

(1)

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(ECE 611)

Assignment 05

$$\frac{d^2y}{dx^2} = -1000 \frac{dy}{dx} - 1000y$$

We assume the solution of the form  $y(x) = e^{rx}$

Substituting this in the ODE, we get:

$$r^2 e^{rx} - 1001 r e^{rx} - 1000 e^{rx}$$

We can divide through  $e^{rx}$

$$r^2 = -1001r - 1000$$

This is a quadratic equation in  $r$ , we solve it

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here,  $a = 1$ ,  $b = -1001$ , and  $c = -1000$ , solving for  $r$

$$r^2 + 1001r + 1000 = 0$$

$$\Rightarrow r^2 + r + 1000r + 1000 = 0$$

$$\Rightarrow r(r+1) + 1000(r+1) = 0$$

$$\Rightarrow (r+1000)(r+1) = 0$$

Two possible values for  $r$

$$r_1 = -1000$$

$$r_2 = -1$$

These are the roots of the characteristic equation. We can write the general solution for  $y(x)$  as a linear combination of these roots:

$$y(x) = c_1 e^{-x} + c_2 e^{-1000x}$$

With initial conditions:

$$\begin{aligned} y(0) &= 1 \\ \Rightarrow c_1 + c_2 &= 1 \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} y'(0) &= 0 \\ \frac{dy}{dx} &= -c_1 e^{-x} - 1000 c_2 e^{-1000x} \end{aligned}$$

$$\begin{aligned} \text{Now } x &\geq 0 \\ \Rightarrow -c_1 - 1000 c_2 &= 0 \quad \text{--- (2)} \end{aligned}$$

Adding eqn (1) and (2)

$$\begin{aligned} (c_1 - c_1) + (c_2 - 1000 c_2) &= 1 \\ \Rightarrow -999 c_2 &= 1 \end{aligned}$$

$y(x) = \left(1 + \frac{1}{999}\right) e^{-x} - \frac{1}{999} e^{-1000x}$

$c_2 = \frac{1}{-999}$

$c_1 - \frac{1}{999} = 1$

$c_1 = 1 + \frac{1}{999}$

Euler's Explicit method  $\rightarrow$  Forward Euler's Method

$$y_1 = y_0 + h \left[ f(x_0, y_0) \right]$$

Backward Euler's method  $\rightarrow$  Implicit Method

$$y_1 = y_0 + h \left[ f(x_1, y_1) \right]$$

$$\frac{d^2y}{dx^2} = -100x^2 - 1000y$$

$$\text{let, } \frac{dy}{dx} = z \Rightarrow z = 0$$

$$\frac{d^2}{dx^2} = -100z^2 - 1000y$$

$$\Rightarrow 0 = 0 - 1000y$$

$$\Rightarrow y = 0$$

$$f(x, y) \approx y = 0$$

$$y(0) = 1$$

$$y'(0) = 0$$

$$h = 0.5$$

$$y'(0) = 0$$

$$\frac{dy}{dx} = 0$$

$$y_1 = y_0 + hy_1$$

$$y_{n+1} = y_n + h \star (y_{n+1} - y_n)$$

$$\Rightarrow y_1 - hy_1 = y_0$$

$$\Rightarrow y_1(1-h) = y_0$$

$$y_1 = \frac{y_0}{1-h}$$

Euler method (implicit).

The implicit euler method updates  $y$  at the next time step  $(y_{n+1})$  based on its current time step  $y_n$

$$y_{n+1} = y_n + h \cdot \frac{dy_{n+1}}{dx}$$

To estimate  $\frac{dy_{n+1}}{dx}$ , backward difference approximation

$$\frac{dy_{n+1}}{dx} \approx \frac{y_{n+1} - y_n}{h}$$

$$y_{n+1} \Rightarrow y_n + h \cdot \frac{y_{n+1} - y_n}{h}$$

$$y_{n+1} = y_n + (y_{n+1} - y_n)$$

The term  $(y_{n+1} - y_n)$  on the right-hand side represents the change in  $y$  over the time interval  $h$

$$\boxed{y_{n+1} = y_n + h \cdot (y_{n+1} - y_n)}$$

$$x = 0 \rightarrow 5$$

$$h = 0.5$$

$$y(0) = 1$$

$$y'(0) = 0$$

$$y_{i+1} = y_i + f(x_i, y_i) \Delta x$$

$$\frac{d^2y}{dx^2} = -100 \frac{dy}{dx} - 1000y$$

Decompose into 1st order DE

$$\frac{dy}{dx} = z$$

$$\frac{dz}{dx} = -100z - 1000y$$

$$\begin{aligned} y'(0) &= 0 \\ \Downarrow \\ z(0) &= 0 \end{aligned}$$

$$\frac{d^2y}{dx^2} = y'' = f(x, y, y')$$

$$\frac{dy}{dx} = y' = u$$

$$\frac{du}{dx} = u' = y'' = f(x, y, u)$$

$$\begin{array}{ccc} y' = u & \xrightarrow{\quad} & \frac{dy}{dx} = z \\ u' = f(x, y, u) & \rightarrow & \frac{dz}{dx} = -100z - 1000y \end{array}$$