# ALGEBRAIC STACKS

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## Overview

These notes roughly correspond to an attempt to learn stack theory that began in the winter of 2023. We will begin with the categorical preliminaries as laid out in [3] before developing the basic theory per the text of Olsson [1] and conclude with a sampling of the more advanced topics in [2, Part 7].

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**Grothendieck Topologies, Sites, and Fibered Categories**

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## 2. Fibered Categories

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## 4. Algebraic Spaces

## 5. Quotients in Algebraic Spaces

## 6. Quasicoherent Sheaves on Algebraic Spaces

**Algebraic Stacks**

## 7. Algebraic Stacks

## 8. Quasicoherent Sheaves on Algebraic Stacks

**The Geometry of Stacks**

## 9. Geometric Properties of Stacks

## 10. Coarse Moduli Spaces

## 11. Gerbes

## 12. Cohomology of Stacks

## 13. Derived Categories of Stacks

## End Material

## References

[1] Martin Olsson. *Algebraic spaces and stacks*. English. Vol. 62. Colloq. Publ., Am. Math. Soc. Providence, RI: American Mathematical Society (AMS), 2016. ISBN: 978-1-4704-2798-6; 978-1-4704-2865-5.
[2] The Stacks project authors. *The Stacks project*. https://stacks.math.columbia.edu. 2023.
[3] Angelo Vistoli. *Notes on Grothendieck topologies, fibered categories and descent theory*. 2007. arXiv: math/0412512 [math.AG].

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