

ALGEBRAIC STACKS

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OVERVIEW

These notes roughly correspond to an attempt to learn stack theory that began in the winter of 2023. We will begin with the categorical preliminaries as laid out in [4] before developing the basic theory per the text of Olsson [2] and conclude with a sampling of the more advanced topics in [3, Part 7]. The standard texts are [1] and [2]. The compendium [3] is encyclopedic.

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Grothendieck Topologies, Sites, and Fibered Categories

1. GROTHENDIECK TOPOLOGIES

2. FIBERED CATEGORIES

3. DESCENT

Algebraic Spaces

4. ALGEBRAIC SPACES

5. QUOTIENTS IN ALGEBRAIC SPACES

6. QUASICOHERENT SHEAVES ON ALGEBRAIC SPACES

Algebraic Stacks**7. ALGEBRAIC STACKS**

8. QUASICOHERENT SHEAVES ON ALGEBRAIC STACKS

The Geometry of Stacks

9. GEOMETRIC PROPERTIES OF STACKS

10. COARSE MODULI SPACES

11. GERBES

12. COHOMOLOGY OF STACKS

13. DERIVED CATEGORIES OF STACKS

End Material

REFERENCES

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- [2] Martin Olsson. *Algebraic spaces and stacks*. English. Vol. 62. *Colloq. Publ.*, Am. Math. Soc. Providence, RI: American Mathematical Society (AMS), 2016. ISBN: 978-1-4704-2798-6; 978-1-4704-2865-5.
- [3] The Stacks project authors. *The Stacks project*. <https://stacks.math.columbia.edu>. 2023.
- [4] Angelo Vistoli. *Notes on Grothendieck topologies, fibered categories and descent theory*. 2007. arXiv: [math/0412512](https://arxiv.org/abs/math/0412512) [[math.AG](#)].

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