The CAPM and investment styles

William Mann

From portfolio theory to the CAPM

Portfolio theory tells us what a mean-variance investor should do, given their predictions about μ, σ, and ρ. The Capital Asset Pricing Model (CAPM) asks what would happen, if all investors actually followed this advice, and made the same forecasts:

- With the same forecasts, they choose the same risky portfolio.
- The only portfolio they can all choose is the market portfolio.
- So, it must in fact have the highest possible Sharpe ratio.
- Then all investments have zero α with respect to it.
- More precisely, the best *prediction* of α is always zero.
  We will always find investments with nonzero α in past data.
  But the CAPM says you could not have picked them in advance.

The Capital Asset Pricing Model (CAPM)

Conclusion: The best prediction about every investment is that

$$\mathbb{E}[r_i - r_f] = \beta_{im} \times \mathbb{E}[r_m - r_f]$$

This appears similar to our earlier formula, but says much more:

- Portfolio theory says what an individual should do, but does not make any predictions about what will actually happen.
- The CAPM does: It predicts that all investments have zero α. If some investment had a positive α, then all investors should try to buy it, and prices should simply adjust until that α disappears.
- In other words, the CAPM describes an equilibrium.

The CAPM uses ideas from portfolio theory

The logic behind the CAPM formula is really just portfolio theory, applied to the case of an investor who holds the market portfolio:

- β_{im} is a (rescaled) correlation with the market portfolio. It is also the slope of the best-fit line from regressing r_i on r_m.
- α_{im} measures whether investment i beats its hurdle rate. It is also the intercept of the same best-fit regression line.
- Suppose we start from the market portfolio and add more of i.
  $\beta_{im} \lessgtr 1$ tells us whether portfolio volatility grows or shrinks,
  $\alpha_{im} \lessgtr 0$ tells us whether portfolio Sharpe ratio grows or shrinks.

Again, the new thing is the prediction that α_{im} is always zero.

This also means $IR_{im} = 0$, so indexing is the best strategy.

The market portfolio in theory and in practice

- The CAPM gives a special role to the "market portfolio," which theoretically includes *every* risky investment.

- To use the CAPM, we need to measure the market portfolio's return, in order to estimate β_{im}. This is clearly a difficult task.

- Most investment in the US is allocated to the stock market, and the average stock investor holds a value-weighted portfolio.

- So, *in practice,* people typically represent the market portfolio with a large, value-weighted portfolio of US stocks.

- Be aware that this is a big simplification of the theory. The "true" market portfolio would include not only stocks, but also corporate bonds, real estate, commodities, etc…
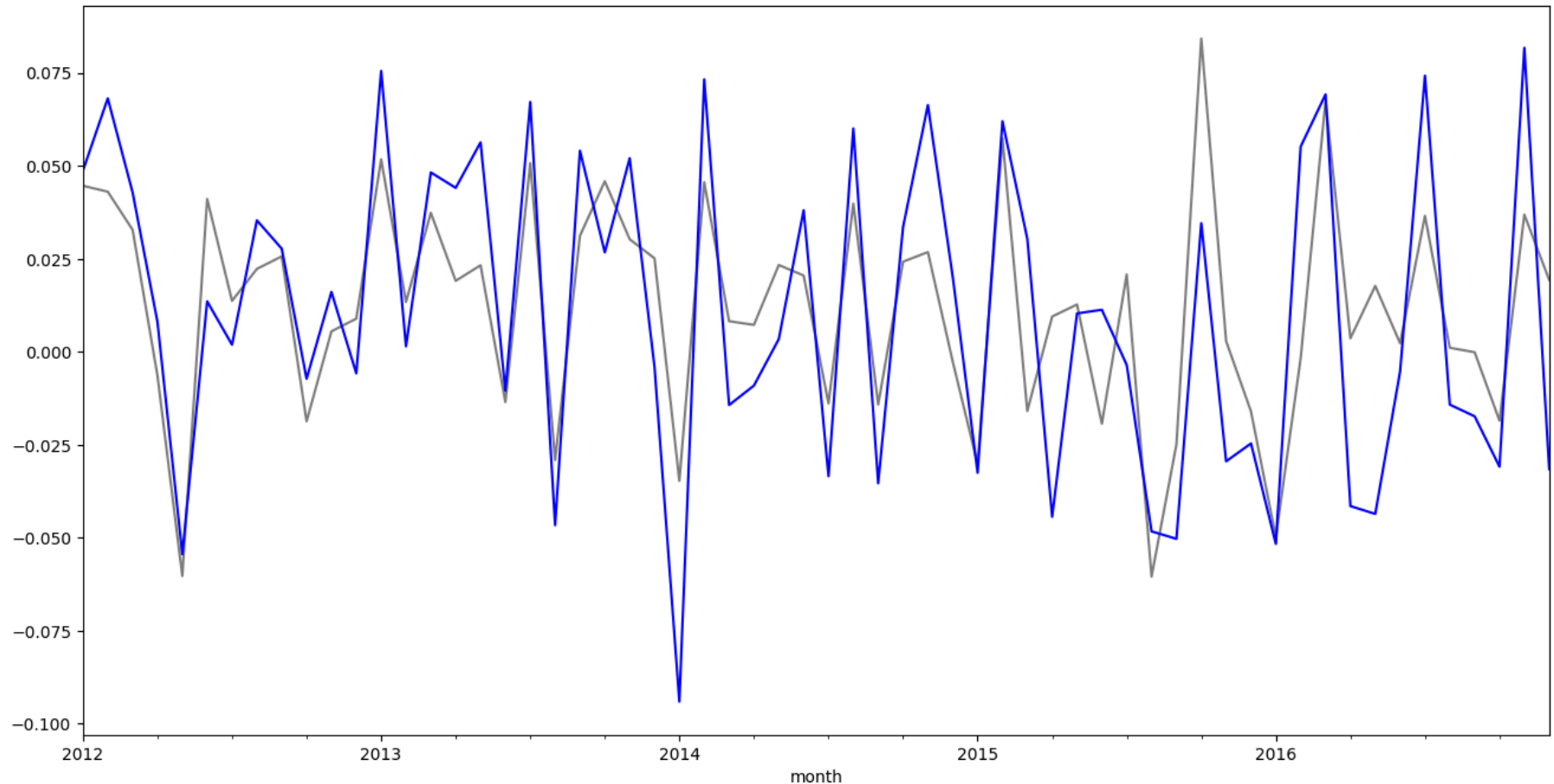
What does market risk look like?

Here's a view of the market portfolio, as it is usually measured: This figure plots monthly returns on the VFINX index fund.
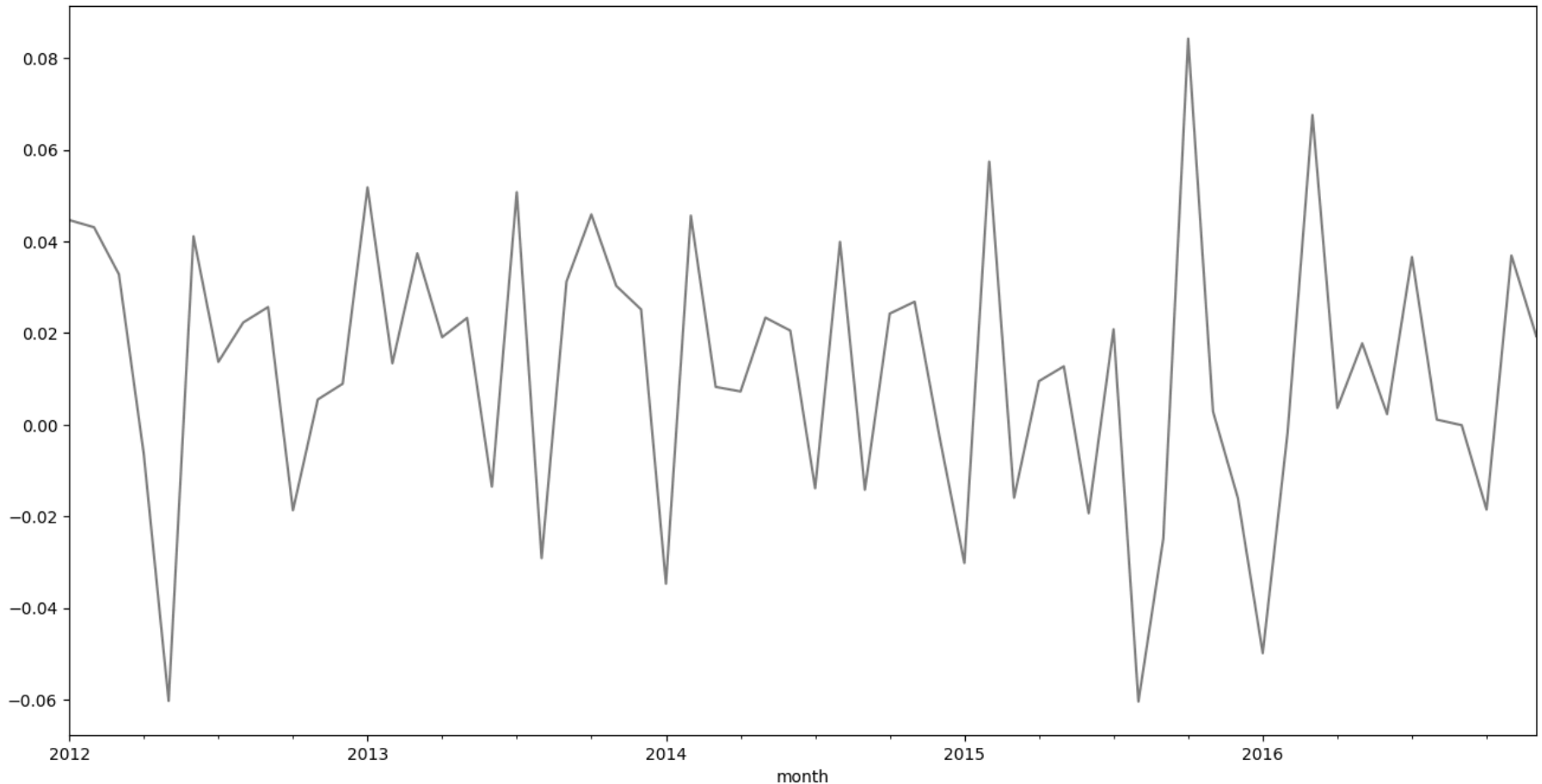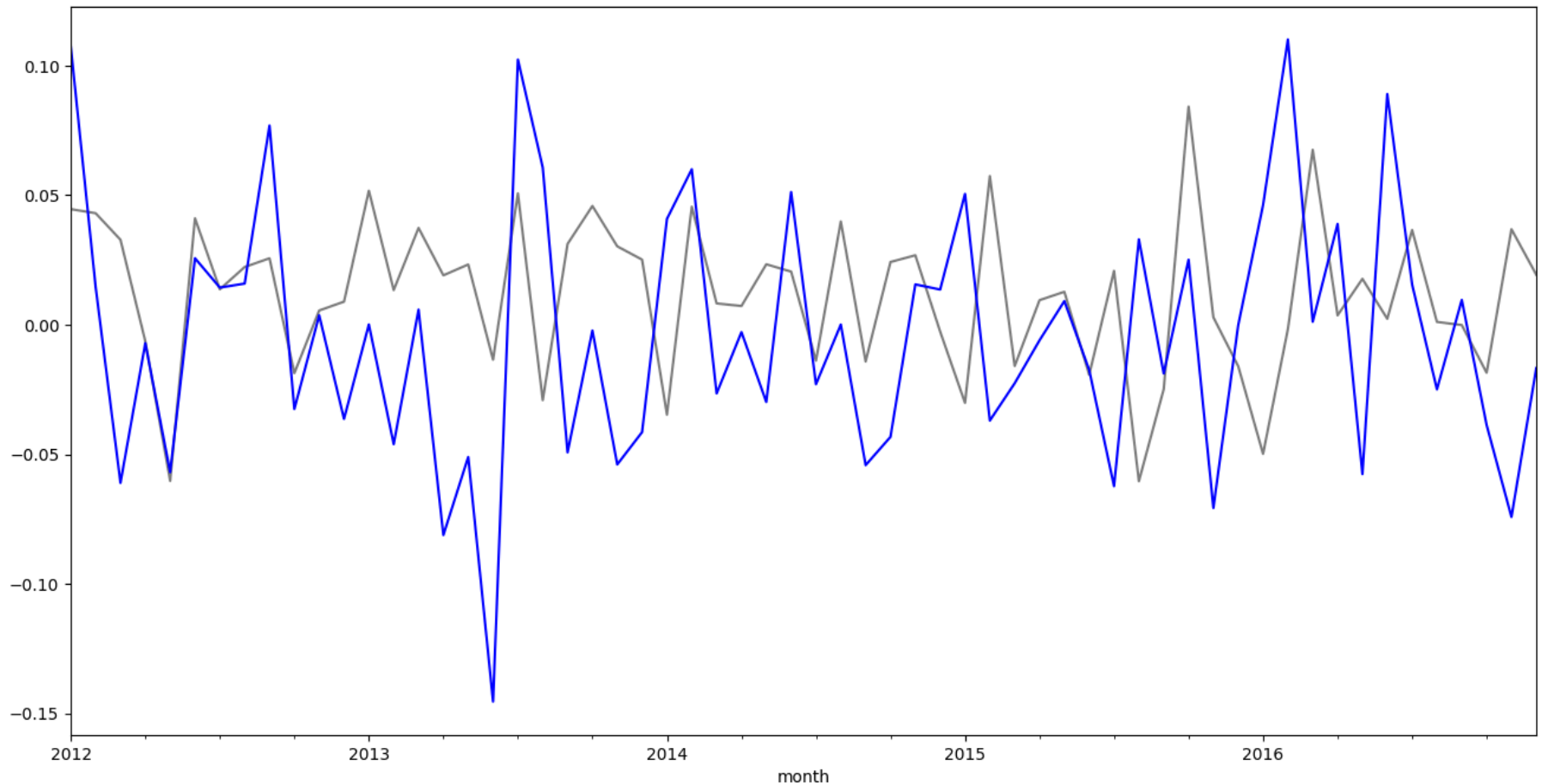
What does market risk look like?

Add in the return of an ETF of retail stocks, with dividends reinvested. Its monthly return has 3% volatility and a market β of 1.09.
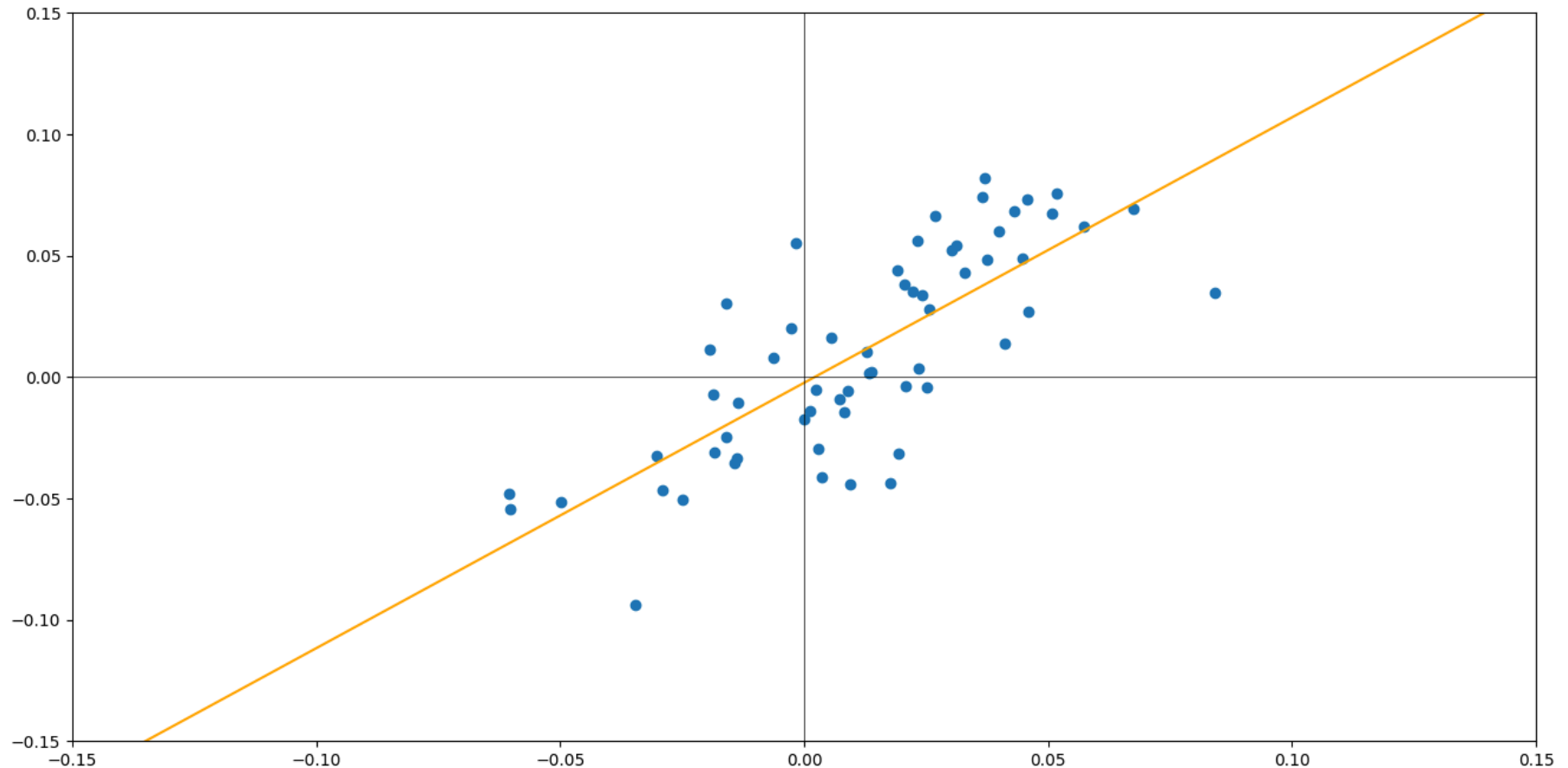
What does market risk look like?

Replace the retail ETF with a gold ETF. This ETF's monthly return has 5% volatility, but a beta of only 0.09.

What does market risk look like?

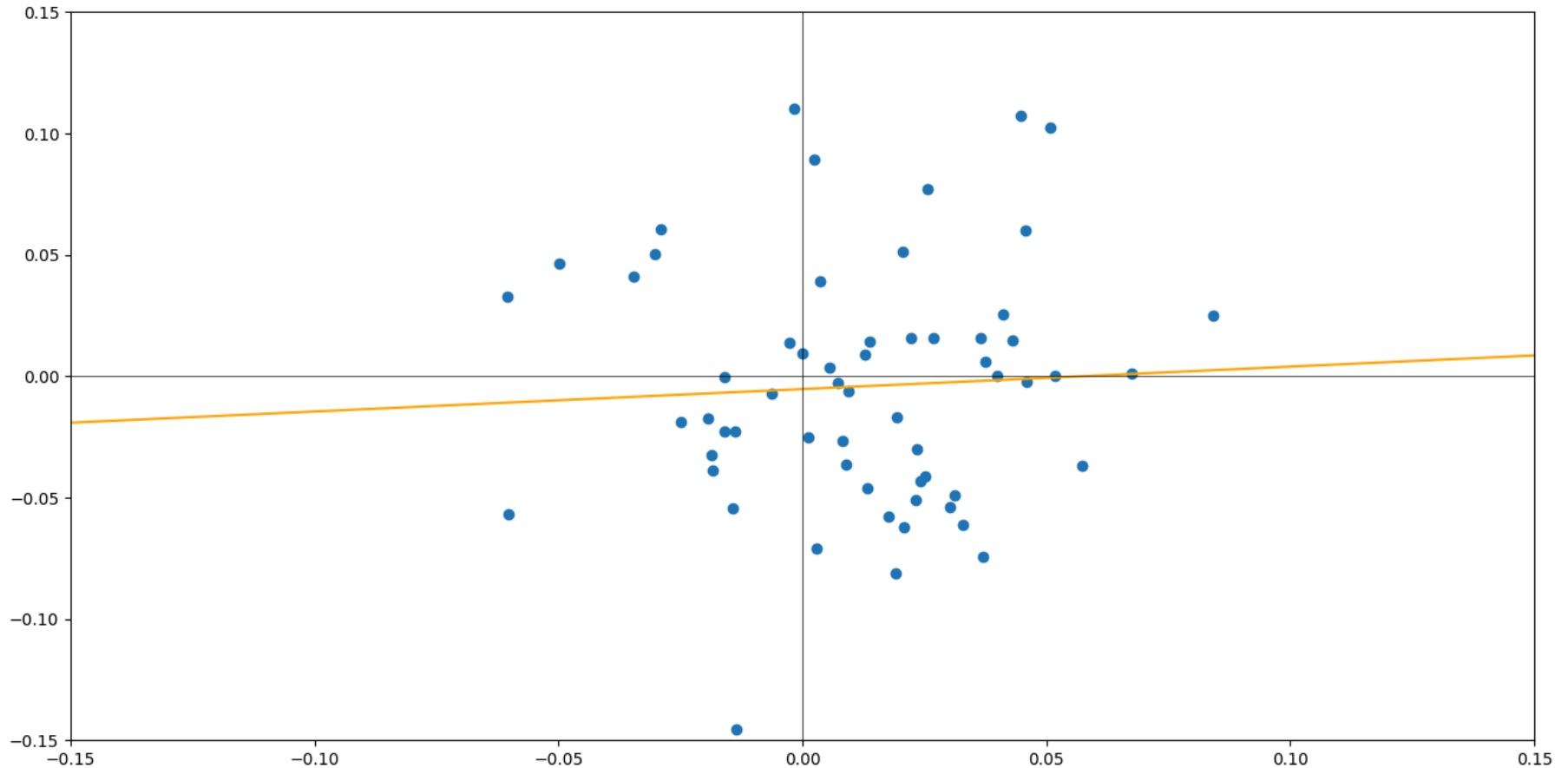Here's a different view of the same data on the retail ETF return:
β is the slope of the best-fit line in the figure. Here, β = 1.09.

What does market risk look like?

A similar figure using the gold ETF:
The line here is almost flat, with slope (beta) of 0.09

β as a correlation

β is the slope of a regression line like the prior ones.

Again, one way to interpret this number is that it measures a relationship between the investment and the market.

- β = 2: Moves twice as far as the market, on average.
- β = 0.5: Moves half as far as the market, on average.
- The market portfolio itself has a beta of 1.
- A risk-free investment has a beta of zero.
- A risky investment can also have a beta of zero! Gold is a risky investment with approximately zero beta.

The CAPM focuses only on β, so it predicts (correctly) that gold earns low average returns despite having high volatility.

β as non-diversifiable risk

A related interpretation is that β measures "non-diversifiable" risk.

- Gold has a low beta because its returns, while very volatile, have very little economic connection to the overall market.
- Adding gold decreases the market portfolio's volatility (β < 1). We call this *idiosyncratic* risk or *diversifiable* risk.
- By contrast, retail stocks have a very high beta, because the performance of these stocks is closely tied to the overall market.
- When you increase holdings of retail stocks in your market portfolio, their risk reinforces the volatility you were already exposed to, and your portfolo volatility slightly increases (β > 1). We call this *market* risk, *non-diversifiable* risk, or *systematic* risk.

The CAPM says that investors only care about *non*-diversifiable risk.

Diversifiable risk does not matter to them, because it disappears when the investment is added to their portfolio.

Application: The analysis of Homework 2

We will reproduce the results of that assignment, and then see how the formulas from this week are another path to the same answers.

I have already downloaded the data on VFINX, gold, and risk-free returns into a dataframe. Here are the first four months:

	VFINX	Gold	rf
month			
2009-01	-0.084126	0.061850	0.0000
2009-02	-0.106570	0.027490	0.0001
2009-03	0.087600	-0.026755	0.0002
2009-04	0.095588	-0.032118	0.0001

First step: Calculate excess returns

month	VFINX	Gold	rf	VFINX_excess	Gold_excess
2009-01	-0.084126	0.061850	0.0000	-0.084126	0.061850
2009-02	-0.106570	0.027490	0.0001	-0.106670	0.027390
2009-03	0.087600	-0.026755	0.0002	0.087400	-0.026955
2009-04	0.095588	-0.032118	0.0001	0.095488	-0.032218

In the homework we found the portfolio with the highest Sharpe ratio. Let's quickly calculate that answer again, using the matrix formulas form last week:

```
The portfolio with the highest Sharpe ratio is:
 30.076% Gold,
 69.924% VFINX.
It has a Sharpe ratio of 0.28480.
```

Using the Sharpe ratio formula from above

Now let's look at the first formula from this week. It say that we want to increase allocation to any investment i as long as $SR_i > \rho_{ip} \times SR_p$, where p is the portfolio we currently hold.

Let's apply this formula to the homework analysis, with gold playing the role of investment i and VFINX the portfolio p.

First we calculate the Sharpe ratio of gold:

```
Sharpe ratio of gold: 0.12996
```

Now imagine we start off holding just VFINX, and are considering whether to allocate some to gold. Calculate the Sharpe ratio of VFINX by itself, and the correlation of gold with VFINX.

```
Sharpe ratio of VFINX: 0.26116
Correlation of gold with VFINX: 0.06342
Multiply these two together: 0.01656
```

Because the Sharpe ratio of gold was greater than this, we conclude that we can achieve a higher portfolio Sharpe ratio if we allocate *some* amount to gold.

Now suppose we try an allocation of 20% to gold. We can simulate the returns of this allocation, and then redo the calculations just above with this new allocation playing the role of p, our current portfolio.

```
Sharpe ratio of an allocation 20% gold, 80% VFINX: 0.28117
Correlation of gold with this allocation: 0.30880
Multiply these two together: 0.08683
```

The result is still less than the Sharpe ratio of gold by itself that we calculated earlier. So we conclude that we should allocate more than 20% to gold.

We can repeat this process by trial and error (or by coding up an optimizer to do it for us). It will eventually settle down to the answer we found in Homework 2. Let's verify that once we reach that answer, the two sides of the equation are exactly equal.

```
Sharpe ratio of gold: 0.12996
Sharpe ratio of optimal portfolio from HW2: 0.28480
Correlation of optimal portfolio with gold: 0.45630
Multiply the last two numbers: 0.12996
```

So this allocation to gold achieves the highest possible Sharpe ratio in our historical data.

Formula with alpha

The next formula from this week said that we want to add more allocation to an investment i as long as it has positive α with respect to our current portfolio.

α measures how much an investment outperforms its hurdle rate:

$$\alpha_{ip} = \mathbb{E}[r_i - r_f] - \beta_{ip} \times \mathbb{E}[r_p - r_f]$$

We can estimate it in either of two ways:

1. Substitute the average excess returns on i and p into the expected values above.

2. Get the intercept from a regression of excess returns of i on excess returns of p,

$$(r_{it} - r_{ft}) = \alpha_{ip} + \beta_{ip} \times (r_{pt} - r_{ft}) + \varepsilon_{ipt}$$

Let's see how these formulas show up in the analysis from Homework 2.

First we check that gold has a positive α with respect to VFINX. In the regression table below, the number labeled "Intercept" is our estimate of α. The number labeled VFINX is our estimate of β.

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	0.0052	0.003	1.519	0.130	-0.002	0.012
VFINX_excess	0.0661	0.075	0.876	0.382	-0.083	0.215

We can interpret α as a monthly return. So, we would say that, in our historical data, gold delivers 0.47% per month more than its appropriate hurdle rate based on its β_{ip} with respect to our current portolio of holding only VFINX.

The value of 0.0721 for β is very low and effectively zero. This tells us that gold has almost no correlation with the performance of the overall stock market (represented by VFINX), even though gold is very volatile on its own.

Since we found a positive α for gold in the regression, this tells us that we can increase our Sharpe ratio by adding some allocation to gold to our portfolio, starting from zero allocation, and so we should.

Let's quickly check that we can match the alpha and beta value from the regression above by calculating directly with the formulas from slide 4:

```
Matching the above regression by direct calculations:
  Beta: 0.06611
  Alpha: 0.00516
```

Now we again try our allocation of 20% to gold, 80% to VFINX, and evaluate the α of gold with respect to this allocation:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	0.0020	0.003	0.603	0.547	-0.004	0.008
Gold20	0.3835	0.086	4.475	0.000	0.214	0.552

There is still some evidence of positive α, suggesting that we could do even better by increasing our allocation to gold some more.

Finally, we can check that gold has exactly zero alpha with respect to our optimal solution from Homework 2, so we could find that same answer by tinkering with the portfolio until alpha settles down to zero:

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	4.337e-19	0.003	1.42e-16	1.000	-0.006	0.006
Optimal	0.6065	0.086	7.068	0.000	0.437	0.776

So far we have matched the answers from Homework 2 using two different formulas, one based on Sharpe ratio and one based on α.

These two approaches felt very similar, because they are really the same thing, just stating the math in a different way.

The second approach explains why α is used as a general measure of investment performance.

Analysis based on information ratios

The information ratio is α scaled by a measure of risk. Where α just says whether it's *possible* to improve your Sharpe ratio using a new investment, the information ratio tells us *how much* your performance can improve using that investment, and how much you want to allocate to it.

The information ratio can help investors judge whether it's actually worthwhile to pursue a new investment. By the same token, active managers should focus on achieving a high information ratio with respect to their benchmark, because this is how they will attract more funds.

We start out by calculating the information ratio of gold with respect to VFINX:

```
Information ratio of gold with respect to VFINX: 0.11362
```

There are many ways to use this number. One of the most important is a formula we saw above: For an investor who allocates only between the new investment i (in this case gold), and another portfolio p (in this case VFINX), the maximum Sharpe ratio they can achieve is given by the following formula (slide 6 and textbook formula 8.26),

$$SR_{max} = \sqrt{SR_p^2 + IR_{ip}^2}$$

Using the numbers we have calculated, we calculate a maximum Sharpe ratio that exactly matches the Sharpe ratio of our solution from Homework 2:

```
Max SR: 0.28480
```

The point is that we can decide how attractive this new investment is, without doing an actual optimization.

If a manager has a good guess what portfolio their clients hold on average (it is likely close to the market portfolio!), then they should maximize their information ratio to attract the most investment into their fund.

# Portfolios formed on size, 1926-2024
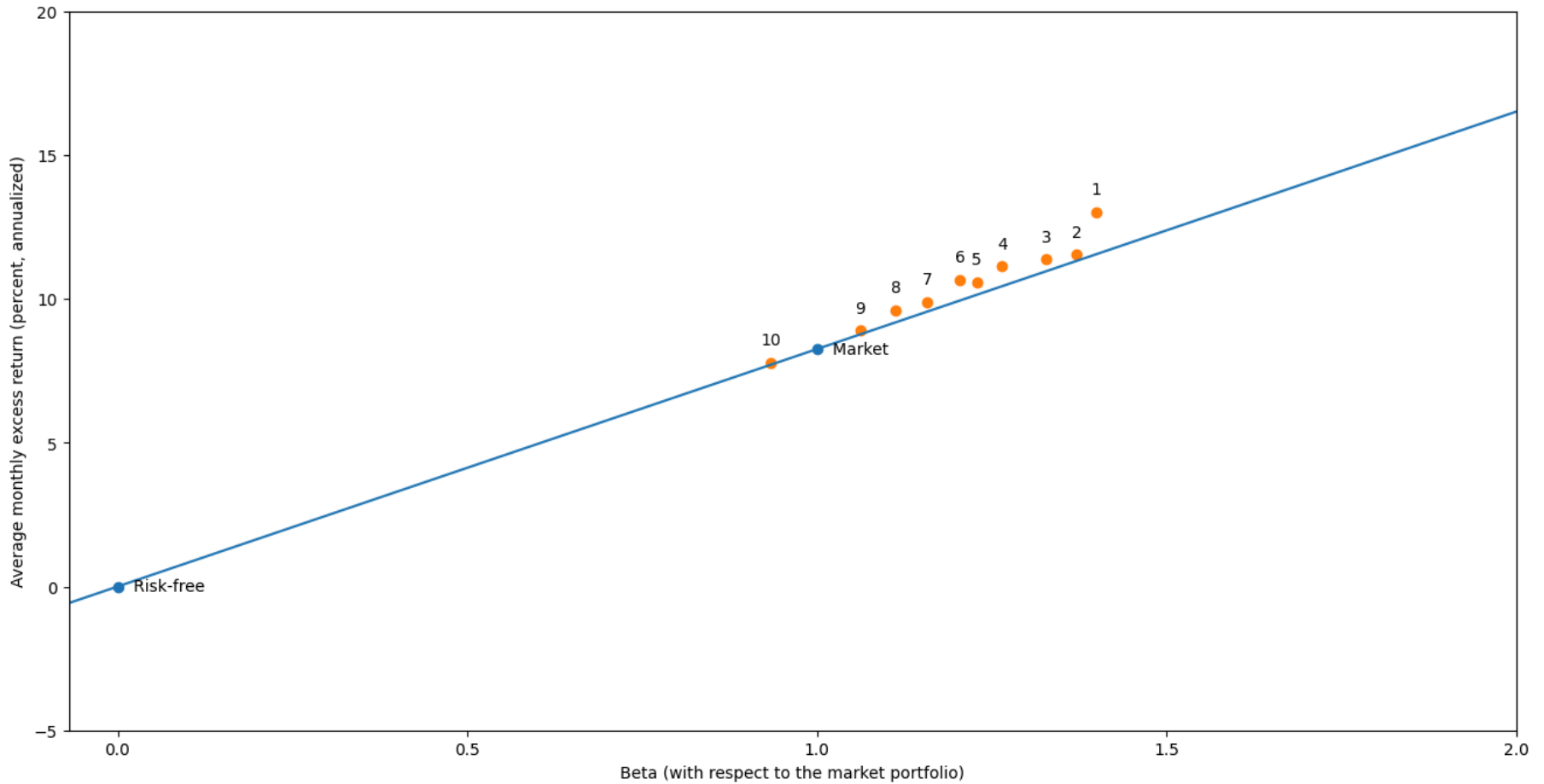
Portfolios formed on value, 1926-2024
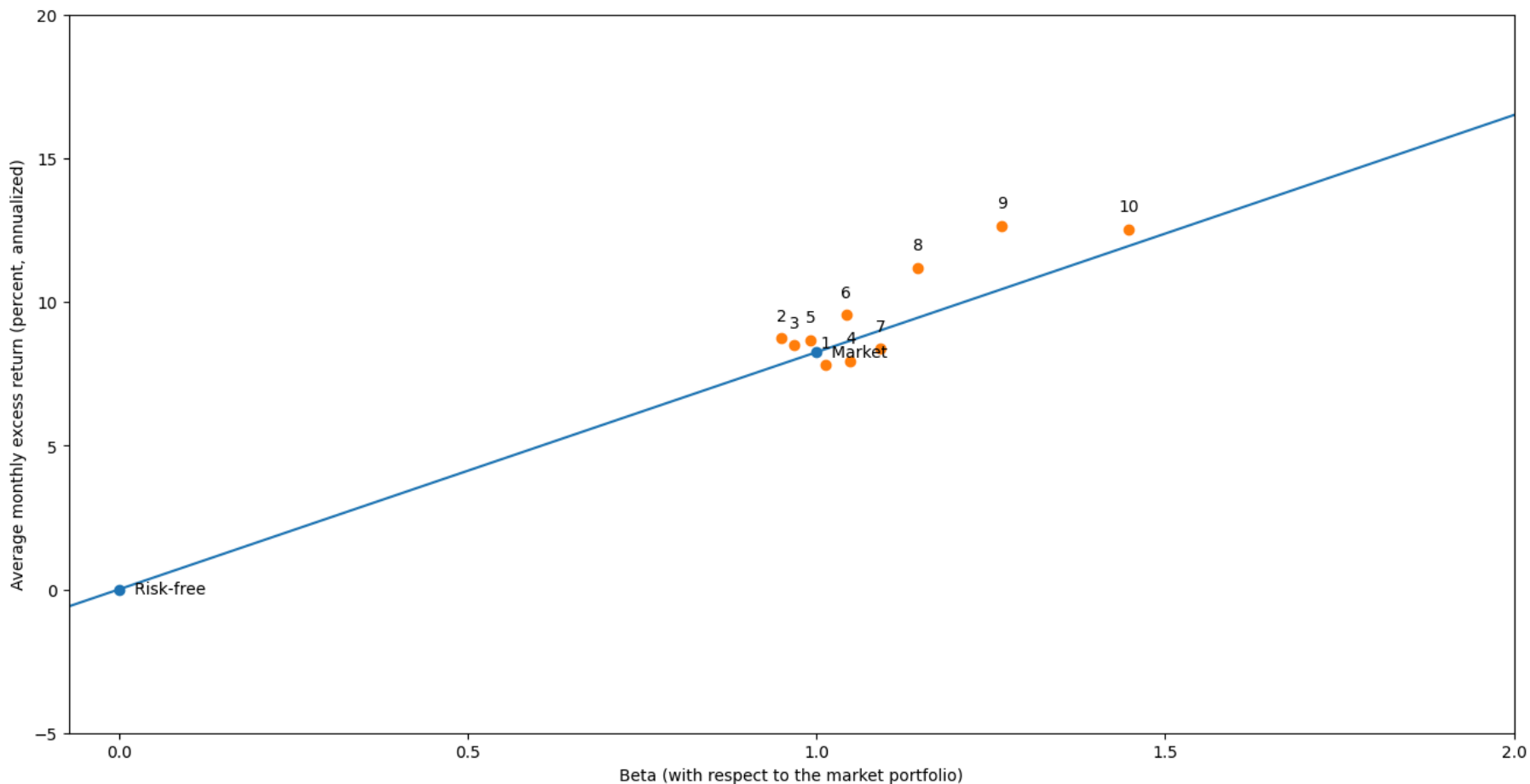
Notes on the historical evidence

- In these figures, size and value strategies seem to offer returns that are roughly fair according to the CAPM. This evidence would not encourage us to depart from a passive strategy.

- However, the early part of this sample includes the Great Depression and the second world war. Many researchers omit these times, and start the analysis in the 1950s.

- It's not clear whether this is theoretically ideal or not. In the next figures, we look at how this changes things.

Portfolios formed on size, 1950-2024
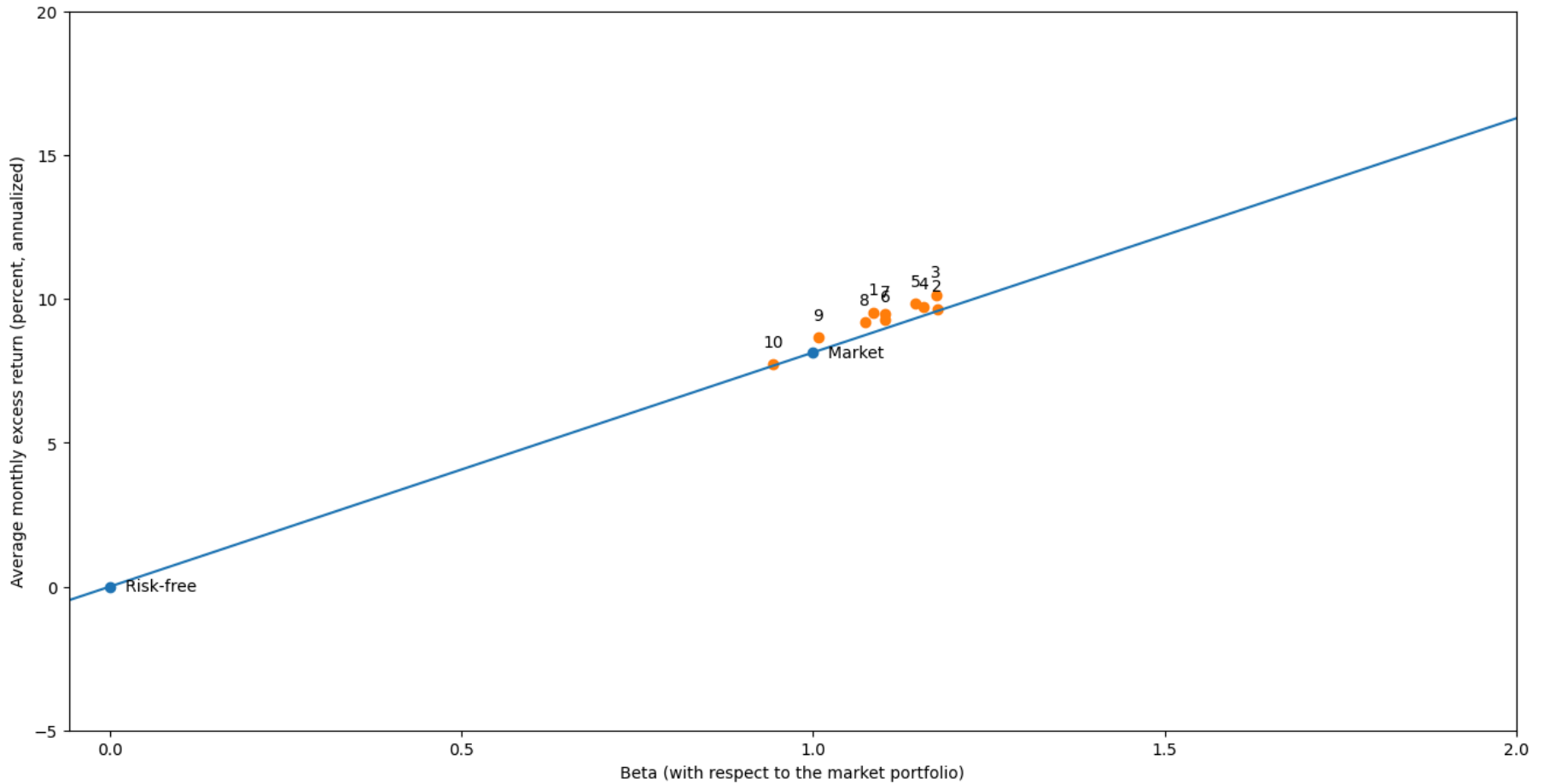
Portfolios formed on value, 1950-2024
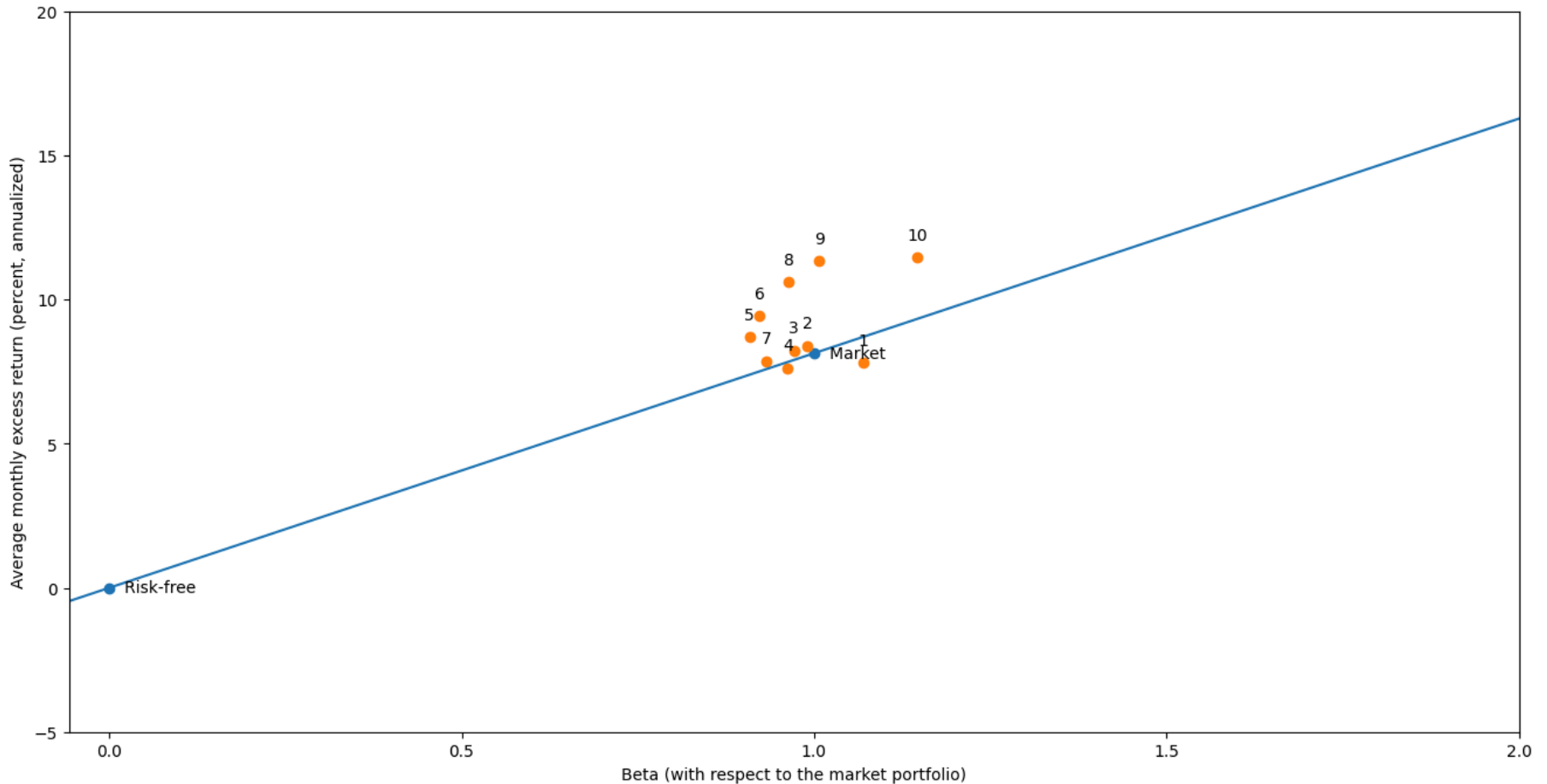
Notes on the figures using 1950–2024

During this shorter time period, returns to value-sorted portfolios did not follow the CAPM.

This suggests opportunities that one could have exploited.

Specifically: Value stocks (those with high BM ratios) offered higher return, with similar beta, to growth stocks. (Compare portfolios #8, 9, 10 with #1, 2, 3.)
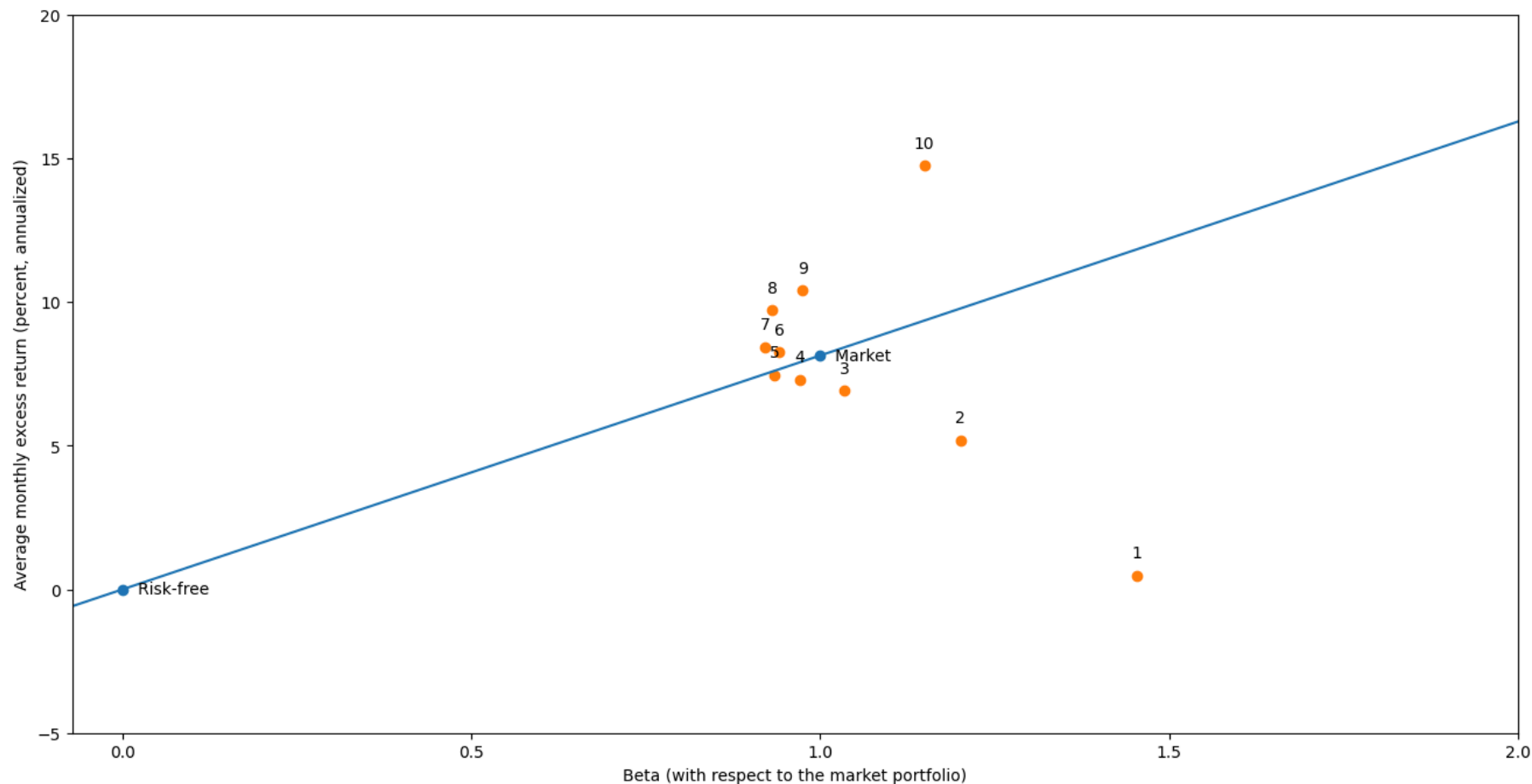
This basic pattern is called the value premium.

Next: Similar conclusion for portfolios sorted on momentum.

Portfolios formed on momentum, 1926-2024
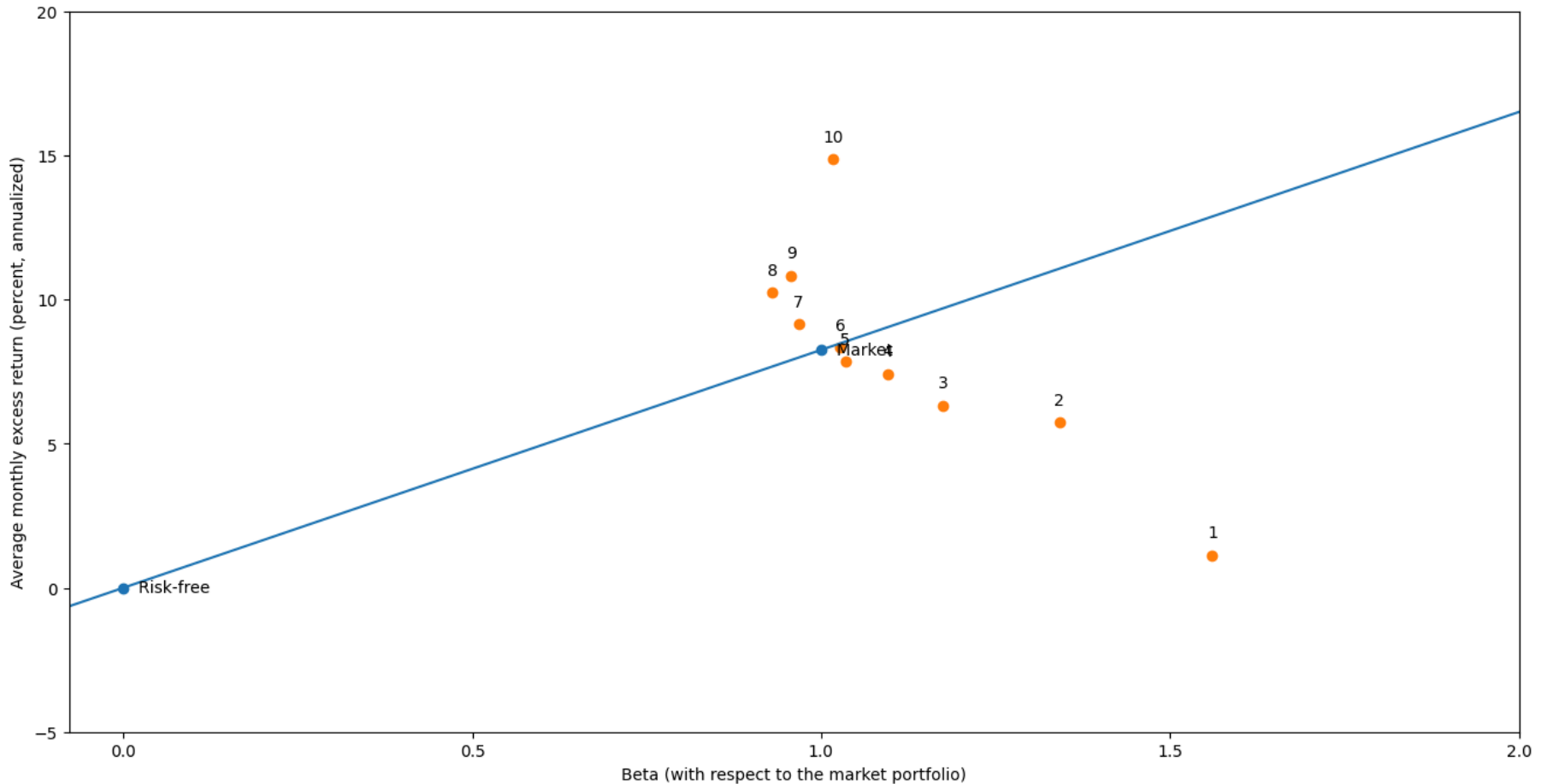
Portfolios formed on momentum, 1950-2024

Notes on value and momentum premiums

By sorting stocks on value and momentum, you could have found portfolios in the past, that would have offered higher returns without taking on more market risk (beta).

In academic literature, these style-based return are called factors. I will use this term going forward.

In response to this evidence, the factor investing industry has grown to meet investor demand for value- and momentum-focused strategies, and to find new factors.

Critical questions to ask about active strategies

- What kind of risk do these strategies require you to bear?

  - The figures just showed average returns and betas.
  - This does not mean you can earn steady, reliable returns.
  - In fact, to earn the averages reported in the prior figures, you had to endure many times of very poor returns.

- How feasible is it to implement these strategies?

  - In the past, how close could you have come to actually achieving the returns in the prior figures?
  - In the future, how reliably can you do what the strategies describe, and how expensive will that be?

Next: Better ways to track returns on strategies

A first step is just to have a better way to track the performance of investment ideas like the ones we studied above.

- So far we've been focused on simple graphs of average return against beta. But these only give us a limited perspective on the performance of an investmen strategy.

- The biggest limitation is that these graphs just show average performance over a long horizon. They don't tell us how much short-term fluctuation the strategy might exhibit relative the long-run average.

- In Module 3, we will build indexes of investment performance that address this issue. The basic idea will be to study the spreads between the returns of different portfolios.

Many ideas just repackage well-known strategies

Many popular investment ideas are tightly connected to well-known strategies like those based on size, value, or momentum.

- An EW index tilts heavily towards small and value stocks.
- Stocks that get added to an index are momentum stocks.

If you know this, you have a better perspective on what is really "new," and what is not, when you evaluate a new strategy.

- Don't overpay for something that is already easily available!
- Again, Module 3 will explore this issue in detail.