# Local Pricing Dynamics in Three Eastern Provinces of the DRC

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## Contents



### Local market pricing dynamics

We focus our initial, provisional, analysis on food price inflation in locally connected markets. This is one of three factors immediately influencing food insecurity, the others being climate change, and political violence. Thus in this preliminary study, we hold climate change and political violence at given, but not expressed, levels, *ceteris paribus*. We relax the political violence assumption toward the end of this initial analysis.

The FEWS site houses a database of staple food, and food market and product support prices, wages, and other per unit data. We examine three local markets in eastern Democratic Republic on the Congo: Maniema, North Kivu, and South Kivu.


They possess contiguous borders with one another and with the interior of the DRC (province Maniema) and the board of Rwanda (provinces North Kivu and South Kivu).

Many staple food prices are collected monthly in the FEWS database. We pick the beans (mixed) all in CDF (Congo Democratique Franc) prices to explore three hypotheses:

1. Each market exhibits price volatility clustering (periods of low volatility followed by periods of high volatility). Food price volatility increases the riskiness households face in providing staple foods for their families especially attempting to predict food and other household essentials.
2. Markets spill their volatility into one another, so that if Maniema experiences a period of high price volatility, then we might expect North or South Kivo to experience a change in the volatility of the mixed bean prices. The degree of market connectivity derives from physical supply chain relationships. These include transportation networks, contracting by vendors across regions, as well as buying and selling cooperatives.
3. Market interactions (dyadic pairs) exhibit not only highly volatile spill over, but highly unpredictable pricing.[1]

We begin by extracting data from the FEWS site, extracting the data to spreadsheets, and transforming the data into analytical views. Next we explore the properties of the transformed data. Specifically we calculate 12 month rolling standard deviations and correlations of prices in and between each province. We then attempt to measure the impact one market's volatility on another market's volatility. Finally we model three markets as three interactive cross-province markets in an attempt to measure the flow of pricing information between markets.

These analyses can, in a principled way amenable to discussion and modification, indicate the most dire of the markets individually and relationally as well in terms of their spillover vulnerabilities and the extreme influence of highly improbable, and uncertain food price movements. In this way, we contribute to an understanding of

[1] By volatile we mean expectations of deviations from trend, and thus the existence of calculable joint probability distributions in the sharing of market information. We measure volatility with statistical variance. By unpredictable, we mean uncertainty, so that meaningful expectations cannot be formed, mainly because they do not exist mathematically or practically in the minds of decision makers. We measure uncertainty with the propensity to observe price movements in the extreme tails of joint price distribution, in excess of pricing thresholds.

which markets, market products, and timing of market events, policy makers might prioritize.

### Some simple summaries

We use tabular and graphical depictions of the shapes of each of the correlation and volatility series. Here is the first routine to generate a tabular summary of the shape of within-month correlations.

```
library(tidyverse)
#
options(digits = 4, scipen = 999999)
#library(learnr)
library(rethinking)
library(rstan)
library(tidybayes)
library(cmdstanr)
#library(psych)
#library(ggplot2)
library(GGally)
library(lubridate)
library(tidyverse)
library(quantreg)
library(forecast)
library(tidyquant)
library(matrixStats)
#
#
symbols <- c("Maniema", "NKivu", "SKivu")
# long format ("TIDY") price tibble for possible other work
price_tbl <- read_csv( "beans-drc.csv" )
return_tbl <- price_tbl %>%
  group_by(symbol) %>%
  tq_transmute(mutate_fun = periodReturn, period = "daily", type = "log", col_rename = "daily_return") %
  mutate(abs_return = abs(daily_return))
#str(return_tbl)
r_2 <- return_tbl %>% select(symbol, date, daily_return) %>% spread(symbol, daily_return)
r_2 <- xts(r_2, r_2$date)[-1, ]
storage.mode(r_2) <- "numeric"
r_2 <- r_2[, -1]
r_3 <- r_2 |>
  na.omit()
r_3 <- r_3[-1,]
#r_corr <- rollapply(r_3, 12, FUN = cor) #[,c(2, 3, 6)]
library(zoo)
r_corr_12 <- rollapply(r_3, width=12, function(x) cor(x[,1],x[,2]), by.column=FALSE)
r_corr_13 <- rollapply(r_3, width=12, function(x) cor(x[,1],x[,3]), by.column=FALSE)
r_corr_23 <- rollapply(r_3, width=12, function(x) cor(x[,2],x[,3]), by.column=FALSE)
r_corr <- cbind( r_corr_12, r_corr_13, r_corr_23 )
colnames(r_corr) <- c("Maniema_NKivu", "Maniema_SKivu", "SKivu_NKivu")
r_vols <- rollapply(r_3, 12, FUN = colSds)
colnames(r_vols) <- c("Maniema", "SKivu", "NKivu")
# de-xts()
#
corr_tbl <- r_corr |>
  as_tibble() |>
```

```r
  mutate(date = index(r_corr))
corr_tbl <- corr_tbl[-(1:11), ] |>
  gather(
    key = market,
    value = corr, -date
    )
vols_tbl <- r_vols |>
  as_tibble() |>
  mutate(date = index(r_vols))
vols_tbl <- vols_tbl[-(1:11),] |>
  gather(
    key = market,
    value = vols, -date
    )
corr_vols <- merge(r_corr, r_vols)
corr_vols_tbl <- corr_vols %>% as_tibble() %>%
  mutate(date = index(corr_vols))
#
#write_csv(corr_vols_tbl, "corr_vols_tbl.csv")
```

```r
corr_tbl %>% group_by(market) %>%
  summarise(mean = mean(corr, na.rm=TRUE),
            sd = sd(corr, na.rm=TRUE), skew = skewness(corr, na.rm=TRUE),
            kurt = kurtosis(corr, na.rm=TRUE),
            min = min(corr, na.rm=TRUE),
            q_25 = quantile(corr, 0.25, na.rm=TRUE),
            q_50 = quantile(corr, 0.50, na.rm=TRUE),
            q_75 = quantile(corr, 0.75, na.rm=TRUE),
            max = max(corr, na.rm=TRUE),
            iqr = quantile(corr, 0.75, na.rm=TRUE) - quantile(corr, 0.25, na.rm=TRUE)
            )
```

```
## # A tibble: 3 x 11
##   market     mean    sd   skew   kurt    min    q_25    q_50    q_75   max   iqr
##   <chr>     <dbl> <dbl>  <dbl>  <dbl>  <dbl>   <dbl>   <dbl>   <dbl> <dbl> <dbl>
## 1 Maniem~ -0.184  0.324 -0.757  0.695 -0.976 -0.349  -0.146  -0.0113 0.533 0.338
## 2 Maniem~ -0.0132 0.338 -0.366 -0.227 -0.812 -0.182   0.0232  0.215  0.717 0.397
## 3 SKivu_~  0.145  0.329 -0.162  0.224 -0.982 -0.0542  0.111   0.392  0.847 0.446
```

We just reuse the code above and substitute `vols` for `corr` to review the volatility data.

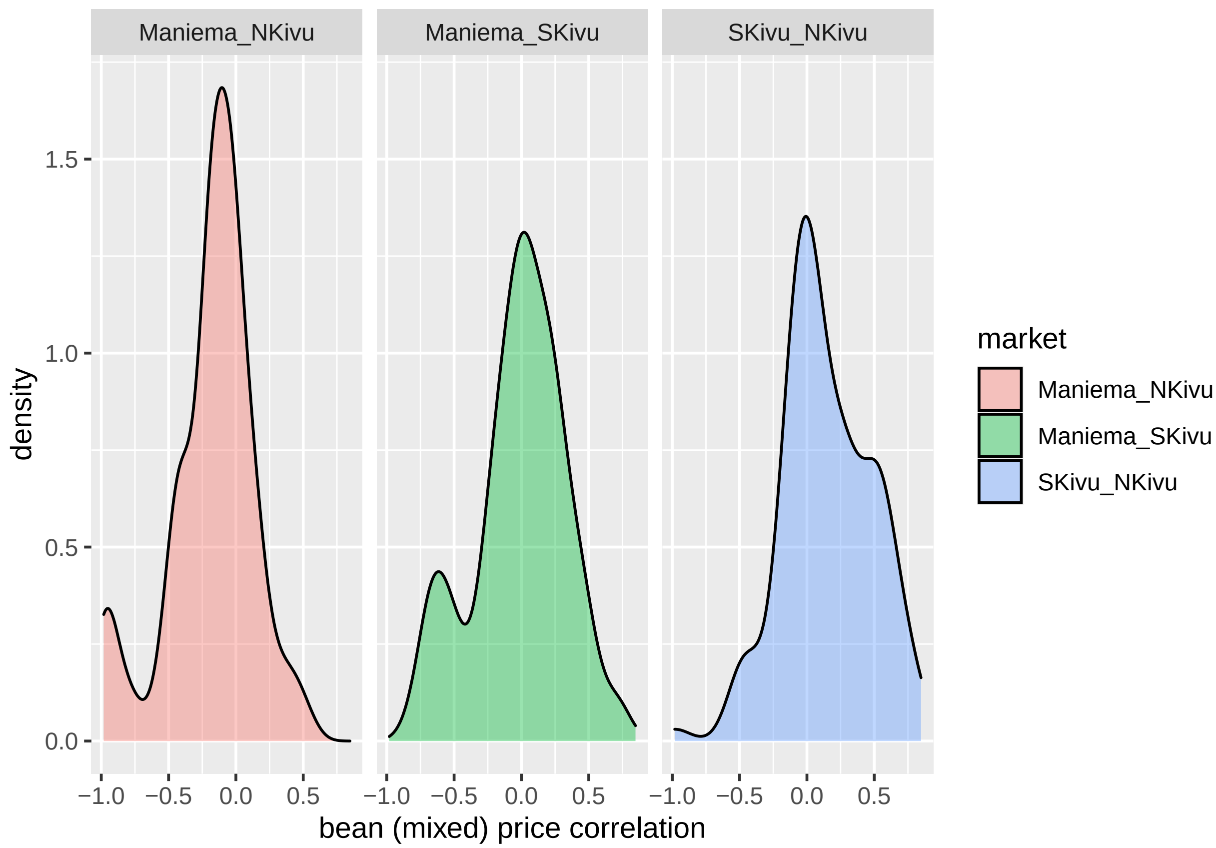
```r
vols_tbl %>% group_by(market) %>%
  summarise(mean = mean(vols, na.rm=TRUE),
            sd = sd(vols, na.rm=TRUE),
            skew = skewness(vols, na.rm=TRUE),
            kurt = kurtosis(vols, na.rm=TRUE),
            min = min(vols, na.rm=TRUE),
            q_25 = quantile(vols, 0.25, na.rm=TRUE),
            q_50 = quantile(vols, 0.50, na.rm=TRUE),
            q_75 = quantile(vols, 0.75, na.rm=TRUE),
            max = max(vols, na.rm=TRUE),
            iqr = quantile(vols, 0.75, na.rm=TRUE) - quantile(vols, 0.25, na.rm=TRUE)
            )
```

```
## # A tibble: 3 x 11
```

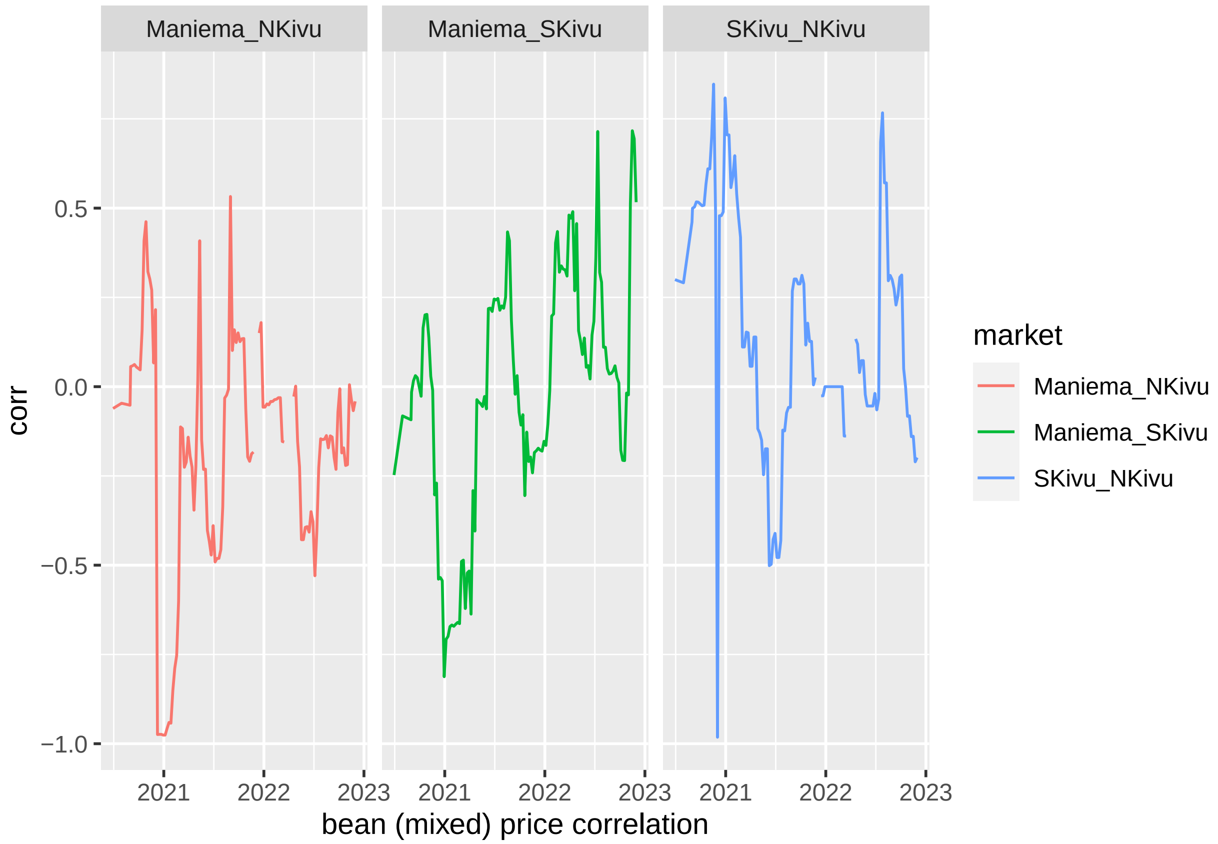
```
##   market    mean     sd  skew  kurt     min   q_25   q_50   q_75   max    iqr
##   <chr>    <dbl>  <dbl> <dbl> <dbl>   <dbl>  <dbl>  <dbl>  <dbl> <dbl>  <dbl>
## 1 Maniema 0.117  0.142   2.50  4.64 0.0147  0.0536 0.0711 0.0954 0.564 0.0418
## 2 NKivu   0.0661 0.0725  2.21  3.64 0.00423 0.0287 0.0441 0.0620 0.283 0.0334
## 3 SKivu   0.0889 0.0796  2.02  3.42 0       0.0498 0.0687 0.0829 0.342 0.0330
```

We view densities and line plots of historical correlations with these routines.

```r
corr_tbl %>% ggplot(aes(x = corr, fill = market)) +
  geom_density(alpha = 0.4) +
  xlab( "bean (mixed) price correlation") +
  facet_wrap(~market)
```




```r
#
corr_tbl %>% ggplot(aes(x = date, y = corr, color = market)) +
  geom_line() +
  xlab( "bean (mixed) price correlation") +
  facet_wrap(~market)
```

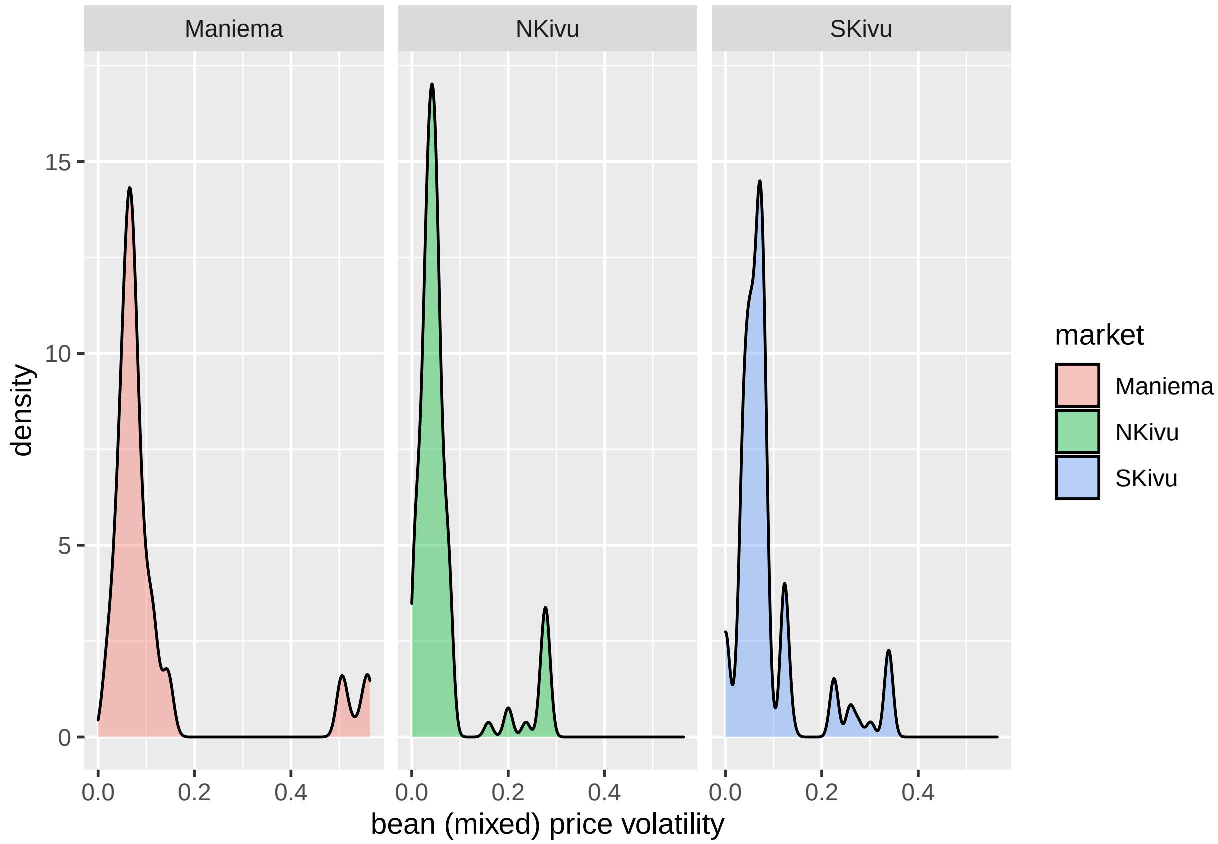



```
#
```

These plots not only support the summary statistics, but they also illustrate the phenomenon of volatility clustering effectively.

We use the right column of `vols_tbl`, namely, `vols`.

```
#
vols_tbl %>% ggplot(aes(x = vols, fill = market)) +
  geom_density(alpha = 0.4) +
  xlab( "bean (mixed) price volatility") +
  facet_wrap(~market)
```




```
#
vols_tbl %>% ggplot(aes(x = date, y = vols, color = market)) +
  geom_line() +
  xlab( "bean (mixed) price volatility") +
  facet_wrap(~market)
```

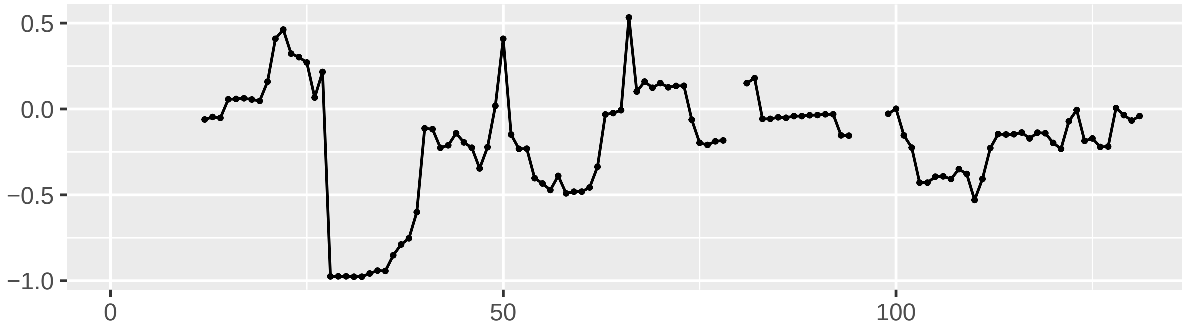

```
#
```

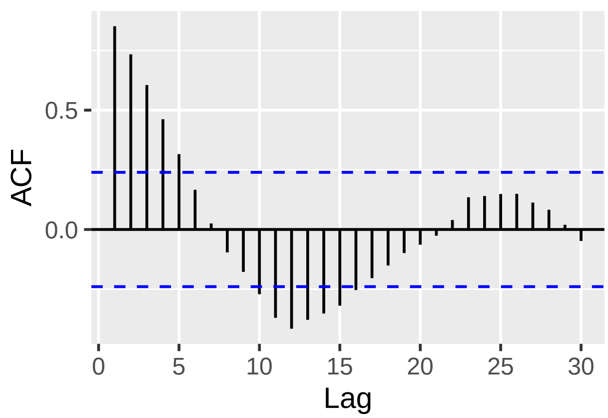
These initial forays into exploring the data clearly indicate the highly volatile nature both of correlation and volatility. The shape of the data shows prominent right skews and potentially thick tails as well. All of these point to the same stylized facts of commodity market returns globally, but now in evidence in local markets.

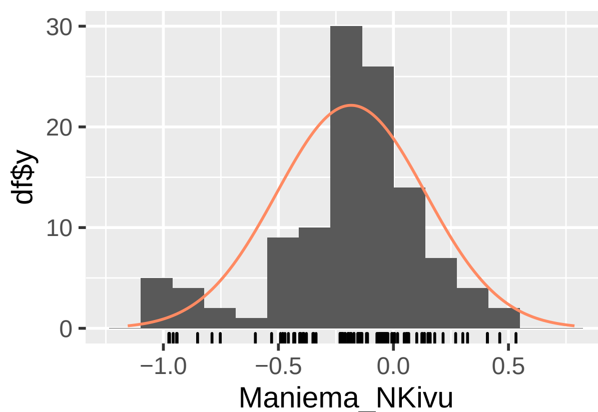
### Do volatility and correlation persist?

with the `Maniema_NKivu` interactions and using the ggplot2 function `ggtsdisplay()` from the `forecast` package, we get all of this at one stop on the way.

```
Maniema_NKivu <- r_corr$Maniema_NKivu
forecast::ggtsdisplay(Maniema_NKivu, lag.max=30, plot.type = "histogram")
```



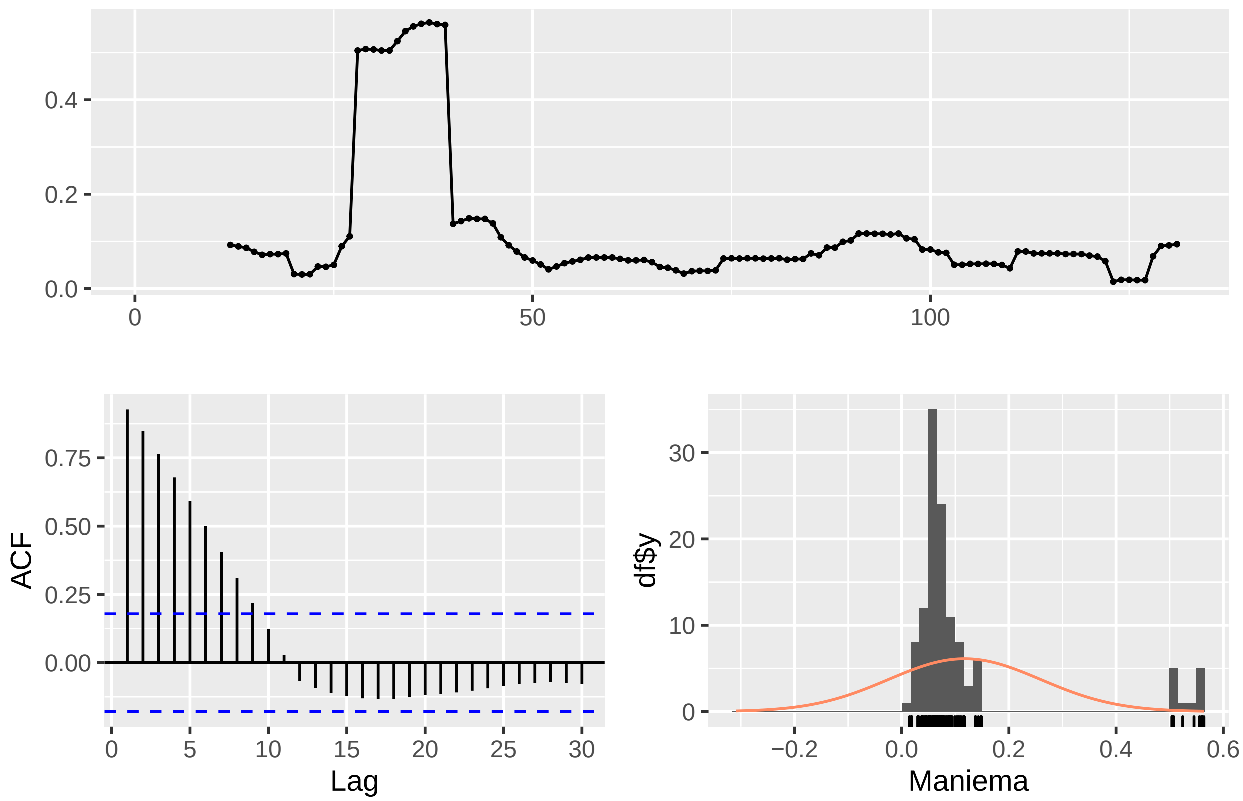




Do correlations persist? There is strong monthly persistence to 10 month lags. The variability of correlation varies only in the negative direction: a low current correlation would seems to be strongly influenced by a high correlation up to 10 months prior.

- The distribution seems skewed to the left and non-normal.

We check the `Maniema` market volatilities next.

```
Maniema <- r_vols$Maniema
forecast::ggtsdisplay(Maniema,lag.max=30, plot.type = "histogram")
```




Does volatility persist? As with correlation, strong and persistent lags over 10 months shows slow decay. Is this some evidence of market memory of risk? Perhaps, but it also indicates in this monthly time interval influences of outliers in the third scatter panel and variability seen in the first time series panel.

Overall, for correlation and volatility we do see recurring patterns which indicate a regularity, an intelligibility we might call stylized in this local market data.

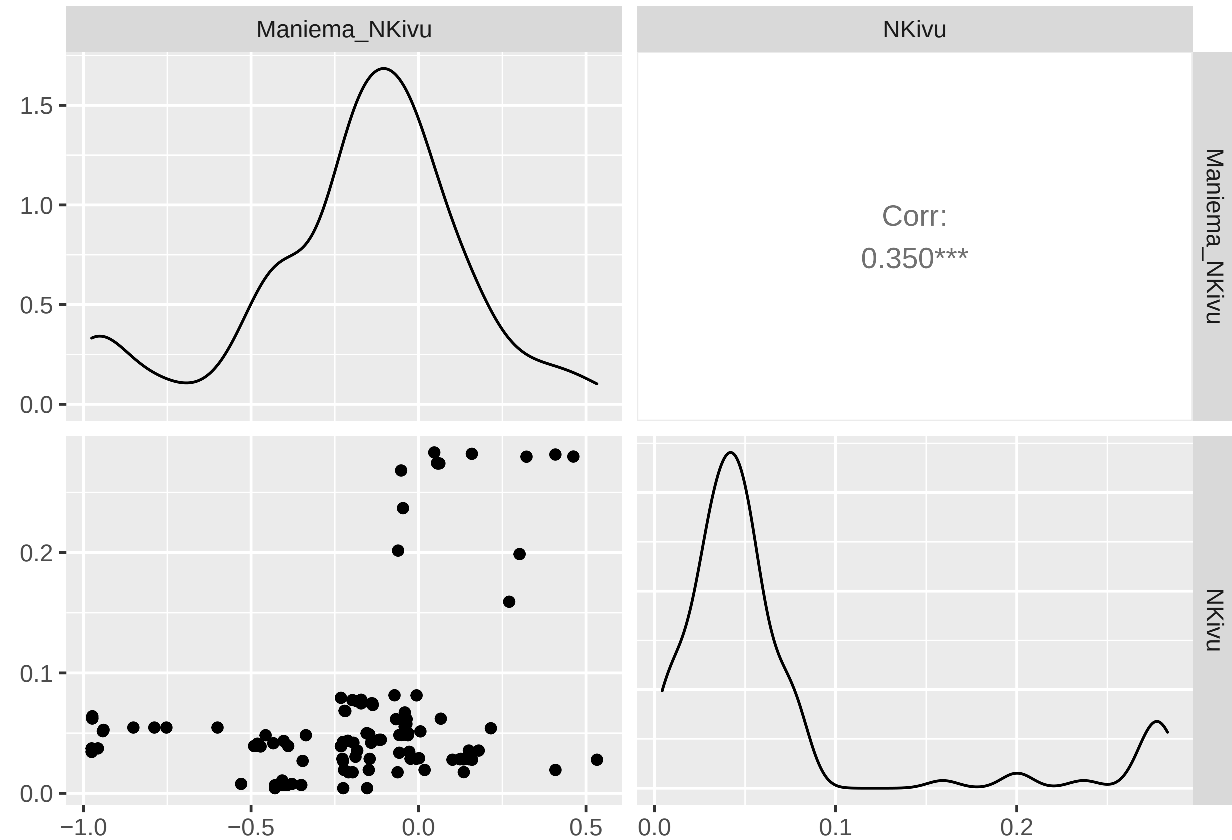
### Do markets spill into one another?

Market spillover occurs when the volatility of one market's prices, through entanglement[2], affects the volatility of another market's prices. We have three individual markets, Maniema, North Kivu (NKivu), and South Kivu (SKivu), all interacting with one another to form dyadic markets informationally, if not also physically with trade among the markets. We are not asking why, just the question of whether we observe spillover. If North Kivu bean prices are volatile, will Maniema be affected? If so, unanticipated trading strategies in one market (North Kivu) will informationally cause unanticipated movements in another (Maniema), here coupled through correlational structures.For example households in Maniema might have very narrow price bands for beans in their household budgets, while households in North Kivu will regularly substitute corn for beans in meal preparation.

Let's examine this idea with a simple scatter matrix.

```
corr_vols <- merge(r_corr, r_vols)
corr_vols_tbl <- corr_vols %>% as_tibble() %>%
  mutate(date = index(corr_vols))
ggpairs(corr_vols_tbl[, c("Maniema_NKivu", "NKivu")])
```

[2]This is a term from quantum mechanics when the state of the system (the market here) is indeterminate but nonetheless components are correlated. See David @Orrell2020quantum to peer into these ideas.




There appears in the scatterplot a mixture of two market interactions. One is a fairly flat cluster, the other much stronger in spillover impact. The overall correlation averages the two behaviors.

What do we observe?

1. Are they apparently normally distributed? Not at all, apparently. We observe a negative skew in correlation and the characteristically positive skew in volatility.
2. What do the outliers look like in a potential relationship between correlations and volatility? The scatter plot indicates potential outliers in very high and very low correlation market environments.
3. Are there potentially multiple regions of outliers? Yes, in very high and low correlation environments. The body of the relation appears to have a positive impact in line with a fairly high correlation over 0.30.

## Quantile regression thoughts

With the existence of outliers in multiple regions we might consider a technique that respects this situation. That technique is quantile regression using the `quantreg` package. Quantile regression (@Koenker2005) can help us measure the impact of high stress episodes on markets, modeled as high and low quantiles.[3]

- Just like `lm()`for Ordinary Least Squares (OLS), we set up `rq()` with left-hand side (correlations) and right hand side variables (volatilities).
- We also specify the quantiles of the left-hand side to identify outliers and the median of the relationship using the `taus` vector. Each value of `tau` will run a separate regression.

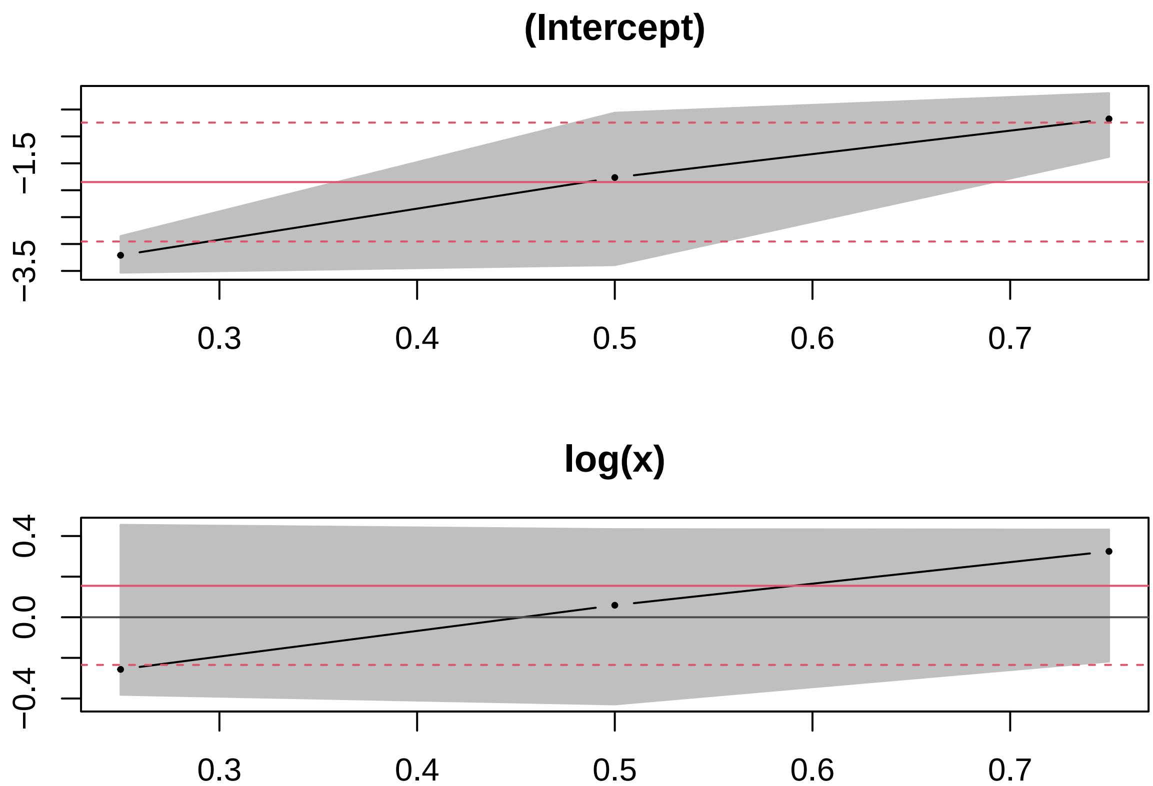
[3] Here is a tutorial on quantile regression that is helpful for the formulation of models and the interpretation of results.

We run this code for one combination of correlations and volatilities. We can modify `y` and `x` for other combinations, and thus, other markets. A log-log transformation can help us understand the relationship between markets as the elasticity of correlation with respect to volatility.

```
library(quantreg)
taus <- c(0.25, 0.50, 0.75) # quantiles of y for a 95% confidence interval
corr_vols_tbl <- na.omit(corr_vols_tbl)
y <- corr_vols_tbl$Maniema_NKivu; x <- corr_vols_tbl$NKivu
fit_corr_vols <- rq(log(y) ~ log(x), tau = taus)
fit_summary <- summary(fit_corr_vols)
fit_summary
```

```
##
## Call: rq(formula = log(y) ~ log(x), tau = taus)
##
## tau: [1] 0.25
##
## Coefficients:
##             coefficients lower bd upper bd
## (Intercept) -3.2091      -3.5311  -2.8526
## log(x)      -0.2567      -0.3814   0.4547
##
## Call: rq(formula = log(y) ~ log(x), tau = taus)
##
## tau: [1] 0.5
##
## Coefficients:
##             coefficients lower bd upper bd
## (Intercept) -1.76431     -3.39146 -0.56092
## log(x)       0.05903     -0.42875  0.43336
##
## Call: rq(formula = log(y) ~ log(x), tau = taus)
##
## tau: [1] 0.75
##
## Coefficients:
##             coefficients lower bd upper bd
## (Intercept) -0.6747      -1.3793  -0.1960
## log(x)       0.3244      -0.2178   0.4313
```

```
plot(fit_summary)
```




The plot depicts the parameter estimate (intercept and slope) on the vertical axis and the quantile of correlation on the horizontal axis. The gray range is the 95% confidence interval of the parameter estimates. The dashed red lines depict the ordinary least squares regression confidence intervals.

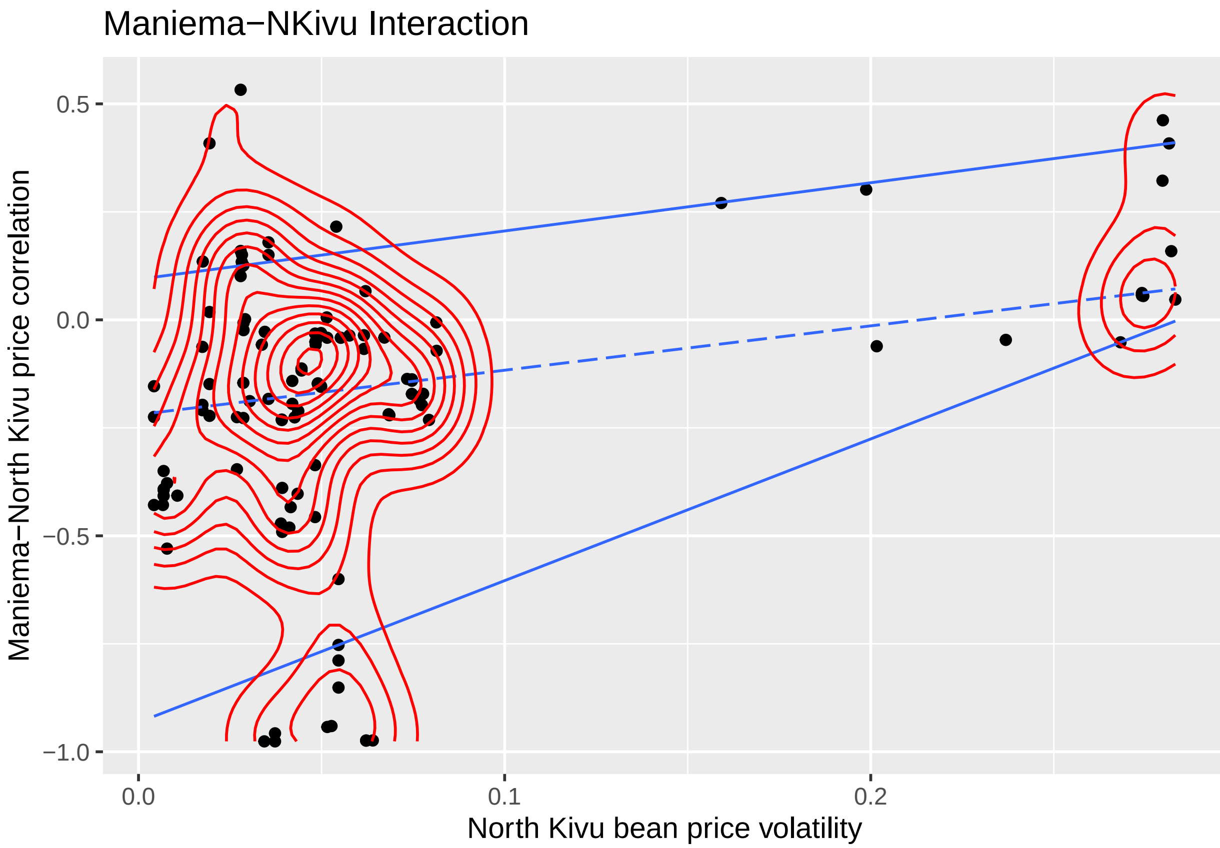
We might ask further questions of this analysis.

1. When is it likely for markets to spill over? Mostly across low to high correlation quantiles.
2. At what likelihood of correlations are market spillovers most uncertain? Again in very low and very high correlation quantile regions.
3. What about the other markets and their spillover effects?
4. What should regional policy makers glean from from these results?

The last two questions deserve further analysis, which means more regressions. The policy maker can get an idea that preparing for market risk is a very high risk-management priority.

One more plot to tie up the market spillover questions.

```
p <- ggplot(corr_vols_tbl,  aes(x = NKivu, y = Maniema_NKivu)) +
    geom_point() +
    ggtitle("Maniema-NKivu Interaction") +
    geom_quantile(quantiles = c(0.10, 0.90)) +
    geom_quantile(quantiles = 0.5, linetype = "longdash") +
    geom_density_2d(colour = "red")  +
    xlab( "North Kivu bean price volatility") +
    ylab( "Maniema-North Kivu price correlation")
p
```




To tailor this picture a bit, we can use `+ ylim(0.25, 1)` to specify the y-axis limits. The dashed line depicts the 50th quantile.

To what degree do our conclusions change when we perform similar analyses on the other market interactions? We just need to re-run the same script with the other market dyads.

## Bayesian thoughts

Alternatively, we can examine the impact of the riskiness of one market on the other using a probabilistic model. Up to this point we have implemented a robust, albeit in a frequentist mood, model of market interaction. This allows us to form a binary hypothesis: $H_0$ no spillover, and $H_1$ spillover. We might question the acceptance or rejection of hypotheses based on the probability emphasis of the null hypothesis. We might also observe overlap of the probability that either might occur.

This objection raises the issue of prior expectations about the hypotheses. If we assume that each is equally probable, perhaps *Beta* distributed then we could let the likelihood of each hypothesis directly impact our inference. If we were to update priors with posteriors, we might also be able to tune the inference further.

Inherently these are not complex models as they have a single regressor acting on a dependent variable. Built into each variate are several assumptions about the variability and co-variability of returns. We might ask next what is the industry structure of spillover effects, at least as represented by these three segments of the renewables market.

We propose this generative model.

$$
\begin{aligned}
\rho_{[i]} &\sim \text{Normal}(\mu_\rho, \sigma_\rho) && (1)\\
\mu_{\rho[i]} &= \alpha_{[i]} + \beta_{[i]}\sigma_{[i]} && (2)\\
\alpha_{[i]} &\sim \text{Normal}(0, 1) && (3)\\
\beta_{[i]} &\sim \text{Normal}(0, 1) && (4)\\
\sigma_{\rho[i]} &\sim \text{Exp}(1) && (5)\\
& && (6)
\end{aligned}
$$

The three markets are indicated by $[i]$, with $\rho$ and $\sigma$ the within-month correlation and market volatility, $\mu_\rho$ and $\sigma_\rho$ the expected value and volatility of the relationship between within-month correlation and market volatility. This formulation allows a mixed-random effects hierarchical model of the probable market risk infrastructure.

```
library(rethinking)
y <- corr_vols_tbl$Maniema_NKivu; x <- corr_vols_tbl$NKivu
d_1 <- tibble(
  Maniema_NKivu = log(y),
  NKivu = log(x)
)
d_1 <- na.omit(d_1)
m_1 <- quap(
  alist(
    Maniema_NKivu ~ dnorm( mu, sigma ),
    mu <- a + b*NKivu,
    a ~ dnorm( 0, 1 ),
    b ~ dnorm( 0, 1 ),
    sigma ~ dexp( 1 )
  ),
  data = d_1
)
precis( m_1 )
```

```
##          mean     sd      5.5%   94.5%
## a     -1.3425 0.5385 -2.203050 -0.4819
## b      0.3173 0.1940  0.007198  0.6273
## sigma  1.2690 0.1695  0.998123  1.5398
```

Priors are proper to the generation of the dependent variable. They can also take on positive or negative signs. Spillover is much in evidence here through size of impact and direction. Let's repeat this for the Maniema-SKivu and SKivu-NKivu markets separately.

```
#library(rethinking)
y <- corr_vols_tbl$Maniema_SKivu; x <- corr_vols_tbl$SKivu
d_2 <- tibble(
  Maniema_SKivu = log(y),
  SKivu = log(x)
)
d_2 <- na.omit(d_2)
m_2 <- quap(
  alist(
    Maniema_SKivu ~ dnorm( mu, sigma ),
    mu <- a + b*SKivu,
    a ~ dnorm( 0, 1 ),
    b ~ dnorm( 0, 1 ),
```

```
    sigma ~ dexp( 1 )
  ),
  data = d_2
)
precis( m_2 )
```

```
##          mean      sd    5.5%    94.5%
## a     -2.7864 0.45811 -3.5186 -2.05427
## b     -0.3519 0.17163 -0.6262 -0.07763
## sigma  0.9565 0.08952  0.8134  1.09953
```

A similar impact is indicated here, but in opposite sign. Now for the last pair, NKivu_SKivu.

```
#library(rethinking)
y <- corr_vols_tbl$SKivu_NKivu; x <- corr_vols_tbl$SKivu
d_3 <- tibble(
  NKivu_SKivu = log(y),
  SKivu = log(x)
)
# there are inf values here
is.na(d_3) <- do.call(cbind,lapply(d_3, is.infinite))
d_3 <- na.omit(d_3)
m_3 <- quap(
  alist(
    NKivu_SKivu ~ dnorm( mu, sigma ),
    mu <- a + b*SKivu,
    a ~ dnorm( 0, 1 ),
    b ~ dnorm( 0, 1 ),
    sigma ~ dexp( 1 )
  ),
  data = d_3
)
precis( m_3 )
```

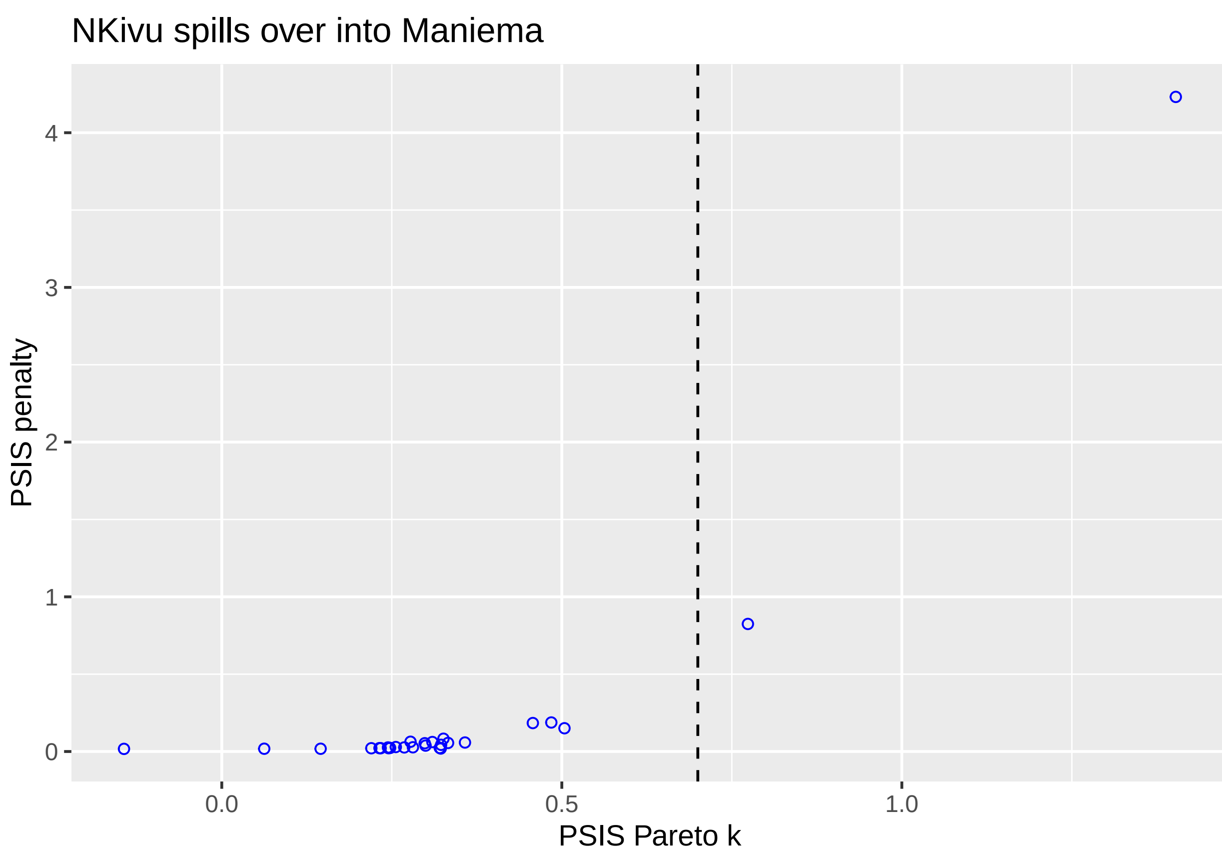
```
##           mean      sd    5.5%  94.5%
## a     -0.02656 0.38676 -0.6447 0.5915
## b      0.53977 0.15559  0.2911 0.7884
## sigma  0.87967 0.07582  0.7585 1.0008
```

North and South Kivu have significant, but not highly elastic spillover effects. All of the models have similarities to the quantile regressions.

Pareto-Smoothed Importance Sampling and cross validation with the Leave-One-Out approach yield a trade-off between spillover variability and bias on the y-axis and uncertainty on the x-axis. Thick-tailed, skewed returns distributions become intelligible with this analysis. The extreme uncertainty of outliers (known-unknowns from $k = 0$ to 0.7; unknown-unknowns for $k > 0.7$ in tests reported by @GelmanHwangVehtari2013 and @VehtariGelmanGabry2015 ) contribute to the ability of the NKivu market to spill its uncertainty into the high variations of the Maniema market all through the naive mechanism of correlation.[4]

[4]Power law distributions notoriously do not possess first, second, third, or even fourth moments analytically across the GPD parameter space, especially for $k$ (see @Embrechts2000 for examples). @Watanabe2009 develops a theory of statistical learning through which singularities in the space of estimation parameters imply that standard inference using Central Limit Theorems, Gaussian distributions, are inadmissable. The existence of divergence, at least as made intelligible through singularity theory, means we should rely on the more robust median, mean absolute deviation, and inter-quartile ranges (probability intervals) to summarize the outcomes of power law distributions. All of this also aligns well with Taleb's many warnings, examples, and inferences by @Taleb2018.

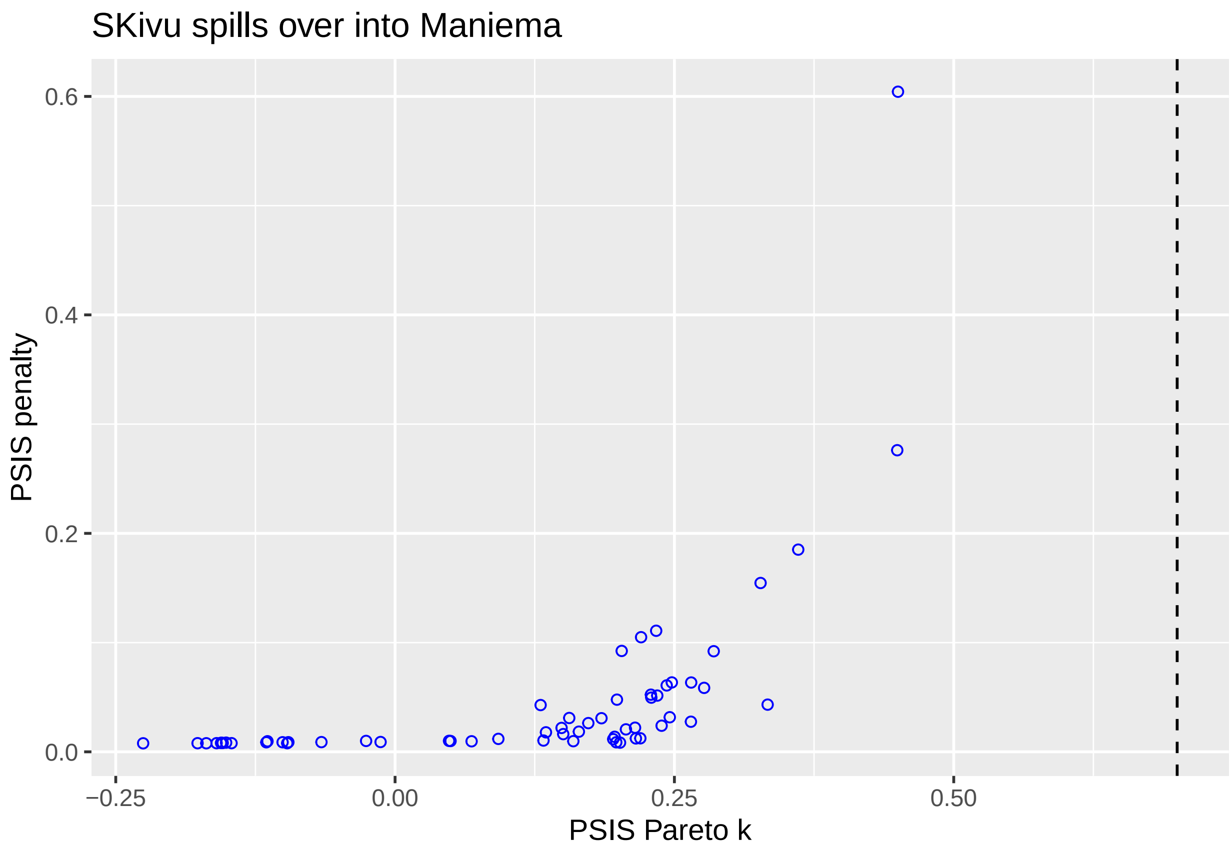
```r
## R code 7.34 McElreath2020
#library( plotly )
options( digits=2, scipen=999999)
d <- d_1
set.seed(4284)
m <- m_1
PSIS_m <- PSIS( m, pointwise=TRUE )
PSIS_m <- cbind( PSIS_m, Maniema_NKivu=d$Maniema_NKivu, NKivu=d$NKivu )
set.seed(4284)
#WAIC_m2.2 <- WAIC(m2.2,pointwise=TRUE)
p1 <- PSIS_m |>
  ggplot( aes( x=k, y=penalty ) ) +
  geom_point( shape=21, color = "blue" ) +
  xlab("PSIS Pareto k") + ylab("PSIS penalty") +
  geom_vline( xintercept = 0.7, linetype = "dashed") +
  ggtitle( "NKivu spills over into Maniema" )
p1 #ggplotly( p1 )
```




Here are the Maniema-SKivu outlier results.

```r
## R code 7.34 McElreath2020
library( plotly )
options( digits=2, scipen=999999)
d <- d_2
set.seed(4284)
m <- m_2
PSIS_m <- PSIS( m, pointwise=TRUE )
```

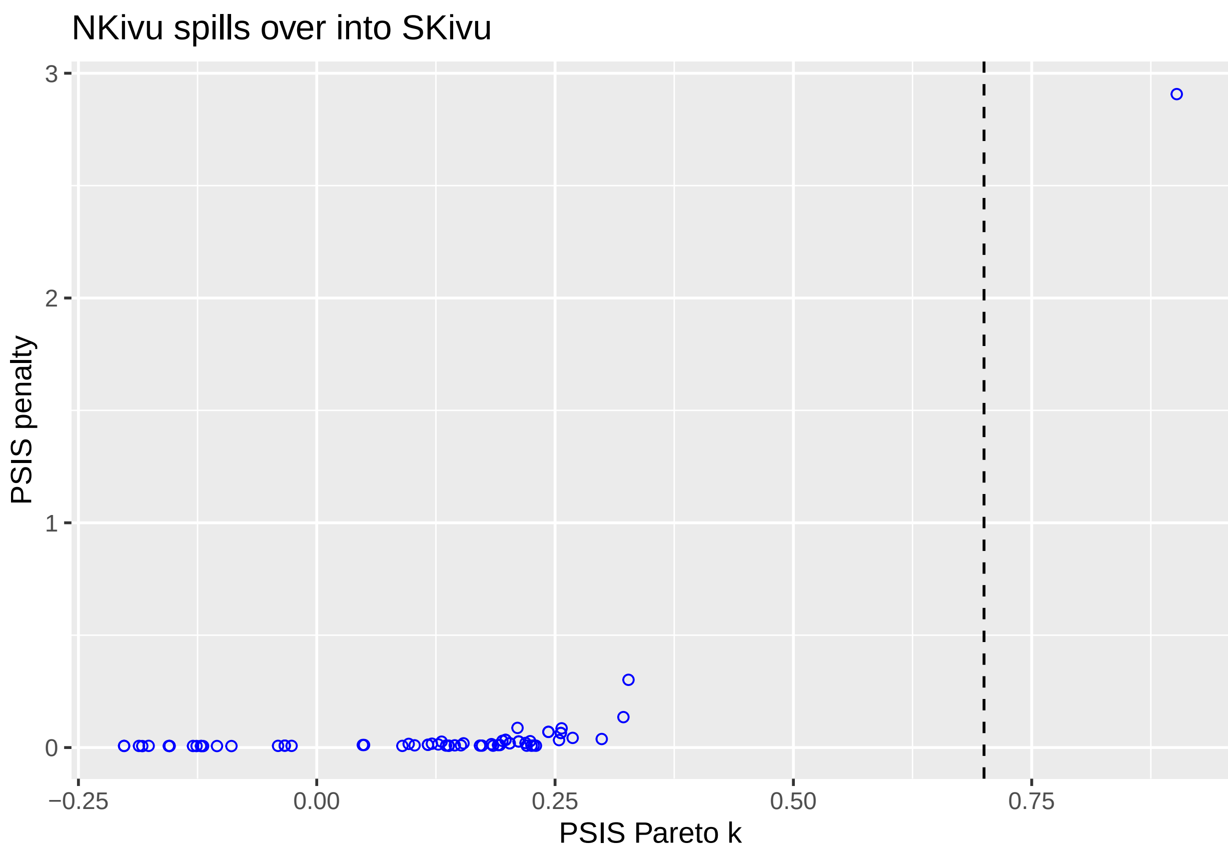
```
PSIS_m <- cbind( PSIS_m, Maniema_SKivu=d$Maniema_SKivu, NKivu=d$SKivu )
set.seed(4284)
#WAIC_m2.2 <- WAIC(m2.2,pointwise=TRUE)
p1 <- PSIS_m %>%
  ggplot( aes( x=k, y=penalty ) ) +
  geom_point( shape=21, color = "blue" ) +
  xlab("PSIS Pareto k") + ylab("PSIS penalty") +
  geom_vline( xintercept = 0.7, linetype = "dashed") +
  ggtitle( "SKivu spills over into Maniema" )
p1 #ggplotly( p1 )
```




and

```
#library( plotly )
options( digits=2, scipen=999999)
d <- d_3
set.seed(4284)
m <- m_3
PSIS_m <- PSIS( m, pointwise=TRUE )
PSIS_m <- cbind( PSIS_m, NKivu_SKivu=d$NKivu_SKivu, SKivu=d$SKivu )
set.seed(4284)
#WAIC_m2.2 <- WAIC(m2.2,pointwise=TRUE)
p1 <- PSIS_m %>%
  ggplot( aes( x=k, y=penalty ) ) +
  geom_point( shape=21, color = "blue" ) +
  xlab("PSIS Pareto k") + ylab("PSIS penalty") +
```

```
  geom_vline( xintercept = 0.7, linetype = "dashed") +
  ggtitle( "NKivu spills over into SKivu" )
p1 #ggplotly( p1 )
```




Our next stop is to look at the joint probability of spillover effects across the three markets.

**Market risk infrastructure**

We now invoke the full generative model, including the jointly considered markets. We index each market and consider all of the impact parameters as jointly determined. They thus share information across markets through the total probability of observing all three markets in the presence of all of the market interaction parameters.

```
#library(rethinking)
corr_vol_spill <- read_csv( "market-connectivity.csv" )
d_4 <- tibble(
    corr = corr_vol_spill$corrs,
    vol = corr_vol_spill$vols,
    mid = corr_vol_spill$mids
  )
d_4 <- na.omit(d_4)
#log vols and (abs) corrs will allow us to interpret coefficients as elasticities
```

Here is Stan code to estimate individual and joint market effects.

```
model_1 <-
"data{
int N;
```

```
int M;
vector[N] y;
vector[N] x;
int cid[N];
}
parameters{
vector[M] a;
vector[M] b;
vector[M] <lower=0> sigma;
}
model{
vector[N] mu;
sigma ~ exponential( 1 );
b ~ normal( 0 , 1 );
a ~ normal( 1 , 1 );
for ( i in 1:N ) {
mu[i] = a[cid[i]] + b[cid[i]] * x;
}
y ~ normal( mu , sigma );
}

"
```

This routine uses the Stan code and our highly transformed data to estimate market spillover between and among pairs of markets.

```
library(rethinking)
m_4 <- ulam(
  alist(
    corr ~ dnorm( mu[mid], sigma[mid] ),
    mu <- a[mid] + b[mid] *vol,
    a[mid] ~ dnorm( 0, 1 ),
    b[mid] ~ dnorm( 0, 1 ),
    sigma[mid] ~ dexp( 1 )
  ),
  data = d_4, log_lik=TRUE
)
precis( m_4, depth = 2 )
#save( m_4, file="spill-all")
#stancode( m_4 )
#options( digits = 2 )
#cov2cor( vcov( m_4 ) )
```

```
load( file="spill-all")
precis( m_4, depth = 2 )
```

```
##              mean    sd   5.5% 94.5% n_eff Rhat4
## a[1]       0.0518 0.087 -0.088  0.18   233     1
## a[2]      -0.0334 0.967 -1.548  1.47  1121     1
## a[3]       0.0228 1.117 -1.660  1.83   809     1
## b[1]      -0.8642 0.925 -2.273  0.63   234     1
## b[2]      -0.0045 0.945 -1.540  1.40   843     1
## b[3]      -0.0047 0.958 -1.552  1.45   749     1
## sigma[1]   0.3635 0.026  0.325  0.41   653     1
## sigma[2]   0.3368 0.020  0.304  0.37   773     1
```

```
## sigma[3]  0.3720 0.028  0.328  0.42   532     1
```

```
#
```

The Bayesian approach integrates the marginal parameters for each of the markets by using the total probability of observing the data across the markets and the range of potential parameter values. Thus the parameters share information across markets for systematic and ideosyncratic components.

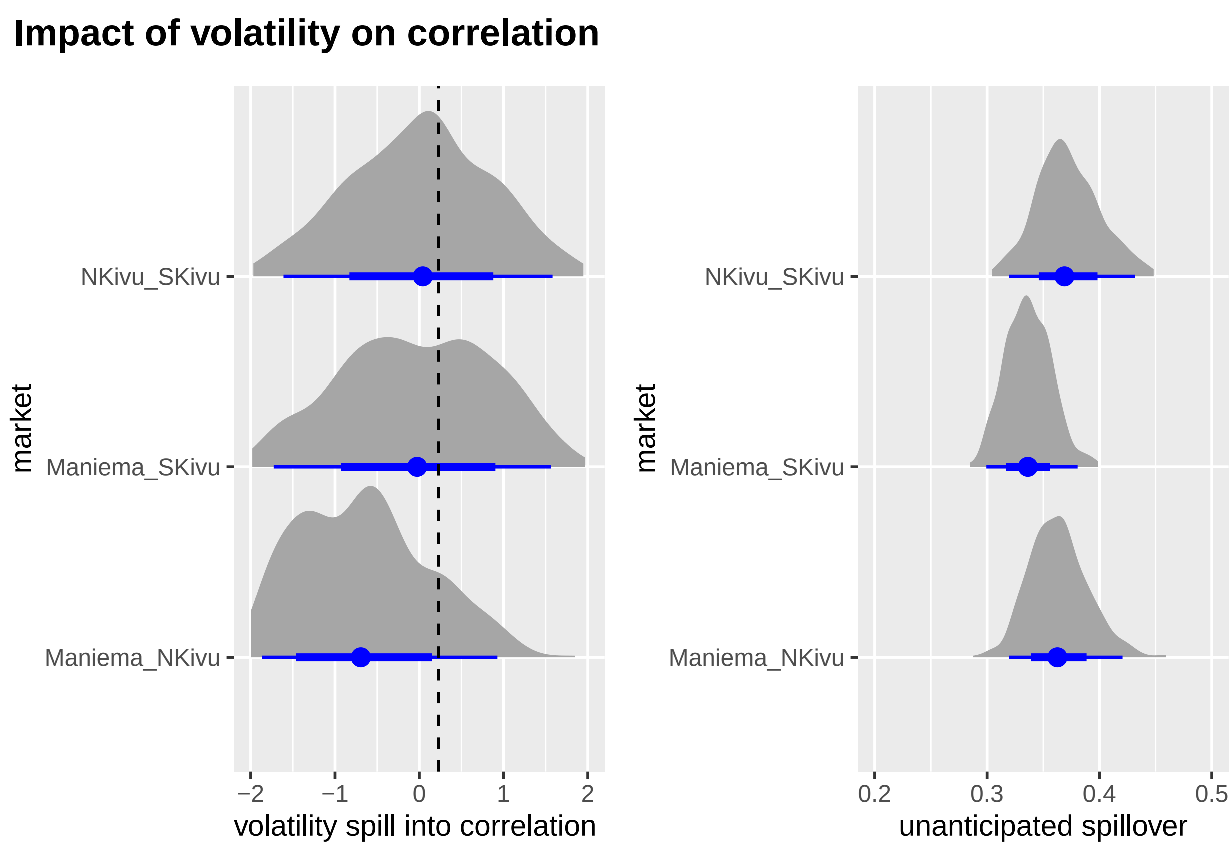
This helper function based on the `cowplot` package will place two plots side-by-side, much like a `ggplot2` facet.

```
#
# draw marginal samples for parameters
#
# ggplot helper
library(cowplot)
plot_2_grid <- function( plot1, plot2, title_1= "Default" ){
    # build side by side plots
    plot_row <- plot_grid(plot1, plot2)
    # now add the title
    title <- ggdraw() +
    draw_label(
      title_1,
      fontface = 'bold',
      x = 0,
      hjust = 0
    ) +
    theme(
      # add margin on the left of the drawing canvas,
      # so title is aligned with left edge of first plot
      plot.margin = margin(0, 0, 0, 7)
    )
    plot_grid(
    title, plot_row,
    ncol = 1,
    # rel_heights values control vertical title margins
    rel_heights = c(0.1, 1)
    )
}
```

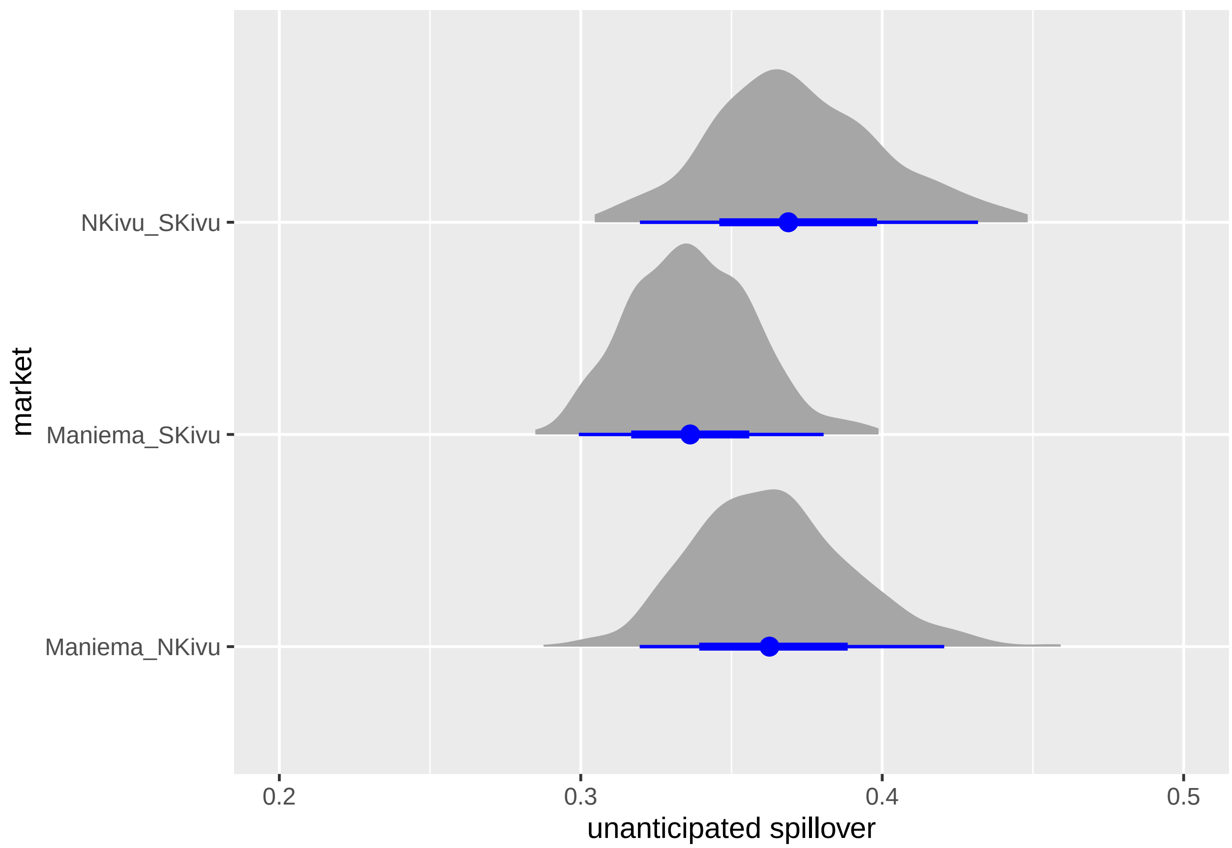
The comparison of slope impact parameters across the three markets in the left panel indicates the high plausibility of information sharing across the markets on a systematic basis. In the right panel are the standard deviations, a measure of the shared information of an unsystematic nature. The joint distribution of these marginal parameters indicate a high degree of sharing of return volatility among the markets.

```
library(tidybayes.rethinking)
m_draws <- m_4 |>
  spread_draws( a[mid], b[mid], sigma[mid] )
mid <- as.factor(m_draws$mid)
levels(mid) <- list( "Maniema_NKivu"=1, "Maniema_SKivu"=2, "NKivu_SKivu"=3 )
m_draws$mid <- mid
# plot grid of two parameters
p1 <- m_draws |>
  ggplot(aes(x = b, y = as.factor(mid) ) ) +
  stat_halfeye( color = "blue") +
  xlab("volatility spill into correlation ") + ylab("market") +
  xlim( -2.0, 2.0) +
```

```
  geom_vline( xintercept = 0.23 , linetype = "dashed") + geom_vline( xintercept = 4.00, linetype = "dash
# build sigma_V comparison
p2 <- m_draws |>
  #filter( sigma < 0.2) %>%
  ggplot(aes(x = sigma, y = as.factor(mid) ) ) +
  stat_halfeye( color = "blue") +
  xlab("unanticipated spillover") + ylab("market") +
  xlim( 0.20, 0.50) +
  geom_vline( xintercept = 0.12 , linetype = "dashed") + geom_vline( xintercept = 0.165, linetype = "da
plot_2_grid( p1, p2, title_1 = "Impact of volatility on correlation")
```




```
#
#ggsave( "compare.png" )
p2
```



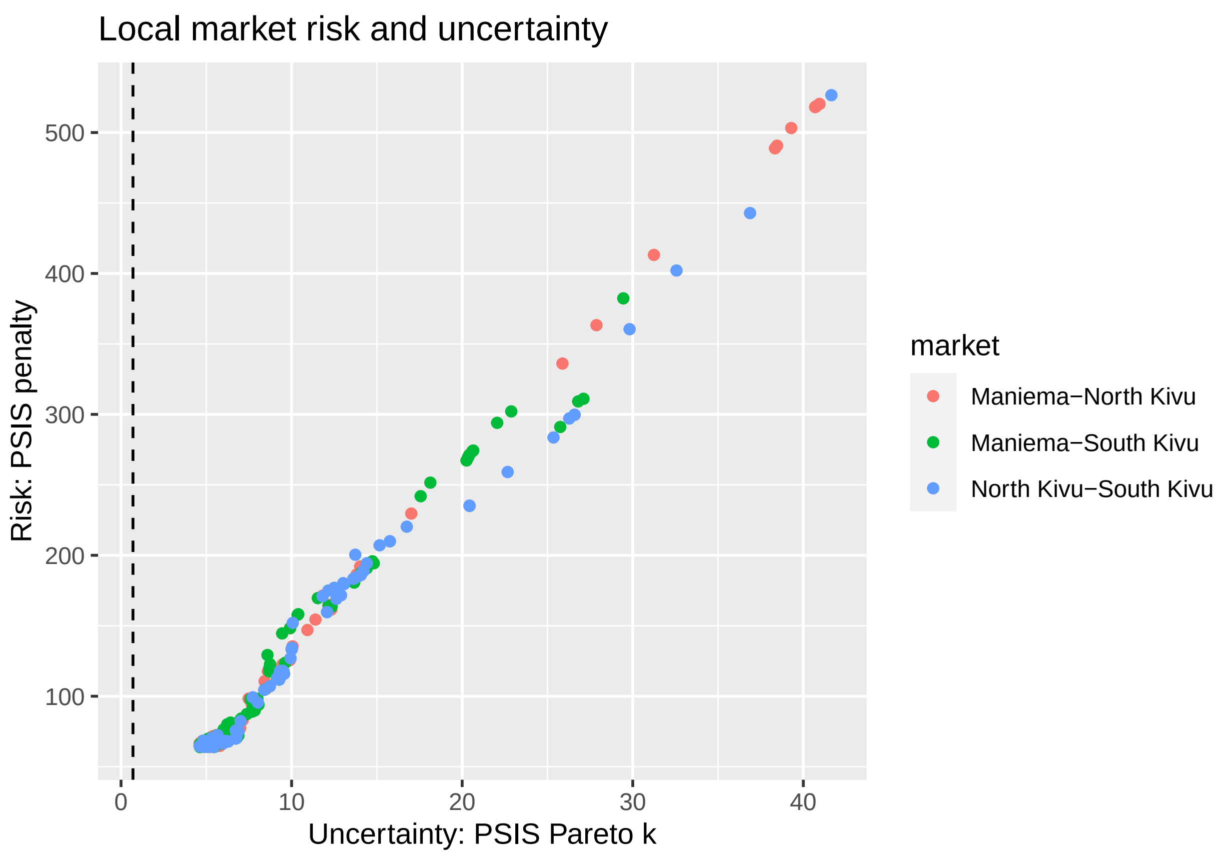

To ground us in a more practical answer. We can say that a 10% move either in NKivu or SKivu volatility will induce at least a 30% in the volatility of Maniema, with moves as low as about 20% and as high as about 45%. A 10% move in NKivu will induce a bit more than an 18% move in SKivu, with moves as low as 6% and as high as over 30%. These intervals are credible with 89% probability.

Again we can review the role of each of the observations on the bias-uncertainty trade-off. High penalty $\sigma$ - high uncertainty $\xi$ observations will obscure efforts to predict correlations. The market to watch in this regard is the NKivu-SKivu pair, which coincides with the high volatilities both of SKivu and NKivu evident in the $\sigma$ distributions we viewed above.

```
## R code 7.34 McElreath2020
#library( plotly )
options( digits=2, scipen=999999)
d <- d_4
set.seed(4284)
m <- m_4
PSIS_m <- PSIS( m, pointwise=TRUE )
PSIS_m$market <- c( rep( "Maniema-North Kivu", 114 ), rep( "Maniema-South Kivu", 114 ), rep( "North Kiv
set.seed(4284)
#WAIC_m2.2 <- WAIC(m2.2,pointwise=TRUE)
p1 <- PSIS_m |>
  ggplot( aes( x=k, y=penalty, fill = market, color = market ) ) +
  geom_point( shape=21 ) +
  xlab("Uncertainty: PSIS Pareto k") + ylab("Risk: PSIS penalty") +
  geom_vline( xintercept = 0.7, linetype = "dashed") +
  ggtitle( "Local market risk and uncertainty" )
p1 #ggplotly( p1 )
```

Table 1: WAIC comparison.

| | WAIC | SE | dWAIC | dSE | pWAIC | weight |
|---|---|---|---|---|---|---|
| m_3 | 179 | 22 | 0 | NA | 4.2 | 1 |
| m_4 | 646841 | 73690 | 646662 | 32419 | 297485.6 | 0 |




Most of the uncertainty is found in North Kivu volatility on Maniema (green). The least uncertain market seems to be evidenced in the relationship between South Kivu and North Kivu.

Finally, at least for this exercise, we can compare the Wide Area Information Criterion, also known as the Watanabe-Akaike Information Criterion, or WAIC for short, of covariance and non-covariance dependent models of market structure.[5]

```
comp <- compare( m_3, m_4 )
knitr::kable(head(comp), format = "latex", caption = "WAIC comparison.") %>%
  kableExtra::kable_styling(font_size = 7)
```

[5]PSIS exploits the distribution of potentially outlying and influential observations using the GPD to model and measure the data point-wise with the shape parameter $k = \xi$. Any point with $k > 0.5$ will have infinite variance and thus contribute to a concentration of points – the thick tail. Related to this idea, WAIC is the log-posterior-predictive density ( *lppd*, that is, the Bayesian deviance) and a penalty proportional to the variance in posterior predictions:

$$WAIC(y, \Theta) = -2(lppd - \underbrace{\Sigma_i var_\theta \, log \; p(y_i|\theta))}_{penalty}$$

The penalty is related to the number of free parameters in the simulations, as @Watanabe2009 demonstrates.

## What have we accomplished?

We can provide provisional answers to a policymaker's initial questions using the work flows developed here.

- Univariate volatility clustering is compatible with high kurtosis, and thus highly volatile volatility.
- Volatile volatility in each asset spills into each market separately. These affects are measured using both quantile regression and Bayesian statistical techniques. The two approaches seem to agree on the probable existence of market spillover.
- We tested the hypothesis that one market's riskiness affects another market, probably. They do, both in systematic, $\beta$, and idiosyncratic, $\sigma$, modes.
- We find that a strongly coupled market risk structure persists across Maniema, SKivu and NKivu assets in each pair of market combinations.
- Maniema is the most sensitive market with highly volatile responses to North Kivu and South Kivu volatility. Both North Kivu and South Kivu are far less sensitive to moves within and across their markets.

The policy maker, the business entrepreneur, the householder should be wary that strong negative movements in one market will probably persist and spill over into negative movements across the three markets. Decisions based on this model will themselves exhibit the results of volatility clustering and market spillover.

Our next step could well be to build volatility clustering, and other thick-tailed outcomes, into the model to derive implied capital requirements through market risk channels, to understand the degree of movement among markets in low, medium, and high volatility price regimes. Such requirements would inform the risk budgeting assignments that might be built into delegations of authority, portfolio limits, even into optimization constraints, let alone the need for individual households to prepare, where possible and capable, for the impact of strong price movements on very tight budgets.

## References