Tutorial 5: Optical Flow

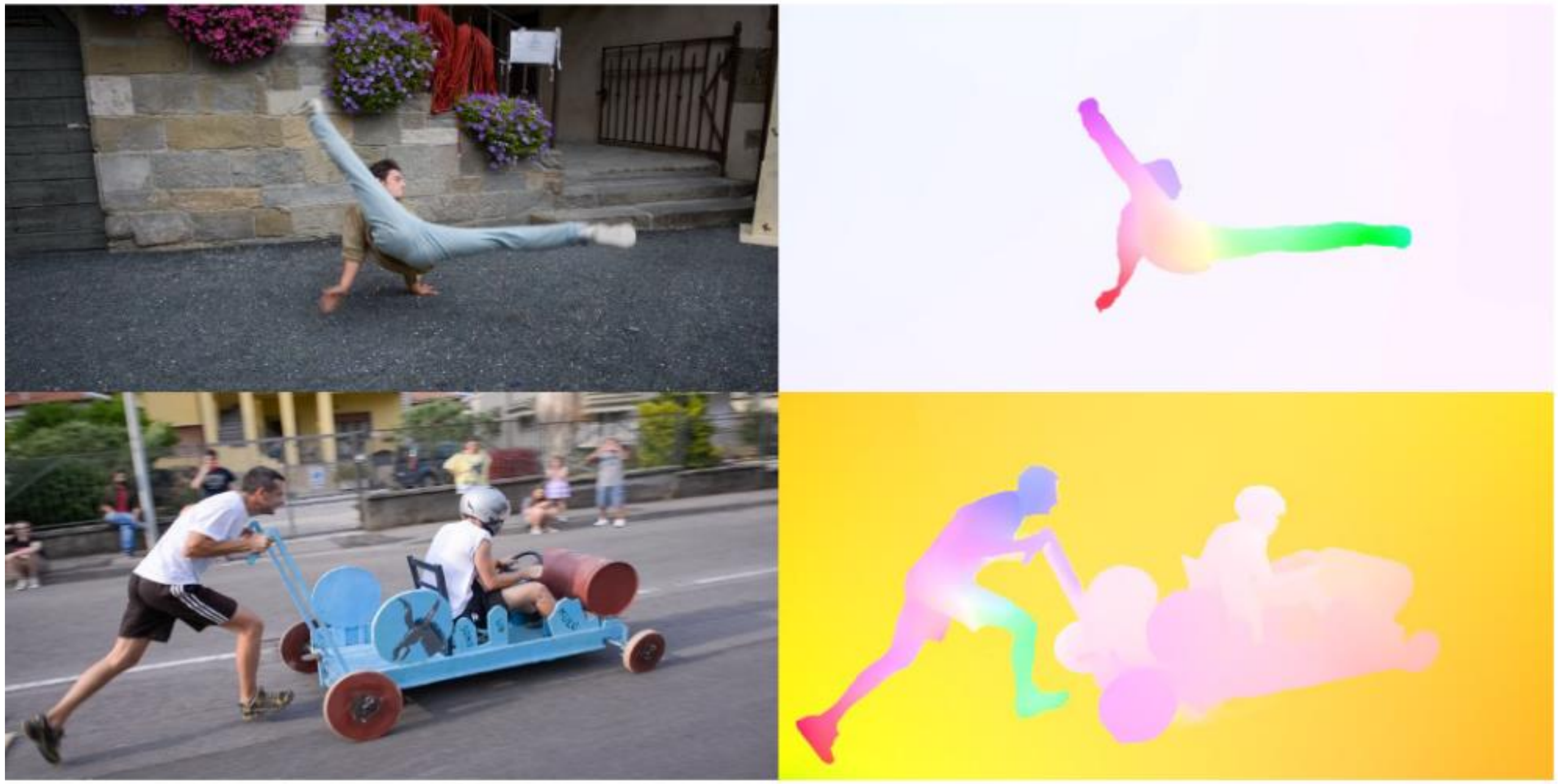

Image Gradient

- Consider the image as a function in terms of x, y, t
- Then by finite difference, then

$$I_x = \frac{f(x + \delta x, y, t) - f(x, y, t)}{\delta x} = I_i(x + 1, y) - I_i(x, y)$$

- In the exercise, to make it smoother, take the average over two images and the neighbor grids

Lucas-Kanade with Pyramids

- One key step for the algorithm to work is that we want to use the smaller image in the pyramid to reduce the optical motion ➔ estimate the flow for warped image
- For the smaller image, estimate the optical flow

$$I'(x, y, t) = I'(x + u', y + v', t + 1)$$
$$\Rightarrow I(x, y, t) \approx I(x + 2u', y + 2v', t + 1)$$
$$I_{warp}(x + 2u', y + 2v') = I(x, y, t)$$

- Then, for the warped image, the optical small is much smaller than the original flow (small motion again!)

Lucas-Kanade with Pyramids (Cont.)

- But what estimated is the optical flow between the warped image and the original image ➔ residual flow

$$I_{warp}(x, y, t) = I(x + u_{warp}, y + v_{warp}, t + 1)$$

- To restore the original optical flow: add back the estimated flow

$$u_{ori} = 2u' + u_{warp}$$
$$v_{ori} = 2v' + v_{warp}$$

- In fact, this is only the approximation (bonus question: how to get a more accurate result?)

Questions

Q: What is the influence of window size and why?

Q: What do you need to change to make it a frame interpolation algorithm?

Questions

e) To determine optical flow uniquely a certain number of equations is necessary. Knowledge about the motion between consecutive frames could reduce the number of equations we need. In the following cases, how many equations do we need at least to determine the optical flow between two 10×10 images uniquely? Explain why.

i) all pixels move in the same direction in 2D but with different magnitudes.

2 pts.

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101: One magnitude per pixel and the angle of the direction.

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Questions

ii) the second frame is obtained by warping the first frame with a 2D affine transformation, i.e. pixel in the first frame at $\begin{bmatrix} x, y \end{bmatrix}$ is moved to $\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} a_5 \\ a_6 \end{bmatrix}$ in the second frame. **2 pts.**

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6. 6 for the global motion parameters.

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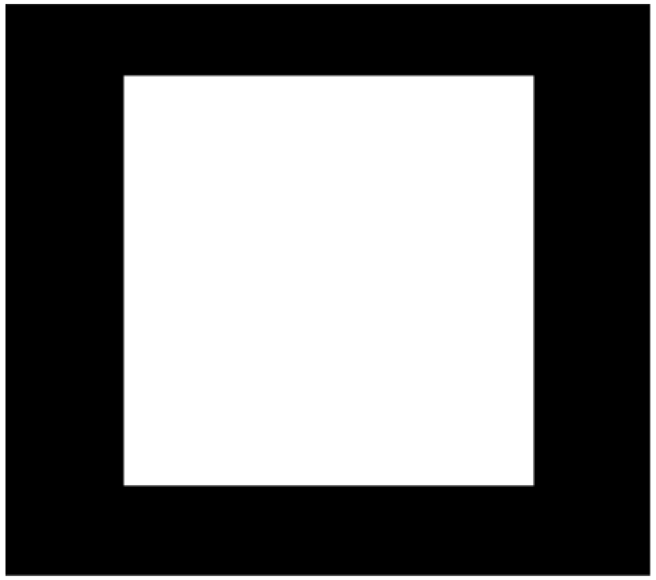
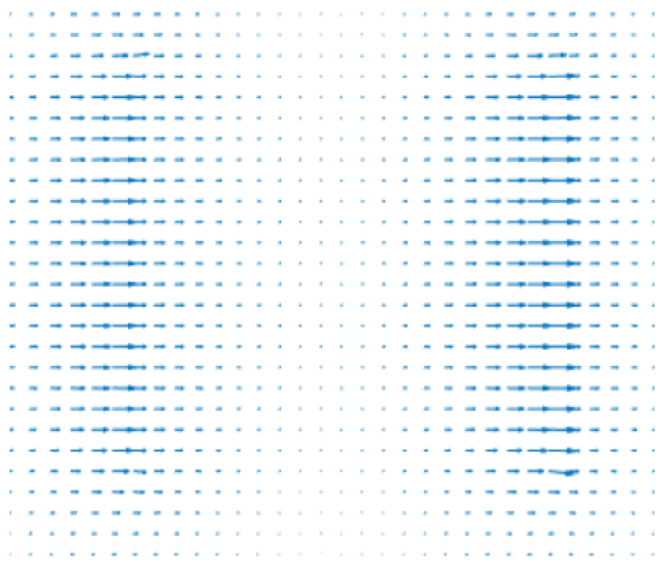
Questions

f) In the following example, the white square in the input frame moves to the right by 1 pixel in the next frame. We apply both Horn-Schunck algorithm and Lucas-Kanade algorithm to estimate the resulting optical flow and obtain different results as shown below (a and b, the small arrow at each location illustrates both the direction and magnitude of the estimated flow). Which of the results is more likely to be produced by Horn-Schunck algorithm and which one is from Lucas-Kanade algorithm? Justify your answer.

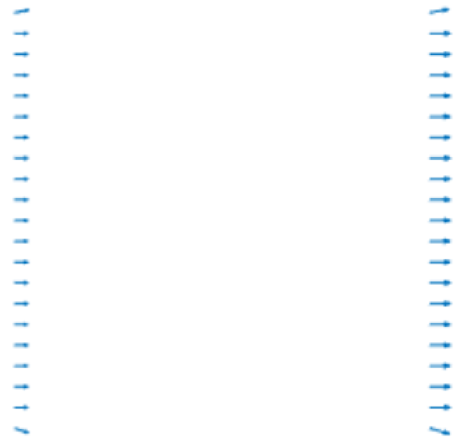
2 pts.



input frame



(a)



(b)

Horn-Schunk Algorithm

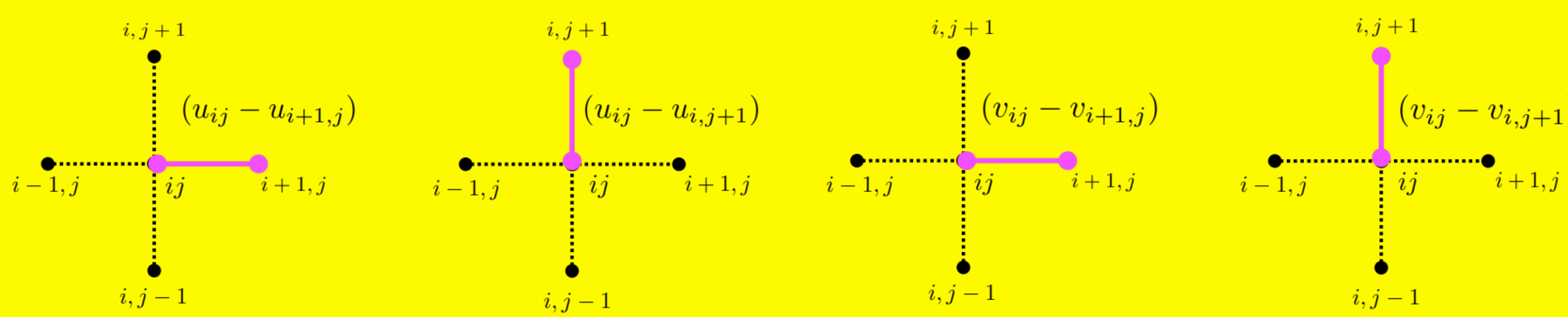
- The overall estimated optical flow should be smooth

$$\min_{\boldsymbol{u},\boldsymbol{v}} \sum_{i,j} \left\{ E_s(i,j) + \lambda E_d(i,j) \right\}$$

Brightness constancy $$E_d(i,j) = \left[I_x u_{ij} + I_y v_{ij} + I_t \right]^2$$

Smoothness

$$E_s(i,j) = \frac{1}{4}\left[(u_{ij} - u_{i+1,j})^2 + (u_{ij} - u_{i,j+1})^2 + (v_{ij} - v_{i+1,j})^2 + (v_{ij} - v_{i,j+1})^2 \right]$$



why not all four neighbors?

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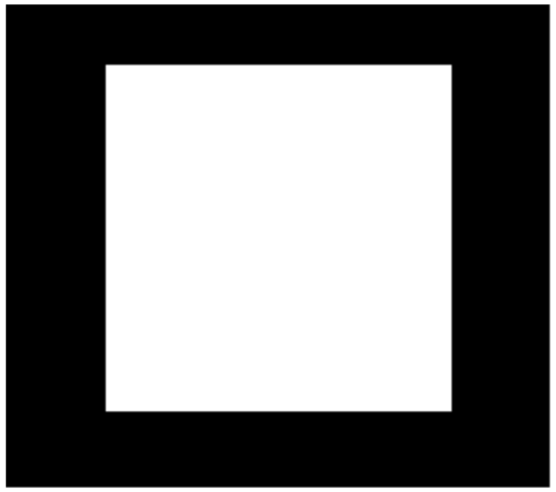
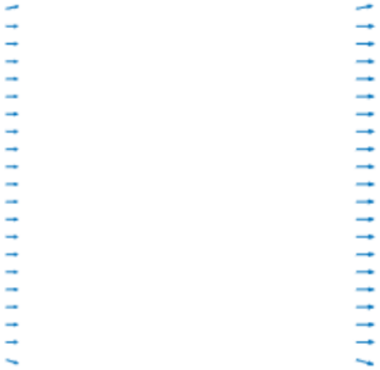


input frame

(a)



(b)

ANSWER

a is the result of Horn-Schunck algorithm since the flow spread from corners and edges.

ANSWER