Assignment #01

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Ⓐ function $f(x) = \dfrac{\ln(1+x)}{x}$

$$f(x) = \frac{1}{x} \cdot \ln(1+x)$$

let
$p(x) = \ln(1+x)$, and do Maclaurin's Series expansion of $p(x)$

now at, $x=0$
$P(0) = \ln(1) = 0$

Now, first derivative

$$p'(x) = \frac{1}{(1+x)} = (1+x)^{-1}$$

$$p'(0) = (1+0)^{-1} = \boxed{1}$$

and

$$p''(x) = -(1+x)^{-2}$$

and at $x=0$

$$p''(0) = -(1)^{-2} = \boxed{-1}$$

Now,

$$p'''(x) = 2(1+x)^{-3}$$

at $x=0$

$$p'''(0) = 2(1+0)^{-3} = 2(1)^{-3} = \boxed{2}$$

and $p^{iv}(x) = -6(1+x)^{-4}$

at $x = 0$

$p^{iv}(0) = -6(1)^{-4} = \boxed{-6}$

and

$p^{v}(x) = 24(1+x)$

and $p^{v}(x) = \boxed{24}$ at $x = 0$

So, Taylor expansion $p(x)$

$$p(x) = p(0) + xp'(0) + \frac{x^2}{2!}p''(0) + \frac{x^3}{3!}p'''(0) + \frac{x^4}{4!}p^{iv}(0) + \frac{x^5}{5!}p^{v}(0) + \cdots\cdots$$

$$p(x) = x(0) + x(1) + \frac{x^2}{2!}(-1) + \frac{x^3}{3!}(2) + \frac{x^4}{4!}(-6) +$$

$$\frac{x^5}{5!}(24) + \cdots$$

$$p(x) = 0 + x - \frac{x^2}{2} + \frac{x^3}{3}$$

$$- \frac{x^4}{4} + \frac{x^5}{5} + \cdots$$

Now, $f(x) = \frac{1}{x} p(x)$

$$f(x) = \frac{1}{x}\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + x^5/5 + \cdots\right)$$

$$\boxed{f(x) = 1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \frac{x^5}{5} + \cdots}$$

now,

Taylor expansion of $f(x)$

So, derivatives of $f(x)$

$$f'(x) = 0 - \frac{1}{2} + \frac{2}{3}x - \frac{3}{4}x^2 + x^4 + \cdots$$

$$f''(x) = \frac{2}{3} - \frac{\cancel{6}^{\,3}}{\cancel{4}_{\,2}}x + 4x^3 + \cdots$$

$$f''(x) = \frac{2}{3} - \frac{3}{2}x + 4x^3 + \cdots$$

$$f'''(x) = -\frac{3}{2} + 12x^2 + \cdots$$

Now at $x = 0$

$$f(0) = 1$$

$$f'(0) = -\frac{1}{2}$$

$f''(0) = \frac{2}{3}$

$f'''(0) = -\frac{3}{2}$

Soo, at $x=0$ Taylor Series expansion of $f(x)$

$$f(x) = f(0) + f'(0)(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3 + \cdots$$

$$f(x) = 1 - \frac{1}{2}x + \frac{2}{3\times 2!}x^2 - \frac{3}{2\times 3!}(x)^3 + \cdots$$

$$\boxed{f(x) = 1 - \frac{1}{2}x + \frac{1}{3}x^2 - \frac{1}{4}x^3 + \cdots}$$

Taylor Series of $f(x)$ upto ∞.

Let's come up with its general term.

$$f(x) = \frac{(-1)^0}{0+1}x^0 + \frac{(-1)^1}{1+1}x^1 +$$

$$\frac{(-1)^2}{2+1}x^2 + \frac{(-1)}{3+1}x^3 +$$

$$\cdots$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$$

Let, $a_n = \dfrac{(-1)^n}{n+1}$

So,

$$\boxed{f(x) = \sum_{n=0}^{\infty} a_n x^n}$$

Showed