A Violation of Bell's Inequality Using Entangled Photons

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Abstract

This experiment was performed to test a version of Bell's inequality now known as the CHSH inequality. As shown by Clauser, Horne, Shimony, and Holt [1], a unitless value (denoted by S) may be constructed such that any local hidden variables theory predicts $S \leq 2$, but quantum mechanics allows for $S \leq 2\sqrt{2}$. In this experiment, an experimental value of S was found through use of measurements on the polarization of pairs of entangled photons. The value found here is $S = 2.45 \pm 0.04$, which is a clear violation of the CHSH inequality.

1 Introduction

The completeness of the theory of quantum mechanics has, throughout history, been called into question by those who find its probabilistic nature unsettling. Before John Bell published his famous paper on the EPR paradox in 1964 it was believed by some that there existed a local "hidden variables" theory, which could reproduce all of the predictions of quantum mechanics while at the same time being deterministic. This was certainly the position held by Einstein, Podolsky, and Rosen, authors of the famous EPR paradox.

In the EPR paper, it was argued that quantum mechanics provides an incomplete description of nature, the alternative situation being that the measurement of a quantum system in one area of space could instantaneously effect the value held by a state in another area of space, regardless of the distance between them, an apparent violation of causality and locality. It was believed by EPR that there existed some hidden variables theory which would preserve locality and reveal deterministic processes behind the probabilistic machinery of quantum mechanics.

Bell, however, showed that any hidden variables theory that reproduces all predictions of quantum mechanics must be nonlocal. Moreover, he showed that any local hidden variables theory would predict, for certain series of measurements, outcomes which differed from those predicted by quantum mechanics. Thus, Bell introduced a testable theory which had the potential to verify the validity of quantum mechanics over any local hidden variables theory.

It is the goal of this experiment to take advantage of a modified version of Bell's argument, first shown to hold by Clauser *et al* [1]. In this form, Bell's argument can be formulated in terms of a physically realizable experiment in which polarization measurements are made on pairs of entangled photons. If a violation of Bell's inequality is realized from the following experiment this would support the theory that any local hidden variables formulation of quantum mechanics is incorrect.

2 Theory

2.1 Measurement of Photon Polarization

The entirety of this experiment rests upon making various types of measurements on the polarization of photons. Thus, before delving into the theory behind violating Bell's inequality, some preliminary explanation on the measurement of photon polarization is in order.

In the quantum mechanical picture, polarization of a photon may be thought of as a normalized state vector, such as the following:

$$|\psi\rangle = \alpha\,|h\rangle + \beta\,|v\rangle \tag{1}$$

Here, the polarization $|\psi\rangle$ is considered to be a superposition of horizontal and vertical polarization states. If the polarization of a photon in this state is measured in the $|h\rangle$-$|v\rangle$ basis, there is a probability $P(h) = |\alpha|^2$ that it will be found to be horizontally polarized, and a probability $P(v) = |\beta|^2$ that it will be found to be vertically polarized.

One way that a measurement in the $|h\rangle$-$|v\rangle$ basis can physically be made is to place a polarizing beam splitter in the path of the photon with the transmission axis aligned along the spatial direction associated with the $|h\rangle$ vector. In this situation, the photon will pass straight through the beam splitter if its polarization is measured as horizontal, and it will be deflected by 90° if its polarization is measured as vertical.

Now, suppose that we would like to make a measurement in a different basis, say $|h'\rangle$-$|v'\rangle$, which is rotated by some angle α with respect to the original basis. To achieve this there are two options. First, one could somehow rotate the actual measuring device (in this case the beam splitter), by α so that its transmission axis is in line with the spacial direction associated with the new basis vector $|h'\rangle$. The second option, which is the one that will be used in this experiment, is to leave the measuring device as it is and instead rotate the polarization state by $-\alpha$. Both of these methods will produce the same values.

By using the second method of measurement, one is able to leave the measurement apparatus stationary and measure in arbitrary bases by simply rotating the states of the photons before measurement. This rotation of the states is quite straightforwardly done through use of a half wave plate, which is explained in detail in Section 3.1

2.2 The CHSH Inequality

Now that the measurement of photon polarization is understood, the goal of this section is to establish the theory behind the CHSH inequality, which is a generalized form of Bell's inequality that is physically testable. This form of Bell's inequality was first suggested by Clauser, Horne, Shimony, and Holt [1].

Let us consider some type of experimental setup in which pairs of photons are produced and subsequently measured for their polarizations. For each pair produced, the photons in the pair are sent down different paths and encounter different detection devices (shown in Fig 1).

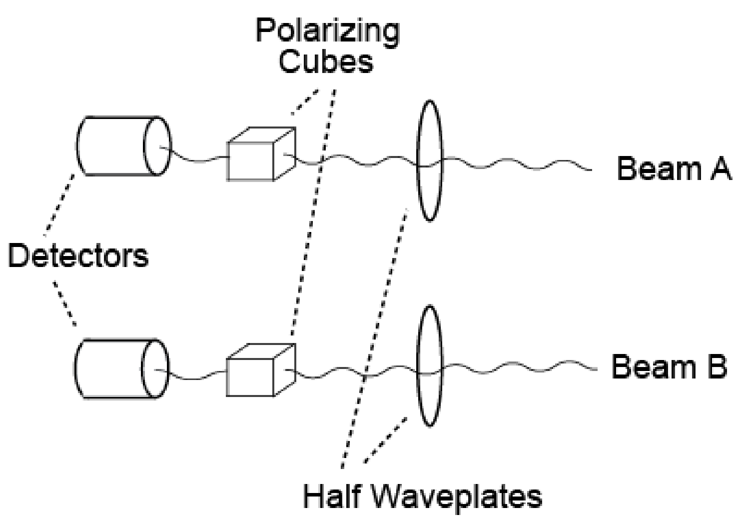

Figure 1: Detector Apparatus

As explained above, the half wave plates in conjunction with the beam splitters can be used to make measurements in whatever basis is desired. Subsequently, the detectors count how many photons are found in the resulting state.

Now, for any given basis that a measurement is performed in there are only two possible measurement results: horizontally polarized or vertically polarized. For notational simplicity, let h represent a horizontal polarization and v represent a vertical polarization. We will use the notation

$$P_{ij}(\alpha, \beta)$$

to denote the probability of simultaneously finding a photon in beam A to be polarized in the i direction when measured in the basis α, and finding a photon in beam B to be polarized in the j direction when measured in the basis β. Clearly, from the discussion above, i and j can only be h or v, respectively.

As an example, the probability of finding photon A horizontally polarized in the basis rotated 0° from the horizontal and simultaneously finding photon B vertically polarized in the basis rotated 90° from the horizontal is denoted:

$$P_{hv}(0^\circ, 90^\circ)$$

Armed with this bit of notation, now consider the following expression:

$$E(\alpha, \beta) = P_{vv}(\alpha, \beta) + P_{hh}(\alpha, \beta) - P_{hv}(\alpha, \beta) - P_{vh}(\alpha, \beta) \tag{2}$$

This quantity E is known as the correlation, and its value ranges from -1 to $+1$. If the photon pairs being measured have like polarizations 100% of the time then $E(\alpha, \beta) = 1$, and the measurements are said to be perfectly correlated. Conversely, if the photon pairs have opposite polarizations 100% of the time then $E(\alpha, \beta) = -1$ and the measurements are said to be anticorrelated.

With this definition of E, the heart of the matter at hand may be reached. Consider the following expression:

$$S = E(a, b) - E(a, b') + E(a', b) + E(a', b') \tag{3}$$

As outlined by Clauser *et al* [1], S is a useful expression because any local hidden variables theory predicts that $S \leq 2$. However, quantum mechanics predicts $S \leq 2\sqrt{2}$. According to the theory of quantum mechanics, the maximal violation of the inequality $S \leq 2$ occurs when $a = -45^\circ$, $a' = 0^\circ$, $b = -22.5^\circ$, and $b' = 22.5^\circ$. This is provided that the photons being measured are in the maximally entangled state:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|vv\rangle + |hh\rangle\right) \tag{4}$$

This state is also known as a Bell state or EPR state. By making measurements on photons in this state, using an experimental setup based upon the one outlined in this section, one can make a direct measurement of S. If the result obtained is definitively greater than 2, then this is in clear violation of any local hidden variables formulation of quantum mechanics.

In this experiment, measurements will be made by rotating the polarization state of the photons, as described in the previous section, such that only photons polarized in the desired directions are allowed to pass through the beam splitter and register as detections. Because the beam splitter allows horizontally polarized light to pass through it a measurement of the number of photons that are found to have horizontal polarization consists simply of rotating the incoming photons by some angle α or β corresponding to the basis in which the measurement is being conducted in. To determine the number of photons that would have been found polarized vertically, one rotates the state by angles $\alpha_\perp$ or $\beta_\perp$. These rotations are made directly before the photons enter the beam splitter.

Thus, the physically realizable method for calculating $E(\alpha, \beta)$ is to use the number of counts detected rather than the probabilities that are shown above. Cast in this form $E(\alpha, \beta)$ is given by:

$$E(\alpha, \beta) = \frac{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) - N(\alpha, \beta_\perp) - N(\alpha_\perp, \beta)}{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) + N(\alpha, \beta_\perp) + N(\alpha_\perp, \beta)} \tag{5}$$

where here, instead of using the probability of finding a photon pair with like or unlike polarizations as in Equation 2, the number of photons found with unlike polarizations is subtracted from the number found with like polarizations, and this result is divided by the total number of incident photons, so that the result is indeed a probability, as is expected of E.

3 Apparatus

3.1 Production of the Bell State

In this experiment a physical realization of the above discussion will be achieved by producing and subsequently measuring entangled photon pairs. The apparatus's functionality is separated into two main categories: the production setup, and the detection setup. The production setup, which is responsible for producing the Bell state photons, will be outlined first and is shown in Fig 2.

Initially, a beam of 405nm photons is produced using a 120mW laser diode. These photons first pass through an optical isolator, which prevents light that

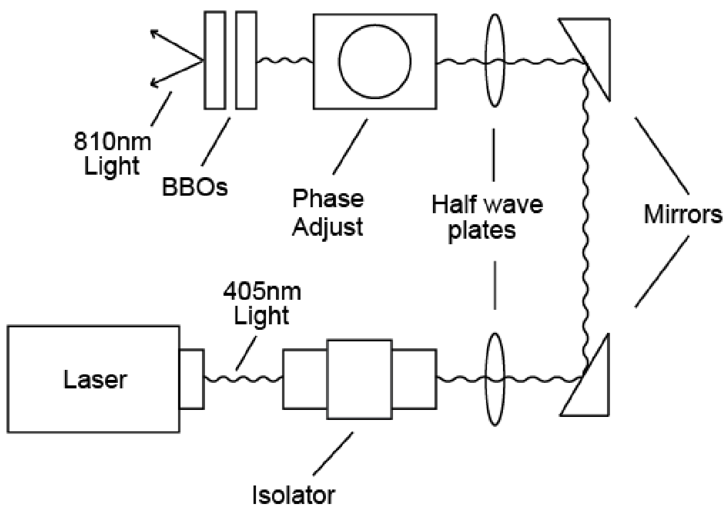

Figure 2: Production of Entangled Photons

is reflected back toward the source from damaging the laser diode. Immediately after this, the beam passes through a half wave plate.

The purpose of this half wave plate, as well as the others in the setup, is to rotate the polarization of the light that enters it. Wave plates are formed from materials which have different indices of refraction for different polarizations of light. The result is that the component of the incident light's polarization which is aligned to the wave plate's optical axis is delayed, while the perpendicular component is unchanged. In particular, half wave plates are constructed so that the phase delay along the optical axis is 90°, which has the effect of reflecting the polarization of the incident beam about the wave plate's optical axis. Careful choice of the wave plate angles, therefore, results in control over the direction of beam polarization.

The half wave plate directly following the isolator is set so that the light exiting it is polarized in the horizontal direction. It is important that the light be horizontally or vertically polarized at this point because of the two mirrors which follow the wave plate. If the light hitting the mirrors is polarized in some direction away from horizontal or vertical, the mirrors can introduce an unwanted elliptical polarization in the light, which the subsequent half wave plate will be unable to remediate due to the fact that it is designed to shift the phase only in increments of 90°.

Next is the half wave plate following the second mirror. The purpose of this wave plate is to rotate the polarization of the beam to 45°, which is needed to produce a proper Bell state. Following this half wave plate is a phase adjustor, which is essentially another half wave plate set so that it may rotate about it's optical axis and, thus, introduce small relative phase shifts in the beam polarization. This is needed because the next elements in this apparatus, the BBOs, introduce a small unwanted relative phase shift to the photon state. This phase adjustor is able to counteract the relative phase shift.

At last the beam reaches the BBOs (barium borate crystals). These elements of the apparatus are what create the entangled photon pairs out of the incoming beam, which will from this point forward be referred to as the pump beam. The BBOs cause what is known as parametric downconversion. In this process incoming 405nm photons are converted into pairs of entangled 810nm photons with correlated polarizations. In other words, each pair of photons exiting the BBOs is either in the state $\left|v\right\rangle_a \left|v\right\rangle_b$ or $\left|h\right\rangle_a \left|h\right\rangle_v$, where a and b denote the different photons.

In the setup, there are two BBOs one after another. One of the BBOs is set to downconvert only horizontally polarized photons, and the other is set to downconvert only vertically polarized photons. Temporarily ignoring the effects of the phase shift introduced by the BBOs and the phase adjustor, if the pump beam is polarized at 45° it is in the state:

$$\left|\psi_p\right\rangle = \frac{1}{\sqrt{2}} \left(\left|h\right\rangle + \left|v\right\rangle\right) \tag{6}$$

This means that a measurement would be equally likely to find a photon in this beam horizontally polarized as it would to find it vertically polarized. Thus, it is to be expected that assuming the downconversion efficiencies of each of the BBOs is equal that half of the photons from the pump beam will be downconverted by each BBO, which means that a measurement on a photon pair exiting the BBOs will be just as likely to find both photons with a horizontal polarization as it would to find them with a vertical polarization. The state of these photons exiting the BBOs is the Bell state:

$$\left|\psi_{EPR}\right\rangle = \frac{1}{\sqrt{2}} \left(\left|hh\right\rangle + \left|vv\right\rangle\right) \tag{7}$$

Realistically, the state exiting the BBOs will not be a perfect Bell state, as outlined above. In reality, there is a small phase difference ϕ introduced by the BBOs, there is a small phase difference φ introduced by the phase adjustor, and the polarization direction θ may not be exactly 45°. Taking these imperfections into account, the evolution of the state is as follows. The pump beam has a state given by:

$$\left|\psi_p\right\rangle = \cos\theta \left|h\right\rangle + e^{i\varphi} \sin\theta \left|v\right\rangle \tag{8}$$

The effect of the BBOs is to take $|h\rangle \to |vv\rangle$ and $|v\rangle \to |hh\rangle$ while adding a relative phase. This results in the state:

$$|\psi\rangle = \cos\theta\,|vv\rangle + e^{i\Delta}\sin\theta\,|hh\rangle \tag{9}$$

where $\Delta = \varphi + \phi$. Thus, it is easy to see that if the phase adjust is correctly set such that $\Delta = 0$, and if the half wave plate following the mirrors is properly adjusted such that $\theta = 45°$, the photons exiting the BBOs will be in the desired Bell state.

3.2 Detection Setup

After the entangled photon pairs are created from the BBOs their polarizations in various bases need to be measured. These measurements are achieved through the detection setup shown in Fig 3.

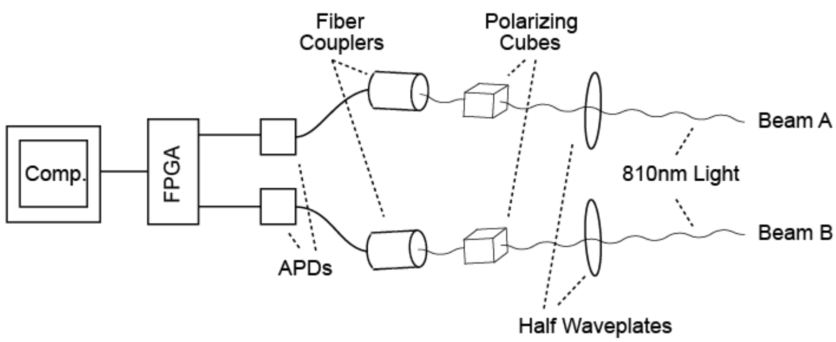

Figure 3: Detection of Entangled Photons

First, the photons in each beam pass through a half wave plate and a polarizing beam splitter. As explained above in Section 2.1, these elements are used to select a basis in which to measure the polarization of the incoming photons. Given careful choice of the half wave plate angle in beam A, one can ensure that only photons polarized vertically in a given basis α are allowed to pass through beam splitter A. Similarly, one could choose a half wave plate angle to ensure only photons polarized horizontally in the basis α are allowed to pass through. The same is true for the photons in beam B.

The photons from each beam which do make it through the beam splitter continue on into a fiber coupler, which captures the incoming photons and sends them into an optical fiber. The photons then travel down the optical fiber and into an avalanche photodiode (APD), which creates a voltage signal for each photon detected. The voltage signal from each APD is sent into the field-programmable gate array (FPGA), which finally sends the detection information to the computer for viewing. The FPGA in this experiment is designed to register coincidence counts, that is it records the number of times that two photons arrive into their respective detectors within a certain time frame, called a coincidence window. For the entirety of this experiment, this coincidence window was set to 5ns.

This coincidence counting system is crucial, because the photodiodes often detect stray light that is not due to entangled photon pairs entering the fiber couplers. The data of interest is specifically from these photon pairs, so only detections that are triggered within a very short time of one another are desired. Even with this setup in place, however, accidental counts are still recorded, and must be accounted for when analyzing the data.

4 Procedure

Before beginning the experiment, the laser power was tested to ensure proper functionality. Using a power meter placed in the beam path directly after the optical isolator, the power of the beam was recorded for values of supply current ranging from 0mA to 100mA (the minimum and maximum supply).

As expected, the output power was related linearly to the supply current, as shown in Fig 4.

4.1 Alignment and Wave Plate Calibration

Before a Bell state could be properly tuned, accurate alignment of the apparatus needed to be achieved. Firstly, it was verified that the violet (405nm) beam was visible near the center of the paper target placed between the detection arms, and that the spot left on the target was reasonably concentrated. These verifications ensured that the beam was passing properly through the BBOs and that the beam was properly focused. No adjustment was needed at this point, because the beam seemed properly aligned with the target, making a concentrated spot.

Next, the alignment of the fiber couplers with the BBOs was checked. This was done by first removing the filters from the faces of the fiber couplers, as well as the half wave plates and polarizing beam splitters from the detection setup. Then, the optical fiber was removed from the APD for beam A and a red laser pointer was attached to the free end of the fiber. Thus, when the laser pointer was turned on, red light was sent out of the fiber coupler and back toward the BBOs.

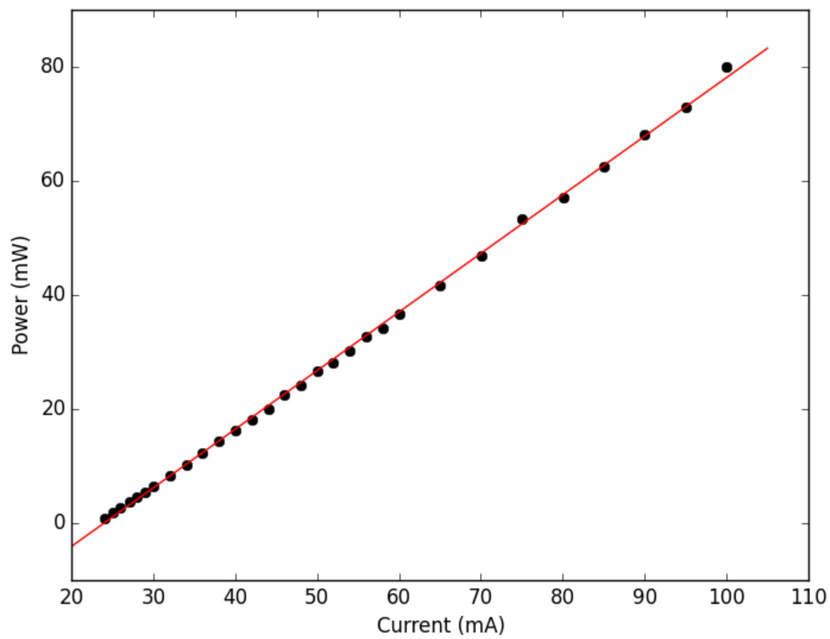

Figure 4: Laser Power

With the red laser pointer turned on, the fiber coupler angle was adjusted until the red laser pointed directly at the center of the BBOs. Irises were placed in the red beam path to help produce a more concentrated spot where the laser pointer light hit the BBOs in order to fine tune the fiber coupler angle.

Once the angle of the fiber coupler was satisfactory, the laser pointer was removed from the optical fiber, the optical fiber was reconnected to the APD, and the filter was replaced on the fiber coupler in detection arm A. The whole process was repeated to align the fiber coupler in detection arm B.

Next, in order to check that the detection arms were set at the proper angles, both detector arms were swung outward so that the angle between them was as large as possible. Then, with the half wave plates and polarizing beam splitters still removed from the detector setup, the laser was powered on and the counts registered in both detectors were individually monitored as the detection arms were slowly moved back toward each other. The angles which maximized counts in both detectors while also producing approximately the same number of counts in each detector were the angles used.

It was found that the number of counts was not extremely sensitive to the detection arm angle, varying by at most approximately 2.0×10^4 counts, from the maximum number of approximately 2.5×10^5 counts.

After ensuring that the detection arm angles were suitably set, the laser was turned off and the beam splitters were replaced on each detection arm. To ensure their proper alignment, the filters were again removed from the faces of the fiber couplers and light from the red laser pointer was sent through the optical fiber of detection arm A. The beam splitter was placed so that the red beam hit roughly at its center. Then, a strip of paper with a pencil sized hole in it was placed in front of the fiber coupler to better collimate the red beam. The angle of the beam splitter was then adjusted until the reflection of the red light from the beam splitter was aimed directly back into the place where it exited the fiber coupler. Once again, the filter was replaced to the fiber coupler and the fiber was reconnected to the APD for detection arm A, and the whole process was repeated to align the beam splitter on detection arm B.

At this point, the beams of entangled photons coming from the BBOs were aligned such that they would pass through the beam splitters and, if found to be horizontally polarized, would continue undeflected and into the fiber couplers for detection.

Next, the polarization of the photons exiting the half wave plate placed before the BBOs needed to be determined. The method chosen for achieving this was to search for the wave plate angle that produced photons in the state $|h\rangle$. Photons exiting the wave plate in this state would emerge from the BBOs in the state $|vv\rangle$. It is expected in this case that all of the photons will be found vertically polarized and will be deflected away from the detectors by the beam splitters. Thus, the wave plate angle was turned until a minimum number of coincidences were found, indicating that the state of the photons exiting the wave plate was indeed $|h\rangle$.

Note that any phase introduced to the state by the phase adjustor and the BBOs has no effect on the probability of measuring a certain polarization in the situation where the state is given by either $|h\rangle$ or $|v\rangle$. This is because the state is not in a superposition, so the phase introduced manifests itself only as a global phase. For this reason the phase difference can be safely ignored in the situation above.

It was found that when the half wave plate angle read 30° the number of coincidences detected per second was as low as 5. This was the smallest value achievable and is to be compared with coincidence rates as high as 150 for other angles. Thus, it was concluded that at the angle of 30° the photons must have been exiting the wave plate in the state $|h\rangle$. The fact that at this angle the coincidence rate was nonzero can be reasonably attributed to accidental counts and the fact that the beam splitters are not 100% efficient.

Finally, the optical axes of both of the detector half wave plates needed to be determined. To achieve this, the half wave plate for detector arm A was placed and aligned by the same method used to align the beam splitters. With the half wave plate before the BBOs set to output light in the state $|h\rangle$, the detector half wave plate was rotated until once again a minimum in the coincidence counts were observed. This was an indication that the detector half wave plate was taking the light entering it in the $|vv\rangle$ state and leaving it in this state. The angle which produced this minimum was recorded, and the half wave plate was again removed. This process was repeated with the half wave plate for detection arm B, and the angle of its optical axis was also noted. Finally, both wave plates were returned to the detection arms and realigned.

With the beams and detection apparatuses properly aligned, and the optical axes of all of the wave plates known, it was then possible to move on to create the desired Bell State.

4.2 Tuning the State

Recall from Section 3.1 that in order to produce an ideal Bell state, the direction of the polarization state emerging from the first half wave plate should be set to $\theta = 45°$, and the total phase difference introduced by the phase adjustor and the BBO should add to $\Delta = 0$. To achieve this, using the information about the wave plates' optical axes already determined, the first wave plate was set to rotate the polarization state of the photons in the pump beam by 45°. This was achieved by rotating the half wave plate by 22.5° from the zero reference of 30°. Thus, the wave plate was set to an angle of

$$\theta_{wp} = 52.5° \tag{10}$$

to produce light polarized at $\theta = 45°$.

With θ set properly, attention was then turned toward adjusting the phase difference, Δ. Now, because of the nature of the complex phase factor, its effects only appear in certain measurements. Because at this point it is assumed that θ is correctly tuned to 45° the state of the photons exiting the BBOs is given by:

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|vv\rangle + e^{i\Delta} |hh\rangle \right) \tag{11}$$

Now, let us ask for the probability of finding photon A horizontally polarized in the α basis and photon B horizontally polarized in the β basis, given that the pair of photons exists in the above state. Using the rules of quantum mechanics, one can derive an expression for this probability as:

$$\begin{aligned} P_{hh}(\alpha,\beta) =& \frac{1}{2}\cos^2\alpha\cos^2\beta + \frac{1}{2}\sin^2\alpha\sin^2\beta \\ &+\frac{1}{4}\sin 2\alpha\sin 2\beta\cos\Delta \end{aligned} \tag{12}$$

Dehlinger and Mitchell [3] give a complete derivation of this formula. For our purposes, we will simply use the result and analyze its implications. Notice that the phase factor Δ only appears in the cross term. This cross term will be identically equal to zero whenever α or β is equal to $0°$ or $90°$. This suggests that in order to observe the behavior introduced by the phase factor, one must measure in a basis other than these two.

Let us imagine, then, measuring in the basis $\alpha = 45°$ and $\beta = 45°_\perp = 135°$. From the above expression it is easy to determine that

$$P_{hh}(45°, 135°) = \frac{1}{4}(1-\cos\Delta) \tag{13}$$

which has a minimum when $\Delta = 0$.

Thus, the detector half wave plates were set to measure in the basis specified above, and the phase adjustor was altered until a minimum number of coincidences were measured.

4.3 Measurement of S

Finally, with the Bell state tuned, the values of θ and Δ were calculated, and a few graphs were produced, demonstrating the purity of the Bell state. These measures were taken to ensure the state actually turned out as predicted from the tuning that was done. The results of these endeavors are shown below in Section 5.

After the purity of the state was mapped out, a measurement of S was made. As explained above in Section 2.2, this consisted of rotating the detector half wave plates properly as to measure the four different values of E. Each value of E required 4 different measurements, as is clear from Equation 5. After a total of 16 measurements, all four values of E along with an experimental value of S was calculated.

5 Results and Analysis

The state produced with the tuning methods outlined above agreed well with expected probabilities for such a state. To show this the graph was fit with the following equation, as suggested by Dehlinger and Mitchel [3]:

$$N(\alpha,\beta) = A\Big(\cos^2\alpha\cos^2\beta\sin^2\theta \\ + \sin^2\alpha\sin^2\beta\cos^2\theta \\ + \frac{1}{4}\sin 2\alpha\sin 2\beta\sin 2\theta\cos\Delta\Big) + C \tag{14}$$

Here, A is the total number of incident photons over a 10 second time period. When multiplied by the probability $P_{hh}(\alpha,\beta)$ introduced in the previous section, this gives the number of coincidence counts expected within 10 seconds.

The parameter C is added to account for a vertical offset when the equation is graphed with respect to β.This offset needs to be introduced in the fit because the detection apparatus measures a nonzero number of coincidences even when, in theory, the number of photons entering the detectors should be zero.

The existence of this offset is completely expected, given that the detectors do register accidental coincidences from background light, and that the beam splitters do not operate 100% ideally. The data shows that this offset is small in comparison to the total number of incident photons, and given that these extra counts are present in every measurement taken, it is assumed that their presence does not effect the value of S measured, given that S depends only on the relative number of counts for each measurement.

To test the purity of the photon state produced, α was held fixed, first at 0° and then at 45°, and β was varied. For each angle β the number of coincidence counts over a 10s interval was recorded. A table of this data is shown below in Table 1.

The data from the above table was fit to Equation 14, using A, C, θ, and Δ as parameters for the fit. This fit is shown in Fig 5.

In this figure, the error bars represent plus or minus one standard deviation, which is given by $\sigma_i = \sqrt{N_i}$ for datapoints $i = 1, 2, \ldots, 19$. Here, N_i is the number of coincidences measured for the ith point.

Using the best fit data from the $\alpha = 45°$ data the values for the parameters are found to be: $A = 1300$, $C = 73$, $\theta = 49°$, and $\Delta = 14°$. Though this is not a pure Bell state, it is fairly close, and further tuning was found to be unneeded.

It is worth noting that for $\alpha = 45°$ the data fits the expected curve very well, whereas for $\alpha = 0°$ each of the data points seem to all be shifted slightly to the left. A likely explanation for this is that when the data was taken

Table 1: Coincidence Counts vs. Basis Angle

β (deg)	Counts $\alpha = 0^\circ$	Counts $\alpha = 45^\circ$	β (deg)	Counts $\alpha = 0^\circ$	Counts $\alpha = 45^\circ$
0	815 ± 28	465 ± 22	100	78 ± 9	244 ± 16
10	751 ± 27	581 ± 24	110	180 ± 13	159 ± 13
20	654 ± 26	667 ± 26	120	314 ± 18	106 ± 10
30	559 ± 24	717 ± 27	130	438 ± 21	82 ± 9
40	407 ± 20	735 ± 27	140	600 ± 24	105 ± 10
50	260 ± 16	709 ± 26	150	703 ± 26	169 ± 13
60	150 ± 12	668 ± 26	160	795 ± 28	251 ± 16
70	58 ± 8	581 ± 24	170	830 ± 29	345 ± 19
80	15 ± 4	475 ± 22	180	828 ± 29	462 ± 21
90	28 ± 5	346 ± 19			

for $\alpha = 0^\circ$, the wave plate setting in detection arm A, which controls α, was slightly off, resulting in an α which was not truly 0°.

This would explain why there is a shift in the data points only for the $\alpha = 0^\circ$ graph, because when $\alpha = 0^\circ$ Equation 14 expects no Δ dependence, but if the measurements were made in a basis given by some nonzero angle, there certainly would be a Δ dependence in the data. Given that the Δ value for the entangled state is not exactly zero, this would affect the data. An examination of Equation 14 reveals that if the term containing Δ is nonzero then this results in a larger or smaller value for $N(\alpha, \beta)$, depending on if the true value of α is positive or negative. This would have an effect like the one seen in the graph for $\alpha = 0$

If the true value of α is shifted from what was expected, this means that the determination of the optical axis for the half wave plate in detection arm A could have been slightly incorrect. This could be a consequence of several things, such as the angle of the detection arms or a slight misalignment of the detection setup during the determination of the optical axis. This could be adjusted by attempting a realignment and redetermination of the optical axis angle of the wave plate in detection arm A, but again, it was found that this was unnecessary.

Finally, the data used to calculate S is shown below in Table 2. Again, each measurement was taken over a 10s interval. So the quantities presented represent the number of coincidences per 10 seconds. The angles used were those that are shown to maximize the value of S: $a = -45^\circ$, $a' = -0^\circ$, $b = -22.5^\circ$, and $b' = 22.5^\circ$. Also, as with the previous data, the uncertainties

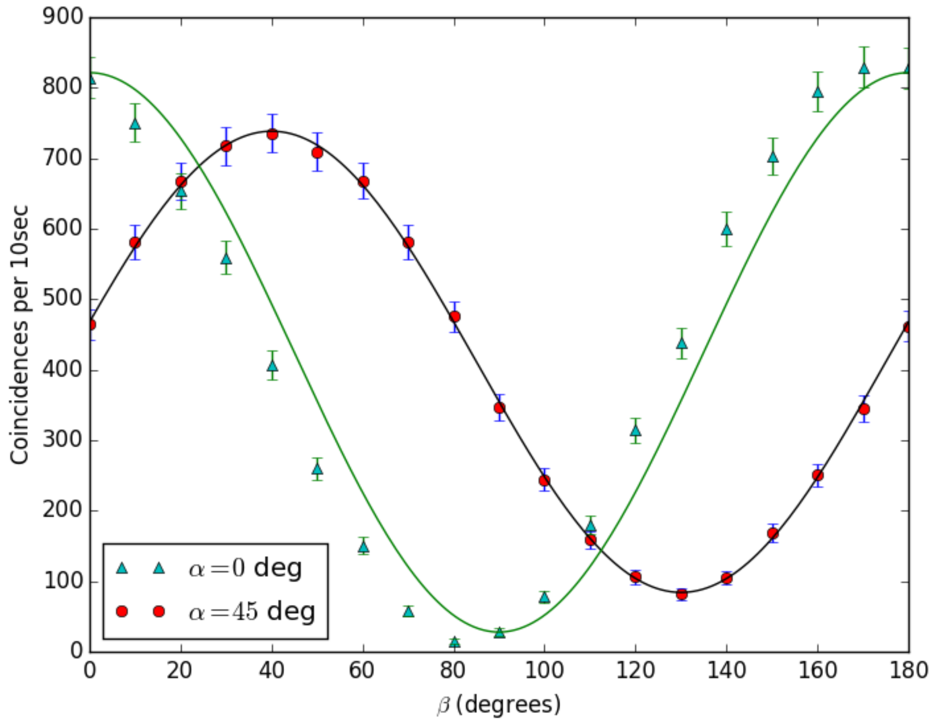

Figure 5: Number of Coincidences vs. Basis Angle, Best Fit

here represent one standard deviation, which, in this case is given by the square root of the number of counts.

This data was into Equation 5 to determine the values $E(a,b)$, $E(a,b')$, $E(a',b)$, and $E(a',b')$. In turn, these values were entered into Equation 3 to determine a value for S. The result of these calculations produces:

$$S = 2.45 \pm 0.04 \tag{15}$$

Here, the uncertainty of 0.04 represents the error propagated through Equation 3 given that each of the 16 measurements of N comes with an uncertainty, as presented in the above table. The expression for this uncertainty involves a 16 term sum, a term for each measured value. If one assigns an order to each of the 16 measurements of N presented above, then it can be said that for measurement N_i the statistical uncertainty is given by $\sigma_i = \sqrt{N_i}$. With this

Table 2: S Measurement Data

Measurement	Coincidences	Measurement	Coincidences
$N(a, b)$	618 ± 25	$N(a', b)$	731 ± 27
$N(a_\perp, b_\perp)$	590 ± 24	$N(a'_\perp, b_\perp)$	730 ± 27
$N(a, b_\perp)$	221 ± 15	$N(a'_\perp, b)$	92 ± 10
$N(a_\perp, b)$	232 ± 15	$N(a', b_\perp)$	71 ± 8
$N(a, b')$	140 ± 12	$N(a', b')$	640 ± 25
$N(a_\perp, b'_\perp)$	147 ± 12	$N(a'_\perp, b'_\perp)$	645 ± 25
$N(a, b'_\perp)$	711 ± 27	$N(a'_\perp, b')$	185 ± 14
$N(a_\perp, b')$	672 ± 26	$N(a', b'_\perp)$	196 ± 14

notation the uncertainty is given by:

$$\delta S = \sqrt{\sum_{i=1}^{16} \left(\frac{\partial S}{\partial N_i} \sigma_i \right)^2} \tag{16}$$

This expression is also used by Dehlinger and Mitchell [3] in their calculation of the statistical uncertainty of S.

On a final note, the issue of accidental coincidences should be addressed. All of the measurements of the number of detected coincidences presented in this report include a small number of accidental counts. The number of these counts are expected to be on the order of magnitude 1–10 per 10 seconds for all of the measurements made here. This is fairly small in comparison with the total number of incident photons per 10 seconds, and in addition, careful examination of Equations 5 and 3 reveal that the presence of accidental counts in the measurements will actually act to decrease S provided that they are fairly constant throughout all of the measurements made.

This holds because the numerator of EquationequationE will remain unchanged if some number of counts is added to each of the measurements, while the denominator will increase. This will in turn decrease S if the decrease in each of the 4 values of E is relatively constant.

Considering this, it is reasonable to assume that the presence of accidental counts cannot degrade our result for S because, presumably if there were no accidental counts, the measured value of S would be even higher.

6 Conclusions

The experimental value of $S = 2.45 \pm 0.04$ is a clear violation of the inequality $S \leq 2$. This result supports the validity of quantum mechanics over any local hidden variables theory. The significance of this result is profound, essentially ruling out the possibility that one can assign definite values of polarization to entangled photons before they are measured, and supporting the idea that the probabilistic nature behind predicting polarizations is fundamental in nature; no further knowledge can be obtained so that one could predict the polarization of a photon with 100% certainty.

References

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