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(ECE 611)

Assignment 05

$$\frac{d^2y}{dx^2} = -1001\frac{dy}{dx} - 1000y$$

We assume the solution of the form $y(x) = e^{rx}$

Substituting this in the ODE, we get:

$$\frac{dy}{dx} = re^{rx}$$

$$\frac{d^2y}{dx^2} = r^2e^{rx}$$

$$r^2e^{rx} = -1001\, re^{rx} - 1000e^{rx}$$

We can divide through e^{rx}

$$r^2 = -1001r - 1000$$

This is a quadratic equation in r, we solve it

$$r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

here, $a = 1$, $b = 1001$, and $c = 1000$, solving for r

$$r^2 + 1001r + 1000 = 0$$

$$\Rightarrow r^2 + r + 1000r + 1000 = 0$$

$$\Rightarrow r(r+1) + 1000(r+1) = 0$$

$$\Rightarrow (r+1000)(r+1) = 0$$

Two possible values for r

$$r = -1000$$

$$r = -1$$

These are the roots of the characeristic equation. We can write the general solution for $y(x)$ as a linear combination of these roots:

$$y(x) = c_1 e^{-x} + c_2 e^{-1000x}$$

With initial conditions:

$$y(0) = 1$$

$$\Rightarrow \quad c_1 + c_2 = 1 \quad \text{---} \quad (1)$$

$$y'(0) = 0$$

$$\frac{dy}{dx} = -c_1 e^{-x} - 1000\, c_2 e^{-1000x}$$

Now $x = 0$

$$\Rightarrow \quad -c_1 - 1000 c_2 = 0 \quad \text{---} \quad (2)$$

Adding Eq^n (1) and (2)

$$(c_1 - c_1) + (c_2 - 1000 c_2) = 1$$

$$\Rightarrow \quad -999 c_2 = 1$$

$$\boxed{c_2 = 1/-999}$$

$$c_1 - \frac{1}{999} = 1$$

$$\boxed{c_1 = 1 + \frac{1}{999}}$$

$$\boxed{y(x) = \left(1 + \frac{1}{999}\right) e^{-x} - \frac{1}{999} e^{-1000x}}$$

Euler's Explicit method $\rightarrow$ Forward Euler's Method

$$y_1 = y_0 + h\left[f(x_0, y_0)\right]$$

Backward Euler's method $\rightarrow$ Implicit Method

$$y_1 = y_0 + h\left[f(x_1, y_1)\right]$$

$$\frac{d^2y}{dx^2} = -1001\frac{dy}{dx} - 1000y$$

$y(0) = 1$

$y'(0) = 0$

$h = 0.5$

let, $\frac{dy}{dx} = z \Rightarrow z = 0$

$$\frac{dz}{dx} = -1001z - 1000y$$

$\Rightarrow 0 = 0 - 1000y$

$\Rightarrow y = 0$

$y'(0) = 0$

$\Downarrow$

$\frac{dy}{dx} = 0$

$f(x, y) \approx y = 0$

$y_1 = y_0 + hy_1$

$\Rightarrow y_1 - hy_1 = y_0$

$\Rightarrow y_1(1-h) = y_0$

$$y_1 = \frac{y_0}{1-h}$$

$$y_{n+1} = y_n + h^*\left(y_{n+1} - y_n\right)$$

Euler method (implicit).

The implicit euler method updates y at the next time step (y_{n+1}) based on its current time step y_n

$$y_{n+1} = y_n + h \cdot \frac{dy_{n+1}}{dx}$$

To estimate $\dfrac{dy_{n+1}}{dx}$, backward difference approximation

$$\frac{dy_{n+1}}{dx} \approx \frac{y_{n+1} - y_n}{h}$$

$$y_{n+1} = y_n + h \cdot \frac{y_{n+1} - y_n}{h}$$

$$y_{n+1} = y_n + (y_{n+1} - y_n)$$

The term $(y_{n+1} - y_n)$ on the right-hand side represents the change in y over the time interval h

$$\boxed{y_{n+1} = y_n + h^* (y_{n+1} - y_n)}$$

$x = 0$ to 5 $\quad h = 0.5$

$y(0) = 1$

$y'(0) = 0$

$$y_{i+1} = y_i + f(x_i, y_i)\,\Delta x$$

$$\frac{d^2y}{dx^2} = -100\,\frac{dy}{dx} - 1000\,y$$

Decompose into 2 1st order DE

$$\frac{dy}{dx} = z$$

$$\frac{dz}{dx} = -100\,z - 1000\,y$$

$$y'(0) = 0$$
$$\Downarrow$$
$$z(0) = 0$$

$$\frac{d^2y}{dx^2} = y'' = f(x, y, y')$$

$$\frac{dy}{dx} = y' = u$$

$$\frac{du}{dx} = u' = y'' = f(x, y, u)$$

$$y' = u \longrightarrow \frac{dy}{dx} = z$$

$$u' = f(x, y, u) \rightarrow \frac{dz}{dx} = -1001z - 1000y$$