

→ Gauss Elimination Method with Partial Pivoting:

$$\begin{array}{l} 2x - 6y - 2z = -38 \\ 2x - y + 7z = -34 \\ -8x + y - 2z = -20 \end{array}$$

$$\text{Step 1: } \left[\begin{array}{ccc|c} 2 & -6 & -2 & -38 \\ 2 & -1 & 7 & -34 \\ -8 & 1 & -2 & -20 \end{array} \right] \xrightarrow{\text{Row } 1 \leftrightarrow \text{Row } 3} \left[\begin{array}{ccc|c} -8 & 1 & -2 & -20 \\ 2 & -6 & -2 & -38 \\ 2 & -1 & 7 & -34 \end{array} \right]$$

$$\text{Step 2: } \left[\begin{array}{ccc|c} 2 & -6 & -2 & -38 \\ -3 & -1 & 7 & -34 \\ -8 & 1 & -2 & -20 \end{array} \right] \xrightarrow{\text{Row } 2 \leftrightarrow \text{Row } 3} \left[\begin{array}{ccc|c} 2 & -6 & -2 & -38 \\ -8 & 1 & -2 & -20 \\ -3 & -1 & 7 & -34 \end{array} \right]$$

Step 3: 1st pivot element "2", therefore the first row remain the first

$$\begin{aligned} \text{Row:} \quad & \left[\begin{array}{ccc|c} 2 & -6 & -2 & -38 \\ -8 & 1 & -2 & -20 \\ -3 & -1 & 7 & -34 \end{array} \right] \\ & \rightarrow \left(R_1 \rightarrow \frac{R_1}{2} \right) \quad \left[\begin{array}{ccc|c} 1 & -3 & -1 & -19 \\ -8 & 1 & -2 & -20 \\ -3 & -1 & 7 & -34 \end{array} \right] \\ & \rightarrow \left(R_2 \rightarrow R_2 + 8R_1 \right) \quad \left[\begin{array}{ccc|c} 1 & -3 & -1 & -19 \\ 0 & -25 & -8 & -156 \\ -3 & -1 & 7 & -34 \end{array} \right] \\ & \rightarrow \left(R_3 \rightarrow R_3 + 3R_1 \right) \quad \left[\begin{array}{ccc|c} 1 & -3 & -1 & -19 \\ 0 & -25 & -8 & -156 \\ 0 & 8 & 4 & -101 \end{array} \right] \end{aligned}$$

4. Write system of equations

5. Get solution by back substitution.

$$\begin{aligned} R_2 &\rightarrow R_2 + 3R_1 \\ R_3 &\rightarrow R_3 + 8R_1 \end{aligned}$$

$$[A:0] \sim \left[\begin{array}{ccc|c} 1 & -3 & -\frac{9}{2} & -19 \\ 0 & -10 & 5.5 & -9.1 \\ 0 & -23 & -6 & -14.2 \end{array} \right]$$

* 2nd pivot element: " = 10 "

$$R_3 \Rightarrow R_3 - R_2 \times \frac{23}{10}$$

$$[A:0] \sim \left[\begin{array}{ccc|c} 1 & -3 & -\frac{9}{2} & -19 \\ 0 & -10 & 5.5 & -9.1 \\ 0 & 0 & -18.65 & 37.3 \end{array} \right]$$

$$-172 + 209.3$$

$$\Rightarrow 2 \cdot 4 - 6 \cdot 8 = (-2)$$

$$\Rightarrow 8 - 48 + 2$$

$$\Rightarrow -40 + 2$$

$$= -38$$

$$\text{Similarly,}$$

$$-3x - \frac{y}{2} + \frac{z}{4} = -3.4$$

$$\Rightarrow$$

$$12 - z - 14$$

$$\Rightarrow 12 - z - 14$$

$$\Rightarrow -34$$

$$\Rightarrow 2 \cdot 4 - 6 \cdot 8 = -20$$

$$\Rightarrow 8 - 48 + 2 = -40$$

$$\Rightarrow -32 + 8 + 4 = -20$$

$$\Rightarrow -20$$

$$\Rightarrow -20$$

$$\Rightarrow x = 4$$

$$\begin{aligned} -10y + 5.5 \times (-2) &= -91 \\ -10y - 11 &= -91 \\ -10y - 11 + 11 &= -91 + 11 \\ -10y &= -80 \\ y &= 8 \end{aligned}$$

Just clearing the answer

Now final step

$$x = 4$$

$$y = 8$$

$$z = 14$$

$$2x - 6y + 2 = -3.8$$

$$2x - 6y + 2 = -3.8$$

$$\Rightarrow 2 \cdot 4 - 6 \cdot 8 = (-2)$$

$$\Rightarrow 8 - 48 + 2$$

$$\Rightarrow -40 + 2$$

$$= -38$$

$$\text{Similarly,}$$

$$-3x - \frac{y}{2} + \frac{z}{4} = -3.4$$

$$\Rightarrow 12 - z - 14$$

$$\Rightarrow 12 - z - 14$$

$$\Rightarrow -34$$

$$\Rightarrow 2 \cdot 4 - 6 \cdot 8 = -20$$

$$\Rightarrow 8 - 48 + 2 = -40$$

$$\Rightarrow -32 + 8 + 4 = -20$$

$$\Rightarrow -20$$

Step 5:
From equation (3)

$$2 = -2$$

From Equation (2)

$$x = 4$$

$$\begin{aligned} -10y + 5.5 \times (-2) &= -91 \\ -10y - 11 &= -91 \\ -10y - 11 + 11 &= -91 + 11 \\ -10y &= -80 \\ y &= 8 \end{aligned}$$