Supplementary material

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I. Gas Turbine

Gas turbine can balance the electric load variation while its wasting heat can be collected to supply the DHS load. We use the classical Rowen's gas turbine model [1], which is shown in Fig. 1. The controller adjusts the input fuel for prescribed exhaust temperature and rotor speed.

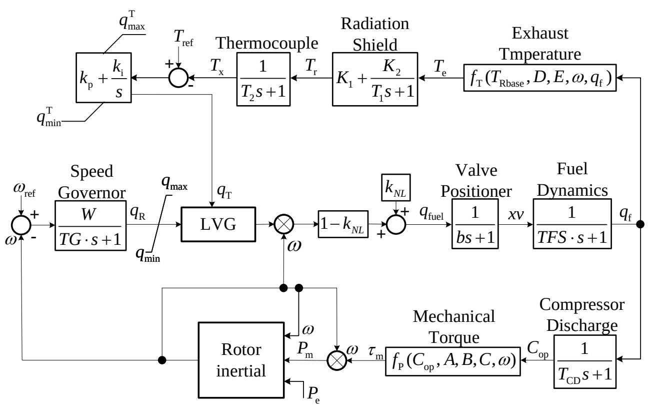

Fig. 1. Diagram of gas turbine.

The model equations of GT are

$$\dot{q}_{\mathrm{R}} = (W(\omega_{\mathrm{ref}} - \omega) - q_{\mathrm{R}})/TG \tag{1a}$$

$$0 = k_{NL} + (1 - k_{NL}) \cdot \min\left(q_{\mathrm{T}}, \mathrm{Sat}(q_{\mathrm{R}}, q_{\min}, q_{\max})\right) - q_{\mathrm{fuel}} \tag{1b}$$

$$\dot{xv} = \frac{q_{\mathrm{fuel}} - xv}{b} \tag{1c}$$

$$\dot{q}_{\mathrm{f}} = \frac{xv - q_{\mathrm{f}}}{TFS} \tag{1d}$$

$$\dot{C}_{\mathrm{op}} = \frac{q_{\mathrm{f}} - C_{\mathrm{op}}}{T_{\mathrm{CD}}} \tag{1e}$$

$$0 = A + BC_{\mathrm{op}} + C(1 - \omega) - \tau_{\mathrm{m}} \tag{1f}$$

$$0 = T_{\mathrm{Rbase}} + D(1 - q_{\mathrm{f}}) + E(1 - \omega) - T_{\mathrm{e}} \tag{1g}$$

$$0 = K_1 T_{\mathrm{e}} + T_{\mathrm{ri}} - T_{\mathrm{r}} \tag{1h}$$

$$\dot{T}_{\mathrm{ri}} = \frac{K_2 T_{\mathrm{e}} - T_{\mathrm{ri}}}{T_1} \tag{1i}$$

$$\dot{T}_{\mathrm{x}} = \frac{T_{\mathrm{r}} - T_{\mathrm{x}}}{T_2} \tag{1j}$$

$$0 = k_{\mathrm{p}}(T_{\mathrm{ref}} - T_{\mathrm{x}}) + k_{\mathrm{i}} q_{\mathrm{Ti}} - q_{\mathrm{T}} \tag{1k}$$

$$\dot{q}_{\mathrm{Ti}} = T_{\mathrm{ref}} - T_{\mathrm{x}} \tag{1l}$$

where ω_{ref} denotes the reference speed value; q_{R} represents the governor output; W and TG are governor parameters; W can be adjusted to modify the droop coefficient; q_{T} indicates the fuel control quantity output by the temperature controller; k_{NL} denotes the fuel consumption of the gas turbine under no-load conditions; the $\min$ function selects the minimum value among its arguments; and Sat represents the saturation function, defined as

$$\mathrm{Sat}(q, q_{\min}, q_{\max}) = \begin{cases} q_{\max} & q \geq q_{\max} \\ q & q_{\min} \leq q \leq q_{\max} \\ q_{\min} & q \leq q_{\min} \end{cases}; \tag{2}$$

xv denotes the valve displacement; q_{f} represents the fuel flow rate; b and TFS are the time constants for the valve control and fuel dynamics, respectively; T_{CD} is the time constant for the compressor discharge process; A, B, and C are calculation parameters related to the mechanical torque of the gas turbine. Furthermore, T_{Rbase} represents the reference temperature, while D and E are calculation parameters associated with exhaust temperature. Moreover, K_1, K_2, and T_1 are calculation parameters related to the radiation field model. Meanwhile, T_2 denotes the thermocouple time constant. Lastly, $k_{\mathrm{p}}/k_{\mathrm{i}}$ represents the proportional/integral control coefficients.

(1a) is the speed governor equation; (1b) is the fuel consumption equation; (1c) and (1d) are respectively the valve positioner and fuel dynamic equations; (1e) and (1f) are respectively the compressor discharge and mechanical torque equations; (1g) is the exhaust temperature equation; (1h)-(1i) are the radiation shield equations; (1j) is the thermocouple equation; (1k)-(1l) are the temperature controller equations.

II. Steam Turbine

Steam turbine (ST) serves as the slack node of DHS while the PV bus of EPS [2]. The main control goal is to maintain the constant temperature of supply water at the slack source node. As illustrated in Fig. 2, its model comprises two main components: a temperature controller and steam volume dynamics [3].

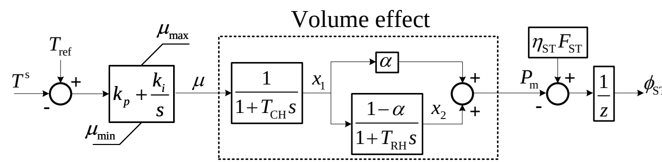

Fig. 2. Diagram of steam turbine.

The model equations of ST are

$$\dot{x}_1 = \frac{\mu - x_1}{T_{\mathrm{CH}}} \tag{3a}$$

$$\dot{x}_2 = \frac{x_1(1-\alpha) - x_2}{T_{\mathrm{RH}}} \tag{3b}$$

$$P_{\mathrm{m}} = \alpha x_1 + x_2 \tag{3c}$$

$$P_{\mathrm{m}} + z\phi_{\mathrm{ST}} = \eta_{\mathrm{ST}} F_{\mathrm{ST}}, \tag{3d}$$

$$\dot{\mu}_1 = k_i (T_{\mathrm{REF}} - T^{\mathrm{s}}) \tag{3e}$$

$$\mu = k_p(T_{\text{REF}} - T^{\text{s}}) + \mu_1 \tag{3f}$$

where μ denotes the valve opening percentage, α represents the percentage of the high-pressure cylinder's steady-state output power to the total output power of the steam turbine, typically around 0.3; T_{CH} is the high-pressure steam volume time constant, generally ranging from 0.1-0.4s; T_{RH} denotes the intermediate-pressure reheated steam volume effect time constant, typically between 4-11s; P_{m} indicates the mechanical power input to the prime mover of the synchronous generator; x_1 and x_2 represent the introduced intermediate variables; ϕ_{ST} denotes the thermal power output of the steam turbine; z represents the power conversion coefficient; η_{ST} indicates the efficiency of the steam turbine; F_{ST} denotes the total power equivalent converted from the fuel input of the steam turbine; μ_1 represents the intermediate variable introduced by the PI controller; T_{REF} denotes the reference temperature of the heat source; T_s indicates the actual outlet temperature of the heat source; k_p/k_i represent the proportional/integral coefficients, respectively.

(3a)-(3c) are the volume dynamic equations; (3d) is the power output equation; (3e)-(3f) are the temperature controller equations.

III. Proof of Convexity

Given hydraulics equations

$$H_j - H_k = K_i \dot{m}_i^2 \operatorname{sign}(\dot{m}_i), \quad \text{where } j,k \text{ are the endpoints of pipe } i,\ i \in \mathbb{E}, \tag{4a}$$

$$\sum_{i\in\mathbb{E}_j^{\text{in}}} \dot{m}_i - \sum_{i\in\mathbb{E}_j^{\text{out}}} \dot{m}_i - \dot{m}_j^{\text{in}} = 0, \quad j \in \mathbb{V}, \tag{4b}$$

$$H_i - H_i^{\text{set}} = 0, \quad i \in V_{\text{slack}}, \tag{4c}$$

and its model

$$\begin{aligned} \min_{\dot{m}_i,\, \dot{m}_i^{\text{in}}} \quad & \mathbf{z} = \sum_{i\in\mathbb{E}} \frac{K_i|\dot{m}_i|^3}{3} - \sum_{i\in V_{\text{slack}}} H_i^{\text{set}} \dot{m}_i^{\text{in}} \\ \text{s.t.} \quad & \sum_{i\in\mathbb{E}_j^{\text{in}}} \dot{m}_i - \sum_{i\in\mathbb{E}_j^{\text{out}}} \dot{m}_i = \dot{m}_j^{\text{in}}, \quad j \in \mathbb{V}. \end{aligned} \tag{5}$$

The following proves that (5) is a convex optimization model and that its global optimal solution satisfies the hydraulic equations (4) under leakage fault scenarios.

1) Convexity

Since the constraints in (5) are linear, according to the definition of convex optimization, we only need to prove that the objective function is convex. We have:

$$\frac{\partial \mathbf{z}}{\partial \dot{m}_i} = K_i |\dot{m}_i|^2 \cdot \operatorname{sign}(\dot{m}_i)$$

$$\frac{\partial^2 \mathbf{z}}{\partial \dot{m}_i^2} = 2K_i|\dot{m}_i| \cdot (\operatorname{sign}(\dot{m}_i))^2 \geq 0,$$

$$\frac{\partial^2 \mathbf{z}}{\partial \dot{m}_i \partial \dot{m}_j} = \frac{\partial^2 \mathbf{z}}{\partial \dot{m}_j \partial \dot{m}_i} = 0,$$

$$\frac{\partial \mathbf{z}}{\partial \dot{m}_i^{\text{in}}} = H_i^{\text{set}}, \quad \frac{\partial^2 \mathbf{z}}{\partial (\dot{m}_i^{\text{in}})^2} = 0.$$

Therefore, the Hessian matrix $\nabla^2 \mathbf{z}$ of the objective function is a positive semidefinite matrix with non-negative diagonal elements, making the objective function convex.

2) Equivalence between global optimal solution and power flow solution

Construct the Lagrangian function of model (5):

$$\begin{aligned} L(\dot{m}) = & \sum_{i\in\mathbb{E}} \frac{K_i|\dot{m}_i|^3}{3} - \sum_{i\in V_{\text{slack}}} H_i^{\text{set}} \dot{m}_i^{\text{in}} \\ & - \sum_{j\in V} \lambda_j \left(\sum_{i\in\mathbb{E}_j^{\text{in}}} \dot{m}_i - \sum_{i\in\mathbb{E}_j^{\text{out}}} \dot{m}_i - \dot{m}_j^{\text{in}} \right). \end{aligned} \tag{6}$$

According to the KKT conditions, the global optimal solution $\dot{m}$ of optimization model (5) and its corresponding Lagrange multipliers λ_i must satisfy:

$$\frac{\partial L(\dot{m})}{\partial \dot{m}_i} = K_i \dot{m}_i^2 \operatorname{sign}(\dot{m}_i) - \lambda_j + \lambda_k = 0, \quad \text{where } j,k \text{ are the endpoints of pipe } i,\ i \in \mathbb{E}, \tag{7}$$

$$\frac{\partial L(\dot{m})}{\partial \dot{m}_i^{\text{in}}} = -H_i^{\text{set}} + \lambda_i = 0, \quad i \in V_{\text{slack}}, \tag{8}$$

$$\frac{\partial L(\dot{m})}{\partial \lambda_j} = \sum_{i\in\mathbb{E}_j^{\text{in}}} \dot{m}_i - \sum_{i\in\mathbb{E}_j^{\text{out}}} \dot{m}_i - \dot{m}_j^{\text{in}}, \quad j \in \mathbb{V}. \tag{9}$$

Let $H_i \triangleq \lambda_i$. Then (7)-(9) can be rewritten as (4). Therefore, the global optimal solution of the convex optimization model (5) must satisfy the hydraulic equations (4) considering multiple slack nodes.

References

[1] M. R. Bank Tavakoli, B. Vahidi, and W. Gawlik, "An Educational Guide to Extract the Parameters of Heavy Duty Gas Turbines Model in Dynamic Studies Based on Operational Data," *IEEE Transactions on Power Systems*, vol. 24, no. 3, pp. 1366–1374, Aug. 2009.

[2] X. Liu, J. Wu, N. Jenkins, and A. Bagdanavicius, "Combined analysis of electricity and heat networks," *Applied Energy*, vol. 162, pp. 1238–1250, Jan. 2016.

[3] Y. Ni, S. Chen, and B. Zhang, *Theory and Analysis of Dynamic Power Systems*, 1st ed. Beijing, China: Tsinghua University Press, 2002.