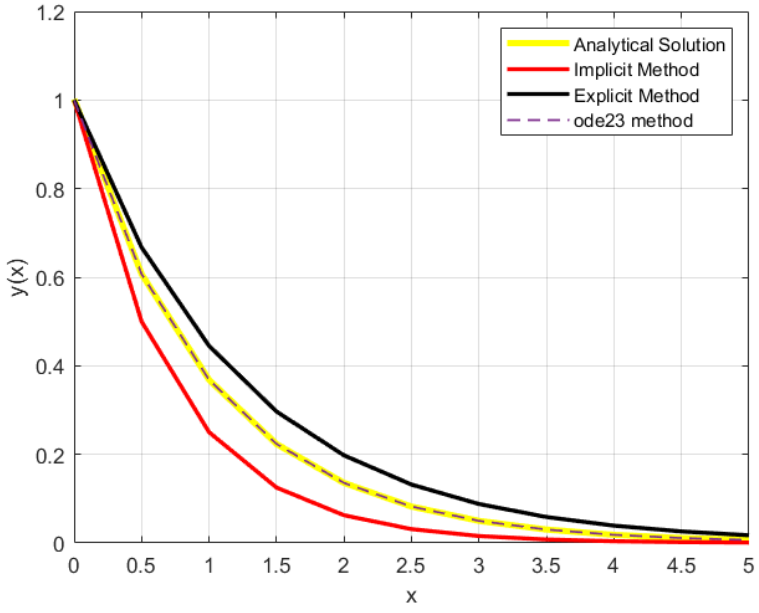
```matlab
%%%Analytical Solution based on the calculation%%%%%%%%
% Define the range of x values
x_values = 0:0.5:5;

% Define the constants C1 and C2
C1 = 1 + 1/999;
C2 = -1/999;

% Calculate the y values for the given x range
y_values = (C1 * exp(-x_values)) - (C2 * exp(-1000 *
x_values));

% Plot the solution
figure;
plot(x_values, y_values, 'yellow', 'LineWidth', 3);
xlabel('x');
ylabel('y(x)');
grid on;
hold on
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%
% Implicit Euler Method
% Define the step size and time span
h = 0.5;
x_span = 0:h:5;

% Initialize arrays to store the results
y_implicit = zeros(size(x_span));

% Set initial conditions
y_implicit(1) = 1;

% Define the parameters for the implicit Euler method
a = 1001;
b = 1000;

% Apply implicit Euler method
for n = 1:length(x_span) - 1
    x_n = x_span(n);

    % Implicit Euler method
    y_implicit(n+1) = (y_implicit(n) + h * (y_implicit(n+1)
- y_implicit(n)));
end
```

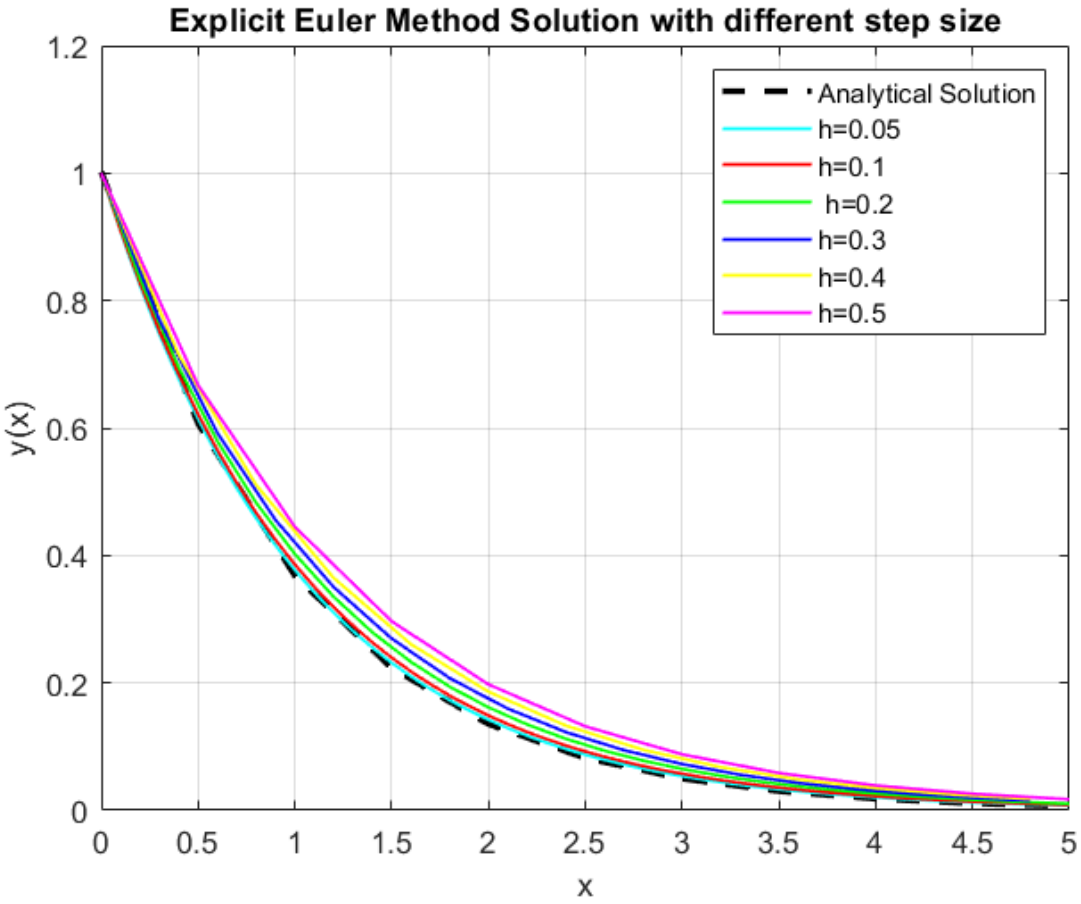
```
plot(x_span, y_implicit, 'r', 'LineWidth', 2);
grid on;
hold on
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%
syms y1 y2 y z t t1 t2 h z1 z2

S1 = z;

S2 =  -1001*z-1000*y;

SS1 = y2- y1 - h*subs(S1,{y z},[y2 z2]);

SS2 = z2- z1 - h*subs(S2,{y z},[y2 z2]);
EQN1 = SS1 ==0;
Y = solve(EQN1,y2);

EQN2 = SS2 ==0;
Z = solve(EQN2,z2);

H=0.5;
T=0:H:5;

YN(1)=1;
ZN(1)=0;

for i=2:length(T)
    EQ1=subs(EQN1,{y1 z1 h},[YN(i-1) ZN(i-1) H]);
    EQ2=subs(EQN2,{y1 z1 h},[YN(i-1) ZN(i-1) H]);
    [solx,soly] = solve(EQ1,EQ2);
    YN(i) = double(solx);
    ZN(i) = double(soly);

end
plot(T, YN, 'black', 'LineWidth', 2);
grid on;
hold on
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%
% Define the ODE as a function
ode = @(x, Y) [Y(2); -1001 * Y(2) - 1000 * Y(1)];

% Define the initial conditions
```

```
initial_conditions = [1; 0]; % y(0) = 1, and dy/dx(0) = 0

% Define the range of x values
x_range = 0:0.5:5; % Adjust the range as needed

% Use ode23 to solve the ODE
[x, Y] = ode23(ode, x_range, initial_conditions);

% Extract the solutions for y and z
y = Y(:, 1);
y=y';
x=x';
% Plot the solution
plot(x, y, '--', 'LineWidth', 1);

legend ('Analytical Solution','Implicit Method','Explicit
Method','ode23 method')

% Print the results
fprintf('-------------------------------------------------
----------\n');
fprintf('Solutions with different methods with step
size,h=0.5 \n')
fprintf('-------------------------------------------------
----------\n');
fprintf('x-value\t  y-analytical\t  y-implicit\t y-
explicit\t y-ode\n');
fprintf('-------------------------------------------------
--------\n');
for i = 1:length(T)
    fprintf('%4.1f\t\t %4.4f\t\t %4.4f\t\t %4.4f\t\t
%4.4f\n', T(i),y_values(i),y_implicit(i),YN(i), y(i));
end
```

Solutions with different methods with step size,h=0.5

x-value	y-analytical	y-implicit	y-explicit	y-ode
0.0	1.0020	1.0000	1.0000	1.0000
0.5	0.6071	0.5000	0.6673	0.6071
1.0	0.3682	0.2500	0.4449	0.3682
1.5	0.2234	0.1250	0.2966	0.2234
2.0	0.1355	0.0625	0.1977	0.1355
2.5	0.0822	0.0313	0.1318	0.0822
3.0	0.0498	0.0156	0.0879	0.0498
3.5	0.0302	0.0078	0.0586	0.0302
4.0	0.0183	0.0039	0.0391	0.0183
4.5	0.0111	0.0020	0.0260	0.0111
5.0	0.0067	0.0010	0.0174	0.0067




Explicit Euler Method Solution with different step size
Analytical Solution
h=0.05
h=0.1
h=0.2
h=0.3
h=0.4
h=0.5
y(x)
x
0
0.2
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5