Predicting Total Residential Building Permits in British Columbia: A Time Series Analysis and Forecast Comparison

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Abstract

This study analyzes and forecasts the number of total residential building permits issued in British Columbia using various time series models. We compare six different forecasting approaches: mean, naive, drift, ARIMA, NNETAR, Exponential Smoothing and Prophet models. The analysis provides insights into the housing construction trends in BC and evaluates which forecasting methods are most effective for predicting building permit activity. Our findings contribute to understanding the dynamics of housing development in the province and can assist stakeholders in urban planning and policy-making.

1 Introduction

Residential building permits are a key leading indicator of housing development and economic health in British Columbia, offering critical insights into market dynamics and policy effectiveness. In a province where housing availability and affordability remain persistent challenges, particularly in urban areas like Vancouver, understanding permit patterns is crucial for informed decision-making.

This analysis examines the temporal patterns of total residential building permits in British Columbia using comprehensive time series techniques. Through statistical analysis and visualization of historical permit data, we aim to identify significant trends, seasonal patterns, and anomalies. Our methodology encompasses exploratory data analysis, time series decomposition, and statistical modeling to develop insights that can guide stakeholders in urban planning, real estate development, and policy formation.

The code and additional resources for this analysis are available in the following **GitHub repository**: https://github.com/wgeronimor/TimeSeries-Predicting-Total-Residential-Building-Permits-for-BC-Canada-TRU-2024

2 Data and Methodology

2.1 Data Description

This analysis uses Statistics Canada's Building Permits Survey (BPER), a monthly census covering building permit activity across 2,400 municipalities (95% of Canada's population). The study focuses on monthly residential building permit counts in British Columbia from 2018 to 2024, encompassing all dwelling unit types. The survey, part of Statistics Canada's Integrated Business Statistics Program, achieves a 98% response rate through mandatory reporting. Data collection occurs within 20 days post-reference month, with preliminary and revised estimates released monthly.

The time series data illustrates residential building permit issuance in British Columbia from 2018 to 2024. The data shows its highest peak in early 2018 at around 3,000 permits, followed by a downward trend with consistent seasonal fluctuations. A significant drop occurred during 2020-2021 (COVID-19 period), with subsequent recovery, though not returning to pre-2018 levels. By 2024, permit numbers stabilized around 1,500-2,000

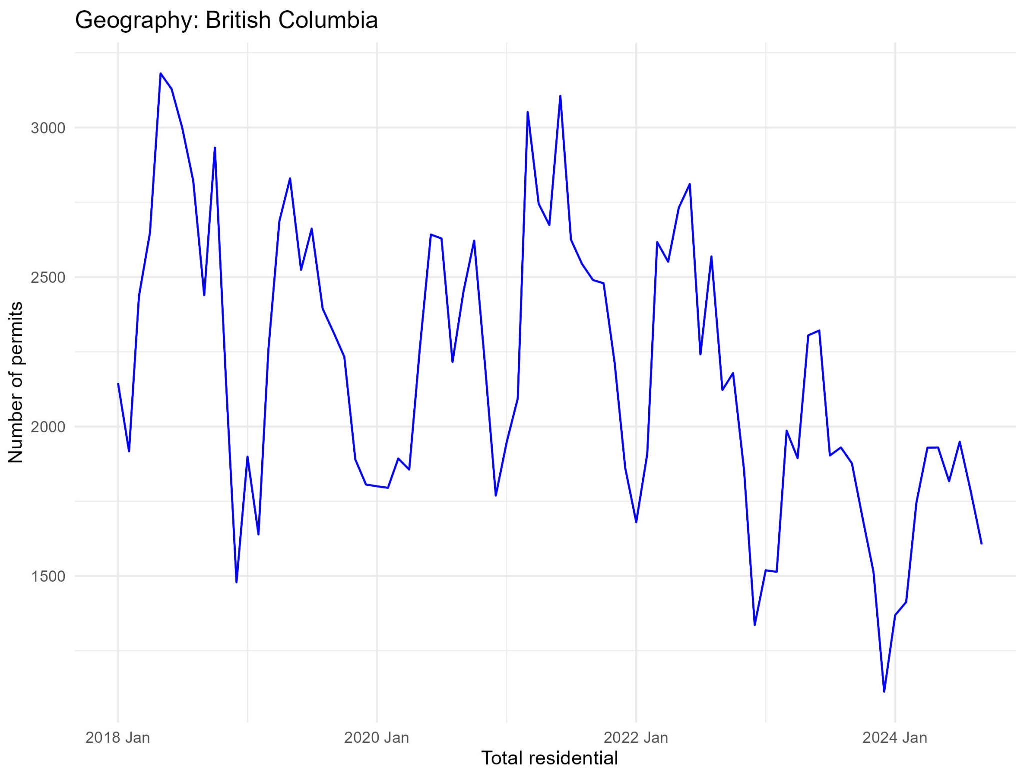

Figure 1: Building Permit from 2018 to 2024

units.

2.2 Forecasting Models

We will now implement and compare six different forecasting approaches:

2.3 Benchmark Models

2.3.1 Mean Model

The mean model assumes that the future values of a time series will be equal to the historical average of all observed values. It can be expressed as:

$$\hat{y}_{T+h|T} = \bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t \tag{1}$$

where:

- $\hat{y}_{T+h|T}$ is the h-step ahead forecast made at time T
- $\bar{y}$ is the sample mean
- T is the total number of observations
- y_t is the observed value at time t

2.3.2 Naive Model

The naive model assumes that the future values of a time series will be equal to the last observed value. It can be expressed as:

$$\hat{y}_{T+h|T} = y_T \tag{2}$$

where:

- $\hat{y}_{T+h|T}$ is the h-step ahead forecast made at time T
- y_T is the last observed value in the time series
- h is the forecast horizon (number of periods ahead to forecast)
- T is the current time period

2.3.3 Drift Model

The drift method, also known as the random walk with drift, extends the naïve method by adding a trend component based on the average change in the historical data. It generates forecasts by adding the average change to the last observed value:

$$\hat{y}_{T+h|T} = y_T + h\left(\frac{y_T - y_1}{T - 1}\right) \tag{3}$$

where:

- $\hat{y}_{T+h|T}$ is the h-step ahead forecast made at time T
- y_T is the last observed value in the time series
- y_1 is the first observed value in the time series
- h is the forecast horizon (number of periods ahead)
- T is the total number of observations
- $\frac{y_T - y_1}{T-1}$ represents the average change over time (slope)

2.4 ARIMA Model

The ARIMA(p,d,q) model combines auto-regression, differencing, and moving average components into a comprehensive framework for time series modeling:

$$\phi(B)(1 - B)^d y_t = \theta(B)\epsilon_t \tag{4}$$

where:

- $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - ... - \phi_p B^p$ is the autoregressive polynomial of order p
- $(1 - B)^d$ is the differencing operator of order d, where B is the backshift operator ($By_t = y_{t-1}$)
- $\theta(B) = 1 + \theta_1 B + \theta_2 B^2 + ... + \theta_q B^q$ is the moving average polynomial of order q

- ϵ_t is white noise with mean zero and constant variance σ^2
- y_t is the observed value at time t
- p is the order of the autoregressive component
- d is the order of integration (differencing)
- q is the order of the moving average component

$$y_t = f(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, ..., \mathbf{y}_{t-p}) + \epsilon_t \tag{5}$$

where:

- y_t is the target variable at time t
- $f(\cdot)$ represents a feed-forward neural network with one hidden layer
- p is the number of lagged inputs (autoregressive order)
- ϵ_t is the error term

2.5 Exponential Smoothing

Exponential smoothing is a time series forecasting method that gives more weight to recent observations and less weight to older ones. The simple exponential smoothing model is suitable for data with no clear trend or seasonality and can be expressed as:

$$\hat{y}t + 1|t = \alpha y_t + (1 - \alpha)\hat{y}t|t - 1 \tag{6}$$

where:

- $\hat{y}t + 1|t$ is the one-step ahead forecast made at time t
- y_t is the observed value at time t
- α is the smoothing parameter ($0 \leq \alpha \leq 1$)
- $\hat{y}t|t - 1$ is the previous forecast made for time t
- $(1 - \alpha)$ represents the weight given to previous forecasts

- α represents the weight given to the most recent observation

The smoothing parameter α determines how quickly the influence of past observations decays. A larger α gives more weight to recent observations, while a smaller α produces smoother forecasts by giving more weight to past observations. The value of α is typically chosen by minimizing the forecast errors on historical data.

2.6 Neural Network Autoregression (NNETAR)

The NNETAR model uses a feed-forward neural network with lagged inputs to model and forecast time series data. The general form of the model is:

$$y_t = f(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, ..., \mathbf{y}_{t-p}) + \epsilon_t \tag{7}$$

where:

- y_t is the target variable at time t
- $f(\cdot)$ represents a feed-forward neural network with one hidden layer
- p is the number of lagged inputs (autoregressive order)
- ϵ_t is the error term

The neural network function f can be expressed more explicitly as:

$$f(\mathbf{x}) = \beta_0 + \sum_{h=1}^{H} \beta_h g(\gamma_{h0} + \sum_{i=1}^{p} \gamma_{hi} y_{t-i}) \tag{8}$$

where:

- H is the number of hidden nodes in the network
- β_h are the weights from the hidden layer to the output
- γ_{hi} are the weights from the input layer to the hidden layer
- $g(\cdot)$ is the activation function, typically sigmoid:

$$g(x) = \frac{1}{1 + e^{-x}} \tag{9}$$

2.7 Prophet Model

The Facebook Prophet model implements an additive regression model that incorporates the following key characteristics:

$$y(t) = g(t) + s(t) + h(t) + \epsilon_t \tag{10}$$

where:

- $g(t)$ is the trend function
- $s(t)$ represents seasonal effects
- $h(t)$ represents holiday effects
- ϵ_t is the error term

3 Results and Discussion

3.1 ARIMA and Benchmark Models

The ARIMA model used is specified as **ARIMA(1,0,1)(2,1,1)[12]**, indicating that the model incorporates both non-seasonal and seasonal components. The model's configuration reflects the following:

- **Non-seasonal parameters:** $p = 1$, $d = 0$, and $q = 1$.
- **Seasonal parameters:** $P = 2$, $D = 1$, $Q = 1$, with a seasonal period of 12 (e.g., monthly data).

The inclusion of seasonal differencing ($D = 1$) ensures that the model accounts for annual patterns in the data, effectively addressing the seasonality present in the series. Additionally, the autoregressive (AR) and moving average (MA) terms for both seasonal and non-seasonal components capture dependencies at various lags, improving the model's ability to reflect the underlying data structure.

Key metrics from the model evaluation are as follows:

- **Residual variance** (σ^2) is estimated at 53,552.
- **Log-likelihood** of the model is -443.54.

Table 1: ARIMA Performance

Model	**RMSE**	**MAE**	**MAPE (%)**
ARIMA(Value)	307.60	281.23	15.69

- **AIC:** 899.09, **AICc:** 900.56, and **BIC:** 912.04.

As shown in Table 2,, the Drift model outperformed the Naive and Mean models with the lowest Root Mean Squared Error (RMSE) of 158.22, Mean Absolute Error (MAE) of 116.17, and Mean Absolute Percentage Error (MAPE) of 6.84%. This indicates that the Drift model is the best-performing benchmark model for the dataset.

Table 2: Benchmark Models Performance

Model	**RMSE**	**MAE**	**MAPE (%)**
Drift	158.22	116.17	6.84
Naive	166.31	120.20	7.10
Mean	412.44	393.63	22.25

Residual diagnostics were conducted to verify that the assumptions of the residuals are met. Figure 2 illustrates the residuals of the Drift model, showing no evident patterns or systematic deviations.

Table 3: Residual Diagnostic Tests (Box-Pierce & Ljung-Box)

Box_Pierce_pvalue	**Ljung_Box_pvalue**
0.1722768	0.1129104

Table 2 presents the results of the residual diagnostic tests for the Drift model. The Box-Pierce test yielded a p-value of 0.172, and the Ljung-Box test produced a p-value of 0.113. Both p-values are above the standard significance threshold of 0.05, indicating that the null hypothesis (H_0) of no autocorrelation in the residuals is accepted. These results confirm that the residuals meet the independence assumption, further validating the performance of the Drift model.

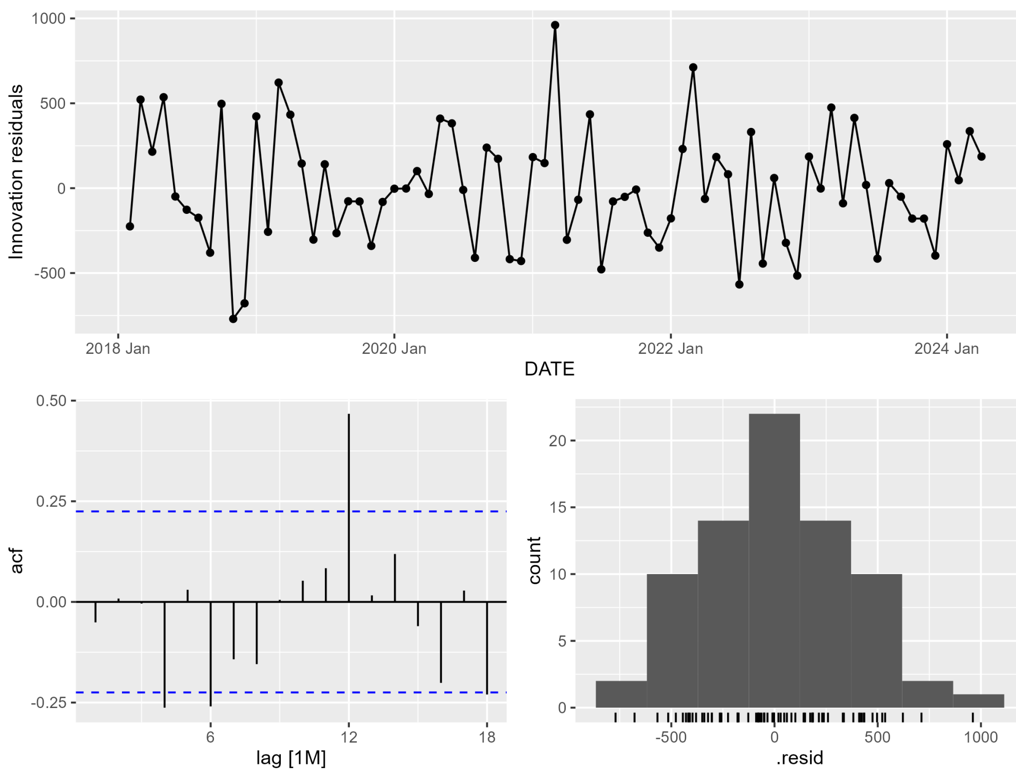

Figure 2: Residuals evaluations of Drift Model

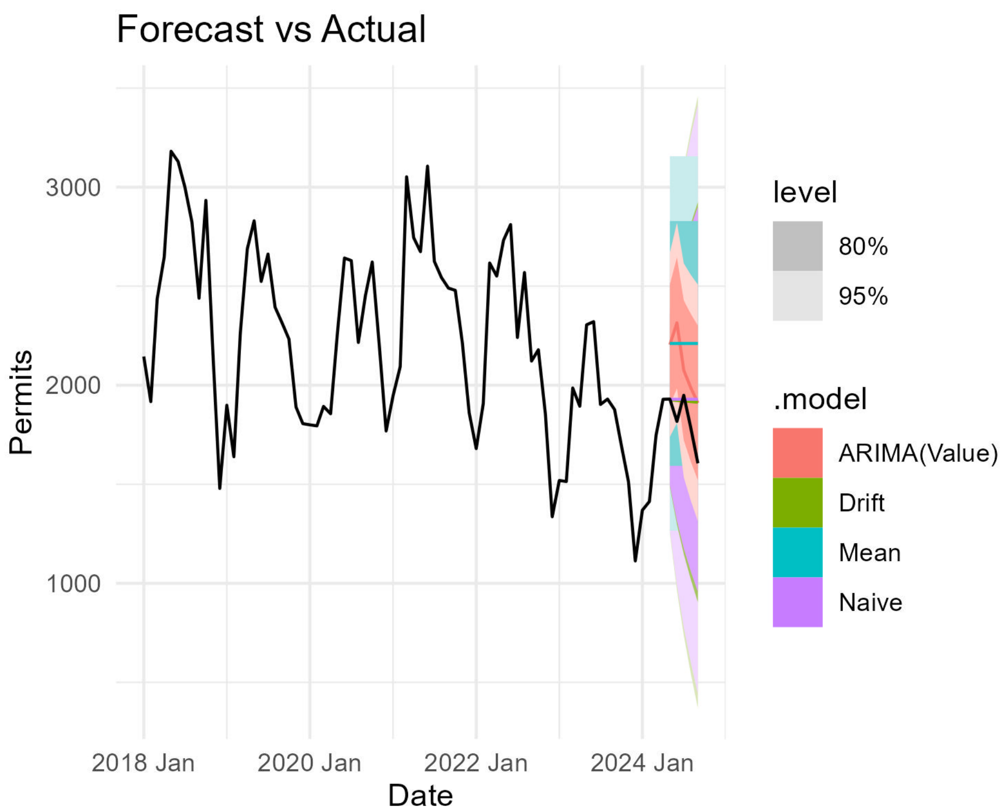

Figure 3: Drift as Best Model by RMSE among ARIMA and Benchmark models

3.2 Exponential Smoothing

Among the exponential smoothing models evaluated, Holt-Winters Multiplicative demonstrated superior accuracy with RMSE of 127.96, MAE of 103.47, and MAPE of 5.89%. Holt (Double Exponential Smoothing) ranked second with RMSE of 151.55, MAE of 113.40, and MAPE of 6.66%, followed by SES (Simple Exponential Smoothing) with RMSE of 159.71, MAE of 118.18, and MAPE of 6.95%. Holt-Winters Additive showed the highest error metrics with RMSE of 215.68, MAE of 177.25, and MAPE of 9.85%.

Table 4: Exponential Smoothing Performance

Model	**RMSE**	**MAE**	**MAPE (%)**
HoltWinters_Multiplicative	127.96	103.47	5.90
Holt	151.55	113.40	6.66
SES	159.71	118.18	6.95
HoltWinters_Additive	215.68	177.25	9.85

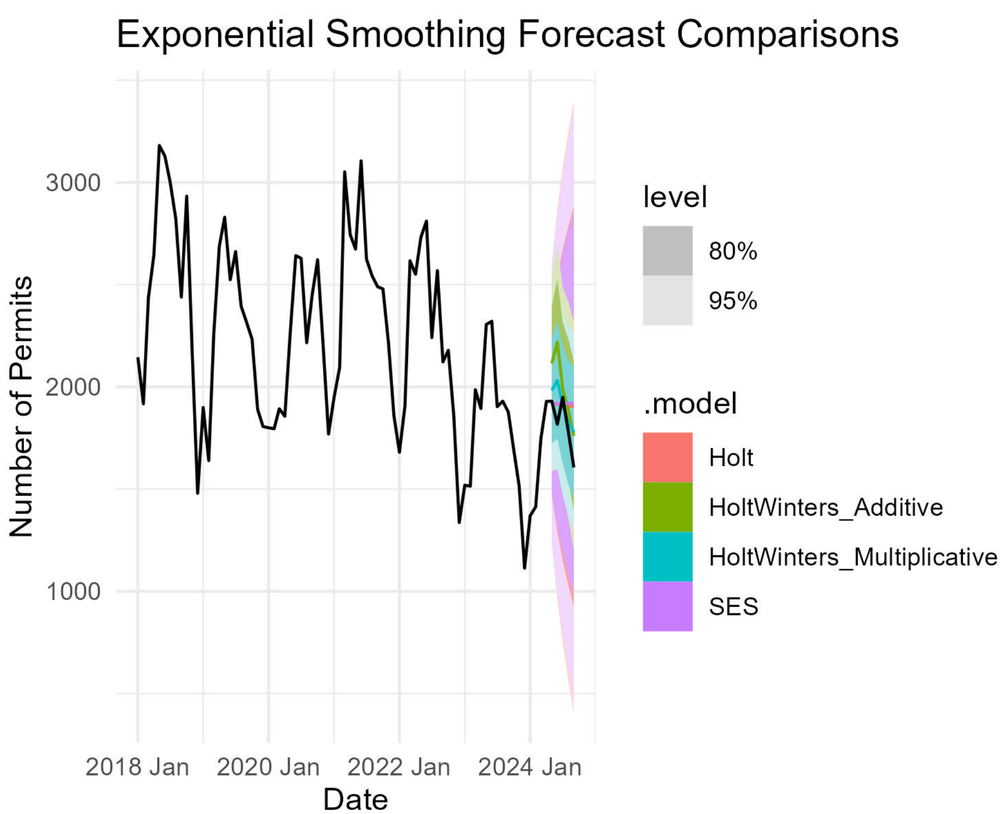

Figure 4: Exponential Smoothing Forecast Comparisons

3.3 NNETAR Model

The NNETAR (Neural Network Autoregression) model yielded an RMSE of 307.61, MAE of 295.82, and MAPE of 16.41%, indicating lower predictive accuracy for the building permits time series. The similarity between RMSE and MAE suggests consistent error distribution without significant outliers.

Table 5: NNETAR Performance

Model	**RMSE**	**MAE**	**MAPE (%)**
NNETAR	325.97	311.69	17.30

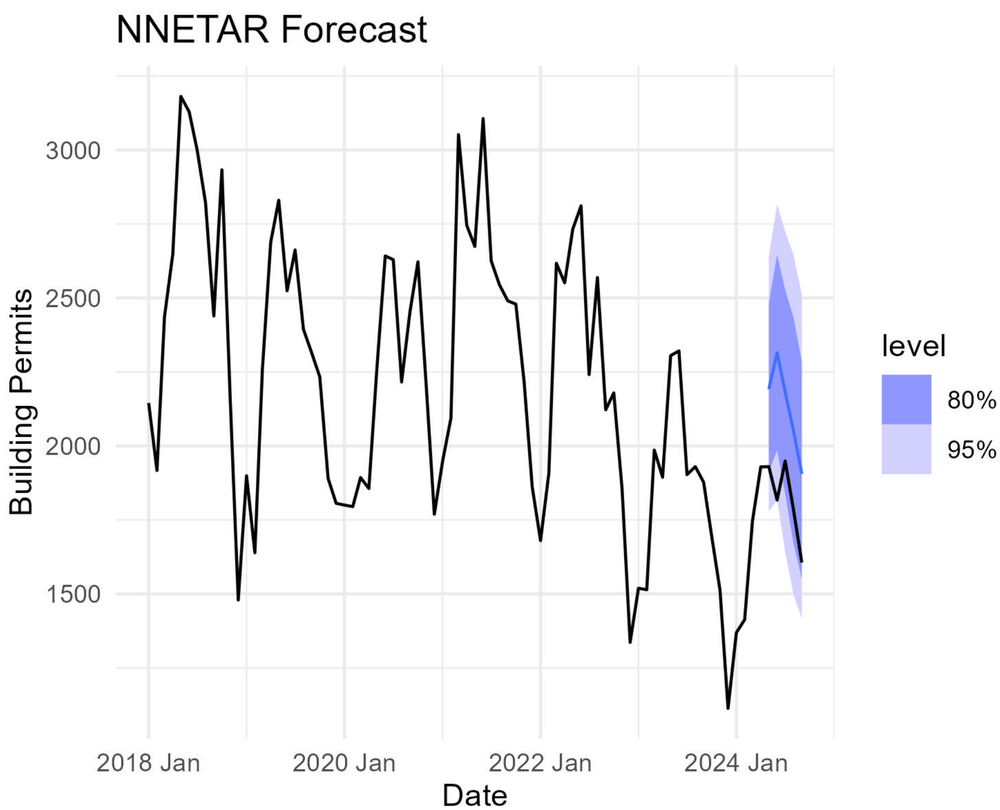

Figure 5: The Forecast of the NNETAR Model

3.4 Prophet Model

The Prophet model generated an RMSE of 283.77, MAE of 193.69, and MAPE of 10.65%, positioning it between the NNETAR and exponential smoothing models in terms of forecast accuracy. The larger difference between RMSE and MAE suggests the presence of some larger forecast errors affecting the overall performance metrics.

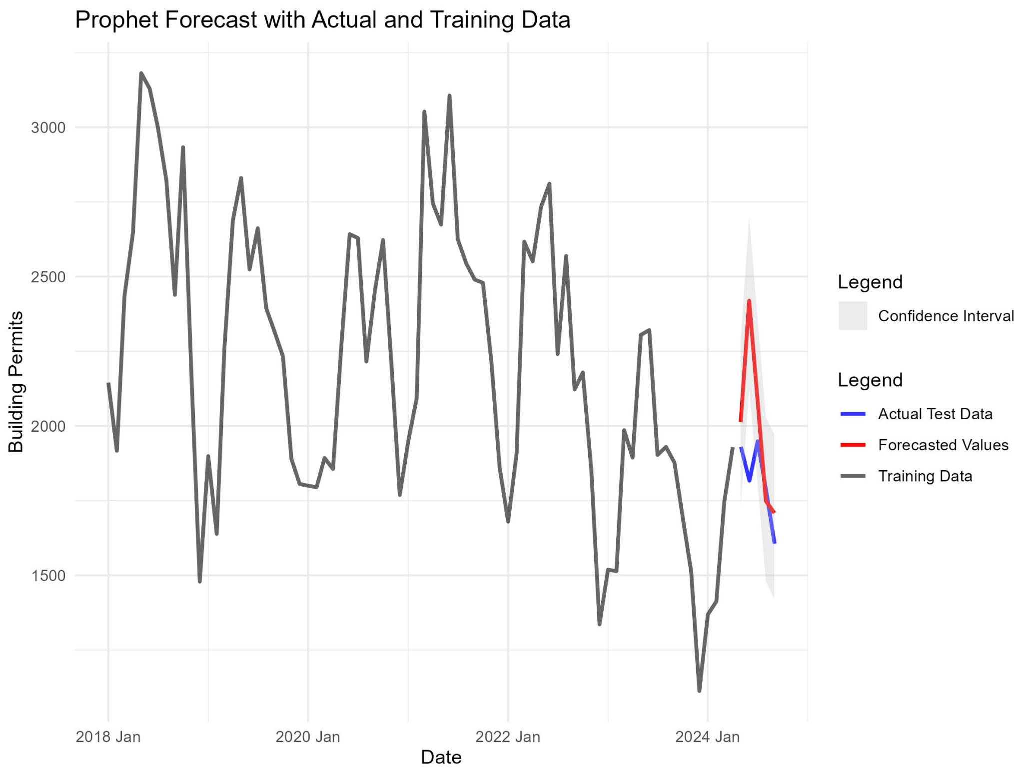

Figure 6: The Forecast using Prophet Model

Table 6: Prophet Performance

Model	**RMSE**	**MAE**	**MAPE (%)**
PROPHET	283.77	193.69	10.65

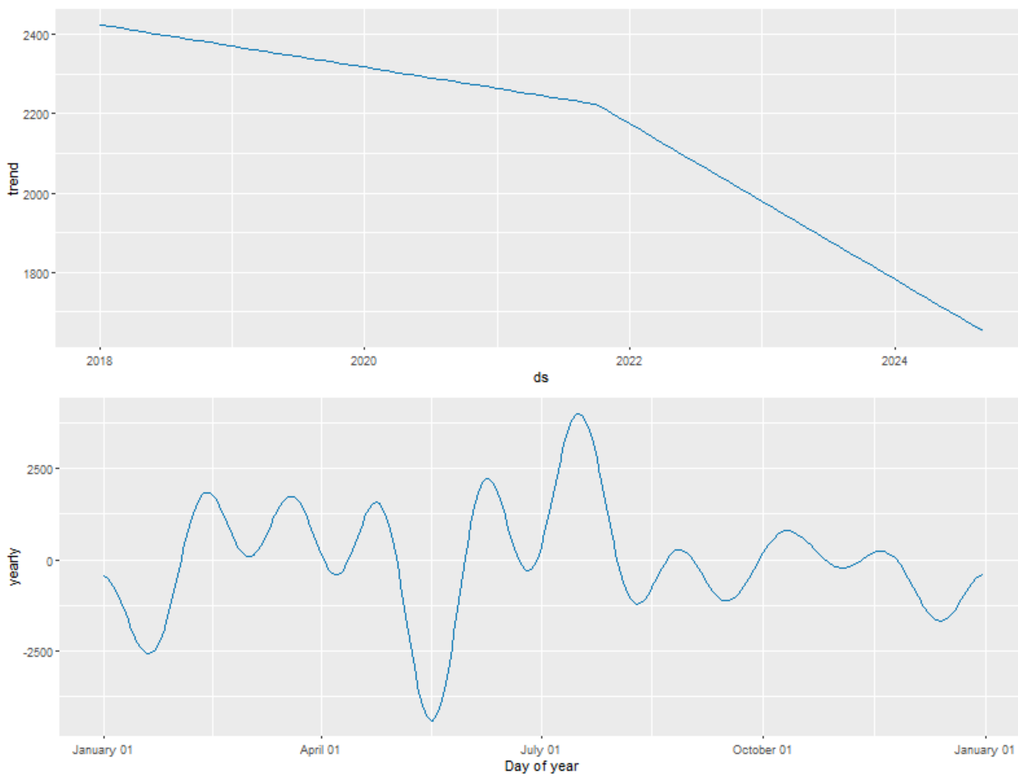

Figure 7: Prophet Components Diagram for trend and yearly

3.5 Model Performance Comparison

Our comparative analysis of forecasting models evaluated performance using three key metrics: Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). Among the benchmark models, the Drift approach demonstrated superior performance with an RMSE of 158.22, MAE of 116.17, and MAPE of 6.8%, outperforming both Simple Exponential Smoothing (RMSE: 159.71) and Naive (RMSE: 166.31) methods. The Mean model exhibited the weakest performance, with an RMSE of 412.43 and MAPE of 22.25%.

The Holt-Winters Multiplicative model emerged as the optimal forecasting method, yielding the lowest error metrics across all measures (RMSE: 127.96, MAE: 103.47, MAPE: 5.89%). While the Holt model showed robust performance (RMSE: 151.55, MAE: 113.40, MAPE: 6.66%), it did not achieve the precision of the Holt-Winters approach. The superior performance of the Holt-Winters Multiplicative model indicates that British Columbia's residential building permits exhibit both seasonal variations and underlying trends, reflecting construction cycles, seasonal weather patterns, and economic variables. These findings suggest the Holt-Winters Multiplicative model is the most suitable approach for forecasting residential building permits in British Columbia.

Table 7: Model Evaluation Summary

Model	**RMSE**	**MAE**	**MAPE (%)**
HoltWinters_Multiplicative	127.96	103.47	5.90
Holt	151.55	113.40	6.66
Drift	158.22	116.17	6.84
SES	159.71	118.18	6.95
Naive	166.31	120.20	7.10
HoltWinters_Additive	215.68	177.25	9.85
PROPHET	283.77	193.69	10.65
ARIMA(Value)	307.60	281.23	15.69
NNETAR	325.97	311.69	17.30
Mean	412.44	393.63	22.25

4 Conclusion

Our comprehensive analysis of residential building permit forecasting models in British Columbia yields several significant conclusions. The Holt-Winters Multiplicative model emerged as the superior forecasting method, demonstrating the lowest error metrics (RMSE: 127.96, MAE: 103.47, MAPE: 5.89%) among all tested approaches. This superior performance indicates that British Columbia's residential building permit patterns exhibit both multiplicative seasonal variations and underlying trends that require sophisticated modeling techniques.

The comparative analysis revealed a clear hierarchy of model effectiveness. While benchmark models like Drift (RMSE: 158.22) and Simple Exponential Smoothing (RMSE: 159.71) showed moderate predictive capability, more advanced approaches including ARIMA and Prophet demonstrated varying degrees of success in capturing the complex dynamics of permit issuance. The NNETAR model, despite its theoretical sophistication, proved less effective for this specific application, suggesting that neural network approaches may not be optimal for this particular time series structure.

These findings have immediate practical implications for stakeholders across the housing sector. For municipal governments, the improved forecast accuracy enables more precise resource allocation for permit processing and better alignment of staffing levels with anticipated demand. Development firms can utilize these predictions to optimize their project pipelines and improve construction scheduling. Additionally, policymakers can leverage this enhanced forecasting capability to evaluate the effectiveness of housing initiatives and adjust interventions based on projected permit activity.

References

[1] S. C. Government of Canada, "Building permits (BPER)," Surveys and statistical programs, https://www23.statcan.gc.ca/imdb/p2SV.pl?Function=getSurvey&Id=1550743 (accessed Dec. 2, 2024).

[2] R. J. Hyndman and G. Athanasopoulos, *Forecasting: Principles and Practice (3rd ed.)*, OTexts, `https://otexts.com/fpp3/bibliography.html` (accessed Dec. 2, 2024).