$