HW#5 Solution

21.8 Integrate the following function using the trapezoidal rule, with n = 1,2,3,4:

$$\int_1^2 (x+1/x)^2\,dx$$

Compute percent relative errors with respect to the true value of 4.8333 to evaluate the accuracy of the trapezoidal approximations.

21.8 as an example for

$n = 4$

$x_0 = 1$ $\quad f(x_0) = 4.0$

$x_1 = 1.25$ $\quad f(x_1) = 4.2025$

$x_2 = 1.5$ $\quad f(x_2) = 4.6944$

$x_3 = 1.75$ $\quad f(x_3) = 5.389$

$x_4 = 2.0$ $\quad f(x_4) = 6.25$

$$I = (2-1)\frac{\left[4+2(4.2025+4.6944+5.389)+6.25\right]}{8}$$

$$= 4.852744$$

$$\epsilon_t = \frac{4.8333-4.8527}{4.8333}\times 100$$

$$= -0.40$$

n	I	ϵ_t %
1	5.125	-6.06
2	4.9097	-1.58
3	4.8677	-0.71
4	4.8527	-0.40

21.9 Integrate the following function both analytically and using Simpson's rules, with n = 4 and 5:

$$\int_{-3}^{5} (4x+5)^3\,dx$$

Discuss the results.

21.9 $\int_{-3}^{5} (4x+5)^3\,dx$

$$= \left.\frac{(4x+5)^4}{16}\right]_{-3}^{5} = 24264$$

$\underline{n = 4}$

$$I = (5-(-3))\frac{\left[-343+4(1+4913)+2(729)+15625\right]}{12}$$

$$= 24264$$

n = 5 use one application of 1/3 and 3/8 Rules

$$I = (.2-(-3))\left[\frac{-343+4(-0.2156)+195.1}{6}\right]$$

$$+ (5-0.2)\left[\frac{195.1+3(1815.8+6434.9)+15625}{8}\right]$$

$$= -79.3344 + 24343.34$$

$$= 24264.006$$

Both are exact (except for roundoff error) because f(x) is 3rd order

22.1 Use Romberg integration to evaluate

$$\int_1^2 (x+1/x)^2\,dx$$

to an accuracy of $\varepsilon_s = 0.5\%$. Your results should be presented in the form of Fig. 22.3. Use the true value of 4.8333 to determine the true error ε_t of the result obtained with Romberg integration. Check that ε_t is less than the stopping criterion ε_s.

22.1

	$\varepsilon_a = 5.3\%$	-0.09%
5.125	4.837963	$\underline{\underline{4.83347}}$
4.909722	4.833751	
4.852744		

$$\varepsilon_t = \left[\frac{4.8333 - 4.83347}{4.8333}\right] \times 100 = -0.0097\%$$

22.4 Obtain an estimate of the integral form Prob. 22.1, but using two-, three-, and four-point Gauss-Legendre formulas. Compute ε_t for each case on the basis of the analytical solution.

22.4

two point:

$$f(x) = x + \frac{1}{x}$$

$$y = \frac{(2+1)}{2} + \frac{(2-1)}{2}x$$

$$dy = \left(\frac{2-1}{2}\right)dx$$

$$\int_1^2 f(x)\,dx = \int_{-1}^{1} f(y)\,dy$$

$$I = 2.074414(1) + 2.75596(1)$$

$$= 4.830374 \qquad \epsilon_t = 0.06\%$$

three point:

$$I = 2.022896(0.555555) + 2.347222(0.88888) + 2.921322(0.555555)$$

$$= 4.833208 \qquad \epsilon_t = 0.0019\%$$

four point:

$$I = 2.009026(0.3478548) + 2.16712(0.6521452) + 2.9977(0.3478548) + 2.573719(0.6521452)$$

$$= 4.833329 \qquad \epsilon_t = -0.0006$$

23.2 Repeat Prob. 23.1, but for $y = \log x$ evaluated at $x = 20$ with $h = 2$.

23.2

	x	$f(x)$
x_{i-2}	16	1.204
x_{i-1}	18	1.256
x_i	20	1.301
x_{i+1}	22	1.342
x_{i+2}	24	1.380

	first order	second order
Foward	0.0205 (5.5%)	0.02125 (2.1%)
Back	0.0225 (−3.7%)	0.02075 (4.4%)
Center	0.0215 (0.9%)	0.02133 (1.7%)

$$\text{true} = \frac{\log_{10} e}{20} = 0.02171$$

23.5 Repeat Prob. 23.4, but for the first derivative of ln x at $x = 4$ using $h_1 = 2$ and $h_2 = 1$.

23.5

	x	$f(x)$
x_{i-2}	2	0.693
x_{i-1}	3	1.099
x_i	4	1.386
x_{i+1}	5	1.609
x_{i+2}	6	1.792

$$D(1) = \frac{1.609 - 1.099}{2} = 0.255$$

$$D(2) = \frac{1.792 - 0.693}{4} = 0.274$$

$$D = \frac{4}{3}(0.255) - \frac{1}{3}(0.274) = 0.2487$$