Introduction to Perturbative QCD

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Abstract

This is a written version of two lectures given at the First School on Field Theory and Gravitation in Vitoria (Brasil) April 15-19, 1997. We discuss the foundation of QCD as the theory of strong interactions and the perturbative analysis of e^+e^- annihilation to hadrons. Typical concepts of perturbative QCD studies, such as collinear singularities, jets, Sudakov form factors, are explained working out this case.

1 Introduction

Quantum Chromodynamics (QCD), introduced by Gell-Mann and Frizsch in 1972 [13], is the current theory of strong interactions. It is a renormalizable nonabelian gauge theory [29] based on the group SU(3), containg quark and gluons as elementary fields,

$$L = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \sum_f \overline{q}_f(i\gamma_\mu D^\mu - m_f)q_f \tag{1}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$ is the field tensor, $D_\mu = \partial_\mu + ig A^a_\mu t^a$ is the covariant derivative, t^a are the Gell-Mann matrices, f^{abc} are the structure constants of SU(3) and f denotes a quark flavor.

Strong interactions present many different phenomena and QCD at present cannot account for all of them. To give an example, QCD is irrelevant to the entire field of low energy nuclear physics. This situation is to be contrasted with QED, where most processes can be computed with high accuracy.

In sec. 2 we discuss the foundation of QCD, i.e. why we believe we have the correct theory (the lagrangian (1)) even though many phenomena are not described or computed inside it.

In sec. 3 we discuss the fundamental property of QCD, asymptotic freedom, according to which QCD approaches a *free* theory in the ultraviolet region. This property explains parton model assumptions and allows perturbative computations as corrections to the free quark behaviour.

In sec. 4 a qualitative discussion is presented of quark confinement, according to which quarks do not exist as isolated particles, but only in bound states, the observed hadrons. This phenomenon cannot be described by definition in perturbative QCD, which assumes that asymptotic states, the hadrons, are infinitesimally close to the free ones, quarks and gluons. There are indications of confinement from lattice (non perturbative) QCD computations, but a rigorous proof is missing. This section describes some qualitative ideas about confinement, to understand why perturbative QCD can be applied despite confinement. We discuss also the production of hadronic jets, a typical phenomeon of high energy hadronic processes. The relation with confinement is explained, and also why we believe that a perturbative QCD computation can describe seminclusive jet properties.

In sec. 5 we consider the production of hadrons in e^+e^- annihilation at high energy as a typical application of perturbative QCD. This is a sim-

pler process than lepton-hadron or hadron-hadron scattering, so it is a good starting point for introducing the reader to perturbative QCD. The total hadronic cross section is perhaps the easiest thing to compute. Furthermore, the assumptions on hadronization to apply perturbative QCD are minimal: we require only that hadronization does not change the probability of the partonic process.

Later we study the structure of the final hadronic state, which reveals many typical perturbative QCD phenomena, like collinear singularities, jets, infrared effects to be factorized, etc. In the latter case a stronger assumption about hadronization has to be made: the momentum flow of the partonic event has to remain substantially the same after hadronization.

There is also a section with the conclusions and an appendix describing the computation of e^+e^- annihilation into three jets.

2 Foundation

QCD is a non abelian gauge theory based on the group SU(3) of color. The main motivation for a non abelian gauge theory came from the parton model of deep inelastic scattering, according to which quarks behave like free particles in hard collisions. This model has been successively extended, with similar success, to describe also hard hadron-hadron collisions and hadronic production in e^+e^- annihilation at high energy (sec. 5). The hystory of deep inelastic scattering and the ideas of the parton model are sketched in sec. 2.1.

The choice of the gauge group and of the quark representation is motivated by various phenomena which are discussed in sec. 2.2.

We believe that all these different phenomena, collectively, give a good support to the structure of QCD as the theory of strong interactions.

2.1 The Parton Model

A series of experiments were performed at SLAC in the sixties to understand the structure of the proton and of the neutron, measuring the so called electric and magnetic form factors. Electrons with an energy up to 20 GeV were sent against a target of hydrogen or deuterium N

$$e + N \to e + X$$
 (2)

where X is any hadronic final state.

The astonishing result was that a larger number than expected of large angle deflections of the electron were observed. Feynman gave a simple phenomenological explanation for this result: the nucleon has to be considered in deep inelastic collisions as a gas of non interacting pointlike particles, the partons; the electron simply suffers an elastic collision with a parton p

$$e + p \to e + p \tag{3}$$

A pointlike cross section has not the form factor suppression of an extended object. We have therefore 'hard' interactions, with large angle deflections of the electron, as observed. To have a physical picture, consider the nucleon in a frame in which it has a relativistic velocity ($\gamma_{Lorentz} \gg 1$), for example the rest frame of the colliding electron. Due to the Lorentz boost, the nucleon looks like a bunch of collinear partons, each carrying a fraction x_q of the nucleon momentum, such that

$$\sum_{q} x_q = 1. (4)$$

A hadron is represented by a function (partonic density) q(x) telling how many partons dn there are with momentum fraction x:

$$dn(x \div x + dx) = q(x)dx \tag{5}$$

The observed hadronic cross section is the convolution of the parton density with the pointlike, partonic cross section.

$$\sigma_{hadr} = q * \sigma_{part} \tag{6}$$

Since the pointlike cross section assumes free partons, all the dynamical effects of strong interactions are contained in the specific form of q(x). We see here the idea of factorization at work: different dynamical processes are represented by separated factors in the cross section. This idea has been gathered and generalized by perturbative QCD, in many different forms: factorization of mass singularites, Operator Product Expansion [31] (for a detailed discussion see for example [21]), Sudakov form factors (see sec. 5.3 and ref. [10]), etc..

Parton model was also able to explain a phenomenological law, the so called Bjorken scaling, i.e the independence of the form factors on Q^2 , which coincide just with the parton densities.

These partons were identified with the quarks introduced to classify the variety of hadrons discovered and their spectroscopy. Partons were assigned a spin s = 1/2, an electric charge e = 2/3, -1/3, a flavor, etc. That gave rise to a series of sum rules relating parton densities of different hadrons, which were experimentally satisfied.

This identification between 'low energy' and 'high energy' entities came out to be non trivial. The valence quarks, the degrees of freedom of the quark model, were not sufficient to account for momentum conservation in e-N collisions. The sum rule (4) seemed to be violated, the right hand side being substantially less than one. It was necessary to postulate the existence of 'sea' quarks, in addition to the valence quarks, short living quark-antiquark pairs in the nucleon, partecipating to the hard processes. This idea was quite reasonable from the quantum field theory point of view, dealing with virtual particles form the very beginning, but was still incomplete. Half of the momentum of the nucleons was not carried neither by the valence quarks nor by the sea quarks because:

$$\sum_{val} x + \sum_{sea} x \simeq 0.5 \tag{7}$$

It was necessary to admit the existence of neutral particles in the nucleon not interacting with the probe (the γ , W or Z), i.e. without electric or weak charge. These components were naturally identified with the gluons after the rise of QCD.

2.2 The Color

Nonabelian gauge theories are a good candidate to describe strong interactions because of asymptotic freedom (they are indeed a *unique* candidate in the framework of relativistic local field theories). The choice of the gauge group is motivated by a variety of phenomena, some of which are listed below (see also [21] and the lectures of B. Mele at this School for a more recent compilation). These phenomena *per se* require only the color group as an (exact) global symmetry. Color enters as a multiplicity factor in the cross sec-

tion (counting of degrees of freedom) or as a quantum number to distinguish quarks.

2.2.1 Spectroscopy

Consider the Δ^{++} in the framework of a constituent quark model. It is a spin 3/2 baryon and is composed of three identical up quarks. Since it is the lowest lying state with these quantum numbers, it is natural to assume that the spatial wave function is the fundamental one. A 3-body potential which gives a reasonable description of the lowest excitations is the harmonic one:

$$V(x_1, x_2, x_3) = \frac{m\omega^2}{6}((x_1 - x_2)^2 + (x_1 - x_3)^2 + (x_2 - x_3)^2)$$
 (8)

where x_i are the quark coordinates, m is the constituent up quark mass and ω is a constant to be determined experimentally.

The ground state of the model defined by the Hamiltonian

$$H = -\frac{1}{2m} \sum_{i=1}^{3} \frac{\partial^{2}}{\partial x_{i}^{2}} + V(x_{1}, x_{2}, x_{3})$$
 (9)

is a gaussian,

$$\psi_0(x_1, x_2, x_3) = \frac{1}{\sqrt{3\sqrt{3}\pi^2}R^3} \exp\left[-\left((x_1 - x_2)^2 + (x_1 - x_3)^2 + (x_2 - x_3)^2\right)/(6R^2)\right]$$
(10)

where $R^2 = 1/(m\omega)$. ψ_0 is symmetric under exchange of any pair of coordinates and it has zero angular momentum (it is invariant under rotations):

$$L = 0. (11)$$

The spin of the baryon

$$\vec{J} = \vec{L} + \vec{S} \tag{12}$$

is enterely carried by the total spin of the quarks S:

$$J = S. (13)$$

The state with $S_z = 3/2$ consists of the up quarks all in the positive z direction

$$|\Delta^{++}, S_z = 3/2\rangle = |u\uparrow, u\uparrow, u\uparrow\rangle$$
 (14)

The spin wave function therefore is also symmetrical under exchange of the spins, so the complete wavefunction is symmetrical under exchange of the quarks. This is in contradiction with the spin-statistics theorem (the old Pauli principle of atomic physics) according to which identical spin 1/2 particles must have an antisymmetric wavefunction. A natural solution of this problem is the introduction of an additional quantum number of the quarks, called the 'color'. There have to be at least three colors. The most economical solution is that quarks come in just three varieties of color, and the baryon wavefunction is antisimetrical under exchange of the color of any two quarks, so it is a color singlet. In color space the wavefunction is therefore

$$|\Delta^{++}\rangle = \frac{1}{\sqrt{6}} \epsilon_{abc} | u_a, u_b, u_c\rangle,$$
 (15)

where ϵ_{abc} is the totally antisymmetric tensor with $\epsilon_{123} = 1$.

Similar problems exist also for the statistics of the Δ^- and Ω^- baryons, which are composed of three identical d (down) and s (strange) quarks respectively. Assuming SU(3) flavor symmetry, the same problem holds for many light baryons and is solved in the same way by the color.

Mesons consist of quark antiquark pairs with the following color singlet wavefunction

$$|M\rangle = \frac{1}{\sqrt{3}} \delta_b^a | \overline{q}_a q'^b \rangle \tag{16}$$

where q and q' denote two arbitrary quark flavors.

2.2.2 $\pi^0 \rightarrow \gamma \gamma \ \text{decay}$

The π^0 has the following valence quark content:

$$|\pi^{0}\rangle = \frac{1}{\sqrt{2}}(|u\overline{u}\rangle - |d\overline{d}\rangle) \tag{17}$$

We may take as an interpolating field for the π^0 (an interpolating field is an operator which excites the particle acting on the vacuum) the divergence of the isospin $I = 1, I_3 = 0$ axial current

$$A_{\mu} = \frac{1}{\sqrt{2}} (\overline{u} \gamma_{\mu} \gamma_5 u - \overline{d} \gamma_{\mu} \gamma_5 d) \tag{18}$$

Due to the triangle anomaly, there is a non zero coupling of the axial current with two electromagnetic currents,

$$\langle 0 \mid TA_{\mu}(x)J_{\nu}(y)J_{\rho}(z) \mid 0 \rangle \neq 0 \tag{19}$$

which produces the decay of the π^0 into two photons. The computation gives

$$\Gamma = \frac{\alpha^2}{64\pi^3} \frac{m_\pi^3}{f_\pi^2} \xi^2 = 7.6 \ \xi^2 \ eV, \tag{20}$$

where f_{π} is a contant determined from leptonic charged π decay (assuming isospin symmetry) and

$$\xi = N(e_u^2 - e_d^2) = 1 (21)$$

with the standard charge assignements and N=3.

The decay amplitude is proportional to N because the pion couples to $q\overline{q}$ pairs of any color.

The measured value is

$$\Gamma_{mis} = 7.7 \pm 0.6 \ eV$$
 (22)

in good agreement with the N=3 rate. Note that this case is very sensitive to the number of colors, due to the quadratic dependence of the rate on N.

2.2.3 Anomaly Cancellation In The Standard Model

A pretty theoretical argument for the color (i.e. not based on any experiment) is the anomaly cancellation in the Standard Model. Anomaly has to cancel because a non vanishing anomaly induces terms of canonical dimension in the lagrangian which are not originally present and are therefore not compatible with gauge invariance. The breaking of gauge invariance renders the theory not renormalizable, destroying the simplicity of the model (the simple ultraviolet structure).

No triangle anomaly is generated if the sum of the charges for every generation is zero. Since

$$e_u + e_d = 1/3$$
 (23)

while

$$e_{\nu} + e_l = -1, \tag{24}$$

a multiplicity for the quark states of 3 is required, which is provided by the color.

3 Asymptotic Freedom

After the success of the parton model, the theoretical problem was that of recoinciling a good phenomenological model with field theory (so successful in the area of electromagnetic and weak interactions). That appeared to be a hard task. Why was a quark behaving as a free particle when involved in an energetic collision? What about his interactions, which actually give rise to his binding in the nucleon?

The resolution of this problem came with the renormalization of non-abelian gauge theories. In such theories the one-loop β -function is negative. The β -function is defined as

$$\beta(\alpha_S) = \mu^2 \frac{\partial \alpha_S}{\partial \mu^2} \tag{25}$$

and represents the variation of the renormalized coupling as we vary the renormalization point μ (see also sec. 5.1.1 to understand its physical meaning). In this variation we have to keep the bare couplings ($\alpha_S^{(0)}$ and the bare masses m_0 's) and the ultraviolet cutoff Λ_0^2 fixed to ensure that we remain within the same physical theory (parametrized in many different ways according to the many different choices of the scale μ to which the parameters of the theory can be defined). Alternatively, we may define the β -function as

$$\beta(\alpha_S^0) = \Lambda_0^2 \frac{\partial \alpha_S^0}{\partial \Lambda_0^2}, \tag{26}$$

i.e. the variation of the bare coupling as we vary the ultraviolet cutoff Λ_0 . The variation is done keeping the renormalized parameters constant, so as to remain in the same physical theory, i.e. to reproduce the same low-energy cross sections (with energy $s \ll \Lambda_0^2$). It can be shown that the two above definitions give the same function up to two loops [4], beyond which details of the regularization (the precise way we cut the ultraviolet region) produce differences in the coefficients. The β -function has a perturbative expansion which we write in the form

$$\frac{\beta(\alpha_S)}{4\pi} = \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_S}{4\pi}\right)^{n+2}$$

$$= \beta_0 \left(\frac{\alpha_S}{4\pi}\right)^2 + \beta_1 \left(\frac{\alpha_S}{4\pi}\right)^3 + \beta_2 \left(\frac{\alpha_S}{4\pi}\right)^4 + \dots$$
(27)

We divided β by 4π because the one-loop beta function is already $O(\alpha_S^2)$. The one-loop value is

$$\beta_0 = -\left(\frac{11}{3}N - \frac{2}{3}n_f\right)$$

$$= -11 + \frac{2}{3}n_f = -7.67 \qquad (for n_f = 5)$$
(28)

where N=3, is the number of colors (as proved in the previous section), and n_f is the number of 'active' flavors at the scale μ . With f<11N/2=16.5 flavors, i.e. $f\leq 16$,

$$\beta_0 < 0 \tag{29}$$

The differential equation (25) can be integrated approximating the β -function with its one-loop value. Denoting the initial integration condition with the scale μ^2 and the final one with the scale Q^2 , we have

$$\alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1 - \beta_0/(4\pi)\alpha_S(\mu^2)\log(Q^2/\mu^2)}$$
(30)

We see that, since $\beta_0 < 0$, the coupling $\alpha_S(Q^2)$ decreases with the energy and approaches zero in the limit of a very large energy (or momentum transfer):

$$\alpha_S(Q^2) \to 0$$
 for $Q^2 \to \infty$ (31)

The theory therefore approaches a free theory in the ultraviolet region (asymptotic freedom [25]), thus explaining the 'free' behaviour of the quark in the hard process. On the other side, the coupling is not small at low energies, explaining quark binding in the hadrons. We have therefore a very simple explanation of the duality of high-energy and low-energy hadronic phenomena with renormalization group ideas. We note that asymptotic freedom is a one-loop property of the β -function because at large enough energies the coupling is small and higher orders are negligible.

Instead of introducing a reference scale μ^2 at which assign the value of the coupling, we can write the running coupling in terms of the so-called Λ_{QCD} parameter, the scale at which the one-loop coupling has a pole (it is an infrared pole because it occurs at low energies). With

$$\Lambda_{QCD}^2 = \mu^2 e^{4\pi/(\beta_0 \alpha_S(\mu^2))}$$
 (32)

the coupling is rewritten as

$$\alpha_S(Q^2) = \frac{1}{-\beta_0/(4\pi)\log(Q^2/\Lambda_{QCD}^2)}.$$
 (33)

We may ask what is the physical meaning of Λ_{QCD} . It is a renormalization group invariant quantity (it does not change if we vary μ^2 in its defining equation) and denotes at which energy renormalization group improved perturbation theory ceases to be valid, pointing therefore to generic nonperturbative phenomena [1]. This scale has to be determined experimentally and turns out to be $\sim 300~MeV$, i.e. of the same order of the hadron masses $(M_{\rho} \sim 770~MeV)$.

We note that the coupling goes to zero only logarithmically with the energy, i.e. very slowly. As long as $\alpha_s \ll 1$, perturbative corrections to the free theory behaviour can be computed and are expected to be sizable. This is the technical justification for perturbative QCD, whose physical justification will be discussed later.

Let us quote also the values of two and three loop coefficients of the β -function in the \overline{MS} scheme [7]:

$$\beta_1 = -102 + \frac{38}{3}n_f = -102 + 12.66 n_f = -38.66$$

$$\beta_2 = \frac{1}{2} \left[-2857 + \frac{5033}{9}n_f - \frac{325}{27}n_f^2 \right]$$

$$= -1428.5 + 279.6n_f - 6.0n_f^2 = -180.5 \qquad (for n_f = 5) (34)$$

We note that $\beta_1 < 0$ if $n_f \le 8$, while $\beta_2 < 0$ if $n_f \le 5$. The (approximate) two-loop solution for the running coupling is given by:

$$\alpha_S(Q^2) = \frac{1}{-\beta_0/(4\pi)\log(Q^2/\Lambda_{QCD}^2)} \left[1 + \frac{\beta_1}{\beta_0^2} \frac{\log(\log(Q^2/\Lambda^2))}{\log(Q^2/\Lambda^2)} \right].$$
 (35)

In QED the β -function is given to one loop by

$$\beta_{QED} = \frac{\alpha^2}{3\pi} > 0 \tag{36}$$

(times n_f if f flavors are considered). Integrating the related differential equation exactly as in the case of QCD, we derive:

$$\alpha(Q^2) = \frac{3\pi}{\log(\Lambda_I^2/Q^2)} \tag{37}$$

where Λ_L , called the Landau pole, is now an ultraviolet scale [19]. The coupling grows with the energy as a consequence of the positive β -function. The QED behaviour of the effective coupling is therefore opposite to that of QCD (eq.(33)). A physical discussion of these different behaviours is given in the next section.

We conclude this section observing that in four space-time dimensions asymptotic freedom is an exclusive property of non abelian gauge theories [8], which therefore emerge as a natural candidate for the strong interactions.

3.1 Antiscreening

In QED the behaviour of the effective charge is controlled by the photon polarization diagram,

$$\gamma^* \to (e^+e^-)^* \to \gamma^* \tag{38}$$

(ultraviolet divergences in the electron field renormalization and in the vertex correction cancel because of the Ward identity).

The physical explanation is the following. Consider a charged particle placed in the vacuum, with bare (i.e. primordial) charge e_0 . Quantum field fluctuations induce short lived e^+e^- pairs. These pairs constitute electric dipoles which align with the test charge so as to minimize the energy. The bare charge is therefore surrounded by a shell of opposite charges (the outer shell is at infinity) with a consequent screening effect. The vacuum behaves as an ordinary medium with material electric dipoles, with dielectric constant

$$\epsilon > 1 \qquad (QED)$$
 (39)

If we measure the charge of the test particle, we find that it is less than the bare charge:

$$e = \frac{e_0}{\epsilon} \tag{40}$$

If we come closer to the test particle, say at distance r, we see an increasing charge, because the outer part of the shell, at distance r' > r, does not contribute to the screening (Gauss theorem). The effective charge (i.e. the observable one) is therefore a function of the distance

$$e = e(r). (41)$$

It increases as we lower the distance, or equivalently, as we increase the momentum transfer $q^2 \sim 1/r^2$.

In QCD the situation is reversed: due to gluon-gluon couplings (three and four gluon vertices), the test charge is surrounded by gluons of the *same* charge. The vucuum behaves as an hypotetical medium with dielectric constant

$$\epsilon < 1 \tag{QCD}$$

so we have an antiscreening effect. As we come closer to the test particle, the effective charge is reduced (see [12, 15] for a more detailed explanation). Asymptotic freedom means that the measured charge approaches zero at an infinitesimal distance.

4 Confinement

Isolated quarks have never been observed, as particles with fractional electric charge, for example. Hence the hypotesis that they are 'confined' in hadrons, i.e. that they do not appear as asymptotic states in any physical process. The current explanation is that the force between two quarks does not vanish as their separation r increases, so that the potential energy of the system V diverges

$$V(r) \to \infty \quad as \quad r \to \infty.$$
 (43)

In any realizable collision, the available energy is finite, making therefore impossible to produce isolated quarks.

A potential which is belived to represent the QCD interaction between a quark and an antiquark in a color singlet state, is the so called 'funnel' potential:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr \tag{44}$$

where k is a constant, called the string tension.

This potential has been extensively used in non relativistic models of $c\overline{c}$ and $b\overline{b}$ bound states, with good results, and is also compatible with lattice (non perturbative) QCD computations. We see that at small distances, $r \ll 1/k$, the potential is coulombic, as it is in perturbative QCD, while at large distances there is a linear grow of the energy with the separation.

The qualitative explanation of the confining potential (44) is the following. When the separation of the quark-antiquark pair is small compared to

the confinement radius, the gluon field has the same form of a dipole field in classical electrodynamics. Chromoelectric field lines spread out in all the space (and the potential decays as 1/r like in QED). When their separation becomes of the order of the confinement radius, there is a reciprocal attraction of the field lines, which makes them to collapse in a tube along the $q\bar{q}$ line. Since the electric flux is the same through any closed surphace containing the quark or the antiquark, and the electric field is non zero only inside the tube, we say that there is the formation of a flux tube (or string). A further increase of the separation does not change the form of the gluon field anymore but only makes the flux tube longer. Since the gluon field is now essentially concentrated in a tube, the energy contained in the tube is proportional to its lenght. In other words, there is a constant energy of the field per unit lenght, which implies a linear term in the potential. In the next section we discuss a model of confinement which induces the above postulated attraction of field lines.

4.1 Dual Meissner effect

A model which induces flux tubes (and therefore confinement via a linear potential) is the so called dual Meissner effect [5].

Consider a superconductor in an external magnetic field \vec{B} above the transition temperature T_C , for definiteness a cilinder with its axis parallel to \vec{B} . As we lower the temperature to $T < T_C$, there is the transition from the normal to the superconducting phase (drop to zero of the electrical resistence). At the same time, the magnetic field lines are expelled outside the sample. The magnetic field inside the cilinder is zero:

$$\vec{B}_{inside} = 0, (45)$$

i.e. the superconductor shows a perfect diamagnetism

$$\chi_M = 0. (46)$$

The explanation is that with the transition electrons pair in particles with charge 2e (Cooper pairs). The latter induce a current \vec{J}_{pairs} along the surphace of the cilinder generating a magnetic field inside the sample opposite to the external one:

$$\vec{B}_{pairs} = -\vec{B} \tag{47}$$

Now imagine of placing a pair of opposite charge magnetic monopoles inside the superconductor. Since the superconductor tends to expel the magnetic field, the field lines of the magnetic dipole are shrunk in a small tube connecting the monopoles. An electric current of Cooper pairs develops along the surphace of the flux tube, effectively squeezing the magnetic dipole field.

As well know, electromagnetism is invariant under the simultaneous exchange of magnetic fields with electric ones and of magnetic charges with electric ones (duality symmetry).

Imagine the dual of an ordinary superconductor, i.e. a system expelling electric fields. If we place a pair of opposite electric charges inside the system, the lines of the electric dipole field will be shrunk into an electric flux tube. The dual superconductor contains magnetic monopoles, while the ordinary superconductor contains 'Cooper pairs, i.e. 'electric monopoles'. Let us describe the phenomenon a little bit more in detail. Soon after the introduction of the electric charges, the system reacts producing a current of magnetic monopoles \vec{J}_m along the surphace of a tube connecting the test charges. \vec{J}_m produces, by duality, a secondary electric field \vec{E}_m cancelling the external field \vec{E} outside the tube

$$\vec{E}_m = -\vec{E}$$
 (outside) (48)

and reinforcing the external field inside the tube. The net effect is the squeezing of the dipole electric field of the pair into a tube.

Confinement has not been rigorously proved in QCD up to now. It is usually assumed that confinement means that hadrons consist of color singlets only.

4.2 Jets

The process of jet formation in high energy collisions is qualitatively related to confinement. As for confinement, there is no rigorous proof of this phenomenon and of the ideas explaining it inside QCD.

Let us consider the simplest case of jet production, e^+e^- collisions, to be discussed in detail in the next section. The electron and the positron annihilate into a photon, which decays into a quark-antiquark pair:

$$e^+ + e^- \rightarrow \gamma \rightarrow q + \overline{q}.$$
 (49)

In the center of mass frame, the quark and the antiquark have the same energy and opposite spatial momenta (back to back). They are originally created in the same spatial point (the fireball) and fly far apart with light velocity. When their separation comes close to the confinement radius,

$$r_{conf} \sim 1 \ fm = 10^{-13} \ cm,$$
 (50)

the dipole color field of the quark pair shrinks to a tube (called flux tube of string). As they separate from each other, the flux tube gets longer and an increasing fraction of the kinetic energy of the pair is converted into field energy. When the energy contained in the flux tube exceeds the mass of a light $q\bar{q}$ pair

$$E_{tube} > 2m, (51)$$

a novel quark-antiquark pair is created which screens the color of the original pair. The string breaks into two shorter strings, each having at his ends an original quark and a novel quark in a color singlet state.

If the relative momentum of the quark-antiquark pair connected to the same string is large enough, the string may break again. We have therefore the production of a parton cascade, i.e. of a large number of quarks and gluons with progressively lower energy.

In high energy processes ($E \gg M_P$, say, where M_P is the proton mass), the flux tube has a large longitudinal momentum ($P_L \sim E$), while it has a limited tranverse momentum ($P_T \sim M_P$). Consequently, the partons produced have large longitudinal momenta and large longitudinal momentum differences, but limited transverse momenta.

After the parton cascade production, we have the recombination of quark and gluons in color singlet states, the observed hadrons. This transition does not induce large momentum transfer between the partons (it is a soft process). This hypotesis is sometimes called local parton-hadron duality and states that hadrons are produced by partons which are close in phase space [22]. This mechanism forbids, for example, that a quark with momentum p coming from one jet combines with a quark with momentum p coming from another jet $(p \cdot p' \gg \Lambda_{QCD}^2)$, destroying the shape of the perturbative momentum distribution. The transition from a partonic jet to a hadronic one therefore does not wash out all the parton information and the longitudinal and transverse momentum distributions are substantially unchanged. The

final particles, the hadrons, are collimated into two small angular regions (jets) opposite to each other see (fig.1).

Furthermore, the angular distribution of the jets relative to the e^+e^- flight direction is expected to coincide with that one of the original $q\overline{q}$ pair. As we will see, this prediction is confirmed by data, giving good support to the above qualitative ideas.

If an energetic gluon is produced at large angle with respect to the original pair (a hard gluon), a new string is formed with a consequent third jet (see fig.2).

From these considerations, it emerges that jets are a universal occurrence of high energy hadronic reactions.

5 Hadron production in e^+e^- collisions

Another evidence for the color comes from the hadronic production in e^+e^- machines (in this section we rest on ref. [12]). Consider the production of a fermion pair In lowest order, the electron and the positron annihilate into a γ or a Z^0 , which decay into a fermion-antifermion pair $(f \neq e)$, (see fig. 3)

$$e^+ + e^- \to \gamma^*, Z^* \to f + \overline{f}$$
 (52)

The differential cross section is given by:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2} \frac{1}{s} \left[(1 + \cos^2\theta) \left(e_f^2 - 2e_f v_e v_f \chi_1(s) + (a_e^2 + v_e^2)(a_f^2 + v_f^2) \chi_2(s) \right) + 4\cos\theta \left(2a_e v_e a_f v_f \chi_2(s) - e_f a_e a_f \chi_1(s) \right) \right]$$
(53)

where θ is the angle between the initial and the final pair,

$$\chi_{1}(s) = k \frac{s(s - M_{Z}^{2})}{(s - M_{Z}^{2})^{2} + \Gamma_{Z}^{2} M_{Z}^{2}},$$

$$\chi_{2}(s) = k^{2} \frac{s^{2}}{(s - M_{Z}^{2})^{2} + \Gamma_{Z}^{2} M_{Z}^{2}},$$

$$k = \frac{\sqrt{2} G_{F} M_{Z}^{2}}{4\pi \alpha} = 1.40$$
(54)

(k is the adimensional ratio of the relevant weak coupling to the electromagnetic one and its numerical value is for $\alpha(M_Z) = 1/128$ [23]). G_F is the Fermi constant, α is the fine structure constant, M_Z and Γ_Z are the mass and the width of the Z boson respectively. v_f and a_f are the vector and axial vector couplings of the Z to the fermions and are given by:

$$v_f = I_{3f} - 2e_f \sin^2 \theta_W$$

$$a_f = I_{3f}$$
(56)

where θ_W is the Weinberg angle, and I_3 is the z-component of the weak isospin: $I_3 = 1/2$ for neutrinos and u-type quarks (e = 2/3), and $I_3 = -1/2$ for charged leptons and d-type quarks (e = -1/3).

The function $\chi_1(s)$ represents the interference of the γ amplitude with the Z one, and the function $\chi_2(s)$ represents the Z amplitude squared.

At small energies compared to the Z mass

$$s \ll M_Z^2 \tag{57}$$

the functions χ_1 and χ_2 are suppressed by the Z mass:

$$\chi_1 \sim -k \frac{s}{M_Z^2} \ll 1, \qquad \chi_2 \sim \chi_1^2 \ll 1$$
(58)

We can therefore neglect both the direct Z effect and the interference one $(\chi_1 = \chi_2 = 0)$, ending up with the pure QED cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2} e_f^2 \frac{1}{s} (1 + \cos^2\theta) \tag{59}$$

The angular dependence is in good agreement with the observed 2-jet cross section and gives support both to the spin=1/2 assignment of the quarks and to the qualitative ideas about jet formation discussed previously (see fig.4).

LEP1 and SLAC machines have operated on the Z peak, i.e. with a center of mass energy $E=M_Z$. In this case the resonant contribution of the Z dominates over the γ and the interference ones. Setting therefore $e_f=0, \ \chi_1=0$ and $\chi_2=k^2M_Z^2/\Gamma_Z^2$ we have for the differential cross section

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2 k^2}{2\Gamma_Z^2} \left[(1 + \cos^2\theta)(a_e^2 + v_e^2)(a_f^2 + v_f^2) + 8\cos\theta a_e v_e a_f v_f \right]$$
(60)

The angular distribution contains an additional $\cos \theta$ term, produced by the interference of the vector and the axial current. This term induces a difference in the number of events in the hemispheres on the electron and the positron side $(\theta : (-\pi/2, \pi/2), (\pi/2, 3/2\pi))$ respectively). This effect is called forward-backward asymmetry and is generated by the C/P violation of weak interactions.

5.1 The Ratio R

Integrating over the polar angle, we derive the total QED cross section:

$$\sigma = \frac{4\pi}{3}\alpha^2 e_f^2 \frac{1}{s} \tag{61}$$

Let us define the ratio

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$
 (62)

This way we get rid of uninteresting factors and concentrate on strong interactions only. On the experimental side, some systematic effects, related to the luminosity determination for example, cancel in taking the ratio.

We assume now that the total hadronic cross section is equal to the total partonic cross section, i.e.:

$$\sigma(e^{+}e^{-} \to hadrons) = \sigma(e^{+}e^{-} \to partons)
= \sigma(e^{+}e^{-} \to q\overline{q}) + \sigma(e^{+}e^{-} \to q\overline{q}g) + \dots$$
(63)

There is a physical justification for that. The time for the partonic process to occur (virtual photon decay) is

$$t_p \sim \frac{1}{E} \tag{64}$$

where E is the center of mass energy, while the time for hadronization is

$$t_h \sim \frac{1}{\Lambda_{QCD}} \tag{65}$$

(or equivalently, the lifetime of a resonance $\tau \sim 10^{-23} sec$). In high energy collisions,

$$E \gg \Lambda_{QCD},$$
 (66)

hadronization occurs therefore much later than the partonic process,

$$t_p \ll t_h,$$
 (67)

so its cross section is substantially unaffected. The above qualitative explanation is not rigorous, but, as we will see, its conclusions turn out to be rigth. We have therefore in lowest order:

$$R = N_c \sum_f e_f^2 \tag{68}$$

where the sum extends to all the quark flavors which are kinematically allowed. The ratio R has therefore the value:

$$R = N \left(e_u^2 + e_d^2 + e_s^2 \right) = 3 \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} \right) = 2$$
 (69)

below the charm-anticharm production threeshold, i.e. below a center of mass energy of ~ 3 GeV, a value of

$$R = N \left(e_u^2 + e_d^2 + e_s^2 + e_c^2 \right) = 3 \left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} \right) = 3.33 \quad (70)$$

between the charm and the beauty production threshold, i.e. between ~ 4 GeV and ~ 10 GeV, and

$$R = N(e_u^2 + e_d^2 + e_s^2 + e_c^2 + e_b^2) = 3(4/9 + 1/9 + 1/9 + 4/9 + 1/9) = 3.67 (71)$$

above the beauty threeshold and well below the Z peak (see fig. 5). The above estimate is basically a counting of the degrees of freedom involved. It is remarkable that it depends only on the electric charges and the colour of the quarks. The above prediction for R is valid only far away from $c\bar{c}$ and $b\bar{b}$ resonances, where hadronization effects cannot be considered negligible because they affect substantially the recombination of partons. We see that R in the resonance regions has peaks and valleys. In these regions we may interpret the QCD estimate (68) as an average over an interval of energy ΔE of order 1 GeV

$$R \rightarrow \langle R \rangle = \frac{1}{\Delta E} \int_{E-\Delta E/2}^{E+\Delta E/2} R(E') dE'$$
 (72)

In the low energy range (a few GeV), the parton estimate is not very accurate because hadron production is still affected by light resonances. Furthermore,

perturbative corrections (to be discussed later) are large, and strongly dependent of unknown higher orders.

At $E = 34 \ GeV$ the experimental value of R is about 3.9, to be compared to 3.67 (see fig.5). The mismatch is only reduced by the Z contribution (it is a O(1%) effect, as can be evaluated inserting the low energy approximation for χ_1 in eq.(53)). As we will see, radiative corrections enhance the value of the lowest order, bringing the theoretical value in the experimental band.

Let us consider now the resonance value $(E = M_Z)$ of R. Integrating over θ we have for the total cross section:

$$\sigma = \frac{4\pi\alpha^2 k^2}{3\Gamma_Z^2} (a_e^2 + v_e^2)(a_f^2 + v_f^2)$$
 (73)

We note however that real photon emission from the initial e^+e^- state (called bremsstrahlung) smears the peak and shifts its position (see fig. 6). So, the previous formula has to be corrected for an accurate quantitative analysis (see the end of sec. 5.3 for a discussion of these effects). The ratio R is given by:

$$R = 3 \frac{\sum_{f=1}^{5} a_f^2 + v_f^2}{a_\mu^2 + v_\mu^2} = 20.09 \tag{74}$$

where we have used eqs.(56) and $\sin^2 \theta_W = 0.2315$ [23]. We note that the QED effects mentioned previously (bremsstrahlung) are the same for the muonic and hadronic channels because they can be factorized as part of the initial e^+e^- annihilation process. So they cancel in taking the ratio of the cross sections. The prediction for the tree level value of R on the peak therefore turns out to be accurate in this respect. Note that R is much larger than in the QED case because quarks have comparable weak charges to that of the μ .

The experimental value is [3]:

$$R_{exp} = 20.80 \pm 0.035 \tag{75}$$

Also in this case, the theoretical value is $\simeq 4\%$ smaller than the experimental one, because of corrections to be discussed in the next section.

5.1.1 Radiative corrections

We have seen in the previous section that the measures of R are so accurate that a comparision with the theory requires the inclusion of perturbative

corrections. Let us consider therefore radiative corrections to R. It is a completely inclusive quantity and, neglecting quark masses, it is characterized by a single energy scale,

$$\sqrt{s}$$
 (76)

(quark masses can be sent to zero because in this limit no singularities are generated). The order α_S corrections are ultraviolet finite because the electromagnetic current which creates the $q\bar{q}$ pair is conserved in QCD (see the appendix). In other words, the UV singularity of the vertex correction is cancelled by the UV singularity of the external quark legs renormalization. In the intermediate stage of the calculation, infrared divergences appear, which cancel between real and virtual diagrams in the completely inclusive process. In this case they offer therefore only a technical problem. Let us see this explicitly in dimensional regularization. Infrared and collinear singularities (see later) are regulated increasing the space-time dimension to n > 4 (see for example [27]). The real corrections, with a gluon in the final state (see fig. 7), give:

$$\sigma(e^{+}e^{-} \to q\overline{q}g) = \sigma_{0} \frac{C_{F}\alpha_{S}}{2\pi} H(\epsilon) \int dx_{1} dx_{2} \frac{x_{1}^{2} + x_{2}^{2} - \epsilon(2 - x_{1} - x_{2})}{(1 - x_{1})^{1 + \epsilon}(1 - x_{2})^{1 + \epsilon}} \\
= \sigma_{0} \frac{C_{F}\alpha_{S}}{2\pi} H(\epsilon) \left[\frac{2}{\epsilon^{2}} + \frac{3}{\epsilon} + \frac{19}{2} + O(\epsilon) \right] \tag{77}$$

where $C_F = \sum t_a t_a = (N^2 - 1)/2N$ for an SU(N) gauge theory ($C_F = 4/3$ in QCD) is the Casimir operator in the fundamental representation, $n = 4 - 2\epsilon$ and

$$H(\epsilon) = \frac{3(1-\epsilon)^2}{(3-2\epsilon)\Gamma(2-2\epsilon)} = 1 + O(\epsilon)$$
 (78)

The virtual corrections, i.e. the diagrams with a reabsorbed gluon, give:

$$\sigma(e^+e^- \to q\overline{q}) = \sigma_0 \frac{C_F \alpha_S}{2\pi} H(\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + O(\epsilon) \right]. \tag{79}$$

Soft singularites appear as poles in ϵ . The double pole $1/\epsilon^2$ is originated by the product of the collinear with the infared singularity (see later), while the simple poles $1/\epsilon$ come from a collinear or an infrared singularity. Both double and simple poles cancel in the sum, giving a finite $O(\alpha_S)$ correction

to the tree level value of R:

$$R = R_{tree} \left(1 + \frac{\alpha_S}{\pi} \right) \tag{80}$$

In the computation to order α_S^2 , we hit (no more cancelling) ultraviolet divergences, of the form

$$\alpha_S^2 \log \frac{\Lambda_0^2}{s},\tag{81}$$

where Λ_0^2 is an ultraviolet cutoff, i.e. an ultraviolet regulator, and $\alpha_S = \alpha_S^{(0)}$ has to be interpreted as the QCD bare coupling. To be more explicit, the correction factor K to R up to $O(\alpha_S^2)$ has the form:

$$K = 1 + C \alpha_S - C \frac{\beta_0}{4\pi} \alpha_S^2 \log \frac{\Lambda_0^2}{s} + C' \alpha_S^2 + \dots$$
$$= 1 + C \alpha_S \left(1 - \frac{\beta_0}{4\pi} \alpha_S \log \frac{\Lambda_0^2}{s} \right) + C' \alpha_S^2 + \dots$$
(82)

(as we have seen $C = 1/\pi$). We can compare this result with the one-loop running coupling of QCD (eq.(30)), in which we set $\mu^2 = \Lambda_0^2$ and we interpret $\alpha_S(\Lambda_0^2)$ as the bare coupling (we also set $Q^2 = s$):

$$\alpha_S(s) = \frac{\alpha_S(\Lambda_0^2)}{1 + \beta_0/(4\pi)\alpha_S(\Lambda_0^2)\log\Lambda_0^2/s}$$

$$= \alpha_S \left(1 - \frac{\beta_0}{4\pi}\alpha_S\log\frac{\Lambda_0^2}{s} + \left(\frac{\beta_0}{4\pi}\right)^2\alpha_S^2\log^2\frac{\Lambda_0^2}{s} + \dots\right)$$
(83)

We may write, up to order α_S^2 :

$$K = 1 + C\alpha_S(s) + C'\alpha_S(s)^2 + \dots$$
 (84)

We see that the logarithmic term in K is simply a renormalization of the bare coupling. The theory is trying to say that we are expanding around a wrong scale, the ultraviolet cut-off Λ_0^2 , very far from the physical scale s. The unfactorized cross section (82) and the factorized one (84) coincide to order α_S^2 included. The terms added to generate the factorized form are indeed of higher order: α_S^3 or more.

We see however that the above factorization is physically reasonable but not unique. We may equally well write:

$$K = 1 + C\alpha_S - C\frac{\beta_0}{4\pi} \alpha_S^2 \log \frac{\Lambda_0^2}{s/4} + \left[C' + C\frac{\beta_0}{4\pi} \log 4 \right] \alpha_S^2 + \dots$$
 (85)

with a consequent factorized formula of the form:

$$K = 1 + C\alpha_S(s/4) + \left[C' + C\frac{\beta_0}{4\pi}\log 4\right]\alpha_S(s/4)^2 + \dots$$
 (86)

We see that the running coupling is evaluated at a different scale, s/4, instead of s, and there is a compensating term of order α_S^2 . So, there is no way in perturbation theory to fix the scale in the running coupling α_S to a unique value. We can take whatever value for the scale, and compensate shifting the finite $O(\alpha_S^2)$ term. Of course, a wird scale setting of $10^{-4}s$ is unreasonable, because it introduces a large logarithmic term $4 \log 10 \sim 10$ in the finite α_S^2 term deteriorating the convergence of the expansion.

The two factorized formulae (84) and (86) agree formally up to order α_S^2 but produce slightly different numerical values. They differ in the way they capture parts of the higher order terms. This phenomenon occurs because of the different treatment of the coupling α_S and of K. The running coupling resums classes of terms of any order in α_S , as it is clear from eq. (83). The group structure of scale tranformations (the renormalization group) allows this resummation, while there is not any known way to resum terms in K to any order in α_S : K is expanded up to a fixed order in α_S , i.e. the series for K is a truncated one. When we change the scale of the coupling, an all-order change, K cannot compensate for the tail of the series. The independence on the (arbitrary) scale choice is therefore not rigorously true in perturbation theory because it is violated by higher orders. This ambiguity never disappear: it can only be shifted to higher orders as we compute more and more terms for K. We have the formal independence from the scale choice of the coupling only in the limit in which the infinite series for K has been computed.

Let us rephrase these facts in a more common language. The UV divergence (81) is converted with traditional renormalization in a similar logarithmic term:

$$\alpha_S^2 \log \frac{s}{\mu^2},\tag{87}$$

where μ is a renormalization point. In dimensional regularization (the only regularization which is effectively used in high order computations), after the subtraction of the $1/\epsilon$ poles, we are left with logs of the above form, where μ is the mass unit introduced to make the coupling dimensionless in dimension $n \neq 4$.

R is therefore written as a function of the renormalized coupling at a fixed scale μ^2 , $\alpha_S(\mu^2)$, and of the above logs:

$$R = R(\alpha_S(\mu), \log s/\mu^2) \tag{88}$$

since R is a physical quantity, it cannot depend on the choice of the renormalization scale μ^2 , which is an arbitrary scale introduced for convenience at an intermediate stage (at the very end we compare R with another process characterized by another physical scale s'). These considerations are formalized with the renormalization group equation

$$\mu^2 \frac{d}{d\mu^2} R = 0, \tag{89}$$

where as usual, one has to vary μ^2 remaining in the same physical theory, i.e. keeping fixed the bare parameters of the theory, or the renormalized parameters at some other fixed scale μ_0^2 . Explicitly eq.(89) reads:

$$\left[\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha_S) \frac{\partial}{\partial \alpha_S}\right] R(\alpha_S, \log s/\mu^2) = 0$$
 (90)

The concrete prediction we are able to give is slightly dependent on the renormalization point, an arbitrary scale which has neither a physical meaning nor a definite value. We need some physical intuition to pick up a good value of μ^2 which minimize the (unknown) higher orders: a natural choice seems to be $\mu^2 = s$, for which R is a (truncated) series in α_S evaluated at the center of mass energy.

$$R = R(\alpha(s)) \tag{91}$$

We have therefore:

$$R = K * R_{tree} (92)$$

where the correction factor (the so called K-factor) has the following series expansion

$$K = 1 + \sum_{n=1}^{\infty} c_n \left(\frac{\alpha_S(s)}{\pi}\right)^n \tag{93}$$

In the \overline{MS} scheme (an off-shell renormalization scheme, specific of dimensional regularization), three correction terms are known [9, 14]:

$$c_{1} = 1$$

$$c_{2} = \frac{365}{24} - 11\zeta(3) + \left(\frac{2}{3}\zeta(3) - \frac{11}{12}\right)n_{f}$$

$$= 1.986 - 0.115 n_{f} = 1.411$$

$$c_{3} = \frac{87029}{288} - \frac{1103}{4}\zeta(3) + \frac{275}{6}\zeta(5) - \frac{\pi^{2}}{48}\beta_{0}^{2} + \left(\frac{55}{72} - \frac{5}{3}\zeta(3)\right)\eta$$

$$- \left(\frac{7847}{216} - \frac{262}{9}\zeta(3) + \frac{25}{9}\zeta(5)\right)n_{f} + \left(\frac{151}{162} - \frac{19}{27}\zeta(3)\right)n_{f}^{2}$$

$$= -6.637 - 1.2n_{f} - 0.005n_{f}^{2} - 1.240\eta$$

$$= -12.80$$
(94)

where the last values of the coefficients are for $n_f = 5$. $\zeta(s)$ is the Rieman ζ -function, defined by the series

$$\zeta(u) = \sum_{n=1}^{\infty} \frac{1}{n^u} \tag{95}$$

and

$$\eta = \frac{1}{3} \frac{(\sum_{f} e_f)^2}{\sum_{f} e_f^2} = \frac{1}{33} = 0.0303 \tag{96}$$

for the electromagnetic diagram (the last value is for $n_f = 5$) and

$$\eta = \frac{(\sum_f v_f)^2}{3\sum_f (v_f^2 + a_f^2)} = 0.0302 \tag{97}$$

for the weak diagram. The different dependence on the electric/weak charges of the η terms is generated by diagrams with two separated fermionic traces (like for example in the process $e^+e^- \to 3g$).

At $\sqrt{s}=34$ GeV, taking $\alpha_s(34~GeV)=0.146\pm0.030$, we have $K\simeq 1.05$, i.e. the required 5% increase of the tree level value for R. At LEP1 energies, $\sqrt{s}=M_Z$, taking $\alpha_S(M_Z)=0.12$ [3], we have $K\simeq 1.039$, i.e. a 4% increase of the tree level value. The experimental measure of R is therefore a clean verification of QCD. Note the large (negative) coefficient of the α_S^3 term,

which makes the perturbative series not so well convergent. For $\alpha_s = 1$, for example, the α_S^3 term is almost three times bigger than the α_S^2 term.

The dependence of the coefficients c_i on the scale, shown explicitly in lowest order at the beginning of this section, can be derived in general imposing the Callan Symanzik equation (90) order by order in α_s .

5.2 Jets in e^+e^- collisions

Perturbative QCD predicts also some properties of the structure of the final state. Of course, the cross section for a single exclusive channel, like for example $e^+e^- \to P\overline{P}$, involves detailed hadronization processes and is therefore outside the reach of perturbative QCD. We can consider seminclusive quantities. Let us consider the emission of a gluon, i.e. the process

$$e^+ + e^- \rightarrow q + \overline{q} + g \tag{98}$$

There are two amplitudes at order α_S , for the emission of the gluon from the quark leg M_a or the antiquark one M_b (fig.7). The amplitudes involve only the QED-like vertex

$$V_{q\overline{q}a} = ig\gamma_{\mu}t_{a}, \tag{99}$$

i.e., the non abelian nature of QCD (three and four gluon couplings) does not fully manifest itself. The only effect of color is in the factor C_F multiplying α_S . This implies that the QCD cross section is identical to the abelian (QED) one with the replacement

$$C_F \alpha_S \to \alpha$$
 (100)

The non-abelian nature of QCD enters instead explicitly in the process

$$e^+e^- \rightarrow q\overline{q}qq,$$
 (101)

which is the dominant contribution to the 4-jet cross section and starts at order α_S^2 . Let's go back to the 3-jet case. We denote with x_i the energy fractions of the quark, the antiquark and the gluon:

$$x_1 = \frac{E_q}{E_h}, \quad x_2 = \frac{E_{\overline{q}}}{E_h}, \quad x_3 = \frac{E_g}{E_h},$$
 (102)

where E_b is the beam energy ($\sqrt{s} = 2E_b$). The conservation of energy and momentum gives:

$$x_1 + x_2 + x_3 = 2$$

 $1 - x_i = \frac{1}{2} x_j x_k (1 - \cos \theta_{jk}), \quad i \neq j \neq k, \quad i \neq k.$ (103)

The kinematical bounds are therefore

$$0 \le x_i \le 1 \tag{104}$$

and

$$x_i + x_j \ge 1, \quad i \ne j \tag{105}$$

In words, a parton has the beam energy as the maximum energy when the other two partons are collinear, and a parton pair has the beam energy as the minimum energy and the total energy as the maximum energy. Since the final state is planar, we have also:

$$\theta_{12} + \theta_{23} + \theta_{13} = 2\pi \tag{106}$$

The differential cross section for having a quark with an energy fraction x_1 and an antiquark with an energy fraction x_2 in the final state is

$$\frac{1}{\sigma} \frac{d\sigma_{q\bar{q}g}}{dx_1 dx_2} = C_F \frac{\alpha_S}{2\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$
(107)

This cross section is derived in the appendix; it coincides with the integrand in eq.(77) in the limit $\epsilon \to 0$. It is symmetric under exchange of x_1 with x_2 , as it should be for the C/P symmetry of QCD (to order α_S , we divide $d\sigma$ simply by σ_0).

If the experiment does not distinguish between the jets formed by the quark, the antiquark and the gluon, it is natural to symmetrize the above cross section: we simply ask what is the cross section for having a jet with energy fraction x_1 , a second jet with energy fraction x_2 and a third jet with energy fraction x_3 . The cross section is therefore:

$$\frac{1}{\sigma} \frac{d\sigma_{3jets}}{dx_1 dx_2} = C_F \frac{\alpha_S}{6\pi} \left(\frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} + \frac{x_1^2 + x_3^2}{(1 - x_1)(1 - x_3)} \frac{x_2^2 + x_3^2}{(1 - x_2)(1 - x_3)} \right)$$
(108)

Since the jets are assumed to be undistinguishble, The energy fractions x_i can be permuted so that

$$x_1 < x_2 < x_3 \tag{109}$$

We therefore say that jet 1 is the less energetic one, jet 2 the intermediate one, and jet 3 the most energetic one. Let's go back to the simpler formula (107). The integration region for the total cross section is

$$0 \le x_1, x_2 \le 1
1 \le x_1 + x_2$$
(110)

There are singularities when the quark energy fractions reach their maximum allowed values (end point singularities):

$$x_i \to 1^- \qquad i = 1, 2 \tag{111}$$

The strongest singularity is a double pole and occurs when the energy fractions reach simultaneously their end point values:

$$x_1 \rightarrow 1^- \qquad and \qquad x_2 \rightarrow 1^- \qquad (112)$$

This happens when theee body kinematics 'resembles' the two body one (the latter is a double spike in the energy fractions $\sim \delta(1-x_1)\delta(1-x_2)$).

As we have seen, the double pole (112) is converted with the integration in dimensional regularization in a double pole $1/\epsilon^2$ (eq.(77)). As we will see, the configuration (112) generates a double logarithmic contribution in the 3-jet cross section, i.e. a term of the form $\alpha_s \log^2 y$.

There is instead a simple pole singularity when only a single energy fraction x_i reaches the singularity,

$$x_i \rightarrow 1^-, \qquad x_j < 1, \qquad j \neq i$$
 (113)

(by $x_j < 1$ we mean less than one by a finite amount, $x_j = 0.5$ for example). This is explicitly seen decomposing the kernel as

$$\frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} = \frac{2}{(1 - x_1)(1 - x_2)} - \frac{1 + x_1}{1 - x_2} - \frac{1 + x_2}{1 - x_1}.$$
 (114)

The configuration (113) is related to simple $1/\epsilon$ poles in dimensional regularization and to single logarithmic terms in the 3-jet cross section, i.e. to terms of the form $\alpha_S \log y$.

Both the leading and the subleading singularities produce divergences in the total cross section, so that

$$\sigma_{tot}(e^+e^- \to q\overline{q}g) = \infty,$$
 (115)

which is an unphysical result. Let us understand the origin of these singularites. They originate technically from the quark and antiquark propagators entering the Feynman amplitudes (the kernel (107) is roughly their product):

$$\frac{1}{(p+k)^2} = \frac{1}{2E_q E_g (1 - \cos \theta_{qg})} = \frac{1}{s} \frac{1}{1 - x_2}
\frac{1}{(p'+k)^2} = \frac{1}{2E_{\overline{q}} E_g (1 - \cos \theta_{\overline{q}g})} = \frac{1}{s} \frac{1}{1 - x_1}$$
(116)

The propagators diverge for two different configurations:

- i) Collinear singularity, $\theta \to 0$, i.e. when the gluon is emitted at a very small—angle with respect to the source (the quark or the antiquark);
- ii) Infrared singularity, $E_g \to 0$, i.e. when the gluon is emitted with a very small energy.

Let us see in detail how the end point singularities of (107) are related to the infrared and the collinear ones. The simple pole singularity

$$x_i \to 1, \quad x_i < 1 \tag{117}$$

implies

$$E_q > 0, \quad \theta_{i3} \to 0, \tag{118}$$

i.e. the collinear emission of a gluon with a finite energy from the quark/antiquark leg. The simple pole singularities (113) are therefore induced by collinears but not soft gluons.

Let us consider now the infrared limit

$$E_g \to 0, \quad \theta_{qg} \neq 0, \quad \theta_{\overline{q}g} \neq 0$$
 (119)

i.e. a gluon with small energy but not collinear to the quark or the antiquark. Kinematics simplifies for

$$x_3 \ll 1 \tag{120}$$

to

$$x_{1} \simeq 1 - x_{3} \frac{1 - \cos \theta_{\overline{q}g}}{2}$$

$$x_{2} \simeq 1 - x_{3} \frac{1 - \cos \theta_{qg}}{2}$$

$$\theta_{qq} + \theta_{\overline{q}g} \simeq \pi$$
(121)

such that

$$\cos \theta_{\overline{q}g} \simeq -\cos \theta_{qg} \tag{122}$$

The infrared contribution to the total cross section is of the form

$$\delta\sigma \sim \alpha_S \int \frac{dx_1 dx_2}{(1 - x_1)(1 - x_2)} \sim \alpha_S \int \frac{dx_1 dx_2}{x_3^2 (1 - \cos\theta_{qq})(1 - \cos\theta_{\overline{q}q})}$$
 (123)

Introducing the new variables

$$\epsilon = x_3
 u = \cos \theta_{qg}$$
(124)

so that $x_1 \simeq 1 - \epsilon(1+u)/2$, $x_2 \simeq 1 - \epsilon(1-u)/2$, we have in the new variables

$$\delta\sigma \sim \alpha_S \int \frac{d\epsilon}{\epsilon} \frac{du}{1 - u^2}$$
 (125)

Note that the change of variables converts the illusory double pole in x_3 into a simple one. Since $u \neq \pm 1$, we have a simple pole singularity for $\epsilon \to 0$. Infrared singularities are therefore associated with simple poles.

We see therefore that both collinear and infrared singularities are associated to simple poles of the differential cross section, i.e. to single logarithmic terms in the 3-jet cross section (see later).

Let us consider finally the case (112), i.e. when the energy fractions x_i reach simultaneously but independently their endpoint values. In this case

$$E_g \rightarrow 0 \quad and \quad \theta_{qg} \rightarrow 0 \quad or \quad \theta_{\overline{q}g} \rightarrow 0$$
 (126)

Eq.(125) is still valid, but in this case

$$\epsilon \to 0 \quad and \quad u \to \pm 1, \tag{127}$$

giving in this case a double pole singularity. We see that the leading singularity is generated by a gluon which is soft and at the same time collinear to the source

$$\delta\sigma \sim \alpha_S \int \frac{d\epsilon}{\epsilon} \int \frac{d\theta}{\theta}$$
 (128)

where we have taken $u \simeq 1 - \theta^2/2$ in eq.(125).

We can integrate the differential cross section (107) as long as we avoid the endpoints. The related cross section will be finite but strongly dependent on the kinematical cuts. We note that these cuts are naturally introduced in an experiment in the form of resolution power. Since detectors have a finite angular resolution $\delta > 0$ we cannot disinguish an isolated quark from a quark accompained by a gluon at an angle $\theta < \delta$. Analogously, the finite energy resolution $\eta > 0$ makes impossible to distinguish a quark from a quark surrounded by a gluon with energy $\epsilon < \eta$. In other words, if the gluon is 'too' collinear and/or soft, the final $q\bar{q}g$ is detected as a $q\bar{q}$ state:

$$|q\overline{q}g; \ \epsilon < \eta, \ \theta < \delta\rangle \rightarrow |q\overline{q}\rangle_{phys}$$
 (129)

The divergent total cross section (115) is therefore unmeasurable. The finite energy and angular resolution of the detectors restrict the integration region of (107) for the observable $q\bar{q}g$ cross section, avoiding the singular regions. Technically: there is a non zero lower limit in the energy and angle integrations in (128). The observable cross section is therefore finite but dependent on the resolution parameters δ and η .

In the same spirit, let us consider now the truly observable cross section for

$$e^+e^- \to q\overline{q}$$
 (130)

to order α_S . The $q\bar{q}g$ cross section (107) has to be integrated in the singular region, i.e. in the region corresponding to an undetectable gluon, but also virtual corrections to order α_S have to be included. It turns out by explicit calculation that collinear and infrared singularites cancel in the sum, giving rise to a finite cross section of order α_S . That is basically the same cancellation which occurs in the completely inclusive $O(\alpha_S)$ correction (eqs.(77) and (79)), which include the same virtual diagrams and the same singular integration regions for the real diagrams.

To summarize, the divergences encountered mean that QCD has the tendency to produce gluons with small energy and with small opening angles with respect to massless sources. Collinear singularities (but not infrared ones) can be regulated with a small quark mass, which changes the propagators according to:

$$\frac{1}{(p+k)^2 - m^2} = \frac{1}{2E_q E_g (1 - \beta_q \cos \theta_{qg})}
\frac{1}{(p'+k)^2 - m^2} = \frac{1}{2E_{\overline{q}} E_g (1 - \beta_{\overline{q}} \cos \theta_{\overline{q}g})}$$
(131)

where $\beta = p/E < 1$ is the velocity. Therefore, the gluons emitted in the production of beauty quarks, i.e. in the process

$$e^+e^- \rightarrow b + \overline{b},$$
 (132)

do not have any collinear singularity. The angular distribution of the gluons is given for small emission angles θ by

$$\frac{dN}{d\cos\theta} \sim \frac{1}{1-\beta\cos\theta} \sim \frac{2\gamma^2}{1+(\gamma\theta)^2} \tag{133}$$

where $\gamma = E/m = 1/\sqrt{1-\beta^2} \gg 1$ is the Lorentz factor. The angular distribution has therefore a fast increase,

$$\frac{dN}{d\theta} \sim \theta \frac{dN}{d\cos\theta} \sim \frac{1}{\theta} \tag{134}$$

when we reduce the emission angle from $\theta \sim 1$ up to

$$\theta_{min} \sim \frac{1}{\gamma}.$$
 (135)

The behaviour (134) is the same as that of massless quarks. At even smaller angles

$$\theta < \theta_{min}$$
 (136)

the angular distribution is essentially constant. That is how the quark mass m acts as a collinear regulator: it cutoffs the increase of the angular distribution. We expect therefore a logarithm of the mass after the angular integration:

$$\int_{1/\gamma} \frac{d\theta}{\theta} \sim \log \frac{E}{m} \tag{137}$$

The angular distribution (133) is similar to the angular distribution of the classical electromagnetic radiation from a relativistic charged particle (see [17]).

The above discussion about physical states and observable cross sections can be rephrased saying that we replaced single particle states with the true 'in' and 'out' states, i.e. the states prepared with accelerators and observed with detectors. These are not anymore single particle states but have components with different number of particles

$$|q\rangle_{phys} = a |q\rangle + b |qg\rangle + c |qgg\rangle + d |qq\overline{q}\rangle + \dots$$
 (138)

The quark state is replaced by a quark surrounded by a cloud of *collinear* and/or *soft* gluons and massless $q\overline{q}$ pairs. In the next section we will give a simple prescription to build these truly observable *out* states.

A remark is in order before ending this section. Up to now we considered the pure perturbative process of gluon radiation, so any angular or energy cut for the gluon is admissible (any y, see next section). The only limitation to the above results inside perturbative QCD is related to the Landau pole: as we lower the momentum transfer going into the soft region, the coupling rapidly increases leaving the perturbative region. But there is also a limitation of the applicability of perturbative QCD results related to confinement. A gluon with an energy of the order of the hadronic scale

$$\epsilon \sim \Lambda_{QCD}$$
 (139)

and/or with a transverse momentum k_T with respect to the source of the order of the hadronic scale,

$$k_T \sim \Lambda_{QCD}$$
 (140)

cannot move on a rectilinear motion neither producing a separate jet (not enough energy) nor flying away as an asymptotic state (confinement). This gluon is trapped by the color source. The perturbative behaviour of such gluon (an almost free motion) is therefore completely different from the 'real behaviour'. If we want perturbative QCD to describe the real hadronic world, we have to discard this conflict situation between perturbative and nonperturbative dynamics, imposing energy and angular cuts well above the hadronic scale:

$$\epsilon \gg \Lambda_{QCD}, \qquad k_T \gg \Lambda_{QCD}.$$
 (141)

That way we assign to different jets partons which can effectively produce different jets.

5.2.1The Jade Algorithm

The definition of an N jet event according to the JADE algorithm is the following [6]. Take the 4-momenta of the particles p_i and compute all the invariant masses:

$$M_{ij}^2 = (p_i + p_j)^2, \quad i \neq j$$
 (142)

If the smallest invariant mass (of particles i and j say) is less than a given threshold,

$$M_{ij}^2 < y s, (143)$$

then particles i and j are combined together to form a 'new' particle' (pseudoparticle) with momentum $p_{ij} = p_i + p_j$. $y \ll 1$ is a constant determining how 'fats' are the jets, such that $ys \gg \Lambda_{QCD}$ to avoid large hadronization effects. If instead

$$M_{ij}^2 \ge y s, \tag{144}$$

particles i and j are not combined together.

We iterate this procedure treating pseudoparticles as particles. At each step two particles are combined into a psedupoparticle, till the reduction stops because all the pair have invariant masses greater than the threshold. The final particles are identified with jets, and their number is the jet multiplicity.

Experimentalists compute n jet fractions combining the meaning momenta of the hadrons in the final state, while theorists compuye n jet fractions with final states composed of partons. If the qualitative ideas about confinement and hadronization discussed in secs. 4 and 4.2 are correct, we can test perturbative QCD comparing hadronic jet fractions with partonic ones.

In the case of a $q\bar{q}q$ final state, a 3-jet event is one in which all the three possible ivariant masses exceed the threshold:

$$(p+k)^2 \ge ys \tag{145}$$

$$(p+k)^2 \ge ys$$

$$(p'+k)^2 \ge ys$$

$$(145)$$

$$(146)$$

$$(p+p')^2 \geq ys \tag{147}$$

which reads in terms of the energy fractions:

$$x_i \le 1 - y \tag{148}$$

and therefore

$$x_i + x_j \ge 1 + y \quad i \ne j \tag{149}$$

We see that the boundary in the integration domain (the dangerous terms) are eliminated.

Let us define the jet fractions f_i as

$$f_{2} = \frac{\sigma_{2jet}}{\sigma_{tot}}$$

$$f_{3} = \frac{\sigma_{3jet}}{\sigma_{tot}}.$$
(150)

where $\sigma_{tot} = \sigma_{2jet} + \sigma_{3jet} = \sigma_0(1 + \alpha_S/\pi)$ is the total cross section to order α_S (to compute f_3 to $O(\alpha_S)$ we divide simply by σ_0). It holds clearly:

$$f_2 + f_3 = 1. (151)$$

We have for f_3

$$f_{3} = \int d\sigma_{q\overline{q}g} = C_{F} \frac{\alpha_{S}}{2\pi} \int_{2y}^{1-y} \frac{dx_{1}}{1-x_{1}} \int_{1+y-x_{1}}^{1-y} \frac{dx_{2}(x_{1}^{2}+x_{2}^{2})}{1-x_{2}}$$

$$= C_{F} \frac{\alpha_{S}}{2\pi} \left(4Li_{2} \left(\frac{y}{1-y} \right) + (3-6y) \log \left(\frac{y}{1-2y} \right) + 2 \log^{2} \left(\frac{y}{1-y} \right) - 6y - \frac{9}{2}y^{2} - \frac{\pi^{2}}{3} + \frac{5}{2} \right)$$

$$(152)$$

where $Li_2(z)$ is the dilogarithmic function (also called the Spence function), defined by the integral representation:

$$Li_2(z) = -\int_0^z du \, \frac{\log u}{1-u}$$
 (153)

We see that there are no infrared singularities in eq.(152) (like the $1/\epsilon$ poles we hit in eq.(77)): the jet fraction is an 'infrared safe' quantity. The jet fractions f_3 and $f_2 = 1 - f_3$ are plotted in fig. 8 as a function of y and in fig. 9 the experimental f_i are represented.

If we vary the center of mass energy \sqrt{s} of the e^+e^- collision keeping y fixed, the jet fractions f_i change because of the scale dependence of the coupling:

$$f_i(s, \alpha_S, y) = f_i(\alpha_S(s), y). \tag{154}$$

We may therefore observe the variation of the QCD coupling measuring jet fractions at different energy with the same cut y. This is also a method for measuring α_S (see the lectures of B. Mele at this School).

5.3 Sudakov Form Factors

Let us assume that we have an experiment with a good resolution power in the measure of invariant masses, i.e. that we can measure very small invariant masses. We can therefore measure the 2-jet fraction $f_2 = 1 - f_3$ up to very small y

$$y \ll 1 \tag{155}$$

The condition (155) is a strong restriction on the phase space of the final hadrons/partons: the final state consists of two very 'thin' jets in the space, which contain almost all the energy of the event.

Neglecting terms which are infinitesimal with y in eq.(152) (of the form y, $y \log y$, $y \log^2 y$, etc., the so called power suppressed corrections), f_2 reads to order α_S :

$$f_2(y) = 1 - C_F \frac{\alpha_S}{2\pi} \left(2\log^2 y + 3\log y - \frac{\pi^2}{3} + \frac{5}{2} + O(y) \right)$$
 (156)

There are three terms. There is a double logarithmic term which is the remnant of the collinear times infrared singularity, a single logarithmic term which is the remnant of the infrared + collinear singularity, and a finite term. Simbolically:

$$f_2^{(1)} \sim \alpha_S \log^2 y + \alpha_S \log y + \alpha_S \tag{157}$$

The y cut, as discussed previously, regulates both kind of singularites. If we use instead two different regulators for collinear and infrared singularities (for example an energy and an angular cut), we have for f_2 an expression of the form:

$$f_2^{(1)} \sim \alpha_S L l + \alpha_S L + \alpha_S l + \alpha_s \tag{158}$$

where L is the collinear logarithm (containing the angle cut) and l the infared logarithm (containing the energy resolutiom).

The $O(\alpha_S^2)$ corrections to f_2 have also been computed [2]:

$$f_2^{(2)} = \left(\frac{\alpha_S}{2\pi}\right)^2 \left(C_F^2 Z_C(y) + C_F N Z_N(y) + T_R Z_T(y)\right)$$
 (159)

where:

$$Z_{C}(y) = 2\log^{4} y + 6\log^{3} y + \left(\frac{13}{2} - \zeta(2)\right)\log^{2} y + \left(\frac{9}{4} - 3\zeta(2) - 12\zeta(3)\right)\log y + \frac{1}{8} - \frac{51}{4}\zeta(2) + 11\zeta(3) + 4\zeta(4)$$

$$Z_{N}(y) = \frac{11}{3}\log^{3} y + \left(2\zeta(2) - \frac{169}{36}\right)\log^{2} y + \left(6\zeta(3) - \frac{57}{4}\right)\log y + \frac{31}{9} + \frac{32}{3}\zeta(2) - 13\zeta(3) + \frac{45}{4}\zeta(4)$$

$$Z_{T}(y) = -\frac{4}{3}\log^{3} y + \frac{11}{9}\log^{2} y + 5\log y + \frac{19}{9} - \frac{38}{9}\zeta(2)$$
(160)

We see that the $O(\alpha_S^2)$ correction to f_2 contains up to four logarithms of y. In general the correction to order α_S^n contains terms up to $\log^{2n} y$, i.e. there at most two logarithms per loop (one collinear times one infrared log for each loop). We have therefore:

$$f_2^{(n)} = \alpha_S^n \sum_{k=0}^{2n} C_k^{(n)} \log^k y = C_{2n}^{(n)} \alpha_S^n \log^{2n} y + C_{2n-1}^{(n)} \alpha_S^n \log^{2n-1} y + \dots$$
 (161)

The perturbative expansion for $f_2(y)$ is not simply an expansion in powers of α_S at the relevant scale, like in the case of R. Every order in α_S is a polynomial in $\log y$. That means that there are logarithms which are not absorbable in the renormalization of the coupling constant, as it does happen instead with R. Let us consider the physical difference between these two observable. R is an inclusive quantity, characterized by a single mass scale, the center of mass energy \sqrt{s} , so the theory can develop only logarithms of the form

$$\log \frac{\Lambda_0^2}{s},\tag{162}$$

where Λ_0^2 is the ultraviolet cutoff, which are absorbed by renormalization of the lagrangian since the theory is renormalizable. The jet fraction f_2 is

instead a seminclusive quantity and its definition involves also another scale, ys. As a result, the theory can develop (and it actually does) logs of the other possible mass ratio,

$$\log \frac{s}{ys} = \log \frac{1}{y} \tag{163}$$

These logs are *final*, i.e. they do not cancel in the final result, as it does happen with the virtual and real diagram infrared singularities in R. The logs (163) have not ultraviolet origin (they are indeed infrared, as we saw), so they are not absorbable by a coupling redefinition.

For $y \ll 1$ these logarithms can become so large

$$\log \frac{1}{y} \gg 1 \tag{164}$$

that

$$\alpha_S \log^2 y \sim 1 \tag{165}$$

even though

$$\alpha_S \ll 1$$
 (166)

The convergence of the expansion in powers of α_S may therefore be spoiled by such large logarithmic coefficients. The solution of this problem is to abandon fixed order perturbation theory, in favour of an expansion according to the degree of singularity of the terms in the limit

$$\log \frac{1}{y} \to \infty. \tag{167}$$

The most singular terms, corresponding to the lowest order in this new asymptotic expansion, are of the form

$$\alpha_S^n \log^{2n} y$$
 $n = 0, 1, 2, 3, ..., k,$ (168)

The sub-leading terms are of the form

$$\alpha_S^n \log^{2n-1} y$$
 $n = 1, 2, 3...k...$ (169)

and are of order $1/\log y \ll 1$ smaller than the leading ones. The sub-sub-leading terms are of the form

$$\alpha_S^n \log^{2n-2} y$$
 $n = 1, 2, 3...k...$ (170)

and are of order $1/\log y$ smaller than the sub-leading ones and so on. Keeping only the leading series if often called double logarithmic approximation (DLA). Let us see how the two series are related. By simple rearrangement:

$$f_{2} = 1 + a_{0}\alpha x^{2} + a_{1}\alpha x + a_{2}\alpha + b_{0}\alpha^{2}x^{4} + b_{1}\alpha^{2}x^{3} + b_{2}\alpha^{2}x^{2} + b_{3}\alpha^{2}x + b_{4}\alpha^{2} + \dots$$

$$= \left(1 + a_{0}\alpha x^{2} + b_{0}\alpha^{2}x^{4} + \dots\right) + \left(a_{1}\alpha x + b_{1}\alpha^{2}x^{3} + \dots\right) + \left(a_{2}\alpha + b_{2}\alpha^{2}x^{2} + \dots\right)$$

$$+ \left(b_{3}\alpha^{2}x + \dots\right) + \dots$$

$$(171)$$

where $x = \log y$, a more confortable notation for the coefficients has been used and we dropped the subscript on α_S .

Since the coefficients of the leading terms (168) are known for any order (they exponentiate, see later), we can extend the validity of our result toward smaller y values factorizing the leading series. Up to order order α^2 we have

$$f_2 = (1 + a_0 \alpha x^2 + b_0 \alpha^2 x^4 + \dots) (1 + a_1 \alpha x + a_2 \alpha + (b_1 - a_0 a_1) \alpha^2 x^3 + (b_2 - a_0 a_2) \alpha^2 x^2 + b_3 \alpha^2 x + b_4 \alpha^2 + \dots)$$

$$(172)$$

We factorized the leading terms subtracting the spurious terms in the remaining factor up to $O(\alpha^2)$ included. The difference with the unfactorized expression is therefore $O(\alpha^3)$. In the first bracket on the right hand side we can add all the higher order terms of the leading series, i.e. resume the whole leading series, without affecting the second bracket up to order α^2 :

$$f_2 = (1 + a_0 \alpha x^2 + b_0 \alpha^2 x^4 + c_0 \alpha^3 x^6 + \dots + f_0 \alpha^n x^{2n} + \dots) (1 + a_1 \alpha x + a_2 \alpha + (b_1 - a_0 a_1) \alpha^2 x^3 + (b_2 - a_0 a_2) \alpha^2 x^2 + b_3 \alpha^2 x + b_4 \alpha^2 + \dots)$$

$$(173)$$

We may factorize the subleading series in the same way:

$$f_2 = (lead.) \left(1 + a_1 \alpha x + b_1' \alpha^2 x^3 + \dots \right) \left(1 + a_2 \alpha + b_2' \alpha^2 x^2 + (b_3 - a_1 a_2) \alpha^2 x + b_4 \alpha^2 \right)$$
(174)

where $b'_1 = b_1 - a_0 a_1$, $b'_2 = b_2 - a_0 a_2$ are the new coefficients shifted by the factorization of the leading series. This procedure can be carried on to any prescribed order in α_S .

Note that the coefficient of the $\log^4 y$ term in (159) is 1/2 of that of the $\log^2 y$ term in (156):

$$f_2 = 1 - C_F \frac{\alpha_S}{\pi} \log^2 y + \frac{1}{2} \left(\frac{C_F \alpha_S}{\pi}\right)^2 \log^4 y + \dots$$
 (175)

It can be proved that the leading terms of any order do exponentiate, i.e. they give a contribution to f_2 of the form

$$f_2 = e^{-\alpha_S C_F/\pi \log^2 y} \tag{176}$$

Expressions of this form are known as Sudakov form factors, who studied similar problems in QED long ago [10, 18, 19, 28]. Since in lowest order $f_2 = 1$, we see that the leading double logarithmic terms suppress the 2-jet cross section for $y \ll 1$. There is a physical explanation for that. For $y \ll 1$ we are considering the quasielastic production of $q\bar{q}$ pairs at high energy, i.e. a final state with little activity around the tree level produced $q\bar{q}$ pair. Accelerated color charges naturally produce radiation, as it happens in classical electrodynamics. The Sudakov represents the suppression of the improbable non radiative channels. We see here an implementation of the factorization idea discussed in sec. 2.1: the basic electromagnetic process is corrected by soft gluon effects which appear as a factor in eq.(176). The important physical information given by the Sudakov form factors is how the measured cross sections depend on the cut and the resolution of the experiments.

Another important property of the Sudakov form factors is the broadening of the sharp structures, like for example peaks in cross sections around resonances. Let us consider a QED case, the Z line shape discussed previously. The Sudakov suppression of non radiative channels implies a relative increase of the radiative cross sections, so that energy is frequently released from the e^+e^- system to the radiation field, through multiple photon emissions:

$$e^+e^- \rightarrow e^+e^- + n\gamma \tag{177}$$

The fluctuations in the energy released by the e^+e^- pair induce equal fluctuations in the energy available for the production of the resonance, so the latter is not anymore produced always on the peak, for any selected beam energy. That smears the peak and shifts its position toward higher center of mass energies $\sqrt{s} > M_Z$.

Let us observe that also the subleading terms proportional to $(C_F)^n$, i.e. the QED-like ones, seem to exponentiate:

$$f_2 = 1 - \frac{C_F \alpha_S}{2\pi} \left(2\log^2 y + 3\log y \right) + \frac{1}{2} \left(\frac{C_F \alpha_S}{2\pi} \right)^2 \left(4\log^2 y + 12\log^3 y \right) + \dots$$
(178)

They do indeed exponentiate, producing the correction factor

$$\delta f_2 = e^{-3\alpha_S C_F/(2\pi)\log y}. (179)$$

The terms (179) however do not contain the whole subleading corrections, as is clear looking to the form of the $Z_N(y)$ and $Z_T(y)$ contributions in (160) (they both contain $\alpha_S^2 \log^3 y$ terms). The factorization of all the subleading corrections is a complicated problem because it involves many different phenomena: the variation of α_S with the scale (if for example $\alpha_S(\mu) \to \alpha_S(y\mu) + O(\alpha_S^2 \log y)$, $\delta(\alpha_S \log^2 y) = O(\alpha_S^2 \log^3 y)$, i.e. a subleading term), separate collinear and infrared singularities.

6 Conclusions

In these lectures we discussed the foundation of perturbative QCD and an important physical application, the high-energy e^+e^- annihilation into hadrons. Even though perturbative QCD is technically similar to the perturbative expansion of QED, the physical content is very different. In QED the physical one-particle states (electrons and photons) are identical to the ones in the free theory (free electrons and photons) after renormalization. One then makes a perturbative expansion of the scattering matrix elements, so only a (weak) convergence assumption of the series is postulated. In perturbative QCD the one-particle states (the hadrons) are not identical at all to the one-particle states in the free theory (free quarks and gluons), after renormalization. We neglect this problem and compute cross sections with quarks and gluons as they were asymptotic states. We relate subsequenty these unphysical processes to the physical ones with a set of qualitative ideas about their connection (parton-hadron duality). Apart from convergence problems, perturbative QCD computations therefore have to be supplemented by assumptions about non-perturbative phenomena (such as confinement, hadronization, jet formation), necessarily accompaining the perturbative process. In these lectures we discussed this point considerably, presenting also qualitative discussions and some models of confinement and jet formation.

In the second part of the paper we considered the e^+e^- annihilation to hadrons at high energy, which has many different dynamical aspects. The simplest observable is the total hadronic cross section. Its perturbative computation relies only on the assumption that hadronization does not change

the probability of the partonic process. Furthermore, the perturbative expansion is very simple: it involves only numerical coefficients times power of α_S evaluated at the center of mass energy. More 'delicate' observables are the jet fractions. Their perturbative computation requires an extra assumption with respect to the total cross section. We have to assume local parton-hadron duality, i.e. that hadrons are formed by partons which are close in phace space. This postulate is necessary because jet fractions contain dynamical effects related to intermediate scales, i.e. momenta p between a cutoff scale Λ ($\gg \Lambda_{QCD}$) and the center of mass energy

$$\Lambda^2 < p^2 < s. \tag{180}$$

These scales have not to be mixed by hadronization. Also the perturbative series for the jet fractions is more complicated: there are large logarithmic corrections of the form

$$\log \frac{s}{\Lambda^2}.\tag{181}$$

The latter is anyway a technical problem that is solved *within* perturbative QCD with resummation techniques.

Comparing with experimental data, we have seen that the perturbative QCD approach, perturbative computations + hadronization assumptions, does indeed work, both qualitatively and quantitatively. A face of the hadronic world seems to be largely understood. This also implies that confinement is very 'hidden' in many processes. The progress in the perturbative QCD direction consists in higher order computations, better resummation techniques, and in finding measurable quantities that characterize some properties of the hadronic states and are little influenced by hadronization.

A Evaluation of the 3-jet cross section

In this section we compute the cross section at order α_S for

$$e^+e^- \rightarrow q\overline{q}g$$
 (182)

We take the gluon propagator in Feynman gauge:

$$S_{ab}^{\mu\nu}(k) = \frac{-ig^{\mu\nu}\delta_{ab}}{k^2 + i\epsilon},$$
 (183)

and the quarks and leptons massless. The Feynman amplitude M is:

$$M = M_a + M_b = \tag{184}$$

$$- ie^2 gt_a \overline{v}_{r'}(l') \gamma_{\mu} u_r(l) \frac{g_{\mu\nu}}{s+i\epsilon} \overline{u}_s(p) \left(\gamma_{\rho} \frac{\hat{p}+\hat{k}}{(p+k)^2} \gamma_{\nu} - \gamma_{\nu} \frac{\hat{p'}+\hat{k}}{(p'+k)^2} \gamma_{\rho} \right) v_{s'}(p') \epsilon_{ah}^{\rho}(k)$$

where $\hat{p} = \gamma_{\mu} p^{\mu}$, $s = q^2$, q = l + l' and l and l' are the 4-momenta of the electron and the positron.

We take now the square of the modulus, average over the initial helicities and sum over the final helicities and polarizations,

$$\frac{1}{4} \sum_{rr'ss'h} |M|^2, \tag{185}$$

using the formulas

$$\sum_{r} u_{r}(p)\overline{u}_{r}(p) = \hat{p}, \qquad \sum_{r} v_{r}(p')\overline{v}_{r}(p') = \hat{p'}$$

$$\sum_{h} \epsilon_{h\mu}(k)\epsilon_{h\nu}(k) = -g_{\mu\nu} + \frac{k_{\mu}k_{\nu}}{k^{2}}$$
(186)

(the $k_{\mu}k_{\nu}/k^2$ term in the photon polarization sum does not contribute because the diagrams are QED-like).

We multiply by the relativistic final states factor

$$\frac{d^3p}{(2\pi)^3 2p_0} \frac{d^3p'}{(2\pi)^3 2p'_0} \frac{d^3k}{(2\pi)^3 2k_0},\tag{187}$$

the relativistic normalization factor for the initial state

$$\frac{1}{2l_0}\frac{1}{2l_0'} = \frac{1}{4E_b^2},\tag{188}$$

the 4-momentum conserving δ function

$$(2\pi)^4 \delta^{(4)}(q - p - p' - k), \tag{189}$$

and divide by the relative velocity v_{rel} of the e^+e^- pair in the center of mass frame (i.e. by the flux for volume V=1):

$$v_{rel} = \frac{l \cdot l'}{l_0 l'_0} = 2, \tag{190}$$

(the two particles have opposite light velocity). We have:

$$d\sigma = \frac{1}{2^{13}\pi^{5}E_{b}^{2}} \sum_{rr'ss'h} |M|^{2} \delta^{(4)}(q - p - p' - k) \frac{d^{3}p}{p_{0}} \frac{d^{3}p'}{p'_{0}} \frac{d^{3}k}{k_{0}}$$

$$= -\frac{C_{F}e^{4}g^{2}}{2^{13}\pi^{5}} \frac{1}{E_{b}^{2}s^{2}} L_{\mu\nu}(l, q) T^{\mu\nu}(q)$$
(191)

where the leptonic and the (completely integrated) hadronic tensors are given by

$$L_{\mu\nu}(q,l) = Tr[\hat{l}\gamma_{\mu}\hat{l}'\gamma_{\nu}]$$

$$T_{\mu\nu}(q) = \int \frac{d^{3}p}{p_{0}} \frac{d^{3}p'}{p'_{0}} \frac{d^{3}k}{k_{0}} \delta^{(4)}(p+p'+k-q) H_{\mu\nu}(p,p',k) \quad (192)$$

and $H_{\mu\nu}$ is the (unintegrated) hadronic tensor

$$H_{\mu\nu} = Tr \left[\hat{p} \left(\gamma_{\rho} \frac{\hat{p} + \hat{k}}{(p+k)^{2}} \gamma_{\mu} - \gamma_{\mu} \frac{\hat{p}' + \hat{k}}{(p'+k)^{2}} \gamma_{\rho} \right) \hat{p}' \left(\gamma_{\nu} \frac{\hat{p} + \hat{k}}{(p+k)^{2}} \gamma^{\rho} - \gamma^{\rho} \frac{\hat{p}' + \hat{k}}{(p'+k)^{2}} \gamma_{\nu} \right) \right]$$
(193)

The tensors are symmetric under exchange of the indices μ and ν and are transverse with respect to q_{μ} because of electromagnetic current conservation

$$q_{\mu} T^{\mu\nu} = 0, \qquad q_{\mu} L^{\mu\nu} = 0, \qquad (194)$$

 $T_{\mu\nu}$ can be parametrized as

$$T_{\mu\nu}(q) = (g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}) T(q^2)$$
 (195)

so that

$$T(q^2) = \frac{1}{3}g^{\mu\nu}T_{\mu\nu}(q). \tag{196}$$

The contraction gives

$$L_{\mu\nu} T^{\mu\nu} = \frac{1}{3} g^{\mu\nu} L_{\mu\nu} g^{\rho\sigma} T_{\rho\sigma} = -\frac{4}{3} q^2 g^{\rho\sigma} T_{\rho\sigma}. \tag{197}$$

We decoupled the tensors with the projector $g_{\mu\nu}g_{\rho\sigma}/3$. Therefore we need only to compute the contraction

$$g^{\mu\nu} H_{\mu\nu}(p, p', k) = Tr \left[\hat{p} \left(\gamma_{\rho} \frac{\hat{q} - \hat{p}'}{(q - p')^{2}} \gamma_{\mu} - \gamma_{\mu} \frac{\hat{q} - \hat{p}}{(q - p)^{2}} \gamma_{\rho} \right) \hat{p}' \left(\gamma^{\mu} \frac{\hat{q} - \hat{p}'}{(q - p')^{2}} \gamma^{\rho} - \gamma^{\rho} \frac{\hat{q} - \hat{p}}{(q - p)^{2}} \gamma^{\mu} \right) \right]$$
(198)

Because of the $\delta^{(4)}(p+p'+k-q)$ we made the replacements

$$p + k = q - p', \quad p' + k = q - p.$$
 (199)

We have:

$$g^{\mu\nu} H_{\mu\nu}(p, p', q - p - p') = \frac{X_{11}}{((q - p')^2)^2} + \frac{X_{12}}{(q - p)^2(q - p')^2} + \frac{X_{21}}{(q - p)^2(q - p')^2} + \frac{X_{22}}{((q - p)^2)^2}$$

$$+ \frac{X_{21}}{(q - p)^2(q - p')^2} + \frac{X_{22}}{((q - p)^2)^2}$$
 (200)

where

$$X_{11} = +Tr \left[\hat{p}\gamma_{\rho}(\hat{q} - \hat{p}')\gamma_{\mu}\hat{p}'\gamma^{\mu}(\hat{q} - \hat{p}')\gamma^{\rho} \right]$$

$$X_{12} = -Tr \left[\hat{p}\gamma_{\rho}(\hat{q} - \hat{p}')\gamma_{\mu}\hat{p}'\gamma^{\rho}(\hat{q} - \hat{p})\gamma^{\mu} \right]$$

$$X_{21} = -Tr \left[\hat{p}\gamma_{\mu}(\hat{q} - \hat{p})\gamma_{\rho}\hat{p}'\gamma^{\mu}(\hat{q} - \hat{p}')\gamma^{\rho} \right]$$

$$X_{22} = +Tr \left[\hat{p}\gamma_{\mu}(\hat{q} - \hat{p})\gamma_{\rho}\hat{p}'\gamma^{\rho}(\hat{q} - \hat{p})\gamma^{\mu} \right]$$

$$(201)$$

The explicit computation gives:

$$X_{11} = 4Tr \left[\hat{p} \left(\hat{q} - \hat{p}' \right) \hat{p}' \left(\hat{q} - \hat{p}' \right) \right] = 4Tr \left[\hat{p} \hat{q} \hat{p}' \hat{q} \right]$$

$$= 16 \left(2p \cdot q \ p' \cdot q - q^2 \ p \cdot p' \right)$$

$$X_{12} = 2Tr \left[\hat{p} \gamma_{\rho} (\hat{q} - \hat{p}') (\hat{q} - \hat{p}) \gamma^{\rho} \hat{p}' \right] = 8(q - p) \cdot (q - p') Tr \left[p \cdot p' \right]$$

$$= 32 \ p \cdot p' \ (q - p) \cdot (q - p')$$

$$X_{21} = X_{12}$$

$$X_{22} = X_{11}$$
(202)

where we used the identities

$$\gamma_{\mu}\gamma_{\rho}\gamma^{\mu} = -2\gamma_{\rho}
Tr[\gamma_{\mu}\gamma_{\nu}\gamma_{\rho}\gamma_{\sigma}] = 4(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\nu\sigma}g_{\mu\rho})
\gamma_{\mu}\gamma_{\alpha}\gamma_{\beta}\gamma_{\rho}\gamma^{\mu} = -2\gamma_{\rho}\gamma_{\beta}\gamma_{\alpha}.$$
(203)

Summing all the terms we derive:

$$g^{\mu\nu} H_{\mu\nu} = 16 \left(2p \cdot q \ p' \cdot q - q^2 p \cdot p' \right) \left[\frac{1}{((q-p)^2)^2} + \frac{1}{((q'-p)^2)^2} \right] + 64 \frac{p \cdot p' (q-p) \cdot (q-p')}{(q-p)^2 (q-p')^2}, \tag{204}$$

The scalar projection of $H_{\mu\nu}$ is much simpler to compute than the original tensor because it is easily reduced to the trace of four gamma matrices, while the computation of $H_{\mu\nu}$ involves the trace of six gamma matrices.

We perform now the integrations in the center of mass frame, where

$$\vec{q} = 0. \tag{205}$$

First we integrate the $\delta^{(3)}(\vec{p}+\vec{p}'+\vec{k})$ over the gluon 3-momentum \vec{k} , which gives

$$\vec{k} = -\vec{p} - \vec{p}', \tag{206}$$

and therefore

$$k_0 = |\vec{p} + \vec{p}'| = \sqrt{p_0^2 + p_0'^2 + 2p_0p_0' \cos\theta_{q\bar{q}}}.$$
 (207)

We choose now the spatial frame in such a way that \vec{p} is directed along the z-axis and integrate over the polar angle of \vec{p} ,

$$\frac{d^3 p'}{p'_0} = p'_0 dp'_0 d\cos\theta' d\phi', \tag{208}$$

The energy-conserving $\delta(k_0 + p_0 + p_0' - q_0)$ -function gives:

$$\int \frac{p_0 dp_0 d\Omega p_0' dp_0' d\phi' d\cos\theta'}{k_0} \delta(q_0 - p_0 - p_0' - k_0) g_{\mu\nu} H^{\mu\nu} = \int dp_0 d\Omega dp_0' d\phi' g_{\mu\nu} H^{\mu\nu}$$
(209)

because

$$\left[\frac{\partial k_0}{\partial \cos \theta'}\right]^{-1} = \frac{k_0}{p_0 p_0'}.$$
 (210)

The integration over $d\Omega d\phi'$ is trivial and gives $8\pi^2$:

$$\int dp_0 dp'_0 d\Omega d\phi' g_{\mu\nu} H^{\mu\nu} = 8\pi^2 E_b^2 dx_1 dx_2 g_{\mu\nu} H^{\mu\nu}$$
 (211)

We express now the hadronic trace (204) in terms of energy fractions x_1 and x_2 . We divide numerators and denominators by E_b^4 , and replace the rescaled scalar products according to the relations

$$p \cdot q = 2x_1, \quad p \cdot q' = 2x_2, \quad q^2 = 4, \quad p \cdot p' = 2(x_1 + x_2 - 1).$$
 (212)

These substitutions give:

$$g^{\mu\nu}H_{\mu\nu} = 8\left[\frac{1-x_1}{1-x_2} + \frac{1-x_2}{1-x_1} + \frac{2(x_1+x_2-1)}{(1-x_1)(1-x_2)}\right]$$
$$= 8\frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$
(213)

We note that the leading singularity $(1-x_1)^{-1}(1-x_2)^{-1}$ in Feynman gauge comes from the interference term (while it comes from a single direct term in a particularly choosen axial gauge [11, 16]).

Putting all the pieces together, we have the final result:

$$d\sigma = \frac{2C_F}{3} \frac{\alpha^2 \alpha_S}{s} dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$
 (214)

which coincides with result (107) multiplied by $\sigma_0 = 4\pi/3\alpha^2/s$. Performing the angular integrations and the Dirac algebra in n dimensions, we derive the dimensionally regularized cross section in eq.(77).

References

- [1] U. Aglietti and Z. Ligeti, Phys. Lett. B 364 (1995) 75-77.
- [2] A. Ali, $QCD \gamma\gamma$ and heavy quark physics, in PHYSICS AT LEP, Vol.2 (Cern Yellow Reports) CERN 86-02 (1986), pag.99; see also the references therein to the original articles.
- [3] G. Altarelli, Cern preprint CERN-TH-95/196. (July 1995);
- [4] G. Altarelli, Phys. Rep 81, n.1 (1982) 1-129.
- [5] N.W. Ashcroft, N.D. Mermin, *Solid State Physics*, HRW International Editions (1976), pag. 731.
- [6] S. Bethke et al. (JADE Collaboration), Phys. Lett. 213B (1988) 235; for improved algorithms, see N. Brown and W.J. Stirling, Durham preprint, RAL-91-049 DTP/91/30 (June 1991).
- W.E. Caswell, Phys. Rev. Lett 33 (1974) 244; D.R.T. Jones, Nucl. Phys. B75 (1974) 531; O.V. Tarasov, A.A. Vladimirov and A. Yu Zharkov, Phys. Lett 93B (1980) 429.
- [8] T.P. Cheng e L.F. Li, *Gauge Theory of Elementary Particles*, Clarendon Press (1984), pag. 280.
- [9] K.G. Chetyrkin, A.L. Kataev and F.V. Tkachov, Phys. Lett. 85B (1979)277; M. Dine and J. Sapirstein, Phys. Rev. Lett. 43 (1979) 668; W. Celmaster and R. Gosalves, Phys. Rev. Lett 44 (1980) 560.
- [10] J.C. Collins, Sudakov Form Factors, in Perturbative Quantum Chromodynamics, ed. A.H. Mueller (World Scientific, Singapore, 1989) 573-614.
- [11] Yu.L. Dokshitzer et al., *Basics of Perturbative QCD*, Editions Frontieres (1991).
- [12] R.K. Ellis, *QCD at Tasy '94*, Fermilab preprint FERMILAB-Conf-94/410-T December 9, 1994.

- [13] H. Fritzsch and M. Gell-Mann in: Proc XVIth Inter. Conf. on High Energy Physics (Chicago-Batavia 1972, Vol.2 pag. 135; H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Lett. B47 (1973) 365.
- [14] S.G. Gorishny, A.L. Kataev and S.A. Larin, Phys. Lett. B259 (1991)144; L.R. Surguladze and M.A. Samuel, Phys. Rev. Lett 66 (1991) 560.
- [15] K. Gottfried and V.F. Weisskopf, *Concepts in Particle Physics*, Oxford University Press (1988); for a discussion on the antiscreening of QCD vacuum pag.380, and for confinement pag. 397.
- [16] W. Greiner and A. Schafer, Quantum Chromodynamics, Springer (1994); this textbook contains many computations described in great detail (it is a german book), among which there is the calculation of the 3-jet cross section in axial gauge, pag. 292.
- [17] J.D. Jackson, Classical Electrodynamics, John Wiley, New York 1975.
- [18] J. Kodaira and L. Trentadue, Phys. Lett. 112 B, n.1 (1982) 66-70; L. Trentadue, SLAC-PUB-2934 June 1982 (T).
- [19] L.D. Landau and E.M. Lifsits, Teoria Quantistica Relativistica, Fisica Teorica 4, Editori Riuntiti Edizioni MIR (1978) (Quantum Field Theory, Vol. IV, A Course in Theoretical Physics).
- [20] T.D. Lee, Particle Physics and Introduction to Field Theory, Harwood Academic Publishers (1988).
- [21] G. Martinelli, *Perturbative QCD*, Proceedings of the 1987 JINR-CERN School of High Energy Physics (near Varna, Bulgaria, 6-19 september 1987).
- [22] P. Nason, M. Magnoli and R. Rattazzi, UPRF-90-292 preprint, October 1990; there is a clear discussion about the assumptions on hadronization on which perturbative QCD relies.
- [23] Particle Data Group, Review of Particle Physics, Phys. Rev. D54 (1996).

- [24] A. Pich, *Quantum Chromodynamics*, lectures given at the 1994 European School of High Energy Physics (Sorrento, Italy, 29 August 11 September 1995).
- [25] H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346; D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343.
- [26] G. Salvini and A. Silverman, Phys. Rep. 171 (n.5 and 6) (1988).
- [27] G. Sterman, An Introduction to Quantum Field Theory, Cambridge University Press (1993).
- [28] L. Trentadue, Phys. Lett. 151 B, n.2 (1985), 171-173.
- [29] C.N. Yang and F. Mills, Phys. Rev. 96, 191 (1954).
- [30] See for example, A. Le Yaouanc et al., *Hadron Transitions in the Quark Model*, Gordon and Breach Science Publishers (1988), pag.44.
- [31] K. Wilson, Phys. Rev. 179 (1969) 1499.

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- Fig.1: A two jet event from hadronic Z decay (DELPHI, taken from ref. [24]).
- Fig.2: A three jet event from hadronic Z decay (DELPHI, taken from ref. [24]).
- Fig.3: Born diagrams for $e^+e^- \to q\overline{q}$.
- Fig.4: Angular distribution of two jet events at 34 GeV center of mass energy. The curve is the QED + parton model prediction $1+x^2$, where $x = \cos \theta$ (TASSO, from ref. [26]).
- Fig.5: Compilation of R values at low energy (upper figure) and at high energy (lower figure), (from ref. [23]).
- Fig.6: Total cross section for $e^+e^- \to \mu^+\mu^-$ +e.m. radiation, around the Z peak. The dotted line is the Born approximation, the dashed line is the Born approximation + leading logs coming from initial state radiation and the solid line is the Born approximation + leading logs + subleading logs (from ref. [26]).
- Fig.7: Diagrams for real corrections (upper figure) and virtual corrections (lower figure) of order α_S to $e^+e^- \to q\overline{q}$.
- Fig.8: Plot of the jet fractions f_2 and f_3 to order α_S (from ref. [12]).
- Fig.9: QCD fits to the jet fractions (OPAL, from ref. [12]).

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