Suggested review problems from the textbook for midterm #2

These are from the **13th** edition of the textbook

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General advice for studying: You should know everything in the slides, understand everything in the homeworks, and be able to work the practice problems listed below. You do not need to know any topics from the textbook that do not appear in the slides, homeworks, or practice problems.

Chapter 6 includes the following material during the chapter that may be helpful to review:

- Example 6.1 (page 169).
- Concept check 6.3 (page 175).
- Concept check 6.4 (page 178).
- Understand Table 6.4 and how it is calculated (page 180).
- Example 6.4 (page 181).
- Concept check 6.6 (page 184).

(The answers to the "concept checks" are at the end of the chapter.)

After that, I recommend the following review problems from the **end** of each chapter.
Solutions are on the following pages.
Note that the "CFA" problems are a separate numbered list that appear after the others.

Chapter 6: 10-21.

Chapter 7: 5, 6, 9, 14, 17. CFA: 4, 5, 6.

Chapter 8: 7 (a, b, d). CFA: 4, 5.

Chapter 9 1, 3, 4, 5, 6, 10, 11, 14, 15, 16, 17, 19. CFA: #5, #7.

- **Note**: In these questions, "expected return" means the CAPM *required* rate of return. That is, the benchmark fair return that the CAPM would set, given the investment's risk.

Note: We have used excess returns whenever possible, but the textbook goes back and forth between using excess and raw returns. This almost never makes a difference to the conclusions of your analysis, but it can change some numbers. For example, in the average/volatility figures, the textbook's values are shifted up (by the risk-free rate) compared to what we would plot. Also, the textbook often (not always) calculates mean-variance utility using raw returns where we use excess returns, so all utility scores are shifted up by the risk-free rate (see Example 6.1 above). When in doubt, obviously you should follow our approach from class, which is based on using excess returns whenever possible.

<u>Solutions to the textbook review problems</u>

Chapter 6

10. The portfolio expected return and variance are computed as follows:

(1) W_{Bills}	(2) r_{Bills}	(3) W_{Inex}	(4) r_{Index}	$r_{Portfolio}$ (1)×(2)+(3)×(4)	$\sigma_{Portfolio}$ (3) × 20%	$\sigma^2_{Portfolio}$
0.0	2%	1.0	10.0%	**10.00%**	20.00%	**0.0400**
0.2	2	0.8	10.0	**8.40%**	16.00%	**0.0256**
0.4	2	0.6	10.0	**6.80%**	12.00%	**0.0144**
0.6	2	0.4	10.0	**5.20%**	8.00%	**0.0064**
0.8	2	0.2	10.0	**3.60%**	4.00%	**0.0016**
1.0	2	0.0	10.0	**2.00%**	0.00%	**0.0000**

11. Computing utility from $U = E(r) - 0.5 \times A\sigma^2$, we arrive at the values in the column labeled $U(A = 2)$ in the following table:

W_{Bills}	W_{Inex}	$r_{Portfolio}$	$\sigma_{Portfolio}$	$\sigma^2_{Portfolio}$	$U(A = 2)$	$U(A = 3)$
0.0	1.0	10.00%	20.00%	0.0400	**0.0600**	0.0400
0.2	0.8	8.40%	16.00%	0.0256	0.0584	0.0456
0.4	0.6	6.80%	12.00%	0.0144	0.0536	**0.0464**
0.6	0.4	5.20%	8.00%	0.0064	0.0456	0.0424
0.8	0.2	3.60%	4.00%	0.0016	0.0344	0.0336
1.0	0.0	2.00%	0.00%	0.0000	0.0200	0.0200

The column labeled $U(A = 2)$ implies that investors with $A = 2$ prefer a portfolio that is invested 100% in the market index to any of the other portfolios in the table.

12. The more risk averse investors (A = 3) prefer the portfolio that is invested **60%** in the market, rather than the 100% market weight preferred by investors with $A = 2$.

13. Expected return = (0.7 × 12%) + (0.3 × 2%) = **9%**

Standard deviation = 0.7 × 28% = **19.6%**

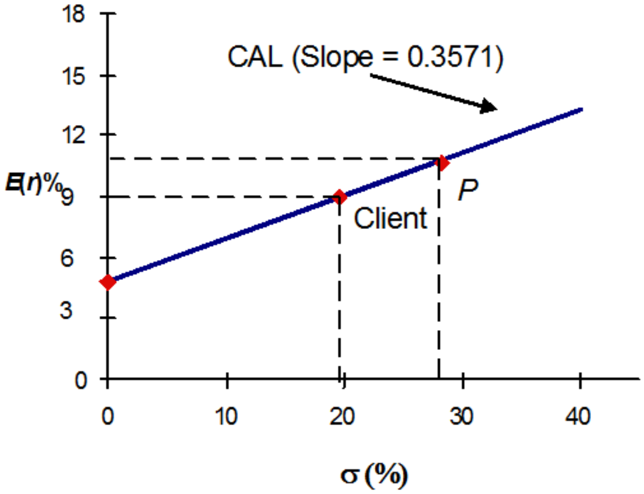
14. Investment proportions:

		30.0% in T-bills
	0.7 × 25% =	17.5% in Stock A
	0.7 × 32% =	22.4% in Stock B
	0.7 × 43% =	30.1% in Stock C

15. Your reward-to-volatility (Sharpe) ratio: $S = \frac{.18 - .08}{.28} = 0.3571$

Client's reward-to-volatility (Sharpe) ratio: $S = \frac{.15 - .08}{.196} = 0.3571$

16.



17.

a.

$$E(r_C) = r_f + y \times (E[r_P] - r_f) = 0.02 + y \times (0.12 - 0.02)$$

If the expected return for the portfolio is 16%, then:

$$10\% = 2\% + 10\% \times y \Rightarrow y = \frac{0.10 - 0.02}{0.10} = \mathbf{0.8}$$

Therefore, in order to have a portfolio with expected rate of return equal to 10%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b.

<u>Client's investment proportions:</u>	<u>**20.0%** in T-bills</u>
	0.8 × 25% = **20.0%** in Stock A
	0.8 × 32% = **25.6%** in Stock B
	0.8 × 43% = **34.4%** in Stock C

c.

$$\sigma_C = 0.8 \times \sigma_P = 0.8 \times 28\% = \mathbf{22.4\%}$$

18. a.

$$\sigma_C = y \times 28\%$$

If your client prefers a standard deviation of at most 12%, then:

$y = 0.12 / 0.28 = 0.4286 = \mathbf{42.86\%}$ invested in the risky portfolio.

b.

$$E(r_C) = 0.02 + 0.1 \times y = 0.02 + (0.1 \times 0.4286) = 0.0629 = \mathbf{6.29\%}$$

19. a. $$y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{0.18 - 0.08}{3.5 \times 0.28^2} = \frac{0.10}{0.2744} = 0.3644$$

Therefore, the client's optimal proportions are: 36.44% invested in the risky portfolio and 63.56% invested in T-bills.

b. $E(r_C) = 0.08 + 0.10 \times y^* = 0.08 + (0.3644 \times 0.1) = 0.1164$ or 11.644%

$\sigma_C = 0.3644 \times 28 = 10.203\%$

20.

a. If the period 1927–2021 is assumed to be representative of future expected performance, then we use the following data to compute the fraction allocated to equity: $A = 4, E(r_M) - r_f = 8.87\%, \ \sigma_M = 20.25\%$ (we use the standard deviation of the risk premium from Table 6.7). Then y^* is given by:

$$y^* = \frac{E(r_M) - r_f}{A\sigma_M^2} = \frac{0.0887}{4 \times 0.2025^2} = 0.5408$$

That is, **54.08%** of the portfolio should be allocated to equity and **45.92%** should be allocated to T-bills.

b. If the period 1975–1998 is assumed to be representative of future expected performance, then we use the following data to compute the fraction allocated to equity: $A = 4, E(r_M) - r_f = 11.00\%, \ \sigma_M = 14.40\%$ and y^* is given by:

$$y^* = \frac{E(r_M) - r_f}{A\sigma_M^2} = \frac{.1100}{4 \times 1440^2} = 1.3262$$

Therefore, **132.62%** of the complete portfolio should be allocated to equity by borrowing **32.62%** in T-bills.

c. In Part b, the market risk premium is expected to be higher than in Part a but market risk is lower than in Part a. Therefore, the reward-to-volatility *ratio* is expected to be much higher Part b, which explains borrowing against T-bills to expand exposure to equity.

21.

a. $$E(r_C) = 5\% = 2\% + y \times (8\% - 2\%) \Rightarrow y = \frac{0.05 - 0.02}{0.08 - 0.02} = 0.50 = \mathbf{50\%}$$

b. $\sigma_C = y \times \sigma_P = .50 \times 15\% = \mathbf{7.5\%}$

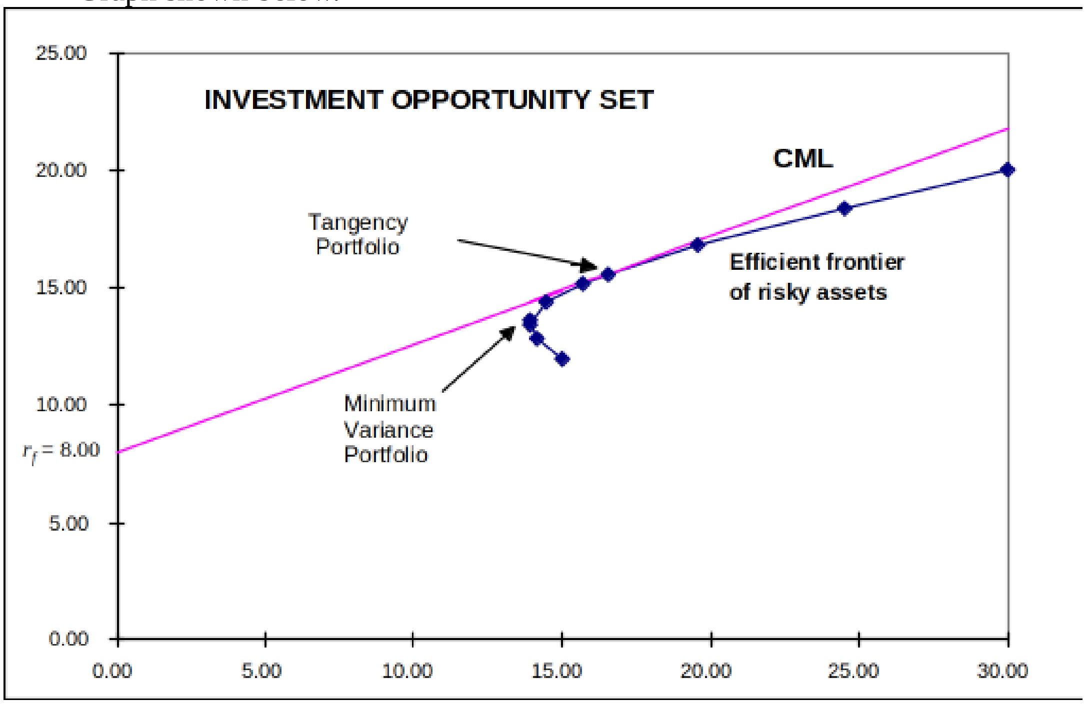
c. The first client is more risk averse, preferring investments that have less risk as evidenced by the lower standard deviation.

Chapter 7

5.

Proportion in Stock Fund	Proportion in Bond Fund	Expected Return	Standard Deviation	
0.00%	100.00%	12.00%	15.00%	
17.39	82.61	13.39	13.92	minimum variance
20.00	80.00	13.60	13.94	
40.00	60.00	15.20	15.70	
45.16	54.84	15.61	16.54	tangency portfolio
60.00	40.00	16.80	19.53	
80.00	20.00	18.40	24.48	
100.00	0.00	20.00	30.00	

Graph shown below.



6. The above graph indicates that the optimal portfolio is the tangency portfolio with expected return approximately 15.6% and standard deviation approximately 16.5%.

9. a. If you require that your portfolio yield an expected return of 14%, then you can find the corresponding standard deviation from the optimal CAL:

$$.14 = E(r_C) = r_f + \frac{E(r_P) - r_f}{\sigma_P}\sigma_C = .08 + \frac{.1561 - .08}{.1654} \times \sigma_c = .08 + 0.4603 \times \sigma_C \rightarrow$$

$$\sigma_C = .1303$$

If $E(r_C) = 14\%$, then the standard deviation of the portfolio is **13.03%**.

b. To find the proportion invested in the money market fund, remember that the mean of the complete portfolio (here, 14%) is an average of the T-bill rate and the optimal combination of stocks and bonds (P). Let y be the proportion invested in the portfolio P. The mean of any portfolio along the optimal CAL is:

$$E(r_C) = (1 - y) \times r_f + y \times E(r_P) = r_f + y \times (E[r_P] - r_f) = .08 + y \times (.1561 - .08)$$

Setting $E(r_c) = 14\%$, we find: y = 0.7881 and (1 − y) = **0.2119** (the proportion invested in the money market fund).

To find the proportions invested in each of the funds, multiply 0.7884 times the respective proportions of stocks and bonds in the optimal risky portfolio:

Proportion of stocks in complete portfolio = 0.7884 × 0.4516 = **0.3559**
Proportion of bonds in complete portfolio = 0.7884 × 0.5484 = **0.4322**

14. False. The portfolio standard deviation equals the weighted average of the component-asset standard deviations *only* in the special case that all assets are perfectly positively correlated. Otherwise, as the formula for portfolio standard deviation shows, the portfolio standard deviation is *less* than the weighted average of the component-asset standard deviations. The portfolio *variance* is a weighted *sum* of the elements in the covariance matrix, with the products of the portfolio proportions as weights.

17. The correct choice is (c). Intuitively, we note that since all stocks have the same expected rate of return and standard deviation, we choose the stock that will result in lowest risk. This is the stock that has the lowest correlation with Stock A.

More formally, we note that when all stocks have the same expected rate of return, the optimal portfolio for any risk-averse investor is the global minimum variance portfolio (G). When the portfolio is restricted to Stock A and one additional stock, the objective is to find G for any pair that includes Stock A and then select the combination with the lowest variance. With two stocks, I and J, the formula for the weights in G is:

$$w_{Min}(I) = \frac{\sigma_J^2 - \text{Cov}(r_I, r_J)}{\sigma_I^2 + \sigma_J^2 - 2\text{Cov}(r_I, r_J)}$$

$$w_{Min}(J) = 1 - w_{Min}(I)$$

Since all standard deviations are equal to 20%:

$$\text{Cov}(r_I, r_J) = \rho\sigma_I\sigma_J = 400\rho \text{ and } w_{Min}(I) = w_{Min}(J) = 0.5$$

This intuitive result is an implication of a property of any efficient frontier, namely, that the covariances of the global minimum variance portfolio with all other assets on the frontier are identical and equal to its own variance. (Otherwise, additional diversification would further reduce the variance.) In this case, the standard deviation of $G(I, J)$ reduces to:

$$\sigma_{Min}(G) = (200 \times [1 + \rho_{IJ}])^{1/2}$$

This leads to the intuitive result that the desired addition would be the stock with the lowest correlation with Stock A, which is **Stock D**. The optimal portfolio is equally invested in Stock A and Stock D, and the standard deviation is **17.03%.**

CFA 4. d. Portfolio Y cannot be efficient because it is strictly dominated by another portfolio. For example, Portfolio X has both higher expected return and lower standard deviation.

CFA 5. c.

CFA 6. d.

Chapter 8

7. a. The two figures depict the stocks' security characteristic lines (SCL). Stock A has higher firm-specific risk because the deviations of the observations from the SCL are larger for Stock A than for Stock B. Deviations are measured by the vertical distance of each observation from the SCL.

   b. Beta is the slope of the SCL, which is the measure of systematic risk. The SCL for Stock B is steeper; hence Stock B's systematic risk is greater.

   d. Alpha is the intercept of the SCL with the expected return axis. Stock A has a small positive alpha whereas Stock B has a negative alpha; hence, Stock A's alpha is larger.

CFA 4. d.

CFA 5. b.

Chapter 9

1. $E(r_P) = r_f + \beta_P \times [E(r_M) - r_f]$

   $.18 = .06 + \beta_P \times [.14 - .06] \rightarrow \beta_P = \dfrac{.12}{.08} = 1.5$

3. a.False. β = 0 implies $E(r) = r_f$, not zero.

   a. False. Investors require a risk premium only for bearing systematic (undiversifiable or market) risk. Total volatility, as measured by the standard deviation, includes diversifiable risk.

   b. False. Your portfolio should be invested 75% in the market portfolio and 25% in T-bills. Then: $\beta_P = (0.75 \times 1) + (0.25 \times 0) = 0.75$

4. The expected return is the return predicted by the CAPM for a given level of systematic risk.

$$E(r_i) = r_f + \beta_i \times [E(r_M) - r_f]$$
$$E(r_{\$1\ Discount}) = .04 + 1.5 \times (.10 - .04) = .13, \text{or } 13\%$$
$$E(r_{Everything\ \$5}) = .04 + 1.0 \times (.10 - .04) = .10, \text{or } 10\%$$

5. According to the CAPM, \$1 Discount Stores requires a return of 13% based on its systematic risk level of β = 1.5. However, the forecasted return is only 12%. Therefore, the security is currently overvalued.

   Everything \$5 requires a return of 10% based on its systematic risk level of β = 1.0. However, the forecasted return is 11%. Therefore, the security is currently undervalued.

6. Correct answer is choice a. The expected return of a stock with a β = 1.0 must, on average, be the same as the expected return of the market which also has a β = 1.0.

10. Not possible. Portfolio A has a higher beta than Portfolio B, but the expected return for Portfolio A is lower than the expected return for Portfolio B.

11. Possible. Portfolio A's offers a lower rate of return with a higher standard deviation compared to portfolio B, but this is not a problem in the CAPM as long as A's <u>beta</u> is less than B's, which it is.

12. Not possible. In the CAPM, the market portfolio must have the highest forecasted Sharpe ratio. Otherwise it would not make sense for all investors to hold the market portfolio as their risky investment portfolio. In the table, A has a higher Sharpe ratio than the market portfolio.

14. Not possible. Portfolio A has a higher beta than the market but a lower rate of return.

15. Not possible. The market excess return here is 8%, and the beta of portfolio A is 0.9, so the required excess return on portfolio A should be 0.9 times 8% = 7.2%. Instead the expected excess return according to the table is 16% minus 10% = 6%.

16. Possible. A has a lower Sharpe ratio than the market, addressing the issue in question 12, and we don't see any information on betas that would lead us to see any problem with the CAPM here.

17. Since the stock's beta is equal to 1.2, its expected rate of return is:

$$.06 + [1.2 \times (.16 - .06)] = 18\%$$

$$E(r) = \frac{D_1 + P_1 - P_0}{P_0} \rightarrow 0.18 = \frac{P_1 - \$50 + \$6}{\$50} \rightarrow P_1 = \$53$$

19. Using the SML: $.04 = .06 + \beta \times (.16 - .06) \Rightarrow \beta = -.02/.10 = -0.2$

CFA 5. d.
CFA 7. d.