The Josephson Effect and Determination of $2e/h$

William Grenard

October 11, 2015

Abstract

This experiment was performed to obtain a measurement of the fundamental ratio $2e/h$. This was done by varying the voltage across a Josephson Junction exposed to microwave radiation, and observing voltage values at which steps in junction current occurred. The value of $2e/h$ was measured in this experiment to be $(453 \pm 12)\frac{MHz}{\mu V}$. This is not in agreement with the value obtained by Parker, Taylor, and Langenberg [1]. It was concluded that the experiment was plagued with a systematic error, which shifted all calculated values of $2e/h$ lower than expected. This error was most likely introduced by incorrect scaling in the voltage axis.

1 Introduction

There are many important reasons one may imagine for an accurate determination of $\frac{2e}{h}$. Virtually all branches of physics and the sciences in general rest upon the accuracy of measured constants, and verification of these values through different means offers insight as well as confirmation or refinement of theories. As Clarke outlines, one specific important result that comes from an accurate determination of $\frac{2e}{h}$ is that it may be used to easily and accurately maintain the standard of the volt [2]. This idea stems from the simple relationship between the voltage and supercurrent frequency across a Josephson junction.

In this experiment, a measurement will be obtained for $\frac{2e}{h}$ through an indirect observation of the AC Josephson effect, which manifests itself as a series of abrupt steps in the I-V curve of the junction. The width of these steps is intimately related to the value of $\frac{2e}{h}$, and examination of these steps will allow for an experimental measurement of this value.

2 Theory

2.1 Superconductivity

As first theorized by Bardeen, Cooper, and Schrieffer, superconductivity occurs because of the occurrence of now-called “Cooper pairs,” in metals at low temperatures [3]. A Cooper pair is a pair of electrons which are attracted together. This occurs because of positive ions from the metal lattice, which are attracted toward the electrons in the metal. These positive ions exhibit an attractive force that is sufficient to overcome the Coulomb repulsion between electrons. The result is that electrons in the superconductor pair together.

The attractive force between the electrons of a Cooper pair is quite weak, however, and this has two main consequences. Firstly, the pairings are very sensitive to thermal fluctuations, and cannot exist at temperatures many degrees above absolute zero. Secondly, the weak interaction means that the electrons in the Cooper pairs have a large spacial separation in comparison to the density of the pairs themselves. In other words, if one were to draw a hypothetical sphere large enough to contain a single Cooper pair, there would be millions of other Cooper pairs packed into the sphere as well, all overlapping each other.

Because of this latter effect, the exclusion principle becomes a major factor in the behavior of the overlapping Cooper pairs. It was shown that in order for the pairs to obey the exclusion principle, their center of mass momenta must be identical [3]. Because a particle’s momentum is dependent upon phase, this implies that the phases of all the Cooper pairs within the same vicinity of each other must have the same phase. Clearly, because of the close proximity of these pairs, this phase locking behavior must extend across the whole superconductor. The result is that the superconductor, a macroscopic object, may be described by a single wavefunction, which all of the pairs obey. It is this concept which accounts for the zero resistance behavior of superconductors: what one electron does, all electrons must do. This ensures that it is very improbable for the electrons to scatter about and disrupt the flow of current once it is started.

2.2 The DC Josephson Effect

One can imagine taking two superconductors separated by a thin insulating barrier, a setup which is known as a Josephson junction. If the insulating barrier is thick enough such that no electrons may pass from one side of the barrier to the other, the superconductors are each described by their own wavefunction, the phases of which need not have any correlation. If the barrier thickness is reduced, there will come a point when Cooper pairs will have a reasonable probability amplitude for tunneling through the barrier. If the conditions are right, this tunneling can give rise to a direct current across the barrier, which is given by the following equations that can be found derived in detail by Feynman [4].

$$j(\Delta\phi) = j_0 \sin \Delta\phi \tag{1}$$

$$\Delta\phi = \Delta\phi_0 + \frac{2e}{\hbar} \int V(t) dt \tag{2}$$

Here, $\Delta\phi$ is the phase difference between the two superconductors, a quantity which is dependent upon the voltage $V(t)$ applied across the junction. j_0 is the *critical current*, the maximum DC current which the junction can sustain without a voltage drop, and is dependent upon the material properties of the junction, as well as the barrier width. Finally $\Delta\phi_0$ is the initial phase difference between the superconductors at time zero.

The first important idea to note from these equations is that if a DC voltage V_0 is applied, the integral term in equation 2 simply becomes $\frac{2eV_0}{\hbar}t$. The coefficient of this expression is very large for any reasonably sized input voltage, which causes this term to increase very rapidly in time. Because of this, the current j_0 oscillates rapidly as well, and averages to zero. On the other hand, if no voltage is applied, then $\Delta\phi = \Delta\phi_0$ and depending upon the value of $\Delta\phi_0$ a nonzero current may appear across the junction. Notice, this current flows in spite of the fact that there is no voltage drop across the junction (in fact, the presence of current is *dependent* on this). This is known as the DC Josephson effect, and it results in an I-V curve that resembles that shown in Figure 1.

In the absence of an applied voltage, the sinusoidal dependence upon the phase difference can be conceptualized quite simply. As Cooper pairs attempt to tunnel through the barrier, only those pairs that change their phase by such an amount as to match the phase of the pairs on the receiving side, will successfully tunnel. Pairs that fail to match their phase will be reflected back. If the phase difference is such that tunneling in one direction is favored over the other, then a current in the favored direction will arise. The direction and net strength of this current varies sinusoidally as the phase difference is altered.

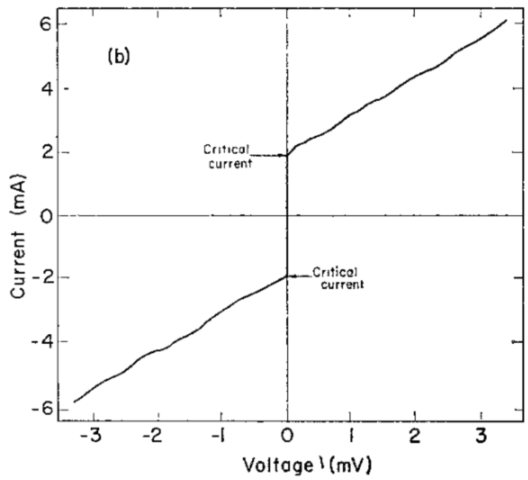

Figure 1: The DC Josephson Effect. *Image from J. Clarke* [2]

2.3 The AC Josephson Effect

If a voltage of large enough size is applied across the junction, then the supercurrent simply oscillates within the junction, as seen in the equations above. Now, when a Cooper pair tunnels through the barrier, it loses potential energy equal to $2eV$. This energy is lost by the emission of a photon of energy equal to this amount. The frequency of this photon is then given by:

$$\nu_p = \frac{2eV}{h} \tag{3}$$

This behavior is known as the AC Josephson effect. One may observe this effect indirectly by pumping microwaves into the junction in addition to applying a voltage across the junction. In addition to the DC voltage, the microwaves induce a weak voltage of their own across the junction, which oscillates with a frequency equal to that of the microwaves themselves. This periodicity in the voltage translates to a periodic fluctuation of the AC supercurrent frequency. It turns out that for certain frequencies (caused by specific voltages), the current through the junction will not average to zero, and as a result, there is a spike in DC current through the junction. These spikes occur at voltage values where the supercurrent frequency is an integer multiple of the input microwave frequency ν_m. This relation is given by:

$$\nu_p = \frac{2eV}{h} = n\nu_m \qquad \text{for } n = \pm 1, \pm 2, \pm 3 \ldots \tag{4}$$

When achieved, this effect looks similar to the graph shown in Figure 2. Because this steplike effect occurs due to the nature of the alternating supercurrent, it's existence is an indirect verification of the AC Josephson effect.

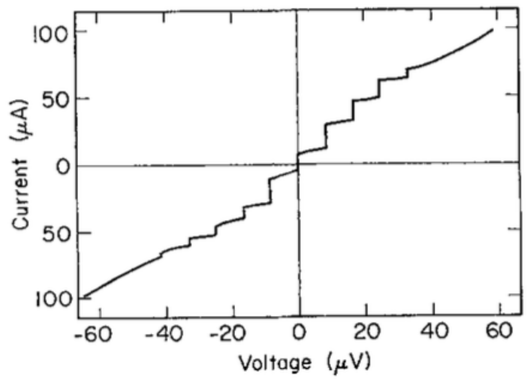

Figure 2: The AC Josephson Effect. *Image from J. Clarke*[2]

3 Apparatus

The Josephson junction in this experiment is created by contacting a niobium needle point with a large niobium screw head, an oxidized layer between the two acting as the insulating barrier for the junction. The junction is housed in an apparatus called the junction assembly (shown in Figure 3). The junction assembly holds the niobium screw on the base, with the niobium needle above it. The needle can be adjusted upwards or downwards by a differential screw to make contact as needed with the niobium base.

This junction assembly is attached to a circuit (shown in Figure 4), containing a low frequency oscillator for application of voltage across the junction, and probes for measuring the voltage across the junction. It is important to note that the leads which are used to measure the voltage (denoted P1 and B1) are placed as close as possible to the junction itself to reduce errors in voltage reading.

In order for niobium to superconduct, it must be cooled to just a few degrees above absolute zero. Thus, to achieve the Josephson effects the junction assembly needs to be submerged in liquid helium. To achieve this, the junction assembly is placed and secured inside the bottom of a long hollowed tube, as shown in Figure 5. The tube containing the junction assembly is then submerged into a dewar of liquid helium when the experiment is ready to be conducted.

Within the tube, there is a long rod with a hex key at its bottom end, which should be inserted into the top of the differential screw in the junction

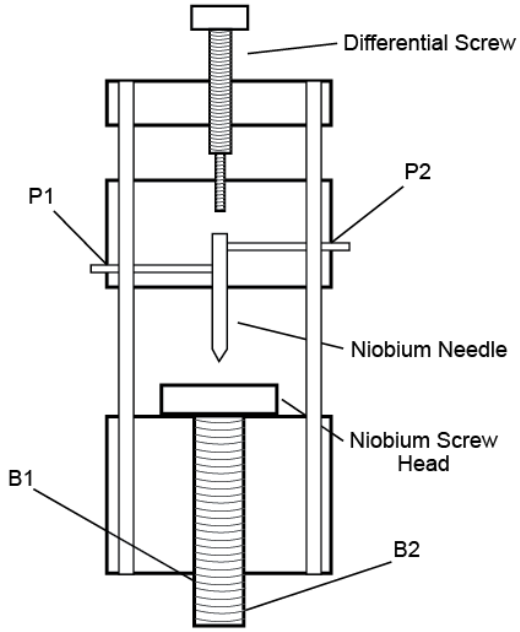

Figure 3: Junction Assembly

assembly. The other end of this rod protrudes from the top of the tube apparatus so that it may be used to turn the differential screw and adjust the height of the niobium needle in the junction assembly. There is also a rubber stopper at the top of the tube, which needs to be in place before submerging into the liquid helium. This stopper seals the gap where the wires from the junction assembly come out of the tube and attach to the BNC ports.

Lastly, a sweep oscillator is used to produce the microwaves that will be pumped into the junction. These microwaves are fed through a doubler, which amplifies the waves and doubles their frequency, and then through an isolator, which protects the oscillator and doubler from reflections. Then, the microwaves are sent to the probe assembly. On there way there, however, they are split, and part of the microwaves enter an absorption cavity, which can be used to very precisely measure the frequency. This final part of the apparatus is shown in Figure 6.

4 Procedure

Initially, seven niobium needles were prepared by filing the tips of niobium wires, about 1 mm in diameter and 1.5 cm in length, to points. These needles

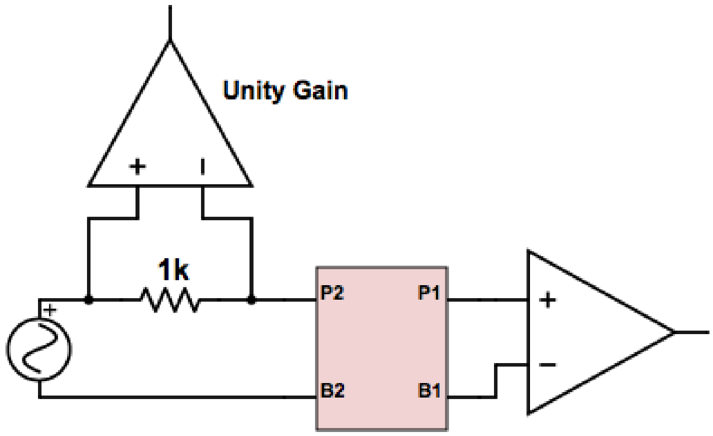

Figure 4: Junction Circuit

were then left over the weekend to form an oxide layer on their surfaces. Once this was finished a needle was selected and the oxide layer from the back half (the side without the point) was scraped away with a file, while leaving the oxide layer on the point intact. The needle was then slid into the junction assembly and secured snuggly by tightening the locking rods in place. Next, the housing which contained the needle was slid downward so that the tip of the needle was within about 1 cm of the niobium base. Finally, the differential screw was screwed in to the top of the junction assembly, far enough to engage the needle housing, but not so far that the differential screw was in danger of coming in contact with the back of the needle, which would cause a short in the circuit.

After ensuring that all of the electrical connections were visibly attached to the junction assembly, two small strips of electrical tape were attached over the sides of the assembly over the connections P1 and P2 to avoid contact with the metal walls of the probe assembly. Then, the junction assembly was carefully slid into the bottom of the probe and secured with the set screws.

To check for proper electrical continuity, a DVM was used to measure the resistance across the various BNC ports at the top of the probe assembly, where the junction assembly's electrical connections were to interface with the rest of the circuit shown in Figure 4. Assuming that the needle had not yet made contact with the niobium base, we expected to see an infinite resistance across the connections P1-B1 and P2-B2. And we expected only a small resistance between the ports P1-P2 and B1-B2. It was verified that the DVM read "OVLD" where infinite resistance was expected, and read 0–1Ω where continuity was expected.

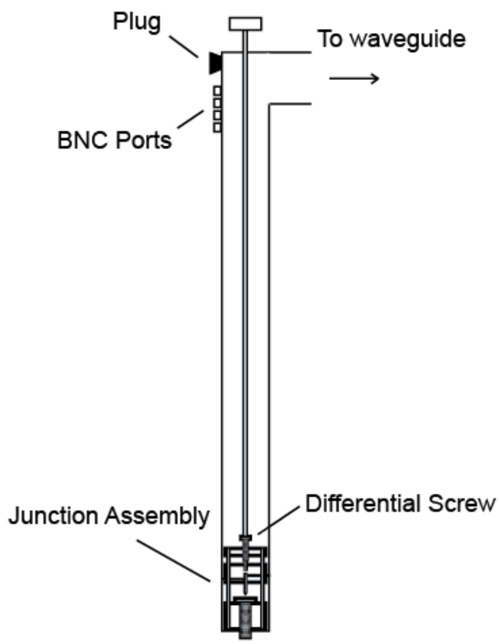

Figure 5: Probe Assembly

Once proper continuity was established, the junction assembly was attached to the circuit by means of BNC cables at the top of the probe assembly. The leads P1 and B1, the closest to the junction itself, were attached to an amplifier initially set to a gain of 500 and bandwidth of 0.03 Hz – 30 kHz, whose output was sent to the X-channel of the oscilloscope. The leads P2 and B2, which would carry the current to the junction, were attached to the low frequency oscillator initially set to output a 5 V peak-to-peak sine wave at 60 Hz. Finally, the Y-channel of the oscilloscope was attached to read the voltage across the $1k\Omega$ resistor within the circuit. This voltage reading is proportional to the current running through the circuit. In fact, 1 V on the vertical axis of the scope translates to 1 mA of current. Thus, with the scope in X-Y mode, the resulting trace can be thought of as the junction's I-V curve.

At this point, it was verified that the scope, in X-Y mode, showed a horizontal line, signifying that there was no current flow. This was expected, because the niobium needle had not yet made contact with the base.

Very carefully, so as not to disturb the position of the needle, the probe assembly was held upright in a vertical position, and the rod within it was gently pressed down toward the junction assembly until an audible click was heard, signifying that the allen head at the end of the rod had attached to the top of the differential screw. At this point, proper electrical continuity was

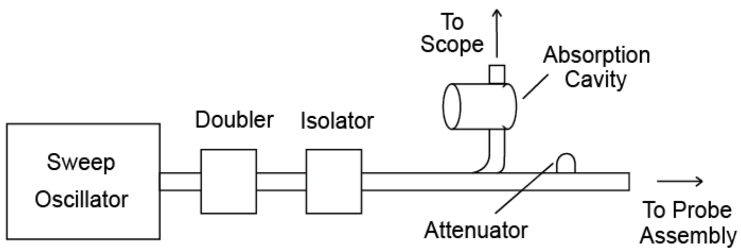

Figure 6: Microwave System

verified once more with the DVM, and the oscilloscope was checked again to verify there was no current flow.

Before inserting the probe assembly, the approximate level of liquid helium in the dewar was noted by inserting a thin metal tube with latex stretched across one end, and lowering the tube until it touched the bottom. Vibrations could clearly be felt moving through the latex. Slowly, the tube was lifted until the frequency of the vibration changed, signaling that the bottom of the tube had reached the surface of the liquid helium. My comparing the height of the tube showing when it was at the bottom of the dewar, versus when it was at the top of the liquid helium, the approximate level was determined. This knowledge gave an idea of when the tube assembly's decent into the dewar should be slowed so as to prevent excess liquid helium boil-off.

At last, it was ensured that the rubber plug was inserted in the probe assembly, and then the assembly itself was gently lowered into the dewar. As it was lowered, it was rotated back and forth constantly to avoid clogs at the mouth of the dewar, which can be caused by helium boil-off freezing water vapor in the air around the insertion point. When the bottom of the probe assembly neared the level of the liquid helium the probe's decent was slowed. When the rushing sound of helium boil-off exiting the dewar was heard, the decent was stopped until the sound subsided. All the while, the assembly was rotated back and forth continuously. This procedure was followed until the probe was fully inserted into the dewar.

To obtain the DC effect the rod at the top of the probe assembly was turned slowly to lower the needle onto the niobium base. Once the oscilloscope showed a trace that resembled the DC effect, a large magnet was brought near the apparatus, and it was verified that the I-V curve fluctuated with the changing magnetic field. This was done to verify that the trace was showing a true DC Josephson effect, and that the step was not due to bridges forming between the junction.

Next, the sweep oscillator, attached to the waveguide assembly as shown in Figure 6 was powered on and initially set to produce a 11 GHz output. The attenuator was initially set to 20 dB (the maximum) to ensure that the power was not so large to damage the waveguide. Using a power meter, the output power after attenuation was measured. This reading, along with the attenuation value was used to calculate the output power of the sweep oscillator and active doubler system.

Finally, the waveguide was attached to the probe assembly, and the search for the AC Josephson effect began. By adjusting the frequency of the microwaves, the power of the microwaves (adjusted by turning the attenuator nob), the vertical position of the niobium needle, the gain of the preamp, and the scale of the oscilloscope the AC effect was obtained. Using the absorption cavity, the precise frequency of the microwaves was measured.

In order to obtain a precise measurement of the gain from the preamp between the junction and the X-channel of the oscilloscope, the leads from P1 and B1 were momentarily removed from the junction and attached to a voltage box, designed to output a DC voltage signal. The output of the voltage box was measured using the DVM, the zero reference of the scope was noted by turning off the power of the voltage box and noting the voltage reading on the scope, and then the amplified voltage reading was noted on the scope after turning on the voltage box. From these parameters, the gain of the preamp was calculated.

5 Results and Analysis

From the niobium Josephson junction, a clear DC effect was achieved. It's shape can be seen from the plot in Figure 7. It is clear from the figure that when the voltage across the junction is zero there is a direct current slightly greater than 1 mA in the circuit. When the magnet was brought near the junction, the waveform on the oscilloscope clearly fluctuated, signifying that the effect seen is a true DC Josephson effect.

Note that the plot in Figure 7 is a scaled form of the plot seen directly on the oscilloscope. Because of the preamp in the circuit, the actual voltage across the junction, which is the value shown in the plot, is given by

$$V_j = \frac{V_{scope}}{G} \tag{5}$$

where G is the gain of the preamp and V_{scope} is the voltage seen on the X-channel of the scope. Based upon the measurements and respective errors of the calibration process described in the previous section, the gain with

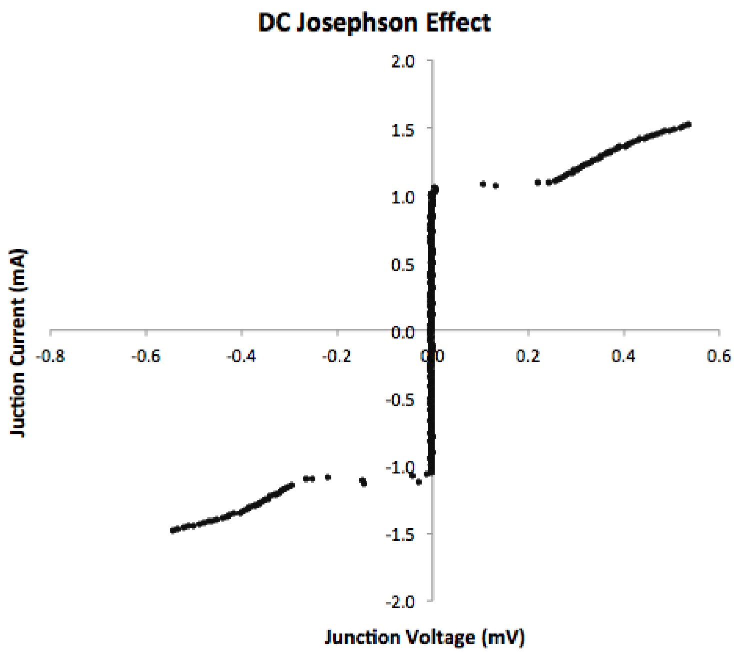

Figure 7: DC Effect

propagated error was determined to be:

$$G = 950 \pm 10 \tag{6}$$

This error in gain propagates to a relative error of approximately 1% for each voltage point. This error is small enough that it is effectively contained within the width of each point on the plot.

Presumably, there is a similar error in the current, due to errors in the unity gain amplifier and 1 kΩ resistor. It has been concluded that these errors can be safely omitted from exact calculation, because they are assumed to be relatively small, and the exact scale of the current axis is unimportant for this experiment; it is the junction voltage (horizontal axis) values that will play a major part in the determination of $\frac{2e}{h}$.

The AC Josephson effect was also observed, and is shown in Figure 8. A clear steplike pattern can be seen, as expected. The steps are much more pronounced for voltages less than zero, but they become almost completely washed out for positive voltage values. A major reason for this occurrence is easier seen from the image of the scope trace, shown in Figure 9.

The steps from the scope trace for positive voltage values are slightly easier to recognize than they are in the scaled plot from the data points, but they

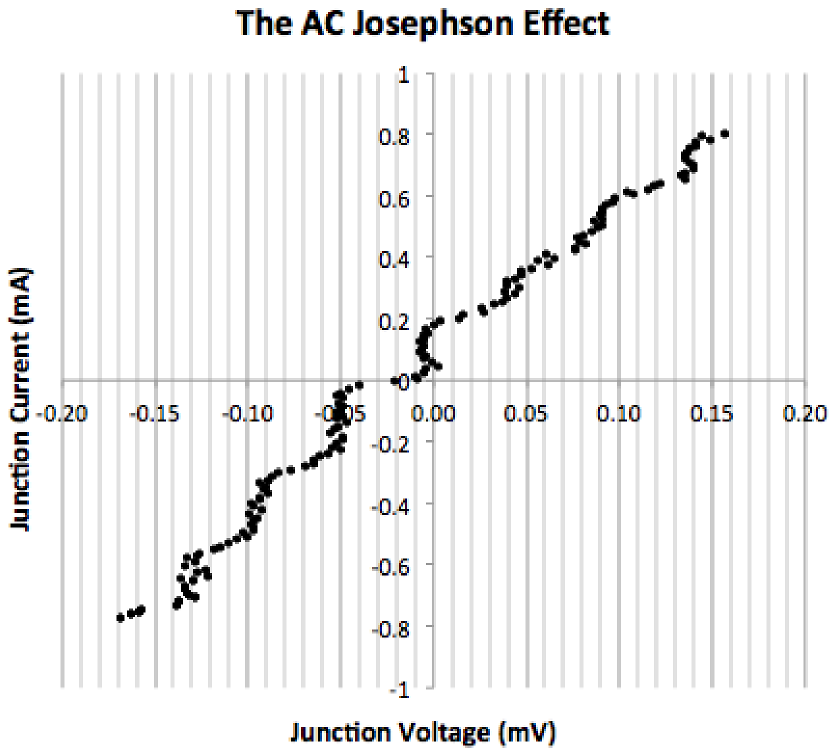

Figure 8: AC Effect

are still far from clear because there seems to be some 60Hz pick-up, as evidenced by the double trace. This double trace could not be eliminated despite attempts by altering the frequency of the input voltage signal. Despite this complication, there are several useable steps in the plot.

It should also be noted that the same considerations of error in horizontal scaling were considered for the AC effect plot as they were for the scaling in the DC plot. Again, the gain was given by the value in equation 6, and the respective error in the voltage of approximately 1% is contained within the size of the plotted data points. The microwave frequency which produced the AC effect was measured in the absorption cavity as $\nu_m = 20.955$ GHz $\pm$ 0.001 GHz. Using this value, along with the relation given in equation 4, a value of $\frac{2e}{h}$ was calculated from the voltage value of each discernible step. The results are outlined in Table 1.

The errors shown for each individual voltage value were estimated solely by the width of the clusters of values at each step. As can be clearly seen, the exact voltage value where each step occurs is somewhat obscured because of

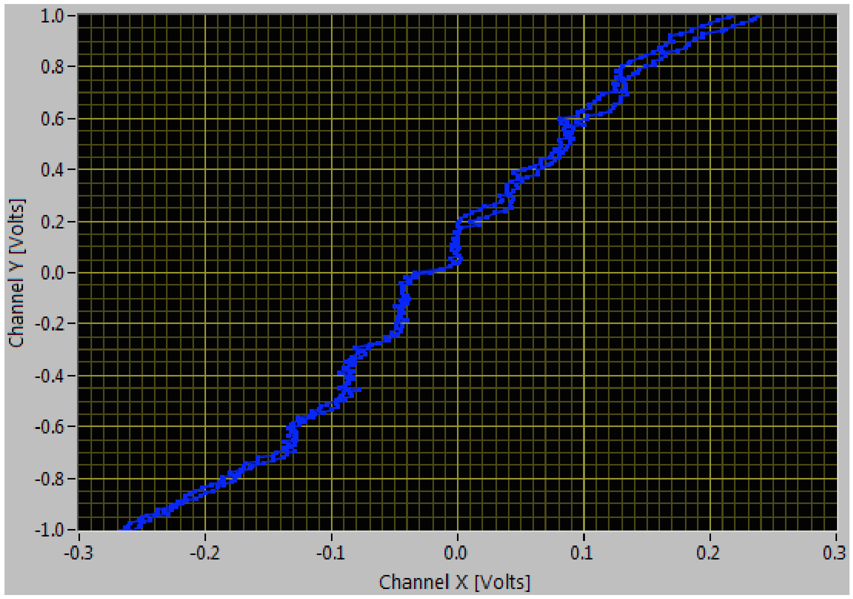

Figure 9: AC Effect Scope Trace

this clustering. The errors in the resulting values of $\frac{2e}{h}$ are errors propagated through equation 4.

Denoting the error in the ith value of $\frac{2e}{h}$ by the symbol σ_i, a weighted average of the above results was performed. The resulting error was found by propagating the errors in each individual value of $\frac{2e}{h}$ through the averaging formula. The resulting value is:

$$\frac{2e}{h} = (453 \pm 12)\frac{MHz}{\mu V} \tag{7}$$

Two things are of note in regards to this quantity. The first is that there is a fairly large discrepancy between it and the value $\frac{2e}{h} = (483.5912 \pm 0.0012)$ MHz/μV obtained by Parker, Taylor, and Langenberg [1]. The second is that the uncertainty in the value found here is approximately 2.6 % of the value itself, which is quite large in comparison Parker, Taylor, and Langenberg's 0.00025% uncertainty.

The source of the large uncertainty in the value of $\frac{2e}{h}$ reported here is due mostly to the large uncertainty in the voltage values where steps occur. Uncertainty in these values, as seen in Table 1 is as high as 7.7%. These large errors are directly attributable to the thickness of the scope trace of the AC effect. One explanation of this thickness, as already discussed, is the presence of the double trace. Another likely explanation is that the tip of the niobium

Table 1: $\frac{2e}{h}$ Calculation Data

Step Number	Junction Voltage (mV)	$\frac{2e}{h}$ (Mhz/μV)
-3	-0.13 $\pm$ 0.01	484 $\pm$ 37
-2	-0.095 $\pm$ 0.005	441 $\pm$ 23
-1	-0.05 $\pm$ 0.005	419 $\pm$ 42
3	0.137 $\pm$ 0.005	459 $\pm$ 17

needle was damaged, resulting in a poor connection and a slightly messy AC effect. This situation is probable, given that the tip was lifted and recontacted several times during the course of trying to obtain the AC effect. In addition, a fresh needle was used after taking the data presented in this report. The scope trace for this needle seemed to be thinner and sharper, supporting the idea that the needle was partly to blame. Unfortunately the AC effect was not obtained with this fresh point in time to take additional data.

A reasonable explanation may also be offered for the discrepancy between the value reported here and that of Parker, Taylor, and Langenberg. It is clear from the values presented in Table 1, as well as the mean value presented in equation 7 that the measurements of $\frac{2e}{h}$ made in this experiment tended to be lower than expected.

Possible explanations for this include that the microwave frequency measured was too low, or that the voltage values where steps occurred were consistently too high. The most probable cause for this latter explanation is that the calibration for the X-axis was incorrect. Because the junction voltage is inversely proportional to the gain of the preamp, if the gain was measured to be lower than its actual value, this would cause all of the voltage plots to shift up in absolute value. This would effectively shift every measurement of $\frac{2e}{h}$ downward.

There is a possibility that the gain was indeed measured too low for the following reason. When the zero voltage reference point of the oscilloscope was recorded, it was done while the DC voltage box was attached. The box was switched off during this process. However, previously it was determined with the DVM that there was a voltage of 0.002 mV across the voltage box when switched off. Because of this, it is possible that the zero reference recorded on the scope was actually the result of this 0.002 mV amplified by the preamp. This would mean that the zero reference was in fact lower than what was recorded, which would mean the true gain of the preamp was larger than measured.

6 Conclusions

The calculation of $\frac{2e}{h}$ presented in this report is not in agreement with what was expected. In addition, the uncertainty associated with the value is unsatisfactory. The disagreement with the expected value can most likely be attributed to an incorrect scaling of the junction voltage axis as a result of an incorrect measurement of the gain. The high degree of uncertainty is largely due to the thickness of the scope trace of the AC Josephson effect, which resulted in large uncertainty in step position. The thickness of the trace may be attributed to unwanted 60 Hz pick-up, and possibly a damaged niobium needle.

References

[1] W. H. Parker, B. N. Taylor, and D. N. Langenberg, Phys. Rev. Letters **18**, 287 (1967).

[2] J. Clarke, Phys. Rev. Letters **38**, 1077 (1970).

[3] D. N. Langenberg, D. J. Scalapino, and B. N. Taylor. "The Josephson Effects." *Scientific American*, pp 30-39 (May 1966).

[4] R. Feynman, R. Leighton, and M. Sands, *"The Feynman Lectures on Physics: Volume 3"*: 1964. *www.feynmanlectures.info*. Web. Sept. 2015.

Note: All experimentation and data taking was performed with the help of Mitchell Maciorski, my lab partner. As a result, scope traces and the raw data from these points were shared between us.