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LING 570 HW #2
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Q1 (2)

These two commands have the same results.
The first command generates the string in `fsa7` with the highest probability calculated from `wfst1`.The second command takes the input string from `wfst1_test`, which has the same content for the file `fsa7`. In the `carmel` command, `-s` switch means take the transducer in the standard input. In this case, it is `wfst1`. `-l` means the standard string input would be passed to the left-hand side of the transduction sequence in the input transducer. `-i` means that the standard input form is a string.

Since the string input has the same content as the FSA input, the results for both commands are the same.

Q2-4

Please refer to the corresponding file in the submission. All coding files are written in `Python 3`. In Q4, I use the first algorithm, converting NFA to DFA for implementation.

Q5

$P(4HEADS) = \binom{5}{4} P(HHHHT) = 5 \times 0.8^4 \times 0.2^4 = 0.4096$
$P(5HEADS) = P(HHHHH) = 0.8^5 = 0.32768$
$P(ATLEAST4HEADS) = P(4HEADS) + P(5HEADS) = 0.4096 + 0.32768 = 0.73728$

Q6

(a) $P(X = 0) = P(X = 0, Y = 1) + P(X = 0, Y = 0) = 0.1 + 0.5 = 0.6$
$P(X = 1) = P(X = 1, Y = 1) + P(X = 1, Y = 0) = 0.15 + 0.25 = 0.4$

	$X = 0$	$X = 1$
$P(X)$	0.6	0.4

(b) $P(Y = 0) = P(X = 0, Y = 0) + P(X = 1, Y = 0) = 0.5 + 0.25 = 0.75$
$P(Y = 1) = P(X = 0, Y = 1) + P(X = 1, Y = 1) = 0.1 + 0.15 = 0.25$

	$Y=0$	$Y=1$
$P(Y)$	0.75	0.25

(c) $P(Y=0|X=0) = P(X=0, Y=0)/P(X=0) = 0.5/0.6 = 5/6$
$P(Y=1|X=0) = P(X=0, Y=1)/P(X=0) = 0.1/0.6 = 1/6$
$P(Y=0|X=1) = P(X=1, Y=0)/P(X=1) = 0.25/0.4 = 5/8$
$P(Y=1|X=1) = P(X=1, Y=1)/P(X=1) = 0.15/0.4 = 3/8$

	$P(Y=y|X=0)$	$P(Y=y|X=1)$
Y=1	1/6	3/8
Y=0	5/6	5/8

(d) No, they are not independent. Suppose they are independent, they should satisfy $P(X=1, Y=1) = P(X=1) \times P(Y=1)$. However,
$P(X=1) \times P(Y=1) = 0.4 \times 0.25 = 0.1 \neq P(X=1, Y=1)$

Q7

(a) In order to calculate the probability of getting a head for one toss, we need to sum up the probability of getting a head for 3 different coins respectively. Since choosing a coin also has its own probability, we need to multiply it respectively by Multiplication Rule. Thus,
$P(1HEAD) = 0.2 \times 0.1 + 0.5 \times 0.4 + 0.7 \times 0.3 = 0.43$

(b) This is the same as calculating the probability of the coin being C1 if the toss is head. In order to calculate the conditional probability, we have to calculate the probability of head for one toss, which has been done in `(a)`. Also, we have to calculate the probability of choosing c1 and a head toss. Thus,
$P(C = c1|Toss = Head) = \frac{P(C=c1, Toss=Head)}{P(Toss=head)} = \frac{0.2 \times 0{,}1}{0.43} = \frac{2}{43} \approx 0.0465$