

# Analytically:  $\int_{-3}^5 (4x-3)^3 dx$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$\Rightarrow \int_{-3}^5 (64x^3 - 144x^2 + 108x - 27) dx$$

Integrate each term separately

$$\begin{aligned} \int (64x^3 - 144x^2 + 108x - 27) dx &= 64 \cdot \frac{x^4}{4} - 144 \cdot \frac{x^3}{3} + 108 \cdot \frac{x^2}{2} - 27x \\ &= 16x^4 - 48x^3 + 54x^2 - 27x \end{aligned}$$

Evaluate the term from -3 to 5:

$$\Rightarrow [16 \cdot (5)^4 - 48 \cdot (5)^3 + 54 \cdot (5)^2 - 27 \cdot 5] - [16 \cdot (-3)^4 - 48 \cdot (-3)^3 + 54 \cdot (-3)^2 - 27 \cdot (-3)]$$

$$\Rightarrow [10000 - 6000 + 1350 - 135] - [1296 + 1296 + 486 + 81]$$

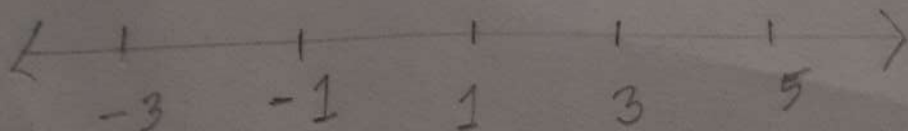
$$\Rightarrow 5215 - 3159$$

$$\Rightarrow \boxed{2056} \quad \text{Ans.}$$

# Trapezoidal  $T_n = \int_a^b f(x) dx$ , here,  $b = 5$   
 $a = -3$

let,  $n = 4$

$$\Delta x = \frac{b-a}{n} = \frac{5 - (-3)}{4} = \frac{8}{4} = 2$$



$$f(x) = (4x-3)^3$$

$$T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

$$= \frac{2}{2} [f(-3) + 2f(-1) + 2f(1) + 2f(3) + f(5)]$$

$$f(-3) = [4(-3)-3]^3 = -3375$$

$$f(5) = (20-3)^3 = 4913$$

$$f(-3) + f(5) = -3375 + 4913 = 1538$$

$$f(-1) = [4(-1)-3]^3 = [-4-3]^3 = -343$$

$$f(1) = 1$$

$$f(3) = 729$$

$$= 1 [1538 + 2(-343) + 2(1) + 2(729)]$$

$$= [1538 + 2 + 1458 - 686]$$

$$= \boxed{2312}$$

# Simpson's  $\frac{1}{3}$  Rule:

$$S_n = \frac{h}{3} \left[ f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n) \right]$$

$$= \frac{2}{3} \left[ f(-3) + 4f(-1) + 2f(1) + 4f(3) + f(5) \right]$$

$$f(-3) + f(5) = 1538$$

$$= \frac{2}{3} \left[ 1538 + 4(-343) + 2(1) + 4(729) \right]$$

$$= \frac{2}{3} \left[ 4456 - 1372 \right]$$

$$= \frac{2}{3} (3084)$$

$$= \boxed{2056}$$