Phase Shift in an Inductor-Resistor Circuit

Increasing coil turns raises phase shift, while higher series resistance lowers it, matching FEM predictions.

18th of september 2025

By William Bowley

Introduction

Phase shift in an inductor-resistor circuit (LR-circuit) occurs because an inductor wants to oppose changes to its magnetic field. This opposition is called inductance and influences the phase shift. The frequency of the voltage source also affects the phase shift, as the inductor reacts more strongly when the current changes more frequently. This is quantified as inductive reactance, measured in ohms, which for a cylindrical solenoid can be approximated as:

$$X_l \approx \frac{2\pi f \mu A}{l} N^2$$

Fig 1A: Analytical approximation for rectance for cylindrical solenoid

Series resistance of the circuit also plays an important role, as it limits the current and controls the rate at which the inductor charges and discharges. This relationship determines the circuit time constant, which directly affects the phase shift. The phase shift can be calculated through the formula presented below:

$$\tan(\theta) = \frac{X_l}{R}$$

Fig 1B: Proposed phase shift calculation based on inductive reactance and series resistance

it is expected that an increase in resistance will decrease the phase shift with a strictly decreasing arc tangent-type curve, whereas increasing the number of turns results in a strictly increasing arctangent-type curve. This is supported by Fig 1B, where phase shift is proportional to the ratio of reactance to series resistance, and by Fig 1A, where resistance is proportional to the square of the number of turns. A simplified relation is proposed below:

$$\theta \propto \frac{1}{R} \qquad \theta \propto N^2$$

Fig 1C: Proposed proportional relationship for the series resistance and number of turns with respect to phase shift.

methodology

The circuit consisted of an inductor and a resistor as a shunt to measure the current waveform. This shunt resistor was just a standard 100 ohm resistor and was also used as the baseline for all trials. The shunt was connected in series with the voltage source of 50 Hz at 10 VAC. The inductor was also connected in series with the source and resistor.

The Rigol DS1054Z was then connected in parallel across the voltage source to measure its waveform on channel 1. It was also connected in parallel across the shunt resistor to measure the current waveform of the circuit. To change the series resistance, a resistor of known value was placed downstream of the shunt. And to change the number of turns, an inductor set with a fixed geometry but different wire diameters was used. This inductor set also had a laminated iron core that was used throughout the runs

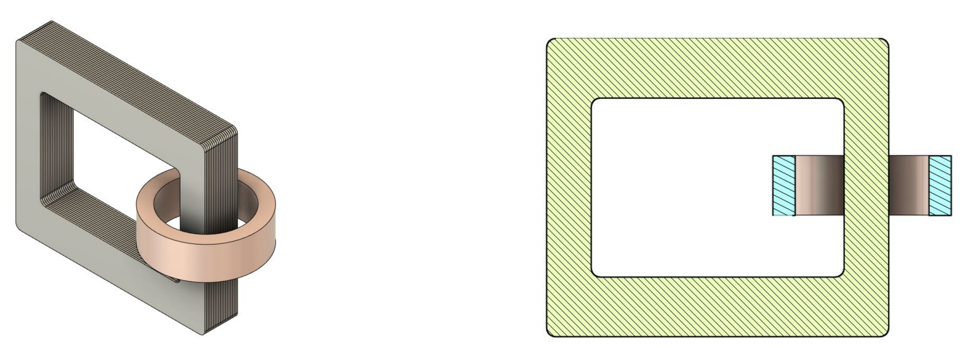

Fig 2A: Approximate 3D model of the inductor in Fusion 360. The full model is shown on the left, and the cross-section used for FEA is shown on the right.

The DS1054Z was calibrated to 10 ms per division for a total of 100 ms bandwidth horizontally and the 5 V per division vertical bandwidth of ± 20 V. Then, for each trial, the oscilloscope measured both channels and the data was saved to a CSV file. The analysis was later performed using a Python script that used Pandas, SciPy, and NumPy to remove high-frequency transient effects and then reported the phase shift for the samples.

The geometry was then modeled in planar FEM software that approximated the inductance and resistance of each inductor setup and then used the equation in Fig. 1B to calculate the expected phase shift.

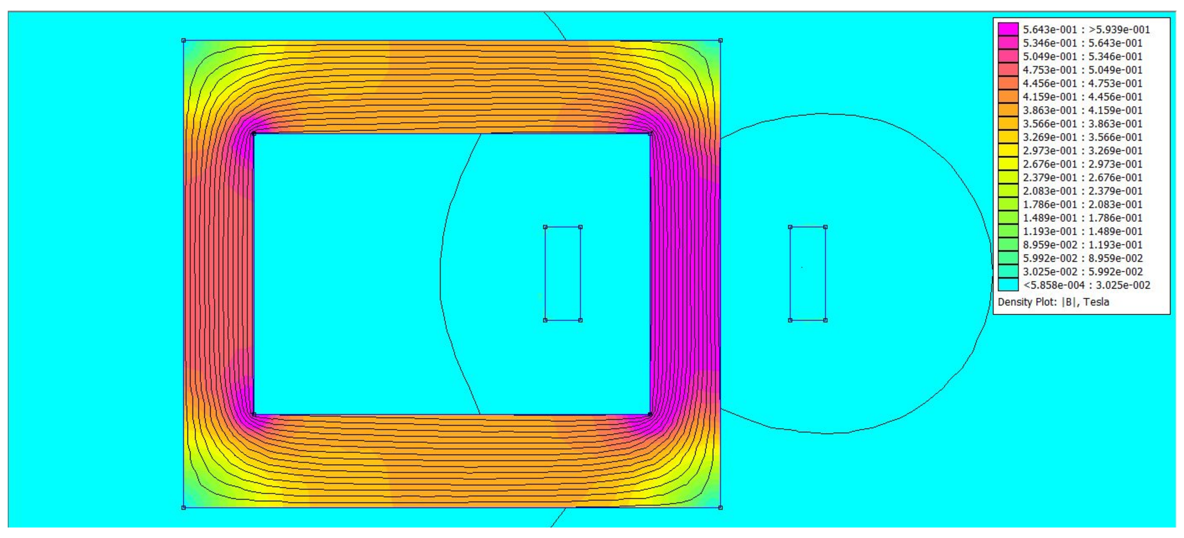

Fig 2B: FEM results for the 1200 turn setup. Shows the strong flux linkage between the coils and the laminated core.

Results

Turns	100Ω	200Ω	300Ω	310Ω
300	2.19	1.11	0.79	0.72
600	12.39	6.57	4.45	4.34
1200	30.02	18.44	13.14	12.82

Fig 3A: Experimental results where phase shift is in degrees. The approximate uncertainty for the raws is ± 2 degrees.

Turns	100Ω	200Ω	300Ω	310Ω
300	2.54	1.27	0.85	0.82
600	9.94	5.05	3.38	3.27
1200	33.76	19.03	13.08	12.68

Fig. 3B: Finite element magnetic (FEM) results from BlueShark-FEA, with phase shift calculated using the equation from Fig. 1B

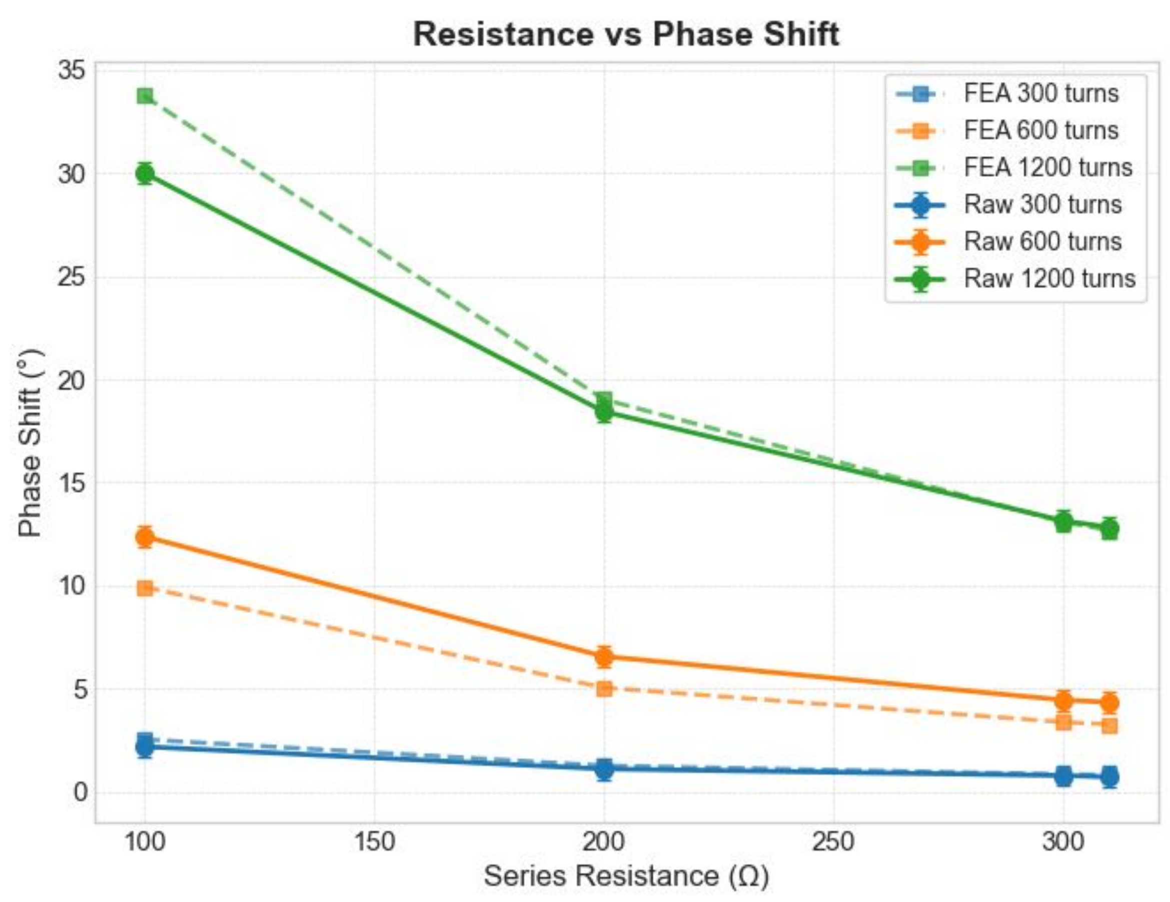

Fig. 4A: Phase shift versus series resistance (fixed turns). The approximate uncertainty in the raw data was ±2°. FEA results do not include uncertainty.

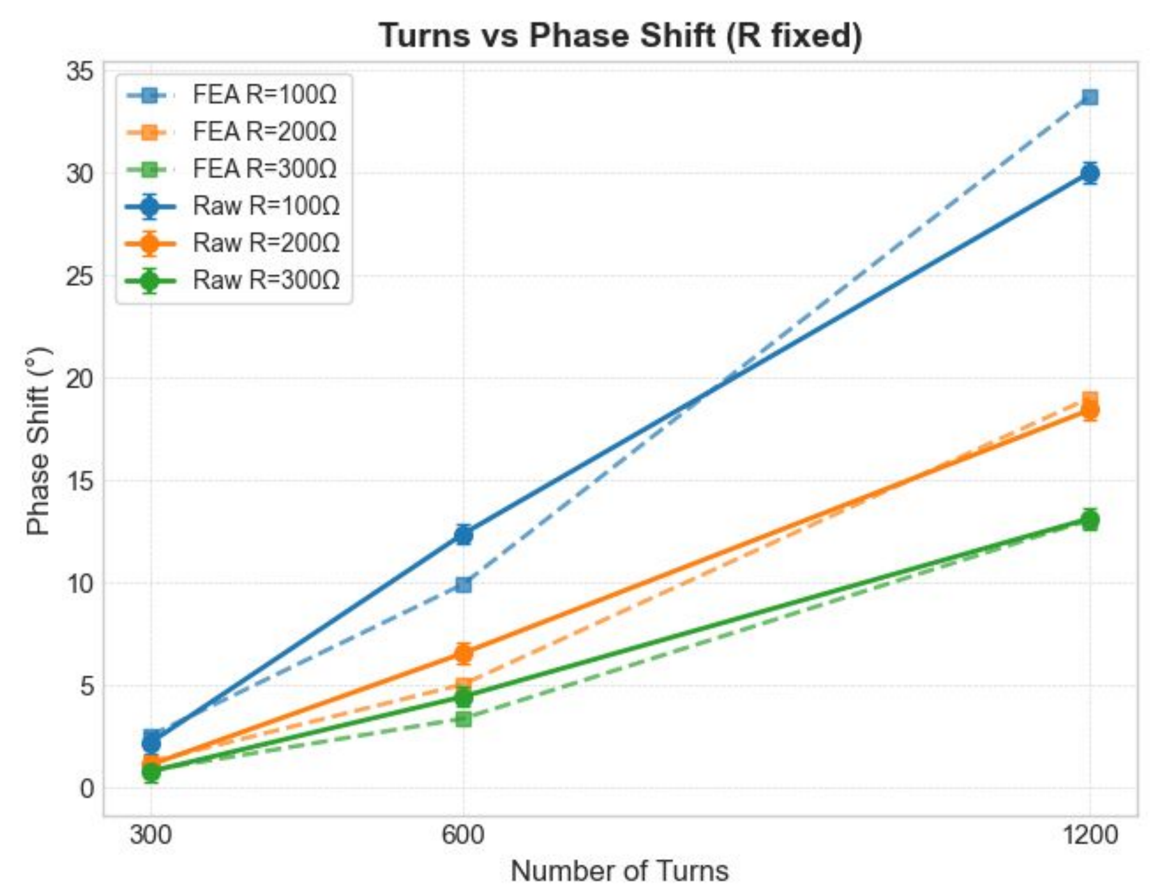

Fig. 4B: Phase shift versus number of turns (fixed resistance). The approximate uncertainty in the raw data was ±2°. FEA results do not include uncertainty.

Discussion

As shown in Figure 4A, there is a relationship between resistance and phase shift. The proposed relation of phase shift being inversely proportional to resistance has an R-squared value of 0.985, which indicates that the relationship is very strong. Figure 4B shows the relationship between number of turns and phase shift. When compared to the proposed squared proportional relationship, it has an R-squared value of 0.999, which is an extremely strong correlation

An interesting observation is that FEM results seem to follow the same relative trend, but the experimental data seems to fluctuate around those results. This is most likely caused by the uncertainty of 2 degrees, which is a large amount. But another factor to consider is that FEM didn't model parasitic capacitance of the carbon film resistor or the inductor, which would have increased the total reactance of the circuit. However, this would have been a systematic error that would shift the FEM results downwards, which from observations seems to be true.

A limitation with the validation of the experimental methodology through FEM is that the magnetic permeability of the core material is unknown. The core was assumed to be homogeneous with the same x and y permeability of 100. This might not be the case, as FEM didn't include AC effects when calculating the inductance and resistance of the inductor. So the permeability might be higher. But this does show that even without AC effect you can still model them, you just have to run a test with that material beforehand. A simple way to improve the methodology is just to measure the permeability of the core material so that it doesn't have to be assumed.

This experiment shows that firstly the relationships in FIG 1C and the formula for phase shift in FIG 1B are correct. It also shows a valid method of getting phase shift from FEA without including AC effects. The implications of this is that phase shift can be modelled under static conditions which decreases computational requirements. Future research should attempt to validate this method for multiphase systems such as Linear Synchronous Motors (LSM) or other AC systems

Conclusion

The data gathered from the experiment does support the hypothesis very strongly as shown by the r-squared values for both number of turns and also resistance. The aim of measuring experimental data and than validating a FEM model of the problem was successful. Producing a method for analysis of phase shift within magnetostatic simulations.

References

- Engineering Toolbox. (2016). Permeability. Retrieved from Engineeringtoolbox.com website: https://www.engineeringtoolbox.com/permeability-d_1923.html
- Series RLC Circuit and RLC Series Circuit Analysis. (2013, June 25). Retrieved from Basic Electronics Tutorials website: https://www.electronics-tutorials.ws/accircuits/series-circuit.html